The Flavor Structure of the Nucleon Sea
Institute for Nuclear Theory
October 5, 2017

Flavor asymmetries in the nucleon from chiral EFT

Wally Melnitchouk

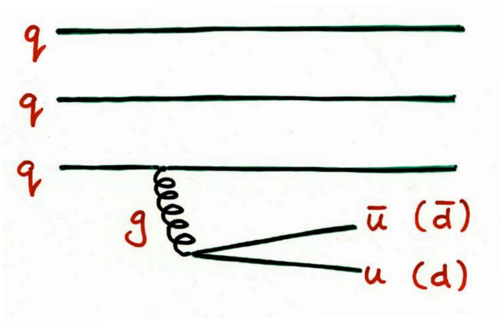
Jefferson Lab

Outline

- Motivation: $\bar{d} - \bar{u}$ asymmetry
- PDF constraints from chiral symmetry in QCD / chiral EFT
- Leading neutron DIS — implications for pion models and pion PDF extraction
- Strange quark asymmetries
- Outlook

Light quark sea

- From perturbative QCD expect symmetric $q\bar{q}$ sea generated by gluon radiation into $q\bar{q}$ pairs (if quark masses are the same)

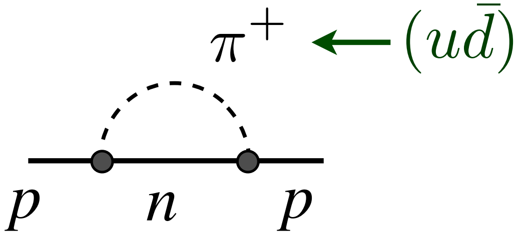


→ since u and d quarks nearly degenerate, expect flavor-symmetric light-quark sea

$$\bar{d} \approx \bar{u}$$

Ross, Sachrajda (1979)

- Thomas suggested that chiral symmetry of QCD (important at low energy) should have consequences for antiquark PDFs in nucleon (measured at high energy)

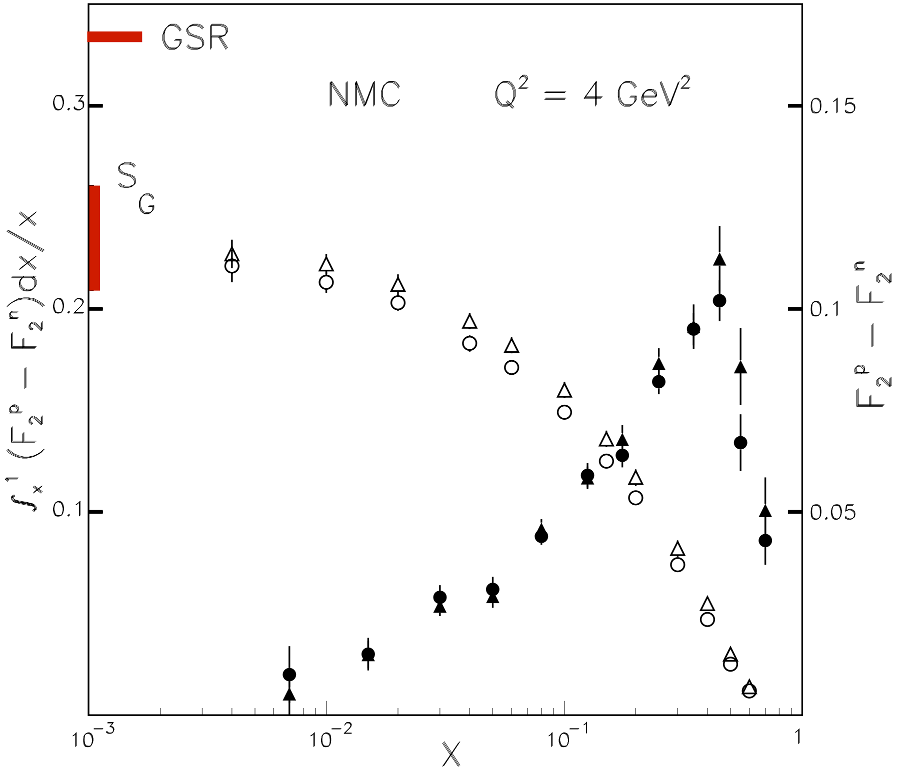


$$\rightarrow \quad \bar{d} > \bar{u}$$

A.W. Thomas (1984)

Light quark sea

- First clear experimental support for $\bar{d} \neq \bar{u}$ came from violation of Gottfried sum rule observed by NMC



$$\frac{1}{x}(F_2^p - F_2^n) = \frac{1}{3}(u^+ - d^+) + \frac{2}{3}(\bar{u} - \bar{d})$$

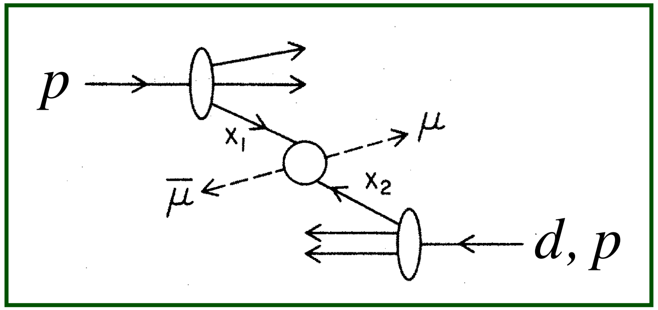
$$S_G \equiv \int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = 0.235 \pm 0.026$$

NMC (1991, 1994)

→ clear evidence for $\bar{d} - \bar{u} > 0$ (or at least integrated value)

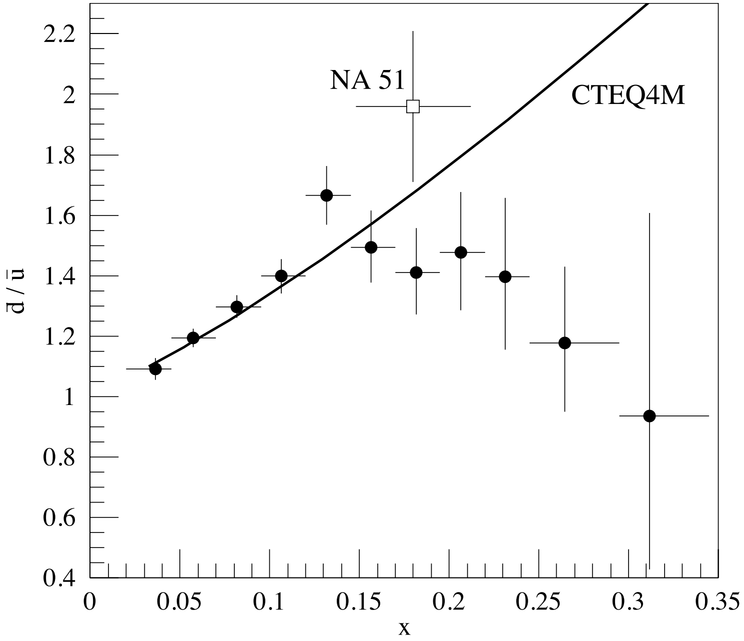
Light quark sea

- x dependence of $\bar{d} - \bar{u}$ asymmetry established in Fermilab E866 pp/pd Drell-Yan experiment



$$\frac{d\sigma}{dx_1 dx_2} \sim \sum_q e_q^2 \, q(x_1) \, \bar{q}(x_2) \; + \; (x_1 \leftrightarrow x_2)$$

$$\frac{\sigma^{pd}}{\sigma^{pp}} \approx 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \quad \text{for } x_1 \gg x_2$$

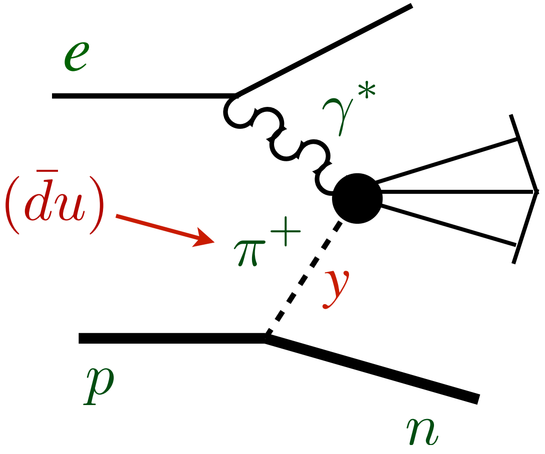


E866 (2001)

⟶ strong enhancement of $\bar{d}$ at $x \sim 0.1 - 0.2$, but intriguing behavior at large x hinting at possible sign change of $\bar{d} - \bar{u}$

Light quark sea

- General agreement with pion loop model calculations

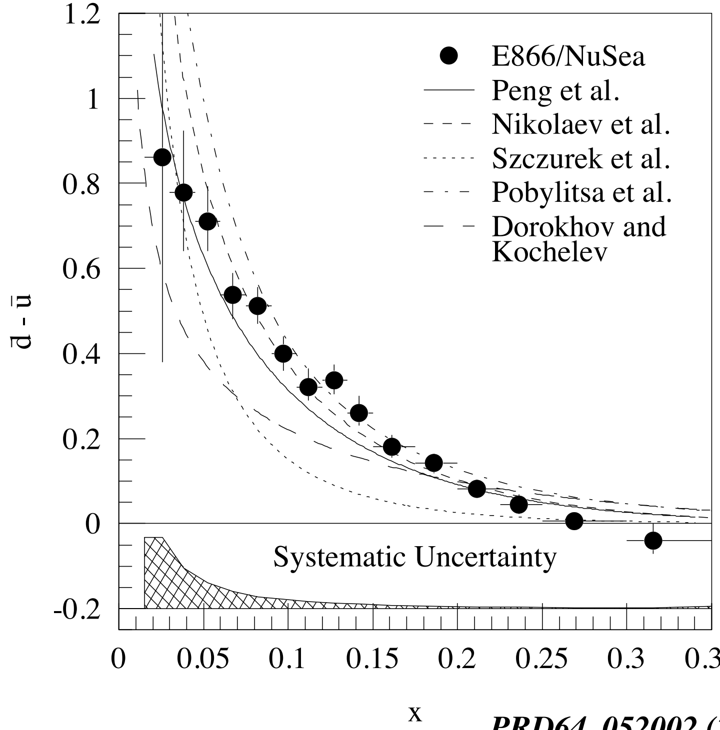


$$(\bar{d}-\bar{u})(x) = \int_x^1 \frac{dy}{y}\, f_{\pi^+ n}(y)\, \bar{q}_v^{\pi}(x/y)$$

$p \to \pi^+ n$ splitting function
("flux factor")

$$\boxed{f_{\pi N}(y) = \frac{3g^2_{\pi NN}}{16\pi^2}\, y \int dk^2 \frac{-k^2}{(k^2-m_\pi^2)^2}\, F^2_{\pi NN}(k^2)}$$

Sullivan (1972)



PRD64, 052002 (2001)

→ shape qualitatively reproduced by many models (except at high x),
— but *is there a direct connection with QCD?*

Chiral EFT

- Expand moments of PDFs in powers of m_π

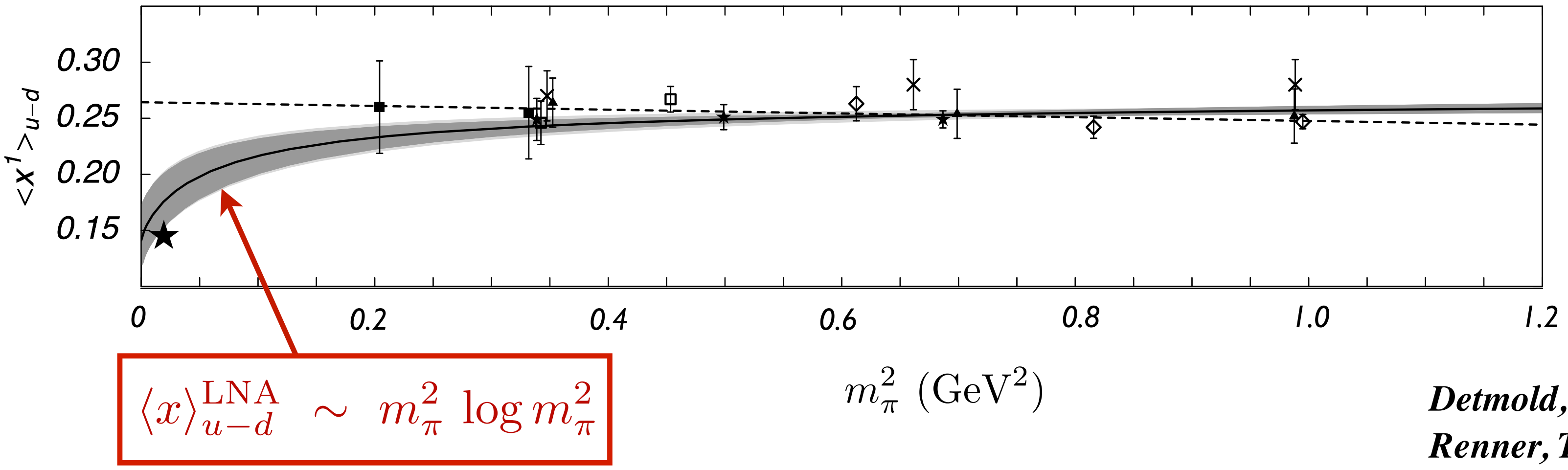
  → coefficients of leading nonanalytic (LNA) terms, reflecting infrared behavior, are model-independent!

  → nonzero LNA term implies nonzero asymmetry from π loops

$$\int_0^1 dx\,(\bar{d}-\bar{u}) = \frac{2g_A^2}{(4\pi f_\pi)^2}\,\log(m_\pi^2/\mu^2) + \text{ terms analytic in } m_\pi^2$$

Thomas, WM, Steffens (2000)

  → chiral nonanalytic behavior provided a way to reconcile (early) lattice data on $(u{-}d)$ momentum fraction with experiment



$$\langle x\rangle_{u-d}^{\mathrm{LNA}} \sim m_\pi^2 \log m_\pi^2$$

Detmold, WM, Negele, Renner, Thomas (2001)

Chiral EFT

- Expand moments of PDFs in powers of m_π

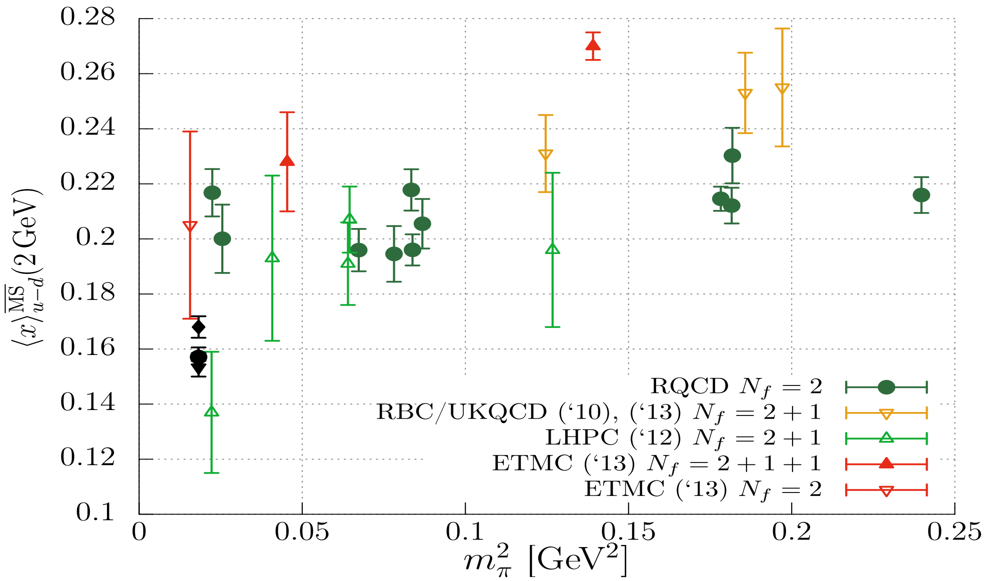
→ coefficients of leading nonanalytic (LNA) terms, reflecting infrared behavior, are model-independent!

→ nonzero LNA term implies nonzero asymmetry from π loops

$$\int_0^1 dx\,(\bar{d}-\bar{u}) = \frac{2g_A^2}{(4\pi f_\pi)^2}\log(m_\pi^2/\mu^2) + \text{ terms analytic in } m_\pi^2$$

Thomas, WM, Steffens (2000)

→ chiral nonanalytic behavior provided a way to reconcile (early) lattice data on $(u-d)$ momentum fraction with experiment



Bali et al. (2014)

Chiral EFT

- Direct calculation of matrix elements of twist-2 operators in EFT

$$\mathcal{L}_{\rm eff} \;=\; \frac{g_A}{2f_\pi}\,\bar{\psi}_N\gamma^\mu\gamma_5\,\vec{\tau}\cdot\partial_\mu\vec{\pi}\,\psi_N \;-\; \frac{1}{(2f_\pi)^2}\,\bar{\psi}_N\gamma^\mu\,\vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\,\psi_N$$

Weinberg (1967)

disagrees with "Sullivan" result!

$$\langle x^n\rangle_{u-d} = a_n\left(1+\frac{(3g_A^2+1)}{(4\pi f_\pi)^2}m_\pi^2\log(m_\pi^2/\mu^2)\right)+\mathcal{O}(m_\pi^2)$$

Chen, X. Ji (2001)
Arndt, Savage (2002)

Sullivan: $4g_A^2$

→ is there a problem with application of EFT or "Sullivan process" to DIS?

→ is light-front treatment of pion loops problematic ($vs.$ covariant/instant form)?

→ consider simple test case: nucleon self-energy

Self-energy

🟩 From lowest order chiral (pseudovector) Lagrangian

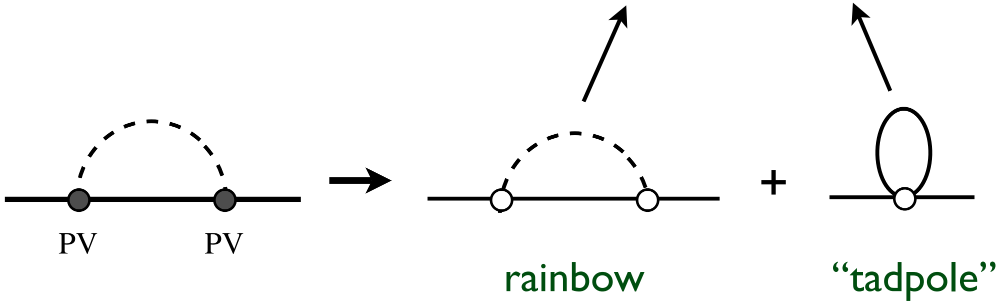
$$\Sigma = i\left(\frac{g_{\pi NN}}{2M}\right)^2 \overline{u}(p)\int\frac{d^4k}{(2\pi)^4}\,(\not{k}\gamma_5\vec{\tau})\,\frac{i\,(\not{p}-\not{k}+M)}{D_N}\,(\gamma_5\not{k}\vec{\tau})\,\frac{i}{D_\pi^2}\,u(p)$$

Goldberger-Treiman $\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M}$

$$\begin{aligned} D_\pi &\equiv k^2 - m_\pi^2 + i\varepsilon \\ D_N &\equiv (p-k)^2 - M^2 + i\varepsilon \end{aligned}$$

⟶ rearrange in more transparent "reduced" form

$$\Sigma = -\frac{3ig_A^2}{4f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{2M}\left[4M^2\left(\frac{m_\pi^2}{D_\pi D_N}+\frac{1}{D_N}\right)+\cancel{\frac{2p\cdot k}{D_\pi}}\right]$$



Self-energy

- 🟩 Covariant (dimensional regularization)

$$\int d^{4-2\varepsilon}k \frac{1}{D_\pi D_N} = -i\pi^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \int_0^1 dx \log \frac{(1-x)^2 M^2 + x m_\pi^2}{\mu^2} + \mathcal{O}(\varepsilon) \right)$$

$$\int d^{4-2\varepsilon}k \frac{1}{D_N} = -i\pi^2 M^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \log \frac{\mu^2}{M^2} + \mathcal{O}(\varepsilon) \right)$$

⟶ combining terms gives well-known m_π^3 LNA behavior (from $1/D_\pi D_N$ term)

$$\Sigma_{\rm cov}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{1}{2\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

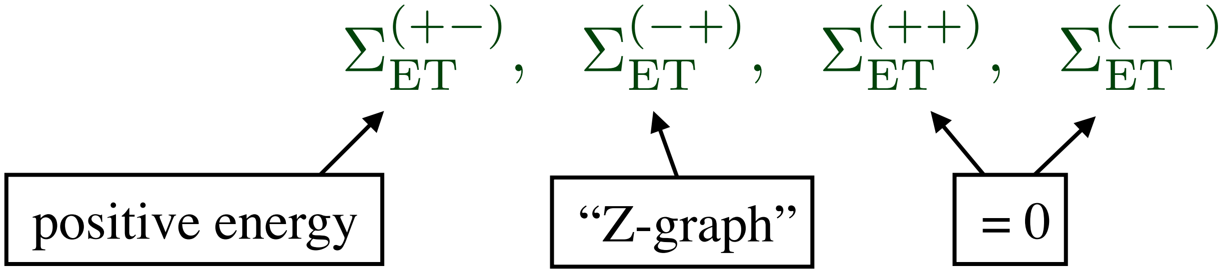
Self-energy

- Equal time (rest frame)

$$\int d^4k \frac{1}{D_\pi D_N} = \int d^3k \int_{-\infty}^{\infty} dk_0 \frac{1}{(-2)(\omega_k - i\varepsilon)} \left(\frac{1}{k_0 - \omega_k + i\varepsilon} - \frac{1}{k_0 + \omega_k - i\varepsilon} \right)$$
$$\times \frac{1}{2(E' - i\varepsilon)} \left(\frac{1}{k_0 - E + E' - i\varepsilon} - \frac{1}{k_0 - E - E' + i\varepsilon} \right)$$

$$\omega_k = \sqrt{\mathbf{k}^2 + m_\pi^2}\ , \quad E' = \sqrt{\mathbf{k}^2 + M^2}$$

⟶ four time-orderings



$$\Sigma_{\rm ET}^{(+-)},\quad \Sigma_{\rm ET}^{(-+)},\quad \Sigma_{\rm ET}^{(++)},\quad \Sigma_{\rm ET}^{(--)}$$

$$\Sigma_{\rm ET}^{(+-)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{3}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$
$$\Sigma_{\rm ET}^{(-+)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(\qquad - \frac{1}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

- Equal time (infinite momentum frame)

$$\Sigma_{\rm IMF}^{(+-)} = -\frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E-E'-\omega_k)}$$

$$= \frac{3g_A^2 M}{32\pi^2 f_\pi^2}\int_0^1 dy \int_0^{\Lambda^2} dk_\perp^2 \frac{m_\pi^2}{k_\perp^2 + M^2(1-y)^2 + m_\pi^2 y}$$

$$p_z \equiv P \to \infty$$

$$y = p_z'/p_z$$

$$\Sigma_{\rm IMF}^{(-+)} = \frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E+E'+\omega_k)} = \mathcal{O}(1/P^2)$$

⟶ nonanalytic behavior as for rest frame expression

$$\Sigma_{\rm IMF}^{\rm LNA} = \Sigma_{\rm IMF}^{(+-)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

Self-energy

- Light front

$$\int dk^+ dk^- d^2k_\perp \frac{1}{D_\pi D_N} = \frac{1}{p^+}\int_{-\infty}^{\infty} \frac{dx}{x(x-1)} d^2k_\perp \int dk^- \left(k^- - \frac{k_\perp^2 + m_\pi^2}{xp^+} + \frac{i\varepsilon}{xp^+}\right)^{-1}$$

$$\times \left(k^- - \frac{M^2}{p^+} - \frac{k_\perp^2 + M^2}{(x-1)p^+} + \frac{i\varepsilon}{(x-1)p^+}\right)^{-1}$$

$$= 2\pi^2 i \int_0^1 dx\ dk_\perp^2\ \frac{1}{k_\perp^2 + (1-x)m_\pi^2 + x^2 M^2}$$

$x = k^+/p^+$

⟶ identical nonanalytic results as covariant & instant form

$$\Sigma_{\rm LF}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

Self-energy

- 🟩 Light front

→ $1/D_N$ "tadpole" term has k^- pole that depends on k^+ and moves to infinity as $k^+ \rightarrow 0$
("treacherous" in LF dynamics)

→ use LF cylindrical coordinates $k^+ = r\cos\phi,\ k^- = r\sin\phi$

$$\int d^4k \frac{1}{D_N} = \frac{1}{2}\int d^2k_\perp \int \frac{dk^+}{k^+}\int dk^- \left(k^- - \frac{k_\perp^2 + M^2}{k^+} + \frac{i\varepsilon}{k^+}\right)^{-1}$$

$$= -2\pi \int d^2k_\perp \left[\int_0^{r_0} dr \frac{r}{\sqrt{r_0^4 - r^4}} + i \lim_{R\to\infty}\int_{r_0}^{R} dr \frac{r}{\sqrt{r^4 - r_0^4}}\right]$$

$$= \frac{1}{2}\int d^2k_\perp \lim_{R\to\infty}\left(-\pi^2 + 2\pi i\ \log\frac{r_0^2}{R^2} + \mathcal{O}(1/R^4)\right)$$

contains $\log(k_\perp^2 + M^2)$
term as required

$r_0 = \sqrt{2(k_\perp^2 + M^2)}$

Self-energy

- Pseudoscalar interaction

$$\Sigma^{\rm PS} = i g_{\pi NN}^2\ \overline{u}(p) \int \frac{d^4k}{(2\pi)^4}\, (\gamma_5 \vec{\tau})\, \frac{i\,(\not{p} - \not{k} + M)}{D_N}\, (\gamma_5 \vec{\tau})\, \frac{i}{D_\pi^2}\, u(p)$$

$$= -\frac{3 i g_A^2 M}{2 f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N} - \frac{1}{D_\pi} \right]$$

→ contains additional ("treacherous") pion "tadpole" term

→ similar evaluation as for $1/D_N$ term

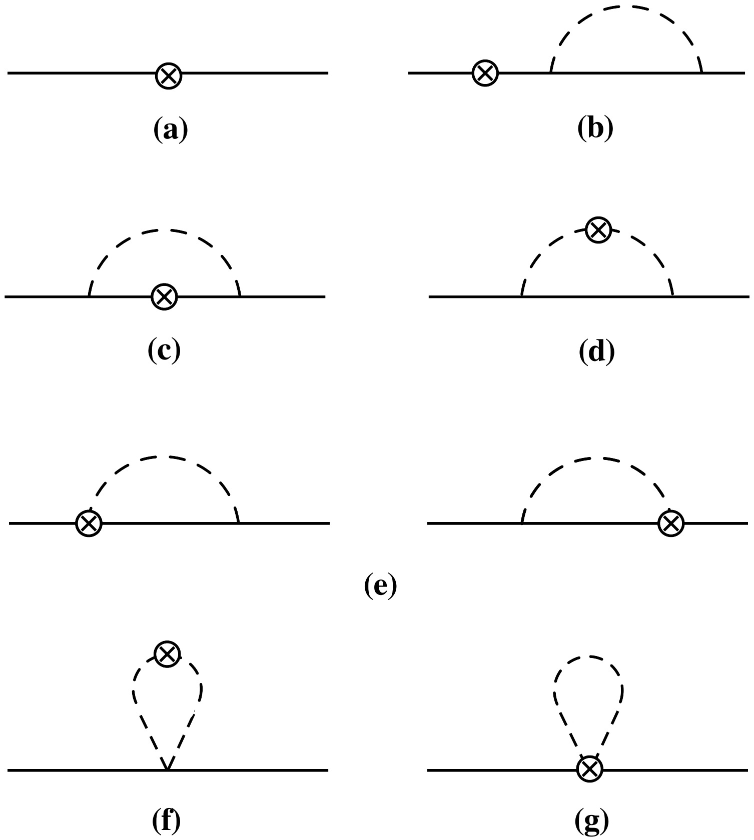
$$\Sigma_{\rm LNA}^{\rm PS} = \frac{3 g_A^2}{32\pi f_\pi^2} \left(\frac{M}{\pi} m_\pi^2 \log m_\pi^2 - m_\pi^3 - \frac{m_\pi^4}{2\pi M^2} \log \frac{m_\pi^2}{M^2} + \mathcal{O}(m_\pi^5) \right)$$

additional *lower order* term in PS theory!

Vertex corrections

- Pion cloud corrections to e.m. coupling to nucleon

  ⟶ wave function renormalization **(b)**,
  N rainbow **(c)**, π rainbow **(d)**,
  Kroll-Ruderman **(e)**,
  bubble **(f)**, tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\, \bar{u}(p)\, \gamma^\mu\, u(p) = \bar{u}(p)\, \Lambda^\mu\, u(p)$$

⟶ taking "+" components:

$$Z_1^{-1} - 1 \approx 1 - Z_1 = \frac{M}{p^+}\, \bar{u}(p)\, \Lambda^+\, u(p)$$

⟶ $e.g.$ for N rainbow contribution, $\Lambda_\mu^N = -\dfrac{\partial \hat{\Sigma}}{\partial p^\mu}$

Vertex corrections

- Nonanalytic behavior of vertex renormalization factors

$$1 - Z_1^N \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2}\left\{ m_\pi^2 \log m_\pi^2 \; - \; \pi \frac{m_\pi^3}{M} \; - \; \frac{2m_\pi^4}{3M^2}\log m_\pi^2 + \; \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^\pi \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2}\left\{ m_\pi^2 \log m_\pi^2 \; - \; \frac{5\pi}{3}\frac{m_\pi^3}{M} \; - \; \frac{m_\pi^4}{M^2}\log m_\pi^2 + \; \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{\text{KR}} \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2}\left\{ \frac{2\pi}{3}\frac{m_\pi^3}{M} \; - \; \frac{m_\pi^4}{3M^2}\log m_\pi^2 + \; \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{\pi\ (\text{tad})} \xrightarrow{\text{NA}} -\frac{1}{2(4\pi f_\pi)^2}\; m_\pi^2 \log m_\pi^2$$

$$1 - Z_1^{\pi\ (\text{bub})} \xrightarrow{\text{NA}} \frac{1}{2(4\pi f_\pi)^2}\; m_\pi^2 \log m_\pi^2$$

⟶ cancellation of $m_\pi^2 \log m_\pi^2$ terms in KR contribution

⟶ demonstration of gauge invariance condition
(in fact, to *all* orders!)

Vertex corrections

- Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1 - Z_1^N$	g_A^2 *	0	$-\frac{1}{2} g_A^2$	$\frac{1}{4} g_A^2$	$\frac{3}{4} g_A^2$	g_A^2
$1 - Z_1^\pi$	0	g_A^2 *	0	$-\frac{1}{4} g_A^2$	$\frac{3}{4} g_A^2$	g_A^2
$1 - Z_1^{\rm KR}$	0	0	$-\frac{1}{2} g_A^2$	$\frac{1}{2} g_A^2$	0	0
$1 - Z_1^{N\,\rm tad}$	0	0	0	$-1/2$	$-1/2$	0
$1 - Z_1^{\pi\,\rm bub}$	0	0	0	$1/2$	$1/2$	0

* also in PS

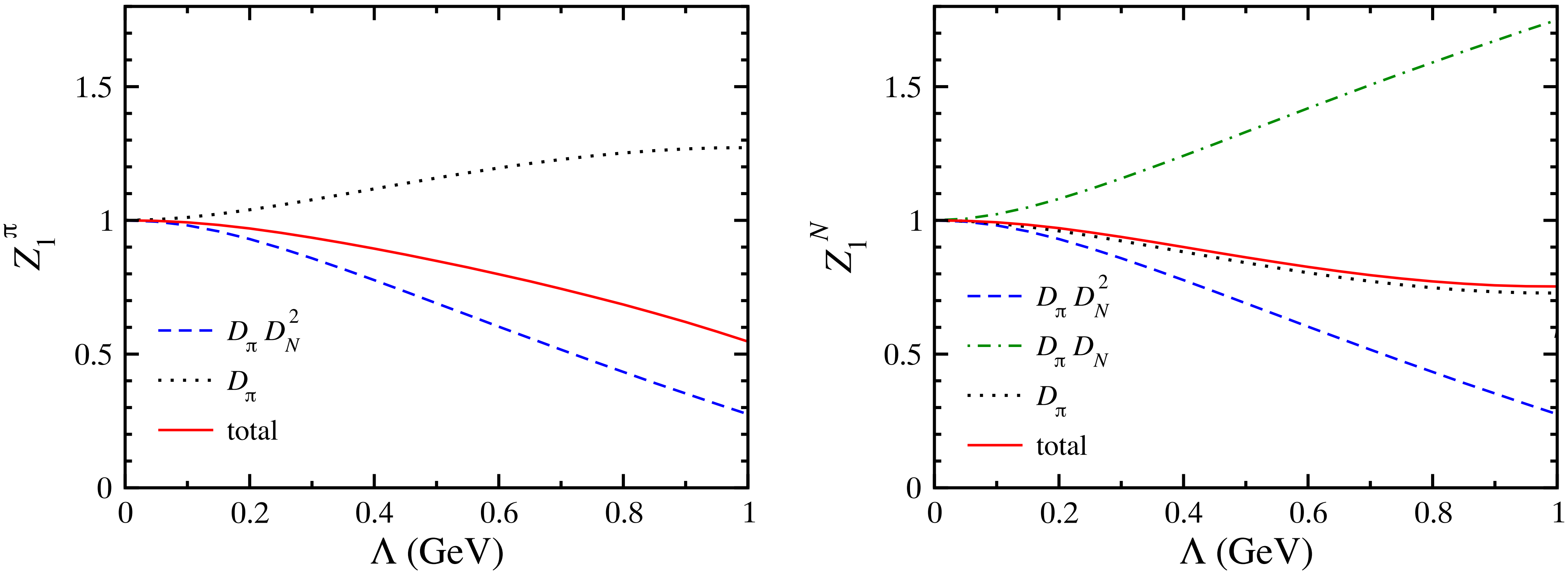
in units of $\dfrac{1}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$

→ origin of EFT *vs*. Sullivan process difference!

$$\left(1 - Z_1^{N\,(\rm PV)}\right)_{\rm LNA} = \frac{3}{4}\left(1 - Z_1^{N\,(\rm PS)}\right)_{\rm LNA}$$

Vertex corrections

- Pion & nucleon rainbow contributions

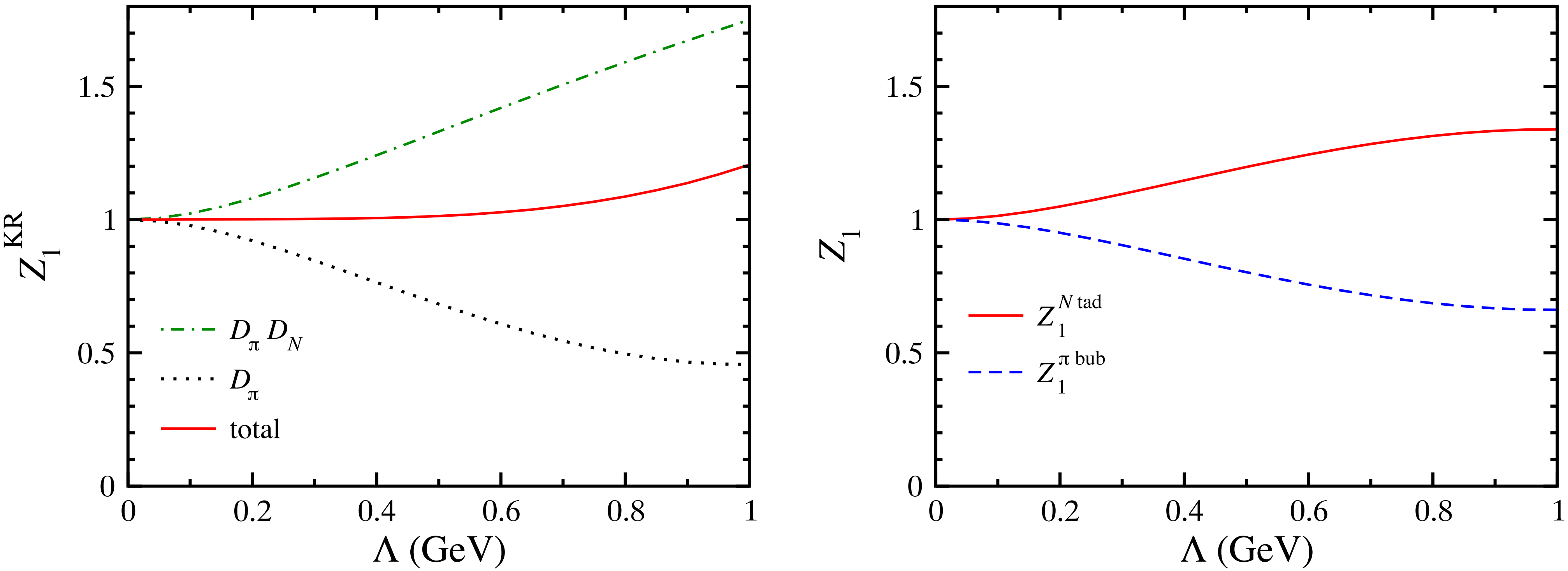


→ δ-function part reduces on-shell pion contribution

→ almost complete cancellation between on-shell & off-shell parts of nucleon contribution

Vertex corrections

- 🟩 Kroll-Ruderman & tadpoles

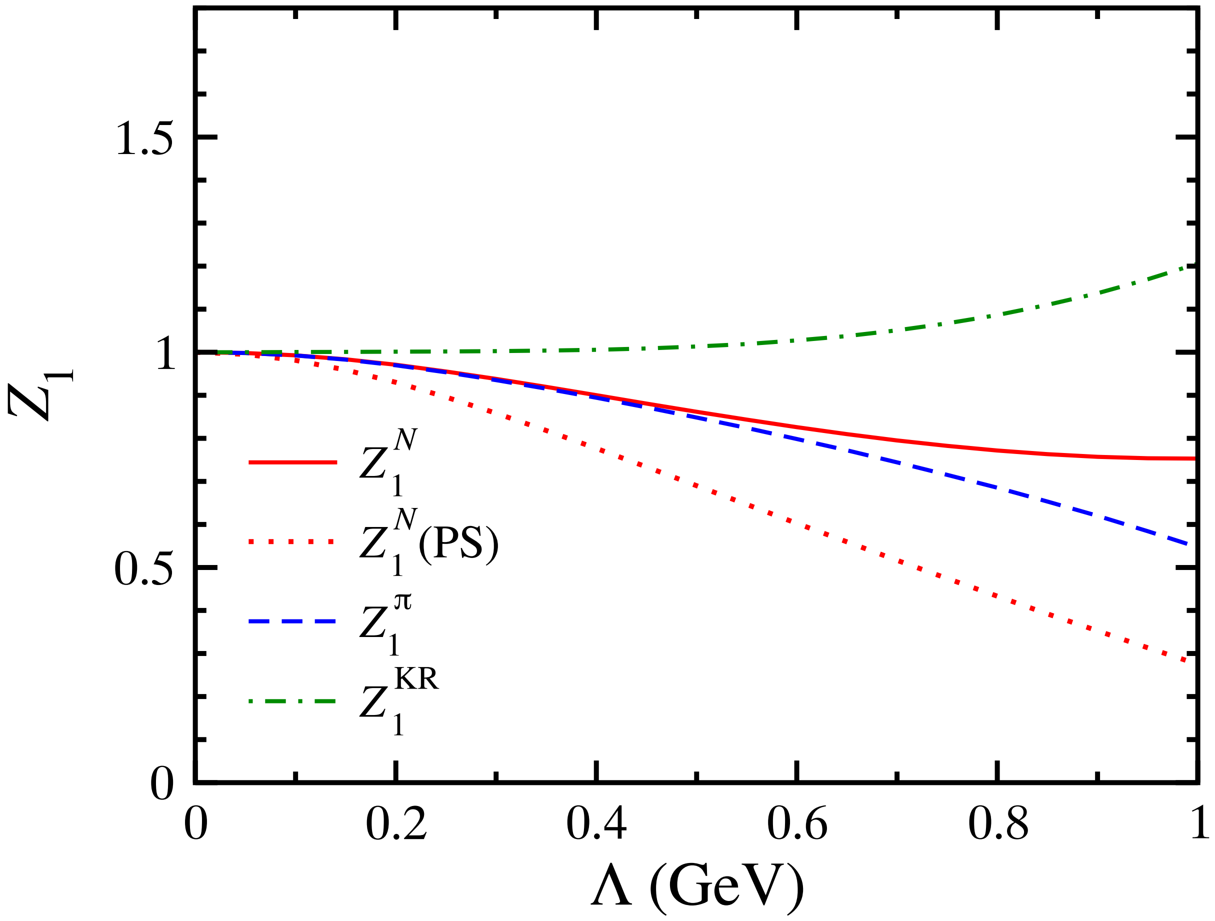


⟶ strong cancellation between off-shell & δfunction parts of KR

⟶ pion & nucleon tadpoles cancel exactly

Vertex corrections

- 🟩 Comparison of all contributions to vertex renormalization



⟶ important differences between PV & PS results (from off-shell & δ-function contributions)

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\mathrm{KR}})$$

Moments of PDFs

- PDF moments related to nucleon matrix elements of local twist-2 operators

$$\langle N|\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n}|N\rangle \;=\; 2\,\langle x^{n-1}\rangle_q\; p^{\{\mu_1}\ldots p^{\mu_n\}}$$

⟶ n-th moment of (spin-averaged) PDF $q(x)$

$$\langle x^{n-1}\rangle_q \;=\; \int_0^1 dx\, x^{n-1}\left(q(x)+(-1)^n\bar{q}(x)\right)$$

⟶ operator is $\widehat{\mathcal{O}}_q^{\mu_1\cdots\mu_n} \;=\; \bar{\psi}\gamma^{\{\mu_1}\,iD^{\mu_2}\ldots iD^{\mu_n\}}\psi \;-\; \text{traces}$

- Lowest (n=1) moment $\langle x^0\rangle_q \equiv \mathcal{M}_N + \mathcal{M}_\pi$ given by vertex renormalization factors $\sim 1 - Z_1^i$

Moments of PDFs

- For couplings involving nucleons

$$\mathcal{M}_N^{(p)} = Z_2 + (1 - Z_1^N) + (1 - Z_1^{N\,(\mathrm{tad})})$$
$$\mathcal{M}_N^{(n)} = 2(1 - Z_1^N) - (1 - Z_1^{N\,(\mathrm{tad})})$$

⟶ wave function renormalization

$$1 - Z_2 = (1 - Z_1^p) + (1 - Z_1^n) \equiv 3(1 - Z_1^N)$$

- For couplings involving only pions

$$\mathcal{M}_\pi^{(p)} = 2(1 - Z_1^\pi) + 2(1 - Z_1^{\mathrm{KR}}) + (1 - Z_1^{\pi\,(\mathrm{bub})})$$
$$\mathcal{M}_\pi^{(n)} = -2(1 - Z_1^\pi) - 2(1 - Z_1^{\mathrm{KR}}) - (1 - Z_1^{\pi\,(\mathrm{bub})})$$

Moments of PDFs

- Nonanalytic behavior

$$\mathcal{M}_N^{(p)} \overset{\text{LNA}}{\longrightarrow} 1 - \frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(p)} \overset{\text{LNA}}{\longrightarrow} \frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_N^{(n)} \overset{\text{LNA}}{\longrightarrow} \frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(n)} \overset{\text{LNA}}{\longrightarrow} -\frac{(3g_A^2 + 1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

⟶ no pion corrections to isoscalar moments

⟶ isovector correction agrees with EFT calculation

$$\mathcal{M}_N^{(p-n)} \overset{\text{LNA}}{\longrightarrow} 1 - \frac{\left(4g_A^2 + [1 - g_A^2]\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \overset{\text{LNA}}{\longrightarrow} \frac{\left(\underbrace{4g_A^2}_{\text{PS (``on-shell'') contribution}} + \underbrace{[1 - g_A^2]}_{\delta\text{-function contribution}}\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

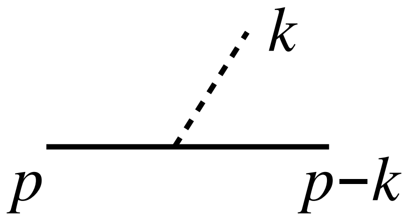
Parton distributions

- Vertex renormalization related to lowest y-moment of splitting function (light cone momentum distribution)

$$1 - Z_1^i = \int dy\, f_i(y)$$

$$y = \frac{k^+}{p^+}$$

i = rainbow, KR, bubble, tadpole



- Matching quark- and hadron-level operators

$$\mathcal{O}_q^{\mu_1 \cdots \mu_n} = \sum_h c_{q/h}^{(n)}\, \mathcal{O}_h^{\mu_1 \cdots \mu_n}$$

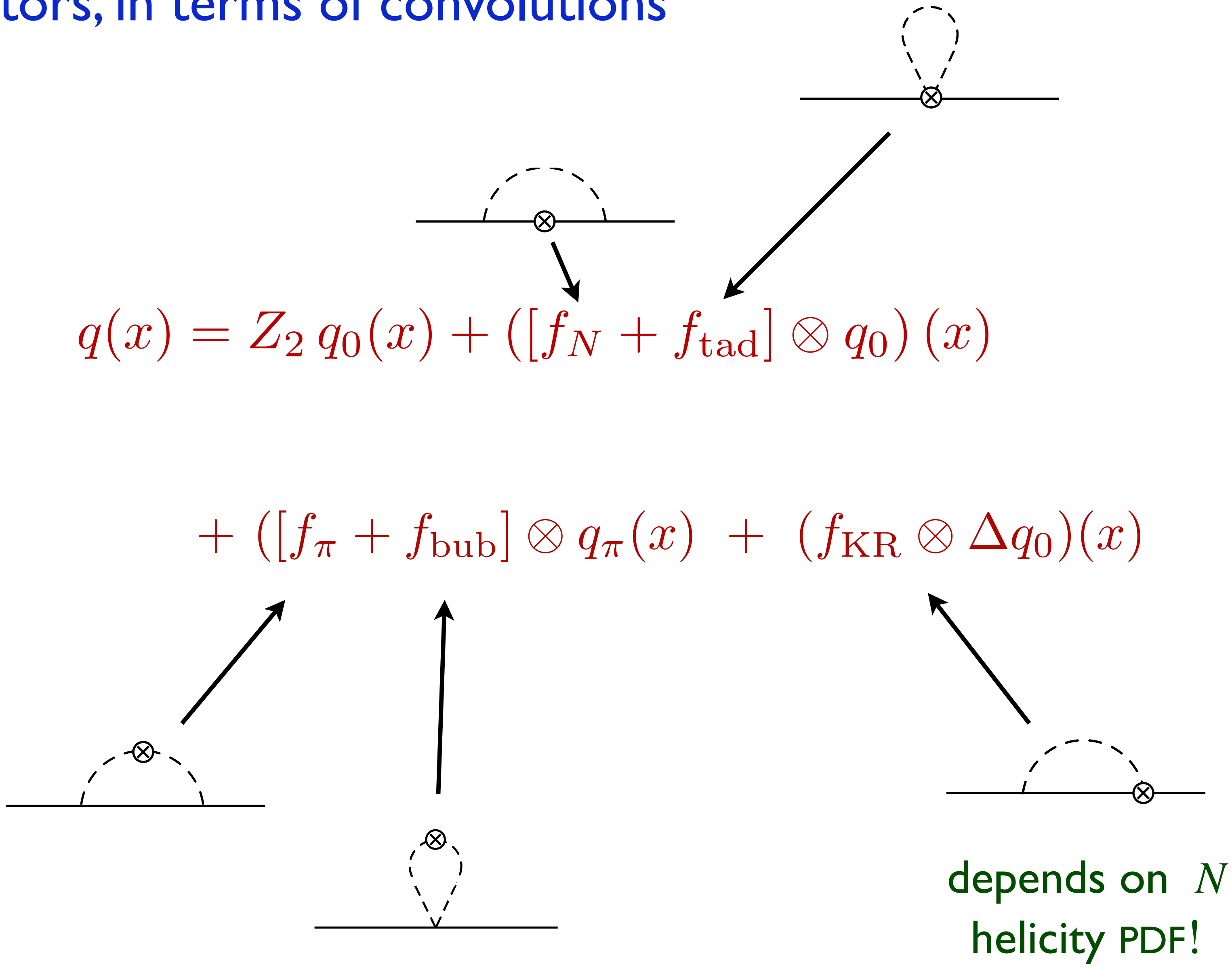
yields convolution representation

$$q(x) = \sum_h \int_x^1 \frac{dy}{y} f_h(y)\, q_v^h(x/y)$$

Chen, X. Ji (2001)

Parton distributions

- Contributions to PDFs related to matrix elements of nonlocal operators, in terms of convolutions



$$q(x) = Z_2\, q_0(x) + \left([f_N + f_{\rm tad}] \otimes q_0\right)(x)$$

$$+ \;([f_\pi + f_{\rm bub}] \otimes q_\pi(x) \;+\; (f_{\rm KR} \otimes \Delta q_0)(x)$$

depends on N
helicity PDF!

Parton distributions

- Contributions to PDFs related to matrix elements of nonlocal operators, in terms of convolutions

→ if "bare" nucleon has symmetric sea, $\bar{d} = \bar{u}$
then only "pion" term contributes

$$(\bar{d} - \bar{u})(x) = \left([f_{\pi^+} + f_{\rm bub}] \otimes \bar{q}_\pi\right)(x)$$

Parton distributions

- Splitting function for pion rainbow diagram itself has on-shell and δ-function contributions!

$$f_\pi(y) = f^{(\mathrm{on})}(y) + f^{(\delta)}(y)$$

Burkardt, Hendricks, C. Ji, WM, Thomas (2013)

$$f^{(\mathrm{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \, \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y) m_\pi^2]^2} \, \mathcal{F}^2$$

$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y) \, \mathcal{F}^2$$

- Bubble diagram contributes only at $y = 0$ (hence $x = 0$)

$$f^{(\mathrm{bub})}(y) = \frac{8}{g_A^2} \, f^{(\delta)}(y)$$

→ contributes to lowest moment, but not at $x > 0$

Parton distributions

- For point-like nucleons and pions, integrals divergent

$\longrightarrow$ finite size of nucleon provides natural regularization scale

$\mathcal{F} = \Theta(\Lambda^2 - k_\perp^2)$ $k_\perp$ cutoff

$\mathcal{F} = \left(\dfrac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t}\right)$ t monopole

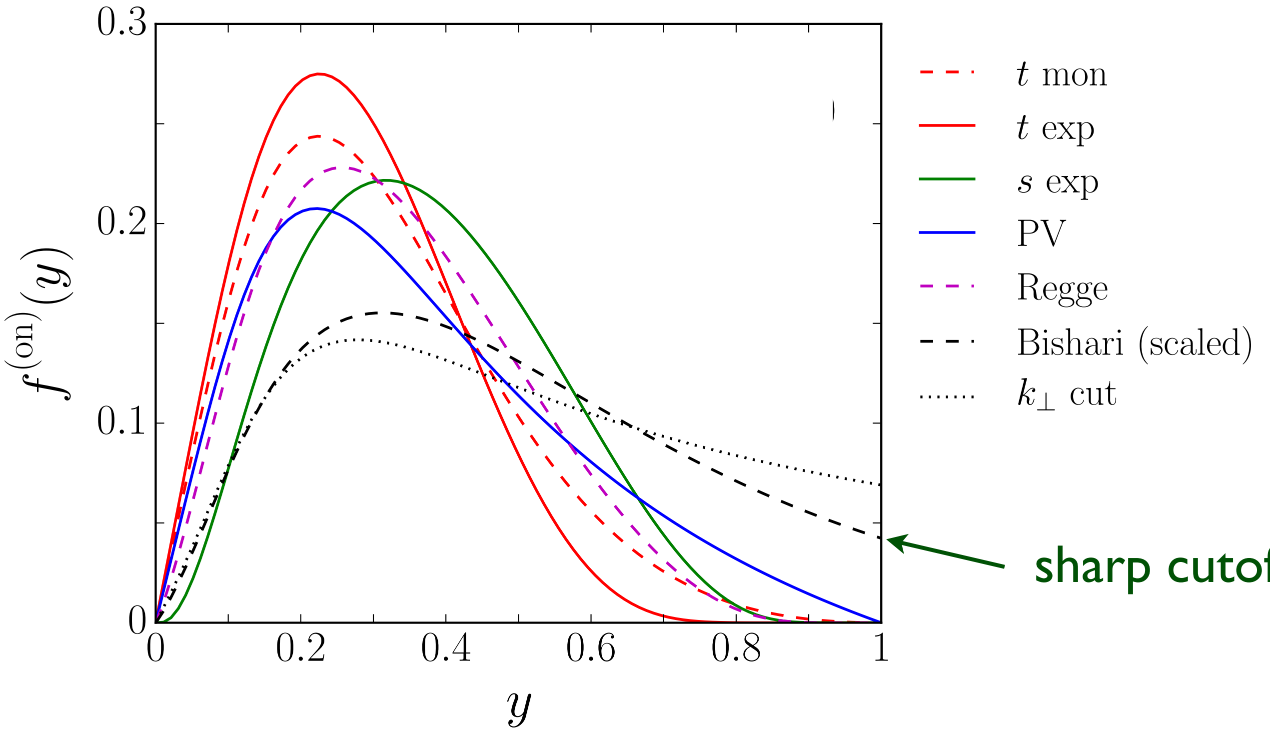
$\mathcal{F} = \exp\left[(t - m_\pi^2)/\Lambda^2\right]$ t exponential

$\mathcal{F} = \exp\left[(M^2 - s)/\Lambda^2\right]$ s-dep. exponential

$\mathcal{F} = \left[1 - \dfrac{(t - m_\pi^2)^2}{(t - \Lambda^2)^2}\right]^{1/2}$ Pauli-Villars

$\mathcal{F} = y^{-\alpha_\pi(t)} \exp\left[(t - m_\pi^2)/\Lambda^2\right]$ Regge

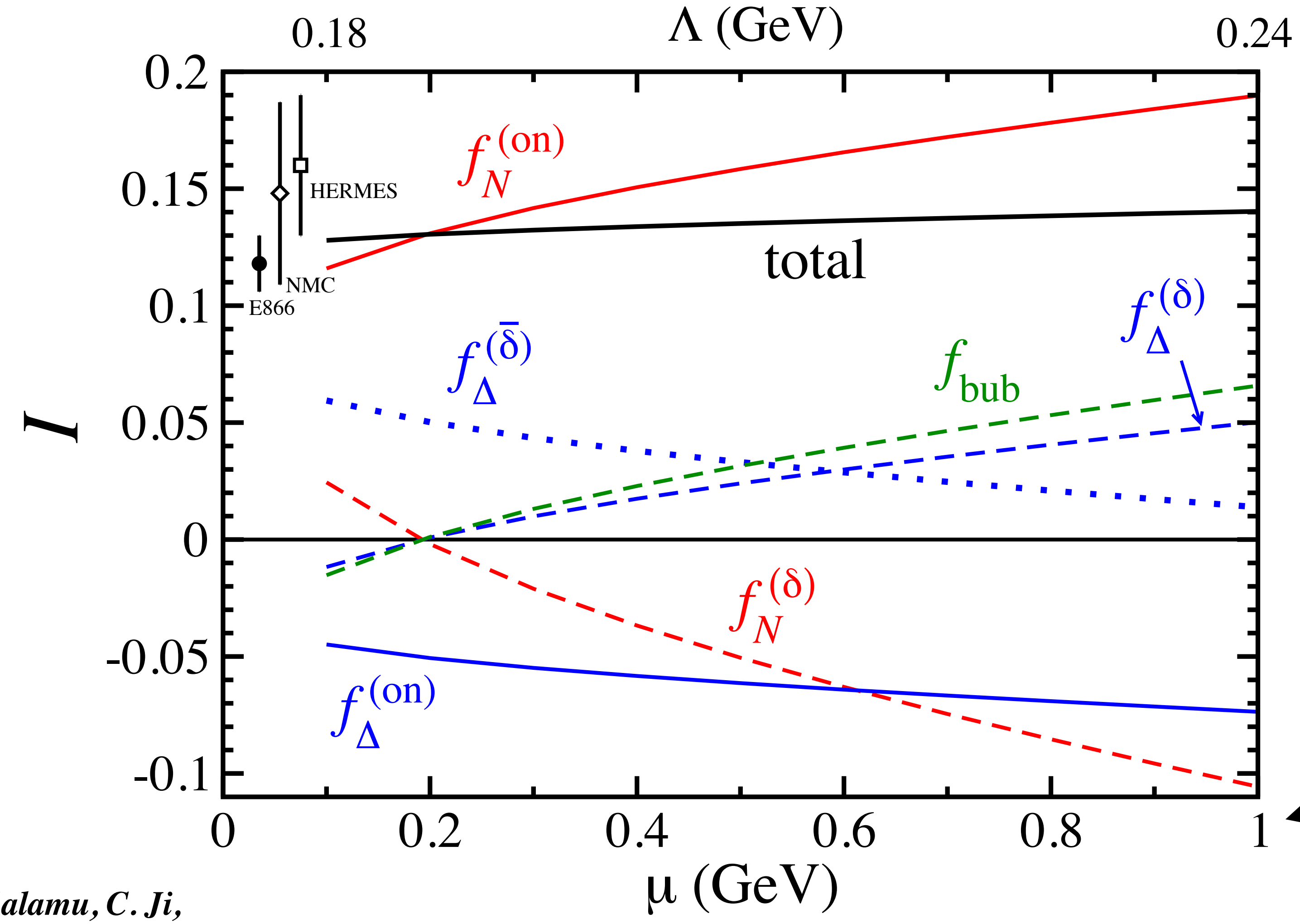
$e.g.$ on-shell function



sharp cutoffs

Parton distributions

- Integrated asymmetry $I = \int_0^1 dx\,(\bar{d} - \bar{u})(x)$



$k_\perp$ cutoff

k^- cutoff

$f_N \equiv f_{\pi(N\,\text{int. state})}$

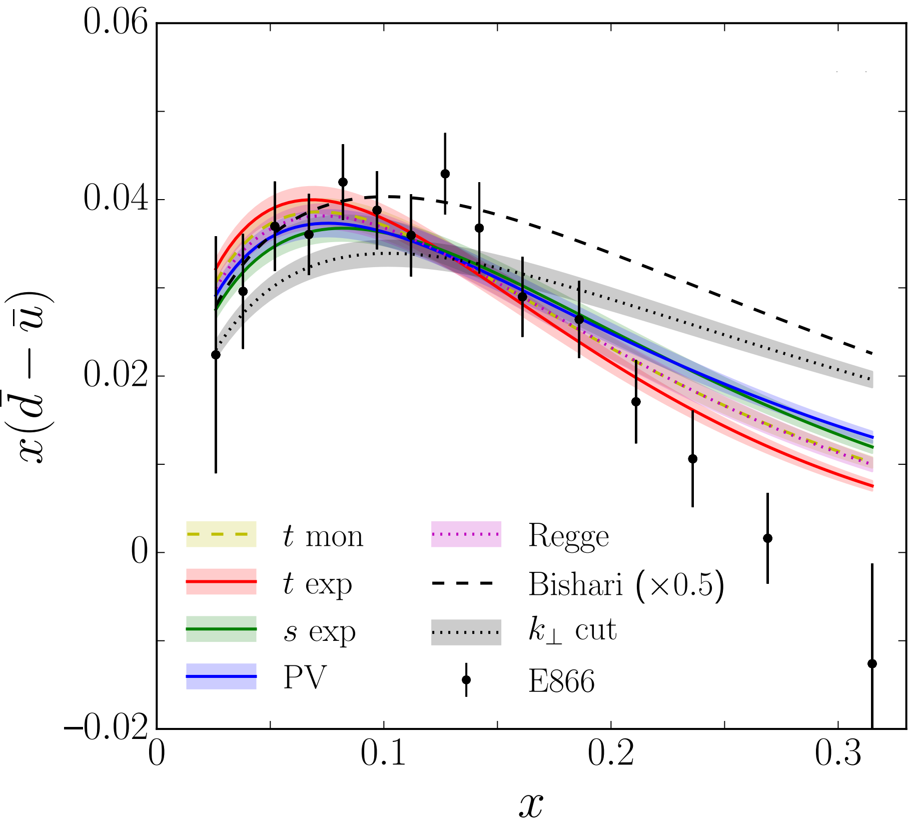
$f_\Delta \equiv f_{\pi(\Delta\,\text{int. state})}$

Salamu, C. Ji,
WM, Wang (2014)

$\longrightarrow$ N on-shell contribution $\approx$ total!

Parton distributions

- E866 $\bar{d} - \bar{u}$ data can be fitted with range of regulators



average pion “multiplicity”

$$\langle n \rangle_{\pi N} = 3 \int_0^1 dy\, f_N^{(\mathrm{on})}(y)$$

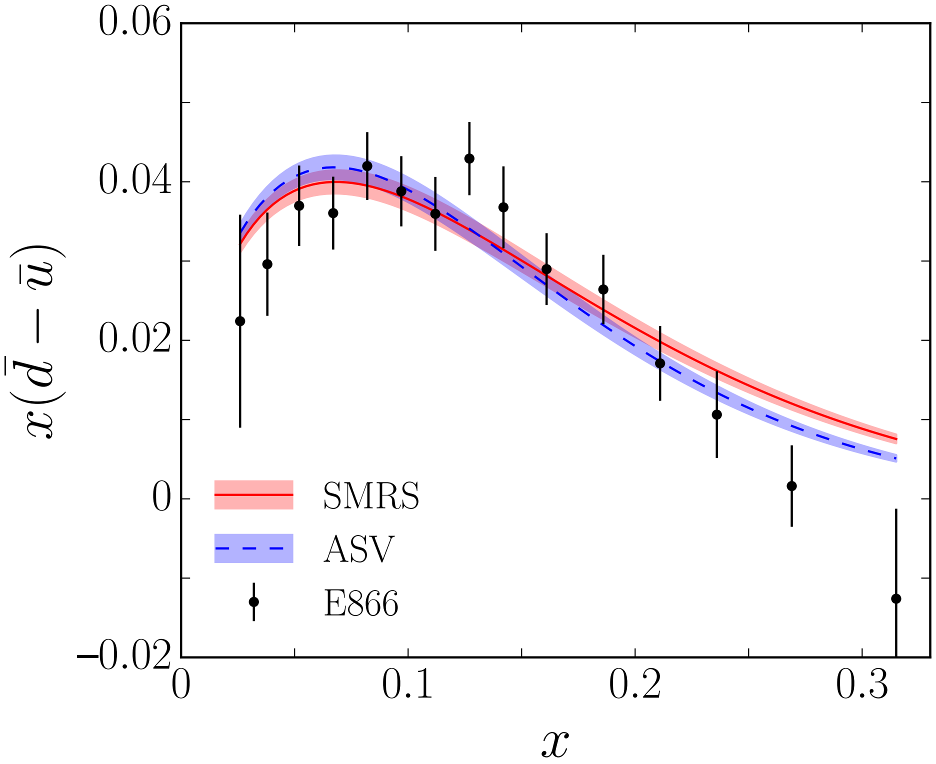
$$\sim\ 0.25 - 0.3$$

→ with exception of $k_\perp$ cutoff and Bishari models, all others give reasonable fits, $\chi^2 \lesssim 1.5$

→ are there other data that can be more discriminating?

Parton distributions

- E866 $\bar{d} - \bar{u}$ data can be fitted with range of regulators



average pion "multiplicity"

$$\langle n \rangle_{\pi N} = 3 \int_0^1 dy\, f_N^{(\text{on})}(y)$$

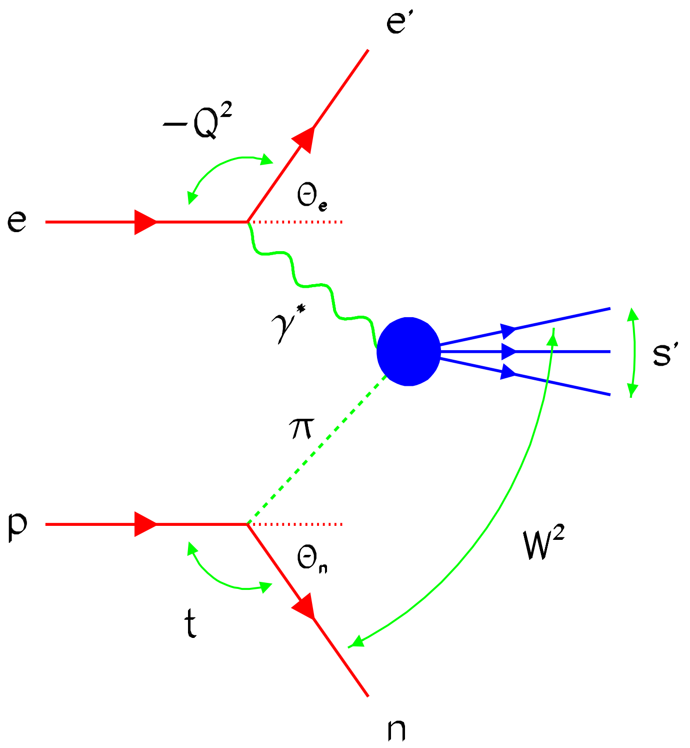
$$\sim \ 0.25 - 0.3$$

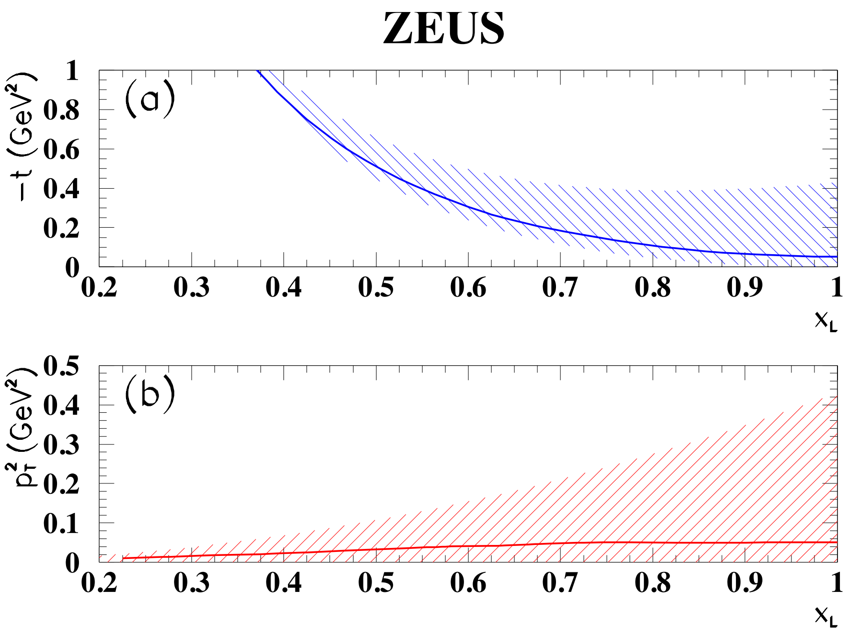
→ with exception of $k_\perp$ cutoff and Bishari models,
all others give reasonable fits, $\chi^2 \lesssim 1.5$

→ are there other data that can be more discriminating?

Leading neutrons at HERA

- ZEUS & H1 collaborations measured spectra of neutrons produced at very forward angles, $\theta_n < 0.8 \text{ mrad}$





$$x_L \approx 1 - y$$

→ can data be described within same framework as E866 asymmetry?

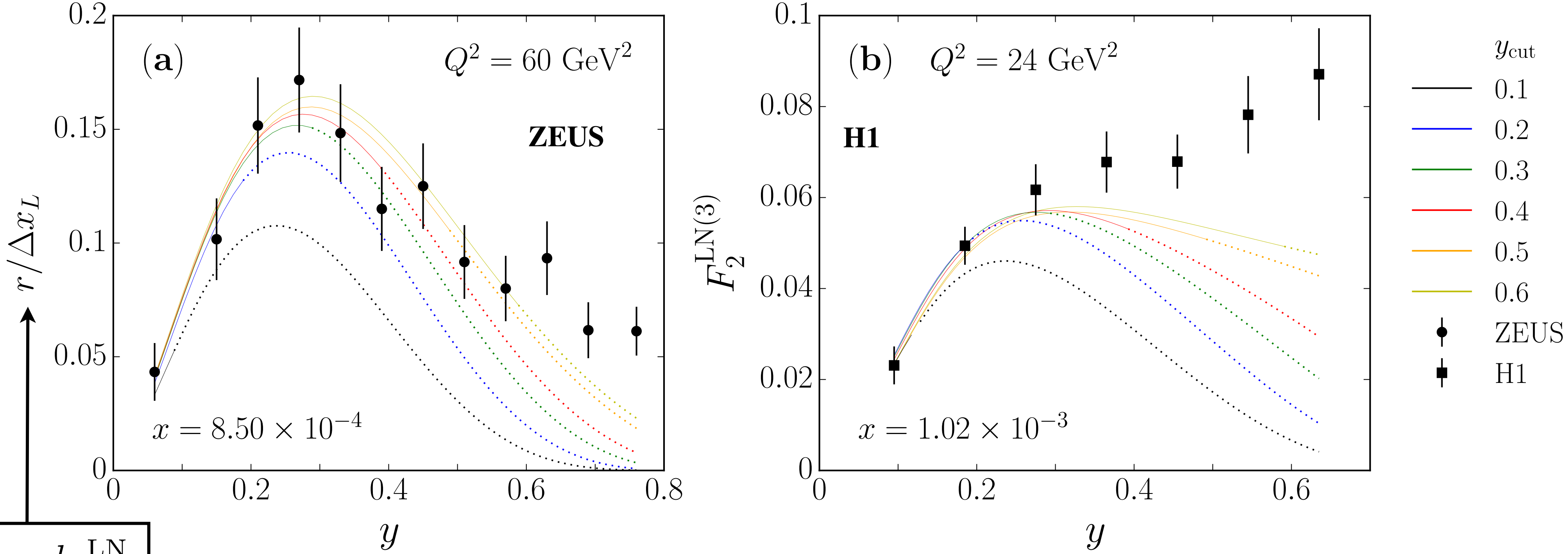
Leading neutrons at HERA

- Measured LN differential cross section (integrated over $p_\perp$)

$$\frac{d^3\sigma^{\rm LN}}{dx\,dQ^2\,dy} \sim F_2^{\rm LN(3)}(x,Q^2,y)$$

$$\boxed{2 f_N^{\rm (on)}(y)\, F_2^\pi(x/y,Q^2)}$$ for π exchange

e.g.

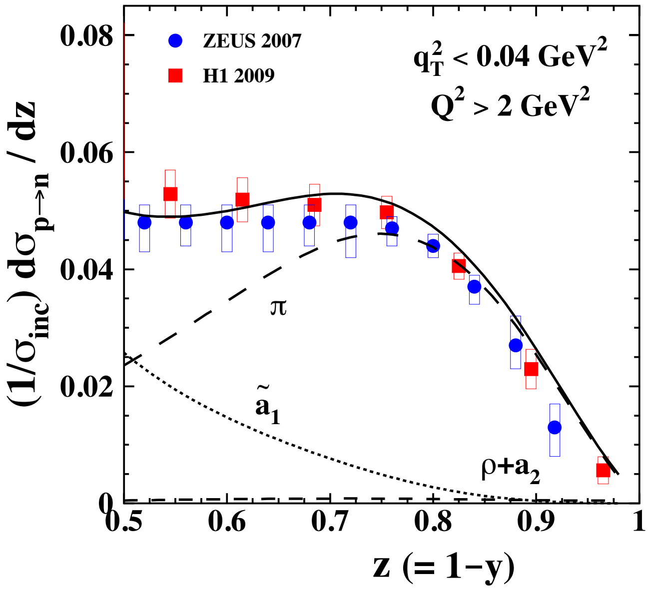


$$\boxed{r = \frac{d\sigma^{\rm LN}}{d\sigma^{\rm inc}}}$$

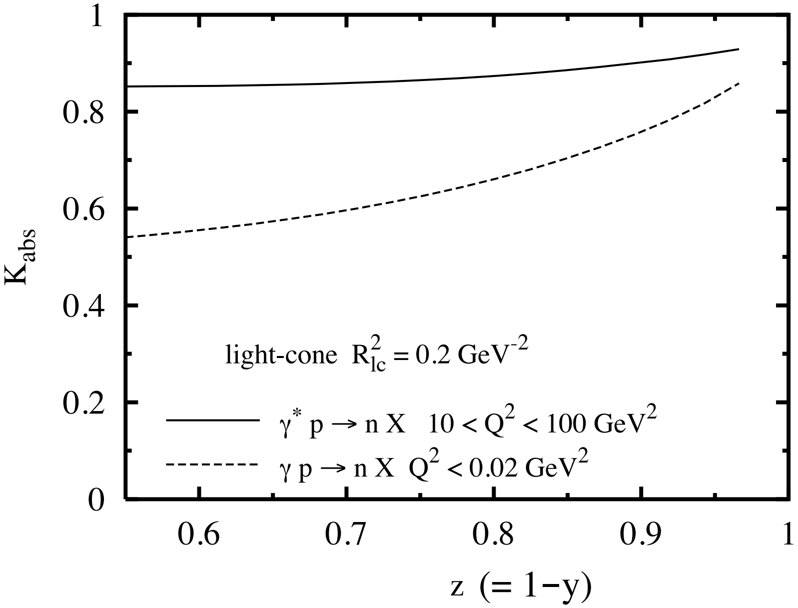
→ quality of fit depends on range of y fitted

Leading neutrons at HERA

- At large *y*, non-pionic mechanisms contribute
  (*e.g.* heavier mesons, absorption)



Kopeliovich et al. (2012)

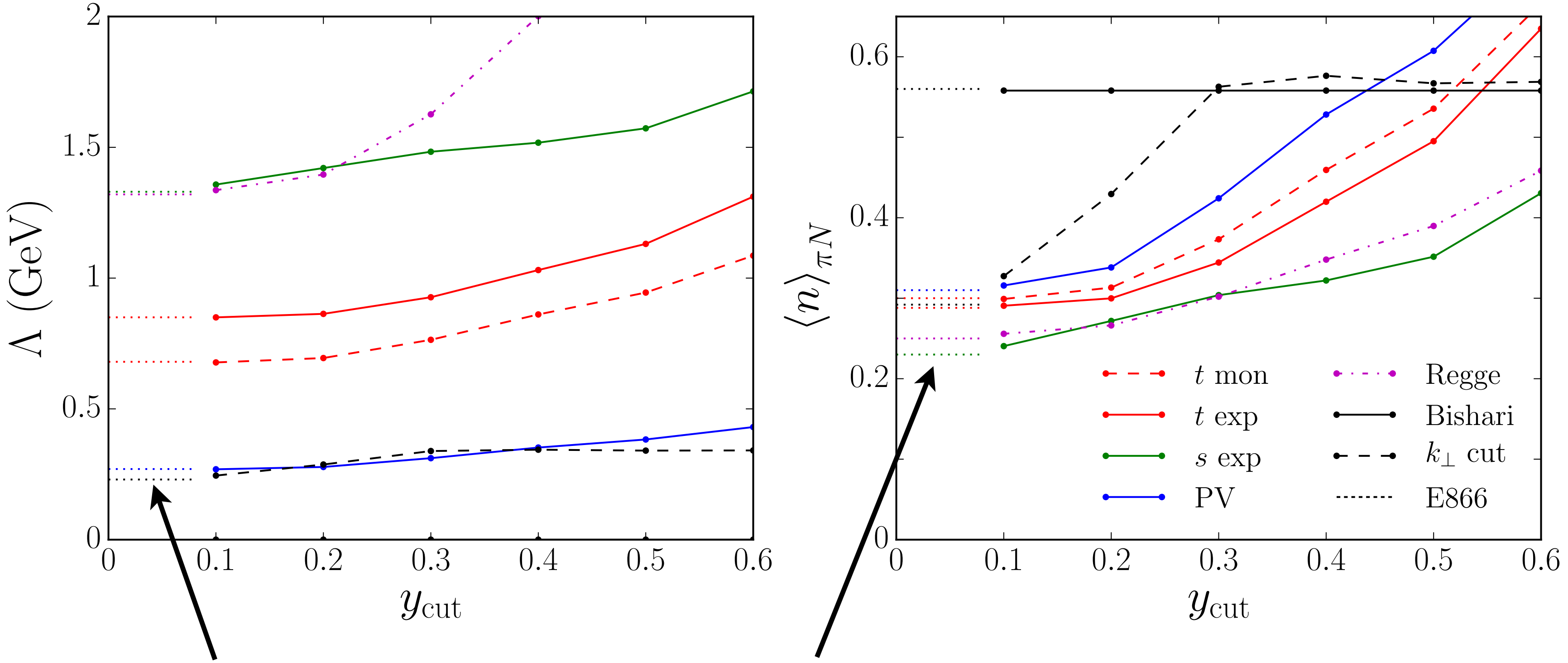


D'Alesio, Pirner (2000)

- To reduce model dependence, fit the value of $y_{\rm cut}$
  up to which data can be described in terms of π exchange

Leading neutrons at HERA

- Fit requires higher momentum pions with increasing y_{cut}

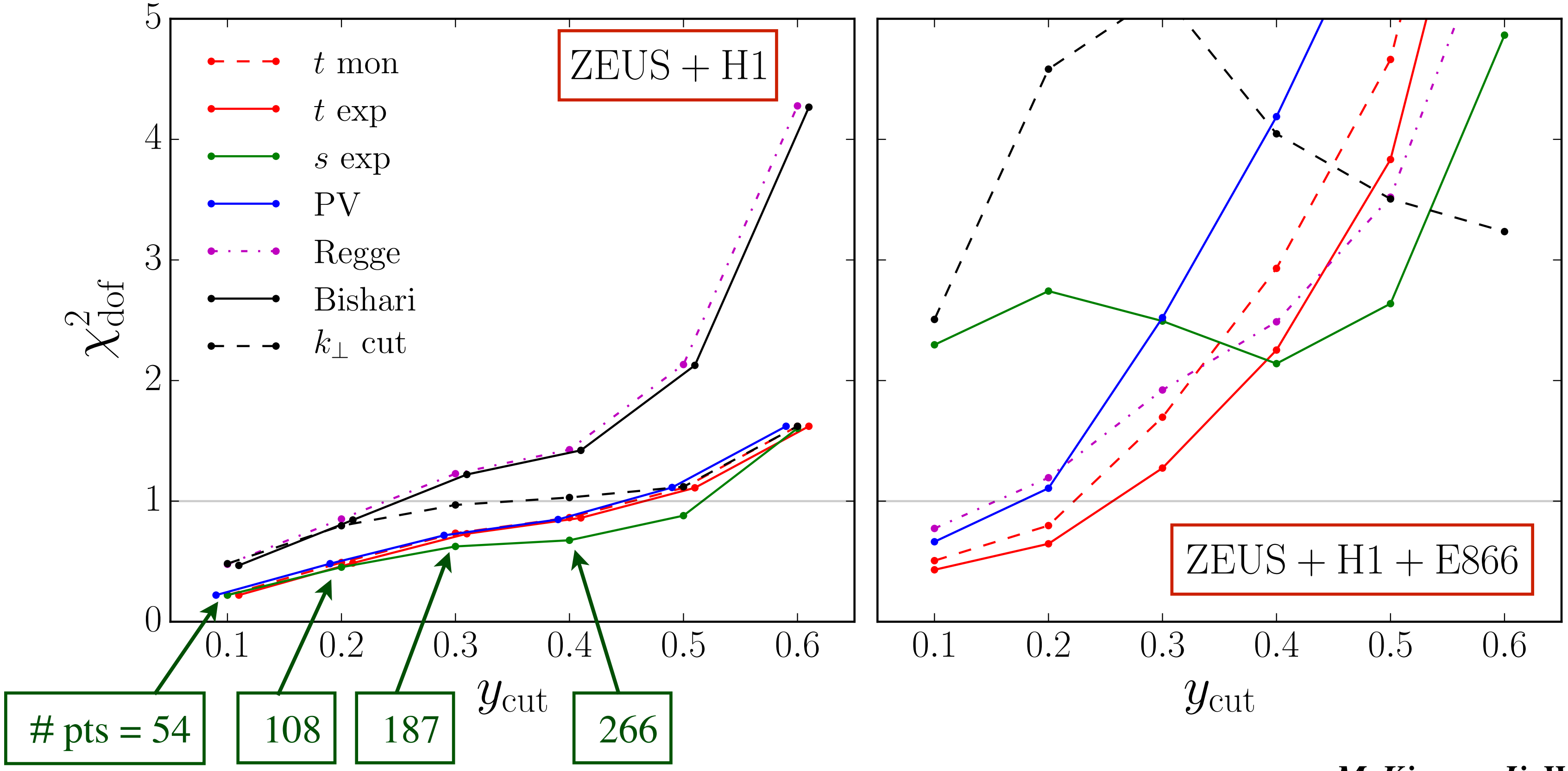


values from fit to E866 data only

→ larger values of y_{cut} more in conflict with E866 data

Leading neutrons at HERA

- Combined fit to HERA LN and E866 Drell-Yan data

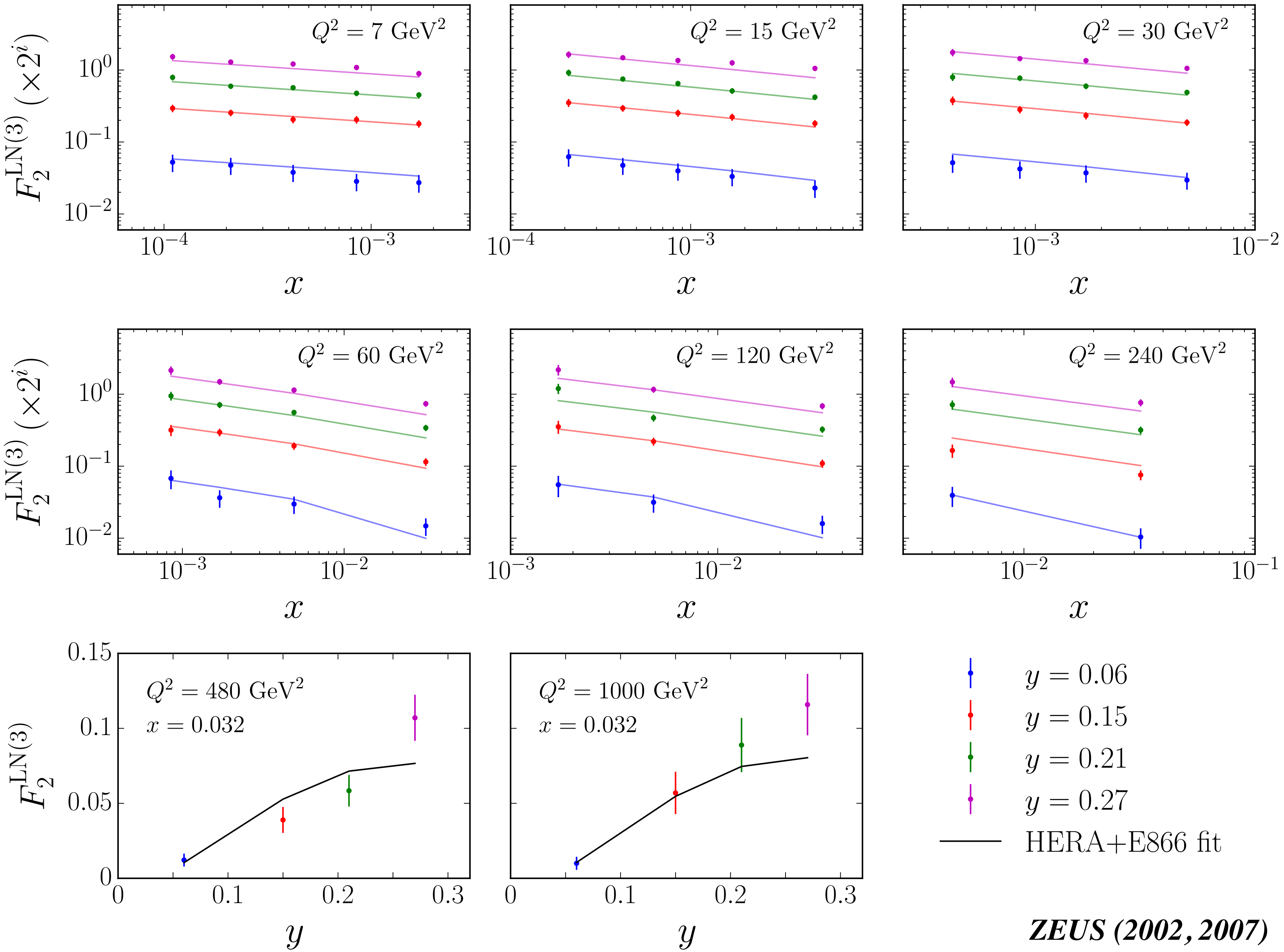


McKinney, Ji, WM, Sato (2016)

→ best fits for largest number of points afforded by t-dependent exponential (and t monopole) regulators

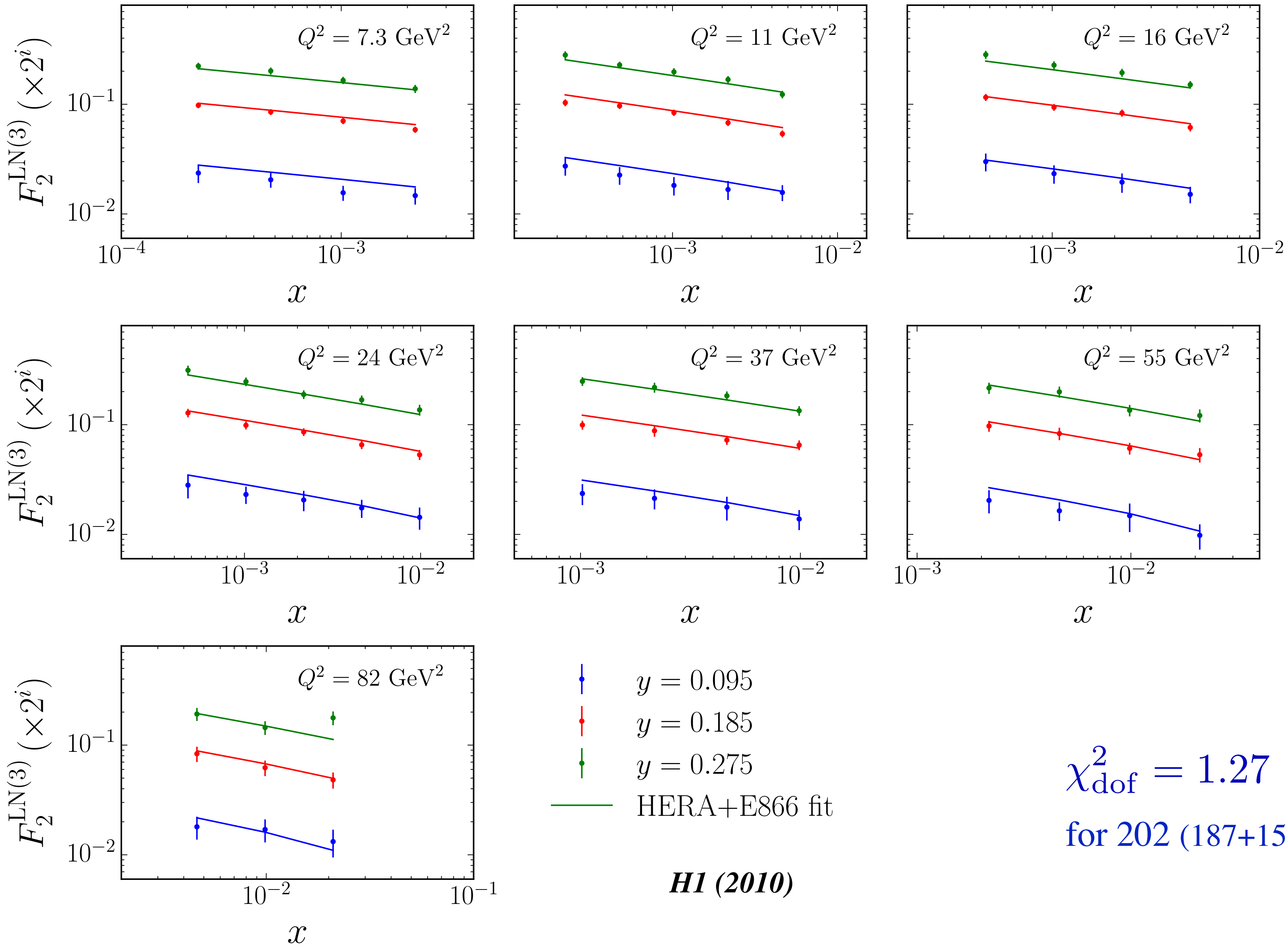
Leading neutrons at HERA

- 🟩 Fit to ZEUS LN spectra for $y_{\rm cut} = 0.3$ (t-dependent exponential)



Leading neutrons at HERA

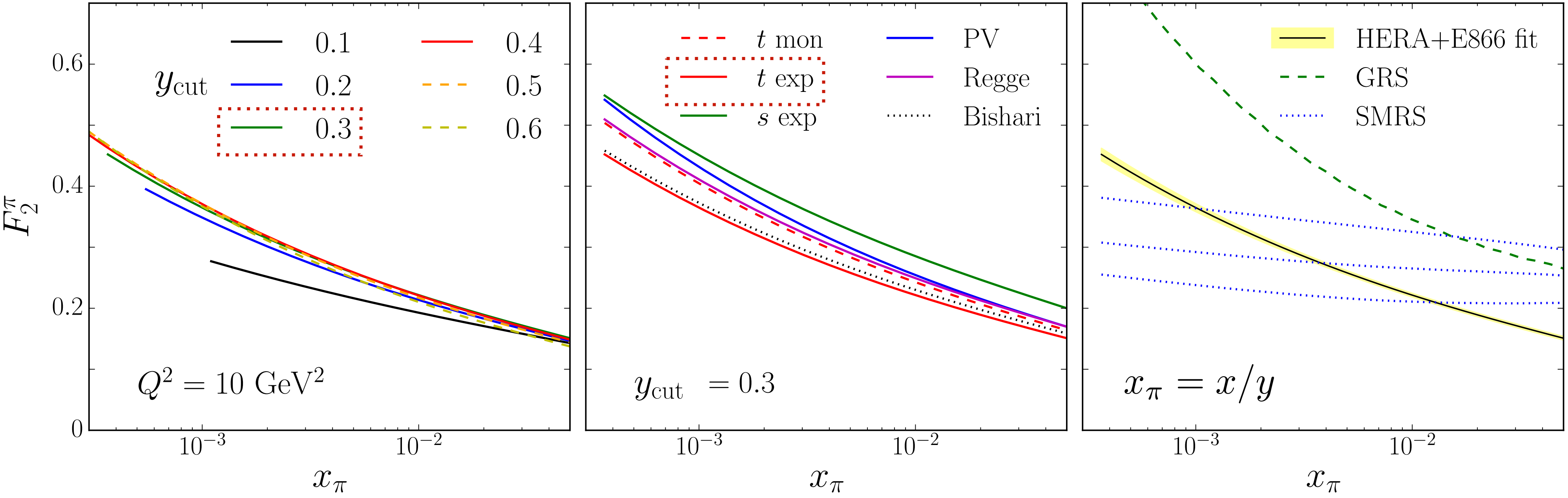
- Fit to H1 LN spectra for $y_{\rm cut} = 0.3$ (t-dependent exponential)



$\chi^2_{\rm dof} = 1.27$

for 202 (187+15) points

Pion structure function



$$F_2^\pi = N\, x_\pi^a\, (1 - x_\pi)^b, \quad a = a_0 + a_1 \eta$$

$$\eta \sim \log(\log Q^2)$$

→ stable values of F_2^π at $4 \times 10^{-4} \lesssim x_\pi \lesssim 0.03$ from combined fit

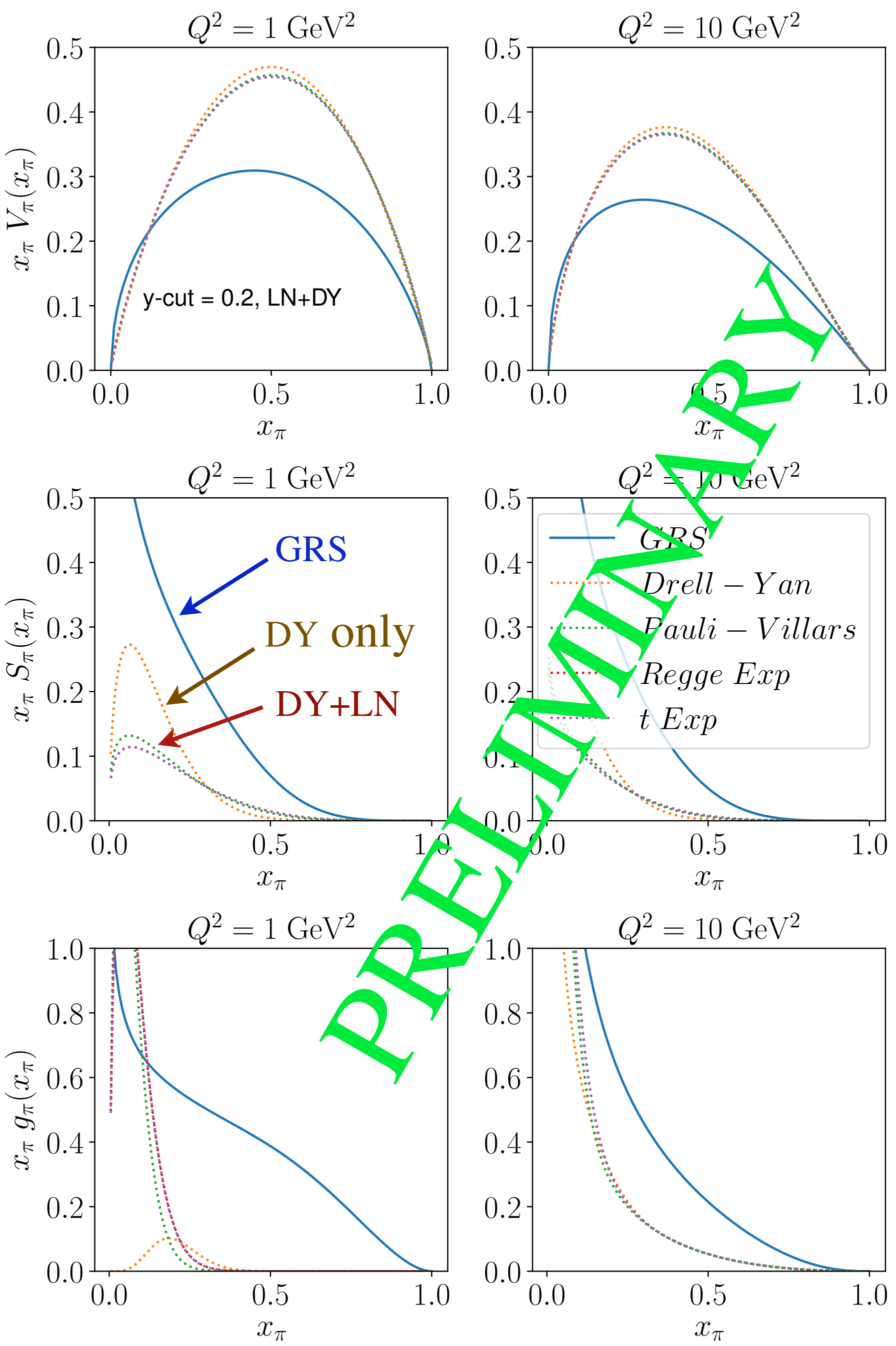
→ shape similar to GRS fit to πN Drell-Yan data (for $x_\pi \gtrsim 0.2$), but smaller magnitude

Pion structure function

- 🟩 Combine "leading neutron" data with πN Drell-Yan data to constrain pion PDFs at low and high x

  → aim: use nested sampling MC algorithm; first determination of pion PDF uncertainties

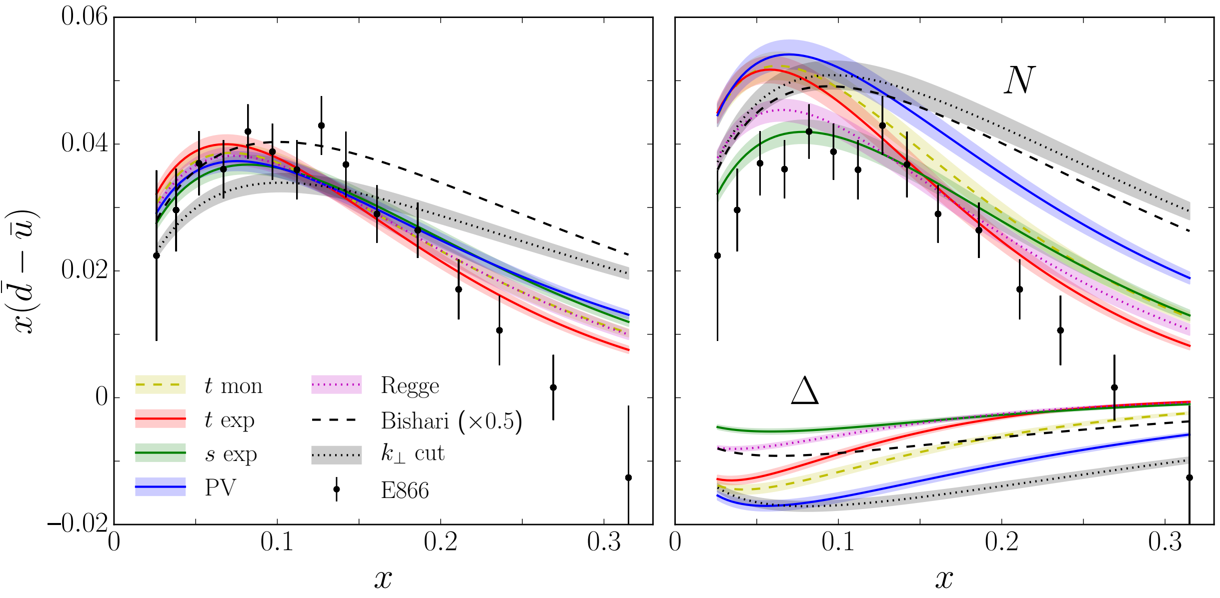
  → preliminary (single-fit!) results suggest much smaller pion sea *cf.* GRS



Barry, Sato, WM, C. Ji (2017)

Sign change at large x?

- E866 data has driven successful phenomenology through interplay of PDFs and chiral physics



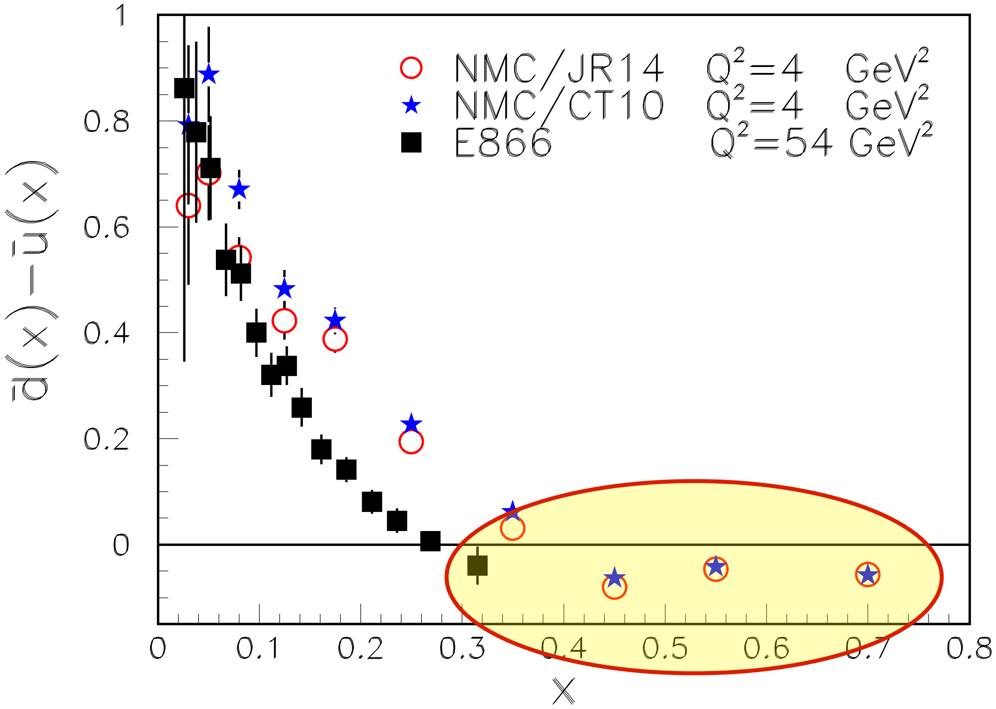
… but lingering question of possible sign change of $\bar{d}-\bar{u}$ at high x

$\longrightarrow$ sign change cannot be accommodated within chiral EFT framework since (negative) Δ contribution << (positive) N contribution

$\longrightarrow$ evidence for other mechanisms?

Sign change at large x?

- "Independent evidence for $\bar{d}-\bar{u}$ sign change at $x \sim 0.3$" from NMC?



Peng et al. (2014)

$$\bar{d}-\bar{u} \;\equiv\; \frac{1}{2}(u_v-d_v)-\frac{3}{2x}(F_2^p-F_2^n)$$

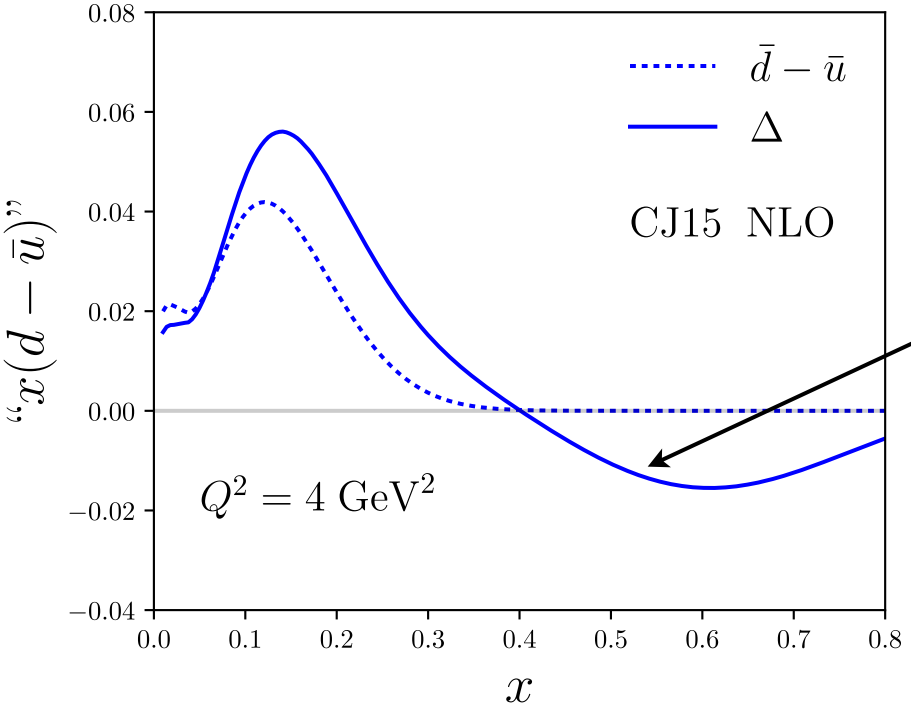
→ conclusions based on LO analysis … how robust?

Sign change at large x?

- At higher order can easily generate zero crossing in

$$\Delta \equiv \frac{1}{2}(u_v - d_v) - \frac{3}{2x}(F_2^p - F_2^n)$$

with no $\bar{d} - \bar{u}$ asymmetry!

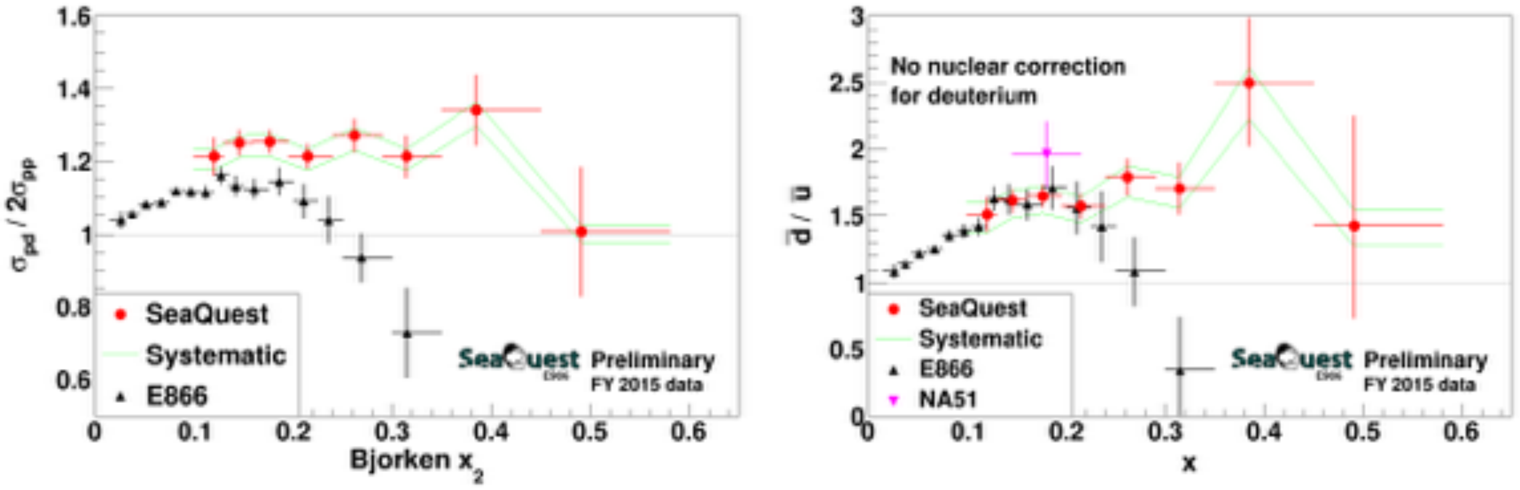


“sign change”
for NMC data

→ no evidence of sign change from DIS data!

Sign change at large x?

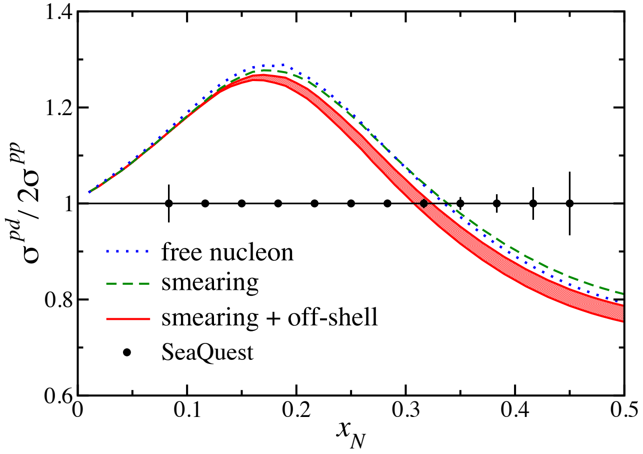
- Preliminary data from SeaQuest (E906) Drell-Yan experiment at Fermilab shows no evidence for sign change



P. Reimer (2016)

→ SeaQuest data consistent with E866 data up to $x \sim 0.2$, remains above unity up to $x \sim 0.5$

→ Results not significantly affected if include nuclear corrections

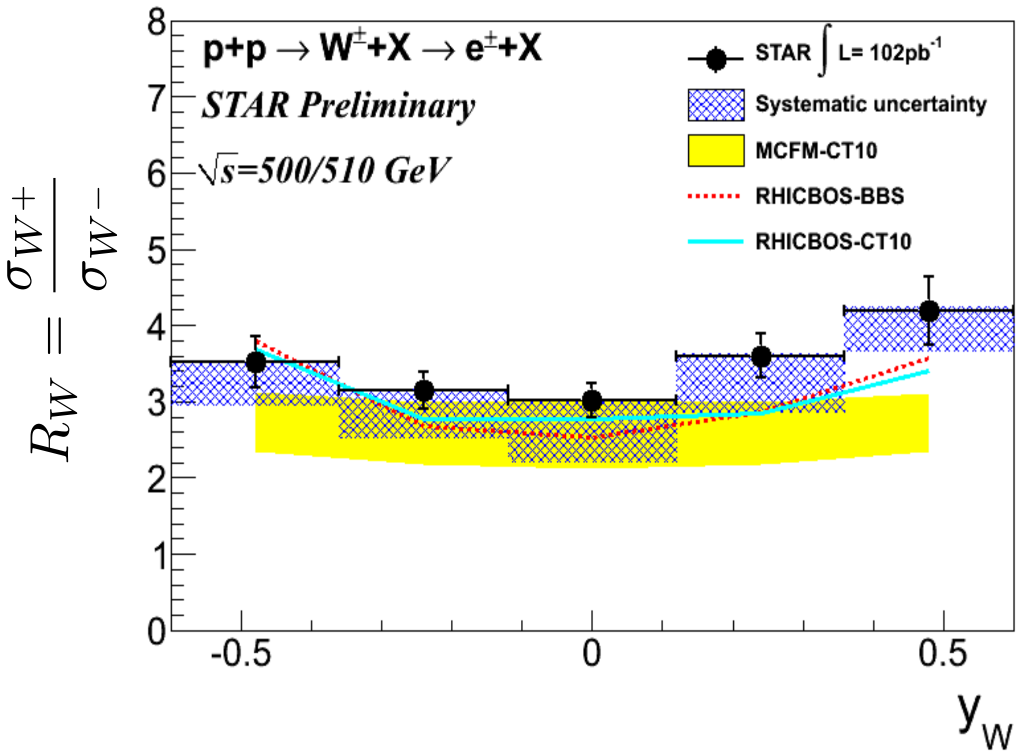


Ehlers, Accardi, Brady, WM (2014)

Sign change at large x?

Results / Status: Cross-section ratio W⁺/W⁻

- STAR: W cross-section ratio measurements at (Run 11 / 500GeV) (Run 12 / 510GeV)



$$y_W \sim 0.5$$

$$\Longrightarrow \quad x_1 \sim 0.26$$

not competitive with SeaQuest for these kinematics — need larger rapidity!

- W boson kinematics can be determined by reconstructing the W kinematics via its recoil
- Combination of data/MC simulations allows W boson rapidity reconstruction
- Critical for transverse single-spin asymmetry result of W production probing Sivers sign change

S. Fazio et al. (STAR Collaboration), DIS 2015.

$\bar{d} - \bar{u}$ asymmetry in Δ^+ ?

- Is there a similar $\bar{d} - \bar{u}$ asymmetry in Δ^+ as in proton?
- Simply on the basis of isospin couplings…

$p \to$	$\pi^+ \; n$	**2/3**
	$\pi^0 \; p$	**1/3**

$p \to$	$\pi^+ \; \Delta^0$	**1/6**
	$\pi^0 \; \Delta^+$	**1/3**
	$\pi^- \; \Delta^{++}$	**1/2**

$\Delta^+ \to$	$\pi^+ \; \Delta^0$	**8/15**
	$\pi^0 \; \Delta^+$	**1/15**
	$\pi^- \; \Delta^{++}$	**2/5**

$\Delta^+ \to$	$\pi^+ \; n$	**1/3**
	$\pi^0 \; p$	**2/3**

⟶ assuming similar $p \to \pi N$ & $\Delta^+ \to \pi \Delta$ splitting functions

$$(\bar{d} - \bar{u})_p \; : \; (\bar{d} - \bar{u})_{\Delta^+} \; = \; 5 : 1$$

⟶ quantitative EFT-based calculation under way;
lattice QCD calculation planned

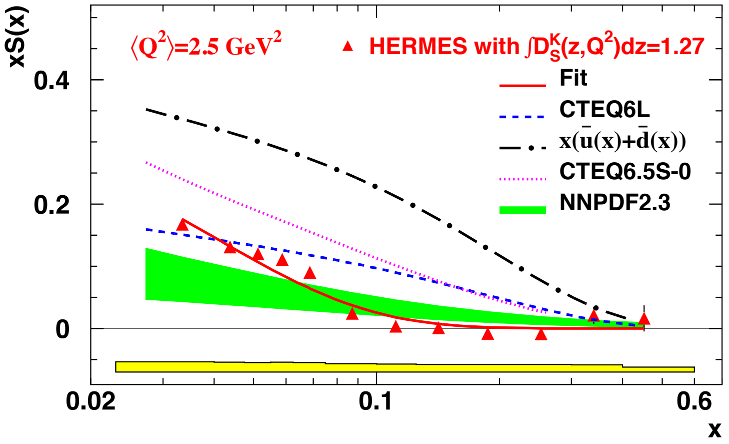
Ethier, WM, Steffens, Thomas (2017)

Strange quarks

- Traditionally, strange quark PDFs most directly determined from $\mu^+\mu^-$ production in $\nu(\bar{\nu})\,A$ DIS $(W^+ s \to c \;/\; W^- \bar{s} \to \bar{c})$

  → but significant uncertainty from nuclear corrections, semileptonic branching ratio uncertainty

  → tension with HERMES semi-inclusive K-production data?



historically, strange to nonstrange ratio

$$\kappa = \frac{s+\bar{s}}{\bar{u}+\bar{d}}$$

$$\sim\; 0.2 - 0.5$$

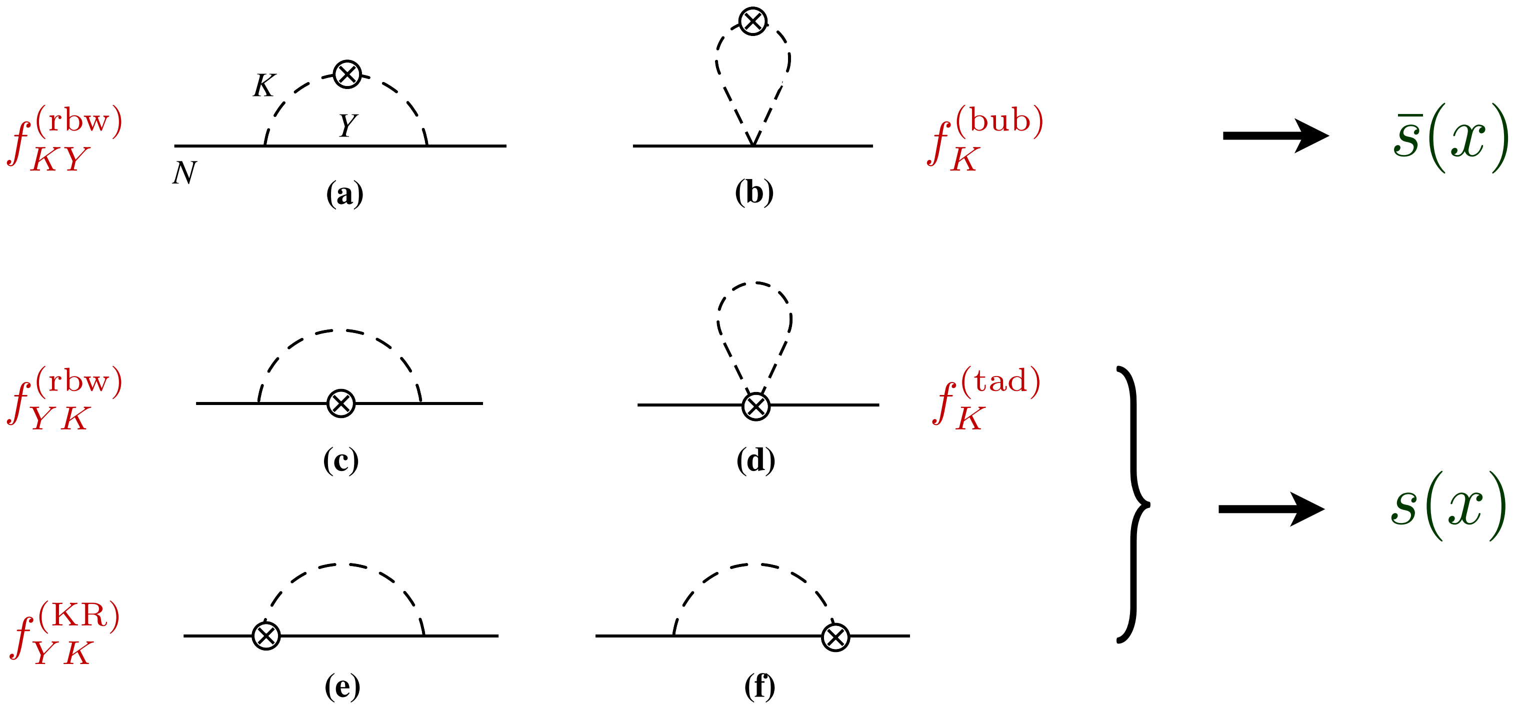
- Some indication of strange–antistrange asymmetry from $\nu/\bar{\nu}$ DIS data

$$S^- = \int_0^1 dx\, x(s-\bar{s}) \;= (2.0 \pm 1.4) \times 10^{-3}$$

NuTeV (2007)

Strange quarks

- Chiral $\mathrm{SU}(3)$ effective theory analysis suggests natural mechanism for generating strange asymmetry



→ gauge invariance requires the relations

$$f_{YK}^{(\mathrm{rbw})} + f_{YK}^{(\mathrm{KR})} = f_{KY}^{(\mathrm{rbw})}$$

$$f_K^{(\mathrm{tad})} + f_K^{(\mathrm{bub})} = 0$$

Strange quarks

Convolution representation

$$\bar{s} = \left(f_{KY}^{(\mathrm{rbw})} + f_{K}^{(\mathrm{bub})} \right) \otimes \bar{s}_K$$

$$s = \left(\bar{f}_{YK}^{(\mathrm{rbw})} \otimes s_Y + \bar{f}^{(\mathrm{KR})} \otimes s_Y^{(\mathrm{KR})} \right) + \bar{f}_K^{(\mathrm{tad})} \otimes s_K^{(\mathrm{tad})}$$

$\bar{f}(y) \equiv f(1-y)$

$\sim \Delta u, \Delta d$

$\sim u, d$

→ KY splitting functions regularized using Pauli-Villars regularization

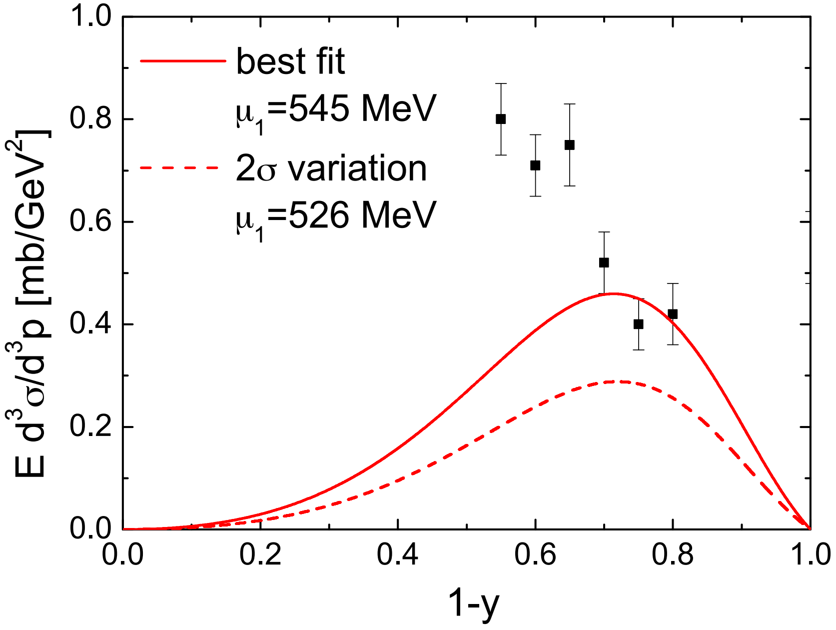
→ δ-function term requires 2 subtractions (parameters μ_1, μ_2)

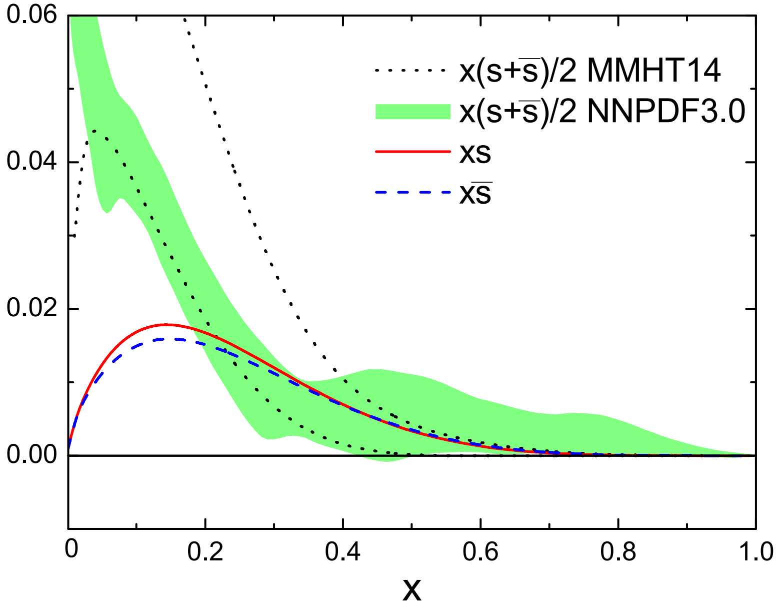
→ since $f_K^{(\mathrm{tad})}(y) \sim \delta(y)$, tadpole term generates *valence-like* strange-quark PDF

$$\sim \; s_K^{(\mathrm{tad})}(x)$$

Strange quarks

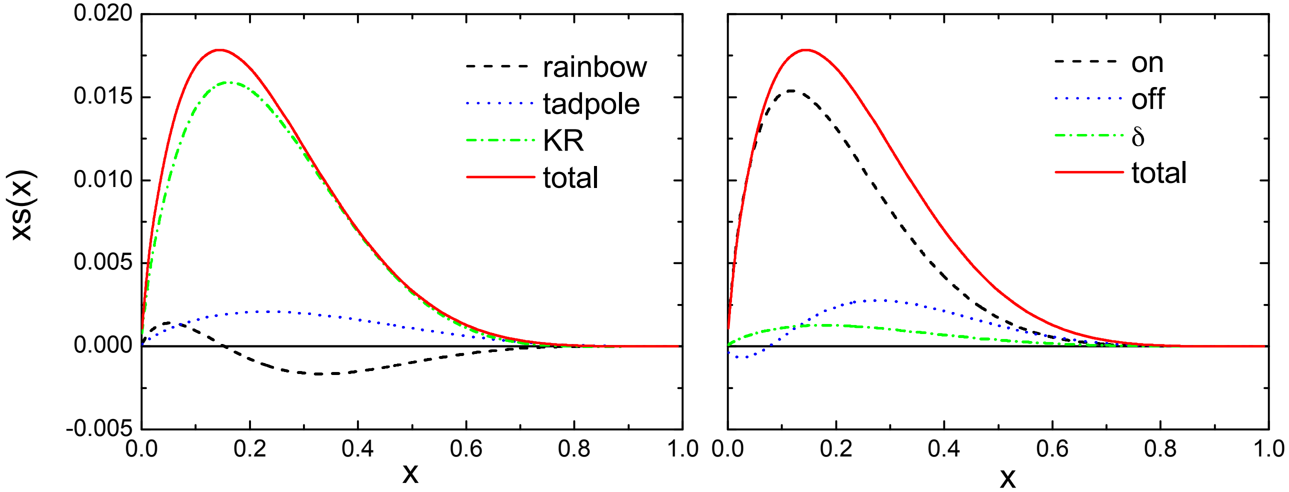
- Constraints on cutoff parameters from $pp \to \Lambda\, X$ and total $(s+\bar{s})_{\mathrm{loops}} \leq (s+\bar{s})_{\mathrm{total}}$





Strange quarks

- Breakdown into individual contributions to $s(x)$

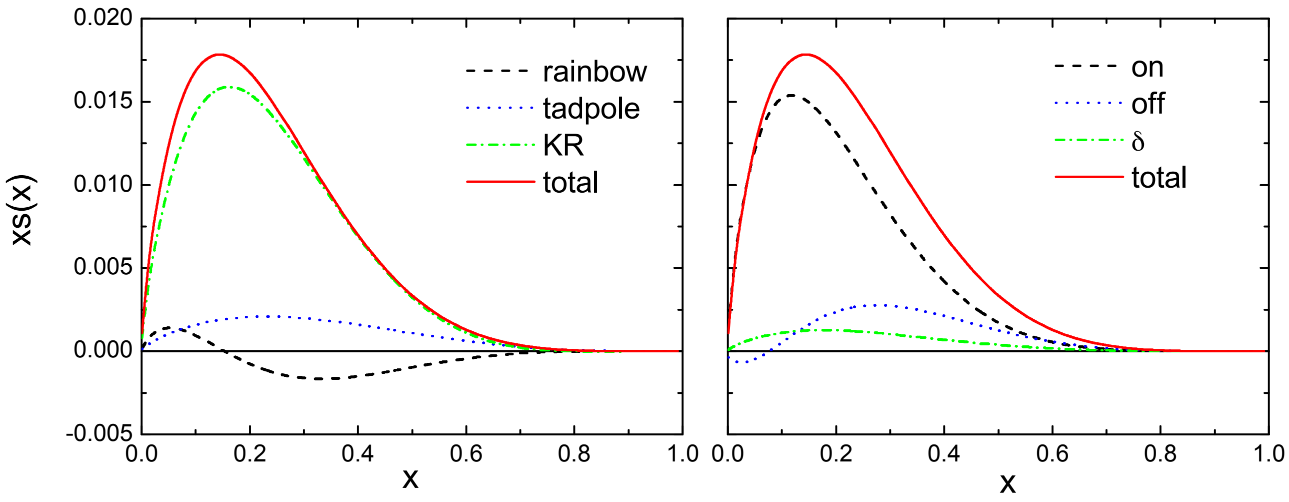


$$\boxed{\begin{aligned} s(x) &= \left(s^{(\mathrm{on})} + s^{(\mathrm{off})} + s^{(\delta)}\right)_{\mathrm{rbw}} + s^{(\delta)}_{\mathrm{tad}} + \left(s^{(\mathrm{off})} + s^{(\delta)}\right)_{\mathrm{KR}} \\ &= \underbrace{s^{(\mathrm{on})}_{\mathrm{rbw}}}_{\text{on-shell}} + \underbrace{s^{(\mathrm{off})}_{\mathrm{rbw}} + s^{(\mathrm{off})}_{\mathrm{KR}}}_{\text{off-shell}} + \underbrace{s^{(\delta)}_{\mathrm{rbw}} + s^{(\delta)}_{\mathrm{tad}} + s^{(\delta)}_{\mathrm{KR}}}_{\delta\text{-function}}, \end{aligned}}$$

$$\boxed{\begin{aligned} \bar{s}(x) &= \left(\bar{s}^{(\mathrm{on})} + \bar{s}^{(\delta)}\right)_{\mathrm{rbw}} + \bar{s}^{(\delta)}_{\mathrm{bub}} \\ &= \underbrace{\bar{s}^{(\mathrm{on})}_{\mathrm{rbw}}}_{\text{on-shell}} + \underbrace{\bar{s}^{(\delta)}_{\mathrm{rbw}} + \bar{s}^{(\delta)}_{\mathrm{bub}}}_{\delta\text{-function}}, \end{aligned}}$$

Strange quarks

- Breakdown into individual contributions to $s(x)$



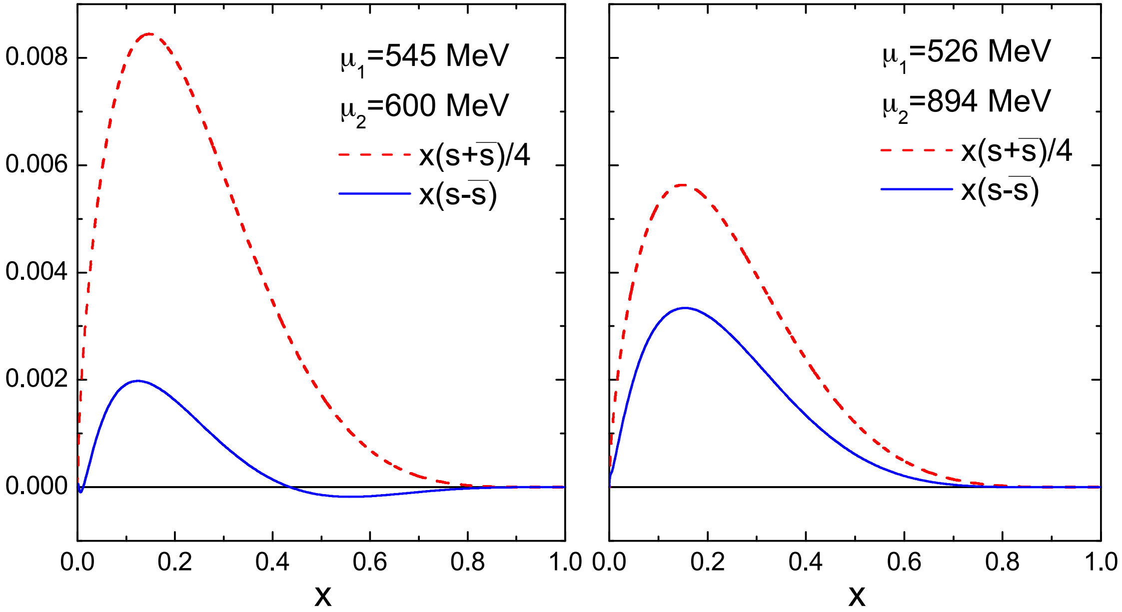
→ large cancellations between off-shell terms in rainbow & KR and between δ-function terms in rainbow, KR and tadpole

→ total $s(x)$ well approximated by on-shell part of rainbow, total off-shell & δ-function terms small

→ explains phenomenological success of earlier loop calculations in terms of on-shell rainbow only

Strange quarks

- Gives rise to small but (mostly) positive $s - \bar{s}$ distribution



→ x-weighted difference $S^- = (0.4 - 1.1) \times 10^{-3}$

X. Wang, C. Ji, WM, Salamu, Thomas, P. Wang (2016)

Outlook

- Discussion of flavor asymmetries in the nucleon now on much firmer theoretical footing

- Ongoing global PDF analysis of "leading neutron" and Drell-Yan data to constrain pion PDFs at low and high x

- Simultaneous analysis of other asymmetries, such as $s - \bar{s}$, $(\bar{d} - \bar{u})_{\Delta}$ $\ldots$ should help reveal nonperturbative origin of the sea