The flavor structure of nucleon sea from lattice QCD

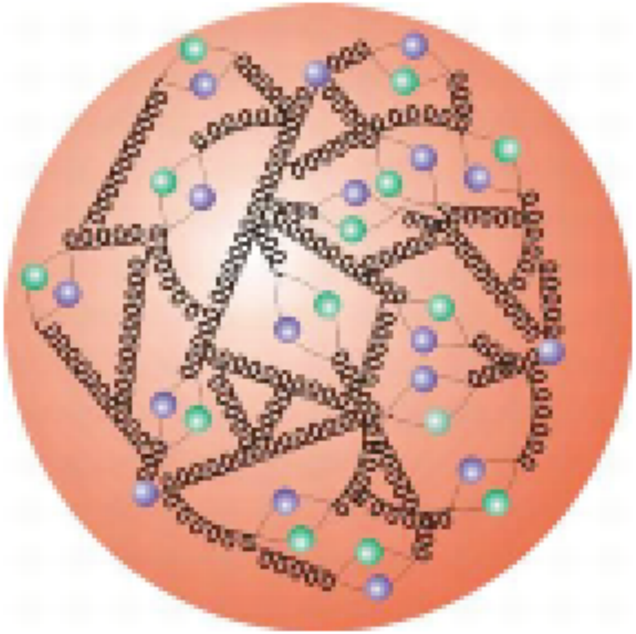
Jiunn-Wei Chen

National Taiwan U.

Collaborators: Saul D. Cohen, Tomomi Ishikawa, Xiangdong Ji, Luchang Jin, Huey-Wen Lin, Peng Sun, Yi-Bo Yang, Jianhui Zhang, Yong Zhao

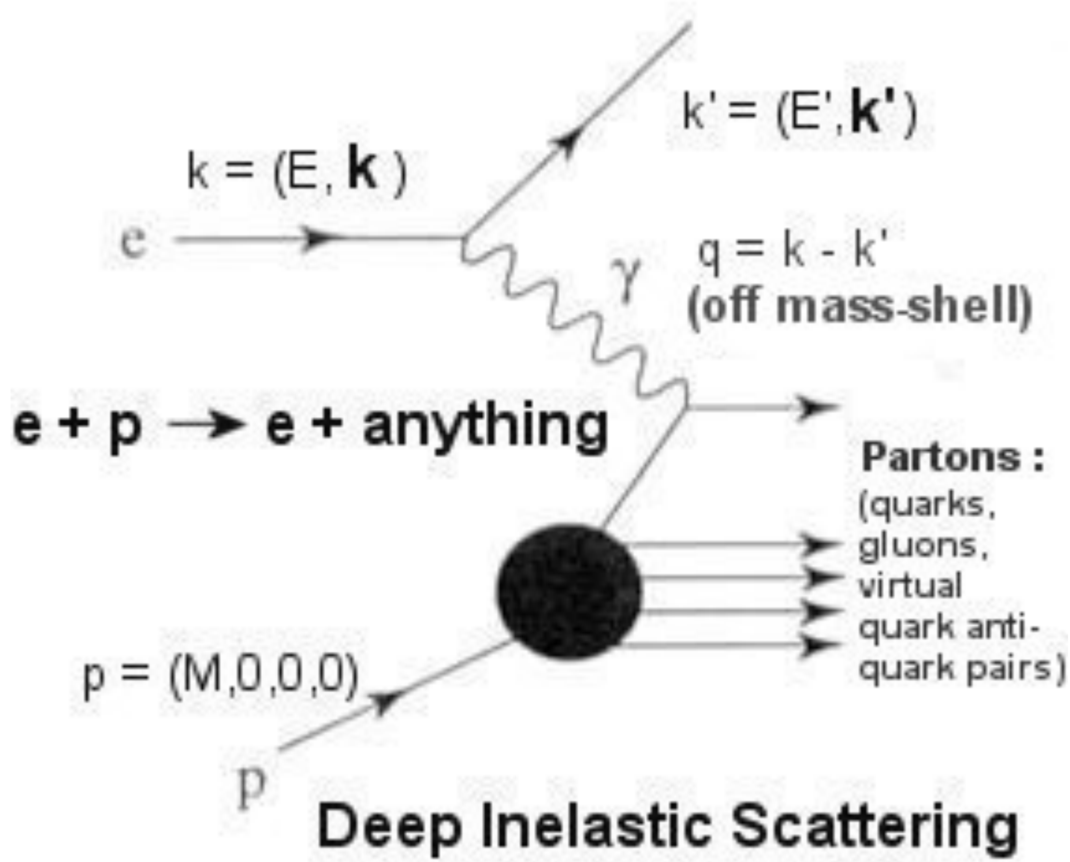
arXiv: 1402.1462 + 1603.06664 + 1609.08102 + 1702.00008 + 1706.01295 + 1708.05301

Feynman’s Parton Model

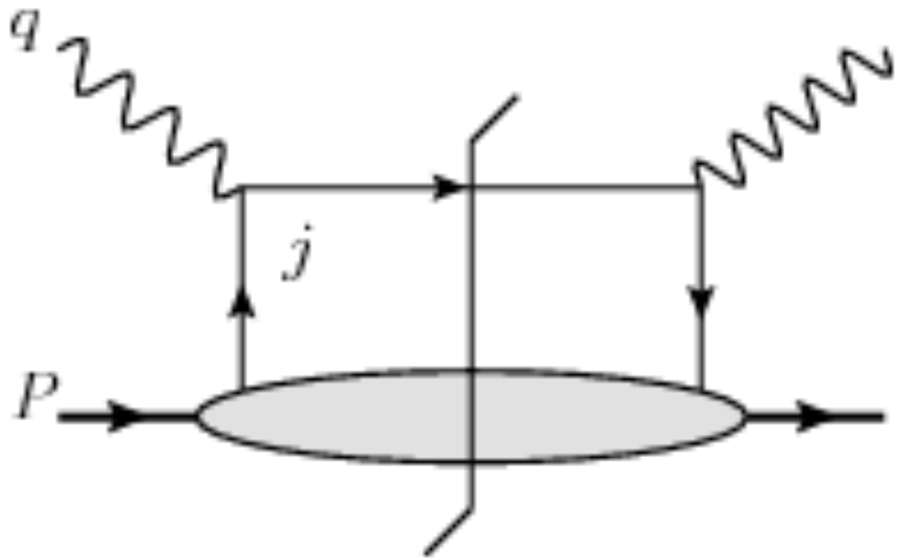


The momentum distributions of partons (quarks, antiquarks and gluons) become one dimensional distributions in the infinite momentum frame.

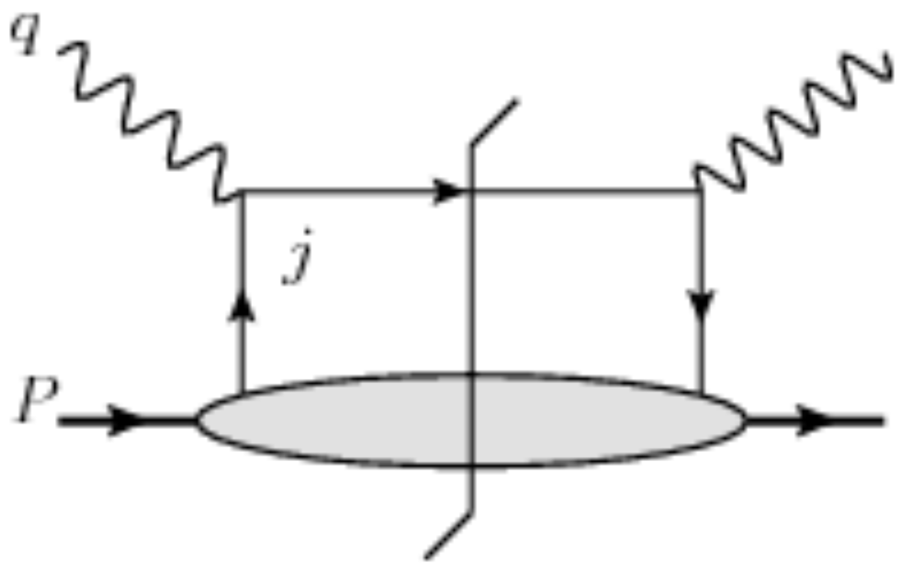
Measuring Parton Distributions Using DIS experiments



Parton Distribution Function (PDF) in QCD



Parton Distribution Function (PDF) in QCD



The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P\rangle$$

Current Status of Proton PDFs

How do momentum and spin distribute among partons?

- Exp: 1d mom. dist. largely mapped out (up to parametrizations of the functional forms); largest sys. uncertainty in Higgs production.

  improve 1d(spin)+3d: BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...

- Theory: Only first few moments could be computed directly from QCD!!!

PDFs from QCD---why is it so hard?

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0)\lambda \cdot \gamma \Gamma \psi(\xi^- \lambda) \right| P \rangle$$

- Non-perturbative, infinite dof, need lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators

$$t^2 - \mathbf{r}^2 = 0$$
$$-t_E^2 - \mathbf{r}^2 = 0$$

- Moments of PDF given by local twist-2 operators (twist = dim - spin); limited to first few moments but carried out successfully

$$a_n = \int_{-1}^{1} dx\, x^{n-1} q(x) \text{ and } q(-x) = -\bar{q}(x)$$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x -dependence directly. (variation: pseudo-PDF, Radyushkin)

Ji's idea

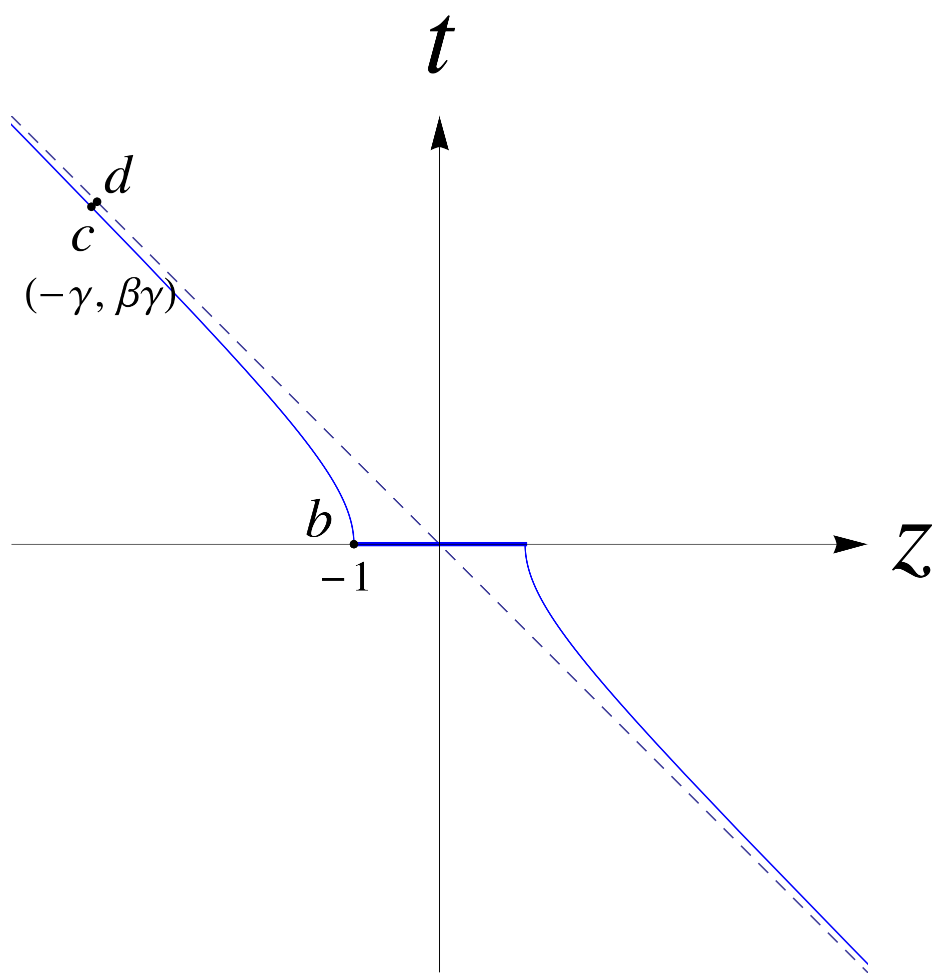
- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \,|\overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda)|\, P\rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.



- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

Review: Ji's LPDF (LaMET)

$$\widetilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda) \right| P \right\rangle$$
$$\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z) P^z$$

$$\lambda^\mu = (0,0,0,1)$$

- Taylor expansion yields

$$\overline{\psi}\lambda\cdot\gamma\Gamma\left(\lambda\cdot D\right)^n\psi = \lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$$

op. symmetric but not traceless

$$\left(\lambda_{\mu_1}\lambda_{\mu_2} - g_{\mu_1\mu_2}\lambda^2/4\right)$$

Review: Ji's LPDF (LaMET)

$$\left\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \right\rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

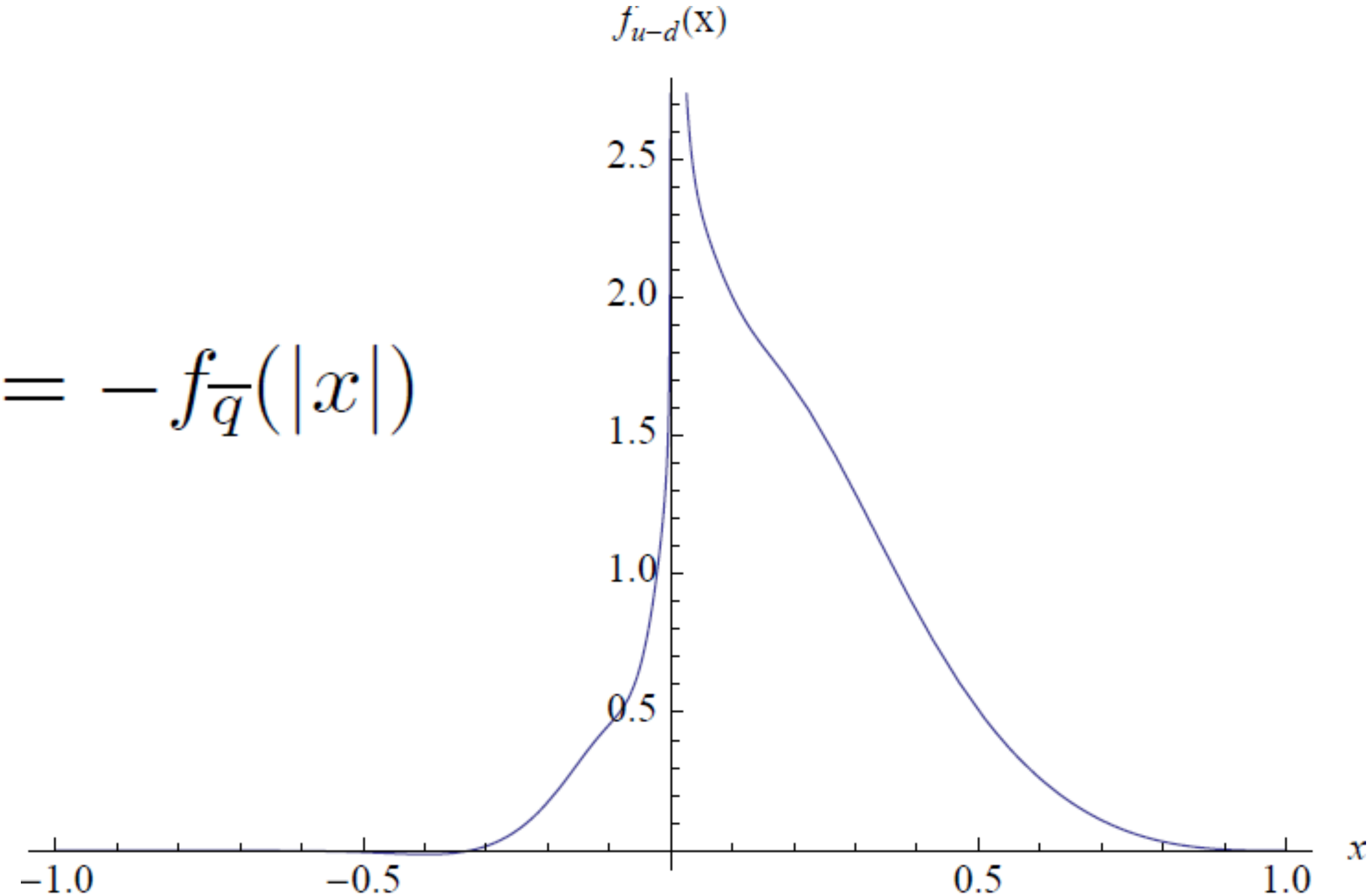
- LHS: trace, twist-4 $\mathcal{O}(\Lambda^2_{QCD}/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots$$

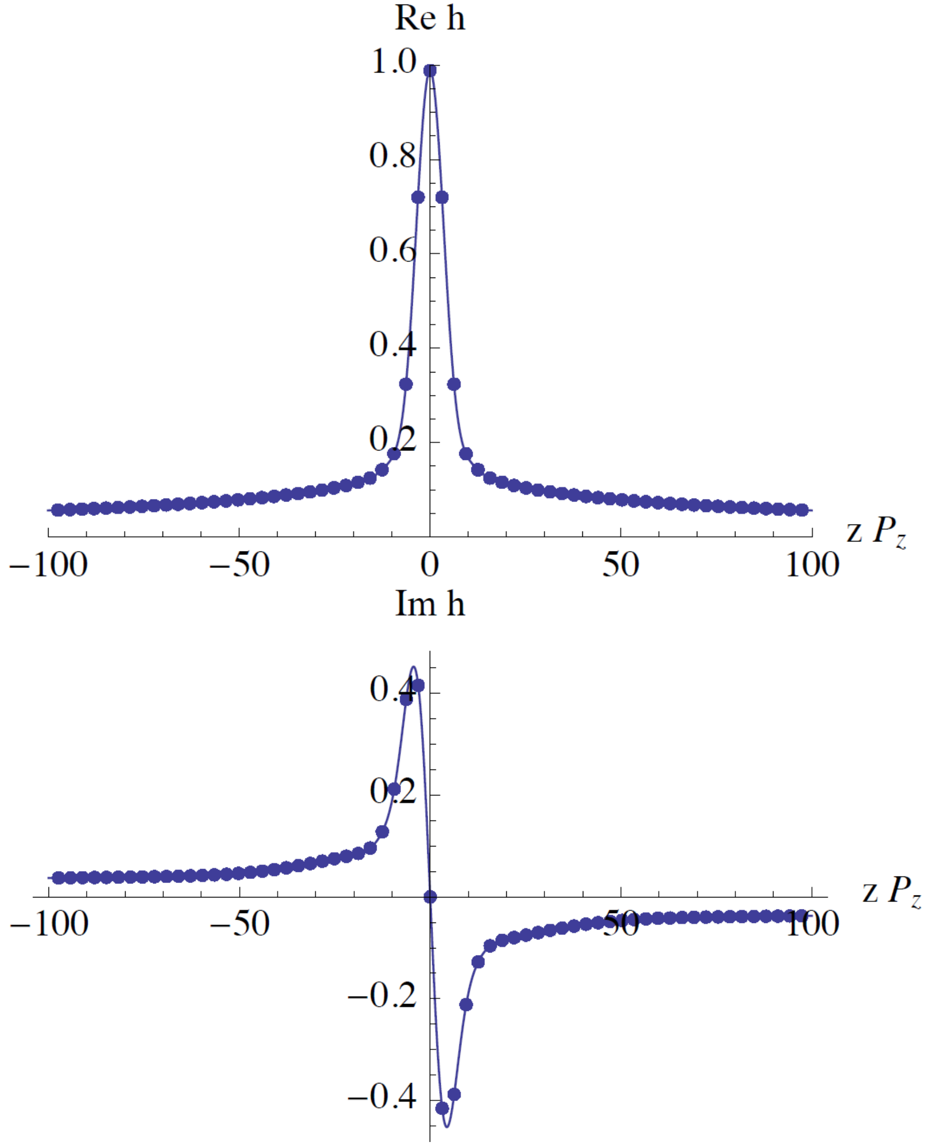
What do we expect to see on the lattice?

- Suppose LPDF were the CTEQ PDF at $P^z \to \infty$

$$f_q(-|x|) = -f_{\overline{q}}(|x|)$$



in the Fourier Space



First (isovector) LPDF Computation

- Lattice: $24^3 \times 64$

  $a \approx 0.12 \text{ fm} \quad L \approx 3 \text{ fm}$

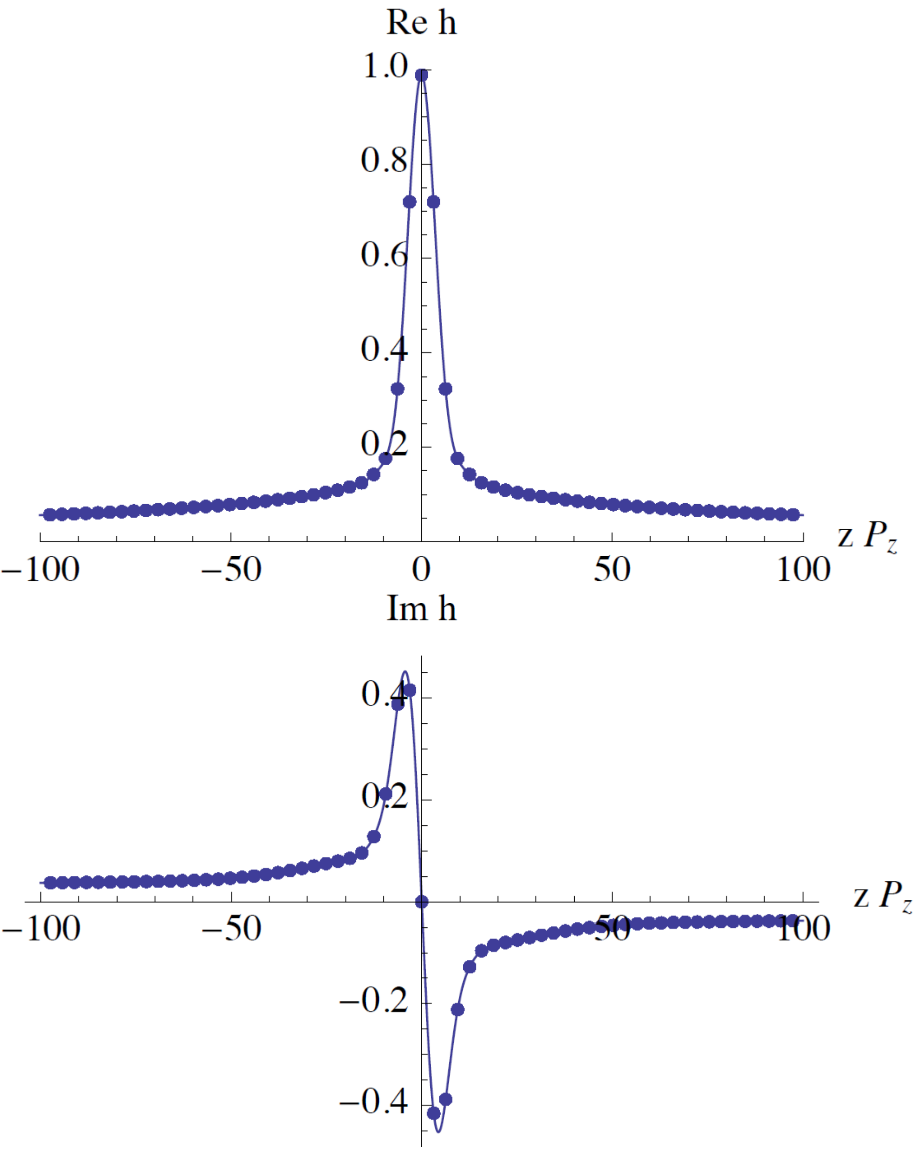
- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

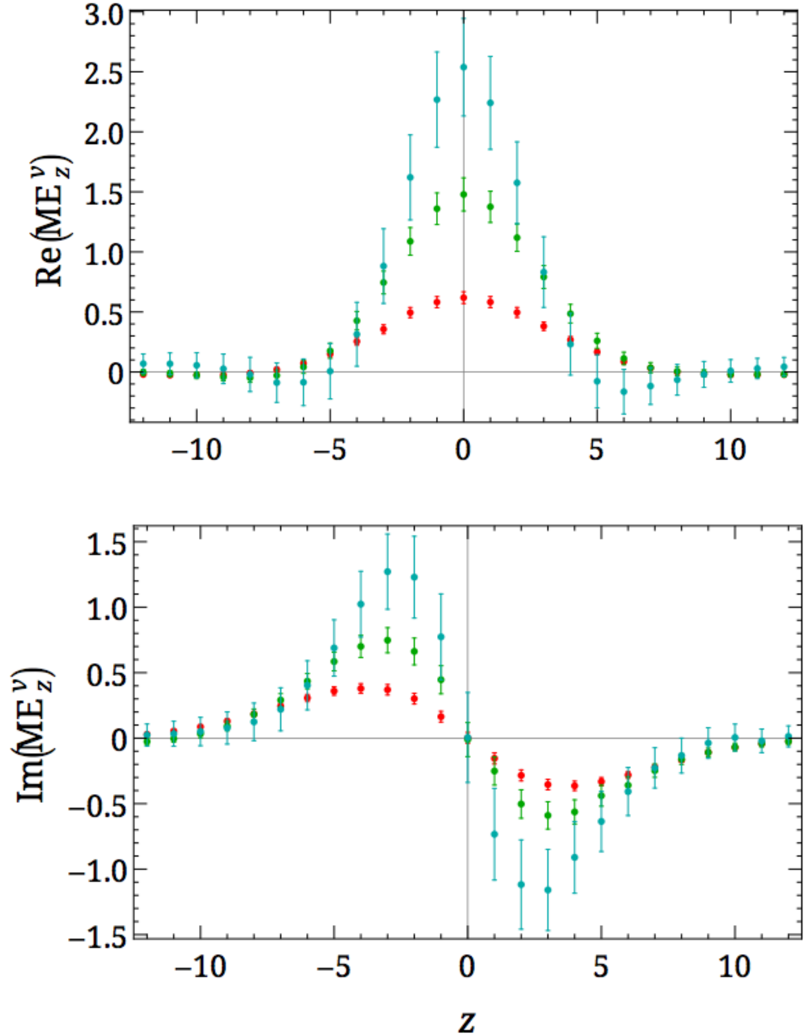
  $N_f = 2 + 1 + 1 \qquad M_\pi \approx 310 \text{ MeV}$

- Gauge fields/links: hypercubic (HYP) smearing, 461 config.

- $P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$ n = 1,2,3…

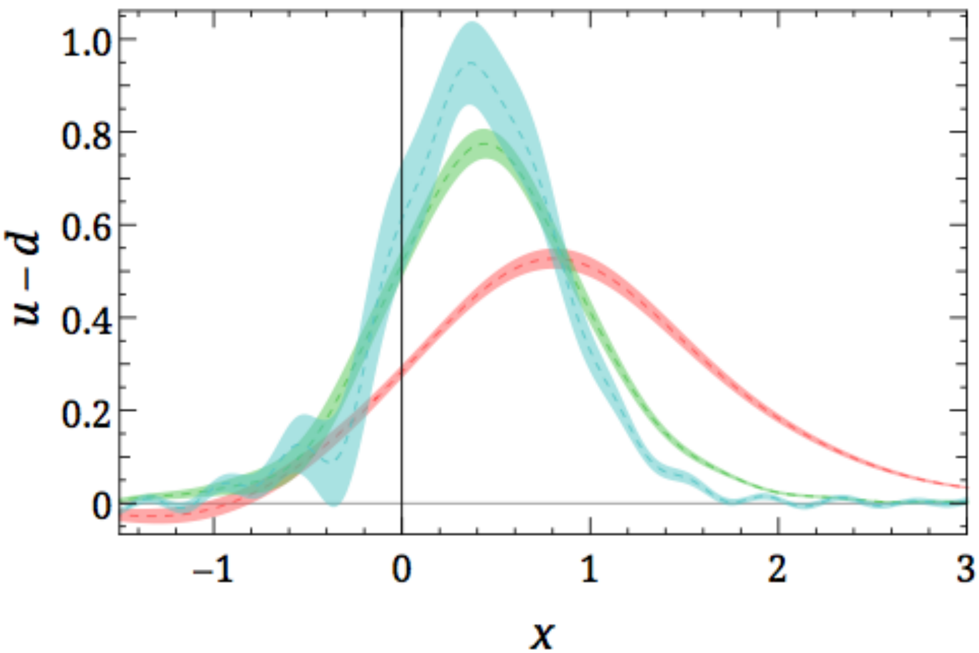
(high momentum smearing: Bali, Lang, Musch, Schafer)





$$P^z = \frac{0.26n}{a},\ n = 1, 2, 3$$

Quasi-PDF (unpolarized)



$$M_\pi \approx 310 \text{ MeV}$$

$$P^z = \frac{2\pi}{L} n = n \times 0.43 \; GeV$$ n = 1, 2, 3.

$\mathcal{O}(M^2/(P^z)^2)$ Corrections

$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

- Computed to all orders in $\mathcal{O}(M^2/(P^z)^2)$.

$$q(x) = \sqrt{1+4c} \sum_{n=0}^{\infty} \frac{f_-^n}{f_+^{n+1}} \left[(1+(-1)^n)\tilde{q}\left(\frac{f_+^{n+1}x}{2f_-^n}\right) + (1-(-1)^n)\tilde{q}\left(\frac{-f_+^{n+1}x}{2f_-^n}\right) \right]$$

$$f_\pm = \sqrt{1+4c} \pm 1 \qquad c = M^2/4P_z^2$$

$\mathcal{O}(\Lambda_{QCD}^2/(P^z)^2)$ Corrections

- Twist-4:

$$q_{tr}(x,\mu^2,P^z) = \frac{\lambda^2}{8\pi}\int_{-\infty}^{\infty} dz \int_0^1 \frac{dt}{t} e^{i\frac{zk^z}{t}} \left\langle P \middle| \widetilde{\mathcal{O}}_{tr}(z) \middle| P \right\rangle$$

$$\widetilde{\mathcal{O}}_{tr}(z) = \int_0^z dz_1 \bar{\psi}(0) \left[\gamma^\nu \Gamma(0,z_1) D_\nu \Gamma(z_1,z) + \int_0^{z_1} dz_2 \lambda\cdot\gamma \Gamma(0,z_2) D^\nu \Gamma(z_2,z_1) D_\nu \Gamma(z_1,z)\right] \psi(z\lambda)$$

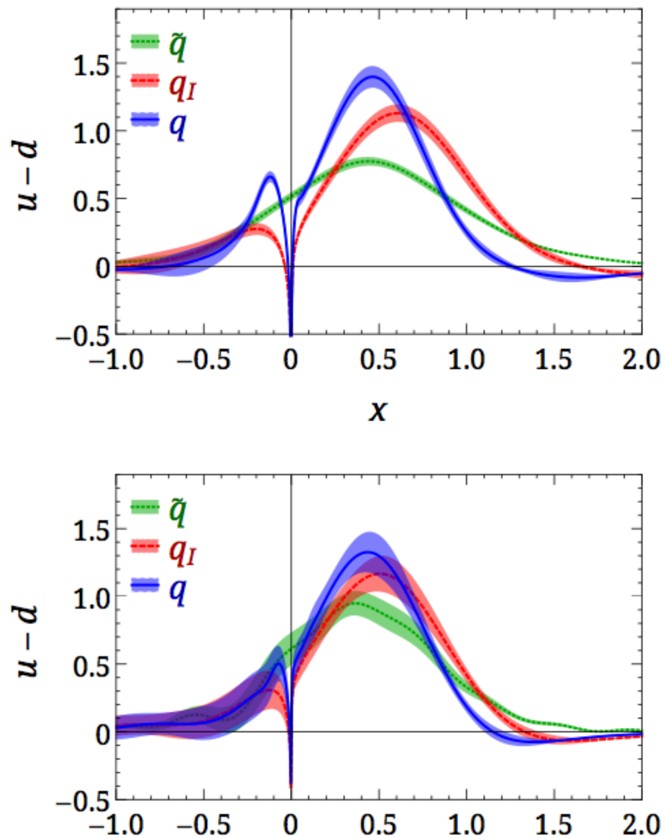
⇨ Parameterized ($\alpha(x) + \beta(x)/P_z^2$)
Additional complications? E.g.
Radyushkin

RG of Wilson Coefficient

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}, \frac{M_N^2}{P_z^2}\right) + \ldots .$$

Xiong, Ji, Zhang, Zhao (GPD: Ji, Schafer, Xiong, Zhang; Xiong, Zhang) Factorization (Ma, Qiu; Li), Linear divergence & LPT (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang; Xiong, Luu, Meissner; Rossi, Testa; Constantinou et al.), RI (Monahan & Orginos; Yong & Stewart; Constantinou et al.), NPR(Constantinou et al.; LP3; Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida; Green, Jansen, Steffens), E vs. M spaces (Carlson et al.; Briceno et al.)

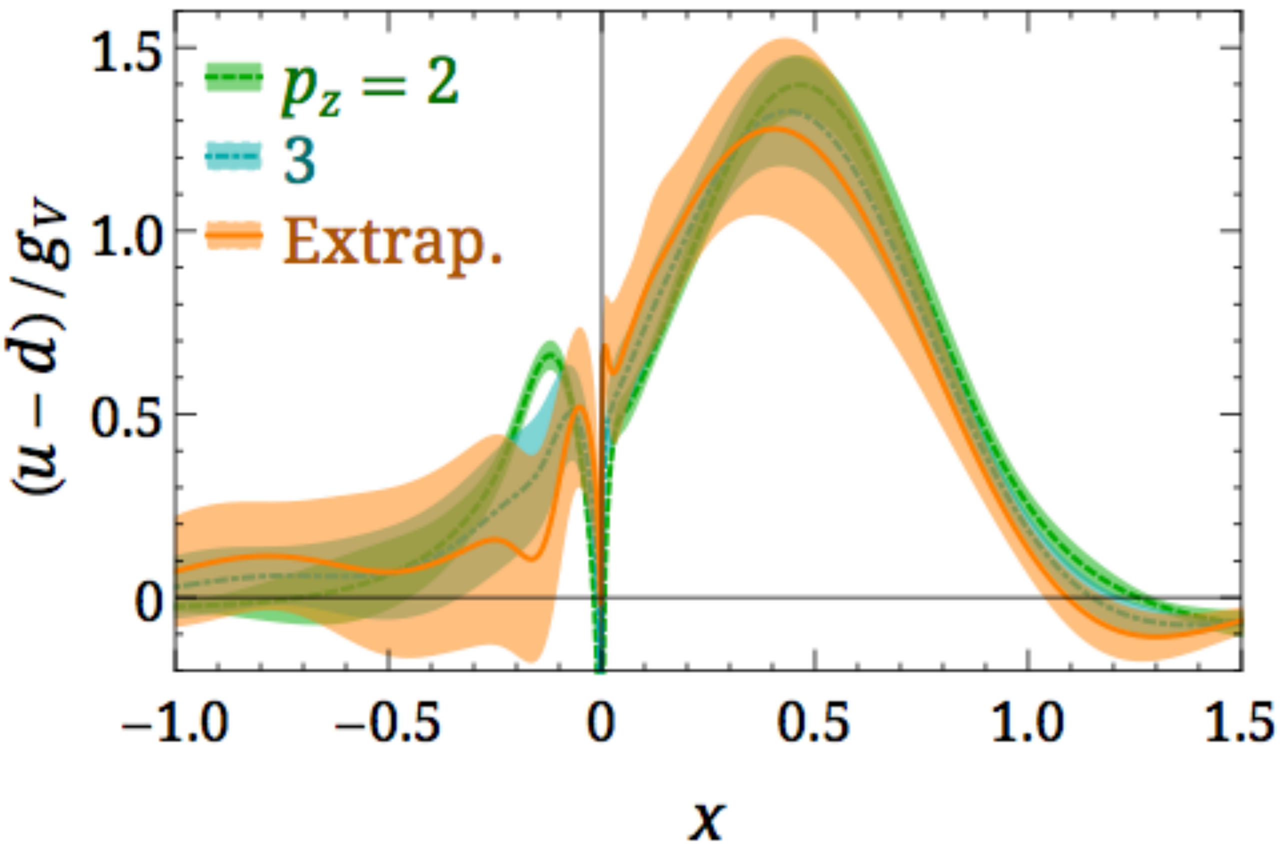
Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)



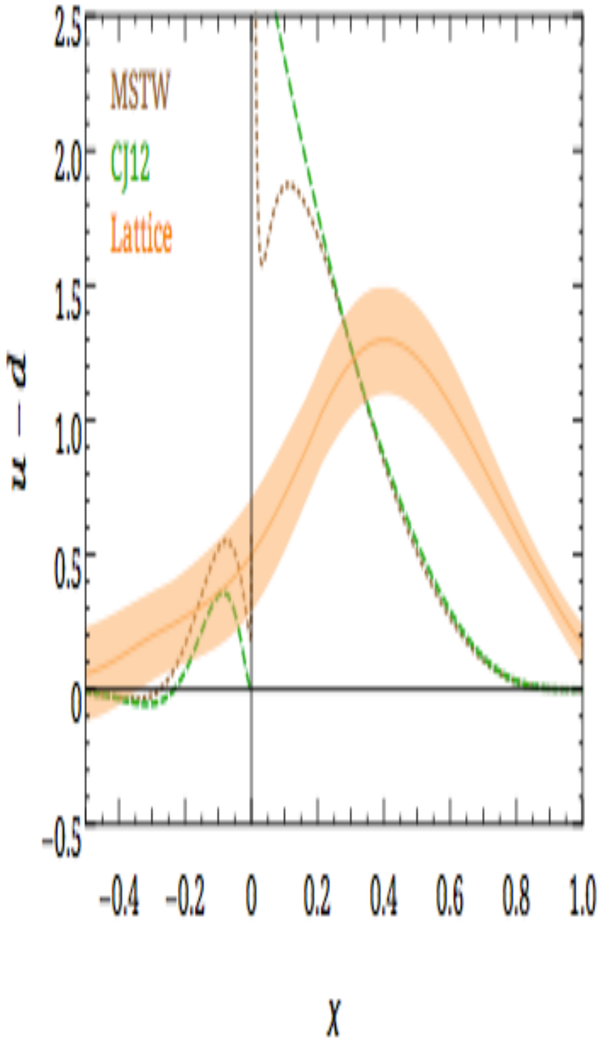
$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

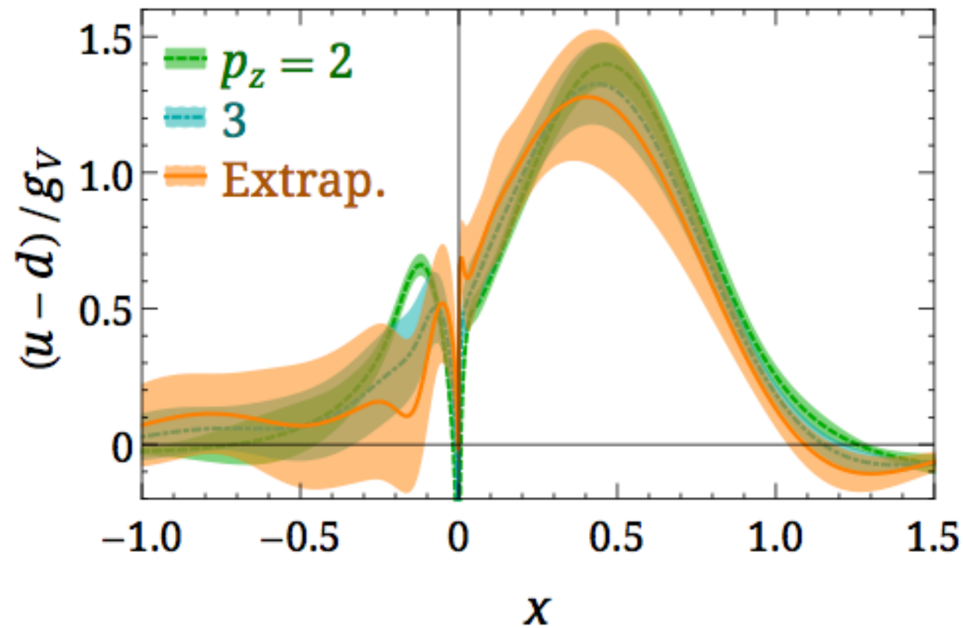
n = 2 (upper) & 3

Unpolarized Isovector Proton PDF



$$\alpha(x) + \beta(x)/P_z^2$$

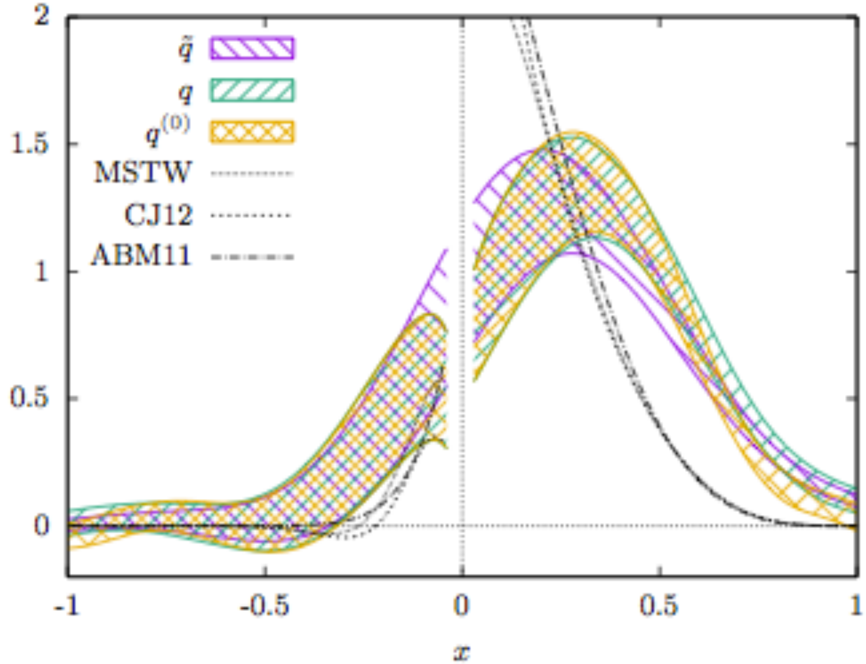


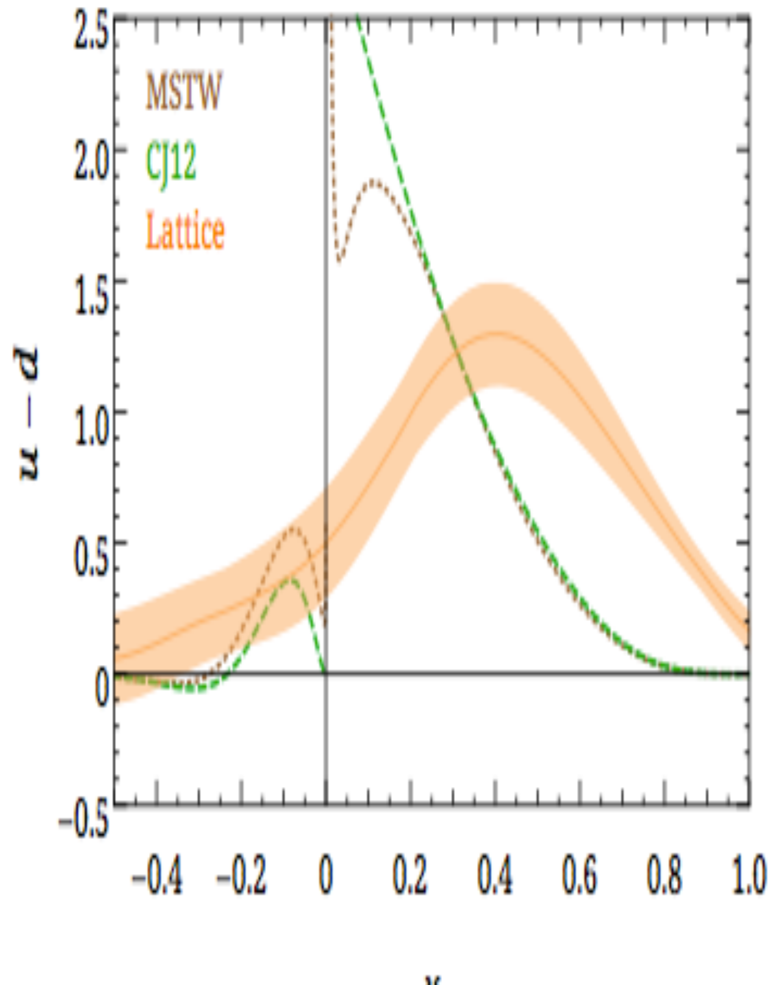


Quark mass effect!

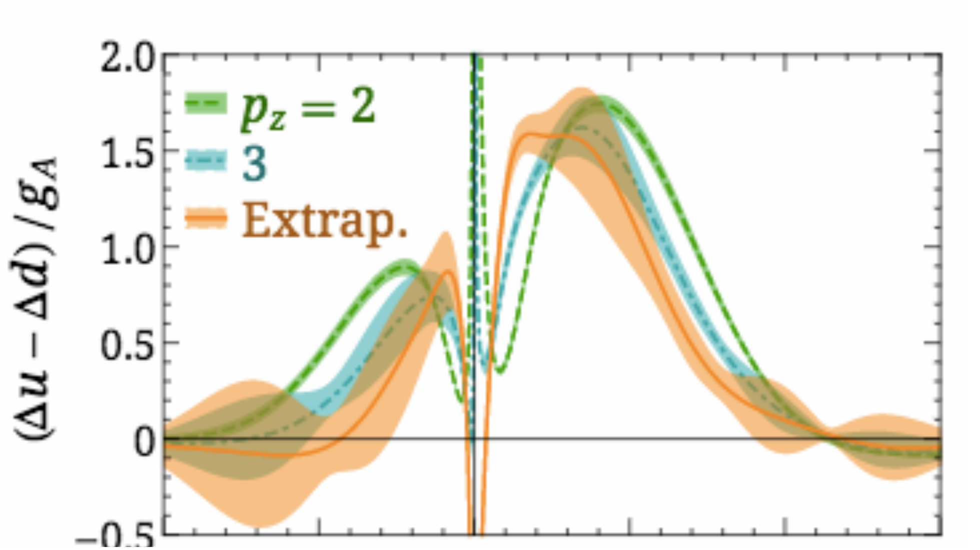
Follow-up works

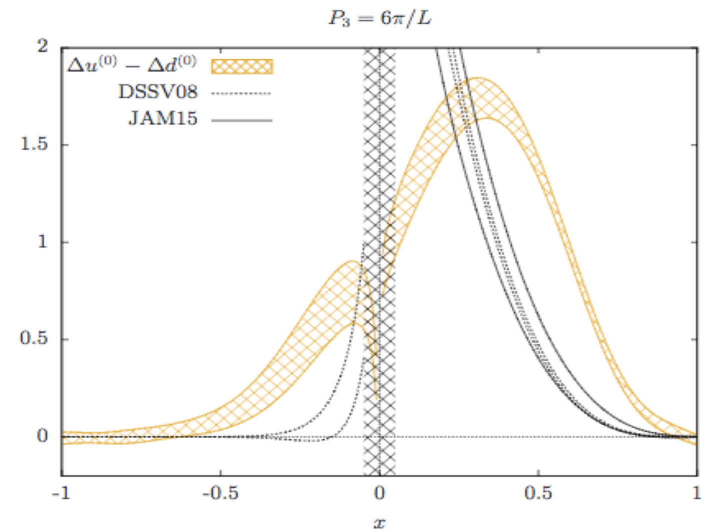
(Alexandrou et. al.:1504.07455+1610.03689)

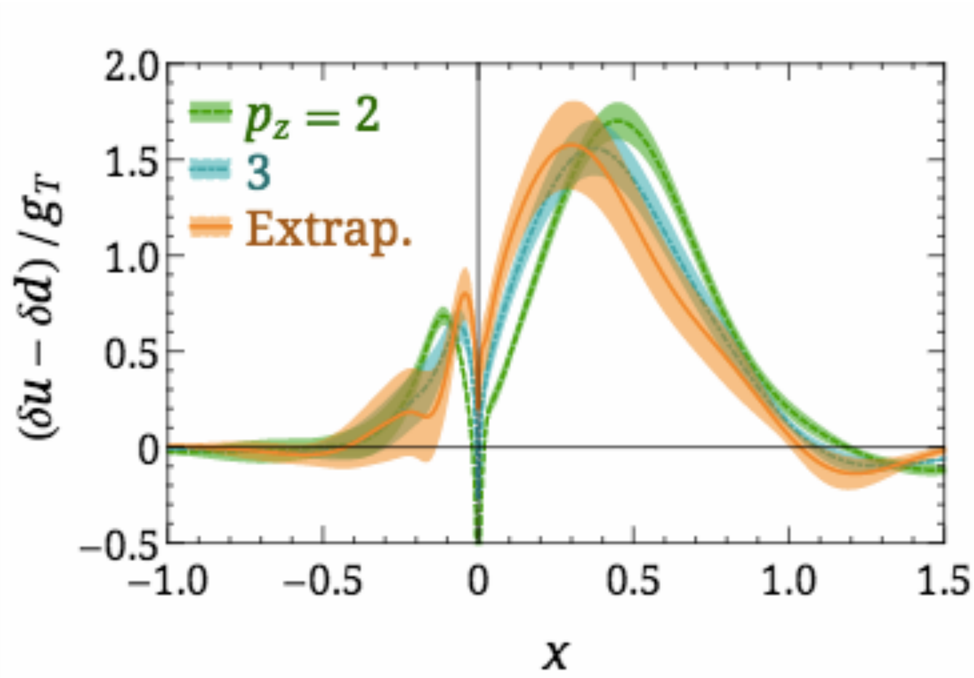


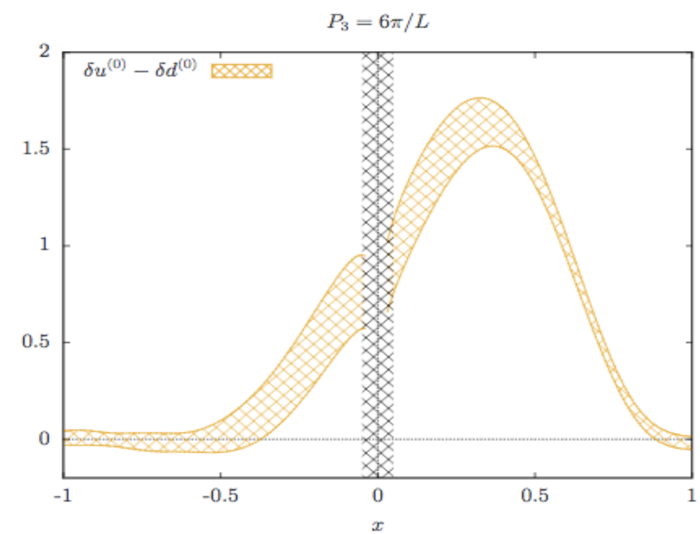


Isovector Proton Helicity and Transversity



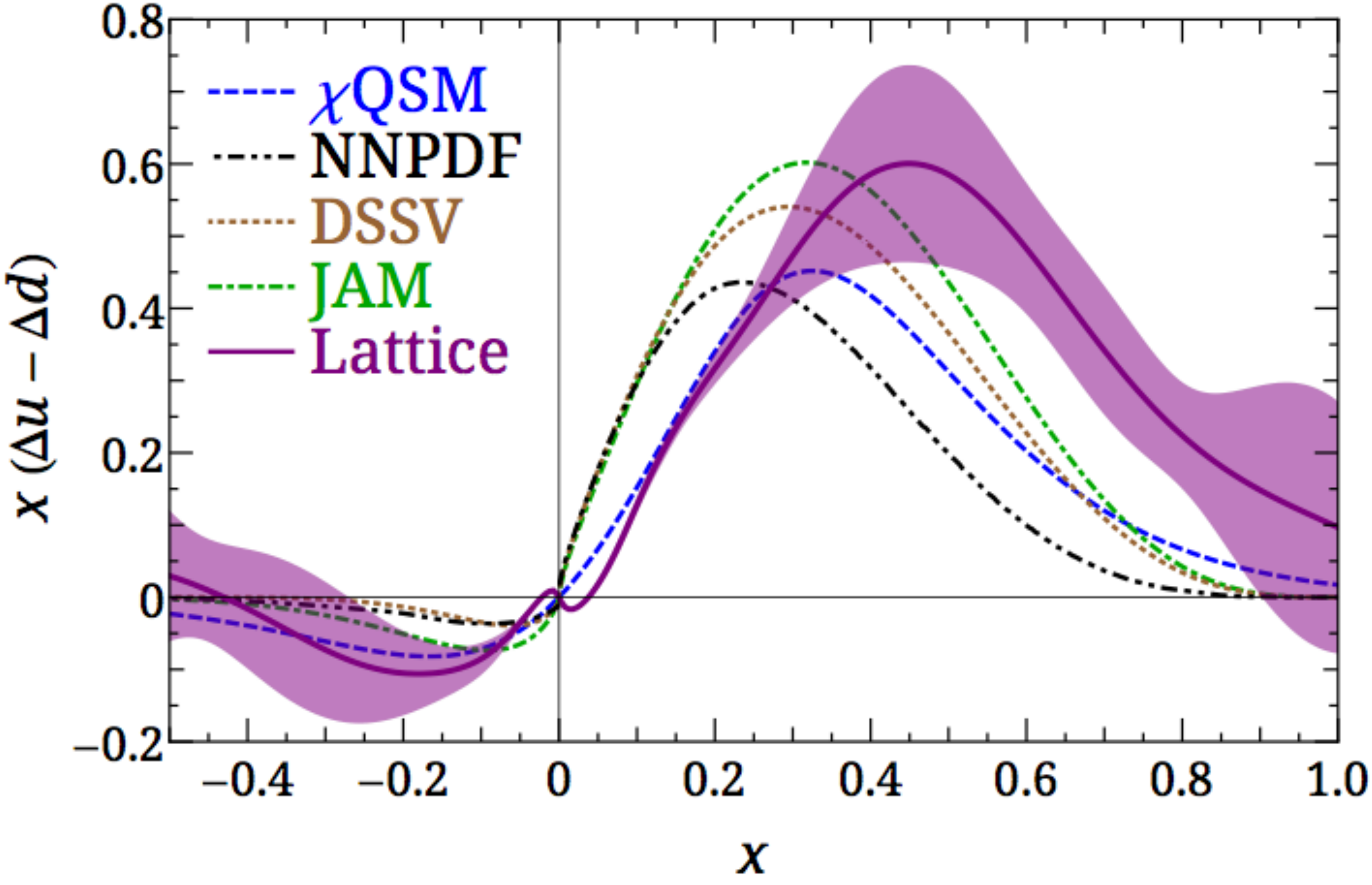




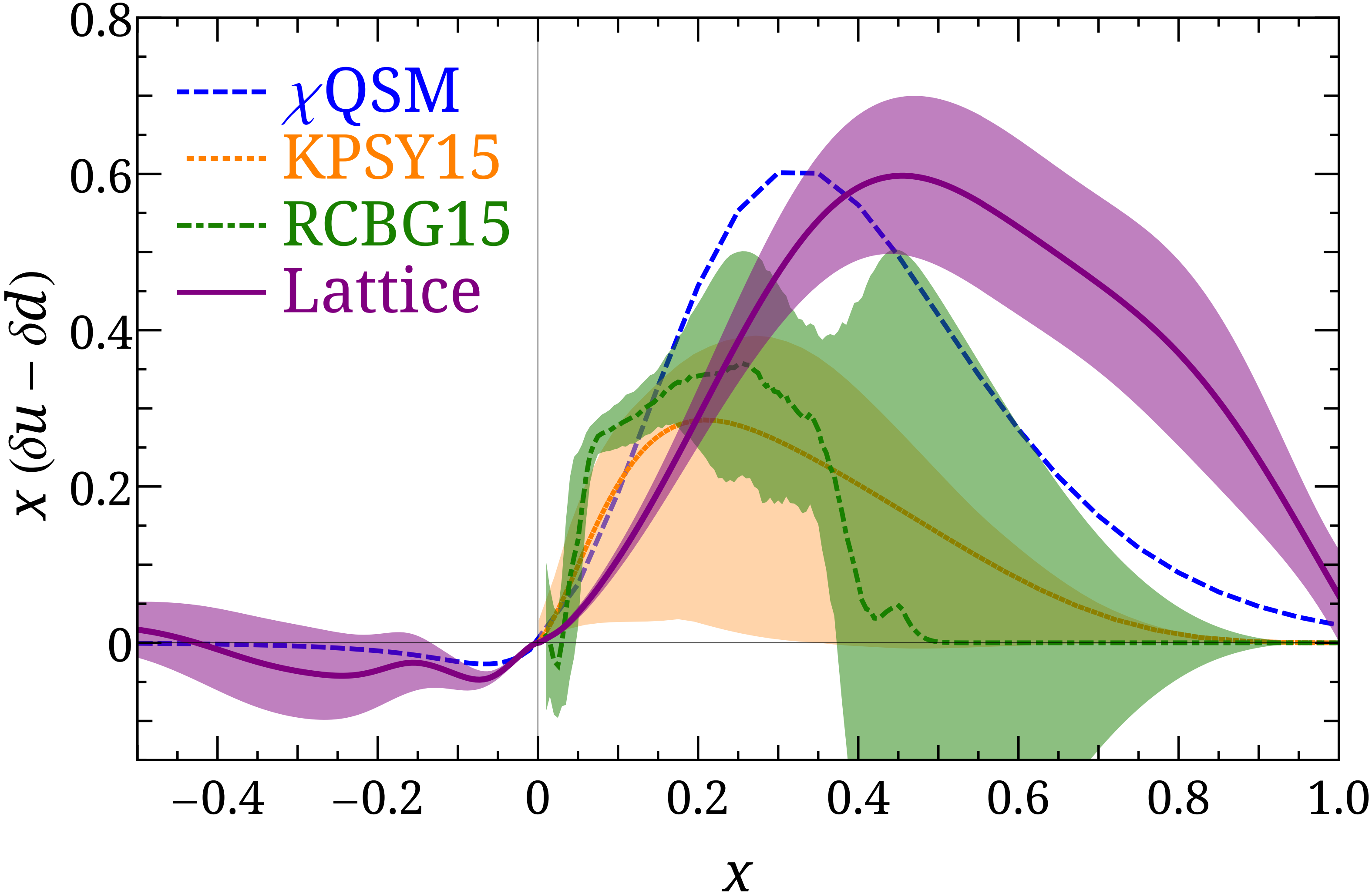


(Alexandrou et.al., 1609.00172)

Isovector Proton Helicity



Isovector Proton Transversity



More on the power divergence in the matching kernel

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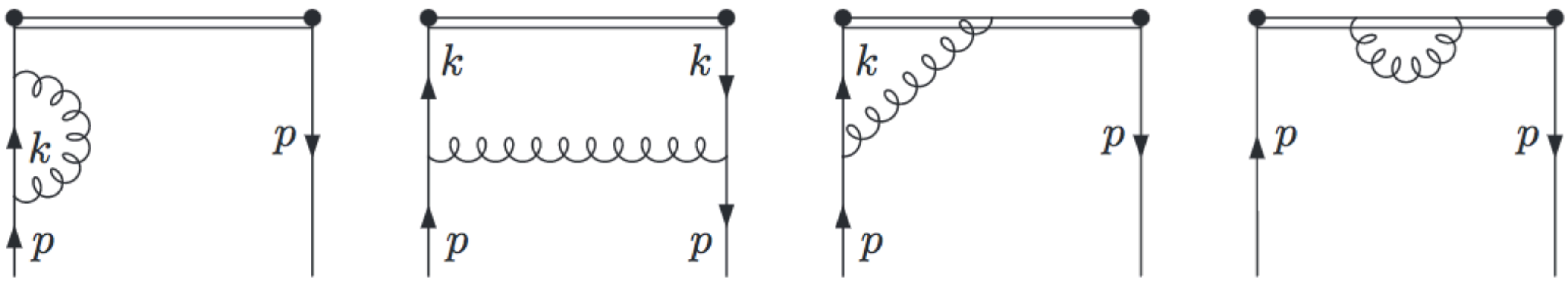
Improved Quasi-PDF's

(Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang)



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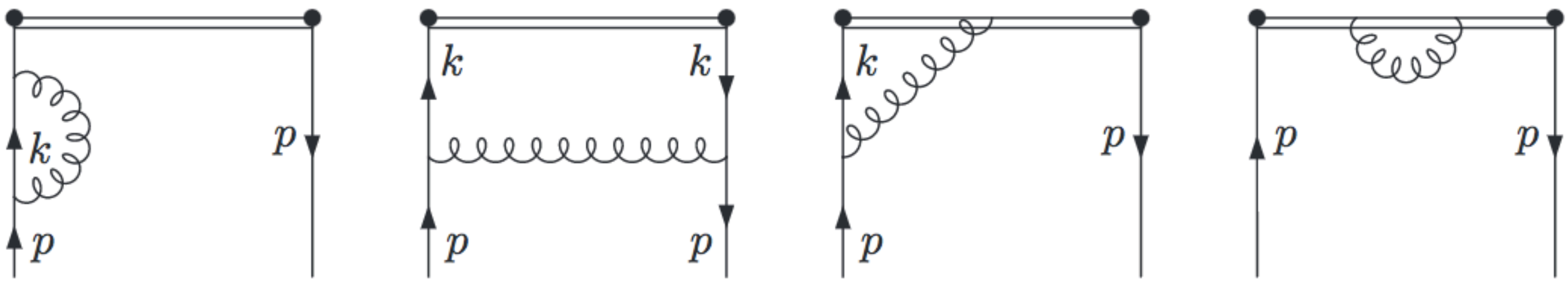
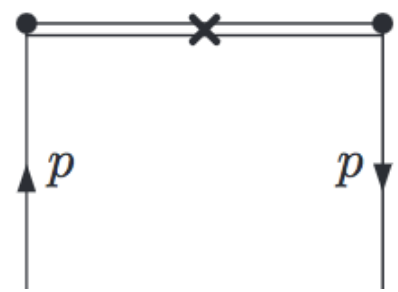
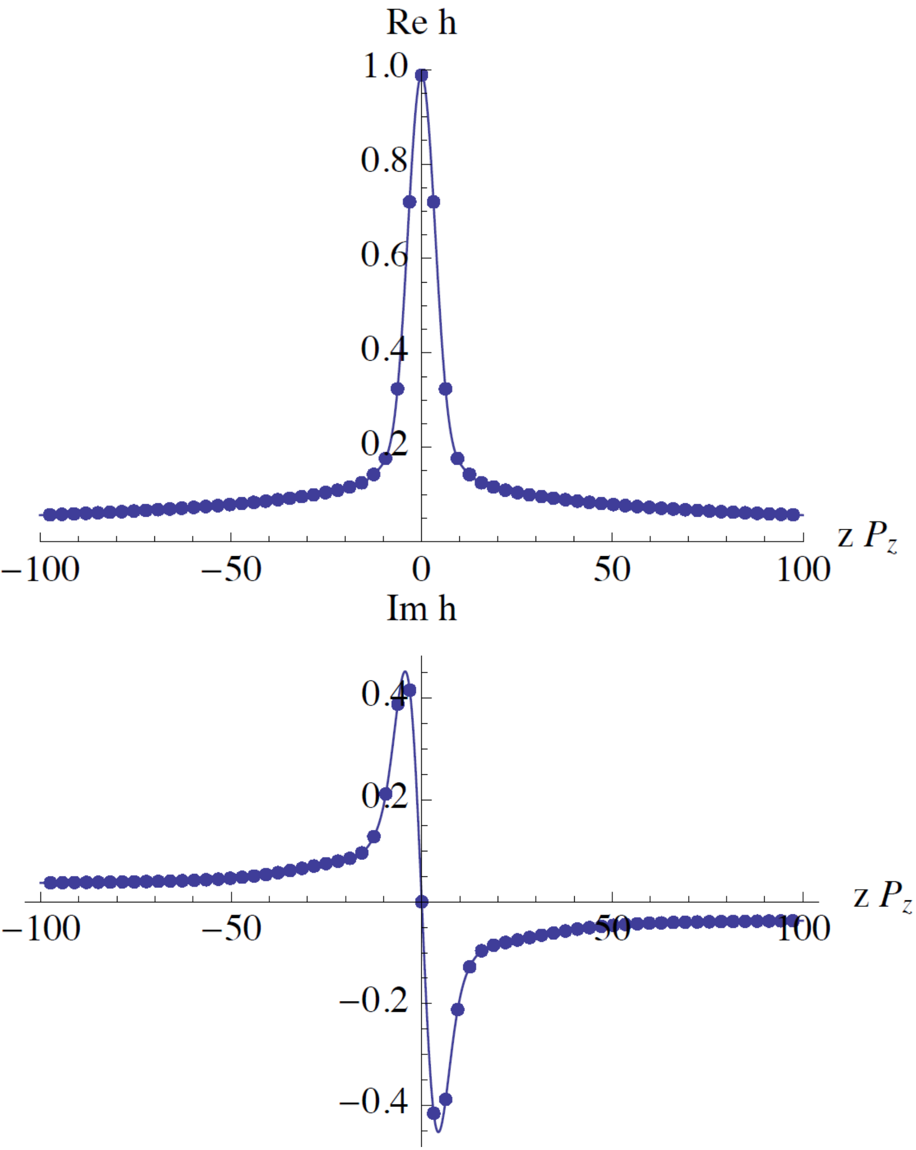
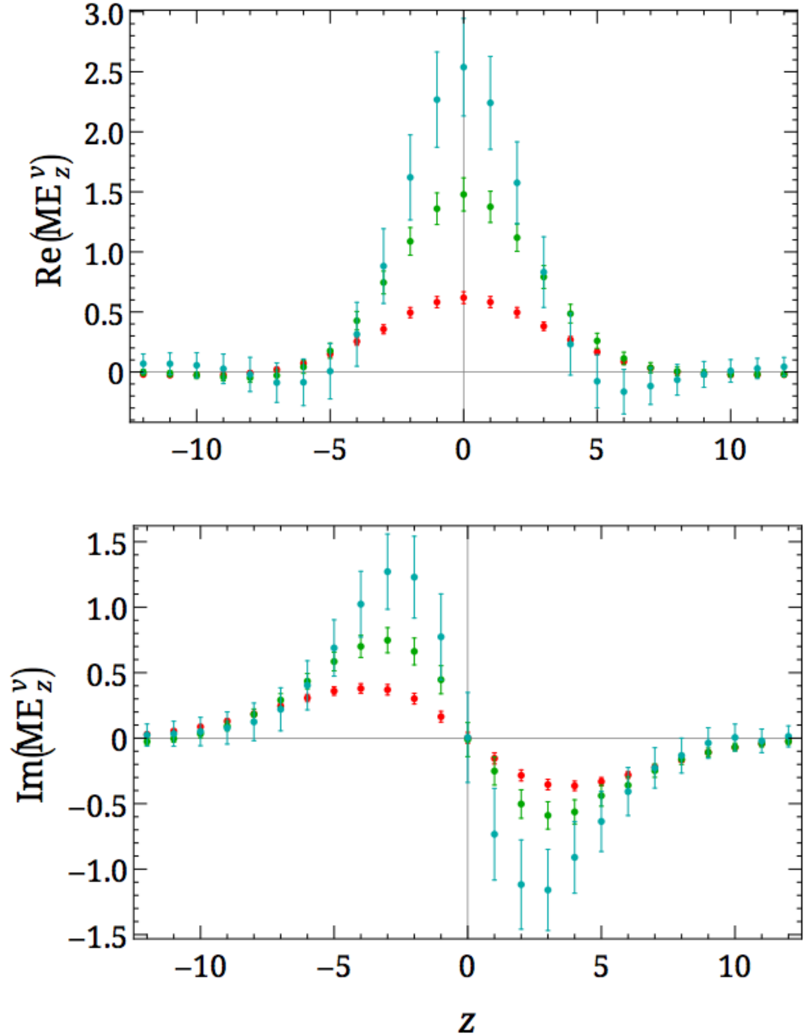


All orders (Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida)



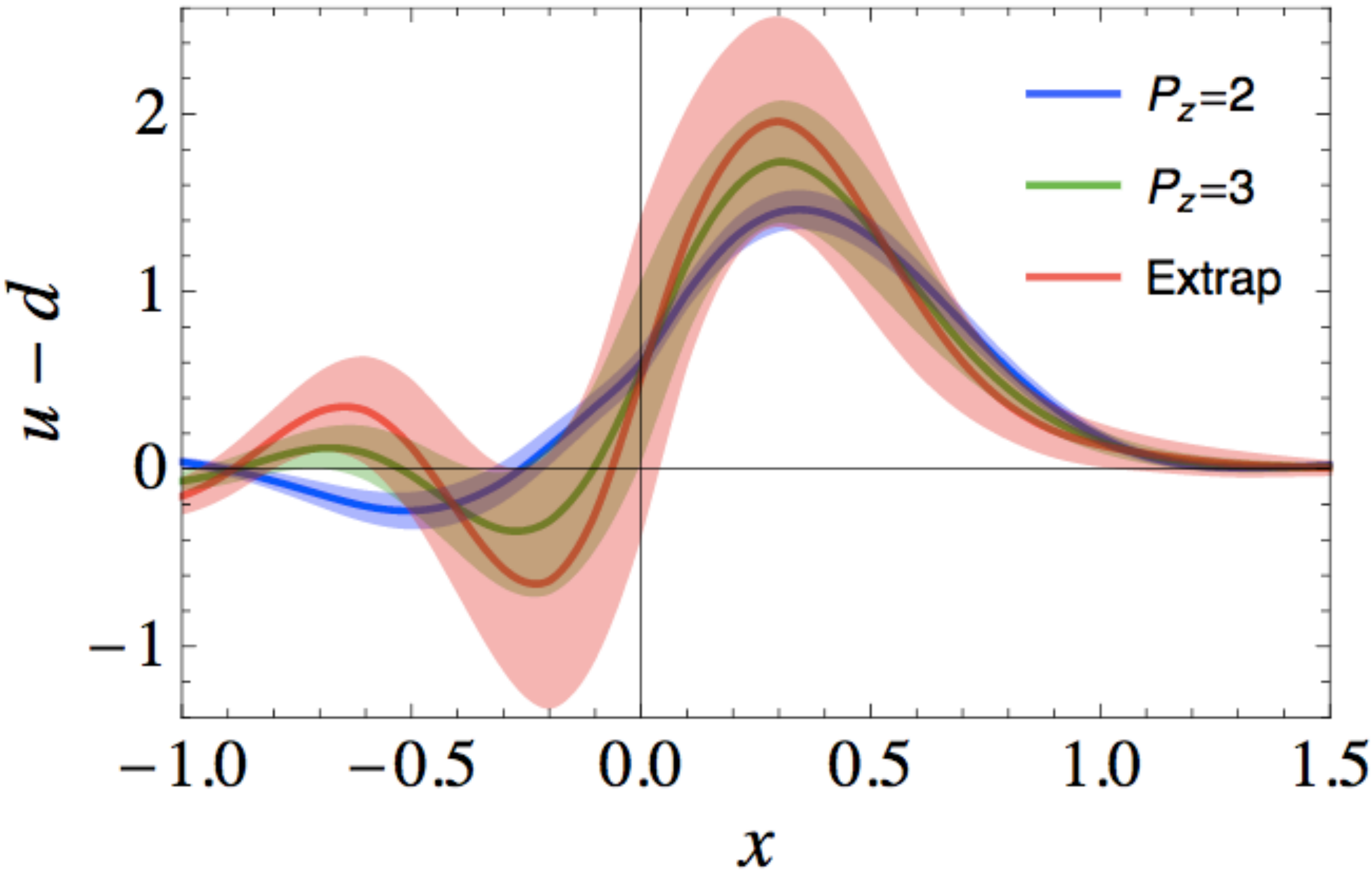
$$\tilde{q}_{\rm imp}(x, \Lambda, p^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z - \delta m|z|} \langle p|\overline{\psi}(0, 0_\perp, z)\gamma^z L(z, 0)\psi(0)|p\rangle$$





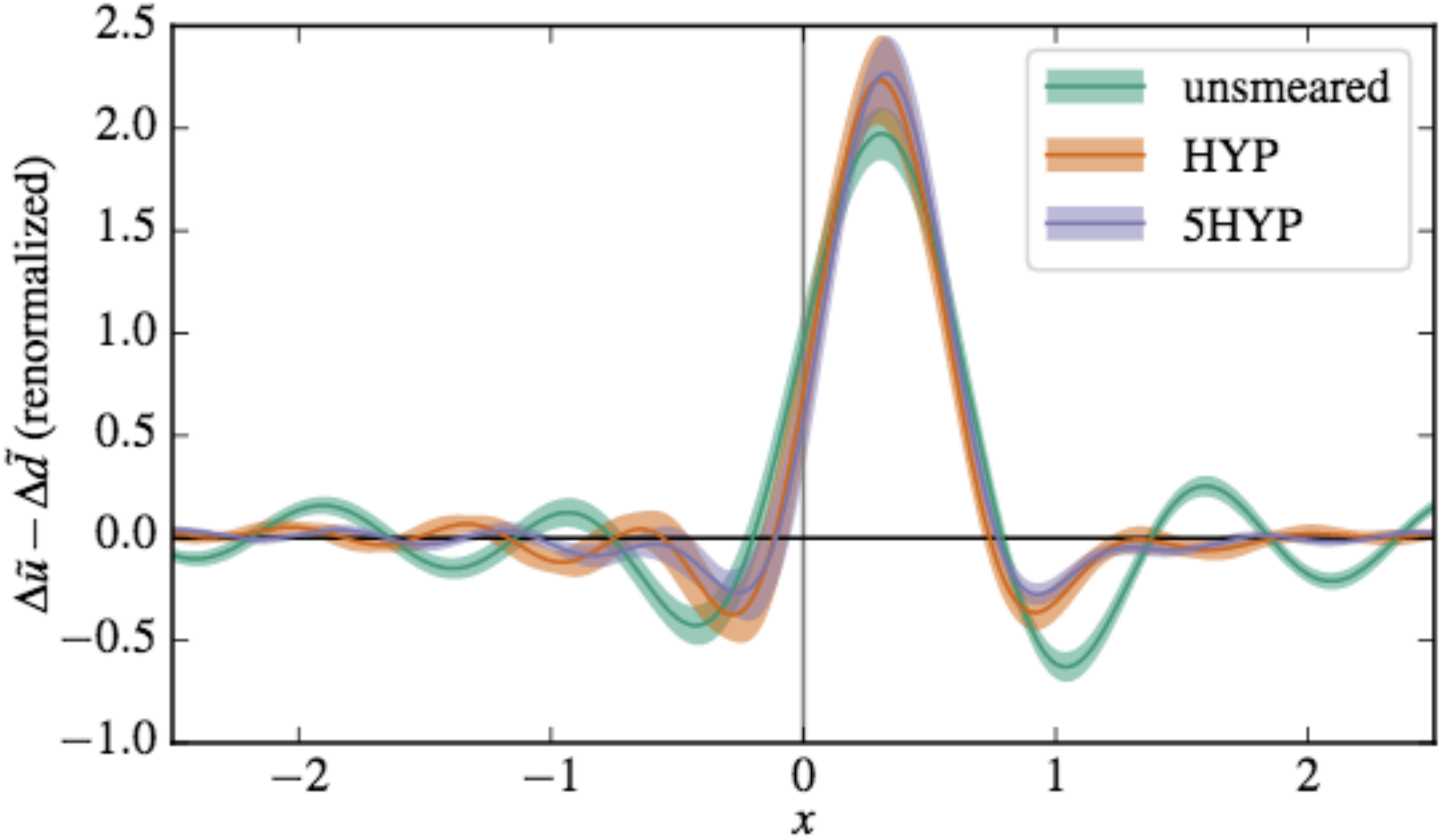
$$P^z = \frac{0.26n}{a},\ n = 1, 2, 3$$

NPR(RI/MOM)+1loop matching



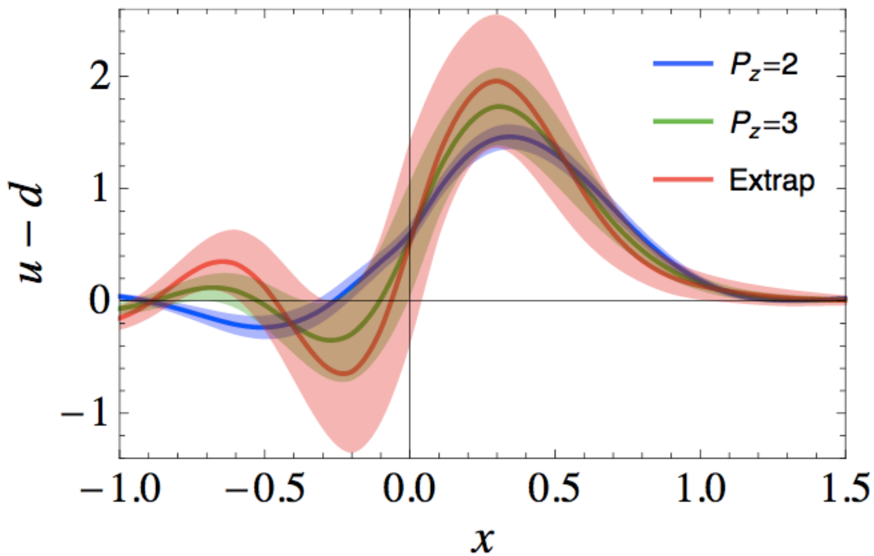
LP3, 1706.01295

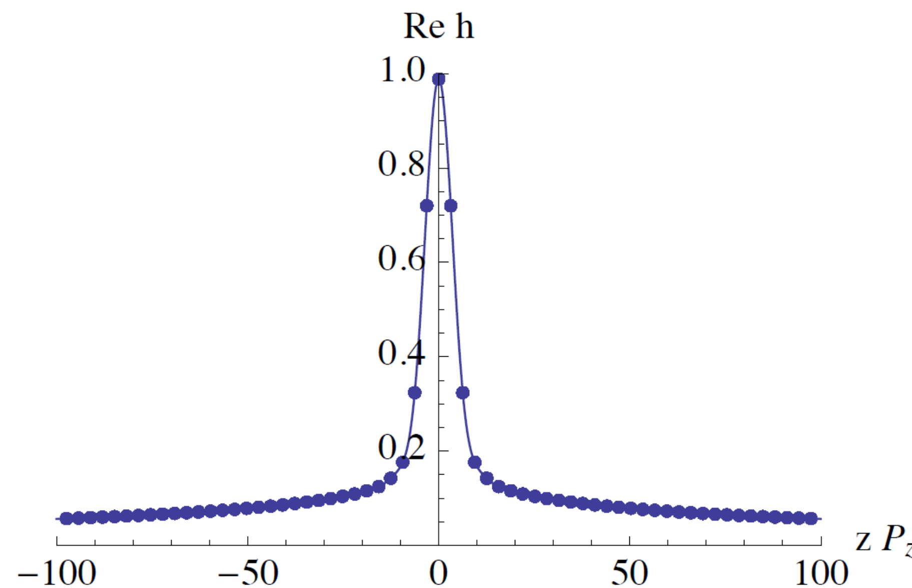
NPR w/o Pz corrections

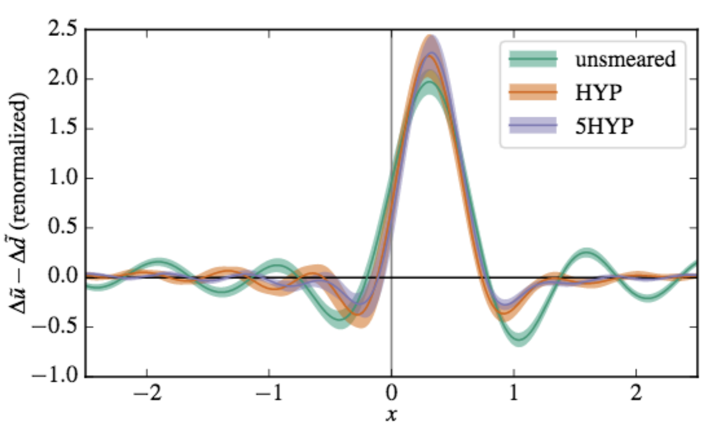


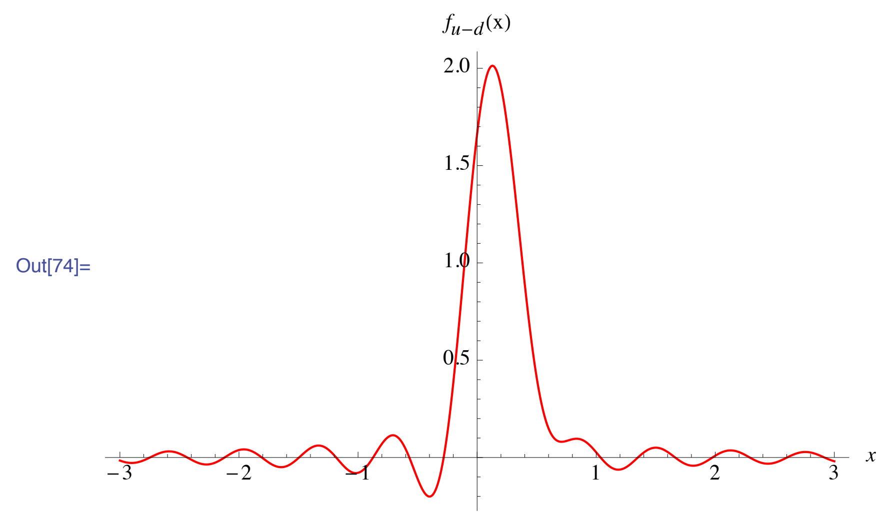
Green, Jansen, Steffens, 1707.07152

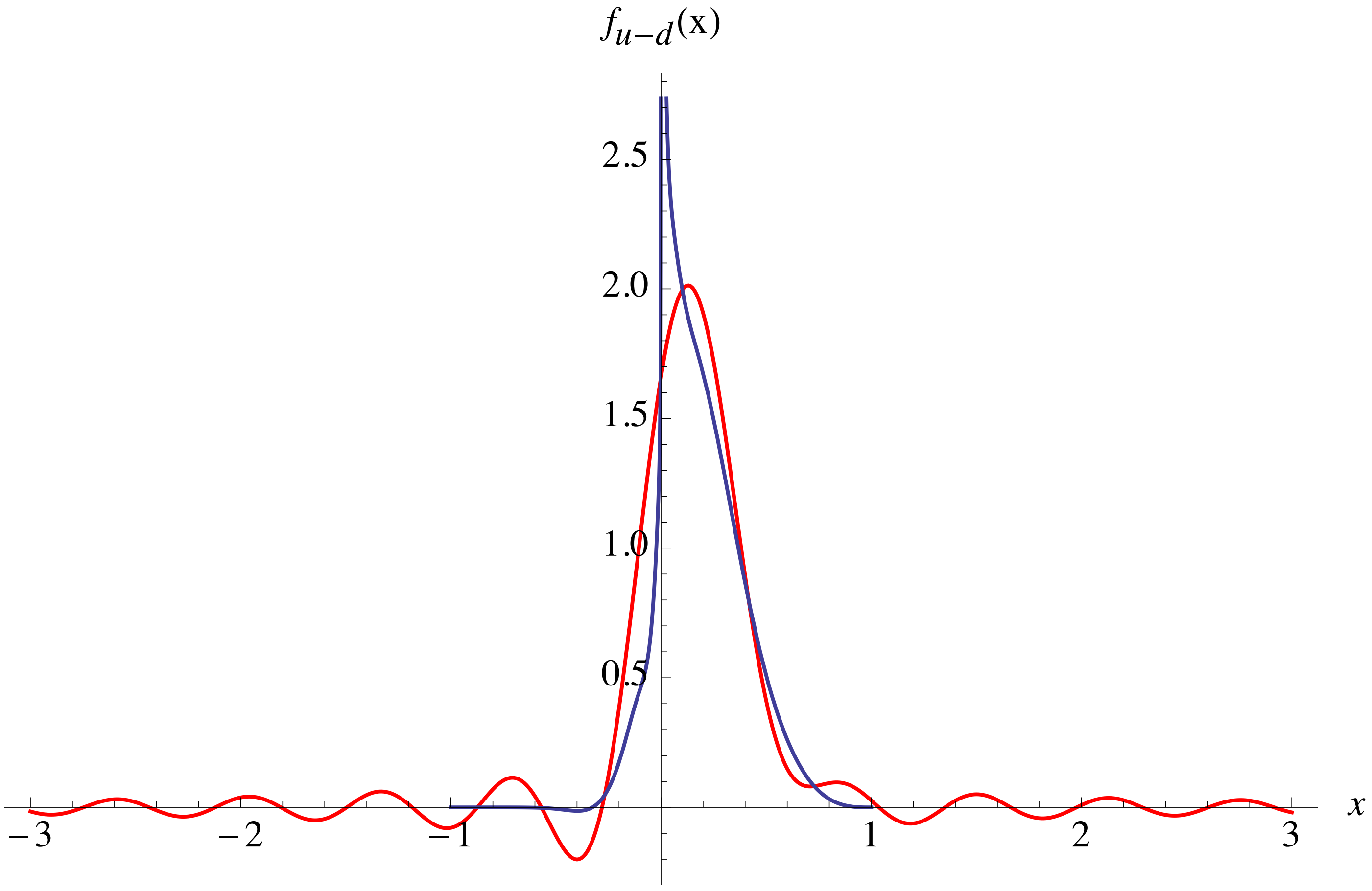
Importance of the Long Tail





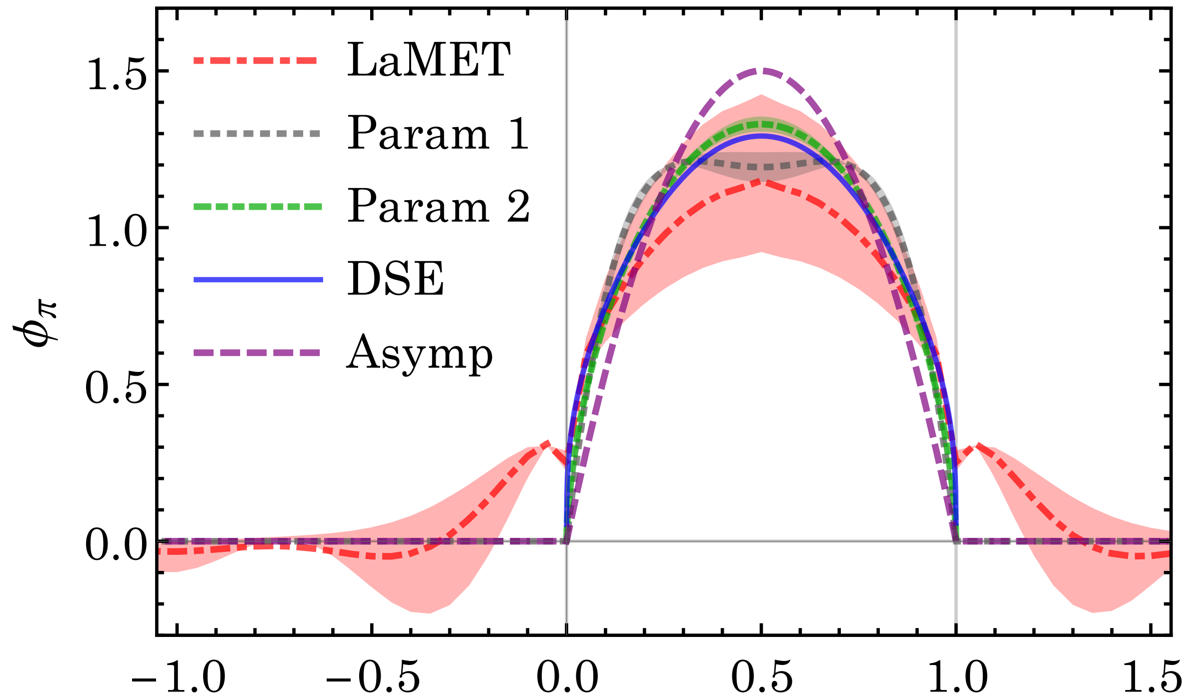






1708.05301(LP3) attempts to address the long tail issue

Pion Light Cone DA-Zhang, JWC, Ji, Jin, Lin



$$\delta m \simeq -260 \pm 200 \text{ MeV}$$

$$\phi_\pi(x,\mu) + 3\phi_\eta(x,\mu) = 2[\phi_{K^+}(x,\mu) + \phi_{K^-}(x,\mu)] = 2[\phi_{K^0}(x,\mu) + \phi_{\overline{K}^0}(x,\mu)],$$

No leading chiral log

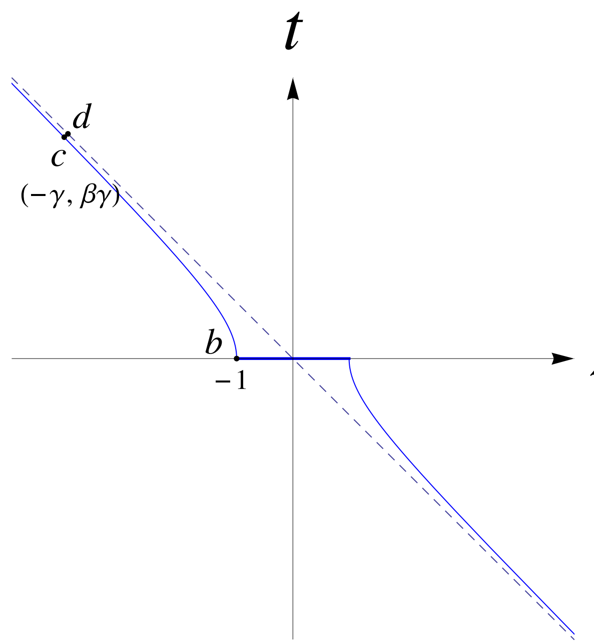
JWC, Iain W. Stewart, Phys.Rev.Lett. 92 (2004) 202001

Comments on Radyushkin's Pseudo-PDF

$$h(z, p_z) = \bar{h}(\nu = zp_z, z^2)$$

$$\mathcal{P}(x, \Lambda, z^2) = \int \frac{d\nu}{2\pi} e^{-ix\nu} \bar{h}(\nu, z^2)$$

$$x = [-1, 1]$$



$$\bar{h}(\nu, z^2)/\bar{h}(0, z^2)$$

Little z dependence

(Orginos, Radyushkin, Karpie, Zafeiropoulos)

Similar but could be complementary to quasi-PDF. Watch out the long tail.

Outlook

- Further tests (non-singlet): long tail (L Pz large by taking Pz large? small x: large Nz); wee partons (smaller quark mass); factorization proof.

  Know whether it works within 5 years (~20%)?

- Singlet PDF's: s, c, b and gluons

  Additional 3-5 yrs?

- If it works, complimentary to exp.: PDF (isov. sea, small and large x's, non-valence partons), DA, GPD, TMD …

Backup slides

Improved Quasi-PDF's

Ishikawa, Ma, Qiu, Yoshida: x-space

JWC, Ji, Zhang: p-space

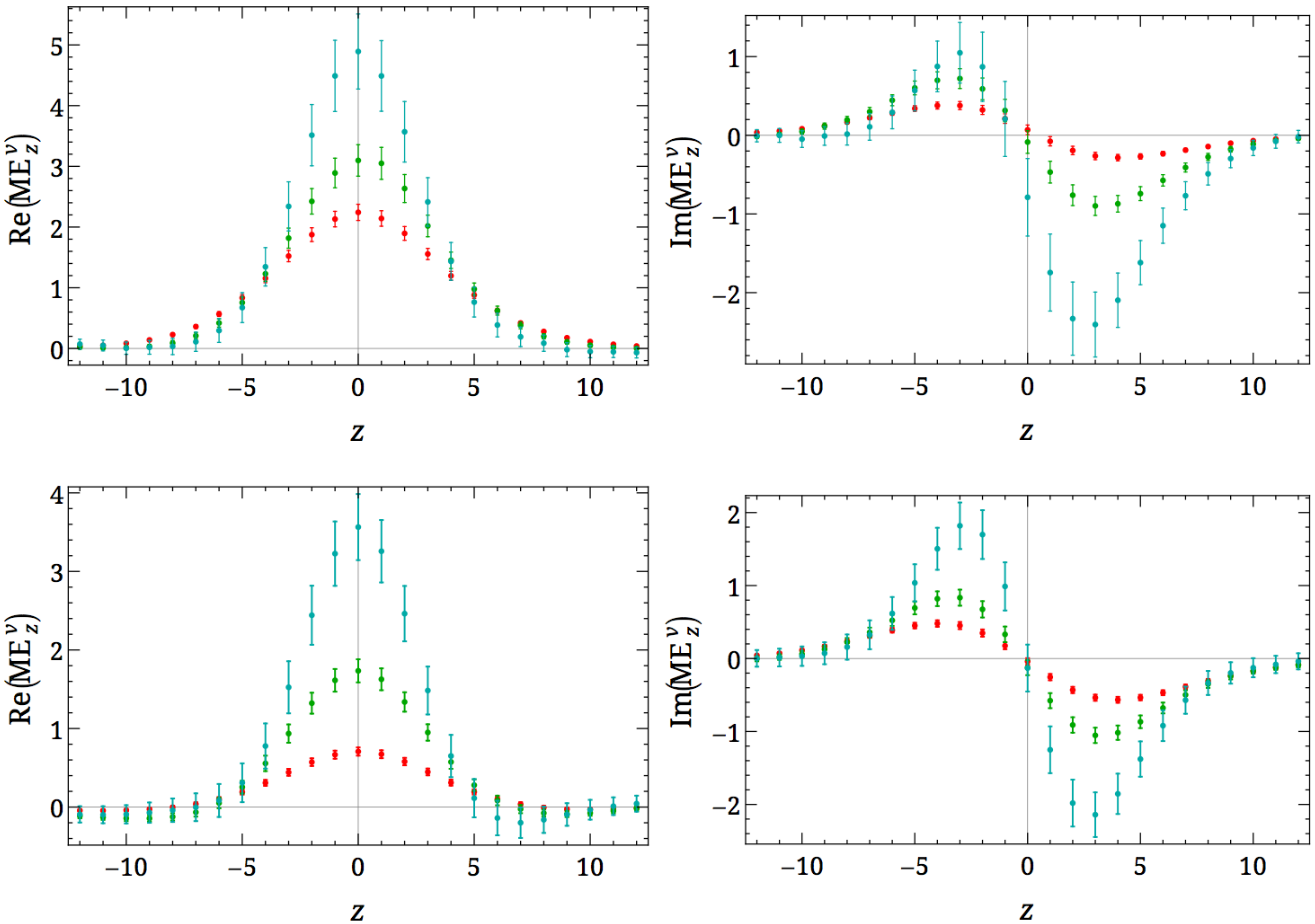
$$\tilde{q}_{\rm imp}(x, a_L, p^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, p^z a_L, \frac{\mu}{p^z}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/(p^z)^2, M^2/(p^z)^2\right)$$

$$Z(\xi) = \delta(\xi - 1) + \frac{\alpha_s}{2\pi}\left[Z^{(1)}(\xi) - \int dy Z^{(1)}(y)\,\delta(\xi - 1)\right] + \ldots$$

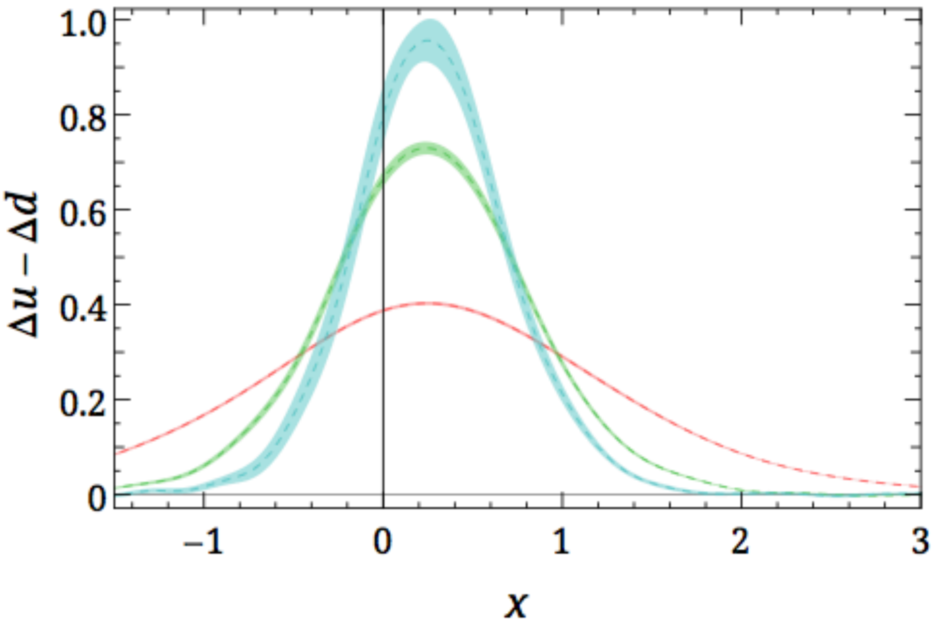
$$Z^{(1)}/C_F = \begin{cases} \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1\,, & \xi > 1\,, \\ \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(p^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln\left[4\xi(1-\xi)\right] - \frac{2\xi}{1-\xi} + 1\,, & 0 < \xi < 1\,, \\ \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi} - 1\,, & \xi < 0\,, \end{cases}$$

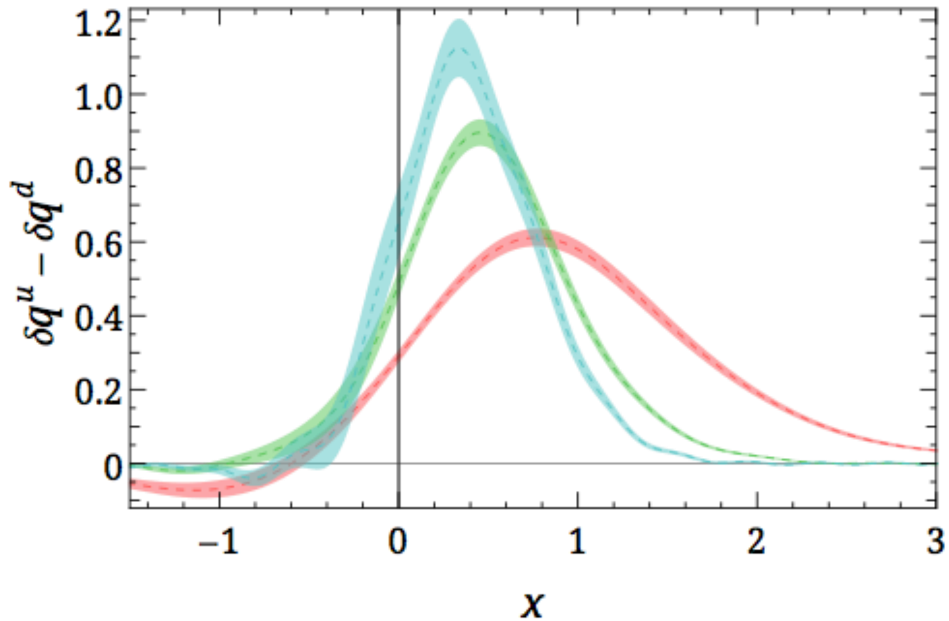
Stewart & Zhang: NP RI/MOM renorm.
+ one-loop RI/MOM MS-bar matching

Helicity and Transversity (isovector)



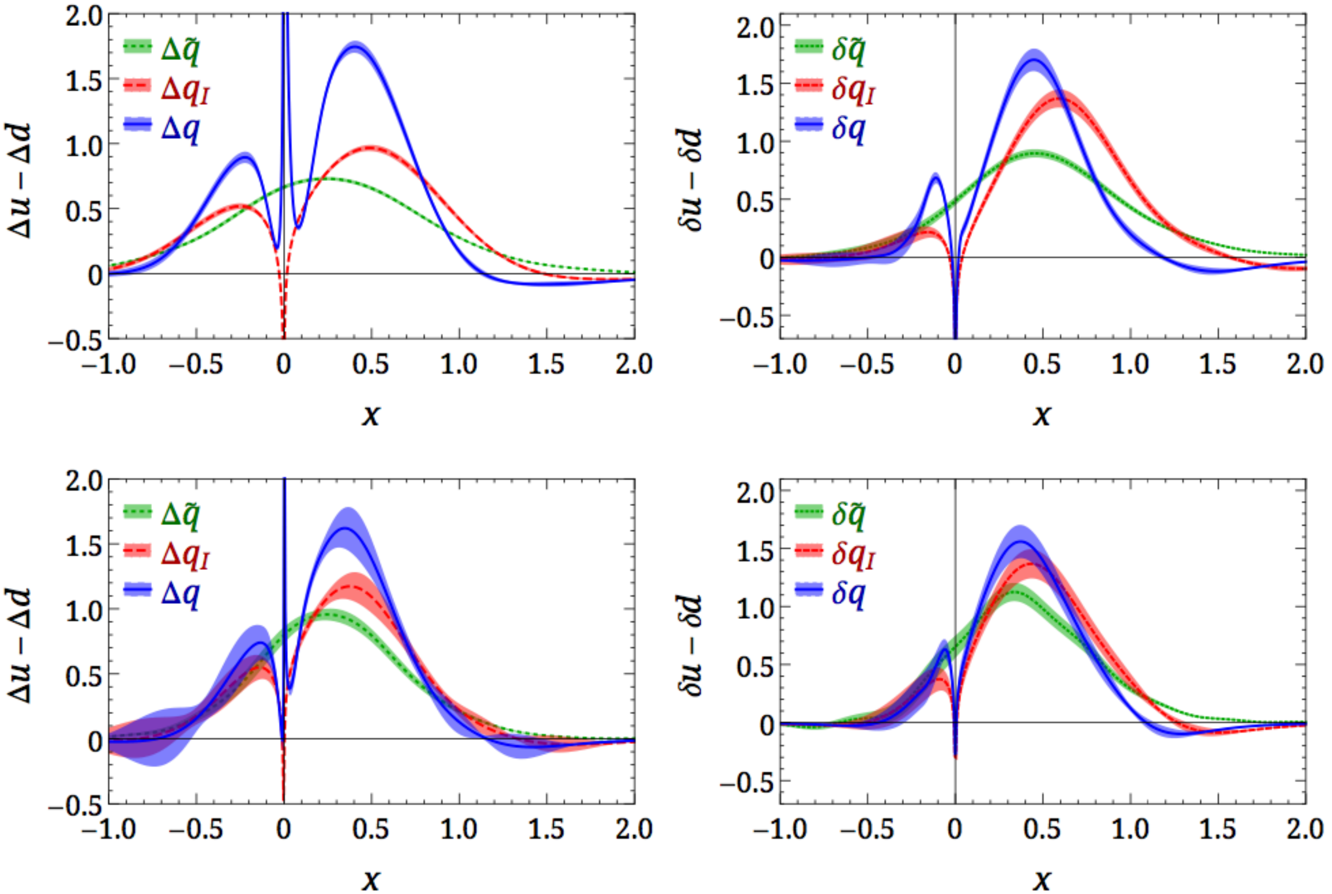
Quasi-PDF (Helicity and Transversity)





$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$ n = 1, 2, 3.

Quasi-PDF (green) w/ loop (red) w/ loop + mass (blue)



$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$

n = 2 (upper) & 3