Nucleon and Pion Structure at Low and High Momenta.

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INT, Seattle

Introduction

- Measures of Hadron Structure and Lattice QCD
- Nucleon Charges and the role of excited states
- Exploring structure at long distances - coordinate-space methods
- Hadron structure at high momentum transfer - Pion Form Factor
- Summary

Lattice QCD

Observables in lattice QCD are then expressed in terms of the path integral as

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \prod_n d\psi(n) \prod_n d\bar{\psi}(n) \mathcal{O}(U, \psi, \bar{\psi}) e^{-\left(S_G[U] + S_F[U,\psi,\bar{\psi}]\right)}$$

Integrate out the Grassmann variables:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \mathcal{O}(U, G[U]) \det M[U] e^{-S_G[U]}$$

Importance Sampling

where $G(U,x,y)^{ij}_{\alpha\beta} \equiv \langle \psi^i_\alpha(x) \bar{\psi}^j_\beta(y) \rangle = M^{-1}(U)$

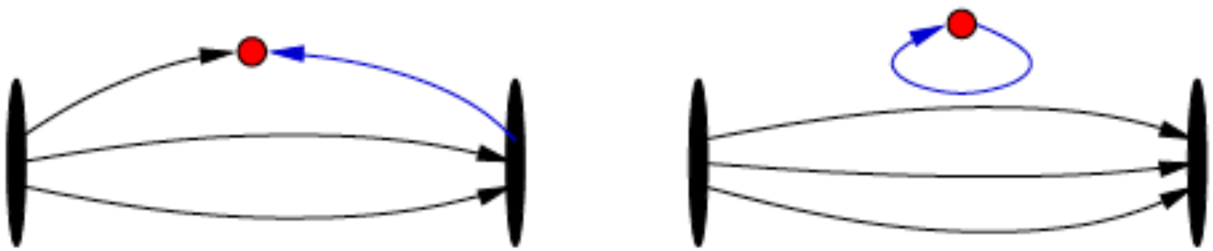
- Generate an ensemble of gauge configurations

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

- Calculate observable

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

Hadron Structure



$$C_{3\text{pt}}(t_{sep}, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 \mid N(\vec{x}, t_{\text{sep}}) V_\mu(\vec{y}, t) \bar{N}(\vec{0}, 0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

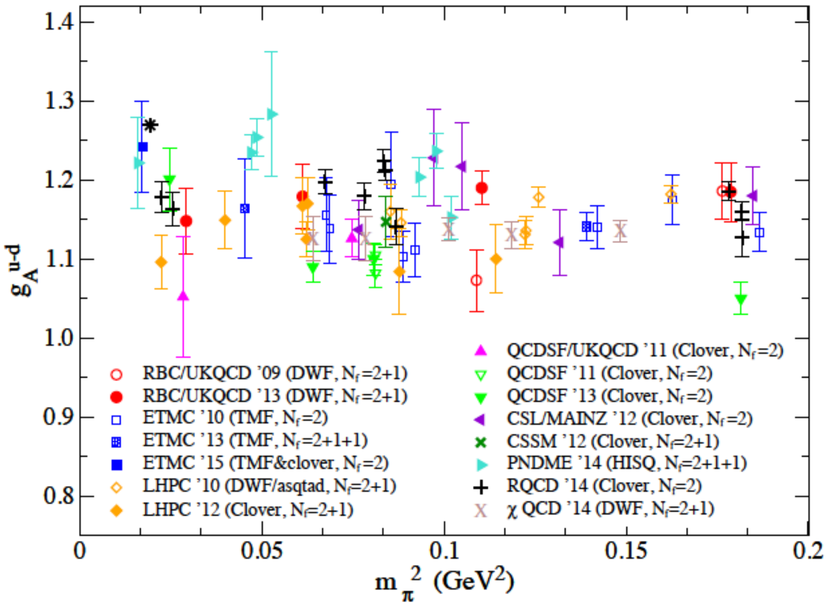
Resolution of unity – insert states

$$\longrightarrow \langle 0 \mid N \mid N, \vec{p} + \vec{q} \rangle \langle N, \vec{p} + \vec{q} \mid V_\mu \mid N \vec{p} \rangle \langle N, \vec{p} \mid \bar{N} \mid 0 \rangle e^{-E(\vec{p}+\vec{q})(t_{\text{sep}} - t)} e^{-E(\vec{p})t}$$

PRECISION ISOVECTOR NUCLEON STRUCTURE

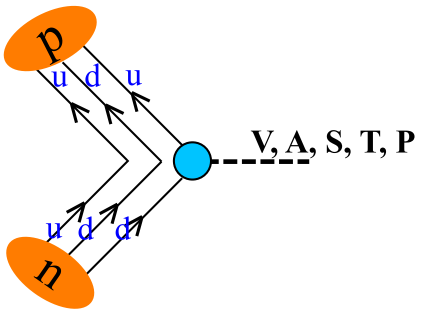
Hadron Structure

M Constantinou, arXiv:1511.00214



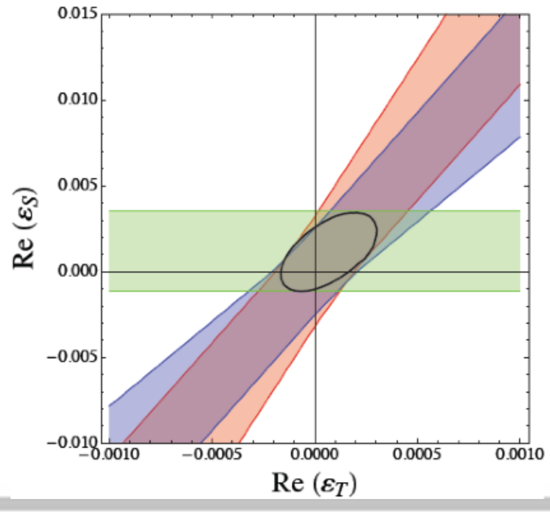
- Governs beta-decay rate
- Important for proton-proton fusion rate in solar models
- Benchmark for lattice QCD calculations of hadron structure

e.g. novel interactions probed in ultra-cold neutron decay



$$H_{eff} \supset G_F \left[\varepsilon_S \, \bar{u} d \times \bar{e}(1-\gamma_5)\nu_e + \varepsilon_T \, \bar{u} \sigma_{\mu\nu} d \times \bar{e} \sigma^{\mu\nu} (1-\gamma_5)\nu_e \right]$$

$$g_S = Z_S \langle p | \bar{u} d | n \rangle \quad g_T = Z_T \langle p | \bar{u} \sigma_{\mu\nu} \, d | n \rangle$$



R Gupta, 2014

Calculation of Physics Observables

Our paradigm: nucleon mass $$C(t) = \sum_{\vec{x}} \langle N(\vec{x},t)\bar{N}(0)\rangle = \sum_n \mid A_n \mid^2 e^{-E_n t}$$

Noise: $$C_{\sigma^2}(t) = \sum_{\vec{x}} \langle \bar{N}N(\vec{x},t)\bar{N}N(0)\rangle \longrightarrow e^{-3m_\pi t}$$

whence

$$C(t)/\sqrt{C_{\sigma^2}(t)} \simeq e^{-(m_N - 3m_\pi/2)t}$$

Use local nucleon interpolating operators $$[uC\gamma_5(1 \pm \gamma_4)d]u$$

Replace quark field by spatially extended (smeared) quark field

$$\psi \longrightarrow (1 - \sigma^2\nabla^2/4N)^N \psi$$

Variational Method

Subleading terms → *Excited* states

Construct matrix of correlators: *different smearing radii*

$$C_{ij}(t) = \sum_{\vec{x}} \langle N_i(\vec{x},t)\bar{N}_j(0)\rangle = \sum_n A_n^i A_n^{j\dagger} e^{-E_n t}$$

Delineate contributions using *variational method*: solve

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$

$$\lambda_N(t,t_0) \to e^{-E_N(t-t_0)}(1 + \mathcal{O}(e^{-\Delta E(t-t_0)}))$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

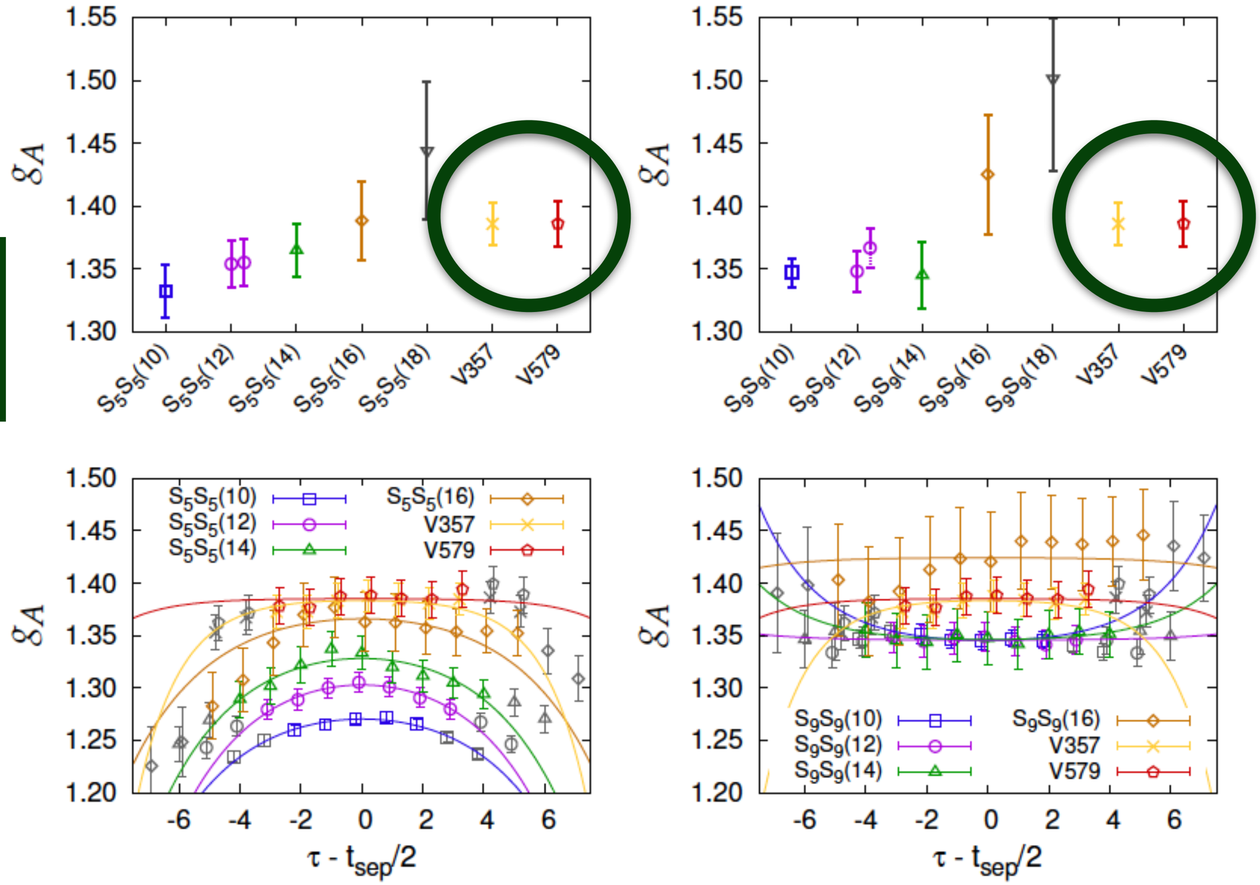
$$v^{(N')\dagger}C(t_0)v^{(N)} = \delta_{N,N'}$$

Systematic Uncertainties

Yoon et al., Phys. Rev. D 93, 114506 (2016)

Failure to isolating **ground state** leads to important systematic uncertainty.

Variational Method



Renormalized Charges

Yoon et al., Phys. Rev. D 95, 074508 (2017)

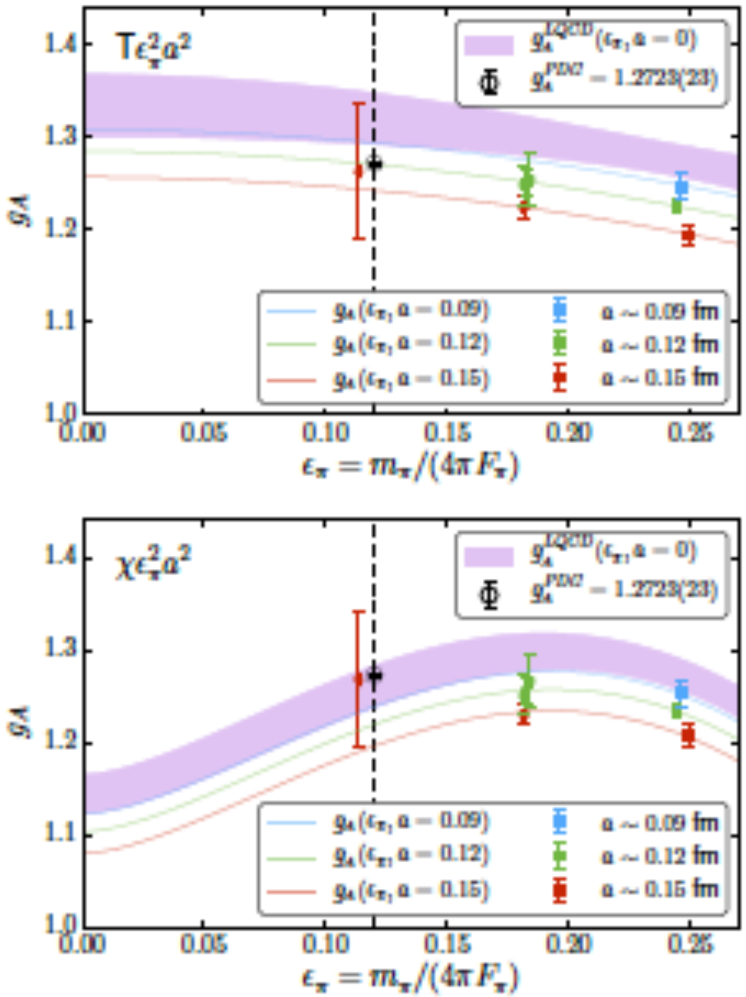
ID	Lattice Theory	a fm	M_π(MeV)	g_A^{u-d}	g_S^{u-d}	g_T^{u-d}	g_V^{u-d}
$a127m285$	2+1 clover-on-clover	0.127(2)	285(6)	1.249(28)	0.89(5)	1.023(21)	1.014(2
$a12m310$	2+1+1 clover-on-HISQ	0.121(1)	310(3)	1.229(14)	0.84(4)	1.055(36)	0.969(2
$a094m280$	2+1 clover-on-clover	0.094(1)	278(3)	1.208(33)	0.99(9)	0.973(36)	0.998(2
$a09m310$	2+1+1 clover-on-HISQ	0.089(1)	313(3)	1.231(33)	0.84(10)	1.024(42)	0.975(3
$a091m170$	2+1 clover-on-clover	0.091(1)	166(2)	1.210(19)	0.86(9)	0.996(23)	1.012(2
$a09m220$	2+1+1 clover-on-HISQ	0.087(1)	226(2)	1.249(35)	0.80(12)	1.039(36)	0.969(3
$a09m130$	2+1+1 clover-on-HISQ	0.087(1)	138(1)	1.230(29)	0.90(11)	0.975(38)	0.971(3

Consistency between different actions

Matrix Elements of 1st excited state?

ID	Type	$\langle 0\|\mathcal{O}_A\|1\rangle$	$\langle 0\|\mathcal{O}_S\|1\rangle$	$\langle 0\|\mathcal{O}_T\|1\rangle$	$\langle 0\|\mathcal{O}_V\|1\rangle$	$\langle 1\|\mathcal{O}_A\|1\rangle$	$\langle 1\|\mathcal{O}_S\|1\rangle$	$\langle 1\|\mathcal{O}_T\|1\rangle$	$\langle 1\|\mathcal{O}_V\|1\rangle$
		−0.179(21)		0.182(16)		−0.9(2.4)		−0.2(1.2)	
			−0.35(4)		−0.014(2)		0.6(1.1)		0.80(34)
a127m285	S_5S_5	−0.172(18)	−0.37(4)	0.210(15)	−0.015(2)	0.75(48)	0.8(9)	0.42(27)	0.87(28)
		−0.295(58)	−0.45(15)	0.167(40)	−0.014(6)	1.5(3.0)	1.8(1.4)	0.54(86)	0.86(55)
		−0.295(57)	−0.45(15)	0.166(47)	−0.014(6)	1.46(54)	1.8(1.4)	0.54(41)	0.86(28)

Feynman-Hellman Method



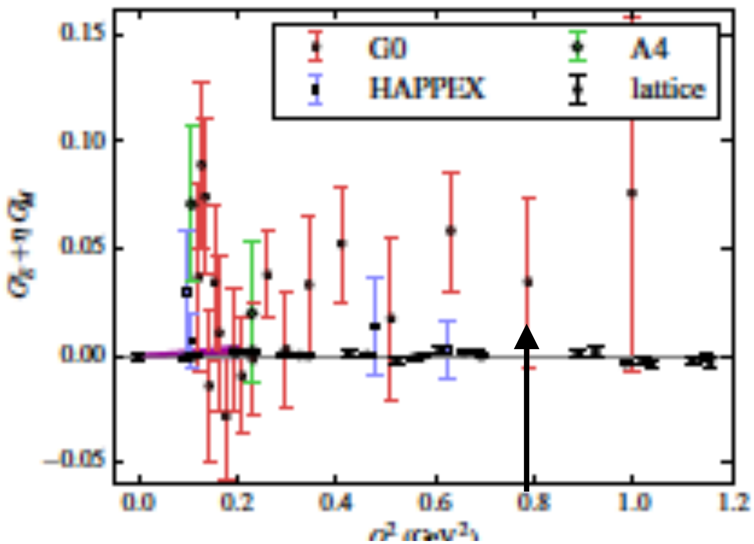
Berkowitz et al, arXiv:1704.01114

Calculation using Feynman-Hellman Theory

$$H = H_0 + \lambda H_\lambda$$
$$\frac{\partial E_n}{\partial \lambda} = \langle n \mid H_\lambda \mid n \rangle$$

Reduces to calculation of energy-shift of two-point functions ***but*** repeat the calculation for each operator

Sea Quark Contributions

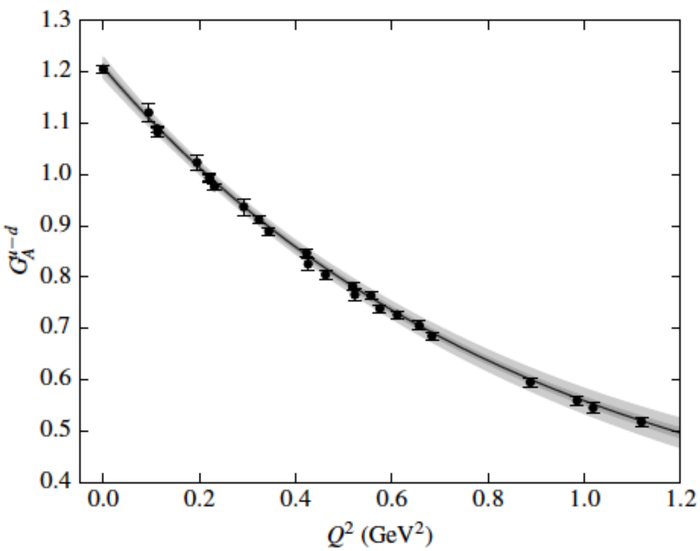


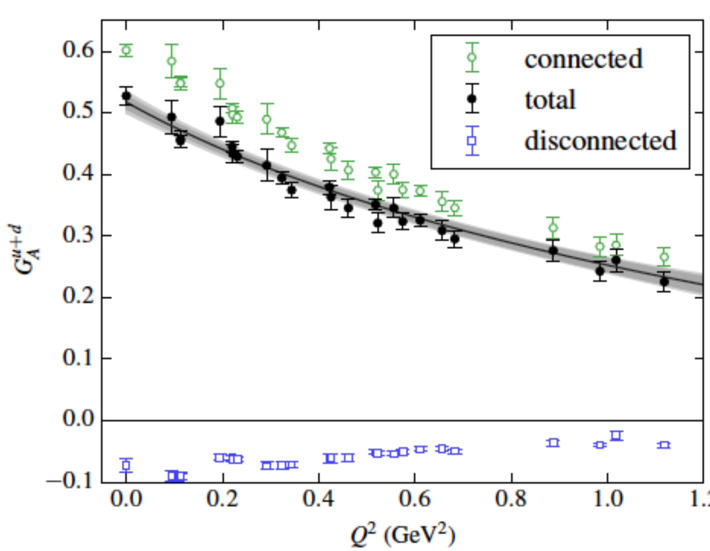
J. Green, K. Orginos *et al.*, Phys. Rev. D 92, 031501 (2015); Phys. Rev. D 95, 114502 (2017

Using *Hierarchical Probing* - A. Stathopoulos, J. Laeuchli, K. Orginos (2013)

Combination *measured* in expt

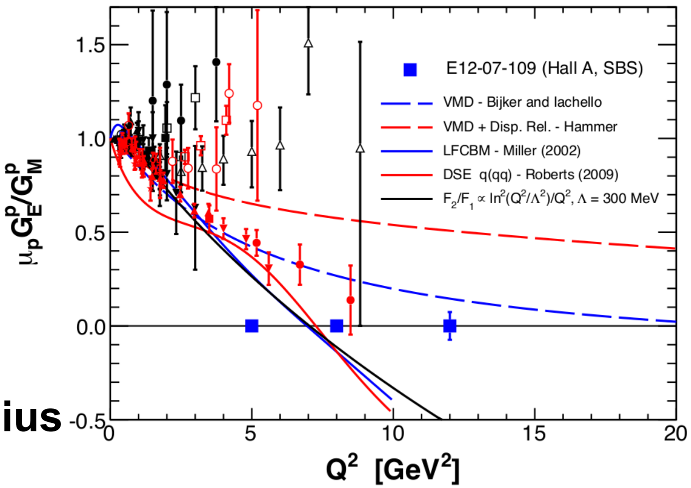
Axial-Vector Form Factors





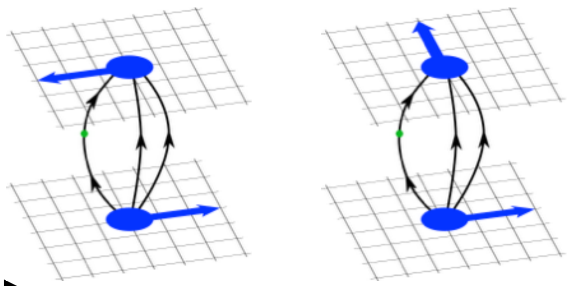
EM Form factors at Low and High Momenta

PRAD: E12-11-106



Approved expt E12-07-109

$Q^2 \lesssim 8.2\ \mathrm{GeV}^2$ $Q^2 \lesssim 4.1\ \mathrm{GeV}^2$

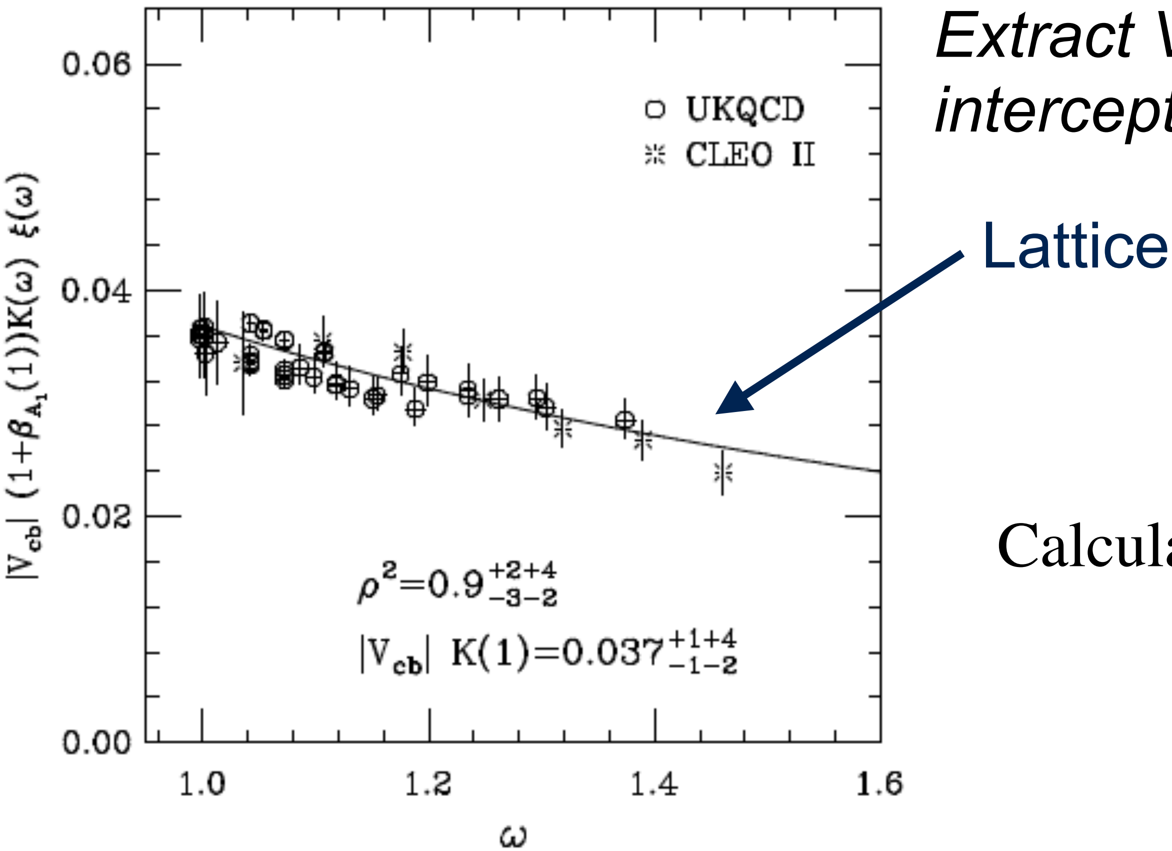


Nucleon Charge Radius

Boosted interpolating operators

See Sergey Syritsyn

Isgur-Wise Function and CKM matrix

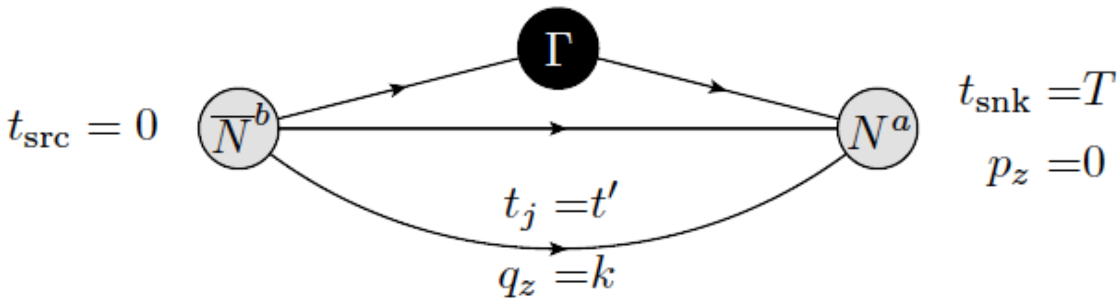


Extract V_{cb} if know intercept at zero recoil

Calculate slope at zero recoil..

UKQCD, L. Lellouch et al., Nucl. Phys. B444, 401 (1995), hep-lat/9410013

Moment Methods



- Introduce three-momentum projected three-point function

$$C^{3\rm pt}(t,t') = \sum_{\vec{x},\vec{x}'} \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle e^{-ikx'_z}$$

- Now take derivative w.r.t. k^2

$$C'_{3\rm pt}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'_z}{2k} \sin\left(kx'_z\right) \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle$$

whence

$$\lim_{k^2\to 0} C'_{3\rm pt}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'^2_z}{2} \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle .$$

Odd moments vanish by symmetry

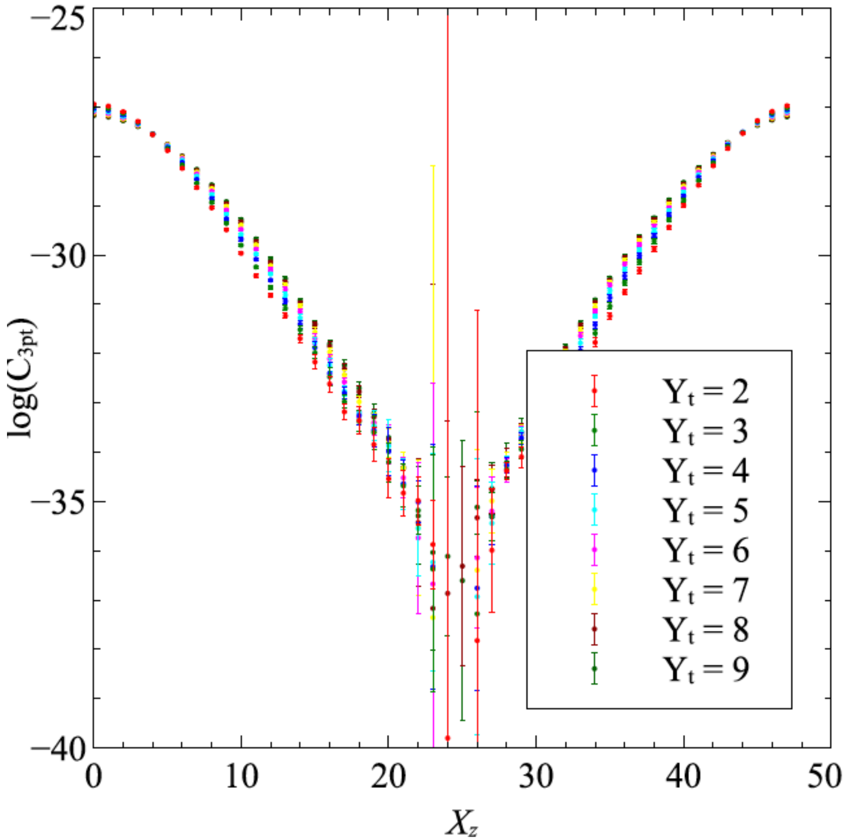
Lattice Details

- Two degenerate light-quark flavors, and strange quark set to its physical value

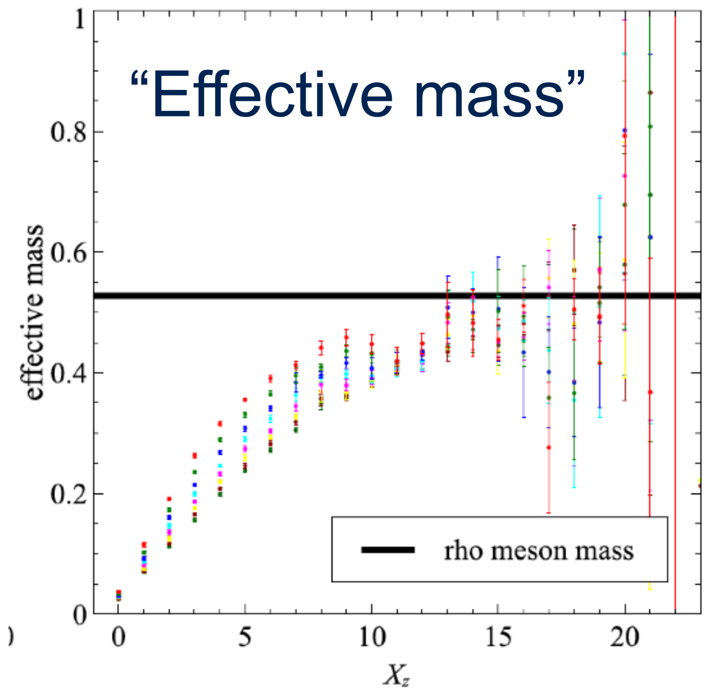
$$
\begin{aligned}
a &\simeq 0.12 \text{ fm} \\
m_\pi &\simeq 400 \text{ MeV} \\
\text{Lattice Size} &: 24^3 \times 64
\end{aligned}
$$

- To gain control over finite-volume effects, replicate in z direction: $24 \times 24 \times 48 \times 64$

Three-point correlator



$\ln\left[C_{3\mathrm{pt}}(t', x'_z)\right]$



- Spatial moments push the peak of the correlator away from origin
- Larger finite volume corrections compared to regular correlators

Fitting the data...

$$C^{3\mathrm{pt}}(t,t') = \sum_{n,m} \frac{Z_n^{\dagger a}(0)\Gamma_{nm}(k^2)Z_m^b(k^2)}{4M_n(0)E_m(k^2)} e^{-M_n(0)(t-t')} e^{-E_m(k^2)t'}$$

$$C_{2\mathrm{pt}}(t) = \sum_m \frac{Z_m^{b\dagger}(k^2)Z_m^b(k^2)}{2E_m(k^2)} e^{-E_m(k^2)t}$$

where

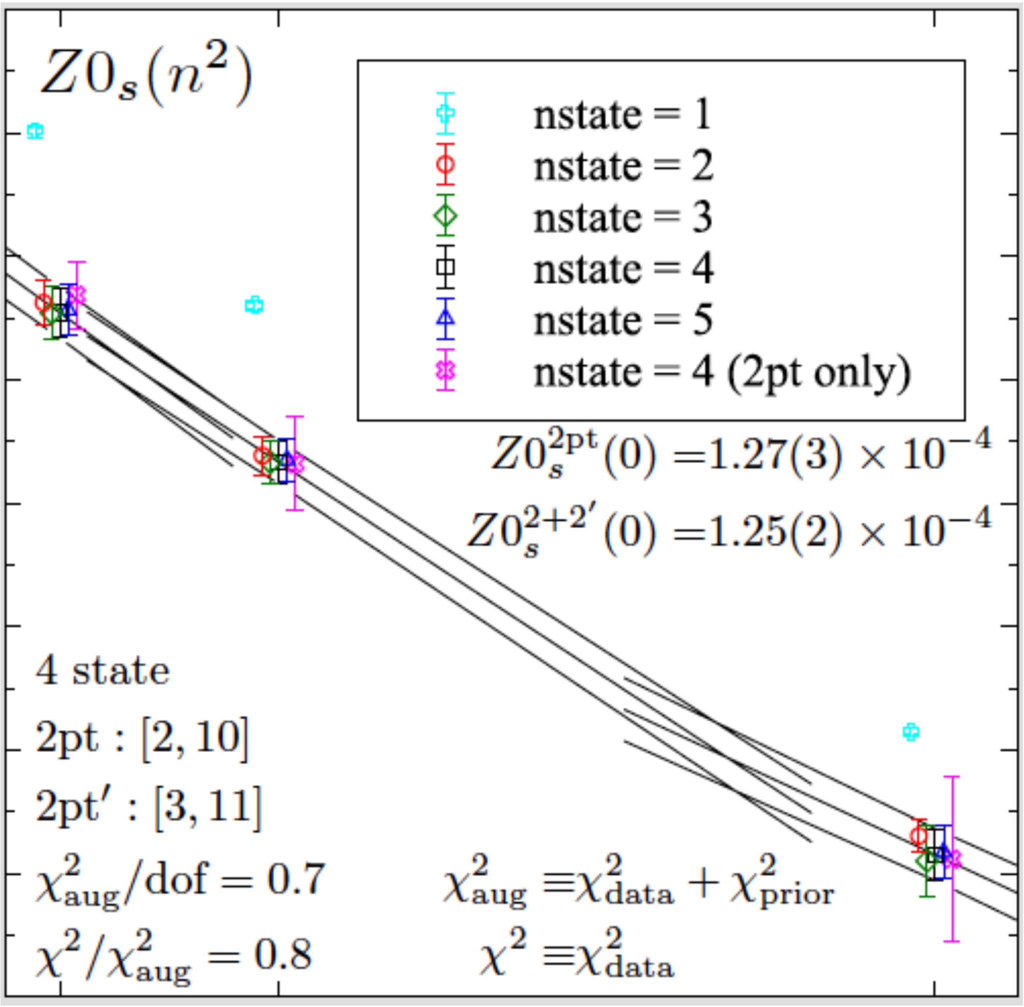
$$Z_n^{\dagger a}(0) \equiv \langle \Omega | N^a | n, p_i = (0,0,0)\rangle$$

$$Z_m^b(k^2) \equiv \langle m, p_i = (0,0,k) | \overline{N}^b | \Omega \rangle$$

$$\Gamma_{nm}(k^2) \equiv \langle n, p_i = (0,0,0) | \Gamma | m, p_i = (0,0,k)\rangle$$

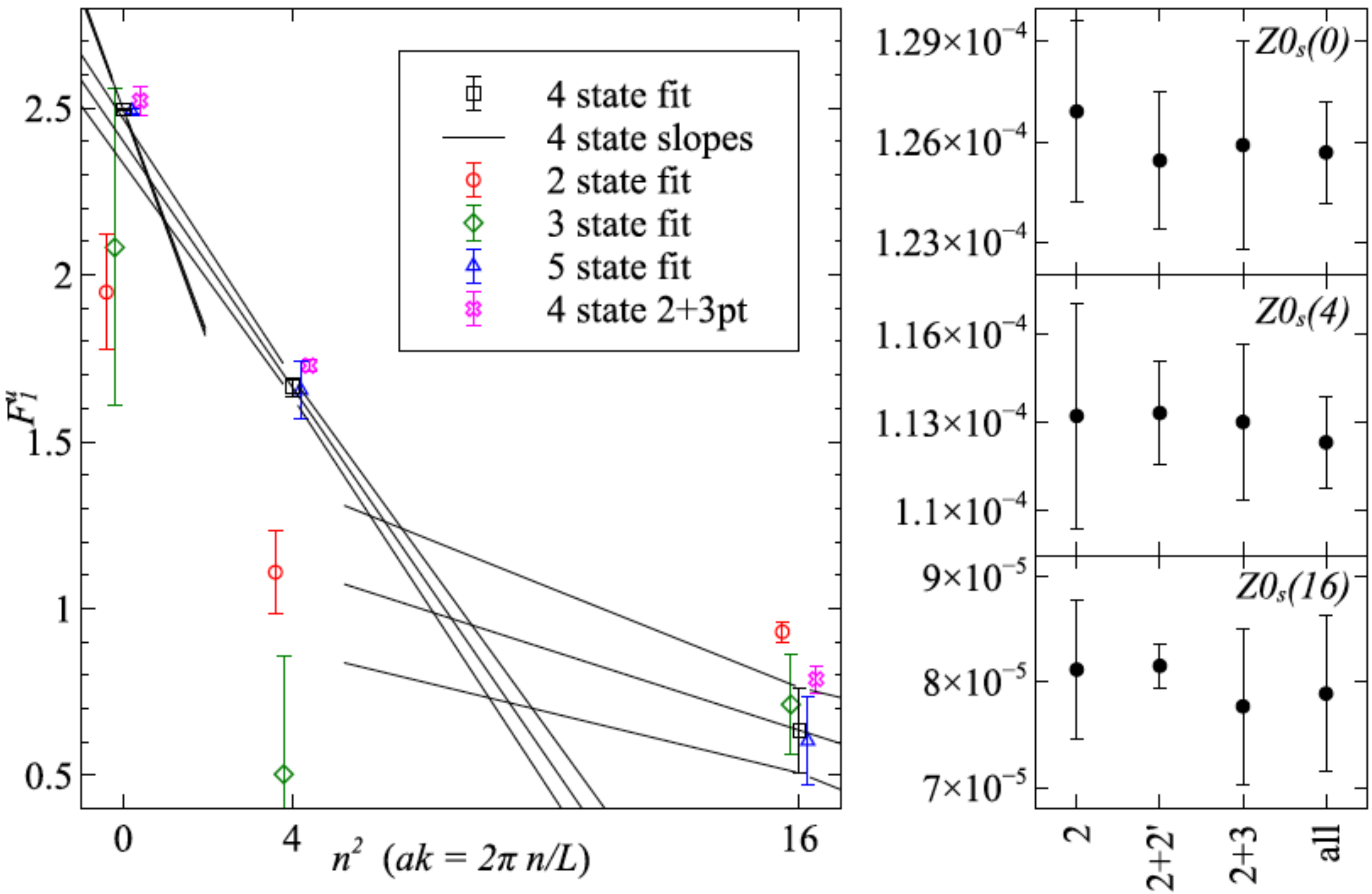
Allow for multi-state contributions in the fit

Fitting - III



In practice we use multi-exponential, Bayesian fits

F_1 Form Factor



Analysis on larger volumes in preparation

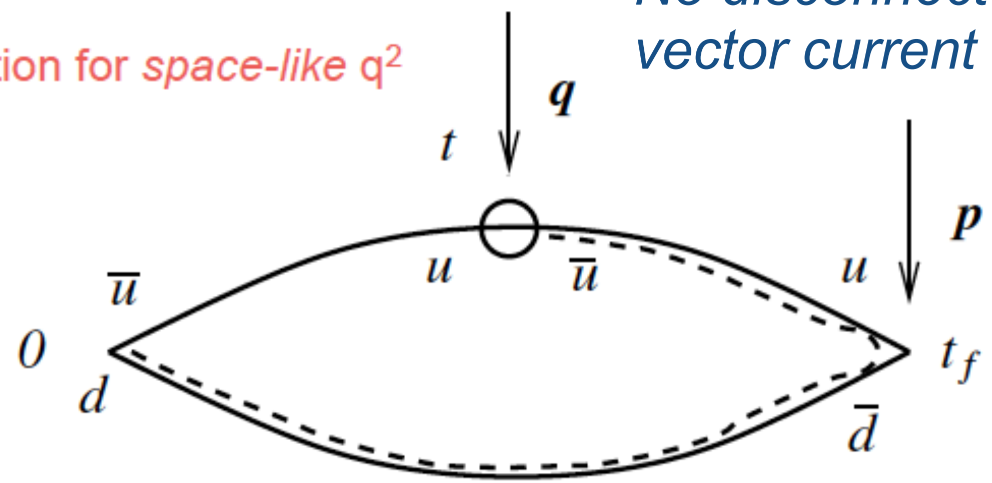
Pion Structure

Raul Briceno, Bipasha Chakraborty, Adithia Kusno, DGR

Pion Structure

No disconnected contributions for vector current

Lattice calculation for *space-like* q^2



$$\Gamma_{\pi^+\mu\pi^+}(t_f,t;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x},t_f)V_\mu(\vec{y},t)\phi^\dagger(\vec{0},0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

Resolution of unity – insert states

$$\to \langle 0\,|\phi(0)|\,\pi(\vec{p}_f)\rangle \frac{e^{-(t_f-t)E_\pi(\vec{p}_f)}}{2E_\pi(\vec{p}_f)} \langle \pi(\vec{p}_f)\,|V_\mu(0)|\,\pi(\vec{p}_i)\rangle \frac{e^{-(t-t_i)E_\pi(\vec{p}_i)}}{2E_\pi(\vec{p}_i)} \langle \pi(\vec{p}_i)\,|\phi\dagger(0)|\,0\rangle\,.$$

$$\langle \pi(\vec{p}_f)\,|V_\mu(0)|\,\pi(\vec{p}_i)\rangle_{\text{continuum}} = Z_V \,\langle \pi(\vec{p}_f)\,\big|V_\mu^{\text{lat}}(0)\big|\,\pi(\vec{p}_i)\rangle = F(Q^2)(p_i+p_f)_\mu$$

Charge conservation

Variational Method Revisited

$$C_{2\text{pt}}(t,\vec{p}) = \sum_{\vec{x}} \langle \phi(\vec{x},t)\phi^\dagger(0)\rangle e^{-i\vec{p}\cdot\vec{x}}$$

Signal-to-noise ratio: $C_{2\text{pt}}(t,\vec{p})/C_{\sigma^2}(t) \longrightarrow e^{-((E(\vec{p})-m_\pi)t)}$

Construct matrix of correlators with *judicious choice of operators*

$$C_{ij}(t) = \frac{1}{V_3}\sum_{\vec{x},\vec{y}} \langle \phi_i(\vec{x},t)\phi_j^\dagger(\vec{y},0)\rangle = \sum_N A_N^i A_N^{j\dagger} e^{-E_N t}$$

Delineate contributions using *variational method*: solve

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$

$$\lambda_N(t,t_0) \to e^{-E_N(t-t_0)}(1+\mathcal{O}(e^{-\Delta E(t-t_0)}))$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

$$v^{(N')\dagger}C(t_0)v^{(N)} = \delta_{N,N'}$$

$$Z_i^N = \sqrt{2m_N}e^{m_N t_0/2}v_j^{(N)*}C_{ji}(t_0).$$

Variational Method: Meson Operators

Aim: interpolating operators of *definite* (continuum) JM: O^{JM}

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point $\bar{\psi}(\vec{x},t)\Gamma D_i D_j \ldots \psi(\vec{x},t)$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x - i\overleftrightarrow{D}_y\right)$$

$$\overleftrightarrow{D}_{m=0} = i\overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x + i\overleftrightarrow{D}_y\right).$$

Straighforward to project to definite spin - *for example J = 0, 1, 2*

$$(\Gamma \times D^{[1]}_{J=1})^{J,M} = \sum_{m_1,m_2} \langle 1, m_1; 1, m_2 \big| J, M \rangle \, \bar{\psi}\Gamma_{m_1} \overleftrightarrow{D}_{m_2} \psi.$$

Caveat: rotational symmetry not a good symmetry of the lattice *but realized in practice for operators of "hadronic size"*

Meson Operators - II

Thomas et al., Phys. Rev. D 85, 014507 (2012)

FINAL STEP: Project to lattice irrep

Lattice Momentum	Little Group (double cover)	Irreps (Λ or Λ^P) (for single cover)
$(0,0,0)$	O_h^D	$A_1^\pm, A_2^\pm, E^\pm, T_1^\pm, T_2^\pm$
$(n,0,0)$	Dic_4	A_1, A_2, B_1, B_2, E_2
$(n,n,0)$	Dic_2	A_1, A_2, B_1, B_2
(n,n,n)	Dic_3	A_1, A_2, E_2
$(n,m,0)$	C_4	A_1, A_2
(n,n,m)	C_4	A_1, A_2
(n,m,p)	C_2	A

…Minimize number of excited energies in lattice irrep

Distillation

M. Peardon *et al.*, PRD80,054506 (2009)

Can we evaluate such a matrix efficiently, for reasonable basis of operators?

Introduce $\tilde{\psi}(\vec{x},t) = L(\vec{x},\vec{y})\psi(\vec{y},t)$ **where L is 3D Laplacian**

Write $L \equiv (1-\kappa\nabla/n)^n = \sum_i f(\lambda_i)\xi^i \times \xi^{*i}$ **where λ_i and ξ_i are eigenvalues and eigenvectors of the Laplacian.**

We now truncate the expansion sufficiently to capture low-energy physics

Insert between each quark field in our correlation function.

Perambulators $\tau^{ij}_{\alpha\beta}(t,0) = \xi^{*i}(t)M^{-1}(t,0)_{\alpha\beta}\xi^j$

$$C_{ij}(t) = \phi_i^{pq}(t)\phi_j^{rs} \times \tau^{pr}(t,0)\tau^{qs\dagger}(t,0)$$

- 3pt functions implemented by replacing one of the perambulators by a so-called generalized perambulator with current inserted.

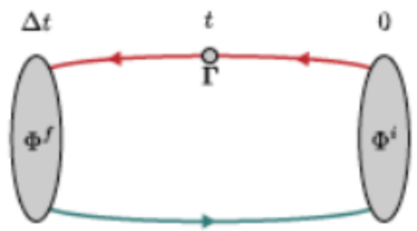
$$S^{ij}(t_f,t,t_i) = \xi^{(i)\dagger}(t_f)M^{-1}(t_f,t)\Gamma(t)M^{-1}(t,t_i)\xi^{(j)}(t_i)$$

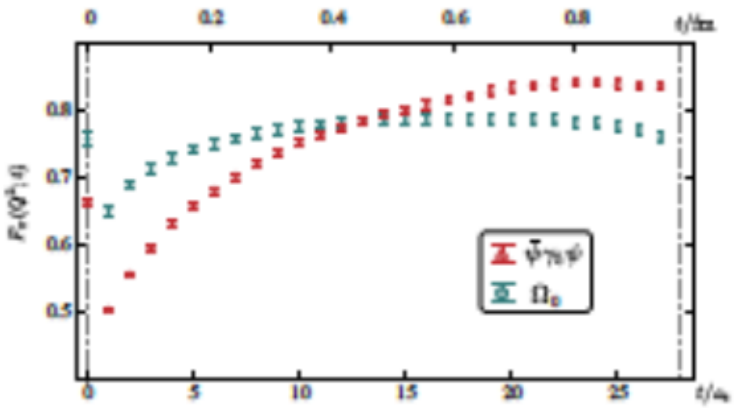
Distillation for Pion FF

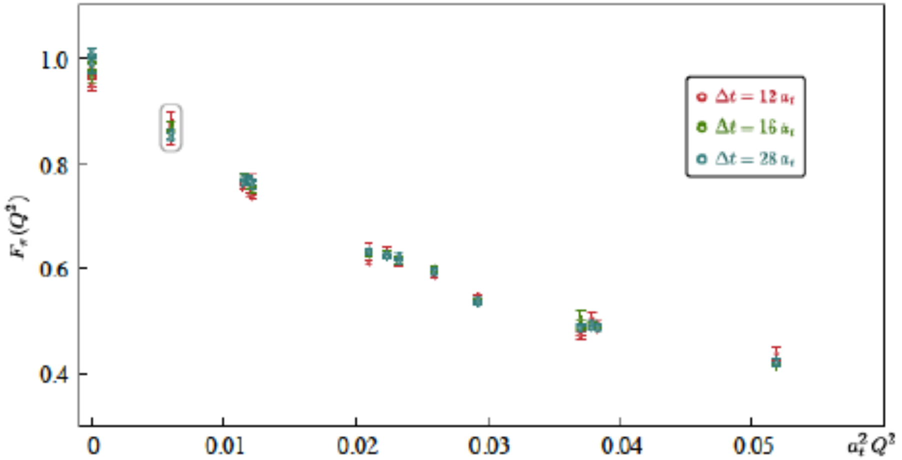
- Formalism for EM matrix elements already demonstrated for mesons by HadSpec collaboration.

$$F(Q^2;t) = F(Q^2) + f_f e^{-\delta E_f(\Delta t - t)} + f_i e^{-\delta E_i t}$$

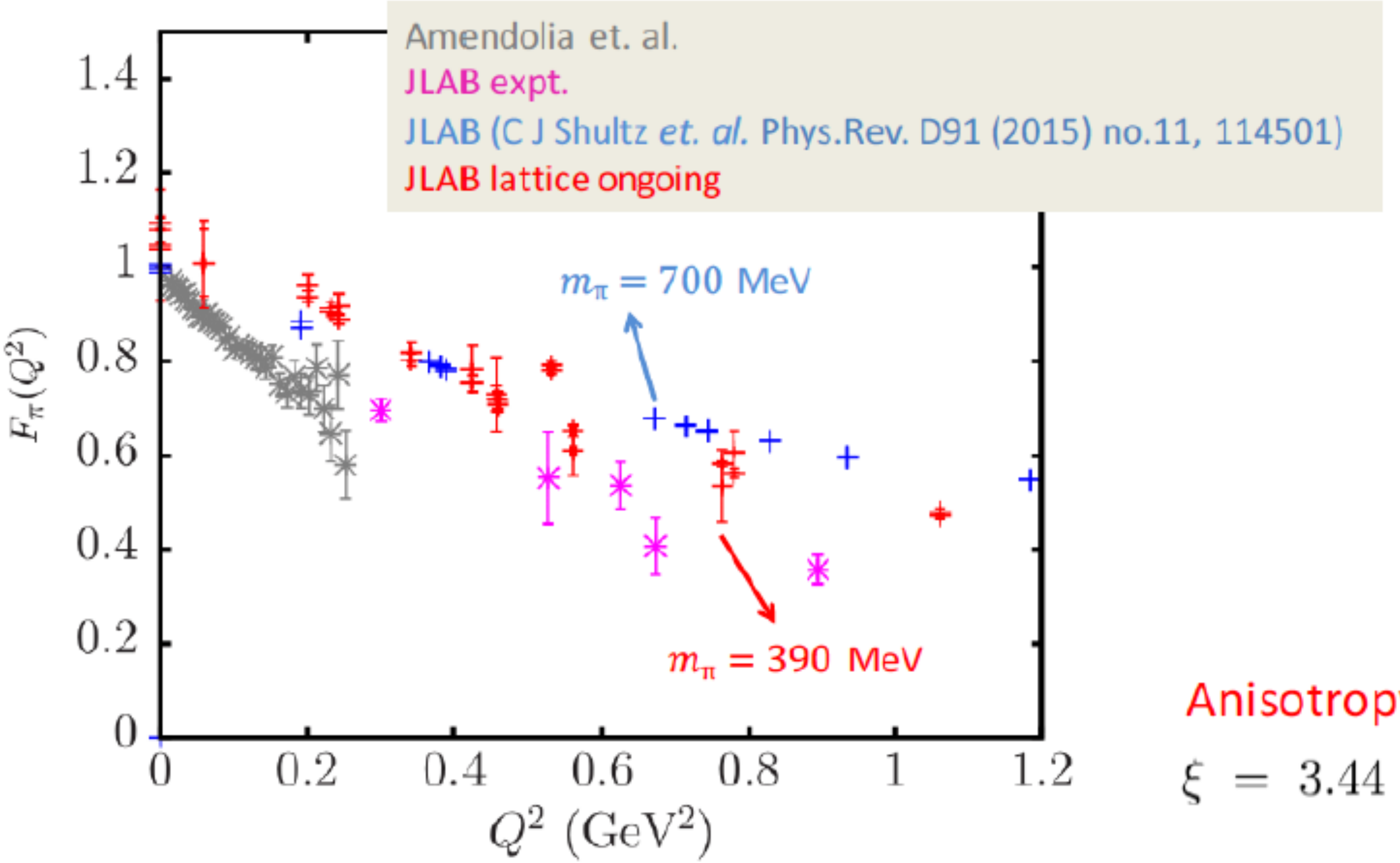
Shultz, Dudek and Edwards, arXiv:1501.07457







Distillation for Pion FF - II



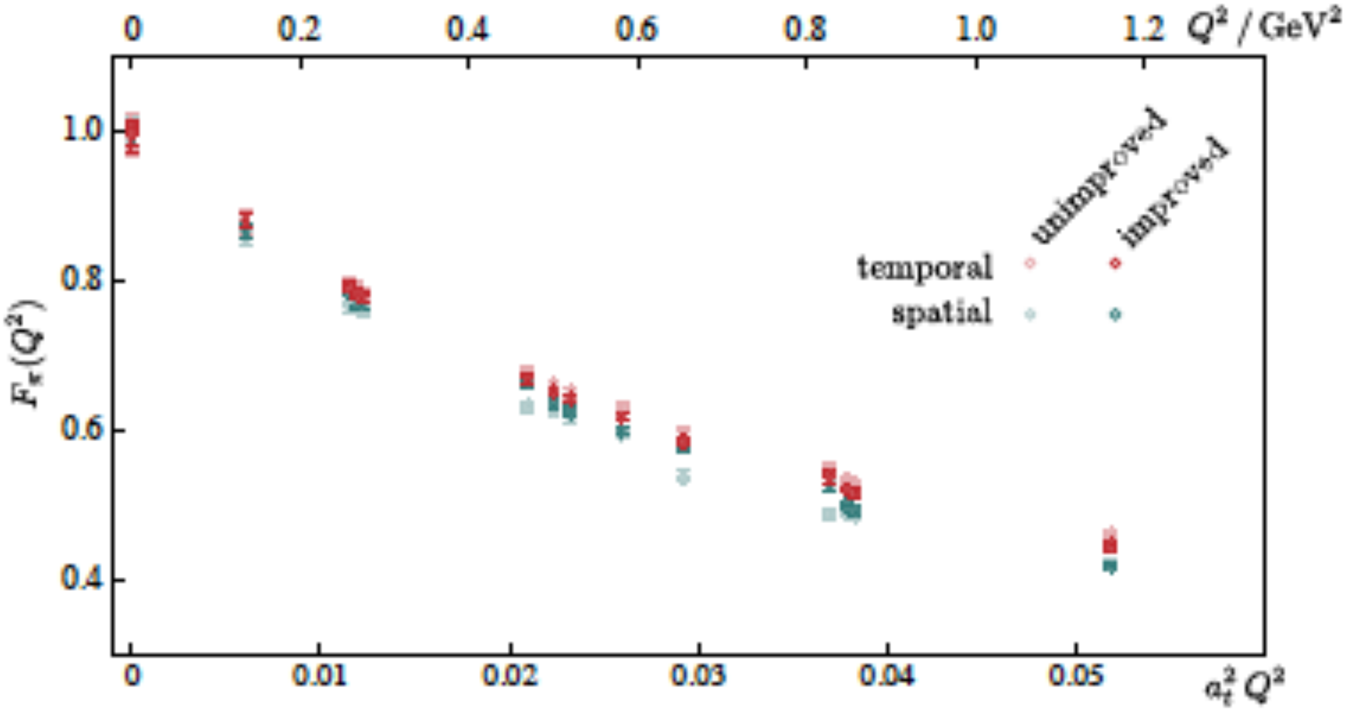
$$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$$

Anisotropy

$$\xi = 3.44$$

$$a_s = 0.12 \text{ fm}, \quad \frac{a_s}{a_t} = 3.44$$

Anisotropic Lattices

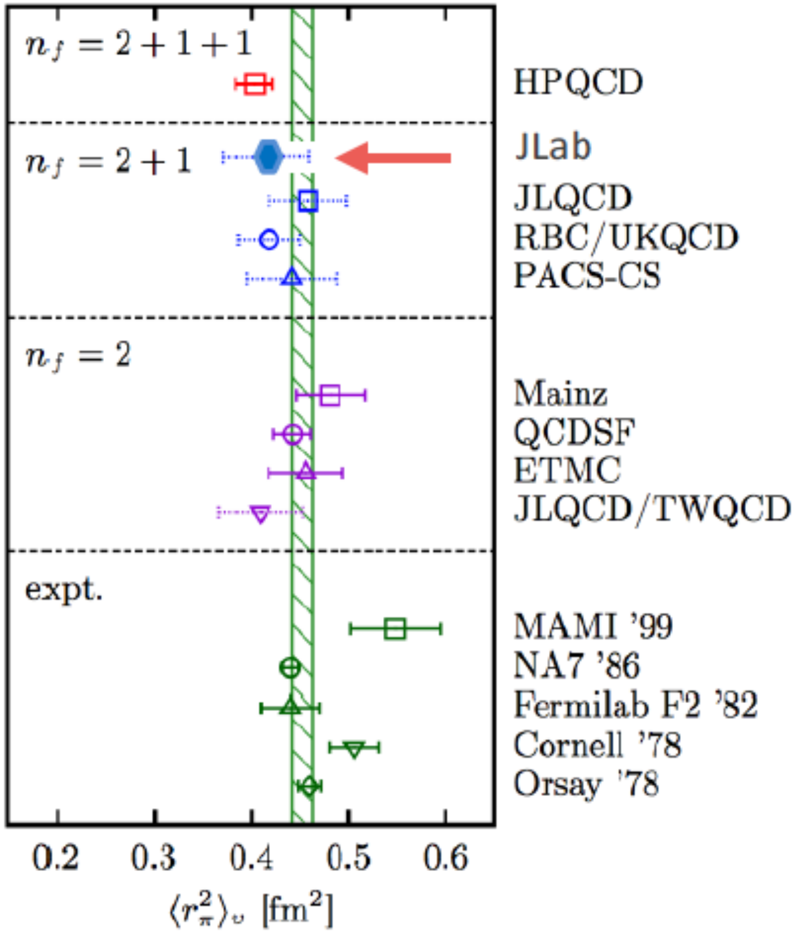


Subtlety:

$$j_4 = \bar{\psi}\gamma_4\psi + \frac{\nu_s}{4\xi}(1-\xi)a_s\partial_j(\bar{\psi}\sigma_{4j}\psi)$$

$$j_k = \bar{\psi}\gamma_4\psi + \frac{1}{4}(1-\xi)a_t\partial_4(\bar{\psi}\sigma_{4k}\psi)$$

Charge Radius

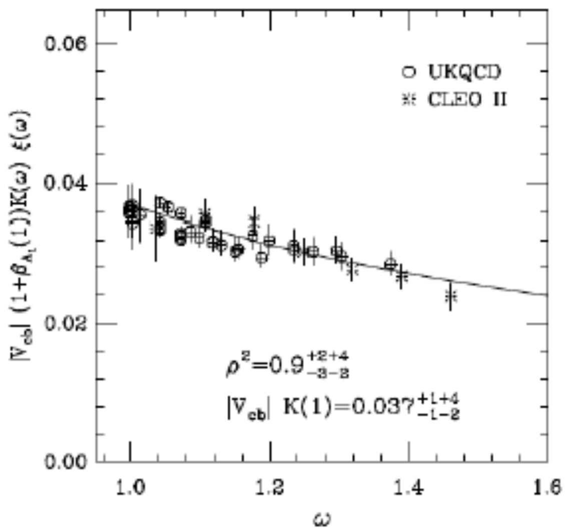


HPQCD, PRD93, 054503 (2017)

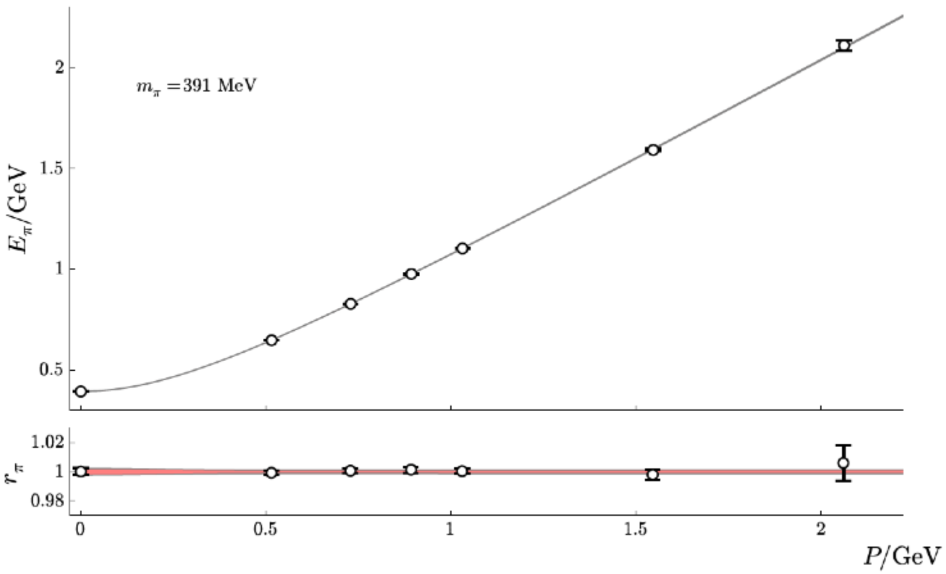
Slope of form factor at $Q^2 = 0$ gives charge radius

Calculate slope at zero recoil..

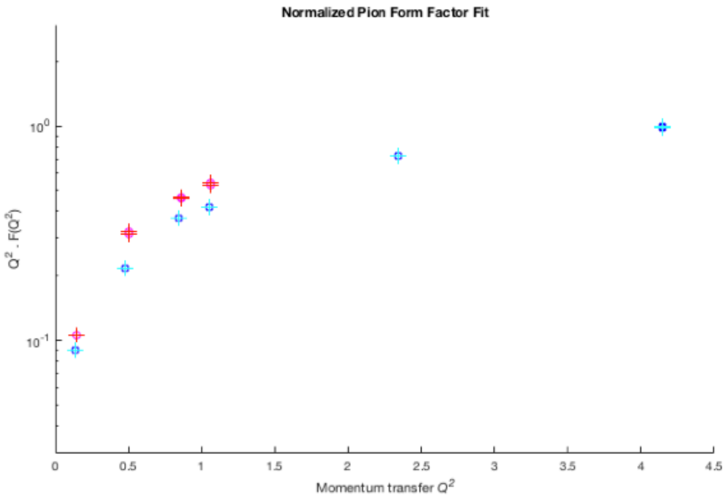
UKQCD, L. Lellouch et al., Nucl. Phys. B444, 401 (1995), hep-lat/9410013



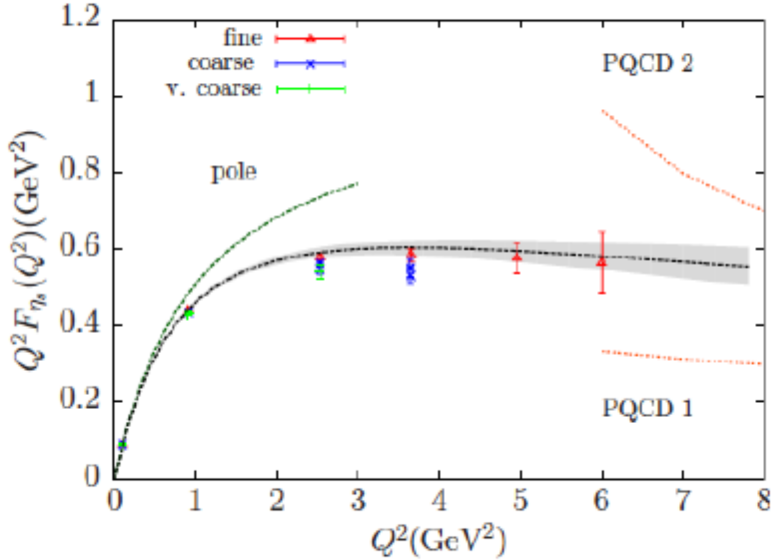
Form Factor at High Momentum



Reach to 4 GeV^2



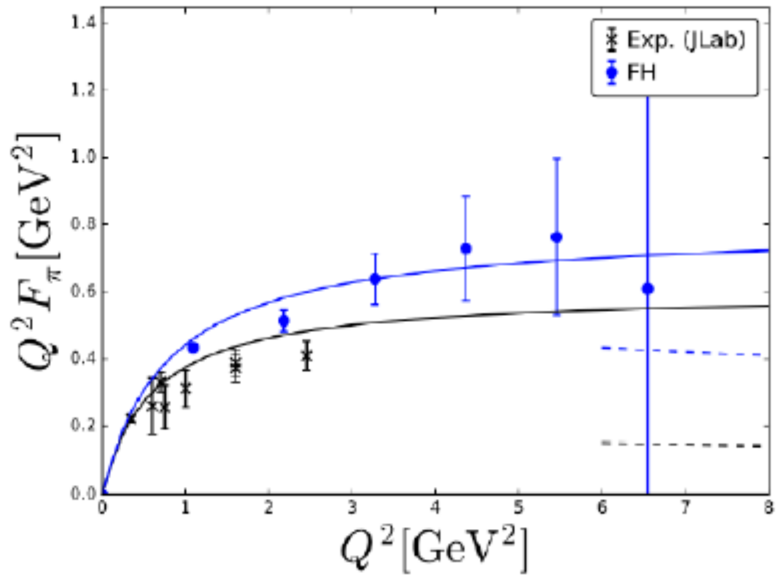
High Momentum - II



Momentum-smearing, at strange-quark masses

Koponen et al (HPQCD), PRD96, 054501 (2017)

Chambers et al (QCDSF/UKQCD/CSSM), arXiv: 1702.01513



Calculation using Feynman-Hellman Theory

$$H = H_0 + \lambda H_\lambda$$

$$\frac{\partial E_n}{\partial \lambda} = \langle n \mid H_\lambda \mid n \rangle$$

Quark Distribution Amplitude

Leading-twist EM form factor at high momentum governed by Quark Distribution Amplitude

$$\phi_\pi(x) = \frac{i}{f_\pi}\int \frac{d\xi}{2\pi} e^{i(x-1)\xi\lambda\cdot P}\langle\pi(P)\mid\bar{\psi}(0)\lambda\cdot\gamma\gamma_5\Gamma(0,\xi\lambda\psi(\xi\lambda)\mid 0>$$

- *Generalized Parton Distributions (off-forward): GPDs*
- *PDFs*
- *(Transverse-Momentum-Dependent Distributions): TMDs*

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use *Operator-Product-Expansion* to formulate in terms of *local operators*

$$\mathcal{O}^{\mu_1\ldots\mu_n} = i^{n-1}\bar{\psi}\gamma_5\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_n\}}\frac{\lambda^a}{2}\psi$$

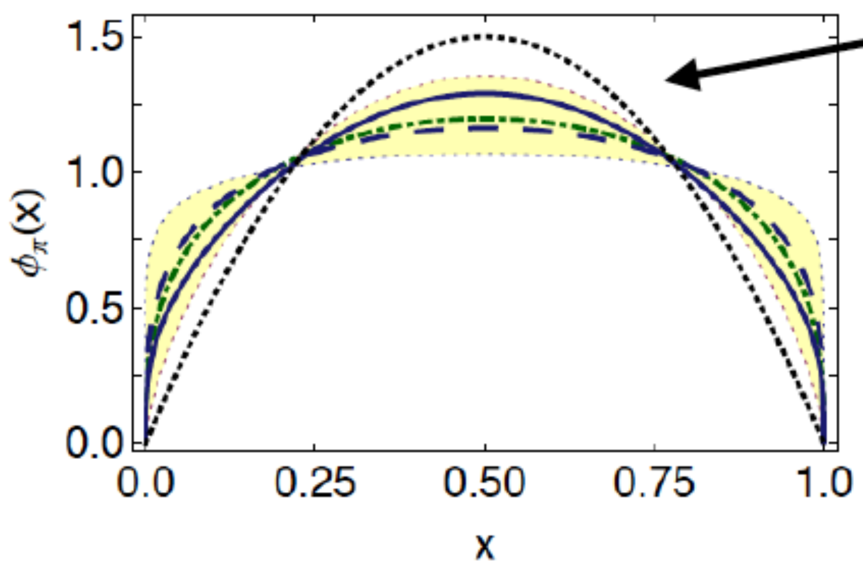
- Discretisation, and hence reduced symmetry of the lattice, introduces power-divergent mixing for N >3 moment.

QDA - II

- Lowest moment: f_π
- 1st moment: vanishes
- 2nd Moment: only constraint…

Martinelli, Sachrajda (87), Daniel, Gupta, Richards (91),…,Braun et al (2015)

Plot from Cloet et al, PRL111, 92001



Asymptotic form

Important for, e.g. DVMP

Direct calculation: A Shaeffer

Quasi Distributions

- A solution, LaMET (Large Momentum Effective Theory) was proposed by X.Ji

X. Ji, Phys. Rev. Lett. 110 (2013) 262002

$$\tilde{\phi}(x,P) = \frac{i}{f_\pi}\int \frac{dz}{2\pi} e^{-i(x-1)Pz}\langle \pi(P) \mid \bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z) \mid 0\rangle$$

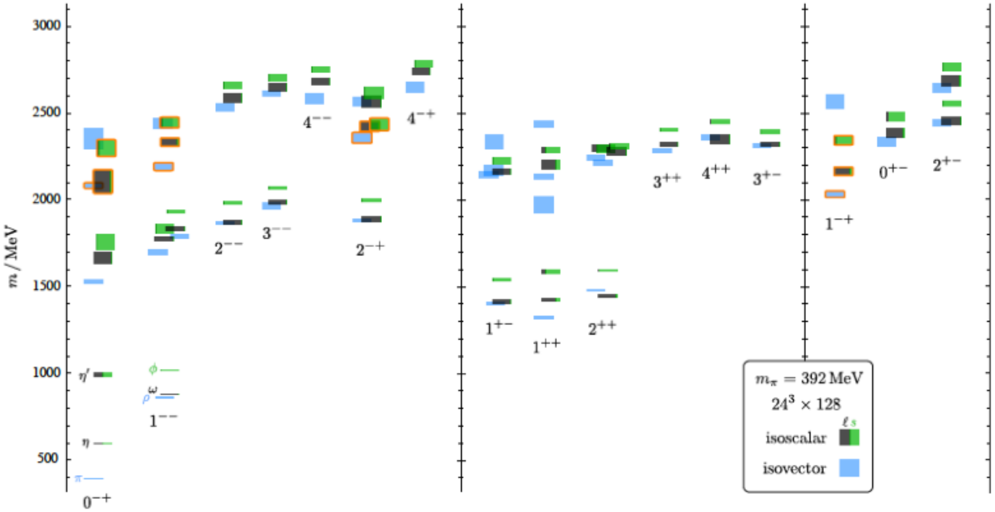
$$\tilde{\phi}(x,\Lambda,P) = \int_0^1 dy\, Z(x,y,\Lambda,\mu,P)\phi(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P^2}, \frac{m_\pi^2}{P^2}\right)$$

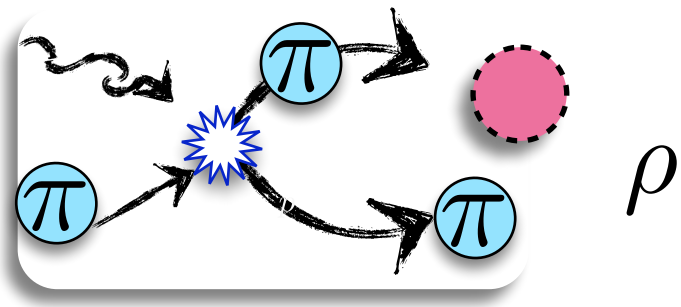
Y-Q Ma and J-W Qiu, arXiv:1404.6860, arXiv:1709.03018

- Matching and evolution of quasi- and light-cone distributions

Carlson, Freid, arXiv:1702.05775
Isikawa et al., arXiv:1609.02018
Monahan and Orginos, arXiv:1612.01584
Orginos, Radyushkin , et al arXiv:1706.05373 (**Pseudo Distributions**)
Briceno, Hansen, Monahan, arXiv:1703.06072 (Euclidean Signature)

Energy-Momentum Tensor?





$$\text{Trace Anomaly: } T_{\mu\mu} = -(1+\gamma_m)\bar{\psi}\psi + \frac{\beta(g)}{2g}G^2$$

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}D_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2; \langle P \mid T_{\mu\nu} \mid P\rangle = P_\mu P_\nu / M$$

Briceno, Hansen and Walker-Loud, PRD 91, 034501 (2015)

Yang, this meeting

Baryon Operators

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point $\quad B = \left(\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}\right)\{\psi_1\psi_2\psi_3\}$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x - i\overleftrightarrow{D}_y\right)$$

$$\overleftrightarrow{D}_{m=0} = i\overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x + i\overleftrightarrow{D}_y\right).$$

Straighforward to project to definite spin: *J = 1/2, 3/2, 5/2*

$$\big|\,[J,M]\big\rangle = \sum_{m_1,m_2} \big|\,[J_1,m_1]\big\rangle \otimes \big|\,[J_2,m_2]\big\rangle \langle J_1 m_1; J_2 m_2 | JM \rangle$$

$D^{[2]}_{J=1}$ is the *signature* of hybrid baryon

Distillation for Baryons?

Measure matrix of correlation functions:

$$C_{ij}(t) \equiv \sum_{\vec{x},\vec{y}} \langle N_i(\vec{x},t)\bar{N}_j(\vec{y},0)\rangle$$

M. Peardon *et al.*, PRD80,054506 (2009)

Perambulators

$$\tau^{ij}_{\alpha\beta}(t,0) = \xi^{*i}(t) M^{-1}(t,0)_{\alpha\beta}\xi^{j}$$

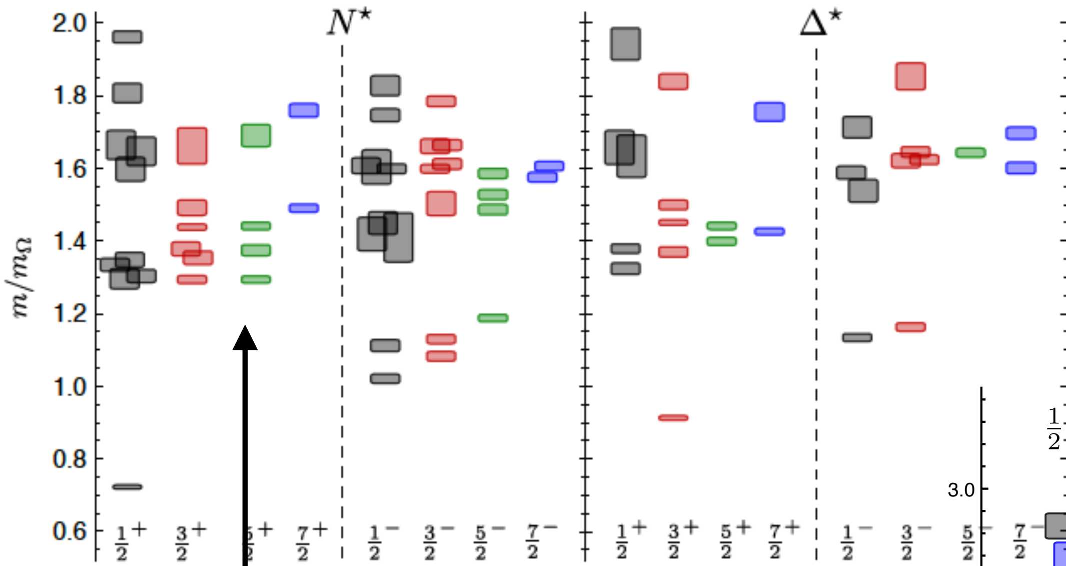
$$C_{ij}(t) = \phi^{i,(pqr)}_{\alpha\beta\gamma)}(t)\phi^{j,(\bar{p}\bar{q}\bar{r})}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}(0) \times \left[\tau^{p\bar{p}}_{\alpha\bar{\alpha}}(t,0)\tau^{q\bar{q}}_{\beta\bar{\beta}}(t,0)\tau^{r\bar{r}}_{\gamma\bar{\gamma}}(t,0) + \ldots\right]$$

- Meson correlation functions N^3
- Baryon correlation functions N^4

Severely constrains baryon lattice sizes

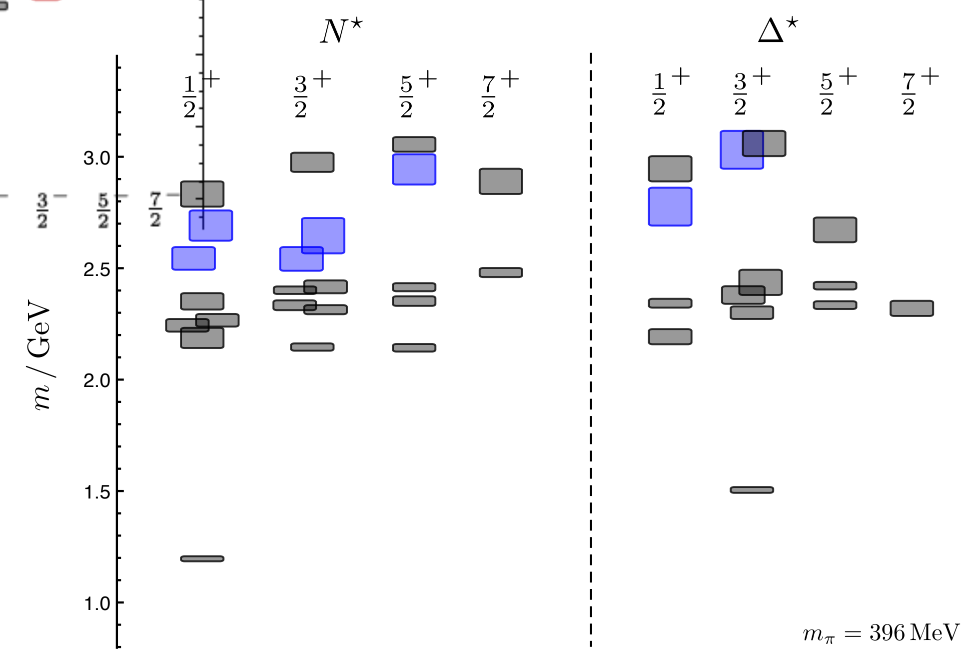
Excited Baryon Spectrum

R.G.Edwards et al., arXiv:1104.5152



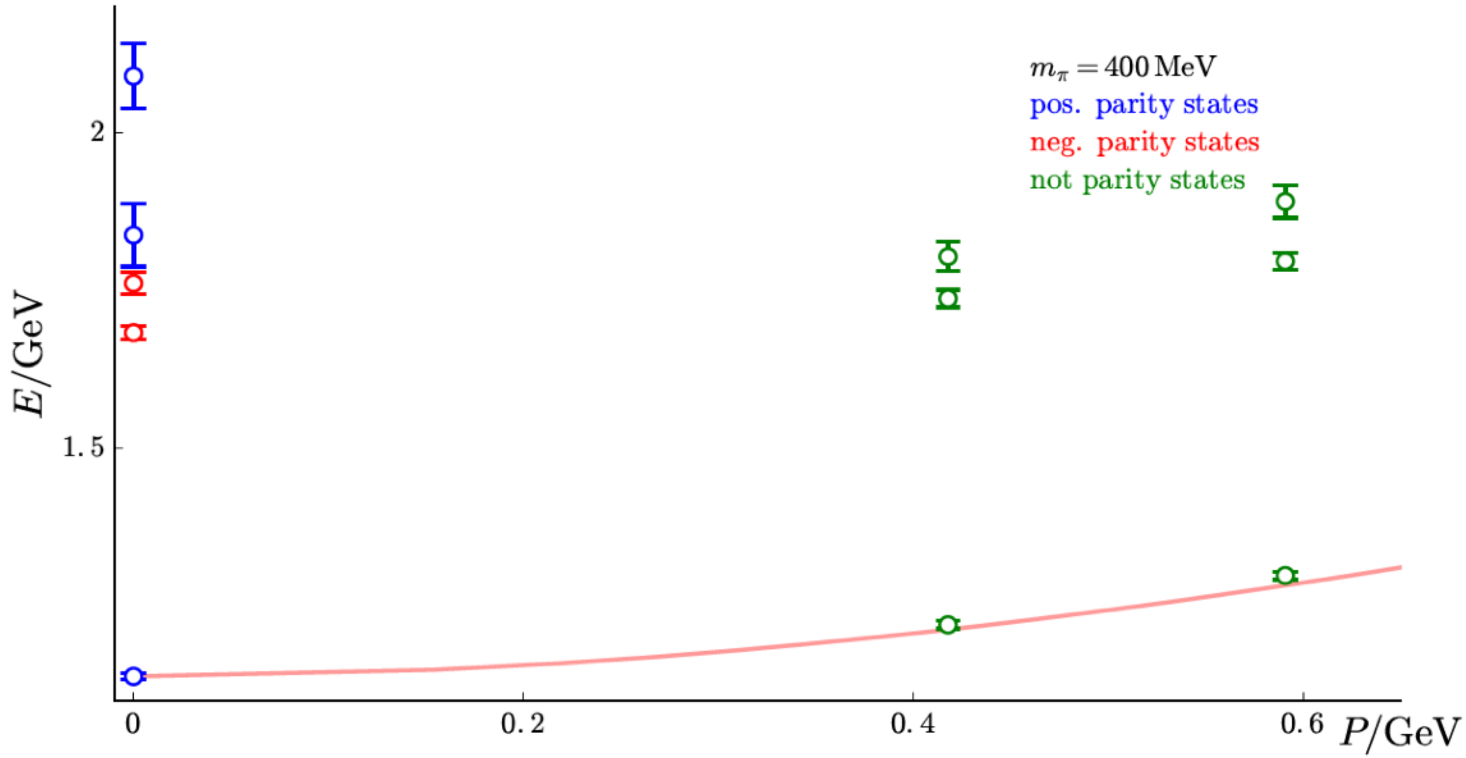
Quark-model Counting

Include hybrid operators of form $D^{[2]}_{l=1,M}$



Dudek, Edwards, arXiv:1201.2349

Nucleon Dispersion Relation



Isotropic Clover Production

ID	m_l	β	a (fm)	M_π (MeV)	L	T	$M_\pi L$	Split at	N_{traj}	On Titan
C12	-0.2800	6.1	0.118	430	48	96	12.4		20000	
C13	-0.2850	6.1	0.114	300	32	96	5.6			1762 - 2104
C13a	-0.2850	6.1	0.114	300	32	96	5.6			1100 - 1870
C13b	-0.2850	6.1	0.114	300	32	96	5.6			1000 - 2618
C13-W	-0.2850	6.1	0.114	300	32	96	5.6			2108 - 3164
C13a-W	-0.2850	6.1	0.114	300	32	96	5.6			1872 - 3564
C13b-W	-0.2850	6.1	0.114	300	32	96	5.6			2620 - 3980
D4	-0.2350	6.3	0.085	400	32	64	5.5		5164	
D5	-0.2390	6.3	0.081	310	32	64	4.0		6020	1000 - 6020
D6	-0.2416	6.3	0.080	210	48	96	3.7		2312 (a)	1000 - 2312
D6a	-0.2416	6.3	0.080	210	48	96	3.7	1000	866 (a)	254 - 866
D6b	-0.2416	6.3	0.080	210	48	96	3.7	1200	956 (a)	284 - 956
D7	-0.2416	6.3	0.080	210	64	128	4.9		1514 (a)	1112 - 1514
D7b	-0.2416	6.3	0.080	210	64	128	4.9	700	640 (a)	330 - 640
D7c	-0.2416	6.3	0.080	210	64	128	4.9	750	732 (a)	288 - 592
D7d	-0.2416	6.3	0.080	210	64	128	4.9	800	762 (a)	328 - 736
D8	-0.2424	6.3	0.080	140	72	196	4.1		370 (b)	

Add third lattice spacing: β = 6.5, a ~0.06

SUMMARY

- Controlling systematic uncertainties key at both low momenta and high momenta
- Pion is an important theatre to test our ideas, and in particular key measure of transition from "soft" to "hard" degrees of freedom in QCD
- Simplest, and computationally least demanding, hadron for lattice structure calculations
- Self-contained confrontation for direct calculation of wave function, and twist expansion in lattice QCD?
- Operators and methods for quark distribution amplitudes provide exploratory theatre for studies of PDFs and GPDs in the nucleon.
- Can we get to high momenta?

Proton EM form factors

- Nucleon Pauli and Dirac Form Factors described in terms of matrix element of vector current

$$\langle N \mid V_\mu \mid N\rangle(\vec{q}) = \bar{u}(\vec{p}_f)\left[F_q(q^2)\gamma_\mu + \sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2m_N}\right]u(\vec{p}_i)$$

- Alternatively, Sach's form factors determined in experiment

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

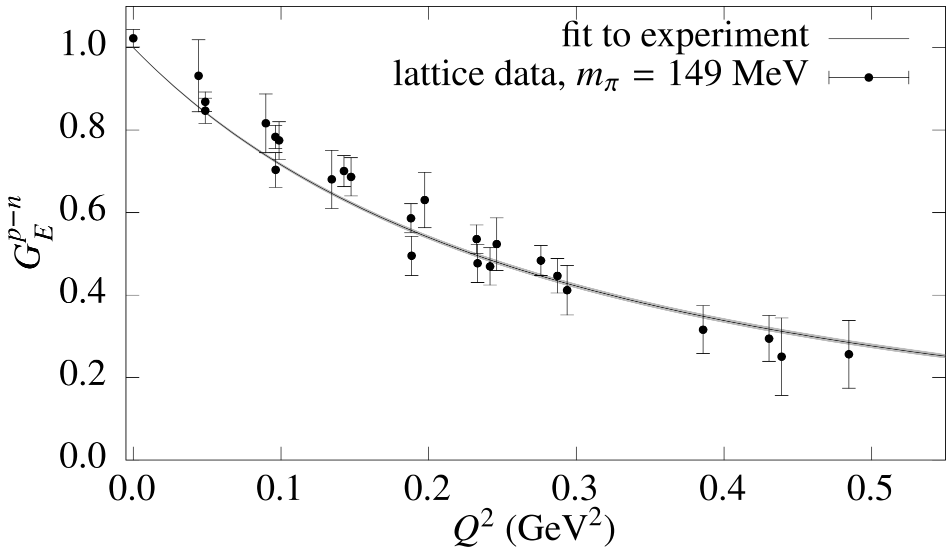
Charge radius is slope at $Q^2 = 0$

$$\left.\frac{\partial G_E(Q^2)}{\partial Q^2}\right|_{Q^2=0} = -\frac{1}{6}\langle r^2\rangle = \left.\frac{\partial F_1(Q^2)}{\partial Q^2}\right|_{Q^2=0} - \frac{F_2(0)}{4M^2}$$

1D Structure: EM Form Factors

Wilson-clover lattices from BMW

Green et al (LHPC), Phys. Rev. D 90, 074507 (2014)



Hadron structure at nearly-physical quark masses

Large Q^2 behavior: Hall C at JLab to 15 GeV^2