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Resummation of azimuthal modulations in SIDIS and Drell-Yan ($\cos\phi$ and $\cos 2\phi$)

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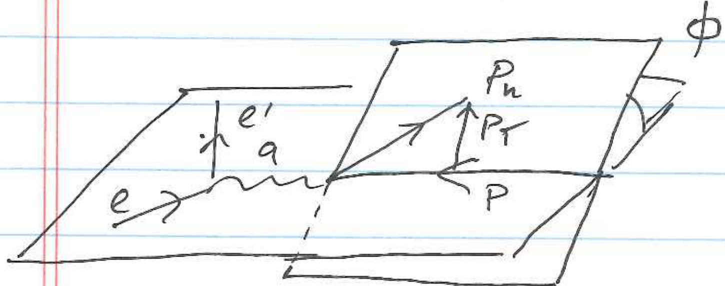
Motivation.

Let us consider SIDIS in $\gamma^* P$ cms frame



$eP \to e'hX$

$$\frac{d^6\sigma}{dx\,dy\,dz\,d\phi_s\,d\phi_h\,dP_T^2} = \frac{\alpha^2}{xyQ^2}\left(1-y+\frac{1}{2}y^2\right) F_{uu} \times$$

$$\times \left\{ 1 + \cos\phi_h \frac{(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2} \underbrace{A_{uu}^{\cos\phi}} + \cos 2\phi \frac{(1-y)}{1-y+\frac{1}{2}y^2} \underbrace{A_{uu}^{\cos 2\phi}} \right\}$$

azimuthal modulations

$$A_{uu}^{\cos\phi} = \frac{F_{uu}^{\cos\phi}}{\underbrace{F_{uu}}_{\text{structure functions}}}, \quad A_{uu}^{\cos 2\phi} = \frac{F_{uu}^{\cos 2\phi}}{F_{uu}}$$

Structure functions can be calculated in large P_T limit by using collinear approximation. $\gamma^* P$ cm frame is used and virtual photon now has transverse momentum

$$q_T \simeq -\frac{P_T}{z}$$

$$F_{UU} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_q x e_q^2 \left(f_1(x) D_1(z) L\left(\frac{Q^2}{q_T^2}\right) + \right.$$

$$\left. + \text{„} f_1 \otimes D_1 \text{ convolutions"} \right)$$

$$F_{UU}^{\cos\phi} = -\frac{1}{Q q_T} \frac{\alpha_s}{2\pi^2 z^2} \sum_q x e_q^2 \left(f_1(x) D_1(z) L\left(\frac{Q^2}{q_T^2}\right) + \right.$$

$$\left. + \text{„} f_1 \otimes D_1 \text{ convolutions"} \right)$$

$$F_{UU}^{\cos 2\phi} = \frac{1}{Q^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_q x e_q^2 \left(f_1(x) D_1(z) L\left(\frac{Q^2}{q_T^2}\right) + \right.$$

$$\left. + \text{„} f_1 \otimes D_1 \text{ convolutions"} \right)$$

where $L\left(Q^2/q_T^2\right) = 2 C_F \ln \frac{Q^2}{q_T^2} - 3 C_F$

$L\left(Q^2/q_T^2\right)$ diverges at $q_T \to 0$ and has to be resummed to all orders in α_s

"$f_1 \otimes D$, convolutions" are different in 3 cases. We will limit our discussion to NLL accuracy and use 0^{th} order of coefficient functions

Using
$$\int d^2b\, e^{-ibq_T} \ln^2 \frac{b^2 Q^2}{b_0^2} = -\frac{8\pi}{q_T^2} \ln \frac{Q^2}{q_T^2}$$
$$\int d^2b\, e^{-ibq_T} \ln \frac{b^2 Q^2}{b_0^2} = -\frac{4\pi}{q_T^2}$$
where $b_0 = 2e^{-\gamma_E}$

we obtain

$$F_{UU} = +\frac{1}{z^2} \sum_q x\, e_q^2 f_1(x) D_1(z) \int \frac{d^2b}{(2\pi)^2} e^{-ibq_T}$$
$$\times \underbrace{e^{-S_p(b)}}_{\text{Sudakov form factor}}$$

where $S_p(b) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n S_p^{(n)}(b)$

$$S_p^{(1)} = \underbrace{C_F \ln^2 \frac{b^2 Q^2}{b_0^2}}_{\text{double log}} - 3C_F \ln \frac{b^2 Q^2}{b_0^2}$$

It is the same as $\quad S_p(b) = -\int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left(A \ln \frac{Q^2}{\mu^2} + B\right)$

$$\begin{matrix} A \\ B \end{matrix} = \sum_{n=1}^{\infty} \begin{matrix} A_k \\ B_k \end{matrix} \left(\frac{\alpha_s}{\pi}\right)^k$$

Let me also rewrite it using

$$\int_0^{2\pi} d\varphi\, e^{-i\vec{q}_T\vec{b}} \equiv \cancel{}\, J_0(q_T b)\,\frac{1}{2\pi}$$

as

$$F_{uu} = \frac{1}{z^2}\sum_a x\, e_a^2 \int \frac{b\,db}{2\pi}\, J_0\!\left(\frac{b P_T}{z}\right) f_1(x) D_1(z)\, e^{-S(b)}$$

Natural question is:

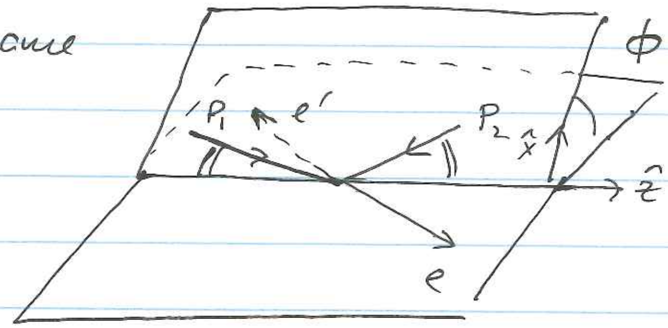
„How to resum $F_{uu}^{\cos\phi}$ and $F_{uu}^{\cos 2\phi}$?"

$F_{uu}^{\cos\phi} = ?$

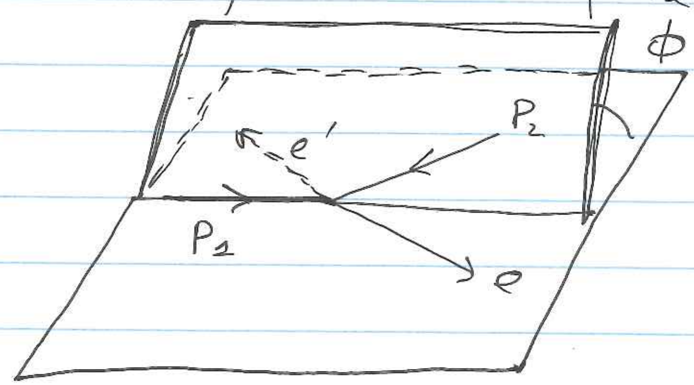
$F_{uu}^{\cos 2\phi} = ?$

Drell-Yan $\quad P_1 P_2 \to e\bar{e}X$

$\cos\phi$ and $\cos 2\phi$ asymmetries are also there
two frames are used, the Collins-Soper frame



The Gottfried-Jackson frame



$$\frac{d\sigma}{d\Omega\, d^4q} = \frac{\alpha^2}{2(2\pi)^4 Q^2 s^2}\Big\{ W_T(1+\cos^2\theta) + W_L(1-\cos^2\theta)$$

$$+ W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta} \sin^2\theta \cos 2\phi \Big\}$$

$$\begin{pmatrix} W_T \\ W_L \\ W_\Delta \\ W_{\Delta\Delta} \end{pmatrix}_{GJ} = \underset{\uparrow}{\begin{pmatrix} \text{matrix} \end{pmatrix}} \begin{pmatrix} W_T \\ W_L \\ W_\Delta \\ W_{\Delta\Delta} \end{pmatrix}_{CS}$$

$$\rho = Q_T/Q$$

Boer, Vogelsang 2006

Small Q_T limit Collins–Soper

$$W_{T,CS} = \frac{\alpha_s}{2\pi}\frac{Q^2}{q_T^2} L\left(\frac{Q^2}{q_T^2}\right) q(x_1)\bar{q}(x_2) +$$

+ „convolutions"

$$W_{L,CS} = 2W_{\Delta\Delta,CS} = \frac{\alpha_s}{2\pi} L(Q^2/q_T^2)\, q_1(x_1)\bar{q}_1(x_2) +$$

+ „convolutions"

$$W_{\Delta,CS} = \frac{\alpha_s}{2\pi}\frac{Q}{q_T}\left(\text{no terms with } L + \text{„convolutions"}\right)$$

„No logarithmic terms in W_Δ!"

Small Q_T limit Gottfried–Jackson

$$W_{T,GJ} = W_{T,CS}$$

$$W_{L,GJ} = 2W_{\Delta\Delta,GJ} = 2\frac{\alpha_s}{2\pi}\left(L(Q^2/q_T^2)\, q_1(x_1)\bar{q}_1(x_2) + \text{„ "}\right)$$

$$W_{\Delta,GJ} = \frac{\alpha_s}{2\pi}\frac{Q}{q_T}\left(L(Q^2/q_T^2)\, q(x_1)\bar{q}(x_2) + \text{„ "}\right)$$

„Logarithmic term is here in this frame!"

<u>Motivation</u>:

<u>Boer, Vogelsang</u>: „Resummation is different from ordinary CS one. Not only splitting functions differ but also W_Δ even does not have Sudakov form factor"

„We do not address full NLL resummation in this work, but note that techniques that go beyond collinear factorization should prove useful."

Berger, Qiu, Rodriguez-Pedraza:

„Is it possible that a different kind of resummation would handle the nonphysical divergence at $q_T = 0$ in W_Δ?
We do not have an answer to this question in the collinear QCD factorization formalism.
However, we might gain insight by investigating the angular distribution from another prospective – starting from transverse momentum dependent quark-antiquark annihilation"

B.V.: „We finally emphasize again that we hope that our study will provide motivation for a developement of the full NLL resummation for the structure functions $W_{L,\Delta,\Delta\Delta}$."

TMD formalism, SIDIS

Structure functions are convolutions of TMDs that obey CS evolution equation.

Convolution:

$$C \equiv \mathcal{C}(\omega f D) = x \sum_q e_q^2 \int d^2k_\perp \, d^2p_\perp \, \delta^{(2)}(z\bar{k}_\perp + \bar{p}_\perp - \bar{P}_T) \, \omega(k_\perp, -p_\perp/z) \, f(x, k_\perp^2) \, D(z, p_\perp^2)$$

For instance $F_{UU} = \mathcal{C}(f_1 D_1) = x \sum_q e_q^2 \int d^2k_\perp \, d^2p_\perp$

$$\delta^{(2)}(z\bar{k}_\perp + \bar{p}_\perp - \bar{P}_T) f_1(x, k_\perp^2) \, D_1(z, p_\perp^2) =$$

$$= x \sum_q e_q^2 \int d^2k_\perp \frac{d^2p_\perp}{z^2} \frac{d^2b}{(2\pi)^2} e^{ib(\bar{k}_\perp + \frac{\bar{p}_\perp}{z} - \frac{\bar{P}_T}{z})}$$

$$f_1(x, k_\perp^2) \, D_1(z, p_\perp^2) = 2\pi \sum_q e_q^2 \, x \int db \, b \, J_0\left(\frac{bP_T}{z}\right)$$

$$\times \tilde{f}_1(x, b^2) \, \tilde{D}_1(z, b^2)$$

~~Where solutions of CS equations using b_* prescription are~~

$$\cancel{\tilde{f}_1(x, b^2) = \frac{1}{2\pi} f_1(x) \, e^{-S(b_*)/2} \, e^{-S_{NP}^{f_1}(b)}}$$

$$\cancel{\tilde{D}_1(z, b^2) = \frac{1}{2\pi} \frac{D_1(z)}{z^2} \, e^{-S(b_*)/2} \, e^{-S_{NP}^{D_1}(b)}}$$

Relevant solutions of CS equation valid at small b (large q_T) are

$$\tilde{f}_1(x,b^2) = \frac{1}{2\pi} f_1(x)\, e^{-S(b)/2}$$

$$\tilde{D}_1(z,b^2) = \frac{1}{2\pi} \frac{D_1(z)}{z^2}\, e^{-S(b)/2}$$

Thus

$$F_{UU} = \sum_q e_q^2 \frac{x}{z^2} f_1(x) D_1(z) \int \frac{db\, b}{2\pi} J_0\left(\frac{P_T b}{z}\right) e^{-S(b)}$$

$\Rightarrow$ the same result as in perturbative QCD

Now let us examine

$$F_{UU}^{\cos\phi} = \frac{2M}{Q} C \left[\frac{\hat{h}\cdot\vec{p}_\perp}{z m_h}\left(x h H_1^\perp + \frac{m_h}{M} f_1 \frac{\tilde{D}_1}{z}\right) - \frac{\hat{h}\cdot\vec{k}_\perp}{M}\left(x f^\perp D_1 + \frac{m_h}{M} h_1^\perp \frac{\tilde{H}}{z}\right)\right]$$

functions include twist-3 functions both for distribution and fragmentation

We can use QCD EOM and Wandzura-Wilczek approximations and derive the following (Bacchetta 2006, Bastami, Schweitzer, AP, 2017)

$$F_{UU}^{\cos\phi} = \frac{2M}{Q} \underbrace{C\left(-\frac{\hat{h}\cdot\vec{k}_\perp}{M} f_1 D_1\right)}_{\text{Also known as Cahn effect}}$$

We obtain

$$F_{UU}^{\cos\phi} = -2 \sum_q e_q^2 x \int d^2k_\perp d^2p_\perp \frac{\vec{k}_\perp \cdot \hat{h}}{Q} f_q(x, k_\perp^2)$$

$$D_q(z, p_\perp^2)\, \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_T) = \text{(after some algebra)}$$

$$= -4\pi \frac{M^2}{Q} \sum_q e_q^2 x \int db\, b^2 J_1(b P_T/z)\, \tilde{f}_1^{(1)}(x, b^2)\, \tilde{D}_1(z, b^2)$$

where

$$\tilde{f}_1^{(1)}(x, b^2) = -\frac{1}{M^2} \frac{\partial}{b\,\partial b} \tilde{f}_1(x, b^2)$$

Gamberg, Boer, Musch, AP 2010 (Bessel weighting)

Expanding at high q_T we obtain

$$F_{UU}^{\cos\phi} = 4\pi \frac{1}{Q} \sum_q e_q^2 x \int db\, b^2 J_1\left(\frac{b P_T}{z}\right) \frac{f_1(x) D_1(z)}{(2\pi)^2 z^2}$$

$$e^{-S(b)} \left(\frac{\partial}{2b\,\partial b} S(b)\right) = \text{(after some algebra)}$$

$$= -\frac{1}{Q q_T} \frac{\alpha_s}{2\pi^2 z^2} \sum_q x e_q^2 f_1(x) D_1(z) L(Q^2/q_T^2)$$

the same as in collinear QCD.

We conclude that we found the resummed result in SIDIS.

W_Δ, $\cos\phi$ in Drell-Yan

Using Lu, Schmidt 2011 we obtain

$$W_\Delta = F_{UU}^{\cos\phi} = \frac{2}{Q}\,\mathcal{C}\Big[(\hat{h}\cdot\vec{k}_{1T})\Big(f^\perp \bar{f}_1 - \frac{M_2}{M_1} h_1^\perp \bar{\hat{h}}\Big)$$

$$- (\hat{h}\cdot\vec{k}_{2T})\Big(f_1 \bar{f}^\perp - \frac{M_1}{M_2}\hat{h}\, h_1^\perp\Big)\Big]$$

where

$$\hat{f} = x_1\big((1-c)f + c\tilde{f}\big)$$

$$\hat{\bar{f}} = x_2\big(c\bar{f} + (1-c)\tilde{\bar{f}}\big)$$

$c = 0$ in Gottfried-Jackson frame

$c = 1/2$ in Collins-Soper frame

Again using EOM & WW we obtain ($x_1 f^\perp = f_1$, $x_2 \bar{f}^\perp = \bar{f}_1$)

$$F_{UU}^{\cos\phi} = \frac{2}{Q}\,\mathcal{C}\Big[(\hat{h}\cdot\vec{k}_{1T})(1-c) f_1 \bar{f}_1 -$$

$$- (\hat{h}\cdot\vec{k}_{2T})\, c\, f_1 \bar{f}_1\Big]$$

Here in case of Drell-Yan we have

$$\mathcal{C}(w f g) = \sum_a e_a^2 \int d^2k_{1T}\, d^2k_{2T}\, \delta^{(2)}(\vec{q}_T - \vec{k}_{1T} - \vec{k}_{2T})$$

$$w(k_{1T}, k_{2T})\,(f\bar{g} + \bar{f}g)$$

$$F_{UU}^{\cos\phi} = \frac{2}{Q}\sum_q e_q^2 \int d^2k_{1T}\, d^2k_{2T}\, \frac{d^2b}{(2\pi)^2}$$

$$e^{ib(\bar q_T - \bar k_{1T} - \bar k_{2T})} \Big((\hat h\cdot \vec k_{1T})(1-c)\, f_1(x_1,k_{1T}^2)\, \bar f_1(x_2,k_{2T}^2)$$

$$- (\hat h\cdot \vec k_{2T})\, c\, f_1(x_1, \bar k_{1T}^2)\, \bar f_1(x_2, k_{2T}^2)\Big) =$$

= (after some algebra)

$$= -\frac{4\pi M^2}{Q}\sum_q e_q^2 \int db\, b^2 J_1(b q_T) \Big[(1-c)\,\tilde f_1^{(1)}\, \bar{\tilde f}_1$$

$$- c\,\tilde f_1\, \bar{\tilde f}_1^{(1)}\Big]$$

Using resummed distributions

$$\tilde f_1(x,b) = \frac{f_1(x)}{2\pi}\, e^{-S(b)/2}$$

we obtain

$$F_{UU}^{\cos\phi} = -\frac{4\pi M^2}{Q}\sum_q e_q^2\, f_1(x_1)\, \bar f_1(x_2)\, (1-2c)\, \frac{\alpha_s}{2\pi q_T}$$

$$\frac{1}{M^2}\cdot\frac{1}{(2\pi)^2}\, L(Q^2/q_T^2) =$$

$$= -\frac{\alpha_s}{2\pi^2 Q q_T}\, L(Q^2/q_T^2)\, (1-2c)\, f_1(x_1)\, \bar f_1(x_2)$$

when $c = 1/2$, CS we have no log terms

$c = 0$, GJ we have log terms

The same as in pert. QCD

We conclude that resummation formulas were found using TMD formalism.

SIDIS

$$F_{UU}^{\cos\phi} = -4\pi \frac{M^2}{Q} \sum_q e_q^2 x \int db\, b^2 J_1\left(\frac{b P_T}{z}\right) \times \tilde{f}_1^{(1)}(x, b^2)\, \tilde{D}_1^q(z, b^2)$$

(without prove here)

$$F_{UU}^{\cos 2\phi} = 2\pi \frac{M^4}{Q^2} \sum_q e_q^2 x \int db\, b^3 J_2(b P_T/z) \times \tilde{f}_1^{(2)}(x, b^2)\, \tilde{D}_1^q(z, b^2)$$

Drell-Yan

$$F_{UU}^{\cos\phi} = -\frac{4\pi M^2}{Q} \sum_q e_q^2 \int db\, b^2 J_1(b q_T) \Big[(1-c)\, \tilde{f}_1^{(1)}(x_1, b^2)\, \tilde{\tilde{f}}_1(x_2, b^2) - c\, \tilde{f}_1^a(x_1, b^2)\, \tilde{\tilde{f}}_1^{q(1)}(x_2, b^2)\Big]$$

$c = 1/2$ CS

$c = 0$ GJ.

f & D obey CS evolution equations