Flavor dependence of fragmentation functions

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Fragmentation functions

- **Our recent works, Comments on flavor separation in e^+e^-**

  (1) M. Hirai, H. Kawamura, S. Kumano, K. Saito,
  Prog. Theor. Exp. Phys. 2016, 113B04.

  (2) N. Sato, J. J. Ethier, W. Melnitchouk, M. Hirai,
  S. Kumano, A. Accardi, Phys. Rev. D 94, 114004 (2016).

Comments

- **Flavor separation and exotic-hadron candidates**

  (3) M. Hirai, S. Kumano, M. Oka, K. Sudoh, PRD 77 (2008) 017504.

interesting future project

Introduction to Fragmentation functions

Purposes of investigating fragmentation functions

Semi-inclusive reactions have been used for investigating

- **origin of proton spin**

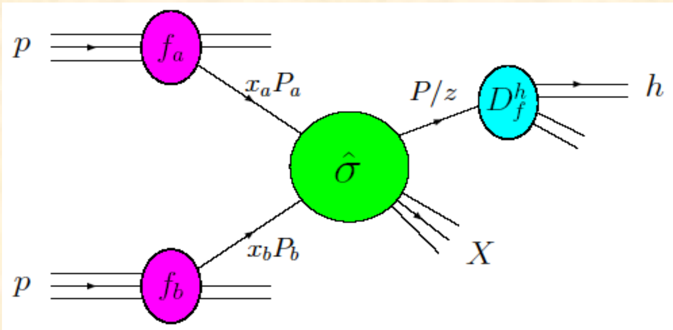
  $\vec{e} + \vec{p} \to e' + h + X, \quad \vec{p} + \vec{p} \to h + X$ **(RHIC-Spin)**

  Quark, antiquark, and gluon contributions to proton spin

  (flavor separation, gluon polarization)

- **properties of quark-hadron matters** $A + A' \to h + X$ **(RHIC, LHC)**

  Nuclear modification (recombination, energy loss, …)

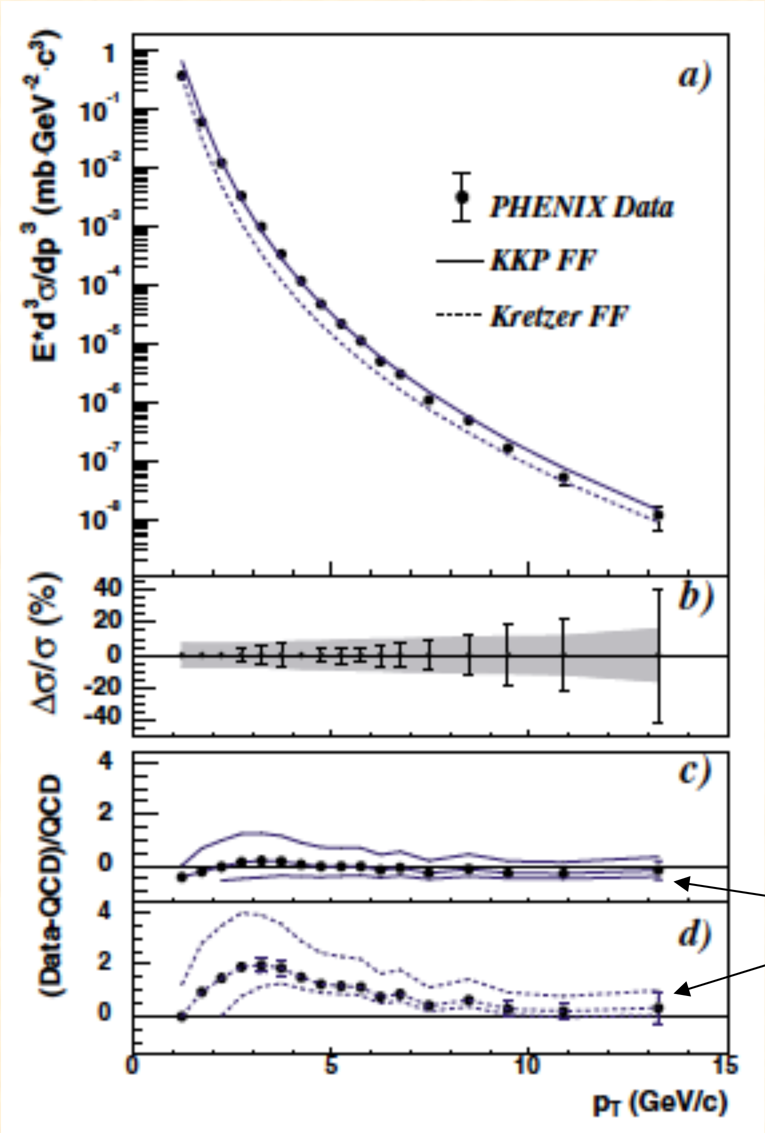


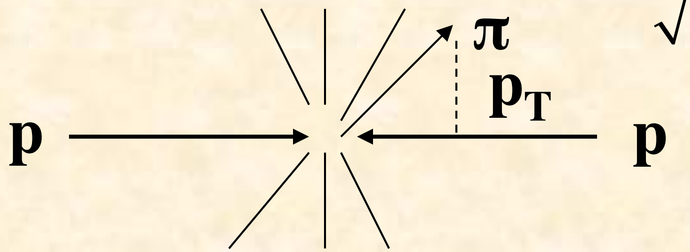
$$\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \to cX) \otimes D_c^{\pi}(z, Q^2)$$

- **Exotic-hadron search**

Pion production at RHIC: p + p→π+ X

S. S. Adler et al. (PHENIX), PRL 91 (2003) 241803





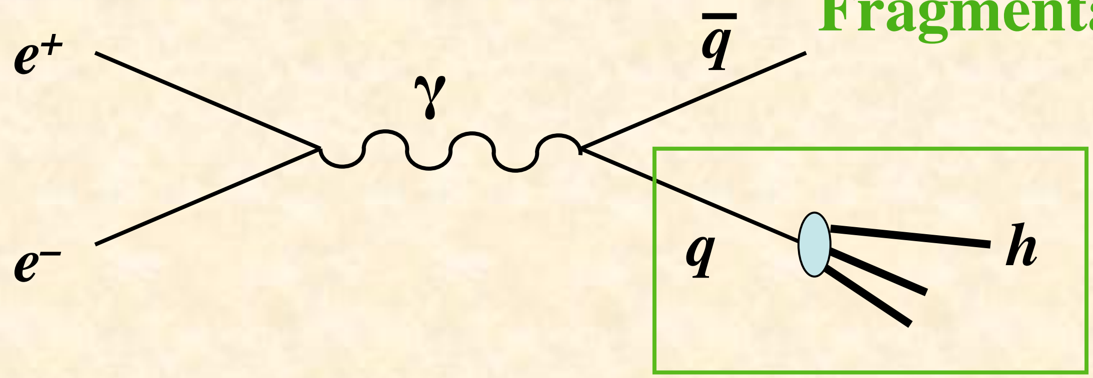
$\sqrt{s} = 200$ GeV

- **Consistent with NLO QCD calculation up to 10^{-8}**
- **Data agree with NLO pQCD + KKP**
- **Large differences between Kretzer and KKP calculations at small p_T**

→Importance of accurate fragmentation functions

Blue band indicates the scale uncertainty by taking $Q=2p_T$ and $p_T/2$.

Fragmentation Function



Fragmentation: hadron production from a quark, antiquark, or gluon

Fragmentation function is defined by

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

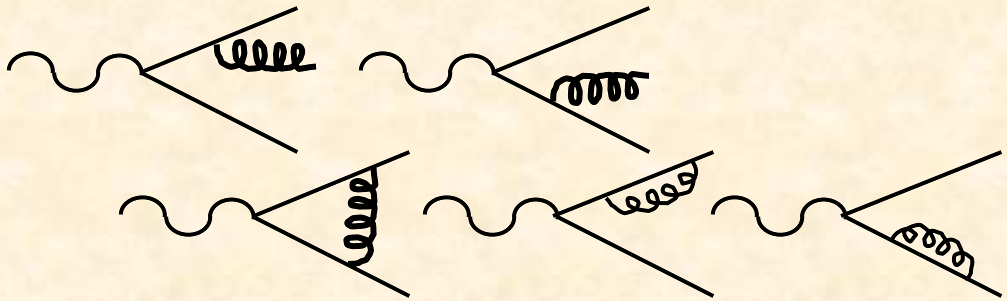
$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q}$$

(Energy fraction = hadron energy scaled to the beam energy)

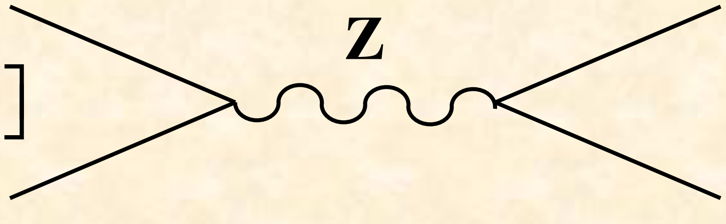
σ_{tot} = total hadronic cross section

$$\sigma_{tot} = \sum_q \sigma_0^q(s) \left[1 + \frac{\alpha_s(s)}{\pi} \right]$$

Higher-order correction



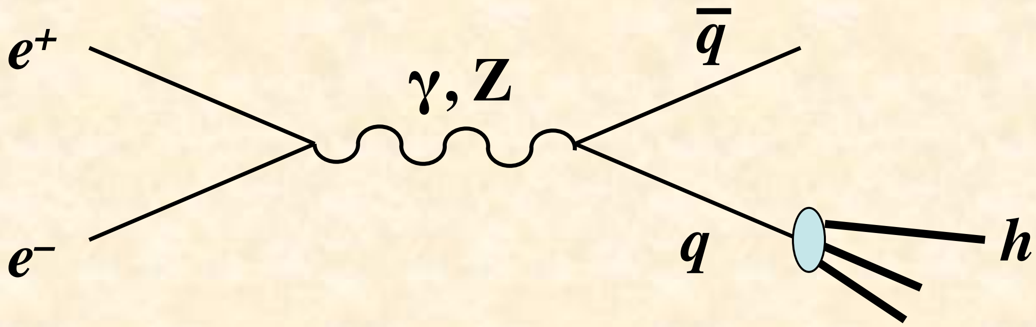
$$\sigma_0^q(s) = \frac{4\pi\alpha^2}{s} \left[e_q^2 + (\gamma\text{-}Z \text{ interference}) + (Z \text{ term}) \right]$$



Variable z

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}$$

note $s = Q^2$



- Hadron energy / Beam energy
- Hadron energy / Primary quark or antiquark energy

Fragmentation Functions

A fragmentation process occurs from quarks, antiquarks, and gluons, so that F^h is expressed by their contributions:

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} \underline{C_i\left(\frac{z}{y},Q^2\right)} \, \underline{D_i^h(y,Q^2)}$$

Calculated in perturbative QCD

Non-perturbative (determined from experiments)

$C_i(z,Q^2)$ = coefficient function

$D_i^h(z,Q^2)$ = fragmentation function of hadron h from a parton i

Energy sum rule

$D_i^h(z,Q^2) =$ probability to find the hadron h from a parton i with the energy fraction z

Energy conservation: $\sum_h \int_0^1 dz\, z\, D_i^h(z,Q^2) = 1$

$h = \pi^+,\ \pi^0,\ \pi^-,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ p,\ \bar{p},\ n,\ \bar{n},\ \cdots$

Quark model: $\pi^+(u\bar{d}),\ \pi^0((u\bar{u} - d\bar{d})/2),\ \pi^-(\bar{u}d),$

$K^+(u\bar{s}),\ K^0(d\bar{s}),\ \bar{K}^0(\bar{d}s),\ K^-(\bar{u}s),$

$p(uud),\ \bar{p}(\bar{u}\bar{u}\bar{d}),\ n(udd),\ \bar{n}(\bar{u}\bar{d}\bar{d}),\ \cdots$

Favored fragmentaion

(from a quark which exists in a naive quark mode)

for example $D_u^{\pi^+},\ D_{\bar{d}}^{\pi^+}$

Unfavored fragmentaion

(from a quark which doe not exist in a naive quark mode)

for example $D_d^{\pi^+},\ D_{\bar{u}}^{\pi^+},\ D_s^{\pi^+}$

Our recent works on Fragmentation functions

(1) M. Hirai, H. Kawamura, S. Kumano, K. Saito,
Prog. Theor. Exp. Phys. 2016, 113B04.

- Impact of Belle and BaBar data
- Flavor separation in e^+e^- data

(2) N. Sato, J. J. Ethier, W. Melnitchouk, M. Hirai,
S. Kumano, A. Accardi, Phys. Rev. D 94, 114004 (2016).

- Monte Carlo analysis of e^+e^- data

Initial functions for the pion

$$D_{u,\bar{d}}^{\pi^+}(z,Q_0^2) = N_u^{\pi^+} z^{\alpha_u^{\pi^+}} (1-z)^{\beta_u^{\pi^+}}$$

$$D_{\bar{u},d,s,\bar{s}}^{\pi^+}(z,Q_0^2) = N_{\bar{u}}^{\pi^+} z^{\alpha_{\bar{u}}^{\pi^+}} (1-z)^{\beta_{\bar{u}}^{\pi^+}}$$

$$D_{c,\bar{c}}^{\pi^+}(z,m_c^2) = N_c^{\pi^+} z^{\alpha_c^{\pi^+}} (1-z)^{\beta_c^{\pi^+}}$$

$$D_{b,\bar{b}}^{\pi^+}(z,m_b^2) = N_b^{\pi^+} z^{\alpha_b^{\pi^+}} (1-z)^{\beta_b^{\pi^+}}$$

$$D_g^{\pi^+}(z,Q_0^2) = N_g^{\pi^+} z^{\alpha_g^{\pi^+}} (1-z)^{\beta_g^{\pi^+}}$$

$$D_q^{\pi^-} = D_{\bar{q}}^{\pi^+}$$

$$D_i^{\pi^0} = \frac{D_i^{\pi^+} + D_i^{\pi^-}}{2}$$

$$n_f = \begin{cases} 3, & \mu_0^2 < Q^2 < m_c^2 \\ 4, & m_c^2 < Q^2 < m_b^2 \\ 5, & m_b^2 < Q^2 < m_t^2 \\ 6, & m_t^2 < Q^2 \end{cases}$$

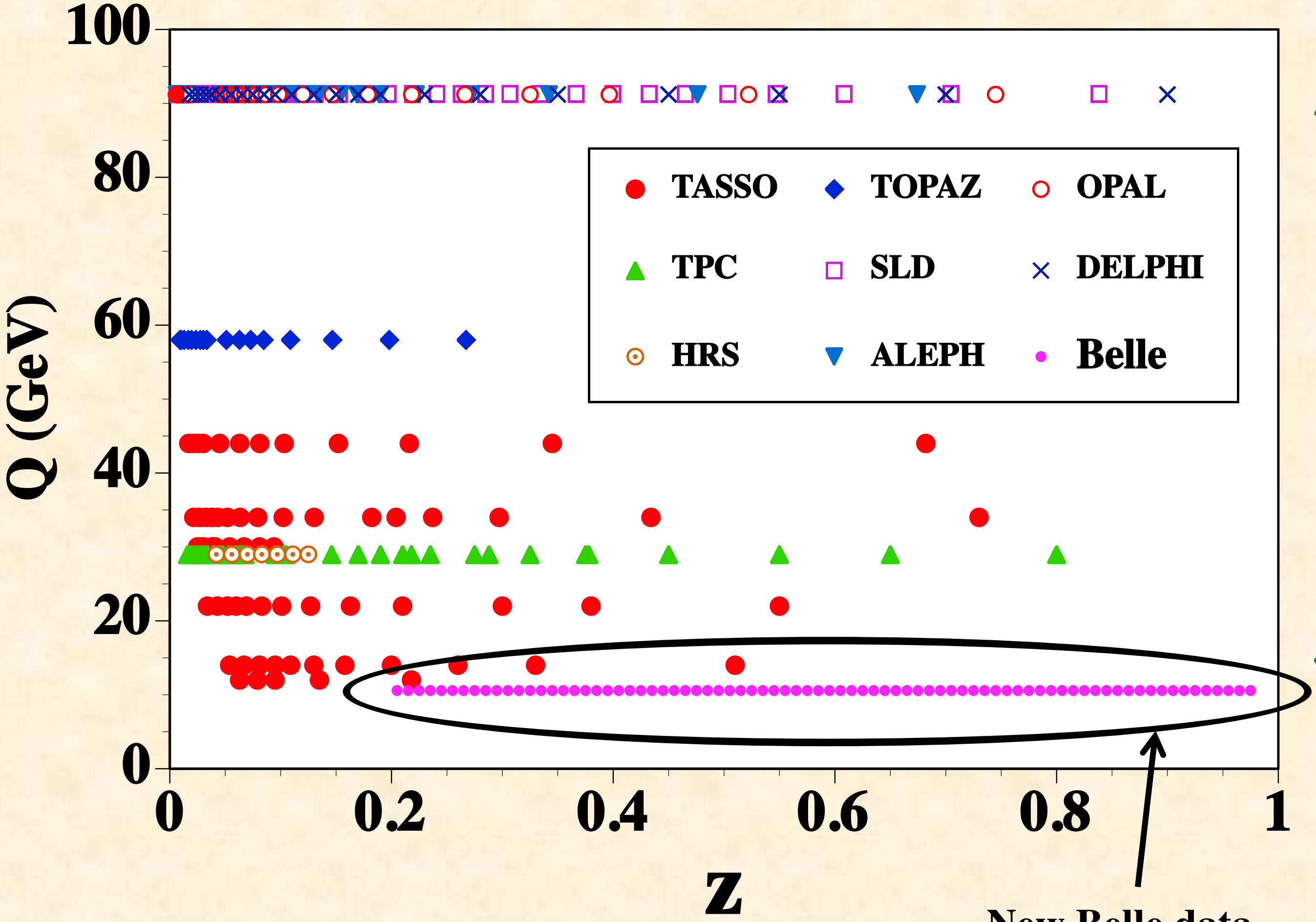
Constraint: 2nd moment should be finite and less than 1

$$N = M^{2\text{nd}} \frac{\Gamma(\alpha+\beta+3)}{\Gamma(\alpha+2)\Gamma(\beta+1)}, \qquad M^{2\text{nd}} \equiv \int_0^1 z D(z)\, dz$$

⇨ $$\alpha_i > -2, \quad \beta_i > -1, \quad 0 < M_i^{2\text{nd}} \left(= \int_0^1 z D_i^h(z) dz \right) < 1$$

New development for an update:

precise Belle (BaBar) measurements $D_i^h(z,Q^2)$



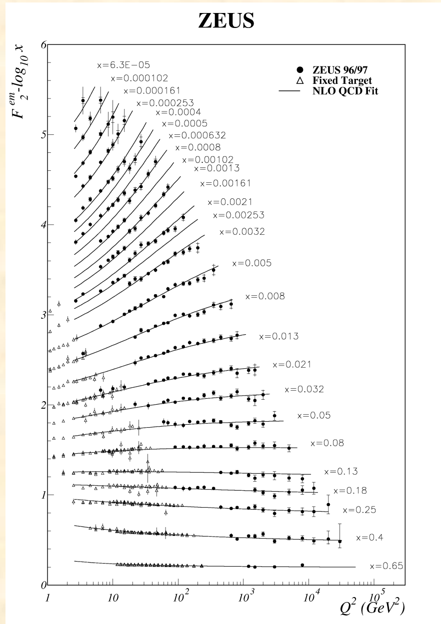
Scale evolution of D_i^h

→ gluon fragmentation function

New Belle data

M. Leitgab *et al.*

arXiv:1301.6183

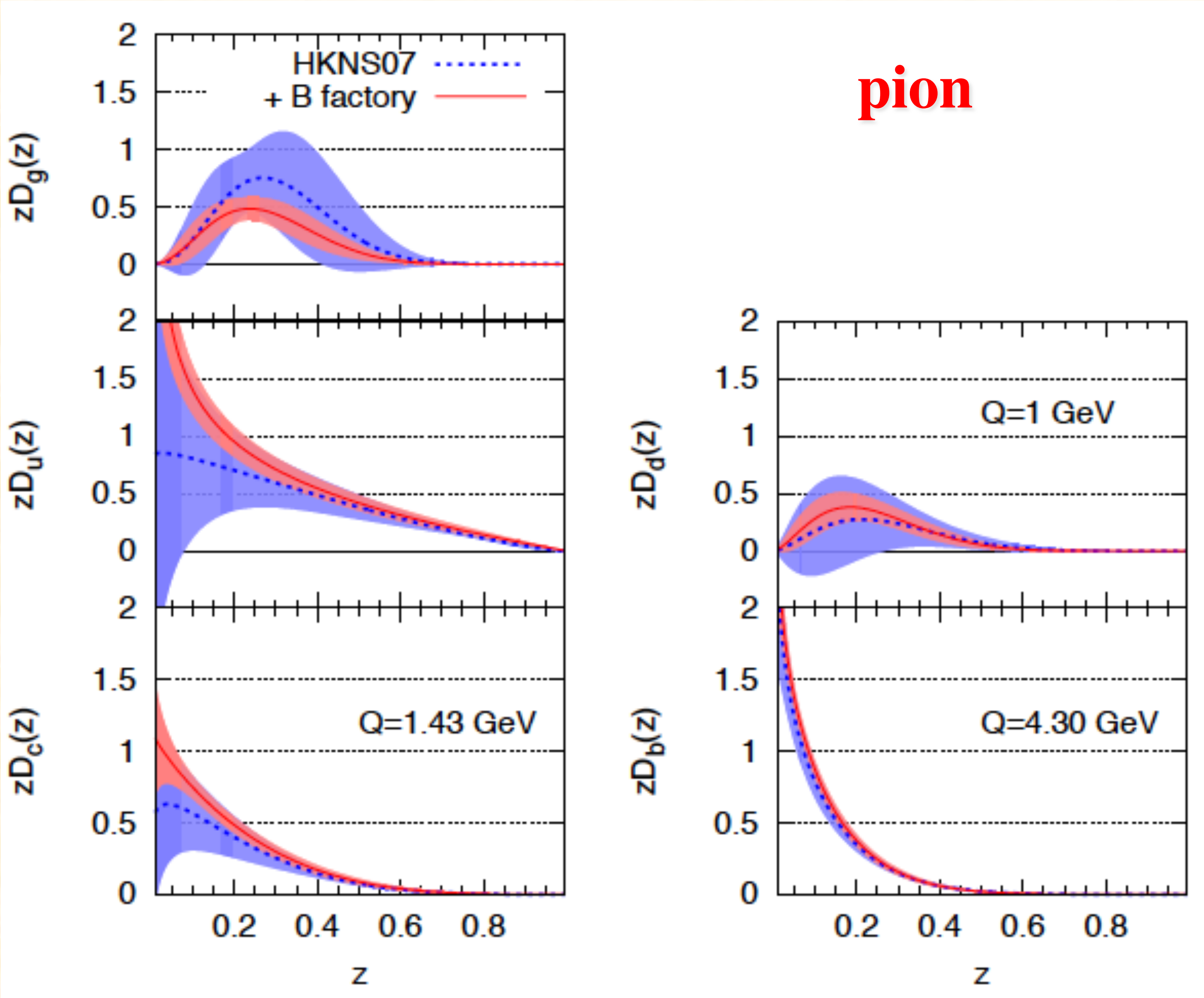


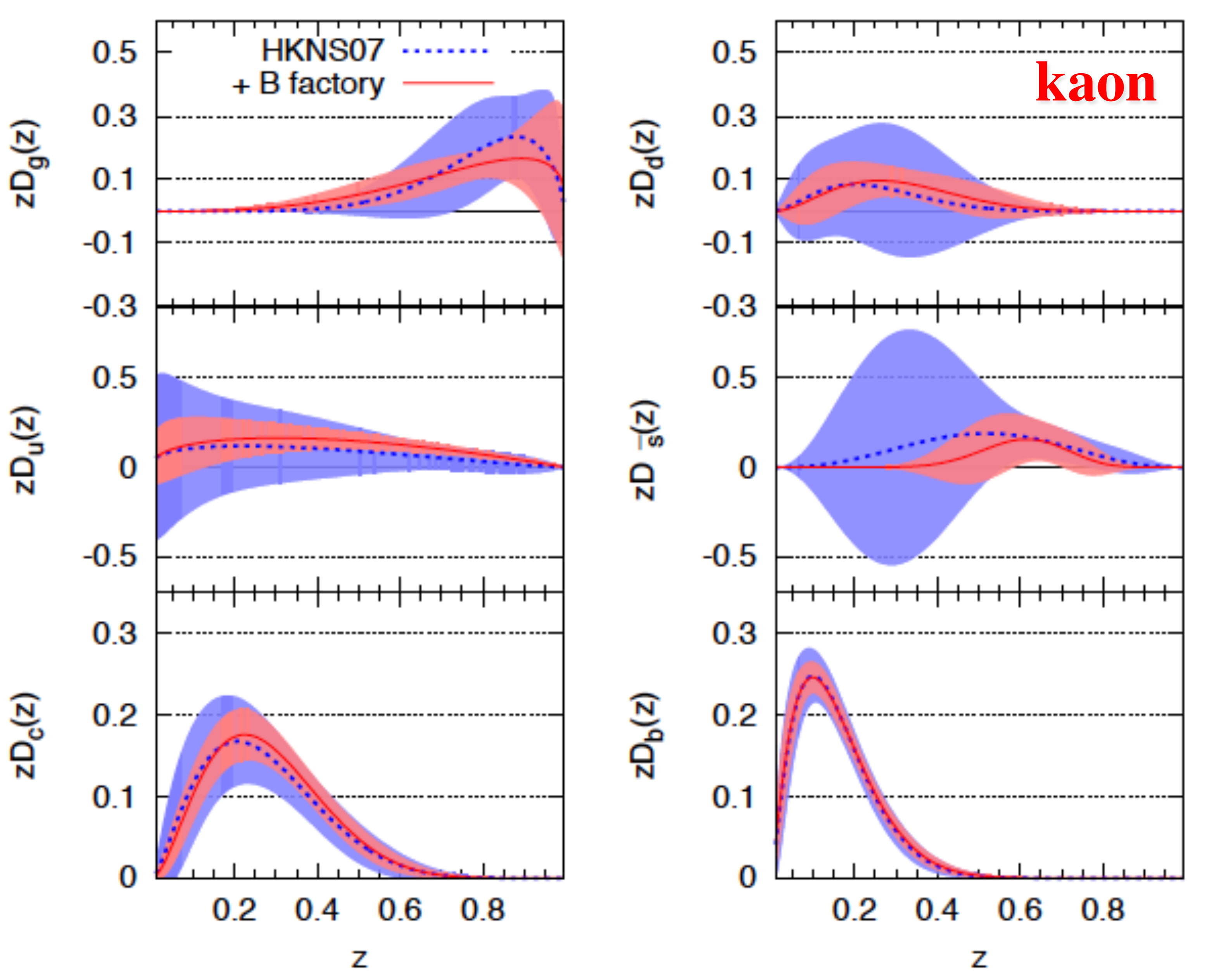
Scale evolution of F_2

→ gluon distribution

Impact of B-factory data

M. Hirai et al., PTEP 2016 (2016) 113B04





kaon
HKNS07
+ B factory
zDg(z)
zDd(z)
zDu(z)
zD s̄(z)
zDc(z)
zDb(z)
0.5
0.3
0.1
-0.1
-0.3
0
-0.5
0.2
0.4
0.6
0.8
z

Z contribution part

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz} = \sum_i \int_z^1 \frac{dy}{y} C_i\left(\frac{z}{y}, Q^2\right) D_i^h(y,Q^2), \quad \sigma_{tot} = \sum_q \sigma_0^q(s)\left[1 + \frac{\alpha_s(s)}{\pi}\right]$$

$$C_q(z)\Big|_Z = [\delta(1-z) + O(\alpha_s)] \frac{4\pi\alpha^2}{s} \left\{(c_V^e)^2 + (c_A^e)^2\right\}\left\{(c_V^q)^2 + (c_A^q)^2\right\} \rho_2(s)$$

$$\rho_2(s) = \left(\frac{1}{4\sin^2\theta_W \cos^2\theta_W}\right)^2 \frac{s^2}{(M_Z^2 - s)^2 + M_Z^2\Gamma_Z^2}$$

$$F^h(z,Q^2) = \tilde{C}_q \otimes \left[\left\{(c_V^u)^2 + (c_A^u)^2\right\}\left\{D_{u^+}^h + D_{c^+}^h\right\} + \left\{(c_V^d)^2 + (c_A^d)^2\right\}\left\{D_{d^+}^h + D_{s^+}^h + D_{b^+}^h\right\}\right] + C_g \otimes D_g^h$$

$$c_V^q = T_3^3 - 2e_q \sin^2\theta_W, \quad c_V^u = +\frac{1}{2} - \frac{4}{3}\sin^2\theta_W, \quad c_V^d = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$$

$$c_V^q = T_3^3, \qquad c_A^u = +\frac{1}{2}, \qquad c_A^d = -\frac{1}{2}$$

$$\sin^2\theta_W = 0.231265$$

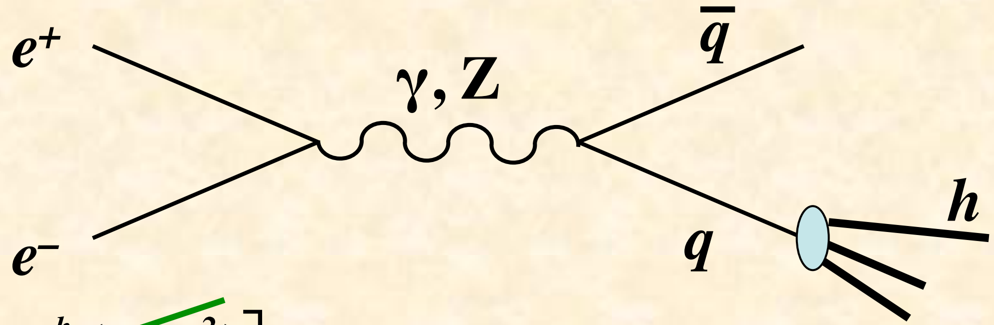
$$(c_V^u)^2 + (c_A^u)^2 = 0.286728, \quad (c_V^d)^2 + (c_A^d)^2 = 0.369594$$

$$\to \left\{(c_V^u)^2 + (c_A^u)^2\right\}\tilde{C}_q = 0.287\tilde{C}_q, \quad \left\{(c_V^d)^2 + (c_A^d)^2\right\}\tilde{C}_q = 0.370\tilde{C}_q$$

If $\left\{(c_V^u)^2 + (c_A^u)^2\right\}\tilde{C}_q = \left\{(c_V^d)^2 + (c_A^d)^2\right\}\tilde{C}_q = 0.33\tilde{C}_q \equiv \tilde{C}'_q$

$$F^h(z,Q^2) \approx \tilde{C}'_q(z,Q^2) \otimes D_\Sigma^h(z,Q^2) + C_g(z,Q^2) \otimes D_g^h(z,Q^2)$$

Flavor separation in e⁺e⁻



M. Hirai et al., PTEP 2016 (2016) 113B04

- At the Z-pole (LEP/SLD)

$$F^h(z,M_Z^2) \approx (c_V^{u\,2} + c_A^{u\,2})\left[D_{u^+}^h(z,M_Z^2) + D_{c^+}^h(z,M_Z^2)\right]$$
$$+ (c_V^{d\,2} + c_A^{d\,2})\left[D_{d^+}^h(z,M_Z^2) + D_{s^+}^h(z,M_Z^2) + D_{b^+}^h(z,M_Z^2)\right]$$
$$\approx\ 0.33 \sum_q D_{q^+}^h(z,M_Z^2) \qquad D_{q^+}^h \equiv D_q^h + D_{\bar{q}}^h$$

flavor singlet combination

- Far from the Z-pole (Belle, TASSO/TPC/HRC/TOPAZ)

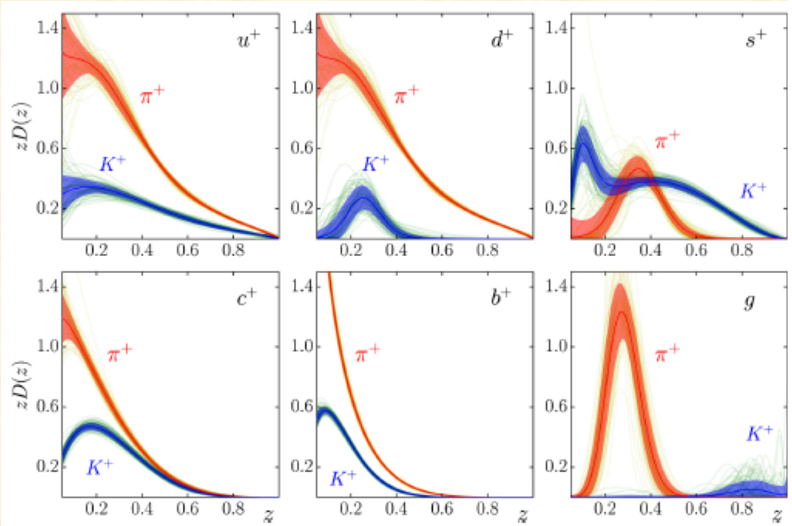
$$F^h(z,Q^2) \approx \frac{4}{9}\left[D_{u^+}^h(z,Q^2) + D_{c^+}^h(z,M_Z^2)\right] + \frac{1}{9}\left[D_{d^+}^h(z,Q^2) + D_{s^+}^h(z,Q^2) + D_{b^+}^h(z,Q^2)\right]$$

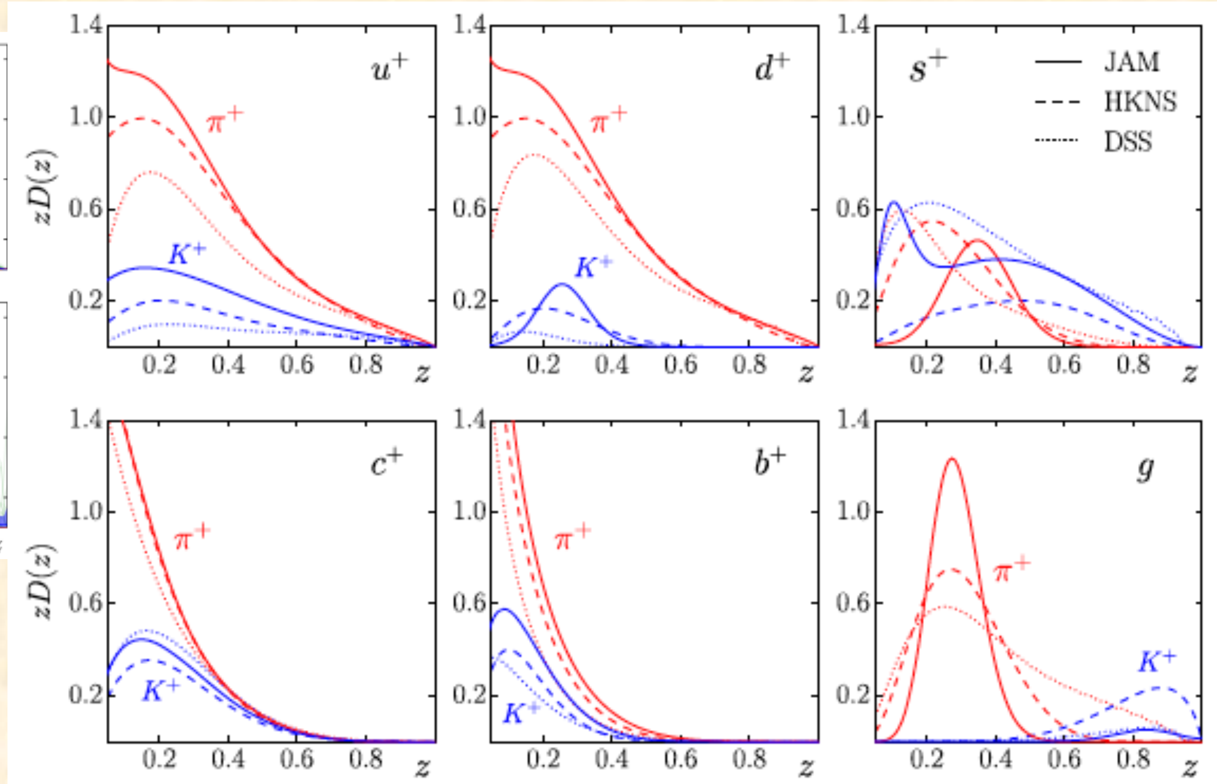
(1) c-quark, b-quark FFs are determined from the flavor tagged data.

(2) If we have very precise data at and far from the Z-pole,
we can determine 2 independent components of the quark FFs.

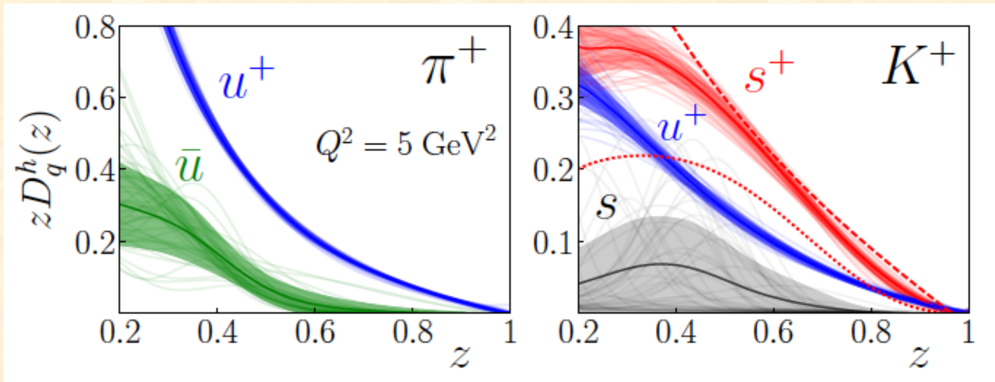
(3) Remaining flavor decomposition & determination of the gluon FF
come from the mixing through scale evolution

N. Sato *et al.*, PRD 94 (2016) 114004





Including semi-inclusive data,
J. J. Ethier, N. Sato, W. Melnitchouk,
PRL 119 (2017) 132001

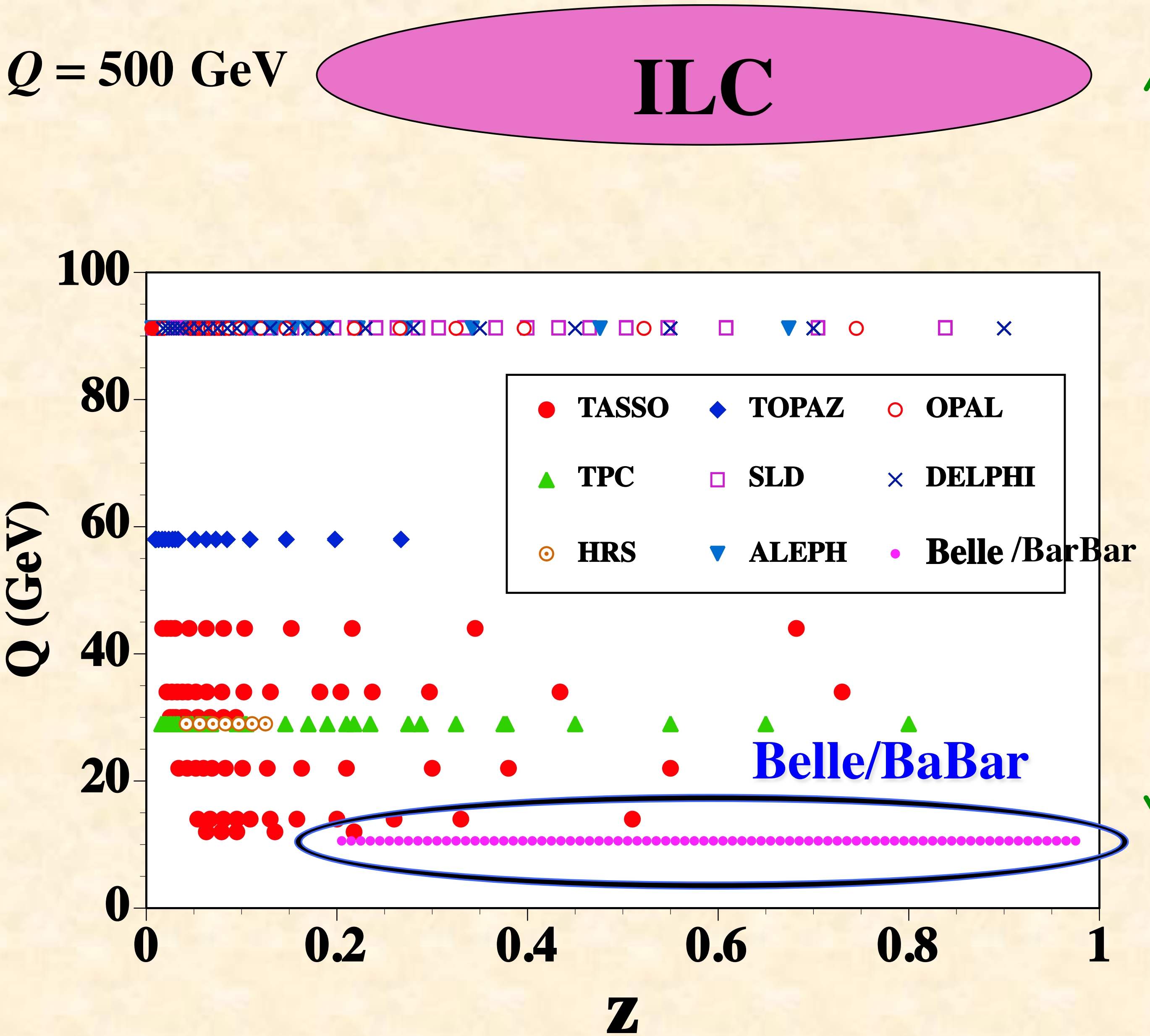


→ Ethier's talk

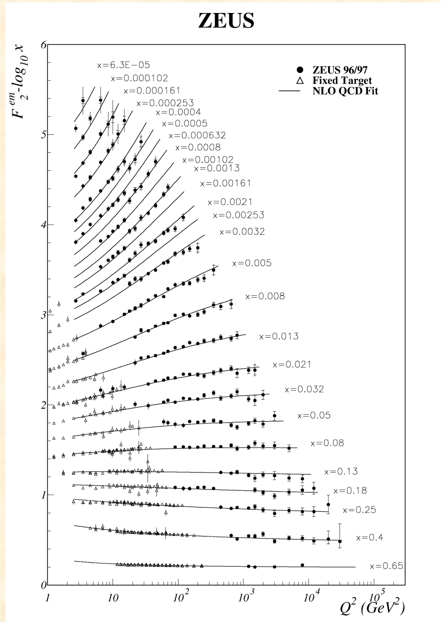
Fragmentation function measurements

$D_i^h(z,Q^2)$

$Q = 500$ GeV



Scale evolution of D_i^h
→ gluon fragmentation function

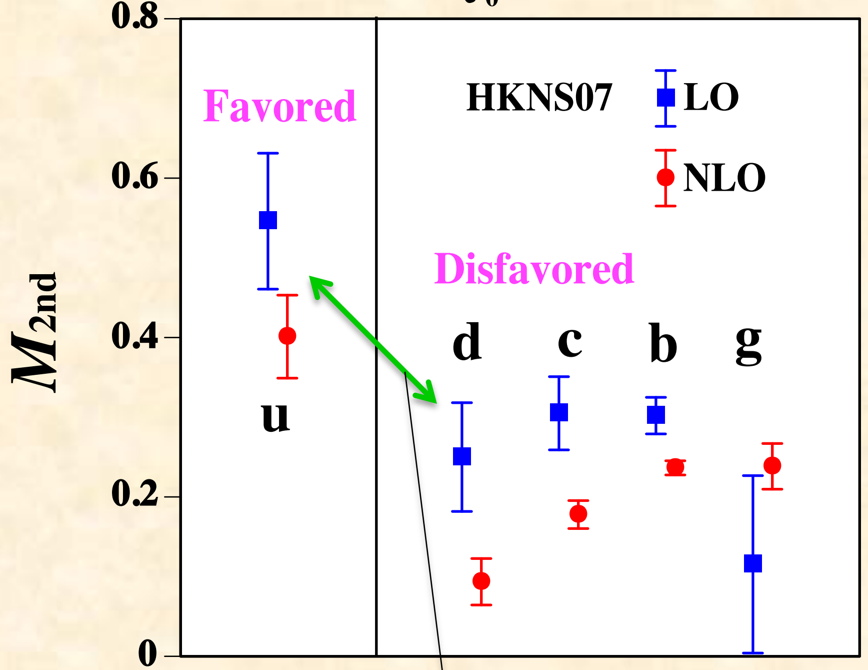


Scale evolution of F_2
→ gluon distribution

Flavor dependence of fragmentation functions for finding internal structure of exotic hadron candidates

2nd moments of pion fragmentation functions

$$M_{2nd} = \int_0^1 dz\, z\, D_i^{\pi^+}(z)$$



2nd moments of
M. Hirai, SK, T.-H. Nagai, K. Sudoh,
PRD 75 (2007) 094009.

There are distinct differences between
the favored and disfavored 2nd moments.

→ It could be used for exotic-hadron studies.

Progress in exotic hadrons

$q\bar{q}$ Meson
q^3 Baryon

$q^2\bar{q}^2$ Tetraquark
$q^4\bar{q}$ Pentaquark
q^6 Dibaryon
…
$q^{10}\bar{q}$ e.g. Strange tribaryon
…
gg Glueball
…

- $\Theta^+(1540)$???: LEPS
  Pentaquark?
  $uudd\bar{s}$?
- Kaonic nuclei?: KEK-PS, …
  Strange tribaryons, …
  $K^- pnn,\ K^- ppn$?
  $K^- pp$?
- X (3872), Y(3940): Belle
  Tetraquark, $D\bar{D}$ molecule
  $c\bar{c}$
  $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
  $D^+(c\bar{d})D^-(\bar{c}d)$?
- $D_{sJ}(2317)$, $D_{sJ}(2460)$: BaBar, CLEO, Belle
  Tetraquark, DK molecule
  $c\bar{s}$
  $D^0(c\bar{u})K^+(u\bar{s})$
  $D^+(c\bar{d})K^0(d\bar{s})$?
- Z (4430): Belle
  Tetraquark, …
  $c\bar{c}u\bar{d}$, D molecule?
- $P_c(4380)$, $P_c(4450)$: LHCb
- …
  $u\bar{c}udc$, $\bar{D}(u\bar{c})\Sigma_c^*(udc)$, $\bar{D}^*(u\bar{c})\Sigma_c(udc)$ molecule?

Scalar mesons $J^P=0^+$ at $M\sim 1$ GeV

Naïve quark-model

$a_1(1230)$

1.0 GeV

$a_0(980)$ $f_0(980)$

$\rho(770)$

0.5 GeV

$f_0(600)=\sigma$

$$\sigma = f_0(600) = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$$

$$f_0(980) = s\bar{s} \quad \rightarrow \text{denote } f_0 \text{ in this talk}$$

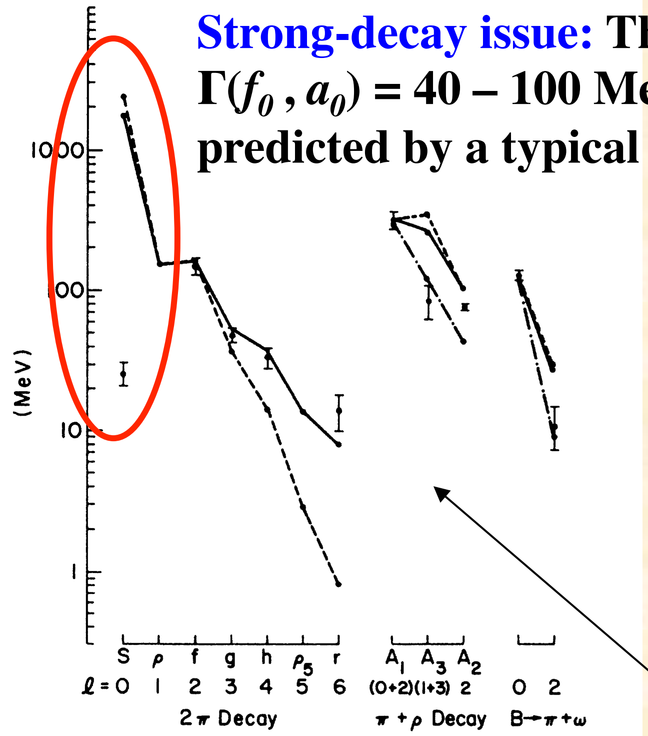
$$a_0(980) = u\bar{d},\quad \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}),\quad d\bar{u}$$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

⇳ **contradiction**

Experiment: $m(\sigma) < m(a_0) \sim m(f_0)$

Strong-decay issue: The experimental widths $\Gamma(f_0, a_0)$ = 40 – 100 MeV are too small to be predicted by a typical quark model.



These issues could be resolved

if f_0 is a tetraquark ($qq\bar{q}\bar{q}$) or a $K\bar{K}$ molecule,

namely an "exotic" hadron.

SK and V. R. Pandharipande, Phys. Rev. D38 (1988) 146.

Determination of $f_0(980)$ structure by electromagnetic decays

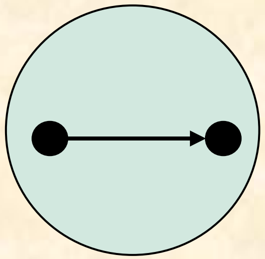
F. E. Close, N. Isgur, and SK, Nucl. Phys. B389 (1993) 513.

Radiative decay: $\phi \to S\gamma$

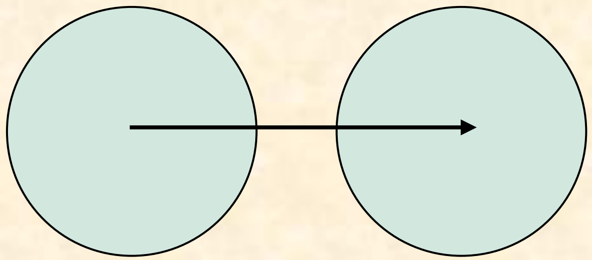
$S = f_0(980), a_0(980)$

$J^p = 1^- \to 0^+$ E1 transition

Electric dipole:
$e\vec{r}$ (distance!)



$q\bar{q}$ model:
Γ = small



$K\bar{K}$ molecule
or $qq\bar{q}\bar{q}$: Γ = large

Experimental results of VEPP-2M and DAΦNE suggest that f_0 is a tetraquark state (or a $K\bar{K}$ molecule?).

CMD-2 (1999): $B(\phi \to f_0\gamma) = (1.93 \pm 0.46 \pm 0.50) \times 10^{-4}$
SND (2000): $(3.5 \pm 0.3^{+1.3}_{-0.5}) \times 10^{-4}$
KLOE (2002): $(4.47 \pm 0.21_{\text{stat+syst}}) \times 10^{-4}$

For some discussions,
N. N. Achasov and A. V. Kiselev, PRD 73 (2006) 054029; D74 (2006) 059902(E); D76 (2007) 077501;
Y. S. Kalashnikova *et al.*, Eur. Phys. J. A24 (2005) 437.

See also Belle (2007)
$\Gamma(f_0 \to \gamma\gamma) = 0.205^{+0.095}_{-0.083}(\text{stat})^{+0.147}_{-0.117}(\text{syst})$ keV

Criteria for determining f_0 structure by its fragmentation functions

M. Hirai, S. Kumano, M. Oka, K. Sudoh, PRD 77 (2008) 017504.

Possible configurations of $f_0(980)$

(1) ordinary u,d - meson $\quad \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

(2) strange meson, $\quad s\bar{s}$

(3) tetraquark ($K\bar{K}$), $\quad \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$

(4) glueball $\quad gg$

Contradicts with experimental widths (for (1))

$$\Gamma_{\text{theo}}(f_0 \to \pi\pi) = 500 - 1000 \text{ MeV} \gg \Gamma_{\text{exp}} = 40 - 100 \text{ MeV}$$

$$\Gamma_{\text{theo}}(f_0 \to \gamma\gamma) = 1.3 - 1.8 \text{ keV} \gg \Gamma_{\text{exp}} = 0.205 \text{ keV}$$

Contradicts with lattice-QCD estimate (for (4))

$$m_{\text{lattice}}(f_0) = 1600 \text{ MeV} \gg m_{\text{exp}} = 980 \text{ MeV}$$

There could difference in fragmentation functions for f_0 depending on its internal structure.

- Favored and disfavored fragmentation functions
- 2nd moments and functional forms

$s\bar{s}$ picture for $f_0(980)$

$$M \equiv \int_0^1 z D(z)\, dz \quad \text{(2nd moment)}$$

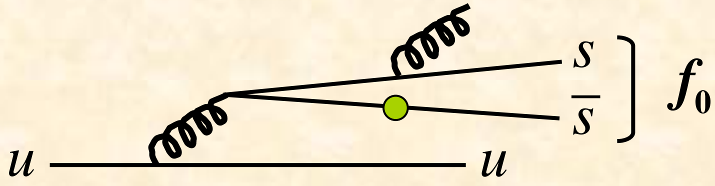
2nd moment: $M(u) < M(s) \lesssim M(g)$

Peak of function: $z_{\max}(u) < z_{\max}(s) \simeq z_{\max}(g)$

More energy is transferred to f_0 from the parent s or g.

Naive estimates!

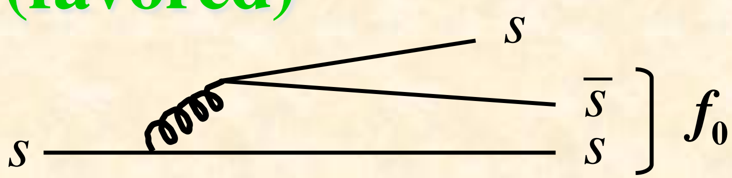
u (disfavored)



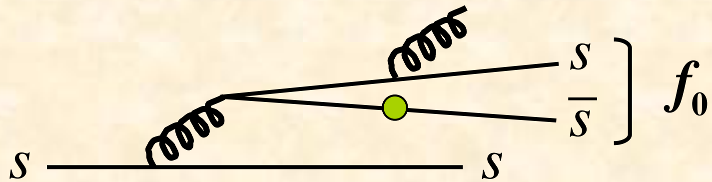
O (g^3)

+ one O (g^3) term of gluon radiation from the antiquark

s (favored)



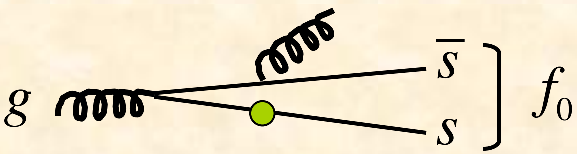
O (g^2)



O (g^3)

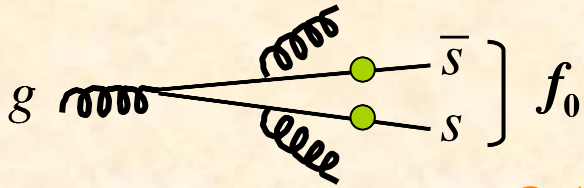
+ one O (g^3) term of gluon radiation from the antiquark

g



O (g^2)

+ one O (g^2) term of gluon radiation from the quark



O (g^3)

+ two O (g^3) terms of gluon radiation from the quark or antiquark

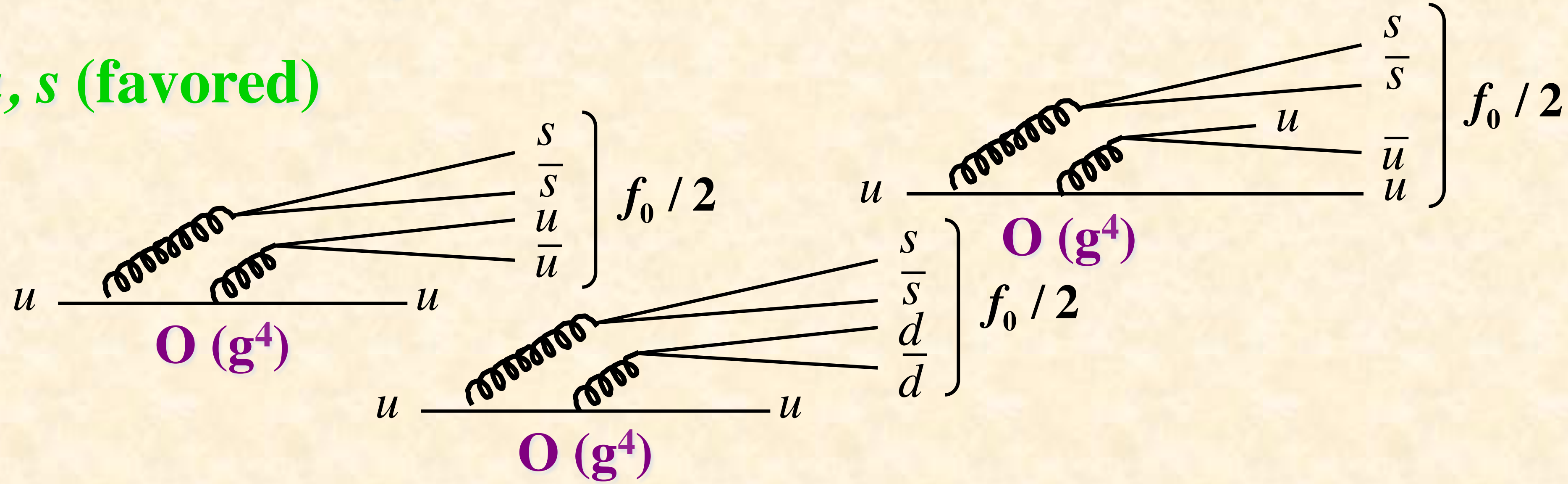
$n\bar{n}s\bar{s}$ picture for $f_0(980)$

$$f_0 = (u\bar{u}s\bar{s} + d\bar{d}s\bar{s}) / \sqrt{2}$$

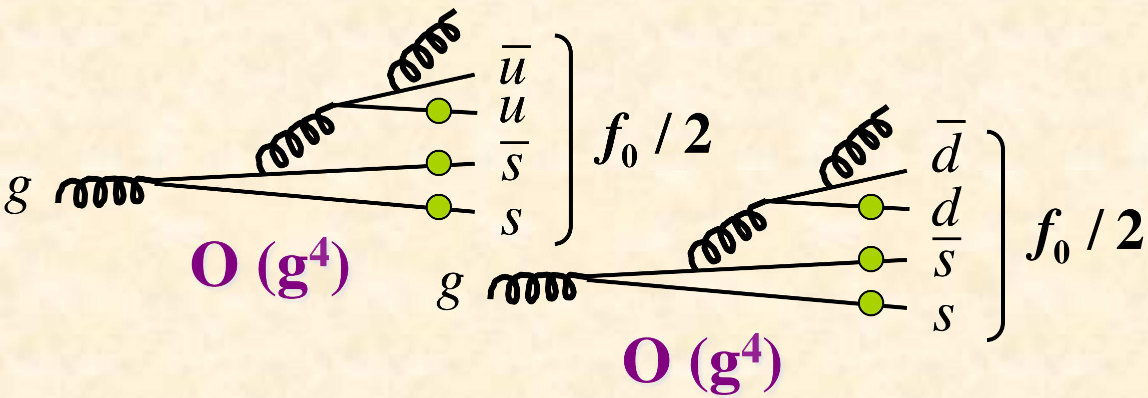
$K\bar{K}$ picture for $f_0(980)$

$$f_0 = \left[K^+(u\bar{s})K^-(\bar{u}s) + K^0(d\bar{s})\bar{K}^0(\bar{d}s) \right] / \sqrt{2}$$

u, s (favored)



g



+ six O (g^4) terms of gluon radiation from other quarks ●

2nd moment: $M(u) = M(s) \lesssim M(g)$

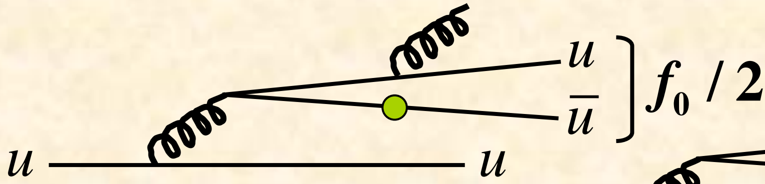
Peak of function: $z_{\max}(u) = z_{\max}(s) \simeq z_{\max}(g)$

$n\bar{n}$ picture for $f_0(980)$

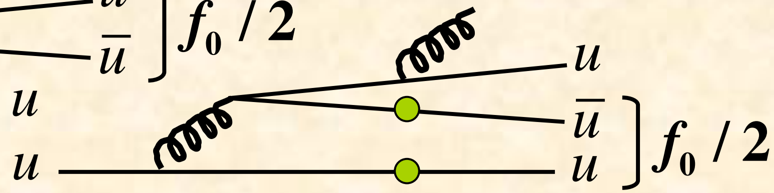
$$f_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$$

u (favored)

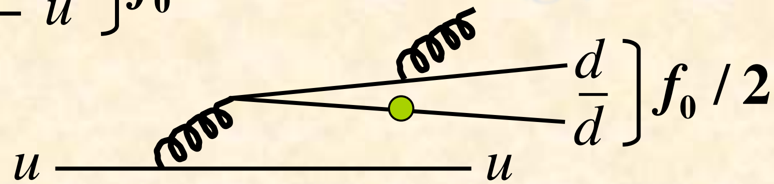
O (g²)



O (g³)



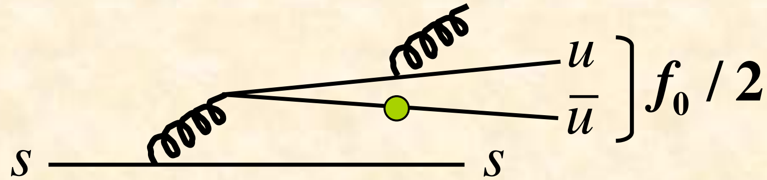
O (g³)



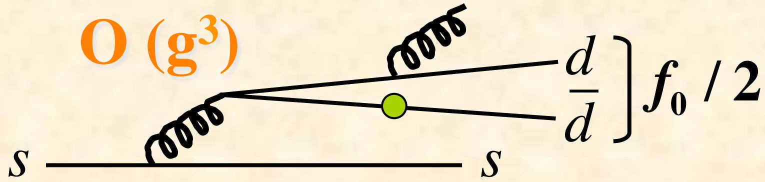
O (g³)

+ four O (g³) terms of gluon radiation from other quarks ●

s (disfavored)



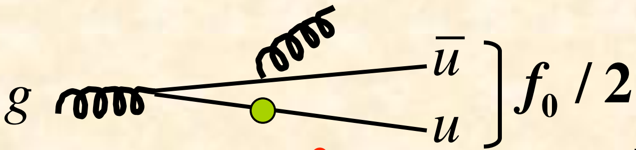
O (g³)



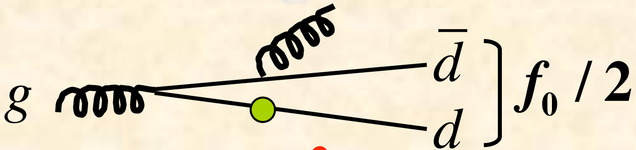
O (g³)

+ two O (g³) terms of gluon radiation from other quarks ●

g



O (g²)



O (g²)

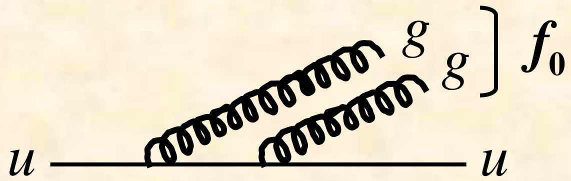
+ Two O (g²) terms of gluon radiation from other quarks ●

2nd moment: $M(s) < M(u) < M(g)$

Peak of function: $z_{max}(s) < z_{max}(u) \simeq z_{max}(g)$

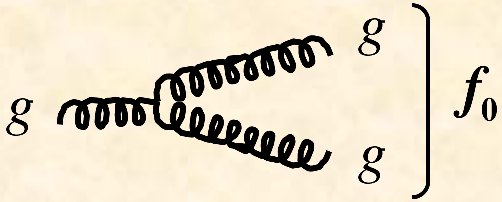
gg picture for $f_0(980)$

u, s (disfavored)



O (g^2)

g (favored)



O (g^1)

2nd moment: $M(u) = M(s) < M(g)$

Peak of function: $z_{max}(u) = z_{max}(s) < z_{max}(g)$

Naive Judgment

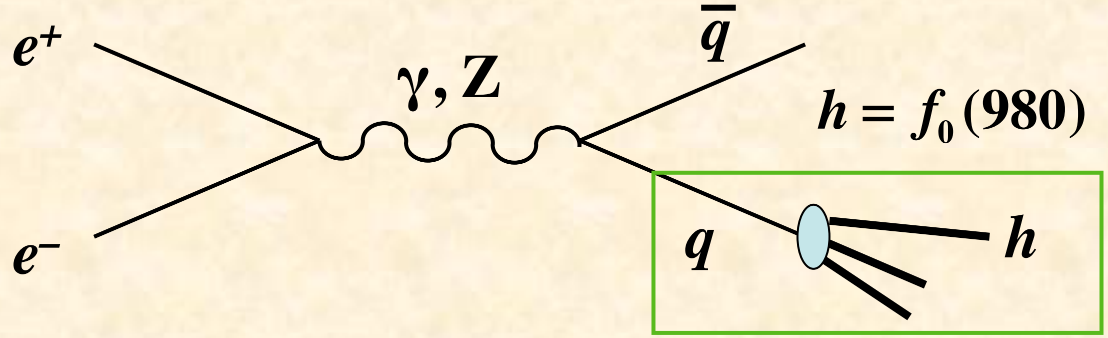
Type	Configuration	2nd Moment	Peak z
Nonstrange $q\bar{q}$	$(u\bar{u}+d\bar{d})/\sqrt{2}$	$M(s)<M(u)<M(g)$	$z_{\max}(s)<z_{\max}(u)\simeq z_{\max}(g)$
Strange $q\bar{q}$	$s\bar{s}$	$M(u)<M(s)\lesssim M(g)$	$z_{\max}(u)<z_{\max}(s)\simeq z_{\max}(g)$
Tetraquark	$(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)\simeq z_{\max}(g)$
$K\bar{K}$ Molecule	$(K^+K^-+K^0\bar{K}^0)/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)\simeq z_{\max}(g)$
Glueball	gg	$M(u)=M(s)<M(g)$	$z_{\max}(u)=z_{\max}(s)<z_{\max}(g)$

Since there is no difference between $D_u^{f_0}$ and $D_d^{f_0}$ in the models, they are assumed to be equal. On the other hand, $D_s^{f_0}$ and $D_g^{f_0}$ are generally different from them, so that they should be used for finding the internal structure. Therefore, simple and "model-independent" initial functions are

$$D_u^{f_0}(z,Q_0^2)=D_{\bar{u}}^{f_0}(z,Q_0^2)=D_d^{f_0}(z,Q_0^2)=D_{\bar{d}}^{f_0}(z,Q_0^2),\quad D_s^{f_0}(z,Q_0^2)=D_{\bar{s}}^{f_0}(z,Q_0^2),$$

$$D_g^{f_0}(z,Q_0^2),\qquad D_c^{f_0}(z,m_c^2)=D_{\bar{c}}^{f_0}(z,m_c^2),\qquad D_b^{f_0}(z,m_b^2)=D_{\bar{b}}^{f_0}(z,m_b^2).$$

Fragmentation functions for $f_0(980)$



$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2$$

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

σ_{tot} = total hadronic cross section

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i\left(\frac{z}{y}, Q^2\right) D_i^h(y,Q^2)$$

Initial functions

$$D_u^{f_0}(z,Q_0^2) = D_d^{f_0}(z,Q_0^2) = N_u^{f_0} z^{\alpha_u^{f_0}} (1-z)^{\beta_u^{f_0}}$$

$$D_s^{f_0}(z,Q_0^2) = N_s^{f_0} z^{\alpha_s^{f_0}} (1-z)^{\beta_s^{f_0}}$$

$$D_g^{f_0}(z,Q_0^2) = N_g^{f_0} z^{\alpha_g^{f_0}} (1-z)^{\beta_g^{f_0}}$$

$$D_c^{f_0}(z,m_c^2) = N_c^{f_0} z^{\alpha_c^{f_0}} (1-z)^{\beta_c^{f_0}}$$

$$D_b^{f_0}(z,m_b^2) = N_b^{f_0} z^{\alpha_b^{f_0}} (1-z)^{\beta_b^{f_0}}$$

- $D_q^{f_0}(z,Q_0^2) = D_{\bar{q}}^{f_0}(z,Q_0^2)$
- $Q_0 = 1$ GeV
  $m_c = 1.43$ GeV
  $m_b = 4.3$ GeV

$$N = M \frac{\Gamma(\alpha+\beta+3)}{\Gamma(\alpha+2)\Gamma(\beta+1)}, \quad M \equiv \int_0^1 zD(z)dz$$

Experimental data for f_0

Total number of data : only 23

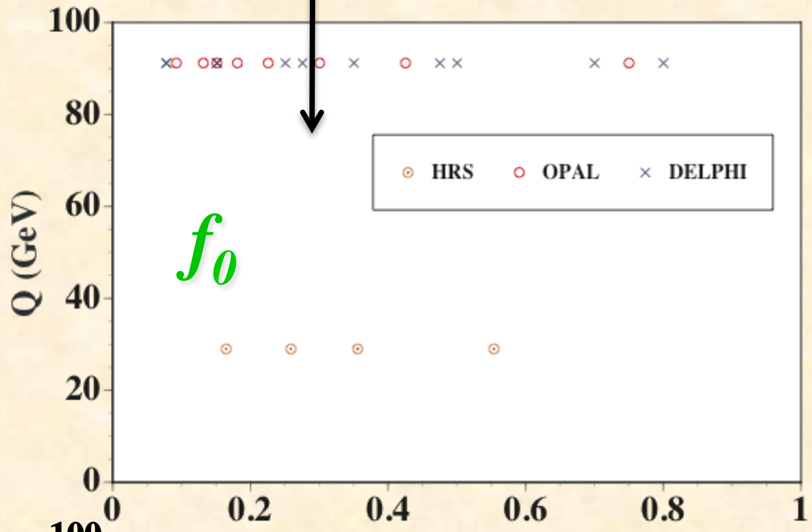
Exp. collaboration	$\sqrt{s}$ (GeV)	# of data
HRS	29	4
OPAL	91.2	8
DELPHI	91.2	11

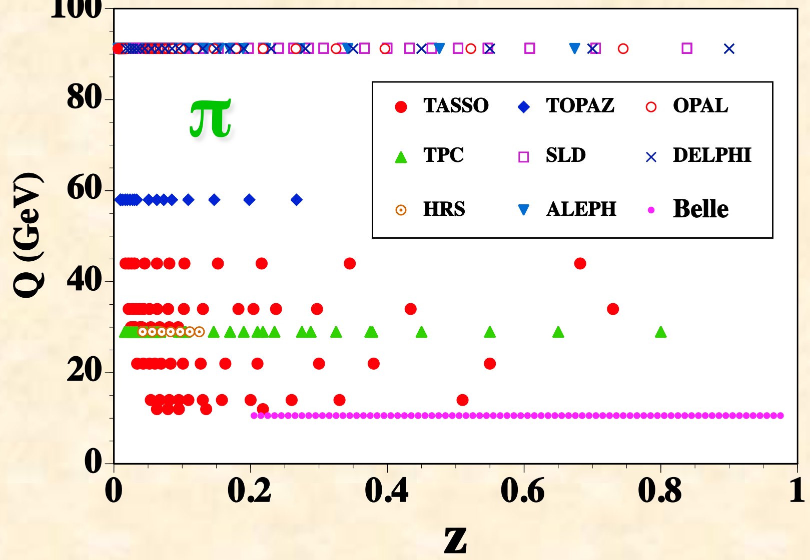
pion

Total number of data : 342

Exp. collaboration	$\sqrt{s}$ (GeV)	# of data
Belle-preliminary	10.58	78
TASSO	12,14,22,30,34,44	29
TCP	29	18
HRS	29	2
TOPAZ	58	4
SLD	91.2	29
SLD [light quark]		29
SLD [c quark]		29
SLD [b quark]		29
ALEPH	91.2	22
OPAL	91.2	22
DELPHI	91.2	17
DELPHI [light quark]		17
DELPHI [b quark]		17

One could foresee the difficulty in getting reliable FFs for f_0 *at this stage.*





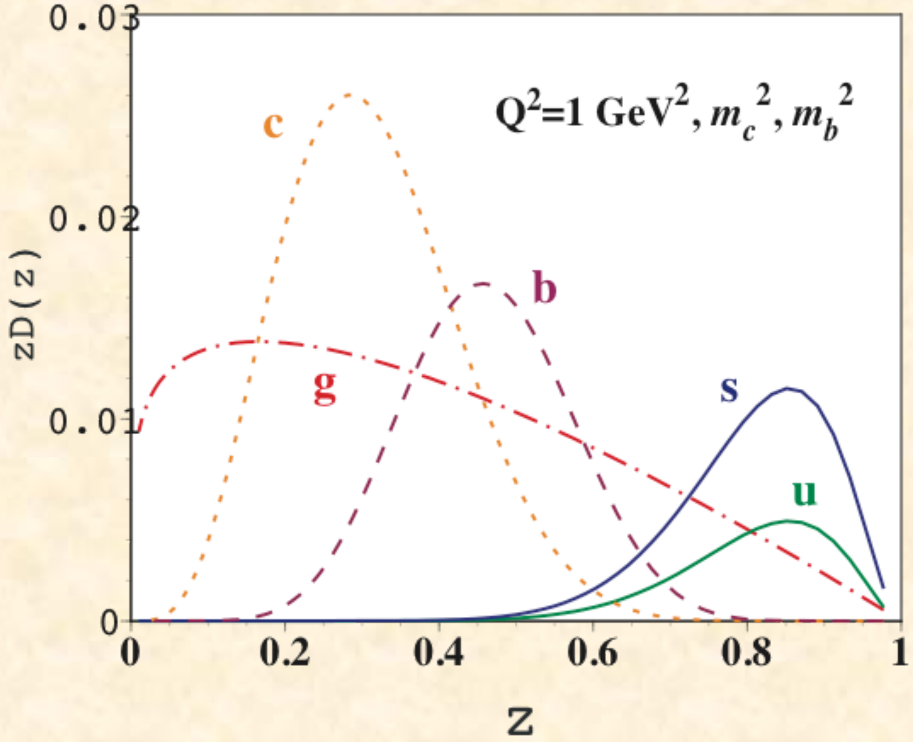
Results on the fragmentation functions

- **Functional forms**

  (1) $D_u^{f_0}(z)$, $D_s^{f_0}(z)$ have peaks at large z
  (2) $z_u^{\max} \sim z_s^{\max}$

(1) and (2) indicate tetraquark structure

$$f_0 \sim \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s})$$



- **2nd moments:** $\frac{M_u}{M_s} = 0.43$

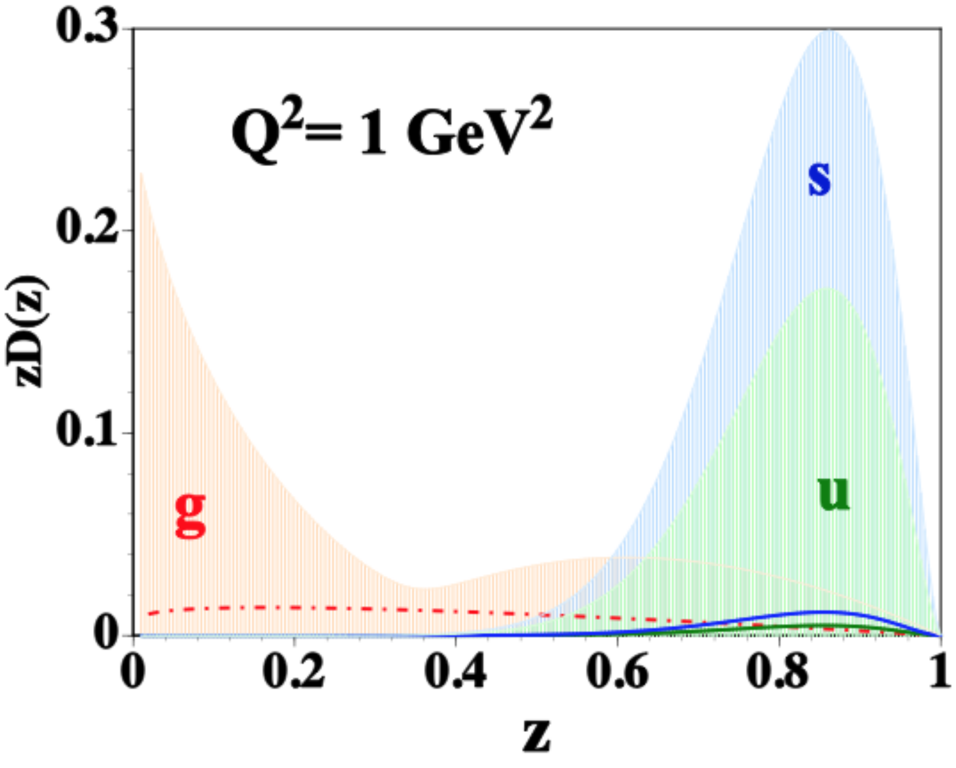
This relation indicates $s\bar{s}$-like structure (or admixture)

$$f_0 \sim s\bar{s}$$

$\Rightarrow$ **Why do we get the conflicting results?**

$\rightarrow$ **Uncertainties of the FFs should be taken into account (next page).**

Large uncertainties



2nd moments

$$M_u = 0.0012 \pm 0.0107$$
$$M_s = 0.0027 \pm 0.0183$$
$$M_g = 0.0090 \pm 0.0046$$

$\rightarrow M_u/M_s = 0.43 \pm 6.73$

The uncertainties are order-of-magnitude larger than the distributions and their moments themselves.

At this stage, the determined FFs are not accurate enough to discuss internal structure of $f_0(980)$.

$\rightarrow$ Accurate data are awaited not only for $f_0(980)$ but also for other exotic and "ordinary" hadrons.

Summary on exotic fragmentation functions

Exotic hadrons could be found by studying fragmentation functions. As an example, the $f_0(980)$ meson was investigated.

(1) We proposed to use 2nd moments and functional forms as criteria for finding quark configuration.

(2) Global analysis of $e^+ + e^- \rightarrow f_0 + X$ data

The results *may* indicate $s\bar{s}$ or $qq\bar{q}\bar{q}$ structure. However, …

- Large uncertainties in the determined FFs

→ The obtained FFs are not accurate enough to discuss the quark configuration of $f_0(980)$.

(3) Accurate experimental data are important

→ Small-Q^2 data as well as large-Q^2 (M_z^2) ones

→ *c*- and *b*-quark tagging

The End

The End