Resummation for (un)polarized hard scattering processes

Felix Ringer

Lawrence Berkeley National Laboratory

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BERKELEY LAB

Outline

- Motivation
- Small jet radius resummation
- Threshold resummation
- Small-z resummation
- Conclusions

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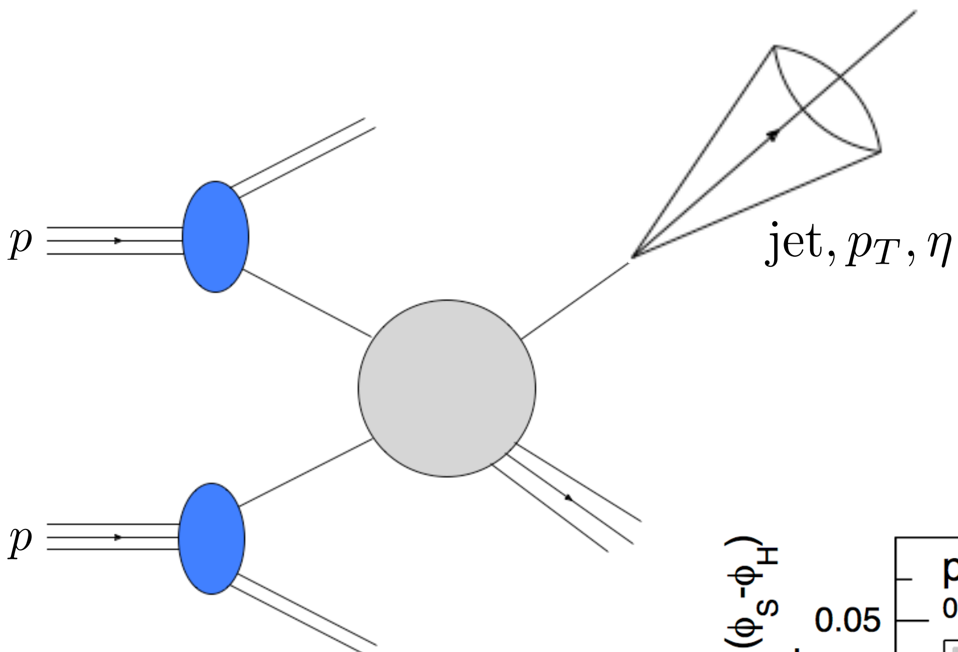
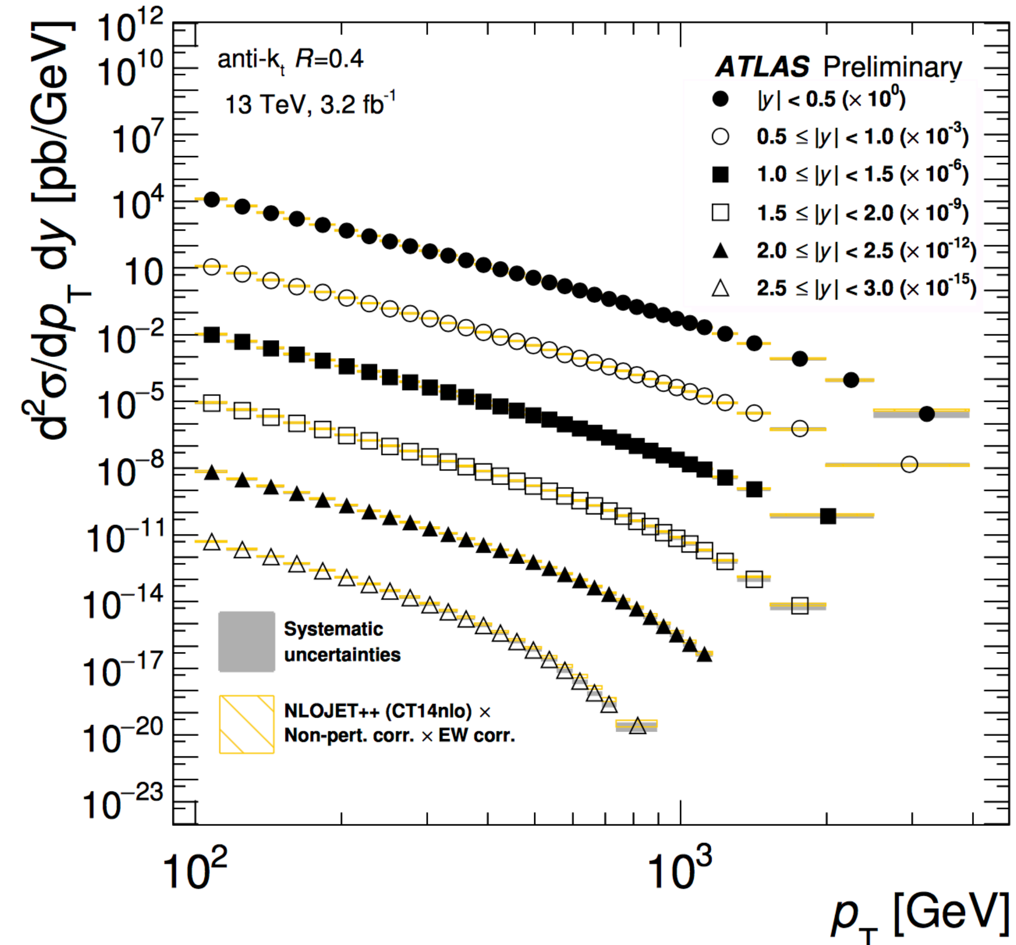
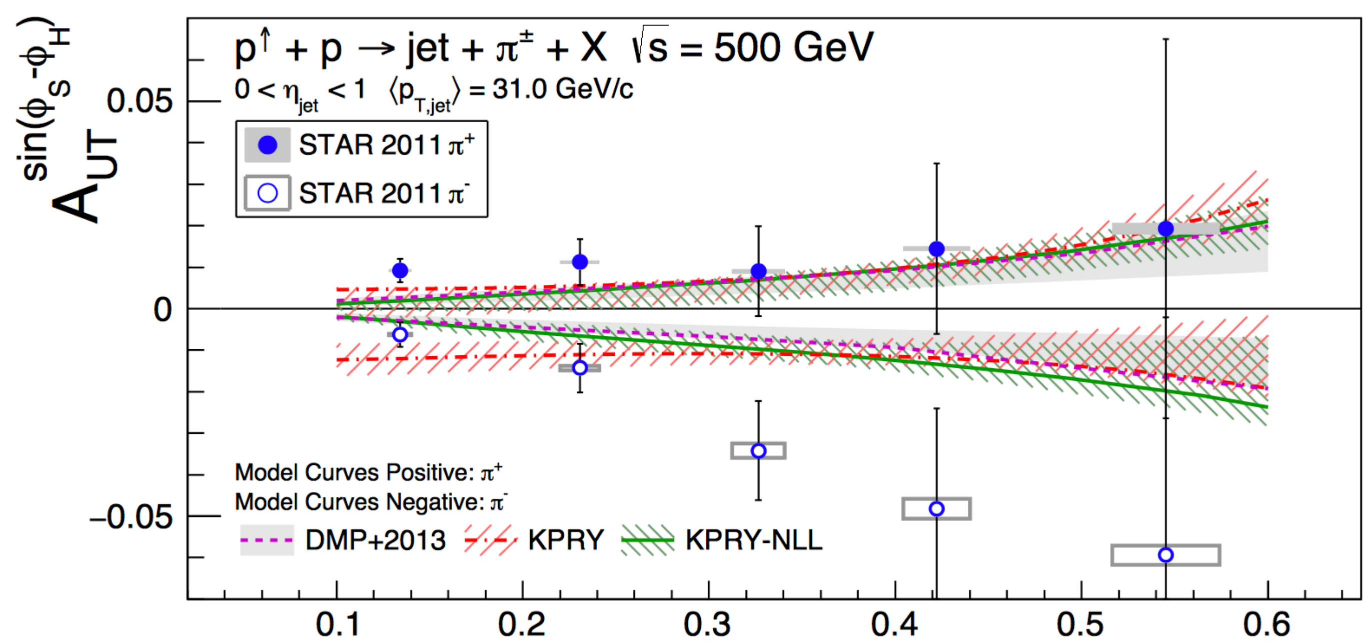
Inclusive Jet Production $pp \to \text{jet}X$

- Baseline process for the extraction of PDFs and α_s
- High precision calculations required at the percent level
- Framework for jet substructure like in-jet TMD FFs



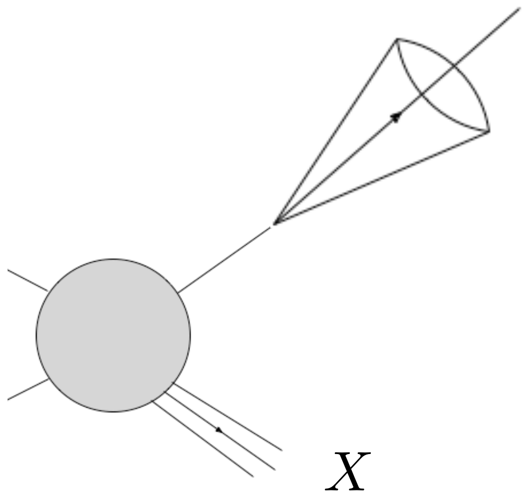




STAR Collaboration 1708.0708

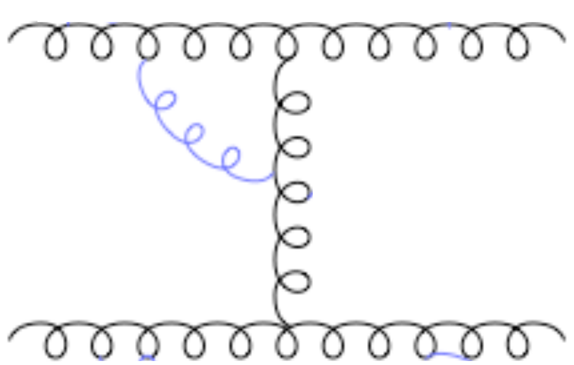
Current state of the art for $pp \to \text{jet}X$

$$\frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}$$



partonic hard-scattering cross section

$$H_{ab} = \alpha_s^2 \left(H_{ab}^{(0)} + \alpha_s H_{ab}^{(1)} + \alpha_s^2 H_{ab}^{(2)} + \dots \right)$$

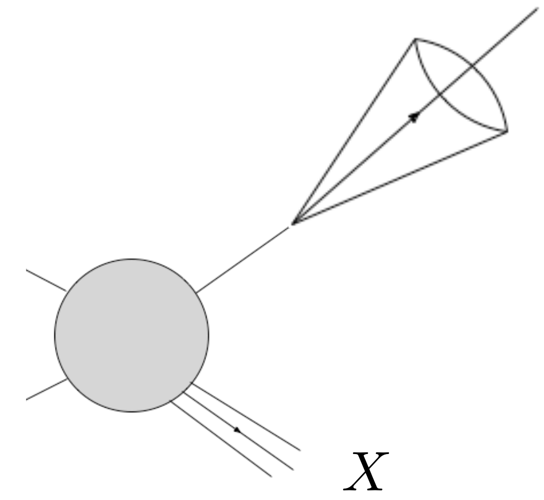


NLO 1990

Ellis, Kunszt, Soper `90

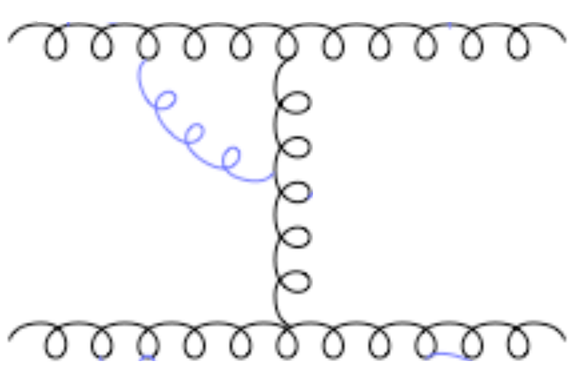
Current state of the art for $pp \to \mathrm{jet}X$

$$\frac{d\sigma^{pp\to \mathrm{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}$$



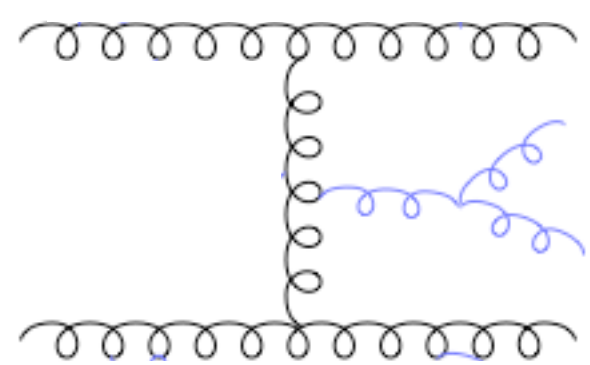
↑ partonic hard-scattering cross section

$$H_{ab} = \alpha_s^2 \left(H_{ab}^{(0)} + \alpha_s H_{ab}^{(1)} + \alpha_s^2 H_{ab}^{(2)} + \dots \right)$$



NLO 1990

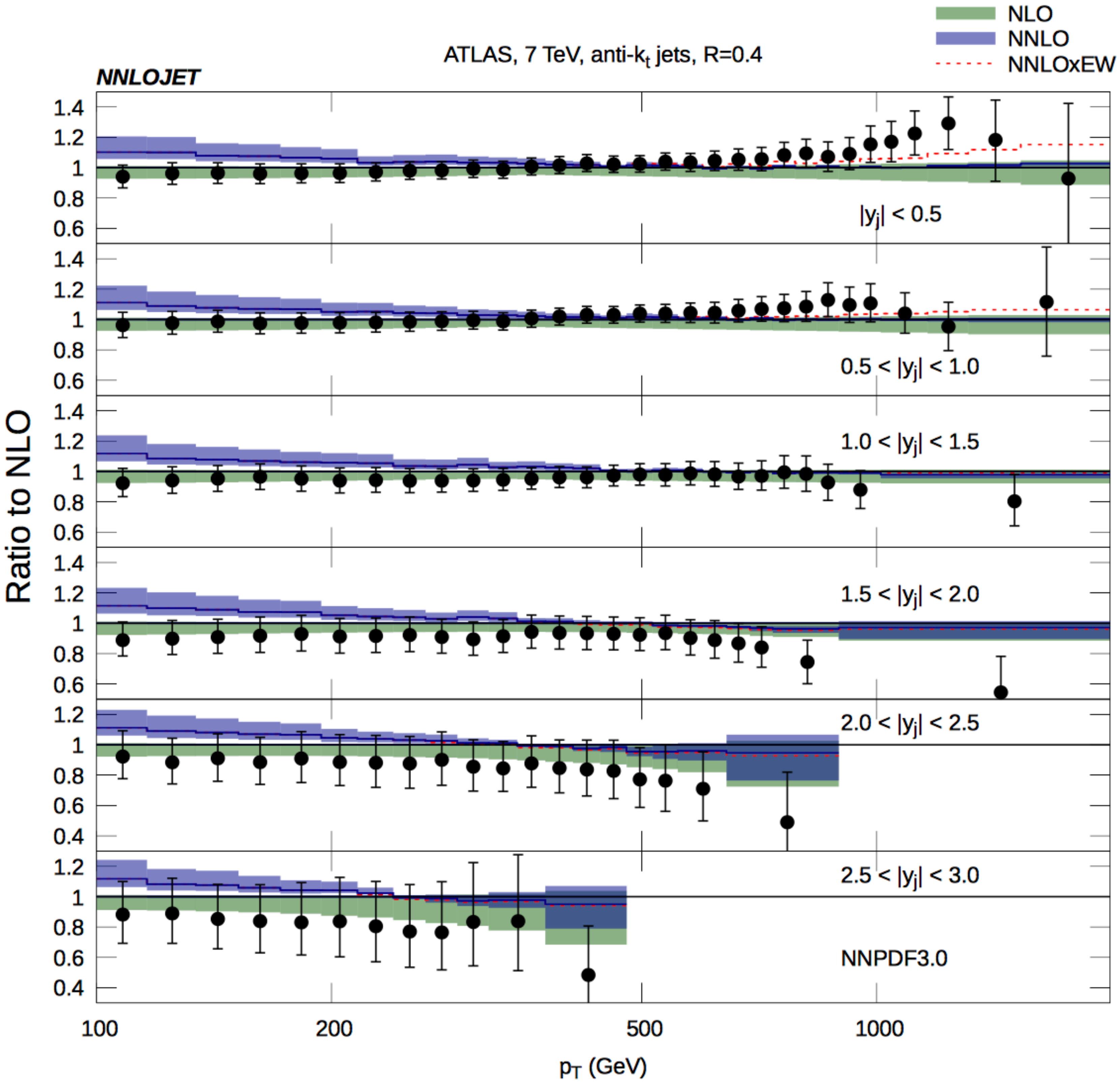
Ellis, Kunszt, Soper `90



NNLO 2016 …

Currie, Glover, Pires `16

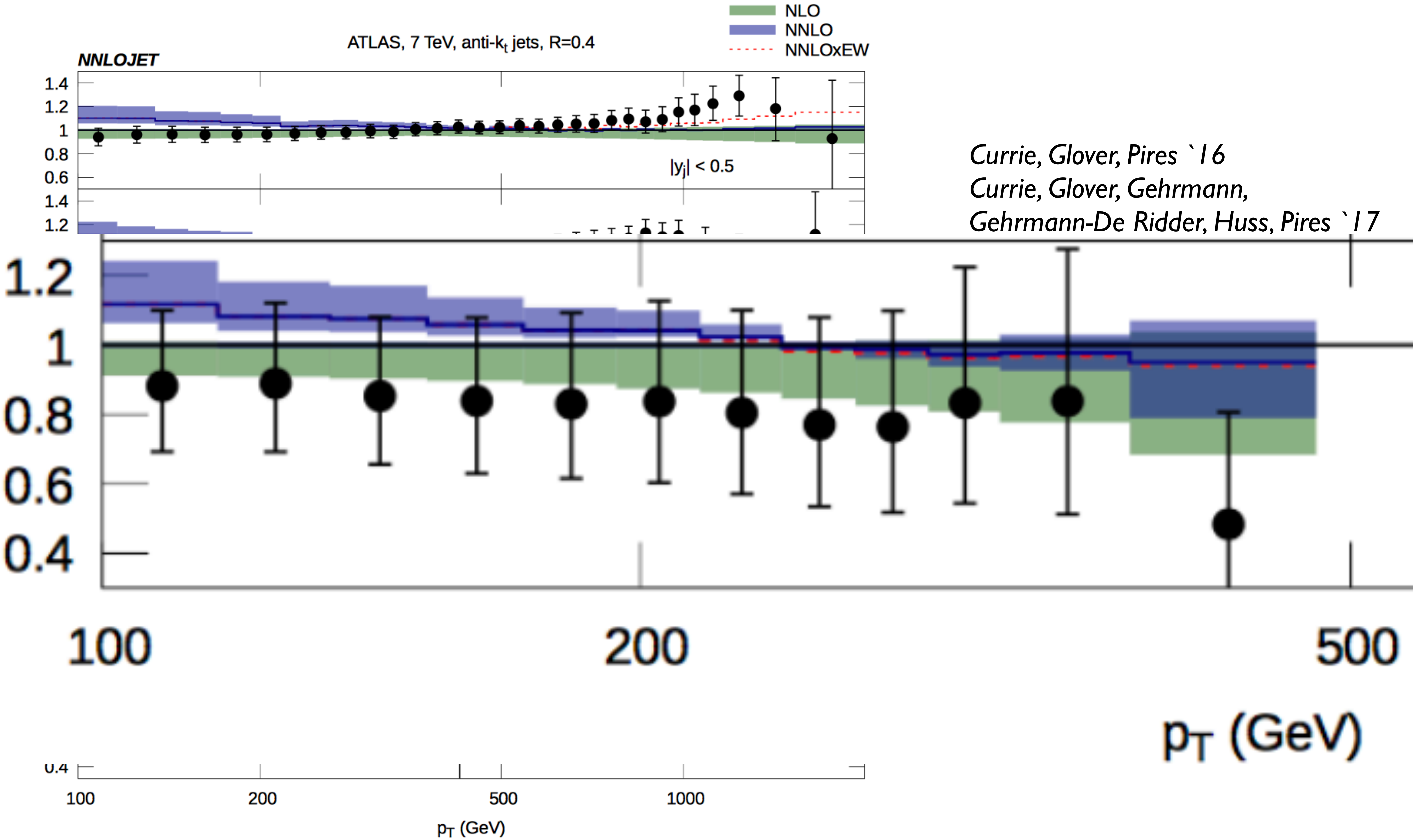
Inclusive jet production $pp \to \mathrm{jet}X$ @ NNLO



Currie, Glover, Pires `16
Currie, Glover, Gehrmann, Gehrmann-De Ridder, Huss, Pires `17

leading color approximation

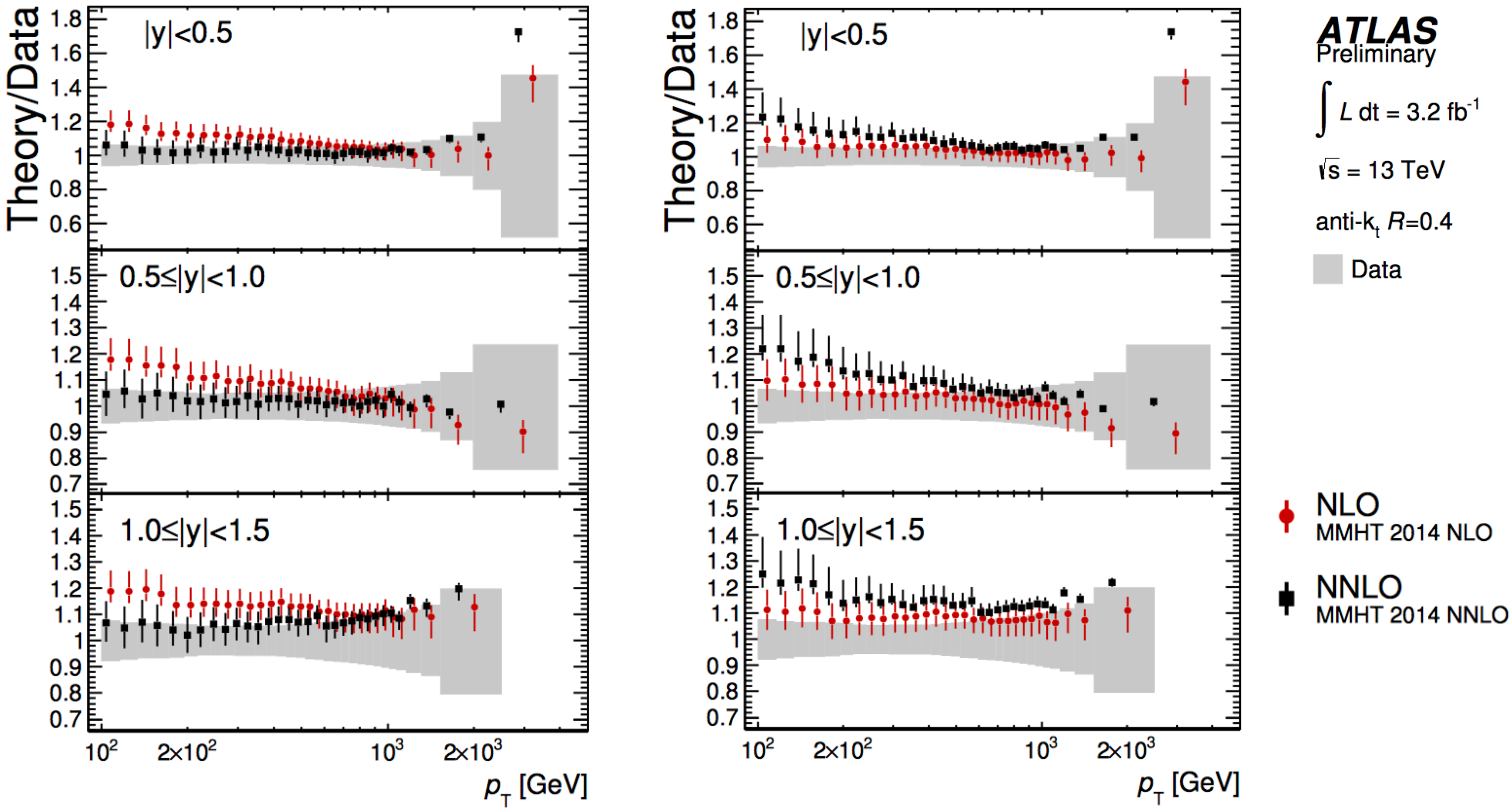
Inclusive jet production $pp \to \mathrm{jet}X$ @ NNLO



Currie, Glover, Pires `16
Currie, Glover, Gehrmann, Gehrmann-De Ridder, Huss, Pires `17

Inclusive jet production $pp \to \mathrm{jet} X$ @ NNLO

$\mu = p_T$ $\mu = p_T^{\max}$



ATLAS-CONF-2017-048

Inclusive di-jet production $pp \to j_1 j_2 X$

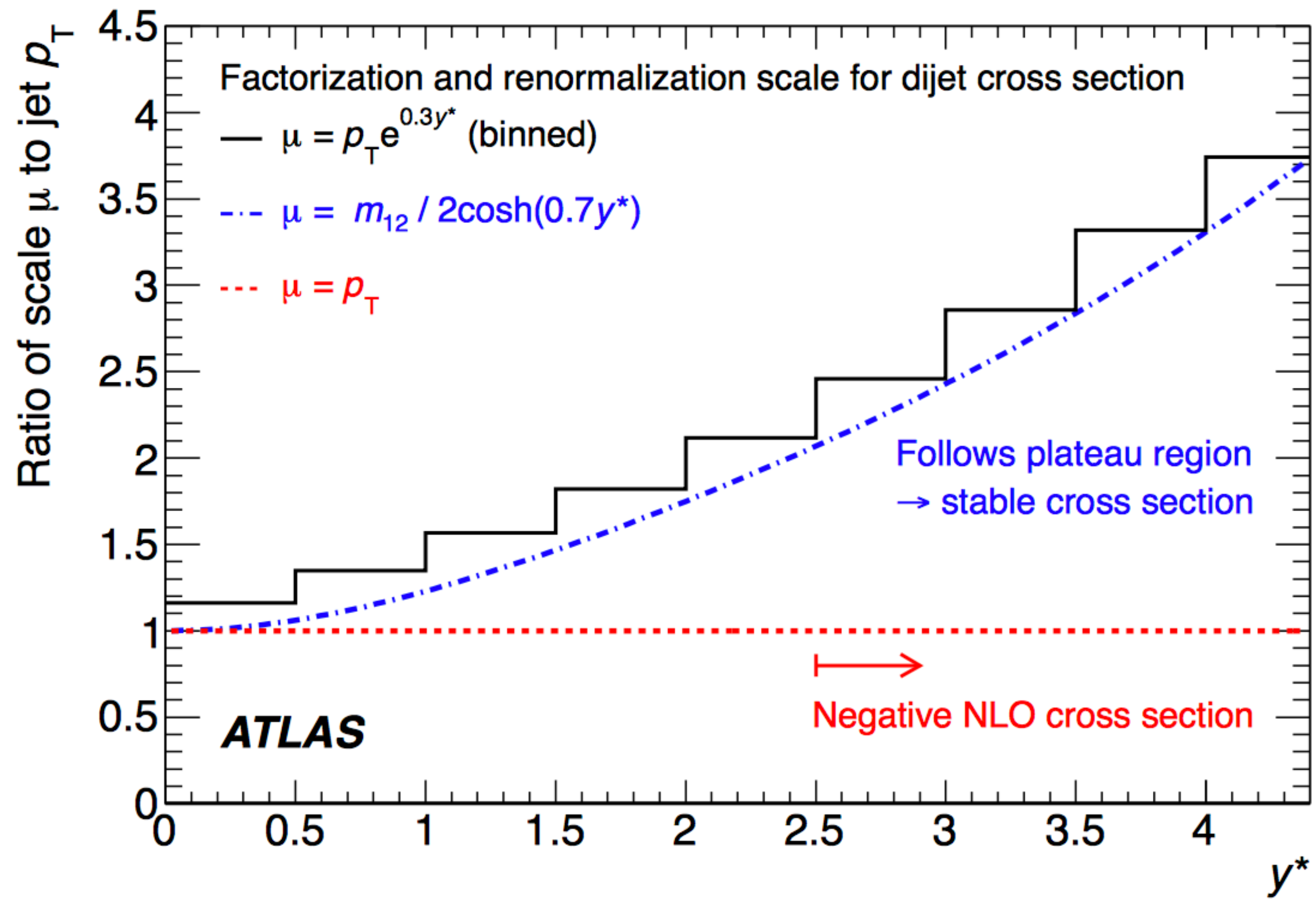
Many possible scale choices exist:

Here: $y^* = |y_1 - y_2|/2$

$p_{T,1} > 30 \text{ GeV}$

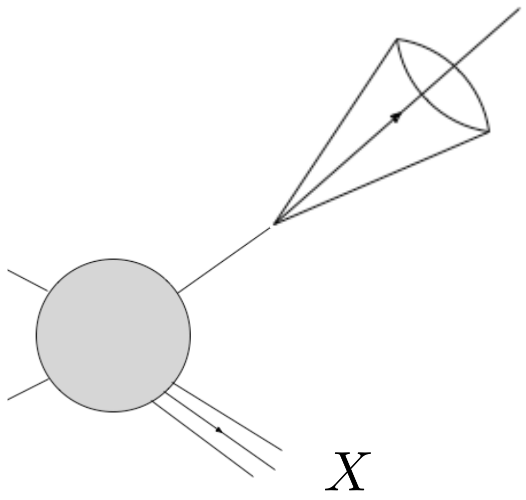
$p_{T,2} > 20 \text{ GeV}$

(1,2 are the two leading jets)



ATLAS PRD 86 (2012) 014022, ATLAS-CONF-2017-048

Inclusive Jet Production $pp \to \mathrm{jet}X$



$$\frac{d\sigma^{pp\to\mathrm{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}$$

partonic hard-scattering cross section

$$H_{ab} = \alpha_s^2 \left(H_{ab}^{(0)} + \alpha_s H_{ab}^{(1)} + \alpha_s^2 H_{ab}^{(2)} + \ldots \right)$$

Cross check + resummation of large logarithms found in analytical calculations:

- Jet radius parameter $\alpha_s^n \ln^n R$
- threshold $\alpha_s^n \left(\frac{\ln^{2n-1}(1-x)}{1-x} \right)_+$
- small-z $\alpha_s^n \ln^{2n}(-t/s)$

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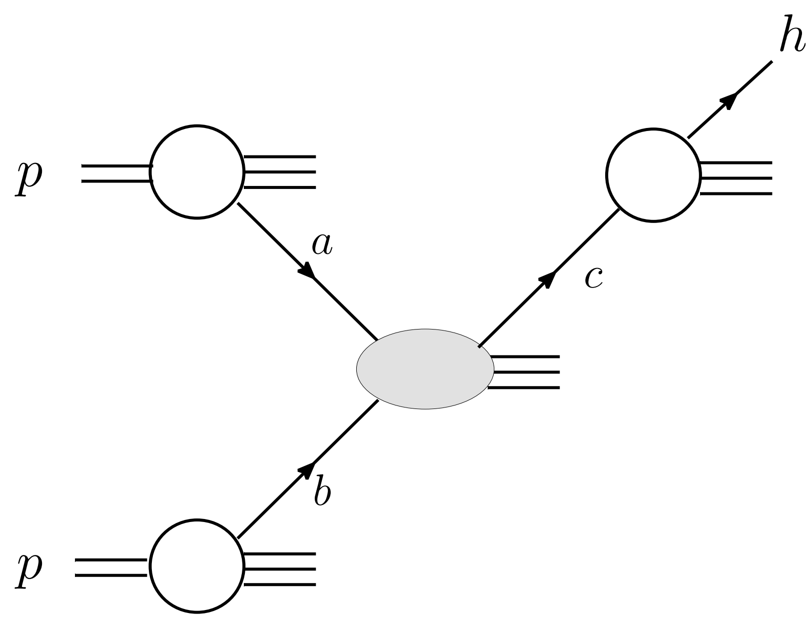
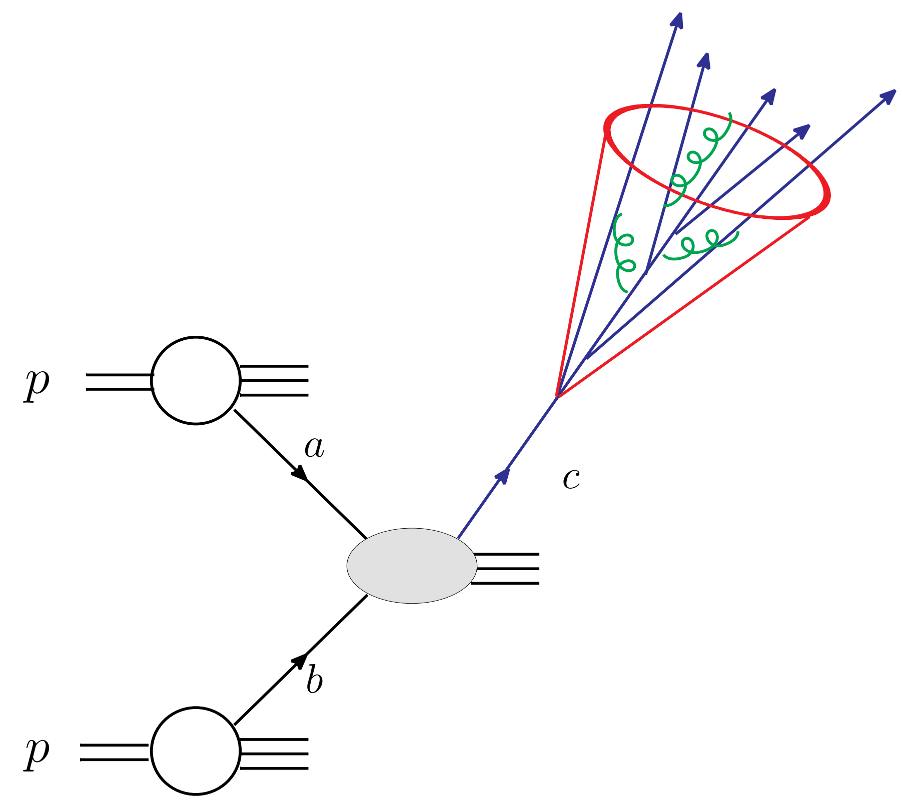
Analogy of hadron and jet cross sections





Factorization

Evolution

Jet

$$\frac{d\sigma^{pp\to \mathrm{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c + \mathcal{O}(R^2)$$

$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

Hadron

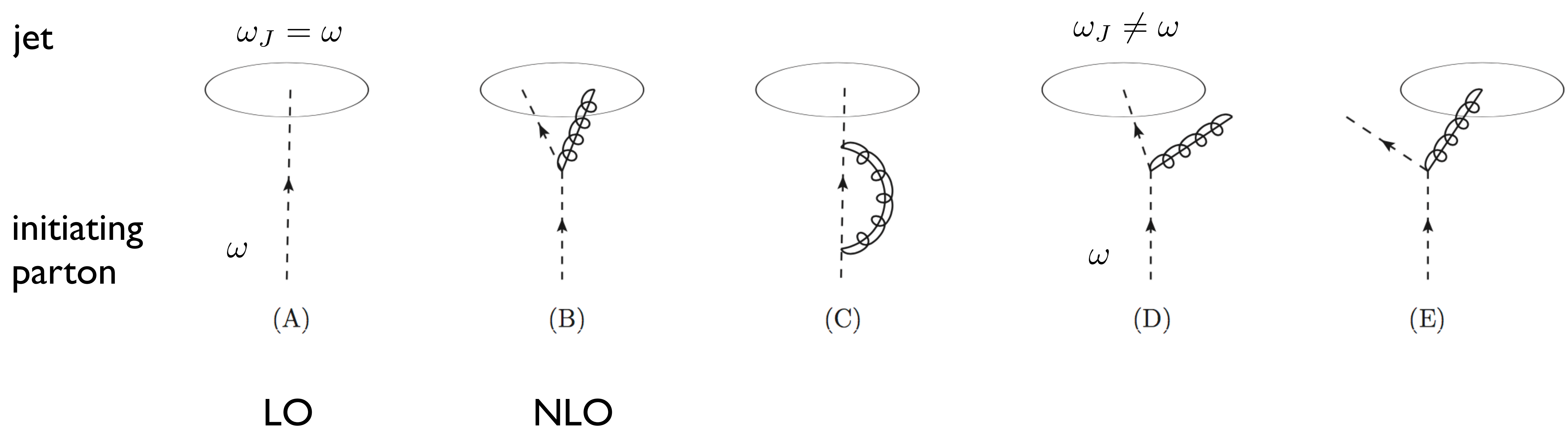
$$\frac{d\sigma^{pp\to hX}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes D_c^h$$

$$\mu \frac{d}{d\mu} D_i^h = \sum_j P_{ji} \otimes D_j^h$$

Kaufmann, Mukherjee, Vogelsang `15
Kang, FR, Vitev `16
Dai, Kim, Leibovich `16

Semi-inclusive jet function in SCET

- The siJFs describe how a parton is transformed into a jet with radius R and carrying an energy fraction z



where $z = \omega_J / \omega$

momentum sum rule: $\int_0^1 dz\, z J_i(z, \omega R, \mu) = 1$

Kaufmann, Mukherjee, Vogelsang `15
Kang, FR, Vitev `16
Dai, Kim, Leibovich `16

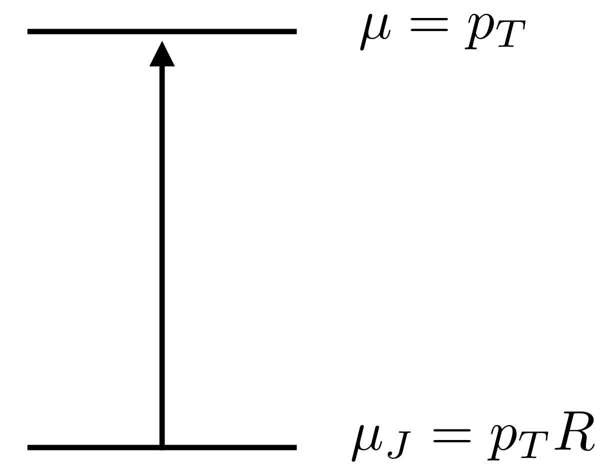
Semi-inclusive jet function in SCET

- NLO result

UV

$$J_q^{(1)}(z, p_T R, \mu) = \frac{\alpha_s}{2\pi}\left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{p_T^2 R^2}\right)\right)\left[P_{qq}(z) + P_{gq}(z)\right]$$
$$- \frac{\alpha_s}{2\pi}\left\{C_F\left[2\left(1+z^2\right)\left(\frac{\ln(1-z)}{1-z}\right)_+ + (1-z)\right] - \delta(1-z) d_J^{q,\mathrm{alg}}\right.$$
$$\left. + P_{gq}(z) 2\ln(1-z) + C_F z\right\},$$

$\overline{\mathrm{MS}}$ scheme



- RG equation
  timelike DGLAP for semi-inclusive jet function

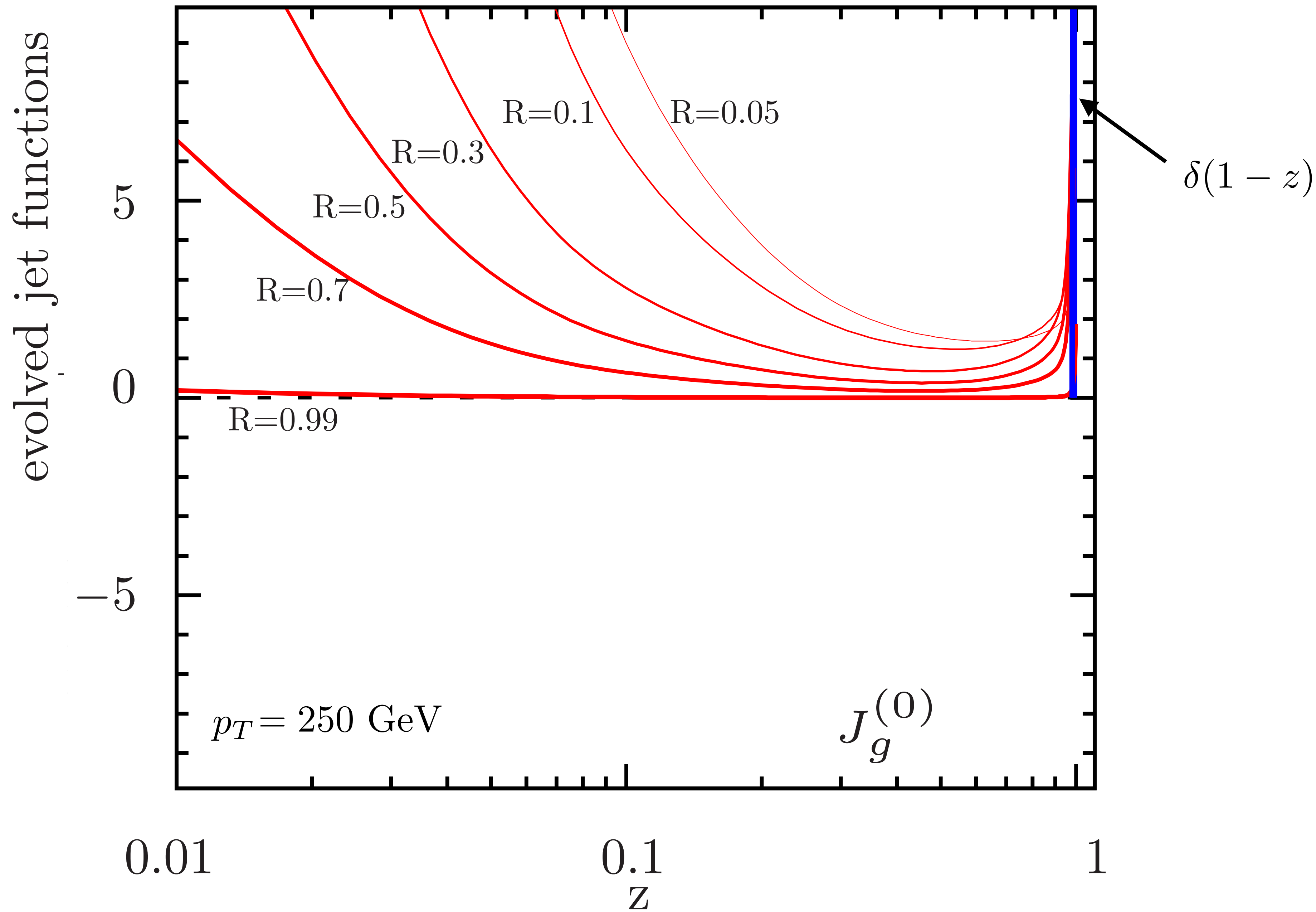
$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

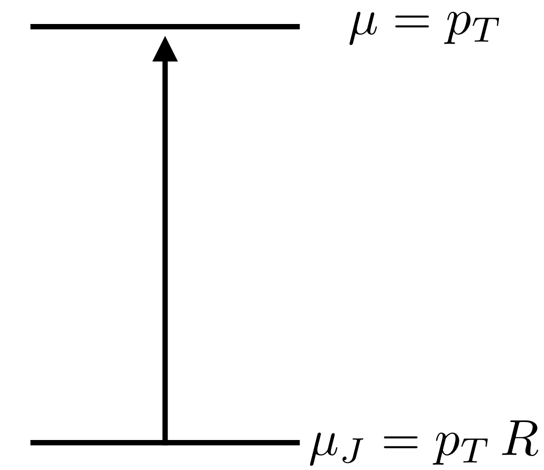
resummation of $\alpha_s^n \ln^n R$

→ solve in Mellin moment space

see also: Dasgupta, Dreyer, Salam, Soyez `16

NLL_R DGLAP evolution





see:

Vogt `04 (Pegasus),
Anderle, FR, Stratmann `15

$$\frac{d\sigma^{pp\to \mathrm{jet}X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c$$

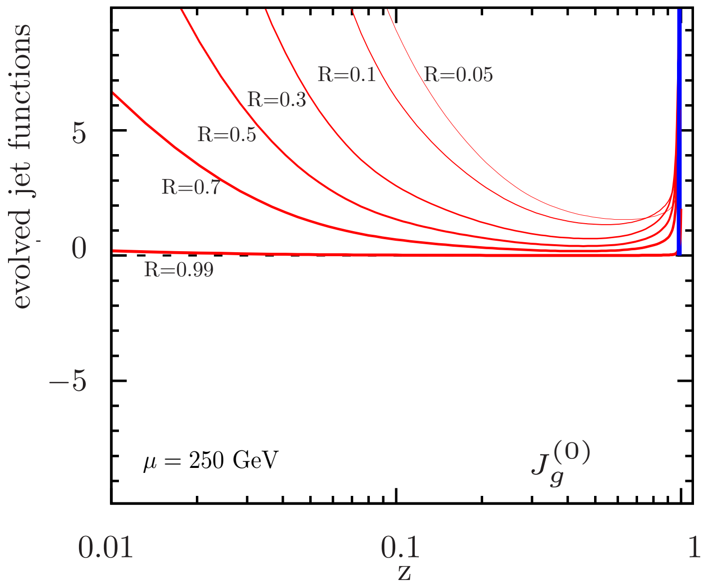
- Adopt a prescription used for quarkonium fragmentation functions

  Bodwin, Chao, Chung, Kim, Lee, Ma `16

$$\frac{d\sigma}{d\eta dp_T} = \frac{2p_T}{\sqrt{s}} \sum_{abc} \int_{z_0}^{1} \frac{dz_c}{z_c^2} J_c(z_c, p_T R, \mu) \int_{VW/z_c}^{1-(1-V)/z_c} \frac{dv}{v(1-v)} \int_{VW/vz_c}^{1} \frac{dw}{w} f_a(x_a,\mu) f_b(x_b,\mu) H_{ab}^c(s,v,w,\mu)$$

$$= \sum_c \int_{z_0}^{1-\varepsilon} \frac{dz_c}{z_c^2} J_c(z_c, p_T R, \mu)\, H_c'\left(\frac{z_0}{z_c}, \eta, p_T, \mu\right) + \underline{\sum_c \int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} J_c(z_c, p_T R, \mu)\, H_c'\left(\frac{z_0}{z_c}, \eta, p_T, \mu\right)}$$

where $z_0 = 2p_T/\sqrt{s}\cosh\eta$



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$$= \sum_c \int_{z_0}^{1-\varepsilon} \frac{dz_c}{z_c^2} J_c(z_c, p_T R, \mu)\, H'_c\left(\frac{z_0}{z_c}, \eta, p_T, \mu\right) + \underline{\sum_c \int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} J_c(z_c, p_T R, \mu)\, H'_c\left(\frac{z_0}{z_c}, \eta, p_T, \mu\right)}$$

where $z_0 = 2p_T/\sqrt{s}\cosh\eta$

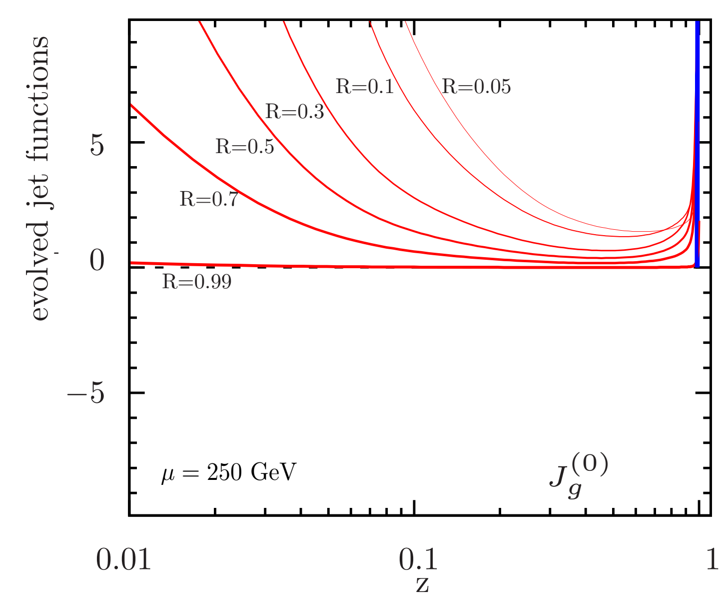
and

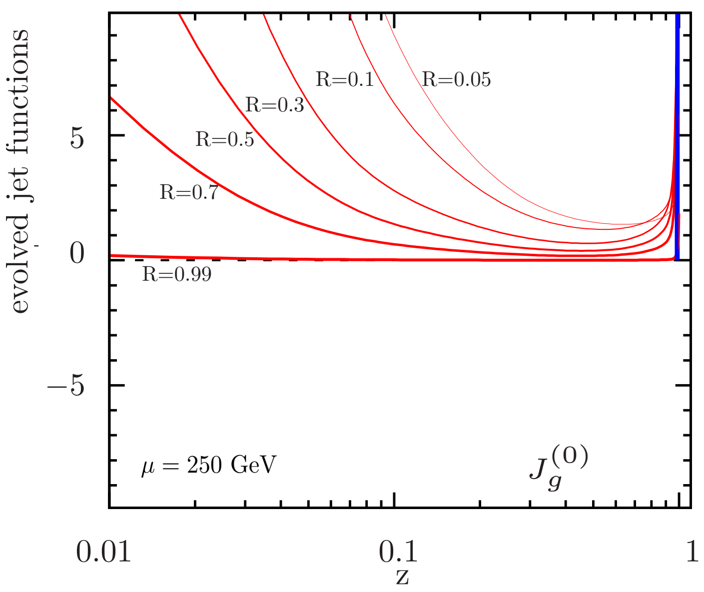
$$\blacksquare = \sum_c \int_{1-\varepsilon}^{1} \frac{dz_c}{z_c^2} z_c^N J_c(z_c, p_T R, \mu)\, z_c^{-N} H'_c\left(\frac{z_0}{z_c}, \eta, p_T, \mu\right)$$

$$\approx \sum_c H'_c(z_0, \eta, p_T, \mu) \int_{1-\varepsilon}^{1} dz_c\, z^{N-2}\, J_c(z_c, p_T R, \mu)$$

$$= \sum_c H'_c(z_0, \eta, p_T, \mu) \left[\int_0^1 dz_c\, z^{N-2}\, J_c(z_c, p_T R, \mu) - \int_0^{1-\varepsilon} dz_c\, z^{N-2}\, J_c(z_c, p_T R, \mu)\right]$$

Requirements for parameters: $\varepsilon \ll 1,\ N > 2$
but final result should be independent of the choice





- Adopt a prescription used for quarkonium fragmentation functions
  Bodwin, Chao, Chung, Kim, Lee, Ma `16
- Mellin space implementation
  FR, Sato, Yuan - in preparation

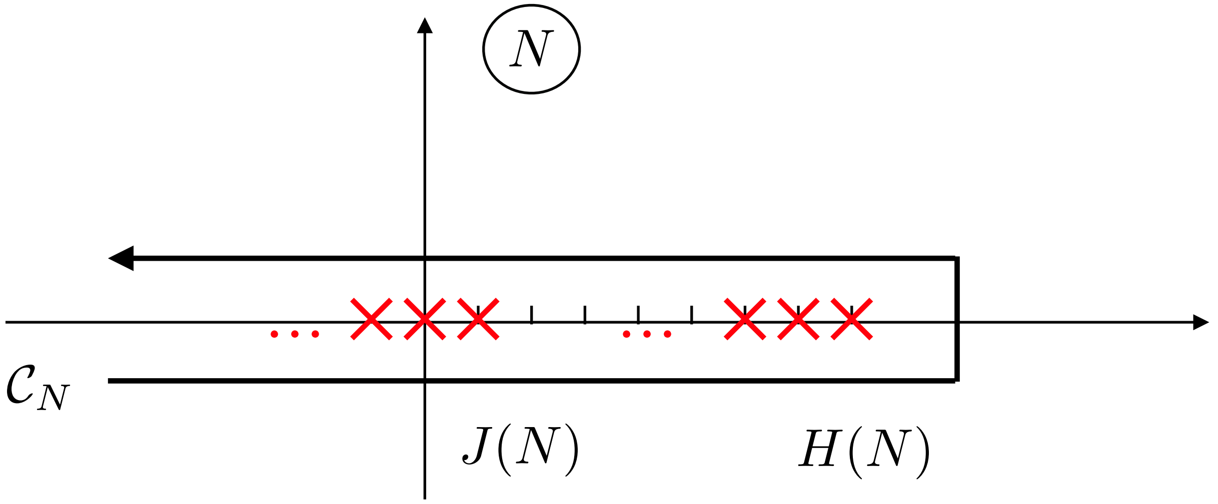
$$z_0 = 2p_T/\sqrt{s}\cosh\eta$$

$$\int_0^1 dz_0\, z_0^{N-1} \frac{d\sigma}{d\eta dp_T} = \int_0^1 dz_0\, z_0^{N-1} \sum_c \int_{z_0}^1 \frac{dz_c}{z_c^2} J_c(z_c, p_T R, \mu)\, H_c'\left(\frac{z_0}{z_c}, \eta, p_T, \mu\right)$$

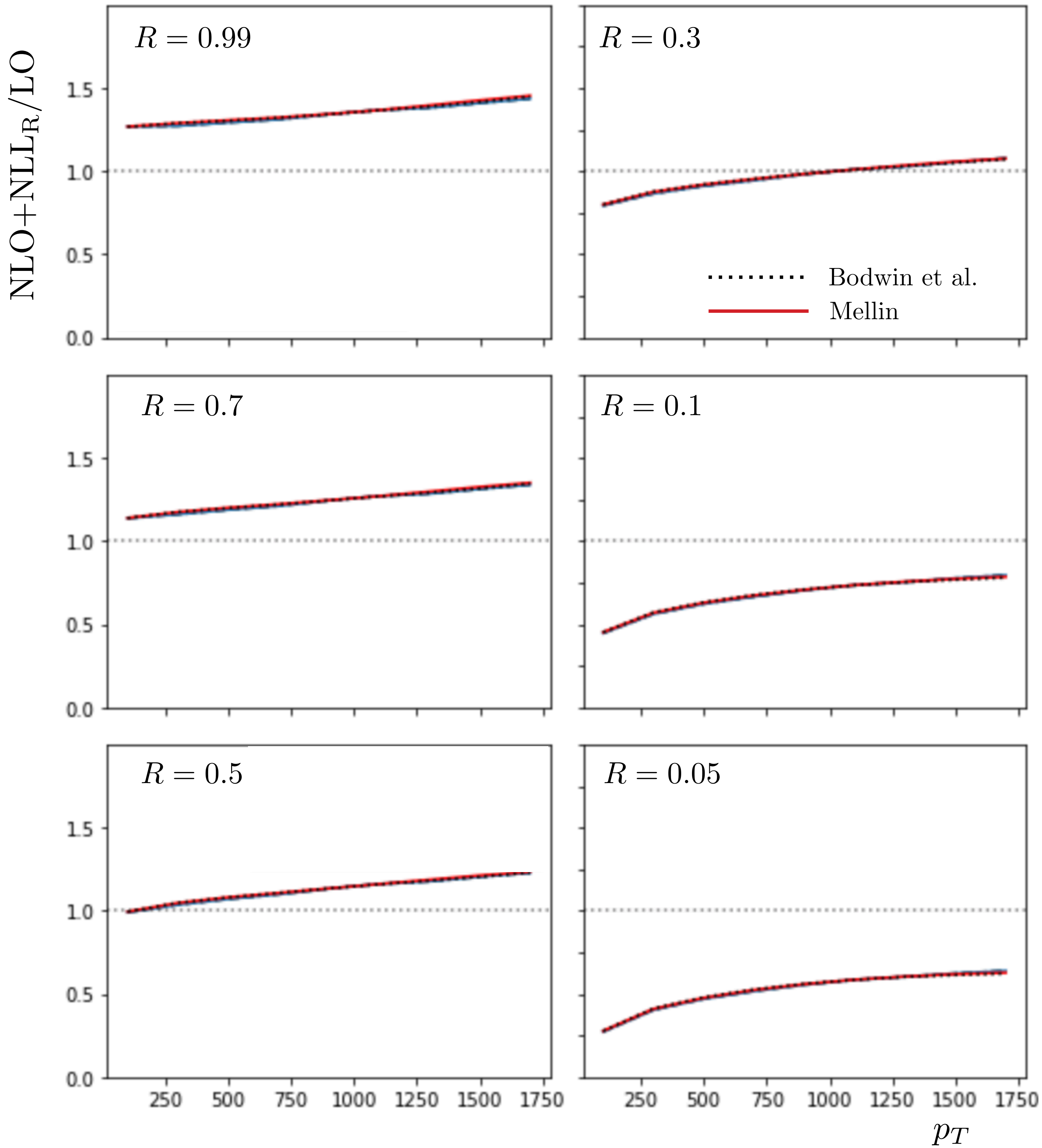
$$= \sum_c J_c(N-1, p_T R, \mu)\, H_c'(N, \eta, p_T, \mu)$$

↑ Mellin transform of fitted function

Inverse: $$\frac{d\sigma}{d\eta dp_T} = \sum_c \int_{\mathcal{C}_N} \frac{dN}{2\pi i} z_0^{-N} J_c(N-1, p_T R, \mu)\, H_c'(N, \eta, p_T, \mu)$$



- Mellin grid technique used for PDF fits can not be applied
- Applications also to di-jets and photon+jet

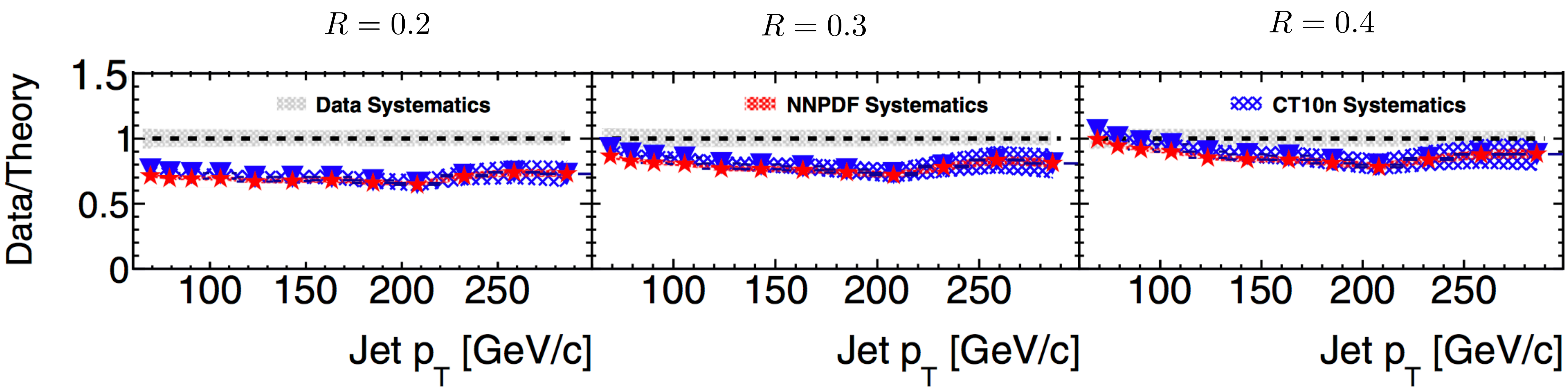


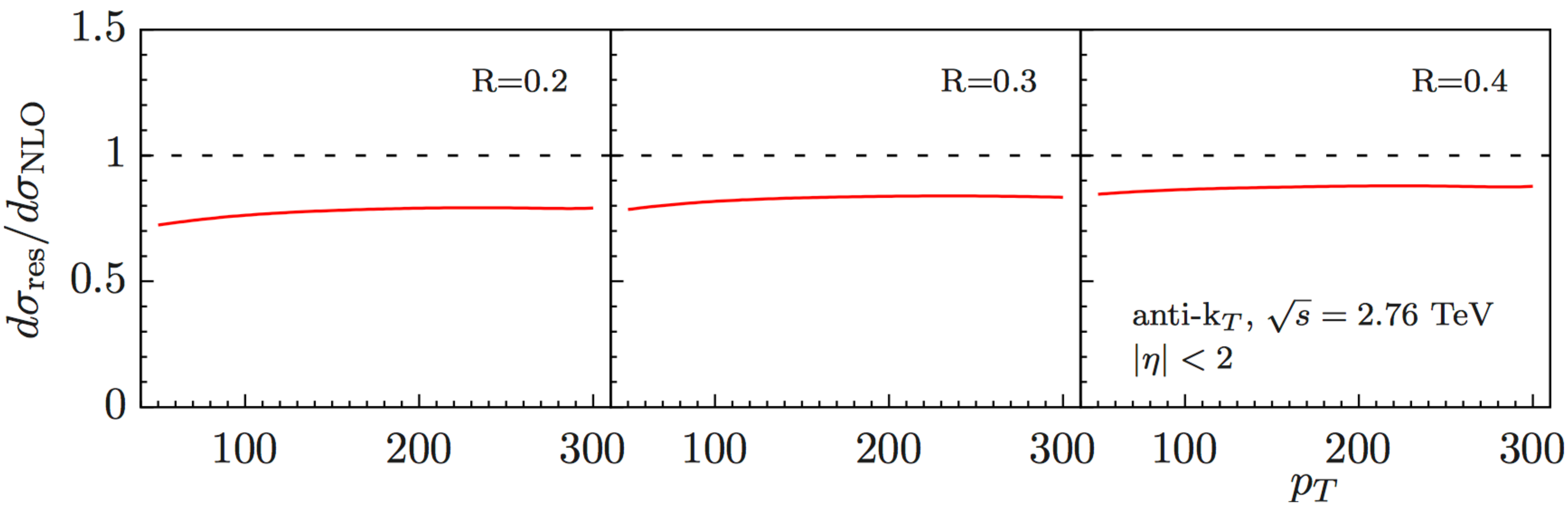
$\sqrt{s} = 8$ TeV

$|\eta| < 0.5$

(Same for jet substructure)

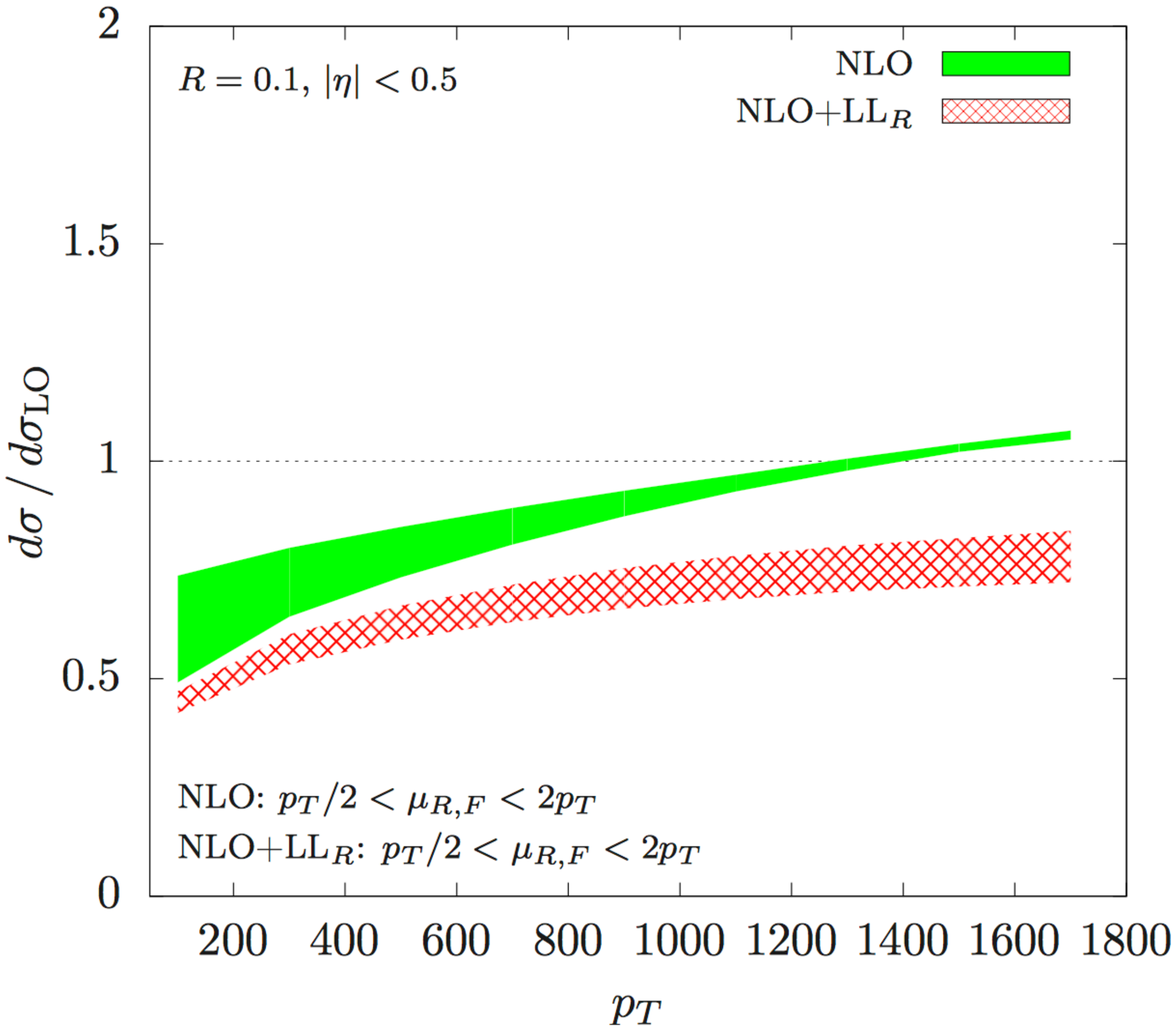
Comparison to LHC data

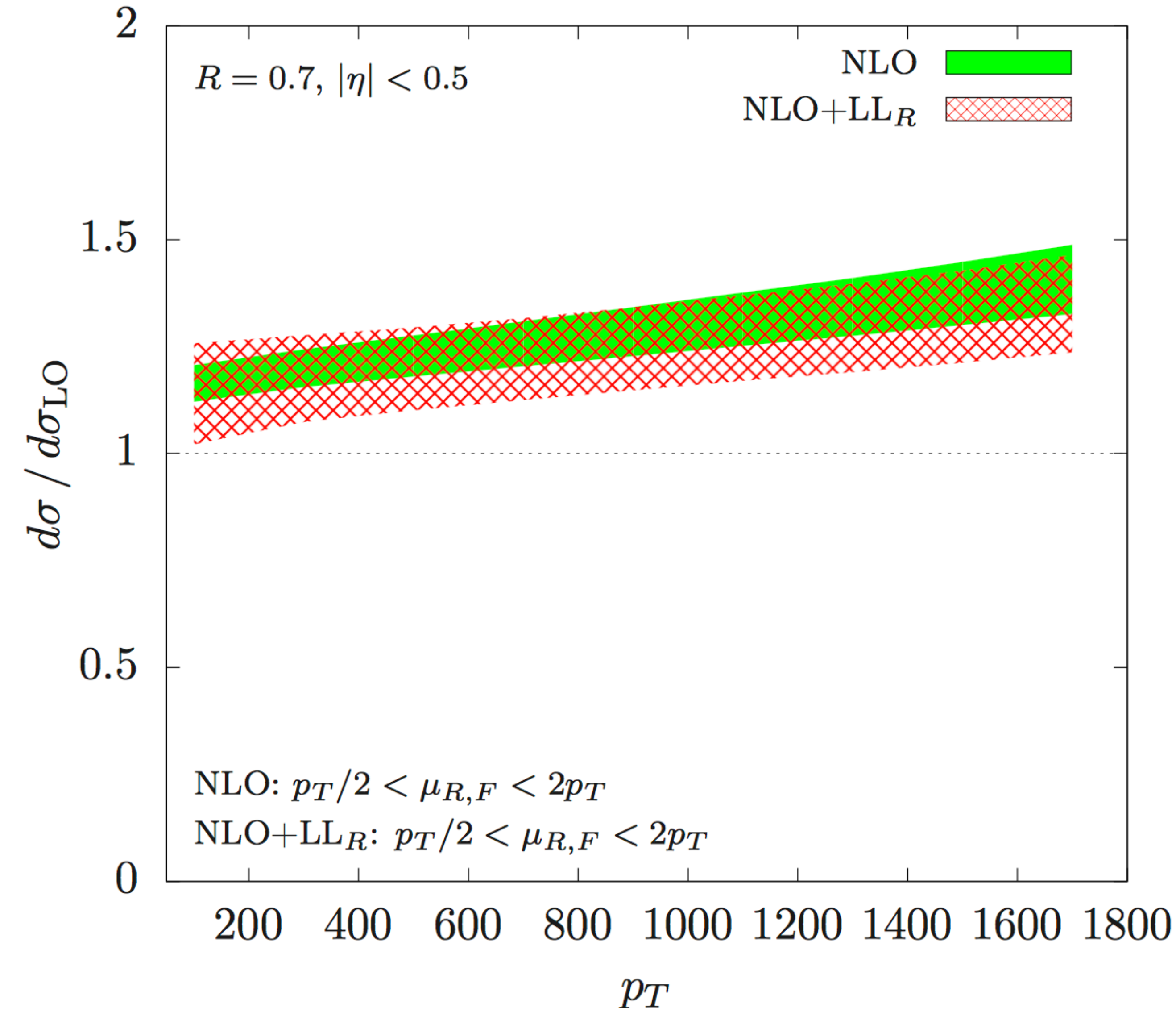




CMS Phys.Rev. C96 015202 (2017)

QCD scale dependence





see also: *Dasgupta, Dreyer, Salam, Soyez `15, `16*

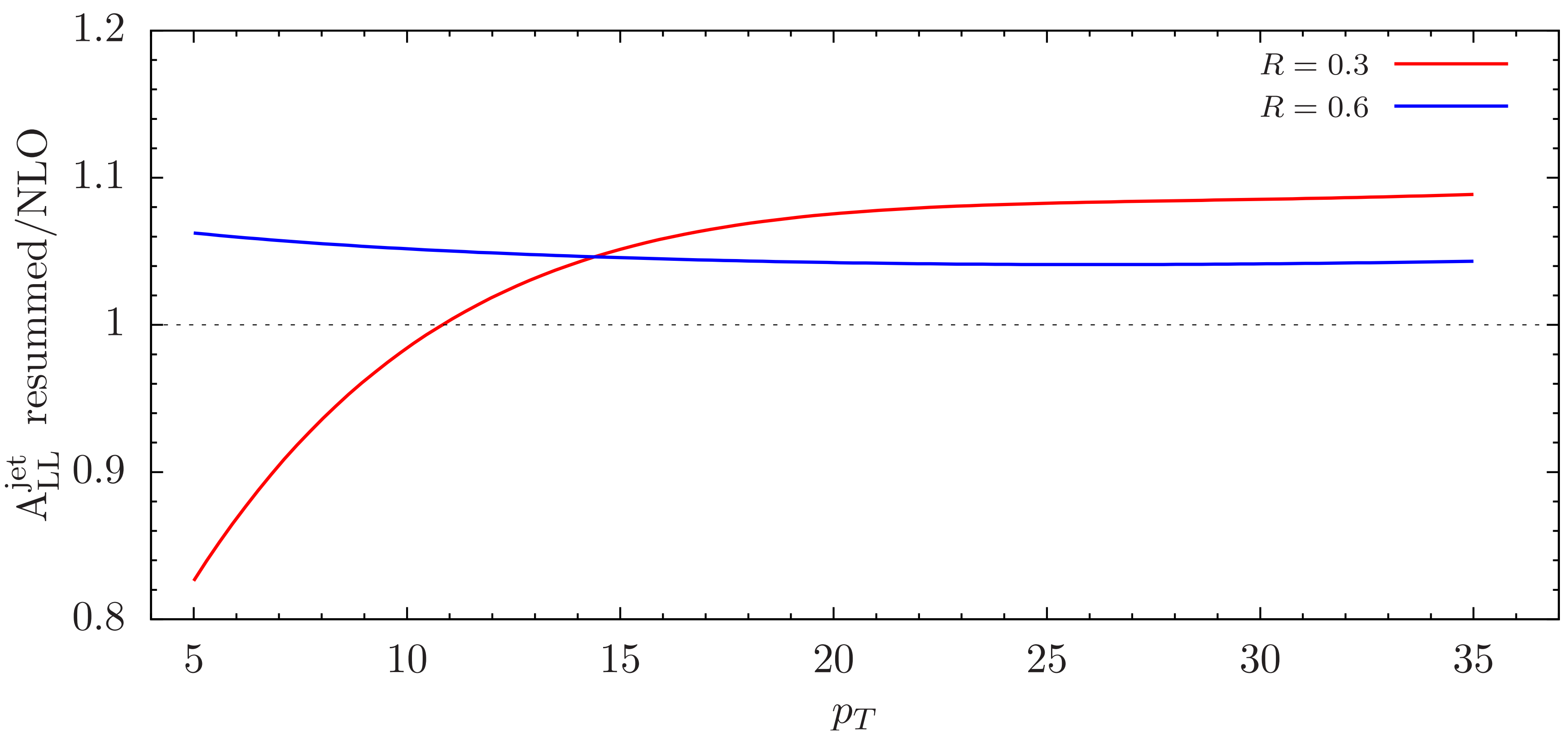
Spin asymmetries at RHIC

Polarized cross section:

Kang, FR, Vogelsang - in preparation

$$\frac{d\Delta\sigma}{d\eta dp_T} = \sum_{abc} \Delta f_a \otimes \Delta f_b \otimes \Delta H_{ab}^c \otimes J_c$$

Same jet functions as for unpolarized case



$\sqrt{s} = 200$ GeV

$|\eta| < 0.5$

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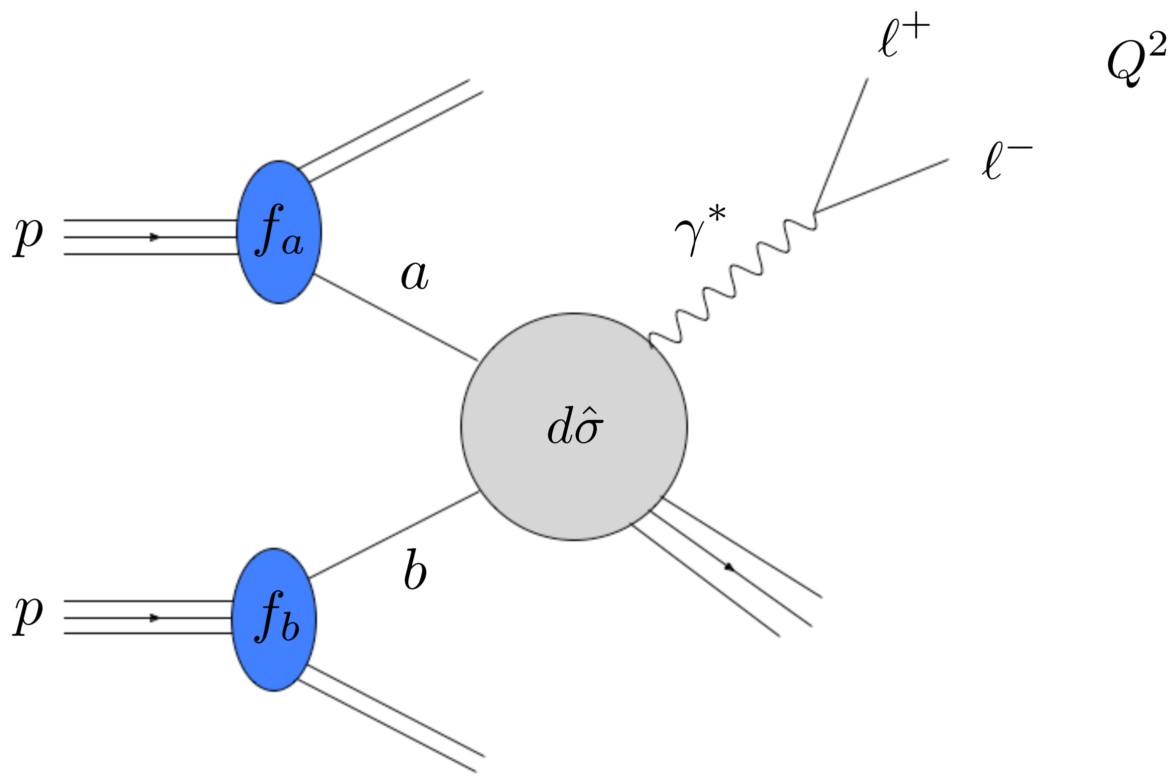
Threshold resummation

Drell-Yan



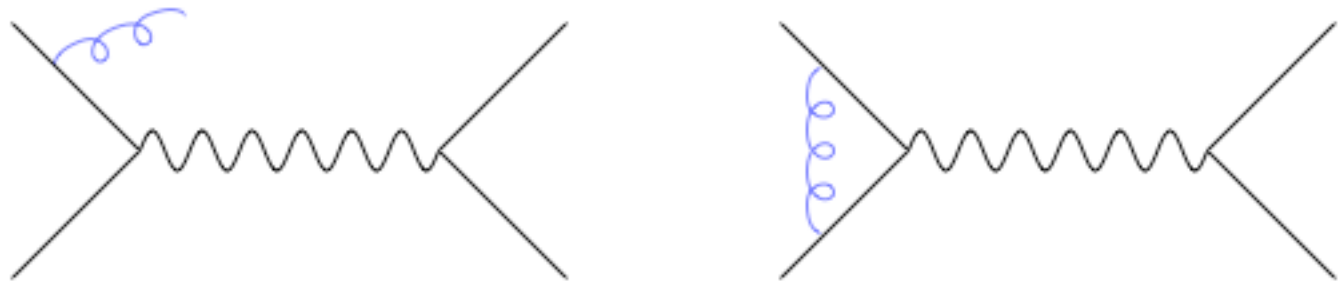
Cross section

$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} f_{a/p} \otimes f_{b/p} \otimes d\hat{\sigma}$$

Hard-scattering part
is calculable perturbatively

$$d\hat{\sigma} = \omega^{(\mathrm{LO})} + \alpha_s\, \omega^{(\mathrm{NLO})} + \alpha_s^2\, \omega^{(\mathrm{NNLO})} + \ldots$$

NLO



$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \sim \frac{\alpha_s}{2\pi} C_F \left[4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z + \left(\frac{2}{3}\pi^2 - 8\right)\delta(1-z) + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2} \right]$$

threshold logarithm

where $z = \dfrac{Q^2}{\hat{s}}$

Sterman `87
Catani, Trentadue `89

$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} f_{a/p} \otimes f_{b/p} \otimes d\hat{\sigma}$$

$$\int_0^1 d\tau\, \tau^{N-1}\; Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} f_{a/p}^N \cdot f_{b/p}^N \cdot d\hat{\sigma}^N$$

$$\alpha_s^k \left(\frac{\ln^{2k-1}(1-z)}{1-z}\right) \to \alpha_s^k \ln^{2k} \bar{N}$$

All order resummation can be achieved by solving RGEs in SCET *Manohar `03*

Accuracy of threshold resummation

$$\mathcal{O}(\alpha_s^k): \qquad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k$$

Fixed Order						
LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
$\mathrm{N^kLO}$	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

$L = \ln \bar{N}$

Accuracy of threshold resummation

$$\mathcal{O}(\alpha_s^k): \qquad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k$$

Fixed Order →

Resummation ↓

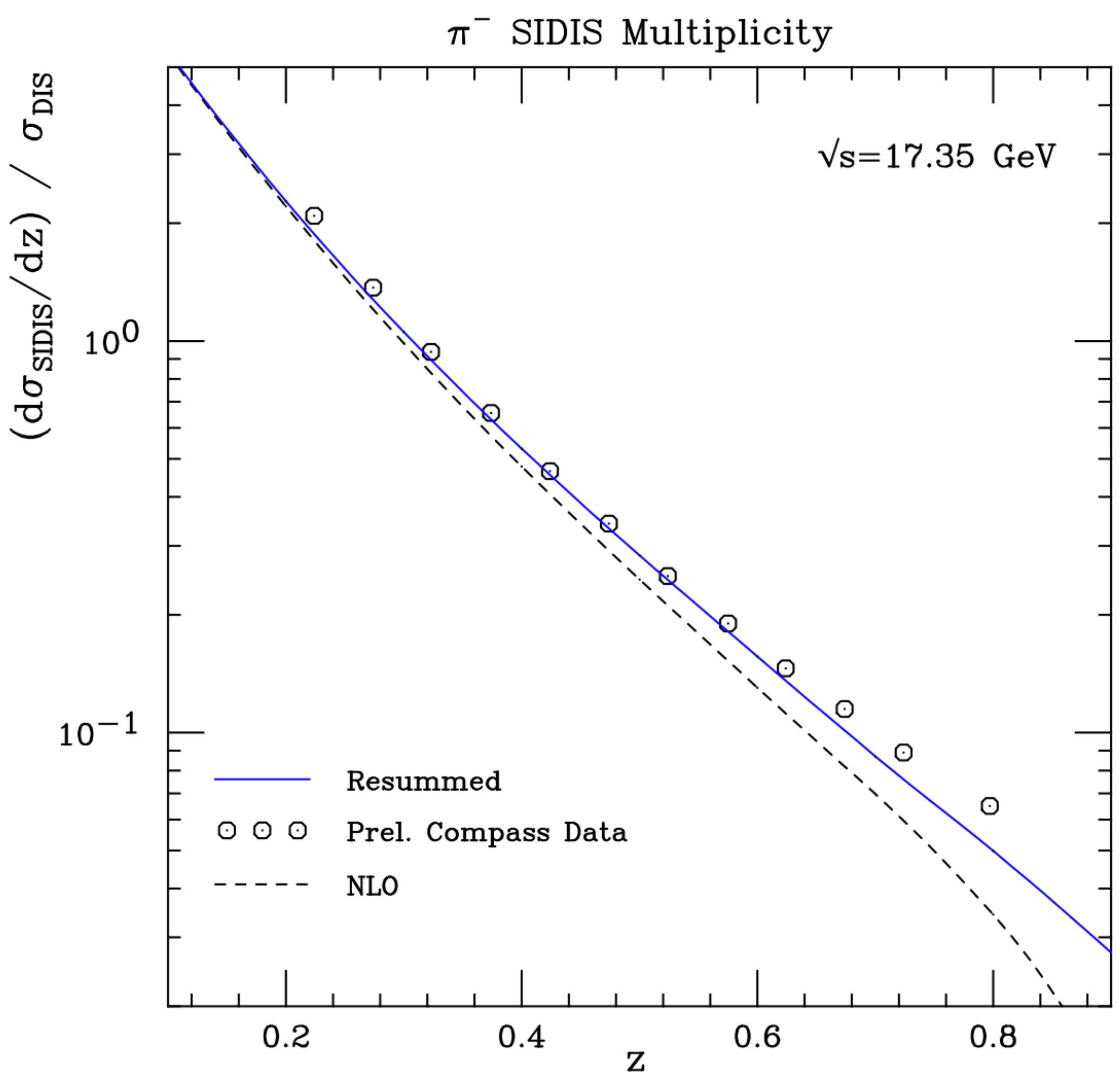
LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

↓ LL ↓ NLL ↓ NNLL

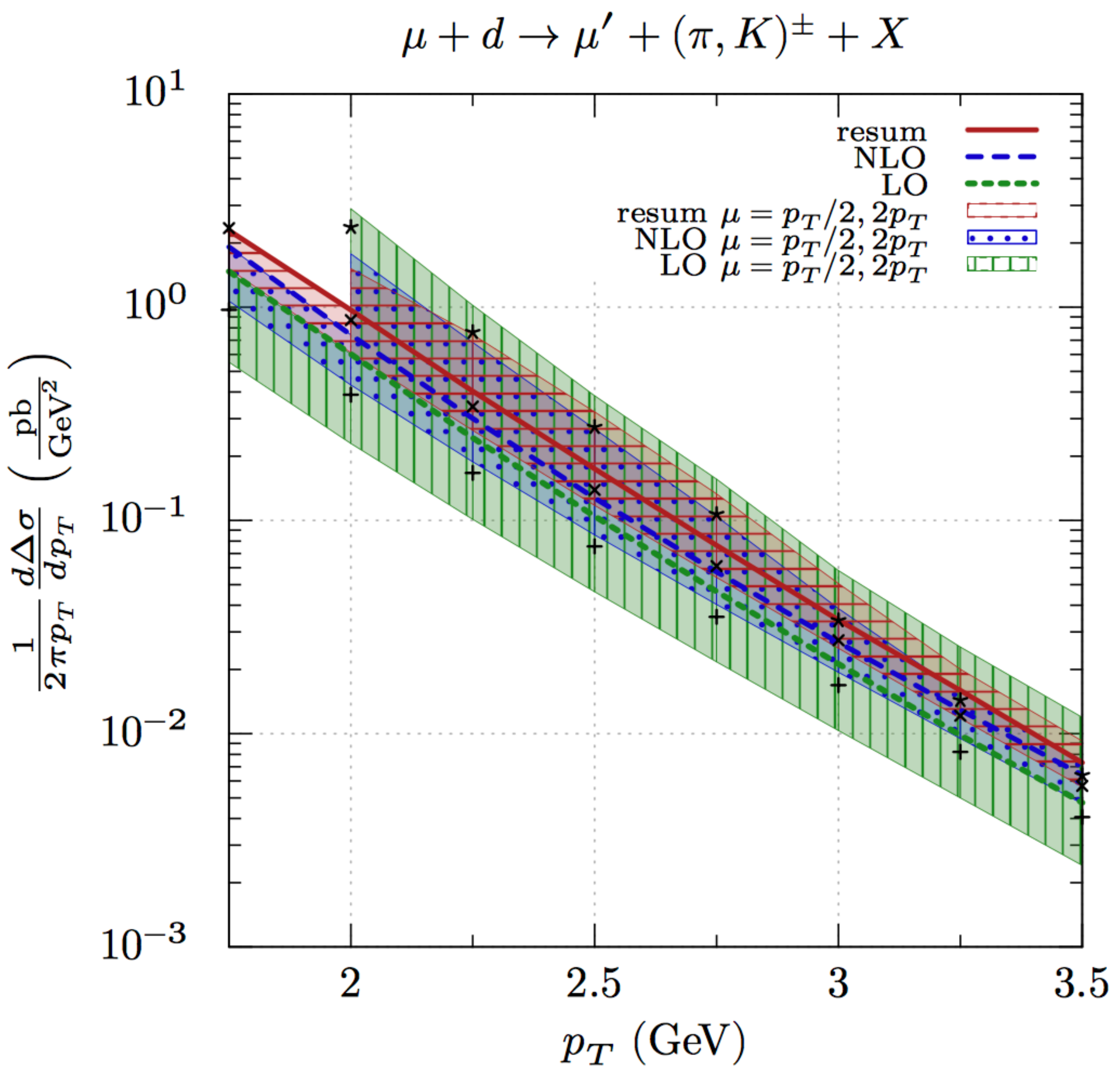
$L = \ln \bar{N}$

$ep \to ehX$

$\gamma N \to hX$



Anderle, FR, Vogelsang `12



Uebler, Schäfer, Vogelsang `17

Inclusive jet production at threshold

- Non-trivial color structure *Kidonakis, Sterman `97, Kidonakis, Oderda, Sterman `98*
- Previously unsolved problems with the inverse transformation

  only approximate NNLO results available *Kidonakis, Owens `01, Kumar, Moch `13, de Florian, Hinderer, Mukherjee, FR, Vogelsang `14*

$$\frac{p_T^2 d^2\sigma}{dp_T^2 d\eta} = \sum_{ab} \int_0^{V(1-W)} dz \int_{\frac{VW}{1-z}}^{1-\frac{1-V}{1-z}} dv\, x_a f_a(x_a,\mu_f)\; x_b f_b(x_b,\mu_f) \frac{d\hat{\sigma}_{ab}}{dvdz}(v,z,p_T,\mu_r,\mu_f,R)$$

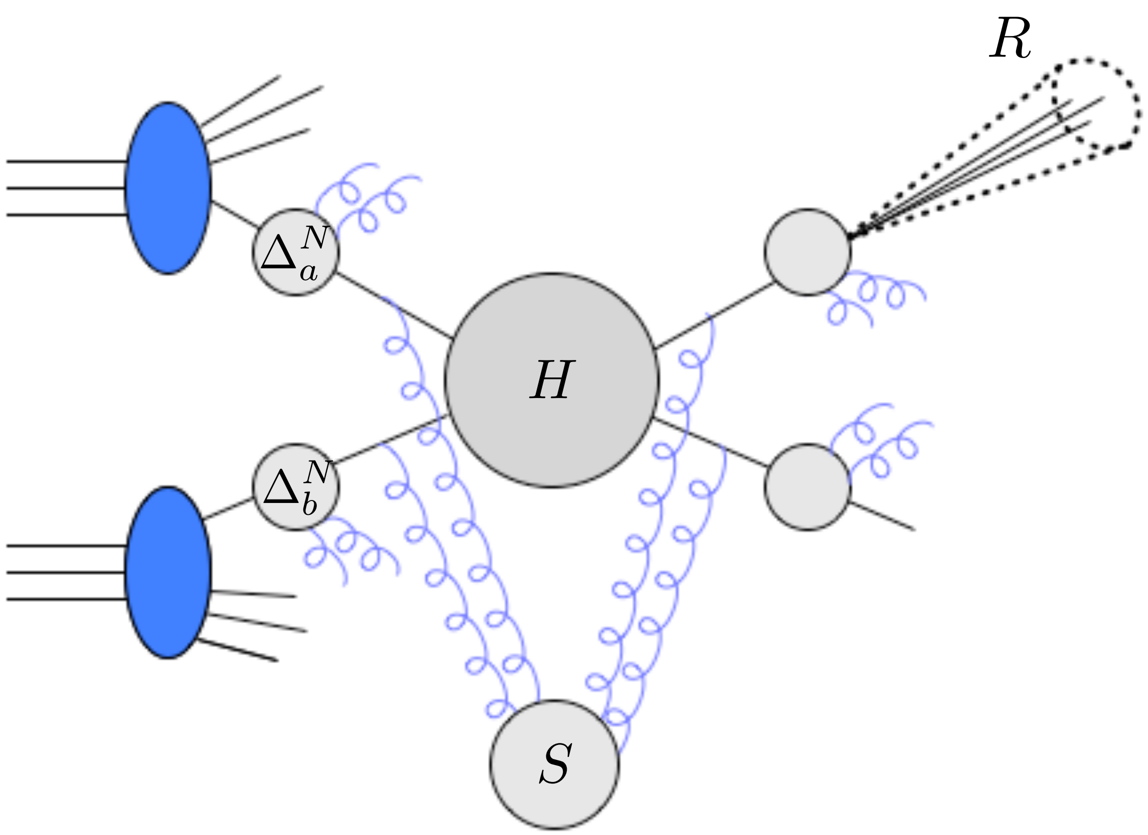
where

$V = 1 - p_T e^{-\eta}/\sqrt{S}$ $\quad$ $VW = p_T e^{\eta}/\sqrt{S}$

$s = x_a x_b S$ $\quad$ $v = \dfrac{u}{u+t}$ $\quad$ $z = s_4/s$

threshold $\quad z \to 0$

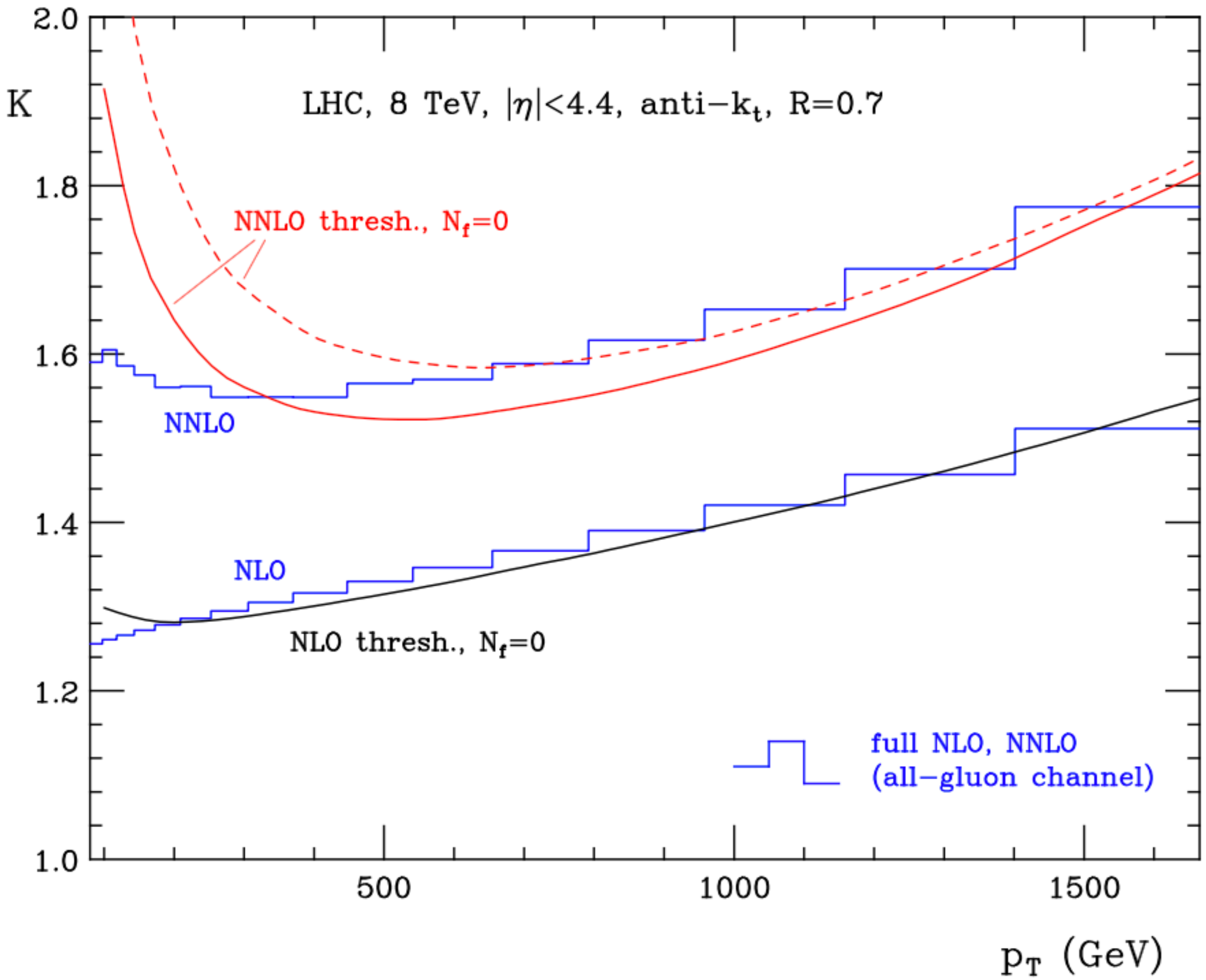
logarithms $\quad \left(\dfrac{\ln z}{z}\right)_+$



Inclusive jet production at threshold

- Non-trivial color structure *Kidonakis, Sterman `97, Kidonakis, Oderda, Sterman `98*
- Previously unsolved problems with the inverse transformation

  only approximate NNLO results available *Kidonakis, Owens `01, Kumar, Moch `13, de Florian, Hinderer, Mukherjee, FR, Vogelsang `14*



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- Requires joint resummation $\alpha_s^n \ln^{2n} \bar{N}$, $\alpha_s^n \ln^n R$ → e.g. *Dai, Kim, Leibovich `17*

Fixed Order →

Resummation ↓

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

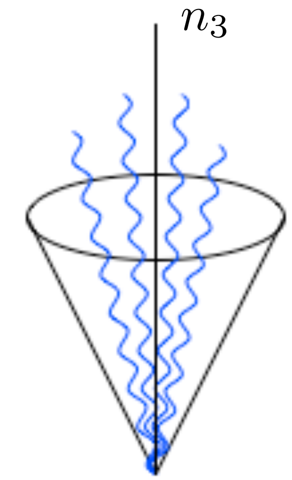
↓ LL ↓ NLL ↓ NNLL

Threshold resummation within SCET

Liu, Moch, FR `17

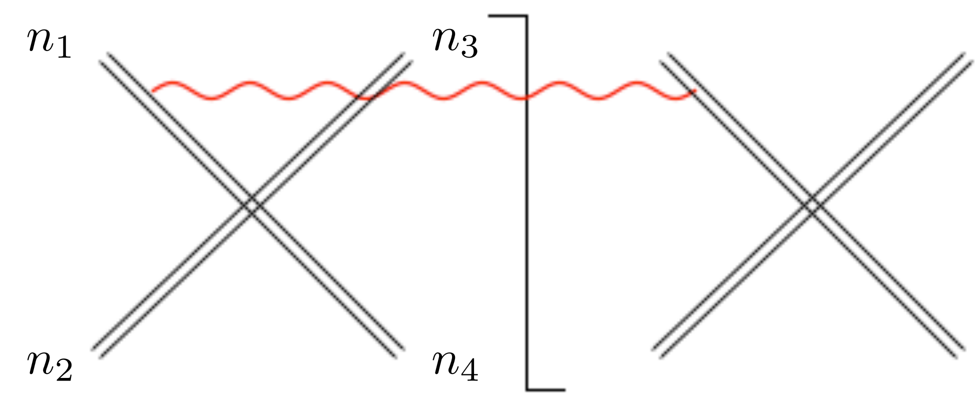
- Joint resummation $\alpha_s^n \ln^{2n} \bar{N}, \quad \alpha_s^n \ln^n R$

Refactorization of the soft sector and the observed jet function: "$J^{\text{obs}}\, S$"



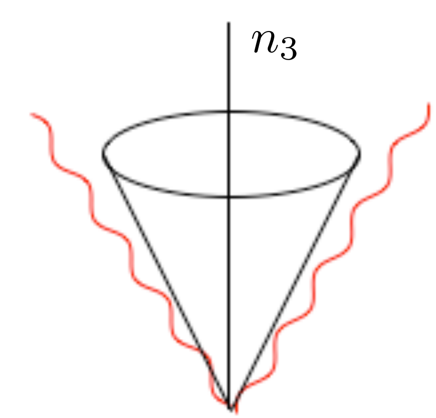
- Jet function

$$J_q(R) = 1 + \frac{\alpha_s}{2\pi} C_F \left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + \frac{1}{\epsilon} L_R + \frac{1}{2} L_R^2 + \frac{3}{2} L_R + \frac{13}{2} - \frac{3\pi^2}{4} \right]$$



- Global-soft function

$$\mathbf{S}_G^{(1)} = \frac{\alpha_s}{\pi\,\epsilon} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_{i \neq j \neq 4} \mathbf{T}_i \cdot \mathbf{T}_j \frac{n_{ij}}{\mu_{sG}} \left(\frac{s_G n_{ij}}{\mu_{sG}} \right)^{-1-2\epsilon}$$



- Soft-collinear function

$$S_c^{(1)} = \mathbf{T}_3^2 \frac{\alpha_s}{\pi\,\epsilon} \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \frac{p_T R}{s \mu_{sc}} \left(\frac{s_c\, p_T R}{s\, \mu_{sc}} \right)^{-1-2\epsilon}$$

Threshold resummation within SCET

- Inverse transformation e.g. Drell-Yan soft function *Becher, Neubert `06*
  - Bare soft function $S^{\rm DY}_{\rm bare}(N,\mu) = 1 + \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{\bar{N}^2}{\mu^2} + \ln^2 \frac{\bar{N}^2}{\mu^2} + \frac{\pi^2}{6} \right]$

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  - Solution of RG equation $S^{\rm DY}(N,\mu) = \exp[-4C_F S(\mu_s,\mu) + 2A_{\gamma_W}(\mu_s,\mu)] S^{\rm DY}(N,\mu_s) \left(\frac{\bar{N}^2}{\mu_s^2} \right)^{\eta}$

    where $\eta = 2C_F A_{\gamma_{\rm cusp}}(\mu_s,\mu)\,, \quad A_{\gamma_{\rm cusp}}(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$

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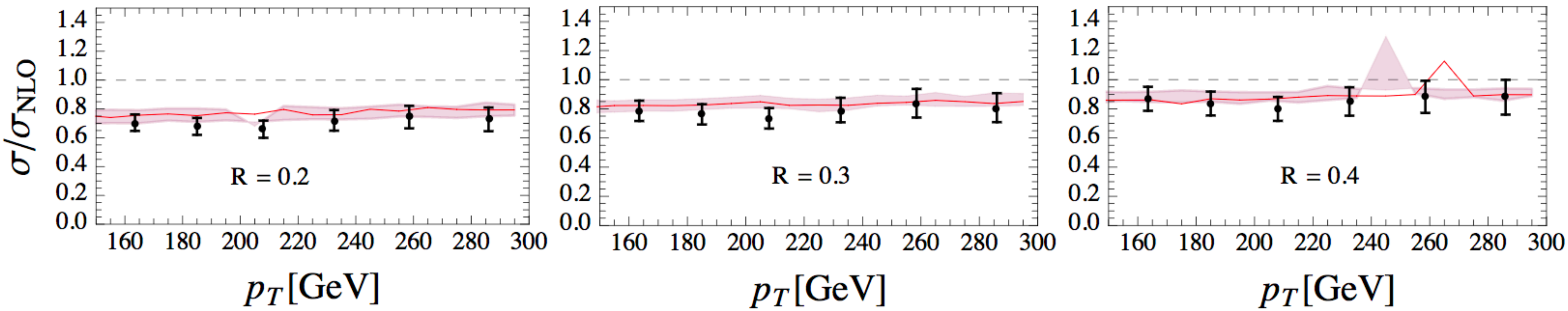
  - Inverse with $$L^m \left(\frac{\bar{N}^2}{\mu_s^2} \right)^\eta = \partial_\eta^{(m)} \left(\frac{\bar{N}^2}{\mu_s^2} \right)^n , \quad L = \ln \frac{\bar{N}^2}{\mu_s^2}$$

→ $$S^{\rm DY}(z,\mu) = \exp[-4C_F S(\mu_s,\mu) + 2A_{\gamma_W}(\mu_s,\mu)] S^{\rm DY}(\partial_\eta,\mu_s) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{1}{Mz} \left(\frac{Mz}{\mu_s} \right)^{2\eta}$$

Threshold resummation within SCET

- Numerical results

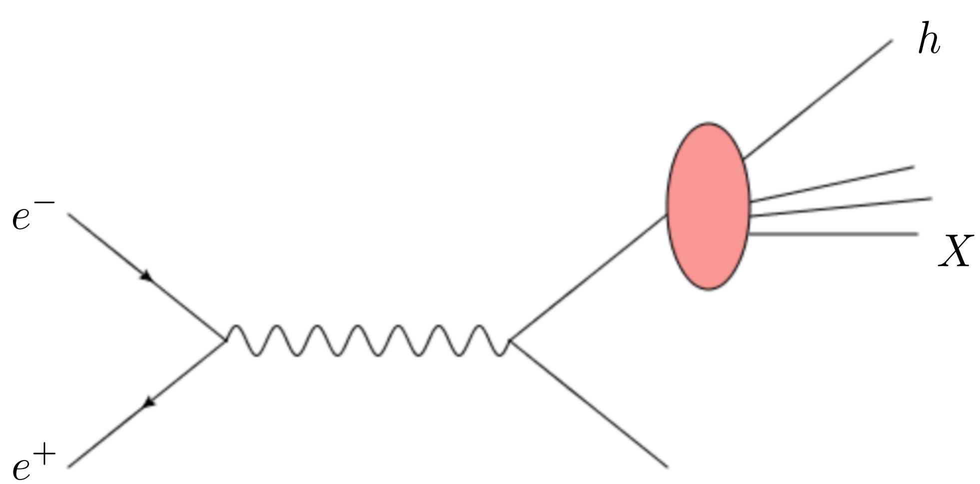
$|\eta| < 2$ $\sqrt{s} = 2.76\ \mathrm{TeV}$ *CMS Phys.Rev. C96 015202 (2017)*

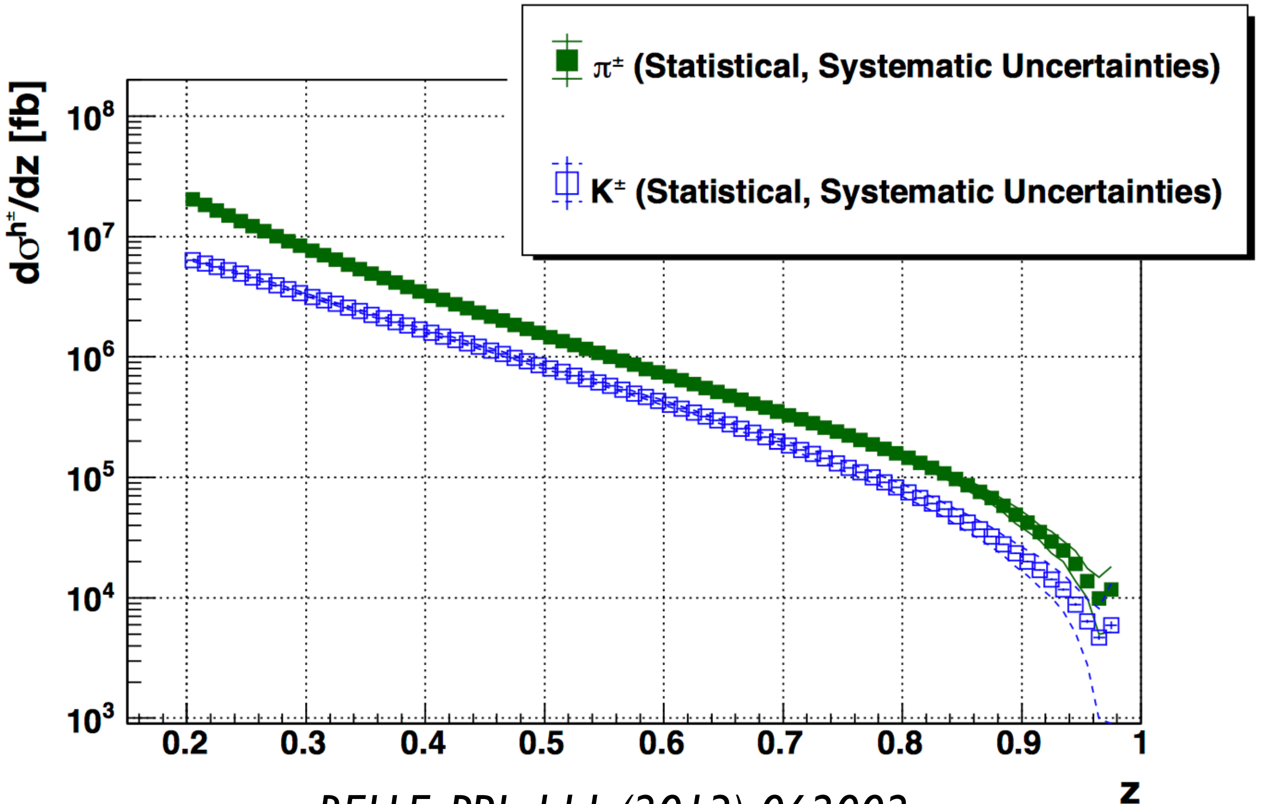


Outline

- Motivation
- Small jet radius resummation
- Threshold resummation
- Small-z resummation
- Conclusions

Small-z resummation $e^+e^- \to hX$





BELLE, PRL 111 (2013) 062002

$$\frac{d\sigma_k^h}{dz} = \sigma_{\rm tot}^{(0)} \left[D_{\rm S}^h(z,\mu^2) \otimes \mathbb{C}_{k,q}^{\rm S}\left(z, \frac{Q^2}{\mu^2}\right) + D_g^h\left(z,\mu^2\right) \otimes \mathbb{C}_{k,g}^{\rm S}\left(z, \frac{Q^2}{\mu^2}\right)\right] + \sum_q \sigma_q^{(0)} D_{{\rm NS},q}^h(z,\mu^2) \otimes \mathbb{C}_{k,q}^{\rm NS}\left(z, \frac{Q^2}{\mu^2}\right)$$

$$z \equiv \frac{2P_h \cdot q}{Q^2} \overset{\rm c.m.s.}{=} \frac{2E_h}{Q}$$

Small-z resummation $e^+e^- \to hX$

All order structure:

coefficient function

$$\mathbb{C}_{T,g}^{S,(k)} \propto a_s^k \frac{1}{z} \log^{2k-1-a}(z)$$

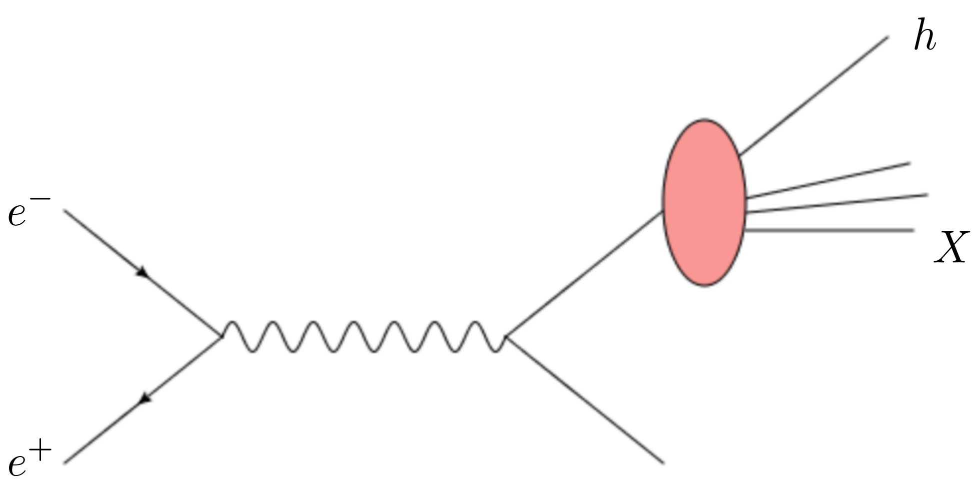
$$\mathbb{C}_{L,g}^{S,(k)} \propto a_s^k \frac{1}{z} \log^{2k-2-a}(z)$$

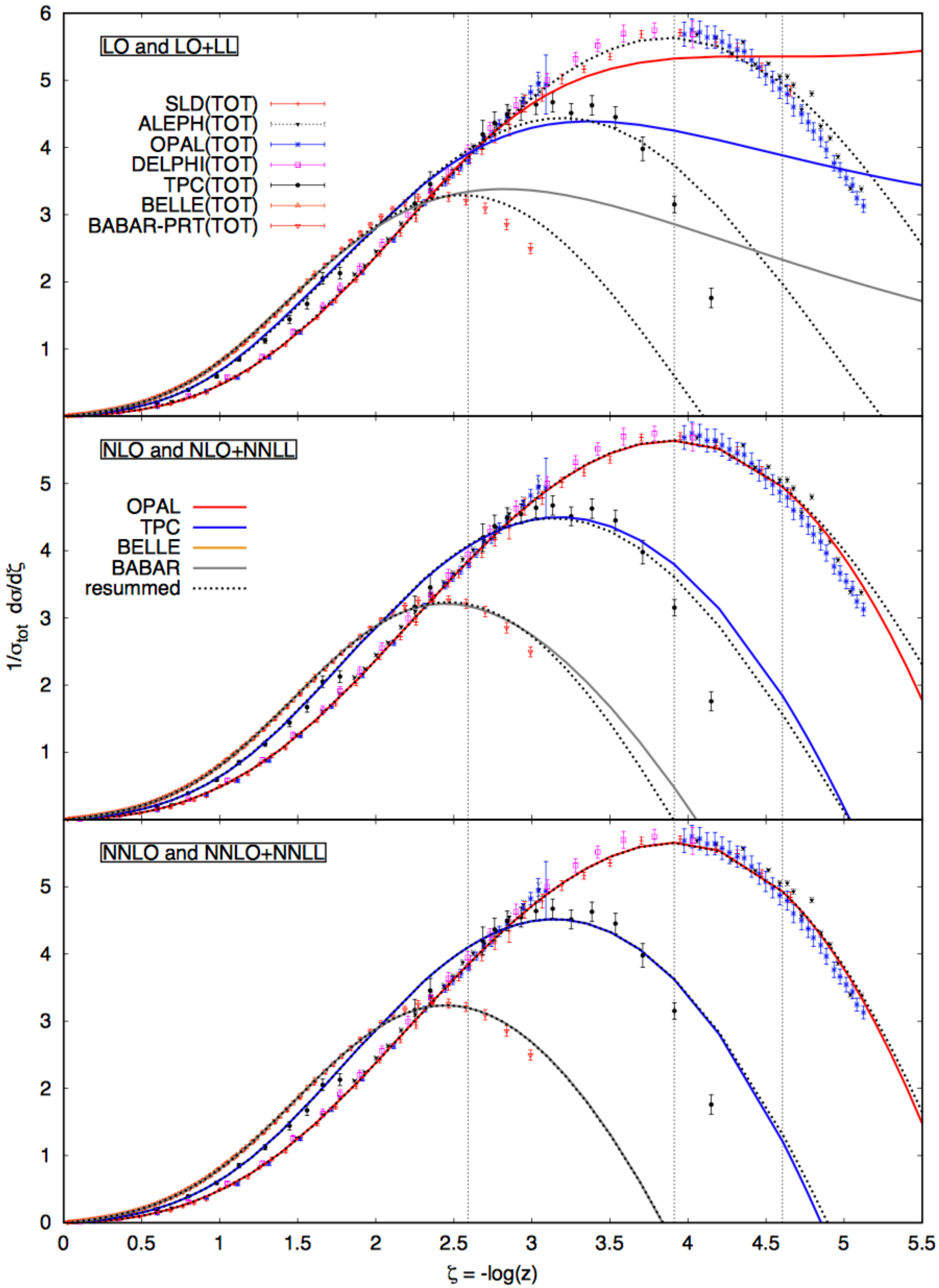
DGLAP kernel

$$P_{gi}^{T,(k)} \propto a_s^{(k+1)} \frac{1}{z} \log^{2k-a}(z) \qquad i = q, g$$

NNLL resummation (a=0,1,2) achieved in Mellin space using algebraic recursion relations

Vogt `11; Kom, Vogt, Yeats `12





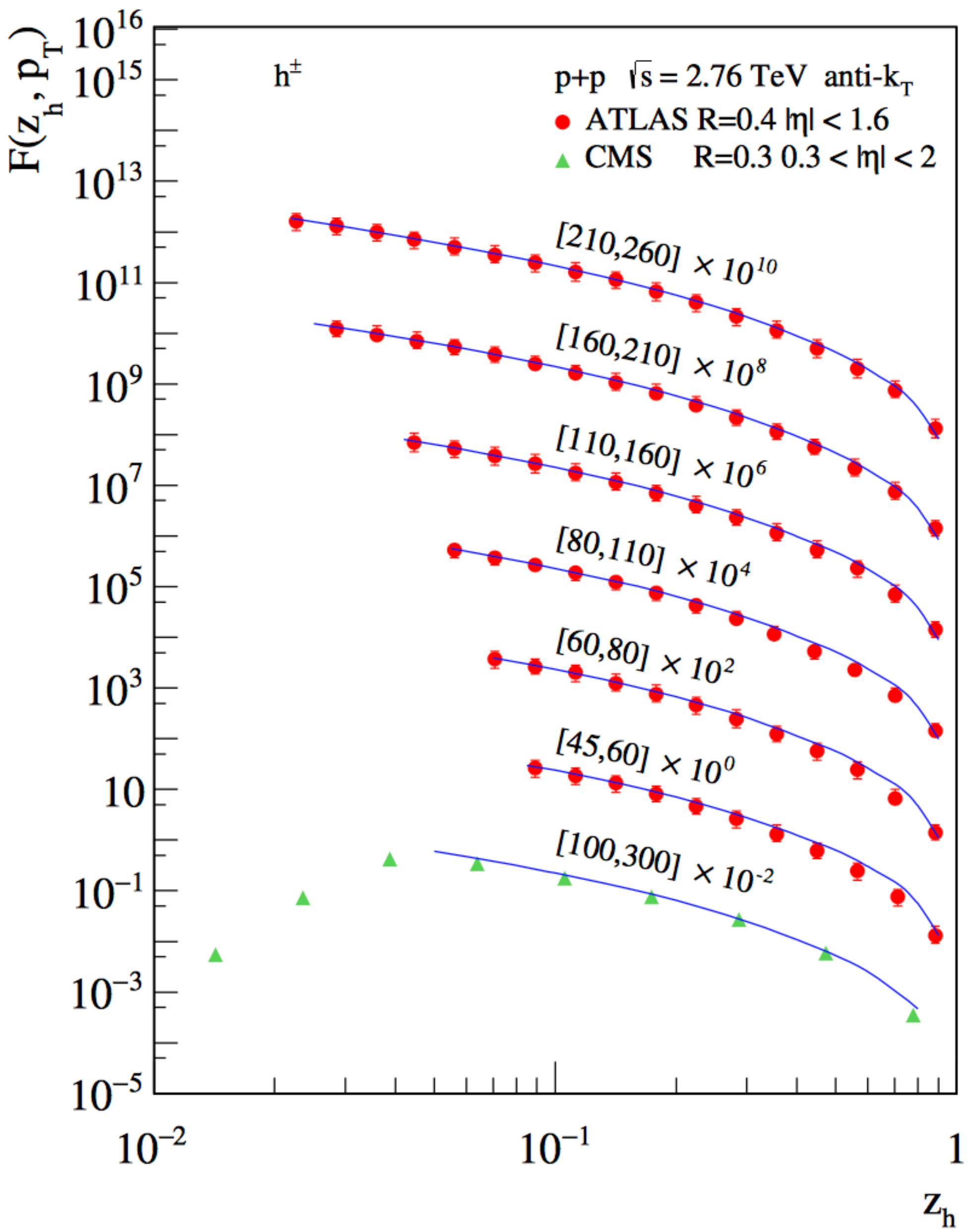
LO (+ LL)

NLO (+ NNLL)

NNLO (+ NNLL)

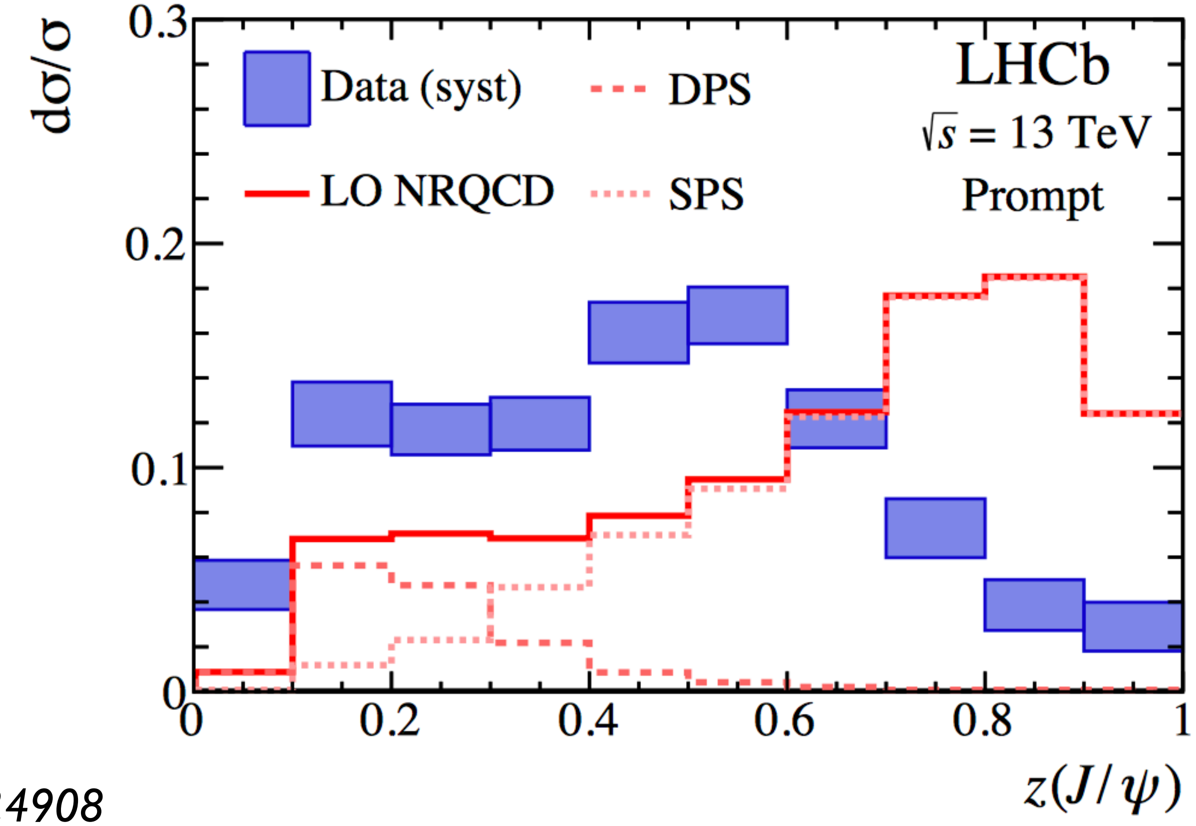
Anderle, Kaufmann, FR, Stratmann `16

Hadron in-jet production



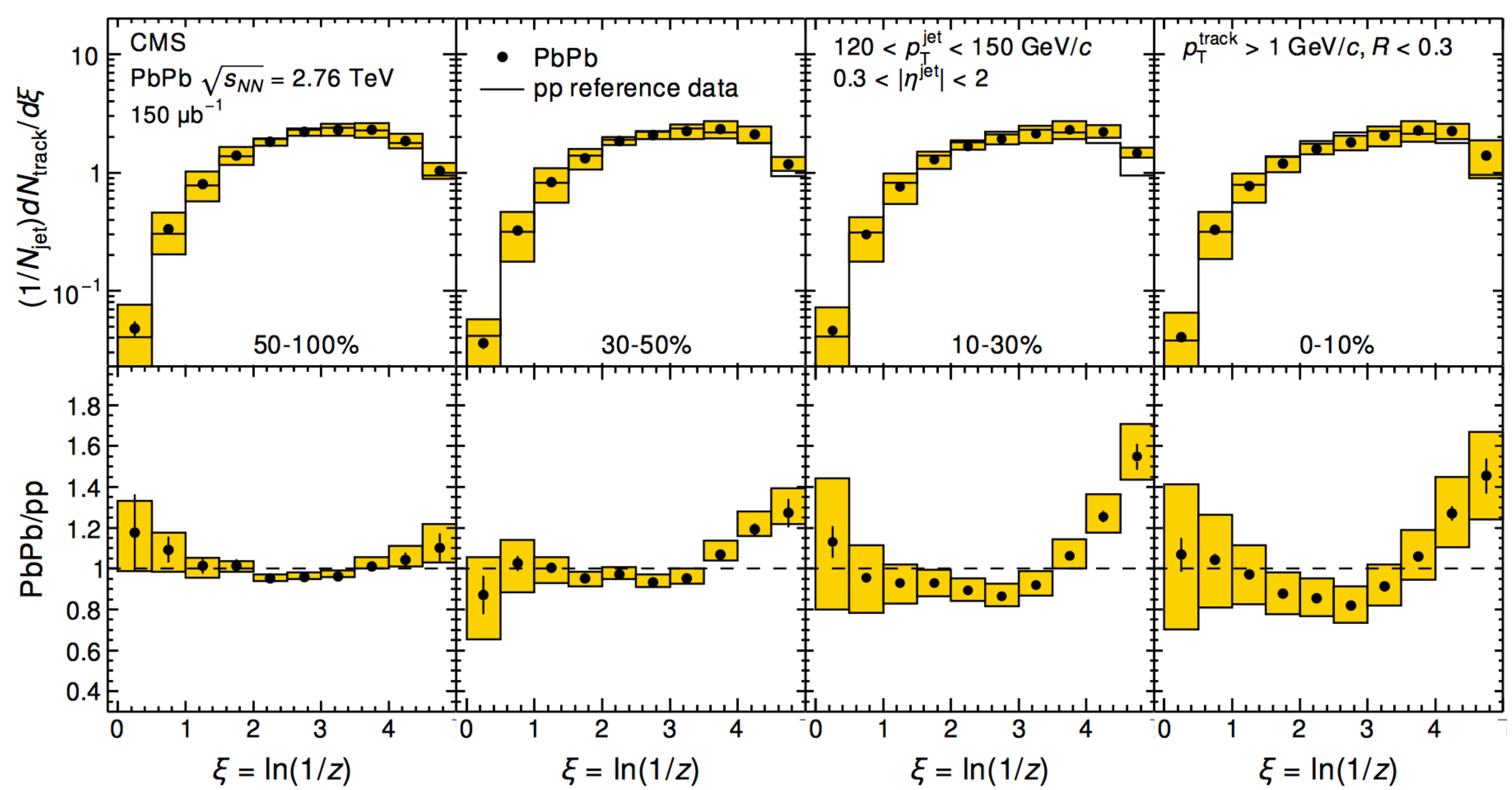
Heavy flavor

LHCb, PRL 118 (2017) 192001

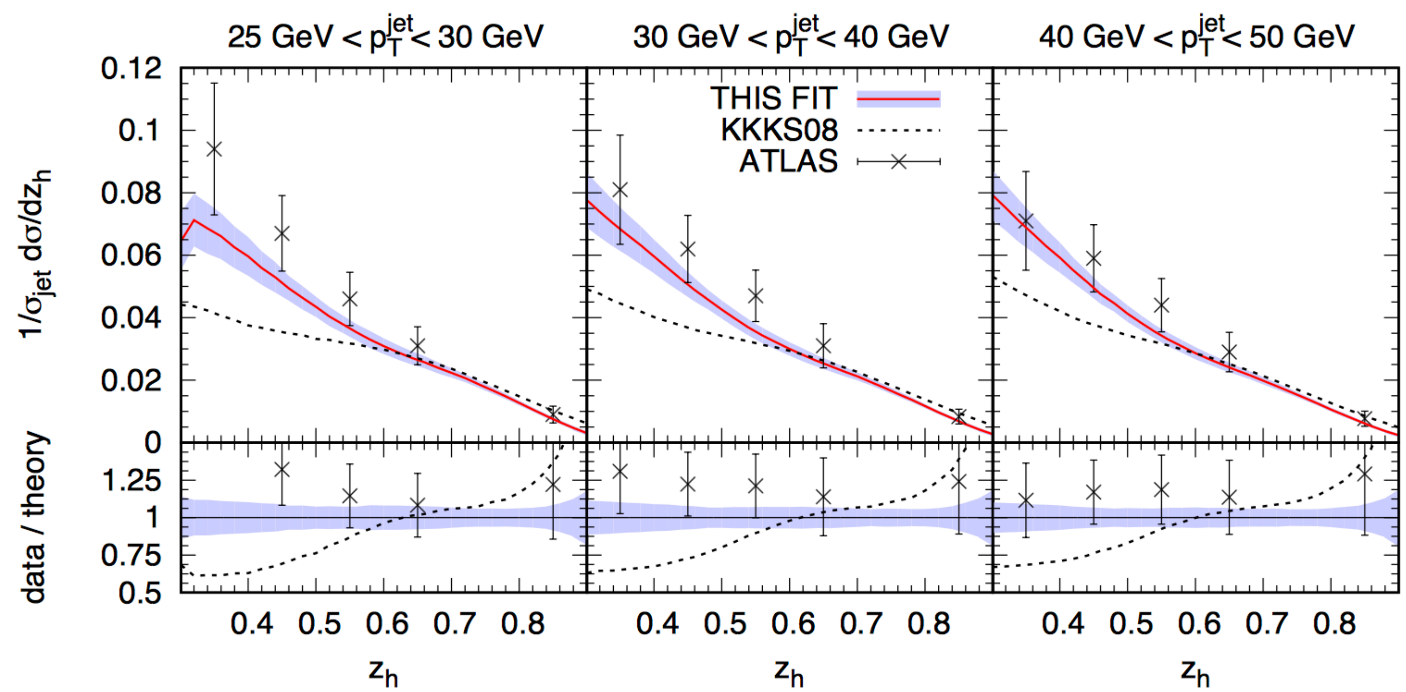


Heavy ion

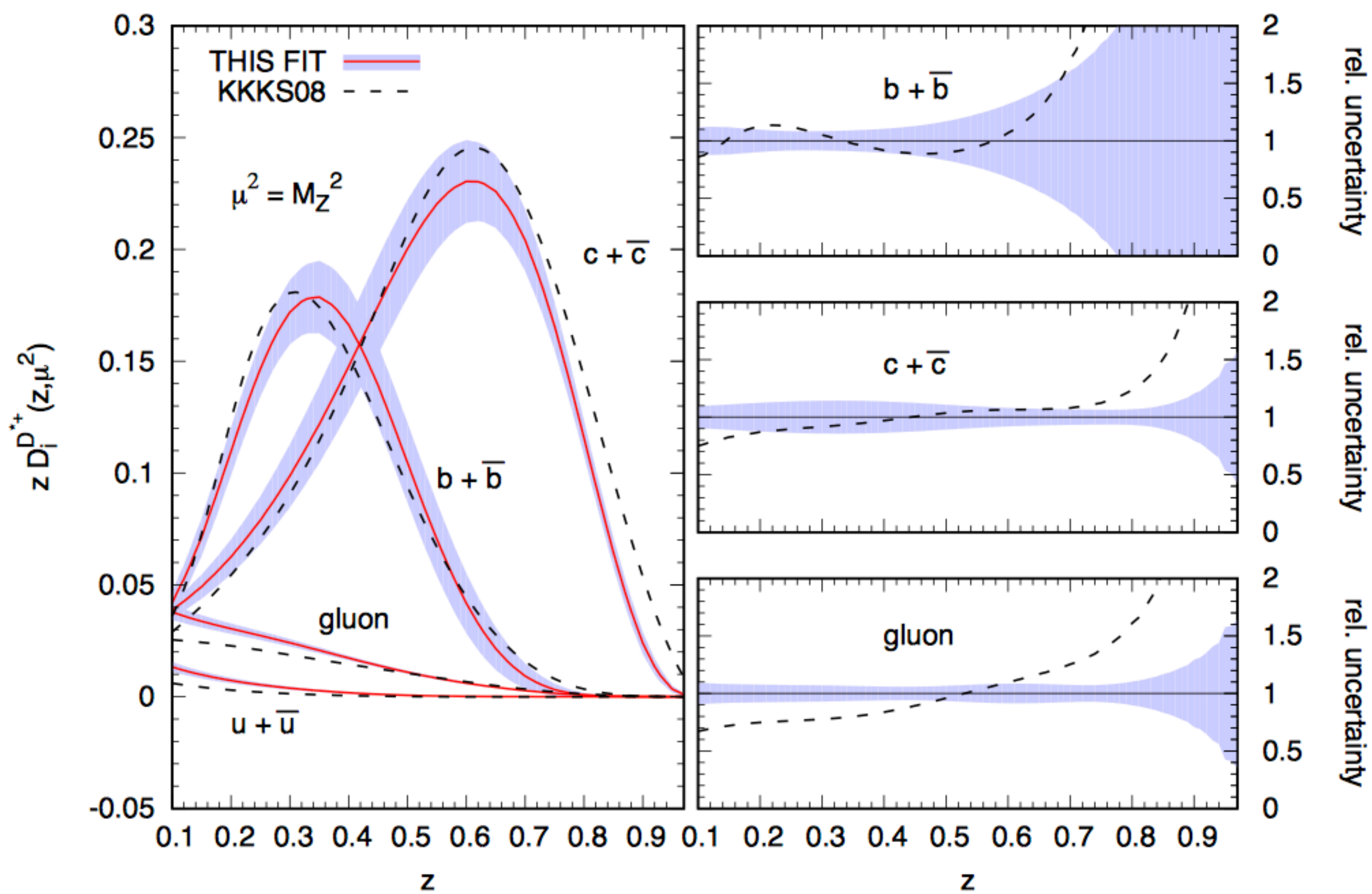
CMS, PRC 90 (2014) 024908



D*-in-jet data



D* meson FFs



Anderle, Kaufmann, FR, Stratmann, Vitev `17

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Conclusions

- All order resummation important for precision phenomenology
- Joint resummation of small jet radius and threshold logarithms
- Extraction of fragmentation functions including resummation

- Extension to NNLL accuracy and thr. NNLO
- Di-jet, photon + jet production
- New applications and tests of large and small-z resummation in the context of jet substructure and ep scattering