

Axial structure of light nuclei from lattice QCD



Silas Beane
U. Washington



Emmanuel Chang
U. Washington



Zohreh Davoudi
MIT



Will Detmold
MIT



Kostas Orginos
William & Mary



Assumpta Parreno
Barcelona



Martin Savage
U. Washington



Phiala Shanahan
MIT



Brian Tiburzi
CCNY/RBC



Mike Wagman
U. Washington

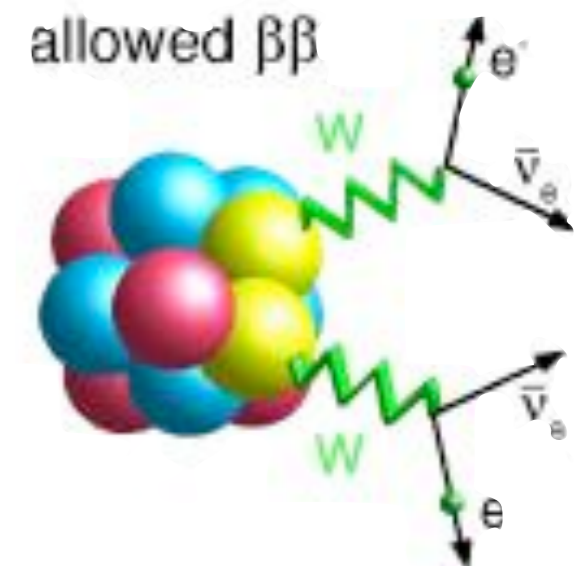


Frank Winter
Jefferson Lab

Phiala Shanahan, MIT

The intensity frontier

- Seek new physics through quantum effects
- Precise experiments
 - Sensitivity to probe the rarest interactions of the SM
 - Search for effects where there is no SM contribution
- Important focus of experimental programs
 - Dark matter direct detection
 - Neutrino physics
 - Charged lepton flavour violation, $\beta\beta$ -decay, proton decay, neutron-antineutron oscillations...
- **Major component is nuclear targets**



Outline

Weak nuclear processes

1. Matrix element determining
 $pp \rightarrow de^+ \nu$
fusion cross-section
2. Gamow-Teller matrix
element in tritium
3. Two-neutrino double-beta
decay matrix element

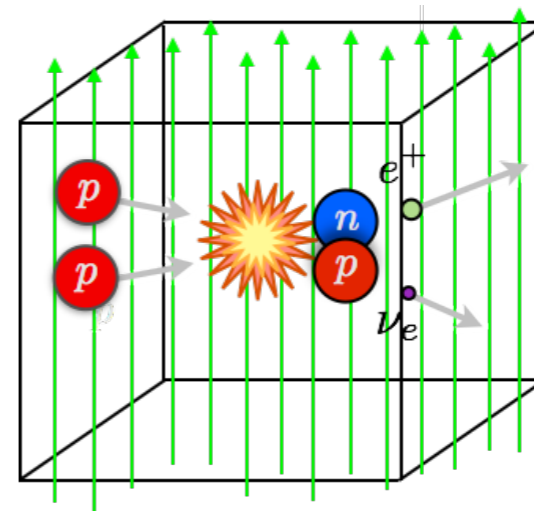
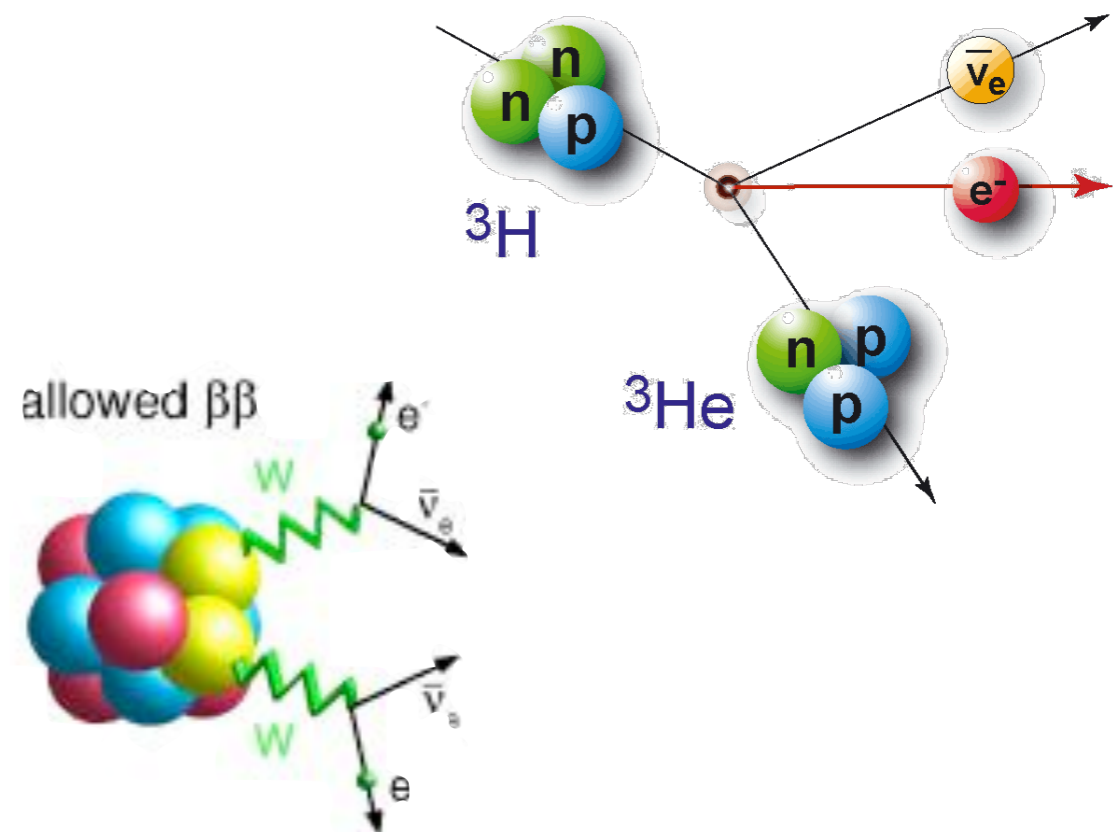


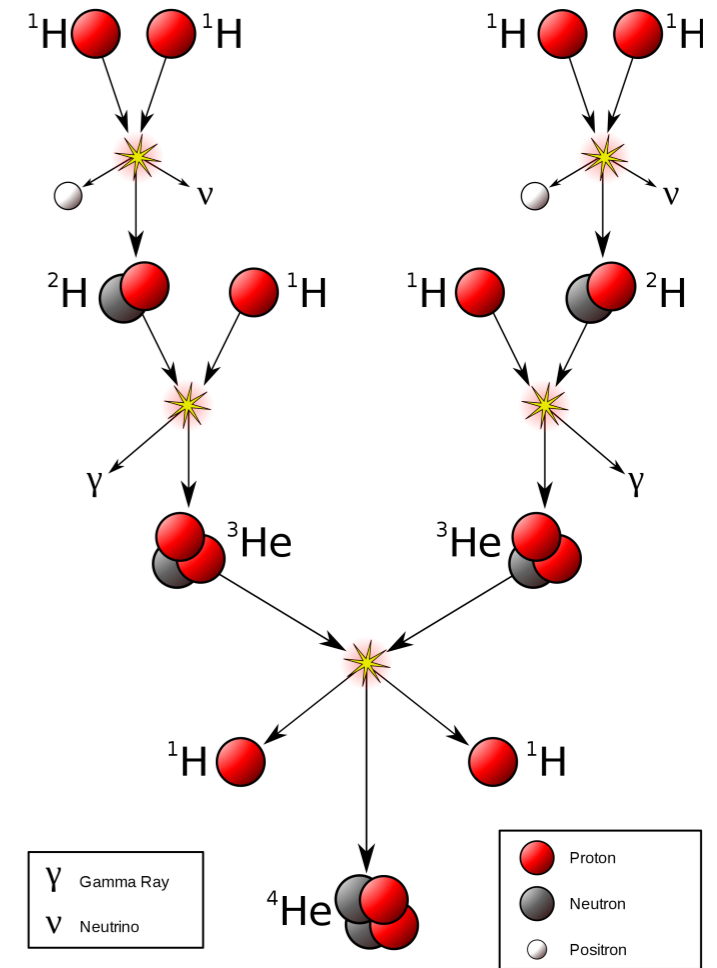
Fig: Z Davoudi



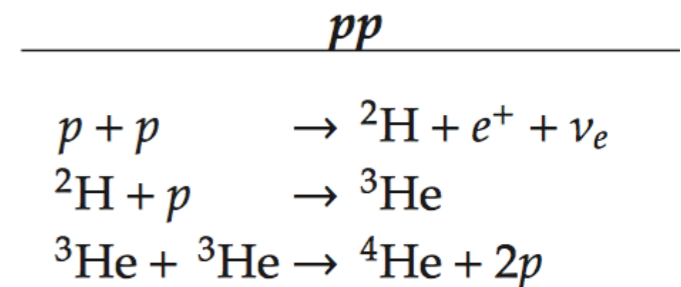
Proton-proton fusion

- Stars emit heat/light from conversion of H to He
- Sun + cooler stars: proton-proton fusion chain reaction

We calculate $\langle d; 3 | A_3^3 | pp \rangle$
 $\rightarrow L_{1,A}, \ell_{1,A}, \bar{L}_{1,A}, \dots$
 $pp \rightarrow de^+ \nu$ cross-section

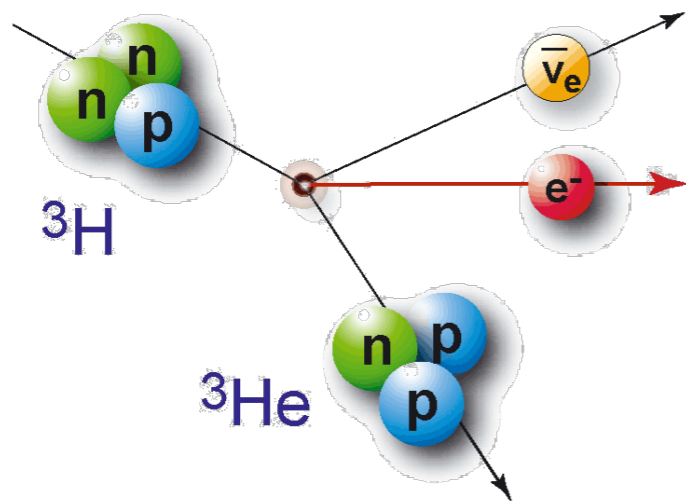


- Related to:
 - Neutrino breakup reaction (SNO)
 - Muon capture reaction (MuSun)



Tritium β -decay

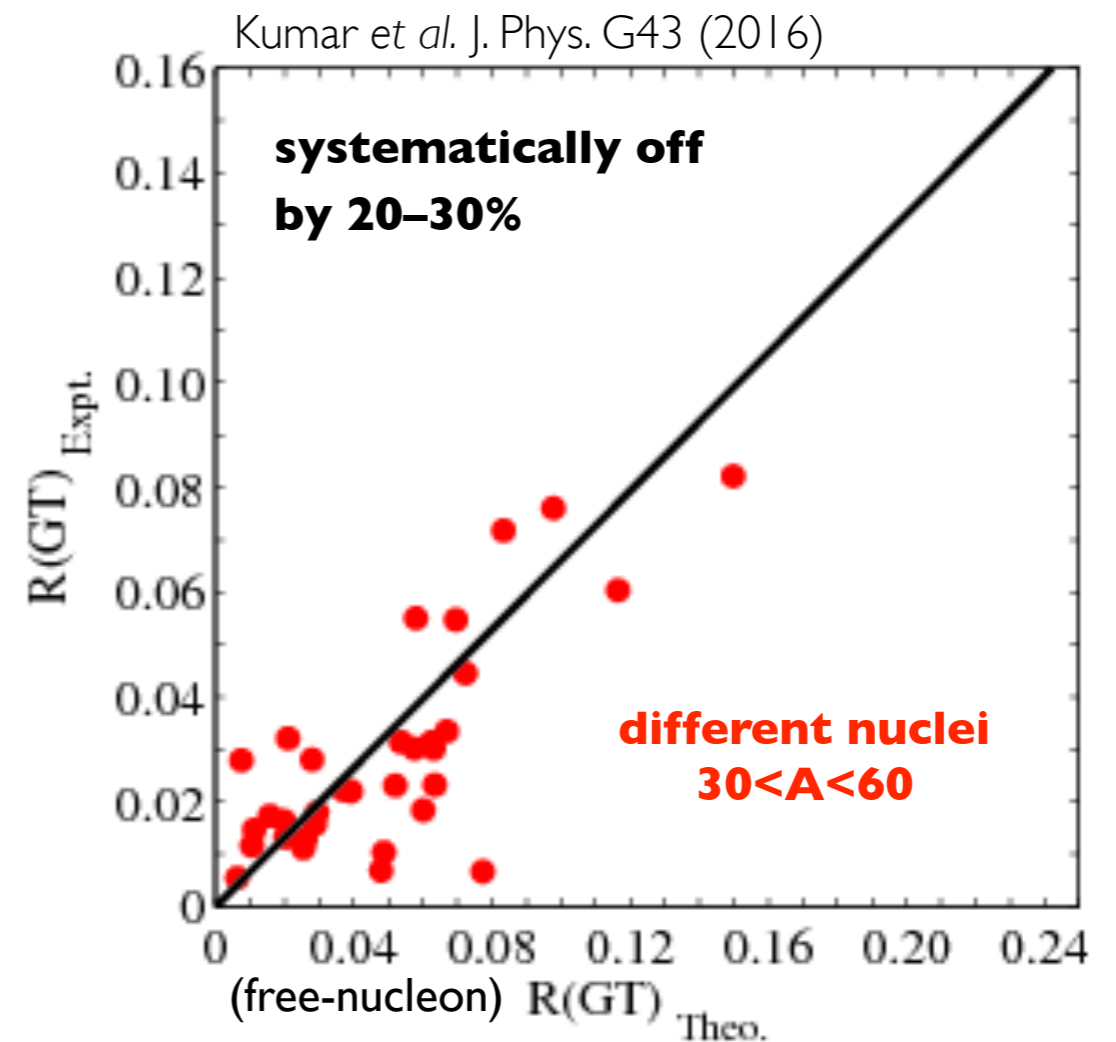
- Simplest semileptonic weak decay of a nuclear system



- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- Understand multi-body contributions to $\langle \mathbf{GT} \rangle \rightarrow$ better predictions for decay rates of larger nuclei

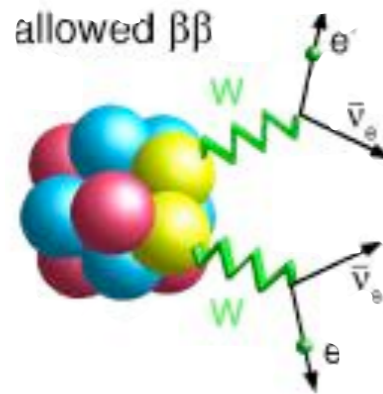
We calculate

$$g_A \langle \mathbf{GT} \rangle = \langle {}^3\text{He} | \bar{q} \gamma_{\mathbf{k}} \gamma_5 \tau^- q | {}^3\text{H} \rangle$$



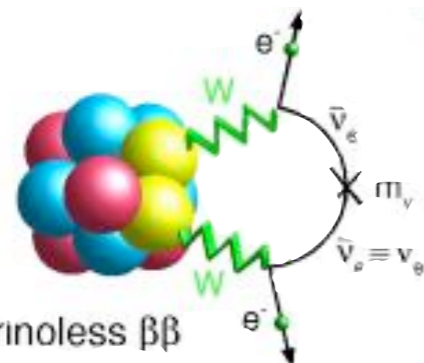
Double β -decay

- Certain nuclei allow observable $\beta\beta$ decay

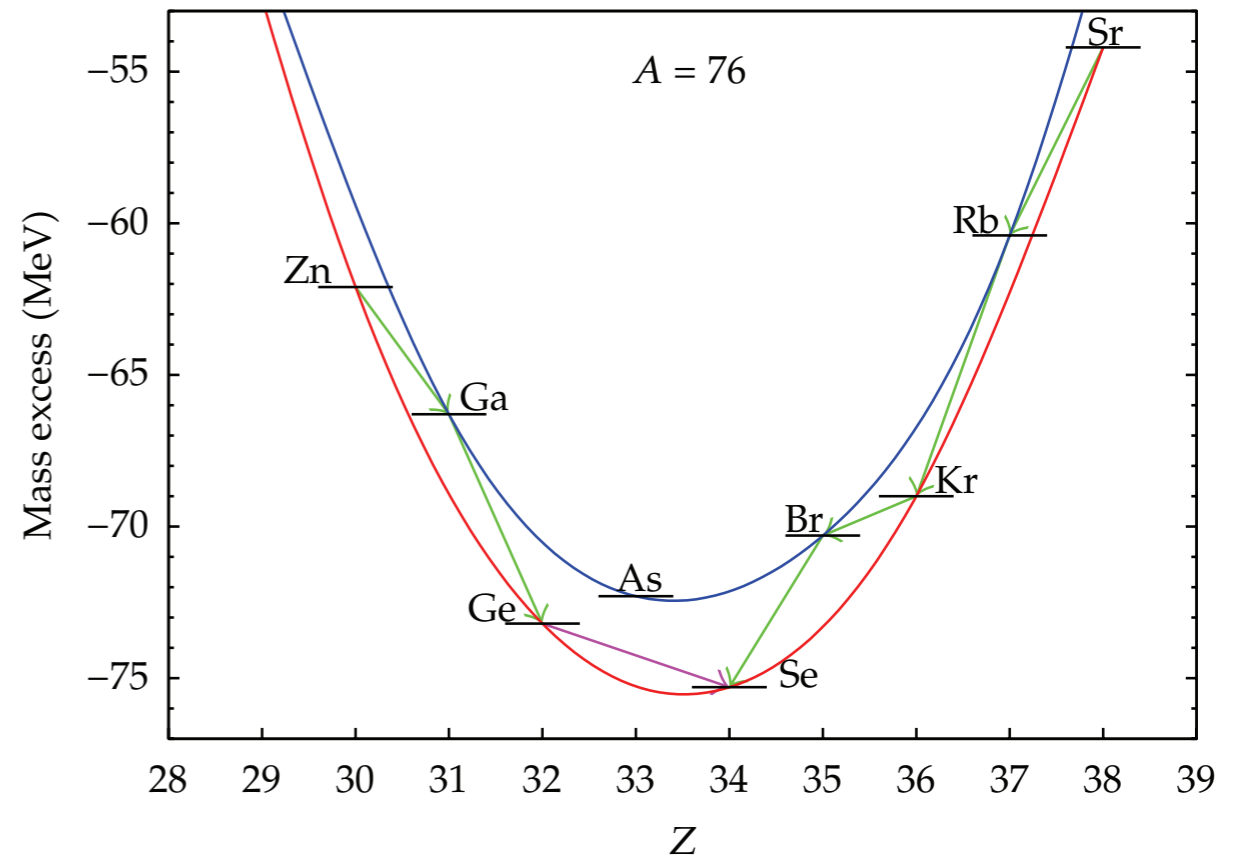


$$T_{1/2}^{2\nu\beta\beta} \gtrsim 10^{19} \text{ y}$$

- If neutrinos are massive Majorana fermions $0\nu\beta\beta$ decay is possible



$$T_{1/2}^{0\nu\beta\beta} > 10^{25} \text{ y}$$



We calculate two-current nuclear matrix elements
 → dictate half-life



Unphysical nuclei

NPLQCD collaboration

- QCD with unphysical quark masses

$m_\pi \sim 800$ MeV, $m_N \sim 1,600$ MeV

$m_\pi \sim 450$ MeV, $m_N \sim 1,200$ MeV

- Spectrum of light nuclei ($A < 5$)

[PRD **87** (2013), 034506]

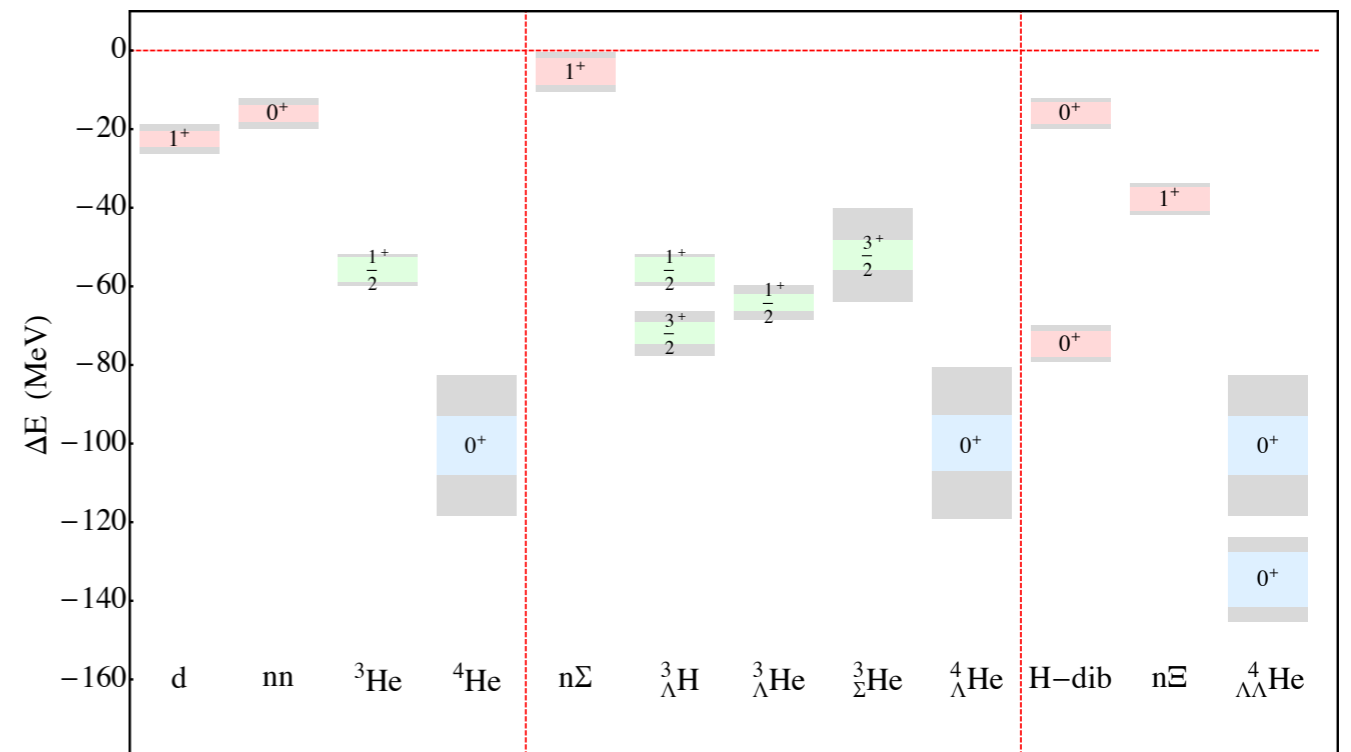
- Nuclear structure: magnetic moments, polarisabilities ($A < 5$)

[PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]

- First nuclear reaction: $np \rightarrow d\gamma$

[PRL **115**, 132001 (2015)]

- Proton-proton fusion and tritium β -decay
 - Double β -decay
- $m_\pi \sim 800$ MeV, $m_N \sim 1,600$ MeV



Background field method

Hadron/nuclear energies are modified by presence of fixed/
constant external fields

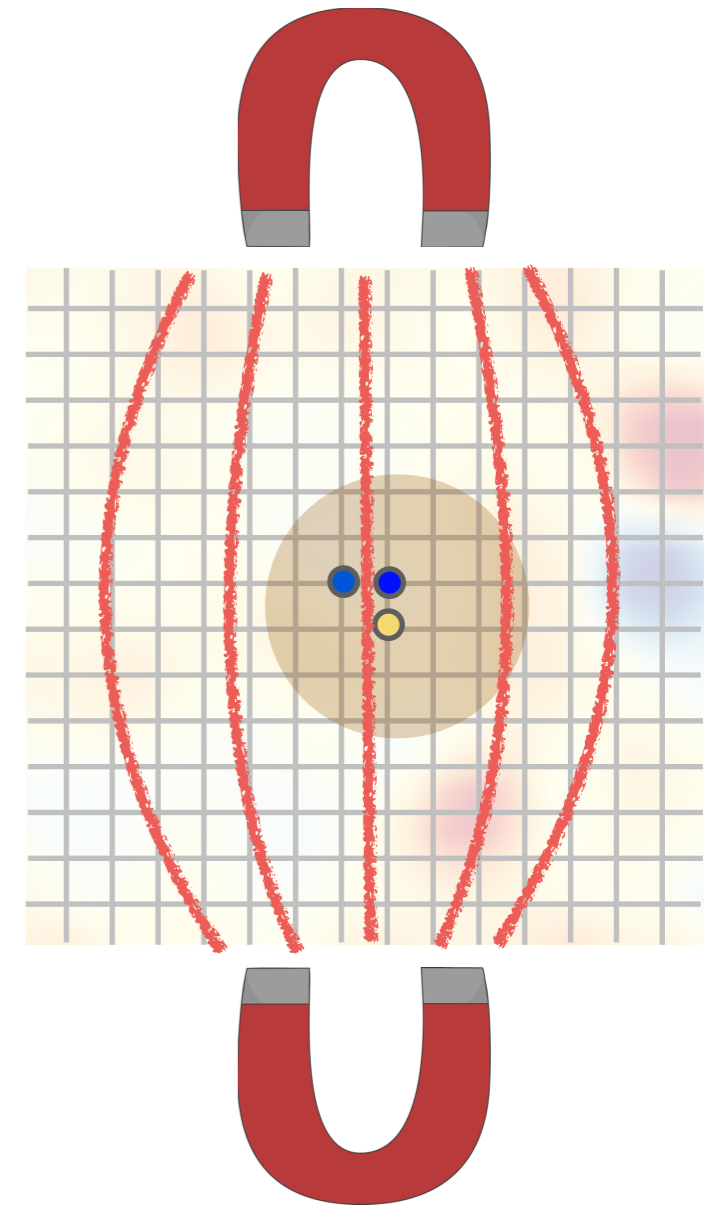
Example: fixed magnetic field

$$E(\vec{B}) = \sqrt{M^2 + (2n + 1)|Qe\vec{B}|} - \vec{\mu} \cdot \vec{B} \\ - 2\pi\beta_{M0}|\vec{B}|^2 - 2\pi\beta_{M2}T_{ij}B_iB_j + \dots$$

landau level mag. mmt

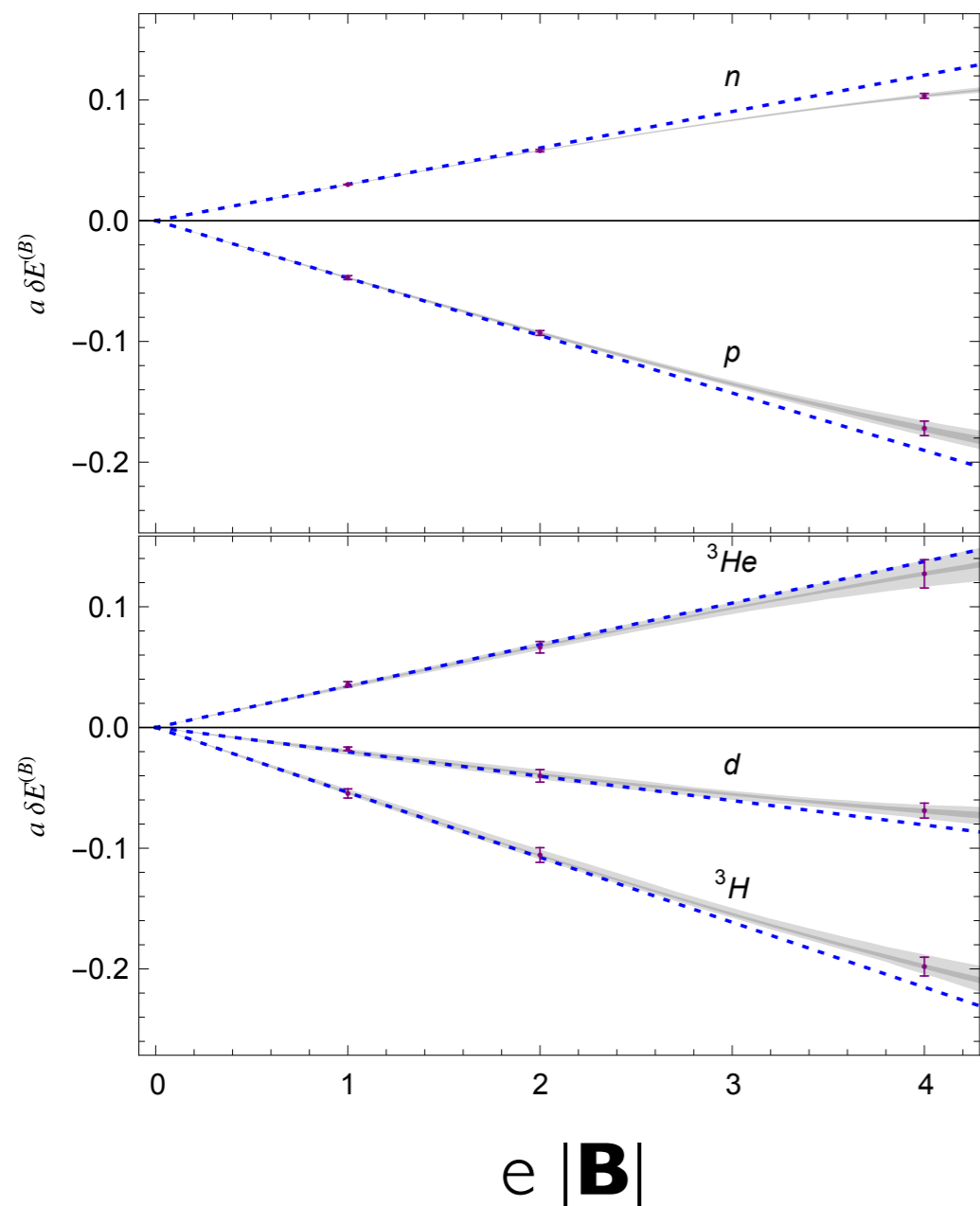
polarisabilities traceless, sym tensor

- Calculations with multiple fields
➔ extract coefficients of response
e.g., magnetic moments, polarisabilities, ...
- Not restricted to simple EM fields
This work: uniform axial background field



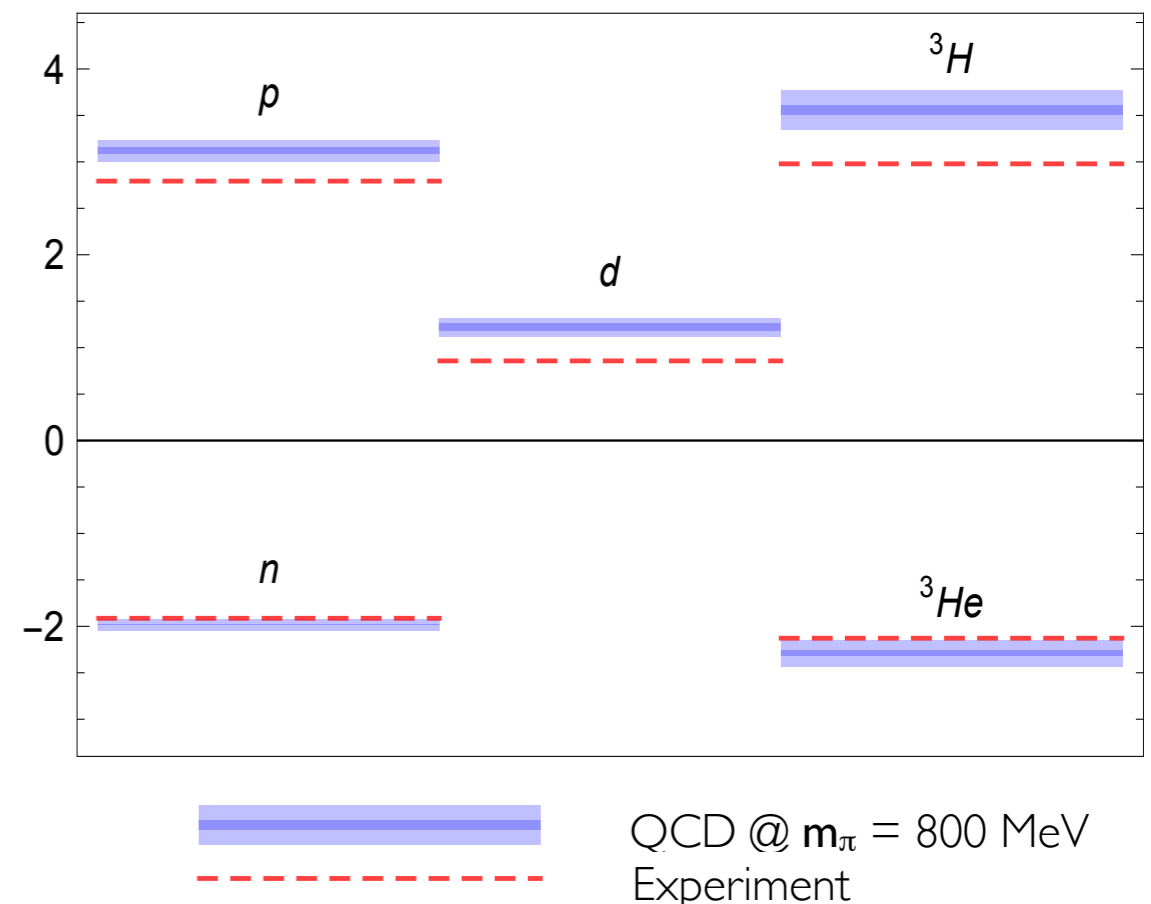
Magnetic moments

Energy shift between ground states
spin-aligned/anti-aligned with \mathbf{B}



linear term

Magnetic moments



Axial background field

Example: fixed magnetic field \rightarrow moments, polarisabilities

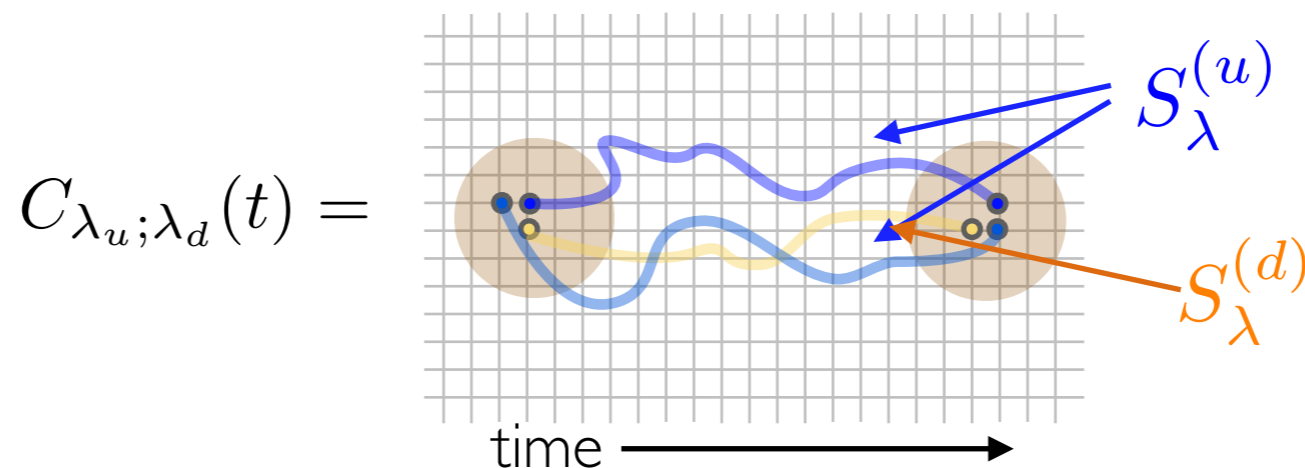
This work: fixed axial background field \rightarrow axial charges, other matrix elts.

Construct correlation functions from propagators modified in axial field

compound propagator

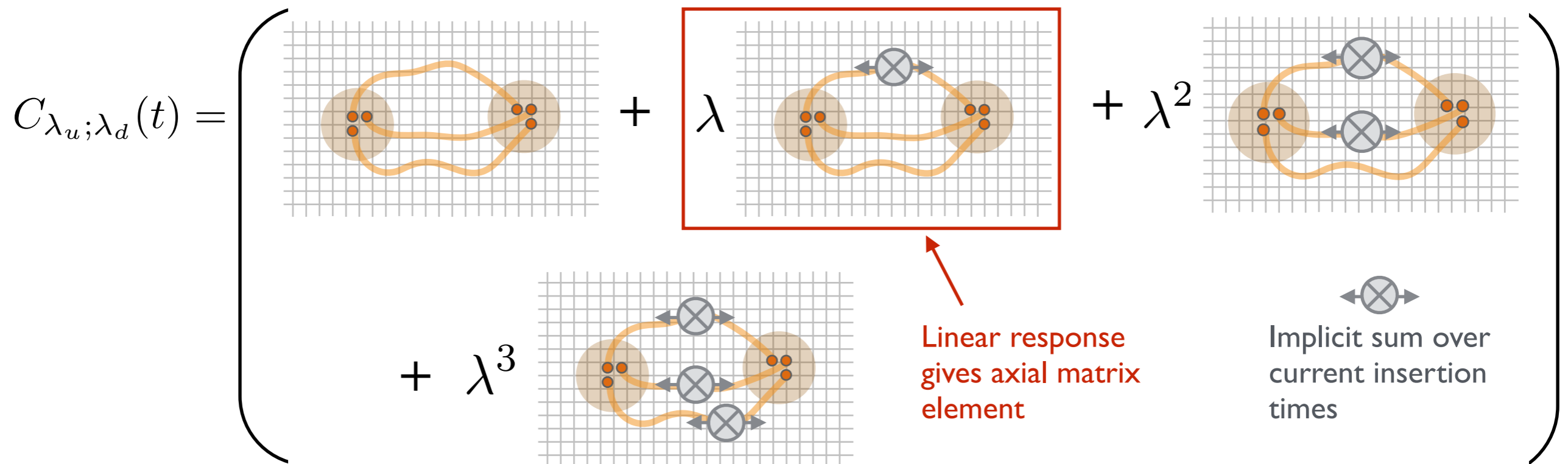
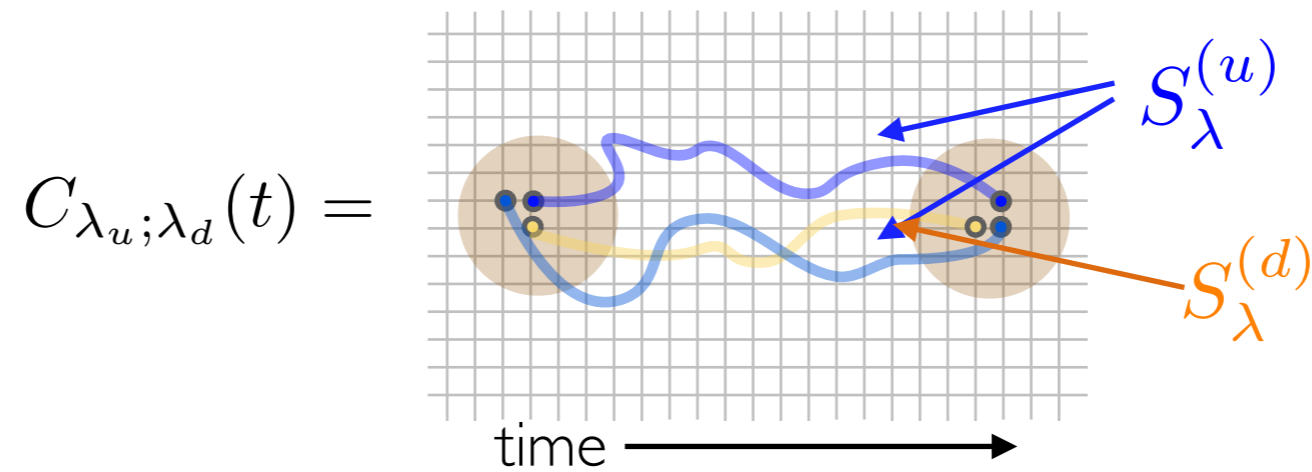
constant

$$S_{\lambda}^{(q)}(x, y) = S^{(q)}(x, y) + \lambda_q \int dz S^{(q)}(x, z) \gamma_3 \gamma_5 S^{(q)}(z, y)$$



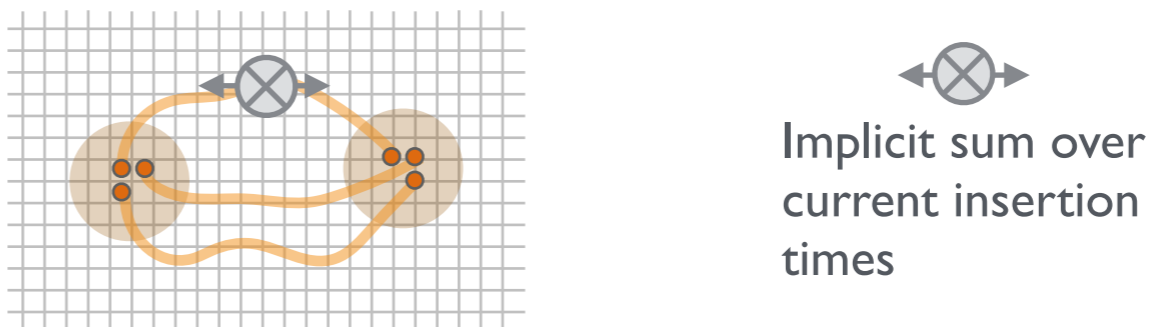
Linear response \longleftrightarrow axial matrix element

Axial background field



Axial background field

Example: determination of the proton axial charge

$$C_{\lambda_u; \lambda_d}(t) \Big|_{\mathcal{O}(\lambda)} =$$


Implicit sum over current insertion times

Time difference isolates matrix element part

$$\begin{aligned}
 C_{\lambda_u; \lambda_d}(t) \Big|_{\mathcal{O}(\lambda)} &= \sum_{\tau=0}^t \langle 0 | \chi^\dagger(t) J(\tau) \chi(0) | 0 \rangle \\
 &= \dots \\
 &= Z_0 e^{-M_p t} \left[C + t \langle p | A_3^{(u)}(0) | p \rangle + \mathcal{O}(e^{-\delta t}) \right]
 \end{aligned}$$

Excited states

Irrelevant constants

Matrix element

$$(C_{\lambda_u; \lambda_d}(t+1) - C_{\lambda_u; \lambda_d}(t)) \Big|_{\mathcal{O}(\lambda)} = Z_0 e^{-M_p t} \langle p | A_3^{(u)}(0) | p \rangle + \mathcal{O}(e^{-\delta t})$$

Proton axial charge

Form ratios to cancel leading time-dependence

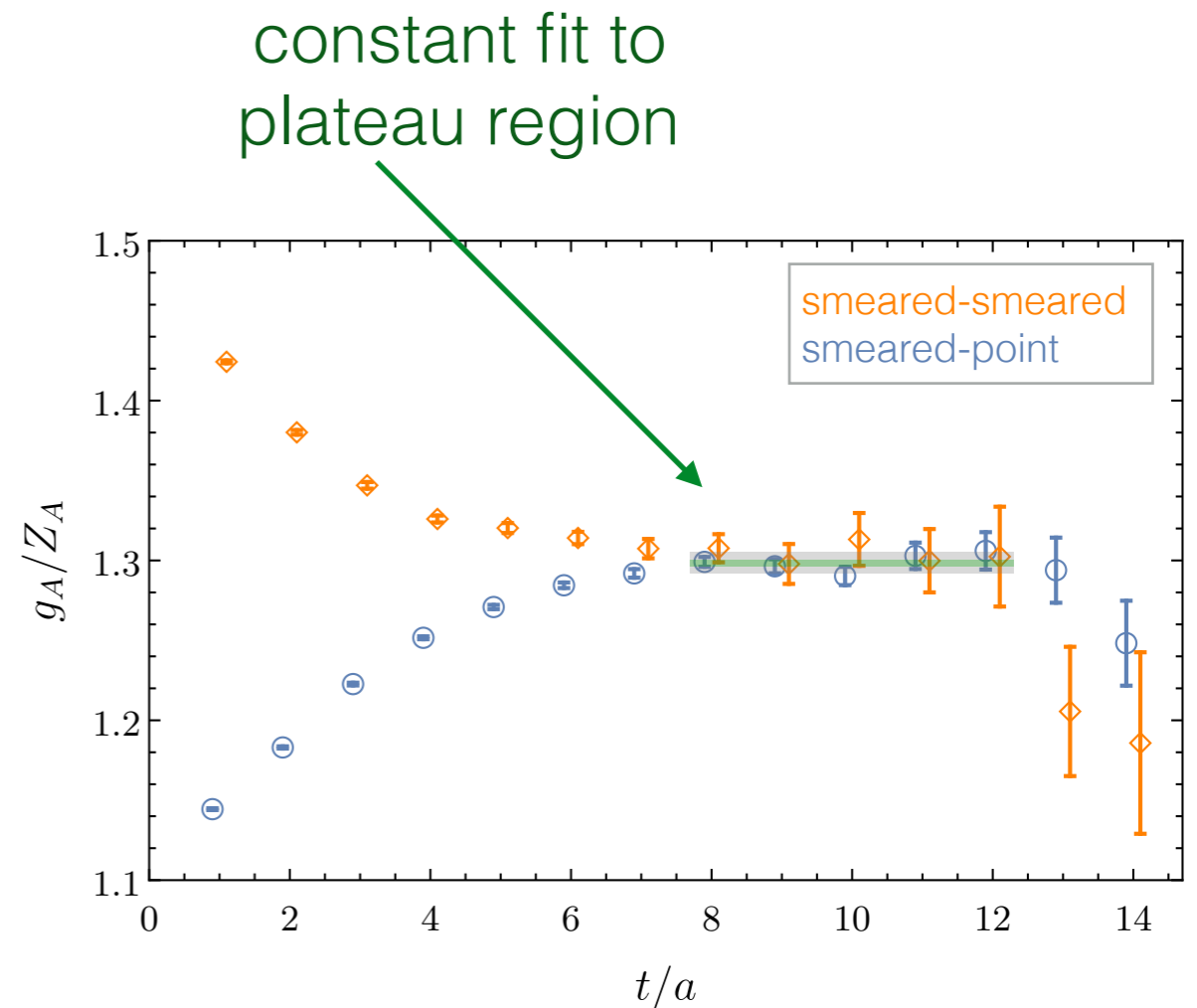
- Extract matrix element through linear response of correlators to the background field
- Form ratios to cancel leading time-dependence

$$R_p(t) = \frac{\left(C_{\lambda_u; \lambda_d=0}^{(p)}(t) - C_{\lambda_u=0; \lambda_d}^{(p)}(t) \right) \Big|_{\mathcal{O}(\lambda)}}{C_{\lambda_u=0; \lambda_d=0}^{(p)}(t)}$$

At late times:

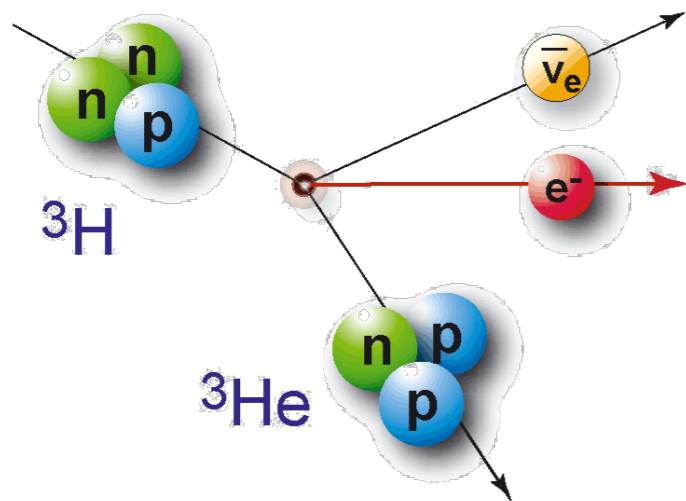
$$R_p(t+1) - R_p(t) \xrightarrow{t \rightarrow \infty} \frac{g_A}{Z_A}$$

- Matrix element revealed through “effective matrix elt. plot”



Tritium β -decay

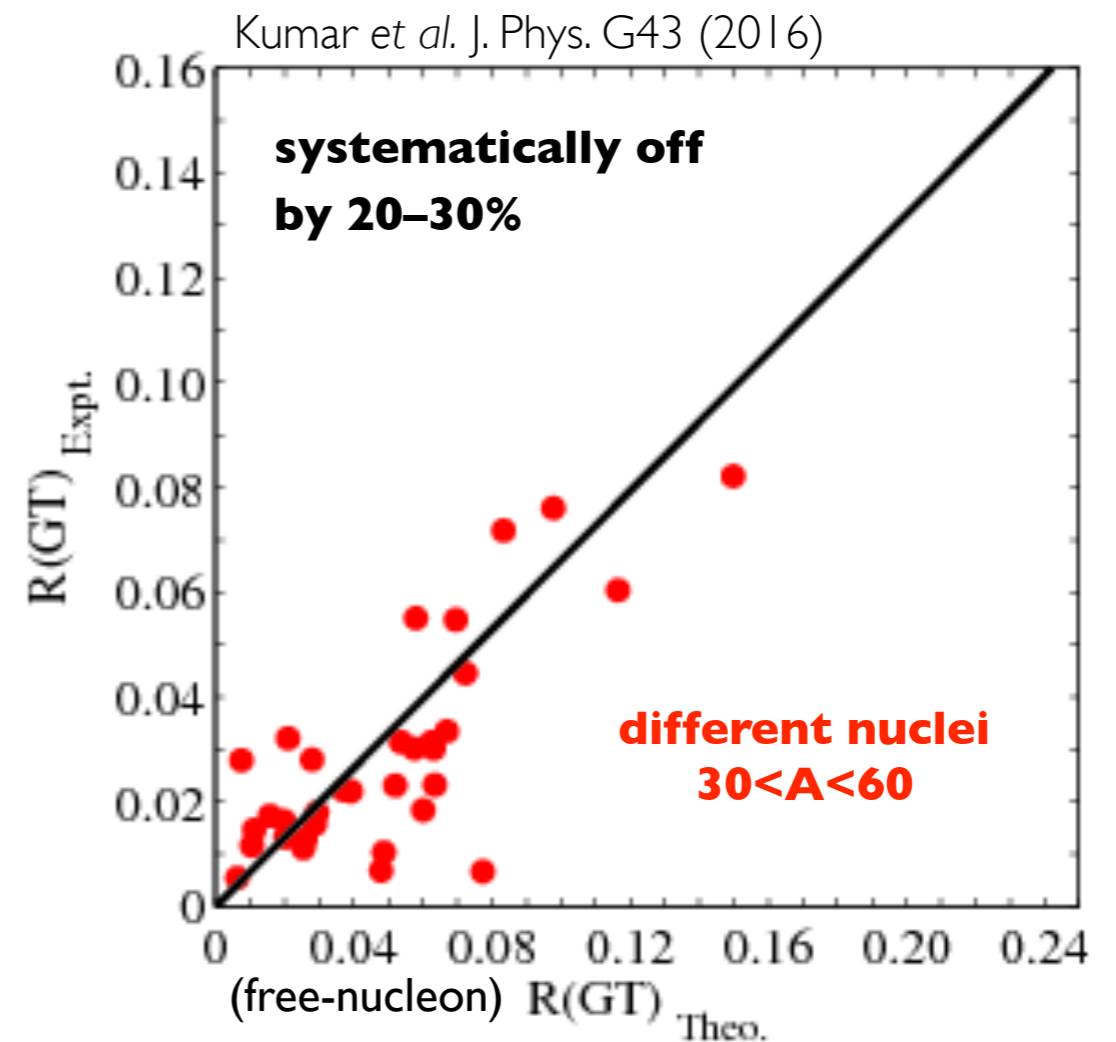
- Simplest semileptonic weak decay of a nuclear system



- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- Understand multi-body contributions to $\langle \mathbf{GT} \rangle \rightarrow$ better predictions for decay rates of larger nuclei

We calculate

$$g_A \langle \mathbf{GT} \rangle = \langle {}^3\text{He} | \bar{q} \gamma_{\mathbf{k}} \gamma_5 \tau^- q | {}^3\text{H} \rangle$$



Tritium β -decay

$$\frac{(1 + \delta_R) f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V g_A^2 \langle \mathbf{GT} \rangle^2}$$

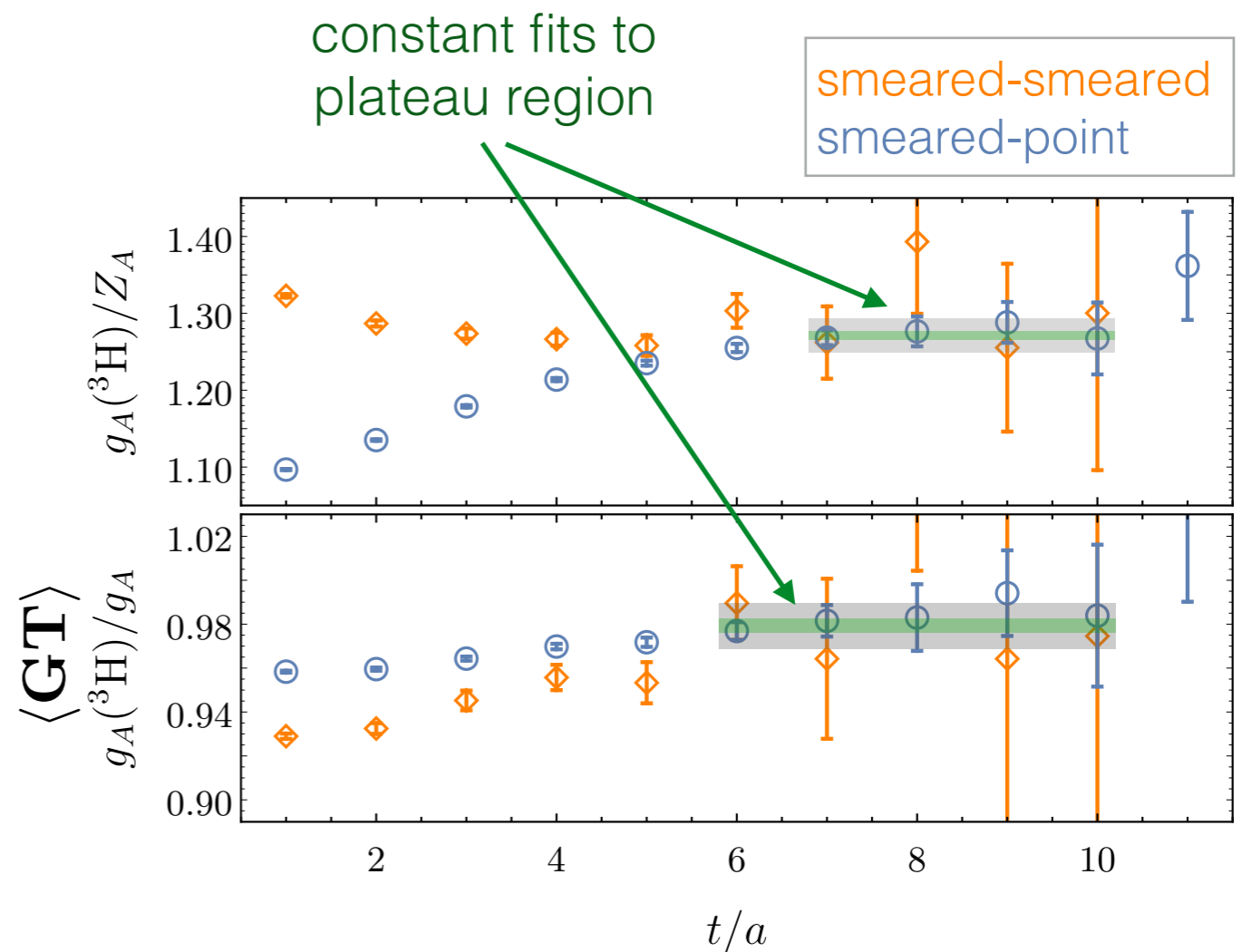
known from theory or expt.

Labels in the equation:
 - $t_{1/2}$: half-life
 - $\langle \mathbf{F} \rangle^2$: vector ME
 - $\langle \mathbf{GT} \rangle^2$: axial ME

- Form ratios of compound correlators to cancel leading time-dependence:

$$\frac{\overline{R}_{3\text{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t \rightarrow \infty} \frac{g_A(^3\text{H})}{g_A} = \langle \mathbf{GT} \rangle$$

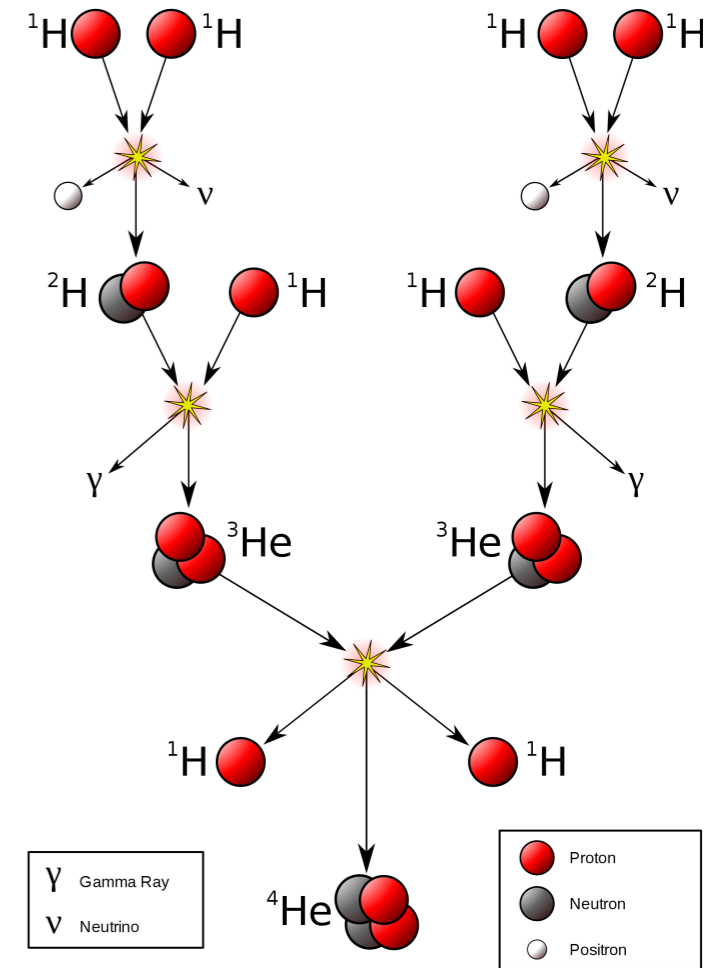
- Ground state ME revealed through “effective ME plot”
- Experiment (physical point)
 $\langle \mathbf{GT} \rangle = 0.9511(13)$



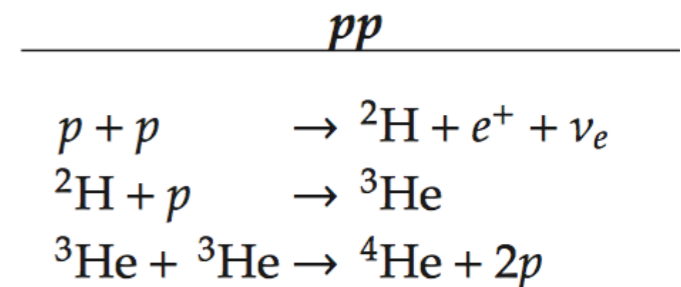
Proton-proton fusion

- Stars emit heat/light from conversion of H to He
- Sun + cooler stars: proton-proton fusion chain reaction

We calculate $\langle d; 3 | A_3^3 | pp \rangle$
 $\rightarrow L_{1,A}, \ell_{1,A}, \bar{L}_{1,A}, \dots$
 $pp \rightarrow de^+ \nu$ cross-section



- Related to:
 - Neutrino breakup reaction (SNO)
 - Muon capture reaction (MuSun)



Proton-proton fusion

- Extract matrix element through linear response of correlators to the background field

matrix elt. is linear in Λ

$$C_{\lambda_u; \lambda_d=0}^{({}^3S_1, {}^1S_0)}(t) = \lambda_u \sum_{\tau=0}^t \sum_{\mathbf{x}} \langle 0 | \chi_{{}^3S_1}^3(\mathbf{x}, t) A_3^u(\tau) \chi_{{}^1S_0}^\dagger(0) | 0 \rangle$$

correlator formed with background field coupling to u quark

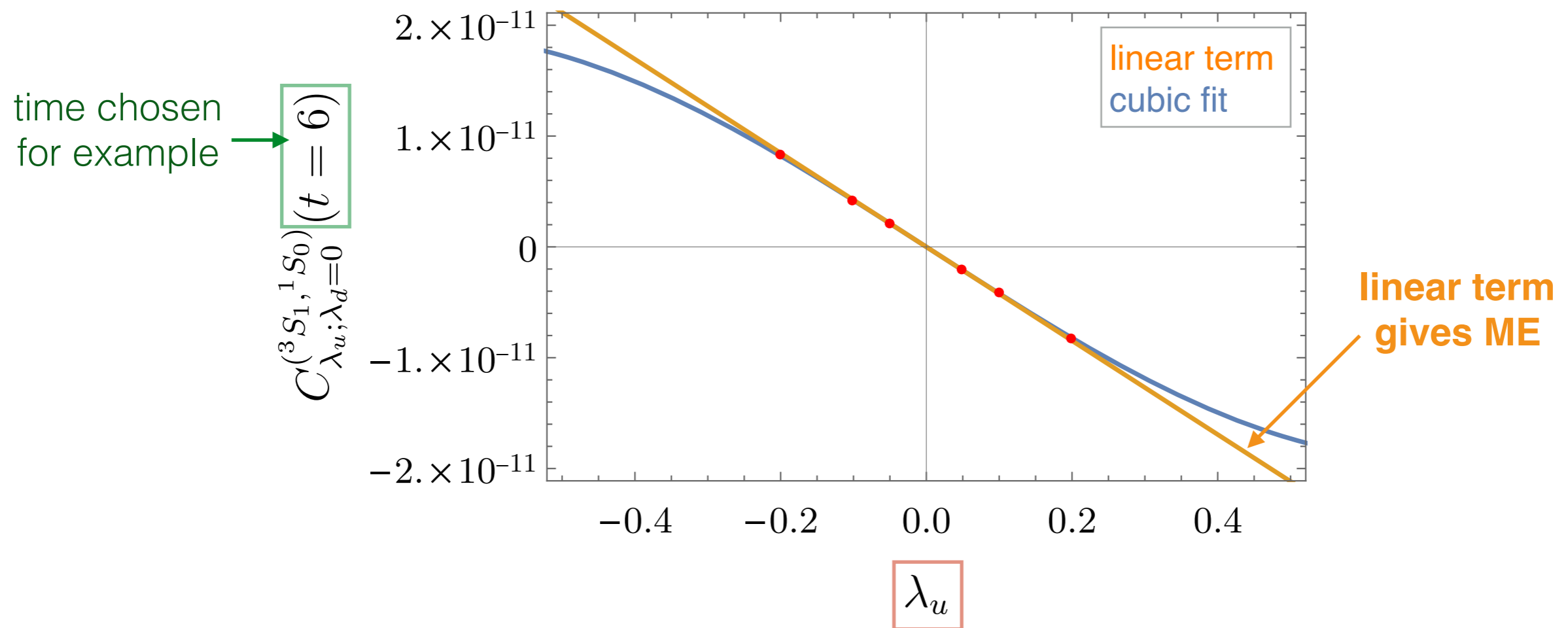
$$+ c_2 \lambda_u^2 + c_3 \lambda_u^3,$$

irrelevant consts.

- Calculate correlators at multiple values of λ_u, λ_d
➔ extract matrix element pieces

Proton-proton fusion

- Example: correlator formed with background field coupling to u quark



six choices of field strength:
can fit up to λ^6

Proton-proton fusion

- Form ratios of compound correlators to cancel leading time-dependence

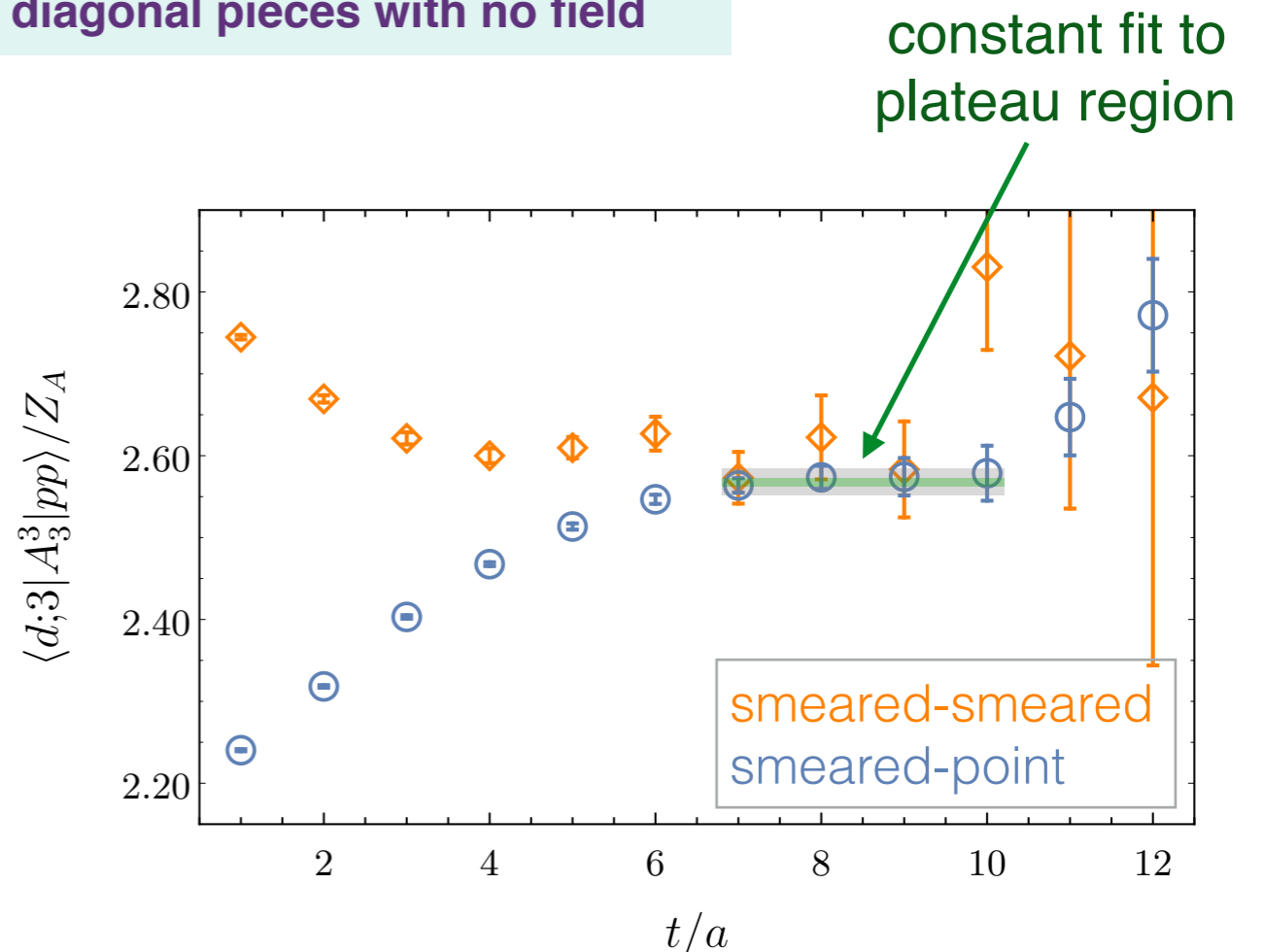
$$R_{3S_1,1S_0}(t) = \frac{\boxed{C_{\lambda_u, \lambda_d=0}^{(3S_1, 1S_0)}(t) \Big|_{\mathcal{O}(\lambda_u)} - C_{\lambda_u=0, \lambda_d}^{(3S_1, 1S_0)}(t) \Big|_{\mathcal{O}(\lambda_d)}}}{\boxed{\sqrt{C_{\lambda_u=0, \lambda_d=0}^{(3S_1, 3S_1)}(t) C_{\lambda_u=0, \lambda_d=0}^{(1S_0, 1S_0)}(t)}}}$$

transition pieces linear in Λ

diagonal pieces with no field

- Fit a constant to the 'effective matrix element plot' at late times

$$\begin{aligned} & \frac{R_{3S_1,1S_0}(t+1) - R_{3S_1,1S_0}(t)}{t \rightarrow \infty} \frac{\langle 3S_1; J_z = 0 | A_3^3 | 1S_0; I_z = 0 \rangle}{Z_A} \\ &= \frac{\langle d; 3 | A_3^3 | pp \rangle}{Z_A} \end{aligned}$$



Proton-proton fusion

Treatment of uncertainties: MEs at $m_\pi \sim 800\text{MeV}$

- Statistical

bootstrap/jackknife over configs.

correlated ratios of correlation functions

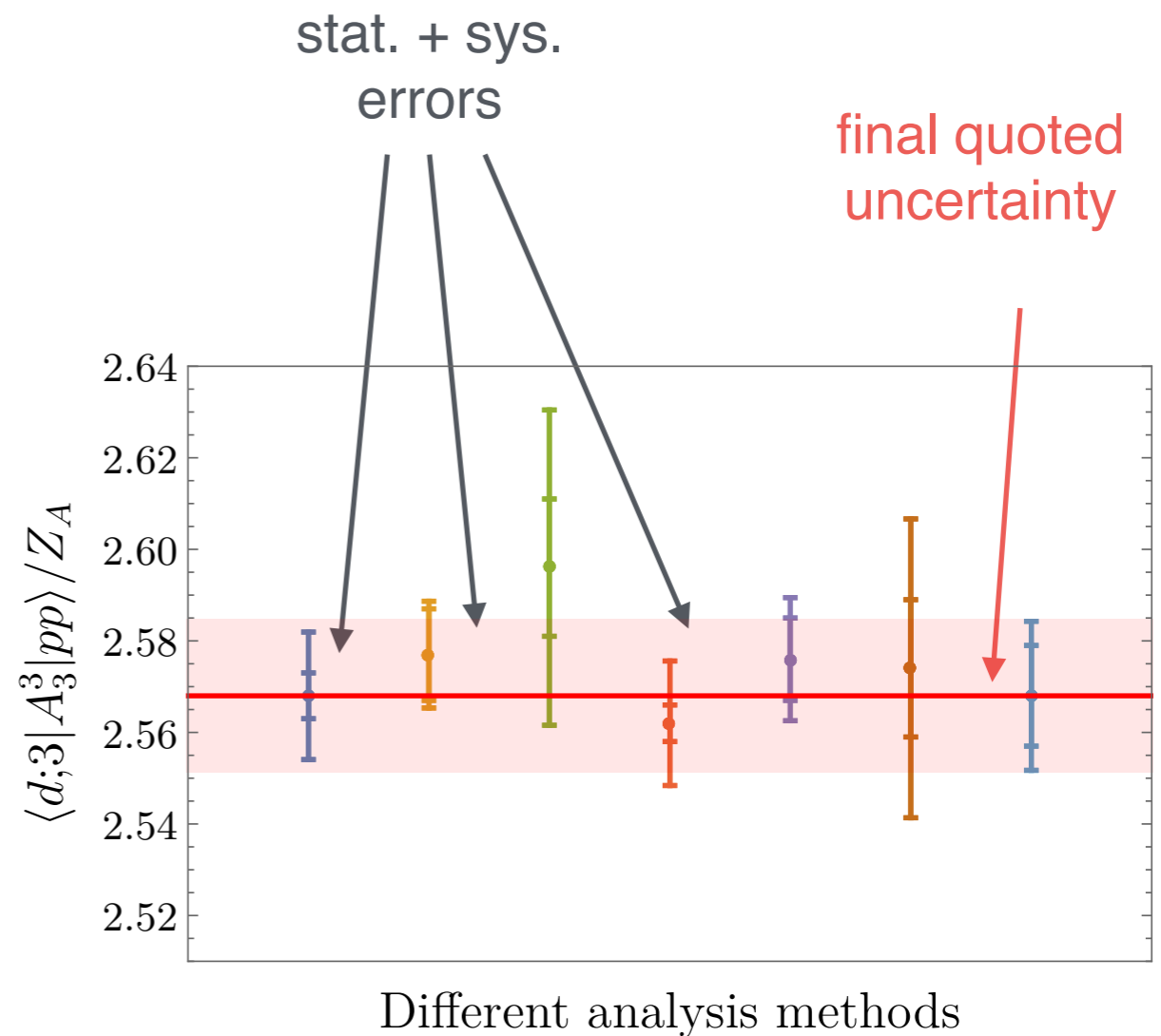
- Systematics in fit

range of field strengths in fit

t-range of plateau fit to ratio

- Systematic in analysis method

range of analysis procedures chosen by different collaboration members



Proton-proton fusion

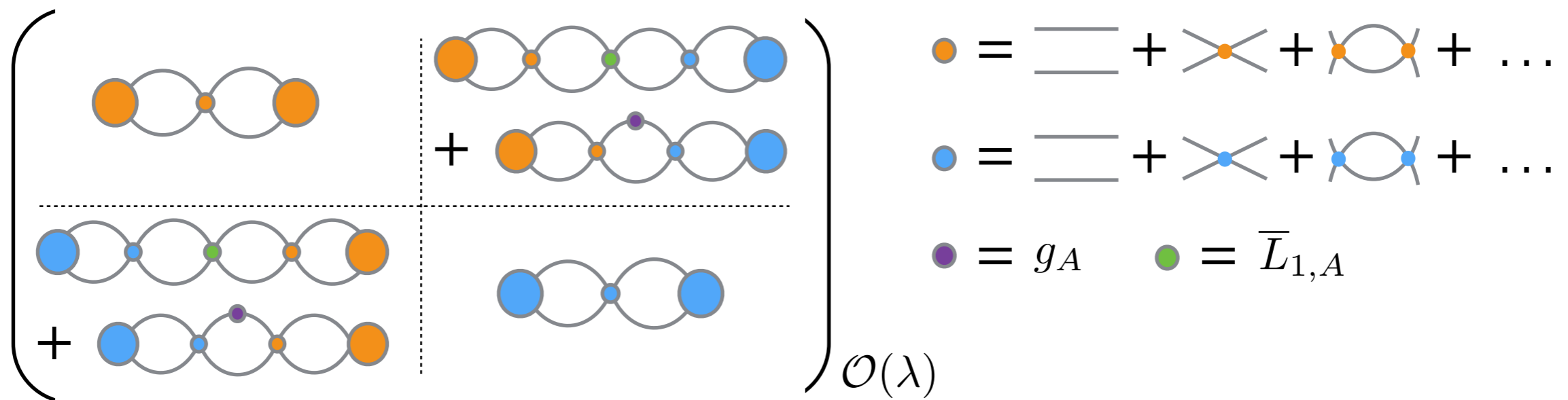
Want to relate lattice QCD ME to

- LECs of EFTs
- pp-fusion cross section

- Finite-volume quantisation condition: relate $\langle d; 3 | A_3^3 | pp \rangle$ to scale-indep. LECs
 - Pionless EFT: $\bar{L}_{1,A}$
 - Dibaryon formalism: $\bar{\ell}_{1,A}$
- Define a new related quantity, $L_{1,A}^{sd-2b}$, which should have mild pion-mass dependence (remove effective range terms in $\bar{L}_{1,A}$)
- Extrapolate $L_{1,A}^{sd-2b}$ to the physical point
 - ➔ Prediction for $\bar{L}_{1,A}, \bar{\ell}_{1,A}$ at the physical point
 - ➔ Prediction for physical cross-section

Finite-volume quantisation

- Axial field splits degeneracy of the nucleon doublet
- 3S_1 and 1S_0 channels mix
- Construct 2x2 inverse scattering amplitude matrix in background field



- Continuum integrals from bubble diagrams \rightarrow discrete sums
- $\text{Det} = 0$ \leftrightarrow poles of scattering amplitude \leftrightarrow eigenenergies

Finite-volume quantisation

- Det of inverse scattering matrix = 0 \longleftrightarrow eigenenergies are solutions of

$$\left[\underbrace{p \cot \delta^{3S_1}}_{\text{from effective range expansion}} + \underbrace{\delta G_0^V(p; L)}_{\text{finite-volume sums}} \right] \left[\underbrace{p \cot \delta^{1S_0}}_{\text{finite-volume sums}} + \underbrace{\delta G_0^V(p; L)}_{\text{finite-volume sums}} \right] = \left[\underbrace{W_3 g_A M \bar{L}_{1,A}}_{\text{two-body LEC}} - \underbrace{W_3 g_A G_1^V(p; L)}_{\text{weak coupling}} \right]^2$$

- ➔ Matrix element related to scale-indep. LEC

$$|\delta E^{3S_1-1S_0}|/W_3 = |\langle {}^3S_1 | A_3^3 | {}^1S_0 \rangle| = Z_d^2 (4g_A \gamma \bar{L}_{1,A} + 2g_A)$$

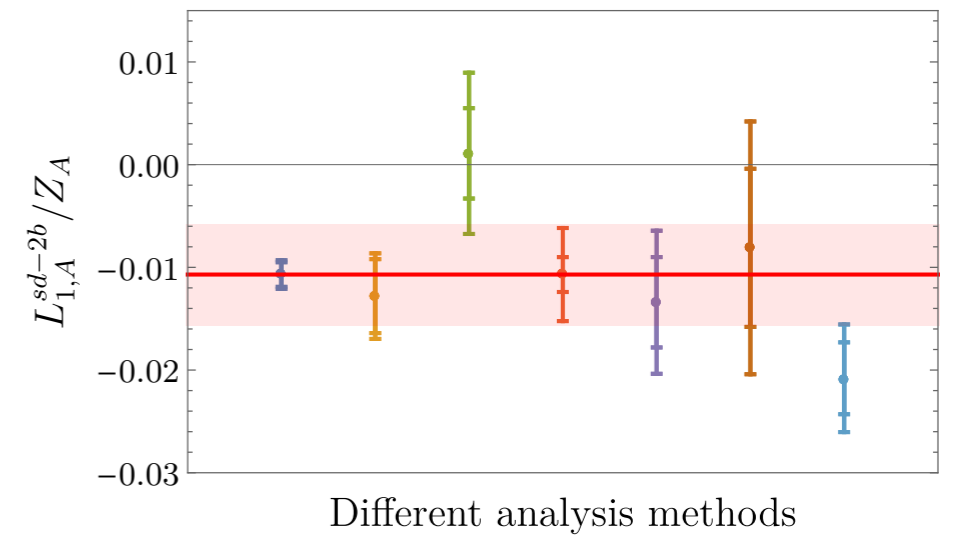
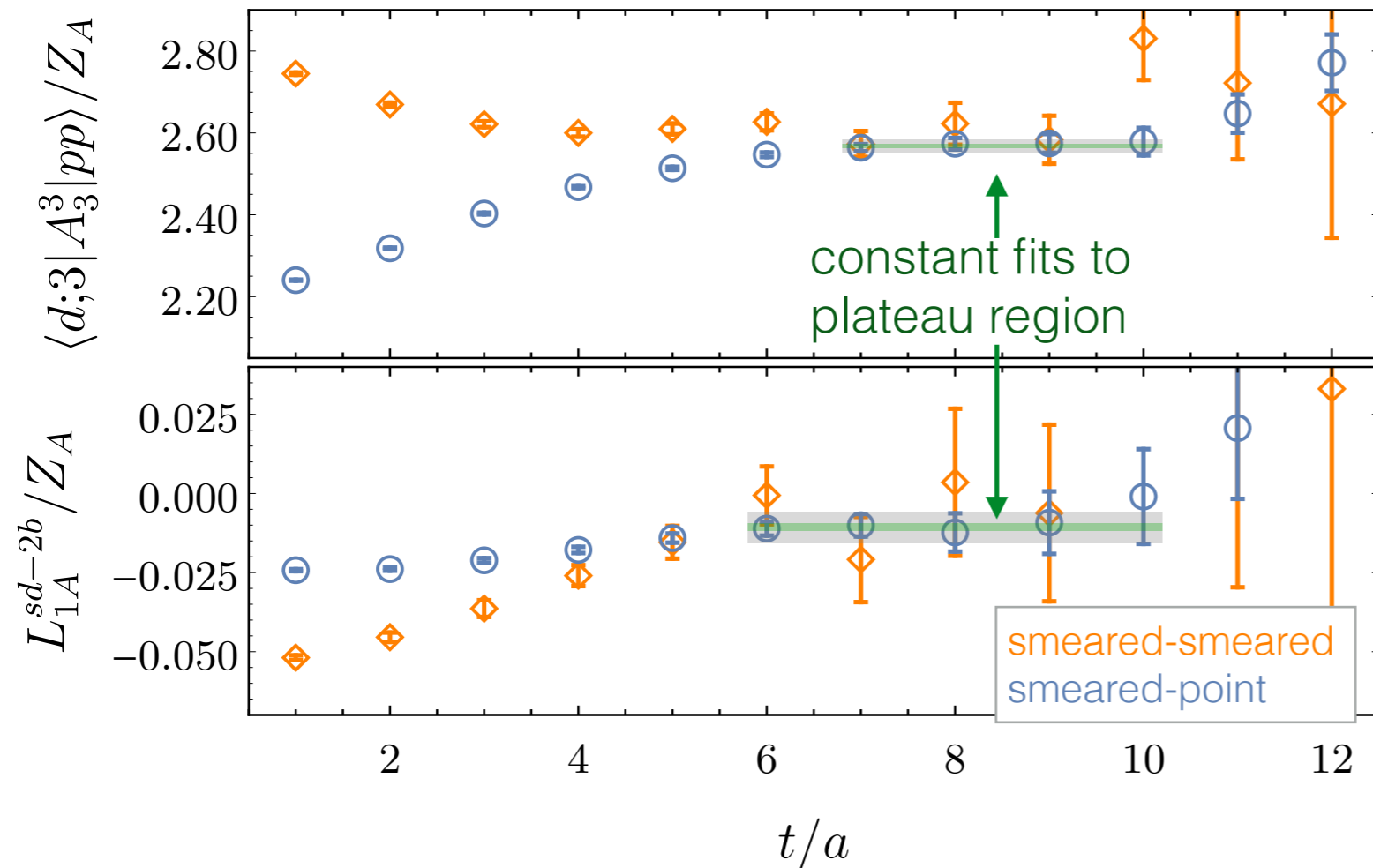
- Define combination that characterises two-nucleon contribution
Expect mild pion-mass dependence ➔ can extrapolate

experience from
 $np \rightarrow d\gamma$

$$L_{1,A}^{sd-2b} \equiv (\langle d; 3 | A_3^3 | pp \rangle - 2g_A)/2$$

$$Z_d = 1/\sqrt{1 - \rho\gamma}$$

Proton-proton fusion



$$\frac{L_{1,A}^{sd-2b}}{Z_A} = -0.0107(12)(49) \quad \longrightarrow$$

Extrapolate,
predict physical
cross-section

Proton-proton fusion

Low-energy cross section for $pp \rightarrow de^+ \nu$ dictated by the matrix element

$$|\langle d; j | A_k^- | pp \rangle| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}} \Lambda(p) \delta_{jk}$$

Relate $\Lambda(0)$ to extrapolated LEC using EFT

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}} \left\{ e^\chi - \gamma a_{pp} [1 - \chi e^\chi \Gamma(0, \chi)] + \frac{1}{2} \gamma^2 a_{pp} \sqrt{r_1 \rho} \right\} - \frac{1}{2g_A} \gamma a_{pp} \sqrt{1-\gamma\rho} L_{1,A}^{sd-2b}$$

extrapolated
lattice value

C_η Sommerfeld factor
 γ Deuteron binding mtm
 r_1, ρ Effective ranges
 a_{pp} pp scattering length
 $\Gamma(0, \chi)$ Incomplete gamma func.
 $\chi = \alpha M_p / \gamma$

N²LO \nrightarrow EFT with effective range contributions
resummed using the dibaryon approach

Butler and Chen, Phys. Lett. B520, 87 (2001)
Detmold and Savage, Nucl. Phys. A743, 170 (2004).

Proton-proton fusion

Physical cross-section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

statistical

systematic

- fitting
- analysis
- uncertainties of phys. mass inputs

quark mass extrap.
(50% additive)

Can also extract

$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9) \text{ fm}^3$$

renormalisation scale $\mu = m_\pi$

higher-order \nrightarrow EFT
corrections
(power-counting)

Proton-proton fusion

Physical cross-section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

$$\Lambda(0) = 2.652(2) \quad (\text{models/EFT})$$

E. G. Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

Can also extract

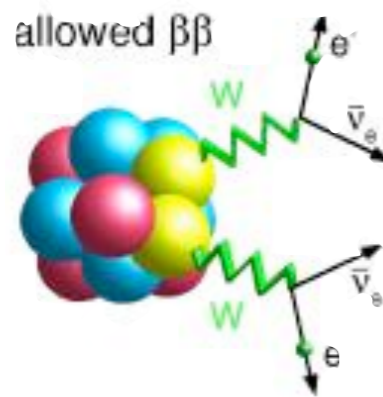
$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9) \text{ fm}^3$$

$$L_{1,A} = 3.6(5.5) \text{ fm}^3 \quad (\text{reactor expts.})$$

M. Butler, J.-W. Chen, and P. Vogel, Phys. Lett. B549

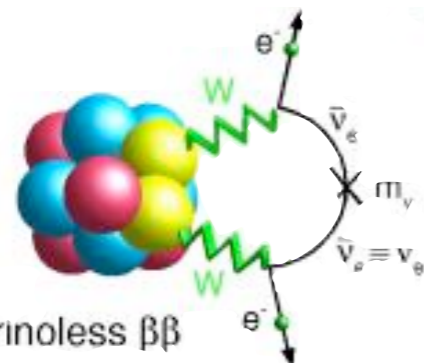
Double β -decay

- Certain nuclei allow observable $\beta\beta$ decay

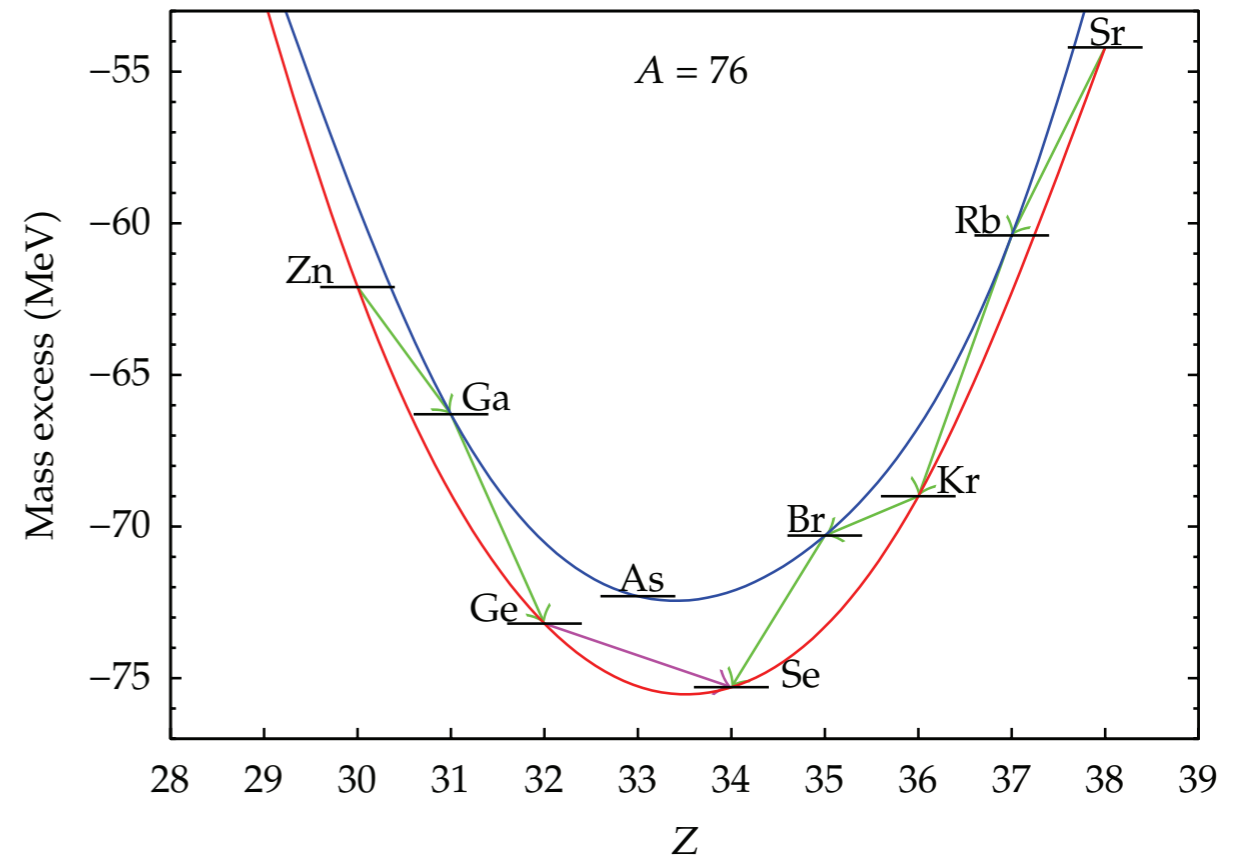


$$T_{1/2}^{2\nu\beta\beta} \gtrsim 10^{19} \text{ y}$$

- If neutrinos are massive Majorana fermions $0\nu\beta\beta$ decay is possible



$$T_{1/2}^{0\nu\beta\beta} > 10^{25} \text{ y}$$



We calculate two-current nuclear matrix elements
 → dictate half-life

Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

$$C_{\lambda_u; \lambda_d}(t) = \left(\begin{array}{c} \text{Diagram 1} + \lambda \text{ Diagram 2} + \lambda^2 \text{ Diagram 3} \\ + \lambda^3 \text{ Diagram 4} \end{array} \right)$$

Linear response gives axial matrix element

Implicit sum over current insertion times


The diagram shows a series of four terms in a large bracket, representing the expansion of the correlation function $C_{\lambda_u; \lambda_d}(t)$. Each term consists of a grid with two clusters of orange dots (representing fermions) and orange paths connecting them. The first term is the background field correlation function. The second term, multiplied by λ , shows one grey circle with a cross and arrows (representing a current insertion) on the path. The third term, multiplied by λ^2 , shows two such current insertions. The fourth term, multiplied by λ^3 , shows three such current insertions. A red arrow points to the second term with the text 'Linear response gives axial matrix element'. A legend below shows a grey circle with a cross and arrows and the text 'Implicit sum over current insertion times'.

Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

$$C_{\lambda_u; \lambda_d}(t) = \left(\begin{array}{l} \text{Diagram 1} + \lambda \text{ Diagram 2} + \lambda^2 \text{ Diagram 3} \\ + \lambda^3 \text{ Diagram 4} \end{array} \right)$$

Quadratic response from two insertions on different quark lines



 Implicit sum over current insertion times

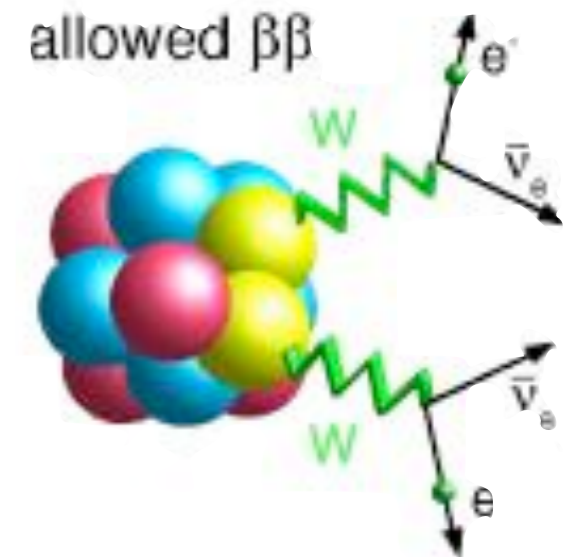
Second order weak interactions

NPLQCD arXiv:1701.03456, 1702.02929

- Background axial field to second order

➔ $nn \rightarrow pp$ transition matrix element

$$M_{GT}^{2\nu} = 6 \int d^4x d^4y \langle pp | T [J_3^+(x) J_3^+(y)] | nn \rangle$$



- $nn \rightarrow ppe\bar{e}\bar{\nu}_e\bar{\nu}_e$ decay not observed in nature because the dinucleon is not bound

BUT

- Nuclear matrix element well-defined and an important subprocess in double-beta decays of large nuclei

Second order weak interactions

- In effective field theory

Effect can be described by 'quenching' g_A

$$i\mathcal{C}_{nn \rightarrow pp} = \text{[diagrams]} + \text{[diagrams]} + \text{[diagrams]} + \text{[diagrams]} + \text{[diagram]} + \text{[diagram]} + \mathcal{O}(\lambda^4)$$

Diagram 1: One-body diagram with g_A label.

Diagram 2: Two-body diagram with $L_{1,A}$ label.

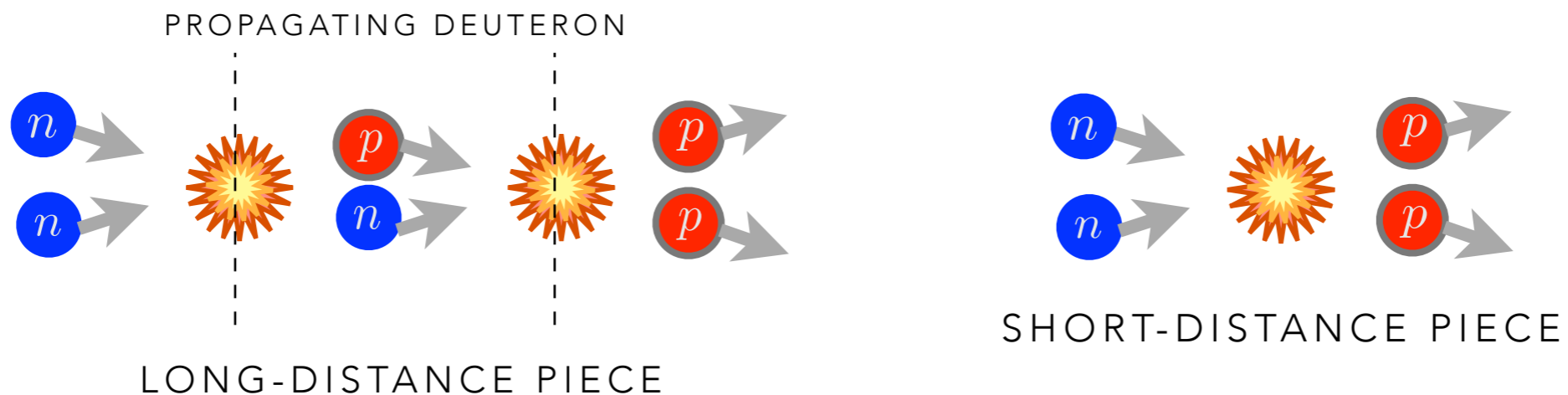
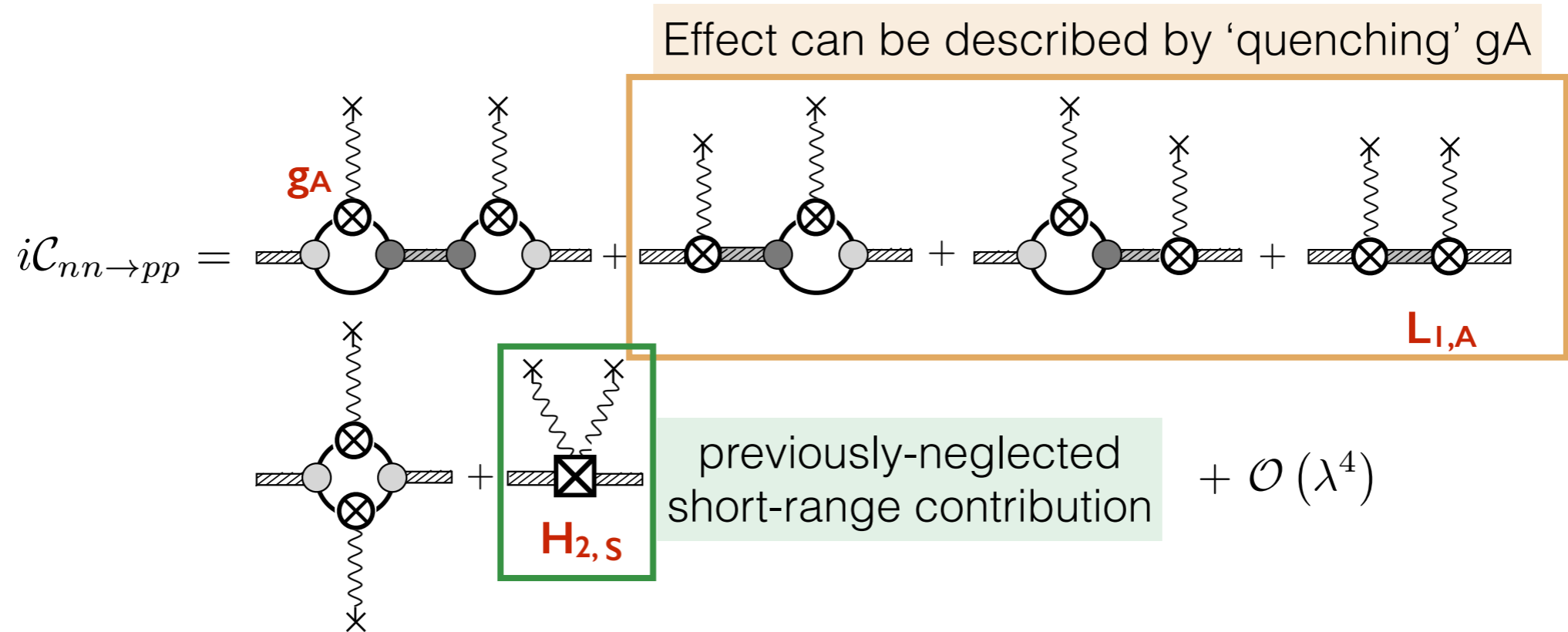
Diagram 3: Two-body diagram with $H_{2,S}$ label, highlighted as 'previously-neglected short-range contribution'.

One body (g_A), two body single-weak current ($L_{1,A}$), two-body second order weak ($H_{2,S}$)

$H_{2,S} \sim$ **isotensor axial polarisability** ignored in all phenomenological analyses (unconstrained except by DBD)

Second order weak interactions

- In effective field theory



Second order weak interactions

From lattice calculations: differentiate terms by their distinct time-dependences in correlation function

Expand contributions to the correlation function

$$\begin{aligned}
 a^2 C_{nn \rightarrow pp}(t) = & 2Z_{pp} Z_{nn}^\dagger e^{-E_{nn}t} \left\{ \left[\frac{e^{\Delta t} - 1}{\Delta^2} - \frac{t}{\Delta} \right] \langle pp | \tilde{J}_3^+ | d \rangle \langle d | \tilde{J}_3^+ | nn \rangle \right. \\
 & + \sum_{l' \neq d} \left[\frac{t}{\delta_{l'}} - \frac{1}{\delta_{l'}^2} \right] \langle pp | \tilde{J}_3^+ | l' \rangle \langle l' | \tilde{J}_3^+ | nn \rangle \\
 & + \sum_{n \neq nn, pp} \left[\frac{e^{\Delta t}}{\Delta(\Delta + \delta_n)} - \frac{1}{\Delta \delta_n} \right] \left(\frac{Z_n}{Z_{pp}} \langle n | \tilde{J}_3^+ | d \rangle \langle d | \tilde{J}_3^+ | nn \rangle + \frac{Z_n^\dagger}{Z_{nn}^\dagger} \langle pp | \tilde{J}_3^+ | d \rangle \langle d | \tilde{J}_3^+ | n \rangle \right) \\
 & + \sum_{n \neq nn, pp} \sum_{l' \neq d} \frac{1}{\delta_{l'} \delta_n} \left(\frac{Z_n}{Z_{pp}} \langle n | \tilde{J}_3^+ | l' \rangle \langle l' | \tilde{J}_3^+ | nn \rangle + \frac{Z_n^\dagger}{Z_{nn}^\dagger} \langle pp | \tilde{J}_3^+ | l' \rangle \langle l' | \tilde{J}_3^+ | n \rangle \right) \\
 & \left. + \sum_{n, m \neq nn, pp} \frac{e^{\Delta t}}{(\Delta + \delta_n)(\Delta + \delta_m)} \frac{Z_n}{Z_{pp}} \frac{Z_m^\dagger}{Z_{nn}^\dagger} \langle n | \tilde{J}_3^+ | d \rangle \langle d | \tilde{J}_3^+ | m \rangle + \mathcal{O}(e^{-\delta t}, e^{-\delta' t}) \right\}.
 \end{aligned}$$

$$\mathcal{R}_{nn \rightarrow pp}(t) = \frac{C_{nn \rightarrow pp}(t)}{2C_{0;0}^{(nn)}(t)} = \left[-t + \frac{e^{\Delta t} - 1}{\Delta} \right] \frac{\langle pp | \tilde{J}_3^+ | d \rangle \langle d | \tilde{J}_3^+ | nn \rangle}{\Delta} + t \sum_{l' \neq d} \frac{\langle pp | \tilde{J}_3^+ | l' \rangle \langle l' | \tilde{J}_3^+ | nn \rangle}{\delta_{l'}}$$

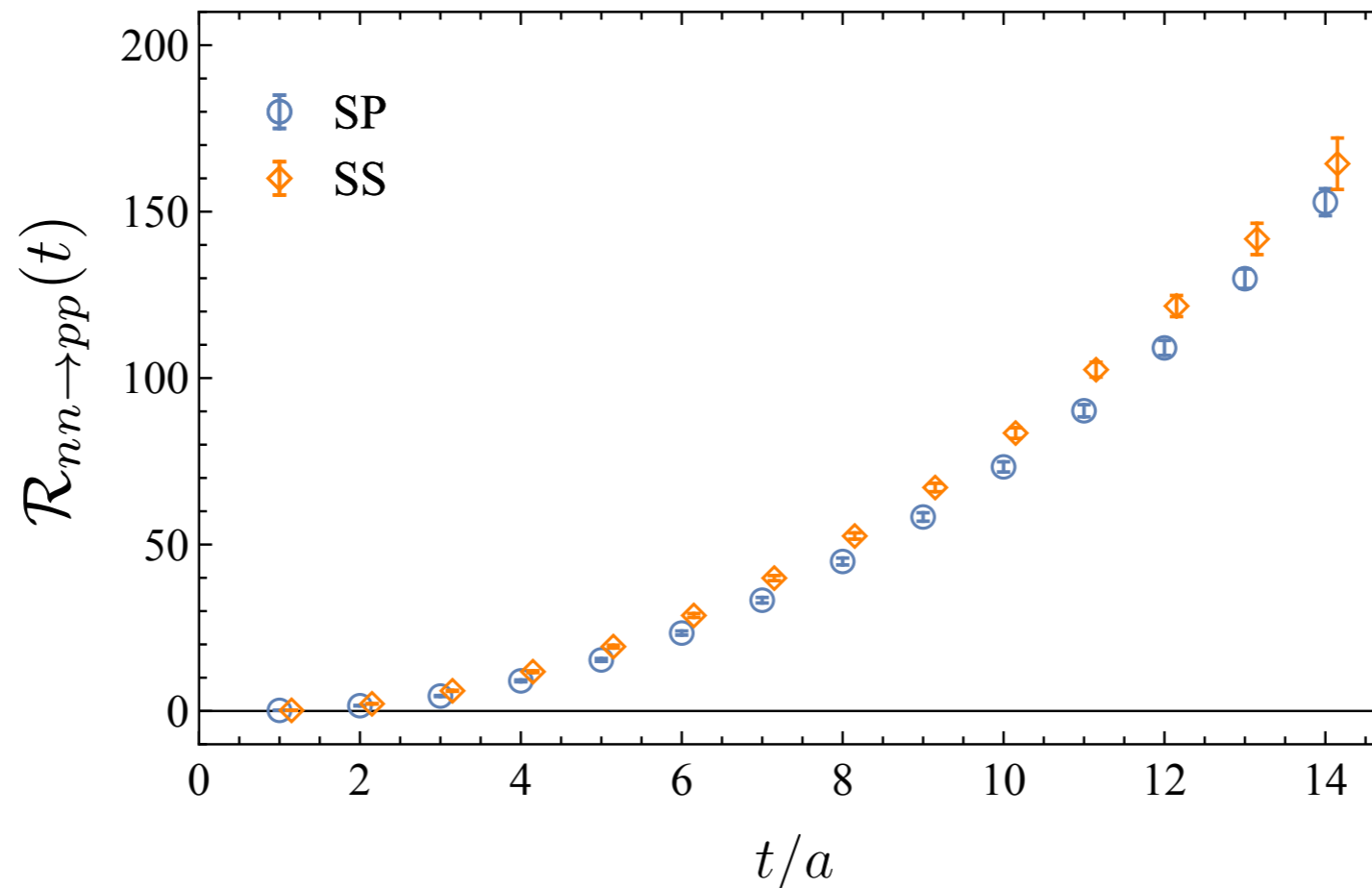
$$+ C + D e^{\Delta t} + \mathcal{O}(e^{-\delta t}, e^{-\delta' t})$$

C, D irrelevant constants

Second order weak interactions

Challenging!

- Correlation function ratio clearly dominated by exponential
- BUT: Deuteron contribution well-determined by calculations with single axial current insertions



Second order weak interactions

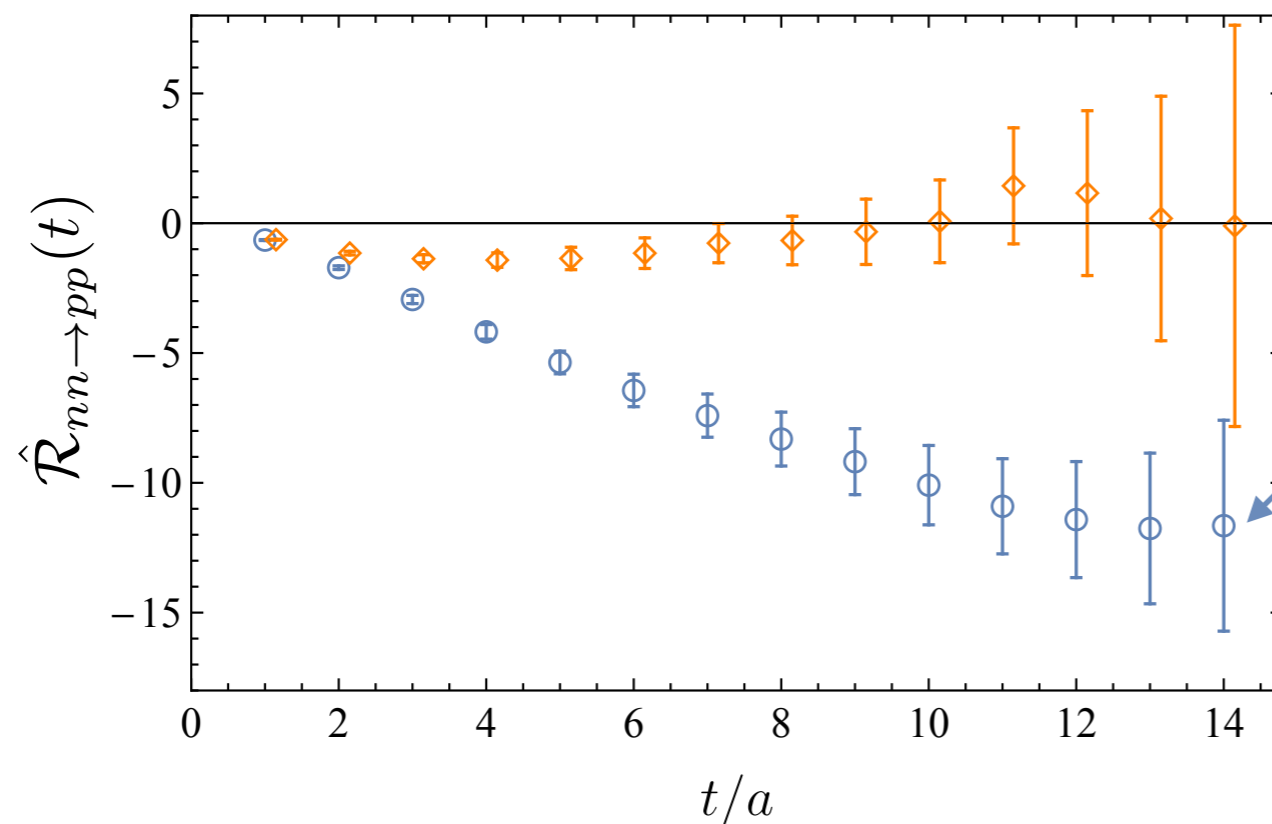
Subtract deuteron pole term determined from (correlated) single-insertion calculations

$$\hat{\mathcal{R}}_{nn \rightarrow pp}(t) = \mathcal{R}_{nn \rightarrow pp}(t) - \frac{|\langle pp | \tilde{J}_3^+ | d \rangle|^2}{a\Delta} \left[-\frac{t}{a} + \frac{e^{\Delta t} - 1}{a\Delta} \right]$$

$$= \frac{t}{a} \sum_{l' \neq d} \frac{\langle pp | \tilde{J}_3^+ | l' \rangle \langle l' | \tilde{J}_3^+ | nn \rangle}{a\delta_{l'}} + c + d e^{\Delta t}.$$

c,d irrelevant constants

$\Delta = E_{nn} - E_d$
determined from
ratios of two-
point functions

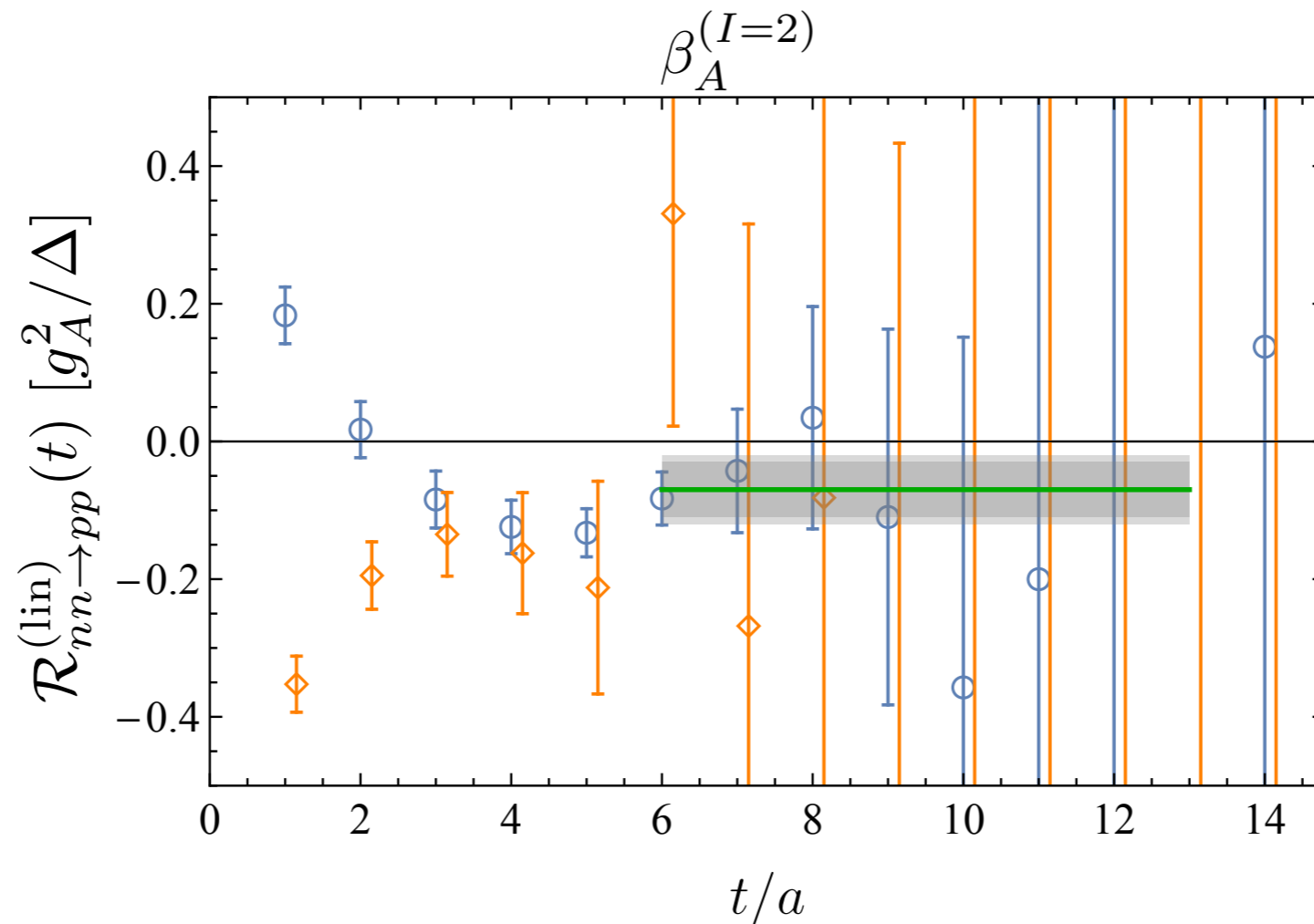


Linear in t:
SP sink has a highly
suppressed overlap onto
the nn scattering states

Second order weak interactions

Form ratios to extract **isotensor axial polarisability**

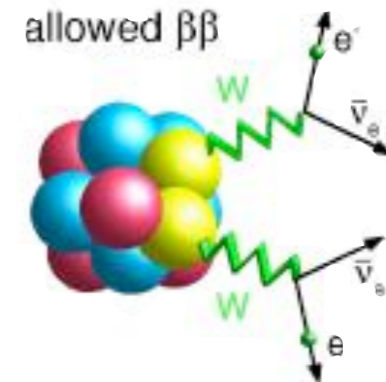
$$\mathcal{R}_{nn \rightarrow pp}^{(\text{lin})}(t) = \frac{(e^{a\Delta} + 1)\hat{\mathcal{R}}_{nn \rightarrow pp}(t + a) - \hat{\mathcal{R}}_{nn \rightarrow pp}(t + 2a) - e^{a\Delta}\hat{\mathcal{R}}_{nn \rightarrow pp}(t)}{e^{a\Delta} - 1} \xrightarrow{t \rightarrow \infty} \frac{1}{aZ_A^2} \frac{\beta_A^{(2)}}{6}$$



Second order weak interactions

NPLQCD arXiv:1701.03456, 1702.02929

- Non-negligible deviation from long distance deuteron intermediate state contribution

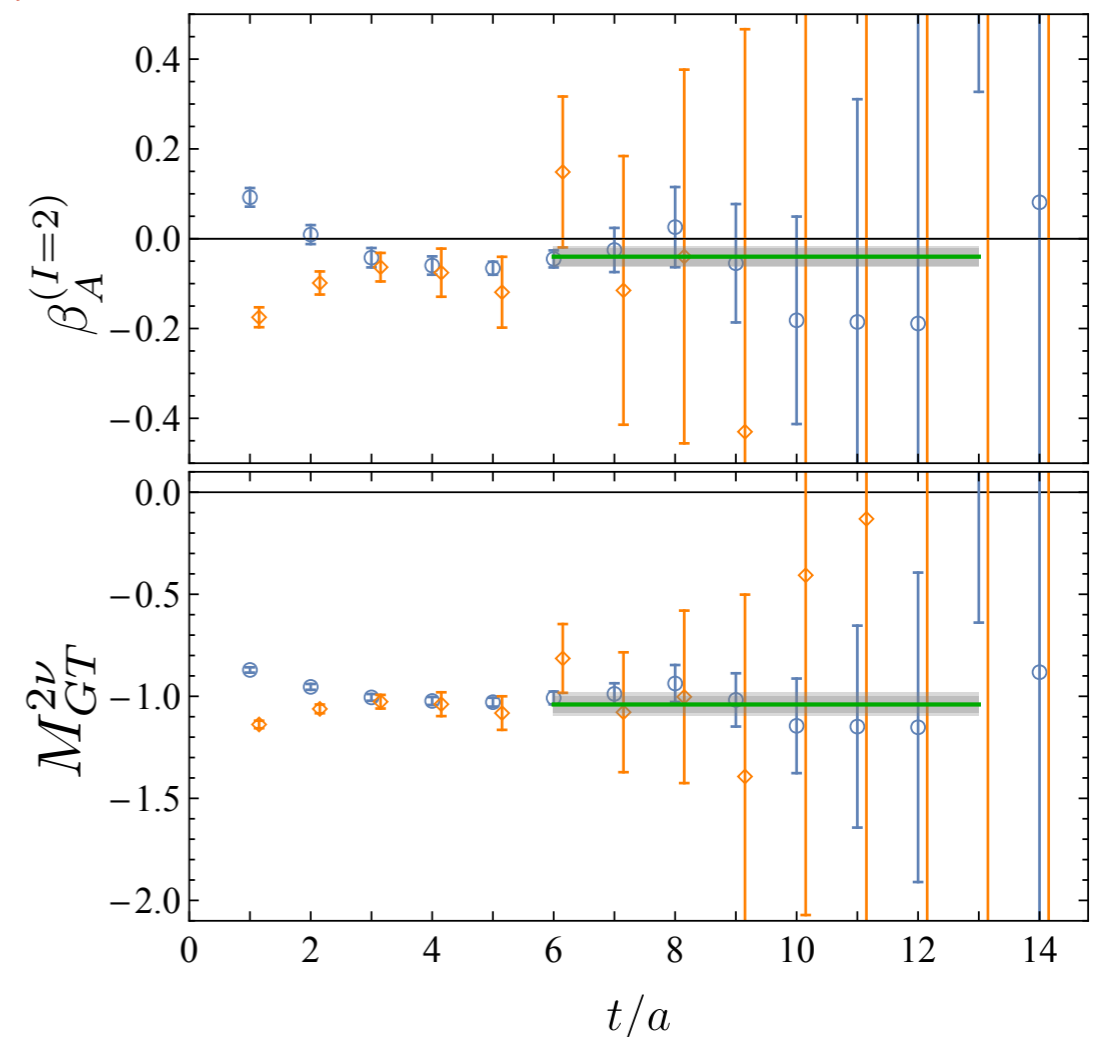


$$M_{GT}^{2\nu} = -\frac{|M_{pp \rightarrow d}|^2}{E_{pp} - E_d} + \beta_A^{(I=2)}$$

Isotensor axial polarisability

➔ Quenching of g_A in nuclei is insufficient!

- Connect to EFT for larger systems



Second order weak interactions

NPLQCD arXiv:1701.03456, 1702.02929

- Match QCD matrix element to low-energy constants of pionless EFT

Correlation function matrix in coupled $nn, np(^3S_1), pp$ channel space

$$\mathcal{C}_{NN \rightarrow NN} \equiv \begin{pmatrix} \mathcal{C}_{nn \rightarrow nn} & \mathcal{C}_{nn \rightarrow np(^3S_1)} & \mathcal{C}_{nn \rightarrow pp} \\ \mathcal{C}_{np(^3S_1) \rightarrow nn} & \mathcal{C}_{np(^3S_1) \rightarrow np(^3S_1)} & \mathcal{C}_{np(^3S_1) \rightarrow pp} \\ \mathcal{C}_{pp \rightarrow nn} & \mathcal{C}_{pp \rightarrow np(^3S_1)} & \mathcal{C}_{pp \rightarrow pp} \end{pmatrix}$$

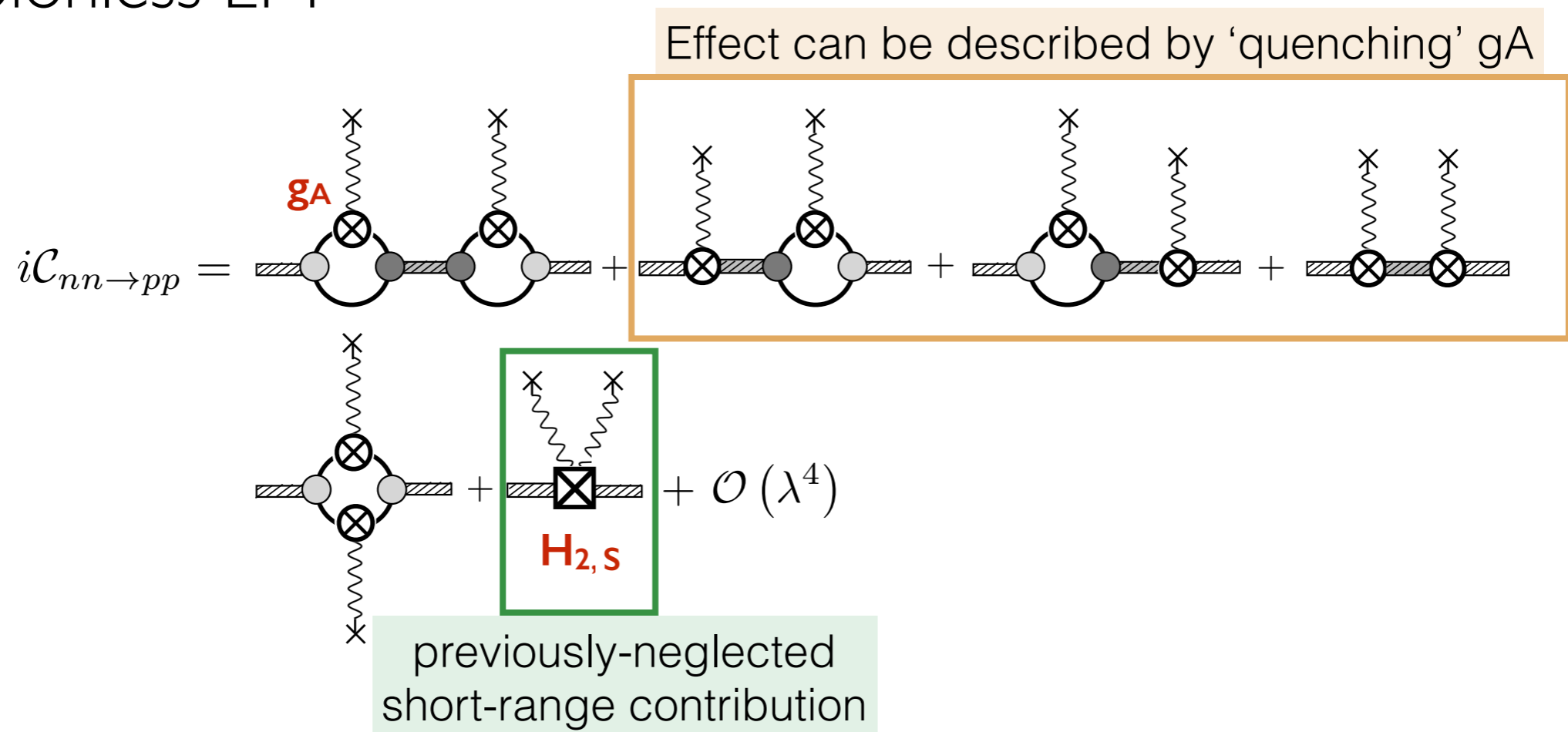
axial background field changes both spin and isospin

➔ no coupling of $np(^1S_0)$

Second order weak interactions

NPLQCD arXiv:1701.03456, 1702.02929

- Match QCD matrix element to low-energy constants of pionless EFT



$$M_{nn \rightarrow pp} = -\frac{|M_{pp \rightarrow d}|^2}{\Delta} + \frac{M g_A^2}{4\gamma_s^2} \mathbb{H}_{2,S}$$

$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8) \text{ fm}$$

$$m_\pi = 806 \text{ MeV}$$

Second order weak interactions

Future challenges:

● Lighter quark masses

- Hierarchy between dinucleon-deuteron mass splitting and gap to excitations of dinucleon changes

➔ Transition matrix elements of excited states not negligible

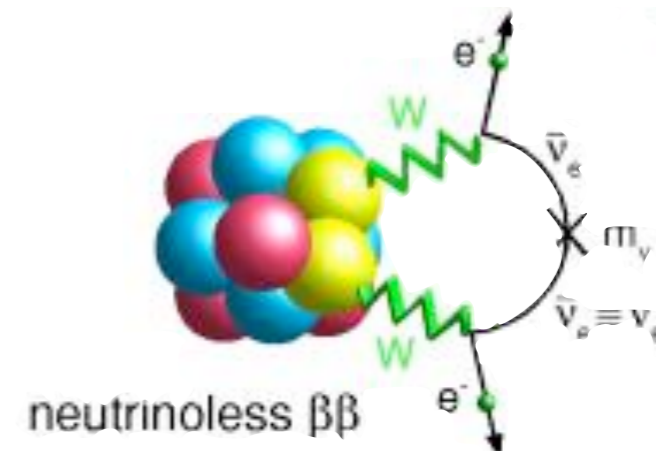
- Need new techniques
e.g., separate source and sink from background field region

- Initial, intermediate, final states no longer bound
➔ need multi-particle finite-volume formalism

Detmold + Savage, Nucl. Phys A743 170 (2004)
Briceno + Davoudi, PRD88 94507 (2013)
Briceno + Hansen PRD94 13008 (2016)
Christ et al. PRD91 114510 (2015)

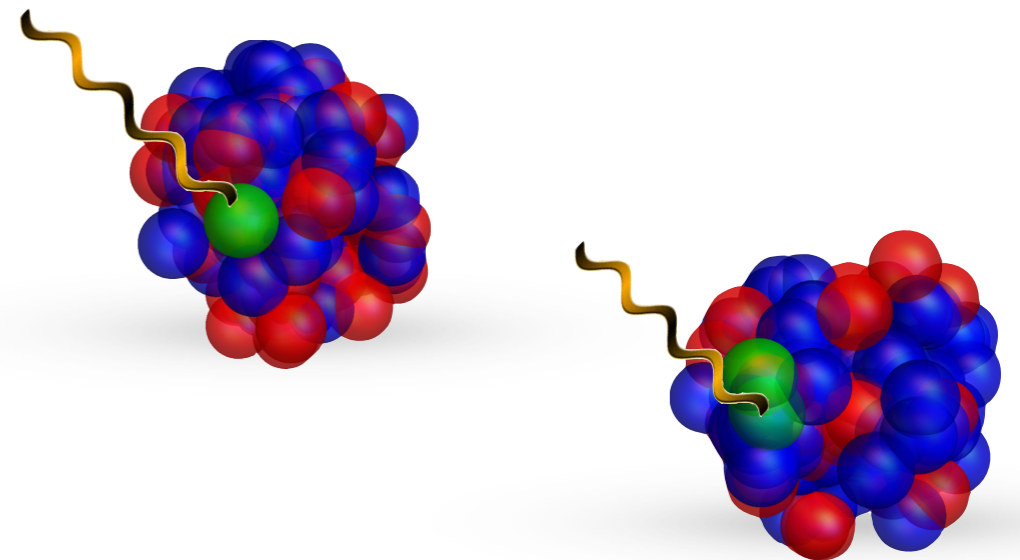
● Neutrinoless double-beta decay

- Contraction of nuclear matrix element with leptonic tensor ➔ integration over the intermediate neutrino momentum in ME
- Both axial and vector currents important



Larger nuclei

- Move to larger (phenomenologically-relevant) nuclei?
- Nuclear effective field theory:
 - 1-body currents are dominant
 - 2-body currents are sub-leading *but non-negligible*
- Determine one body contributions from single nucleon
- Determine few-body contributions from $A=2,3,4\dots$
- Match EFT and many body methods to LQCD to make predictions for larger nuclei



Summary

- Current state-of-the-art: significant systematics but phenomenologically interesting at current precision
 - Spectroscopy of nuclei
 - Structure, i.e., magnetic moments, polarisabilities
 - **Electroweak interactions**
- Nuclear matrix elements important to experimental programs e.g.,
 - Neutrino breakup reaction (SNO)
 - Muon capture reaction (MuSun)
 - Double-beta decay

