Moments of the pion distribution amplitude from lattOPE with a valence heavy quark

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The flavour structure of nucleon sea
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W.Detmold and CJDL, Phys.Rev.D73, 014501 (2006) [hep-lat/0507007]
& work in progress

Collaborators

- William Detmold (MIT)
- Santanu Mondal (Nat’l Chiao-Tung University)

Outline

- Motivation and general strategy.
- Lattice OPE and the structure functions.
- Lattice OPE and the pion light-cone wavefunction.
- (Extremely) exploratory numerical result.
- Outlook

Motivation and general strategy

W.Detmold and CJDL, Phys.Rev.D73, 014501 (2006) [hep-lat/0507007]

Parton distribution from lattice QCD

The "traditional" approach

- Hadronic tensor

$$W_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|\, [J^\mu(x), J^\nu(0)]\, |p,S\rangle$$

optical theorem

Imaginary part → challenging in Euclidean QCD

$$T_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S| T\, [J^\mu(x) J^\nu(0)]\, |p,S\rangle$$

- Light-cone OPE

$$T[J^\mu(x)J^\nu(0)] = \sum_{i,n} \mathcal{C}_i\left(x^2,\mu^2\right)\, x_{\mu_1}\ldots x_{\mu_n} \mathcal{O}_i^{\mu\nu\mu_1\ldots\mu_n}(\mu)$$

local operators, issue of operator mixing

leading moments in practice

Power divergences arising from Lorentz symmetry breaking

Introducing the valence heavy quark

- Valence → not in the action.
- The "heavy quark" is relativistic. → propagating in both space and time
- The current for computing the even moments of the PDF

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)$$

↓ Euclidean Compton tensor

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_{S} \langle p,S| t^{\mu\nu}_{\Psi,\psi}(q) |p,S\rangle = \sum_{S} \int d^4x \; \mathrm{e}^{iq\cdot x} \langle p,S| T\left[J^{\mu}_{\Psi,\psi}(x) J^{\nu}_{\Psi,\psi}(0) \right] |p,S\rangle$$

Strategy for extracting the moments

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_S \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_S \int d^4x\ \mathrm{e}^{iq\cdot x} \langle p,S|T\left[J^{\mu}_{\Psi,\psi}(x)J^{\nu}_{\Psi,\psi}(0)\right]|p,S\rangle$$

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)$$

- Simple renormalisation for quark bilinears.

- Work with the hierarchy of scales

$$\Lambda_{\mathrm{QCD}} \ll m_{\Psi} \sim \sqrt{q^2} \ll \frac{1}{\hat{a}}$$ → Need very fine lattices

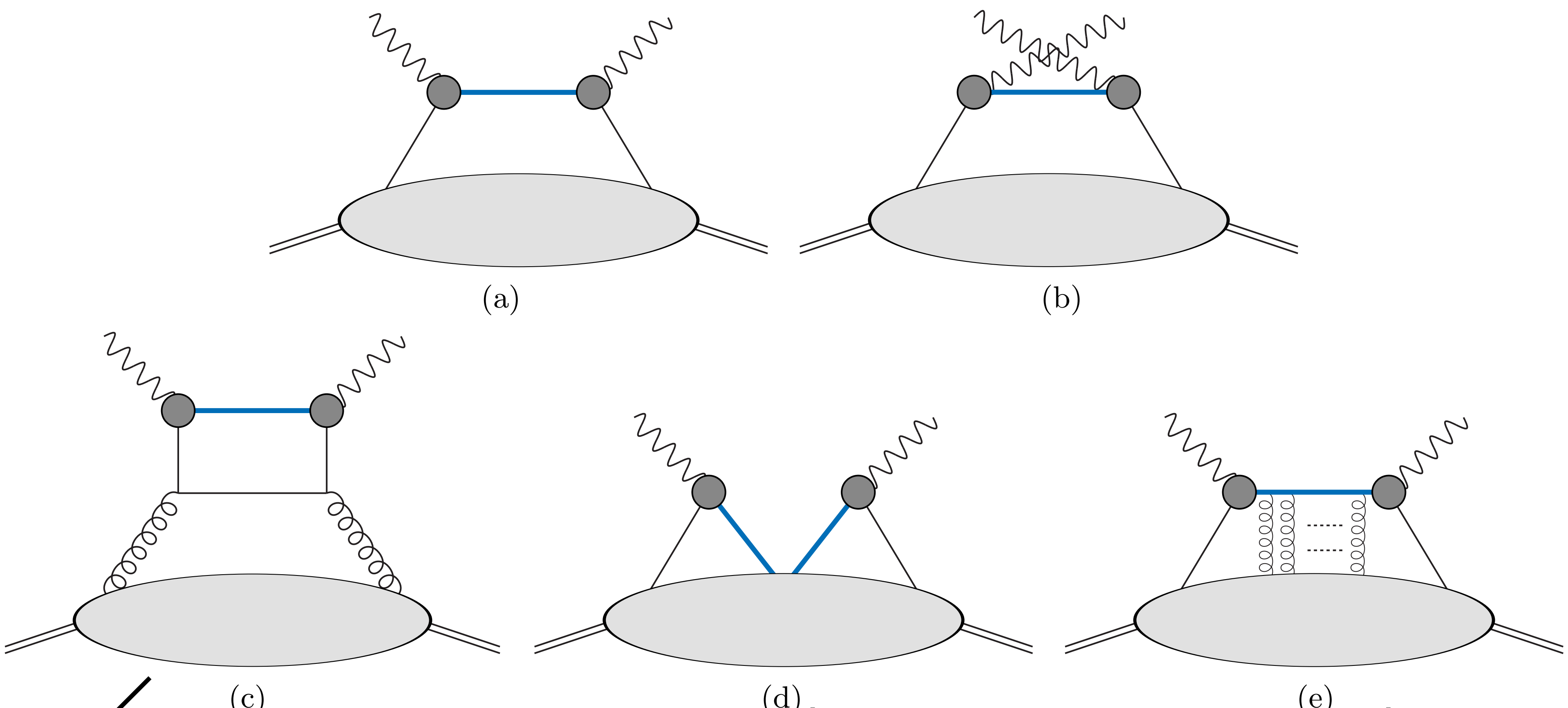
- Extrapolate $T^{\mu\nu}_{\Psi,\psi}(p,q)$ to the continuum limit first.

→ Then match to the Euclidean OPE results.

→ Extract the moments without power divergence.

Euclidean OPE and valence heavy quark

These are the leading-twist contributions that we are after.



leading twist, absent in $T^{\mu\nu}_{\Psi,v} = T^{\mu\nu}_{\Psi,u} - T^{\mu\nu}_{\Psi,d}$

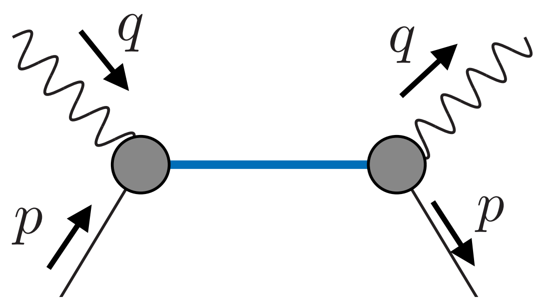
higher twist, absent

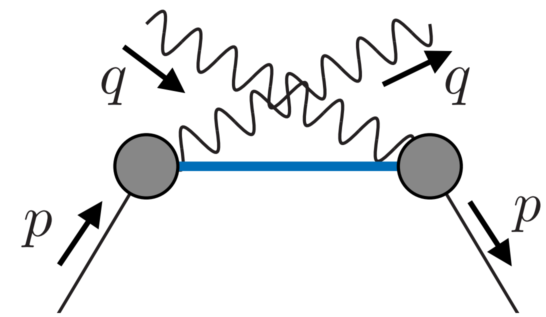
leading and higher twist

Lattice OPE and structure functions

W.Detmold and CJDL, Phys.Rev.D73, 014501 (2006) [hep-lat/0507007]

Key features of the Euclidean OPE



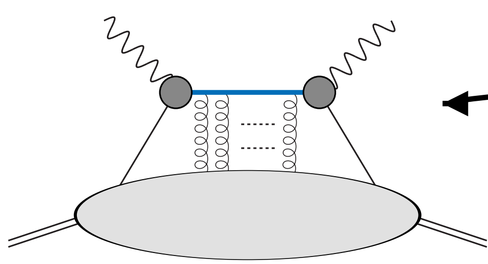


$$t^{\mu\mu}_{\Psi,\psi} = \bar{\psi}\gamma^{\mu}\frac{-i(\not{D} + \not{q}) + m_\Psi}{(iD+q)^2 + m_\Psi^2}\gamma^{\nu}\psi + \bar{\psi}\gamma^{\nu}\frac{-i(\not{D} - \not{q}) + m_\Psi}{(iD-q)^2 + m_\Psi^2}\gamma^{\mu}\psi$$

Standard way to proceed, just like the Minkowskian case

$$\frac{-i(\not{D} \pm \not{q}) + m_\Psi}{(iD \pm q)^2 + m_\Psi^2} = -\frac{-i(\not{D} \pm \not{q}) + m_\Psi}{Q^2 + D^2 - m_\Psi^2}\sum_{n=0}^{\infty}\left(\frac{-2iq\cdot D}{Q^2 + D^2 - m_\Psi^2}\right)^n$$

higher-twist contributions



(e)

Taylor expand *a'la* $\left(\frac{-2iq\cdot D + D^2}{Q^2 - m_\Psi^2}\right)$ → $\Lambda^2_{\mathrm{QCD}}/(q^2 + m_\Psi^2)$

Also $m_\Psi = M_\Psi - \alpha/2$ → $\tilde{Q}^2 = Q^2 + D^2 - m_\Psi^2 = Q^2 - M_\Psi^2 + \alpha + \beta$

physical

in principle can fit

Results for structure function

Simplifying *via* choosing $p=(0,0,0,i\,M)$ & $q=(0,0,\sqrt{q_0^2-Q^2},i\,q_0)$

$$T^{\{34\}}_{\Psi,\psi}(p,q) \;=\; \sum_{\substack{n=2\\ \text{even}}}^{\infty} A^n_\psi(\mu^2) f(n)$$

$$\sum_S \langle p,S|\mathcal{O}^{\mu_1\cdots\mu_n}_\psi|p,S\rangle \;=\; A^n_\psi(\mu^2)\,[p^{\mu_1}\ldots p^{\mu_n} - \text{traces}]$$

$$\mathcal{O}^{\mu_1\cdots\mu_n}_\psi = \overline{\psi}\gamma^{\{\mu_1}(iD^{\mu_2})\ldots(iD^{\mu_n\}})\psi - \text{traces}$$

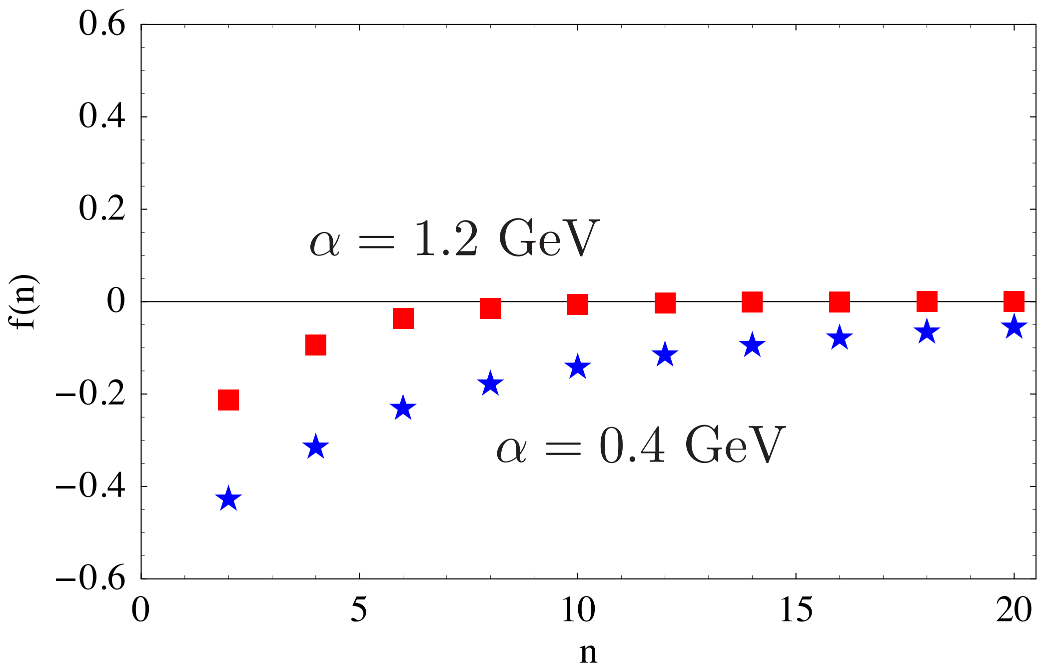
$$f(n) \;=\; -\sqrt{q_0^2-Q^2}\,\zeta^n\left\{\frac{2}{q_0}\left[\mathcal{C}_n\frac{\widetilde{Q}^2}{Q^2}\frac{(n-1)\eta C^{(2)}_{n-1}(\eta)-4\eta^2C^{(3)}_{n-2}(\eta)}{n(n-1)}+\mathcal{C}''_n\frac{\eta}{n}C^{(2)}_{n-1}(\eta)\right]\right.$$

$$\left.+\frac{q_0}{Q^2}\left[\mathcal{C}_n\frac{\widetilde{Q}^2}{Q^2}\frac{n(n-2)C^{(1)}_n(\eta)-2\eta(2n-3)C^{(2)}_{n-1}(\eta)+8\eta^2C^{(3)}_{n-2}(\eta)}{n(n-1)}+2\mathcal{C}''_n\left(C^{(1)}_n(\eta)-2\frac{\eta}{n}C^{(2)}_{n-1}(\eta)\right)\right]\right\}$$

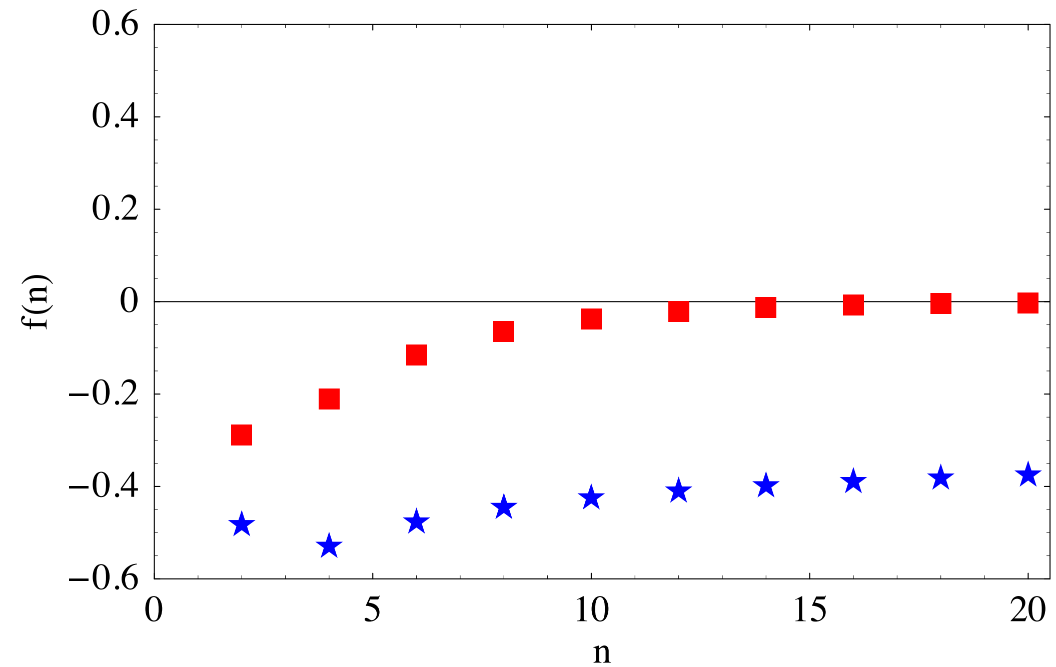
$$\zeta=\frac{\sqrt{p^2q^2}}{\widetilde{Q}^2},\qquad \eta=\frac{p\cdot q}{\sqrt{p^2q^2}}$$

Results for f(n)

$$T^{\{34\}}_{\Psi,\psi}(p,q) \;=\; \sum_{\substack{n=2\\ \text{even}}}^{\infty} A^n_\psi(\mu^2) f(n)$$



$M_\Psi = 3.54$ GeV, $Q^2 = 1.5$ GeV^2 and $q_0 = 2.76$ GeV



$M_\Psi = 2.1$ GeV, $Q^2 = -3.85$ GeV^2 and $q_0 = 1.98$ GeV

Lattice OPE
and the pion light-cone wavefunction

W.Detmold, CJDL, S.Mondal, work in progress

Pion light-cone wavefunction

Important input for flavour physics

$$\langle \pi^+(p)|\bar{u}(z/2)\gamma_5\gamma_\mu d(-z/2)|0\rangle = -ip_\mu f_\pi \int_0^1 d\xi \; \mathrm{e}^{i\left(\bar{\xi}p\frac{z}{2} - \xi p\frac{z}{2}\right)} \phi_\pi(\xi)$$

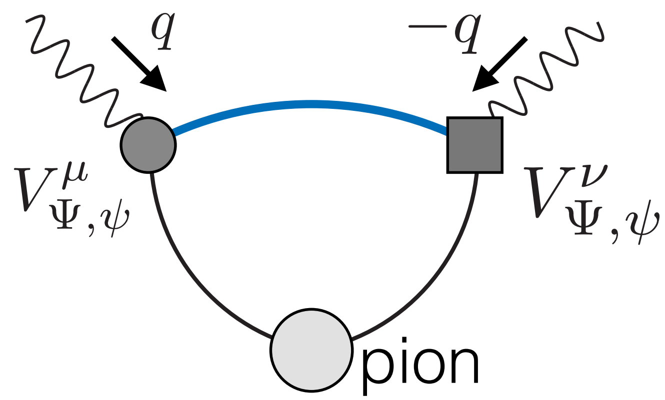
OPE

$$a_n = \int_0^1 d\xi \; \xi^n \phi_\pi(\xi)$$

$$\langle \pi^+(p)|O_\psi^{\mu_1\ldots\mu_n}|0\rangle = f_\pi a_{n-1}[p^{\mu_1}\ldots p^{\mu_n} - \text{traces}]$$

$$O_\psi^{\mu_1\ldots\mu_n} = \overline{\psi}\gamma_5\gamma^{\{\mu_1}(iD^{\mu_2})\ldots(iD^{\mu_n\}})\psi - \text{traces}$$

Lattice OPE, VV type setup



$$+ \begin{pmatrix} q \leftrightarrow -q \\ \mu \leftrightarrow \mu \end{pmatrix}$$

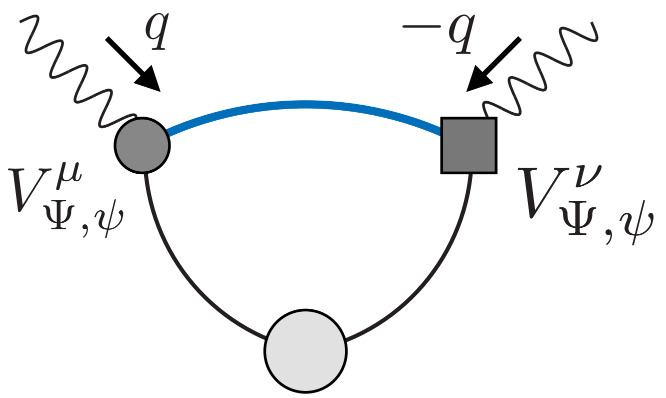
$$V^{\mu}_{\Psi,\psi} = \overline{\Psi}\gamma^{\mu}\psi + \overline{\psi}\gamma^{\mu}\Psi$$

$$V^{\nu}_{\Psi,\psi} = \overline{\Psi}\gamma^{\nu}\psi + \overline{\psi}\gamma^{\nu}\Psi$$

$$U^{\mu\nu}_{\Psi,\psi}(q) = \int d^4x \; e^{iqx} \; \langle \pi^+(p)|T[V^{\mu}_{\Psi,\psi}(x)V^{\nu}_{\Psi,\psi}]|0\rangle$$

μ and ν anti-symmetrised

Lattice OPE, VV type result



$$+ \left(\begin{matrix} q \leftrightarrow -q \\ \mu \leftrightarrow \mu \end{matrix} \right)$$

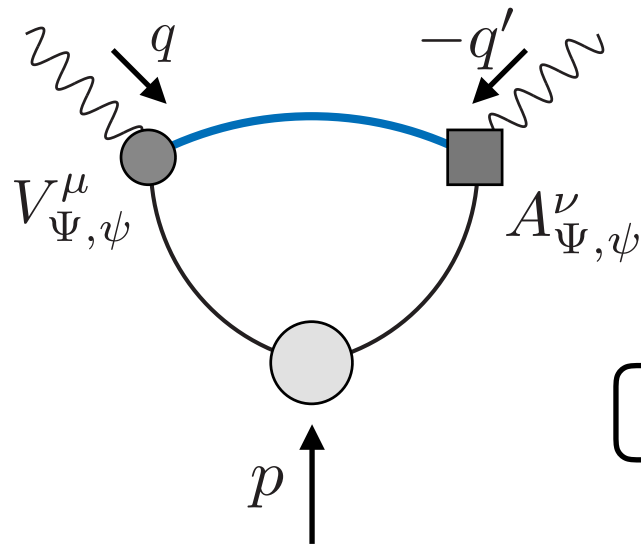
For simplicity, set the Wilson coefficients to unity

$$U^{[\mu\nu]}_{\Psi,\psi} = \sum_{n=0,\text{even}}^{\infty} \frac{\zeta^{n+1} a_n f_\pi}{n+1} \left[\frac{2\eta C^2_n(\eta) \left(p^{[\mu} q^{\nu]}\right)}{p \cdot q} \right]$$

Pion momentum

$$\zeta = \frac{\sqrt{p^2 q^2}}{\widetilde{Q}^2}, \qquad \eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}$$

Lattice OPE, VA type setup



$$+ \left(\begin{array}{c} q \leftrightarrow -q' \\ V^{\mu}_{\Psi,\psi} \leftrightarrow A^{\nu}_{\Psi,\psi} \end{array} \right)$$

$$\boxed{q' = q + p}$$

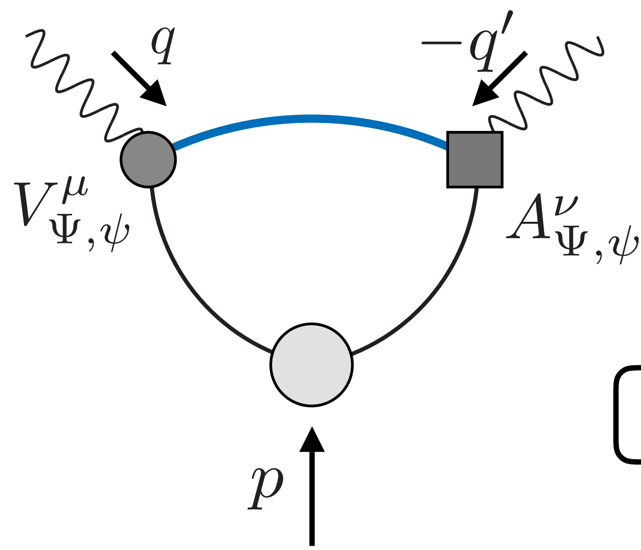
$$V^{\mu}_{\Psi,\psi} = \overline{\Psi}\gamma^{\mu}\psi + \overline{\psi}\gamma^{\mu}\Psi$$

$$A^{\mu}_{\Psi,\psi} = \overline{\Psi}\gamma^{\mu}\gamma^{5}\psi + \overline{\psi}\gamma^{\mu}\gamma^{5}\Psi$$

$$S^{\mu\nu}_{\Psi,\psi}(p,q) = \int d^4x\, e^{i\,q\cdot x} \langle \pi^{+}(p) | T[V^{\mu}_{\Psi,\psi}(x) A^{\nu}_{\Psi,\psi}(0)] | 0 \rangle$$

μ and ν symmetrised

Lattice OPE, VA type result



$$+ \left(\begin{array}{c} q \leftrightarrow -q' \\ V^{\mu}_{\Psi,\psi} \leftrightarrow A^{\nu}_{\Psi,\psi} \end{array} \right)$$

$$q' = q + p$$

$$S^{\{\mu\nu\}}_{\Psi,\psi} = \frac{i\tilde{Q}^2}{2} \sum_{n=0}^{\infty} \frac{\zeta^{n+2} a^n f_\pi}{(n+1)(n+2)} (1 + I_n)$$

For simplicity, set the Wilson coefficients to unity

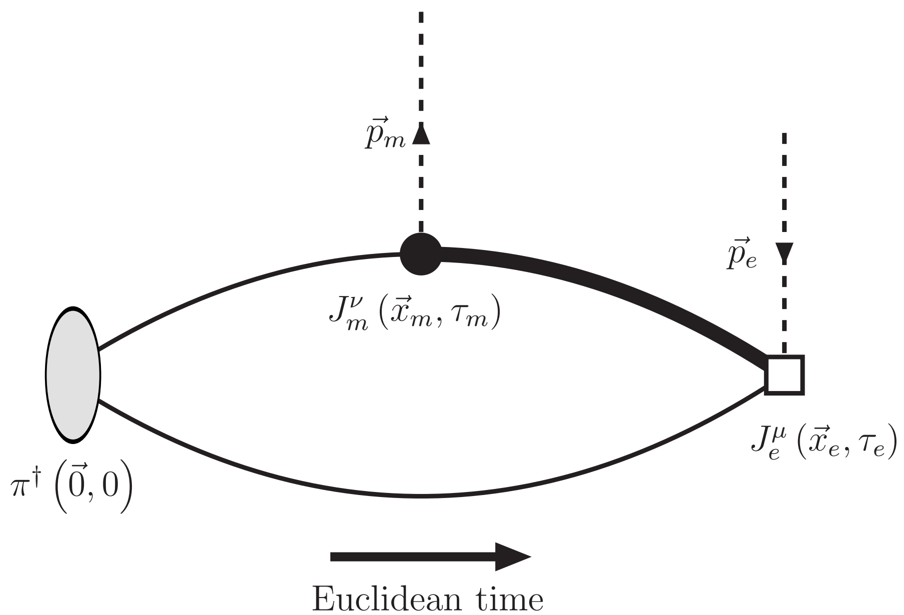
$$\times \{ 8\eta^2 C^3_n(\eta) p^\mu p^\nu (p.q)^{-2} + 4[(n+1)\eta C^2_{n+1}(\eta) - 4\eta^2 C^2_n(\eta)] p^{\{\mu} q^{\nu\}} (q^2)^{-1} (p.q)^{-1}$$
$$+ [(n+2) C^1_{n+2}(\eta) - 2\eta C^2_{n+1}(\eta)] \delta^{\mu\nu} (q^2)^{-1}$$
$$+ [n(n+2) C^1_{n+2}(\eta) - 2\eta(2n+1) C^2_{n+1}(\eta) + 8\eta^2 C^3_n(\eta)] q^\mu q^\nu (q^2)^{-2} \}$$

$$I_n = \sum_{m=0}^{\infty} C^{m+n}_n \left(\frac{2p \cdot q}{\tilde{Q}^2} \right)^m, \qquad \zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}, \qquad \eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}$$

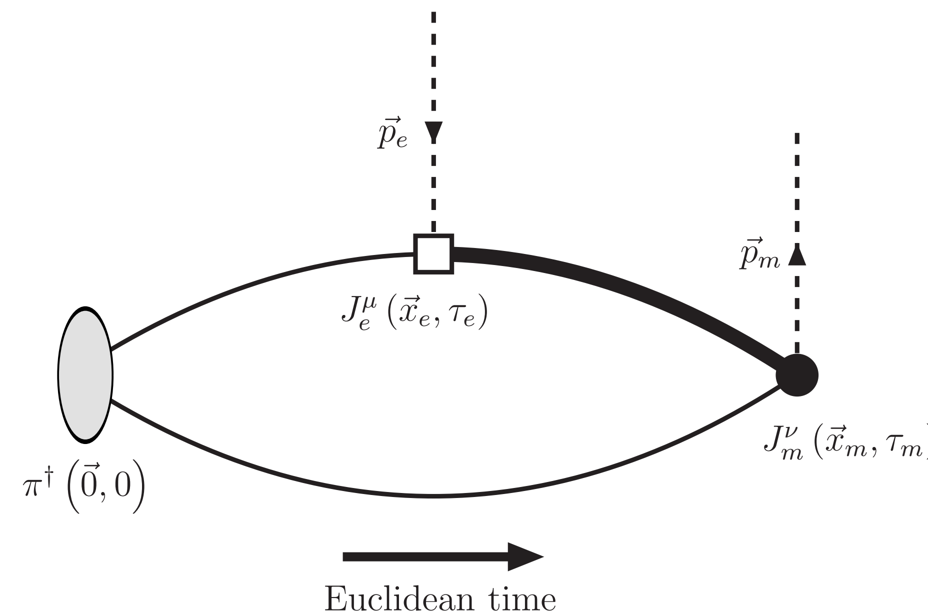
(Extremely) exploratory numerical result

W.Detmold, CJDL, S.Mondal, work in progress

The correlators



$$\tau_e > \tau_m >> 0$$

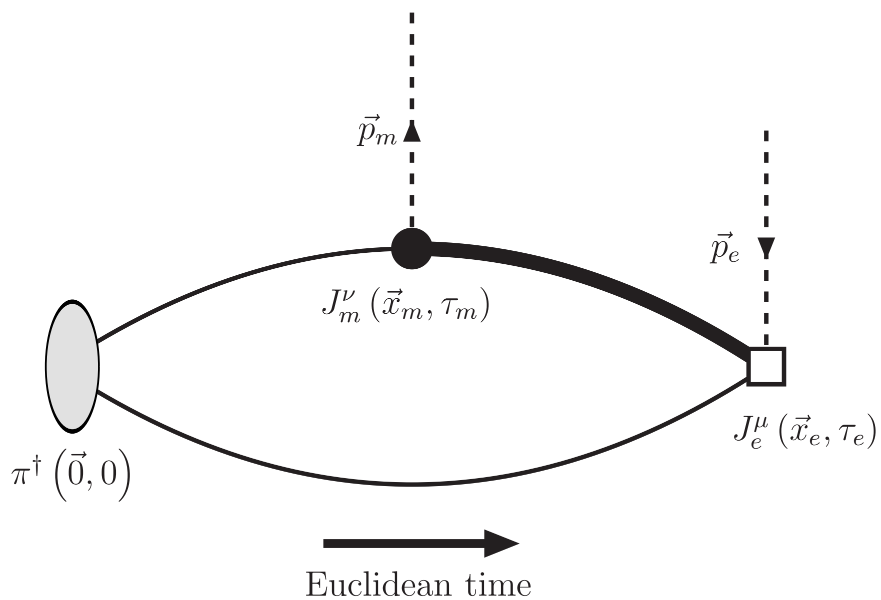


$$\tau_m > \tau_e >> 0$$

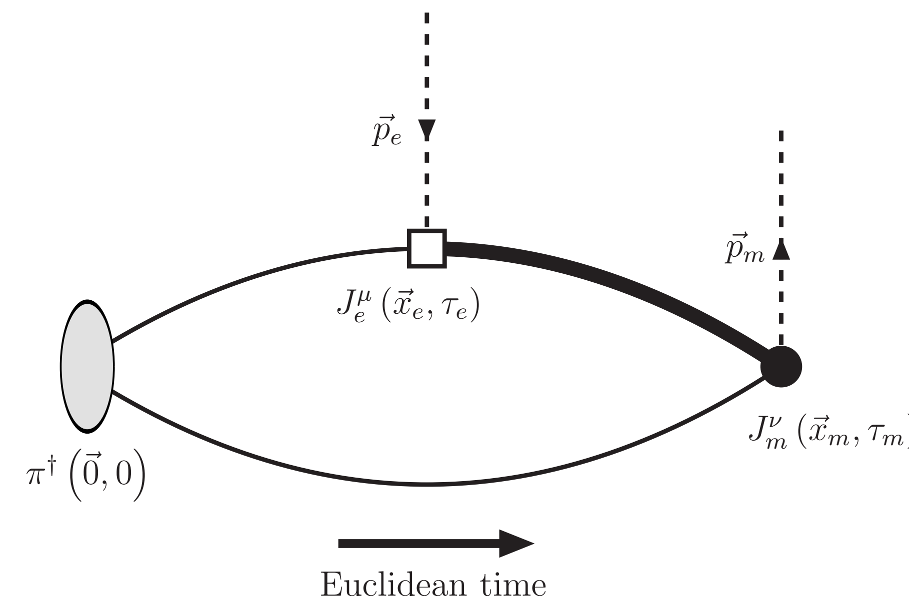
$$C_3^{\mu\nu}\left(\tau_e, \tau_m; \vec{p}_e, \vec{p}_m\right) = \int \mathrm{d}^3 x_e \int \mathrm{d}^3 x_m \; \mathrm{e}^{i\vec{p}_e \cdot \vec{x}_e} \mathrm{e}^{-i\vec{p}_m \cdot \vec{x}_m} \left\langle 0 \middle| \mathrm{T}\left[J_e^\mu\left(\vec{x}_e, \tau_e\right) J_m^\nu\left(\vec{x}_m, \tau_m\right) \mathcal{O}_\pi^\dagger(\vec{0}, 0)\right] \middle| 0 \right\rangle$$

$$C_\pi\left(\tau_\pi; \vec{p}_\pi\right) = \int \mathrm{d}^3 x \; \mathrm{e}^{i\vec{p}_\pi \cdot \vec{x}} \left\langle 0 \middle| \mathcal{O}_\pi(\vec{x}, \tau) \mathcal{O}_\pi^\dagger(\vec{0}, 0) \middle| 0 \right\rangle$$

Extracting the VV and VA matrix elements



$\tau_e > \tau_m >> 0$



$\tau_m > \tau_e >> 0$

$$C_\pi(\tau_\pi;\vec{p}_\pi) \xrightarrow{\tau_\pi\to\infty} \frac{\left|\left\langle \pi(\vec{p}_\pi)\left|\mathcal{O}^\dagger_\pi(\vec{0},0)\right|0\right\rangle\right|^2}{2E_\pi} \times \mathrm{e}^{-E_\pi\tau_\pi}$$

$$R^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_m-\tau_e;\vec{p}_e,\vec{p}_\pi) = \frac{C^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_e-\vec{p}_\pi)}{C_\pi(\tau_e;\vec{p}_\pi)} \times \left\langle \pi(\vec{p}_\pi)\left|\mathcal{O}^\dagger_\pi(\vec{0},0)\right|0\right\rangle$$
$$= \int \mathrm{d}^3x\ \mathrm{e}^{-i\vec{p}_m\cdot\vec{x}} \left\langle 0\left|J^\nu_m(\vec{x},\tau_m-\tau_e)\,J^\mu_e(\vec{0},0)\right|\pi(\vec{p}_\pi)\right\rangle_{\tau_m>\tau_e}$$

$$R^{\mu\nu}_{3;\tau_e>\tau_m}(\tau_e-\tau_m;\vec{p}_e,\vec{p}_\pi) = \frac{C^{\mu\nu}_{3;\tau_e>\tau_m}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_m-\vec{p}_\pi)}{C_\pi(\tau_m;\vec{p}_\pi)} \times \left\langle \pi(\vec{p}_\pi)\left|\mathcal{O}^\dagger_\pi(\vec{0},0)\right|0\right\rangle$$
$$= \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{p}_e\cdot\vec{x}} \left\langle 0\left|J^\mu_e(\vec{x},\tau_e-\tau_m)\,J^\nu_m(\vec{0},0)\right|\pi(\vec{p}_\pi)\right\rangle_{\tau_e>\tau_m}$$

$\tau \equiv \tau_m - \tau_e$ and perform Fourier transform $\int d\tau\ \exp(i\tau q_4)$

Simulation details

testing our approach with fine quenched lattices

- In this talk: exploratory study with 24 configs:

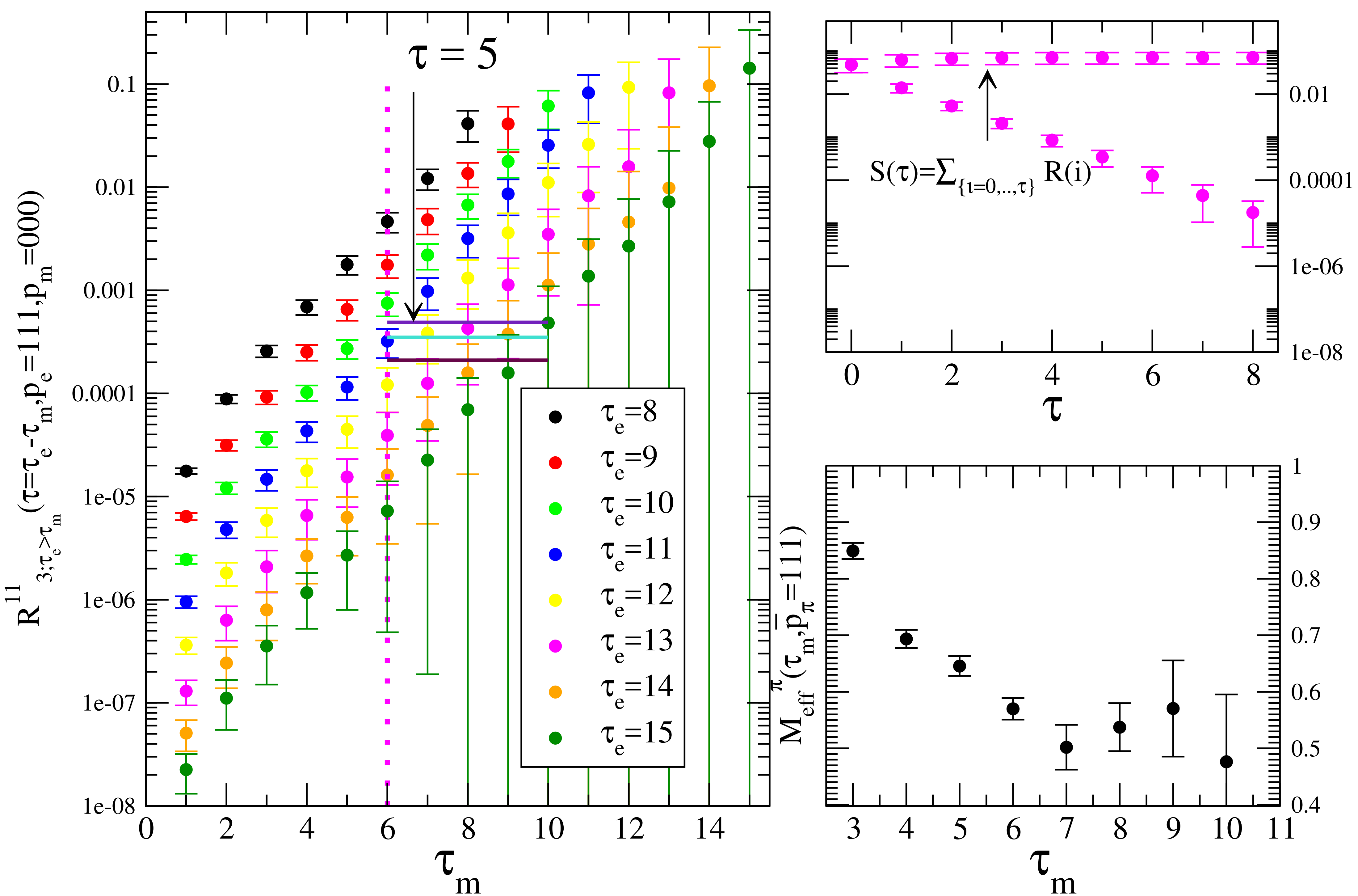
$$a^{-1} \sim 2 \text{ GeV}, \; L^3 \times T = 24^3 \times 48.$$

$$m_\Psi \sim 1.1 \text{ GeV}, \; M_\pi \sim 370 \text{ GeV}.$$

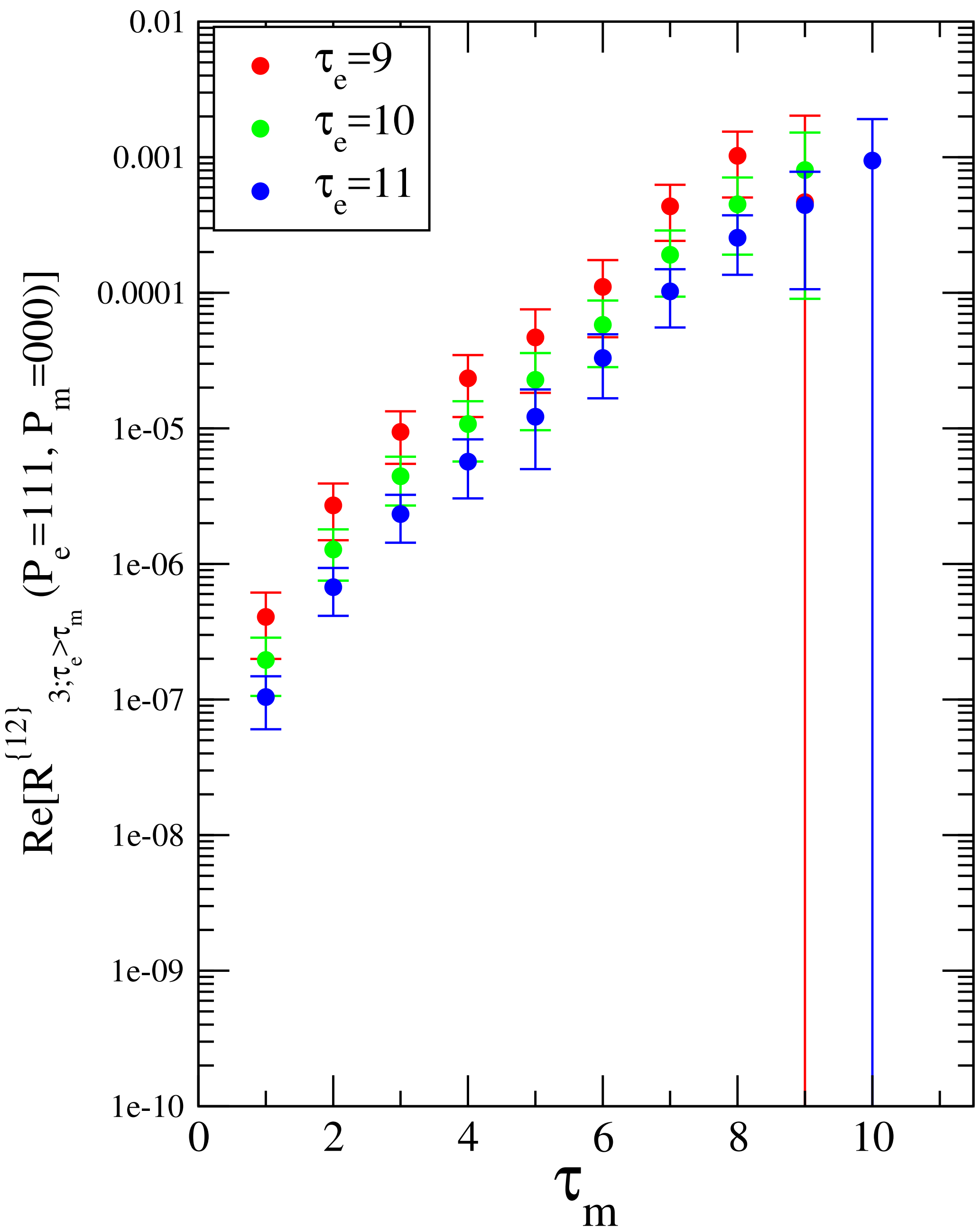
- Available (~200 each) configs at $L = 32, 48, 64, 96.$

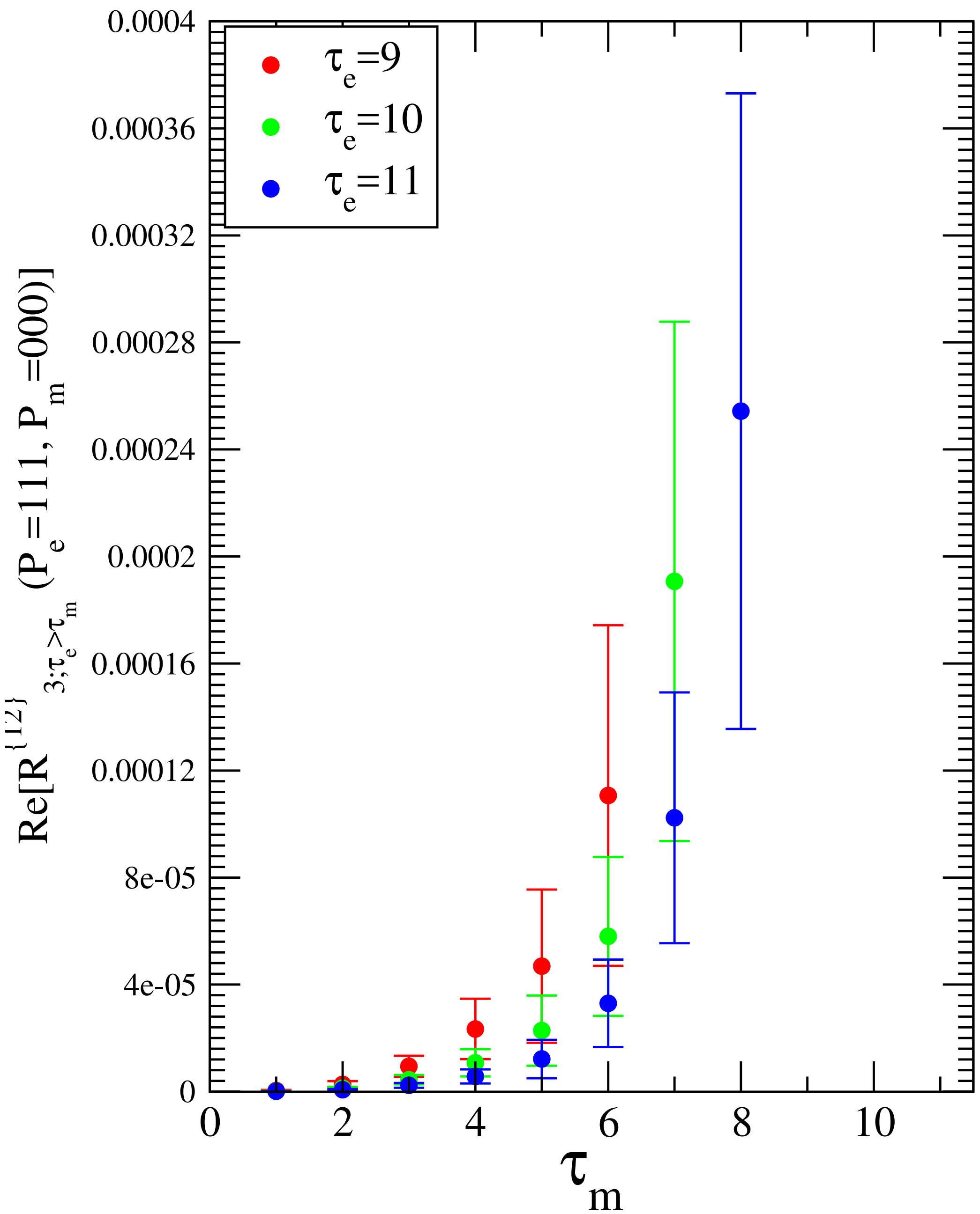
→ Cut-off scale as high as 8 GeV.

VA type, “diagonal”

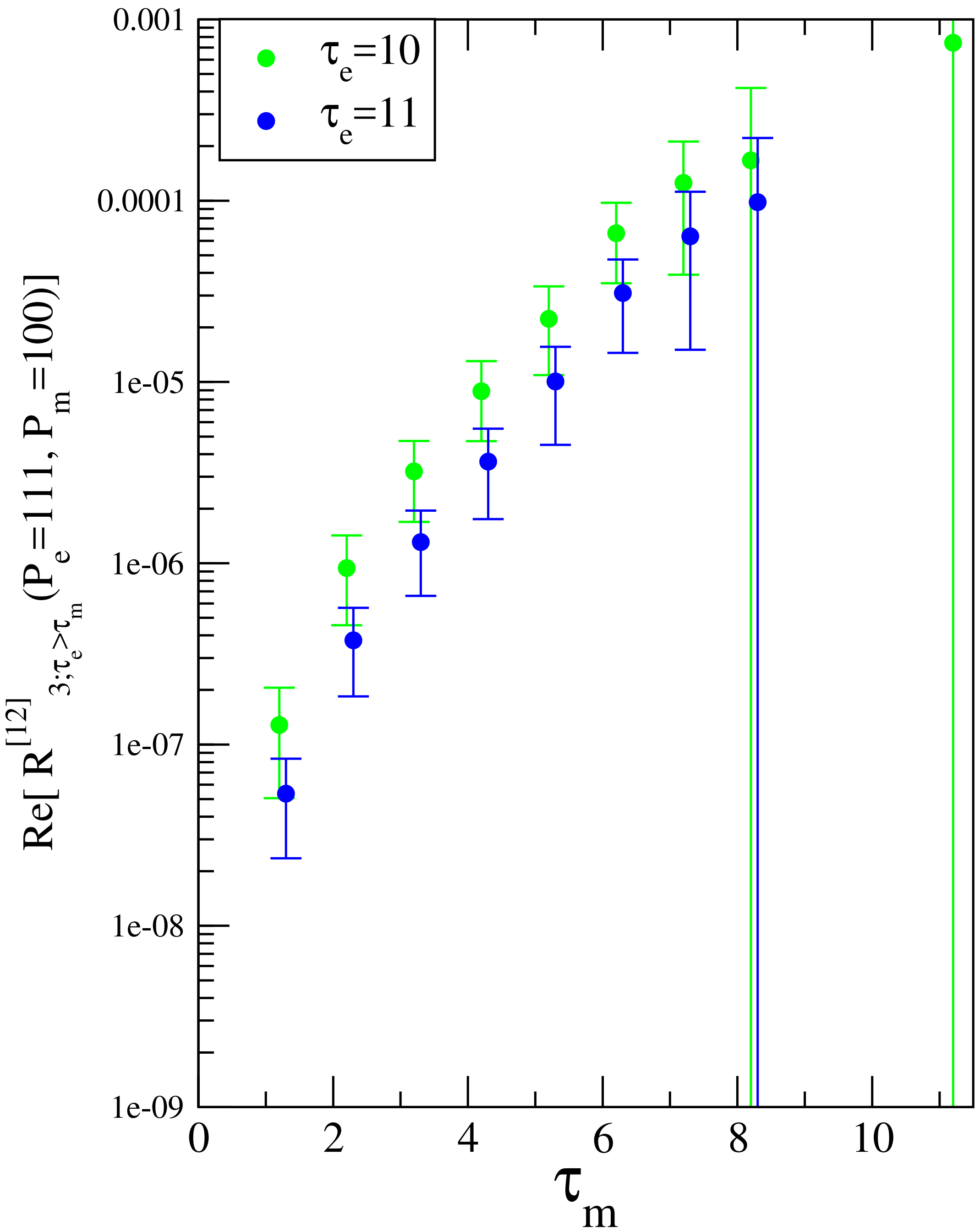


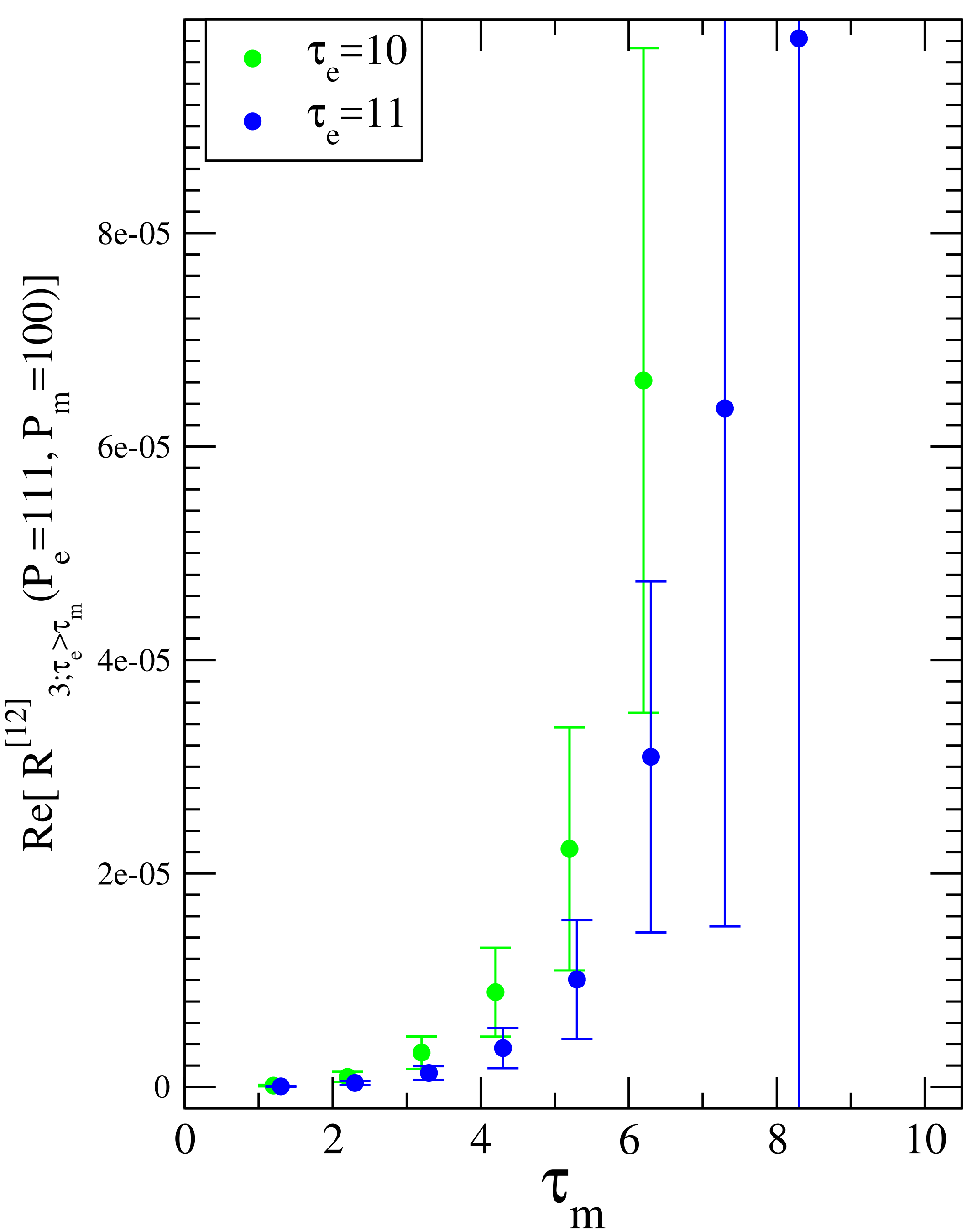
VA type, symmetric





VV type, anti-symmetric





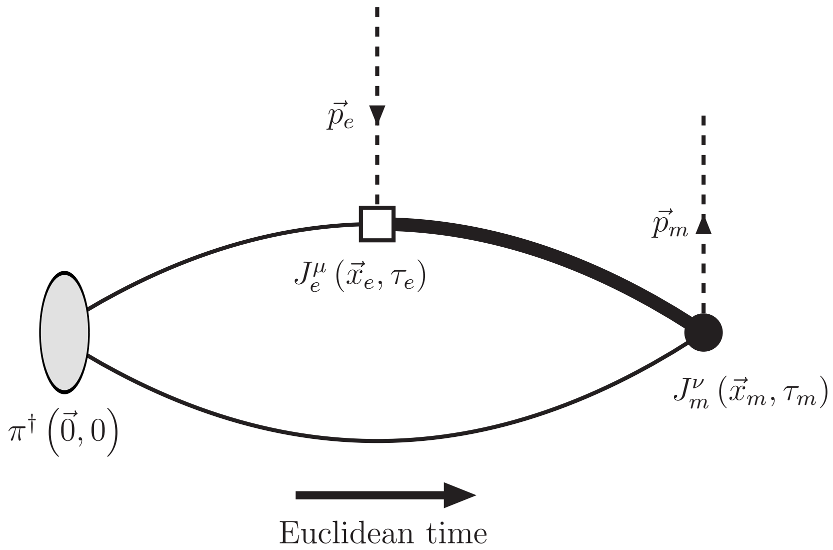
Outlook

- Finishing the quenched numerical study of the pion LC wavefunction.
- Parton distribution of the pion.
- Parton distribution of the nucleon.

Backup slides

W.Detmold, CJDL, S.Mondal, work in progress

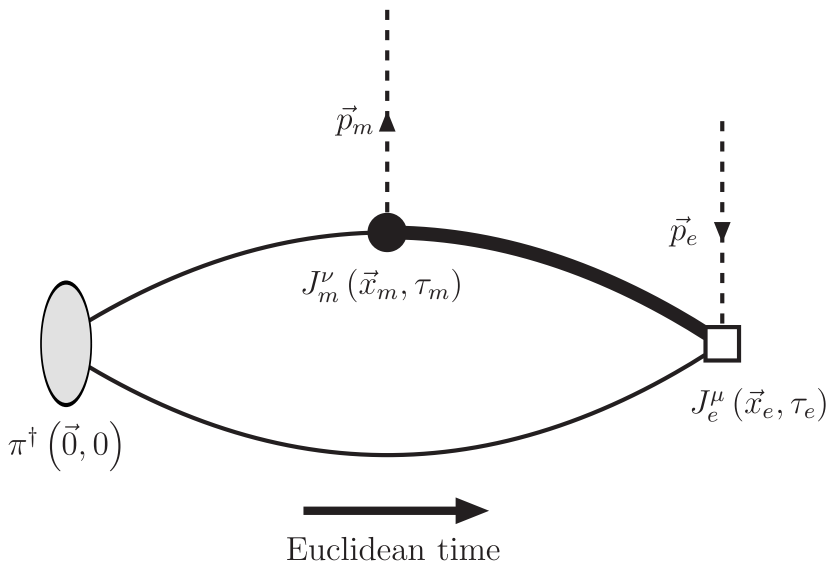
Behaviour of the 3-point function



$$\tau_m > \tau_e >> 0$$

$$
\begin{aligned}
C^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_m) \quad &= \quad \sum_n \frac{1}{2E_n}\int d^3x_e \int d^3x_m \; e^{i\vec{p}_e\cdot\vec{x}_e} e^{-i\vec{p}_m\cdot\vec{x}_m} \\
&\quad \times \left\langle 0 \left| J_m^\nu(\vec{x}_m-\vec{x}_e,\tau_m-\tau_e)\, J_e^\mu(\vec{0},0)\; e^{-i\hat{p}\cdot\vec{x}_e} e^{-\hat{H}\tau_e} \right| n \right\rangle \left\langle n \left| \mathcal{O}_\pi^\dagger(\vec{0},0) \right| 0 \right\rangle \\
&\xrightarrow{\tau_e\to\infty} \frac{1}{2E_\pi}\int d^3x_e \int d^3x_m \; e^{i\vec{p}_e\cdot\vec{x}_e} e^{-i\vec{p}_m\cdot\vec{x}_m} e^{-i\vec{p}_\pi\cdot\vec{x}_e} e^{-E_\pi\tau_e} \\
&\quad \times \left\langle 0 \left| J_m^\nu(\vec{x}_m-\vec{x}_e,\tau_m-\tau_e)\, J_m^\mu(\vec{0},0) \right| \pi(\vec{p}_\pi) \right\rangle \left\langle \pi(\vec{p}_\pi) \left| \mathcal{O}_\pi^\dagger(\vec{0},0) \right| 0 \right\rangle \\
&\overset{\vec{x}_m=\vec{x}+\vec{x}_e}{=} \frac{1}{2E_\pi}\int d^3x_e \; e^{i\vec{p}_e\cdot\vec{x}_e} e^{-i\vec{p}_m\cdot\vec{x}_e} e^{-i\vec{p}_\pi\cdot\vec{x}_e} e^{-E_\pi\tau_e} \left\langle \pi(\vec{p}_\pi) \left| \mathcal{O}_\pi^\dagger(\vec{0},0) \right| 0 \right\rangle \\
&\quad \times \int d^3x \; e^{-i\vec{p}_m\cdot\vec{x}} \left\langle 0 \left| J_m^\nu(\vec{x},\tau_m-\tau_e)\, J_e^\mu(\vec{0},0) \right| \pi(\vec{p}_\pi) \right\rangle \\
&= \quad \frac{1}{2E_\pi}\delta_{\vec{n}_\pi,\vec{n}_e-\vec{n}_m} \times e^{-E_\pi\tau_e} \left\langle \pi(\vec{p}_\pi) \left| \mathcal{O}_\pi^\dagger(\vec{0},0) \right| 0 \right\rangle \\
&\quad \times \int d^3x \; e^{-i\vec{p}_m\cdot\vec{x}} \left\langle 0 \left| J_m^\nu(\vec{x},\tau_m-\tau_e)\, J_e^\mu(\vec{0},0) \right| \pi(\vec{p}_\pi) \right\rangle
\end{aligned}
$$

Behaviour of the 3-point function



$$\tau_e > \tau_m >> 0$$

$$
\begin{aligned}
C^{\mu\nu}_{3;\tau_e>\tau_m}(\tau_e, \tau_m; \vec{p}_e, \vec{p}_m) \quad &= \quad \sum_n \frac{1}{2E_n} \int \mathrm{d}^3x_e \int \mathrm{d}^3x_m \; \mathrm{e}^{i\vec{p}_e\cdot\vec{x}_e} \mathrm{e}^{-i\vec{p}_m\cdot\vec{x}_m} \\
&\quad \times \left\langle 0 \left| J^{\mu}_{e}(\vec{x}_e - \vec{x}_m, \tau_e - \tau_m)\, J^{\nu}_{m}(\vec{0}, 0)\; \mathrm{e}^{-i\hat{p}\cdot\vec{x}_m} \mathrm{e}^{-\hat{H}\tau_m} \right| n \right\rangle \left\langle n \left| \mathcal{O}^{\dagger}_{\pi}(\vec{0}, 0) \right| 0 \right\rangle \\
&\xrightarrow{\tau_m\to\infty} \frac{1}{2E_\pi} \int \mathrm{d}^3x_e \int \mathrm{d}^3x_m \; \mathrm{e}^{i\vec{p}_e\cdot\vec{x}_e} \mathrm{e}^{-i\vec{p}_m\cdot\vec{x}_m} \mathrm{e}^{-i\vec{p}_\pi\cdot\vec{x}_m} \mathrm{e}^{-E_\pi\tau_m} \\
&\quad \times \left\langle 0 \left| J^{\mu}_{e}(\vec{x}_e - \vec{x}_m, \tau_e - \tau_m)\, J^{\nu}_{m}(\vec{0}, 0) \right| \pi(\vec{p}_\pi) \right\rangle \left\langle \pi(\vec{p}_\pi) \left| \mathcal{O}^{\dagger}_{\pi}(\vec{0}, 0) \right| 0 \right\rangle \\
&\overset{\vec{x}_e=\vec{x}+\vec{x}_m}{=} \frac{1}{2E_\pi} \int \mathrm{d}^3x_m \; \mathrm{e}^{i\vec{p}_e\cdot\vec{x}_m} \mathrm{e}^{-i\vec{p}_m\cdot\vec{x}_m} \mathrm{e}^{-i\vec{p}_\pi\cdot\vec{x}_m} \mathrm{e}^{-E_\pi\tau_m} \left\langle \pi(\vec{p}_\pi) \left| \mathcal{O}^{\dagger}_{\pi}(\vec{0}, 0) \right| 0 \right\rangle \\
&\quad \times \int \mathrm{d}^3x \; \mathrm{e}^{i\vec{p}_e\cdot\vec{x}} \left\langle 0 \left| J^{\mu}_{e}(\vec{x}, \tau_e - \tau_m)\, J^{\nu}_{m}(\vec{0}, 0) \right| \pi(\vec{p}_\pi) \right\rangle \\
&= \quad \frac{1}{2E_\pi} \delta_{\vec{n}_\pi, \vec{n}_e - \vec{n}_m} \times \mathrm{e}^{-E_\pi\tau_m} \left\langle \pi(\vec{p}_\pi) \left| \mathcal{O}^{\dagger}_{\pi}(\vec{0}, 0) \right| 0 \right\rangle \\
&\quad \times \int \mathrm{d}^3x \; \mathrm{e}^{i\vec{p}_e\cdot\vec{x}} \left\langle 0 \left| J^{\mu}_{e}(\vec{x}, \tau_e - \tau_m)\, J^{\nu}_{m}(\vec{0}, 0) \right| \pi(\vec{p}_\pi) \right\rangle
\end{aligned}
$$