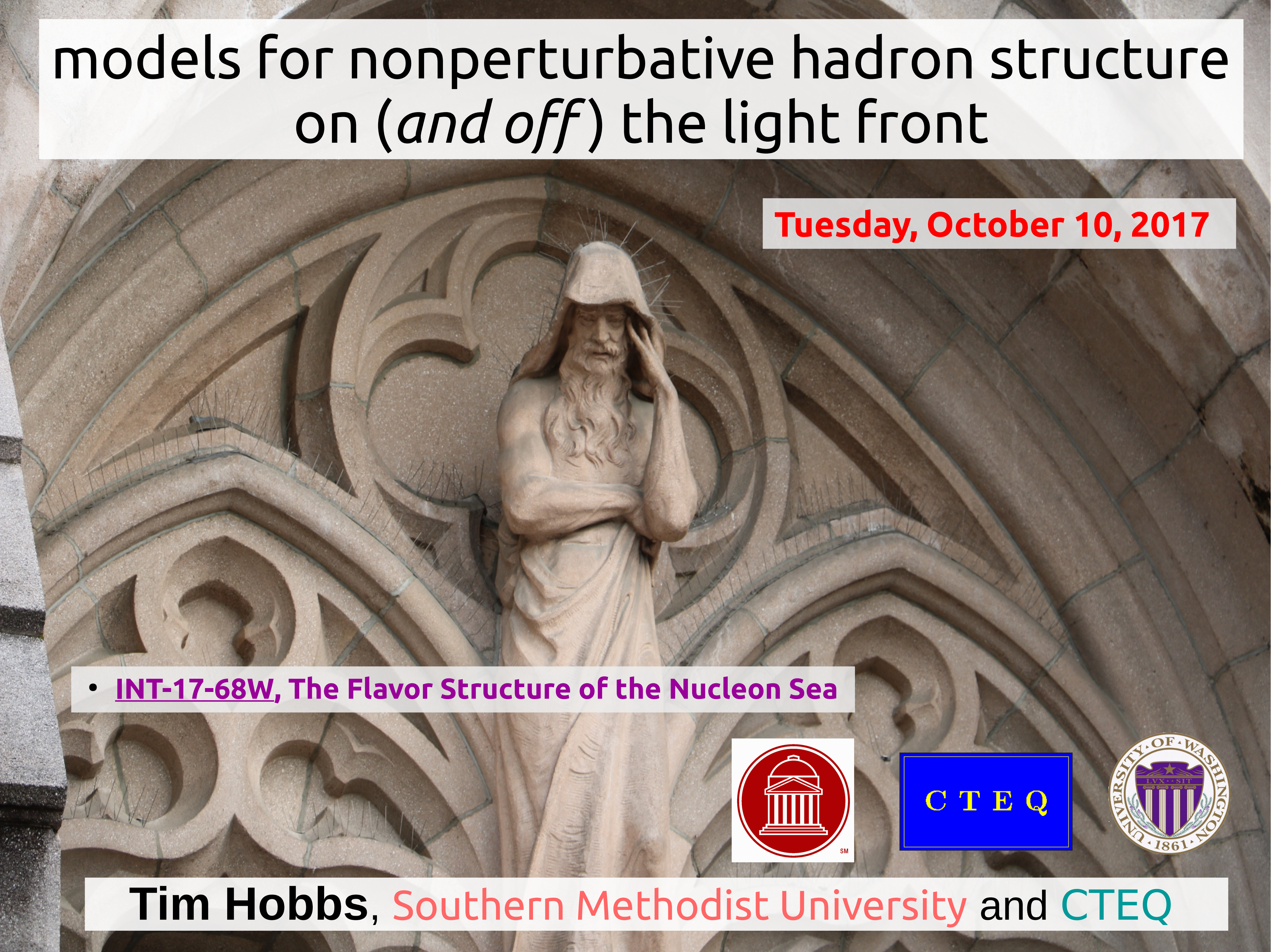


models for nonperturbative hadron structure on (*and off*) the light front

Tuesday, October 10, 2017

- **INT-17-68W, The Flavor Structure of the Nucleon Sea**

Tim Hobbs, Southern Methodist University and CTEQ

motivation and direction

- a thorough, quantitative grasp of the nucleon sea (here, broadly defined to include various non-valence contributions) is vital to tomography

- recent calculations in several flavor sectors highlight the usefulness of light-front dynamics

→ charm in the proton wave function; ‘Intrinsic charm’ and the nucleon’s HQ sigma term

(crucial to BSM searches – e.g., WIMP direct detection)

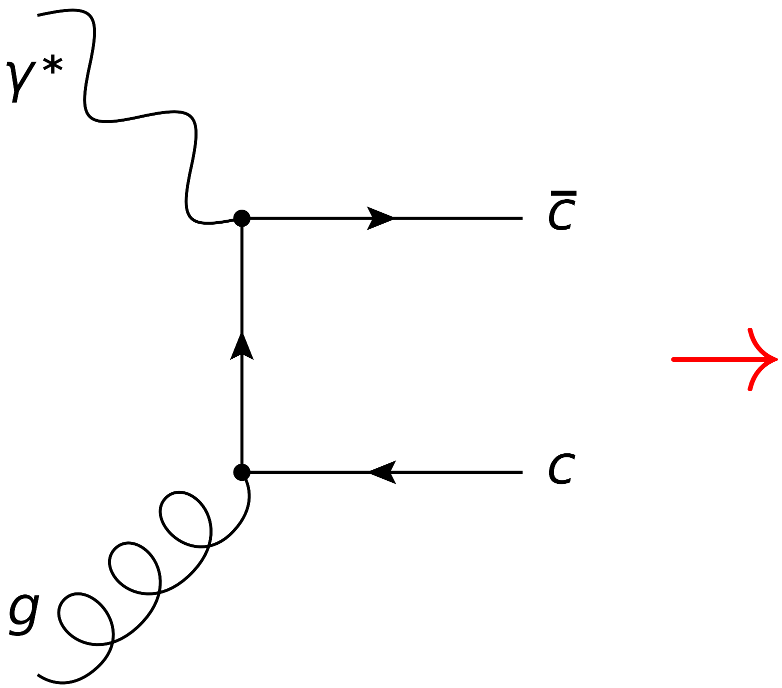
→ strange in DIS and elastic form factors

- in a somewhat different area, light-front constituent quark models can guide lattice calculations

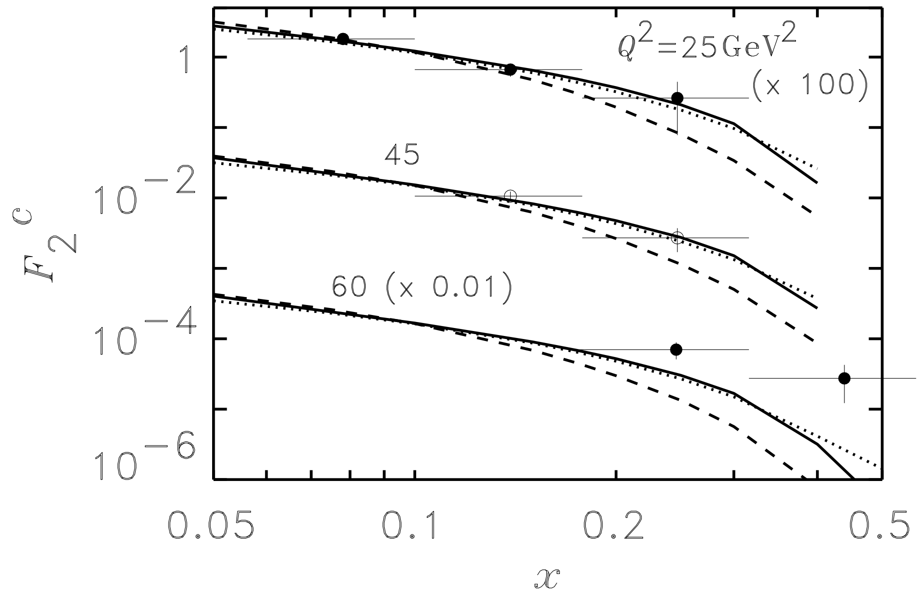
→ the valence **quasi-PDF** of the **pion** may be relatively cleanly measured on the lattice

charm in *perturbative* **QCD** (pQCD)

- $c(x, Q^2 \leq m_c^2) = \bar{c}(x, Q^2 \leq m_c^2) = 0$



F. M. Steffens, W. Melnitchouk and A. W. Thomas,
Eur. Phys. J. C **11**, 673 (1999) [hep-ph/9903441].



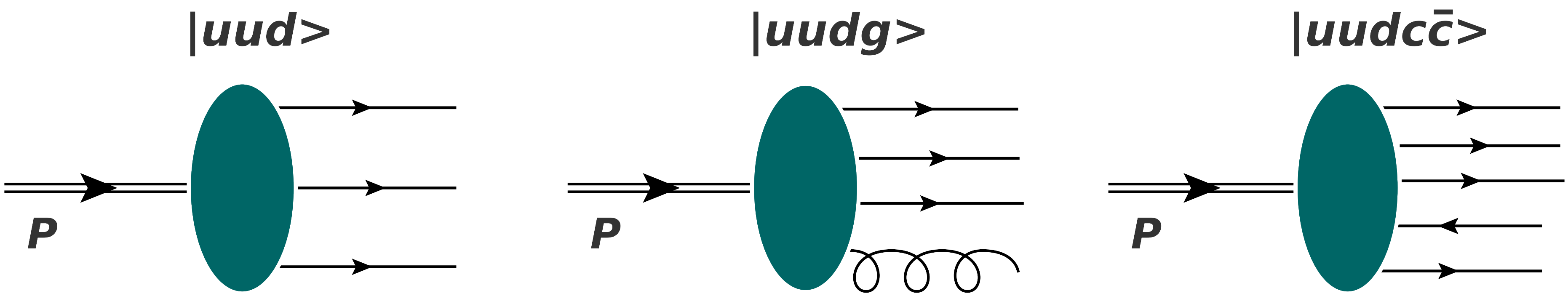
- *intermediate* Q^2:

$$F^c_{2,\ \mathrm{PGF}}(x, Q^2) \;=\; \frac{\alpha_s(\mu^2)}{9\pi}\; \int_x^{z'}\; \frac{dz}{z}\; C^{\mathrm{PGF}}(z, Q^2, m_c^2) \cdot xg\left(\frac{x}{z}, \mu^2\right)$$

- *high* Q^2:

massless **DGLAP** (i.e., *variable flavor-number* schemes)

simplest *nonperturbative* **model** calculations



→ original models possessed *scalar* vertices...

- Brodsky et al. (1980):

$$P(p \to uudc\bar{c}) \sim \left[M^2 - \sum_{i=1}^{5} \frac{k_{\perp i}^2 + m_i^2}{x_i} \right]^{-2}$$

→ produces *intrinsic* PDF, $c^{\rm IC}(x) = \bar{c}^{\rm IC}(x)$

- Blümlein (2015):

$$\tau_{life} = \frac{1}{\sum_i E_i - E} = \frac{2P}{\left(\sum_{i=1}^{5} \frac{k_{\perp i}^2 + m_i^2}{x_i} - M^2 \right)} \Bigg|_{\sum_j x_j = 1} \quad \textbf{vs.} \quad \tau_{int} = \frac{1}{q_0}$$

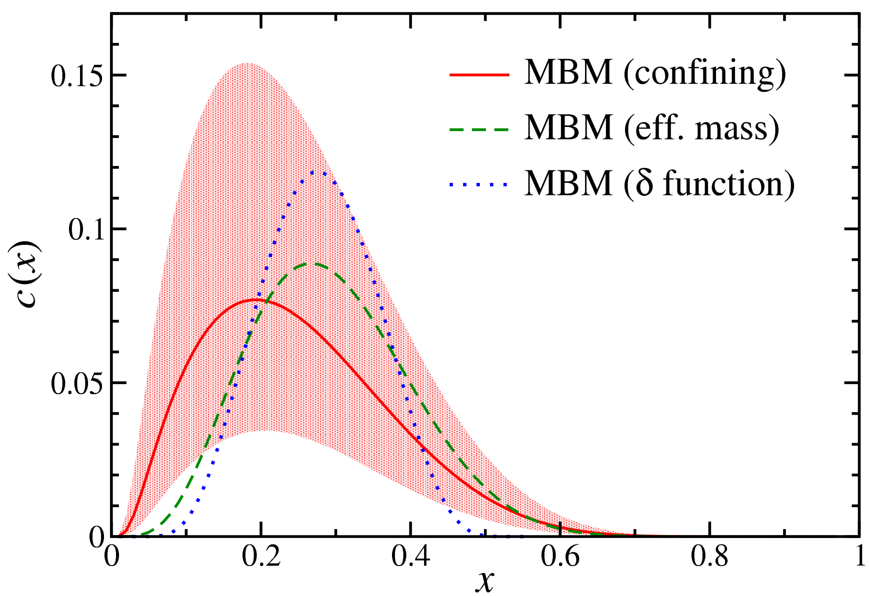
→ comparison constrains $x - Q^2$ space over which IC is observable

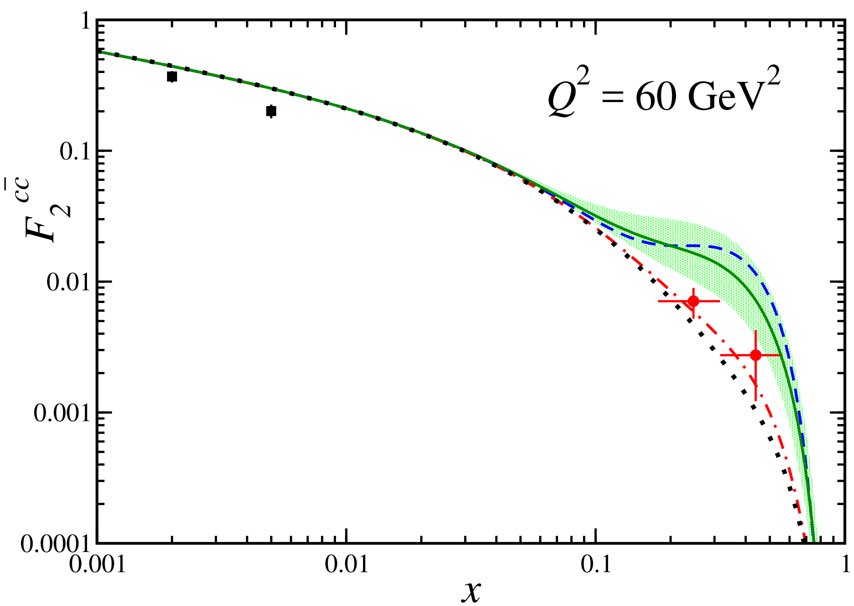
charm in the nucleon

- tune **universal** cutoff $\Lambda = \hat{\Lambda}$ to fit **ISR** $pp \to \Lambda_c X$ collider data

multiplicities, **momentum sum**:

$$\langle n \rangle_{MB}^{(\mathrm{charm})} = 2.40\%\ ^{+2.47}_{-1.36}; \qquad P_c := \langle x \rangle_{\mathrm{IC}} = 1.34\%\ ^{+1.35}_{-0.75}$$





$$F_2^{c\bar{c}}(x, Q^2) = \frac{4x}{9}\left[c(x, Q^2) + \bar{c}(x, Q^2)\right]$$

$\to$ evolve to EMC scale, $Q^2 = 60$ GeV^2

low-x H1/ZEUS data check *massless* **DGLAP** evolution

systematics of global QCD analysis

extract/constrain quark densities:

$$F^{\gamma}_{qh}(x,Q^2) \; = \sum_f \int_0^1 \frac{d\xi}{\xi} \, C_i^{\gamma f}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \frac{\mu_F^2}{\mu}, \alpha_S(\mu^2)\right) \cdot \phi_{f/h}(\xi, \mu_F^2, \mu^2)$$

- $C_i^{\gamma f}$: pQCD Wilson coefficients
- $\phi_{f/h}(\xi, \mu_F^2, \mu^2)$: *universal* parton distributions

(...here, $\mu_F^2 = 4m_c^2 + Q^2$)

$\Longrightarrow$ exploit properties of QCD to constrain models:

$$\sum_q \int_0^1 dx \; x \cdot \{f_{q+\bar{q}}(x,Q^2) + f_g(x,Q^2)\} \equiv 1 \quad \text{(mom. conserv.)}$$

- DGLAP: **couples** Q^2 evolution of $f_q(x,Q^2)$, $f_g(x,Q^2)$

constraints from **global** fits...

P. Jimenez-Delgado, **TJH**, J. T. Londergan and W. Melnitchouk; PRL 114, no. 8, 082002 (2015).

26 sets:

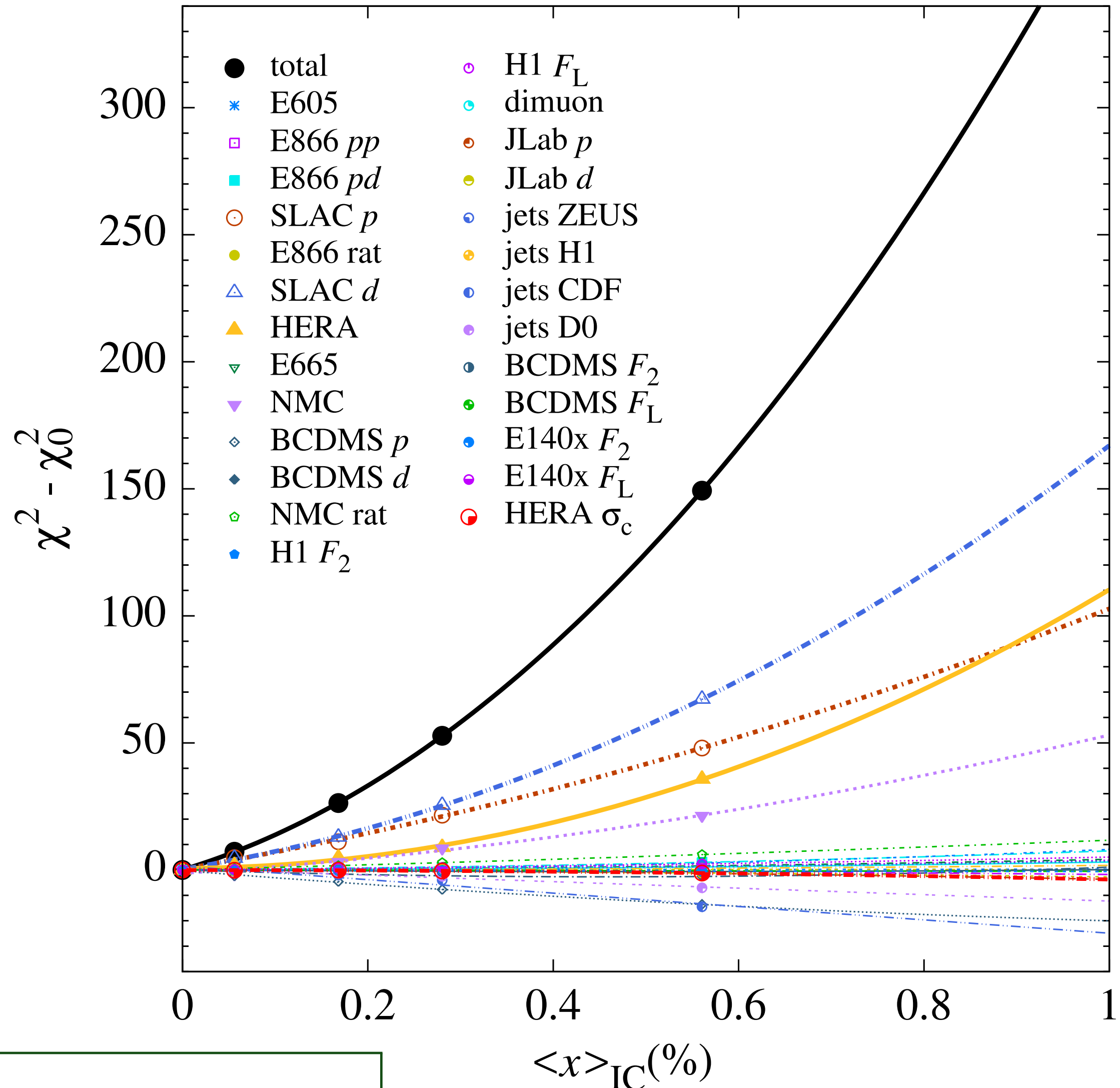
$N_{dat} = 4296$

$Q^2 \geq 1$ GeV^2

$W^2 \geq 3.5$ GeV^2

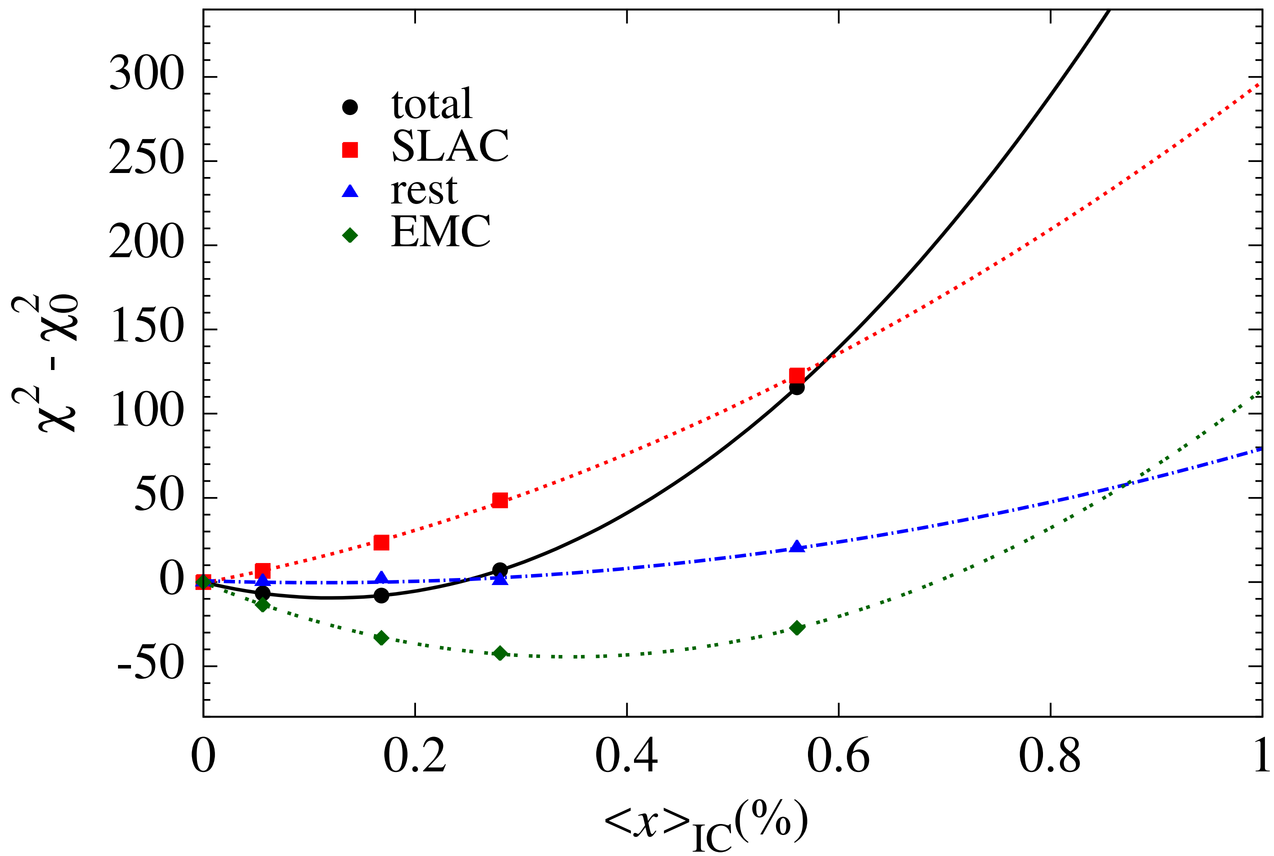
$\Longrightarrow$

** **HTs, TMCs, smearing...**



- constrain: $$\langle x \rangle_{\rm IC} = \int_0^1 dx\, x \cdot [c + \bar{c}](x)$$ ... '*total* IC momentum'

...and constrained by **EMC**



EMC alone: $\langle x \rangle_{\rm IC} = 0.3 - 0.4\%$

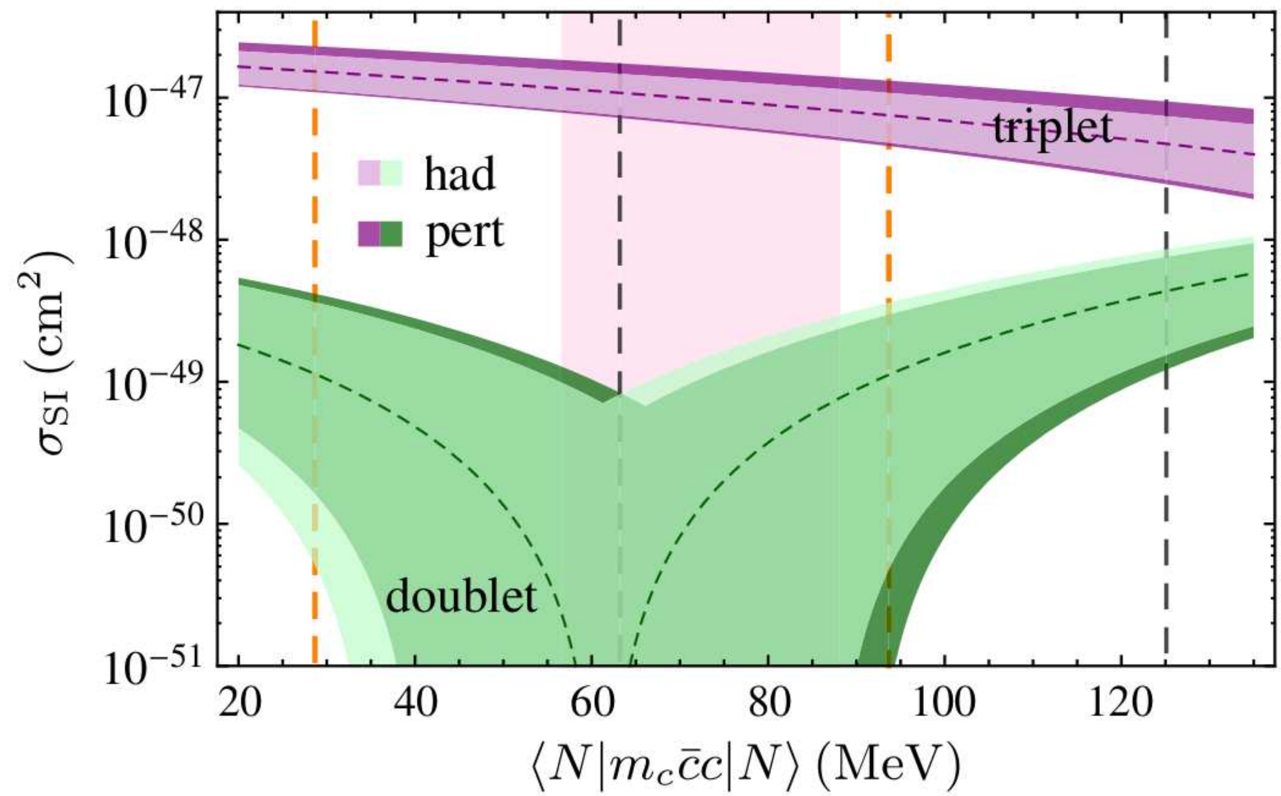
+ SLAC/'REST': $\boldsymbol{\langle x \rangle_{\rm IC} = 0.13 \pm 0.04\%}$

...but $F_2^{c\bar{c}}$ poorly fit — $\chi^2 \sim 4.3$ per datum!

in progress: charm **sigma term** and DM?

- heavy-particle EFT: after integrating away WIMP scale,
  $\sigma_c = m_c \langle p|\bar{c}c|p\rangle$ dominant DM cross section contribution

  Hill and Solon, Phys. Rev. Lett. **112**, 211602 (2014).



- what might $F_2^{c\bar{c}}(x, Q^2 = m_c^2)$ imply for σ_c ??

… need models for *both* the charm PDF *and* $\sigma_{c\bar{c}}$

- **light-front wave functions (LFWFs)** are one such approach
- they deliver a **frame-independent** description of hadronic bound state structure

  - the light front represents physics *tangent* to the light cone:

$$x^{\mu} = (x^0, \mathbf{x}) \longrightarrow (x^+, x_\perp, x^-)$$

$$x^{\pm} = x^0 \pm x^3, \quad x_\perp = (x^r)\,; \quad r = \{1, 2\}$$

- with them, many matrix elements (GPDs, TMDs) are calculable via the same **universal** objects:

$$c(x) \sim \langle \bar{c}\,\gamma^+ c \rangle \longleftrightarrow \sigma_{c\bar{c}} = m_c \, \langle p \big| \bar{c} c \big| p \rangle$$

- in fact, have already developed this technology for **nucleon strangeness**!

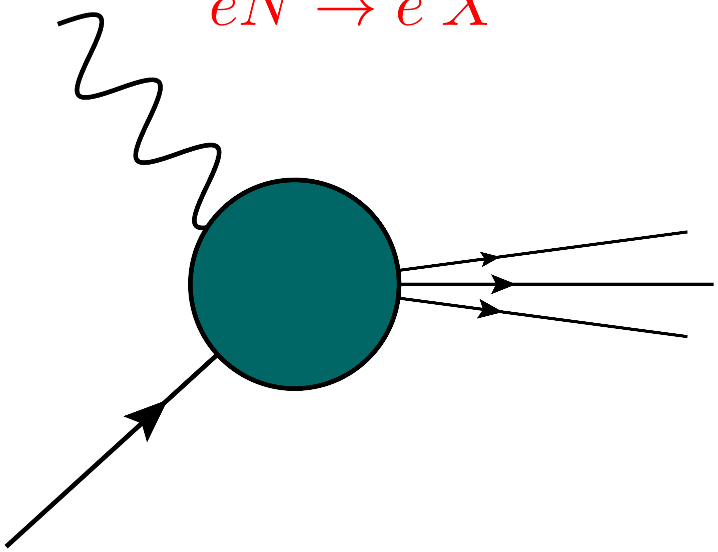
TJH, M. Alberg, and G. A. Miller; PRC91, 035205 (2015).

DIS and elastic strangeness

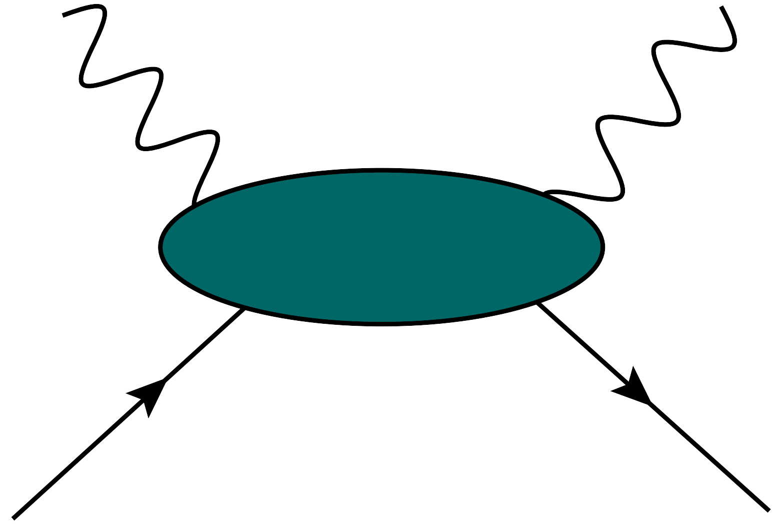
- *predict* inelastic <u>and</u> elastic observables?

  $\rightarrow$ requires knowledge of quark-level proton **wave function**

$eN \rightarrow e'X$



$eN \rightarrow e'N'$



$$xS^+ = \int_0^1 dx\; x[s(x) + \bar{s}(x)]$$

$$xS^- = \int_0^1 dx\; x[s(x) - \bar{s}(x)]$$

$$F_1(Q^2) \sim \langle P', \uparrow | J^+_{EM} | P, \uparrow \rangle$$

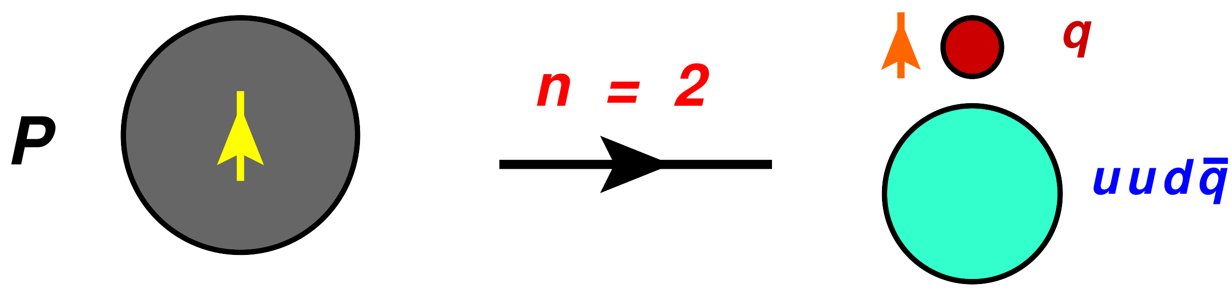
$$F_2(Q^2) \sim \langle P', \downarrow | J^+_{EM} | P, \uparrow \rangle$$

$$J^\mu_{EM} = \gamma^\mu F_1(Q^2) + i\frac{\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2)$$

hadronic **light-front** wave functions **(LFWFs)**

- **S. J. Brodsky, D. S. Hwang, B. Q. Ma and I. Schmidt; Nucl. Phys. B 593, 311 (2001).**

$$|\Psi_P^\lambda(P^+, \mathbf{P}_\perp)\rangle = \sum_n \int \prod_{i=1}^{n} \frac{dx_i d^2\mathbf{k}_{\perp i}}{\sqrt{x_i}(16\pi^3)} \, 16\pi^3 \, \delta\left(1 - \sum_{i=1}^{n} x_i\right)$$
$$\times \; \delta^{(2)}\left(\sum_{i=1}^{n} \mathbf{k}_{\perp i}\right) \psi_n^\lambda(x_i, \mathbf{k}_{\perp i}, \lambda_i) \, |n; k_i^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle$$



$$|\Psi_P^\lambda(P^+, \mathbf{P}_\perp)\rangle = \frac{1}{16\pi^3} \sum_{q=s,\bar{s}} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}} \psi_{q\lambda_q}^\lambda(x, \mathbf{k}_\perp)$$
$$\times \; |q; xP^+, x\mathbf{P}_\perp + \mathbf{k}_\perp\rangle$$

$\rightarrow$ **3D** helicity WF $\psi_{q\lambda_q}^\lambda(x, \mathbf{k}_\perp)$; **light-front fraction**: $x = k^+/P^+$

electromagnetic form factors

- the quark q contribution from any 5-quark state is then:

$$F_1^q(Q^2) = e_q \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \sum_{\lambda_q} \psi^{*\lambda=+1}_{q\lambda_q}(x, \mathbf{k}'_\perp)\ \psi^{\lambda=+1}_{q\lambda_q}(x, \mathbf{k}_\perp)$$

$$F_2^q(Q^2) = e_q \frac{2M}{[q^1 + iq^2]} \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \sum_{\lambda_q} \psi^{*\lambda=-1}_{q\lambda_q}(x, \mathbf{k}'_\perp)\ \psi^{\lambda=+1}_{q\lambda_q}(x, \mathbf{k}_\perp)$$

- for strangeness, $q \to s$; total strange: $s + \bar{s}$

$$F^{s\bar{s}}_{1,2}(Q^2) = F^{s}_{1,2}(Q^2) + F^{\bar{s}}_{1,2}(Q^2) \implies$$

Sachs form :

$$G_E^{s\bar{s}}(Q^2) = F_1^{s\bar{s}}(Q^2) - \frac{Q^2}{4M^2} F_2^{s\bar{s}}(Q^2)$$

$$G_M^{s\bar{s}}(Q^2) = F_1^{s\bar{s}}(Q^2) + F_2^{s\bar{s}}(Q^2)$$

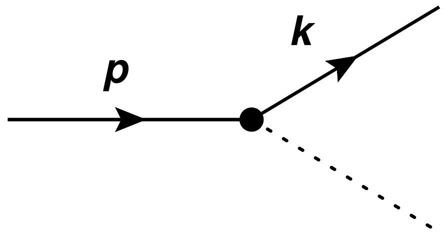
$$\mu_s = G_M^{s\bar{s}}(Q^2 = 0)$$

$$\rho_s^D = \left.\frac{dG_E^{s\bar{s}}}{d\tau}\right|_{\tau=0}$$

where $\tau = Q^2 \big/ 4M^2$

strangeness **wave functions**

- require a **proton** $\rightarrow$ **quark** + **scalar tetraquark** LFWF:



I. C. Cloët and G. A. Miller; Phys. Rev. C 86, 015208 (2012).

$$\boxed{\psi^{\lambda}_{\lambda_s}(k,p) \;=\; \bar{u}^{\lambda_s}_s(k)\;\phi(M_0^2)\;u^{\lambda}_N(p)}$$

$\phi(M_0^2)$: scalar function $\rightarrow$ quark-spectator interaction

(M_0^2 = quark-tetraquark invariant mass^2!)

$$\text{e.g.,}\qquad \psi^{\lambda=+1}_{s\lambda_s=+1}(x,\mathbf{k}_\perp) \;=\; \frac{1}{\sqrt{1-x}}\left(\frac{m_s}{x}+M\right)\;\phi_s$$

$$\text{gaussian}:\;\; \phi_s \;=\; \frac{\sqrt{N_s}}{\Lambda_s^2}\exp\left\{-M_0^2(x,\mathbf{k}_\perp,\mathbf{q}_\perp)\Big/2\Lambda_s^2\right\}$$

$$F_1^s(Q^2) \;=\; \frac{e_s N_s}{16\pi^2\Lambda_s^4}\int\frac{dx\,dk_\perp^2}{x^2(1-x)}\left(k_\perp^2+(m_s+xM)^2-\frac{1}{4}(1-x)^2Q^2\right)$$

$$\times\;\exp(-s_s/\Lambda_s^2) \qquad \boxed{s_s = (M_0^2+M_0'^2)/2} \qquad \textit{sim. for } F_2^s(Q^2)!$$

$s\bar{s}$ distribution functions

- s quark distribution $\equiv$ x-**unintegrated** $F_1^s(Q^2=0)$ form factor (up to e_s!):

$$s(x) \;=\; \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \sum_{\lambda_s} \psi^{*\lambda=+1}_{s\lambda_s}(x,\mathbf{k}_\perp)\; \psi^{\lambda=+1}_{s\lambda_s}(x,\mathbf{k}_\perp)$$

$\rightarrow$ again inserting **helicity wave functions** $\psi^{\lambda=+1}_{q\lambda_q}(x,\mathbf{k}_\perp)$ ($Q^2=0 \implies \mathbf{k}'_\perp = \mathbf{k}_\perp$):

$$\boxed{s(x) \;=\; \frac{N_s}{16\pi^2\Lambda_s^4} \int \frac{dk_\perp^2}{x^2(1-x)} \left(k_\perp^2 + (m_s + xM)^2\right)\; \exp(-s_s/\Lambda_s^2)}$$

$$s_s \;=\; \frac{1}{x(1-x)}\left[k_\perp^2 + (1-x)m_s^2 + x m_{S_p}^2 + \frac{1}{4}(1-x)^2 Q^2\right]$$

$\rightarrow$ total of **eight** model parameters!

(N_s, Λ_s, m_s, and m_{S_p} ... **AND** anti-strange)

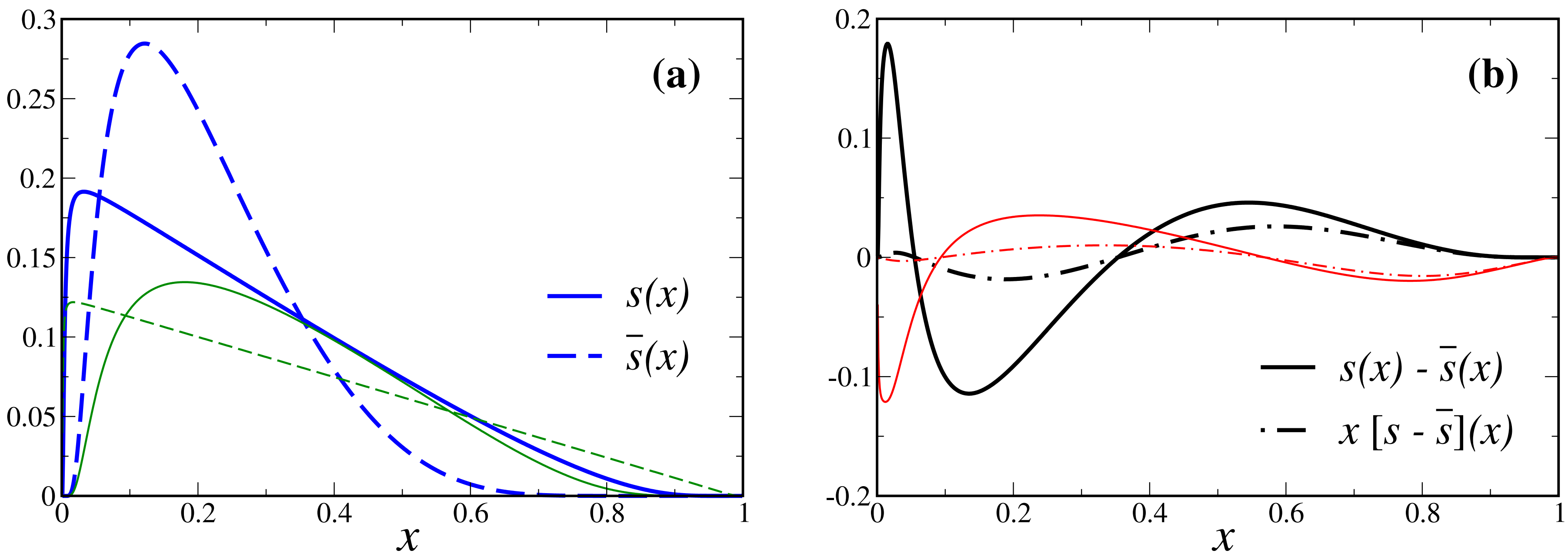
limits from **DIS** measurements

- DIS measurements have placed limits on the PDF-level total strange momentum xS^+ and asymmetry xS^-

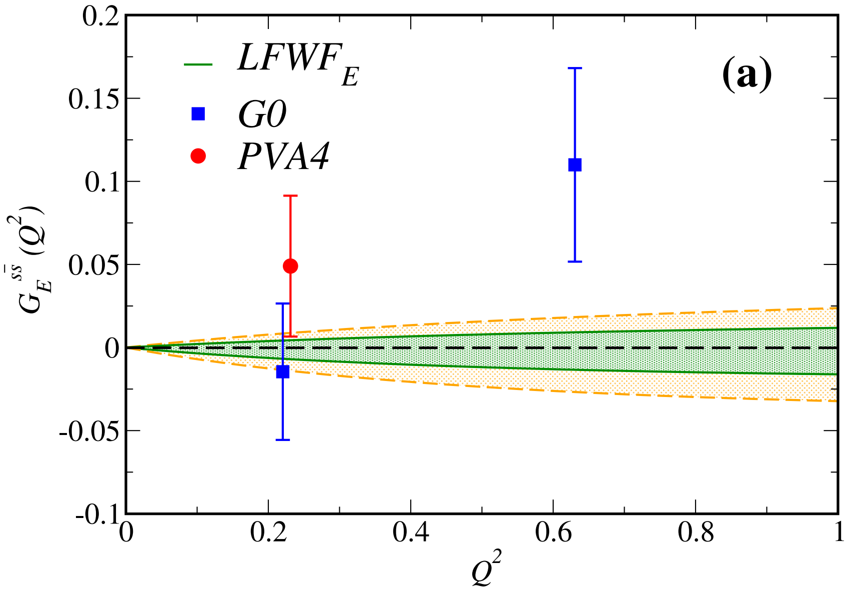
CTEQ6.5S:

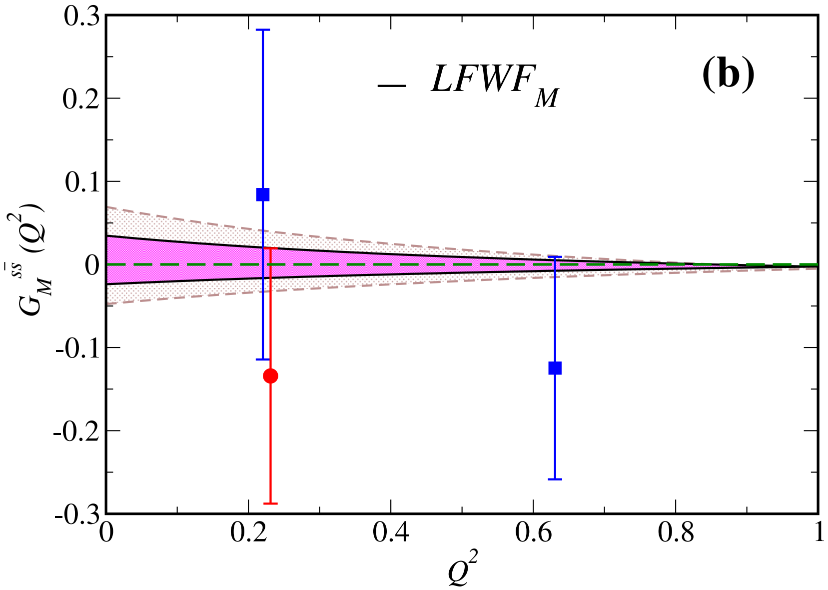
$$\boxed{0.018 \leq xS^+ \leq 0.040} \qquad \boxed{-0.001 \leq xS^- \leq 0.005}$$

- **SCAN** the available parameter space subject to the DIS limits; **SEARCH** for extremal values of μ_s, ρ_s^D



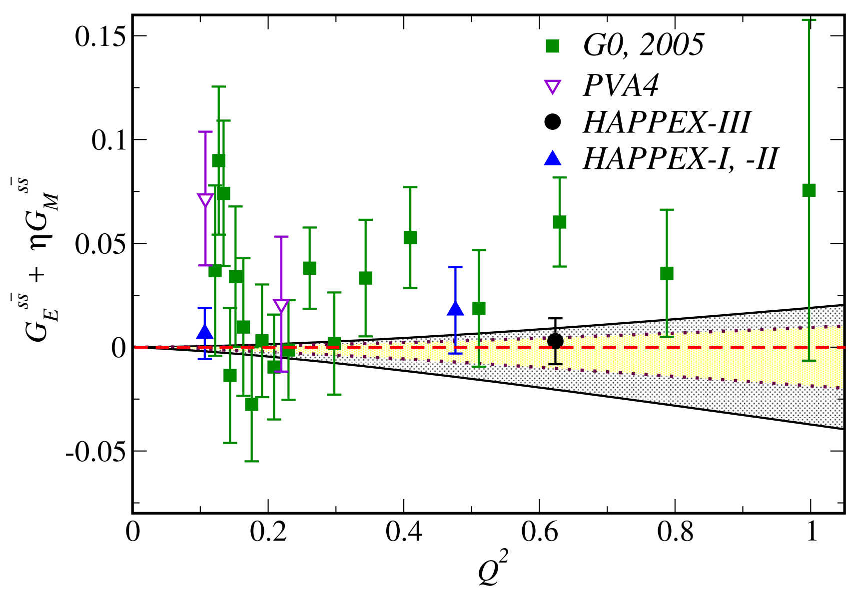
constraints on **elastic** form factors





- **DIS-driven limits** to elastic FFs are significantly **more stringent** than current experimental precision

$$\eta(Q^2) \sim 0.94\ Q^2$$ →

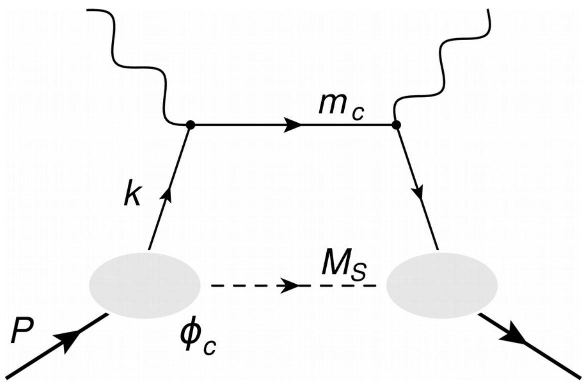


we build an analogous model for charm... first the PDF

- use a similar scalar spectator picture; details generalize:

$$|\Psi_P^\lambda(P^+, \mathbf{P}_\perp)\rangle = \frac{1}{16\pi^3}\sum_{q=c,\bar{c}}\int \frac{dx\, d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}}$$

arXiv:1707.06711



$$\times\, \psi_{q\lambda_q}^\lambda(x, \mathbf{k}_\perp)\, |q; xP^+, x\mathbf{P}_\perp + \mathbf{k}_\perp\rangle$$

$$F_2^{c\bar{c}}(x, Q^2 = m_c^2) = \frac{4x}{9}\left(c(x) + \bar{c}(x)\right)$$

$$c(x) = \frac{1}{16\pi^2}\int \frac{dk_\perp^2}{x^2(1-x)}\left[\frac{k_\perp^2 + (m_c + xM)^2}{(M^2 - s_{cS})^2}\right] \left|\phi_c(x, k_\perp^2)\right|^2$$

use a power-law (γ=3) covariant vertex function, $\phi_c(x, k_\perp^2) = \sqrt{g_c}\left(\frac{\Lambda_c^2}{t_c - \Lambda_c^2}\right)^\gamma$

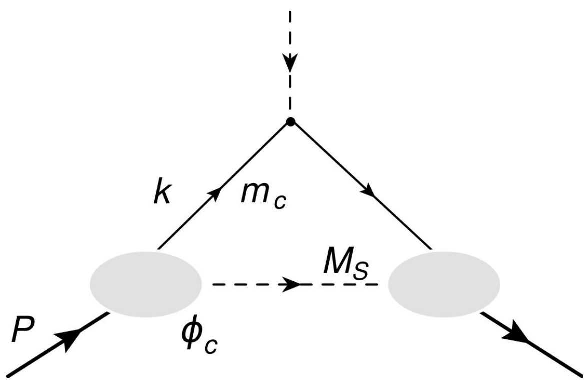
$$\begin{cases} s_{cS}\left(x, k_\perp^2\right) = \dfrac{1}{x\,(1-x)}\left(k_\perp^2 + (1-x)\, m_c^2 + x\, M_S^2\right) & \text{invariant mass} \\ t_c\left(x, k_\perp^2\right) = \dfrac{1}{1-x}\left(-k_\perp^2 + x\left[(1-x)M^2 - M_S^2\right]\right) & \text{covariant } k^2 \end{cases}$$

then, a covariant formalism gives the **sigma term**:

- IF the LFWFs can be constrained with information from the DIS sector, we may evaluate $\sigma_{c\bar{c}}$

$$\sigma_c = \frac{i g_c}{2M} \int \frac{d^4k}{(2\pi)^4}\, \overline{u}(p) \left(\frac{1}{\not{k} - m_c + i\epsilon} \right) [m_c \mathcal{I}_4] \left(\frac{1}{\not{k} - m_c + i\epsilon} \right) u(p)$$

$$\times \left(\frac{1}{[p-k]^2 - M_S^2 + i\epsilon} \right) \left(\frac{\Lambda^2}{k^2 - \Lambda_c^2 + i\epsilon} \right)^{2\gamma}$$



$$\sigma_{c\bar{c}} = \sigma_c + \sigma_{\bar{c}}$$

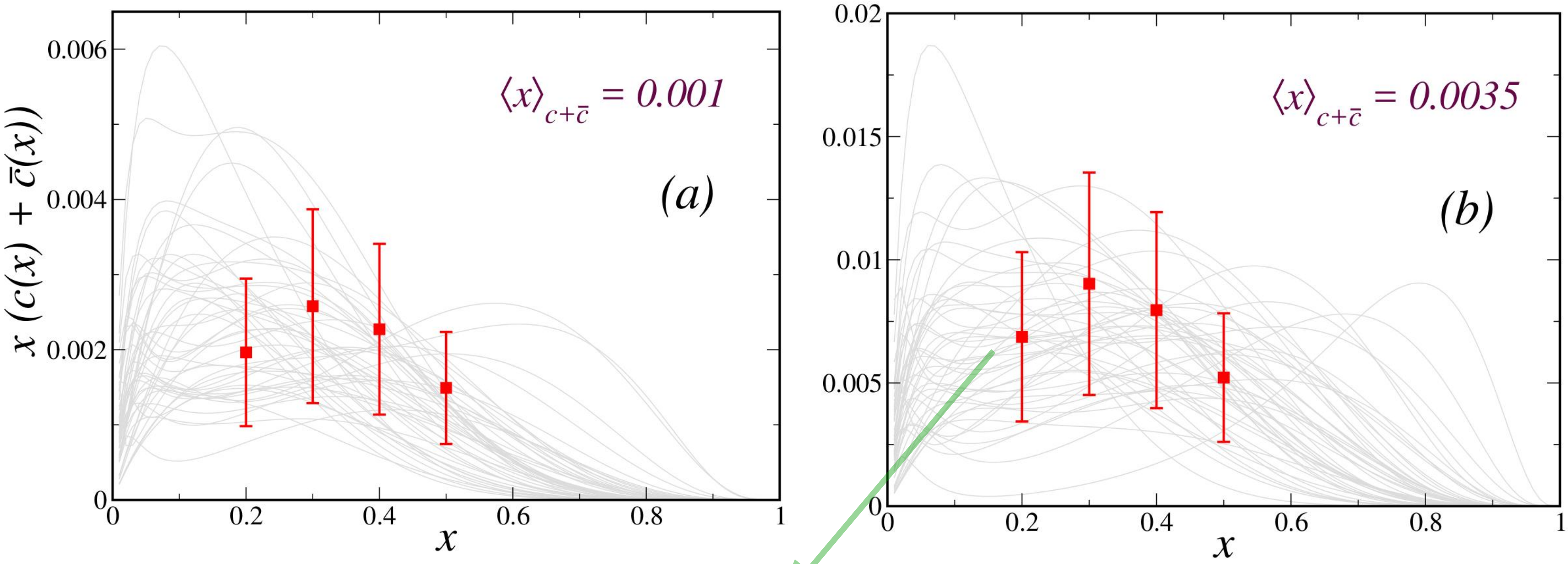
…we determine **probability distribution functions** (p.d.f.s) for this quantity

- this formalism is required because the LFWFs contain noncovariant parts:

$$i \frac{\sum_\lambda u_\lambda(k)\, \overline{u}_\lambda(k)}{k^2 - m^2 + i\epsilon} = \frac{i}{\not{k} - m + i\epsilon} - i \frac{\gamma^+}{2k^+}$$

- **it remains to determine the (free) *parameters* of the light-front model,**

$$\left(g_c,\ m_c,\ \Lambda_c,\ \Lambda_{\bar{c}},\ M_S,\ M_{\bar{S}} \right)$$



- we constrain the model with hypothetical **pseudo-data** (taken from the `confining' MBM) of a given $\langle x \rangle_{\rm IC} \pm 50\%$

(input data normalizations are inspired by the just-described global analysis)

$\langle x \rangle_{\rm IC} = 0.001$ [upper limit tolerated by the full fit/dataset]

$\langle x \rangle_{\rm IC} = 0.0035$ [central value preferred by EMC data alone]

- rather than traditional χ^2 minimization, the model space is instead explored using **Bayesian methods**

model simulations with **m**arkov **c**hain **m**onte **c**arlo (MCMC)

- specifically, use a **D**elayed-**R**ejection **A**daptive **M**etropolis (DRAM) algorithm

Haario et al., Stat. Comput. (2006) **16**: 339–354.

construct a Markov chain consisting of $n_{sim} \approx 10^5 - 10^6$ simulations, sampling the ***joint posterior distribution***

$$p(\vec{\theta}\,|x) \sim p(x|\vec{\theta}\,)\, p(\vec{\theta}\,)$$

x	: input data
$\vec{\theta}$	: parameters

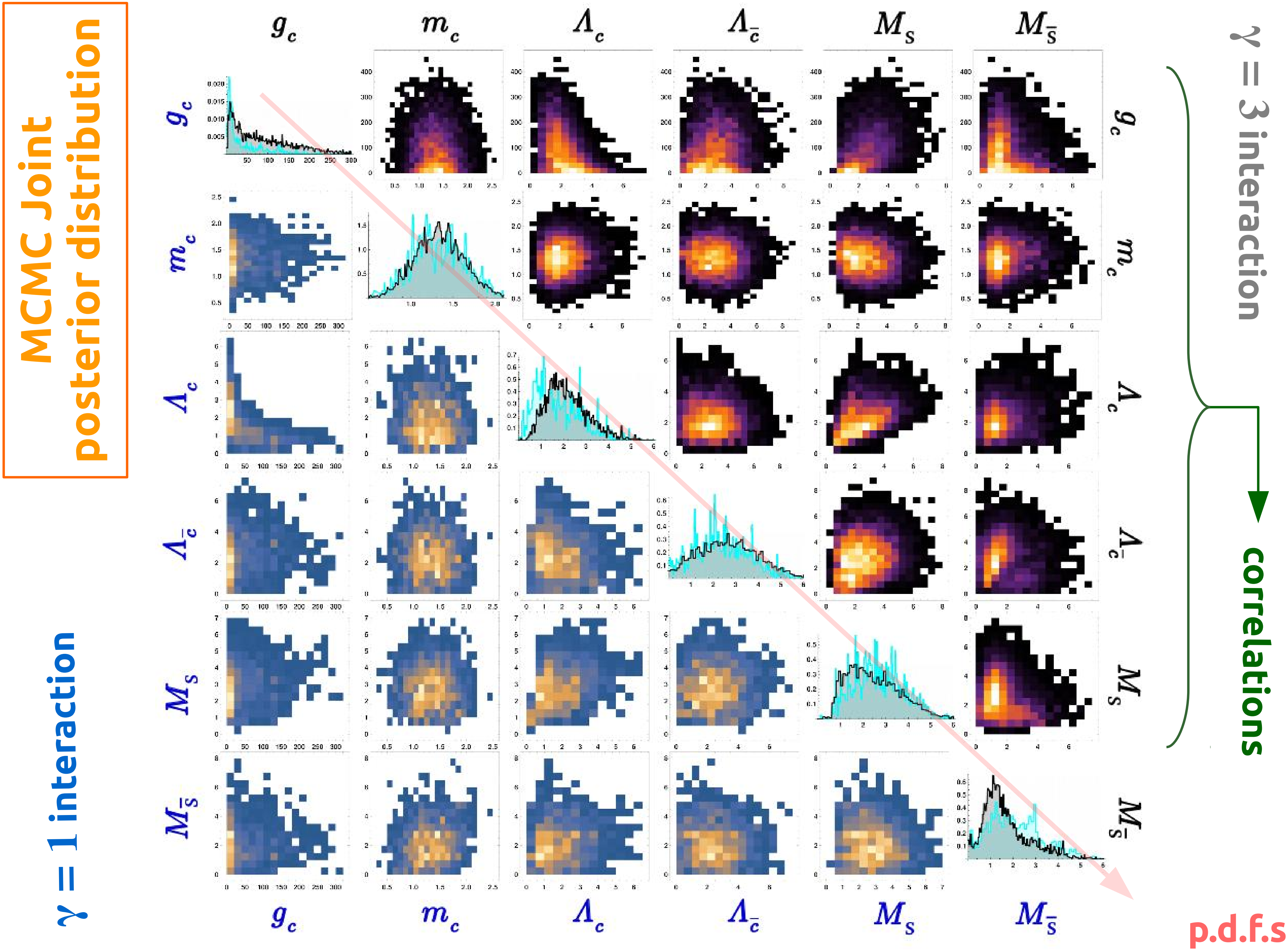
BROAD gaussian priors

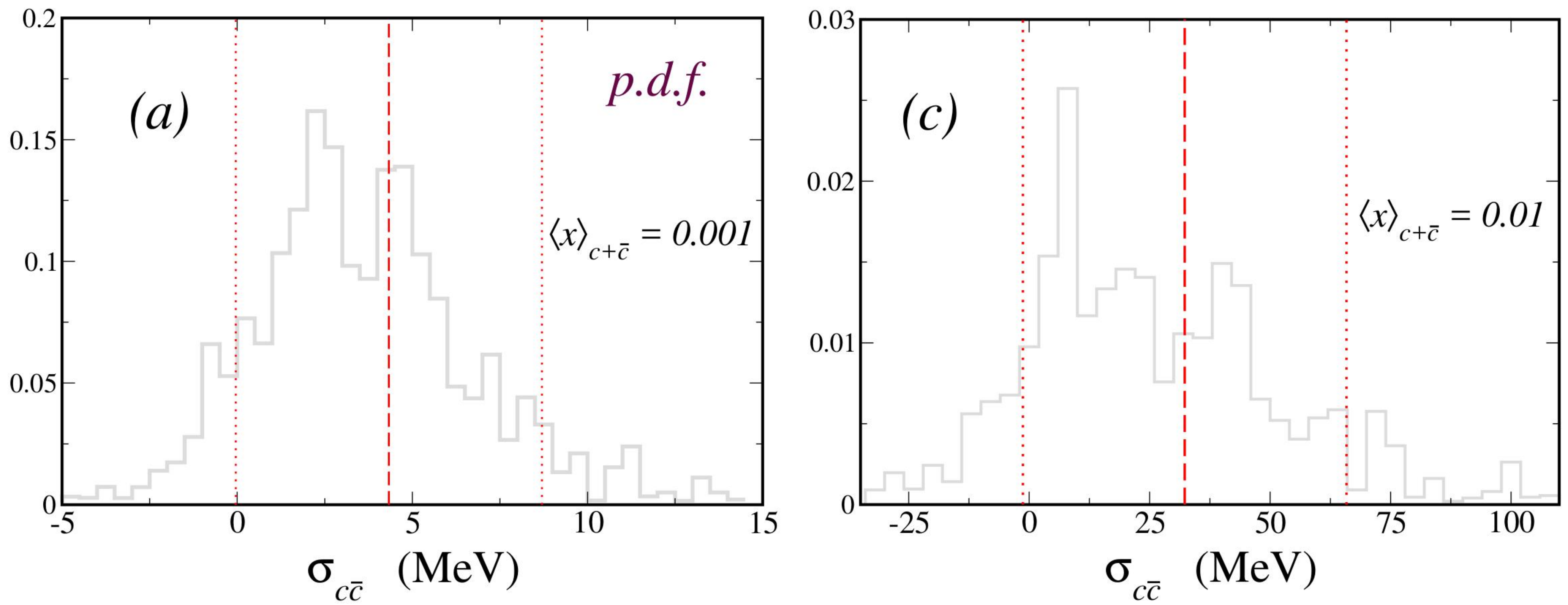
likelihood function $p(x|\vec{\theta}\,) = \exp\left(-\chi^2/2\right)$

$$\chi^2 = \sum_i \left(\frac{1}{\sigma_i^{data}}\right)^2 \left| F_2^{c\bar{c}}(x_i,\vec{\theta}\,) - F_2^{c\bar{c},\,data}(x_i)\right|^2$$

- asymptotically, the MCMC chain fully explores the joint posterior distribution

✔ from this, we extract **probability distribution functions (p.d.f.s)** for the model parameters and derived quantities, **including** $\sigma_{c\bar{c}}$


MCMC Joint posterior distribution
$\gamma = 3$ interaction
correlations
$\gamma = 1$ interaction
p.d.f.s
g_c
m_c
Λ_c
$\Lambda_{\bar{c}}$
M_S
$M_{\bar{S}}$



$$\sigma_{c\bar{c}} \;=\; 4.3 \pm 4.4\,\mathrm{MeV} \quad (\,\gamma = 3 \text{ interaction}\,) \quad \sigma_{c\bar{c}} \;=\; 32.3 \pm 33.6\,\mathrm{MeV}$$

- we find better concordance cf. existing **lattice determinations**, for somewhat larger IC magnitudes; also, close correlation with the DIS sector –

$$\sigma_{c\bar{c}} \;=\; 94\,(31)\,\mathrm{MeV}$$ (χQCD)[1]

$$=\; 67\,(34)\,\mathrm{MeV}$$ (MILC)[2]

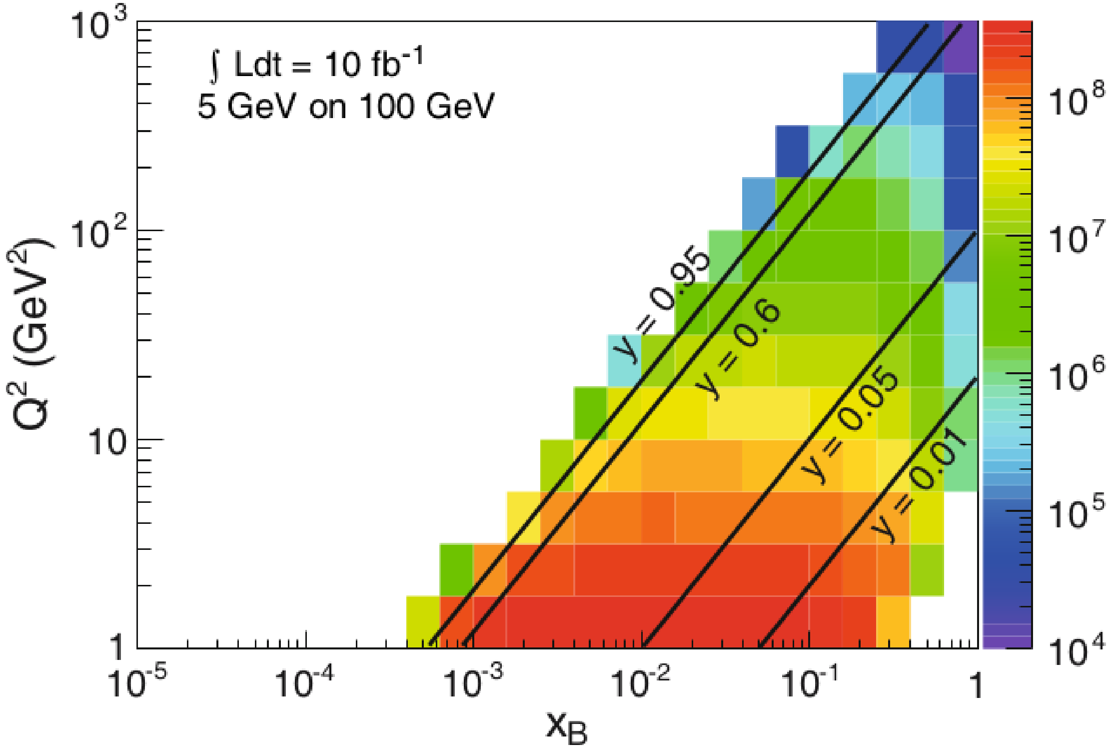
$$\sigma_{c\bar{c}} \;=\; 79\;(21)\binom{12}{8}\,\mathrm{MeV}$$ (AR)[3]

$\mathcal{O}\left(\alpha_s^3\right)$ pQCD is similar…

[1] Gong et al., Phys. Rev. **D88**, 014503 (2013).

[2] Freeman and Toussaint, Phys. Rev. **D88**, 054503 (2013).

[3] Abdel-Rehim et al., Phys. Rev. Lett. **116**, 252001 (2016).



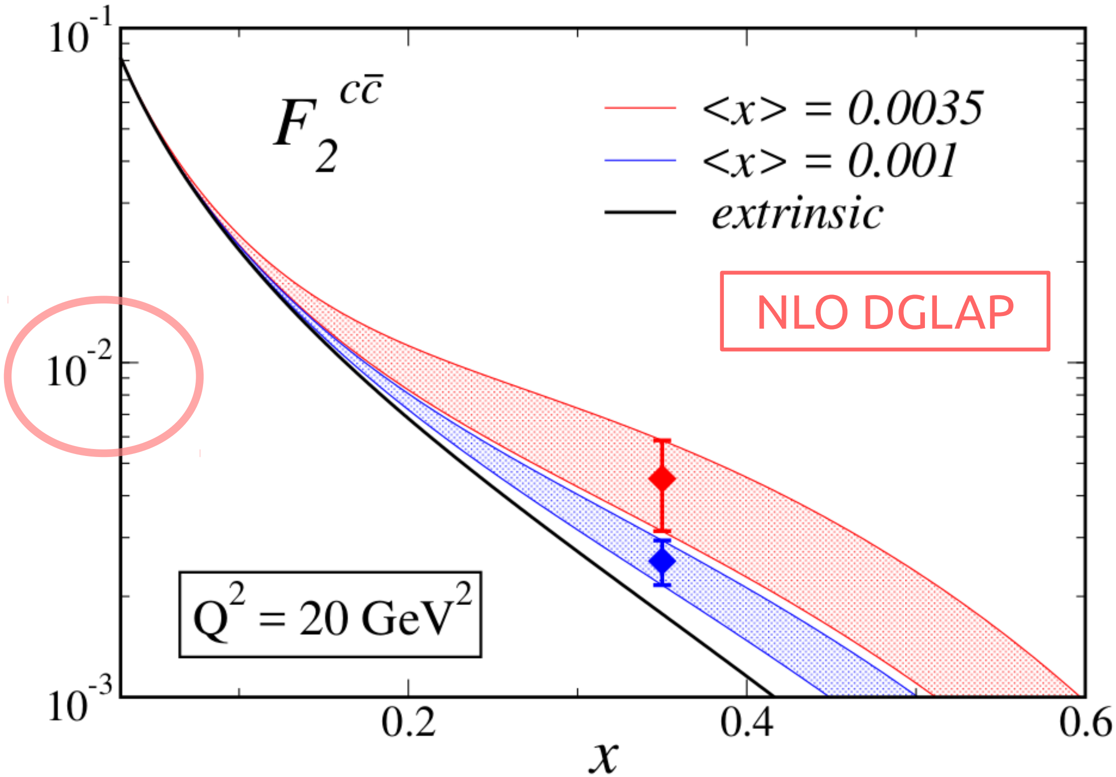
EIC Whitepaper, Eur. Phys. J. A (2016) **52**: 268

- e.g., MEIC-like scenario:

$$\sqrt{s} = 45\,\mathrm{GeV}$$

- a definitive measurement would simply **reprise the EMC observation of $F^{c\bar{c}}_2$**

- still, considerable precision will be needed to be sensitive at the necessary level



a future, unified description of the proton wave function may have the potential to provide the charm PDF and sigma term within a more comprehensive tomography

epilogue: LaMET and the pion structure function

- knowledge of the pion structure function is crucial to unraveling the nucleon's light quark sea (e.g., $\bar{d} - \bar{u}$); LaMET techniques may open this quantity to Lattice QCD

TJH, arXiv: **1708.05463** [hep-ph].

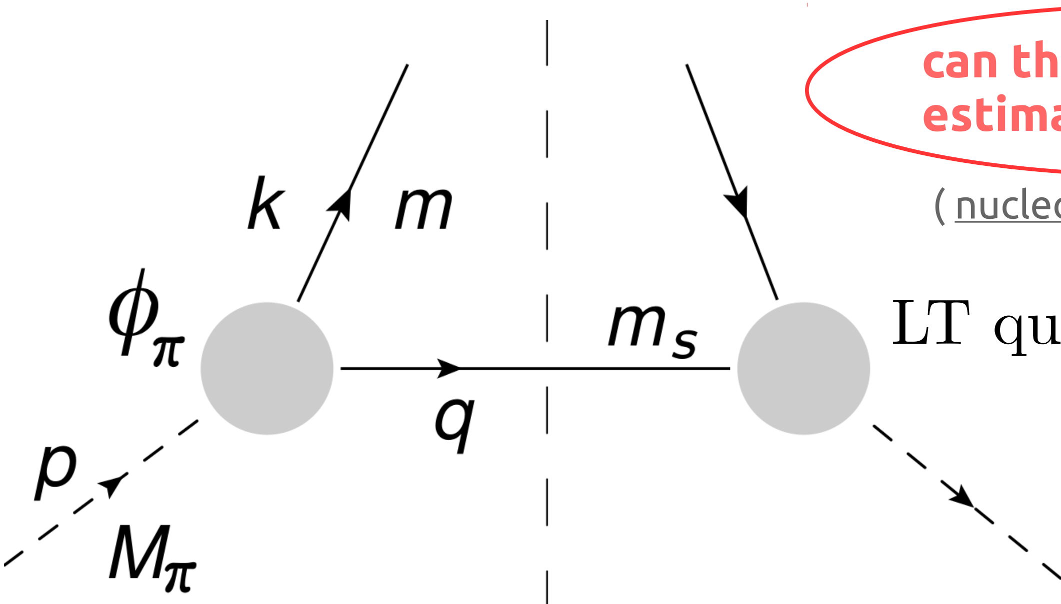
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-i\xi^- k^+} \langle p \left| \overline{\psi}(\xi^-)\gamma^+ \mathcal{U}(\xi^-,0)\psi(0) \right| p \rangle$$

...while matrix elements for lightlike correlations are not accessible on a Euclidean Lattice, *quasi-PDFs* are:

Ji, PRL**110**, 262002 (2013).

$$\widetilde{q}(x,\mu^2,p_z) = \int \frac{d\xi_z}{4\pi} e^{-i\xi_z k_z} \langle p \left| \overline{\psi}(\xi_z)\gamma^z \mathcal{U}(\xi_z,0)\psi(0) \right| p \rangle$$

these differ from the exact PDFs by power-suppressed corrections of order $\mathcal{O}\left(\frac{\Lambda^2}{p_z^2}, \frac{M^2}{p_z^2}\right)$



can these effects be estimated / controlled ?

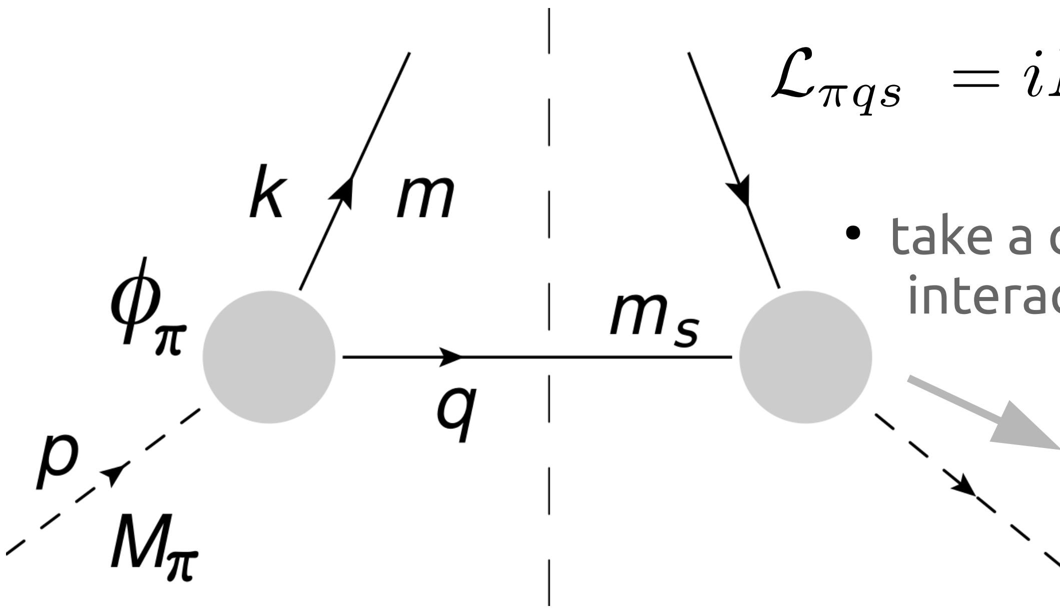
(<u>nucleon</u>: Gamberg et al, PLB743 (2015) 112.)

$$\text{LT quark-line cut} \longrightarrow \frac{\gamma^+}{2p^+}\,\delta\left(x - \frac{k^+}{p^+}\right)$$

$$\frac{\gamma^z}{2p_z}\,\delta\left(x - \frac{k_z}{p_z}\right)$$

the “exact” pion light-front PDF via a constituent quark model

- first evaluate the LF pion valence PDF using a minimal model that couples the pion to its constituent quarks



$$\mathcal{L}_{\pi q s} = i N_\pi^{1/2} \, \overline{\psi}_q \gamma_5 \varphi_\pi \psi_s + \text{h.c.}$$

- take a covariant vertex factor for the quark-pion interaction inspired by power counting ($n_s = 1$),

$$\left[\phi_\pi(k^2)\right]^2 \equiv \Lambda_\pi^2 \Big/ \left(k^2 - \Lambda_\pi^2\right)$$

$$q_\pi^{\mathrm{LF}}(x) = \frac{N_\pi}{2\,(2\pi)^4} \int dk^+ dk^- d^2 k_\perp \left(\frac{1}{2p^+}\right) \delta\left(x - \frac{k^+}{p^+}\right)$$

$$\times\ tr\Big(\gamma_5\,(\not{k} + m)\,\gamma^+ (\not{k} + m)\gamma_5(-\not{q} + m_s)\Big)\ 2\pi\,\delta\big(q^2 - m_s^2\big) \left[\frac{\phi_\pi(k^2)}{(k^2 - m^2)}\right]^2$$

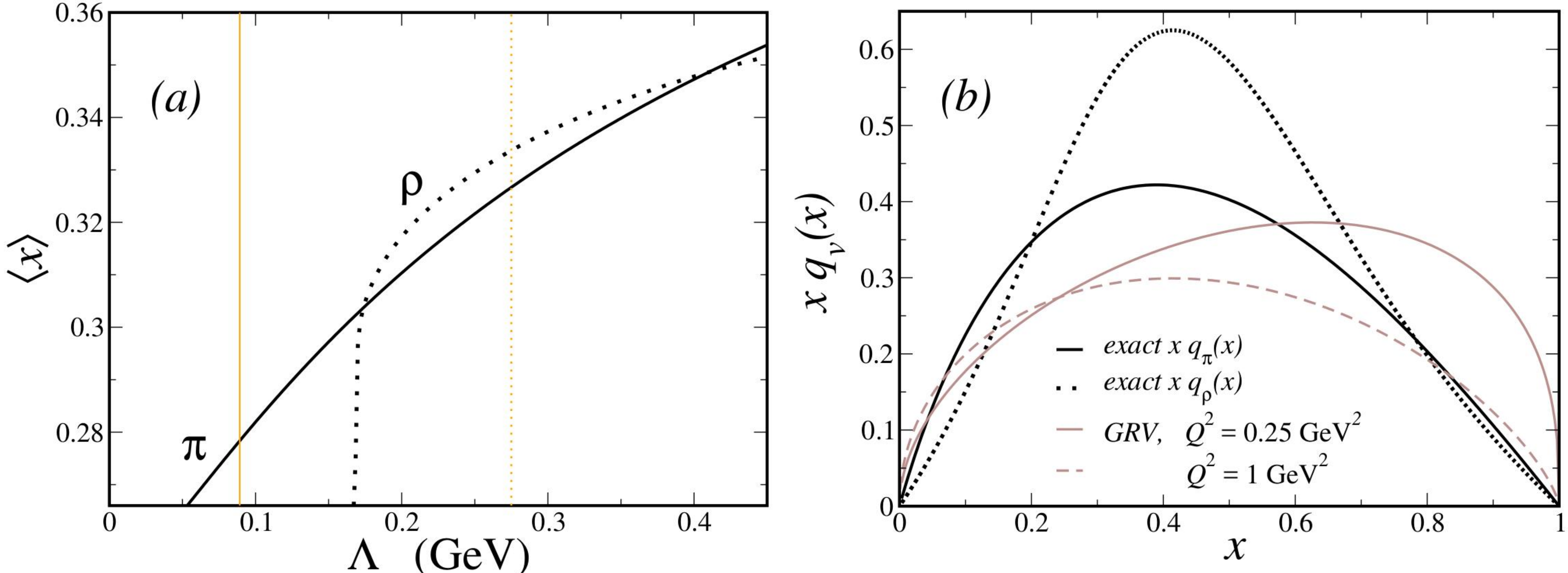
$$q = p - k$$

$$q_\pi^{\mathrm{LF}}(x) = \frac{N_\pi}{8\pi^2} \int \frac{dk_\perp^2}{x^2(1-x)} \left\{k_\perp^2 + \big(x m_s + (1-x)\,m\big)^2\right\} \left[\frac{\phi_\pi(t_\pi)}{(M_\pi^2 - \hat{s})}\right]^2$$

determining the pion SF model parameters

- for the pion, masses can be fixed to physical or constituent values:

$$M_\pi = 0.139\,\mathrm{GeV},\ m = M/3 \approx 0.33\,\mathrm{GeV}$$



$$\langle x \rangle_\pi \;=\; \int_0^1 dx\, x q_\pi^{\mathrm{LF}}(x) \;=\; 0.279(83)^* \quad\longrightarrow\quad \Lambda_\pi \;=\; 0.0892\,\mathrm{GeV}$$

*LQCD 1st moment calculation: Best et al., PRD**56**, 2743 (1997).

- the overall strength is set by a **normalization condition** such that the model is then completely determined

$$N_\pi \;=\; 1 \Big/ \int dx\, q_\pi^{\mathrm{LF}}(x)$$

the corresponding pion quasi-PDF may then be found:

$$\widetilde{q}_\pi(x,p_z) \;=\; \frac{N_\pi}{(2\pi)^4}\int dk^0 dk_z\, d^2k_\perp \left(\frac{1}{2p_z}\right)\,\delta\left(x-\frac{k_z}{p_z}\right)$$

$$\times\; tr\Big(\gamma_5\,(\not{k}+m)\,\gamma^z(\not{k}+m)\gamma_5(-\not{q}+m_s)\Big)\;2\pi\,\delta\big(q^2-m_s^2\big)\left[\frac{\phi_\pi(k^2)}{(k^2-m^2)}\right]^2$$

- now, integrating delta functions introduces explicit dependence on p_z ——

$$\delta\left(q^2-m_s^2\right) \;=\; \frac{1}{2\left(p^0-k^0\right)}\,\delta\left(p^0-k^0-\sqrt{m_s^2+k_\perp^2+(1-x)^2p_z^2}\right)$$

$$\widetilde{q}_\pi(x,p_z) \;=\; \frac{N_\pi}{4\pi^2}\int\frac{dk_\perp^2}{2(1-x)\mu_s}\;\left\{2x\left(mm_s+(\tilde{q}\cdot\tilde{k}_\pi)\right)+(m^2-\tilde{k}_\pi^2)\,(1-x)\right\}$$

$$\times\left[\frac{\phi_\pi(\tilde{k}_\pi^2)}{(M_\pi^2+m_s^2-m^2+2(1-x)\big(1-\mu_\pi\,\mu_s\big)}\right]^2$$

(the main result for the pion quasi-PDF)

$$\mu_\pi \;\equiv\; \sqrt{1+\frac{M_\pi^2}{p_z^2}}$$

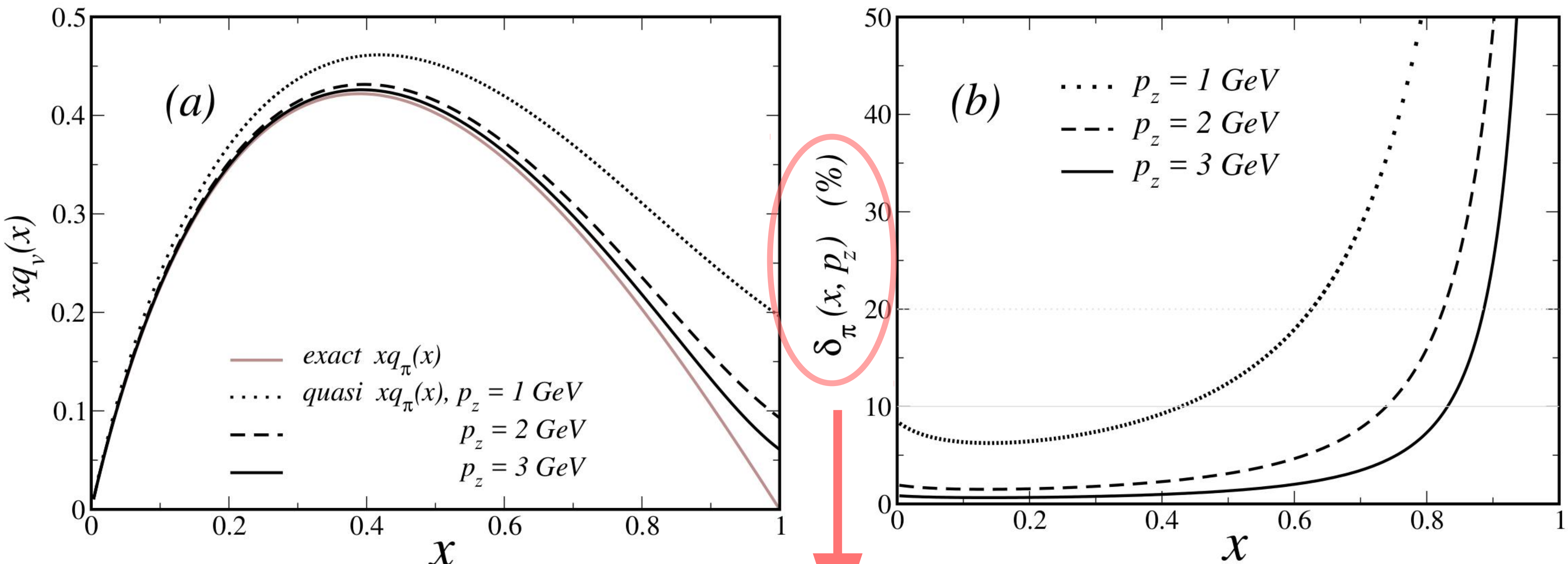
$$\mu_s \;\equiv\; \sqrt{1+\frac{m_s^2+k_\perp^2}{(1-x)^2p_z^2}}$$

$$\tilde{k}_\pi^2 \;=\; M_\pi^2+m_s^2+2\,(1-x)\big(1-\mu_\pi\,\mu_s\big)\,p_z^2$$

→ compare π quasi-/PDFs for several p_z

- we observe the expected behavior: at infinite boost, meson quasi-PDFs match onto the exact result,

$$\lim_{p_z \to \infty} \widetilde{q}_M(x, p_z) = q_M^{\mathrm{LF}}(x)$$



- away from this limit, we compute the LaMET deviations from the LF PDF:

$$\delta_M(x, p_z) \equiv \frac{\widetilde{q}_M(x, p_z)}{q_M^{\mathrm{LF}}(x)} - 1$$

→ even at fairly modest p_z these corrections can be $\lesssim 10\%$!

conclusions

- understanding the nucleon's non-valence structure remains a challenge for the field, but **light-front methods** can help

  → can construct interpolating models that access the flavor structure of the proton wave function

  → we thereby quantify the relationship between strangeness in the nucleon's **elastic form factors** and **structure function**

  (searches for strange in $G_{E,M}(Q^2)$ have some distance to go)

  → this can be extended to charm!

- we have established a close connection between $F^{c\bar{c}}_{2,\mathrm{IC}}$ and $\sigma_{c\bar{c}}$

  → **to exploit this connection, more experimental information is required, but diverse channels are/will be available (e.g., at EIC)**

- LaMET techniques hold promise for computing the valence **quasi-distributions** of the pion, $\widetilde{q}_\pi(x)$

  → invaluable for studies of light sea flavor asymmetries!

– THANKS –

meson-baryon *models* (MBMs)

- we implement a framework which *conserves* **spin/parity**

- **nonperturbative** mechanisms are needed to break
$$c(x, Q^2 \le m_c^2) = \bar{c}(x, Q^2 \le m_c^2) = 0!$$

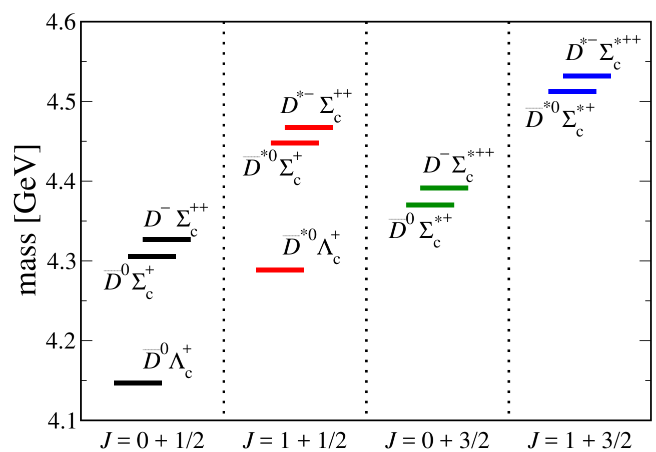
We build an **EFT** which connects IC to properties of the hadronic spectrum: **[TJH, J. T. Londergan and W. Melnitchouk, Phys. Rev. D89, 074008 (2014).]**

- $|N\rangle \; = \; \sqrt{Z_2}\; |N\rangle_0 \; + \; \sum_{M,B} \int dy\, \boldsymbol{f_{MB}(y)}\, |M(y); B(1-y)\rangle$

$$y = k^+/P^+ \text{: } k \text{ meson, } P \text{ nucleon}$$

$$c(x) = \sum_{B,M} \left[\int_x^1 \frac{d\bar{y}}{\bar{y}}\, f_{BM}(\bar{y})\, c_B\!\left(\frac{x}{\bar{y}}\right) \right]$$

- a similar *convolution* procedure may be used for $\bar{c}(x)$ . . .



amplitudes from hadronic **EFT**

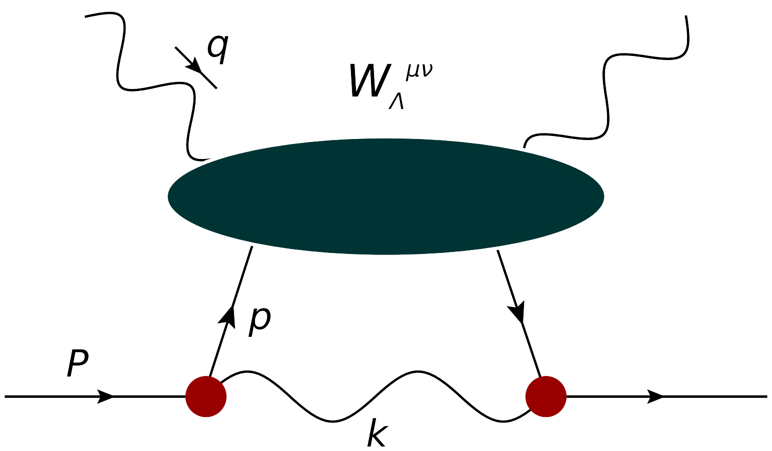
- *e.g.*, for the **dominant** contribution to $c(x)$, i.e., $\boxed{\Lambda_c D^*}$:

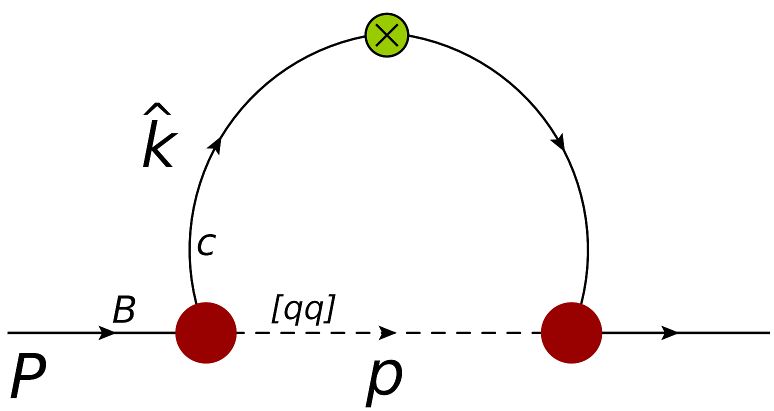
$$c(x) = \int_x^1 \frac{d\bar{y}}{\bar{y}}\, f_{\Lambda D^*}(\bar{y}) \cdot c_\Lambda\left(\frac{x}{\bar{y}}\right):$$

$$\mathcal{L}_{D^*\Lambda N} = g\,\bar{\psi}_N \gamma_\mu\, \psi_\Lambda\, \theta^\mu_{D^*} \;+\; \frac{f}{4M}\bar{\psi}_N \sigma_{\mu\nu} \psi_\Lambda\, F^{\mu\nu}_{D^*} \;+\; \text{h.c.}$$

$$\mathcal{L}_{c[qq]\Lambda} \;=\; g\,\bar{\psi}_\Lambda\, \psi_c\, \phi_{[qq]} \;+\; \text{h.c.}$$

quark model $\rightarrow$ had. g, f



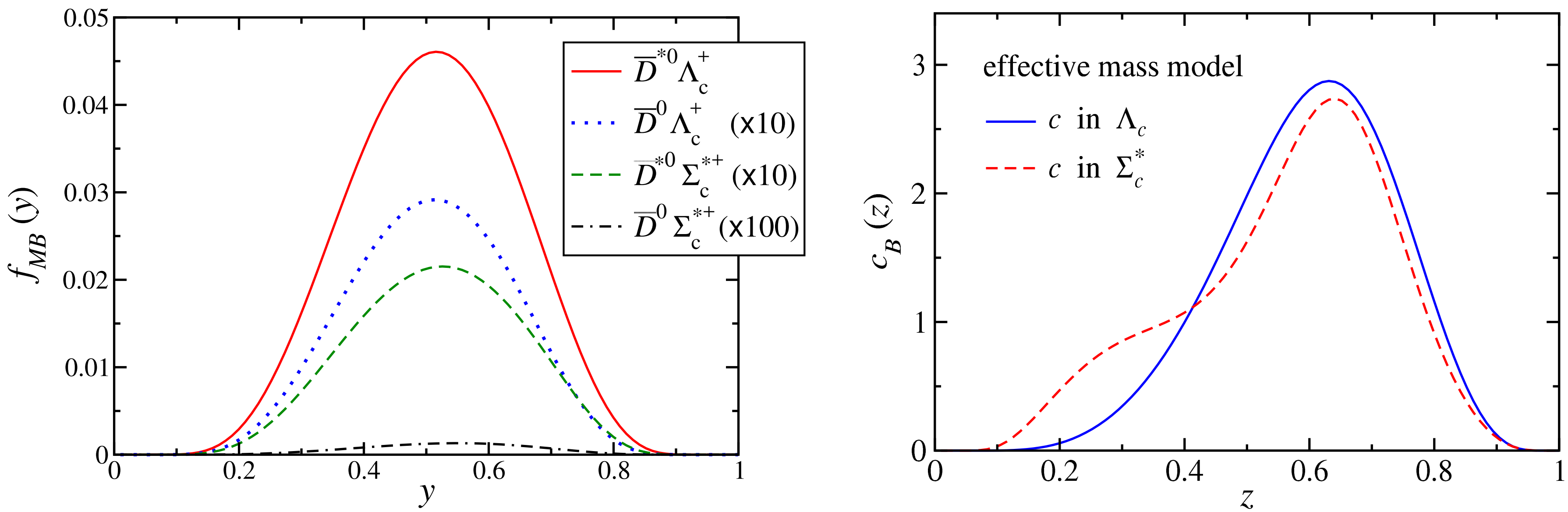


$\rightarrow$ evaluate forward-moving **TOPT** diagrams

hadron/parton **distributions**

$$f_{BD^*}(\bar{y}) = T_B \, \frac{1}{16\pi^2} \int dk_\perp^2 \, \frac{|\mathcal{F}(s_{BM})|^2}{(s_{BM}-M^2)^2} \, \frac{1}{\bar{y}(1-\bar{y})} \times \left[g^2 \, G_v(\bar{y}, k_\perp^2) \; + \; \frac{gf}{M} \, G_{vt}(\bar{y}, k_\perp^2) \; + \; \frac{f^2}{M^2} \, G_t(\bar{y}, k_\perp^2) \right]$$

$$c_B(z) = N_B \, \frac{1}{16\pi^2} \int d\hat{k}_\perp^2 \, \frac{1}{z^2(1-z)} \, \frac{|\mathcal{F}(\hat{s})|^2}{(\hat{s}-M_B^2)^2} \left[\hat{k}_\perp^2 + (m_c + z M_B)^2 \right]$$

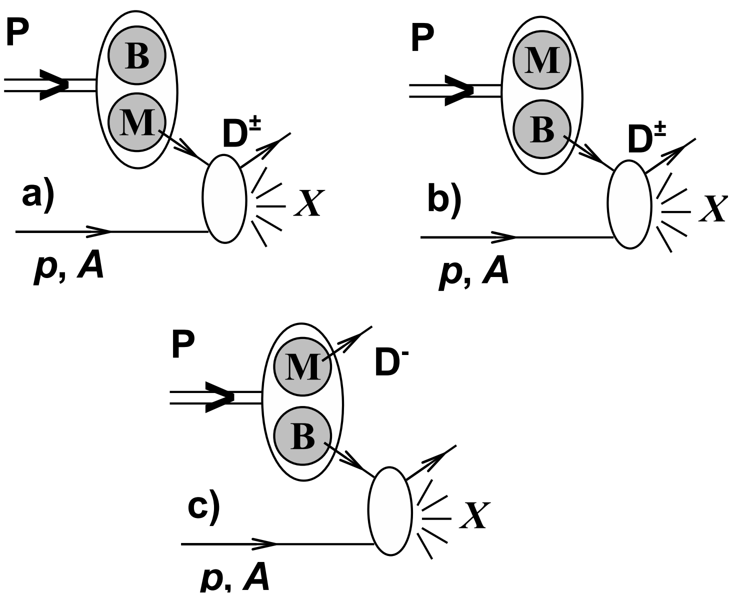


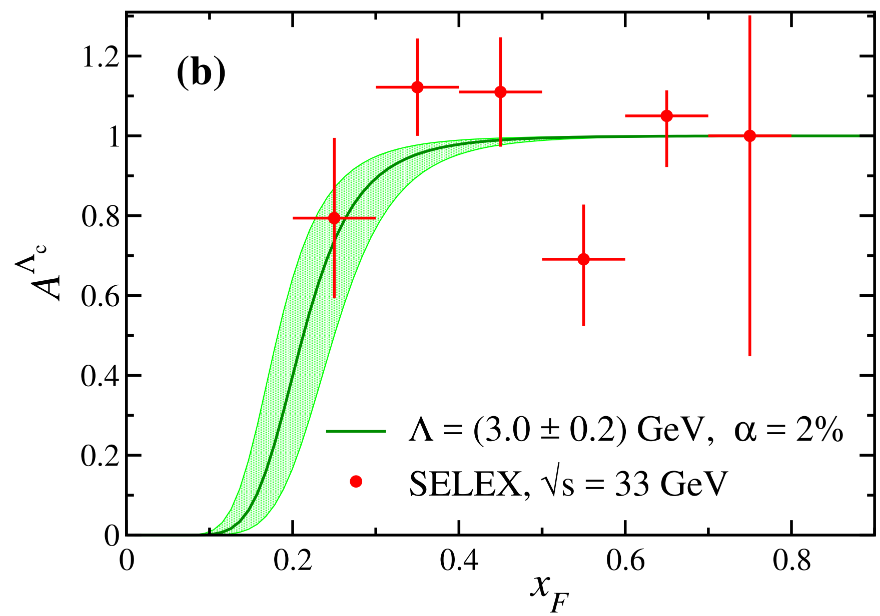
$\rightarrow$ **model dependence** mainly from $\mathcal{F}(s)$,

$$s(\bar{y}, k_\perp^2) \;=\; (M_\Lambda^2 + k_\perp^2)/\bar{y} \;+\; (m_D^2 + k_\perp^2)/(1-\bar{y})$$

production **asymmetries**?

$$A^{\Lambda_c}(x_F) = \frac{\sigma^{\Lambda_c}(x_F)-\sigma^{\bar{\Lambda}_c}(x_F)}{\sigma^{\Lambda_c}(x_F)+\sigma^{\bar{\Lambda}_c}(x_F)} \qquad (\sigma^{\Lambda_c}(x_F) \equiv d\sigma^{\Lambda_c}/dx_F)$$





$$\frac{d\sigma^{\Lambda_c}}{dx_F} = \frac{d\sigma^{\Lambda_c}_{(\mathrm{val})}}{dx_F} + \frac{d\sigma^{\Lambda_c}_{(\mathrm{sea})}}{dx_F}$$

$$\frac{d\sigma^{\Lambda_c}_{(\mathrm{val})}}{dx_F} \approx \sigma_0 \sum_M f_{\Lambda_c M}(x_F)$$

$$\frac{d\sigma^{\Lambda_c}_{(\mathrm{sea})}}{dx_F} \equiv \frac{d\sigma^{\bar{\Lambda}_c}}{dx_F} \approx \bar{\sigma}_0(1-x_F)^{\bar{n}}$$

$$\rightarrow \quad \boxed{A_{\Lambda_c}(x_F) = \frac{\sum_M f_{\Lambda_c M}(x_F)}{\sum_M f_{\Lambda_c M}(x_F)+2\alpha(1-x_F)^{\bar{n}}}} \quad (\alpha = \bar{\sigma}_0/\sigma_0,\ \bar{n} = 6.8)$$

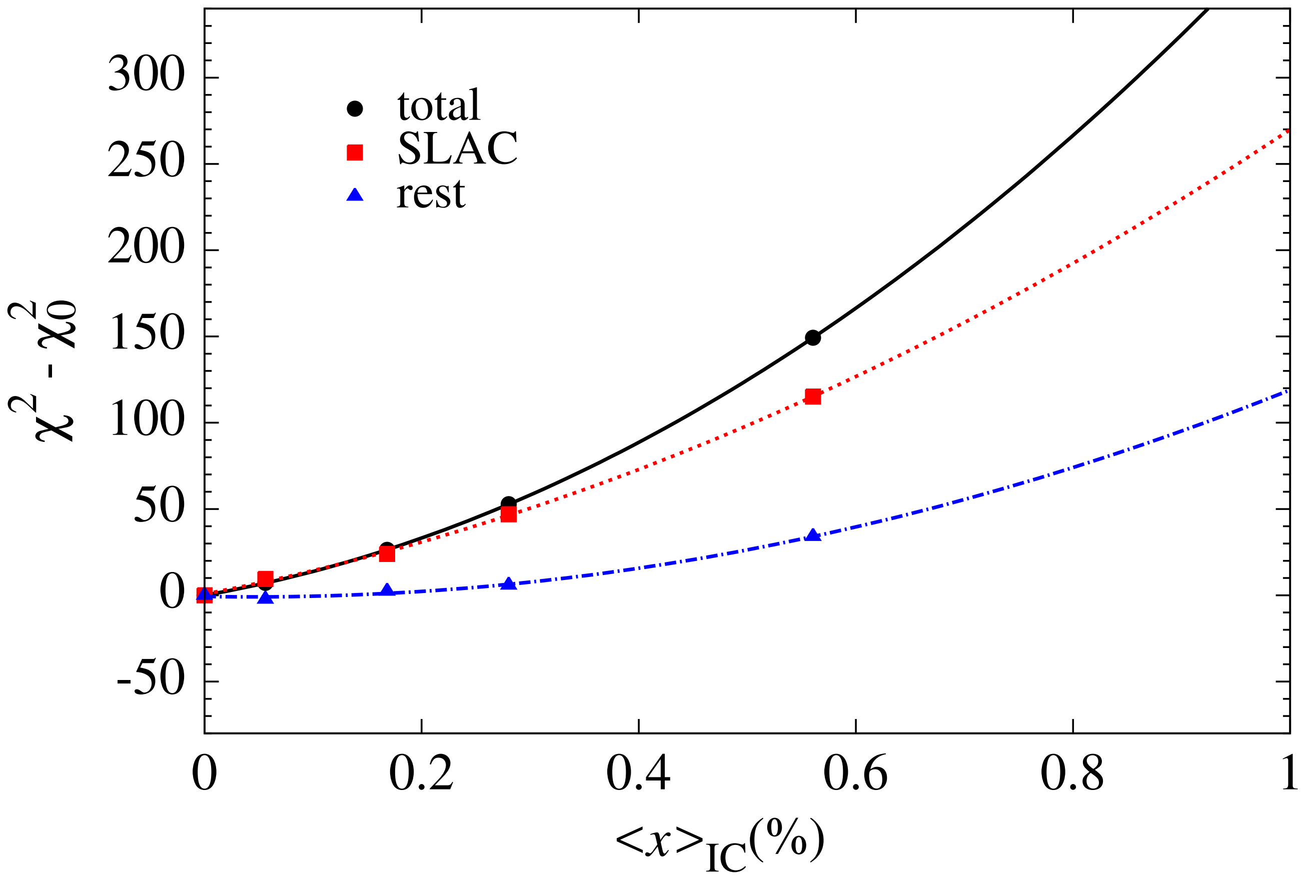
...without **EMC** $F_2^{c\bar{c}}$...

SLAC ep, ed data!

$\langle Q^2 \rangle \sim 15$ GeV^2

$0.06 \leq x \leq 0.9$

$(\chi^2/N_{dat} \sim 1.25)$



'SLAC + REST' $\Longrightarrow$ $\langle x \rangle_{\rm IC} < 0.1\%$; at 5σ !

'REST' only $\Longrightarrow$ $\langle x \rangle_{\rm IC} < 0.1\%$; at 1σ

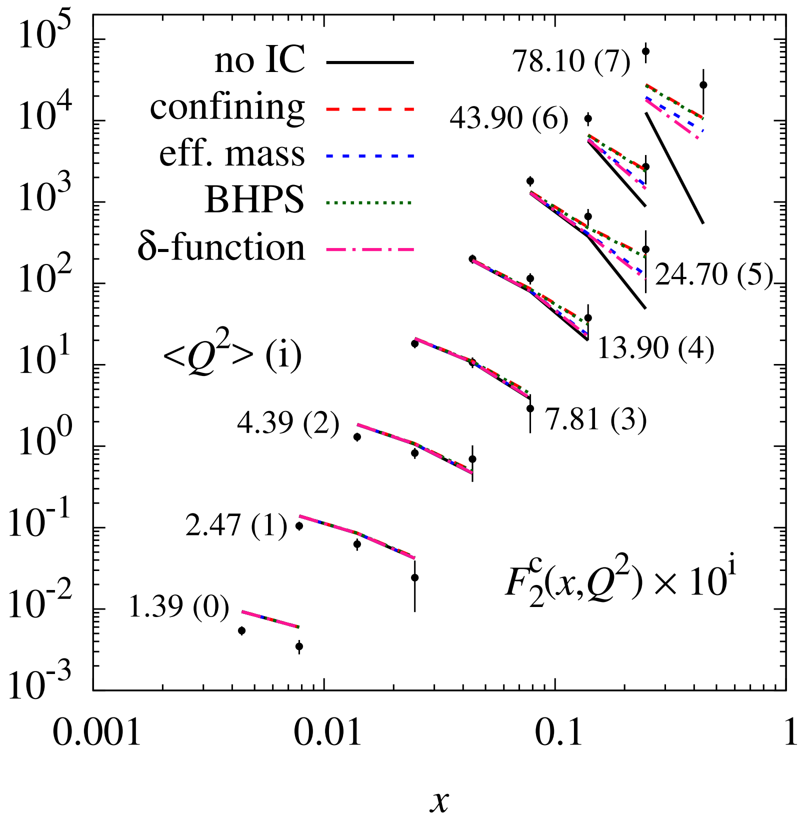
cf., $\langle x \rangle_{\rm IC} \sim 2 - 3\%$

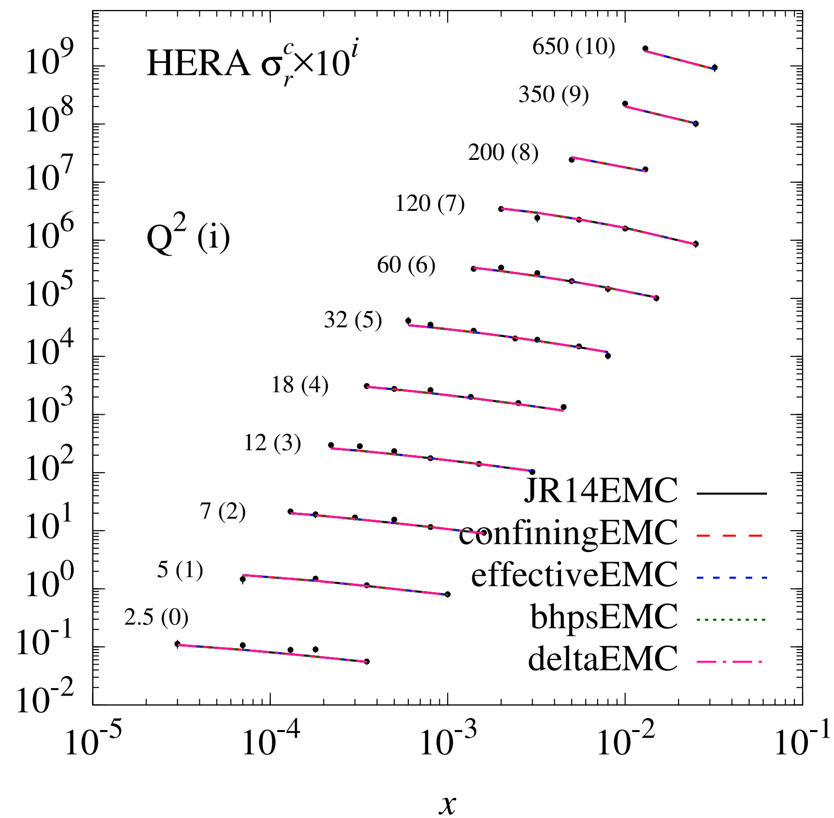
e.g., **[S. Dulat et al., Phys. Rev. D 89, 073004 (2014).]**

N.B.: different *tolerances*: $\Delta\chi^2 = 1$ vs. $\Delta\chi^2_{CT} = 100$

data comparisons:

...*full* fits, constrained by EMC $F_2^{c\bar{c}}$ measurements:





- EMC: low-x/low-Q^2 tension with **HERA** σ_r^c
- $\frac{\tau_{life}}{\tau_{int}} = 5 \rightarrow$ for $Q^2 = 170$ GeV^2, EMC sensitive to IC at $x \lesssim 0.01$

$\rightarrow$ **more** $F_2^{c\bar{c}}$ **data** are needed!

new/ongoing global analyses

- **NNPDF3**: not anchored to specific parametrizations/models

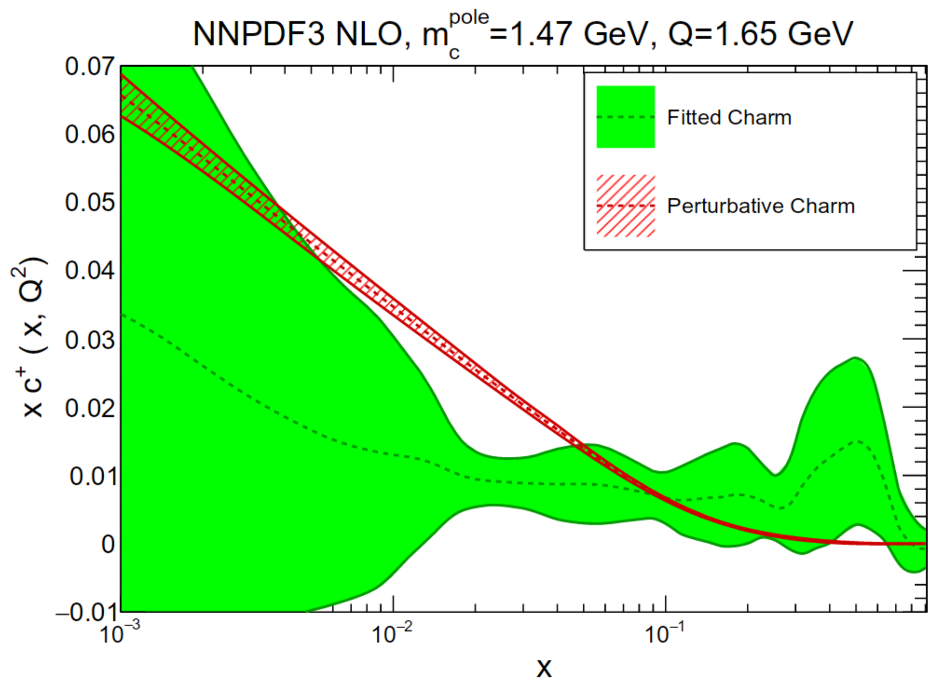
  see: Ball *et al.* Eur. Phys. J. **C76** (2016) no.11, 647

- *included* EMC:

$\langle x \rangle_{\rm IC} = 0.7 \pm 0.3\%$ at $Q \sim 1.5$ GeV

$\rightarrow$ drove a **very** hard $c(x) = \bar{c}(x)$ distribution

- peaked at $x \sim 0.5$
- AND, required a *negative* IC component to describe EMC $F_2^{c\bar{c}}$!



- complementary analyses for possible intrinsic **bottom**

  see: Lyonnet *et al.* JHEP**07** (2015) 141.

$\rightarrow$ would be negligible based on the analysis presented here...

future experimental prospects?

- jet hadroproduction: $pp \to (Zc) + X$ at **LHCb**

  e.g., Boettcher, **Ilten**, Williams, PRD**93**, 074008 (2016).

$\to$ a "direct" measure in the forward region, $2 < \eta < 5$

...sensitive to $c(x)$, $x \sim 1$ for *one* colliding proton

$\to$ can discriminate $\langle x \rangle_{\mathrm{IC}} \gtrsim 0.3\%$ ("valencelike"), 1% ("sealike")

- prompt atmospheric neutrinos?

  see: **Laha** & Brodsky, 1607.08240 (2016).

  $\to$ IceCube ν spectra may constrain IC normalization

- possible impact upon hidden charm **pentaquark**, P_c^+?

  e.g., Schmidt & Siddikov, PRD**93**, 094005 (2016).

- AFTER@LHC? ... fixed-target pp at $\sqrt{s} = 115$ GeV

  Brodsky *et al.* Adv. High Energy Phys. **2015**, 231547 (2015). **[Signori]**