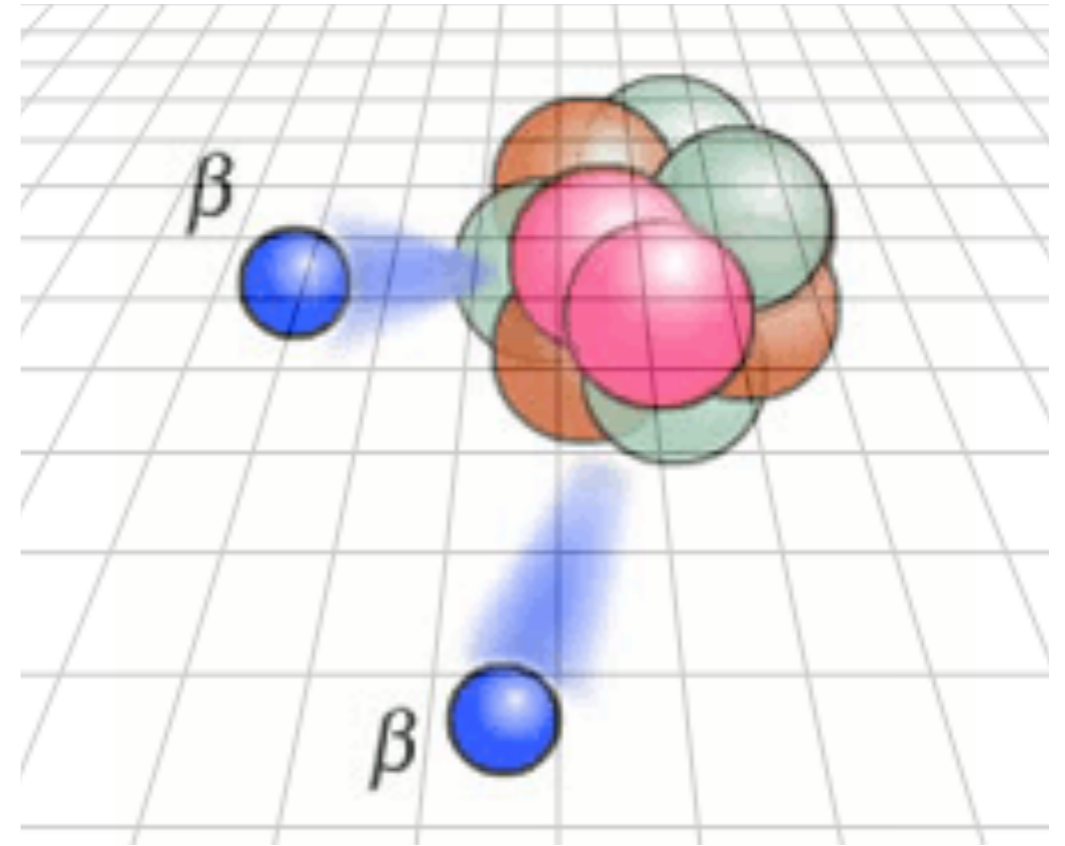


Status of LQCD calculations of double-beta decay operators



Amy Nicholson
UC Berkeley/UNC

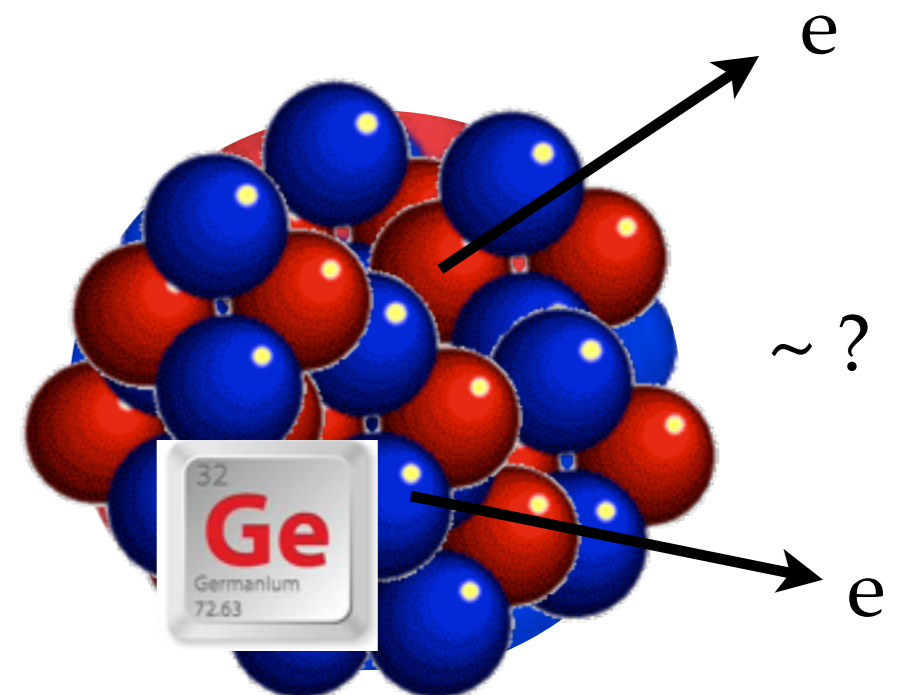
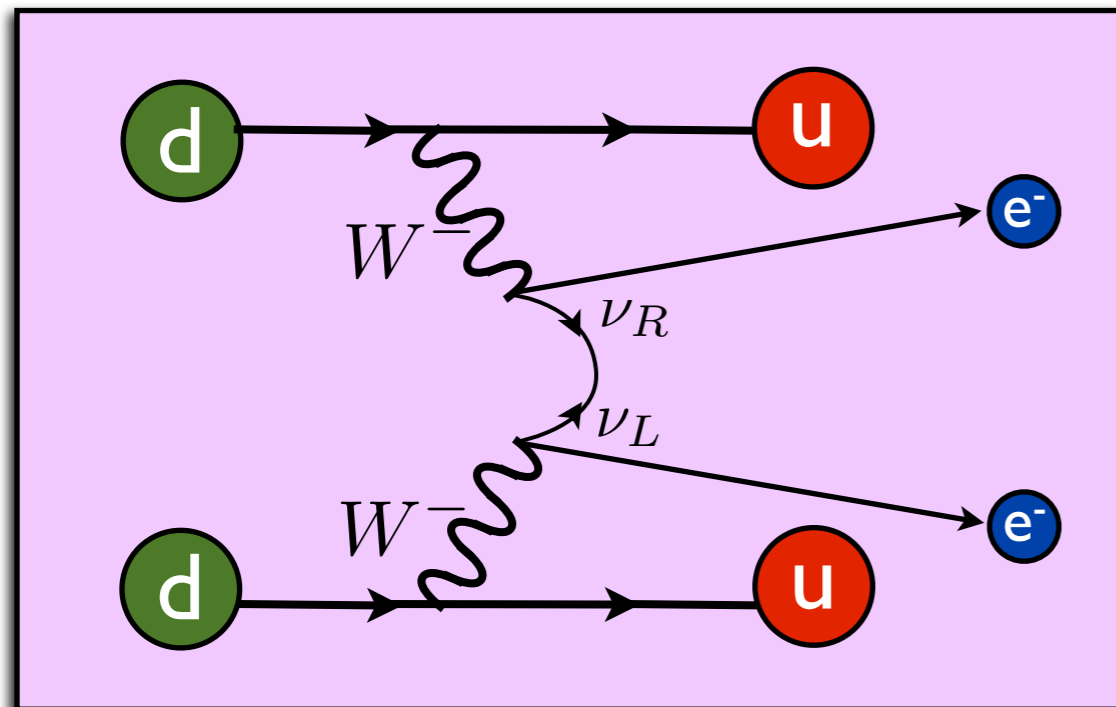
Institute for Nuclear Theory, Seattle, WA

Lattice QCD Input for Neutrinoless Double-Beta Decay

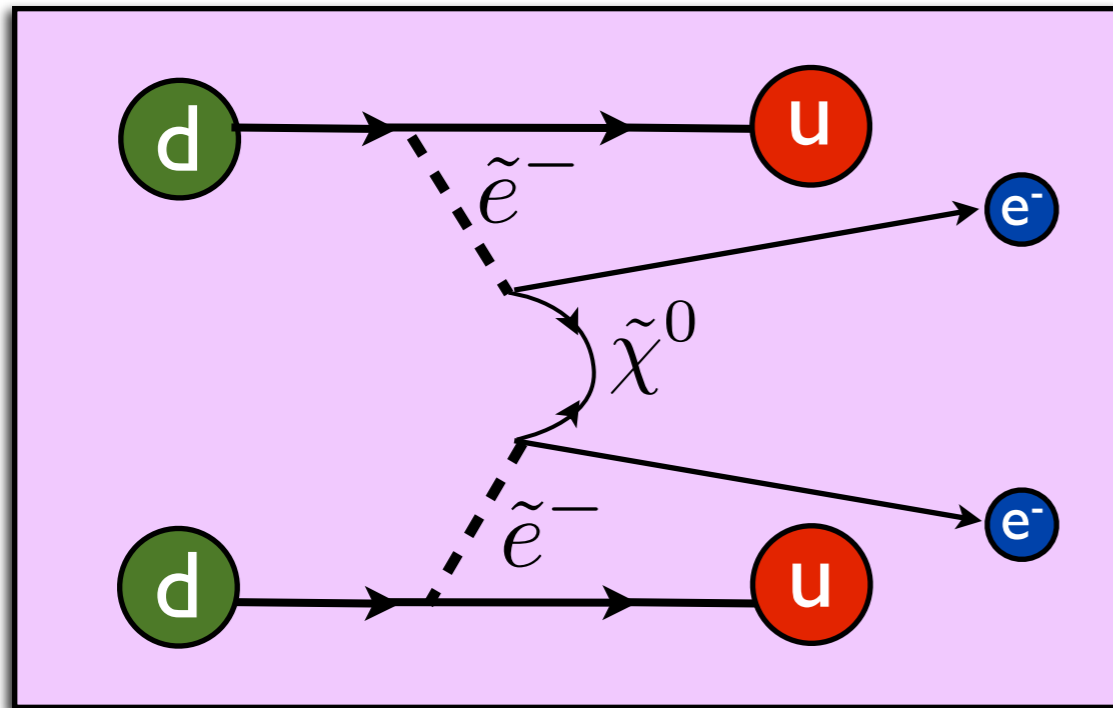
July 6, 2017



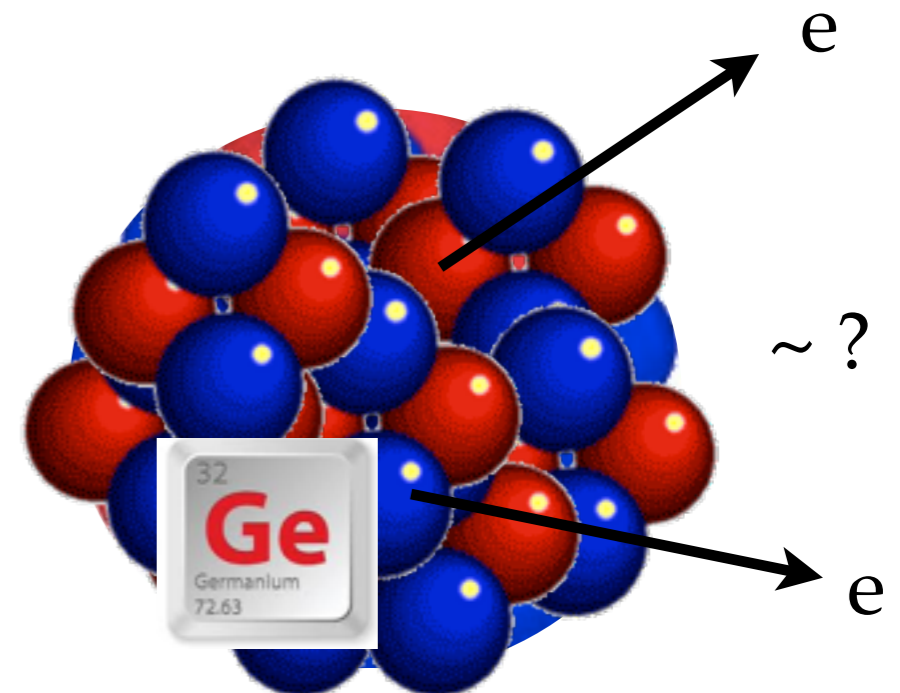
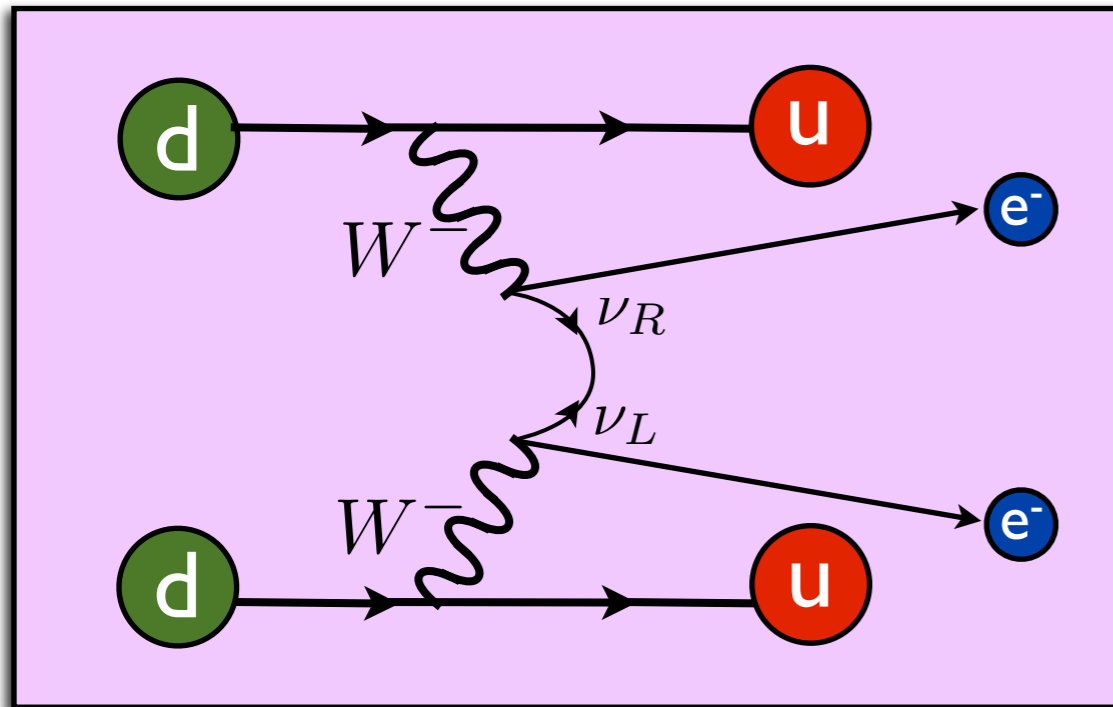
Relating Theory to Experiment



Relating Theory to Experiment

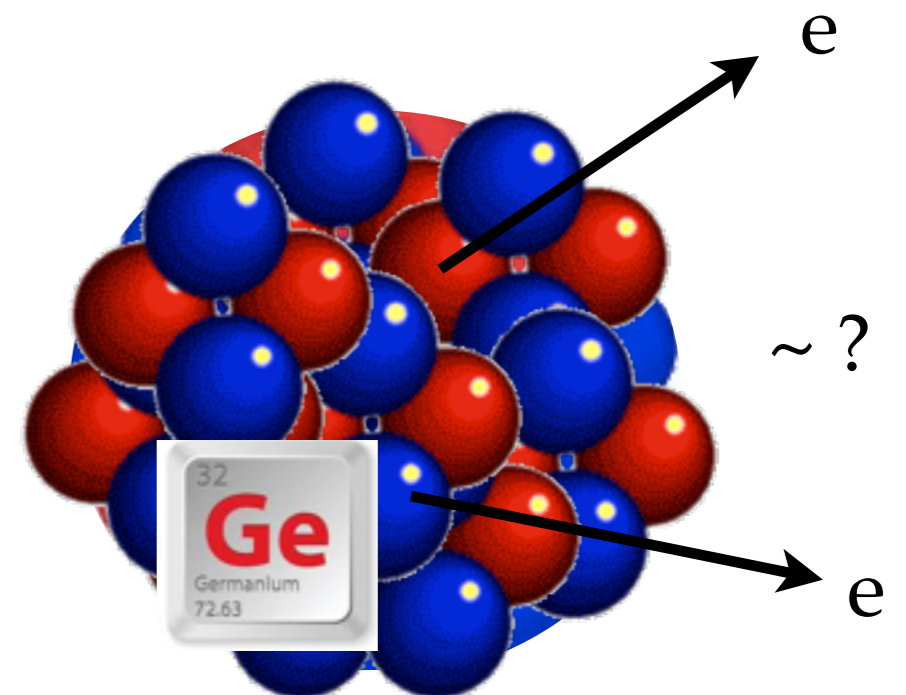
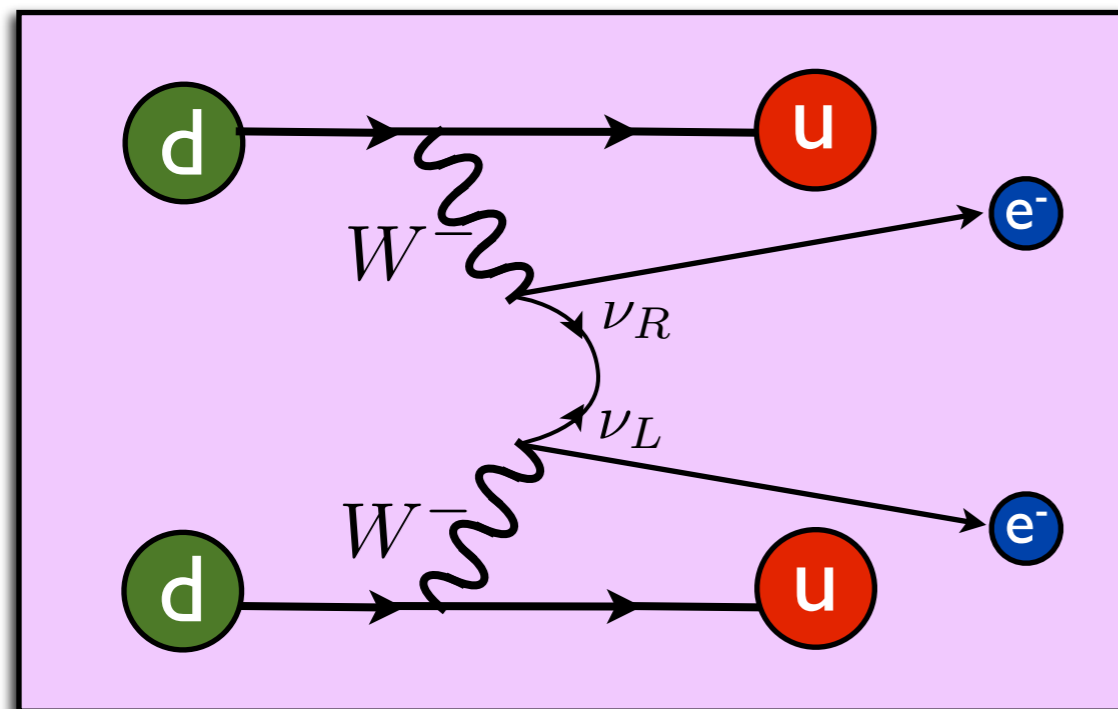


$0\nu\beta\beta$ experiments
could help
constrain R-parity
violating
coefficients



Relating Theory to Experiment

- LQCD: formulation of QCD in discretized, finite spacetime
- All errors are quantifiable and may be systematically removed
 - Extrapolations to continuum, infinite volume, physical quark mass
- LQCD can't directly calculate your favorite $0\nu\beta\beta$ isotope!



Why?

- Need enormous lattices
- Tiny energy splittings
- Large range of scales

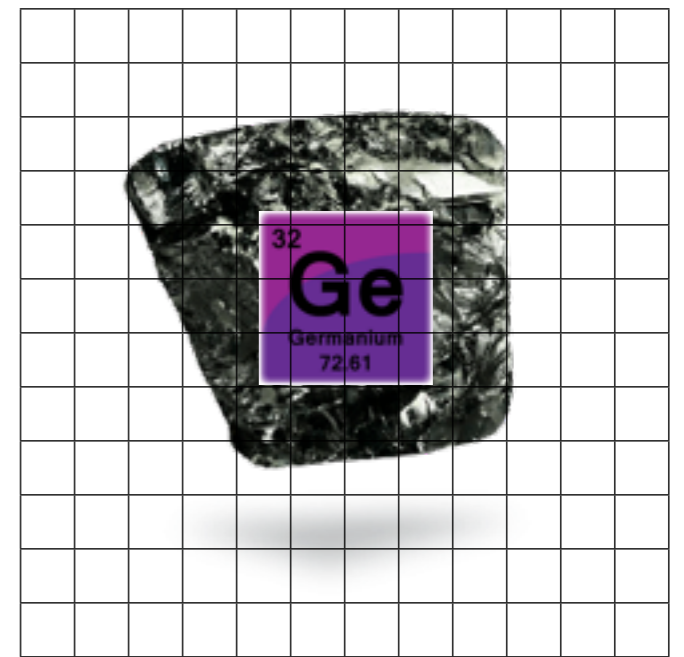
- Wick contractions:

$$(A+Z)! \times (2A-Z)! \quad \boxed{\text{He}^4 : 518400}$$

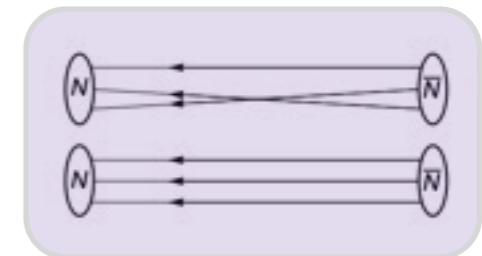
- **Nucleon noise/sign problem**

signal/noise \sim

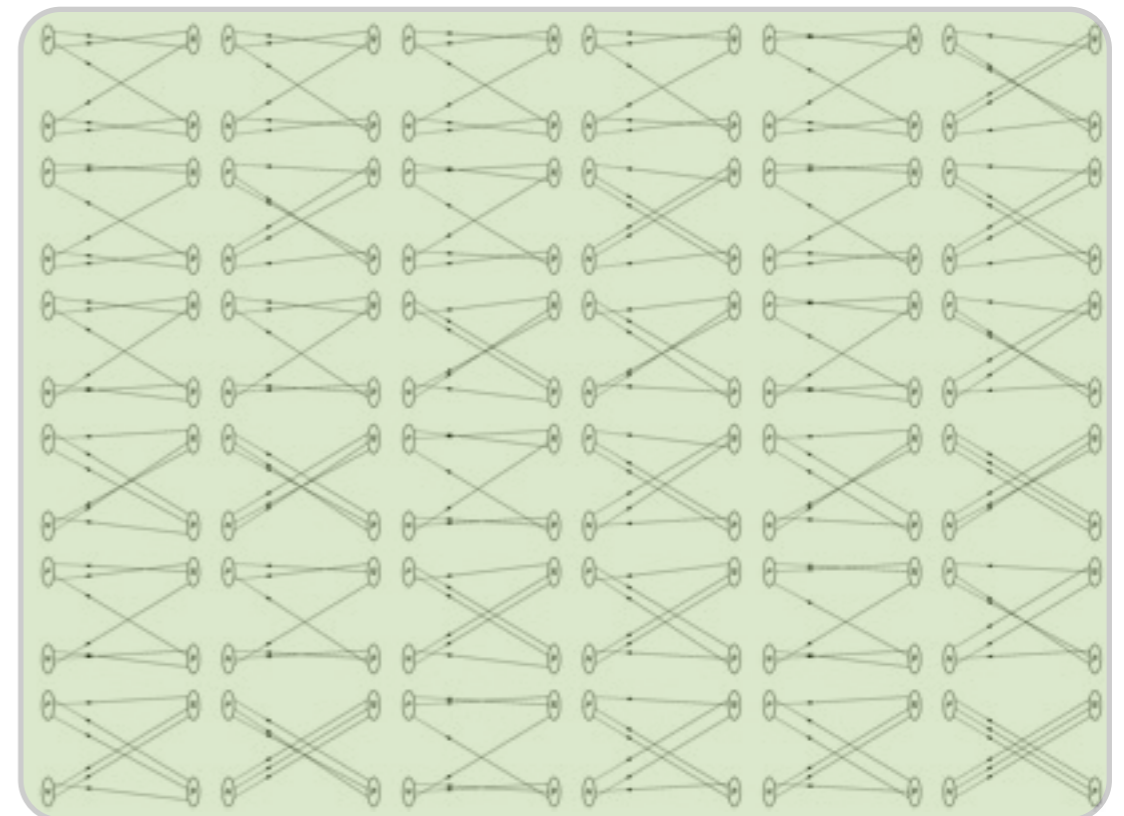
$$e^{-A(m_N - 3/2m_\pi)t}$$



Nucleon:



Deuteron:



Why?

- Need enormous lattices
- Tiny energy splittings
- Large range of scales

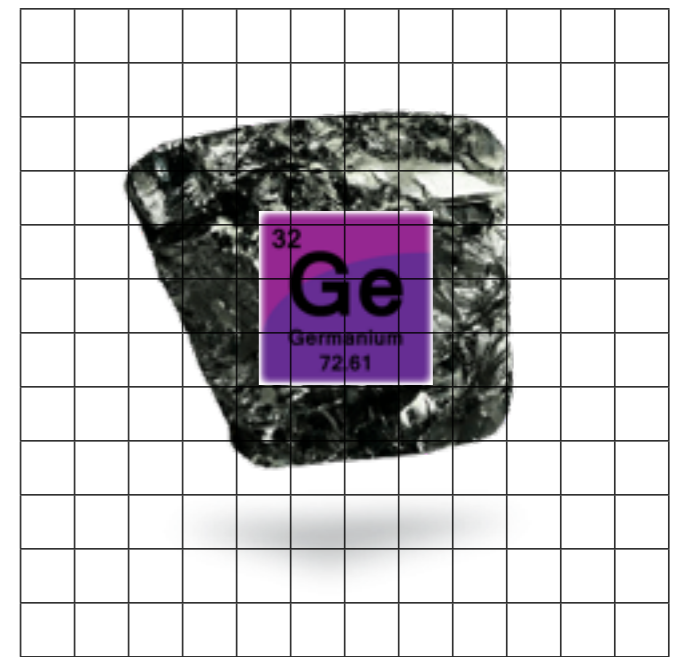
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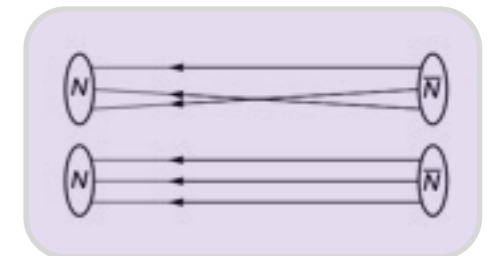
- **Nucleon noise/sign problem**

signal/noise \sim

$$e^{-A(m_N - 3/2m_\pi)t}$$



Nucleon:

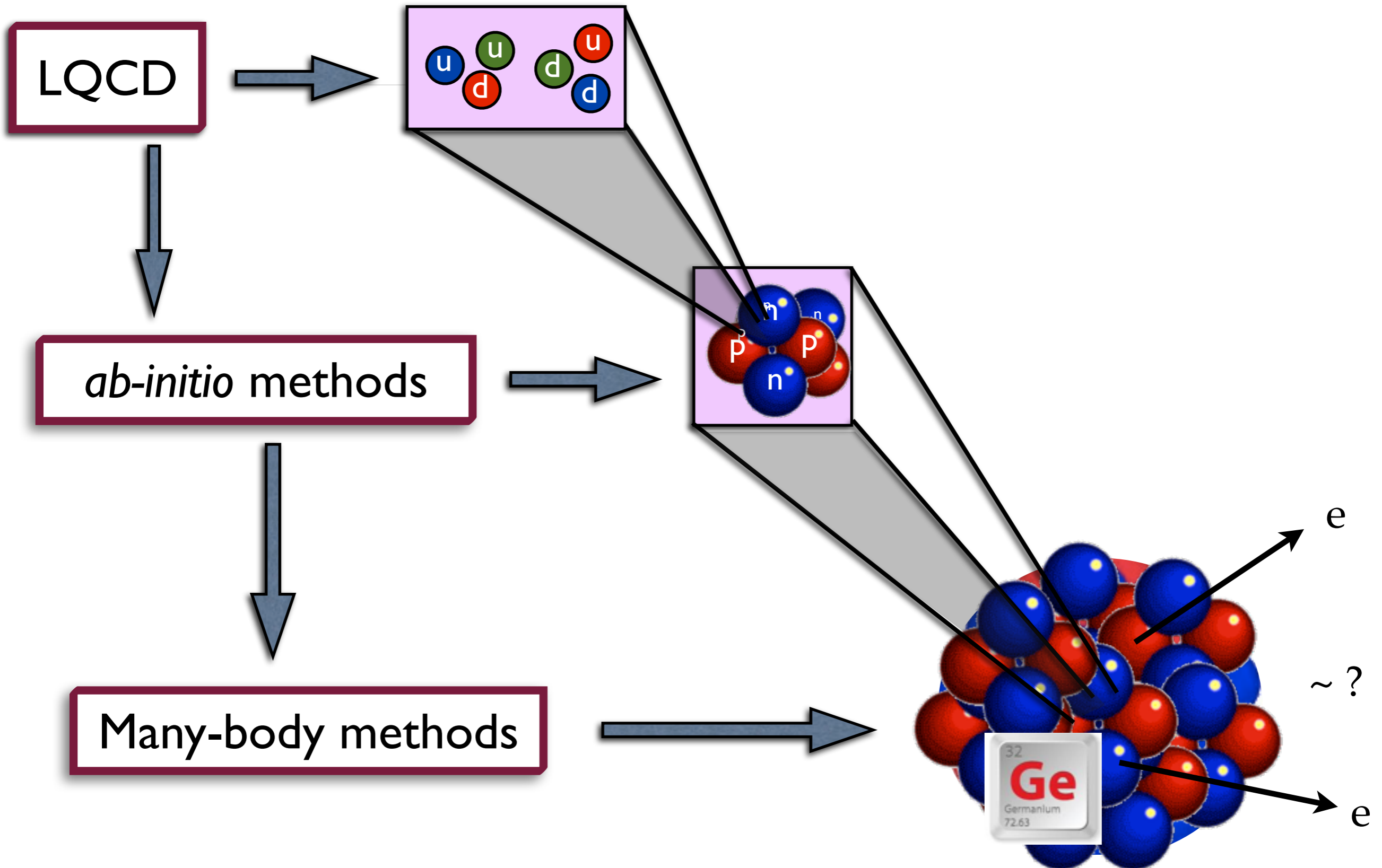


Deuteron:



Most calculations done at unphysically heavy quark (pion) masses - need theory to extrapolate in m_π

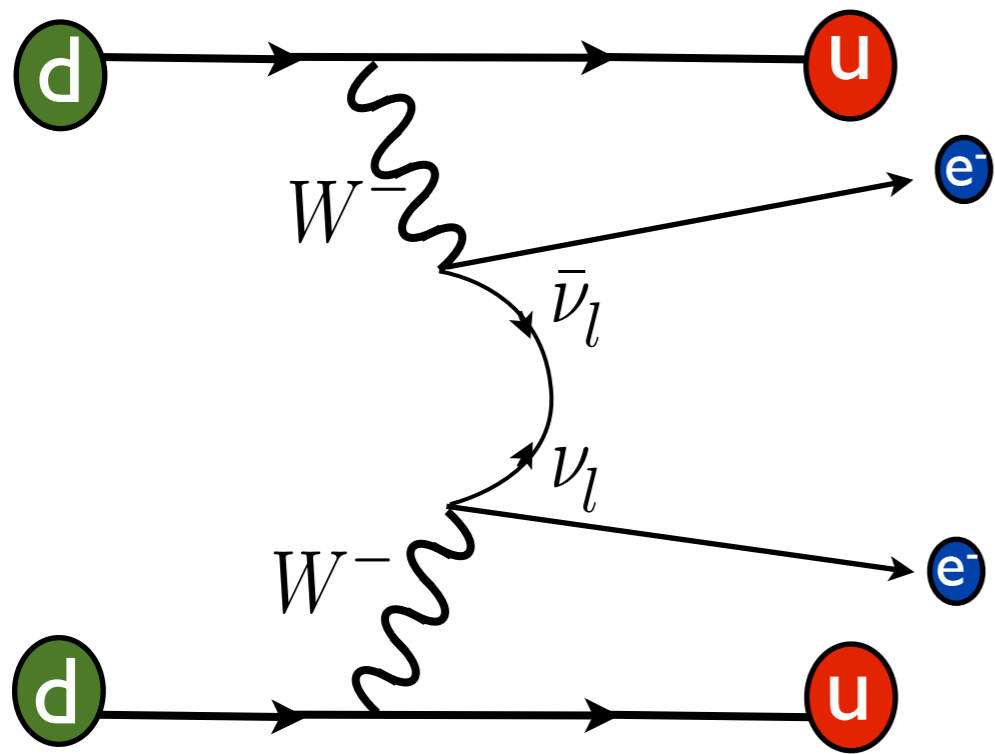
Relating Theory to Experiment



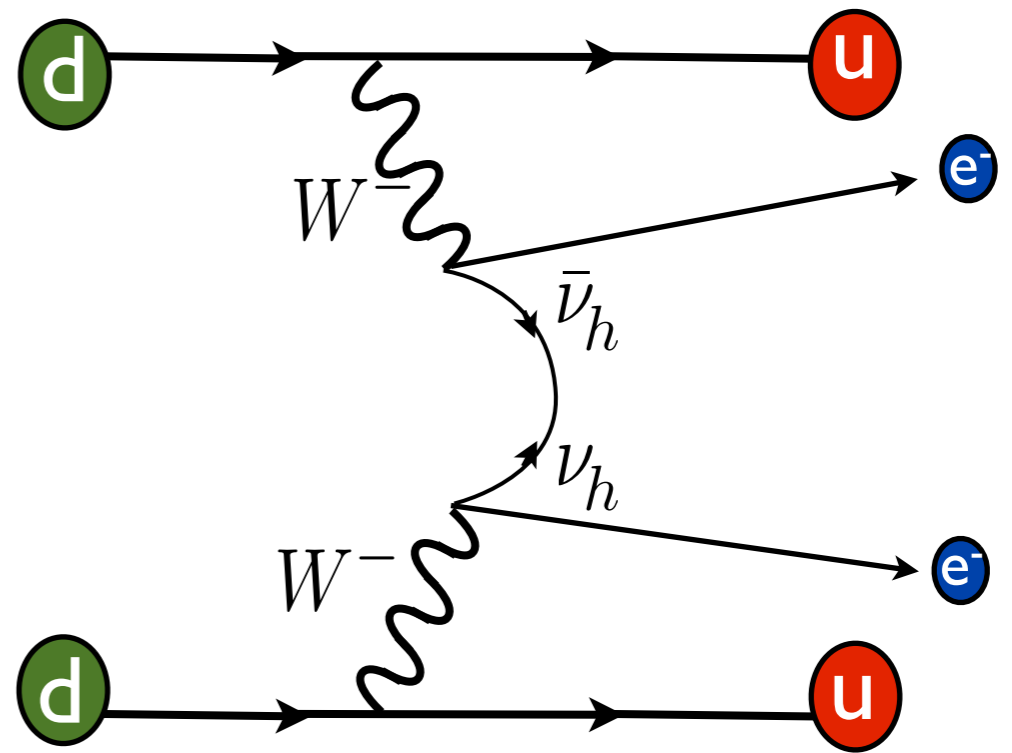
Lattice QCD

contributions to $0\nu\beta\beta$

- Long-range
 - Axial charge of the nucleon
- Short-range
 - Leading order single pion exchange contribution
 - Two-nucleon matrix elements



Long-range

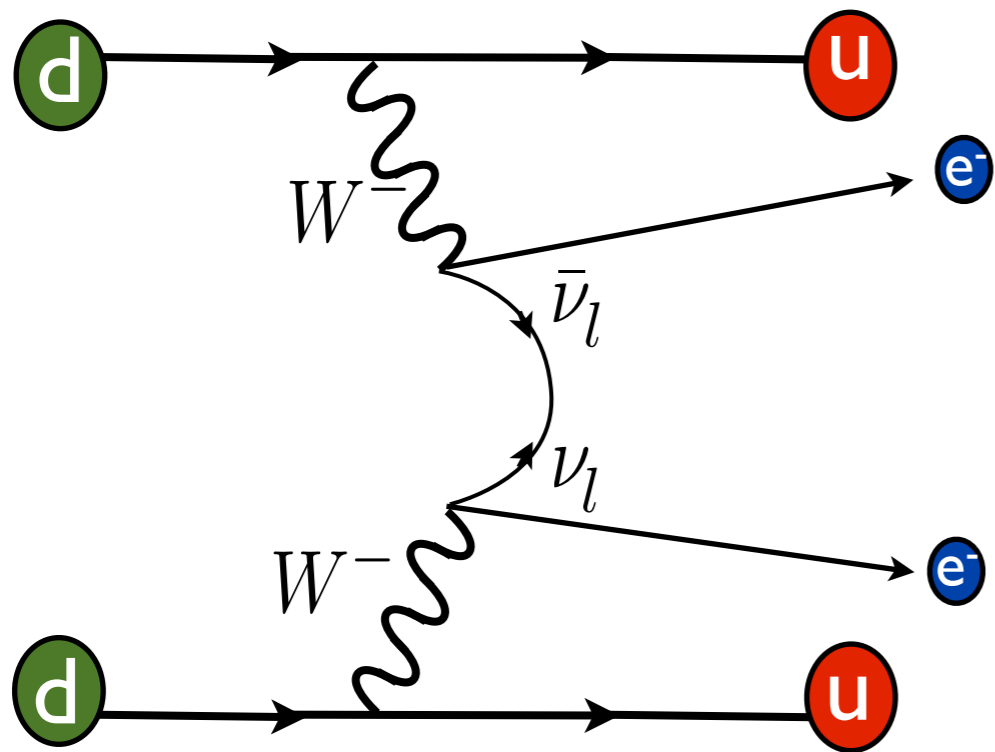


Short-range

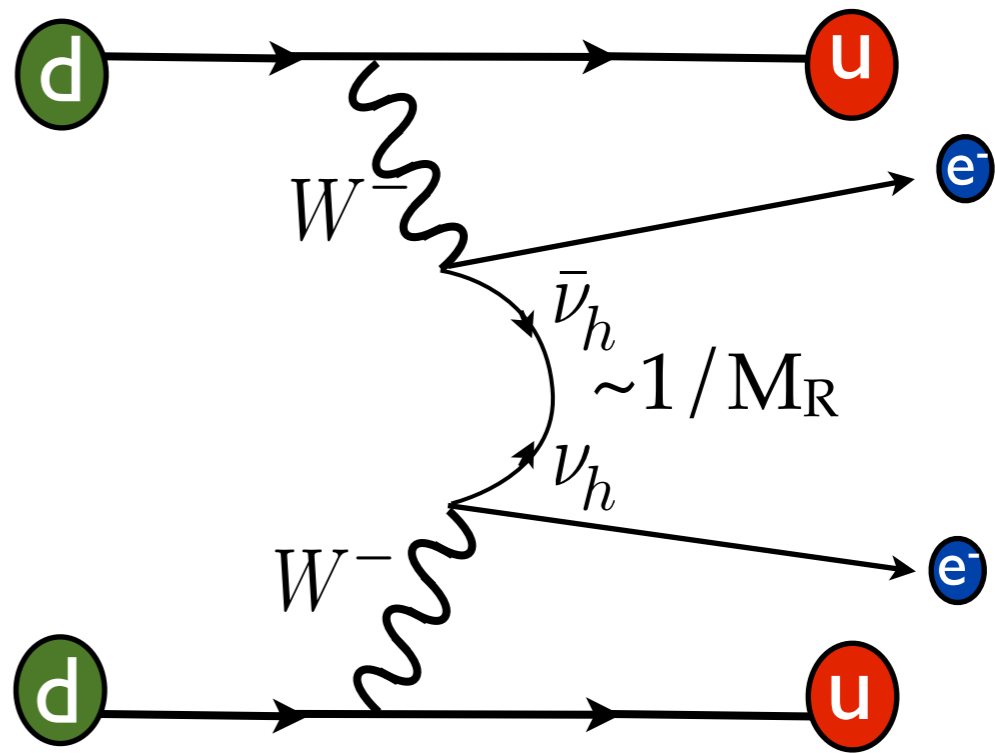


$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

$$m_l \sim M_D^2 / M_R \quad m_h \sim M_R$$



Long-range

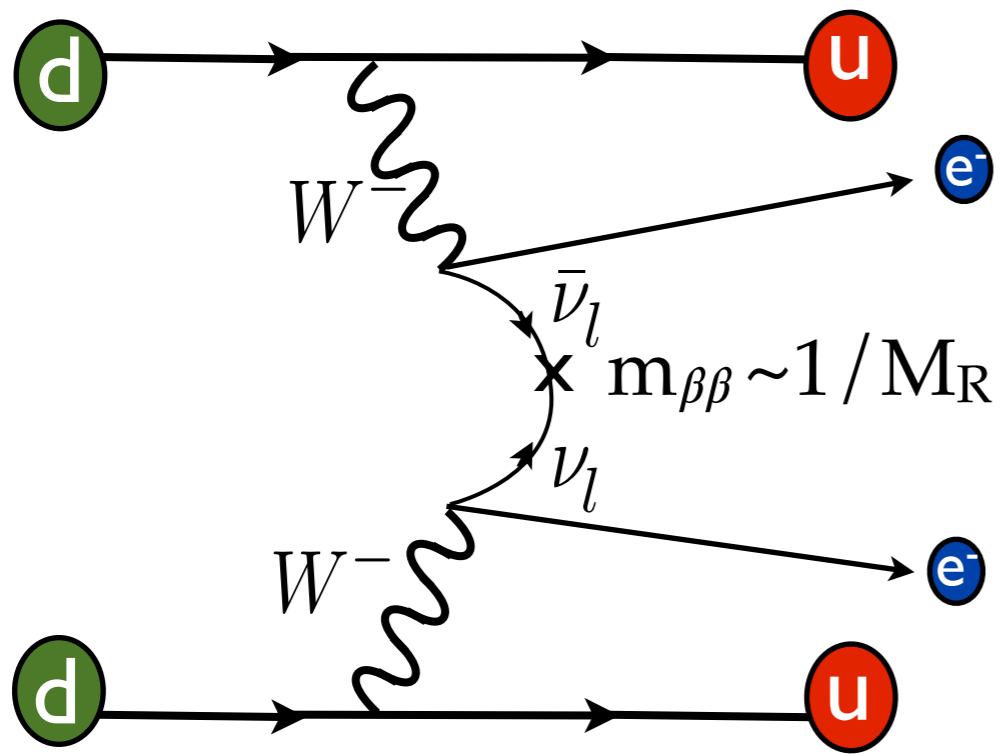


Short-range

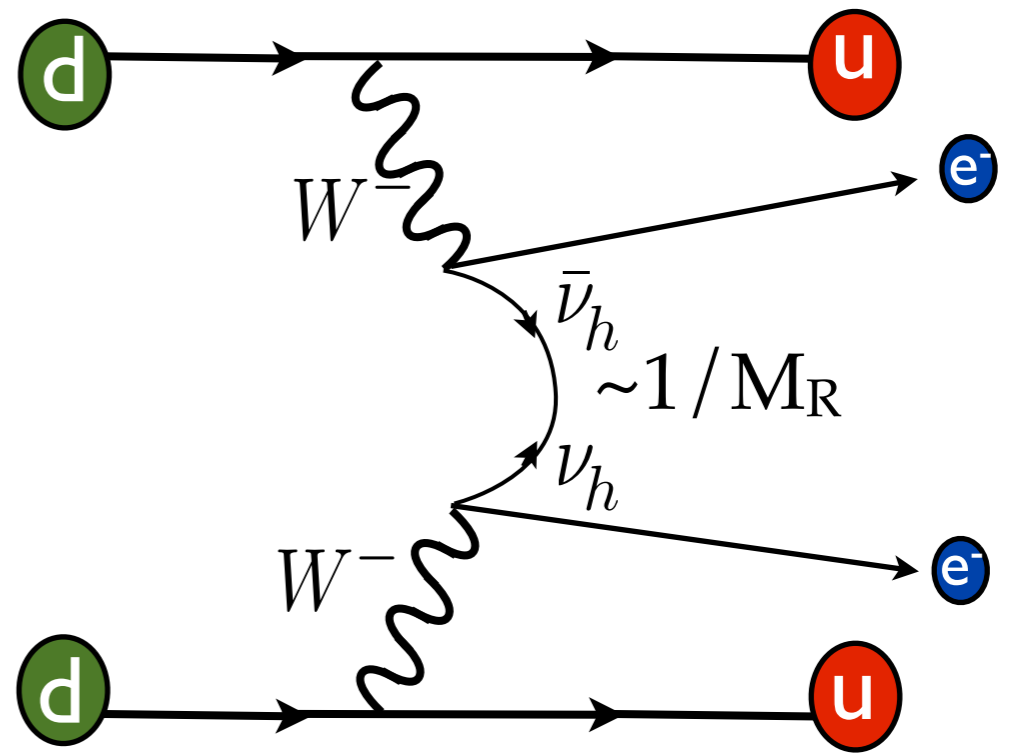


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Long-range

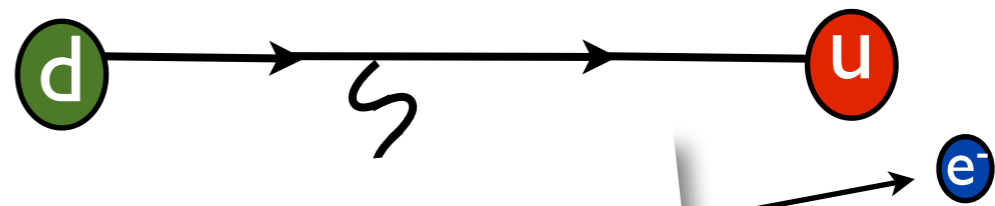
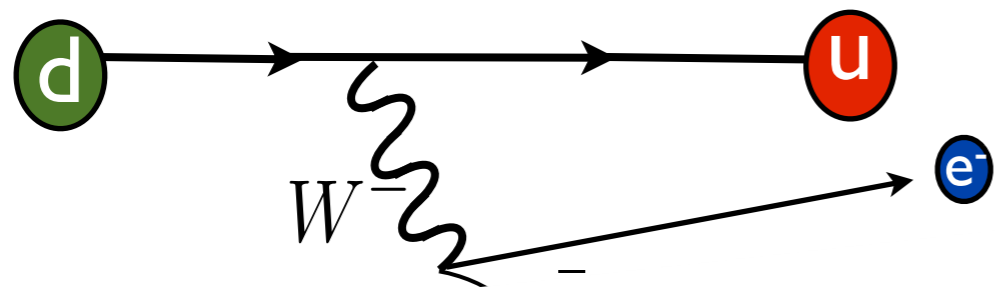


Short-range



$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

$$m_l \sim M_D^2/M_R \quad m_h \sim M_R$$



Which type dominates depends on details of BSM model



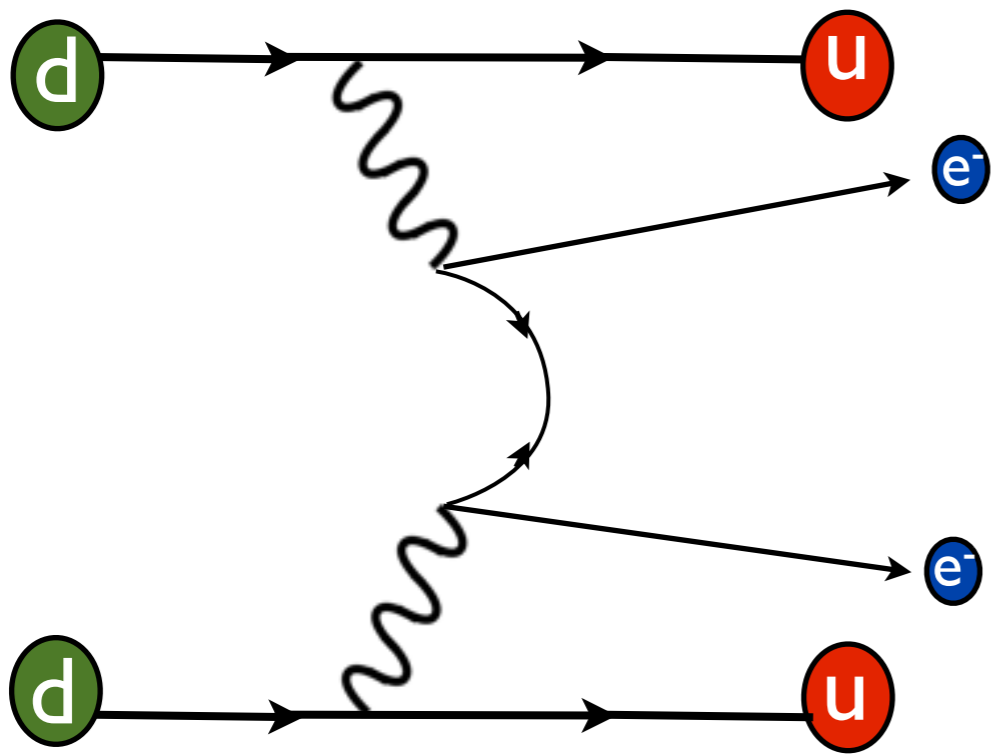
Long-range

Short-range

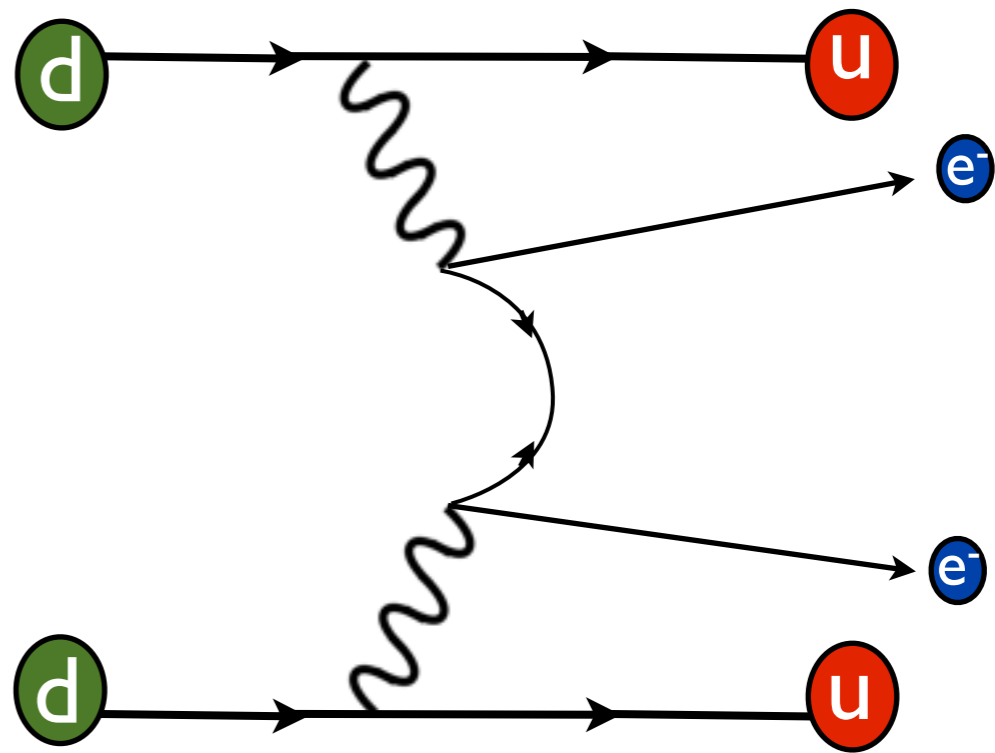


$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

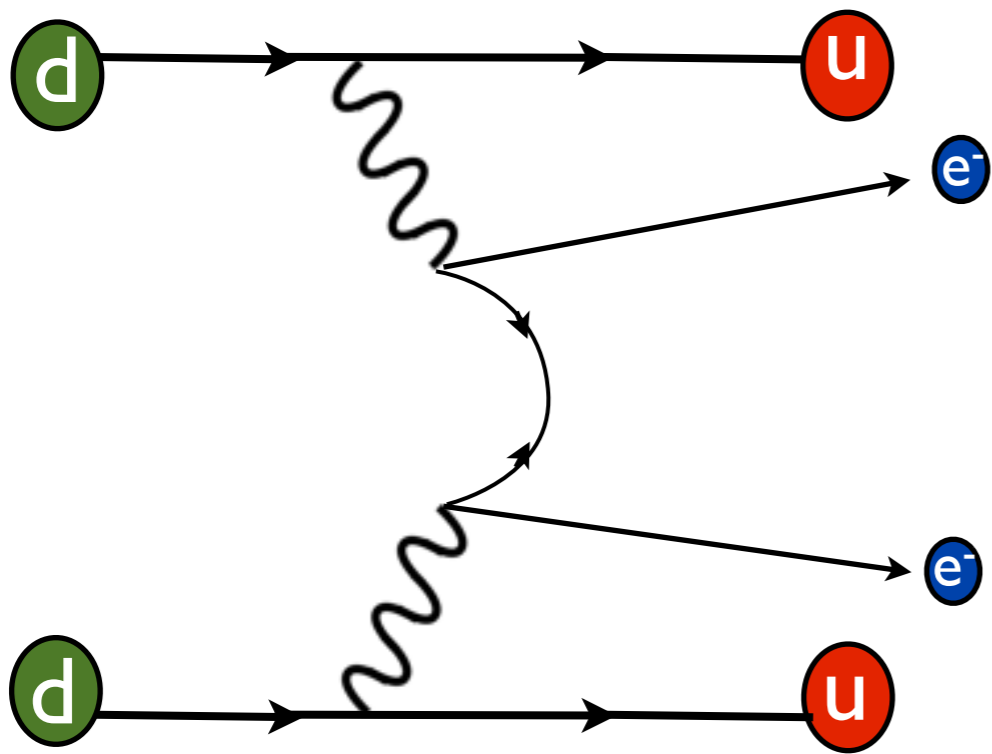
$$m_l \sim M_D^2 / M_R \quad m_h \sim M_R$$



Long-range

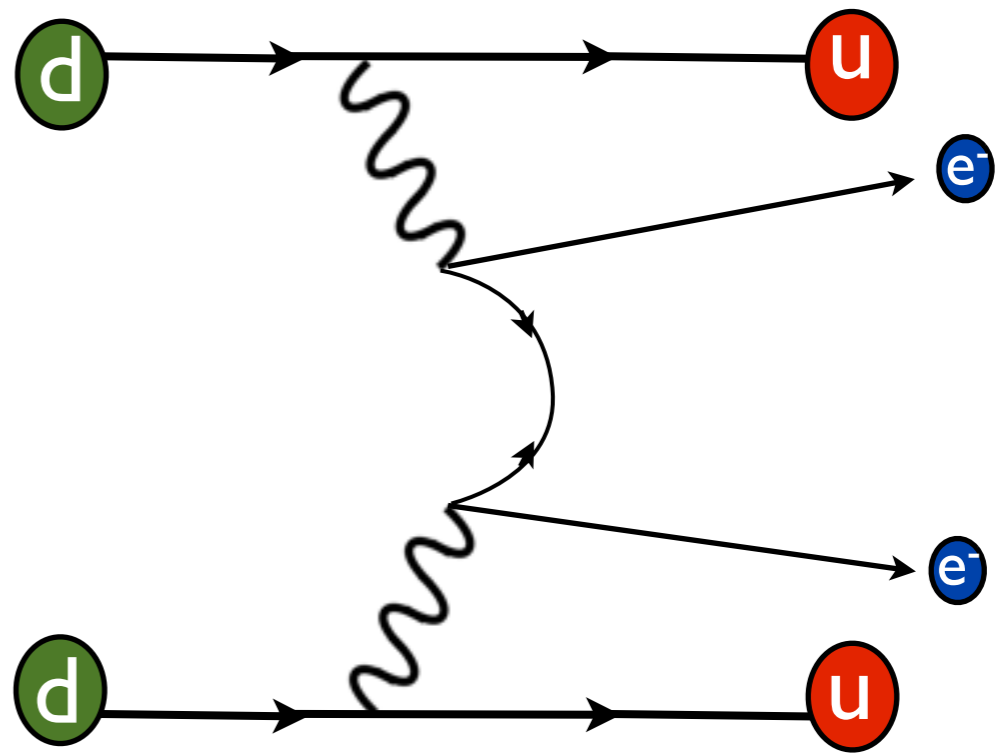


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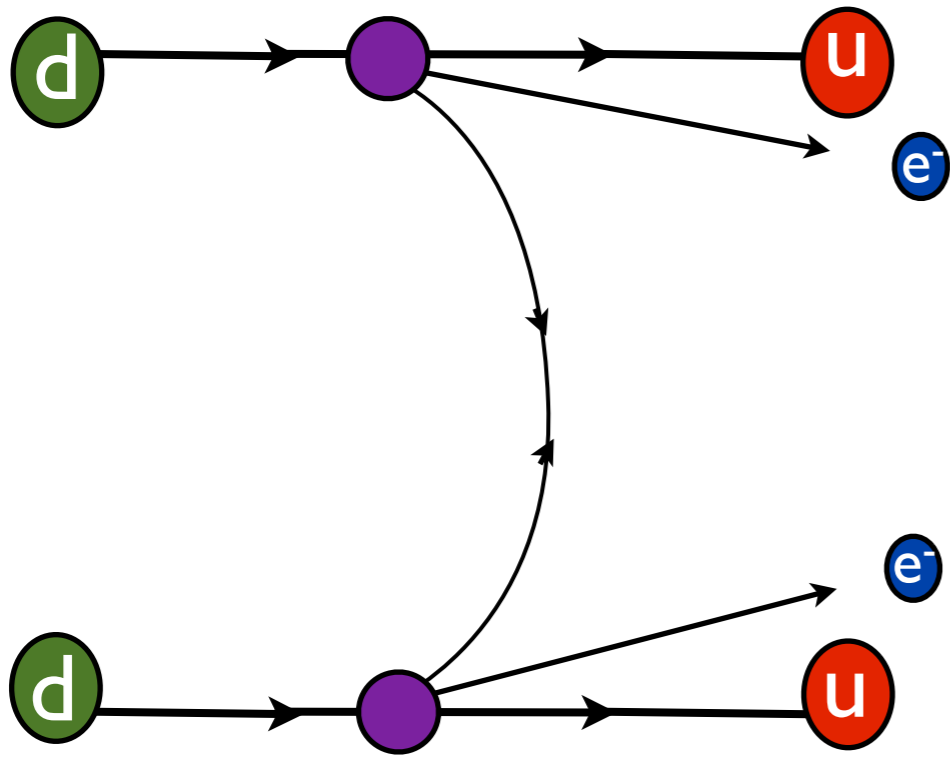


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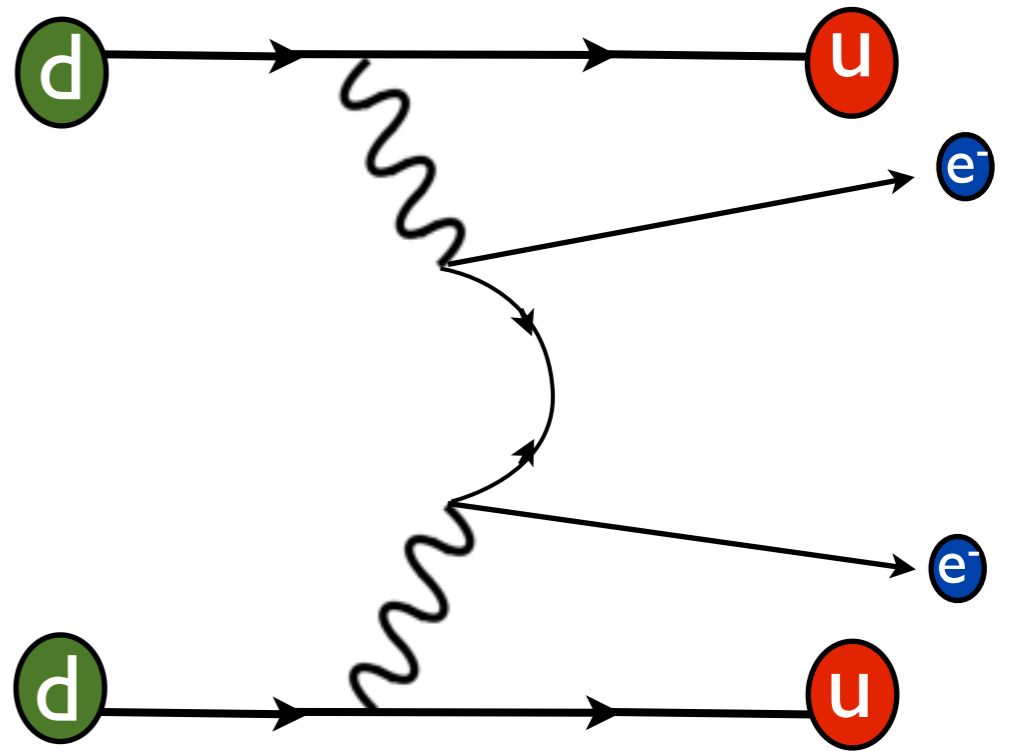
$$\Lambda \ll M_W$$



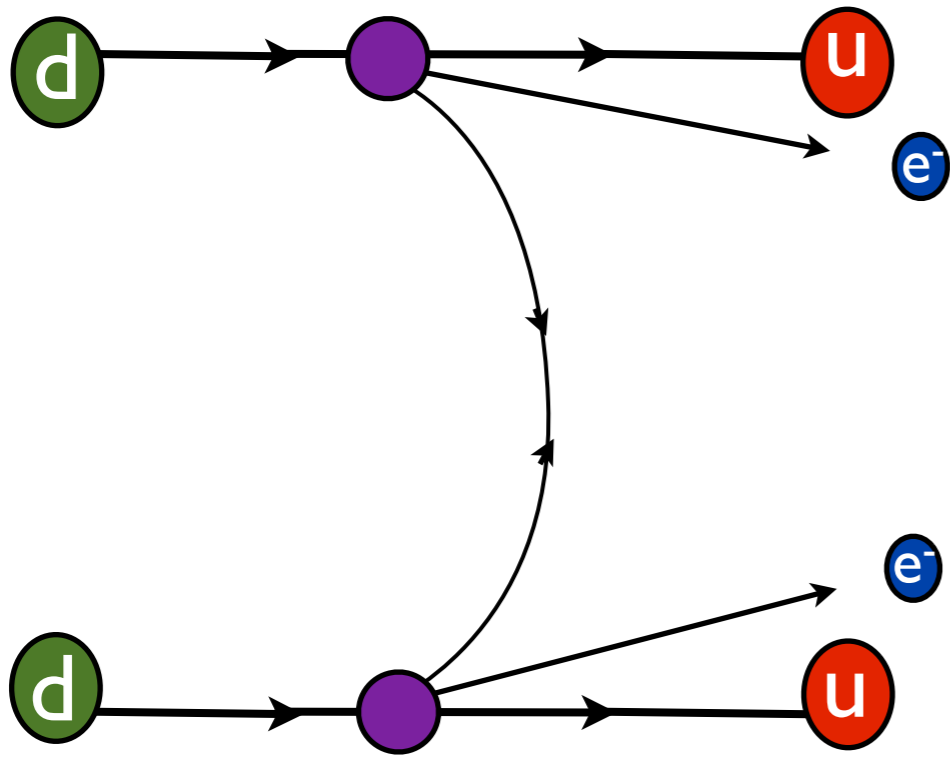
Short-range



Long-range

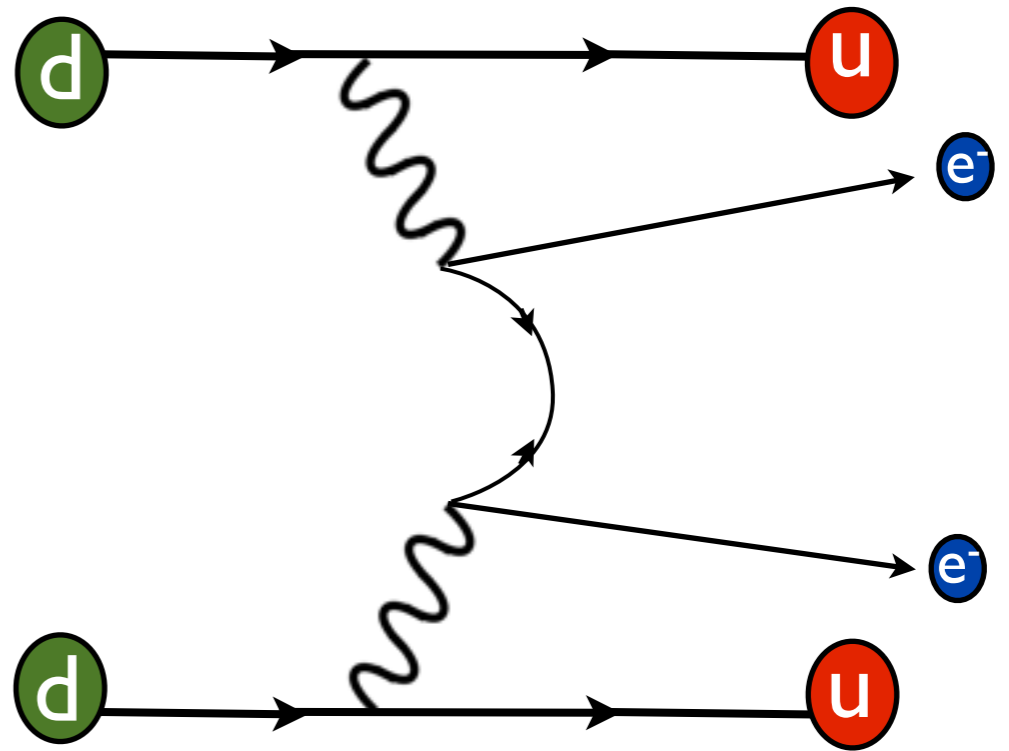


Short-range

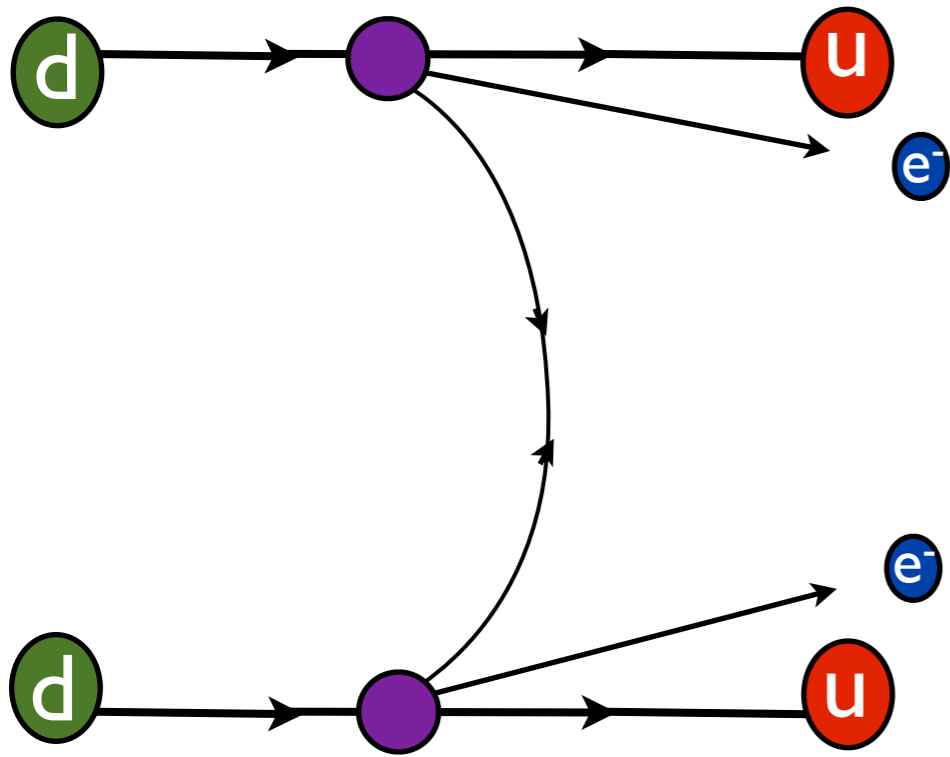


Long-range

$$\Lambda \ll \Lambda_{\text{QCD}}$$

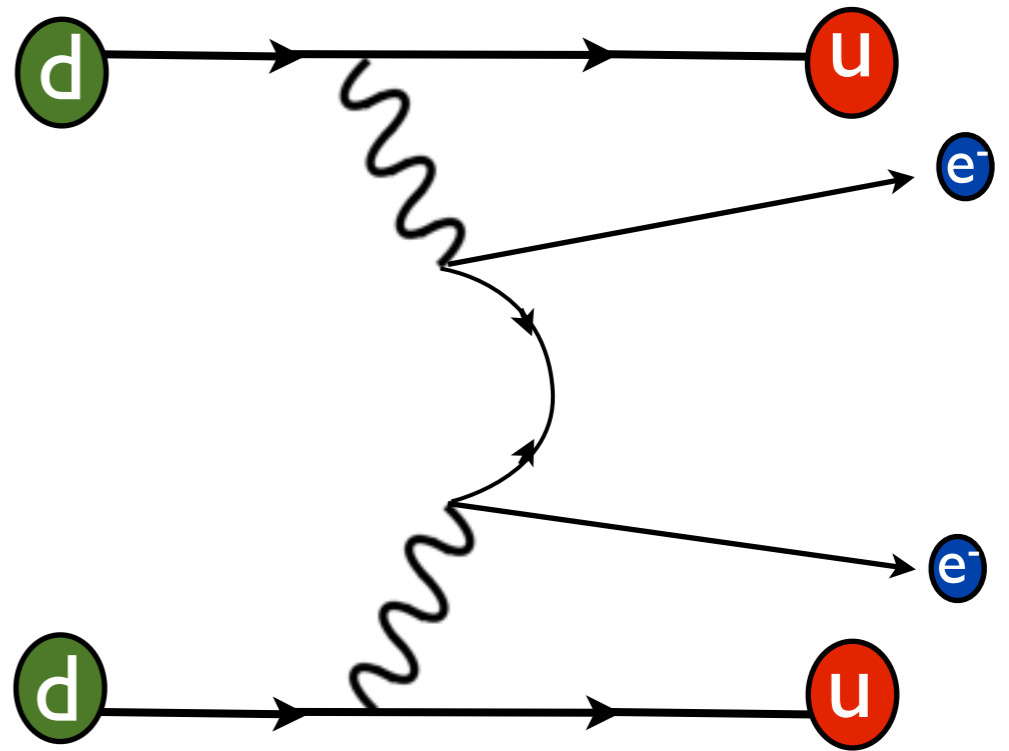


Short-range

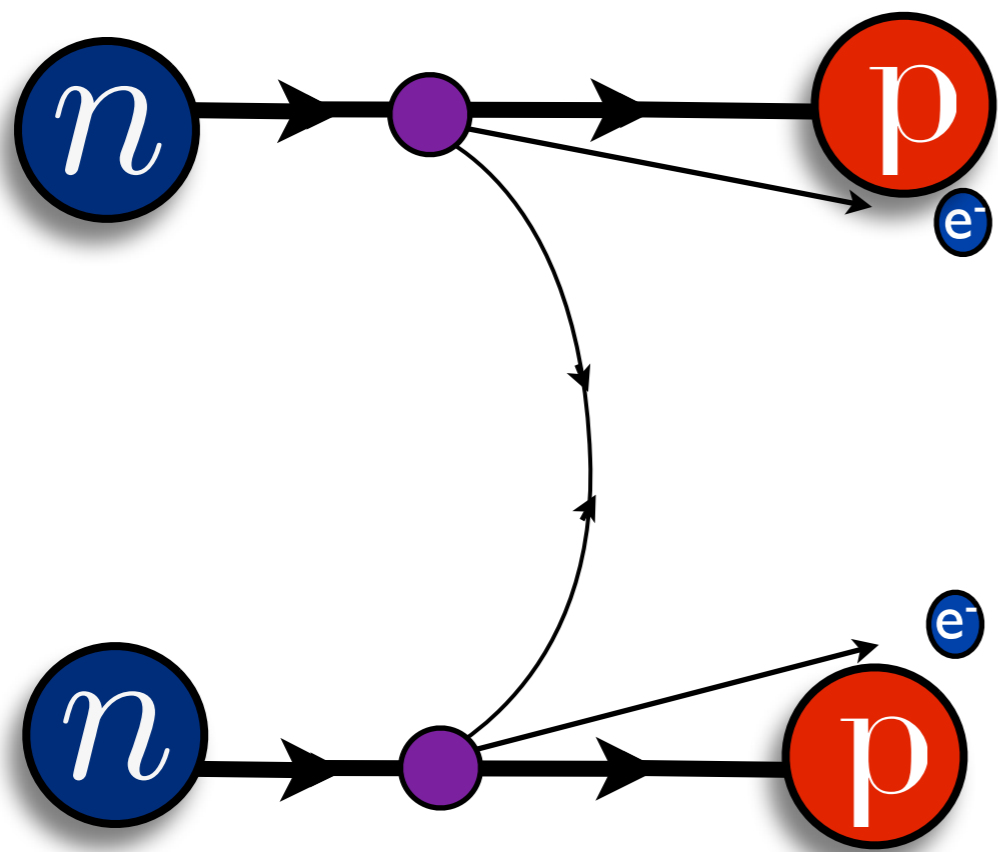


Long-range

$\Lambda \ll \Lambda_{\text{QCD}}$

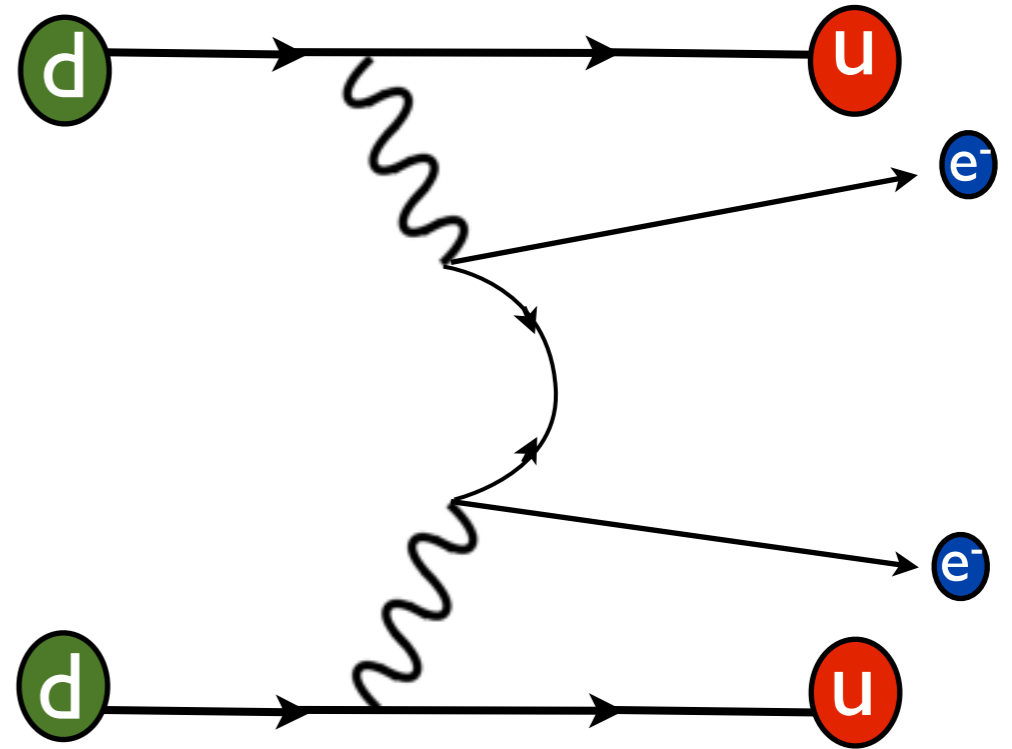


Short-range

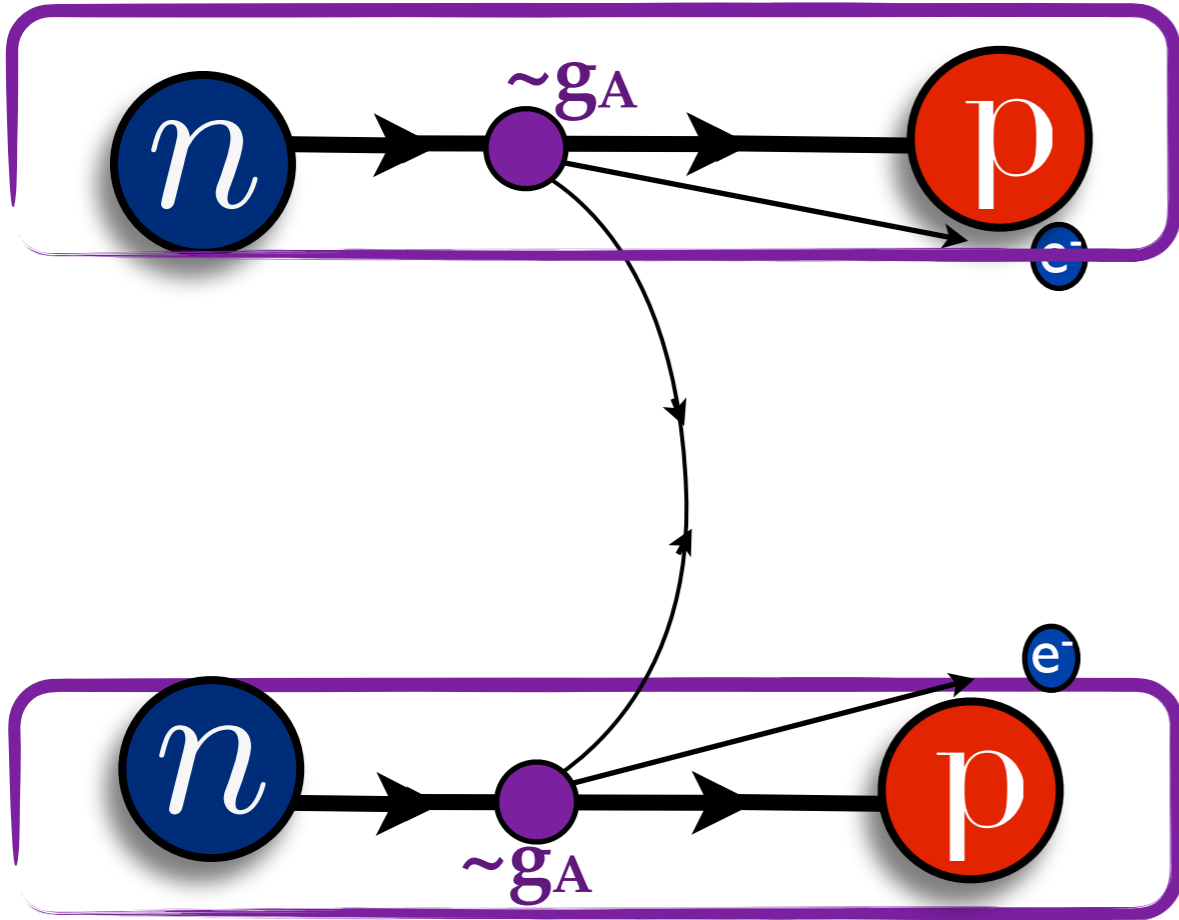


Long-range

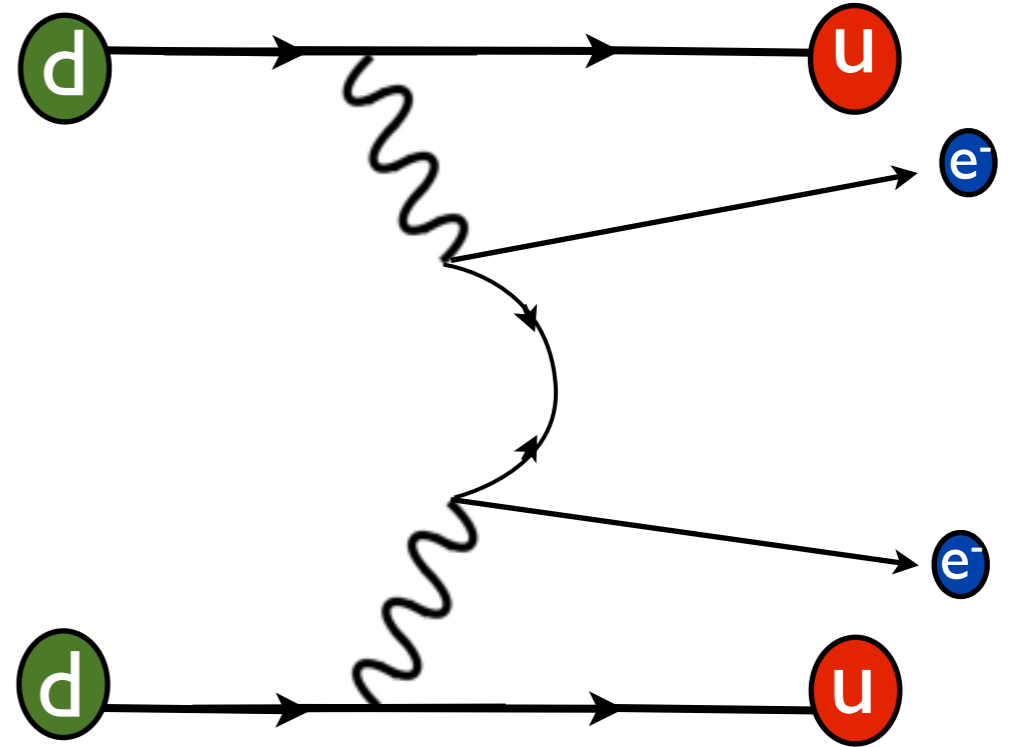
$$\Lambda \ll \Lambda_{\text{QCD}}$$



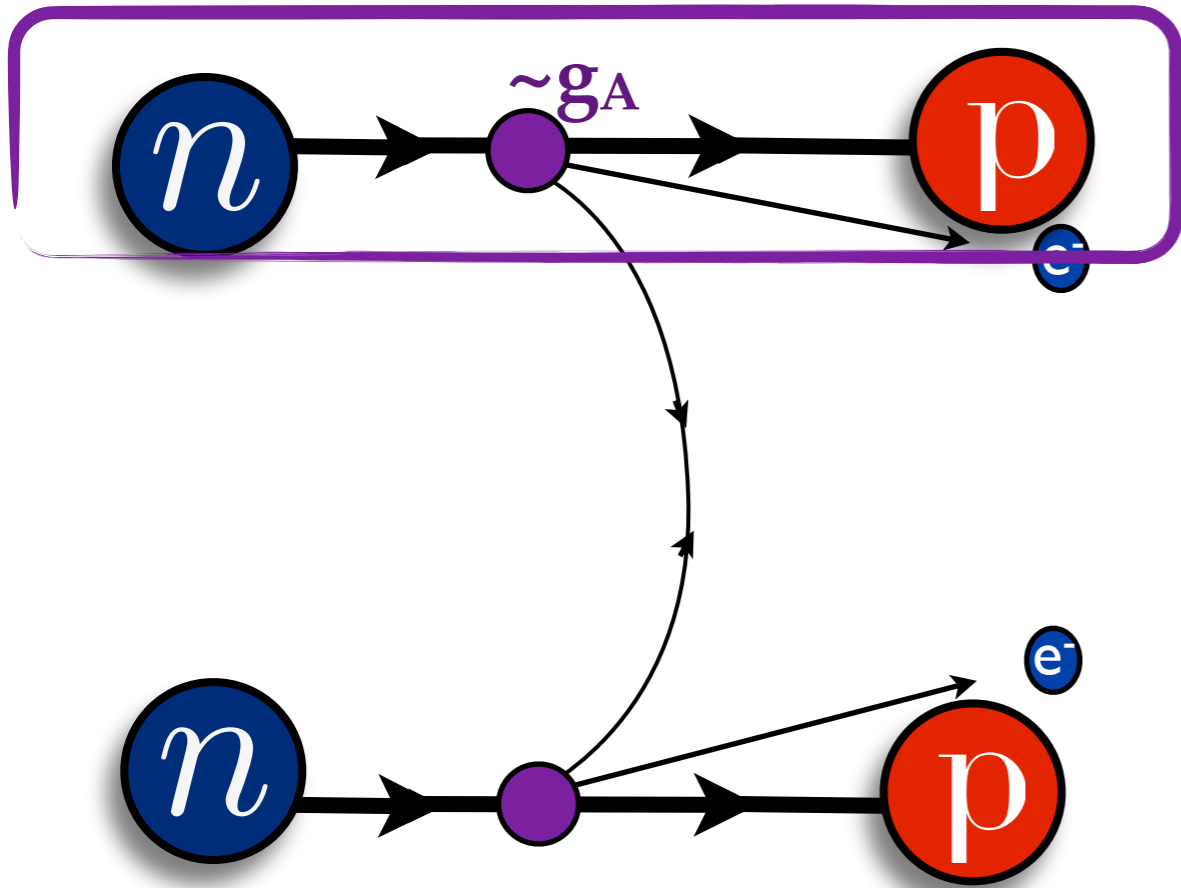
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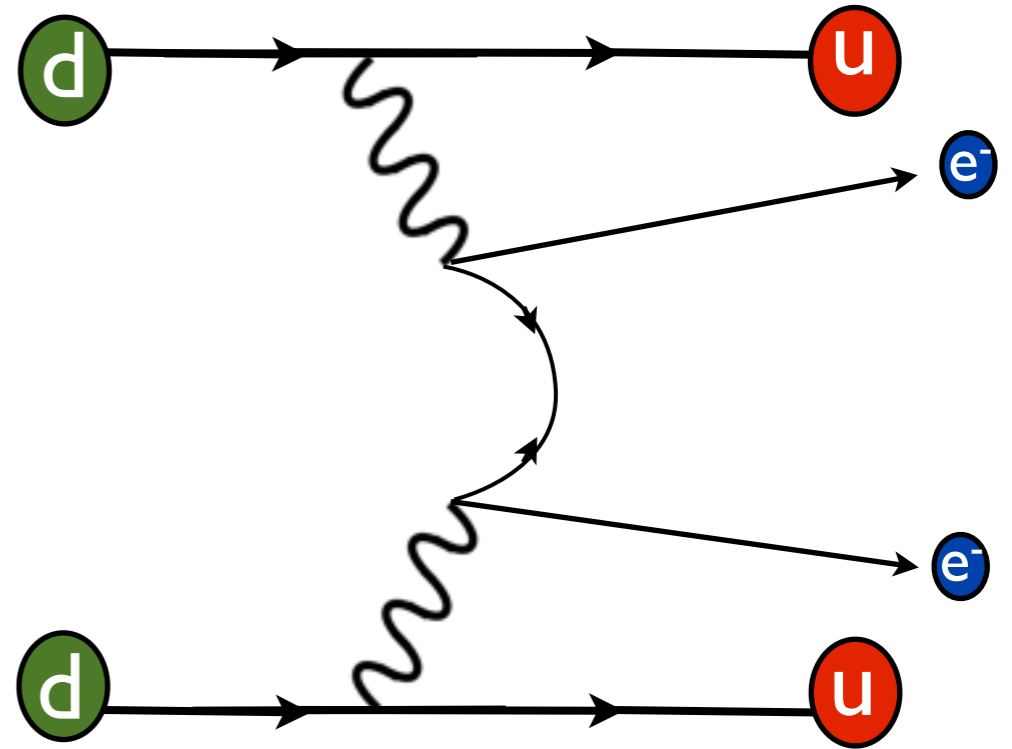
Long-range



Short-range

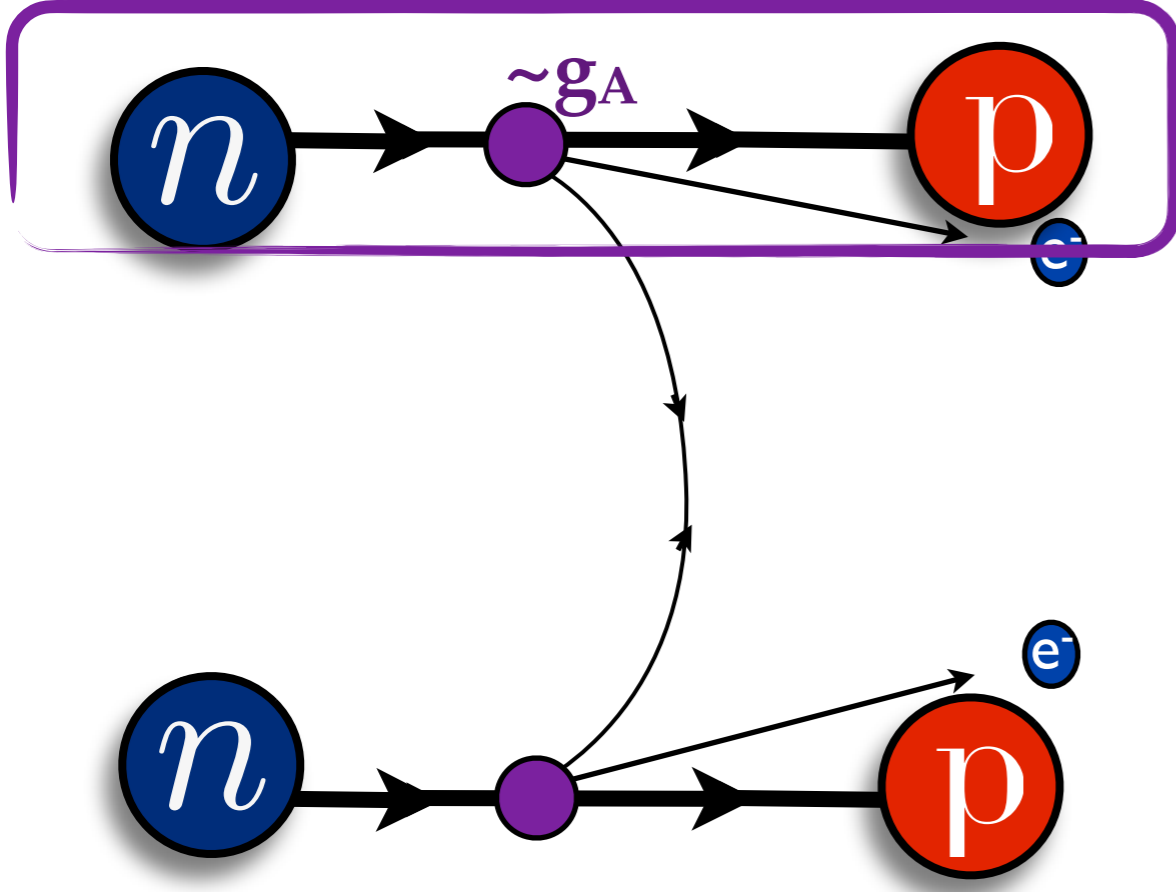


Long-range

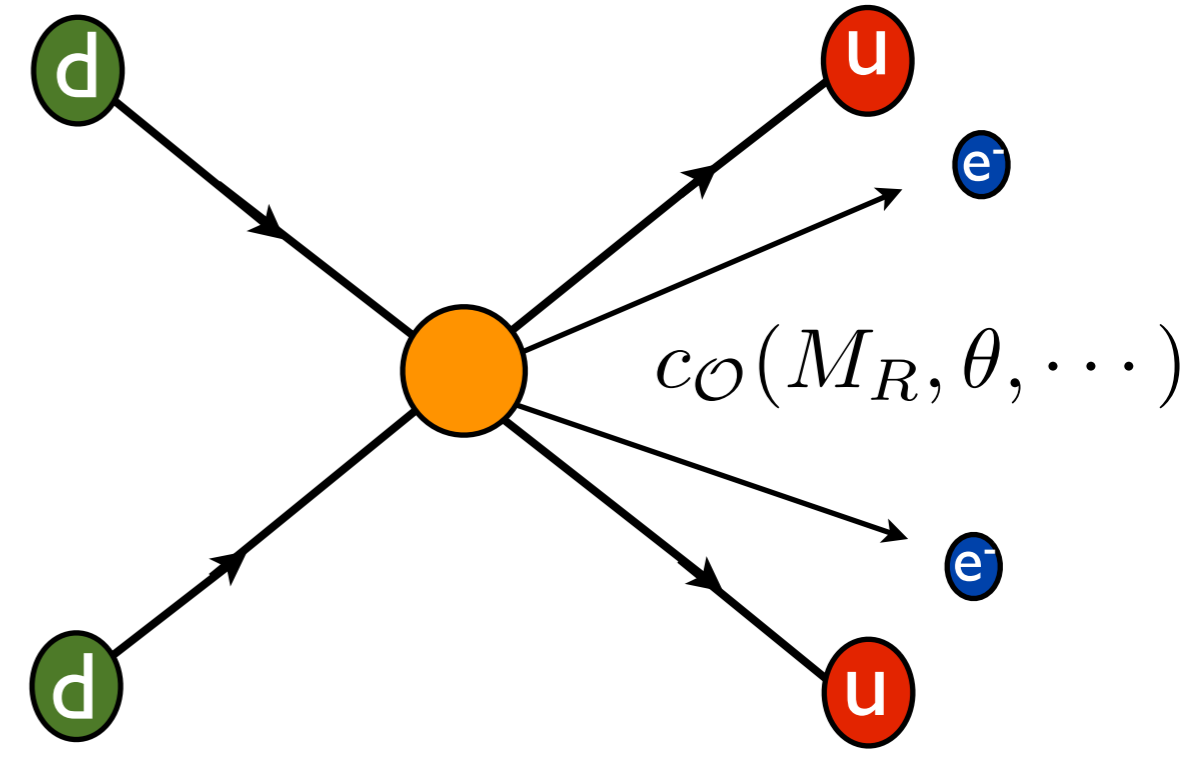


Short-range

$$\Lambda \ll M_W$$



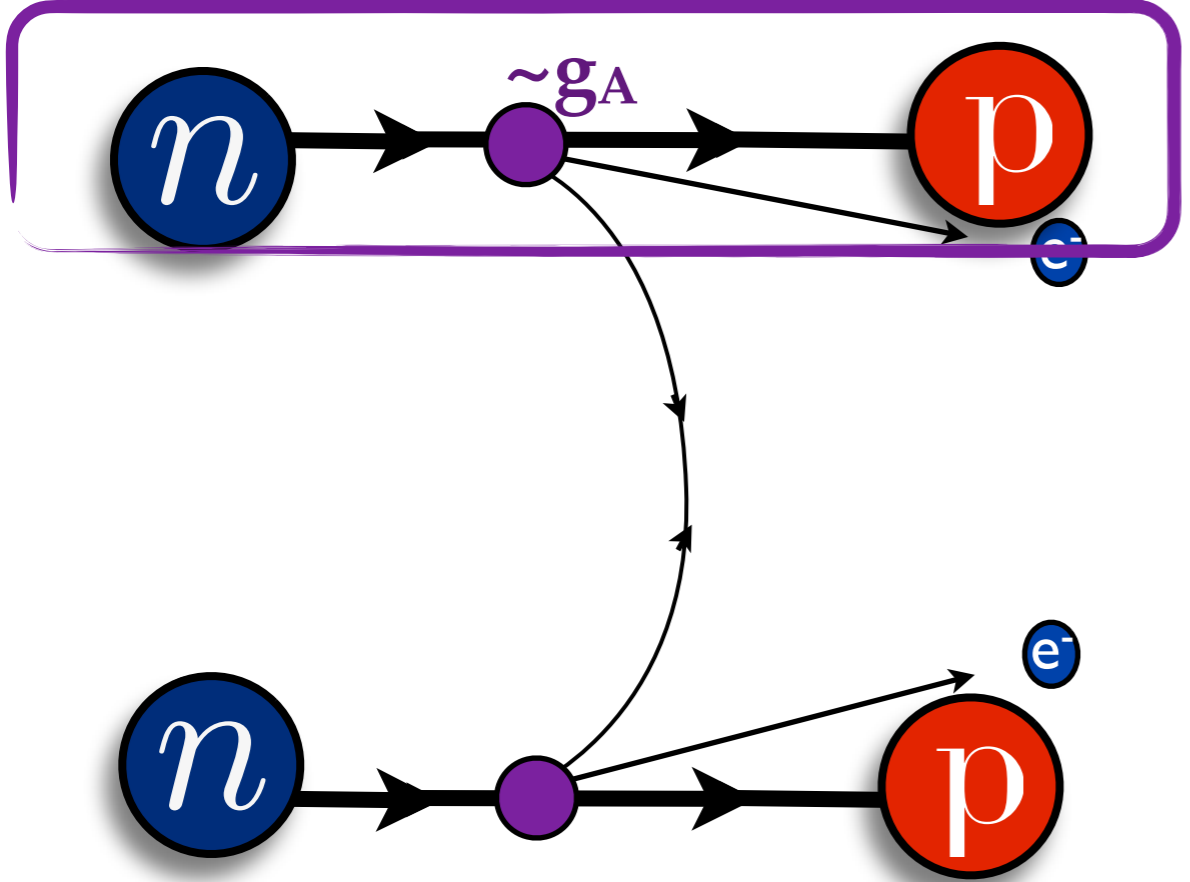
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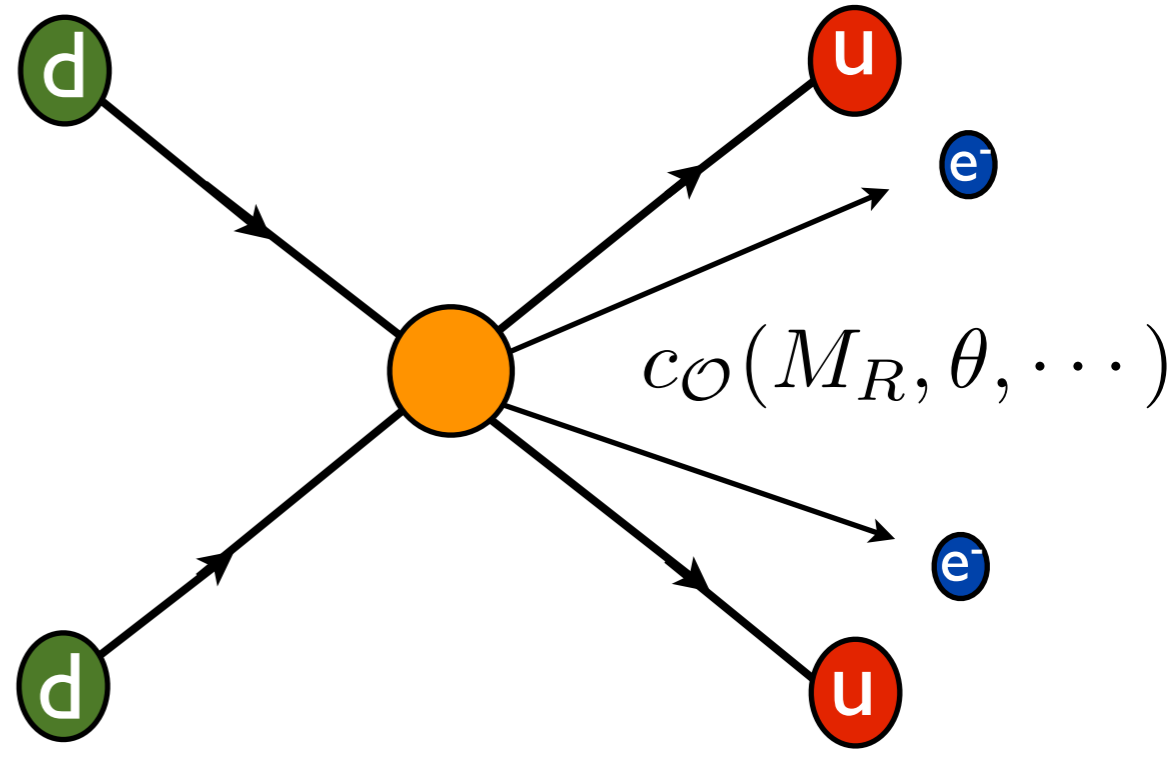
Short-range

$$\begin{aligned} \mathcal{O}_{1+}^{ab} &= (\bar{q}_L \tau^a \gamma^\mu q_L) (\bar{q}_R \tau^b \gamma_\mu q_R), \\ \mathcal{O}_{2\pm}^{ab} &= (\bar{q}_R \tau^a q_L) (\bar{q}_R \tau^b q_L) \pm (\bar{q}_L \tau^a q_R) (\bar{q}_L \tau^b q_R), \\ \mathcal{O}_{3\pm}^{ab} &= (\bar{q}_L \tau^a \gamma^\mu q_L) (\bar{q}_L \tau^b \gamma_\mu q_L) \pm (\bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_R \tau^b \gamma_\mu q_R), \\ \mathcal{O}_{4\pm}^{ab,\mu} &= (\bar{q}_L \tau^a \gamma^\mu q_L \mp \bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_L \tau^b q_R - \bar{q}_R \tau^b q_L), \\ \mathcal{O}_{5\pm}^{ab,\mu} &= (\bar{q}_L \tau^a \gamma^\mu q_L \pm \bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_L \tau^b q_R + \bar{q}_R \tau^b q_L). \end{aligned}$$

Prezeau, Ramsey-Musolf, Vogel (2003)

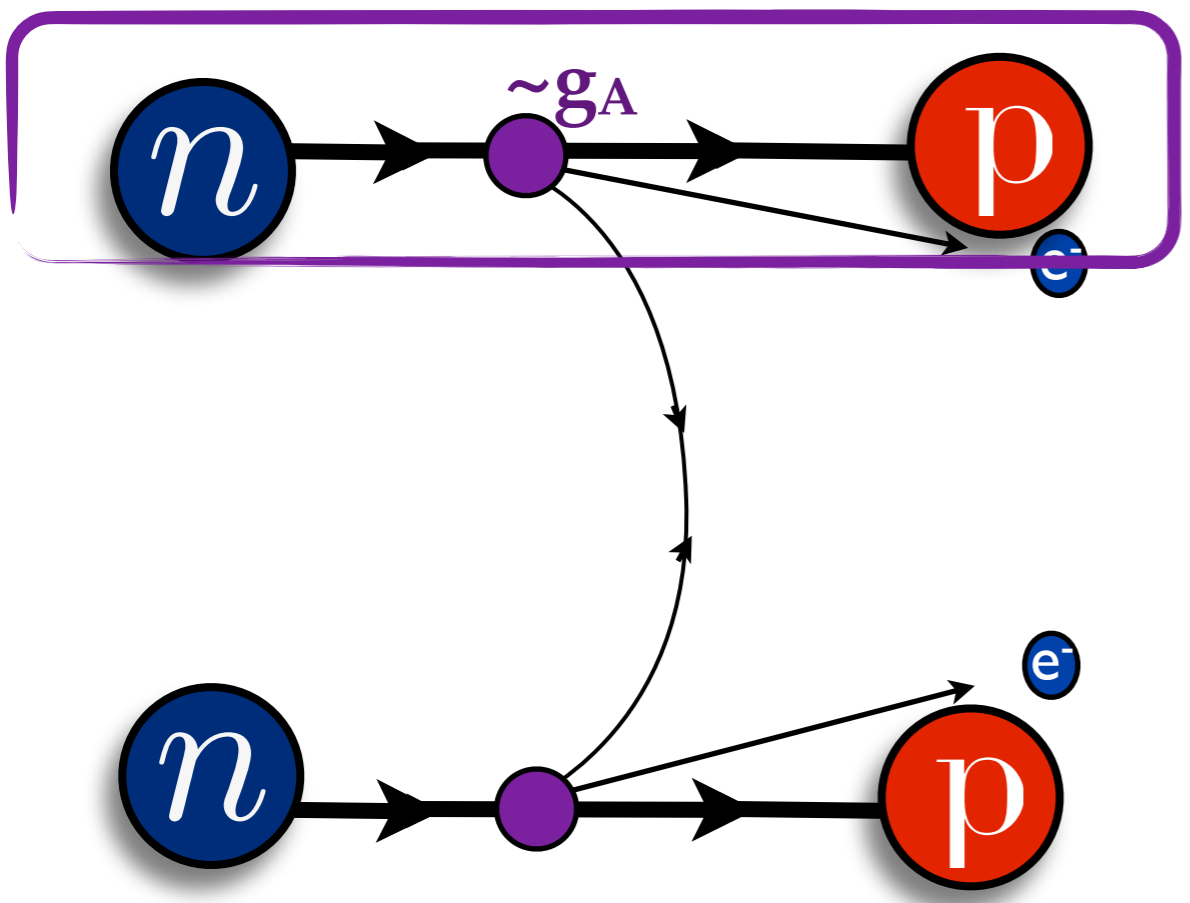


Long-range

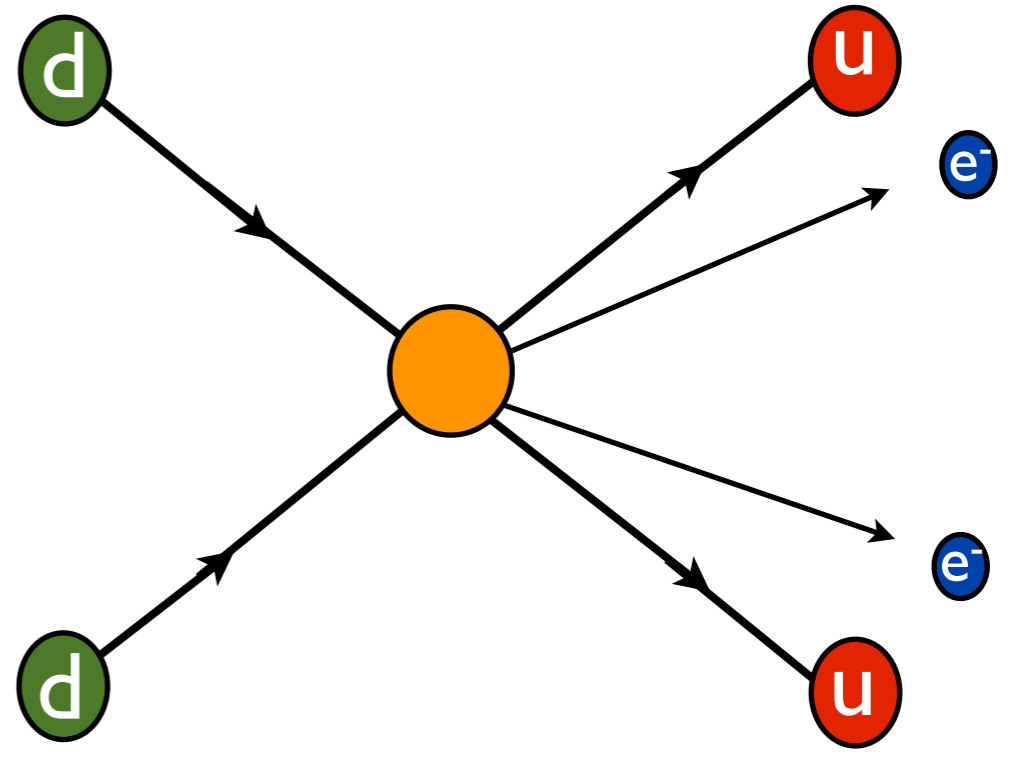


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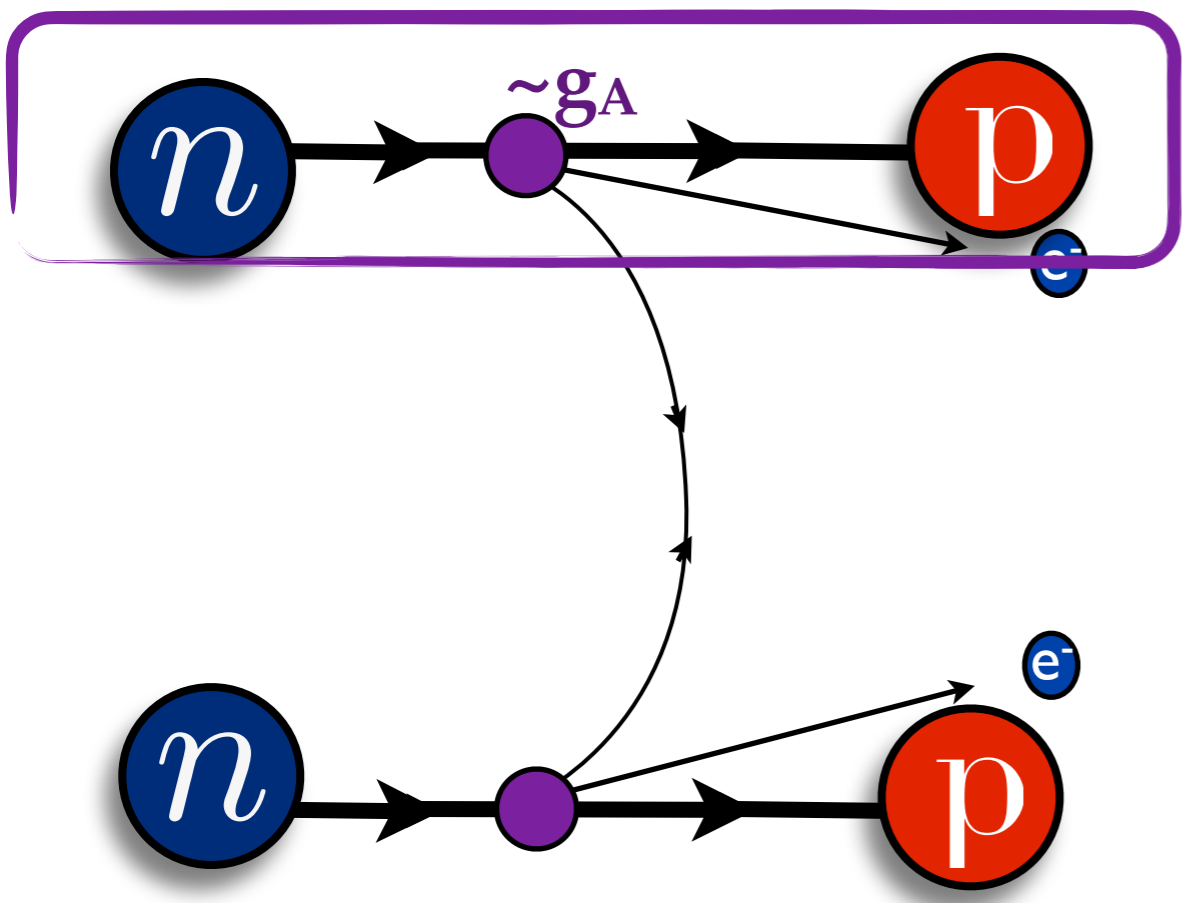
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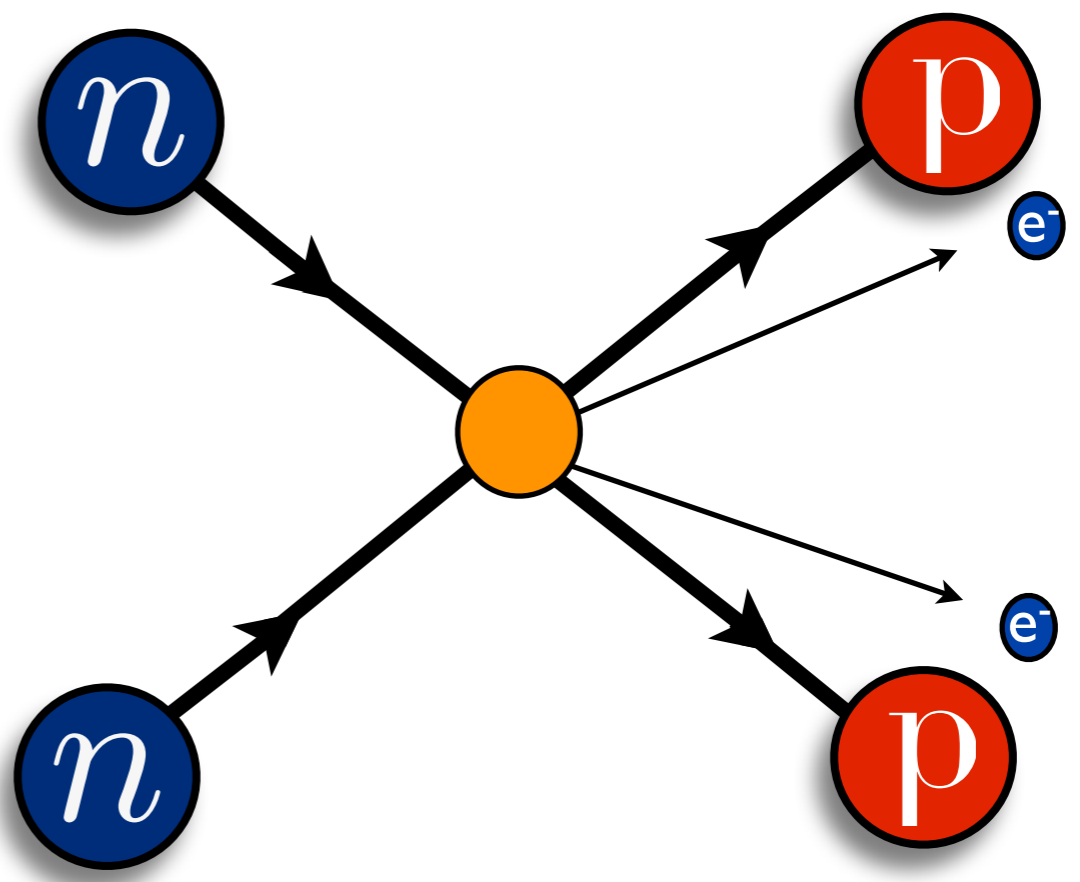
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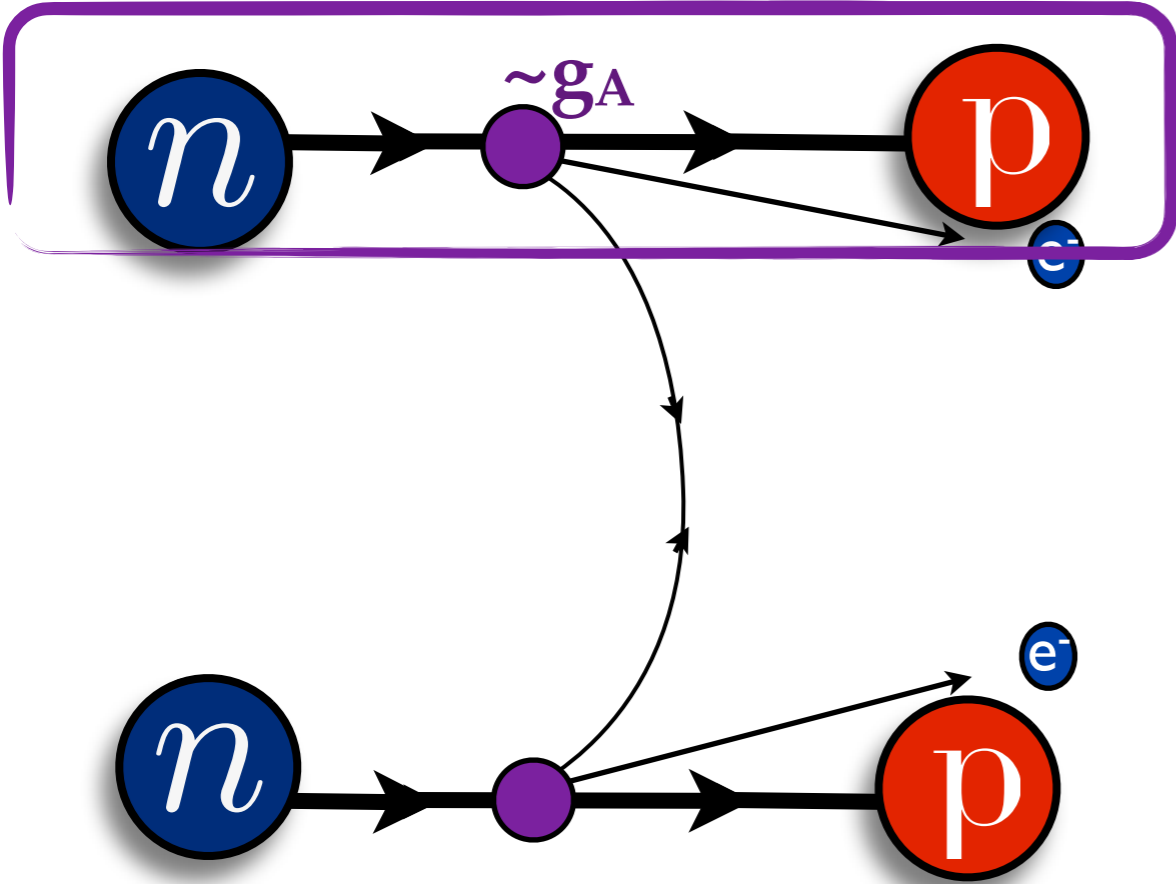
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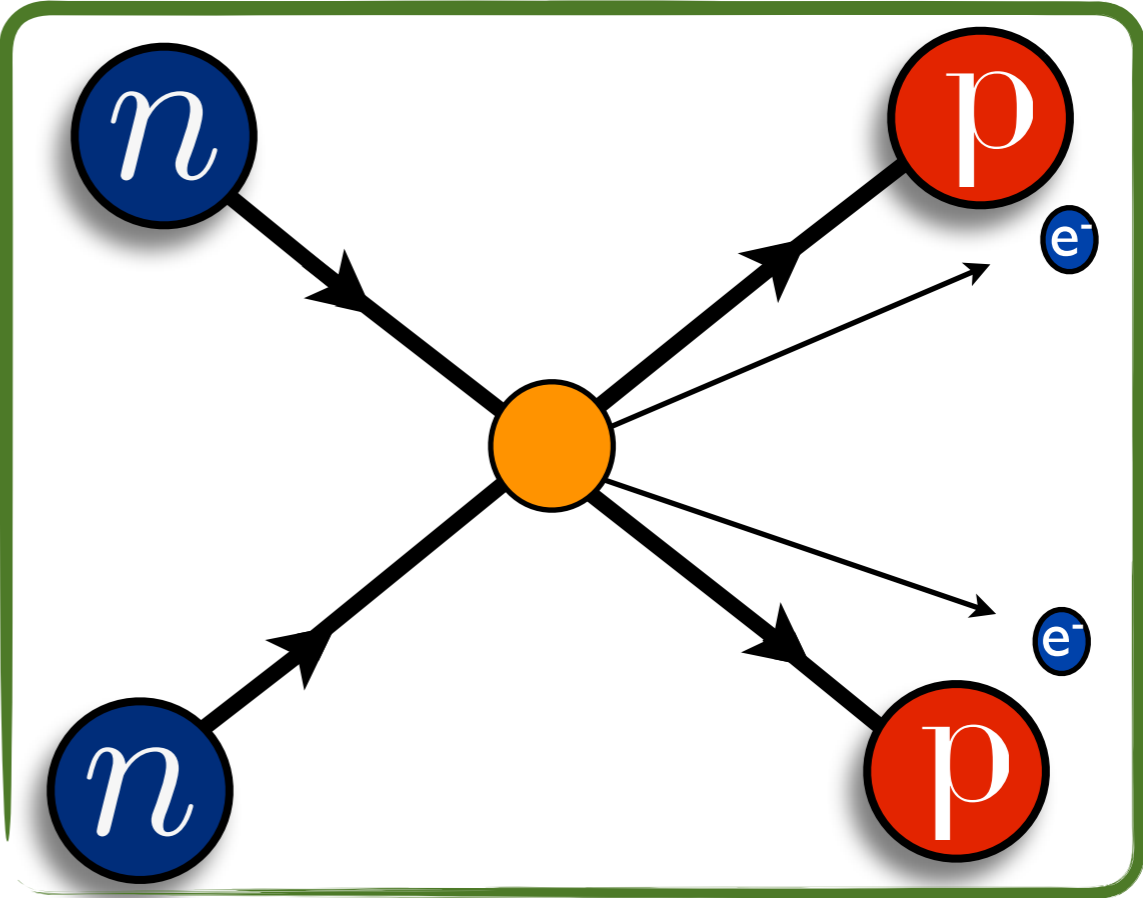
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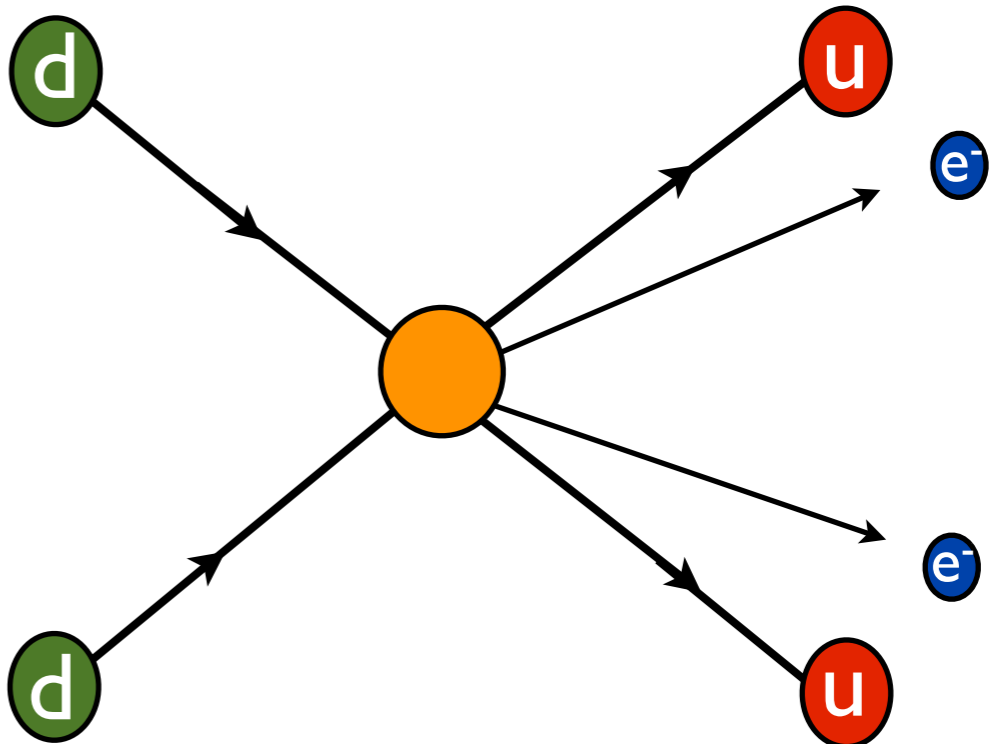
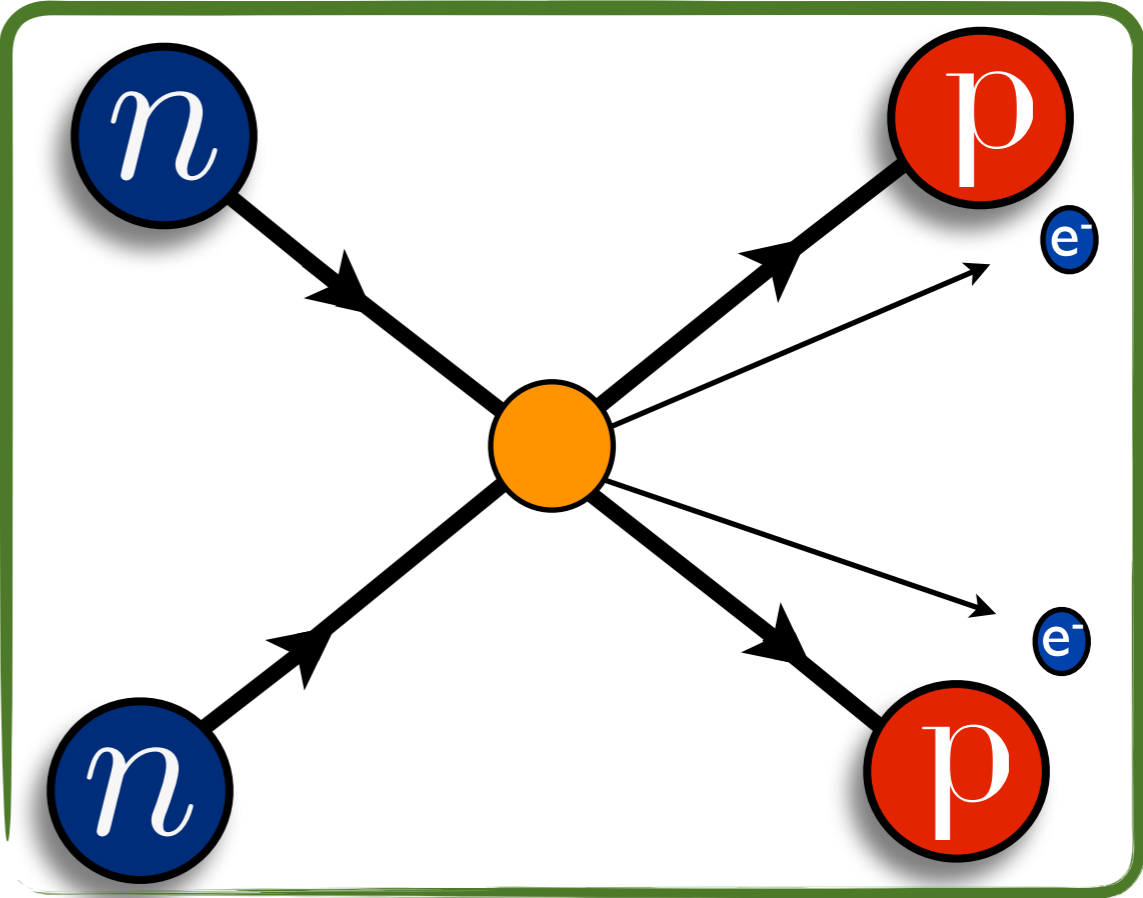
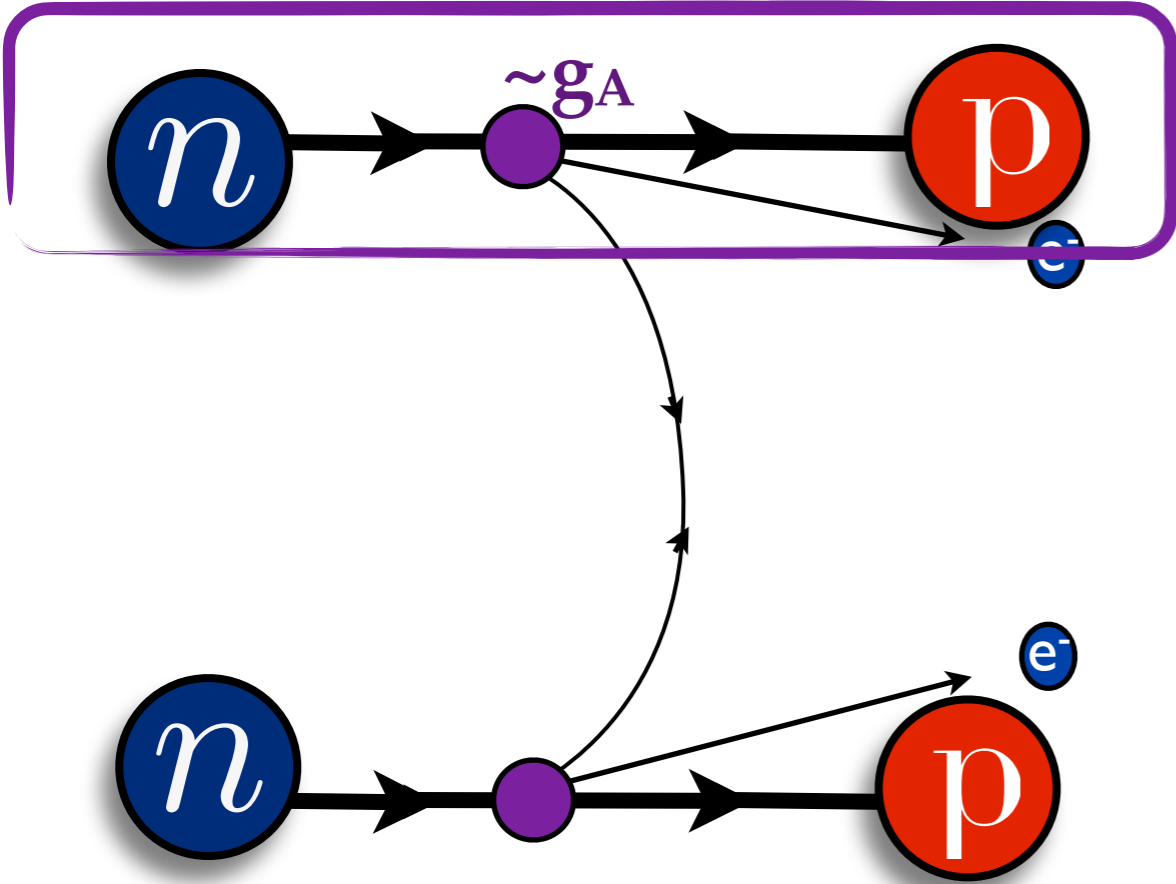


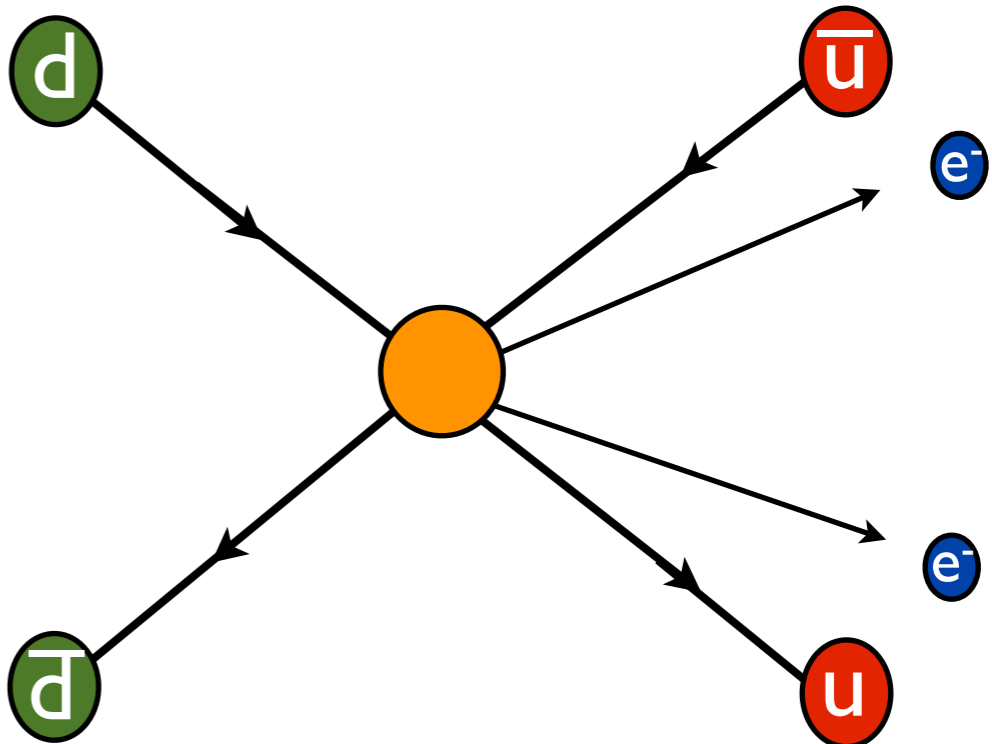
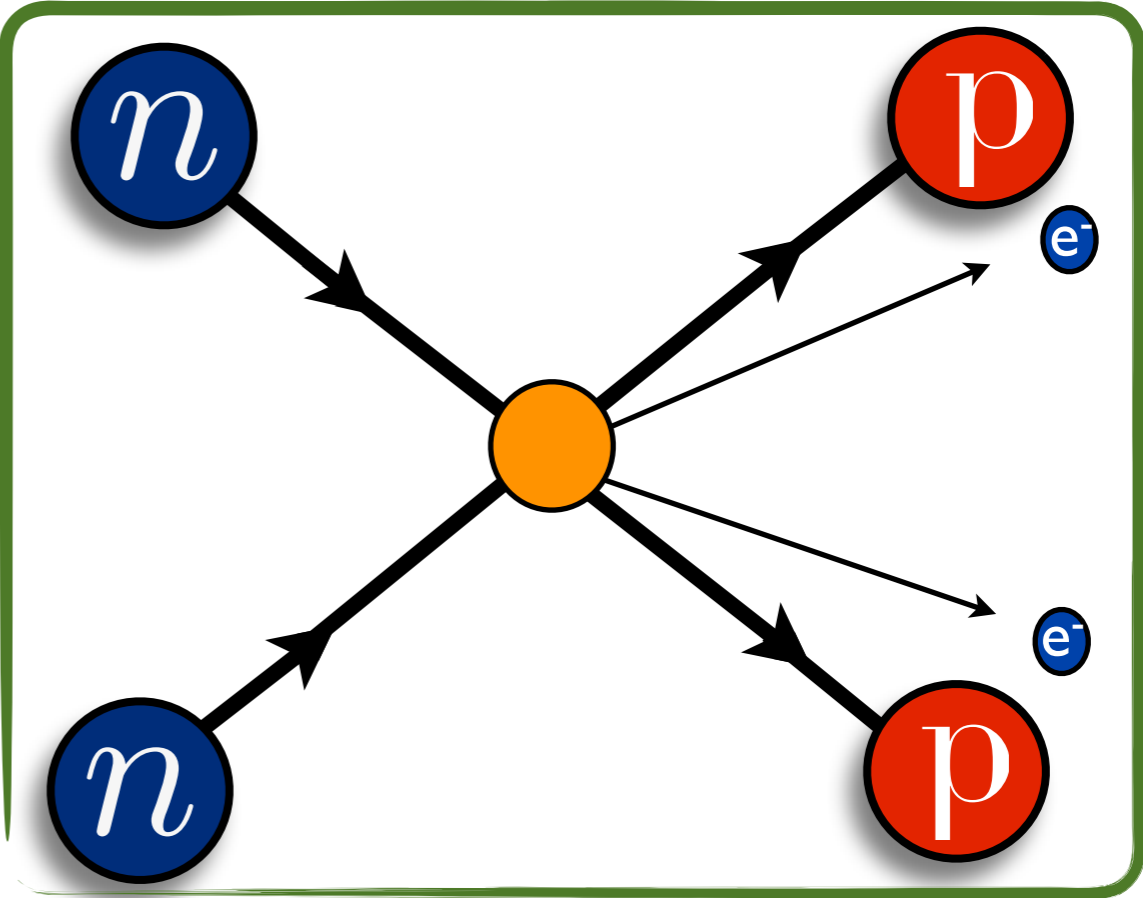
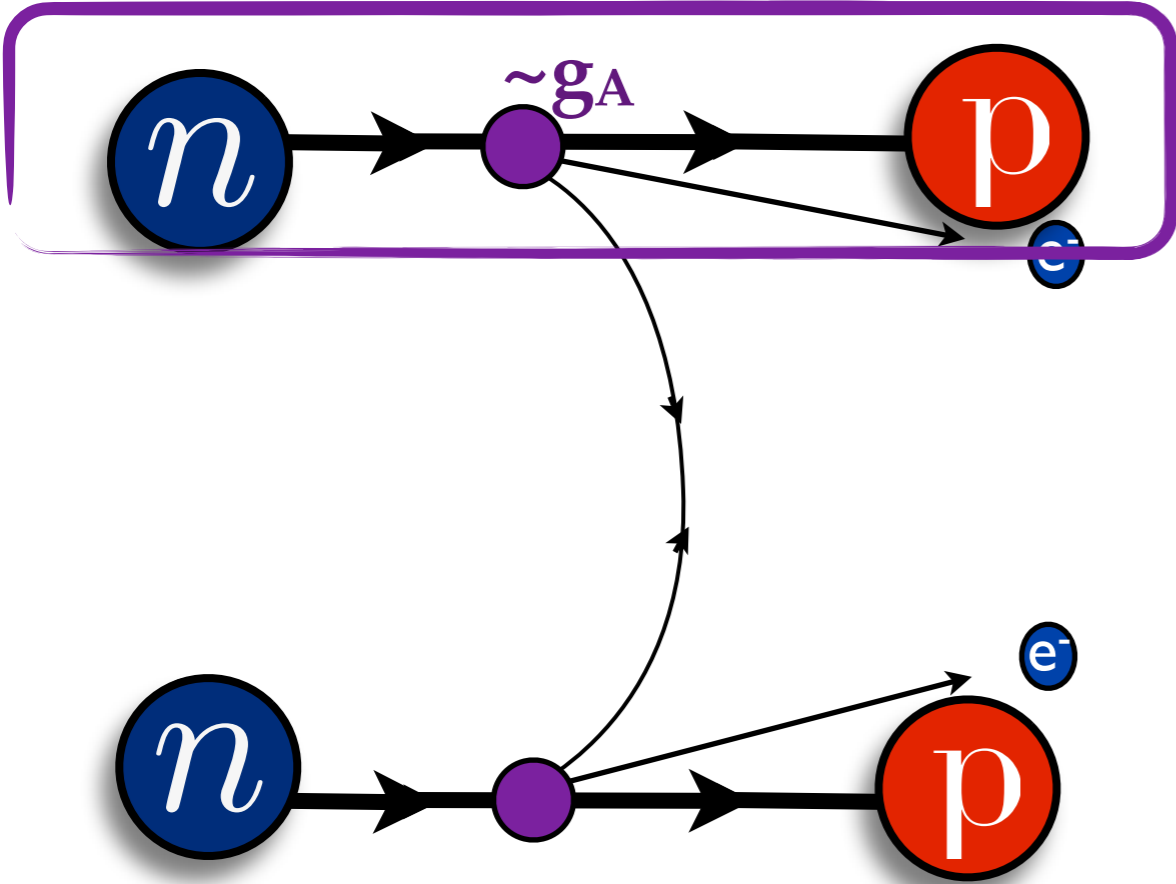
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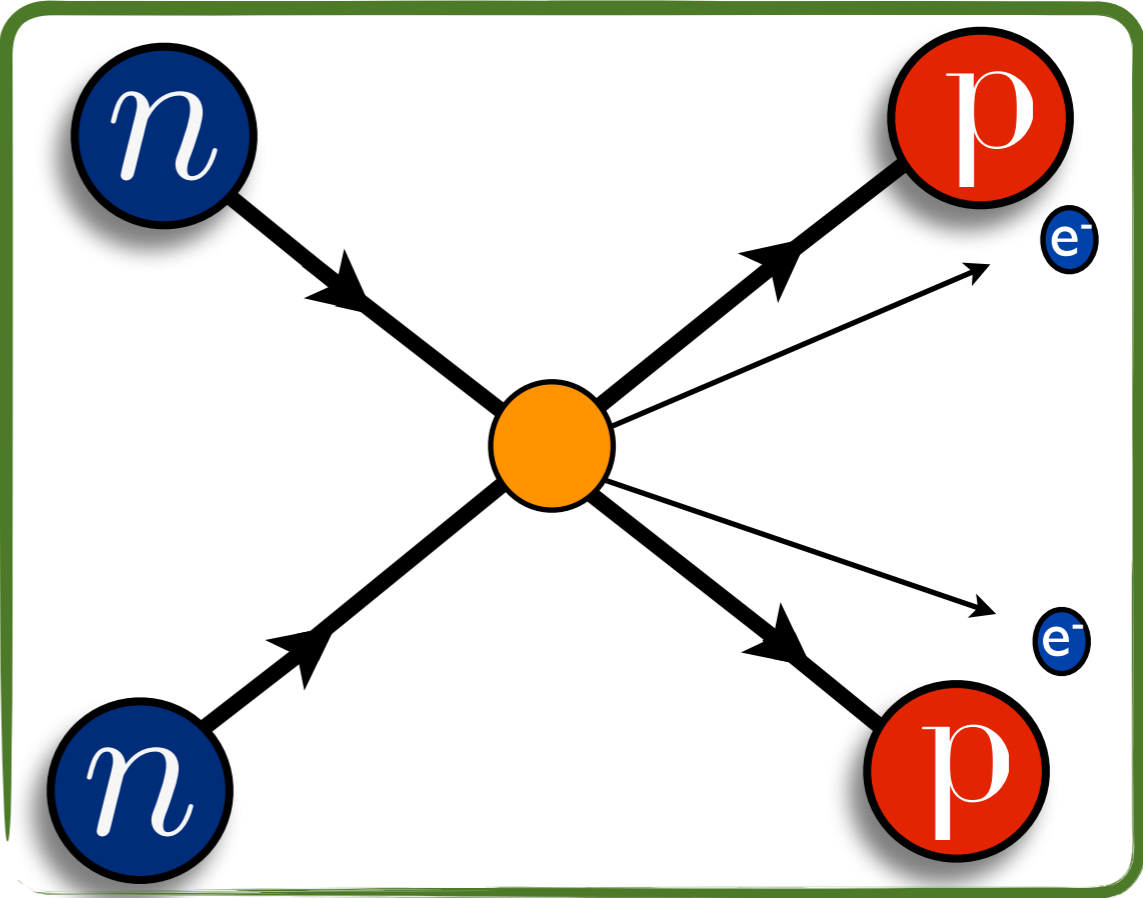
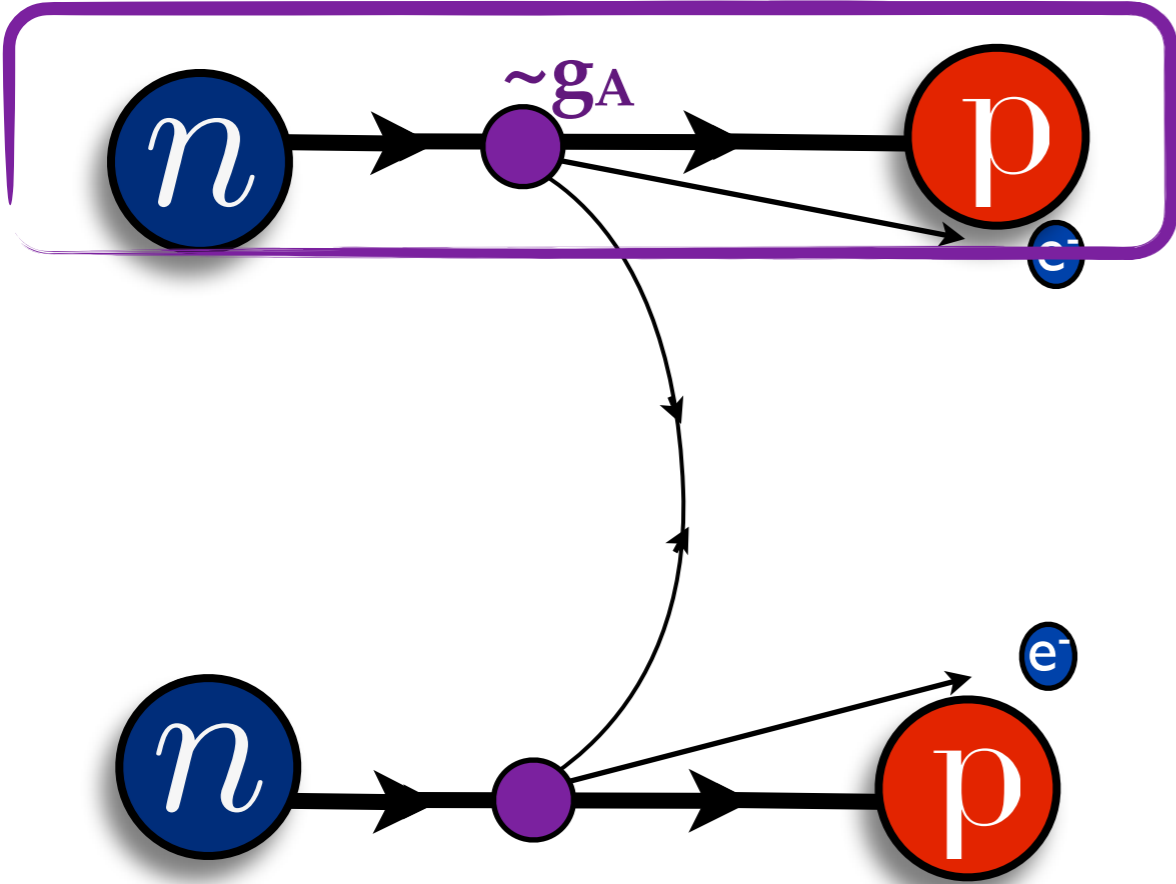
Short-range

Prezeau, Ramsey-Musolf,
Vogel (2003)

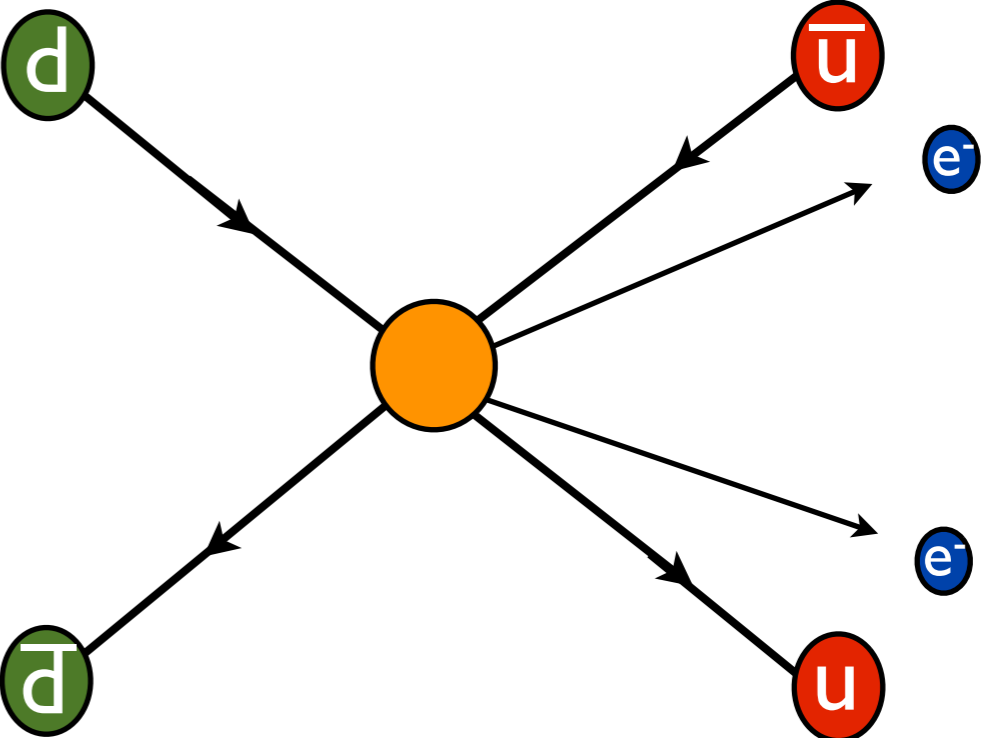


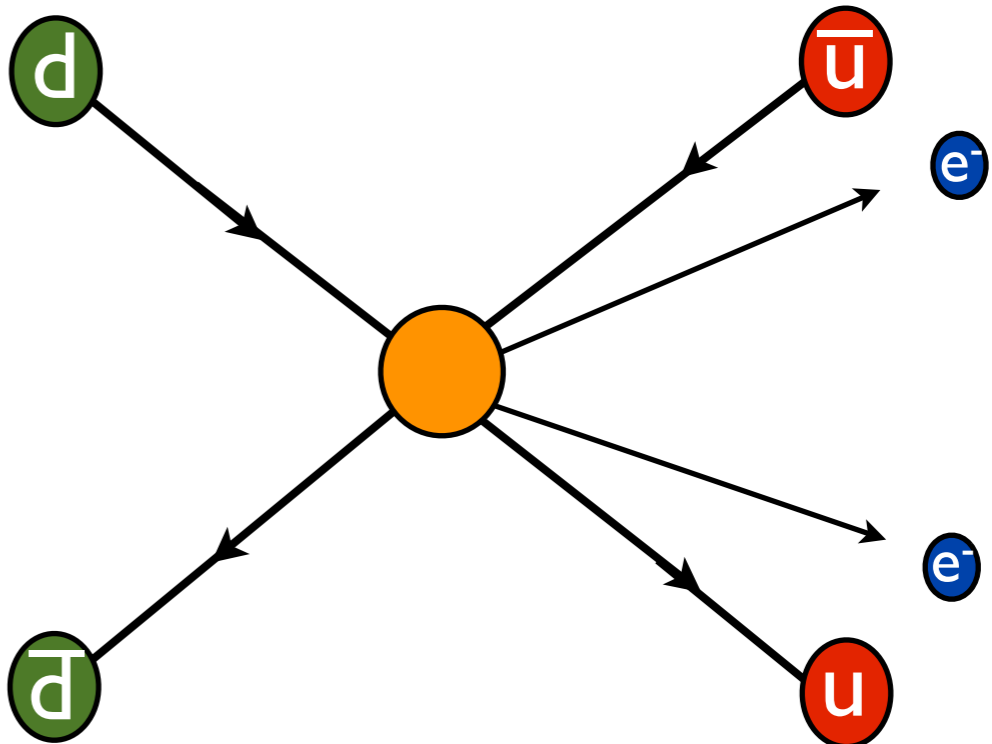
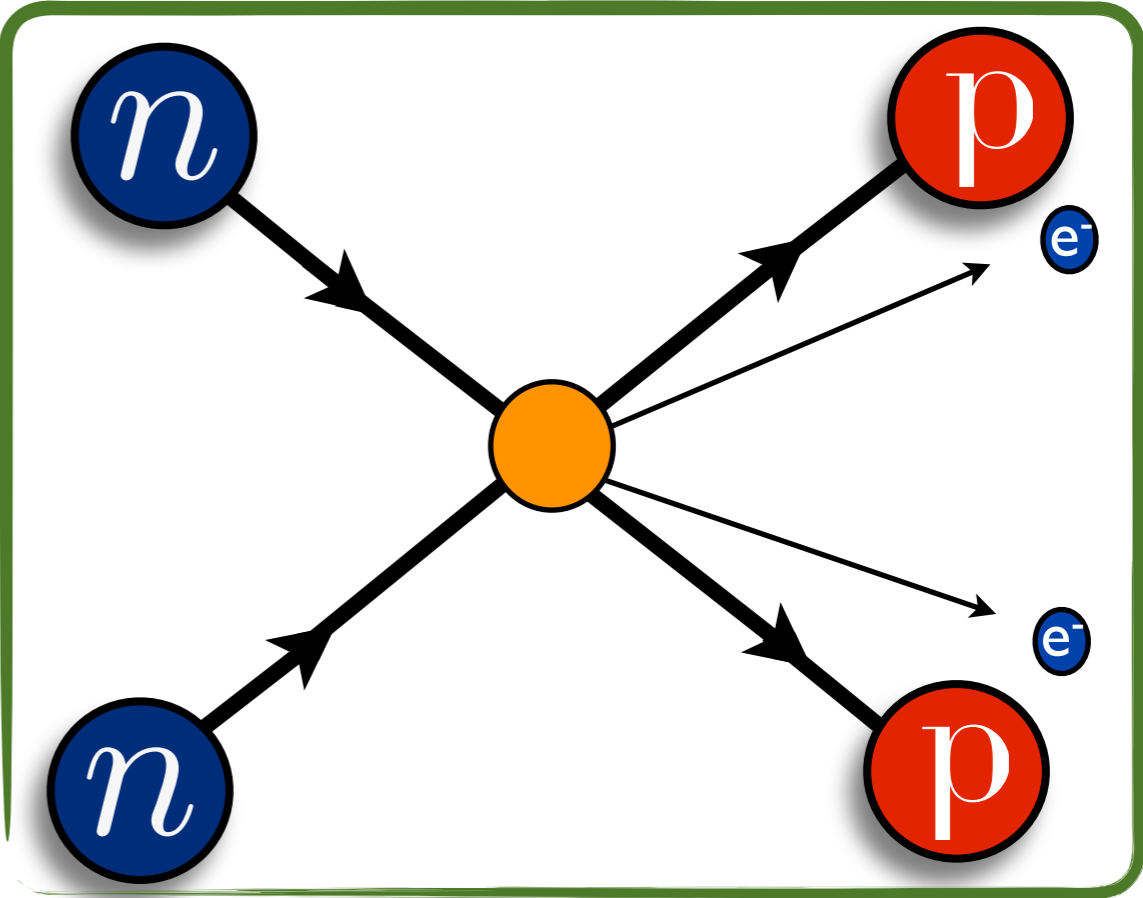
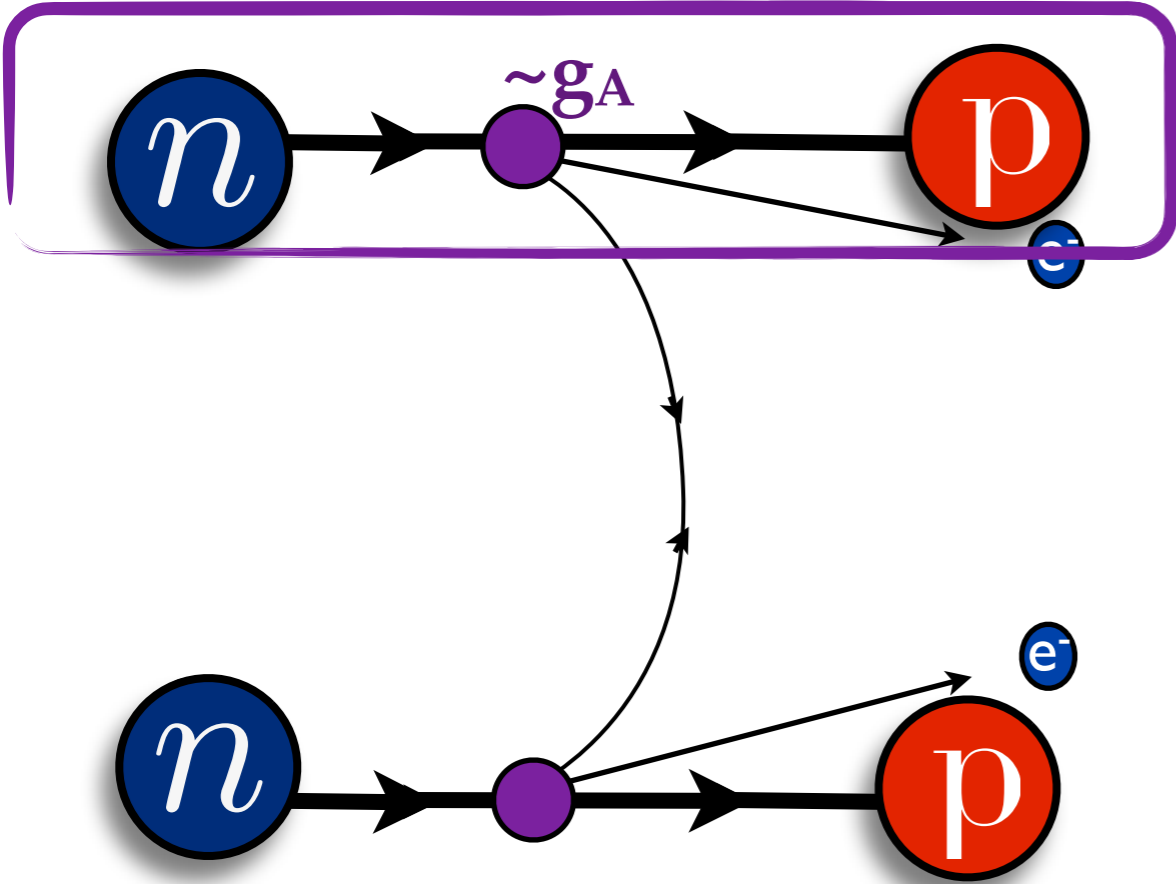


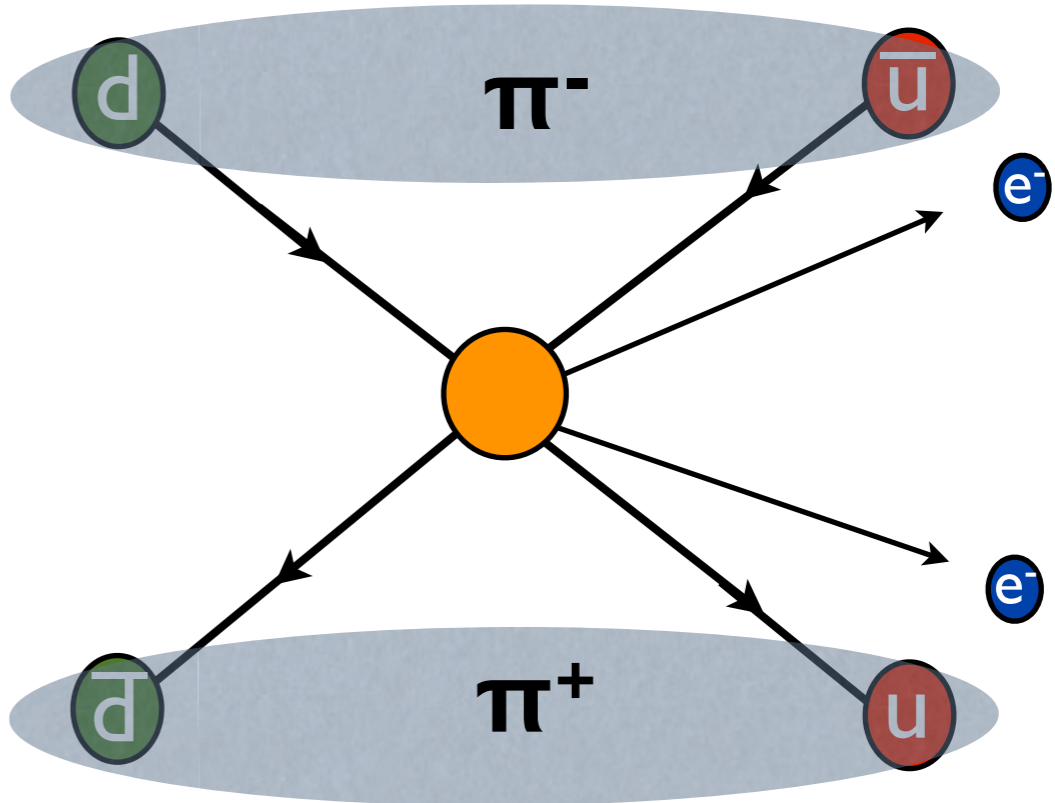
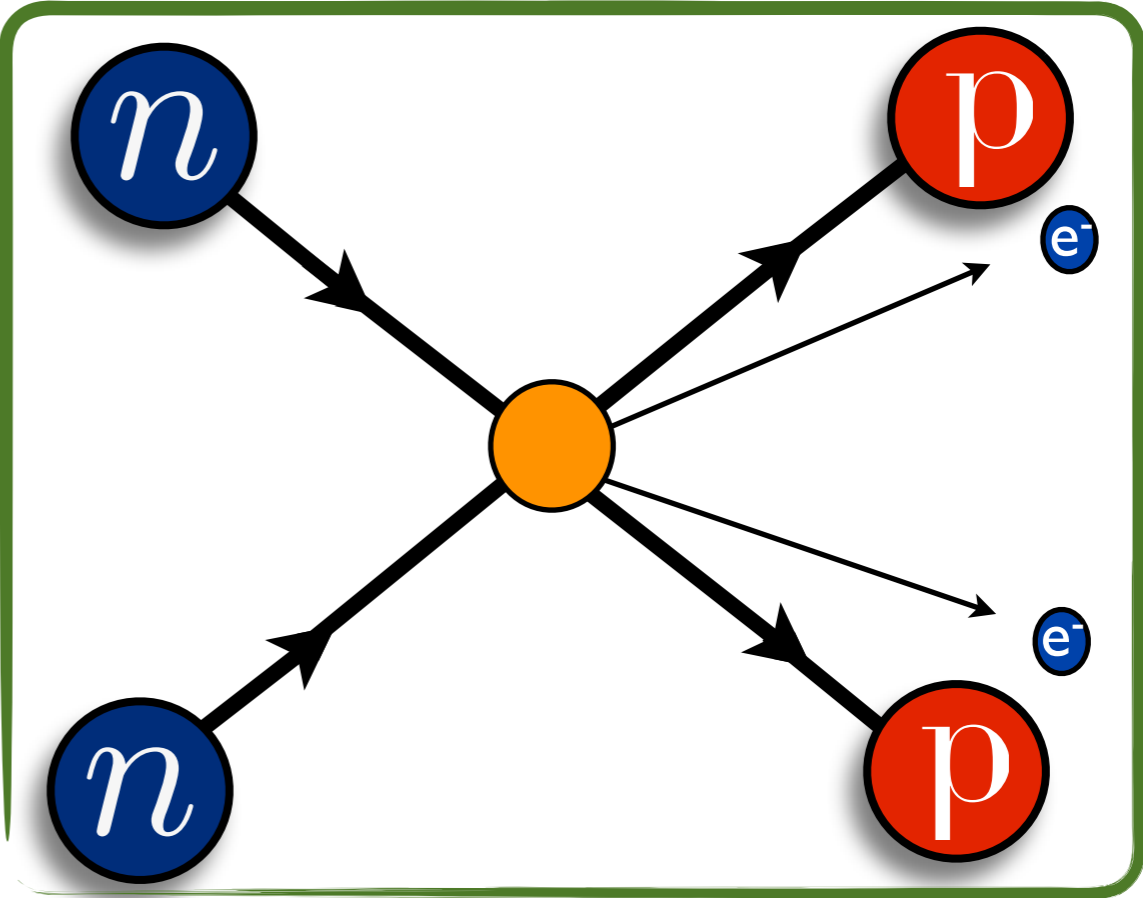
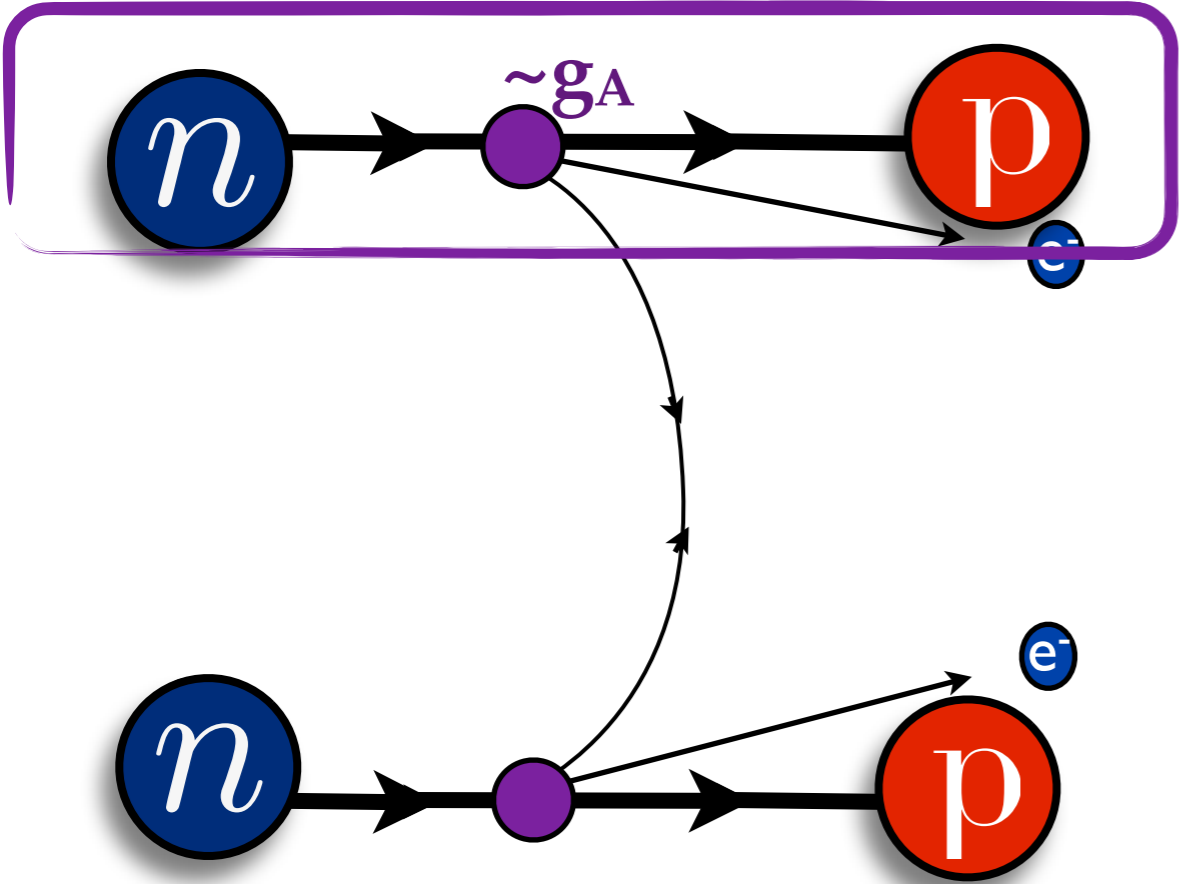
Prezeau, Ramsey-Musolf,
Vogel (2003)



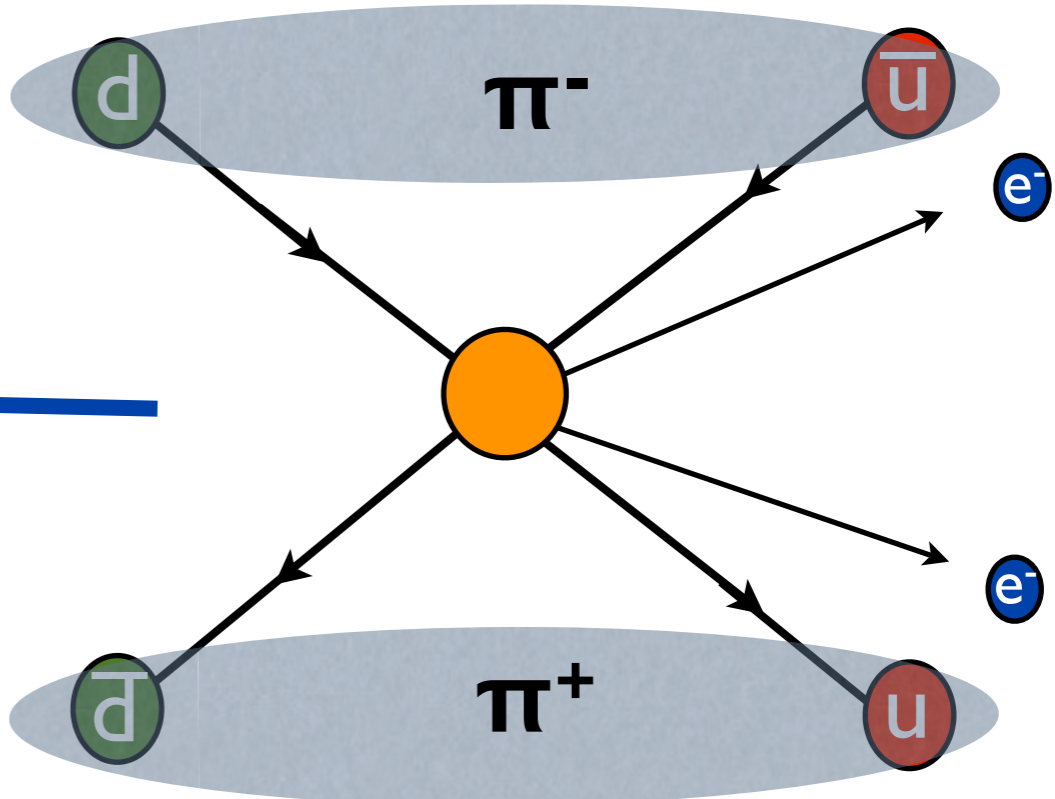
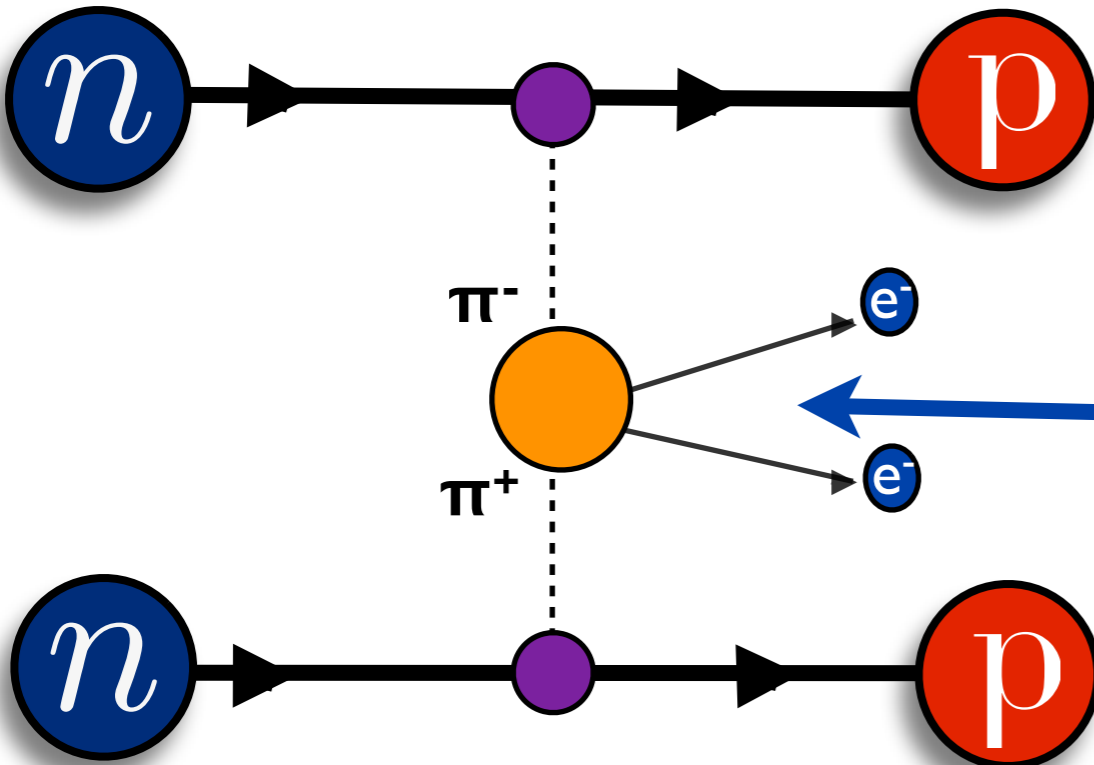
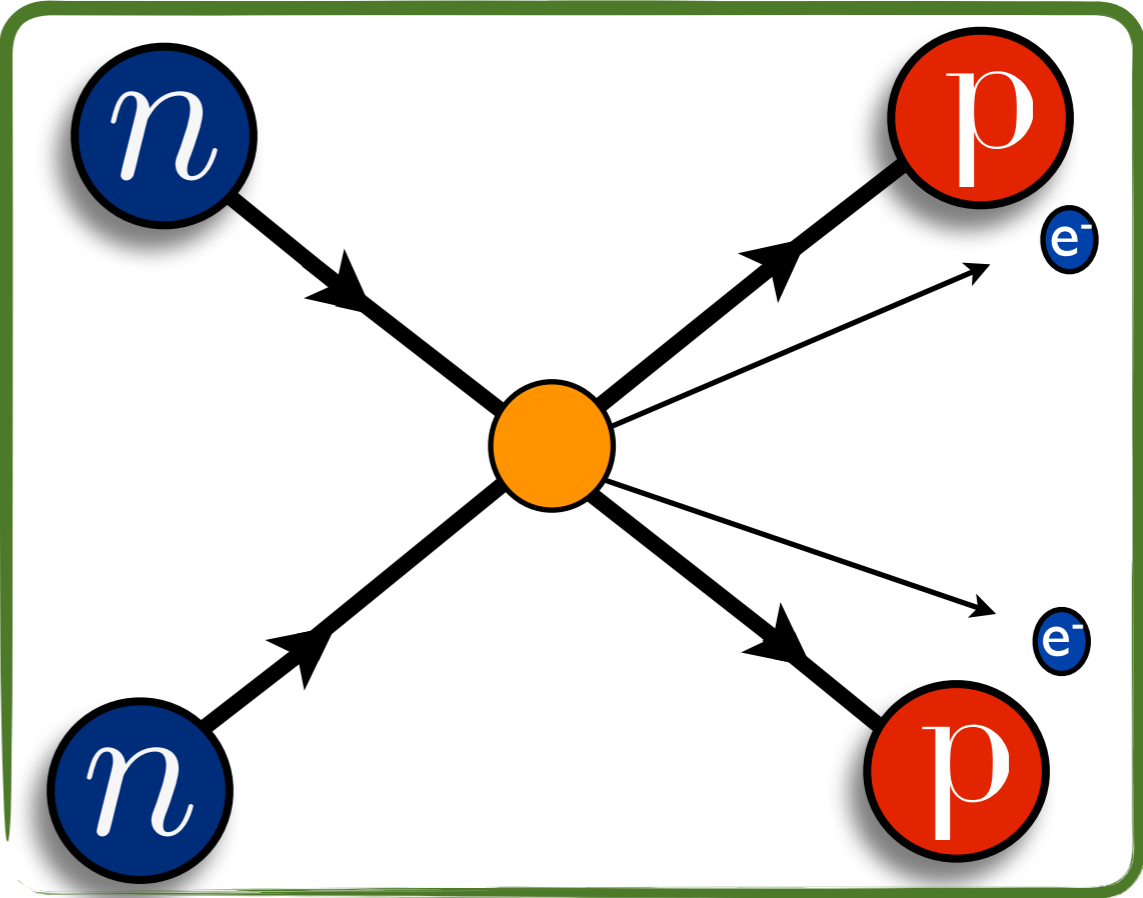
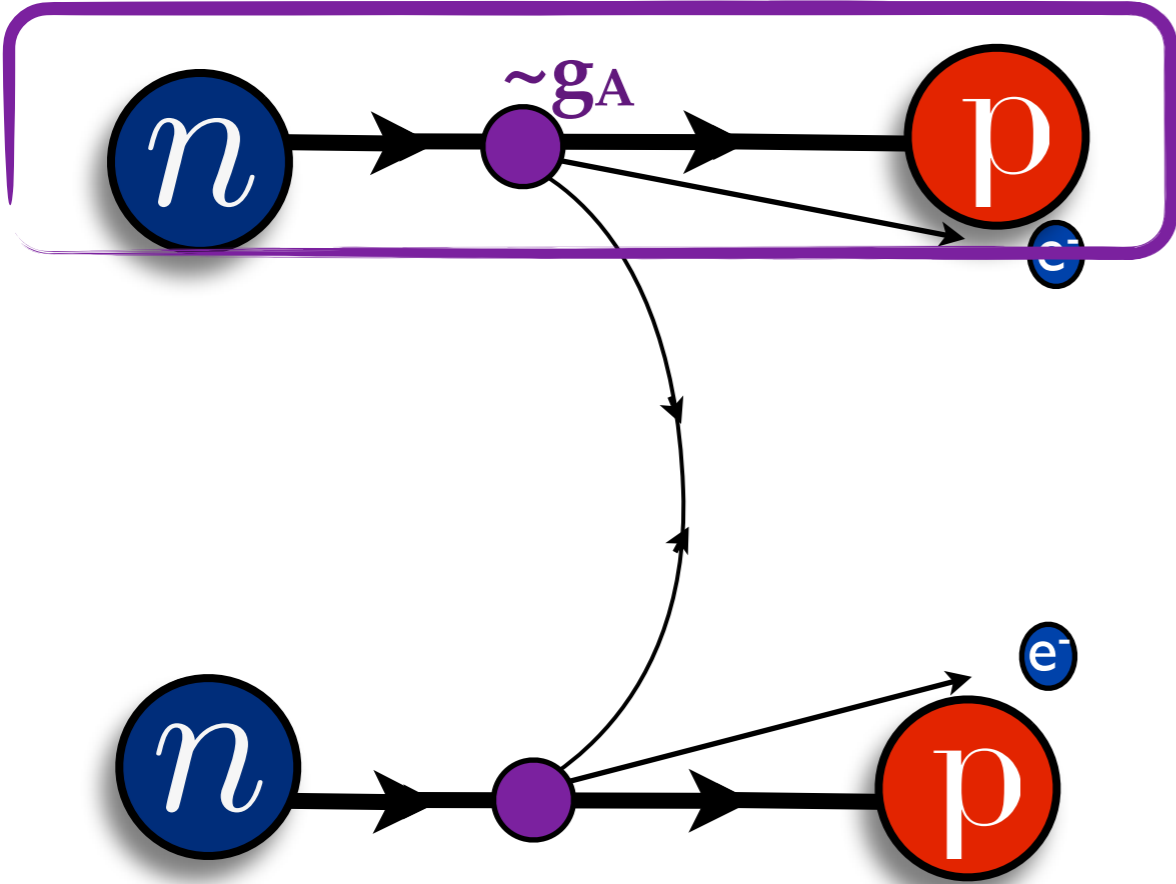
$\Lambda \ll \Lambda_{\text{QCD}}$



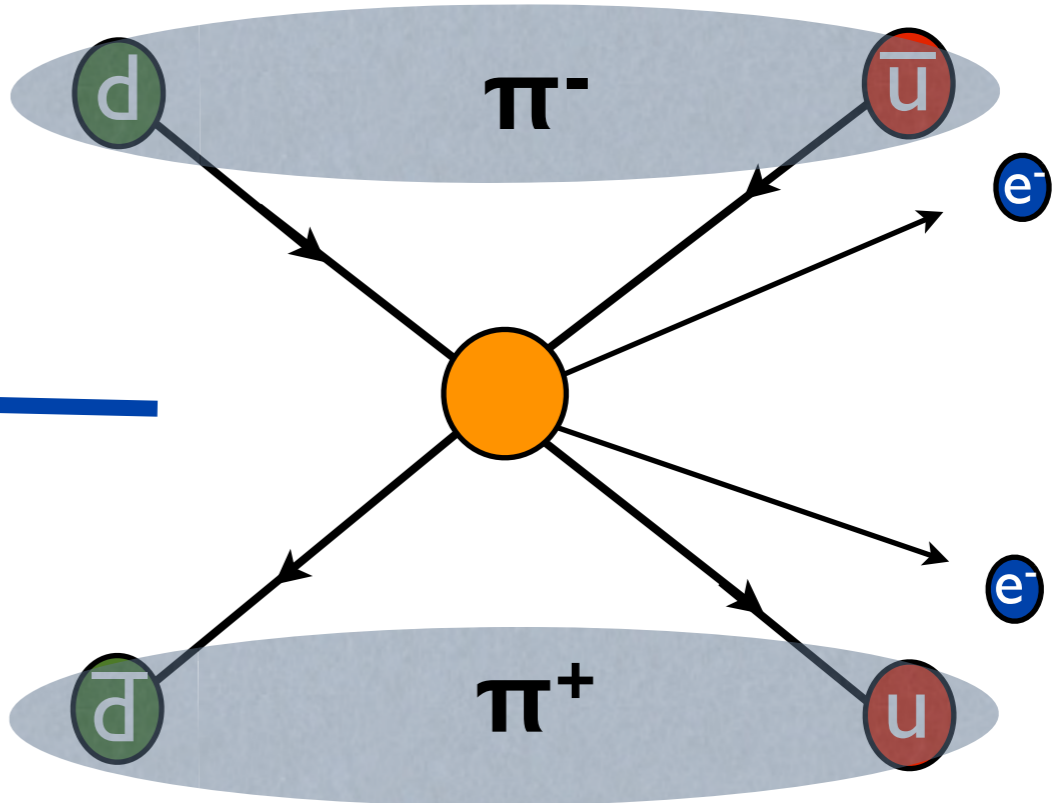
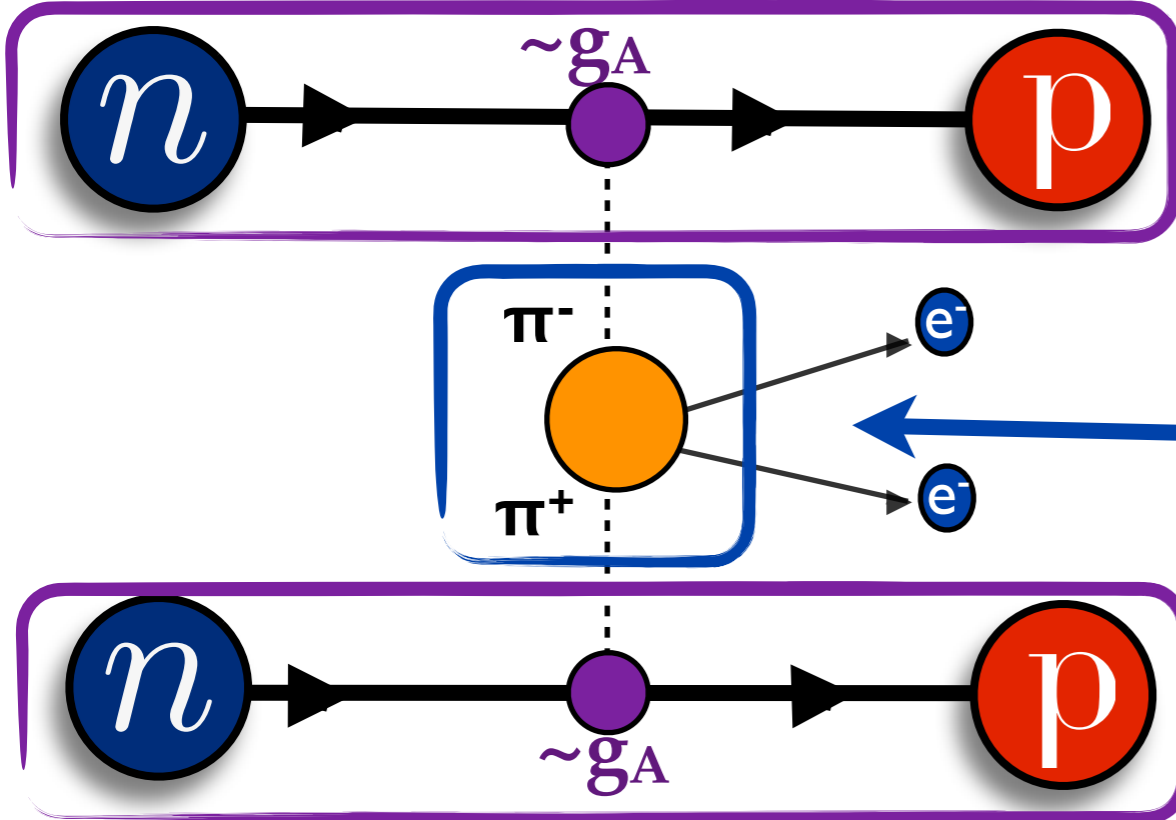
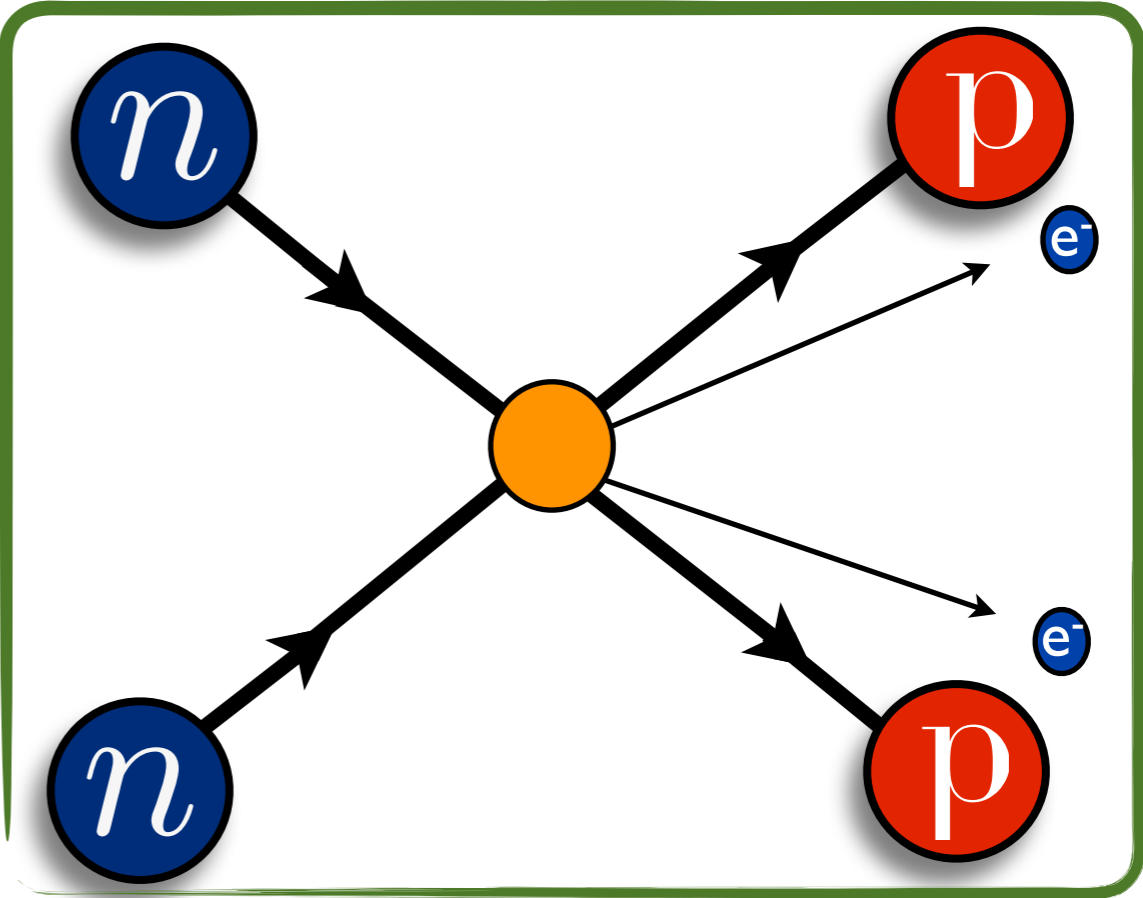
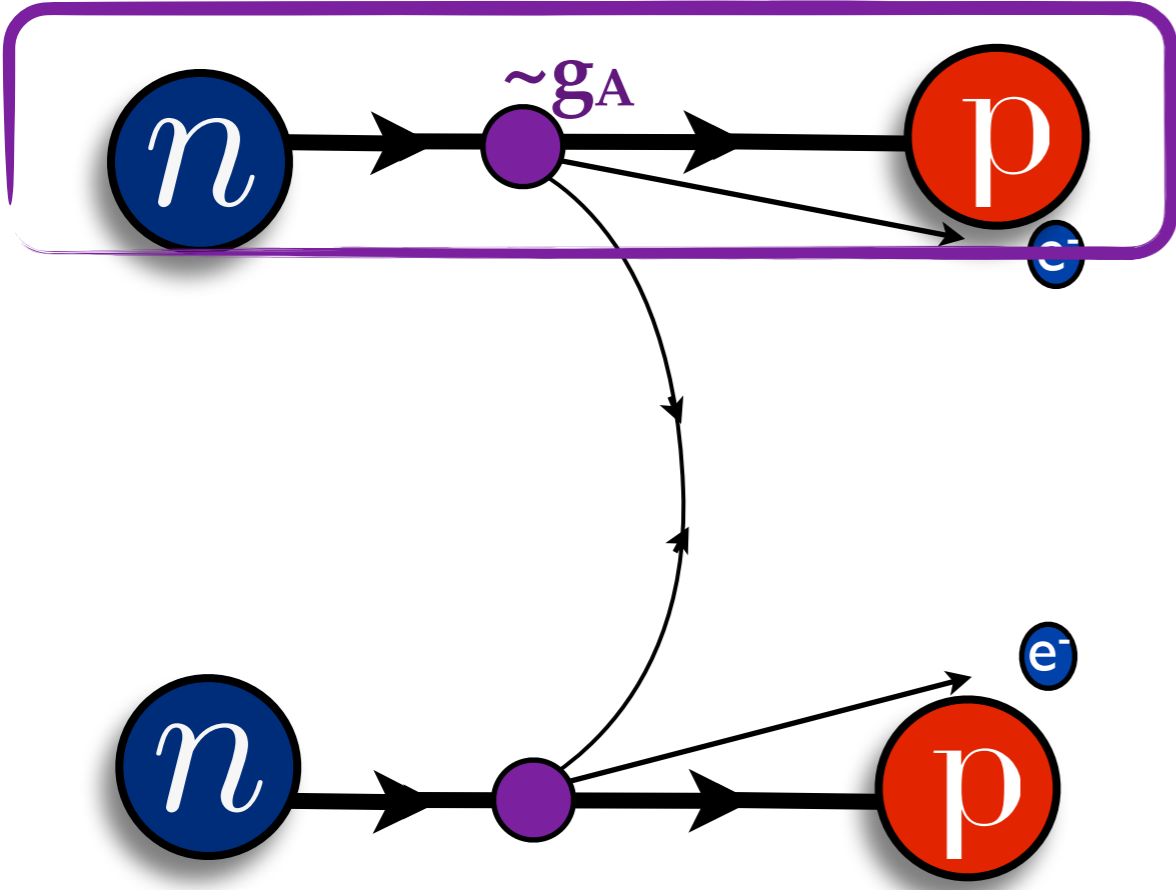


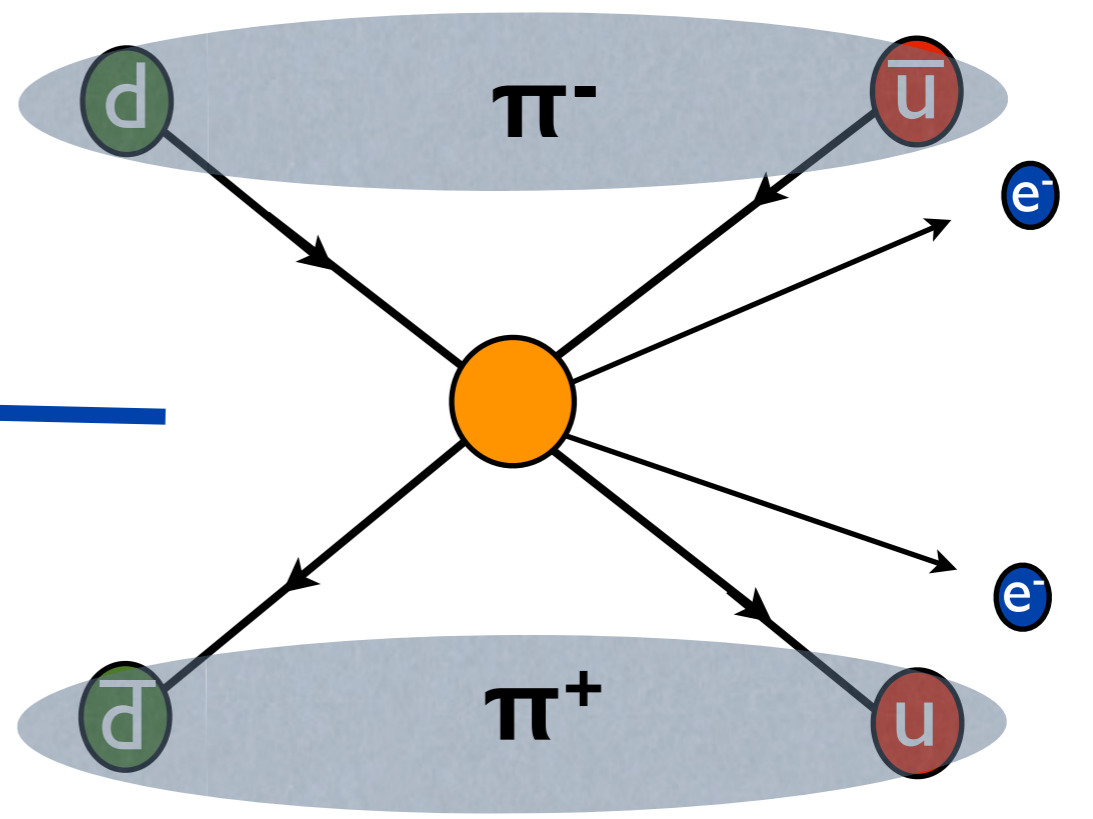
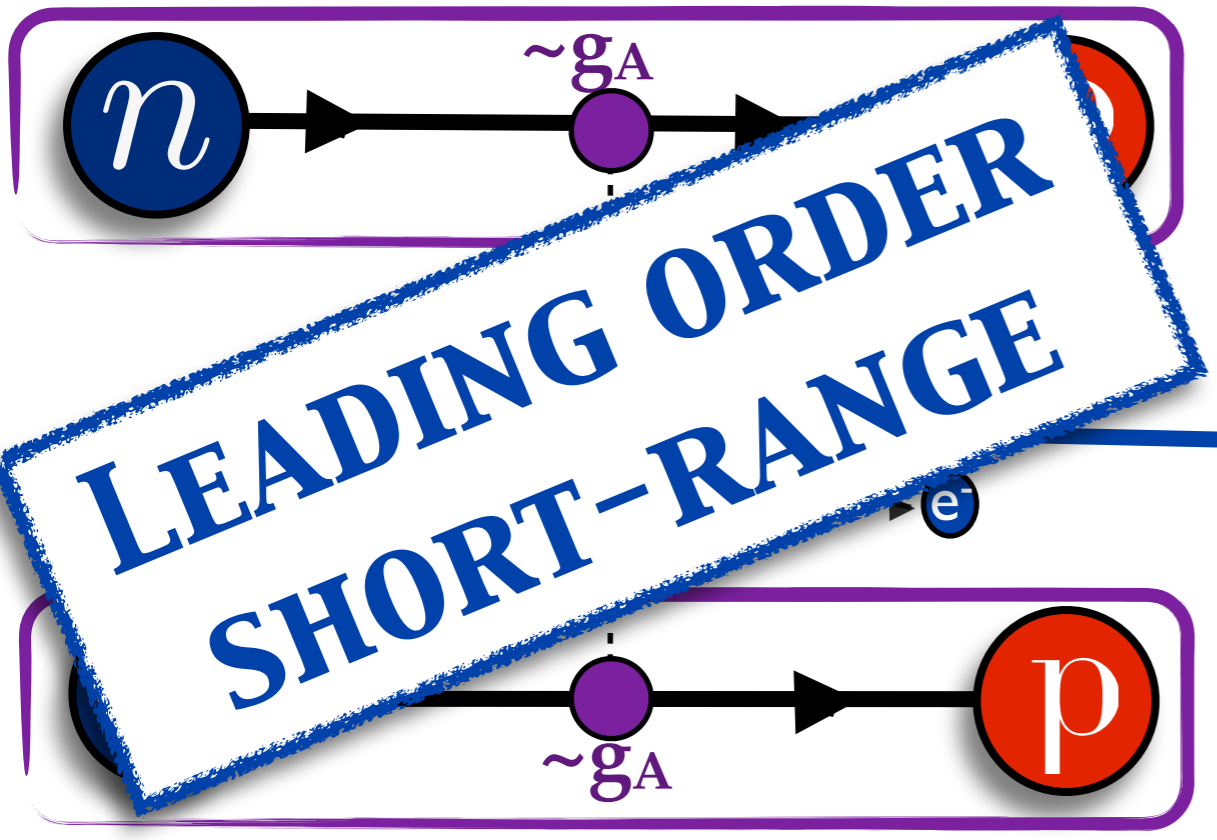
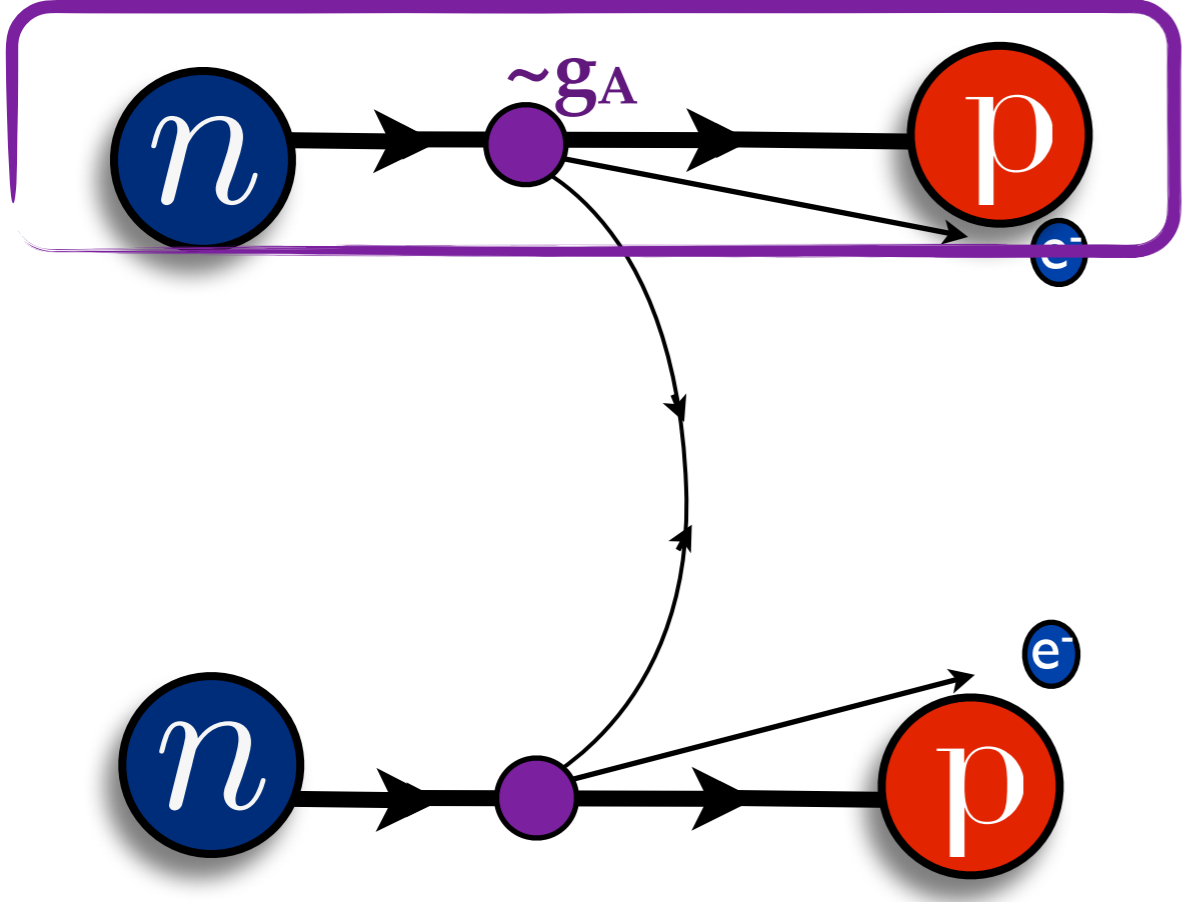


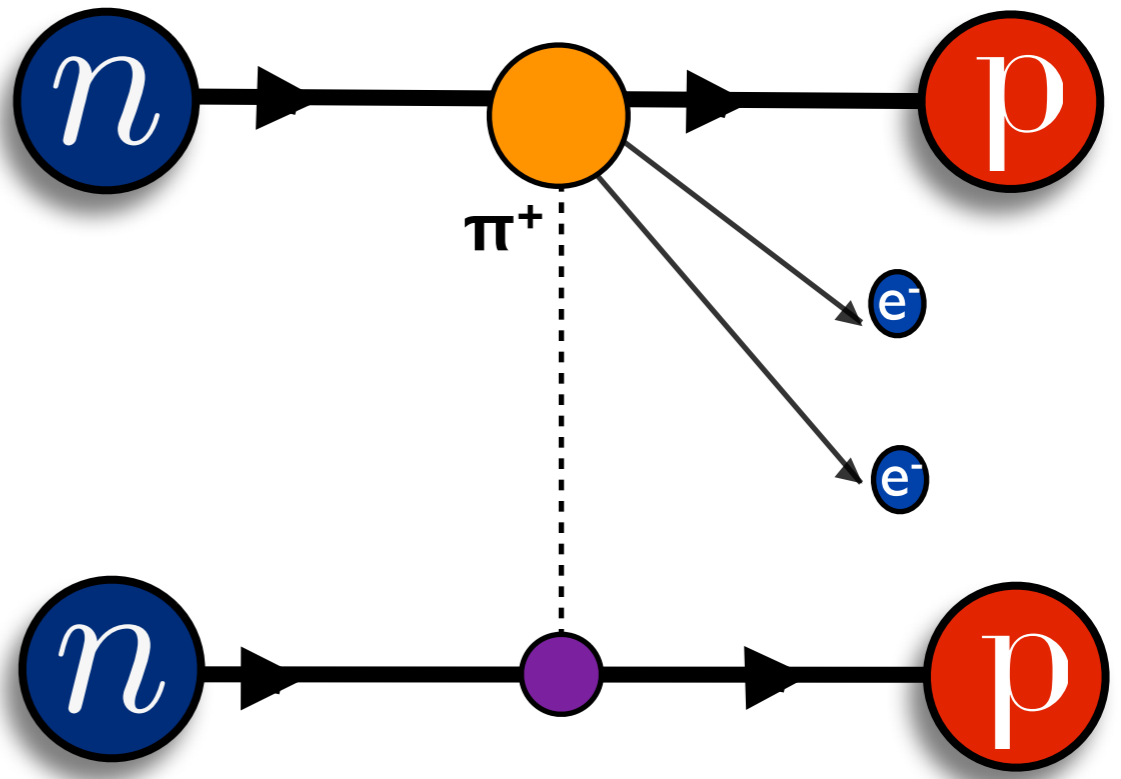
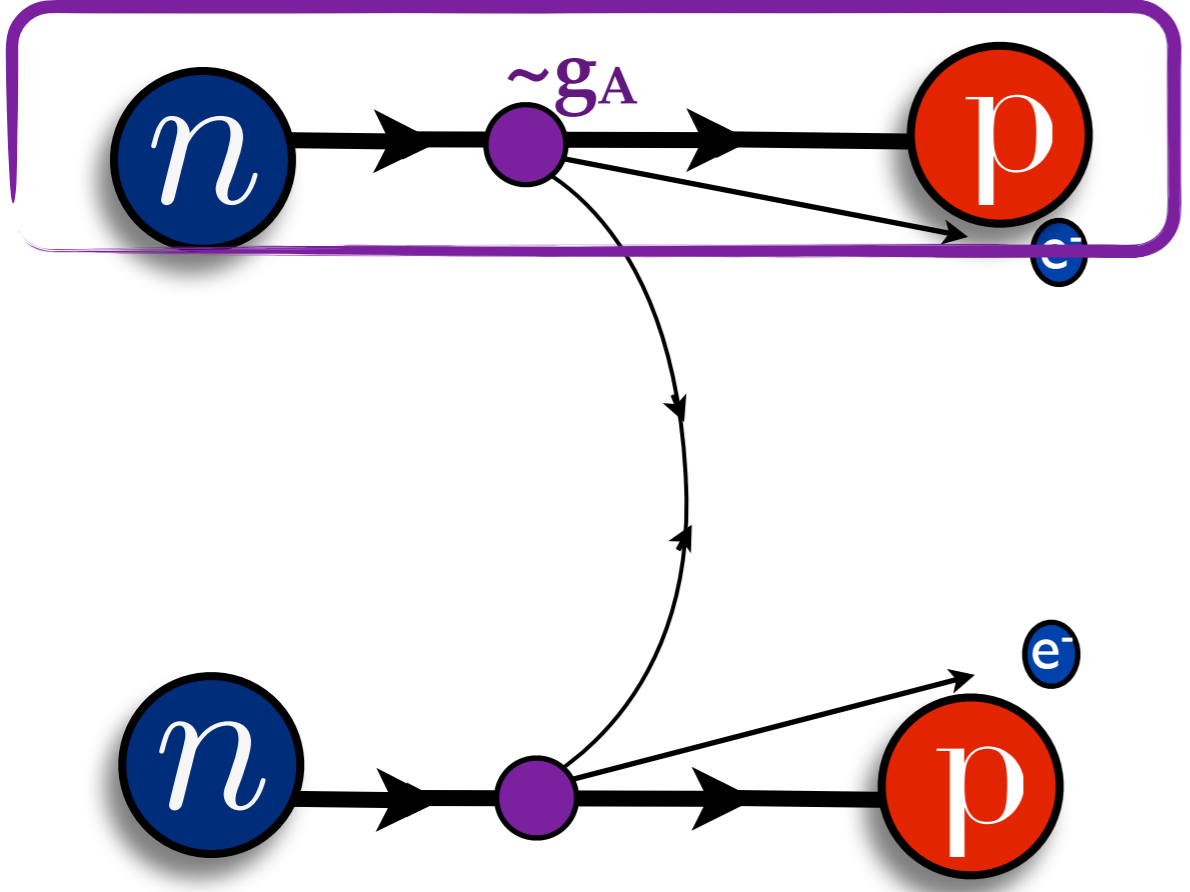
Prezeau, Ramsey-Musolf, Vogel (2003)

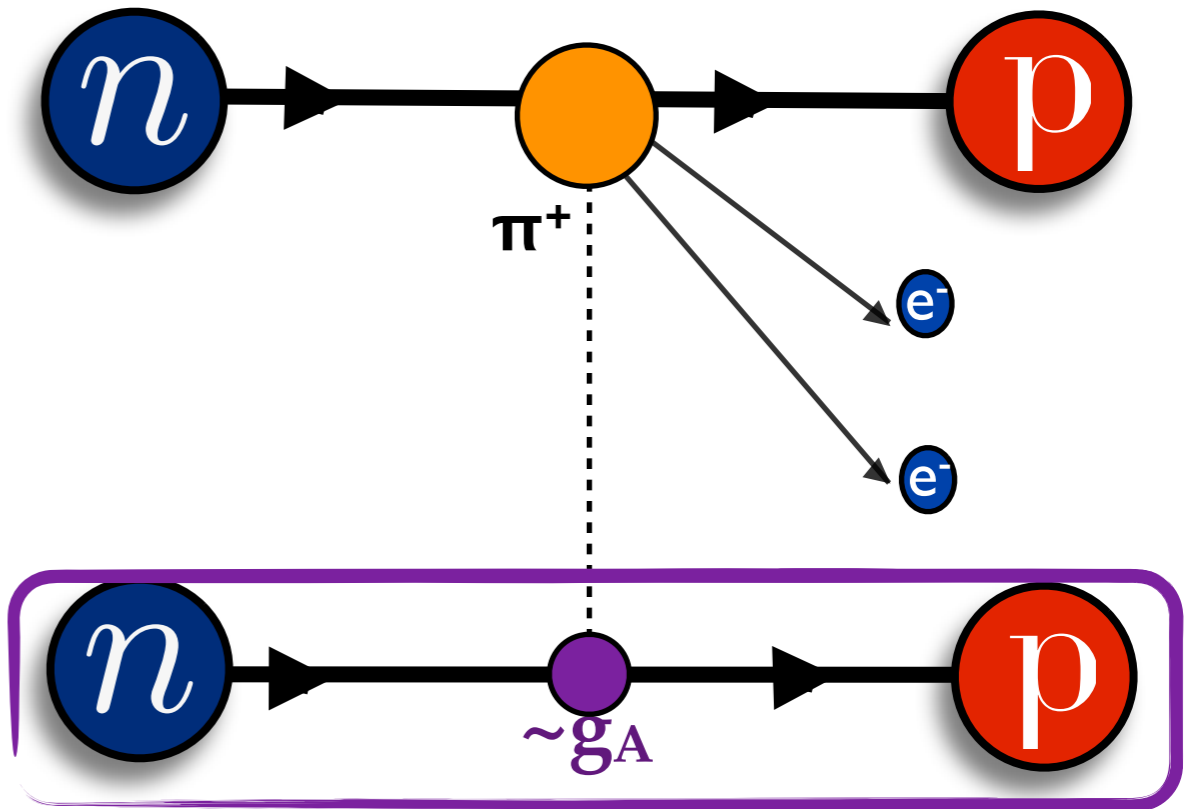
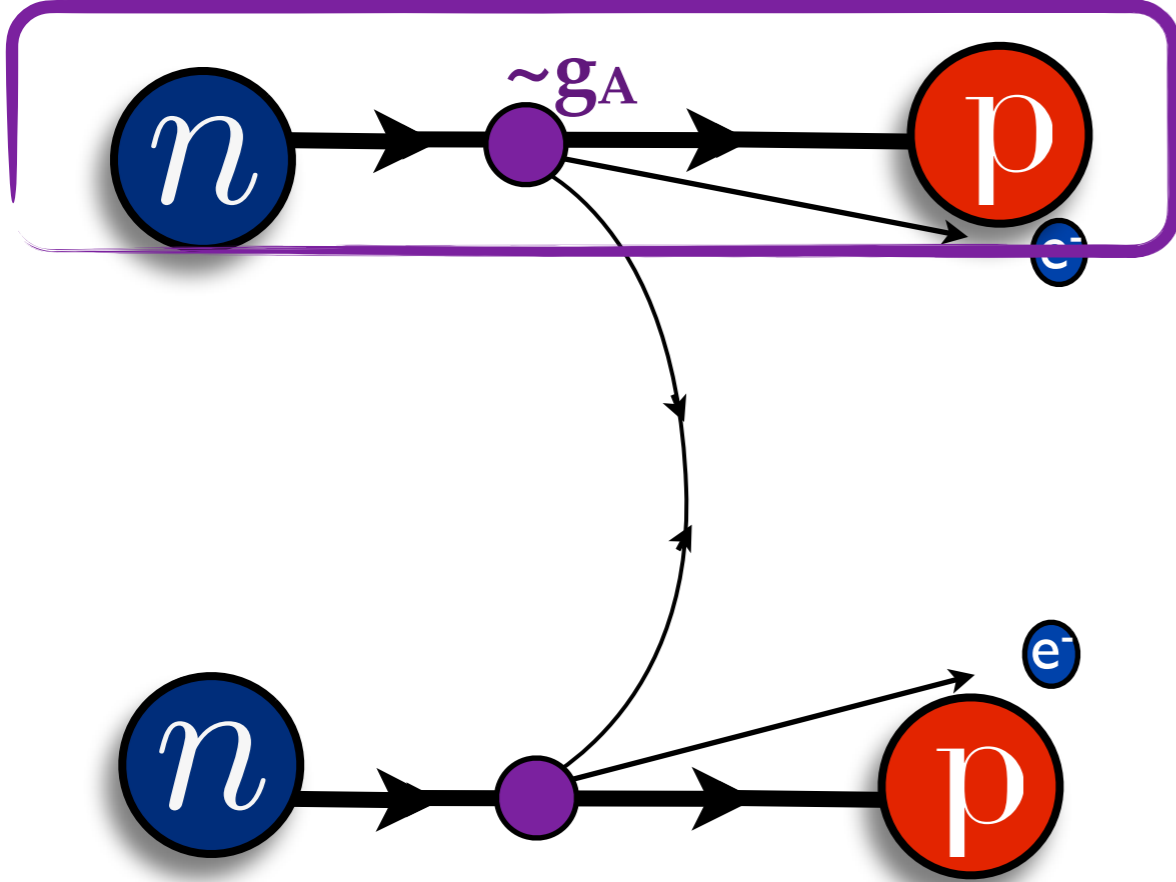


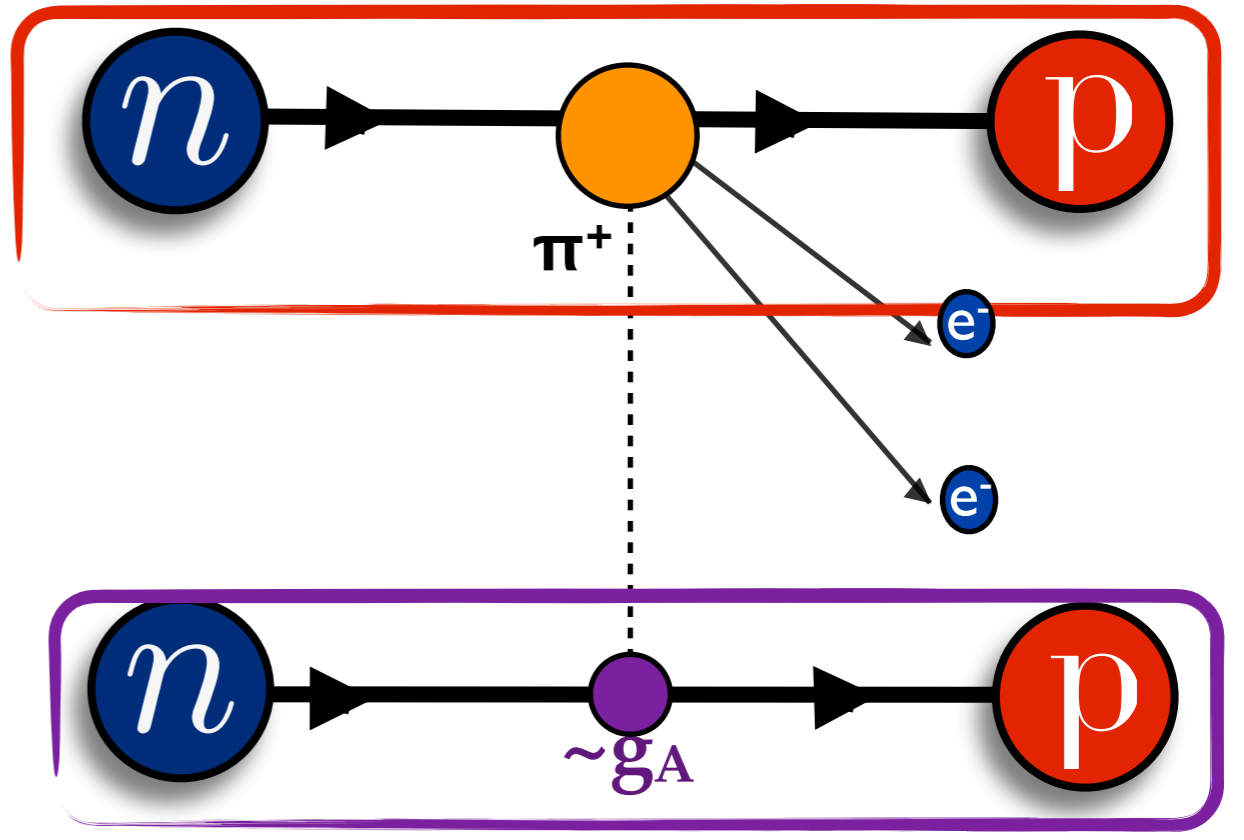
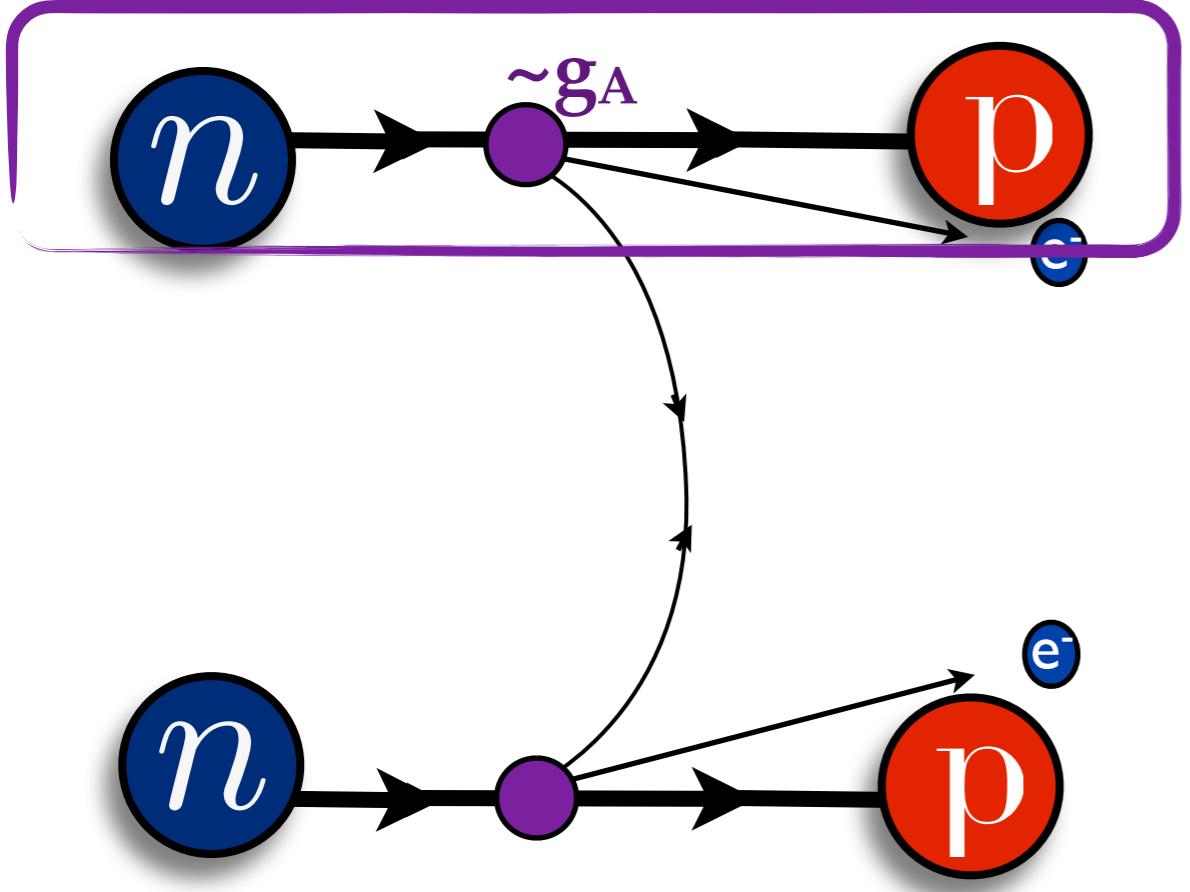
Prezeau, Ramsey-Musolf, Vogel (2003)

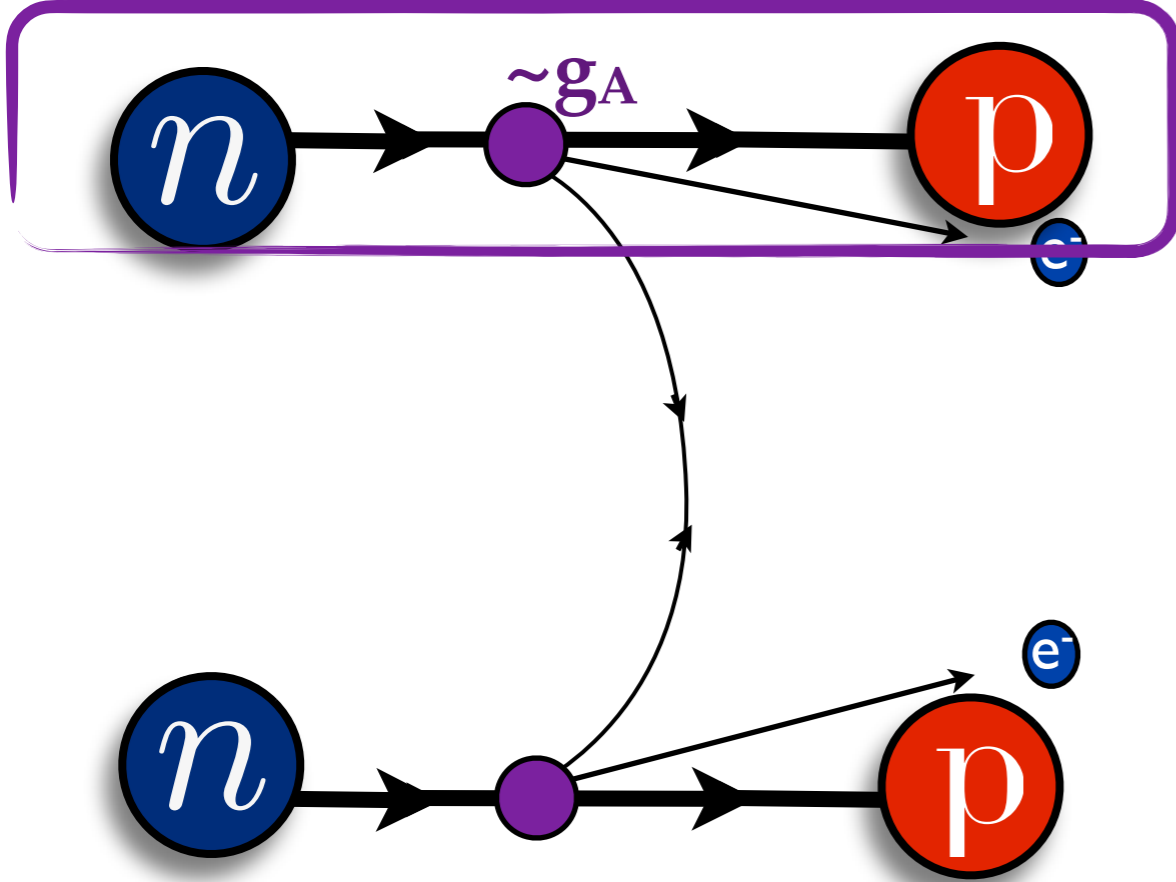


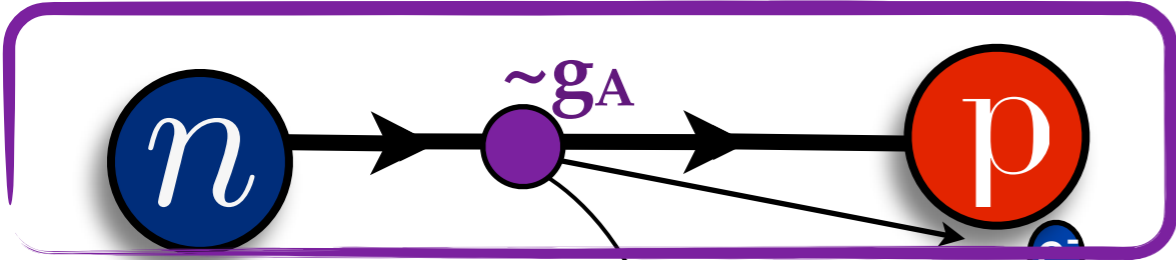




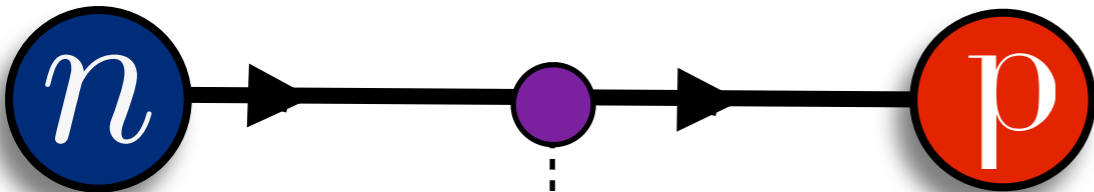
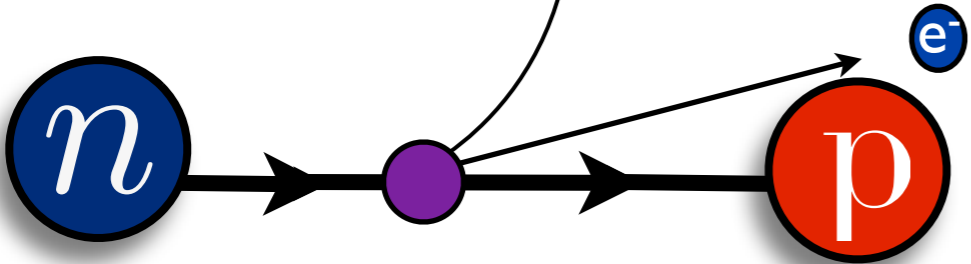




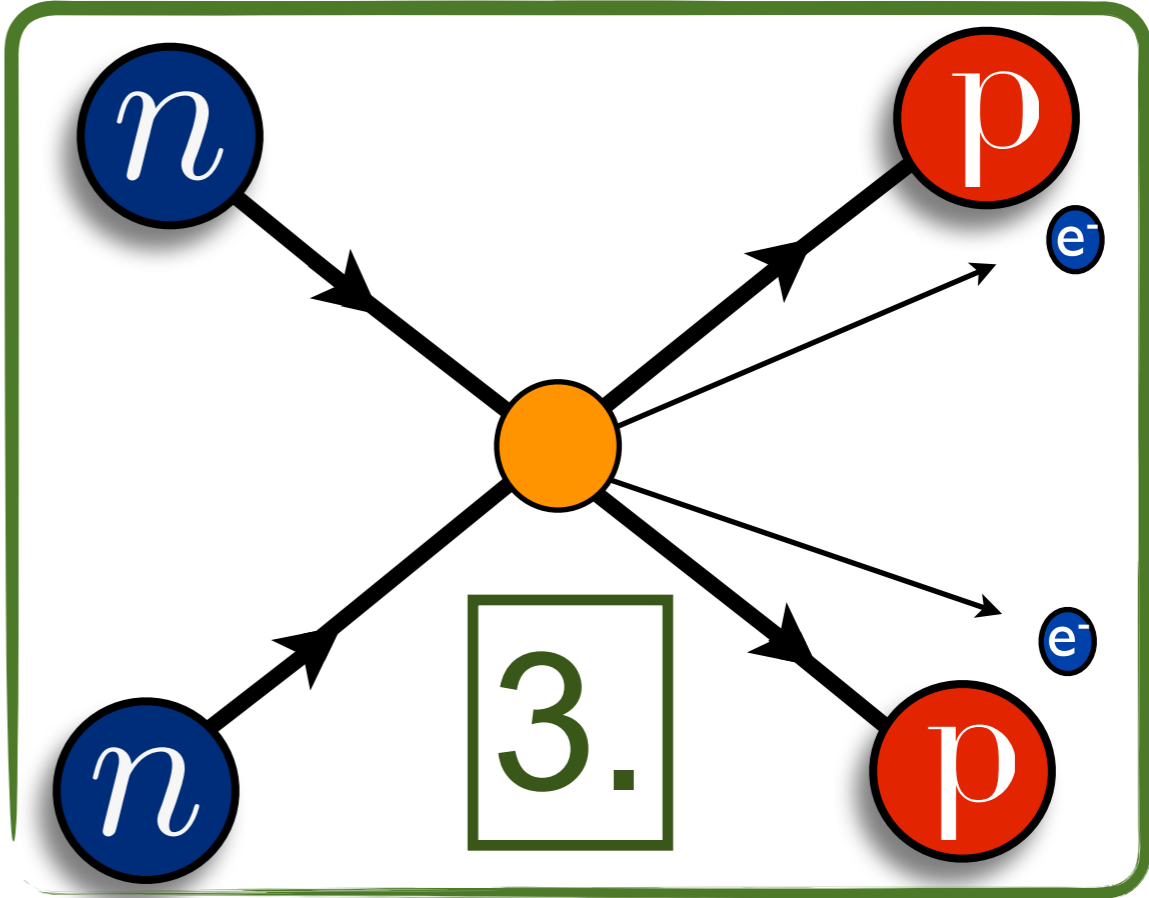
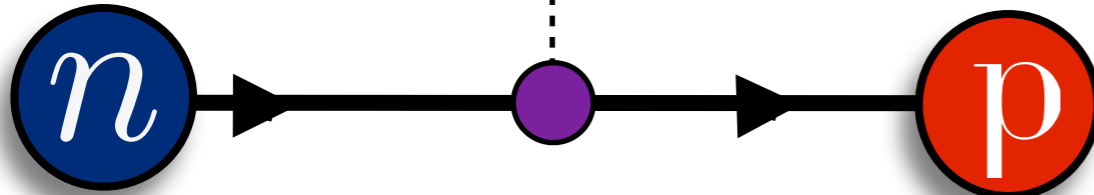
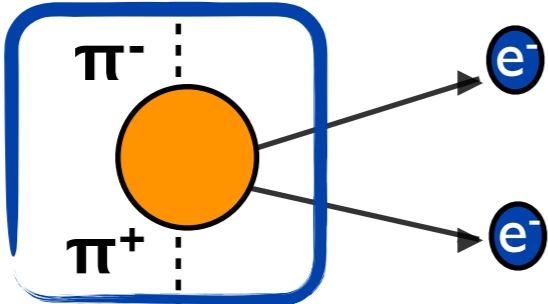




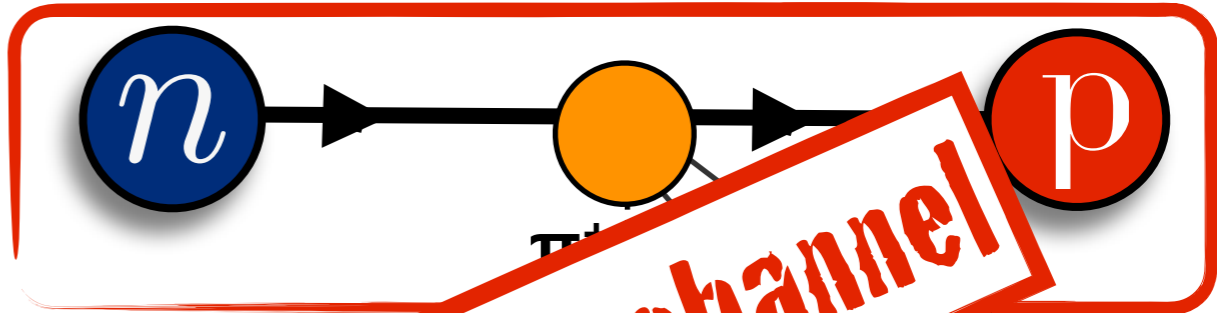
1.



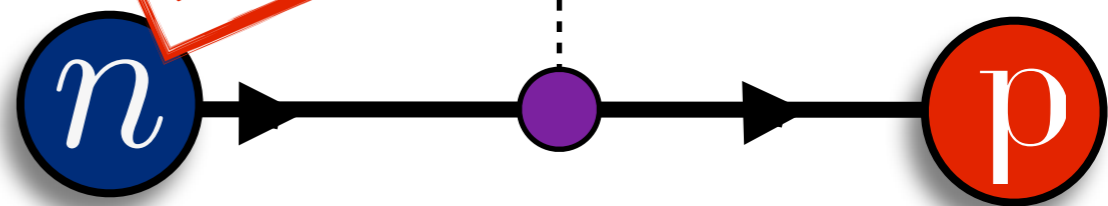
2.



3.

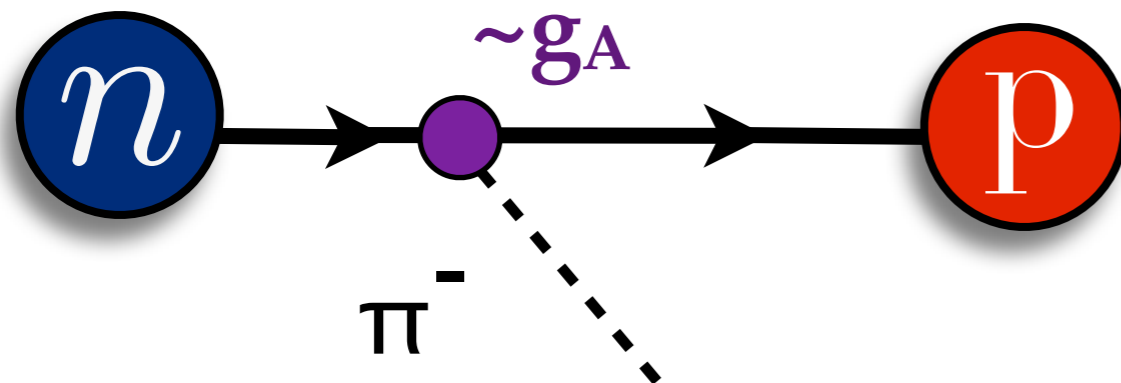
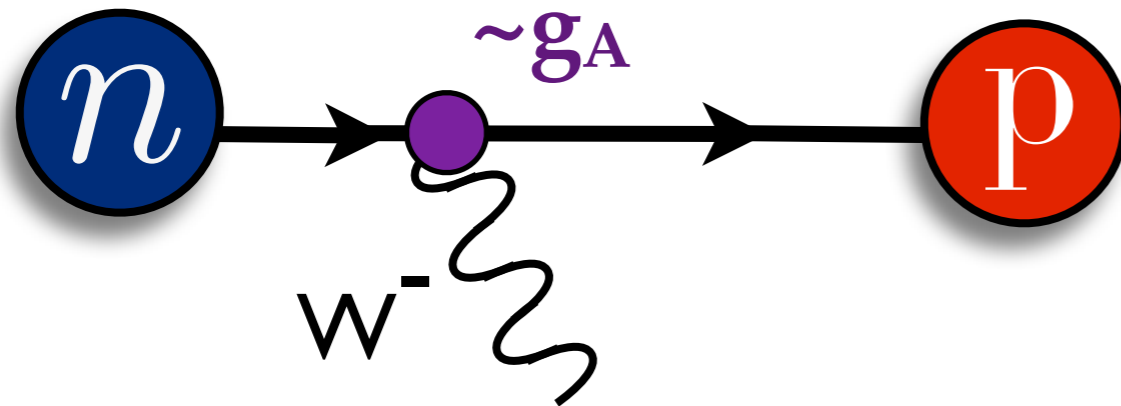


Difficult channel



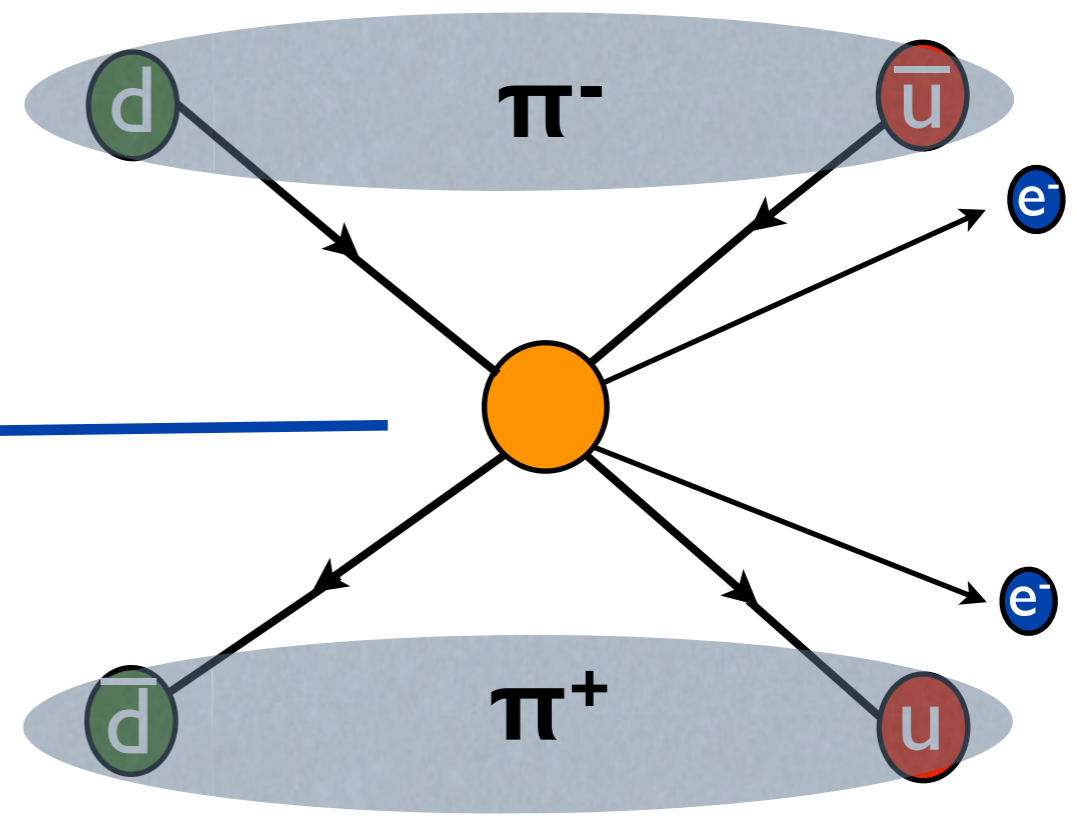
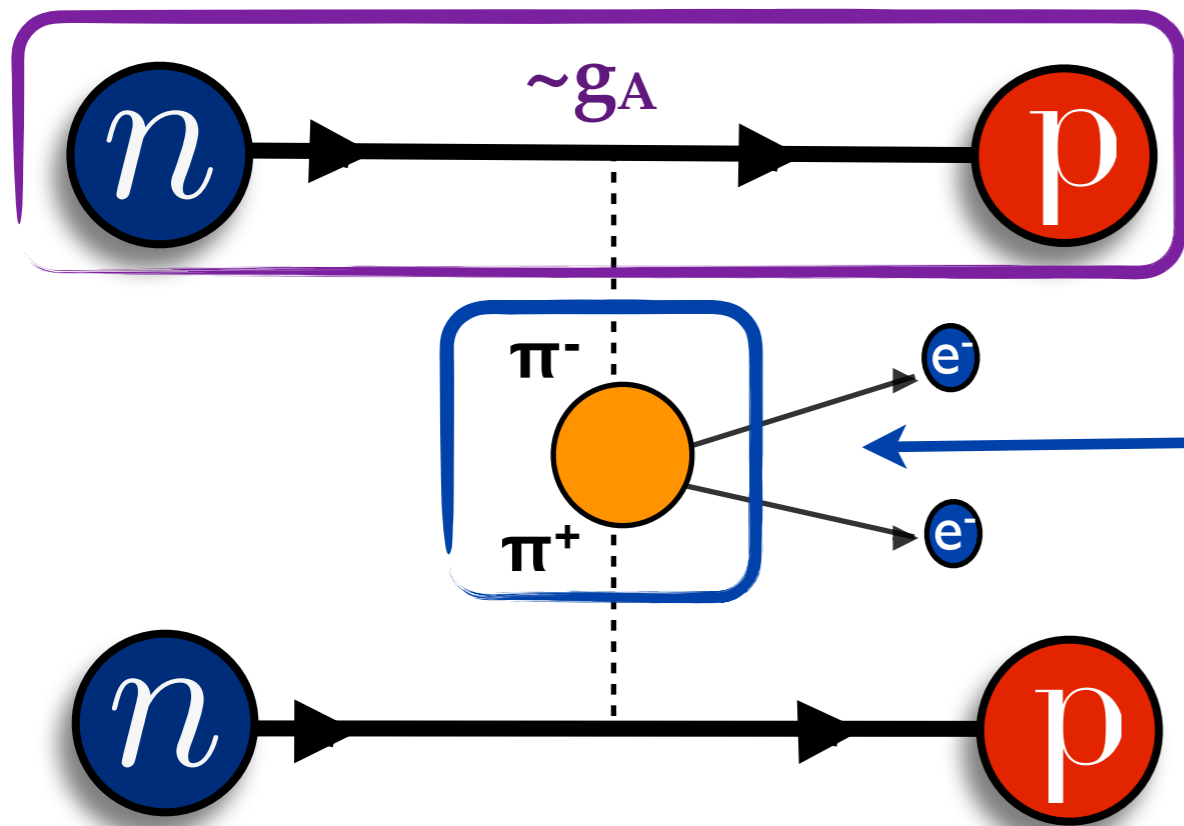
1.

Nucleon axial charge, g_A



- Very well-tested experimentally
 - $g_A^{\text{exp}} = 1.2723(23)$
 - good place to look for BSM physics
- Benchmark for nuclear physics on the lattice
- Would like to understand medium modifications (Z. Davoudi, P. Shanahan)

See R. Gupta's talk for status



2.

$\pi^- \rightarrow \pi^+$ Transition:
no direct experimental input

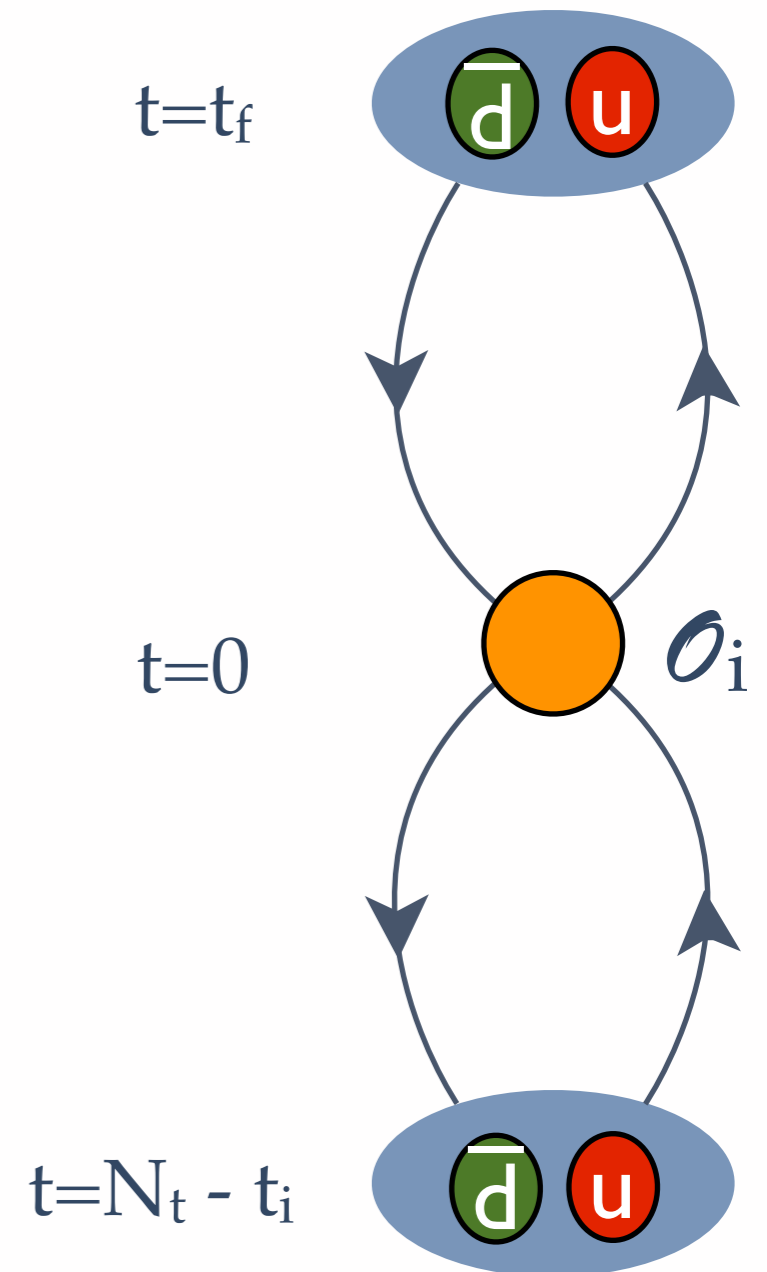
Long-range pion calculation

- Evolution in Euclidean time leads to exponential damping of excited states

$$\begin{aligned} \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle &= \langle \mathcal{O}(0) e^{-Ht} \mathcal{O}(0) \rangle \\ &= \sum_n |\langle 0 | \mathcal{O} | n \rangle|^2 e^{-E_n t} \xrightarrow{t \rightarrow \infty} \langle 0 | \mathcal{O} | 0 \rangle e^{-E_0 t} \end{aligned}$$

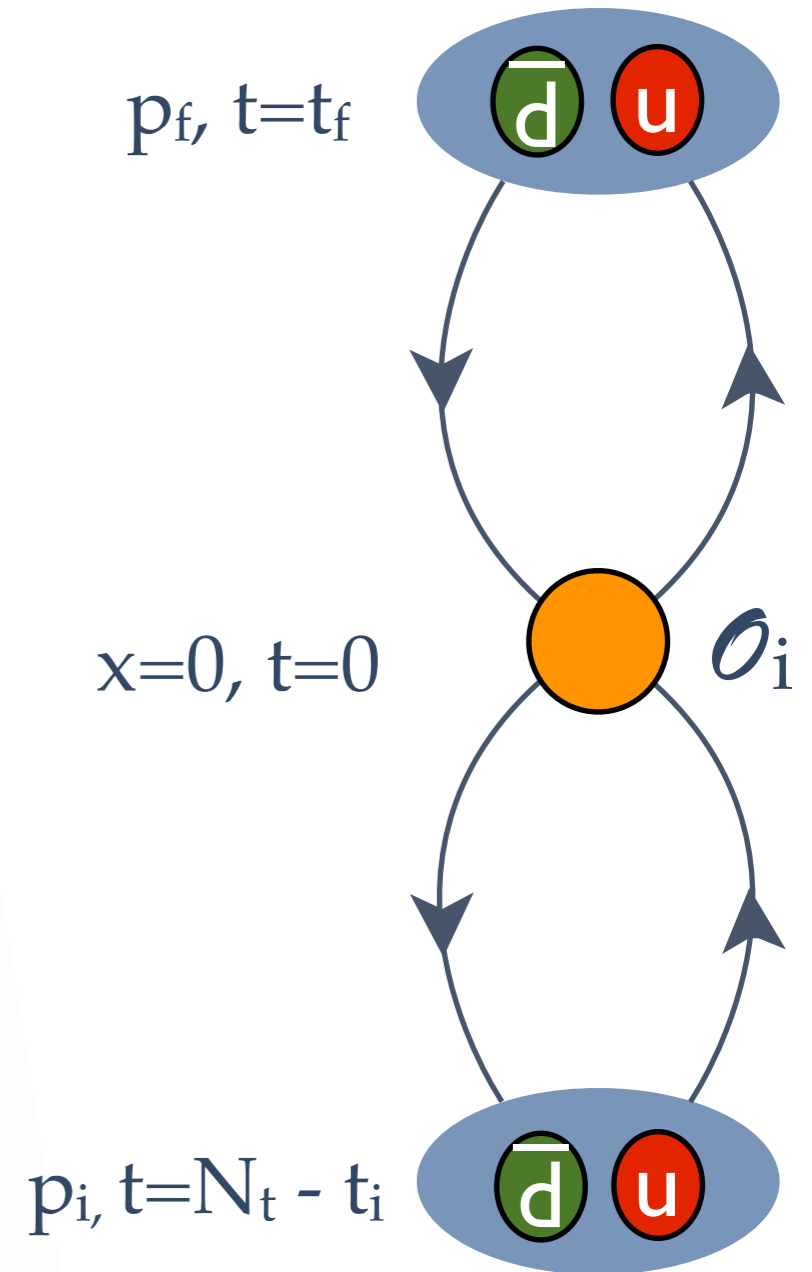
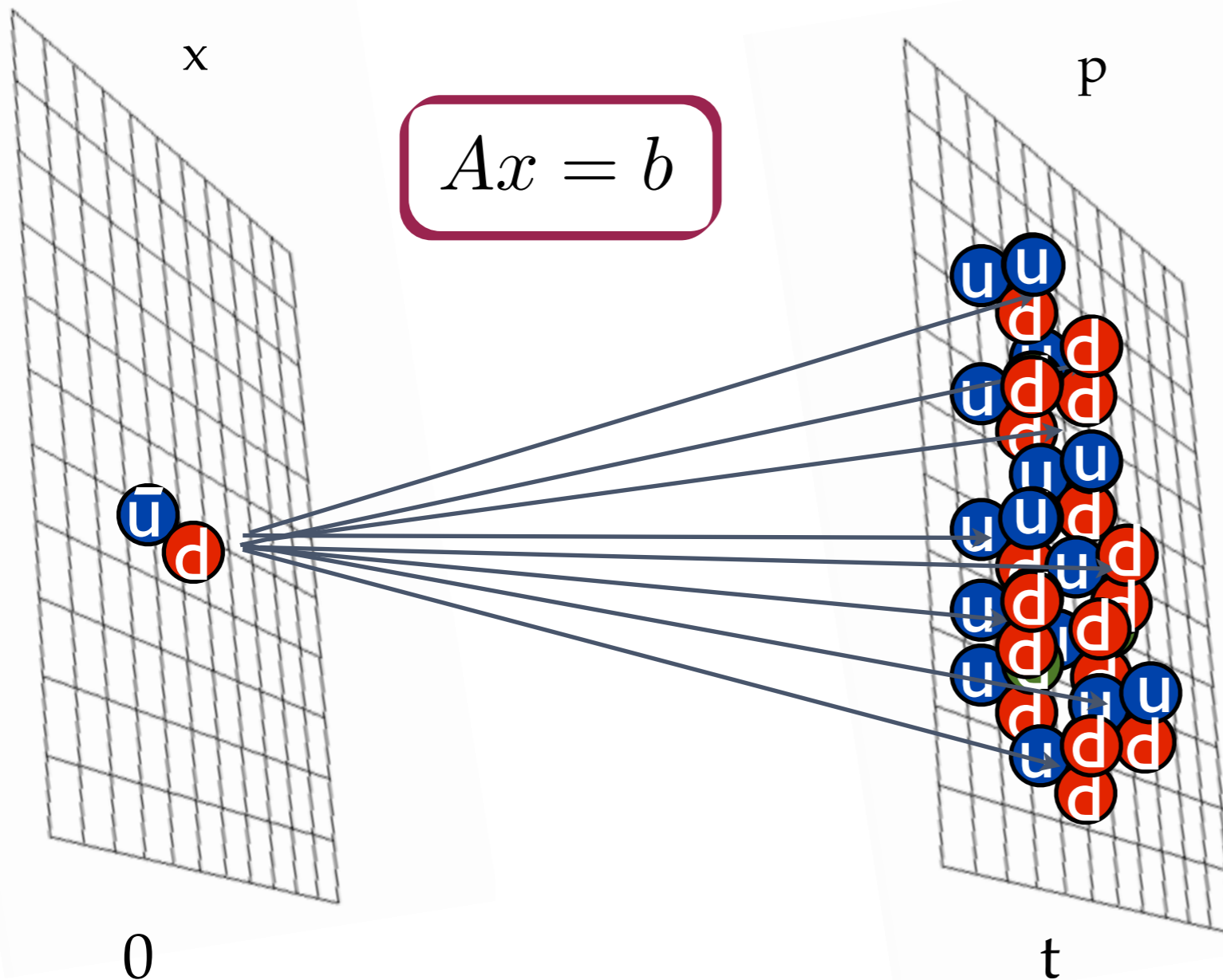
- Easy to compute pion physics on the lattice

- Clean signals
- Single particle



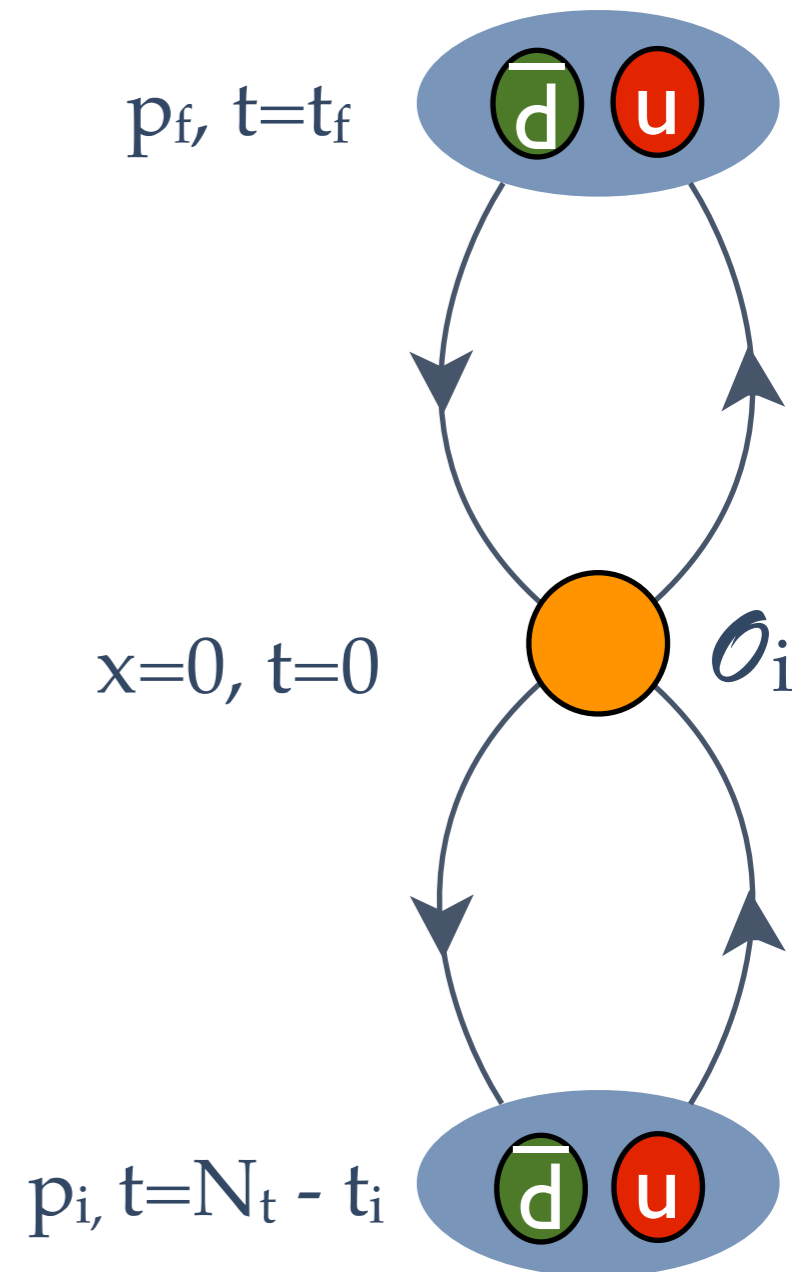
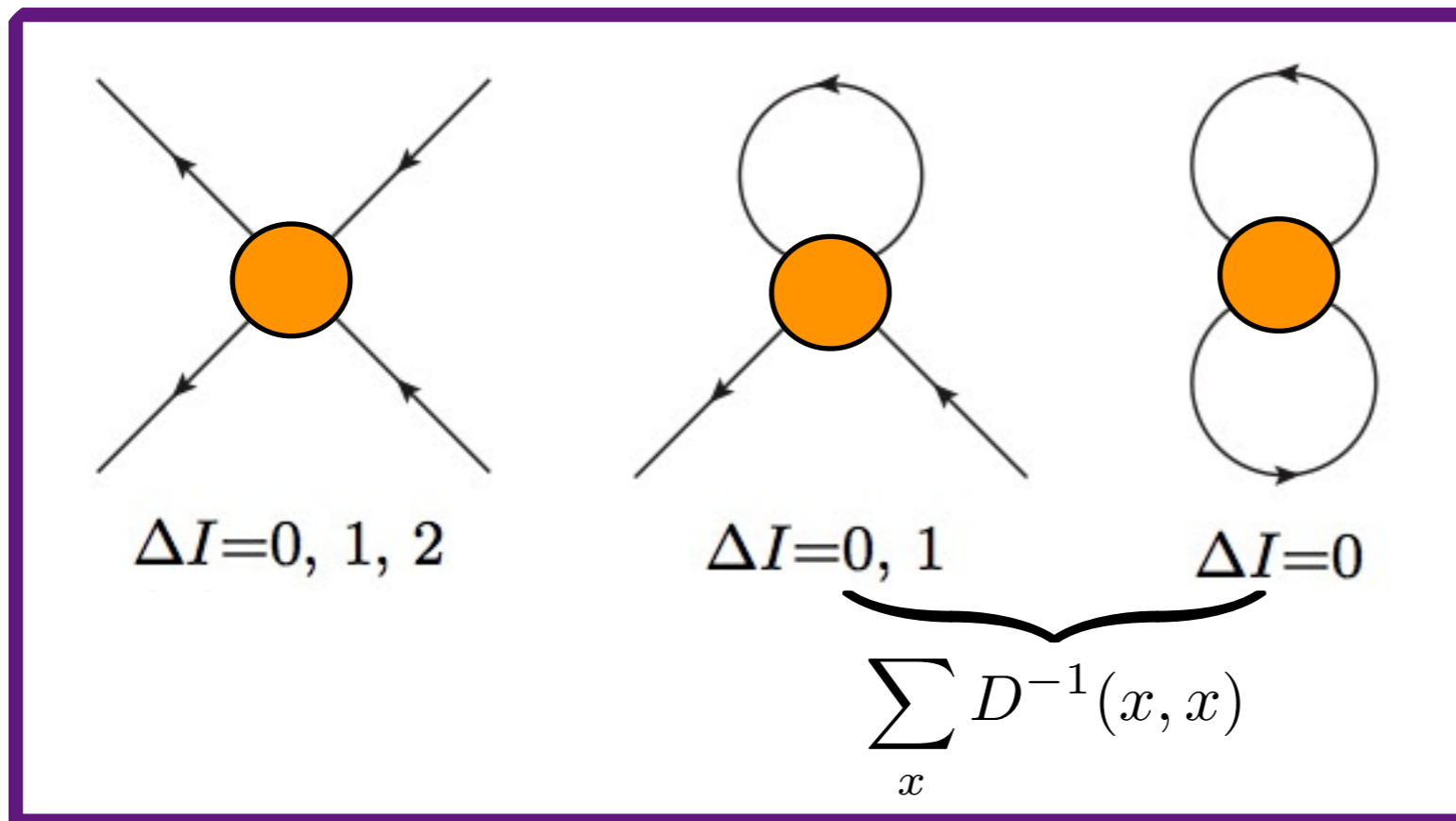
Long-range pion calculation

- Can perform exact momentum projection at source and sink



Long-range pion calculation

- Can perform exact momentum projection at source and sink
- $\Delta I = 2$ no disconnected pieces from operators



XPT:

$0\nu\beta\beta$ -decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\pi\pi ee$ LO	✓	✓	X	X	X	X	X	X	X
$\pi\pi ee$ NNLO	✓	✓	X	✓	X	X	X	X	X
$NN\pi ee$ LO	X	X	✓	X	X	✓	✓	✓	✓
$NN\pi ee$ NLO	X	✓	X	✓	X	✓	✓	✓	✓
$NNNNe e$ LO	✓	✓	X	✓	X	✓	✓	✓	✓

$$\mathcal{O}_{1+}^{ab} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_R \tau^b \gamma_\mu q_R),$$

$$\mathcal{O}_{2\pm}^{ab} = (\bar{q}_R \tau^a q_L)(\bar{q}_R \tau^b q_L) \pm (\bar{q}_L \tau^a q_R)(\bar{q}_L \tau^b q_R),$$

$$\mathcal{O}_{3\pm}^{ab} = (\bar{q}_L \tau^a \gamma^\mu q_L)(\bar{q}_L \tau^b \gamma_\mu q_L) \pm (\bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_R \tau^b \gamma_\mu q_R),$$

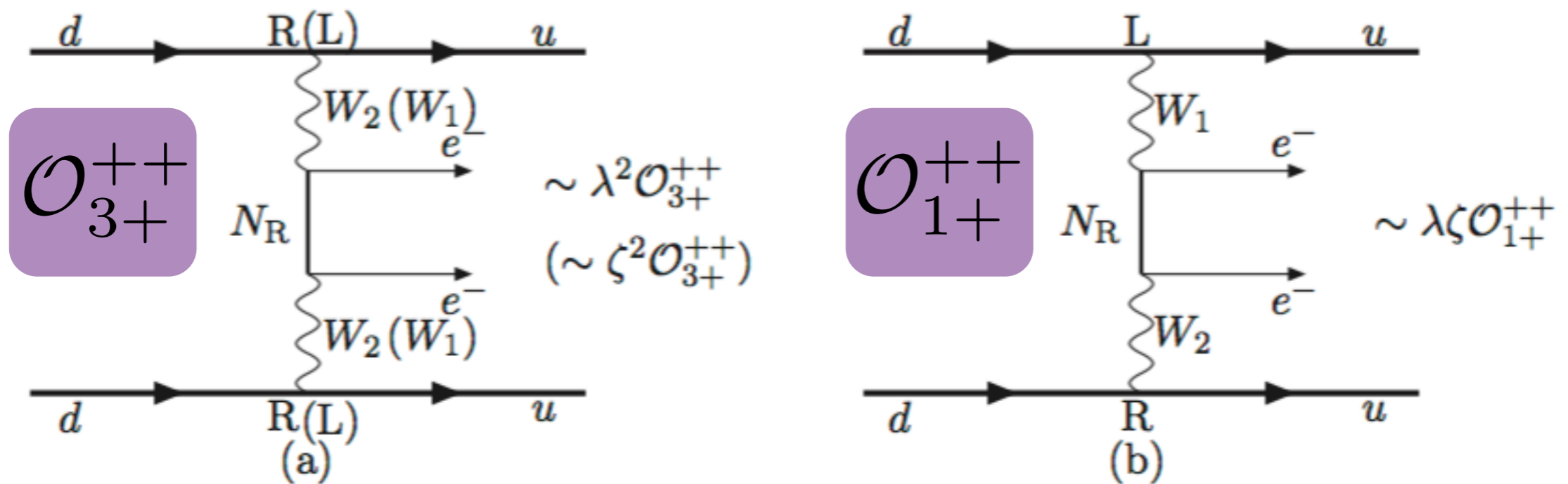
$$\mathcal{O}_{4\pm}^{ab,\mu} = (\bar{q}_L \tau^a \gamma^\mu q_L \mp \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R - \bar{q}_R \tau^b q_L),$$

$$\mathcal{O}_{5\pm}^{ab,\mu} = (\bar{q}_L \tau^a \gamma^\mu q_L \pm \bar{q}_R \tau^a \gamma^\mu q_R)(\bar{q}_L \tau^b q_R + \bar{q}_R \tau^b q_L).$$

XPT:

$0\nu\beta\beta$ -decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\pi\pi ee$ LO	✓	✓	X	X	X	X	X	X	X
$\pi\pi ee$ NNLO	✓	✓	X	✓	X	X	X	X	X
$NN\pi ee$ LO	X	X	✓	X	X	✓	✓	✓	✓
$NN\pi ee$ NLO	X	✓	X	✓	X	✓	✓	✓	✓
$NNNNe e$ LO	✓	✓	X	✓	X	✓	✓	✓	✓

Left-right symmetric models



Contractions

- QCD interactions can mix colors below the electroweak scale
- Must add color mixed versions of Prezeau, Ramsey-Musolf, Vogel ops 1&2

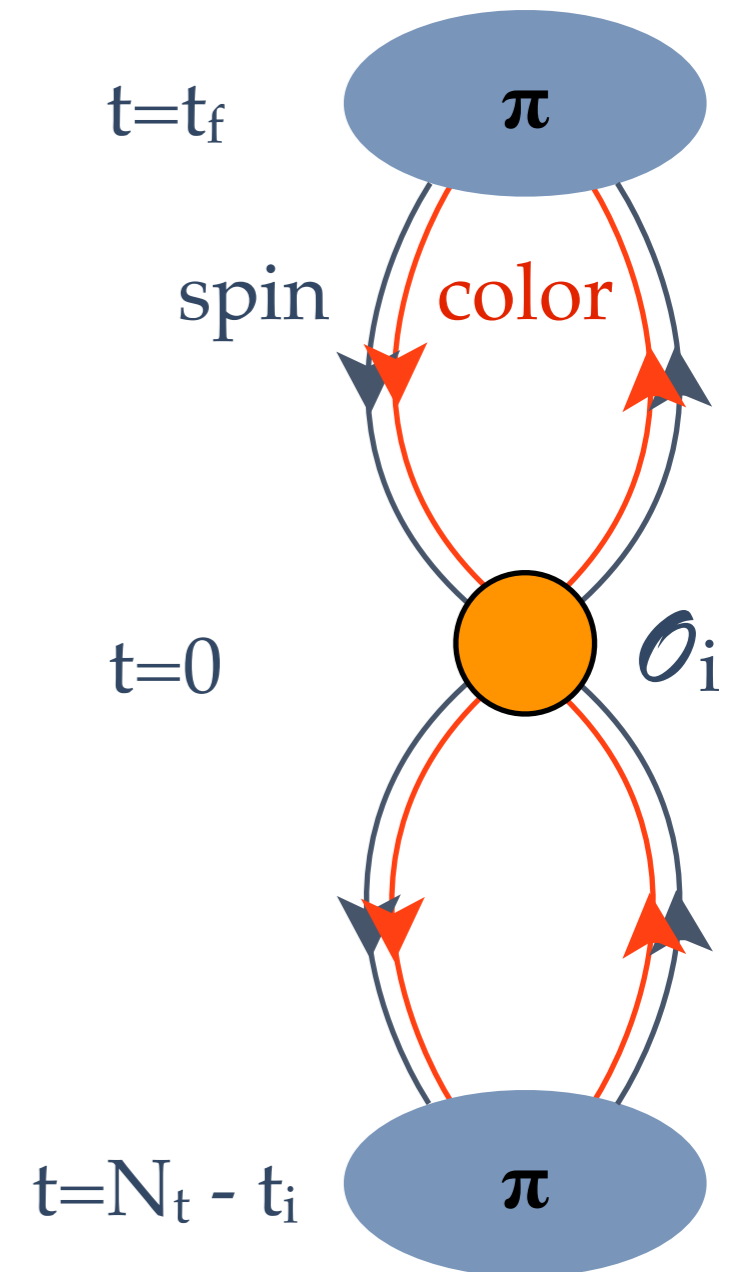
$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}'_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}'_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}_{3+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_L \tau^- \gamma_\mu q_L] + (\bar{q}_R \tau^- \gamma^\mu q_R) [\bar{q}_R \tau^- \gamma_\mu q_R]$$



Contractions

- QCD interactions can mix colors below the electroweak scale
- Must add color mixed versions of Prezeau, Ramsey-Musolf, Vogel ops 1&2

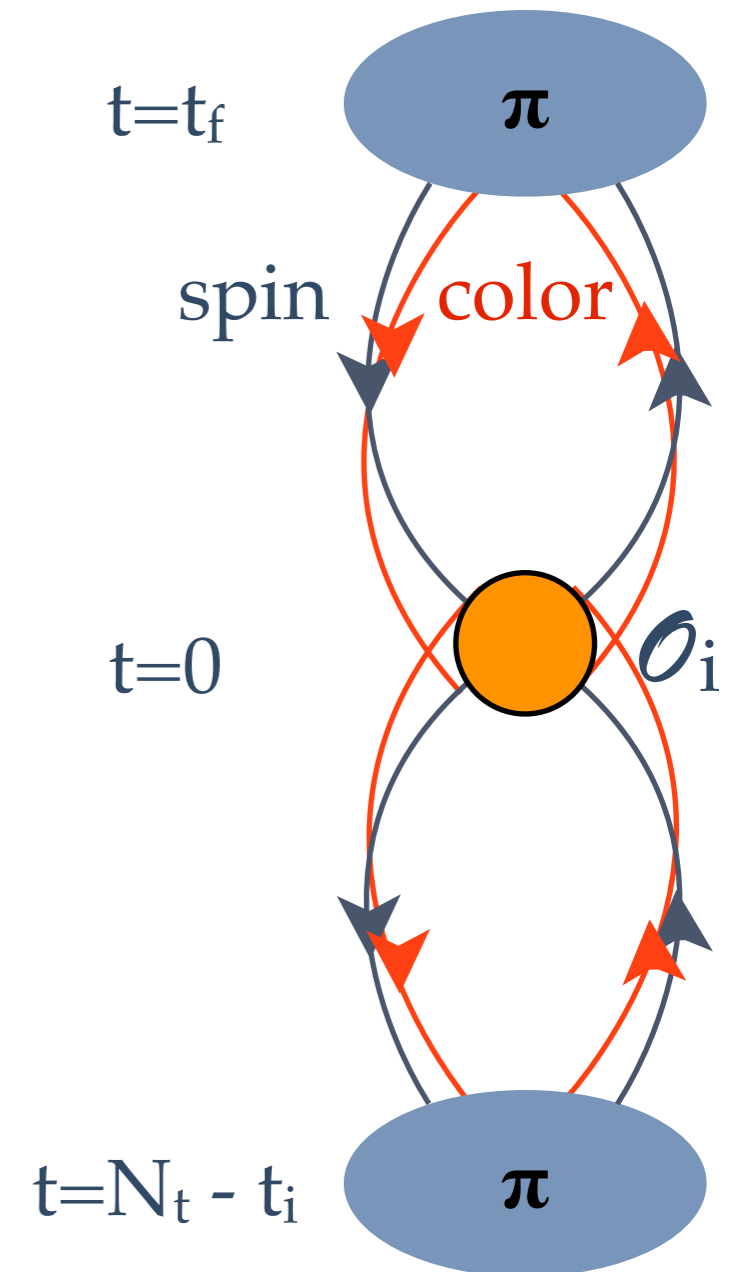
$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

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$$\mathcal{O}_{3+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_L \tau^- \gamma_\mu q_L] + (\bar{q}_R \tau^- \gamma^\mu q_R) [\bar{q}_R \tau^- \gamma_\mu q_R]$$



Lattice Ensembles

HISQ ensembles

$a[fm] : m_\pi[MeV]$	310	220	135
0.15	$16^3 \times 48, m_\pi L \sim 3.78$	$24^3 \times 48, m_\pi L \sim 3.99$	$32^3 \times 48, m_\pi L \sim 3.25$
0.12		$24^3 \times 64, m_\pi L \sim 3.22$	
0.12	$24^3 \times 64, m_\pi L \sim 4.54$	$32^3 \times 64, m_\pi L \sim 4.29$	$48^3 \times 64, m_\pi L \sim 3.91$
0.12		$40^3 \times 64, m_\pi L \sim 5.36$	
0.09	$32^3 \times 96, m_\pi L \sim 4.50$	$48^3 \times 96, m_\pi L \sim 4.73$	

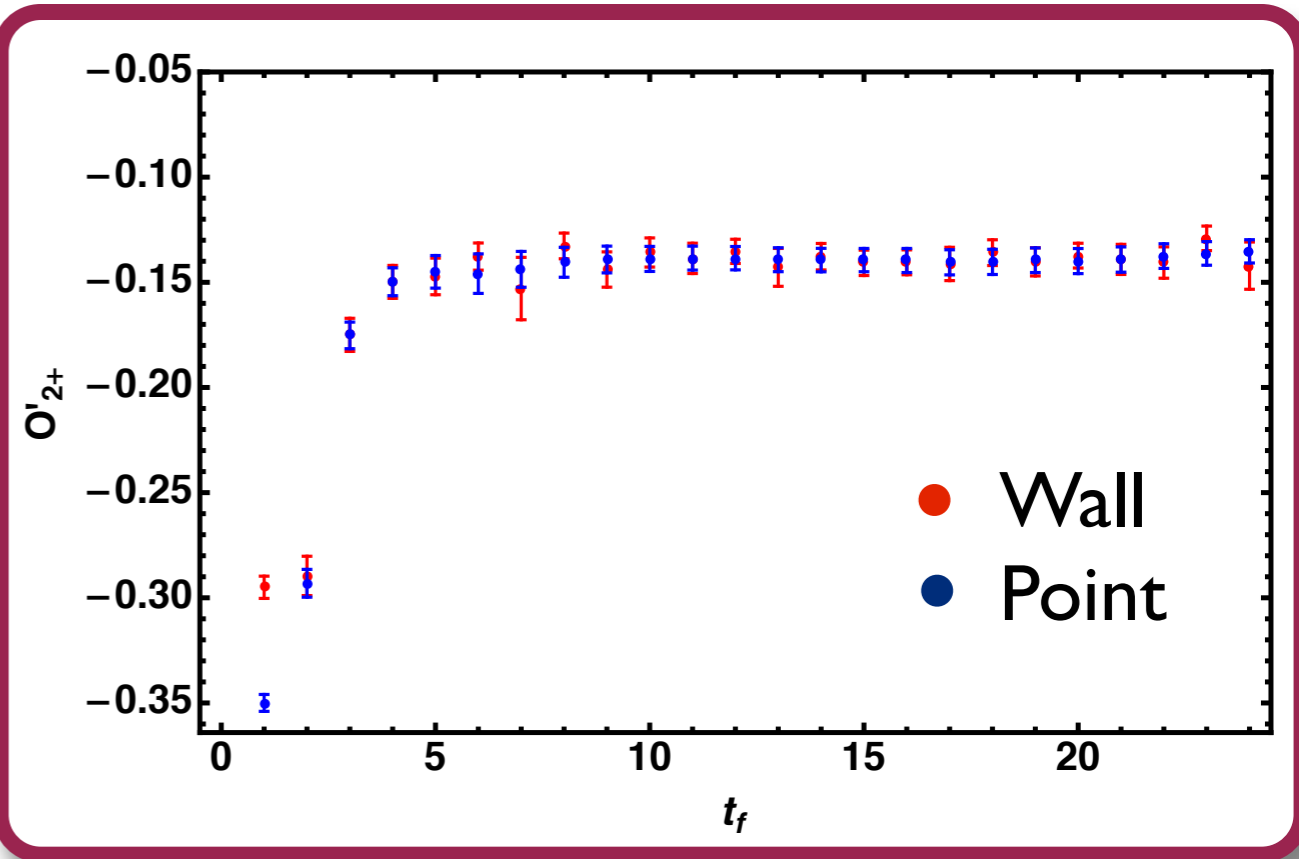
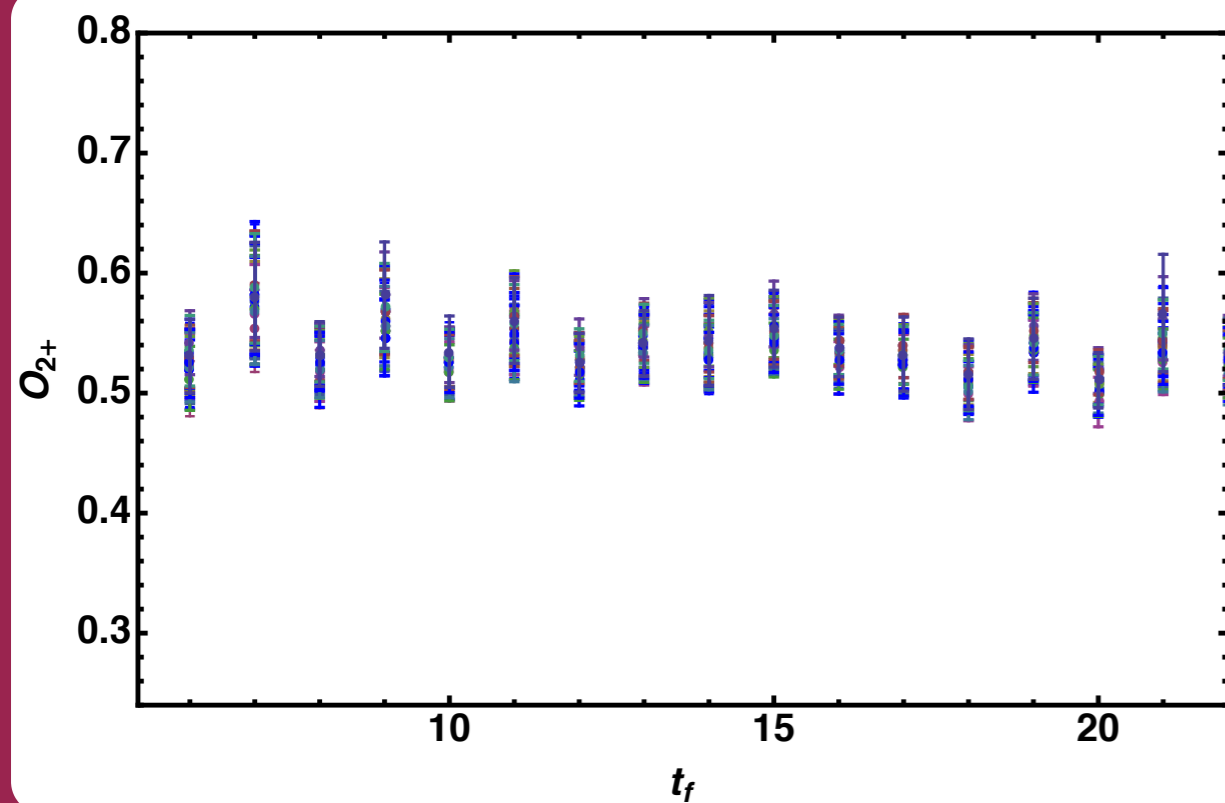
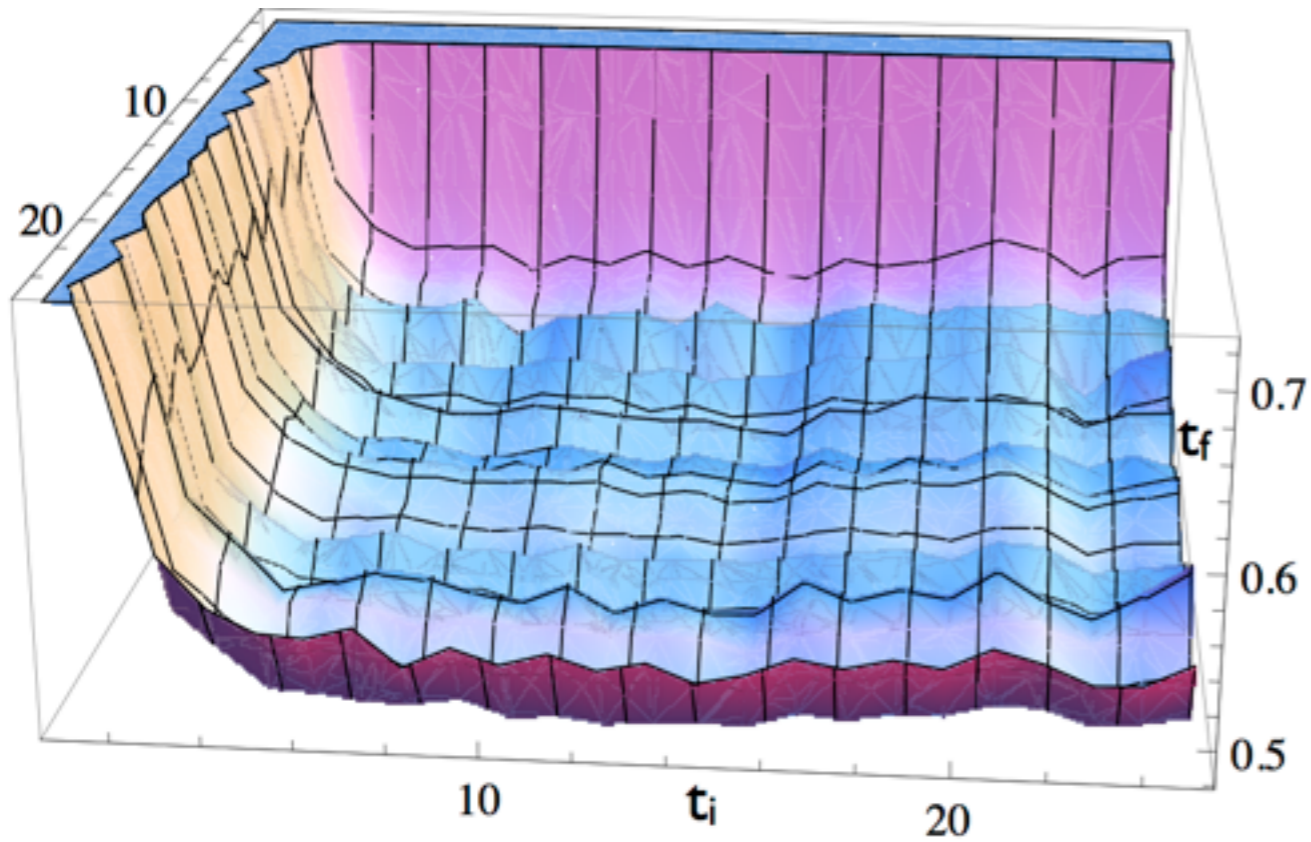
- Möbius DWF on HISQ
- Gradient flow method for smearing configs
 - $m_{\text{res}} < 0.1 m_\ell$ for moderate L_5

MILC Collaboration Phys.
Rev. D87 (2013) 054505

Narayanan, Neuberger
(2006), Luscher (2010)

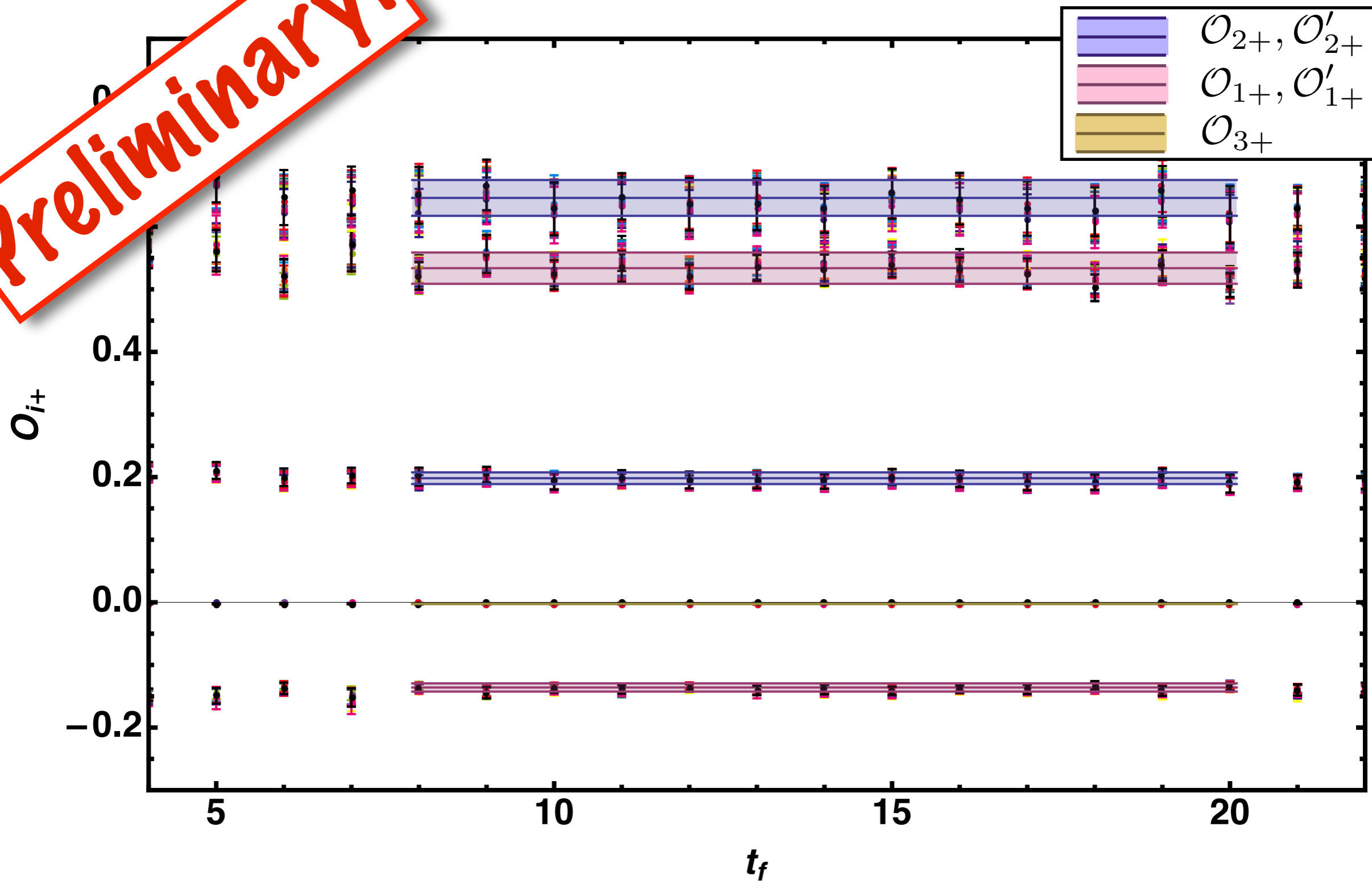
Callat arXiv:1701.07559

Signals

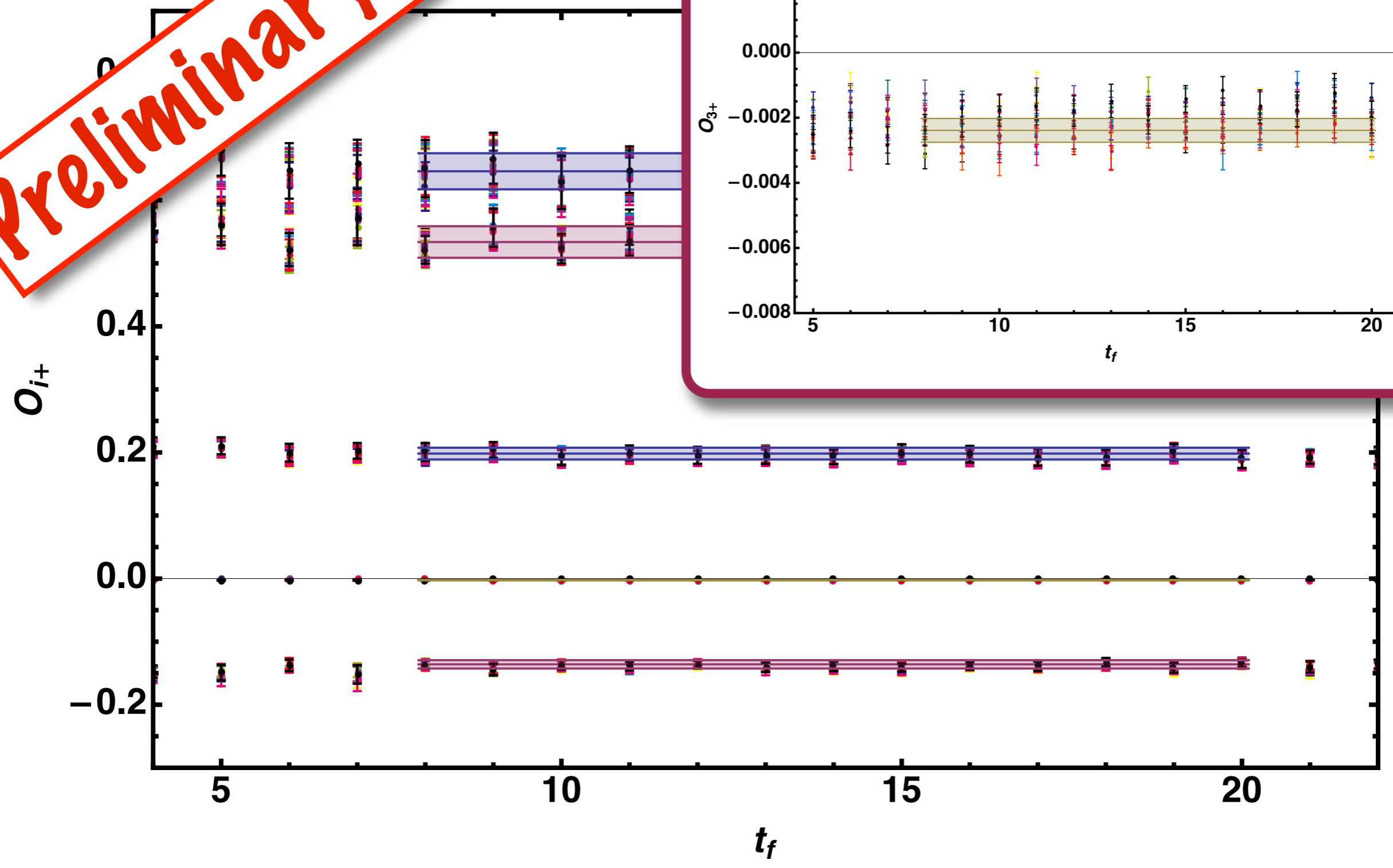


- $m_\pi \sim 135$ MeV
- $L = 5.76$ fm
- $a = 0.12$ fm

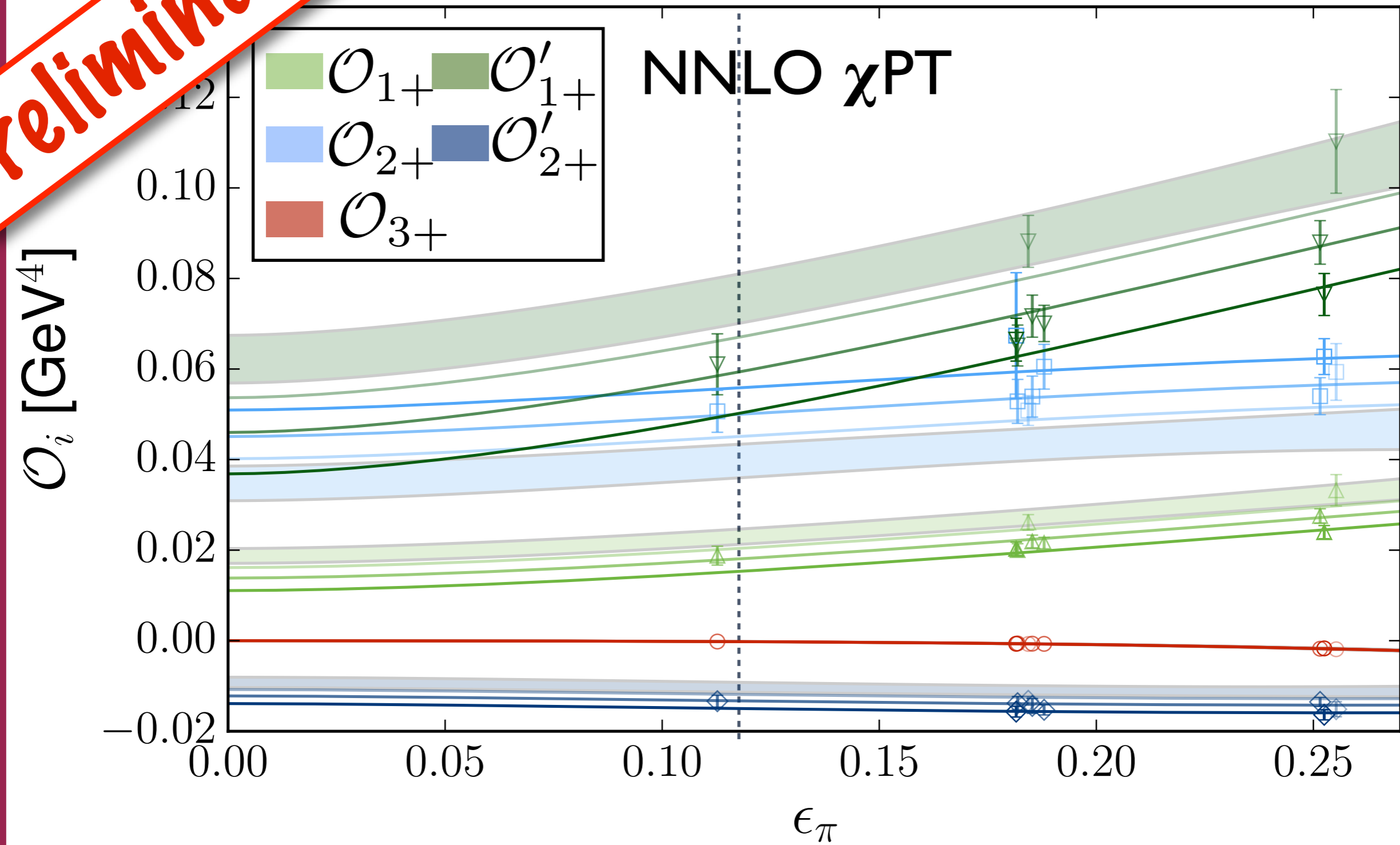
Preliminary!



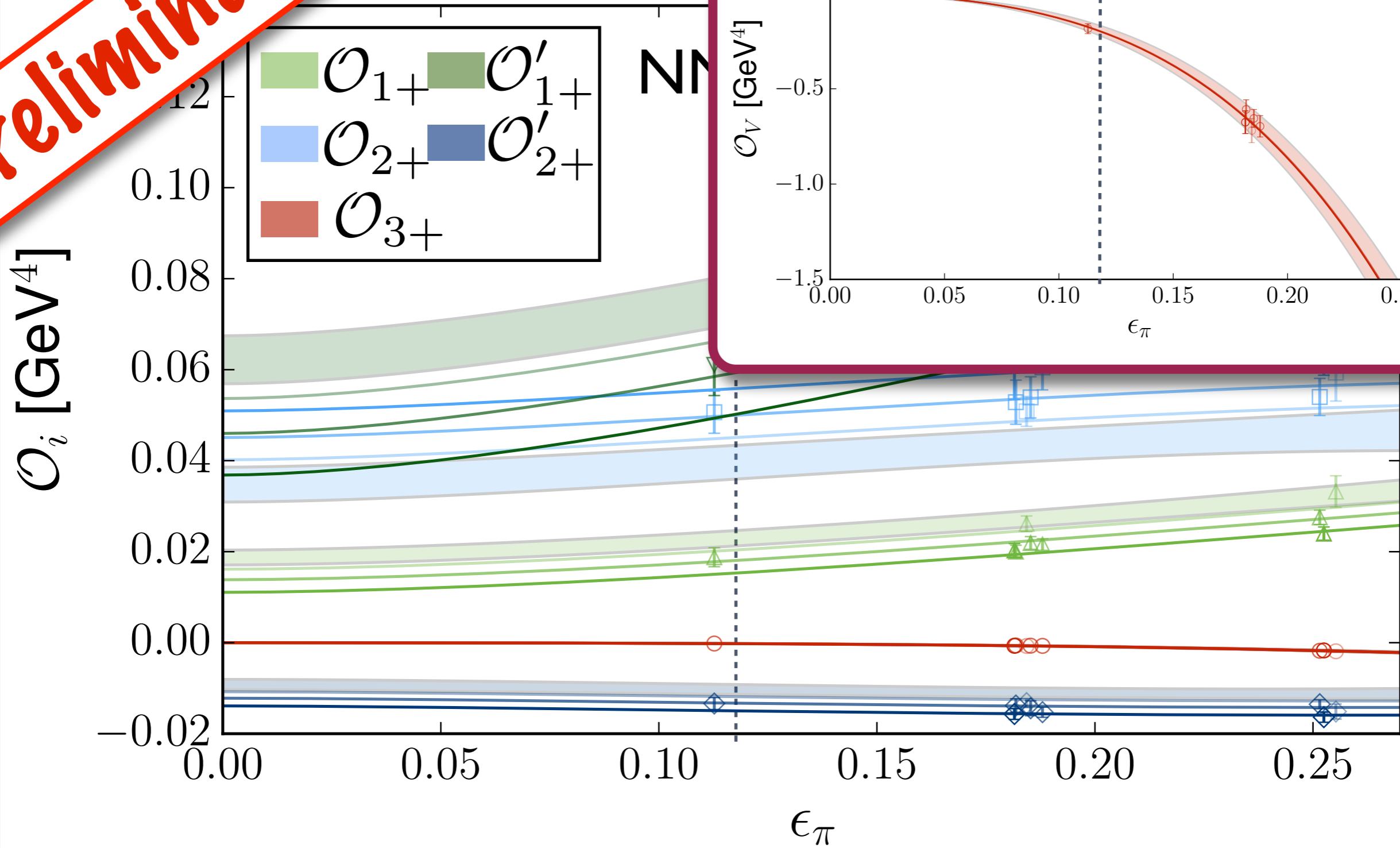
Preliminary!



Preliminary!

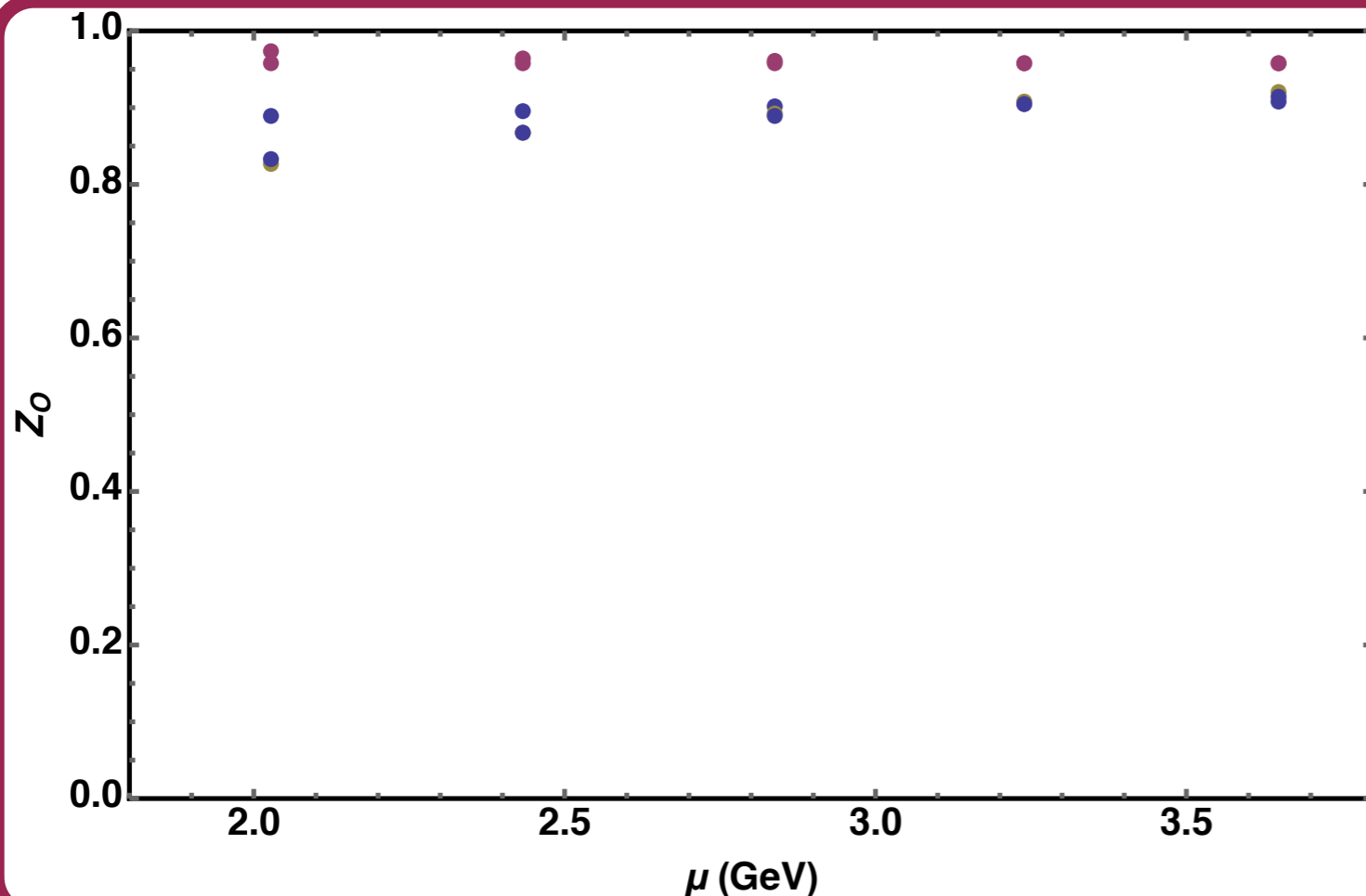


Preliminary!

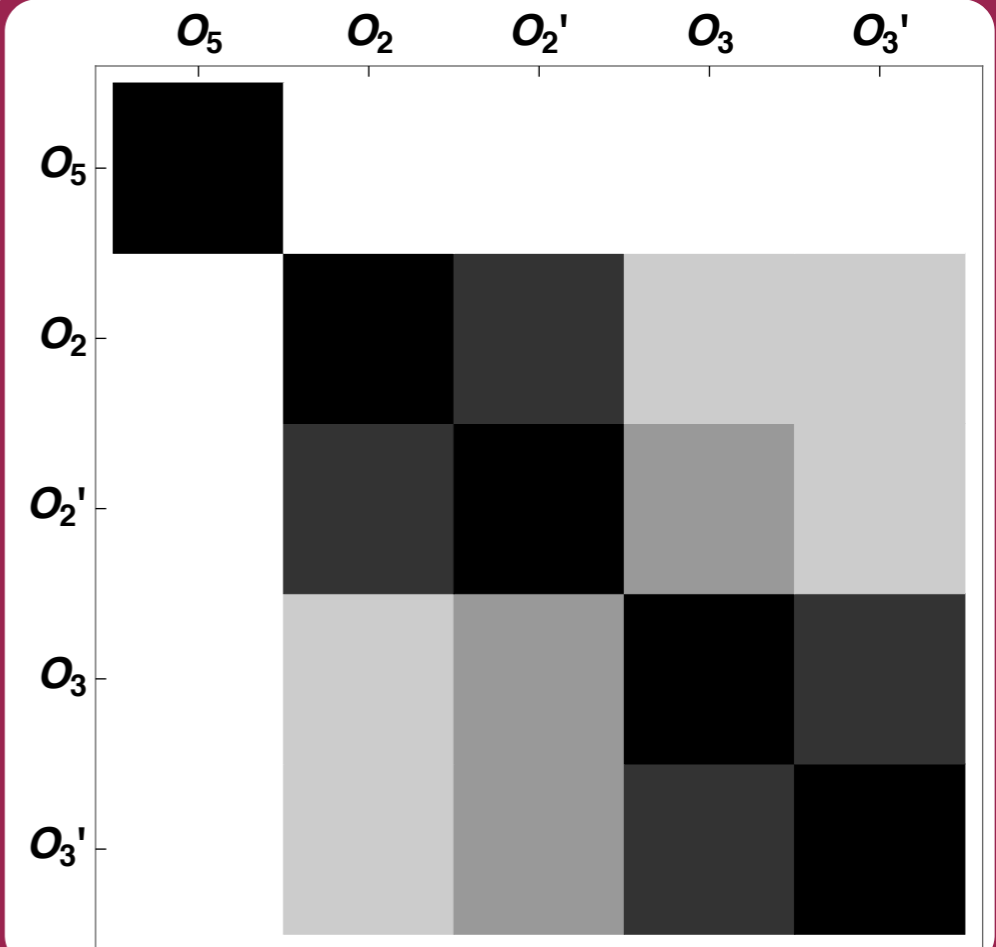


Renormalization

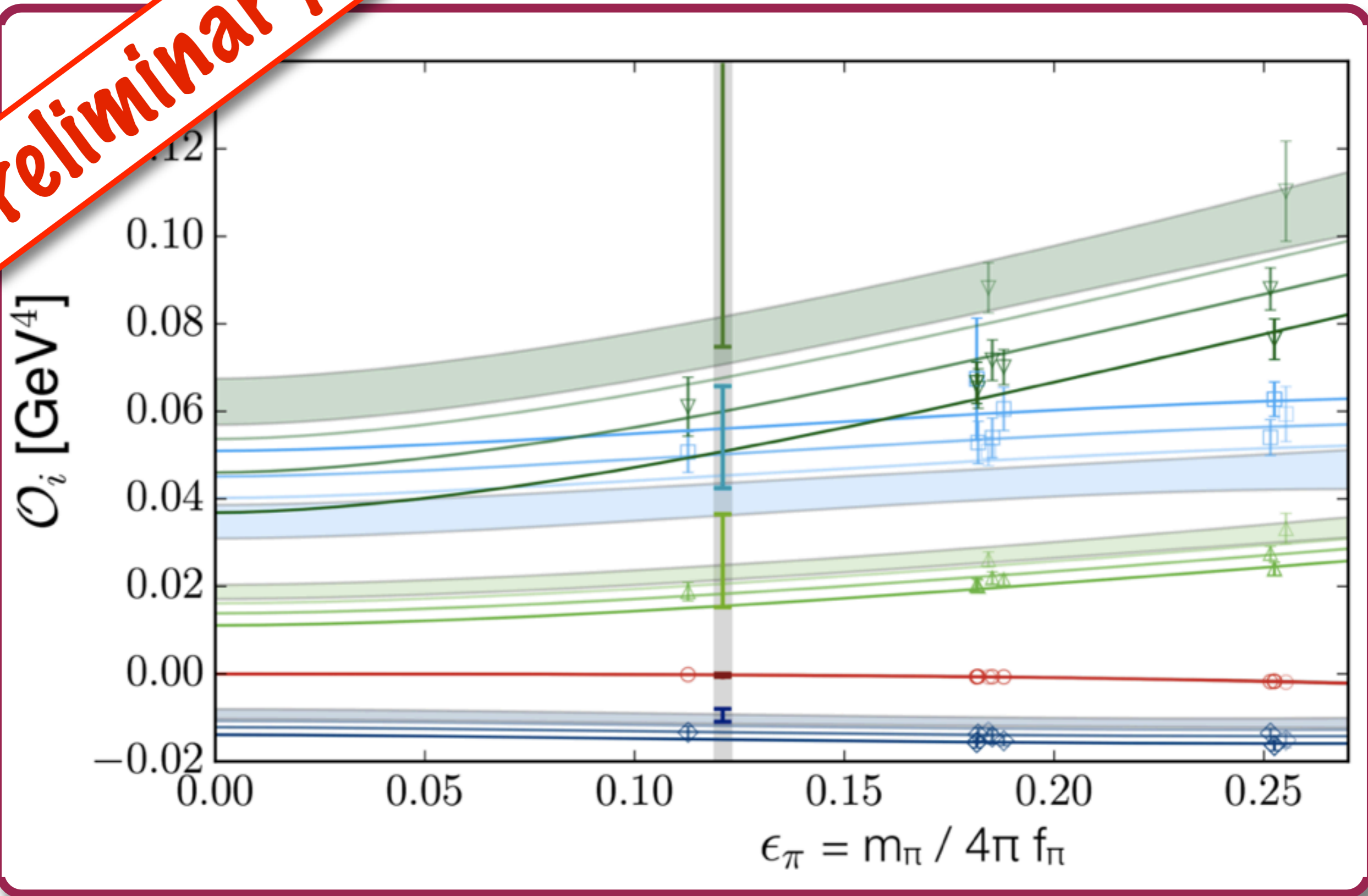
- Lattice perturbation theory is difficult and poorly convergent
- Nonperturbative running (RI-SMOM) to match onto \overline{MS}

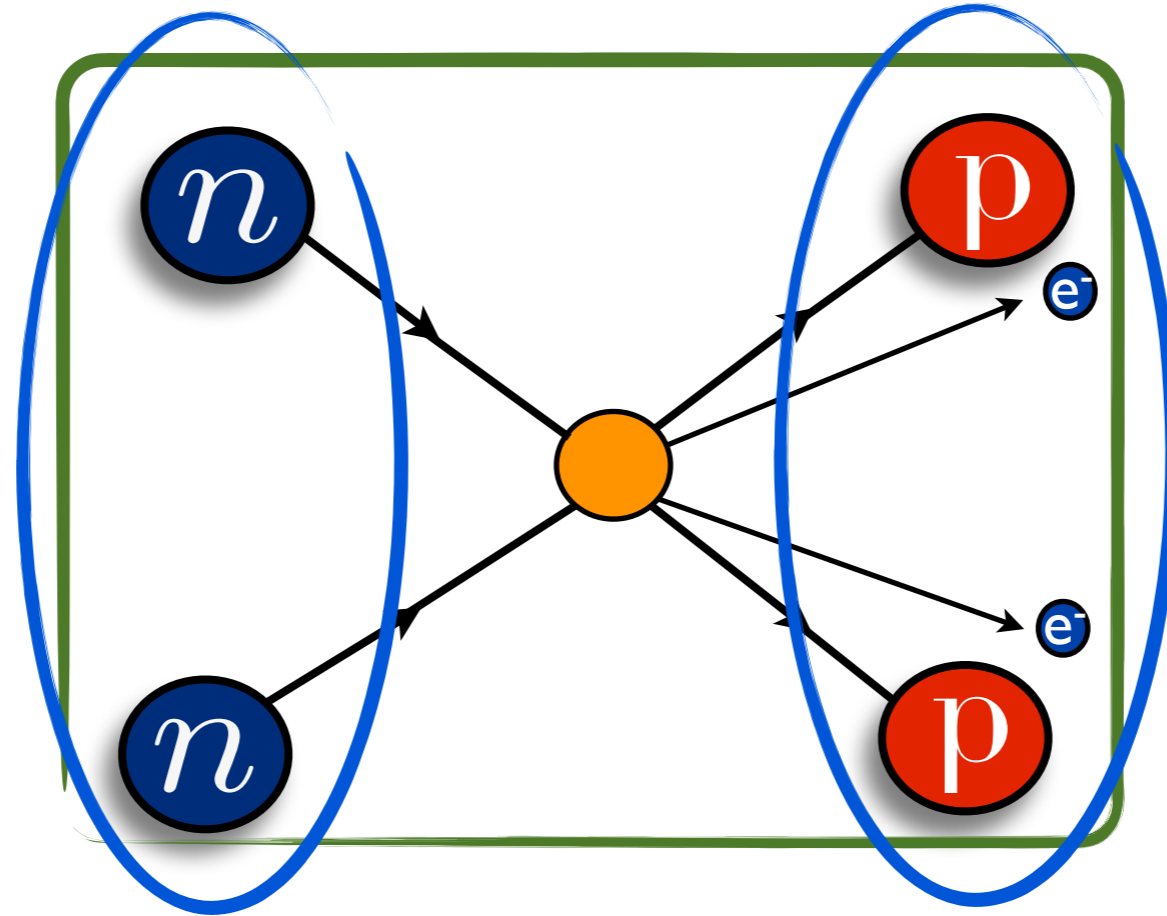


Mixing matrix



Preliminary!





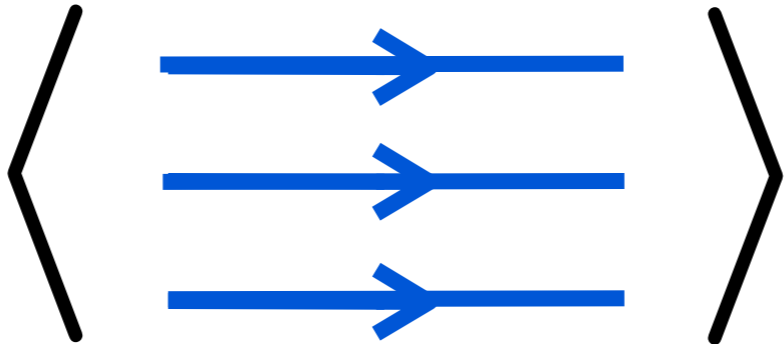
3.

Two-nucleon
contact



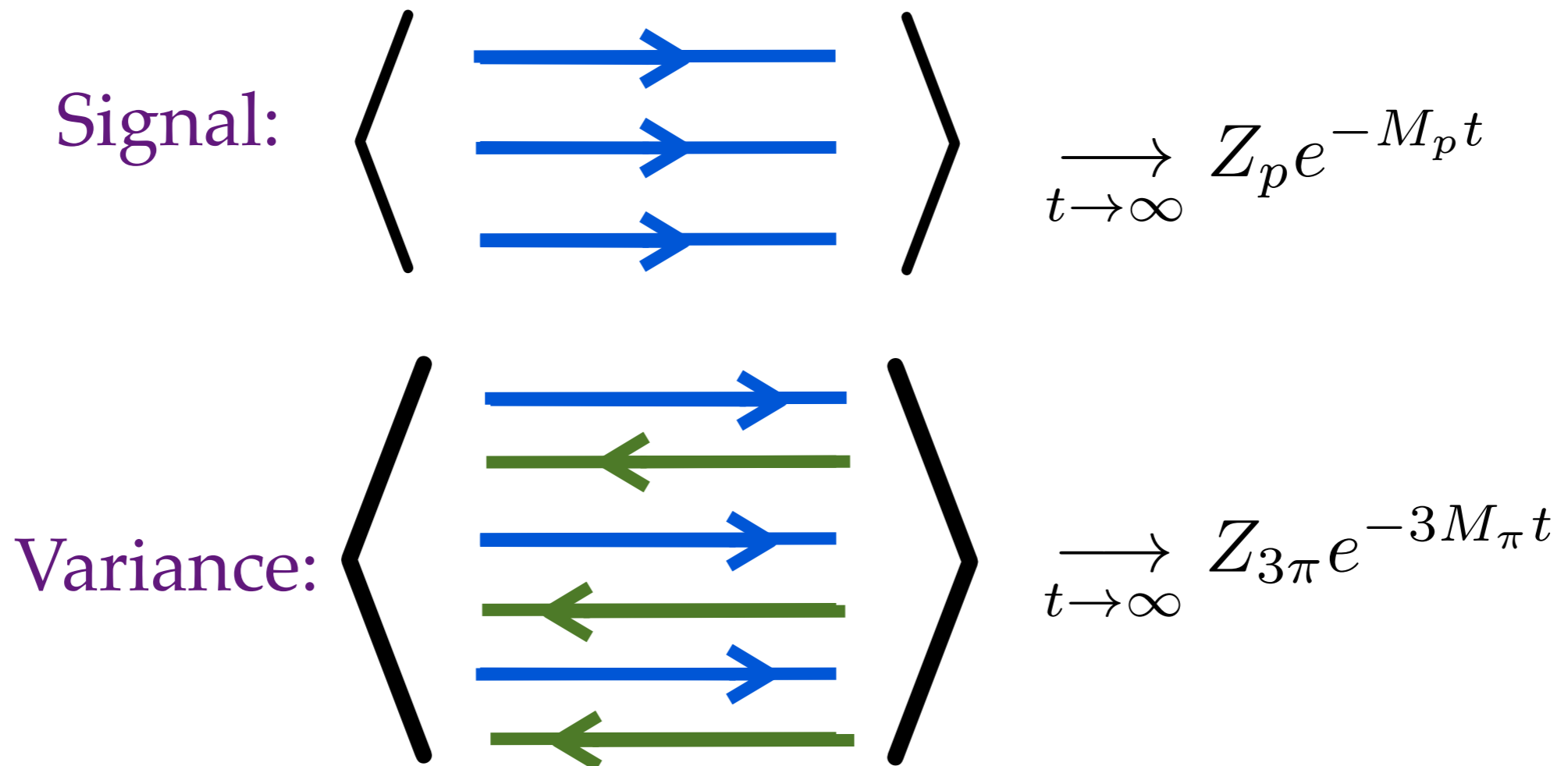
Two-nucleon contact

- Nucleons and multi-particle states are much more difficult!
- exponentially poor signal-to-noise problem, small excited state energy splittings,

Signal:  $\xrightarrow[t \rightarrow \infty]{} Z_p e^{-M_p t}$

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$$\frac{\text{Signal}}{\text{Noise}} \xrightarrow{t \rightarrow \infty} \sqrt{N_{\text{cfgs}}} \frac{Z_{Ap}}{\sqrt{Z_{3A\pi}}} e^{-A(M_p - 3/2m_\pi)t}$$

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Exponentially
large ensembles

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Exponentially
large ensembles

Heavy quark
mass

Two-nucleon contact

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Exponentially large ensembles

Maximize overlap

Heavy quark mass

Two-nucleon contact

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$\frac{\text{Signal}}{\text{Noise}} \xrightarrow{t \rightarrow \infty} \frac{\sqrt{N_{\text{cfgs}}} Z_{Ap}}{\sqrt{Z_{3A\pi}}} e^{-A(M_p - 3/2m_\pi)t}$

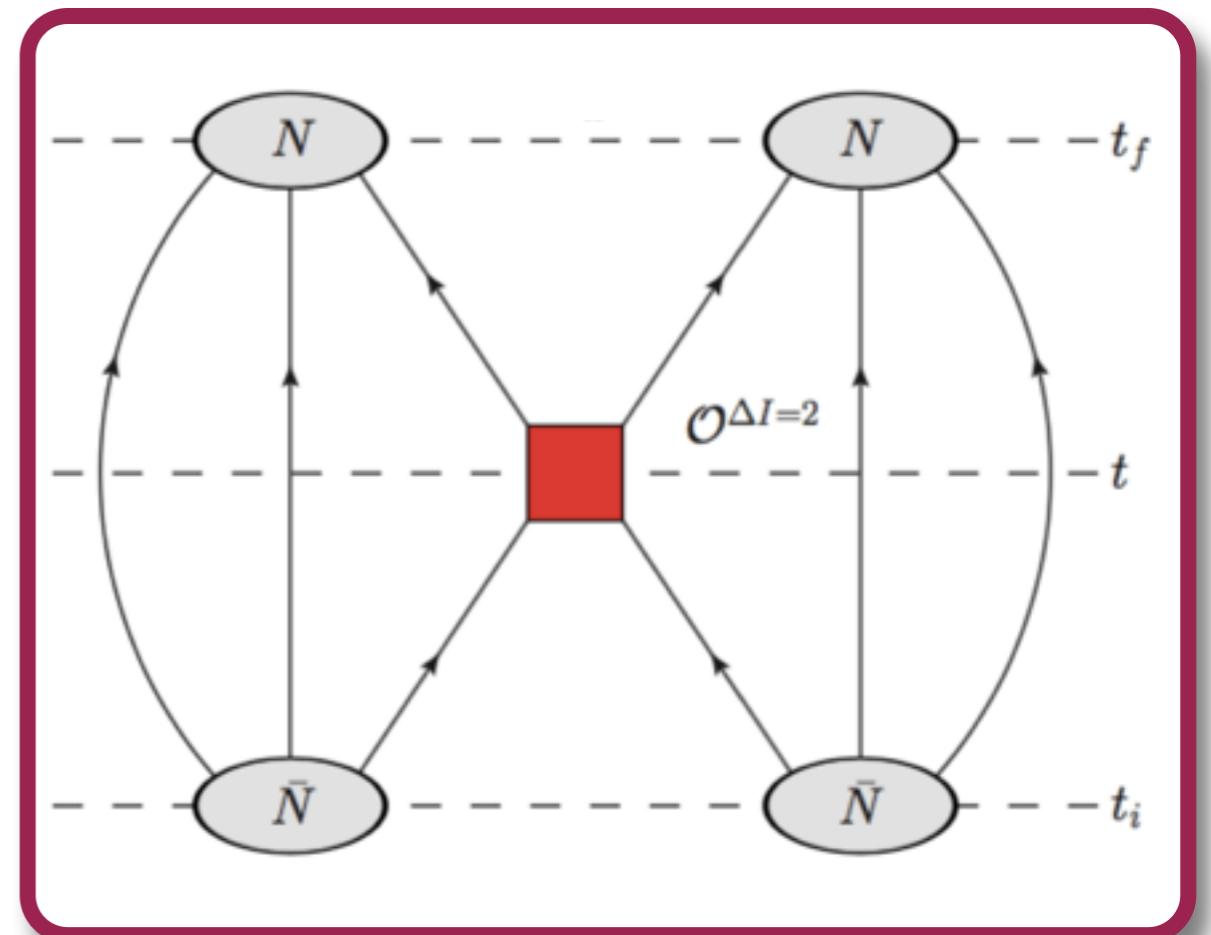
Exponentially large ensembles

Maximize overlap

Heavy quark mass

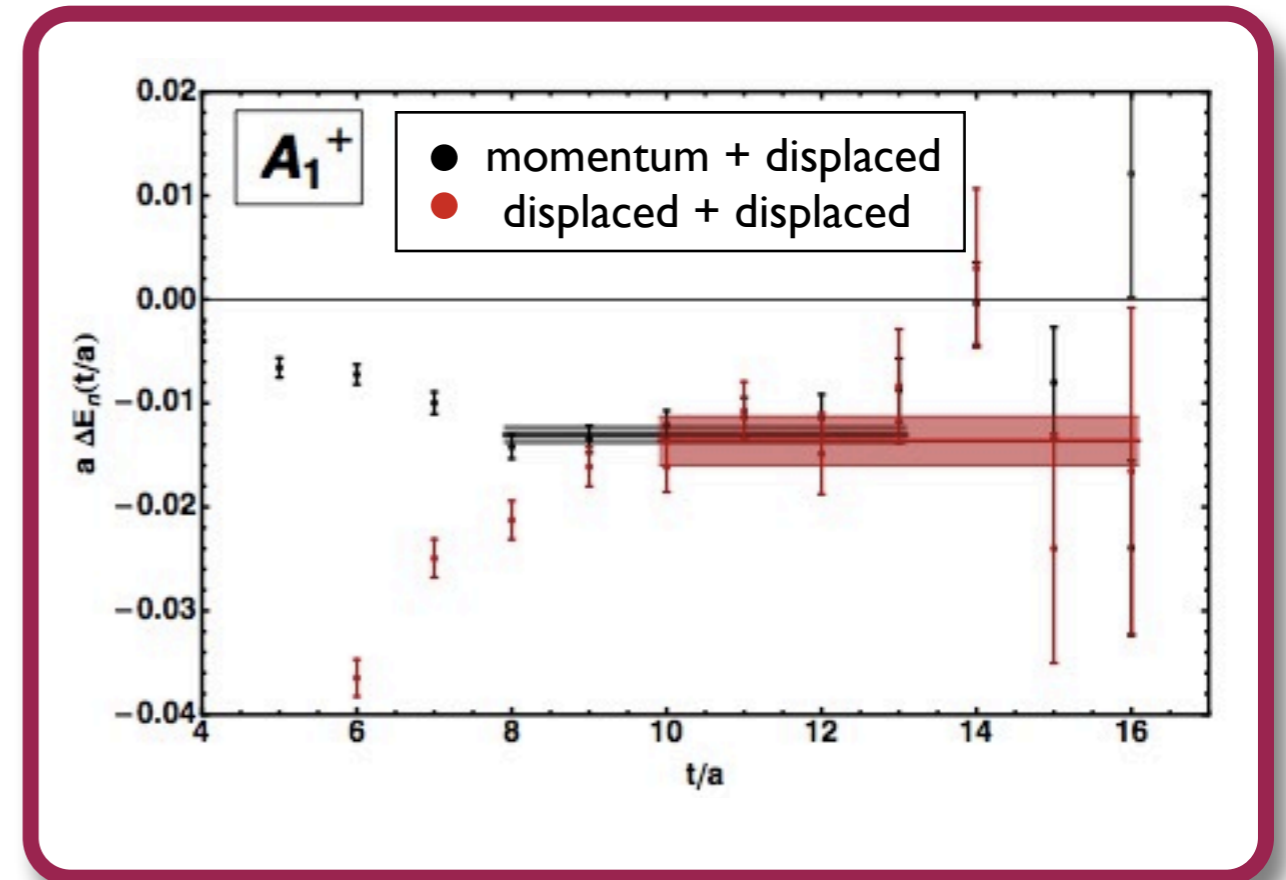
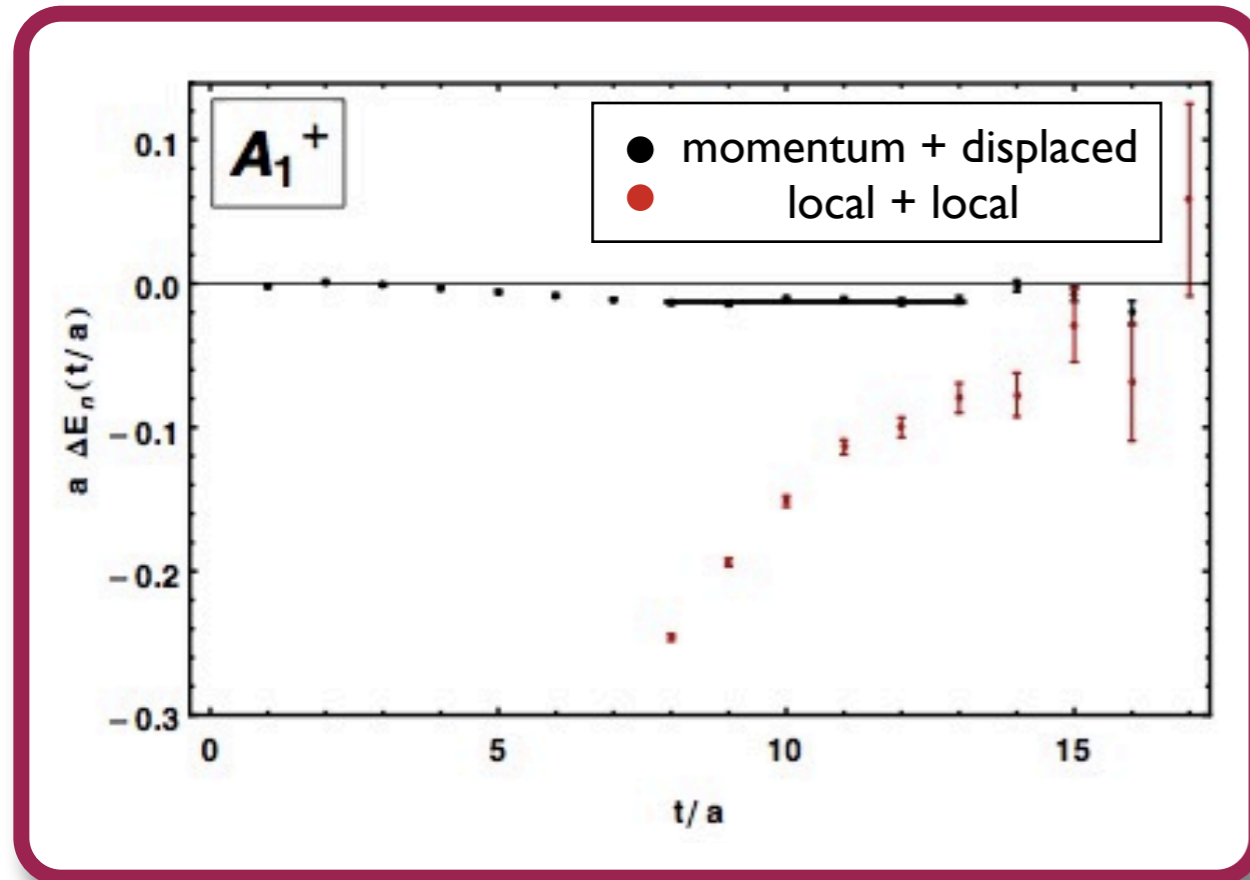
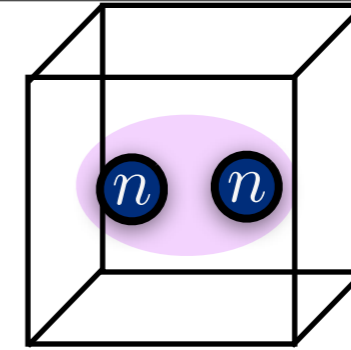
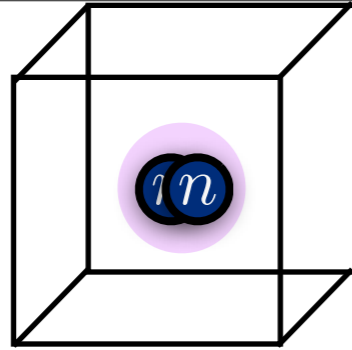
Two-nucleon contact

- Nucleons and multi-particle states are much more difficult!
 - exponentially poor signal-to-noise problem, small excited state energy splittings,
- Isospin limit: 576 contractions*
- Must deal with multi-particle states in a finite volume*
- Ops must be in position space
 - otherwise all-to-all propagators connect to 4-quark operator



*Doi & Endres, Originos et. al., Günther et. al.

*R. Briceno, M. Hansen Phys.Rev. D94 (2016) no.1, 013008

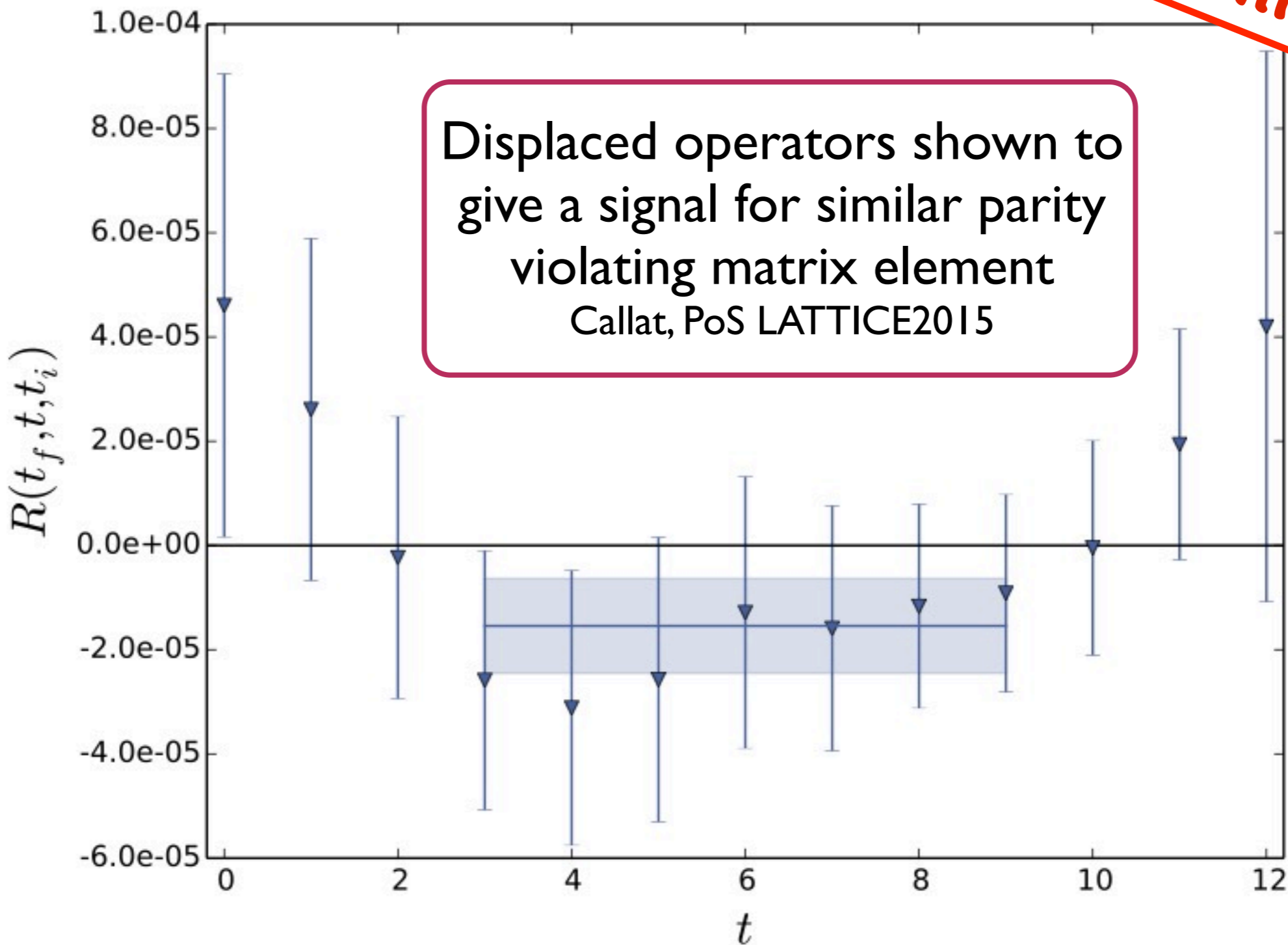


Need displaced operators

Callat arXiv:1508.00886 (2015)

Iso-clover cfgs, $m_\pi \sim 800$ MeV
(W. Detmold, R. Edwards, D. Richards, K. Orginos)

Preliminary

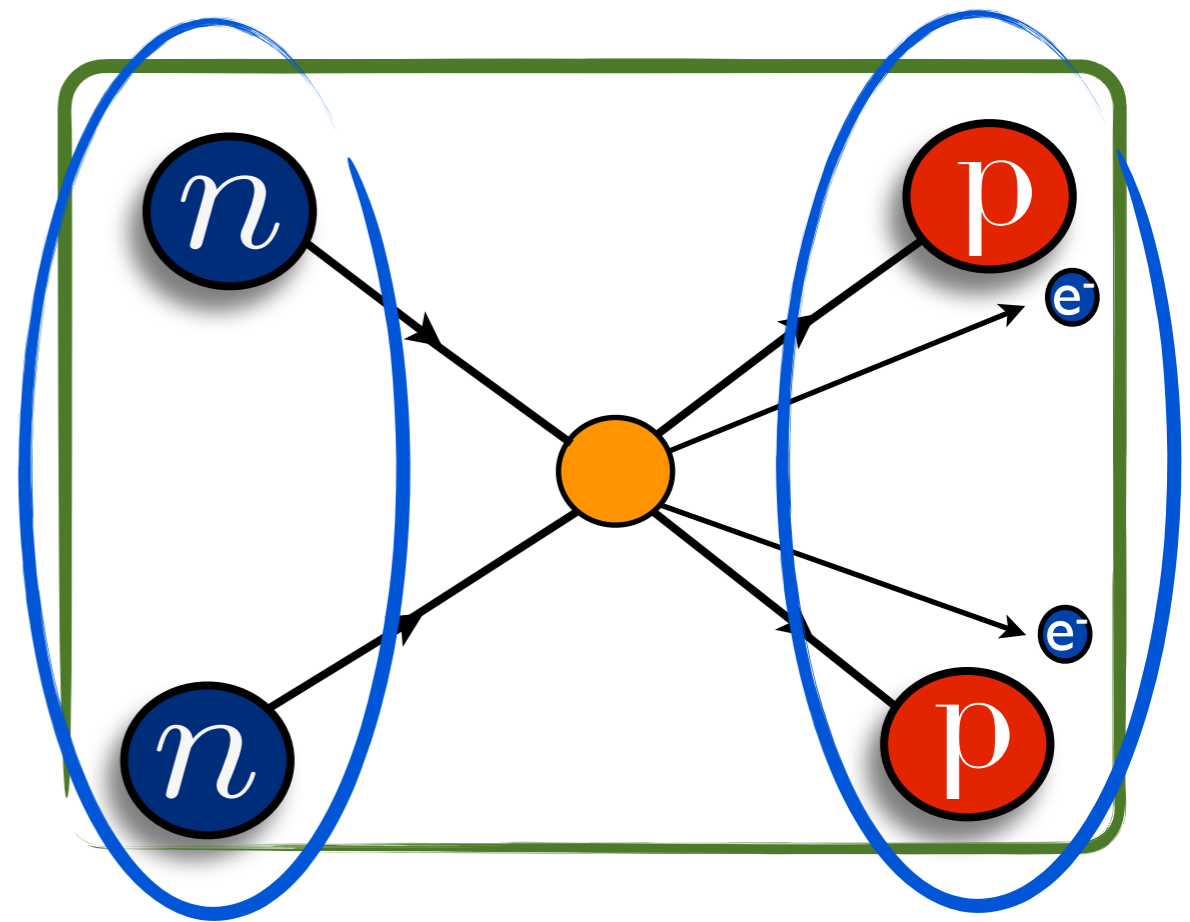


Callat arXiv:1508.00886 (2015)

Iso-clover cfigs, $m_\pi \sim 800$ MeV
(W. Detmold, R. Edwards, D. Richards, K. Orginos)

3.

Two-nucleon contact

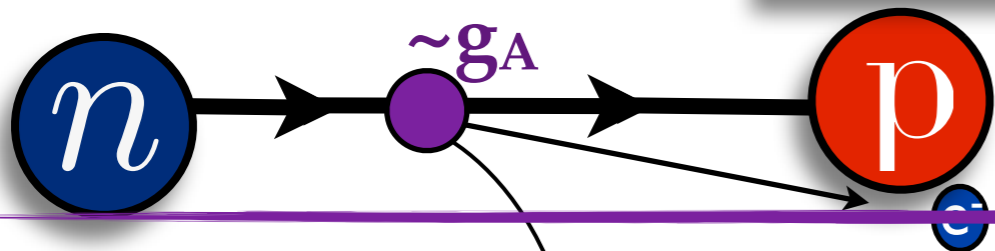


- Some new developments:
 - Exponentially improved NN operators
 - will allow us to lower the pion mass
 - HOBET in a periodic box
 - more direct path from finite volume lattice results to nuclear many-body techniques (W. Haxton & K. McElvain)

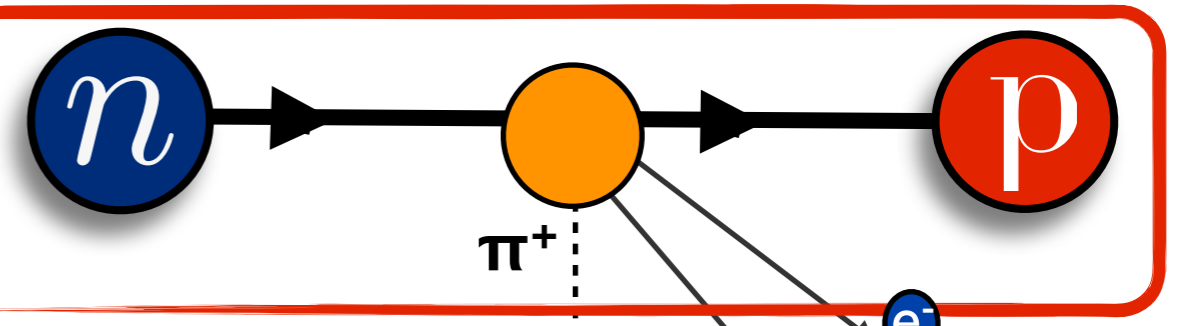
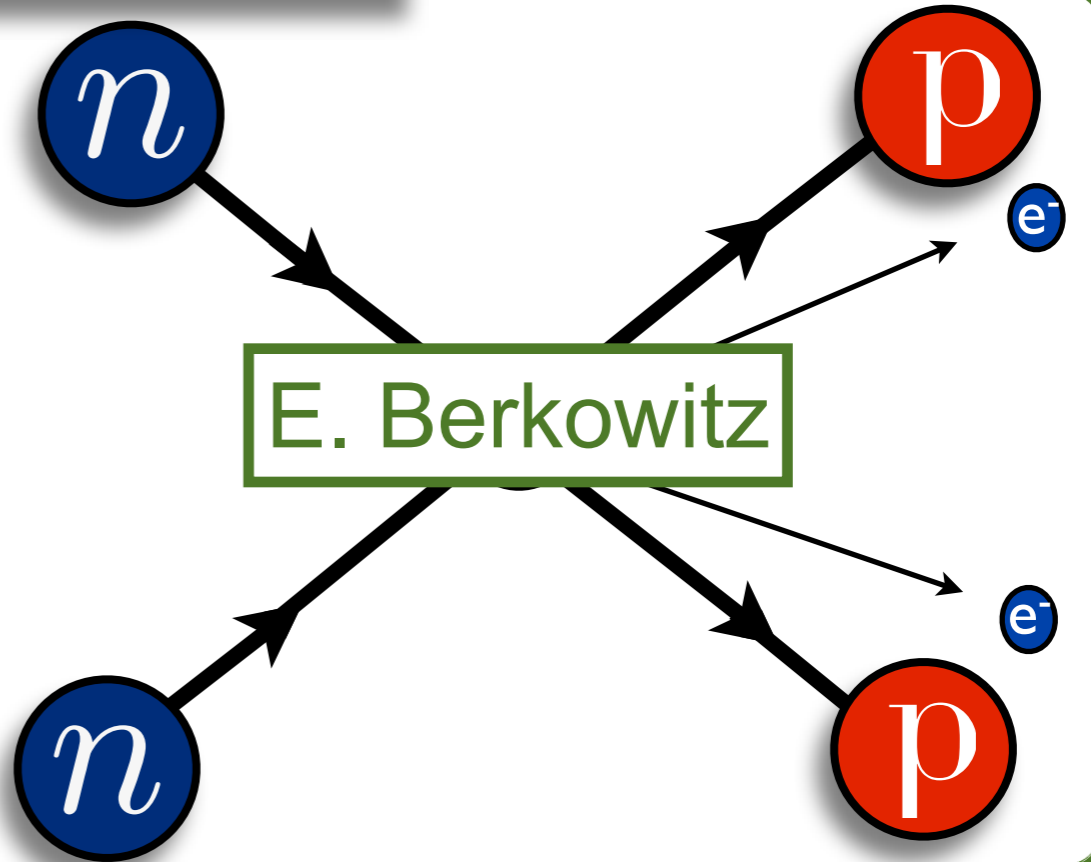
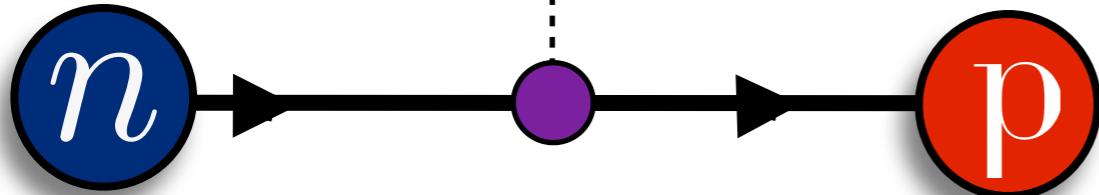
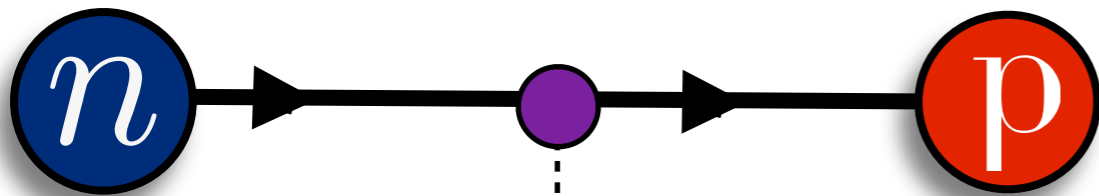
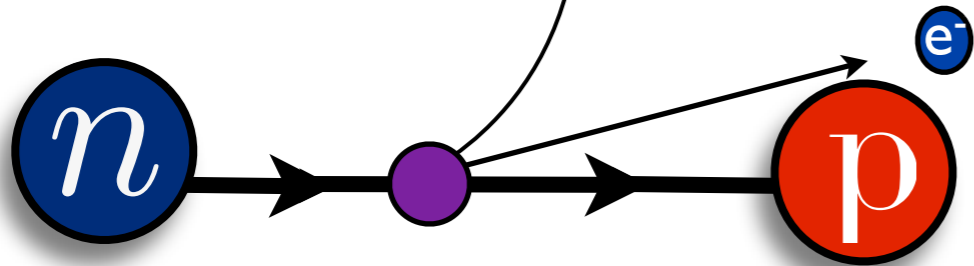
See E. Berkowitz's talk for updates



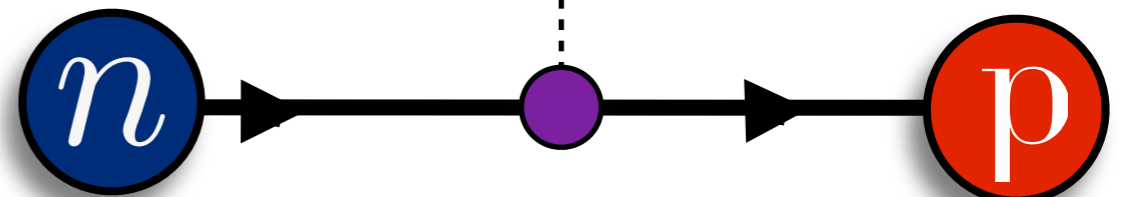
Summary



H.-W. Lin, Z. Davoudi,
P. Shanahan



Difficult: future work



- LBL/UCB: C.C. Chang, AN, A. Walker-Loud
- LLNL: P. Vranas
- NERSC: T. Kurth
- Jülich: E. Berkowitz
- BNL: E. Rinaldi
- nVidia: M.A. Clark
- JLab: B. Joo
- Plymouth: N. Garron
- WM/LBL: D. Brantley, H. Monge-Comacho
- CCNY: B. Tiburzi

