

# Tensor-polarized Parton Distribution Functions for Spin-1 Deuteron

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University of Washington, Seattle, October 12, 2017.

Reference: Shunzo Kumano and Qin-Tao Song, Phys. Rev. D **94**, 054022 (2016)

# Outline

- ◆ **Motivation**
- ◆ **Theoretical and experimental status**
- ◆ **Results**

# Spin Puzzle the proton

In Quark Model

The proton is S wave

$$\frac{1}{2}(\Delta u_v + \Delta d_v) = \frac{1}{2}$$

$$\Delta L = 0$$

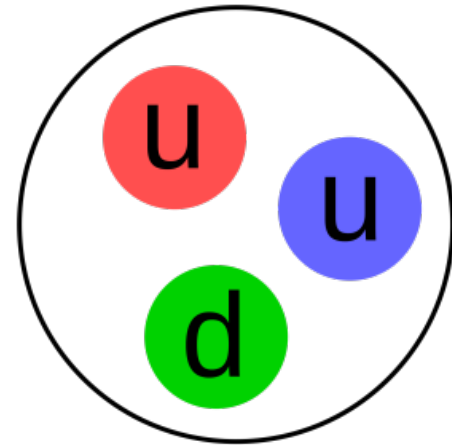
In patron picture, proton spin composites

$$\frac{1}{2}(\Delta u^+ + \Delta d^+ + \Delta s^+) + \Delta g + \Delta L = \frac{1}{2}$$

However, recent research  
shows

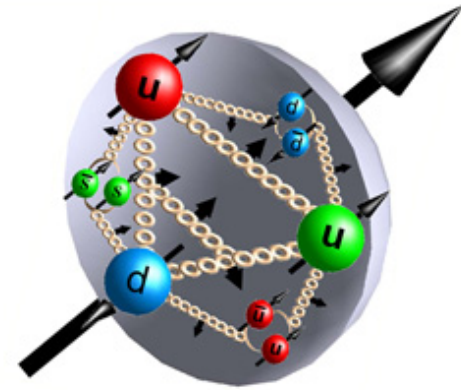
$$\Delta u^+ + \Delta d^+ + \Delta s^+ \approx 0.3$$

$$\Delta g + \Delta L \neq 0$$



proton in quark model

**The quarks only contribute 20-30% of the spin, the rest should come from gluons and orbital angular momentum**



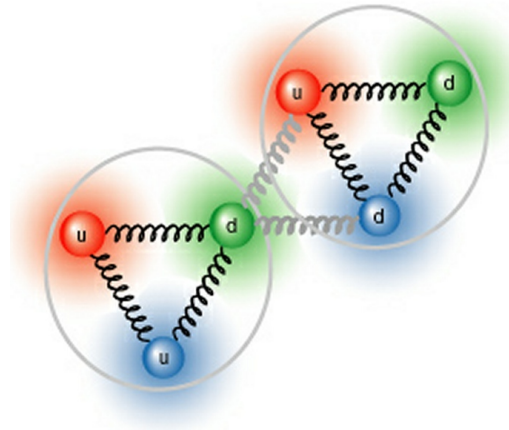
In order to solve the spin puzzle of the proton, it is necessary to investigate the proton structure function to know the gluon-spin and orbital angular momentum contribution in the proton.

**We investigate another spin observable in the spin-1 deuteron.**

# Tensor-polarized structure in deuteron

The deuteron was originally considered as proton and neutron in S wave.

Experimental magnetic moment of deuteron is consistent with the S-wave proposal, while the existence of **electric quadrupole moment** indicates that the deuteron should also contain D wave.



Magnetic Moment(D)  $\approx$  Magnetic Moment(p)+Magnetic Moment(n)

$$\text{S wave: } \delta_T q_i(x, Q^2) = q_i^0 - \frac{q_i^1 + q_i^{-1}}{2} = 0, \quad b_1 = \frac{1}{2} \sum_i e_{ii}^2 (\delta_T q_i(x, Q^2) + \delta_T \bar{q}_i(x, Q^2)) = 0$$

$$\text{S-D Mix: } \delta_T q_i(x, Q^2) = q_i^0 - \frac{q_i^1 + q_i^{-1}}{2} \neq 0, \quad b_1 = \frac{1}{2} \sum_i e_{ii}^2 (\delta_T q_i(x, Q^2) + \delta_T \bar{q}_i(x, Q^2)) \neq 0$$

where  $q^m$  is parton distribution function in hadron **spin-m** state.

# Puzzle of deuteron tensor structure

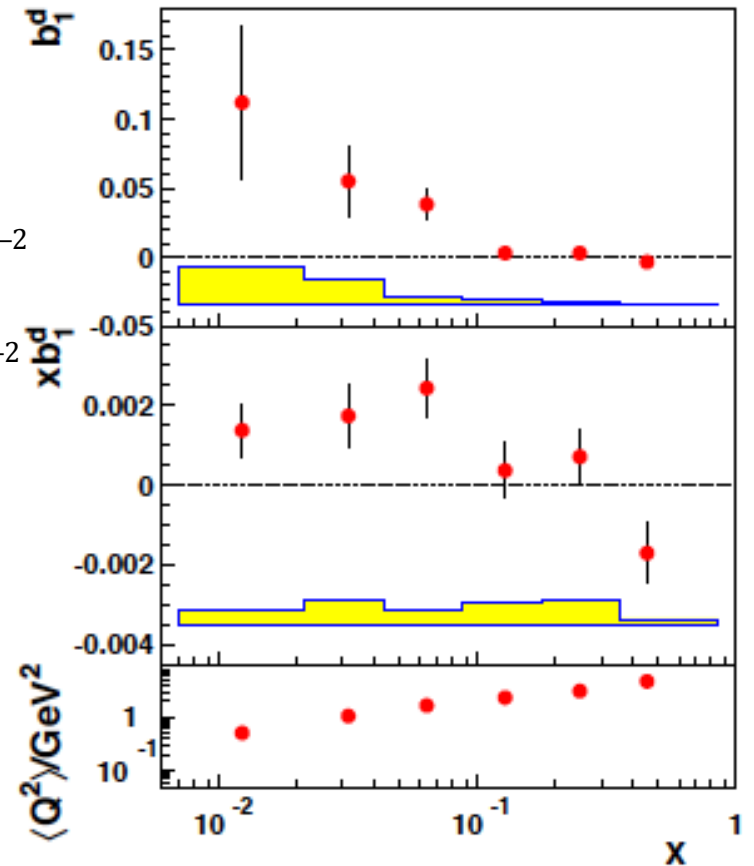
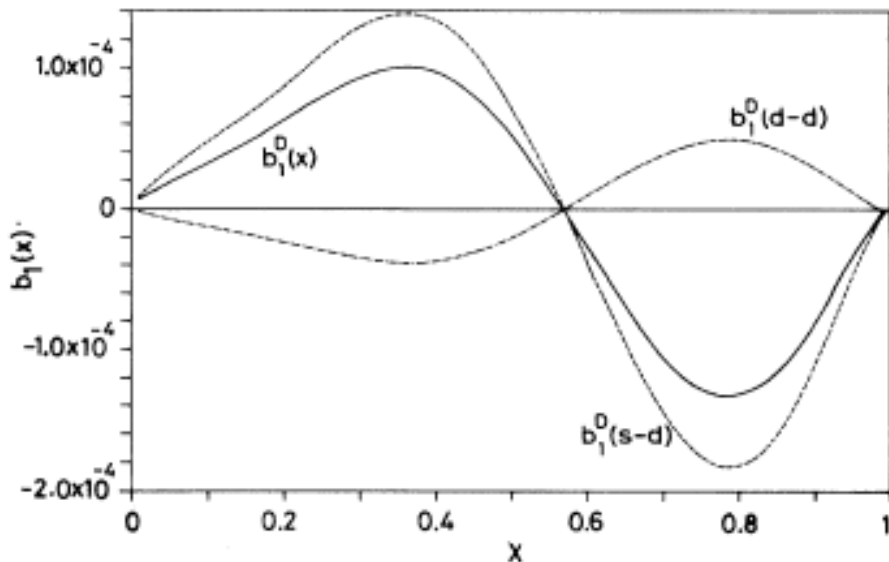
Hermes data show that  $b_1$  is not as small as the prediction for the S-D mixture proposal.

$$\int_{0.002}^{0.85} b_1(x) dx = [1.05 \pm 0.34(stat) \pm 0.35(sys)] \times 10^{-2}$$

$$\int_{0.02}^{0.85} b_1(x) dx = [0.35 \pm 0.10(stat) \pm 0.18(sys)] \times 10^{-2}$$

H.Khan and P. Hoodhoy, PRC 44 (1991) 1219

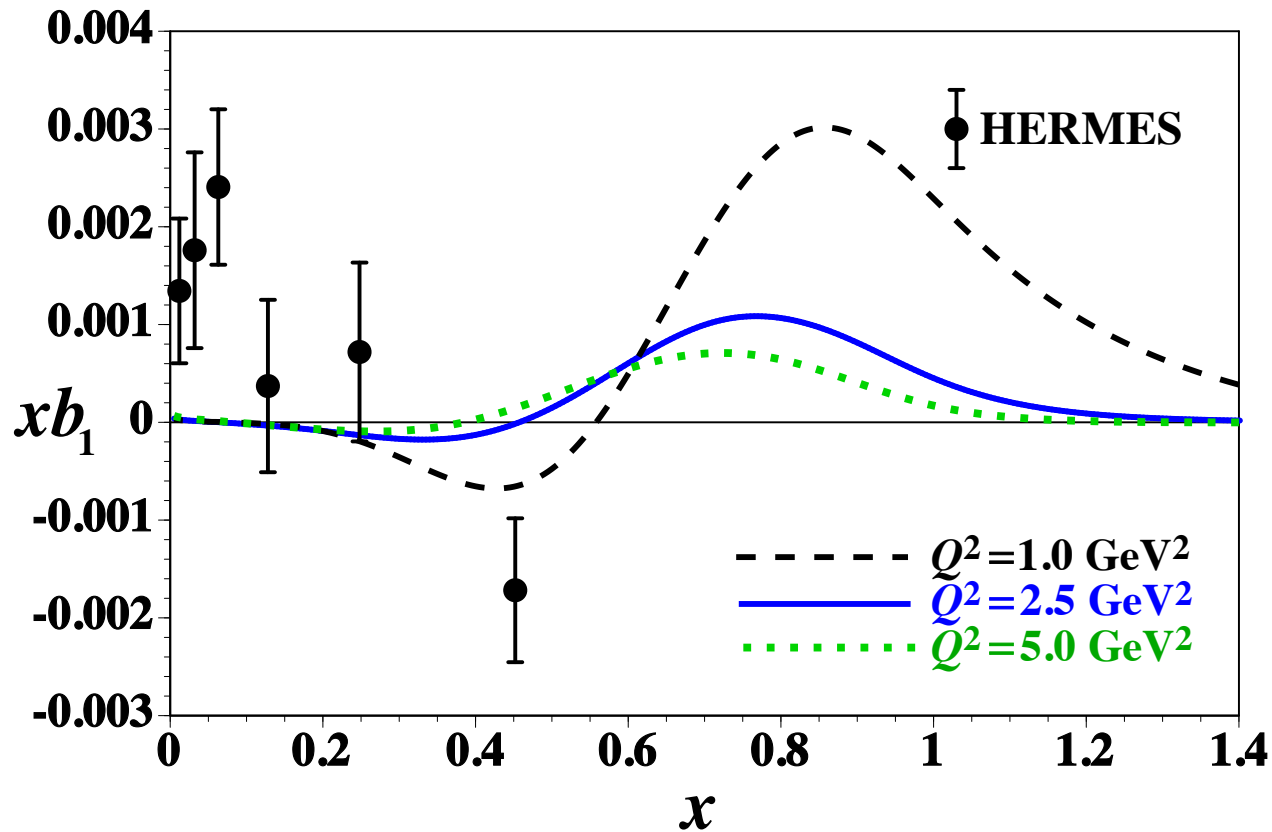
A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.



$$xb_1 \sim 10^{-4}$$

↕ Order of magnitude difference

$$xb_1 \sim 10^{-3} \text{ in HERMES data}$$



W. Cosyn, Yu-Bing Dong, S. Kumano and M. Sargsian, PRD 95 (2016) 074036

**Standard S-D mixture proposal can not explain the experimental data.**

Possible explanations for the unexpected  $b_1$  of the deuteron:

- ◆ Six quarks configuration of the deuteron
- ◆ Shadowing effects of the nucleus

G. A. Miller, PRC 89 (2014) 045203

N. N. Nikolaev and W. Schafer, PLB 398 (1997) 245

J. Edelmann, G. Piller, and W. Weise, Z. Phys. A 357, 129 (1997)

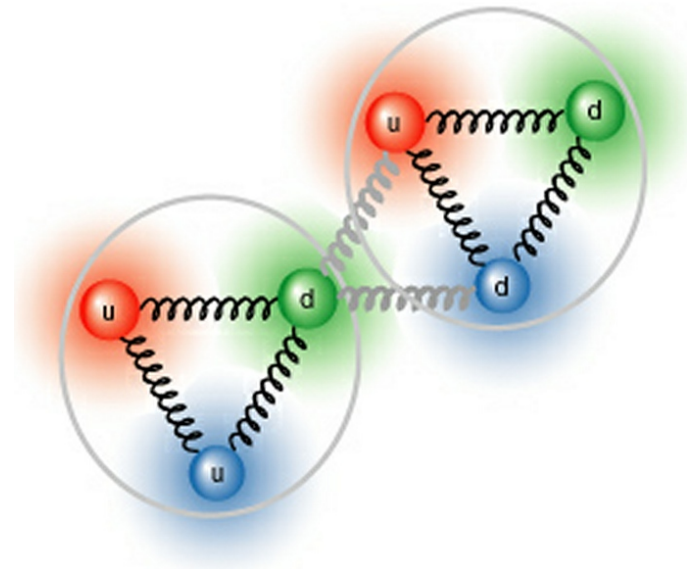
K. Bora and R. L. Jaffe, PRD 57 (1998), 6906

**The structure of the deuteron is not well understood!**

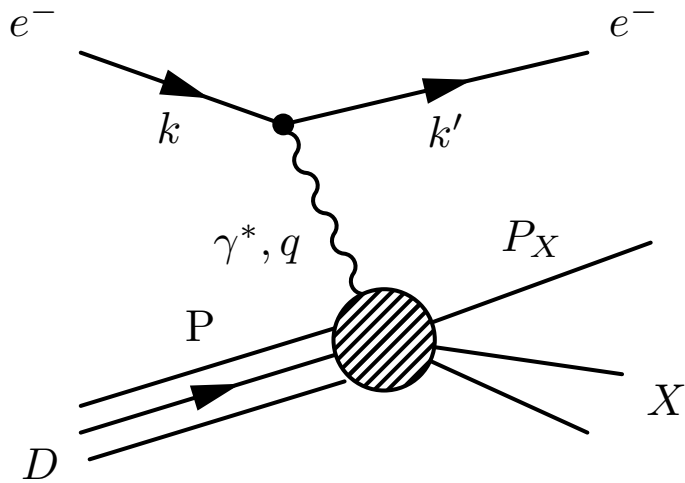


# An introduction to tensor structure of deuteron: theory and experiment

Two ways to investigate the structure of deuteron are  
**Deep Inelastic Scattering** and **Drell-Yan Process**



# Deuteron Structure Function in DIS



DIS for deuteron

$$W_{\mu\nu}^{\lambda_i\lambda_f} = \int \frac{d^4x}{4\pi} e^{iqx} \langle p, \lambda_f | J_\mu(x) J_\nu(0) | p, \lambda_i \rangle$$

$$W_{\mu\nu}^{\lambda_i\lambda_f} = -F_1 \hat{g}_{\mu\nu} + F_2 \frac{\hat{p}_\mu \hat{p}_\nu}{M\nu} + g_1 \frac{i}{\nu} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{\nu^2} \varepsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma) - b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu})$$

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NP B312 (1989) 571

L. L. Frankfurt and M. I. Strikman, NP A405 (1983) 557.

$F_1$ ,  $F_2$ ,  $g_1$  and  $g_2$  exist in spin-1/2 hadron, while  $b_1$ ,  $b_2$ ,  $b_3$  and  $b_4$  are the new quantities for spin-1 hadron. In total, there are 8 structure functions for deuteron.

## In Parton Model

$$F_1 = \frac{1}{2} \sum_i e_i^2 (q_i(x, Q^2) + \bar{q}_i(x, Q^2))$$

$$b_1 = \frac{1}{2} \sum_i e_i^2 (\delta_T q_i(x, Q^2) + \delta_T \bar{q}_i(x, Q^2))$$

$$\delta_T q_i(x, Q^2) = q_i^0 - \frac{q_i^1 + q_i^{-1}}{2}$$

Where  $q^m$  is parton distribution function in spin- $m$  hadron state, and index  $i$  is the quark flavor. **The tensor –polarized distributions** will disappear if the deuteron is S wave.

$$\int dx b_1(x) = \frac{5}{36} \int dx [\delta_T u_v(x) + \delta_T d_v(x)] + \frac{1}{18} \int dx [8\delta_T \bar{u}(x) + 2\delta_T \bar{d}(x) + \delta_T s(x) + \delta_T \bar{s}(x)]$$

$$\int dx \delta_T q_v(x) = \int dx [q_v^0 - \frac{q_v^1 + q_v^{-1}}{2}] = 0$$

$$\int dx b_1(x) = 0 + \frac{1}{18} \int dx [8\delta_T \bar{u}(x) + 2\delta_T \bar{d}(x) + \delta_T s(x) + \delta_T \bar{s}(x)]$$

# Experiment Status: the measurement of $b_1$

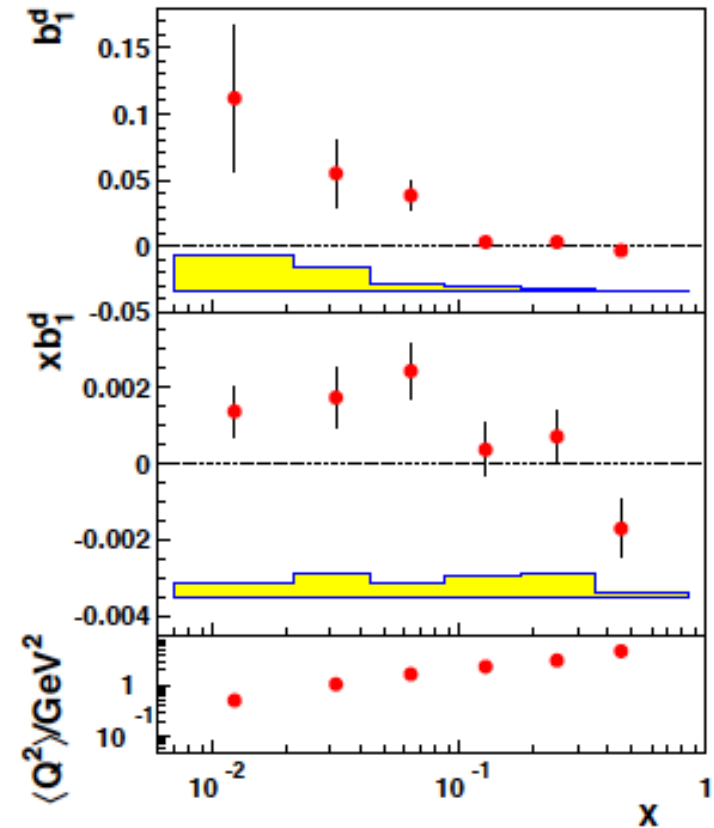
$$\int dx b_1(x) = 0 + \frac{1}{18} \int dx [8\delta_T \bar{u}(x) + 2\delta_T \bar{d}(x) + \delta_T s(x) + \delta_T \bar{s}(x)]$$

$$\int_{0.002}^{0.85} dx b_1(x) = [1.05 \pm 0.34(stat) \pm 0.35(sys)] \times 10^{-2}$$

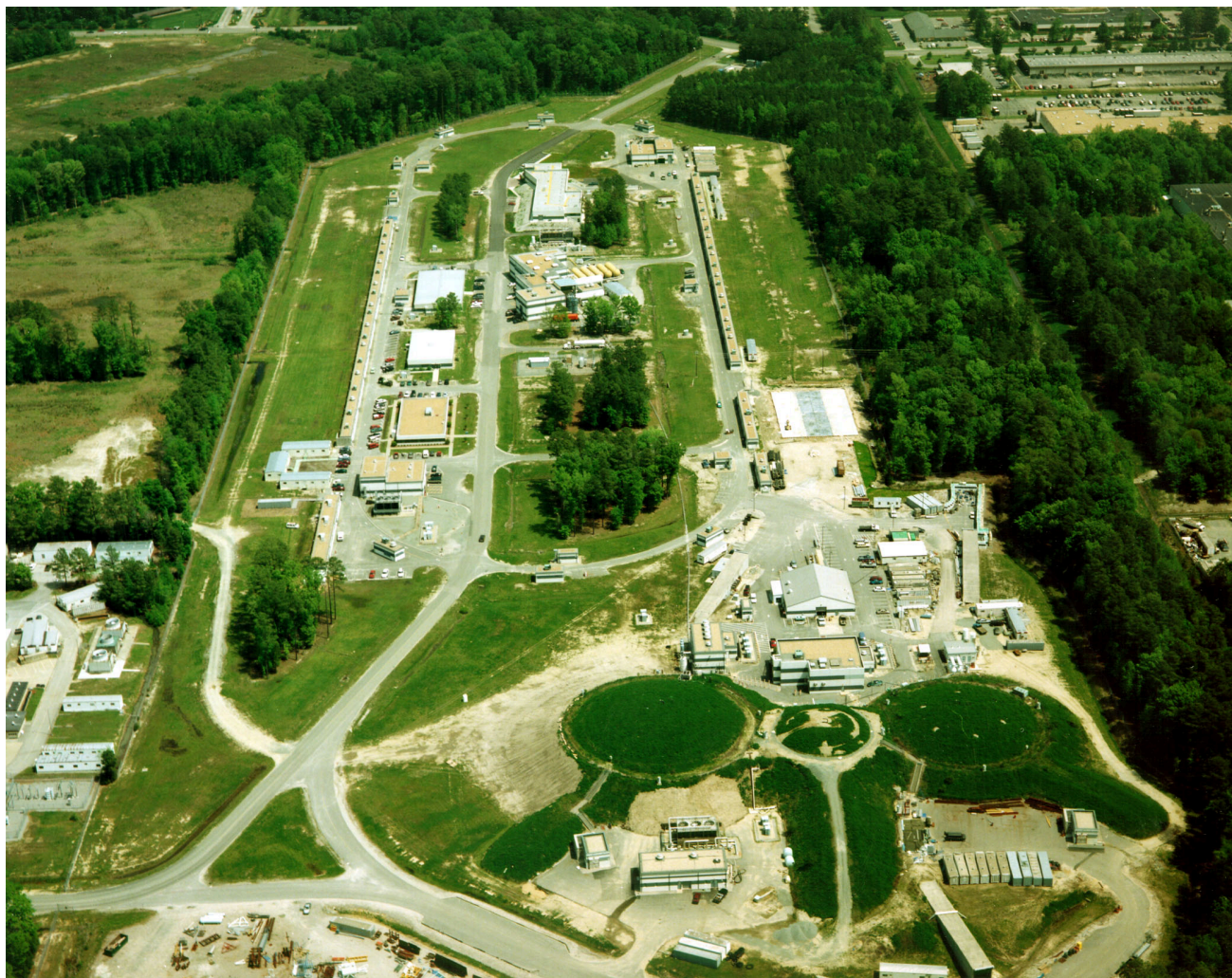
$$\int_{0.02}^{0.85} dx b_1(x) = [0.35 \pm 0.10(stat) \pm 0.18(sys)] \times 10^{-2}$$

The derivation from  $0$  of the integration will indicate the existence of **tensor – polarized distributions** for antiquark(sea) quarks.

A. Airapetian *et al.* (HERMES), PRL 95 (2005) 242001.

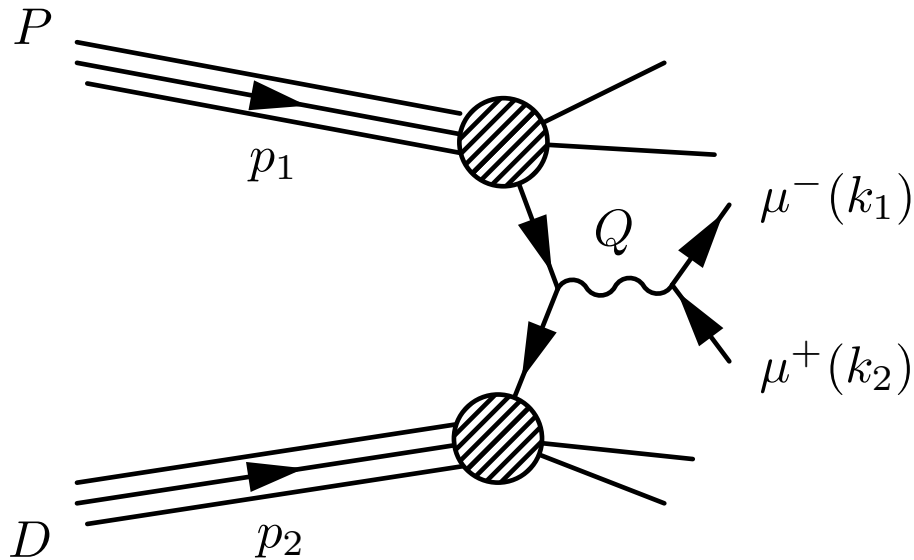


There is an approved experiment to measure  $b_1$  at JLab (Thomas Jefferson National Accelerator Facility), and this will help us to understand the tensor structure of deuteron.



# Drell-Yan process for proton and deuteron

$$P + D \rightarrow \gamma^* \rightarrow \mu^- \mu^+ + X$$



$$W_{\mu\nu} = \int \frac{d^4\xi}{(2\pi)^4} e^{iQ\xi} \langle P_1 S_1 P_2 S_2 | J_\mu(0) J_\nu(\xi) | P_1 S_1 P_2 S_2 \rangle$$

There are 108 structure functions for the hadron tensor of unpolarized proton-polarized deuteron Drell-Yan Process, and the spin asymmetry  $A_{UQ_0}$  is measured with the tensor polarized deuteron.

$$A_{UQ_0} = \frac{1}{2\langle\sigma\rangle} \left[ \sigma(\bullet, 0) - \frac{\sigma(\bullet, +1) + \sigma(\bullet, -1)}{2} \right]$$

S. Hino and S. Kumano, PRD 59 (1999) 094026;  
PRD 60 (1999) 054018

In Parton Model

$$A_{UQ_0} = \frac{\sum_i e_i^2 (q_i(x_1) \delta_T \bar{q}_i(x_2) + \bar{q}_i(x_1) \delta_T q_i(x_2))}{2 \sum_i e_i^2 (q_i(x_1) \bar{q}_i(x_2) + \bar{q}_i(x_1) q_i(x_2))}$$

The spin asymmetry  $A_{UQ_0}$  will indicate that existence of tensor –polarized distributions  $\delta_T q$  and  $\delta_T \bar{q}$ , which are only available in D-wave deuteron. In experiment, the tensor –polarized distributions have been confirmed by **Hermes measurements for  $b_1$  of electron-deuteron DIS.**

At Large  $x_F = x_1 - x_2 : q_i(x_1)\delta_T \bar{q}_i(x_2) \gg \bar{q}_i(x_1)\delta_T q_i(x_2)$

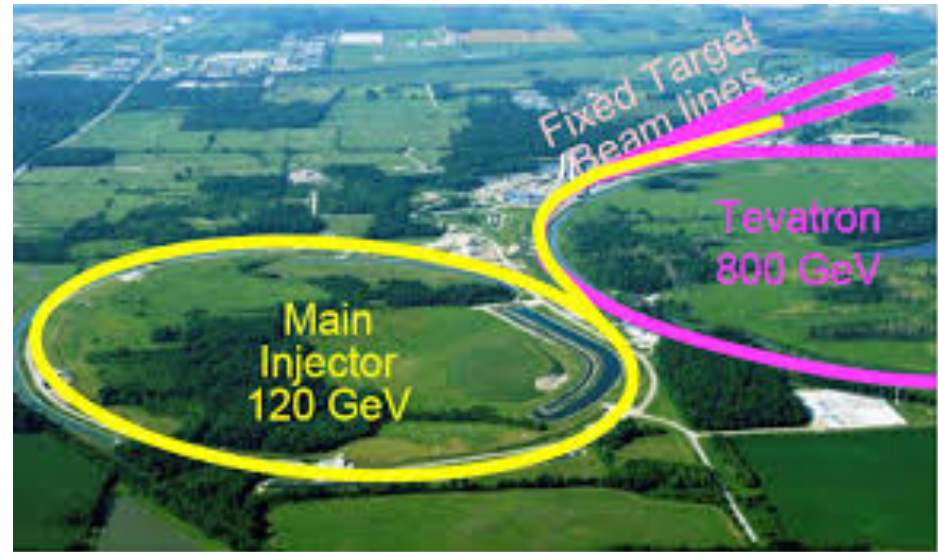
**If the tensor –polarized distributions of the quarks are neglected.**

$$A_{UQ_0} = \frac{\sum_i e_{ii}^2 (q_i(x_1)\delta_T \bar{q}_i(x_2))}{2 \sum_i e_{ii}^2 (q_i(x_1)\bar{q}_i(x_2))}$$



The asymmetry  $A_{UQ0}$  at large  $x_F$  reflects antiquark tensor-polarized distribution, and it is easier to get the antiquark tensor-polarized distribution from the measurement of the asymmetry  $A_{UQ0}$ .

The asymmetry could be measured by Fermilab E-1309 experiment through proton-deuteron Drell-Yan Process. The beam is unpolarized proton (120 GeV, Fermilab Main-Injector) and the target is (tensor) polarized deuteron.



See Kun Liu's talk at INT-17-68W workshop, October 5, 2017

Fermilab Drell-Yan process

## Results:

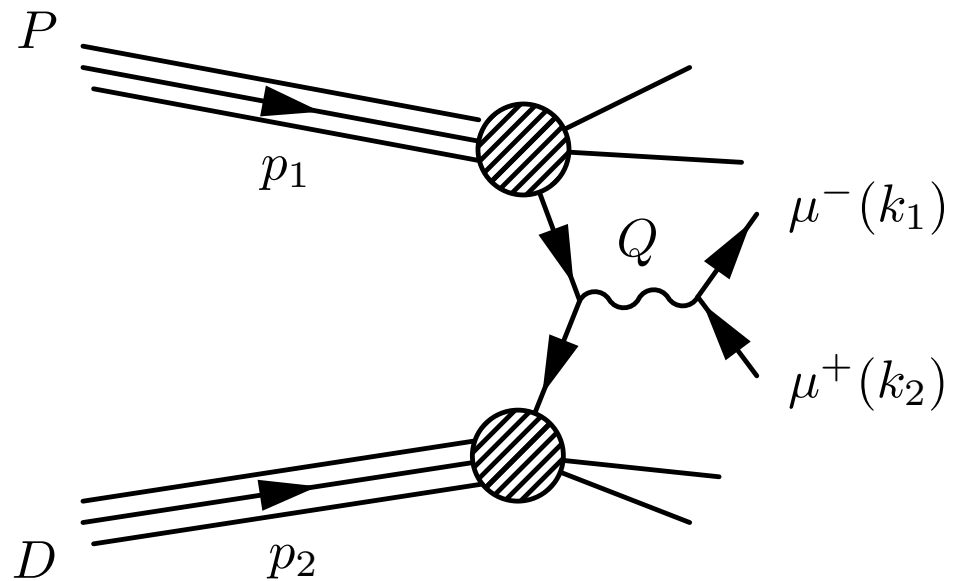
# Estimate on tensor-polarized asymmetry for the proton-deuteron Drell-Yan Process

$$P + D \rightarrow \mu^- \mu^+ + X$$

$$E_p = 120 \text{ GeV}$$

$$s = (p_1 + p_2)^2 = M_p^2 + M_d^2 + 2M_d E_p$$

$$Q^2 = x_1 x_2 s$$



Where  $p_1$  and  $p_2$  are the momenta of proton and deuteron, respectively.

In order to get  $A_{UQ}(x_1, x_2)$ , we need unpolarized distributions of proton and and tensor-polarized distributions of deuteron.

$$A_{UQ_0} = \frac{\sum_i e_i^2 (q_i(x_1) \delta_T \bar{q}_i(x_2) + \bar{q}_i(x_1) \delta_T q_i(x_2))}{2 \sum_i e_i^2 (q_i(x_1) \bar{q}_i(x_2) + \bar{q}_i(x_1) q_i(x_2))}$$

The unpolarized distributions of proton and deuteron  $q(x, Q^2)$  can be obtained by **MSTW**. We use the **functional form of parameterizations** for the initial tensor-polarized distributions of deuteron ( $Q^2=2.5\text{GeV}^2$ ) based on **Hermes data**.

A.D. Martin *et al* EPJC 63 (2009) 189

S. Kumano, PRD 82 (2010) 017501

## Parameterizations for initial tensor-polarized distributions of deuteron

$$\delta_T q^D(x, Q_0^2) = \delta_T w(x) \times q^D(x, Q_0^2) = \delta_T w(x) \times \frac{u_v(x, Q_0^2) + d_v(x, Q_0^2)}{2}$$

$$\delta_T \bar{q}^D(x, Q_0^2) = \bar{\alpha} \times \delta_T w(x) \times \bar{q}^D(x, Q_0^2) = \bar{\alpha} \times \delta_T w(x) \times \frac{2\bar{u}(x, Q_0^2) + 2\bar{d}(x, Q_0^2) + s(x, Q_0^2) + \bar{s}(x, Q_0^2)}{6}$$

$$\delta_T w(x) = ax^b(1-x)^c(x_0 - x)$$

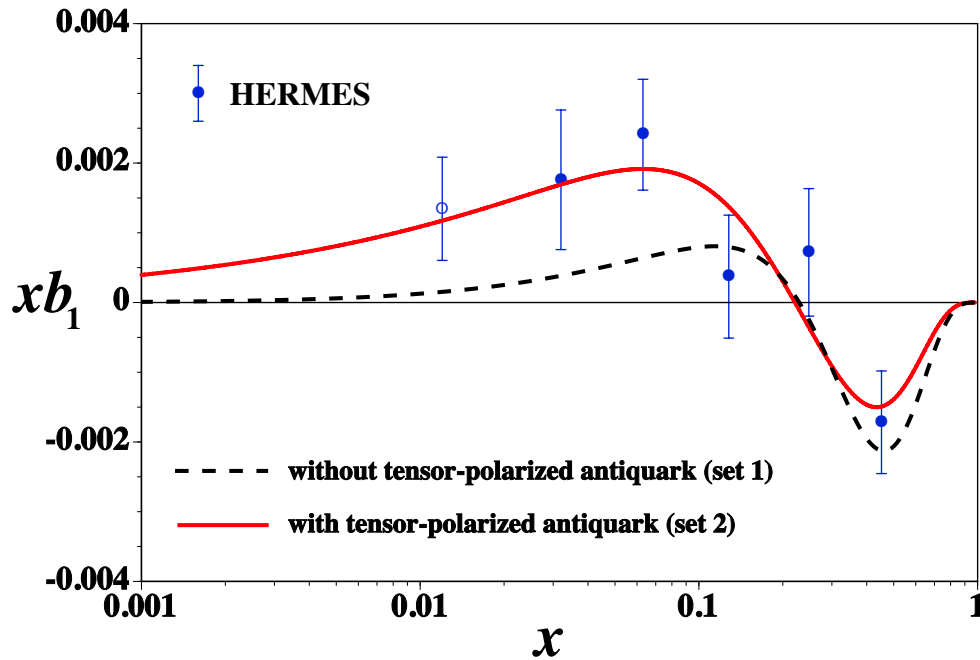
$$Q_0^2 = 2.5 \text{ GeV}^2$$

The existence of the node  $x_0$  satisfies the sum rule  $\int dx (b_1)_{\text{valence}} = 0$

$u_v(x)$  and  $d_v(x)$  are the valence quark distributions for the proton,  
 $\bar{u}(x)$ ,  $\bar{d}(x)$  and  $\bar{s}(x)$  are antiquark distributions for the proton.

**Set 1:**  $\delta_T \bar{q}^D(x) = 0$  no tensor-polarized antiquark distributions ( $\alpha_{\bar{q}} = 0$ ),

**Set 2:**  $\delta_T \bar{q}^D(x) \neq 0$  finite tensor-polarized antiquark distributions ( $\alpha_{\bar{q}} \neq 0$ ).



Set-1 results of  $xb_1$  can not explain the Hermes data at small  $x$  ( $x < 0.1$ ).

Set-2 results can fit the data well enough.

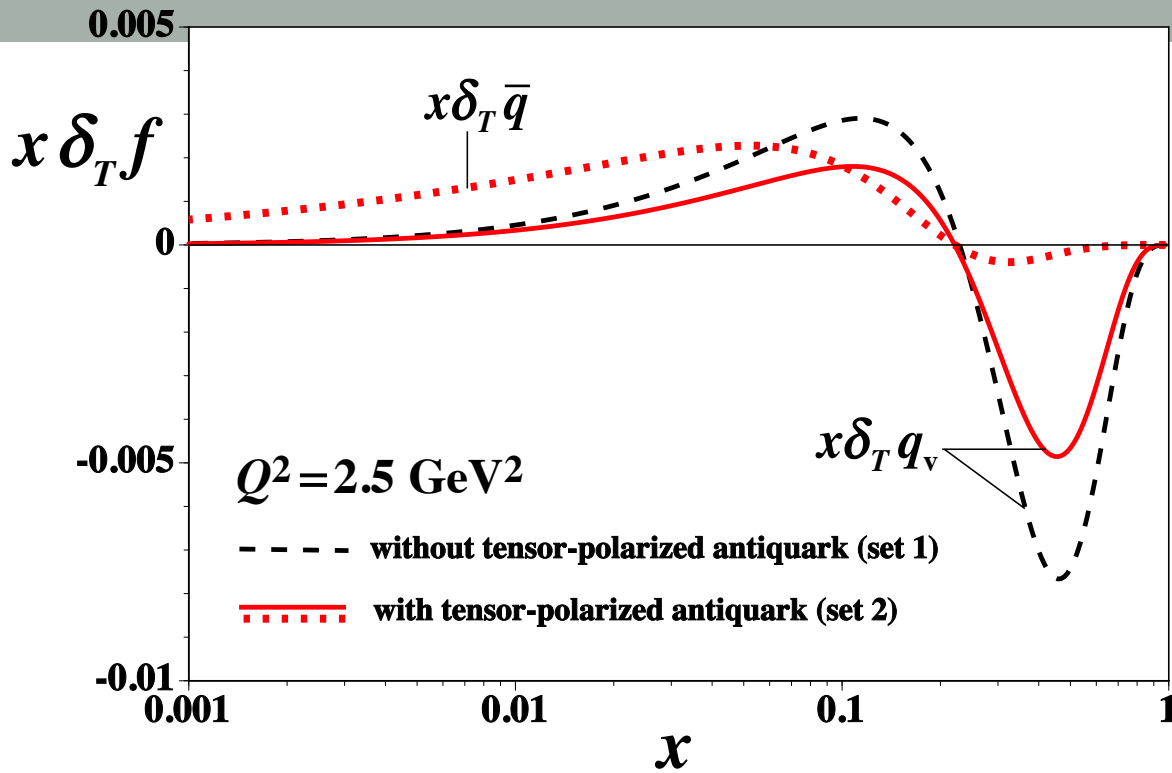
It is better to consider the antiquark tensor-polarized distributions at  $Q^2 = 2.5 \text{ GeV}^2$ .

$$\int_{0.002}^{0.85} b_1(x) dx = [1.05 \pm 0.34(\text{stat}) \pm 0.35(\text{sys})] \times 10^{-2}$$

$$\int_{0.02}^{0.85} b_1(x) dx = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2}$$

$$\int dx b_1(x) = 0 + \frac{1}{18} \int dx [8\delta_T \bar{u}(x) + 2\delta_T \bar{d}(x) + \delta_T s(x) + \delta_T \bar{s}(x)]$$

**Finite antiquark tensor-polarized distributions are necessary!**



Tensor-Polarized distributions at  $Q^2=2.5 \text{ GeV}^2$ , the set-2 antiquark tensor-polarized distribution is dominant at small  $x$  region ( $x < 0.02$ ). There is a node at  $x_0=0.229$  for set 1 and  $x_0=0.221$  for set 2, and this node is also predicted by standard S-D mixture proposal for deuteron.

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, NPB 312 (1989) 571

H. Khan and P. Hoodbhoy, PRC 44 (1991) 1219

W. Cosyn Yu-Bing Dong, S. Kumano and M. Sargsian, PRD 95 (2016) 074036

# The tensor-polarized distributions at other energy scale

The tensor-polarized distributions can be obtained by evolving initial tensor-polarized distributions to any energy scale  $Q^2$ . The gluon tensor-polarized distribution is set to be 0 at  $Q^2=2.5 \text{ GeV}^2$ .

$$\delta_T q^D(x_2, Q_0^2) \rightarrow \delta_T q^D(x_2, Q^2)$$

$$\delta_T g^D(x_2, Q_0^2) \rightarrow \delta_T g^D(x_2, Q^2)$$

$Q^2$  is determined by  $\mathbf{x}_1$  and  $\mathbf{x}_2$

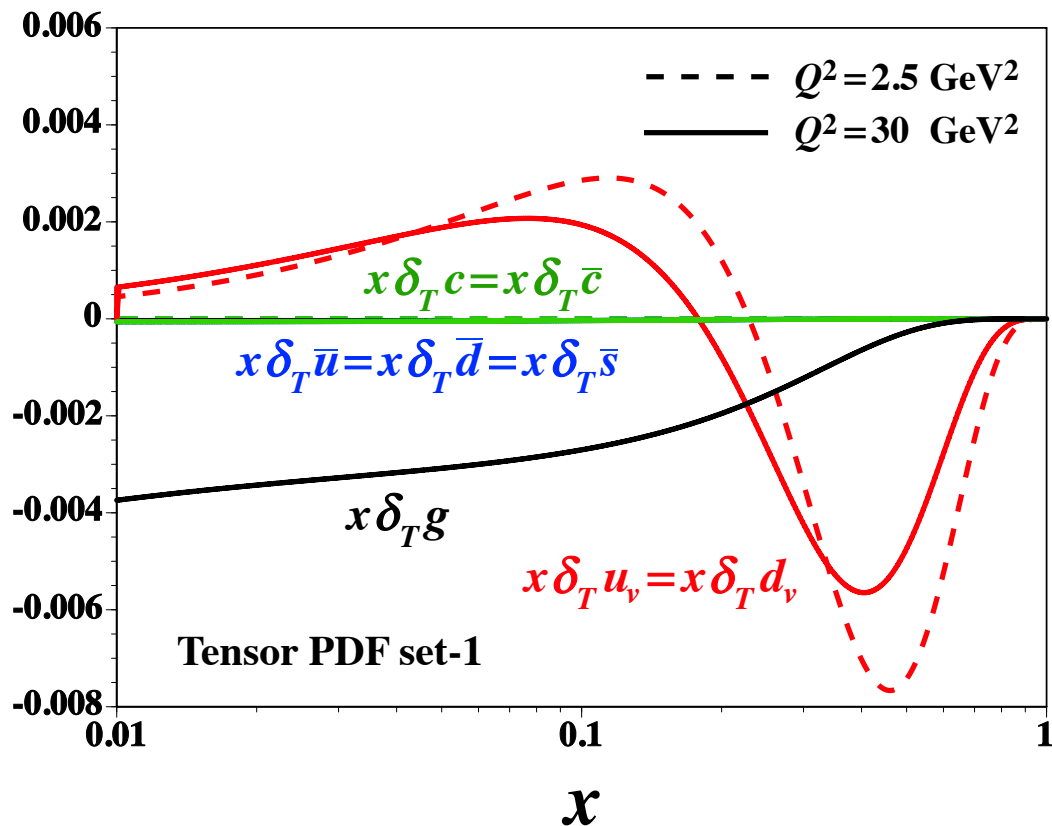
$E_p = 120 \text{ GeV}$  Fermilab Main Injector Proton Beam

$$s = (p_1 + p_2)^2 = M_p^2 + M_d^2 + 2M_d E_p = 454.545 \text{ GeV}^2$$

$$Q^2 = M_{\mu\mu}^2 = x_1 x_2 (2p_1 p_2) = x_1 x_2 s$$

**DGLAP evolution**

$$x \delta_T f(x)$$

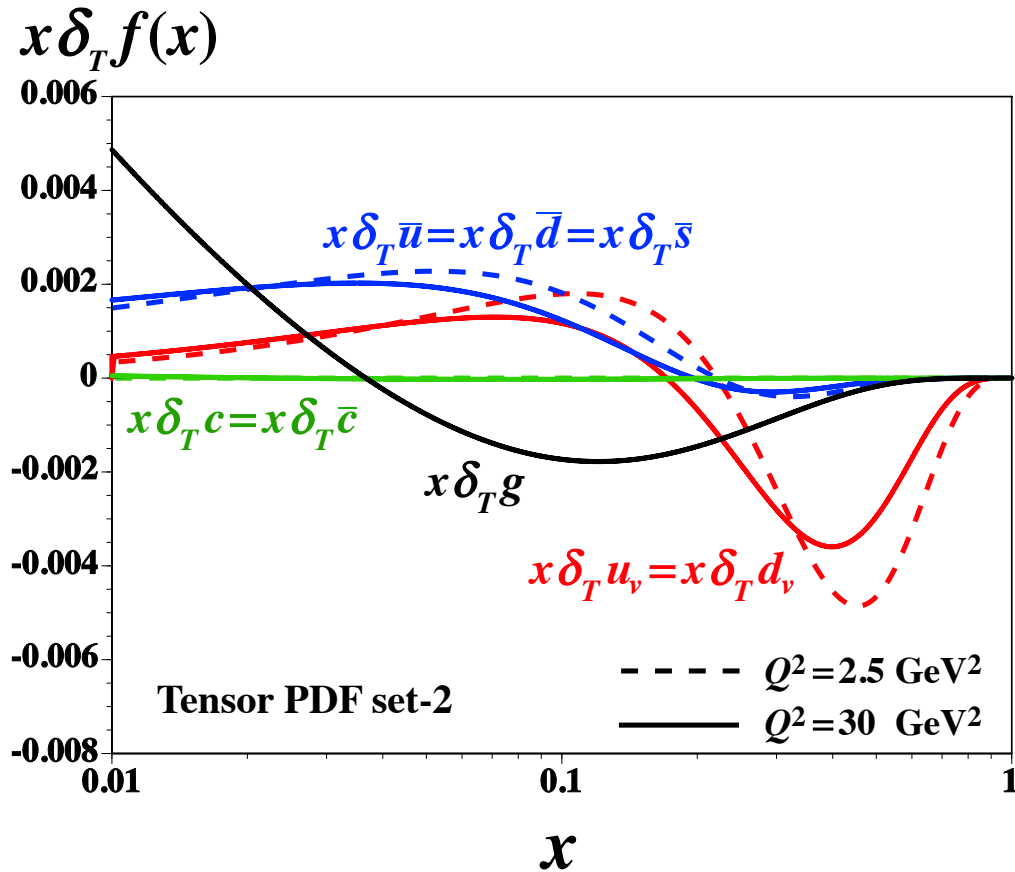


symmetry for antiquarks

$$\delta_T \bar{u} = \delta_T \bar{d} = \delta_T \bar{s} = \delta_T \bar{c}$$

The **set-1** tensor-polarized distributions at  $Q^2=2.5 \text{ GeV}^2$  and  $Q^2=30 \text{ GeV}^2$ . There also exists the tensor-polarized distribution for gluon, even though it is set to be zero at the initial energy scale  $Q^2=2.5 \text{ GeV}^2$ . Because there are no antiquark tensor-polarized distributions at  $Q^2=2.5 \text{ GeV}^2$ , so the symmetry for antiquarks will hold for any energy scale (leading order).



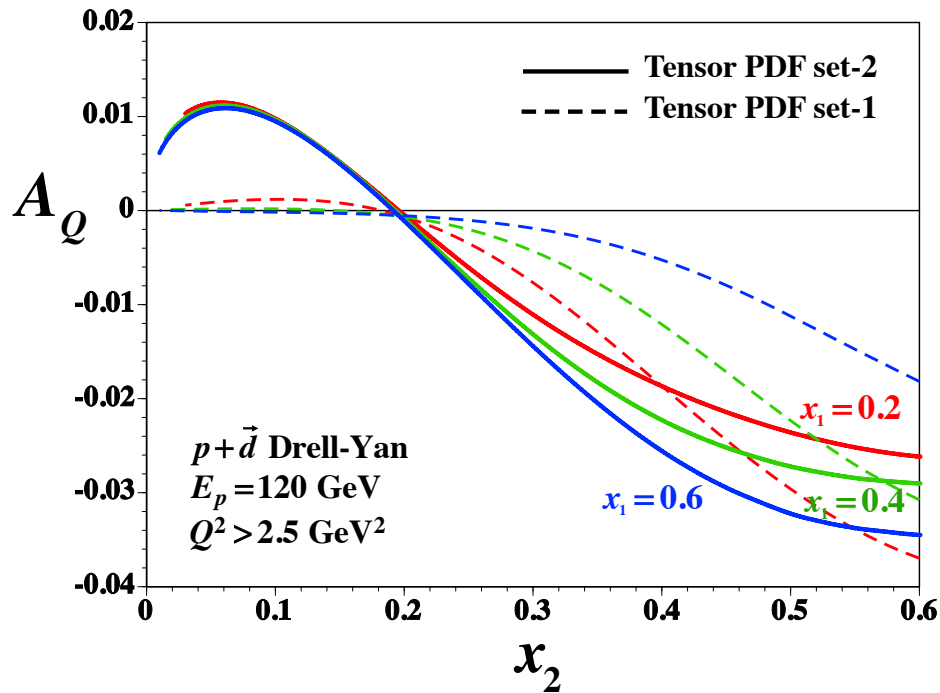


symmetry for antiquarks

$$\delta_T \bar{u} = \delta_T \bar{d} = \delta_T \bar{s} \neq \delta_T \bar{c}$$

The set-2 tensor-polarized distributions at  $Q^2 = 2.5 \text{ GeV}^2$  and  $Q^2 = 30 \text{ GeV}^2$ .

Because there are antiquark tensor-polarized distributions ( $\delta_T \bar{u} = \delta_T \bar{d} = \delta_T \bar{s} \neq 0$ ) at  $Q^2 = 2.5 \text{ GeV}^2$ , so the antiquark tensor-polarized distributions are SU(3) flavor symmetric for any energy scale (leading order).



In the figure, tensor-polarized asymmetry  $A_Q$  is shown at typical values of  $x_1=0.2, 0.4$  and  $0.6$ .

$$A_Q(x_1, x_2) = 2A_{UQ_0}(x_1, x_2)$$

$$A_{UQ_0} = \frac{\sum_i e_i^2 (q_i(x_1) \delta_T \bar{q}_i(x_2) + \bar{q}_i(x_1) \delta_T q_i(x_2))}{2 \sum_i e_{ii}^2 (q_i(x_1) \bar{q}_i(x_2) + \bar{q}_i(x_1) q_i(x_2))}$$

- ◆ The values of set-1 and set-2 are both **a few percent**.
- ◆ The set-1 results are so different from those of set-2 at small region of  $x_2$ , and this is because that **antiquark tensor-polarized distributions** are more important when  $x_2$  is small.
- ◆ The set-2 results should be **more reliable**, since the tensor-polarized distributions can also explain the Hermes data well.

# Error bands estimate

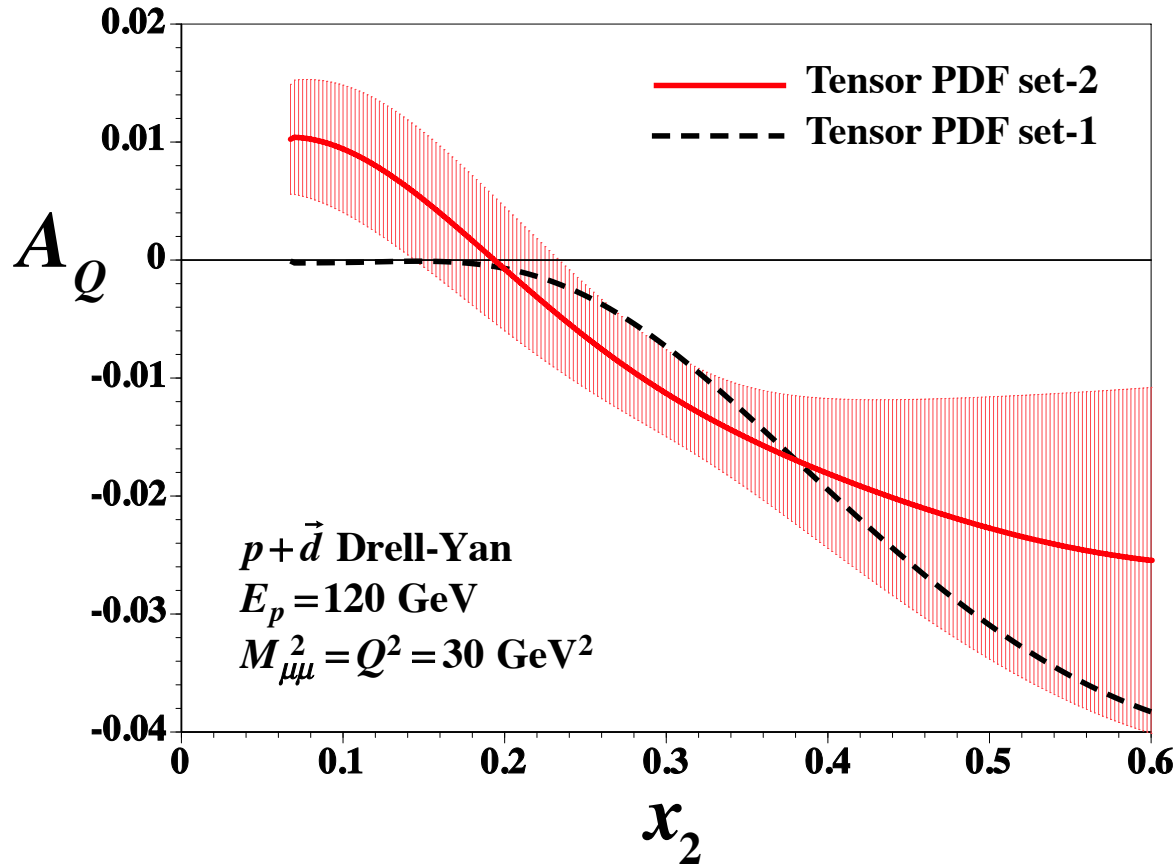
Hessian Matrix

$$\chi^2(\hat{\xi} + \delta\hat{\xi}) - \chi^2(\hat{\xi}) = \sum_{ij} H_{ij} \delta\xi_i \delta\xi_j$$

The error of a physics quantity  $f(x)$  is

$$[\delta f(x)]^2 = \Delta\chi^2 \sum_{ij} \left[ \frac{\partial f(x)}{\partial \xi_i} \right]_{\hat{\xi}} H_{ij}^{-1} \left[ \frac{\partial f(x)}{\partial \xi_j} \right]_{\hat{\xi}}$$

$$\Delta\chi^2 = 1$$



Spin asymmetry  $A_Q$  at typical energy scale ( $Q^2=30$  GeV<sup>2</sup>) with the uncertainties estimate.

# Summary

The new structure function  $b_1$  (DIS) and spin asymmetry  $A_Q$  (Drell-Yan) of deuteron reflect the tensor-polarized distributions, which have a close relationship with the orbital angular momentum in spin-1 hadrons. In this talk, we give the theoretical estimate of the spin asymmetry  $A_Q$ , and it is of the order of a few percent. In the future, those quantities could be measured by Jlab ( $b_1$ ) and Fermilab ( $A_Q$ ), which may reveal the puzzle of deuteron.

**Thank you very much**