Status of g_A Calculations and Future Prospects

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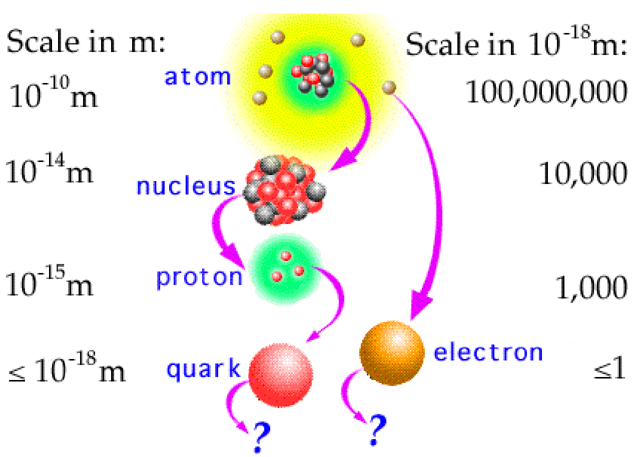

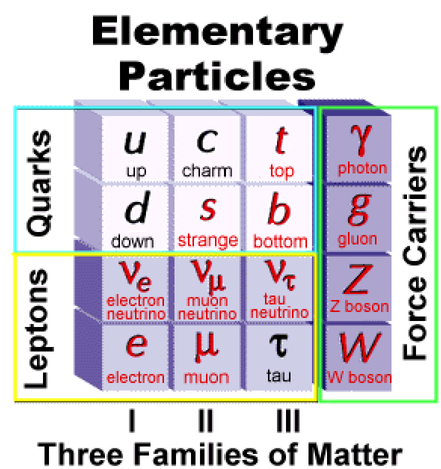

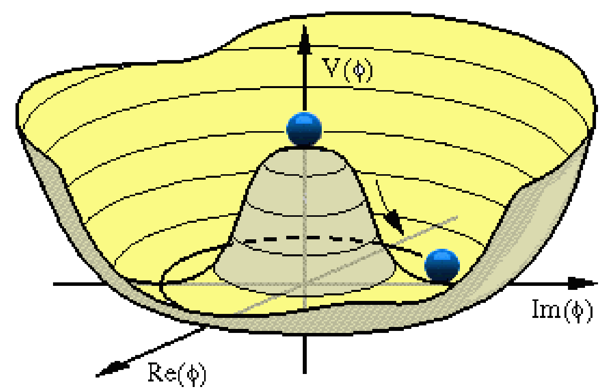

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g_A
encapsulates the strength of the interaction of the weak current with the nucleon

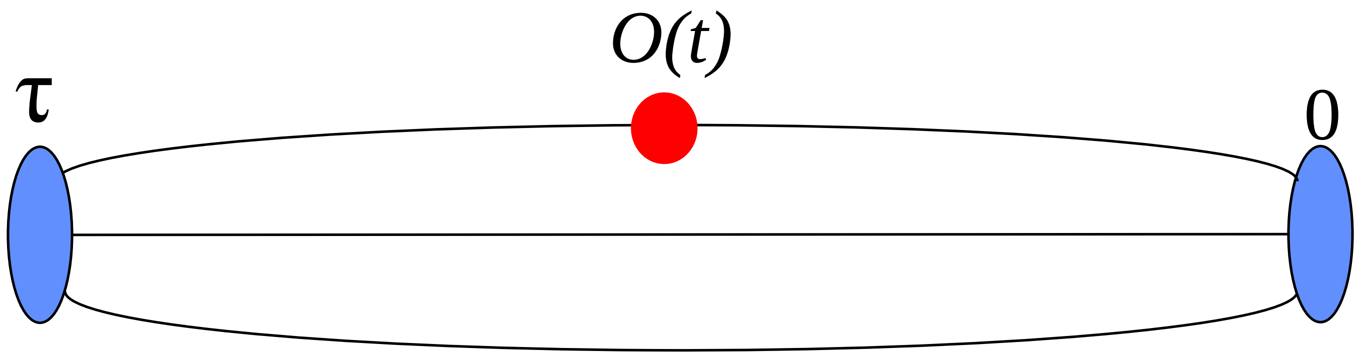
- Need to calculate the *QCD (binding)* corrections to $g_A = 1$, *i.e.*, $g_A/g_V \neq 1$ *for nucleons accounts for them being bound states of quarks and gluons*

- Isovector charges g_A, g_S, g_T are the simplest calculations using lattice QCD of properties of nucleons

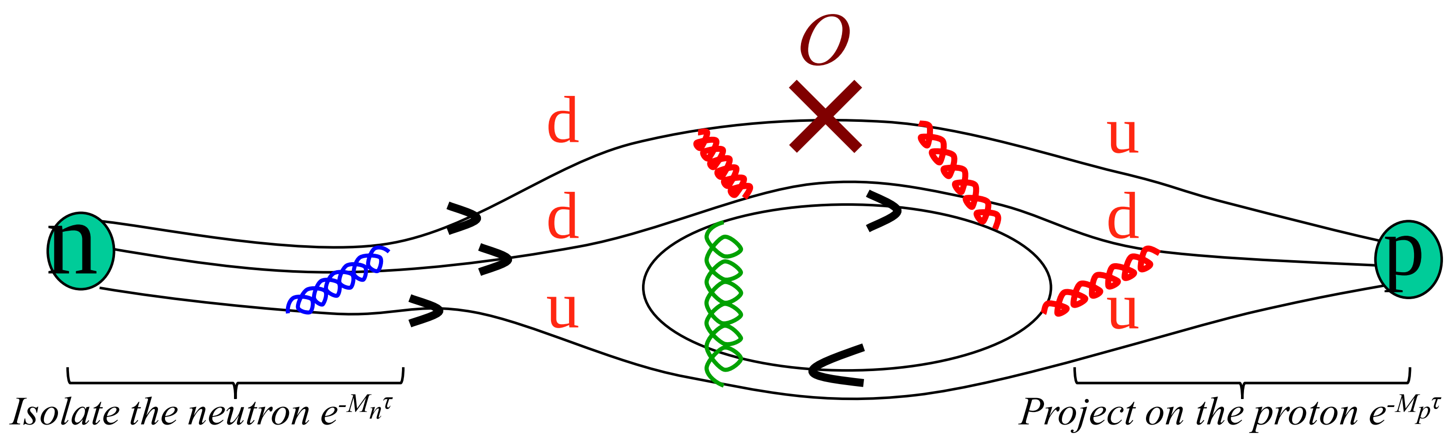
Nucleon 3-point Functions

$$\langle N[\tau]O[t]\overline{N}[0]\rangle = \left\langle \epsilon^{abc}\left(u^{Ta}\, C\gamma_5 d^b\right)u^c\,[\tau] \quad \bar{u}\Gamma d\,[t] \quad \epsilon^{ijk}\overline{u}^i\left(\overline{d}^j C\gamma_5 \overline{u}^{Tk}\right)[0]\right\rangle$$

Wick contraction gives the following diagram in terms of $S_F = D^{-1}$



LQCD calculation includes the full non-perturbative 3-point function



LQCD gives correlation functions via path integral formulation of field theory

Perturbation theory

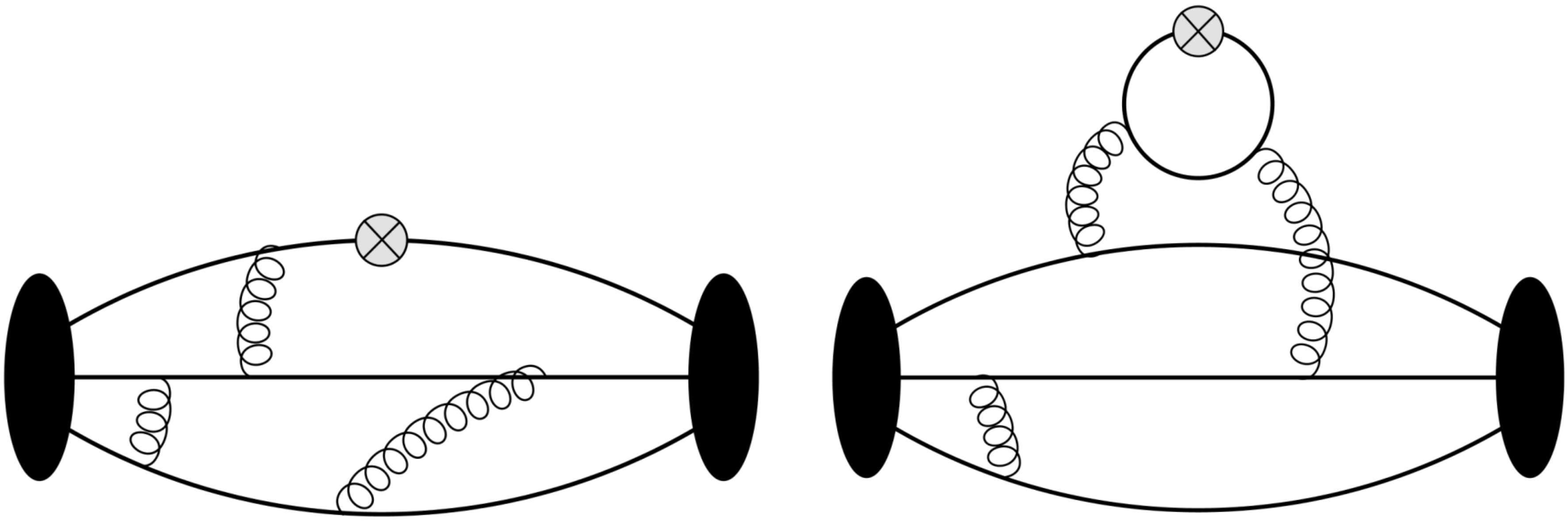
- Perturbative vacuum
- Free propagator $(p+m)^{-1}$
- Calculate every possible diagram if $\alpha_s \sim 1$
- Contribution of excited states have oscillating phase $\sim e^{iEt}$

LQCD

- Background configuration distributed as e^{-S}
- Feynman propagator D^{-1} evaluated on each config.
- Stochastic evaluation of the path integral (average over gauge configurations)
- Discretization errors
- States propagate as $\sim e^{-Et}$

  $\Rightarrow$ Excited states damped exponentially by the mass-gaps

If we can extract the matrix elements of quark bilinear operators within the nucleon state by calculating the "connected" and "disconnected" correlation functions with high precision, we can address a number of physics questions.



Connected **Disconnected**

We need the matrix elements of a number of quark bilinear operators within nucleon states

- Isovector charges g_A, g_S, g_T $\langle p | \bar{u} \Gamma d | n \rangle$
- Axial vector form factors $\langle p(q) | \bar{u} \gamma_\mu \gamma_5 d(q) | n(0) \rangle$
- Vector form factors $\langle p(q) | \bar{u} \gamma_\mu d(q) | n(0) \rangle$
- Flavor diagonal matrix elements $\langle p | \bar{q} q | p \rangle$
- Quark EDM and quark chromo EDM
- Generalized Parton Distribution Functions

Challenges to obtaining high precision results for matrix elements within nucleon states

- **High Statistics: < 1% statistical errors**
- **Demonstrate Control Over All Systematic Errors:**
  - Non-perturbative renormalization of bilinear operators (RI_{smom} scheme)
  - Contamination from excited states
  - Finite volume effects
  - Chiral extrapolation to physical m_u and m_d (simulate at physical point)
  - Extrapolation to the continuum limit (lattice spacing $a \rightarrow 0$)

Perform simulations on ensembles with multiple values of

- Lattice size $M_\pi L \rightarrow \infty$
- Light quark masses $\rightarrow$ physical m_u and m_d
- Lattice spacing $a \rightarrow 0$

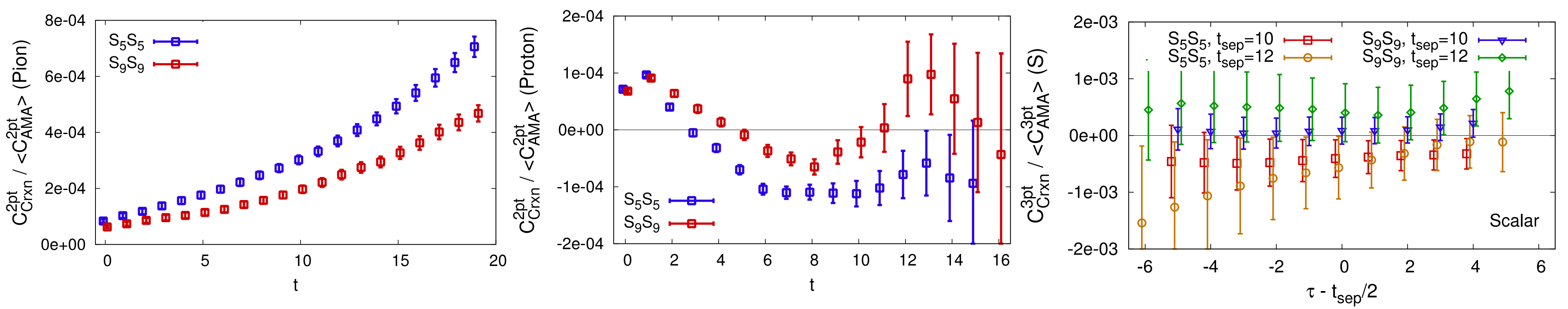
Toolkit

- Multigrid Dirac invertor → propagator $S_F = D^{-1}\eta$
- Truncated solver method with bias correction (AMA)
- Coherent source sequential propagator
- Deflation + hierarchical probing (for disconnected)
- 3-5 values of t_{sep} with smeared sources
- 2-state (3-state fit) to multiple values of t_{sep}
- Non-perturbative renormalization constants (RI-sMOM)
- Combined extrapolation in a, M_π, $M_\pi L$
- Uncertainty due to extrapolation ansatz

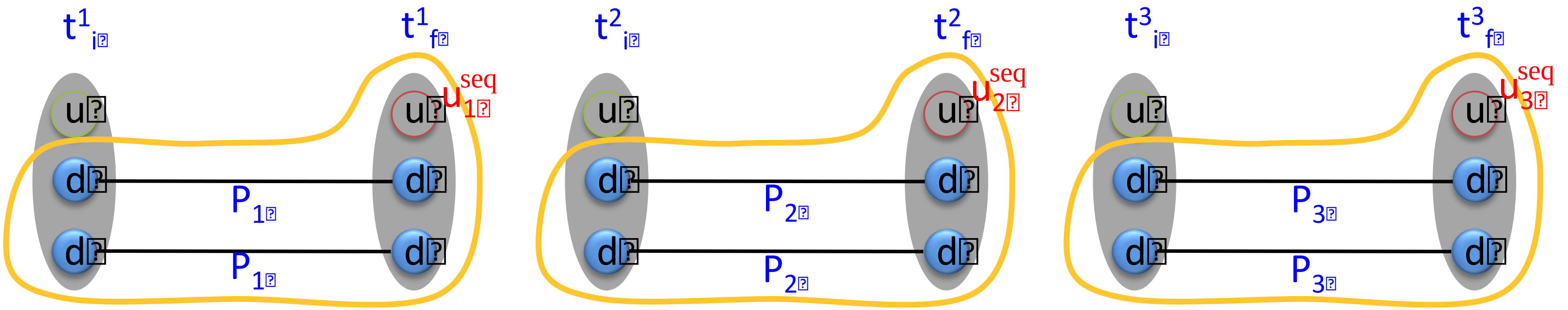
Truncated solver + bias correction (AMA)

$$C^{AMA} = \frac{1}{N_{LP}} \sum_{i=1}^{N_{LP}} C_{LP}(x_i^{LP}) + \frac{1}{N_{HP}} \sum_{i=1}^{N_{HP}} \{C_{HP}(x_i^{HP}) - C_{LP}(x_i^{HP})\}$$

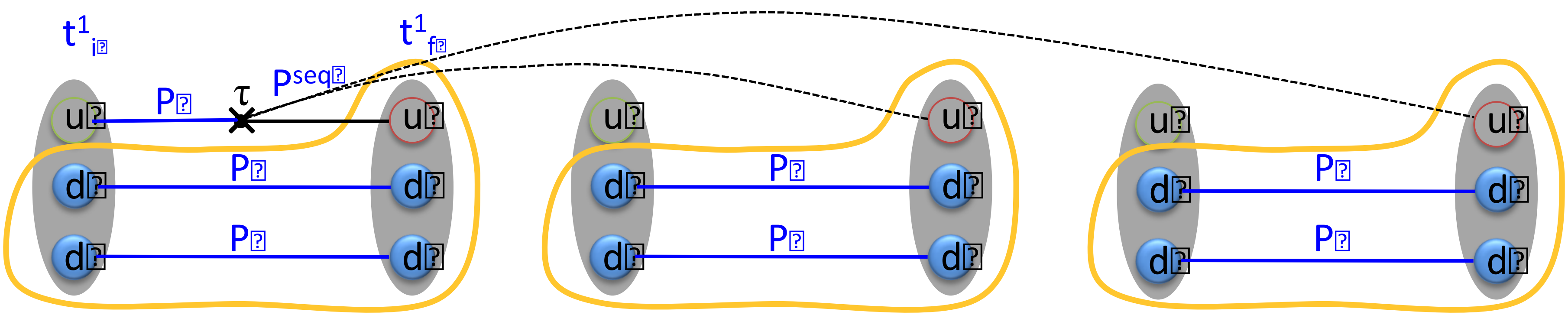
- Use multigrid inverter with
  - $r_{LP} = 10^{-3}$
  - $r_{HP} = 10^{-7} – 10^{-10}$
  - N_{LP} = 64–160, N_{HP} = 3-5 per configuration
- The bias term is negligible (~1% of the error)
- The AMA error is <15% larger than LP



Coherent Source Sequential Propagator



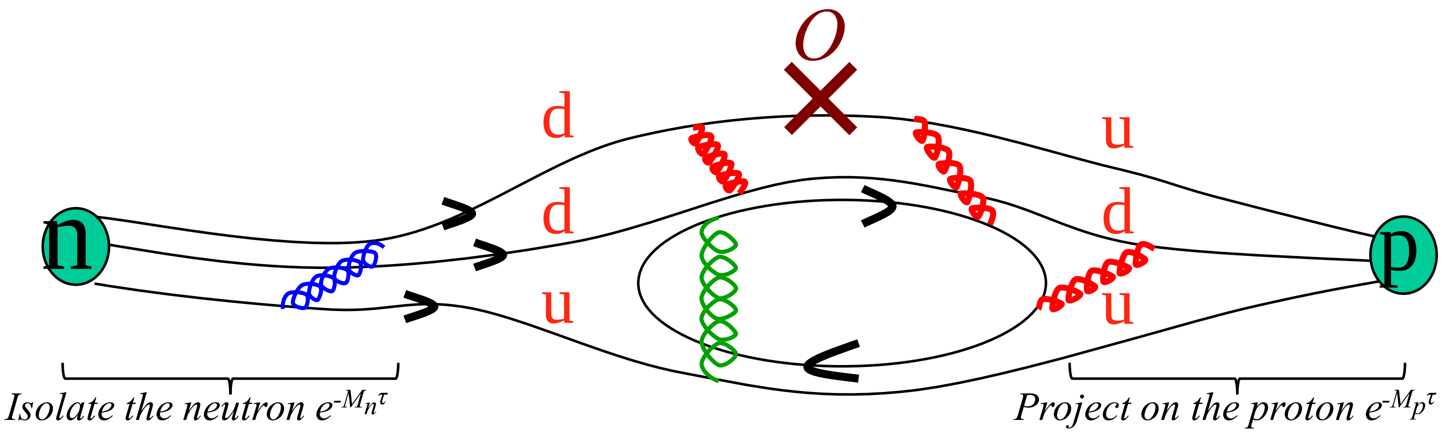
3 measurements being done in a single computer job



Bias = 0 after gauge integral: Increase in Variance ???

Spectral decomposition of 3-pt function:

All states that couple to the nucleon interpolating operator N contribute



$$\langle \Omega | \hat{N}(t,p')\hat{O}(\tau,p'-p)\hat{N}(0,p)|\Omega\rangle =$$

$$\sum_{i,j}\langle \Omega|\hat{N}(p')|N_j\rangle e^{-\int dt\, H}\langle N_j|\hat{O}(\tau,p'-p)|N_i\rangle e^{-\int dt\, H}\langle N_i|\hat{N}(p)|\Omega\rangle =$$

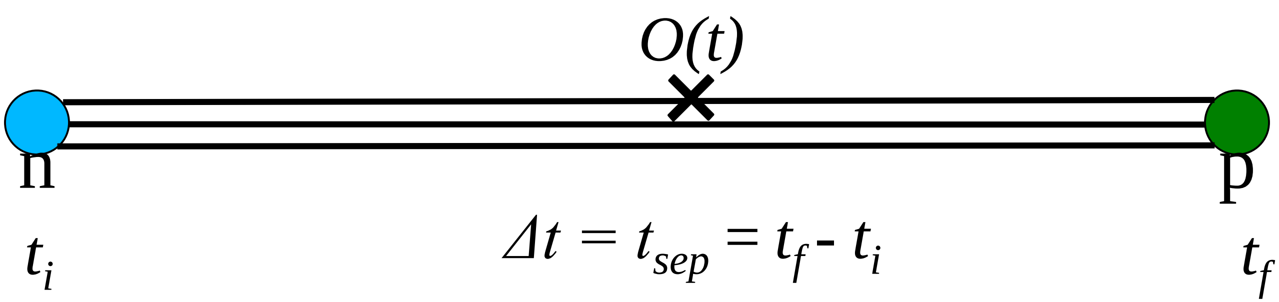
$$\sum_{i,j}\langle \Omega|\hat{N}(p')|N_j\rangle e^{-E_j(t-\tau)}\langle N_j|\hat{O}(\tau,p'-p)|N_i\rangle e^{-E_i\tau}\langle N_i|\hat{N}(p)|\Omega\rangle$$

Controlling excited-state contamination: n-state fit

$$G^2(t) = |A_0|^2 e^{-M_0 t} + |A_1|^2 e^{-M_1 t} + |A_2|^2 e^{-M_2 t} + |A_3|^2 e^{-M_3 t} + \ldots.$$

$$G^3(t, \Delta t) = |A_0|^2 \langle 0|O|0\rangle e^{-M_0 \Delta t} + |A_1|^2 \langle 1|O|1\rangle e^{-M_1 \Delta t} +$$
$$A_0 A_1^* \langle 0|O|1\rangle e^{-M_0 \Delta t} e^{-\Delta M(\Delta t - t)} + A_0^* A_1 \langle 1|O|0\rangle e^{-\Delta M t} e^{-M_0 \Delta t} + \ldots$$

M_0, M_1, … masses of the ground & excited states
A_0, A_1, … corresponding amplitudes



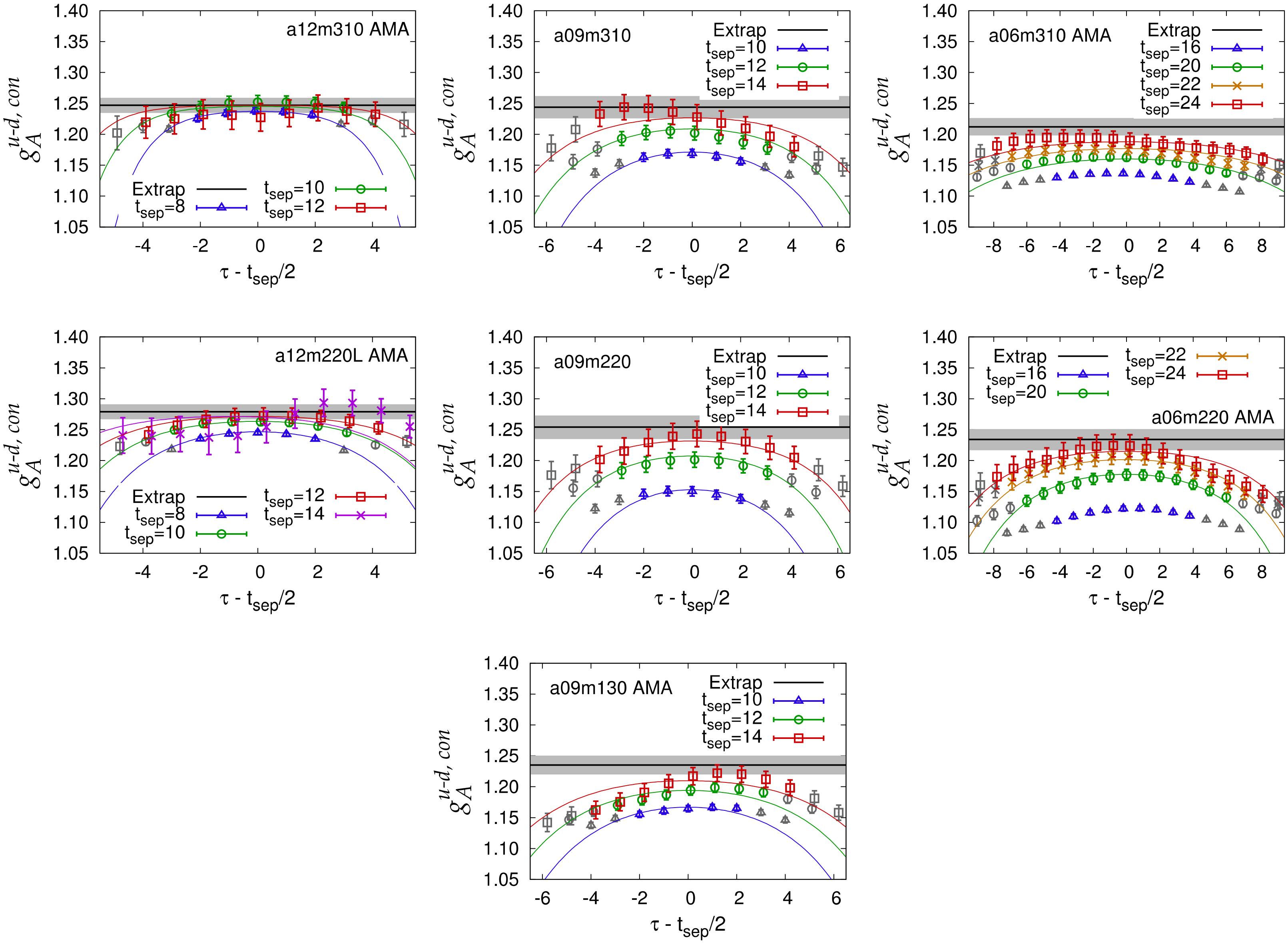
Make a simultaneous fit to data at multiple $\Delta t = t_{sep} = t_f - t_i$

Controlling excited-state contamination

- Reduce A_n/A_0 in an n-state fit
  - Tune source smearing size σ
  - Tune the interpolating operator
  - Variational method
- Generate data at multiple values of t_{sep}
- $2 \rightarrow 3 \rightarrow \ldots$ state fits to data at multiple values of t_{sep}
- CalLat method

Yoon et al, PRD D93 (2016) 114506
Yoon el al, PRD D95 (2017) 074508

g_A: Excited State Contamination



Data and fits on 7 clover-on-HISQ ensembles: Bhattacharya et al, PRD94 (2016) 054508

Analyzing lattice data $\Omega(a, M_\pi, M_\pi L)$:

Extrapolations in $a, M_\pi^2, M_\pi L$

We use lowest order corrections when fitting lattice data w.r.t.

- Lattice spacing: a
- Dependence on light quark mass: $m_q \sim M_\pi^2$
- Finite volume: $M_\pi L$

$$g_{A,T}(a, M_p, L) = g + A\,a + B\,M_p^2 + C\,M_p^2 e^{-M_p L} + \ldots$$

$$g_S(a, M_p, L) = g + A\,a + B\,M_p + C\,M_p e^{-M_p L} + \ldots$$

2+1+1 flavor HISQ lattices from MILC

M_s tuned to its physical value using M_{ss}

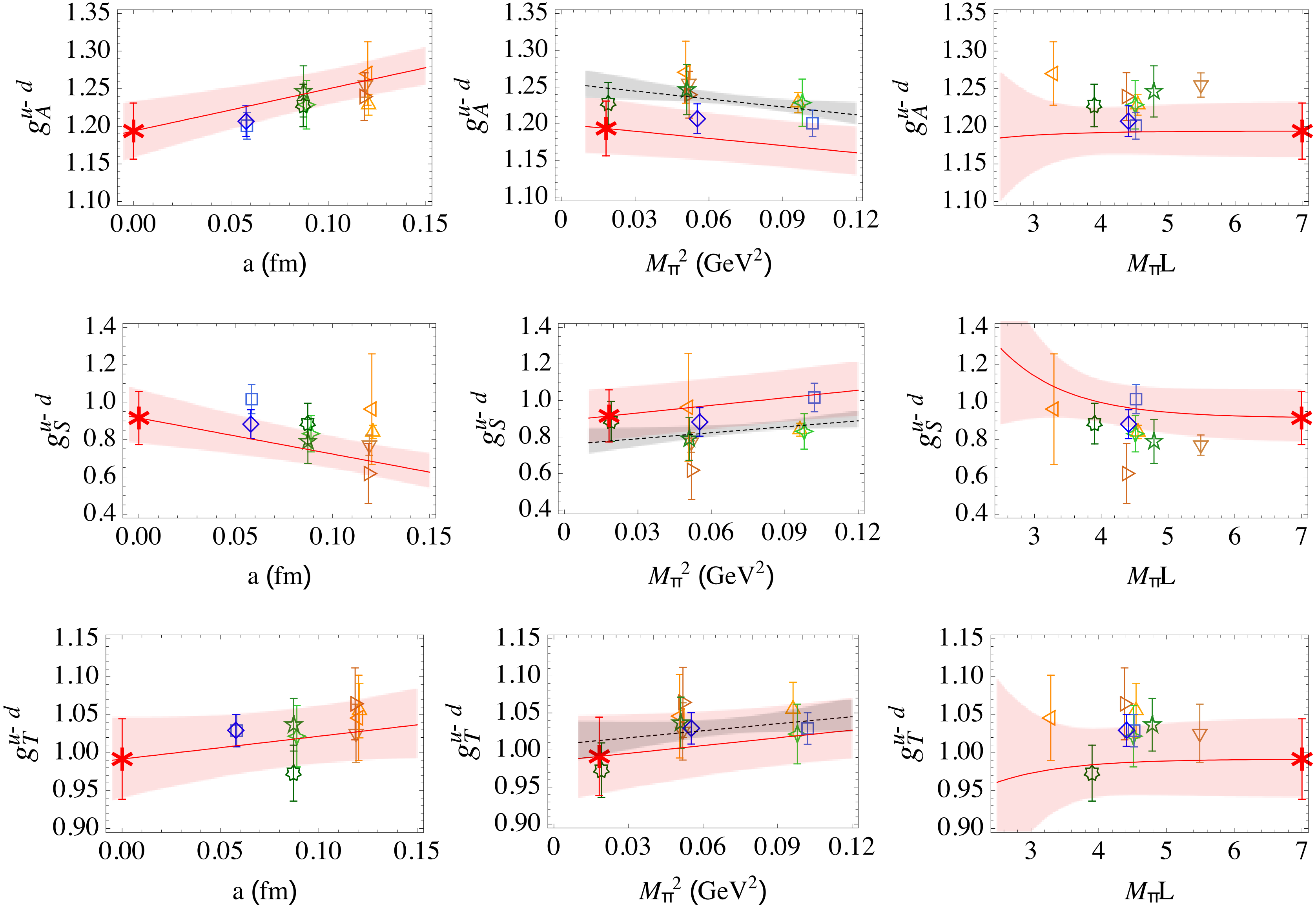
a (fm)	m_l/m_s	Volume	$M_\pi L$	M_π (MeV)	# Configs	HP	AMA
0.12 □	0.2	$24^3 \times 64$	4.55	310	1013	8,104	64,832
0.12 △	0.1	$24^3 \times 64$	3.29	225	1000	24,000	
0.12 ◆	0.1	$32^3 \times 64$	4.38	228	958	7,664	
0.12 ▽	0.1	40^3 x 64	5.49	228	1010	8,080	68,680
0.09 □	0.2	$32^3 \times 96$	4.51	313	881	7,048	
0.09 ◆	0.1	$48^3 \times 96$	4.79	226	890	7,120	
0.09 ○	0.037	$64^3 \times 96$	3.90	138	883	7,064	56,512
0.06 □	0.2	$48^3 \times 144$	4.52	320	1000	8,000	64,000
0.06 ◆	0.1	$64^3 \times 144$	4.41	235	650	2,600	41,600

Update: 2+1+1 flavor HISQ lattices from MILC

M_s tuned to its physical value using M_{ss}

a (fm)	m_l/m_s	Volume	$M_\pi L$	M_π (MeV)	# Configs	HP	AMA
0.12	0.2	$24^3 \times 64$	4.55	310	1013	8,104	64,832
0.12	0.1	$24^3 \times 64$	3.29	225	1000		60,544
0.12	0.1	$32^3 \times 64$	4.38	228	958		47,616
0.12	0.1	40^3 x 64	5.49	228	1010	8,080	68,680
0.09	0.2	$32^3 \times 96$	4.51	313	881		42,176
0.09	0.1	$48^3 \times 96$	4.79	226	890		53,312
0.09	0.037	$64^3 \times 96$	3.90	138	883	7,064	56,512
0.06	0.2	$48^3 \times 144$	4.52	320	1000	8,000	64,000
0.06	0.1	$64^3 \times 144$	4.41	235	650	2,600	41,600
0.06	0.037	$96^3 \times 192$	3.7	135	322	1,610	51,520

Simultaneous extrapolation in $a, M_\pi^2, M_\pi L$



9 clover-on-HISQ ensembles: Bhattacharya et al, PRD94 (2016) 054508

Results on isovector charges of the proton (clover-on-HISQ)

(Bhattacharya et al, PRD94 (2016) 054508)

Isovector charges

***** $g_T = 0.987(51)$**
**** $g_A = 1.195(33)$**
**** $g_S = 0.97(12)$**

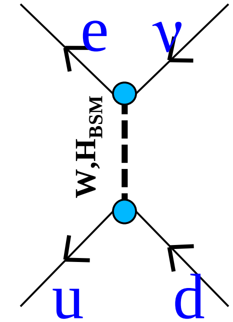
Flavor diagonal charges

$g_T^u = 0.792(42)$
$g_T^d = -0.194(14)$

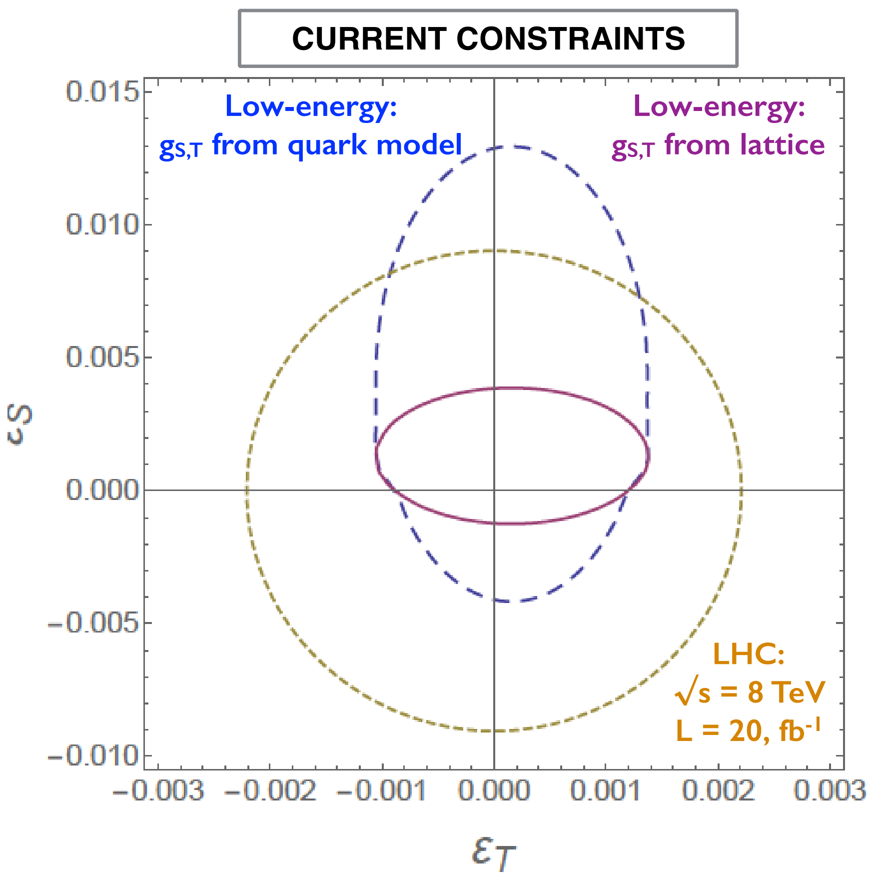
2-state fits to 2-point and 3-point functions

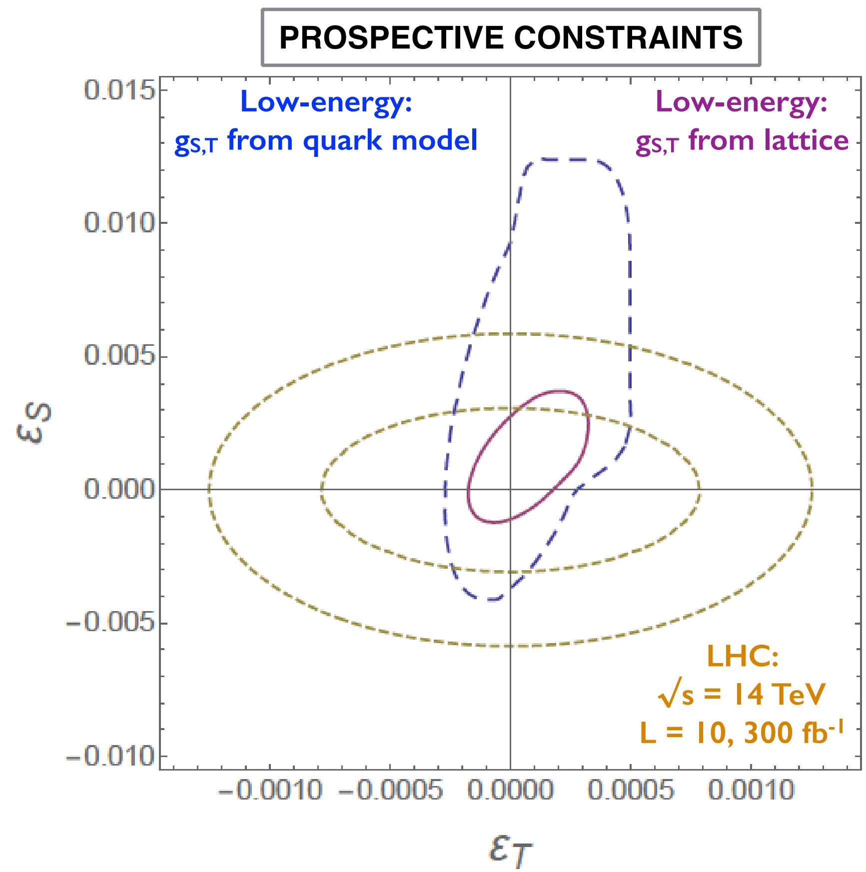
Constraints on [ε_S ,ε_T]: β-decay versus LHC

- LHC: (u+d → e+ν) look for events with an electron and missing energy at high transverse mass
- low-energy experiments + lattice with $\delta g_S/g_S$ ~10%



LHC

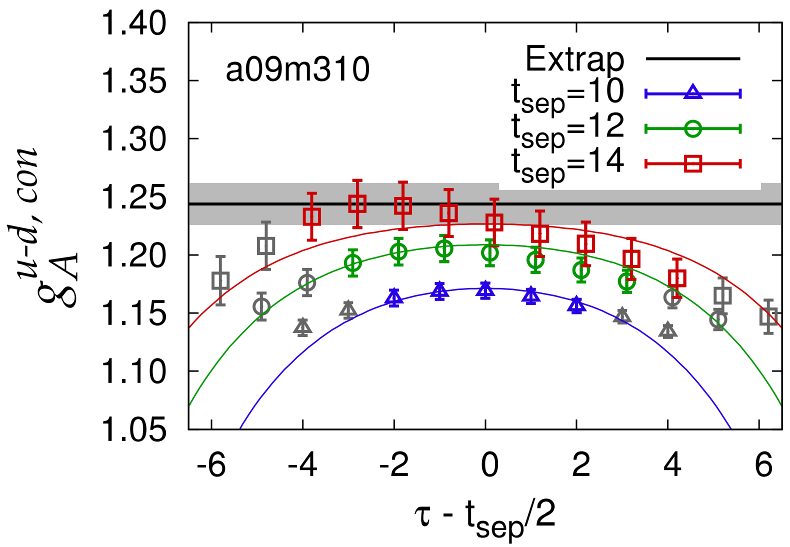




Developments since PNDME PRD94 (2016) 054508)

PNDME (Preliminary):

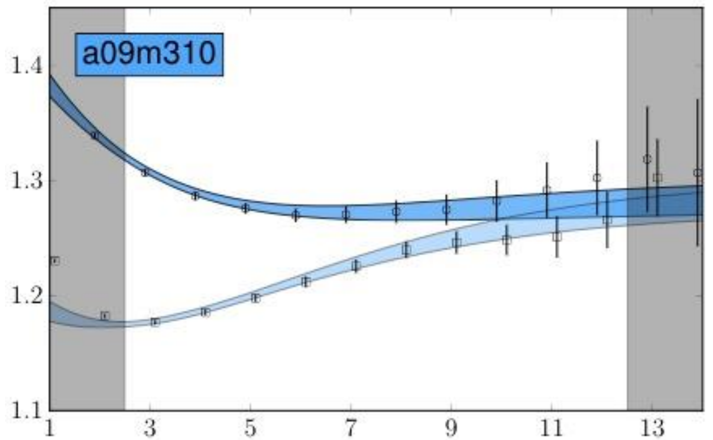
- Added a06m135 ensemble
- 4 ensembles HP → AMA
- Better smearing on a09
- 4-state fits to 2-point functions
- 3-state fits to 3-point functions
- Covariant error matrix



CalLAT (DWF-on-HISQ)

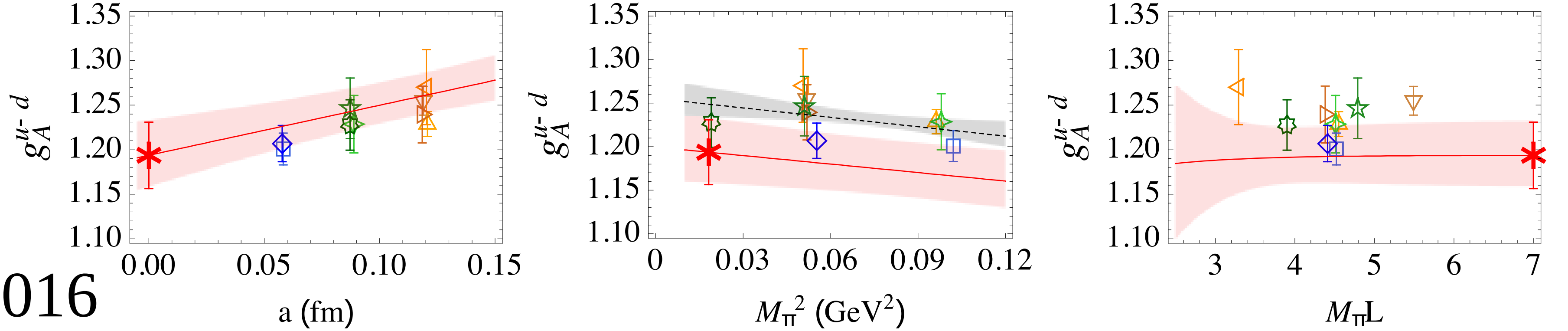
Multistate fit to

$$\frac{C_3(\tau+1)}{C_2(\tau+1)} - \frac{C_3(\tau)}{C_2(\tau)}$$



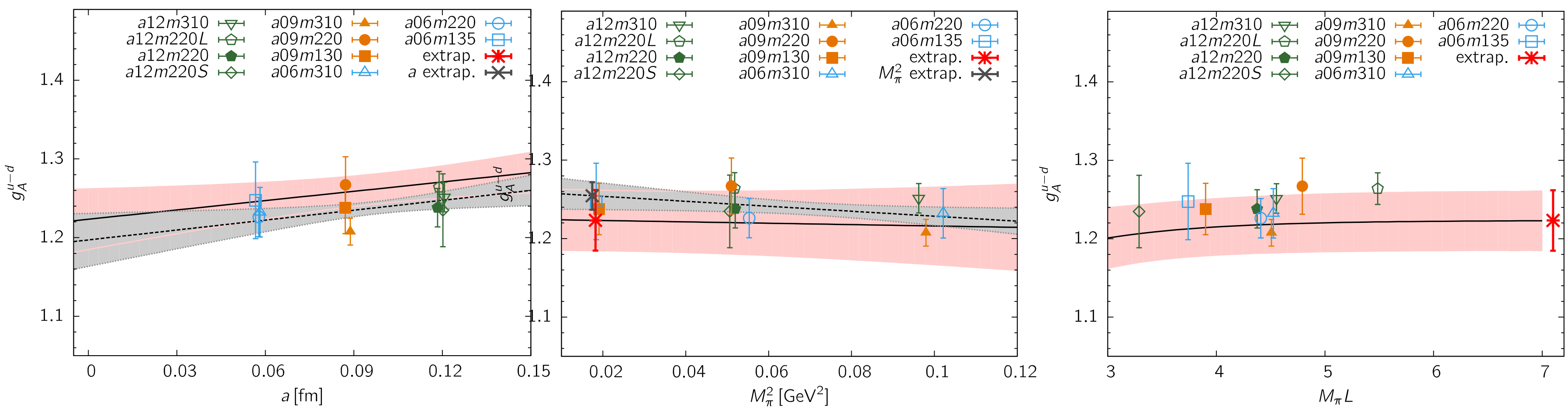
$\mathbf{g_A = 1.278(21)(26)}$

PNDME g_A: Simultaneous extrapolation in a, M_π^2, $M_\pi L$



2016

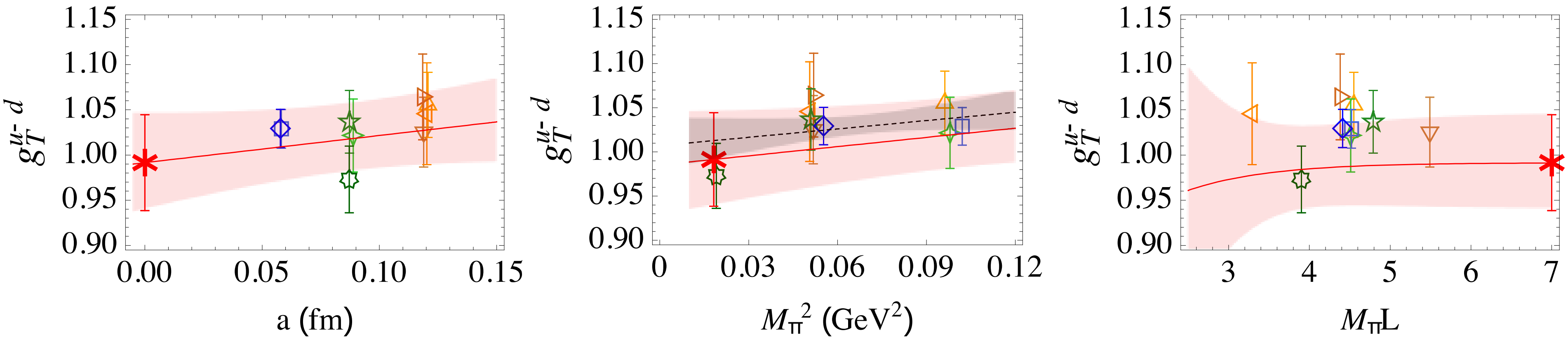
↓



2017: 10 clover-on-HISQ ensembles

g_A = 1.195(33) → 1.223(39)

g_T: Simultaneous extrapolation in $a, M_\pi^2, M_\pi L$



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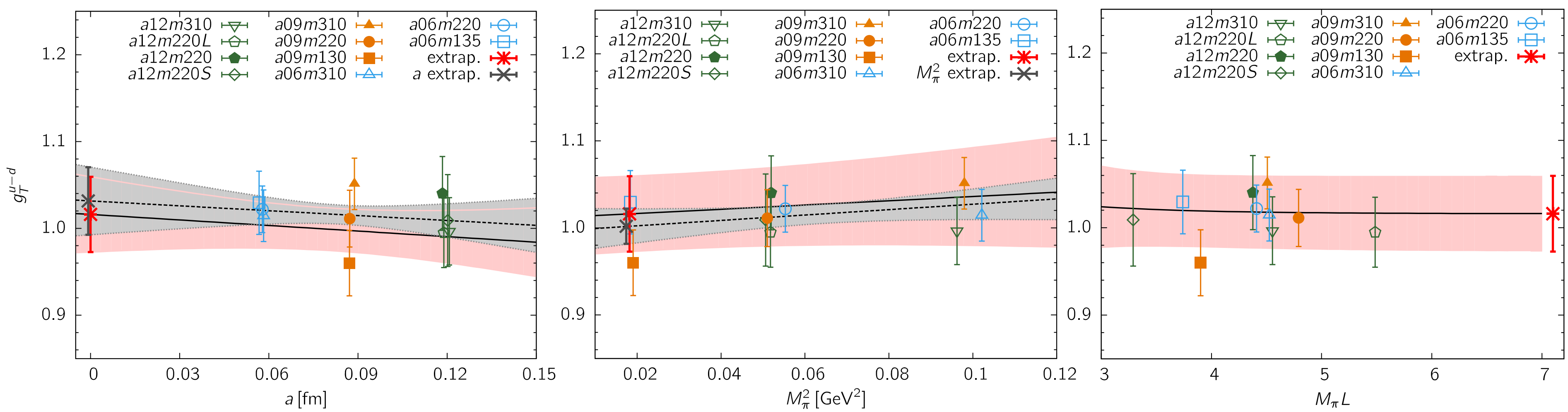
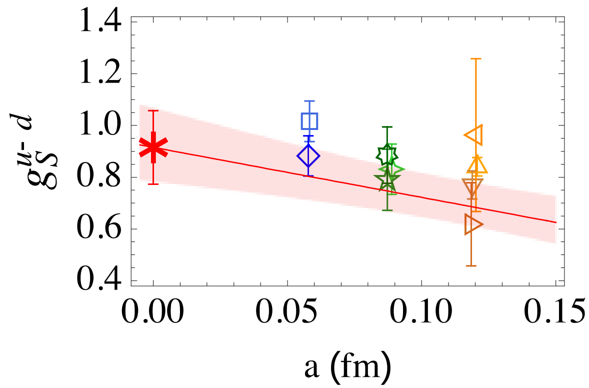


$$g_T = 0.987(51) \quad \Longrightarrow \quad 1.016(43)$$

PNDME (2017 in preparation): 10 clover-on-HISQ ensembles

g_S: Simultaneous extrapolation in $a, M_\pi^2, M_\pi L$



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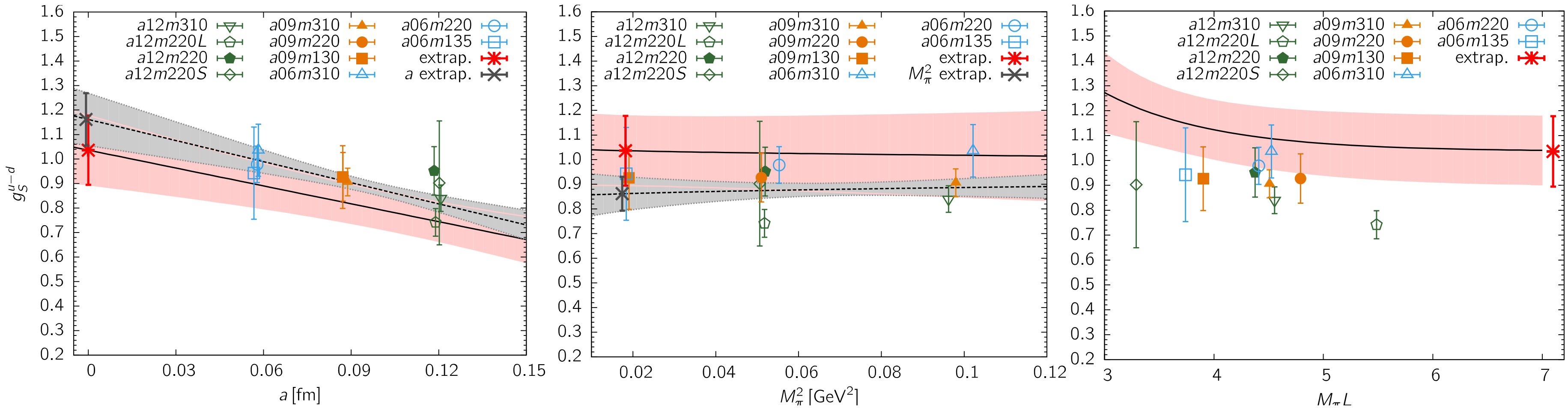


$$g_S = 0.97(12) \quad \Longrightarrow \quad 1.03(14)$$

PNDME (2017): 10 clover-on-HISQ ensembles

g_A: What is different in the 2 calculations

PNDME

- Clover-on-HISQ
  - $Z_A/Z_V \rightarrow \sim 1\%$ error
- 1-HYP smearing of lattice
- Sequential propagator calculated with insertion of nucleon at sink at fixed τ:
  - all charges and FF in one go
  - Each τ needs new calculation
- n-state fit to data at different t and τ to get g_A

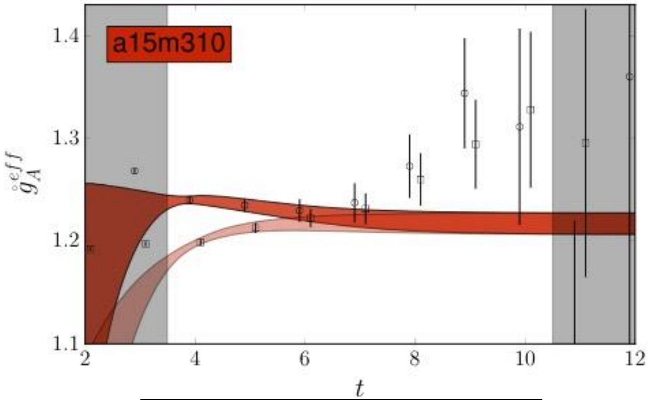
CalLat

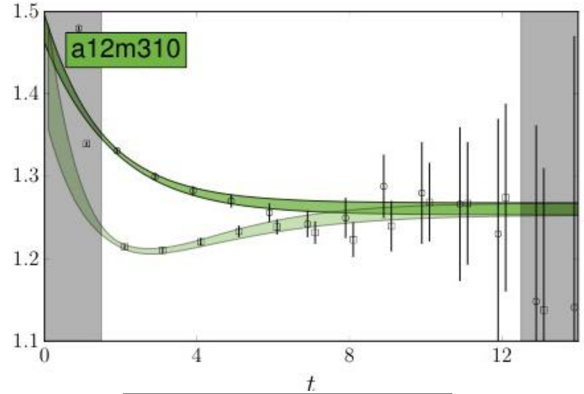
- DWF-on-HISQ
  - $Z_A/Z_V = 1 \rightarrow$ no error
- Wilson flow $t_{wf}/a^2 = 1$
- Sequential propagator calculated with insertion of operator at all $t \in [0,T]$:
  - Get all τ at the same time
  - Get data for any sink smearing
  - Contact terms & data outside [0,τ]
- Data summed over all t (+ artifacts) Make 2-state fit to $g_A^{\text{eff}} = \frac{C_3(\tau+1)}{C_2(\tau+1)} - \frac{C_3(\tau)}{C_2(\tau)}$

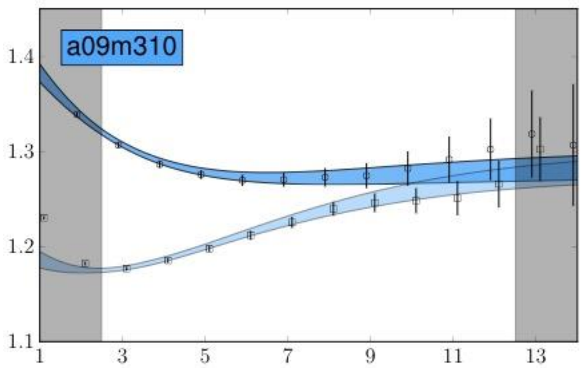
PNDME: 10 clover-on-HISQ ensembles

CalLAT: 8→12 DWF-on-HISQ ensembles

Multistate fits: CalLat versus PNDME







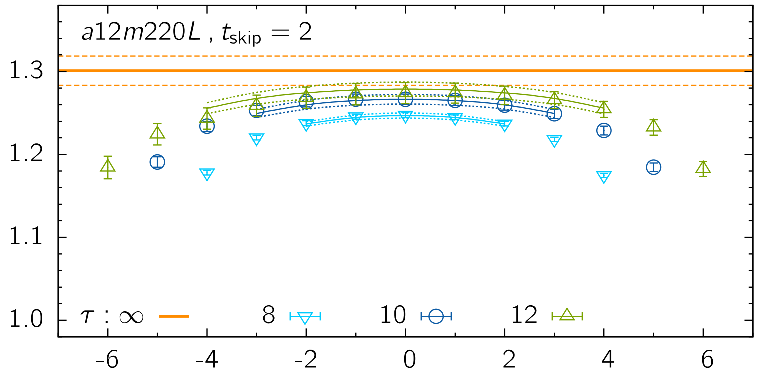
$4 \leq t \leq 6$

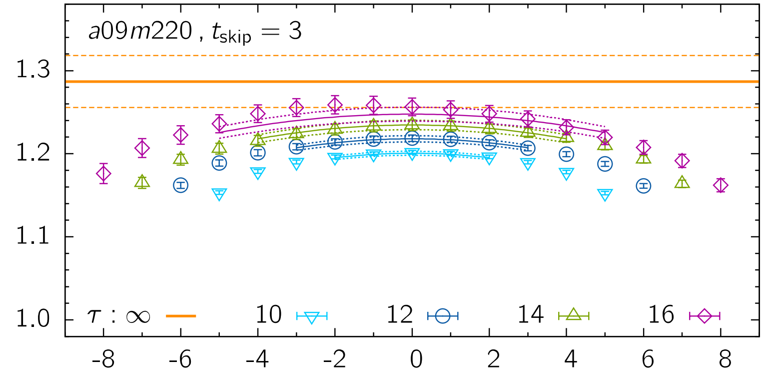
$2 \leq t \leq 7$

$3 \leq t \leq 10$

t_{sep} = 8, 10, 12

t_{sep} = 10, 12, 14





CalLat: Gain in signal comes from including much smaller t_{sep} in fit for extracting g_A

Data agree within errors

	PNDME	CalLat
a12m310	1.229(14) 1.251(19)	1.237(07)
a12m220S	1.270(40) 1.264(20)	1.272(28)
a12m220	1.240(32) 1.238(24)	1.259(15)
a12m220L	1.255(16) 1.235(46)	1.252(21)
a09m310	1.231(33) 1.208(17)	1.258(14)

In the CalLat calculatios

$g_A^{eff} = g_A + \Delta(\mathrm{ESC}) + \Delta(\mathrm{artifacts}=\mathrm{contact}+\mathrm{outside})$

Is $\Delta(\mathrm{artifacts}) = 0$???

chiral-continuum fit function

SU(2) NNLO baryon χPT

m_π^2 analytic	$g_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4$
non-analytic	$-\epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + g_0 c_3 \epsilon_\pi^3$
a^2 analytic	$a_2\epsilon_a^2 + b_4\epsilon_\pi^2\epsilon_a^2 + a_4\epsilon_a^4$
NLO FV	$(8/3)\epsilon_\pi^2[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)]$

parameterize with

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \qquad \epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{\omega_0^2}$$

additional

$\mathrm{O}(m_{\mathrm{res}})$	$a_1 a/\omega_0$
gen. one-loop	$s_2\alpha_s\epsilon_a^2$

fit strategy

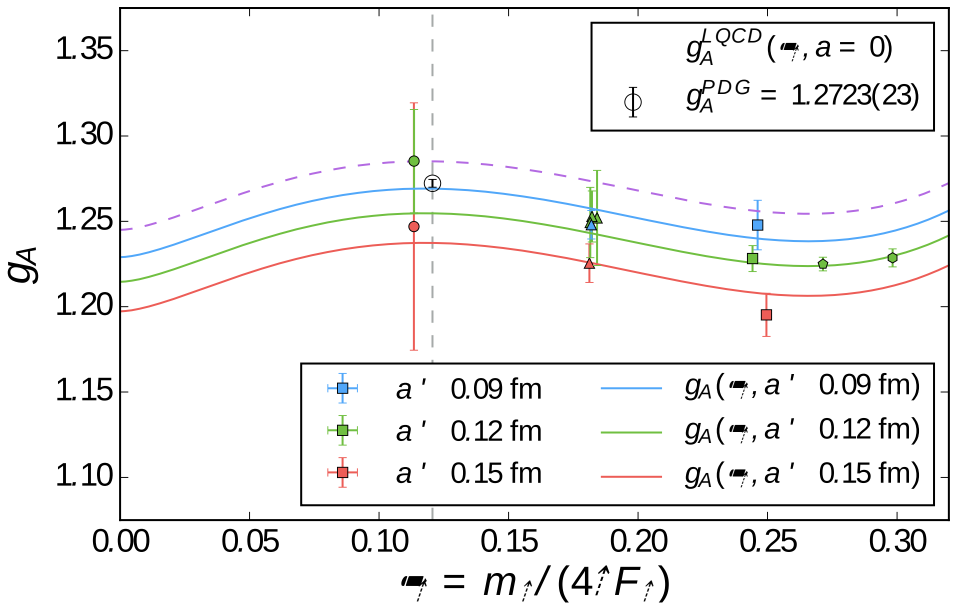
use Bayes Theorem as method of uncertainty quantification

LEC priors motivated by χPT power counting

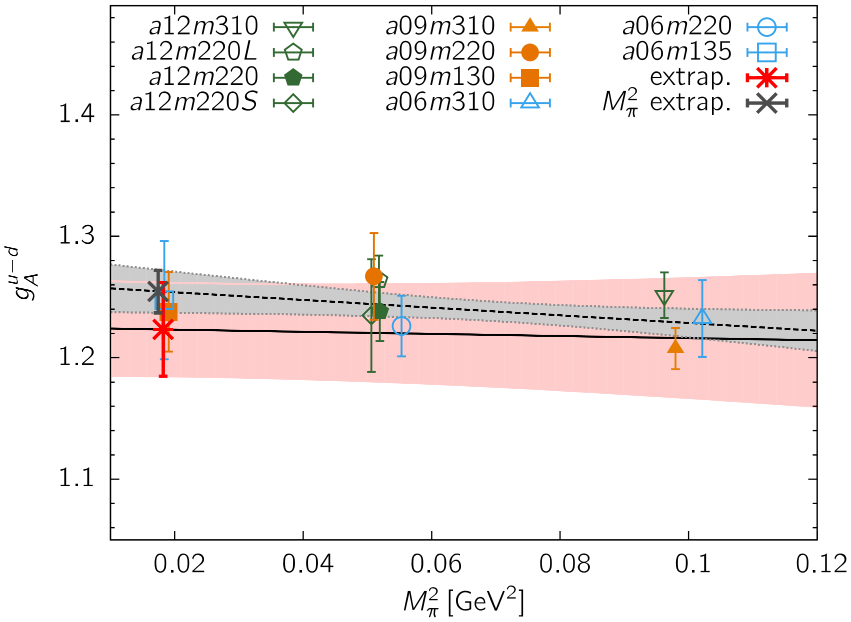
show stability of chiral-continuum extrapolation under varying models

Chiral Extrapolation

CalLat



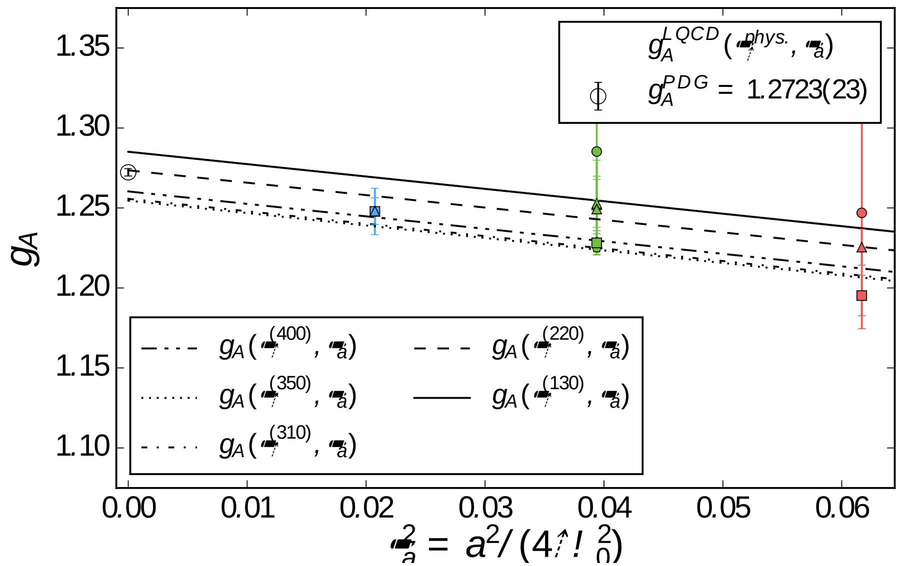
PNDME

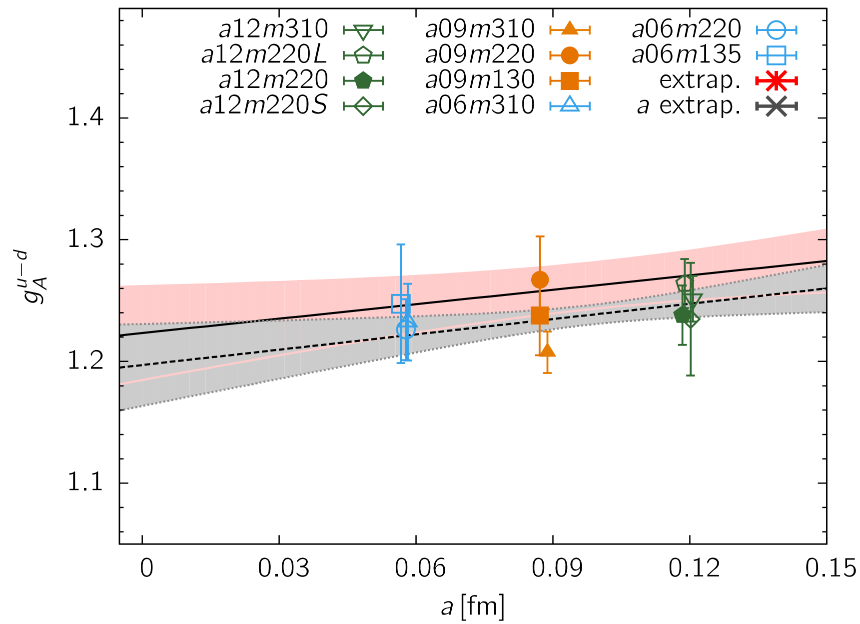


Similar trend: small increase in g_A as M_π goes from 310 → 135MeV

CalLat: chiral log term consistent with prediction when fit includes ϵ_π^3

Main difference in the 2 results arises due to the continuum extrapolation





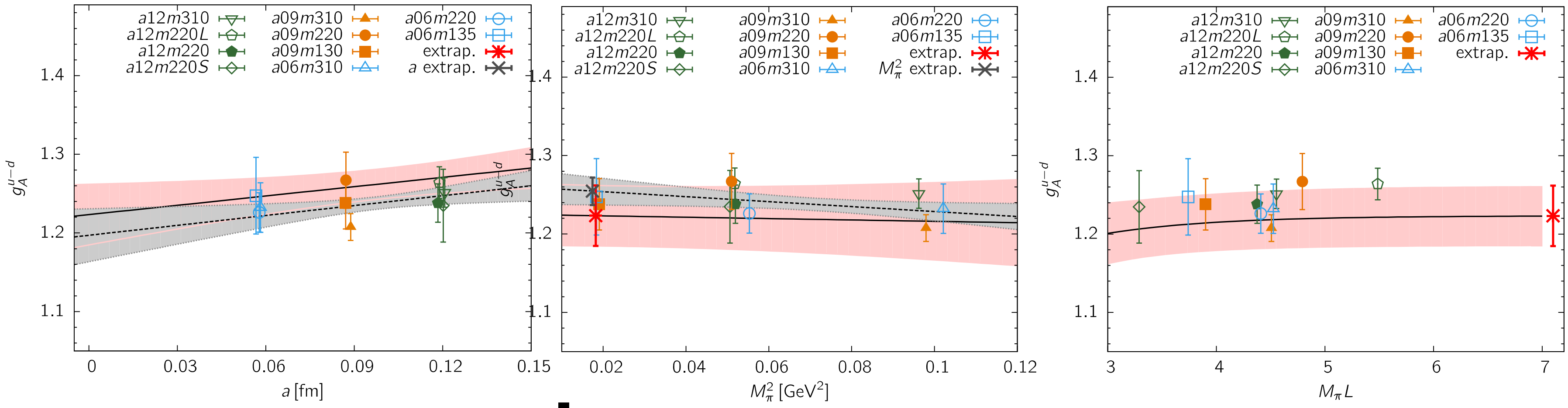
CalLat at Lattice2017 showed additional data

1.285(17)

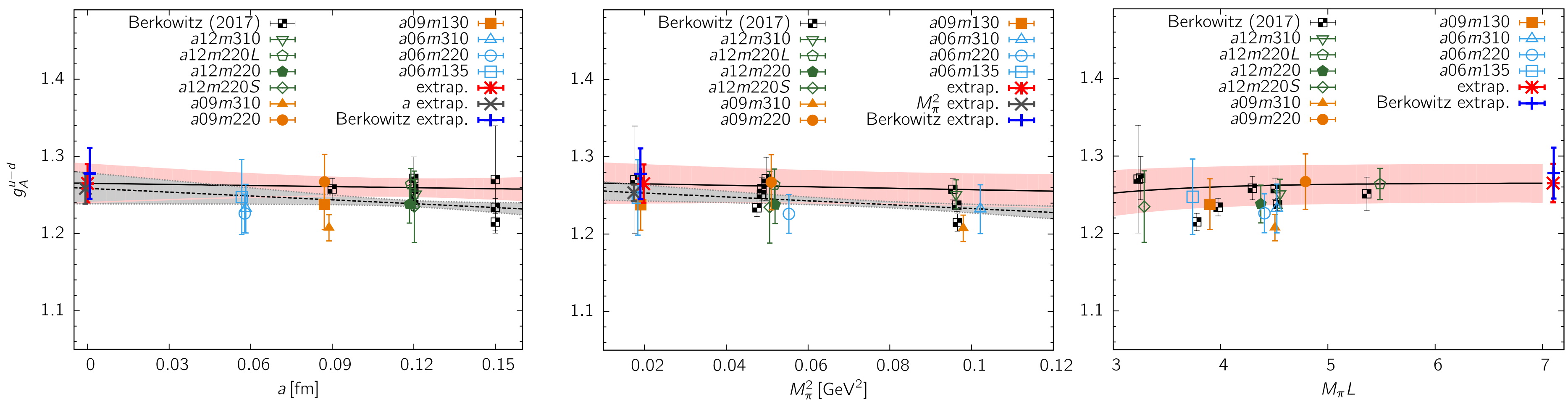
PNDME 2017:

g_A = 1.223(39)

g_A: Simultaneous extrapolation in $a, M_\pi^2, M_\pi L$



↓ PNDME 2017 + CalLat 2016



$$g_A = 1.265(25)$$

10 clover-on-HISQ ensembles: PNDME

8 DWF-on-HISQ ensembles: CalLAT

Summary

	Ensembles	
PNDME 2016	9	1.195(33)(20)
PNDME 2017	10	1.223(39)
CalLat 2016	8	1.278(21)(26)
CalLat 2017	12	1.285(17)
Combined fit	10+8	1.265(25)

PNDME 2017

- Added a06m135 ensemble
- 4 ensembles HP $\rightarrow$ AMA
- Better smearing on a09
- 4-state fits to 2-point functions
- 3-state fits to 3-point functions
- Covariant error matrix

CalLat 2017

- Added 4 ensembles
  - a12m130
  - a12m450
  - a12m400
  - a09m220

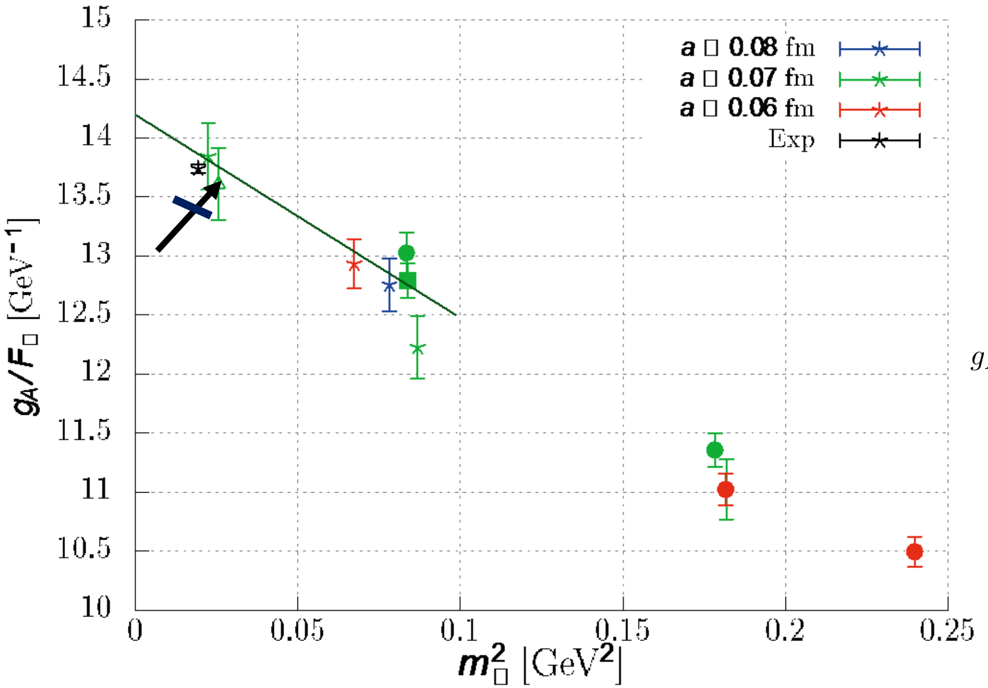
Other Calculations

- RQCD' 2015 (n_f=2)
  - 11 ensembles (7 with M_π<300; 1 with M_π<250)
  - Data points for g_A are all below 1.22
  - Linear fit in M_π^2 to g_A/F_π gives g_A =1.280(44)(46)
- Mainz'2017 (n_f=2)
  - 11 ensembles similar to RQCD (7 with $M_\pi \leq 330$)
  - Linear fit g_A vs M_π^2 using 7ens: $g_A = 1.278(68)_{-0.087}$
- ETMC'2017 (n_f=2)
  - One ensemble: a=0.094fm, M_π=130 MeV, $M_\pi L$=3
  - g_A = 1.212(33)(22)

Chiral extrapolation

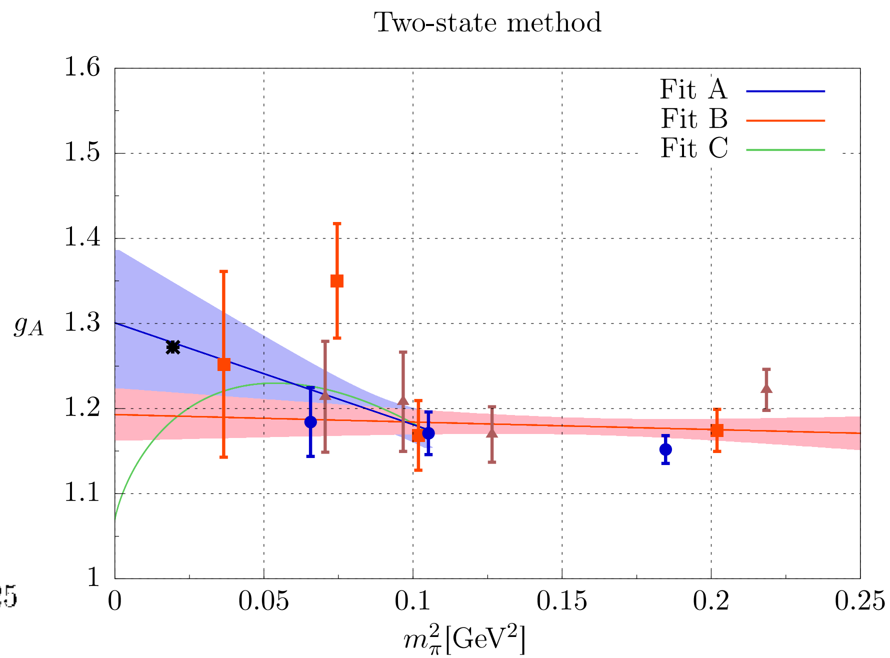
RQCD’14

g_A =1.280(44)(46)



Mainz’17

g_A = 1.278(68)$_{-0.087}$



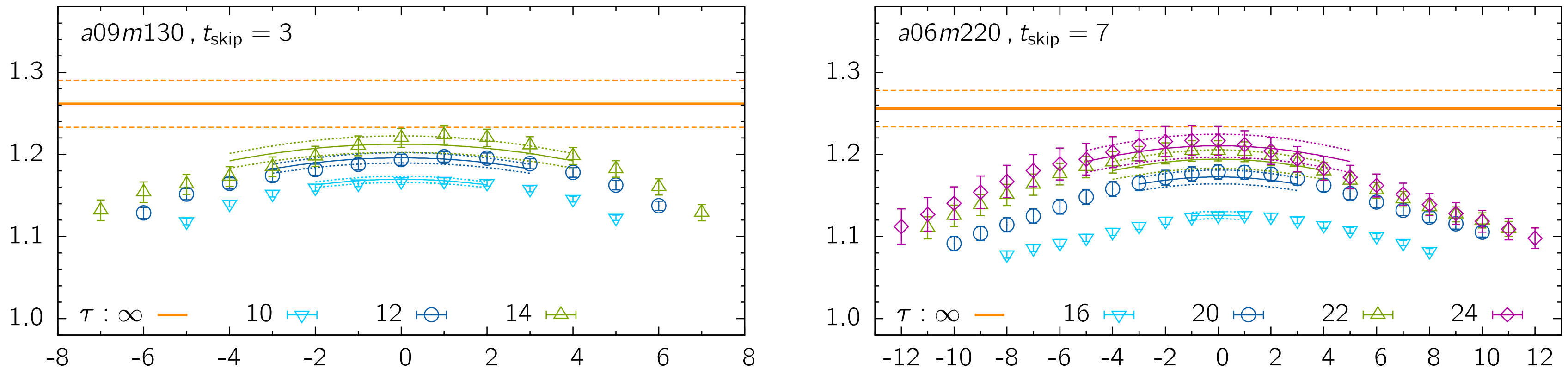
g_A=1.18 at the M_π=150MeV point!

Fit A: A + B M_π^2 (M_π<330MeV)

Fit B: A + B M_π^2 (No Cut)

CalLat: Excited State Contamination

$$g_A^{eff} = g_A + \Delta(\text{ESC}) + \Delta(\text{artifacts=contact+outside})$$



Does using propagators with tuned smeared sources on lattices smoothed by Wilson flow reduce artifacts?
and τ dependence?

What is the leading excited-state?

Oliver Baer Lattice 2017

- Excited states contributing to the nucleon 2-pt function:
  - Resonance states: $N^*(1440)$, $N^*(1710)$, ...
  - Multi-particle states: $N\pi$, $N\pi\pi$, $\Delta\pi$, ...

- Multi-particle states become important for physical pion masses

Sketch:

ignore interaction

Finite volume, periodic bc

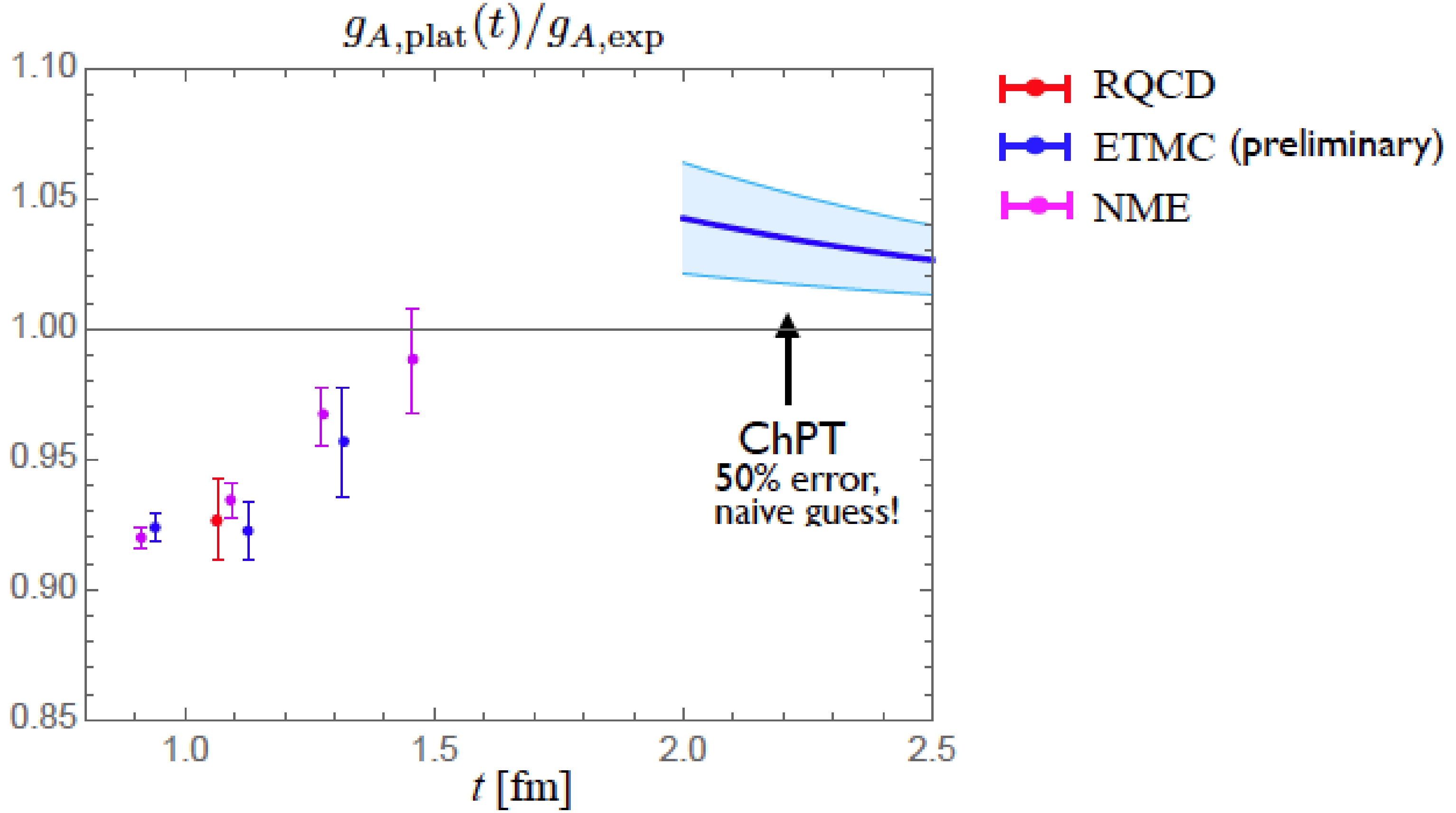
⇒ discrete momenta

$$\vec{p}_k = \frac{2\pi}{L}\vec{k}$$

$M_{\pi,\text{phys}}L = 4$ $\quad \#N(\vec{p}_k)\pi(-\vec{p}_k) = 3$ ◆ (orange)

$M_{\pi,\text{phys}}L = 6$ $\quad \#N(\vec{p}_k)\pi(-\vec{p}_k) = 7$ ◆ (purple)

Comparison with lattice data: axial charge



Lattice data + ChPT suggests <u>accidental agreement</u> for some $t \approx 1.5\mathrm{fm}$

➡ needs to be checked !

Work done in collaboration with

PNDME collaboration
(Clover-on-HISQ)

- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Yong-Chull Jang
- Huey-Wen Lin
- Boram Yoon

Bhattacharya et al, PRD85 (2012) 054512
Bhattacharya et al, PRD89 (2014) 094502
Bhattacharya et al, PRD92 (2015) 114026
Bhattacharya et al, PRL 115 (2015) 212002
Bhattacharya et al, PRD92 (2015) 094511
Bhattacharya et al, PRD94 (2016) 054508
Gupta et al, arXiv:1705:06834

NME collaboration
(Clover-on-Clover)

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- Jeremy Green
- Yong-Chull Jang
- Bálint Joó
- Huey-Wen Lin
- Kostas Orginos
- David Richards
- Sergey Syritsyn
- Frank Winter
- Boram Yoon

Yoon et al., PRD D93 (2016) 114506
Yoon el al., PRD D95 (2017) 074508

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- Institutional Computing at Los Alamos National Laboratory.
- The USQCD Collaboration, funded by the Office of Science, U.S. DOE

- Clover-on-HISQ
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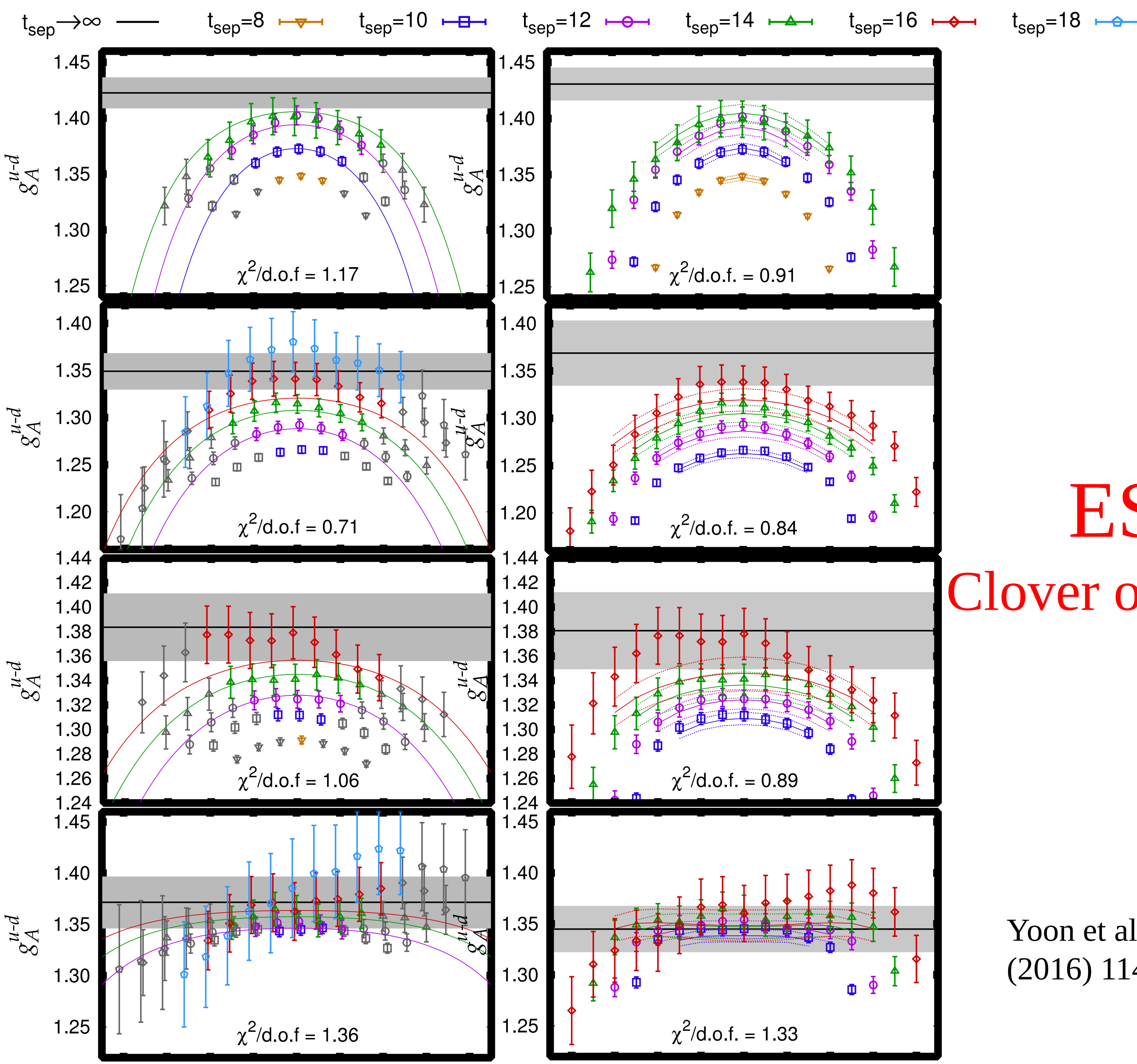
2+1 flavor Clover lattices (Jlab/W&M)

M_s tuned to its physical value using $(2M^2_{K^+} - M^2_{p^0})/M^2_W$

a fm	M_π MeV	Lattice Volume	$M_\pi L$	t_{sep}	Smearing σ	# of Configs	HP Src.	LP Src
0.114	316	$32^3 \times 96$	5.85	8,10,12,14	5	1000	4000	128,480
0.081	312	$32^3 \times 64$	4.11	10,12,14,16,18	5	1005	3,015	96,480
0.081	312	$32^3 \times 64$	4.11	8,10,12,14,16	7	1005	3,015	96,480
0.081	312	$32^3 \times 64$	4.11	10,12,14,16,18	9	1005	3,015	96,480
0.081	312	$32^3 \times 64$	4.11	12	V357, V579	443	0,1329	42,528
0.079	192	$48^3 \times 96$	3.7	8,10,12,14,16	7	629	2,516	80,512
0.079	198	$64^3 \times 128$	5.08	8,10,12,14,16	7	467	2,335	74,720



ESC

Clover on Clover

Yoon et al., PRD D93 (2016) 114506