Moments of the pion distribution amplitude from lattOPE with a valence heavy quark

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W.Detmold and CJDL, Phys.Rev.D73, 014501 (2006) [hep-lat/0507007] & work in progress

Collaborators

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Outline

- Motivation and general strategy.
- Lattice OPE and the structure functions.
- Lattice OPE and the pion light-cone wavefunction.
- (Extremely) exploratory numerical result.
- Outlook

Motivation and general strategy

W.Detmold and CJDL, Phys.Rev.D73, 014501 (2006) [hep-lat/0507007]

Parton distribution from lattice QCD Lattich quotification in din function ζ P arton distribution from lattice \bigcap provide stringent tests of \mathcal{L} \mathbf{h} and \mathbf{h} are discussed by \mathbf{h} asufoundle nom faulte ged

other high-energy experiments from first principles. By comparing to accurate experimental data, such calculations from first principles. By comparing to accurate experimental data, such calculations in the calculations o The "traditional" approach τ available from experiment, e.g., the transversity distribution τ of \mathbb{R}^n . The structure functions described the structure functions describe the structure functions describe the structure functions described where p and S are the momentum and spin of the external state, q is the momentum transfer between the momentum
S are the momentum transfer between the momentum transfer between the lepton and the lepton and the lepton and

· Hadronic tensor

Parton distribution from lattice QCD
The "traditional" approach
Hadronic tensor
$W_S^{\mu\nu}(p,q) = \int d^4x e^{iq \cdot x}(p,S[[J^{\mu}(x), J^{\nu}(0)] p,S \rangle)$
$\underbrace{\text{Initial theorem}}_{T_S^{\mu\nu}(p,q)} \left\{ \underbrace{\text{Imaginary part}}_{\text{Imaginary part}} \right\} \underbrace{\text{Challenging in Euclidean QCD}}_{\text{Challenging in Euclidean QCD}}$
Light-cone OPE
$T[J^{\mu}(x)J^{\nu}(0)] = \sum_{i,n} C_i (x^2, \mu^2) x_{\mu_1} \dots x_{\mu_n} C_i^{\mu\nu\mu_1 \dots \mu_n}(\mu)$
$\underbrace{\text{local operators, issue of operator mixing}}_{\text{Power divergences arising from Lorentz symmetry breaking}}$

• Light-cone OPE \blacksquare Tight-cone OPE challenging because of the analytical continuation to Minkowski space that is required [1, 2]. In addition, such $f \circ 1$ calculations of \overline{O} (CPE) construction of the currents of the current of the current of the curren

$$
T[J^{\mu}(x)J^{\nu}(0)] = \sum_{i,n} C_i (x^2, \mu^2) x_{\mu_1} \dots x_{\mu_n} O_i^{\mu \nu \mu_1 \dots \mu_n}(\mu)
$$

local operators, issue of operator mixing
Geading moments in practice
Power divergences arising from Lorentz symmetry breaking

Introducing the valence heavy quark $\mathbf r$ scaling of the structure functions $\mathbf r$, $\mathbf r$, $\mathbf r$, $\mathbf r$, $\mathbf r$, then they result in significant limit, they result in significant limit, they result in significant limit, they result in significant limit contributions which are operated the OPE belonging an expansion in terms of α representations of the Lorentz group. These contributions scale as powers of M²/Q², where M is the target mass and

- Valence **not** in the action. Ω_{α} Velemes Ω_{α} and in the setien valence **investigate investigated and altitude**.
- The "heavy quark" is relativistic. in Eq. (4). In this section we present the OPE in Euclidean space relevant for computing higher moments of partons of partons of particle.

distributions on the lattice with the lattice with the lattice with the lattice with the lattice \sim

extending that couple and time \longrightarrow (propagating in both space and time) Fig. 1: Contributions to the Compton scattering tensor. Diagrams (c), (c), (c) correspond to the leading twist contributions.

• The current for computing the even moments of the PDF $T1$ involves gluonic operators gluonic operators and vanishes for the isovector combination, Eq. (7). Diagram (d) The current for computing the even moments of the PDF

 \bigcap

$$
J_{\Psi,\psi}^{\mu}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)
$$

$$
T_{\Psi,\psi}^{\mu\nu}(p,q) \equiv \sum_{S} \langle p, S | t_{\Psi,\psi}^{\mu\nu}(q) | p, S \rangle = \sum_{S} \int d^{4}x \ e^{iq \cdot x} \langle p, S | T \left[J_{\Psi,\psi}^{\mu}(x) J_{\Psi,\psi}^{\nu}(0) \right] | p, S \rangle
$$

Strategy for extracting the moments $\boldsymbol{\alpha}$ thereby enablished extraction of matrix elements of $\boldsymbol{\alpha}$ operators with a simple renormalisation procedure. The moments of the moments of the moments of the Mellin moments structure functions which are identical in Euclidean space and Minkowski space and their analytical continuation is analyt \sim and \sim Etrotogy for extracting the moments ditatesy for extracting in

$$
\begin{aligned}\n\begin{bmatrix}\nT^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_{S} \langle p, S | t^{\mu\nu}_{\Psi,\psi}(q) | p, S \rangle &= \sum_{S} \int d^4x \ e^{iq \cdot x} \langle p, S | T \left[J^{\mu}_{\Psi,\psi}(x) J^{\nu}_{\Psi,\psi}(0) \right] | p, S \rangle \\
J^{\mu}_{\Psi,\psi}(x) &= \overline{\Psi}(x) \gamma^{\mu} \psi(x) + \overline{\psi}(x) \gamma^{\mu} \Psi(x)\n\end{bmatrix}\n\end{aligned}
$$

- Simple renormalisation for quark bilinears. ompressed conditions due distance contributions between the currents in a similar way to a similar way to a la \cdot Simple renormalication for quark hilinears strupte renormandanon for quark omneard. $\begin{array}{ccc} \n\mathbf{C}^* & 1 & \mathbf{I}^* & \mathbf{I}^* & \mathbf{C}^* & 1 & \mathbf{I}^* \mathbf{I}^* & \mathbf{I}$ Sumple renormalisation for quark omnears.
- Work with the hierarchy of scales • Work with the hierarchy of scal ales and the set of α \mathbf{r} and \mathbf{r} and

 $\Lambda_{\rm QCD} \ll m_\Psi \sim \sqrt{q^2} \ll \frac{1}{\hat{\pi}}$ \hat{a} $\Lambda_{\rm QCD} \ll m_\Psi \sim \sqrt{q^2} \ll \frac{1}{\gamma}$ and $\gamma_{\rm QCD} \ll m_\Psi \sim \sqrt{q^2 \ll m_\Psi}$ (Need very fine lattices) $\Lambda_{\rm QCD} \ll m_\Psi \sim \sqrt{q^2 \ll \frac{1}{\hat{a}}}$ \longrightarrow Need very fine lattices

• Extrapolate $T_{\Psi,\psi}^{\mu\nu}(p,q)$ to the continuum limit first. Then match to the Euclidean OPE results. \equiv Extract the moments without power divergence. $\frac{1}{2}$ the distribution distribution in the correlation in Section II. Finally in Section II. Finally in Section II. $A = \frac{1}{2}$ and m $\frac{1}{2}$ and multipute. However, the non-dynamical nature of the fiction of the fic Extrapolate $T^{\mu\nu}_{\Psi,\psi}(p,q)$ to the continuum limit first. Γ Then motels to the Γ uelidean ODE requise the two operators in Equidence of the results. Extract the moments with Euclidean ODE racu \mathbf{r} $st.$ \sim Laude in municing without μ is given by the diagrams.

Euclidean OPE and valence heavy quark

Lattice OPE and structure functions

W.Detmold and CJDL, Phys.Rev.D73, 014501 (2006) [hep-lat/0507007]

Key features of the Euclidean OPE $id_{\alpha\alpha} \cap \mathbb{D}$ \blacksquare

Results for structure function n and real of the C(N) in Eq. (14) the C(N) the C(N) the C(N) the C(N) in Eq. (14) the C(N) in E \mathbf{D}_{α} and \mathbf{f}_{α} for atrusture function Results for structure function $\mathcal{P}_{\mathbf{H}}$ are Gegenbauer polynomials that arise from $\mathcal{P}_{\mathbf{H}}$ S medication in Succession $\overline{}$

Simplifying *via* choosing $p = (0, 0, 0, i, M)$ & $q = (0, 0, \sqrt{q_0^2 - Q^2}, i q_0)$ combination of {µ, ν} = {3, 4} is \overline{h} \log ing $p = (0,0,0,i|M)$ & $q = (0,0,\sqrt{q_0^2-Q^2}, i\ q_0)$ Simplifying via choosing $p = (0, 0, 0, i M)$ & $q = (0, 0, \sqrt{q_0^2 - Q^2}, i q_0)$ $c_{1, 4}$ is $c_{2, 4}$ in $c_{3, 4}$ is $c_{4, 4}$ is $c_{5, 4}$ is $c_{6, 4}$ is $c_{7, 4}$ is $c_{8, 4}$ is $c_{9, 4}$ is $c_{1, 4}$ is $Cimplif v$

$$
T_{\Psi,\psi}^{\{34\}}(p,q) = \sum_{n=2}^{\infty} A_{\psi}^n(\mu^2) f(n)
$$

$$
\sum_{S} \langle p, S | \mathcal{O}_{\psi}^{\mu_1 \dots \mu_n} | p, S \rangle = A_{\psi}^n(\mu^2) [p^{\mu_1} \dots p^{\mu_n} - \text{traces}]
$$

$$
\mathcal{O}_{\psi}^{\mu_1 \dots \mu_n} = \overline{\psi} \gamma^{\{\mu_1} (iD^{\mu_2}) \dots (iD^{\mu_n}\}) \psi - \text{traces}
$$

$$
f(n) = -\sqrt{q_0^2 - Q^2} \zeta^n \left\{ \frac{2}{q_0} \left[C_n \frac{\tilde{Q}^2}{Q^2} \frac{(n-1)\eta C_{n-1}^{(2)}(\eta) - 4\eta^2 C_{n-2}^{(3)}(\eta)}{n(n-1)} + C_n'' \frac{\eta}{n} C_{n-1}^{(2)}(\eta) \right] \right\}
$$

$$
+ \frac{q_0}{Q^2} \left[C_n \frac{\tilde{Q}^2}{Q^2} \frac{n(n-2)C_n^{(1)}(\eta) - 2\eta(2n-3)C_{n-1}^{(2)}(\eta) + 8\eta^2 C_{n-2}^{(3)}(\eta)}{n(n-1)} + 2C_n'' \left(C_n^{(1)}(\eta) - 2\frac{\eta}{n} C_{n-1}^{(2)}(\eta) \right) \right] \right\}
$$

$$
\zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}, \qquad \eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}
$$

Results for f(n) (a) (b) (c) (a) (b) (c) $\mathbf{R}_{\text{a}\text{c}11}$ and \mathbf{R}_{a} for $\mathbf{f}(n)$ choose the proton of \mathbf{r} $=$ $\frac{1}{2}$ $\frac{1}{2$

 $M_{\Psi} = 3.54 \text{ GeV}, Q^2 = 1.5 \text{ GeV}^2 \text{ and}$ $\overline{}$ $M_{\Psi} = 3.54 \text{ GeV}, Q^2 = 1.5 \text{ GeV}$

 $H_{\Psi} = 3.54 \, \text{GeV}, \, Q^2 = 1.5 \, \text{GeV}^2 \, \text{ and } \, q_0 = 2.76 \, \text{GeV} \, \qquad M_{\Psi} = 2.1 \, \text{GeV}, \, Q^2 = -3.85 \, \text{GeV}^2 \, \text{ and } \, q_0 = 1.98 \, \text{GeV}$ q⁰ = 2.76 GeV while on the right we have chosen M^Ψ = 2.1 GeV, Q² = −3.85 GeV² and q⁰ = 1.98 GeV. The stars and boxes $q_0 = 2.76 \text{ GeV}$ $M_{\Psi} = 2.1 \text{ GeV}, Q^2 = -3.85 \text{ GeV}^2 \text{ and } q_0 = 1.98 \text{ GeV}.$ c_{max} and c_{max} and c_{max} and c_{max} and c_{max} and c_{max} and c_{max} 54 GeV, $Q^2 = 1.5$ GeV² and $q_0 = 2.76$ GeV $M_{\Psi} = 2.1$ GeV, $Q^2 = -3.85$ GeV² and $q_0 = 1.98$ GeV. α and α and α and α and we have chosen M α and β and β GeV and β GeV and β GeV and β GeV and β

Lattice OPE and the pion light-cone wavefunction

W.Detmold, CJDL, S.Mondal, work in progress

Pion light-cone wavefunction \overline{C} ω Pion light-cone wavefunction

Important input for flavour physics

$$
\langle \pi^+(p)|\bar{u}(z/2)\gamma_5\gamma_\mu d(-z/2)|0\rangle = -ip_\mu f_\pi \int_0^1 d\xi \ e^{i(\bar{\xi}p\frac{z}{2}-\xi p\frac{z}{2})}\phi_\pi(\xi)
$$

$$
\underbrace{\left(a_n = \int_0^1 d\xi \ \xi^n \phi_\pi(\xi)\right)}_{\text{Q}_{\psi}^{\mu_1}...\mu_n} = \underbrace{\int_0^1 d\xi \ \xi^n \phi_\pi(\xi)}_{\text{Q}_{\psi}^{\mu_1}...\mu_n} = \underbrace{\int_0^1 \phi_n \phi_n(\xi)}_{\text{Q}_{\psi}^{\mu_1}...\mu_n}.
$$

Lattice OPE, VV type setup III. ANATOMY OF OPE IN THE EUCLIDEAN SPACE V ty 0 Lattice OPE. VV type setup $\overline{}$ $d\mathcal{L}_{\mathcal{L}} = \mathbf{Q} \mathbf{D} \mathbf{\Gamma} \mathbf{I} \mathbf{I} \mathbf{I} \mathbf{I}_{\mathcal{L}}$ ا ا dξ ξnφπ(ξ) (10) attice O-

$$
V_{\Psi,\psi}^{\mu} = \overline{\Psi}\gamma^{\mu}\psi + \overline{\psi}\gamma^{\mu}\Psi
$$

$$
V_{\Psi,\psi}^{\nu} = \overline{\Psi}\gamma^{\nu}\psi + \overline{\psi}\gamma^{\nu}\Psi
$$

$$
U_{\Psi,\psi}^{\mu\nu}(q) = \int d^4x \ e^{iqx} \langle \pi^+(p)|T[V_{\Psi,\psi}^{\mu}(x)V_{\Psi,\psi}^{\nu}]|0\rangle
$$

 μ and ν anti-symmetrised

 $=$ $\frac{1}{2}$

q
q

Ψ,ψ(x)A^ν

 \mathbb{R}^n , where \mathbb{R}^n

Lattice OPE, VV type result 0 Lattice OPE. VV type result $\overline{}$ \mathbf{A}^{t} (c) \mathbf{A}^{t} (\mathbf{A}^{t} $\overline{1}$ $\bigcap_{i=1}^n$ (10) attice OF

$$
\left(\begin{array}{c}q\leftrightarrow-q\\\mu\leftrightarrow\mu\end{array}\right)
$$

Enr simplicity set the Wilson coefficients to unity ents to unity
 \overline{y} For simplicity, set the Wilson coefficients to unity τ ≡ τ^m − τ^e and perform Fourier transform \$ dτ exp(iτq4)

$$
U_{\Psi,\psi}^{[\mu\nu]} = \sum_{n=0,\text{even}}^{\infty} \frac{\zeta^{n+1} a_n f_\pi}{n+1} \left[\frac{2\eta C_n^2(\eta) \left(p^{[\mu} q^{\nu]} \right)}{p \cdot q} \right]
$$

$$
\zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}, \qquad \eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}
$$

Lattice OPE, VA type setup **7** dξ ξnφπ(ξ) (10) Lattice OF $\overline{}$ \mathbf{r} α α \overline{u} but $ine \bigcap \mathbf{PF}$ VA type setup Jν = V ^ν ^Ψ,^ψ = Ψγ^νψ + ψγ^νΨ. (11)

$$
\mathcal{S}_{\Psi,\psi}^{\mu\nu}(p,q) = \int d^4x \, e^{i\,q\cdot x} \langle \pi^+(p) | T[V_{\Psi,\psi}^{\mu}(x) A_{\Psi,\psi}^{\nu}(0)] | 0 \rangle
$$

$$
\mu \text{ and } \nu \text{ symmetrised}
$$

Lattice OPE, VA type result $\overline{}$ \mathbf{L} (c) (10) \mathbf{L} (10) \mathbf{L} (10) \mathbf{L} (10) \mathbf{L} (11) \mathbf{L} — Lattice OF $\frac{1}{\epsilon}$ \overline{r} \overline{r} \overline{L} \overline{L} Ω Ω Ω Γ Λ θ θ Ω Ω and the symmetrization of the symmetrization of the first term can be written as \mathcal{L}_{d} term can be written as a symmetrization of the first term can be written as a set of the first term can be written as a set of

$$
S_{\Psi,\psi}^{\{\mu\nu\}} = \frac{i\tilde{Q}^2}{2} \sum_{n=0}^{\infty} \frac{\zeta^{n+2} a^n f_\pi}{(n+1)(n+2)} (1+I_n) \int \text{For simplicity, set the Wilson coefficients to unity}
$$

$$
\times \{8\eta^2 C_n^3(\eta)p^\mu p^\nu (p.q)^{-2} + 4[(n+1)\eta C_{n+1}^2(\eta) - 4\eta^2 C_n^2(\eta)]p^{\{\mu}q^{\nu\}}(q^2)^{-1}(p.q)^{-1}
$$

$$
+ [(n+2)C_{n+2}^1(\eta) - 2\eta C_{n+1}^2(\eta)]\delta^{\mu\nu}(q^2)^{-1}
$$

$$
+ [n(n+2)C_{n+2}^1(\eta) - 2\eta(2n+1)C_{n+1}^2(\eta) + 8\eta^2 C_n^3(\eta)]q^\mu q^\nu(q^2)^{-2}\}
$$

$$
\left(I_n = \sum_{m=0}^{\infty} C_n^{m+n} \left(\frac{2p \cdot q}{\tilde{Q}^2}\right)^m, \qquad \zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}, \qquad \eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}\right)
$$

(Extremely) exploratory numerical result

W.Detmold, CJDL, S.Mondal, work in progress

$$
C_3^{\mu\nu}(\tau_e, \tau_m; \vec{p}_e, \vec{p}_m) = \int d^3x_e \int d^3x_m \ e^{i\vec{p}_e \cdot \vec{x}_e} e^{-i\vec{p}_m \cdot \vec{x}_m} \left\langle 0 \left| \mathcal{T} \left[J_e^{\mu}(\vec{x}_e, \tau_e) J_m^{\nu}(\vec{x}_m, \tau_m) \mathcal{O}_{\pi}^{\dagger}(\vec{0}, 0) \right] \right| 0 \right\rangle
$$

$$
C_{\pi}(\tau_{\pi}; \vec{p}_{\pi}) = \int d^3x \ e^{i\vec{p}_{\pi} \cdot \vec{x}} \left\langle 0 \left| \mathcal{O}_{\pi}(\vec{x}, \tau) \mathcal{O}_{\pi}^{\dagger}(\vec{0}, 0) \right| 0 \right\rangle
$$

#

#

Extracting the VV and VA matrix elements Extracting the VV and VA matrix element n **V** V and $V\Lambda$ ma

$$
C_{\pi}\left(\tau_{\pi};\vec{p}_{\pi}\right)\xrightarrow{\tau_{\pi}\to\infty}\frac{\left|\left\langle\pi\left(\vec{p}_{\pi}\right)\left|\mathcal{O}_{\pi}^{\dagger}(\vec{0},0)\right|0\right\rangle\right|^{2}}{2E_{\pi}}\times\mathrm{e}^{-E_{\pi}\tau_{\pi}}
$$

$$
R^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_m-\tau_e;\vec{p}_e,\vec{p}_\pi)=\frac{C^{\mu\nu}_{3;\tau_m>\tau_e}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_e-\vec{p}_\pi)}{C_\pi(\tau_e;\vec{p}_\pi)}\times\left\langle\pi(\vec{p}_\pi)\left|\mathcal{O}^{\dagger}_\pi(\vec{0},0)\right|0\right\rangle
$$

$$
=\int d^3x \; e^{-i\vec{p}_m\cdot\vec{x}}\left\langle 0\left|J^{\nu}_m(\vec{x},\tau_m-\tau_e)J^{\mu}_e(\vec{0},0)\right|\pi(\vec{p}_\pi)\right\rangle_{\tau_m>\tau_e}
$$

$$
R_{3;\tau_e>\tau_m}^{\mu\nu}(\tau_e-\tau_m;\vec{p}_e,\vec{p}_\pi)=\frac{C_{3;\tau_e>\tau_m}^{\mu\nu}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_m-\vec{p}_\pi)}{C_\pi(\tau_m;\vec{p}_\pi)}\times\left\langle\pi(\vec{p}_\pi)\left|\mathcal{O}^{\dagger}_{\pi}(\vec{0},0)\right|0\right\rangle
$$

$$
=\int d^3x \ e^{i\vec{p}_e\cdot\vec{x}}\left\langle 0\left|J^{\mu}_e(\vec{x},\tau_e-\tau_m) J^{\nu}_m(\vec{0},0)\right|\pi(\vec{p}_\pi)\right\rangle_{\tau_e>\tau_m}
$$

$$
\mathcal{T}\equiv\tau_m-\tau_e \text{ and perform Fourier transform } \int d\tau \ \exp(i\tau q_4)
$$

 \mathcal{L}^{max}

Simulation details τ^m > τ^e >> 0 \sum ππατατοπ συμπερ $\frac{1}{s}$

testing our approach with fine quenched lattices

• In this talk: exploratory study with 24 configs: $a^{-1} \sim 2 \text{ GeV}, L^3 \times T = 24^3 \times 48.$ $× 48.$

> $m_{\Psi} \sim 1.1 \text{ GeV}, M_{\pi} \sim 370 \text{ GeV}.$ $\partial_{\mathcal{C}}V$.

• Available (~200 each) configs at $L = 32, 48, 64, 96$.

Cut-off scale as high as 8 GeV.

VA type, "diagonal"

VA type, symmetric

VV type, anti-symmetric

Outlook

- Finishing the quenched numerical study of the pion LC wavefunction.
- Parton distribution of the pion.
- Parton distribution of the nucleon.

Backup slides

W.Detmold, CJDL, S.Mondal, work in progress

Behaviour of the 3-point function

 $\tau_m > \tau_e >> 0$

$$
C_{3;\tau_{m}>\tau_{c}}^{\mu\nu}(\tau_{e},\tau_{m};\vec{p}_{e},\vec{p}_{m}) = \sum_{n} \frac{1}{2E_{n}} \int d^{3}x_{e} \int d^{3}x_{m} e^{i\vec{p}_{e}\cdot\vec{x}_{e}} e^{-i\vec{p}_{m}\cdot\vec{x}_{m}}
$$

\n
$$
\times \langle 0 | J_{m}^{\nu} (\vec{x}_{m} - \vec{x}_{e}, \tau_{m} - \tau_{e}) J_{e}^{\mu}(\vec{0},0) e^{-i\vec{p}\cdot\vec{x}_{e}} e^{-\hat{H}\tau_{e}} |n\rangle \langle n | \mathcal{O}_{\pi}^{\dagger}(\vec{0},0) | 0\rangle
$$

\n
$$
\xrightarrow{\tau_{c} \to \infty} \frac{1}{2E_{\pi}} \int d^{3}x_{e} \int d^{3}x_{m} e^{i\vec{p}_{e}\cdot\vec{x}_{e}} e^{-i\vec{p}_{m}\cdot\vec{x}_{m}} e^{-i\vec{p}_{\pi}\cdot\vec{x}_{e}} e^{-E_{\pi}\tau_{e}}
$$

\n
$$
\times \langle 0 | J_{m}^{\nu} (\vec{x}_{m} - \vec{x}_{e}, \tau_{m} - \tau_{e}) J_{m}^{\mu}(\vec{0},0) | \pi (\vec{p}_{\pi}) \rangle \langle \pi (\vec{p}_{\pi}) | \mathcal{O}_{\pi}^{\dagger}(\vec{0},0) | 0\rangle
$$

\n
$$
\xrightarrow{\vec{x}_{m} = \vec{x} + \vec{x}_{e}} \frac{1}{2E_{\pi}} \int d^{3}x_{e} e^{i\vec{p}_{e}\cdot\vec{x}_{e}} e^{-i\vec{p}_{m}\cdot\vec{x}_{e}} e^{-i\vec{p}_{\pi}\cdot\vec{x}_{e}} e^{-E_{\pi}\tau_{e}} \langle \pi (\vec{p}_{\pi}) | \mathcal{O}_{\pi}^{\dagger}(\vec{0},0) | 0\rangle
$$

\n
$$
\times \int d^{3}x e^{-i\vec{p}_{m}\cdot\vec{x}} \langle 0 | J_{m}^{\nu} (\vec{x}, \tau_{m} - \tau_{e}) J_{e}^{\mu}(\vec{0},0) | \pi (\vec{p}_{\pi}) \rangle
$$

\n
$$
= \frac{1}{2E_{\pi}} \delta_{\vec{n}_{\pi},\vec{n}_{e}-\
$$

Behaviour of the 3-point function

 $\tau_e > \tau_m >> 0$

$$
C_{3;\tau_{e}>\tau_{m}}^{\mu\nu}(\tau_{e},\tau_{m};\vec{p}_{e},\vec{p}_{m}) = \sum_{n} \frac{1}{2E_{n}} \int d^{3}x_{e} \int d^{3}x_{m} e^{i\vec{p}_{e}\cdot\vec{x}_{e}} e^{-i\vec{p}_{m}\cdot\vec{x}_{m}} \times \langle 0 |J_{e}^{\mu}(\vec{x}_{e}-\vec{x}_{m},\tau_{e}-\tau_{m}) J_{m}^{\nu}(\vec{0},0) e^{-i\vec{p}\cdot\vec{x}_{m}} e^{-\hat{H}\tau_{m}} |n\rangle \langle n | \mathcal{O}_{\pi}^{\dagger}(\vec{0},0) |0\rangle
$$

\n
$$
\xrightarrow{\tau_{m}\to\infty} \frac{1}{2E_{\pi}} \int d^{3}x_{e} \int d^{3}x_{m} e^{i\vec{p}_{e}\cdot\vec{x}_{e}} e^{-i\vec{p}_{m}\cdot\vec{x}_{m}} e^{-i\vec{p}_{\pi}\cdot\vec{x}_{m}} e^{-E_{\pi}\tau_{m}}
$$

\n
$$
\times \langle 0 |J_{e}^{\mu}(\vec{x}_{e}-\vec{x}_{m},\tau_{e}-\tau_{m}) J_{m}^{\nu}(\vec{0},0) | \pi(\vec{p}_{\pi}) \rangle \langle \pi(\vec{p}_{\pi}) | \mathcal{O}_{\pi}^{\dagger}(\vec{0},0) | 0 \rangle
$$

\n
$$
\vec{x}_{e} = \frac{\vec{x}+\vec{x}_{m}}{2E_{\pi}} \frac{1}{2E_{\pi}} \int d^{3}x_{m} e^{i\vec{p}_{e}\cdot\vec{x}_{m}} e^{-i\vec{p}_{m}\cdot\vec{x}_{m}} e^{-i\vec{p}_{\pi}\cdot\vec{x}_{m}} e^{-E_{\pi}\tau_{m}} \langle \pi(\vec{p}_{\pi}) | \mathcal{O}_{\pi}^{\dagger}(\vec{0},0) | 0 \rangle
$$

\n
$$
\times \int d^{3}x e^{i\vec{p}_{e}\cdot\vec{x}} \langle 0 | J_{e}^{\mu}(\vec{x},\tau_{e}-\tau_{m}) J_{m}^{\nu}(\vec{0},0) | \pi(\vec{p}_{\pi}) \rangle
$$

\n
$$
= \frac{1}{2E_{\pi}} \delta_{\vec{n}_{\pi},\vec{n}_{e}-\vec{n}_{m}} \times e^{-E_{\
$$