# RQCD Lattice Simulations

G. Bali, V. Braun, P. Bruns, S. Collins, A. Cox, M. Diehl, M. Engelhardt, B. Gläßle, M. Göckler, M. Gruber, F. Hutzler, P. Korcyl, B. Lang, B. Musch, E. Scholz, J. Simeth, P. Wein, T. Wurm, J.-H. Zhang, C. Zimmermann, ...

- RQCD, CLS and SFB/TRR-55
- Standard objects: masses and decay constants, e.g., $D_{s0}^*(2317)$ and $D_{s1}(2460)$; GPDs; ...
- Coordinate space methods and DAs (Ji 2013, Braun & Mueller 2008)
- TMDs and DPDs
- QCD in strong magnetic fields
- Conclusion
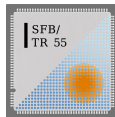
RQCD: The Regensburg (& associates) lattice group

CLS: The main systematic uncertainties are meanwhile due to discretization errors. Rapidly increasing topological autocorrelation times for decreasing $a \Rightarrow$ Lüscher and Schaefer suggested simulations with open boundary conditions $\Rightarrow$ CLS is an international collaboration pooling its computer resources. Regensburg and Wuppertal are members.

SFB/TRR-55: An interdisciplinary research center of Regensburg and Wuppertal University focused on LQCD, Applied Mathematics and HPC plus many external lattice collaborations (e.g. TMDC)

Ensembles with $m_u + m_d + m_s =$ physical
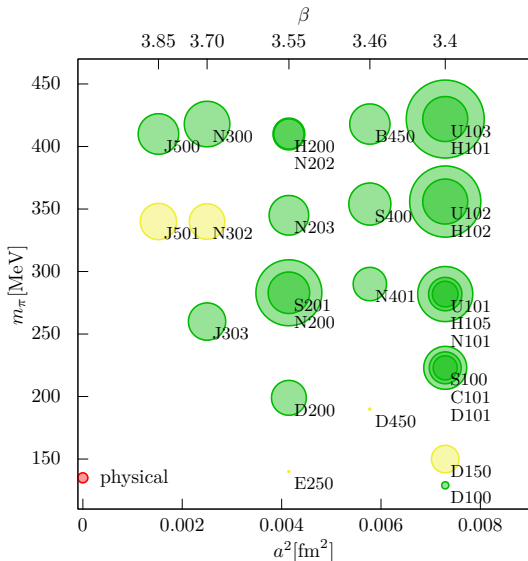
Ensembles with $m_s =$ physical
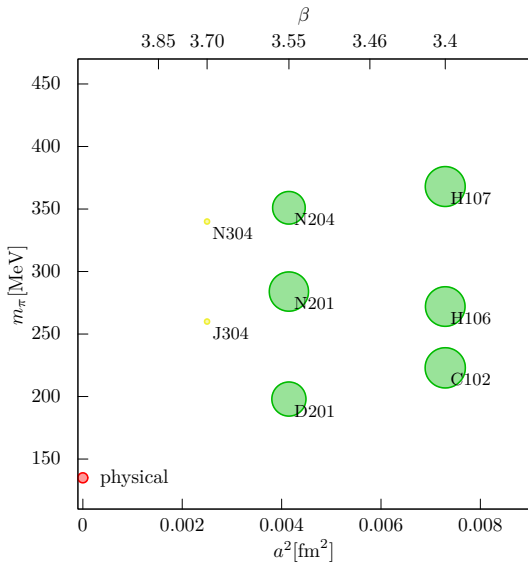
Moments of GPDs

$$h(P_1) + \Gamma^*(q_1) \to h(P_2) + \Gamma(q_2)$$

with $\Delta_\mu = q_{2\mu} - q_{1\mu}$, $t = \Delta^2$, $P_\mu = (P_{1\mu} + P_{2\mu})/2$
and $\xi = -Q^2/2P \cdot q$

Spin $\frac{1}{2}$ - the nucleon (modulo gauge links)

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P_2| \, \bar{q}(-\frac{1}{2}z) \, \gamma^+ q(\frac{1}{2}z) \, |P_1\rangle \Big|_{z^+=0, \, z_\perp=0}$$
$$= \frac{1}{P^+} \left[H_q(x,\xi,t) \, \bar{N}(P_2)\gamma^+ N(P_1) + E_q(x,\xi,t) \, \bar{N}(P_2)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} N(P_1) \right]$$

$$
\begin{aligned}
\langle p'\lambda|\mathcal{O}^{\{\mu_1\mu_2\}}|p\lambda\rangle &= P^{\{\mu_1}\langle\langle\gamma^{\mu_2\}}\rangle\rangle A_{20}(t) \\
&+ \frac{\mathrm{i}}{2m}\,P^{\{\mu_1}\langle\langle\sigma^{\mu_2\}\alpha}\rangle\rangle\Delta_\alpha B_{20}(t) - (2\xi)^2 C_{20}(t) \\
H^{(2)}(\xi,t) &= \int_{-1}^{1} dx x H(x\xi,t) = A_{20}(t) + (2\xi)^2 C_{20}(t) \\
E^{(2)}(\xi,t) &= \int_{-1}^{1} dx x E(x\xi,t) = B_{20}(t) - (2\xi)^2 C_{20}(t) \\
\mathcal{O}^{\{\mu_1\mu_2\}} &= \bar{q}(0)\Gamma^{\{\mu_1}\mathrm{i}\overset{\leftrightarrow}{D}{}^{\mu_2\}} q(0) \\
\Gamma^{\mu_1} &= \gamma^{\mu_1},\ \gamma^{\mu_1}\gamma_5 \\
F(b_T) &= \frac{1}{(2\pi)^2}\int d^2\Delta_T\, e^{-\mathrm{i}b_T\cdot\Delta_T}\, F(\Delta_T)
\end{aligned}
$$

M. Diehl and P. Hägler, hep-ph/0504175
parton density, expressed in terms of GPDs

$$
\frac{1}{(2\pi)^2} \int d^2\Delta_\perp\, e^{ib_\perp \cdot \Delta_\perp} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P_2|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ [1 + \vec{s} \cdot \vec{\gamma})\gamma_5] q(\tfrac{1}{2}z)\, |P_1\rangle \Big|_{z^+=0}^{z_\perp=0}
$$

$$
= \frac{1}{2}\Big[F + s^i F_T^i \Big]
$$

$$
= \frac{1}{2}\bigg[H - S^i \epsilon^{ij} b^j \frac{1}{m} E' - s^i \epsilon^{ij} b^j \frac{1}{m}\Big(E_T' + 2\tilde{H}_T' \Big)
$$

$$
+ s^i S^i \Big(H_T - \frac{1}{4m^2}\, \Delta_b \tilde{H}_T \Big) + s^i (2b^i b^j - b^2 \delta^{ij}) S^j \frac{1}{m^2} \tilde{H}_T'' \bigg]
$$
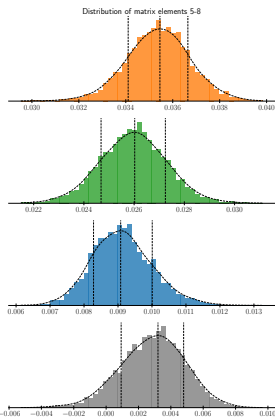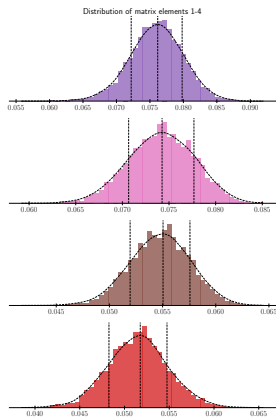
has a simple interpretation:

$S^i \epsilon^{ij} b^j$ coupling of proton spin to quark angular momentum
$s^i \epsilon^{ij} b^j$ coupling of quark spin to quark angular momentum
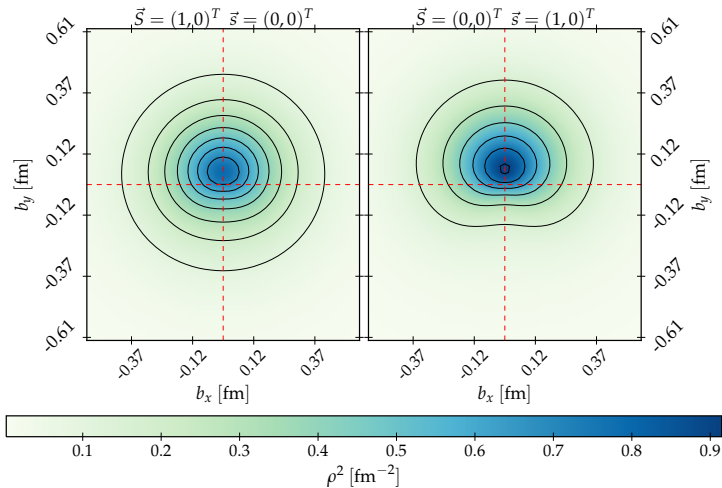$s^i S^i$ coupling of quark spin and proton spin
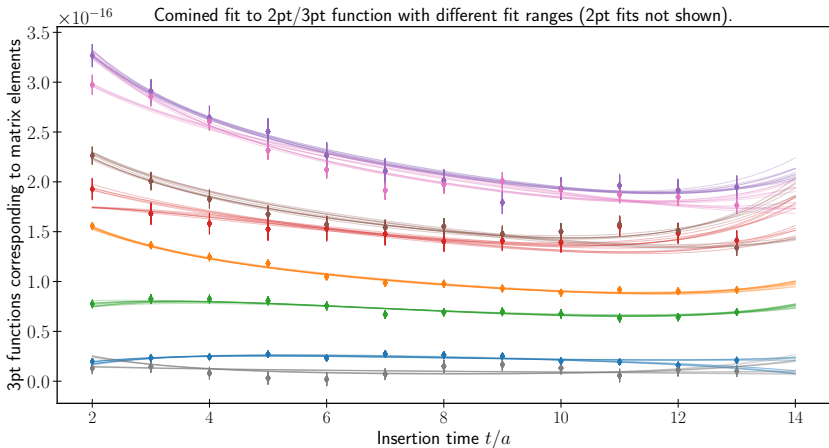
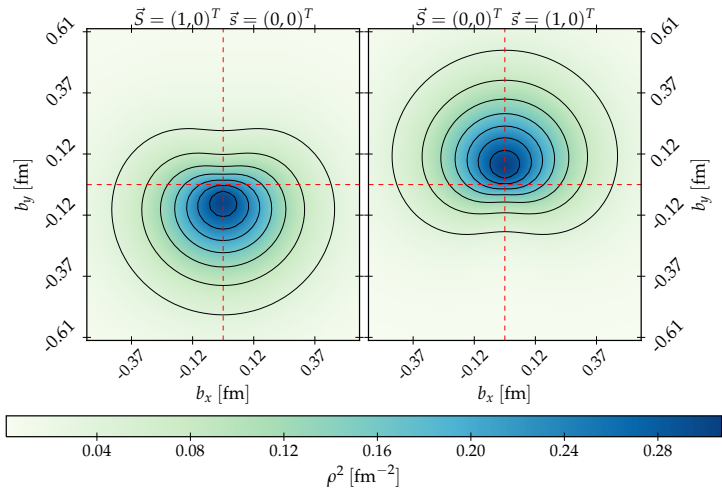results for the different correlators

R. Rödl et al.
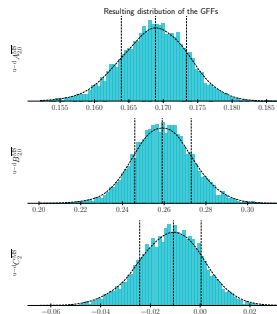
distribution of extrapolated results many fits, defining the
systematic error R. Rödl et al.

transverse densities for u quarks

transverse densities for d quarks

Standard observabels II:
$D_{s0}^*(2317)$ and $D_{s1}(2460)$ masses and decay constants
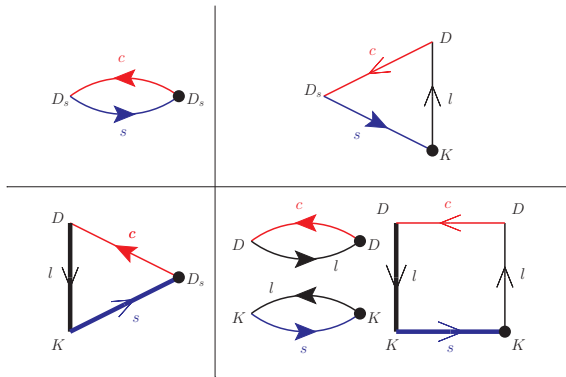
$N_f = 2$; physical masses; large statistics; QPACE 1;
S. Collins and A. Cox et al. 1706.01247

These resonances couple strongly to $D + K$. Therefore one has to work with mixed states, meaning with a matrix of correlators.

Aim:
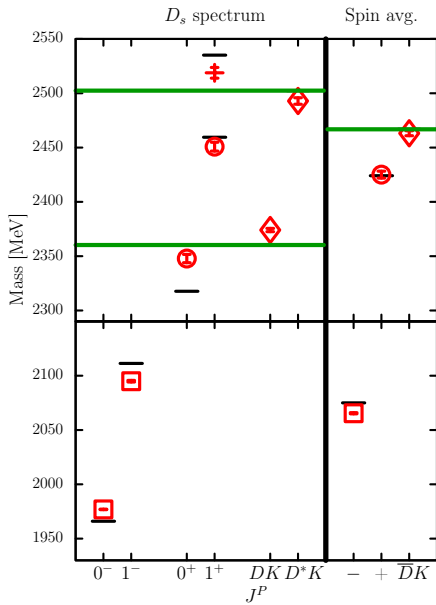(i) FLAG corrected QCD masses: $m_{D_s^*}^{QCD} = m_{D_s^*} - \delta m_{D_s^*}^{QED}$
(ii) decay constants

charm, strange, light: red, blue, black lines
stochastic propagators: lines with filled arrows
sequential stochastic propagators: two lines with open arrows
stochastic source: black dot

	$f_S^{0^+}$	$f_V^{0^+}$	$f_A^{1^+}$
This work	241(4)(2)(+12)(10)	114(2)(0)(+5)(10)	194(3)(4)(+5)(10)
LQCD hep-lat//0604001	340(110)	200(50)	
B-decays+HQS hep-ph/0410301		74(11)	166(20)
B-decays+HQS hep-ph/0305038		67(13)	
B-decays+HQS hep-ph/0605073		58-86	130-200
QM hep-ph/0107047		440	410
QM hep-ph/0312232		122-154	
LF QM hep-ph/0310359		71	117
LF QCDSR hep-ph/0505195*	225(25)		225(25)
DK-molecule 0705.0892		67.1(4.5)	144.5(11.1)
LF QM 1103.2973		$74.4^{+10.4}_{-10.6}$	159^{-36}_{+32}
QM 1203.4362		119	165
QCDSR 1506.01993	333(20)		245(17)

* Colangelo et al. "... favours $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ as ordinary mesons"

Other approaches to calculate hadron structure:

1.) Large Momentum Effective Theory (LaMET) X. Ji et al.

$$\tilde{q}(x, \Lambda, p^z) = \int \frac{dz}{4\pi} e^{ixp^z z} \langle p|\overline{\psi}(0, 0_\perp, z)\gamma^z L(z, 0)\psi(0)|p\rangle$$

gauge link $L(z, 0)$ hadron momentum $p^\mu = (p^0, 0, 0, p^z)$
UV cutoff Λ

Results are plagued by artefacts for $p^z \leq 1\text{GeV}$.

Ideas for improved formulations are needed. Two suggestions:
(i) modified definitions
(ii) momentum smearing

A Gaussian weight suppresses long distance artefacts but does not require recalculation of the matching functions
Jianhui Zhang et al. tbp

$$
\begin{aligned}
\tilde{q}_{GW}(x, \Lambda, p^z) &= \int \frac{dz}{4\pi} e^{ixp^z z - z^2/a^2} \langle p | \overline{\psi}(0, 0_\perp, z) \gamma^z L(z, 0) \psi(0) | p \rangle \\
&= \int_{-1}^{1} \frac{du}{|u|} \int dy \, Z\left(\frac{y}{u}, \frac{\Lambda}{p^z}, \frac{\mu}{p^z}\right) \left[\frac{ap^z}{2\sqrt{\pi}} e^{-\frac{a^2(x-y)^2 p_z^2}{4}} \right] q(u) \\
&+ \mathcal{O}(\Lambda_{\mathrm{QCD}}^2/p_z^2, M^2/p_z^2, 1/ap^z)
\end{aligned}
$$

This is only an illustration. Other modifications can be more effective but will require recalculation of Z.
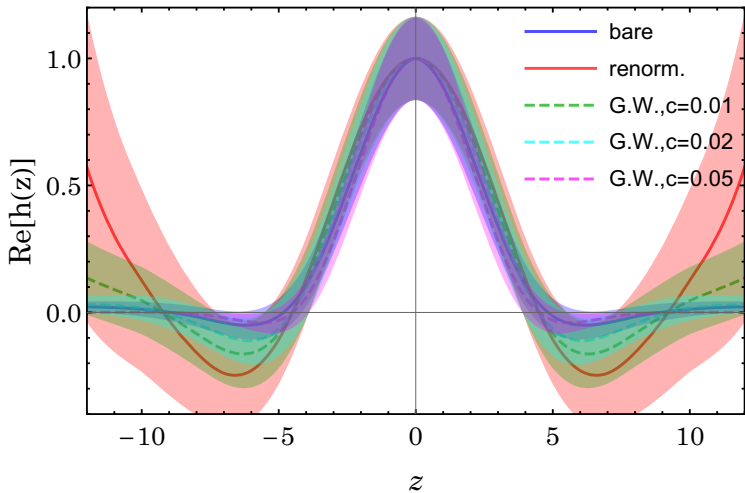
Real part of the correlator
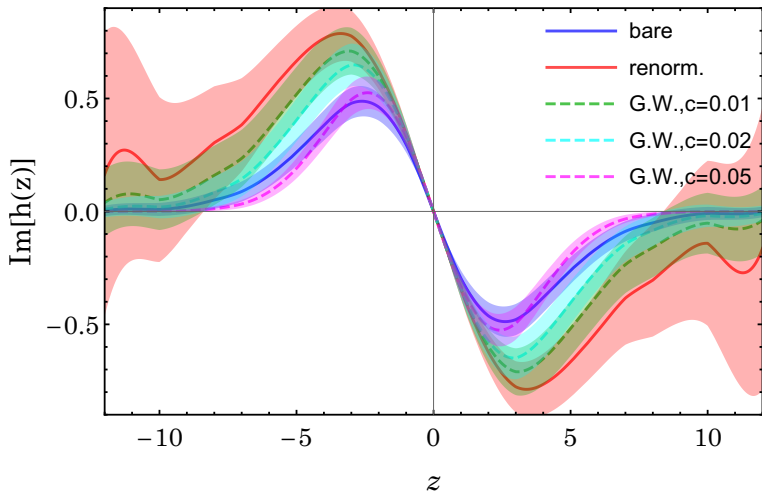
Imaginary part of the correlator
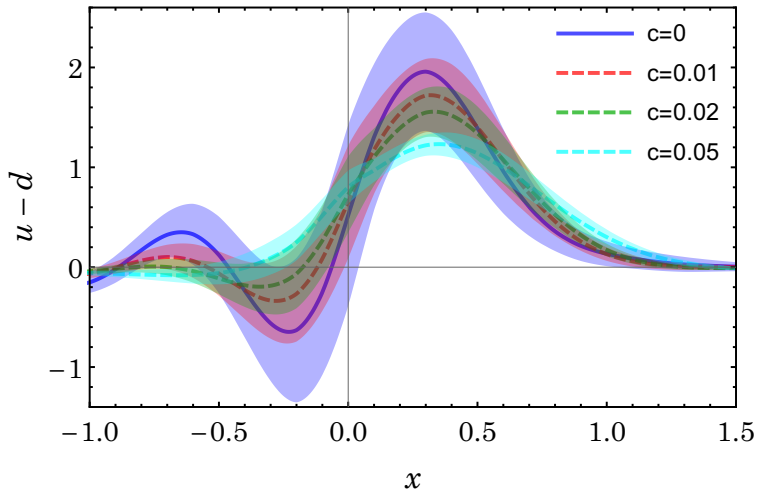
resulting quark pdf. For $p^z \to \infty$ all have to agree

Whatever you do, reaching large momenta is the key

momentum dependent smearing: B. Musch et al., 1602.05525
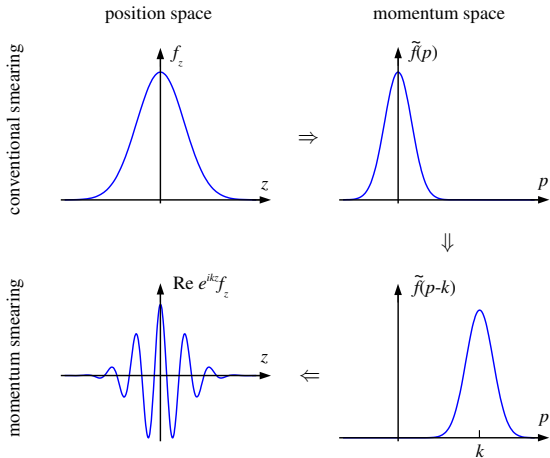
- modified smearing works great
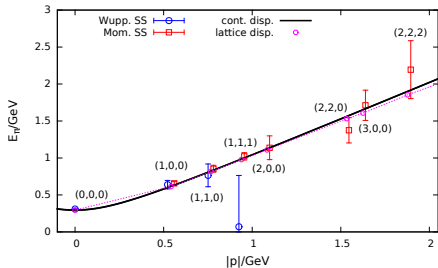
pion
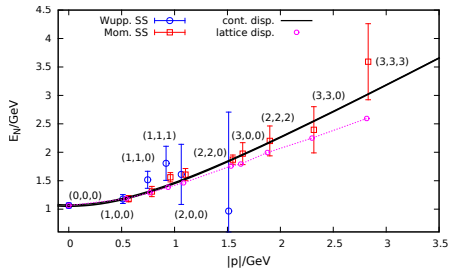
nucleon

This improves also the moment method, e.g., for the pion DA
F. Hutzler et al. 1705.10236

in the continuum

$$\langle 0|\bar{d}(z_2 n)\not{n}[z_2 n, z_1 n]\gamma_5 u(z_1 n)|\pi^+(p)\rangle$$
$$= i f_\pi(p \cdot n) \int_0^1 dx\, e^{-i(z_1 x + z_2(1-x))p \cdot n}\phi(x, \mu^2),$$
$$\langle \xi^n \rangle = \int_0^1 dx\, (2x-1)^n \phi(x, \mu^2)$$
$$\phi(x, \mu^2) = 6x(1-x)\left[1 + \sum_{n=1}^\infty a_n(\mu^2)C_n^{3/2}(2x-1)\right]$$

on the lattice:

$$
\begin{aligned}
C_{\rho\mu\nu}^{\pm}(t, \boldsymbol{p}) &= a^3 \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\boldsymbol{x}} \langle \mathcal{O}_{\rho\mu\nu}^{\pm}(\boldsymbol{x}, t) J^{\dagger}(0) \rangle \\
J(x) &= \bar{d}(x)\gamma_5 u(x) \\
\mathcal{O}_{\rho\mu\nu}^{+}(x) &= \bar{d}(x) \left[\overleftarrow{D}_{\{\mu}\overleftarrow{D}_{\nu} + 2\overleftarrow{D}_{\{\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{\{\mu}\overrightarrow{D}_{\nu} \right] \gamma_{\rho\}}\gamma_5 u(x) \\
\mathcal{O}_{\rho\mu\nu}^{-}(x) &= \bar{d}(x) \left[\overleftarrow{D}_{\{\mu}\overleftarrow{D}_{\nu} - 2\overleftarrow{D}_{\{\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{\{\mu}\overrightarrow{D}_{\nu} \right] \gamma_{\rho\}}\gamma_5 u(x), \\
\mathcal{R}_{4ij}^{\pm} &= \frac{C_{4ij}^{\pm}(t, \boldsymbol{p})}{C_4(t, \boldsymbol{p})} \\
\langle \xi^2 \rangle &= \zeta_{11} R^- + \zeta_{12} R^+ \qquad \text{renormalization}
\end{aligned}
$$
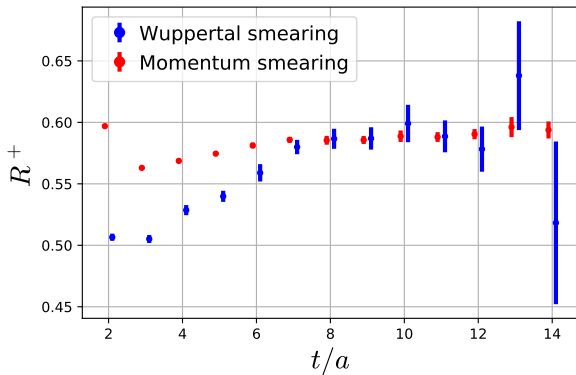
The improved errors for R^+ on H105
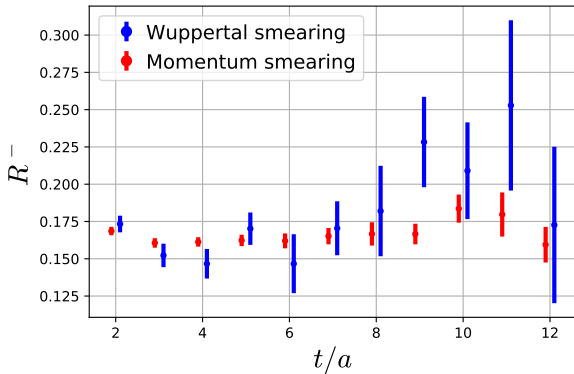
The improved errors for R^- on H105
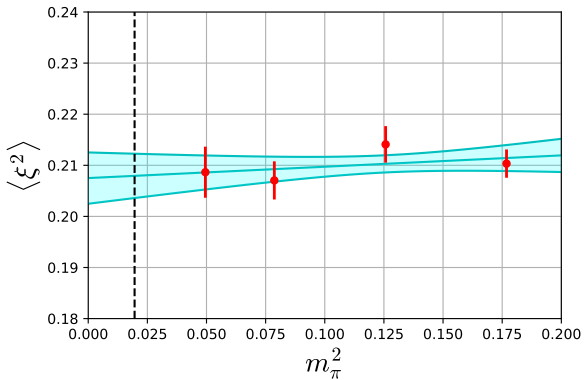
The resulting on H105
Note: No continuum extrapolation yet

(2) We have invested most effort into a formulation other than LaMET:

V. Braun nad D. Müller EPJ **C55** (2008) 349; arXiv:0709.1348

advantages:

- physical current operators; no renormalization of Wilson line needed
- avoids lattices axes, large discretization errors
- controle of higher twist effects
- no matching function needed
- no unclear systematic uncertainties

disadvantage:

- more work is needed to obtain suggestive results. The total amount of work needed to get reliable and phenomenologically relevant results is probably the same for all variants.

Start from non-local correlators to analyse, e.g., the pion distribution amplitude

$$\langle 0|T\{j_\mu(x)j_\nu(-x)|\pi^0(p)\rangle \quad = \quad -\frac{5i}{p}f_\pi\epsilon_{\mu\nu\rho\sigma}\frac{x^\rho p^\sigma}{8\pi^2 x^4}\ T(p\cdot x, x^2)$$

$$T(p\cdot x, x^2)\ =\ \int_0^1 du\ \ e^{i(2u-1)p\cdot x}\ \ H(u,(\mu x)^2,\alpha_s(\mu))\ \Phi_\pi(u,\mu)$$

$$+\quad\ \text{higher twist}$$

H short distance coefficient function, pQCD

It is completely sufficient to study $x^0 = 0$, $\vec{x} \neq 0$ **IF** $|\vec{x}|$ is small enough for pQCD to apply.
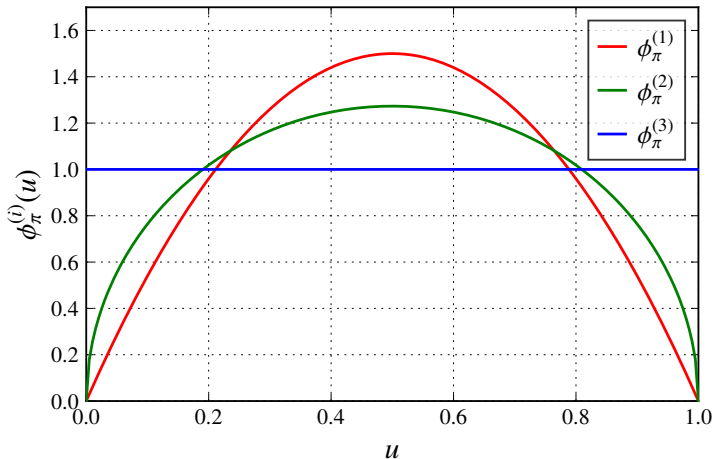
Three model DAs for the pion
$$\phi_\pi^{(1)}(u) = 6u(1-u), \quad \phi_\pi^{(2)}(u) = \frac{8}{\pi}\sqrt{u(1-u)}, \quad \phi_\pi^{(3)}(u) = 1$$

The real part of $T \Leftarrow$ pion distribution amplitude:

$\mu = 0.95$ GeV $= 2/|\mathbf{z}|$

- $|\mathbf{p}| = 1.08$ GeV
- $|\mathbf{p}| = 1.53$ GeV
- $|\mathbf{p}| = 1.88$ GeV

y-axis: $\mathrm{Re}\,T(\mathbf{p} \cdot \mathbf{z}, \mathbf{z}^2)\, a^4/F_\pi$

x-axis: $\mathbf{p} \cdot \mathbf{z}$
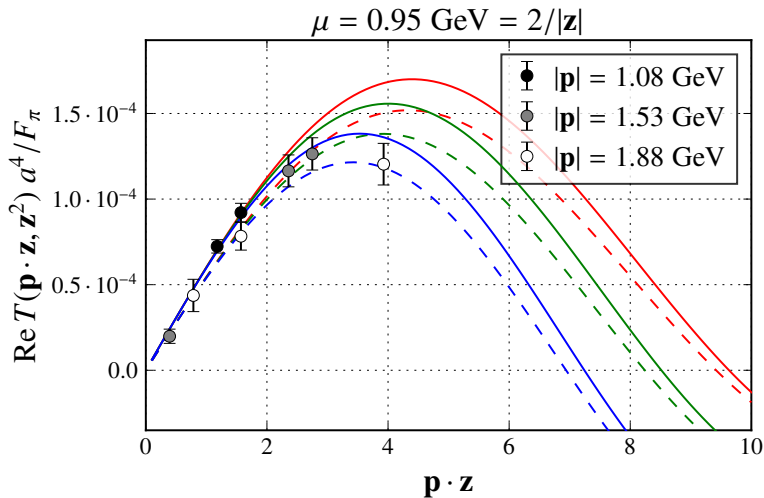
solid/dashed: with/without HT corrections; not yet continuum extrapolated

TMDs   M. Engelhardt et al.

TMDs are related to correlators of the type

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2}\langle P, S| \, \bar{q}(0) \, \Gamma \, \mathcal{U}[0, \eta v, \eta v + b, b] \, q(b) \, |P, S\rangle$$
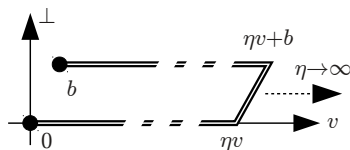
We simulate for spatial, not light-like separations, but the limit $\hat{\zeta} \to \infty$ of

$$\hat{\zeta} := \frac{v \cdot P}{\sqrt{v^2}\sqrt{P^2}}$$

reproduces the light-cone behavior.

We used RBC/UKQCD (domain wall) and W&M (Clover) ensembles, $N_f = 2 + 1$

ID	Clover	DWF
Fermion Type	Clover	Domain-wall
Geometry	$32^3 \times 96$	$32^3 \times 64$
$a(\mathrm{fm})$	0.11403(77)	0.0840(14)
$m_\pi(\mathrm{MeV})$	317(2)(2)	297(5)
# confs.	967	533
# meas.	23208	4264

only connected diagrams, i.e. $u - d$

$$\widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}}(b, P, S, \ldots) = \widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \cdot \mathcal{S} \cdot Z_{\text{TMD}} \cdot Z_2$$

$$\Phi^{[\Gamma]}(x, \boldsymbol{k}_T, P, S, \ldots) = \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{2\pi P^+} e^{ix(b \cdot P) - i\boldsymbol{b}_T \cdot \boldsymbol{k}_T} \left. \widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}} \right|_{b^+ = 0}$$

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_{ij} \boldsymbol{k}_i \boldsymbol{S}_j}{m_N} f_{1T}^\perp$$

$$\Phi^{[\gamma^+ \gamma^5]} = \Lambda g_1 + \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{m_N} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = \boldsymbol{S}_i h_1 + \frac{(2\boldsymbol{k}_i \boldsymbol{k}_j - \boldsymbol{k}_T^2 \delta_{ij}) \boldsymbol{S}_j}{2m_N^2} h_{1T}^\perp + \frac{\Lambda \boldsymbol{k}_i}{m_N} h_{1L}^\perp + \frac{\epsilon_{ij} \boldsymbol{k}_j}{m_N} h_1^\perp$$

$$\tilde{f}^{[m](n)}(\boldsymbol{b}_T^2, \ldots) = n! \left(-\frac{2}{m_N^2} \partial_{\boldsymbol{b}_T^2} \right)^n \int_{-1}^1 dx x^{m-1} \int d^2 \boldsymbol{k}_T e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_T} f(x, \boldsymbol{k}_T^2)$$

$$\langle \vec{k}_y \rangle_{TU}(\boldsymbol{b}_T^2; \ldots) = m_N \frac{\tilde{f}_{1T}^{\perp[1](1)}(\boldsymbol{b}_T^2; \ldots)}{\tilde{f}_1^{[1](0)}(\boldsymbol{b}_T^2; \ldots)}$$

the generalized tensor charge

$$\tilde{h}_1^{[1](0)}(\boldsymbol{b}_{\mathrm{T}}^2;\ldots)/\tilde{f}_1^{[1](0)}(\boldsymbol{b}_{\mathrm{T}}^2;\ldots)$$

$$g_T^{u-d} = \int dx\, d^2\boldsymbol{k}_{\mathrm{T}}\, h_1(x,\boldsymbol{k}_{\mathrm{T}}^2) = \tilde{h}_1^{[1](0)}(\boldsymbol{b}_{\mathrm{T}}^2{=}0)$$
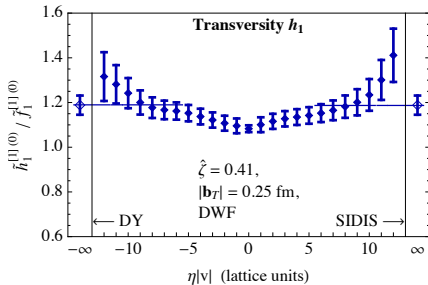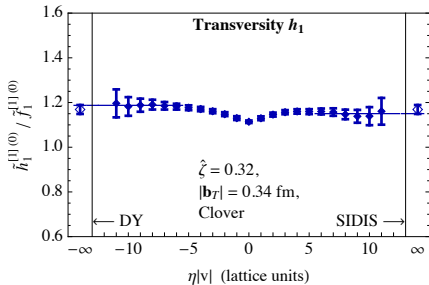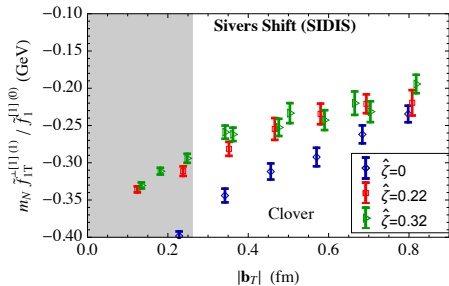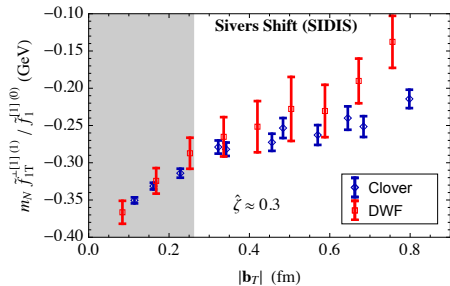
limits:

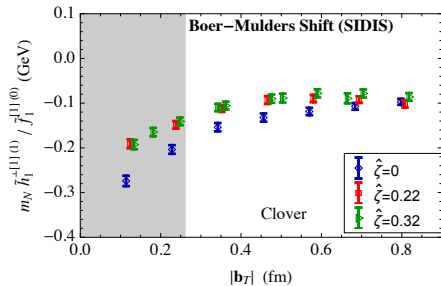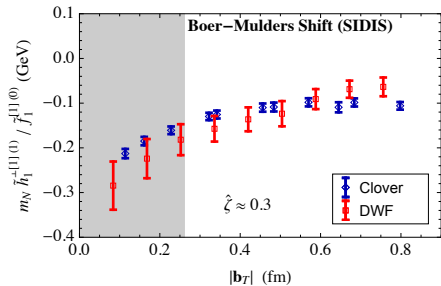$$\eta|v| \to \infty$$
$$b_T \gg a$$
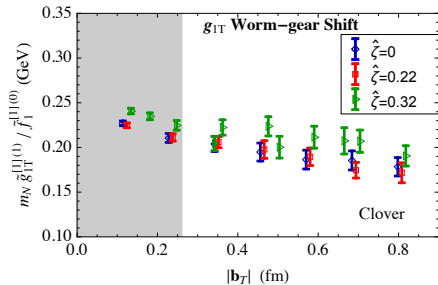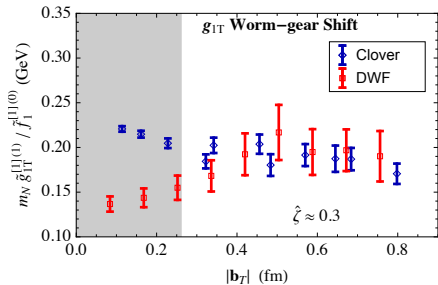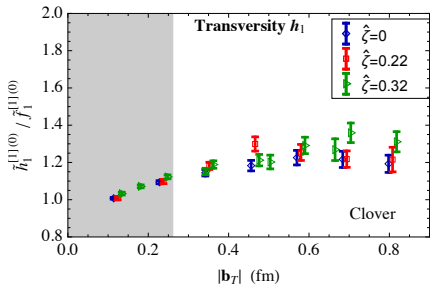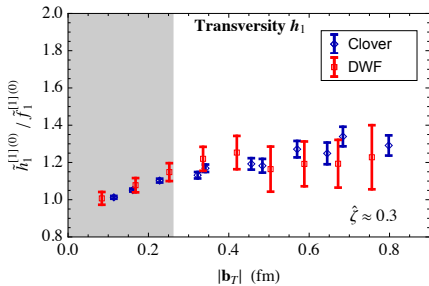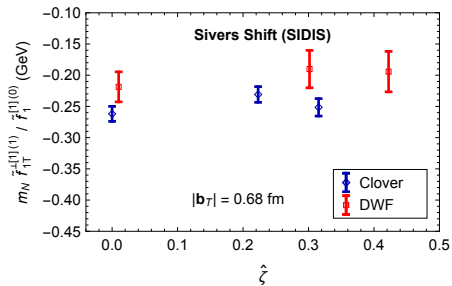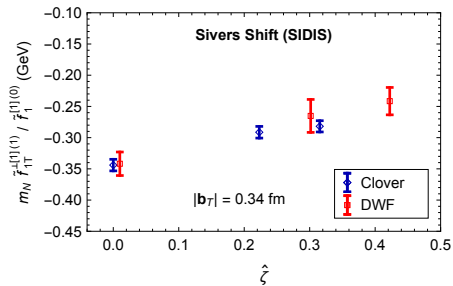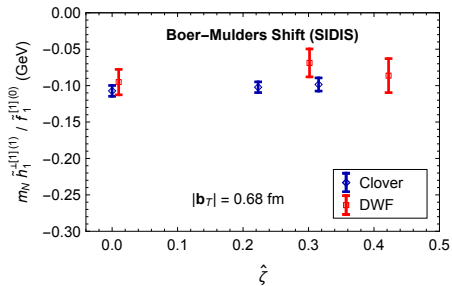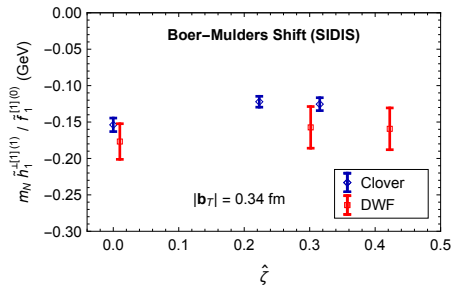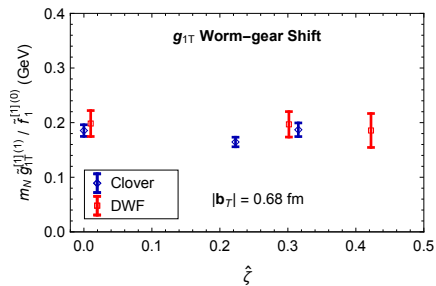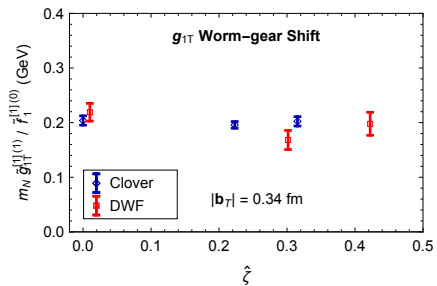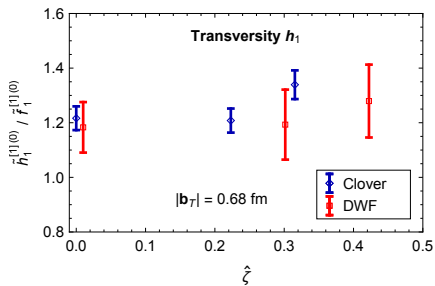$$\hat{\zeta} \to \infty$$
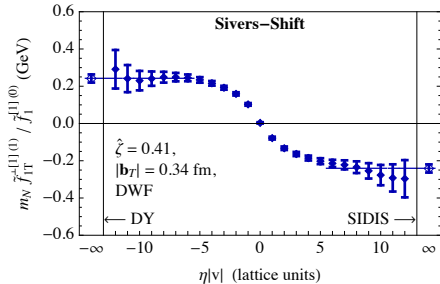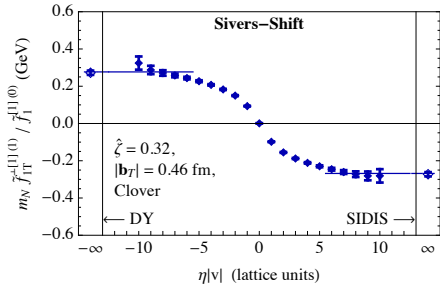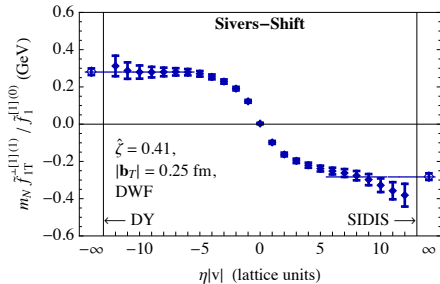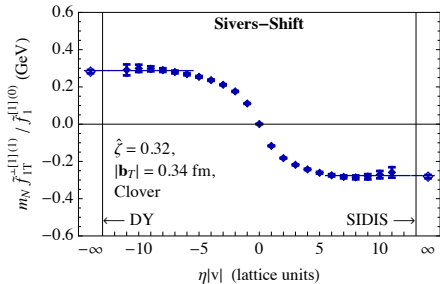
Sivers shift
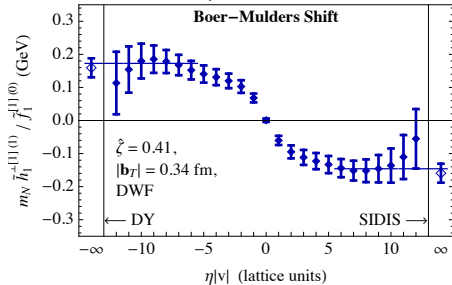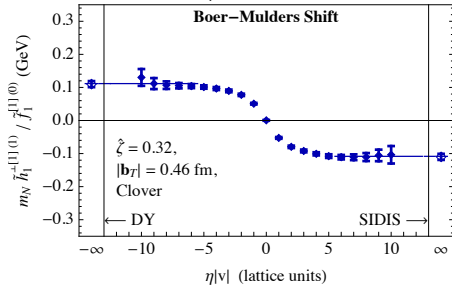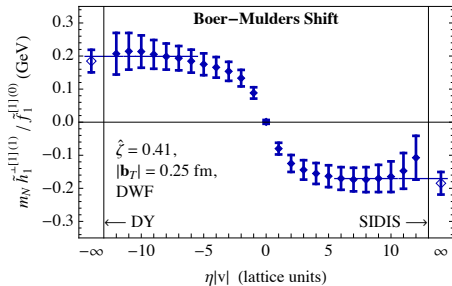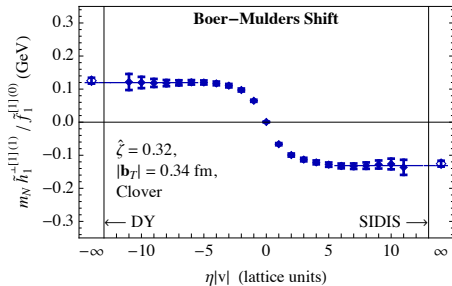
Boer-Mulders shift
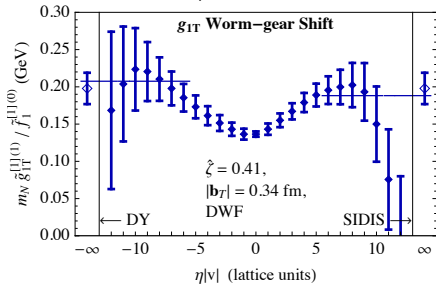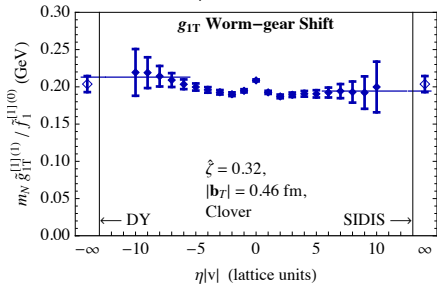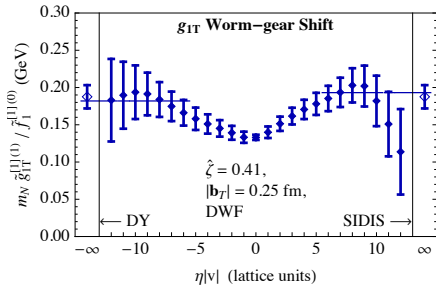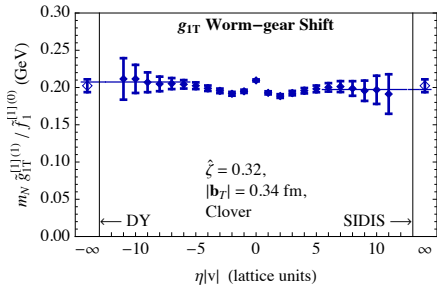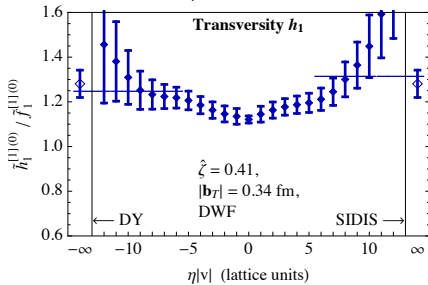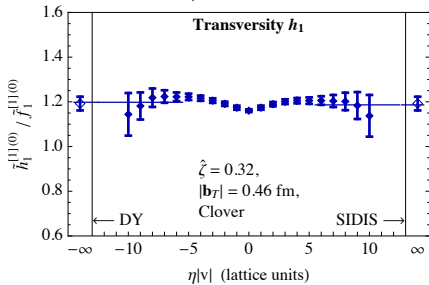
Worm-gear shift, for g_{1T}

transversity ratio

Sivers shift

Boer-Mulders shift

Worm-gear shift, for g_{1T}

transversity ratio
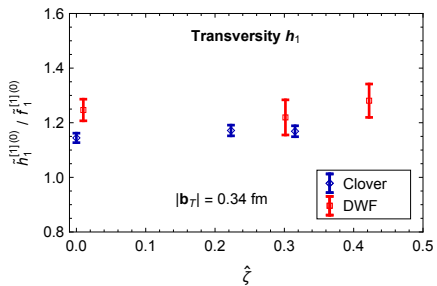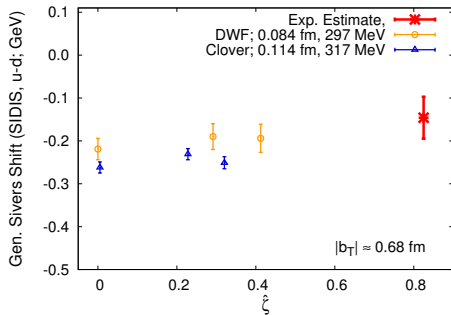
Sivers shift

Boer-Mulders shift

Worm-gear shift, for g_{1T}

transversity ratio
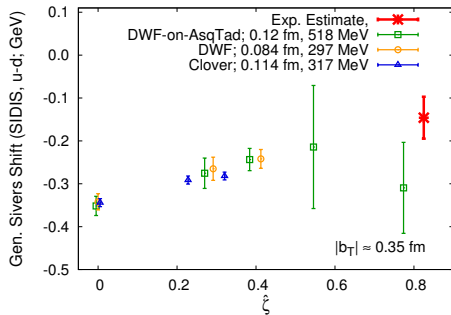
Comparison with experiment

M- Diehl et al.

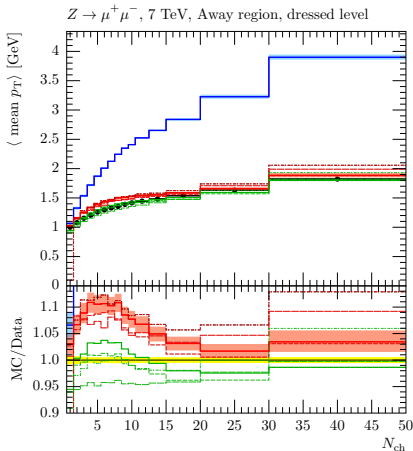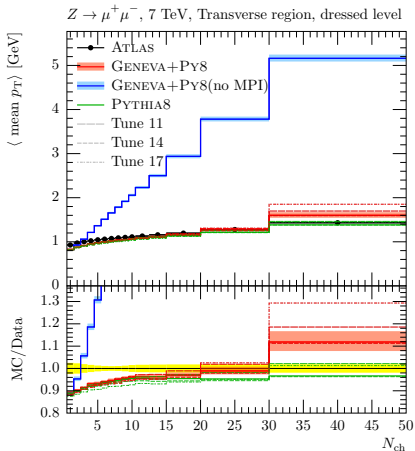Full use of discovery potential requires a better of the "underlying event", example: Double hard interactions
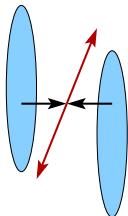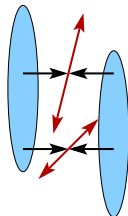
Naive (?) assumption

$$d\sigma_{DPS} = \frac{d\sigma_{SPS}d\sigma_{SPS}}{2\sigma_{eff}}$$

Two plots from Alioli, Bauer, Guns, Tackmann 1605.07192
mean charged particle p_T as function of N_{ch}.
Events with large N_{ch} have a higher MPI contribution. MPIs
produce many particles with p_T of O(1 GeV).

Quark-quark correlations in the pion; RQCD; $N_f = 2$,
Clover-Wilson fermions, down to nearly physical mass
($m_\pi = 150$ MeV).

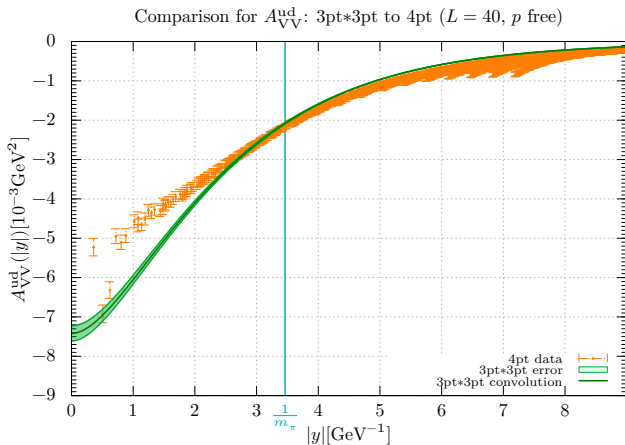Many direct Tests of naive factorization, e.g., Integrals based on

$$\int 4pt - correlator \;\; \overset{?}{=} \;\; \int (formfactor)^2$$

$$\int_{-\infty}^{\infty} d(\vec{p}_\perp \cdot \vec{y}_\perp) \, A_{VV}(y^2, py) \;\; = \;\; \frac{1}{2p^+} \int_{-\infty}^{\infty} dy^- \, \langle \pi(p)|V^+(0)V^+(y)|\pi(p)\rangle \Big|_{y^+=0}$$
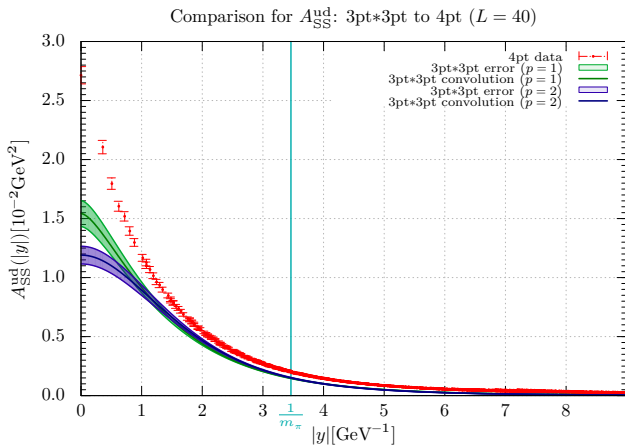
$$\int_{-\infty}^{\infty} d(py) A_{VV}^{ud}(y^2, yp) \;\; \overset{?}{\approx} \;\; - \int \frac{d^2 r_\perp}{(2\pi)^2} \, e^{-i\vec{y}_\perp \cdot \vec{r}_\perp} \, |F_V(r^2)|^2$$

Quark-quark correlations in the pion are sizeable. On
theoretical grounds they should be stronger in nucleons.

Direct tests of naive factorization: Test 1 VV case

Comparison for A_{VV}^{ud}: 3pt*3pt to 4pt ($L = 40$, p free)

Direct tests of naive factorization: Test 1 SS case

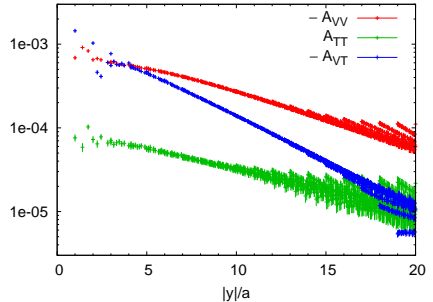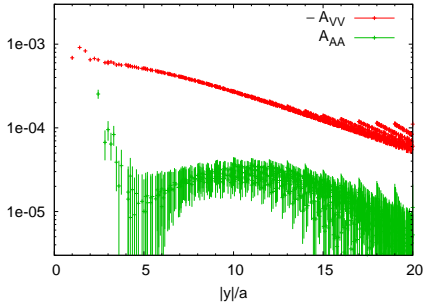Comparison for A_{SS}^{ud}: 3pt∗3pt to 4pt ($L = 40$)
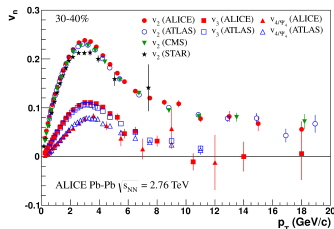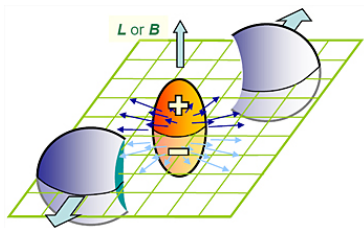
Spin correlations

AA: longitudinal spin correlation $u^\uparrow \bar{d}^\uparrow + u^\downarrow \bar{d}^\downarrow - u^\uparrow \bar{d}^\downarrow - u^\downarrow \bar{d}^\uparrow$

TT: transverse spin correlation $\vec{s}_u \cdot \vec{s}_d$

VT: $\vec{y} \cdot \vec{s}_{\bar{d}}$

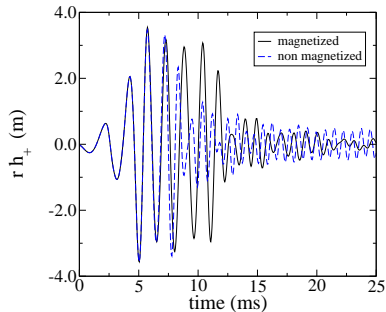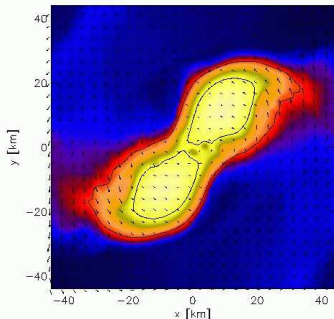Key question of relativistic heavy ion physics:

In HICs one produces the strongest magnetic fields in the universe

- Does this influence thermalization ?
- Does this lead to novel effects like the CME ?

relevance of QCD & magnetic fields for astrophysics

arXiv:0801.4387 gravitational wave signal from neutron star merger

Lee, Fukushima, Kharzeev et al.: The QCD chiral anomaly $G^{\mu\nu}\tilde{G}_{\mu\nu} \sim \vec{E}_{color} \cdot \vec{B}_{color}$ can induce an electromagnetic $\vec{E}$ parallel or antiparallel to $\vec{B}$. STAR 1404.1433 :
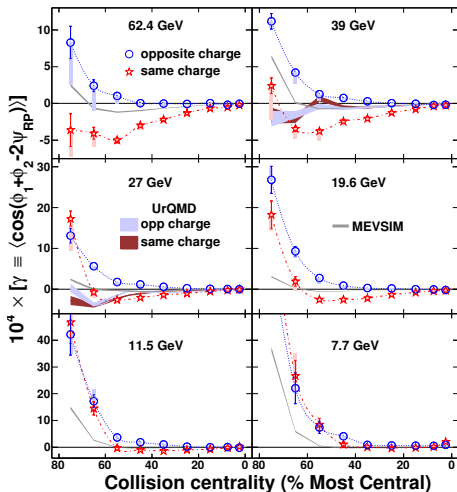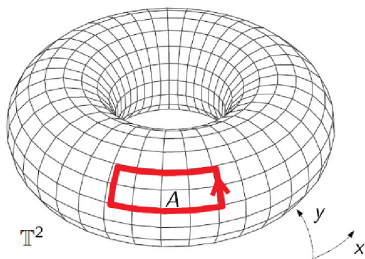
Magnetic field on the Lattice
QCD is contained in the generating functional:

$$
\begin{aligned}
Z[J^a_\mu, \bar{\psi}^i, \eta^i] &= \int \mathcal{D}[A^{a\mu}, \bar{\psi}^i, \psi^i] \\
&\quad \exp\left(\mathrm{i} \int d^4x \left[\mathcal{L}_{\mathrm{QCD}} - J^a_\mu A^a_\mu - \bar{\psi}^i \eta^i - \bar{\eta}^i \psi^i \right] \right)
\end{aligned}
$$

Discretized space time $\Rightarrow$ e.g. the Wilson action

$$
\begin{aligned}
U(l_1) &= \exp\left(-igA^b(l_1)\, \frac{\lambda^b}{2}\, a \right) \\
W_\square &= \mathrm{Tr}\{ U(l_1) U(l_2) U(l_3) U(l_4) \} \\
\sum_\square \frac{2}{g^2}(3 - \mathcal{R}e\, W_\square) &= \frac{1}{4} \int d^4x \left(F^a_{\mu\nu} F^a_{\mu\nu} + O(a^2) \right)
\end{aligned}
$$

Magnetic field on the torus with surface area $L_x L_y$

$\mathbb{T}^2$

- phase factor for a charged particle transported along path
  $\mathcal{C}$: $\exp(iq \oint_{\mathcal{C}} x_\mu A_\mu)$
- Stokes theorem: $\int_{\mathcal{C}} dx_\mu A_\mu = \int \int_A d\sigma B = B \cdot A$
  but also

$$= - \int \int_{\mathcal{T}^2 - A} d\sigma B = -B \cdot (L_x L_y - A)$$

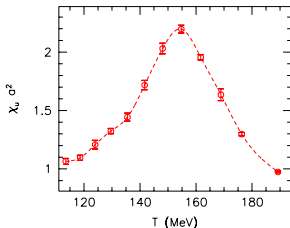- equality of phase factors gives quantization condition
  [Hashimi, Wiese '09]

$$\exp(iqBL_xL_y) = 1 \quad \rightarrow \quad qBL_xL_y = 2\pi \cdot N_b, \quad N_b \in \mathcal{Z}$$
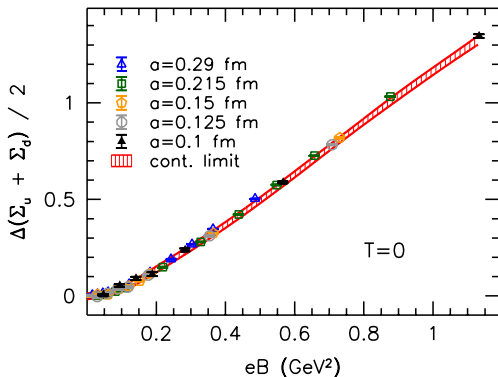
Transition characteristics

- chiral susceptibility
  ($\sim$ specific heat)

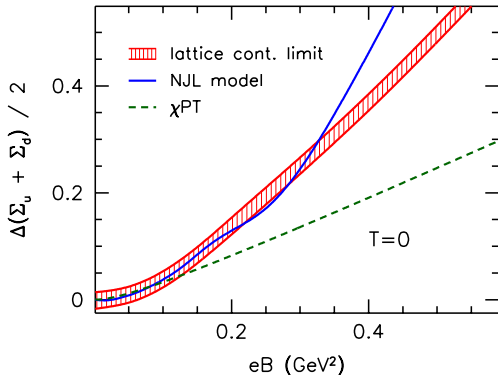 $$\chi = \frac{\partial^2 \log Z}{\partial m^2} \qquad (1)$$

- transition temperature: peak maximum
- order of transition: volume-dependence of height
  $h(V) \propto V^\alpha$
  1st ($\alpha = 1$), 2nd ($0 < \alpha < 1$) or crossover ($\alpha = 0$)

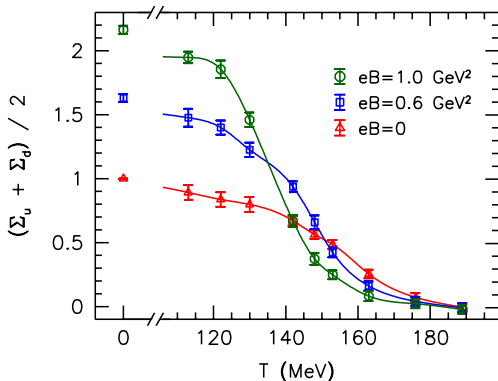- magnetic catalysis at zero temperature is a robust concept: χPT, NJL model, AdS-CFT, linear σ model, . . .

- magnetic catalysis at zero temperature is a robust concept: χPT, NJL model, AdS-CFT, linear σ model, ...
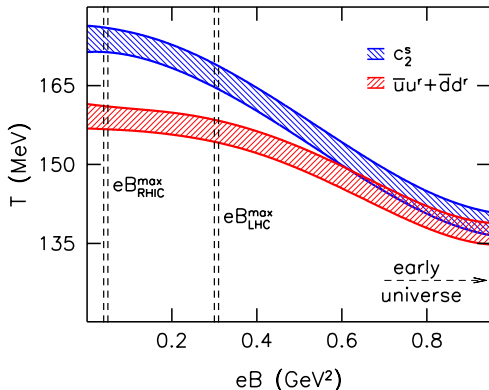
Inverse magnetic catalysis

- lattice QCD, physical m_π, continuum limit [Bali et al '11, '12]

- at $T \approx 150$ MeV the condensate is reduced by B
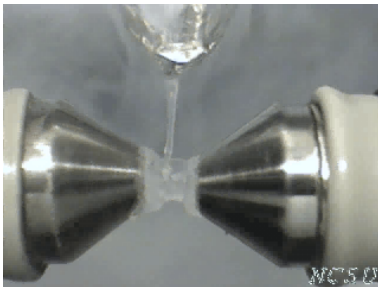 dubbed 'inverse magnetic catalysis'

Phase diagram

- inflection point of $\bar{\psi}\psi(T)$ defines T_c
- significant difference whether IMC is exhibited or not:

lattice QCD, physical m_π, continuum limit [Bali et al '11]

Matter in magnetic fields (linear response)

- paramagnets: attracted by magnetic field
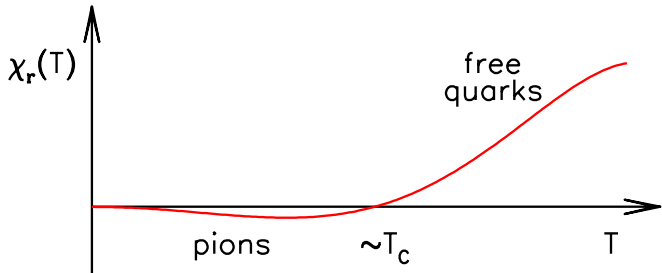- diamagnets: repel magnetic field

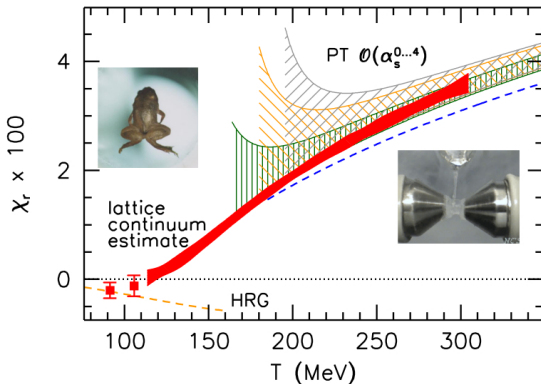paramagnet: liquid oxygen   diamagnet: frog

- is thermal QCD as a medium para- or diamagnetic?

fermions give paramagnetic behaviour, bosons diamagnetic
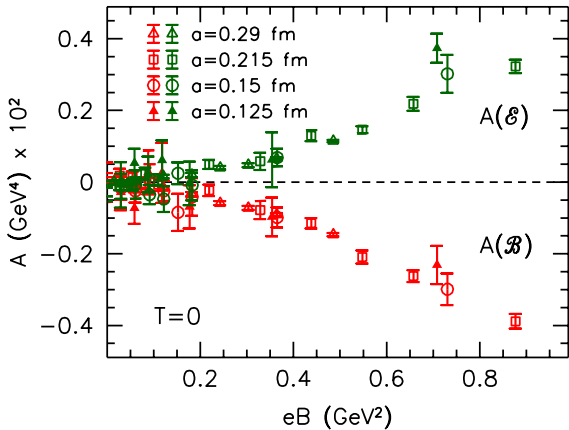⇒ Expectation for the susceptibility

Susceptibility from the lattice

- comparison to Hadron Resonance Gas model (low *T*) and to perturbation theory (high *T*)
- The quark gluon plasma is paramagnetic

$$A(\mathcal{E}) = \frac{T}{V} \left\langle \frac{\beta}{6} \sum_n \left(\text{tr}\,\mathcal{E}_\perp^2(n) - \text{tr}\,\mathcal{E}_\parallel^2(n) \right) \right\rangle$$
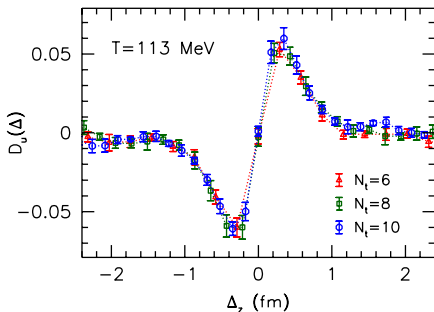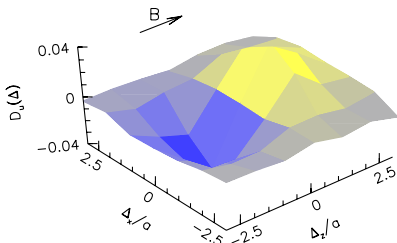
The correlation between topological charge and electric current

1401.4141

$$J_\nu^f(x) = \bar{\psi}_f \gamma_\nu \psi_f(x)$$

$$D_f(\Delta) = \frac{\left\langle q_{\text{top}}(x) \cdot J_t^f(x + \Delta) \right\rangle}{\sqrt{\left\langle q_{\text{top}}^2(x) \right\rangle \left\langle \Sigma_{xy}^f(x) \right\rangle}}$$

lattice results:

Conclusions

- RQCD investigates a very broad spectrum of problems with large computer resources
- PDFs, GPDs, DPDs, TMDs, ...
- The frontier is the continuum limit $\Rightarrow$ CLE open boundary conditions
- No solution yet for $\langle x(u - d) \rangle$
- encouraging results for alternative lattice approaches
- QCD in a magnetic field is a fascinating topic