INT Workshop INT-17-67W
"Lattice QCD input for neutrinoless double-β decay"
July 6-7 2017

EFT approach to $0\nu\beta\beta$ and the impact of Lattice QCD

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Detailed instructions:

Effective operator structures and prioritization of LQCD calculations that can have the most impact on the nuclear double-beta decay program

Outline

- Introduction: $0\nu\beta\beta$ and Lepton Number Violation (LNV)

- LNV in the "Standard Model EFT"

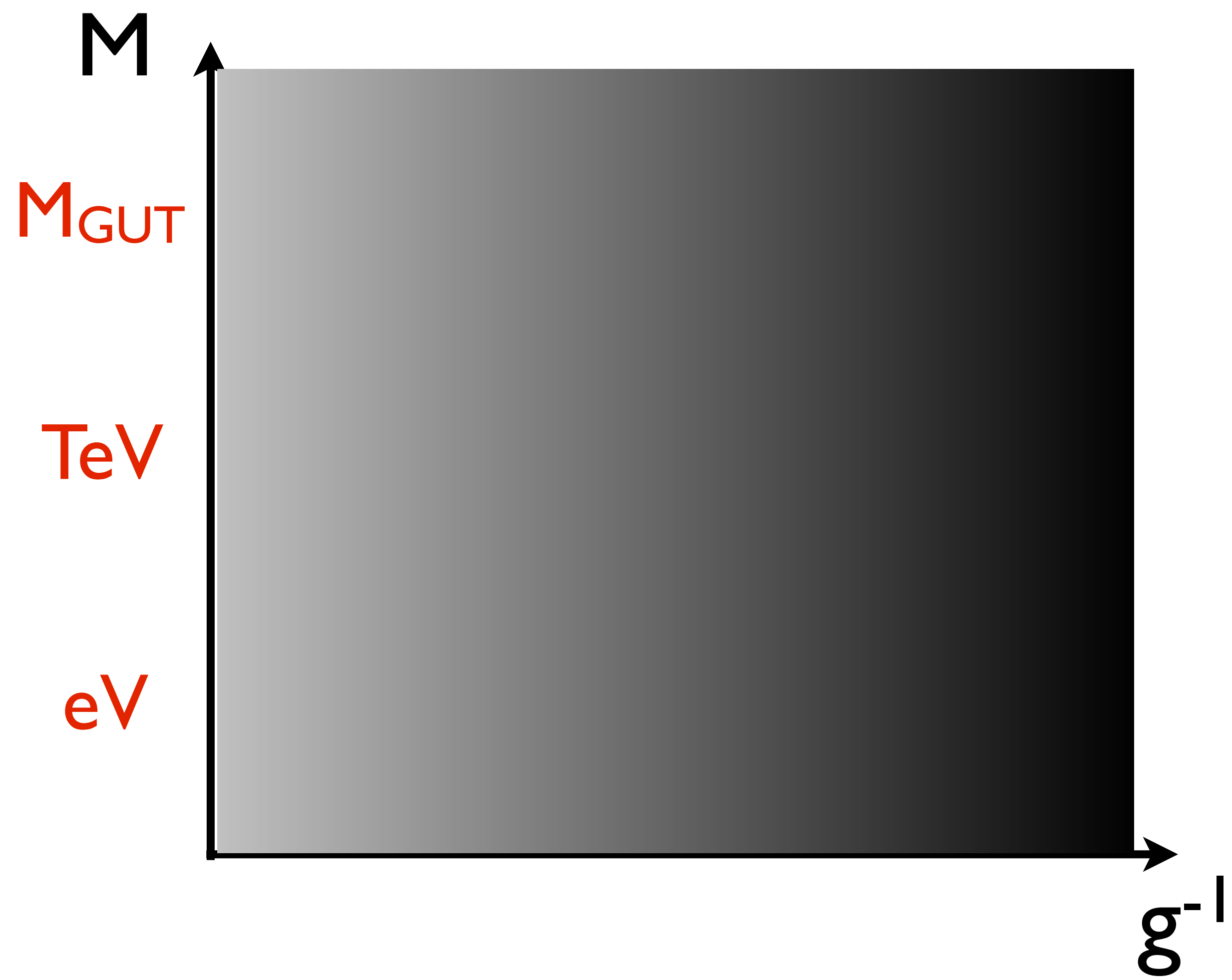
    - $0\nu\beta\beta$ from dim-5 operator ("light neutrino exchange")
    - $0\nu\beta\beta$ from dim-7 and dim-9 operators

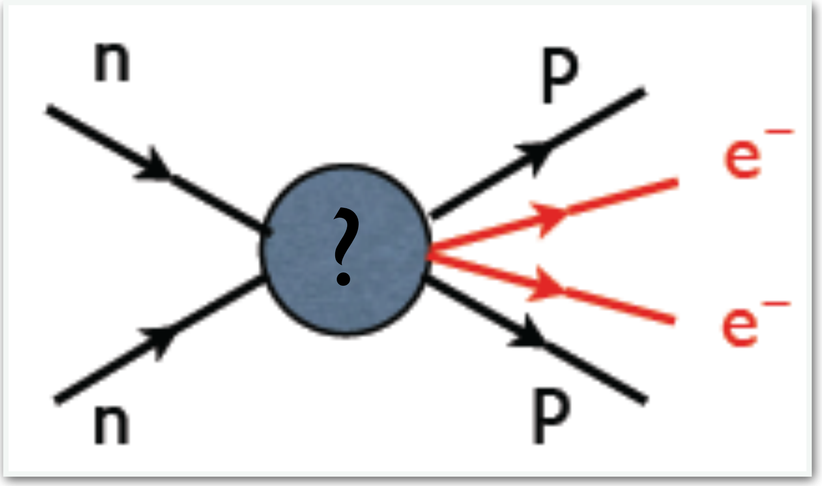
Emphasis on matching to chiral EFT & related LQCD input

** Many thanks to Emanuele Mereghetti for sharing his slides from INT talk on Jun 27

0νββ and Lepton Number Violation

- Ton-scale 0νββ searches ($T_{1/2}$ >$10^{27\text{-}28}$ yr) sensitive to LNV from a variety of mechanisms





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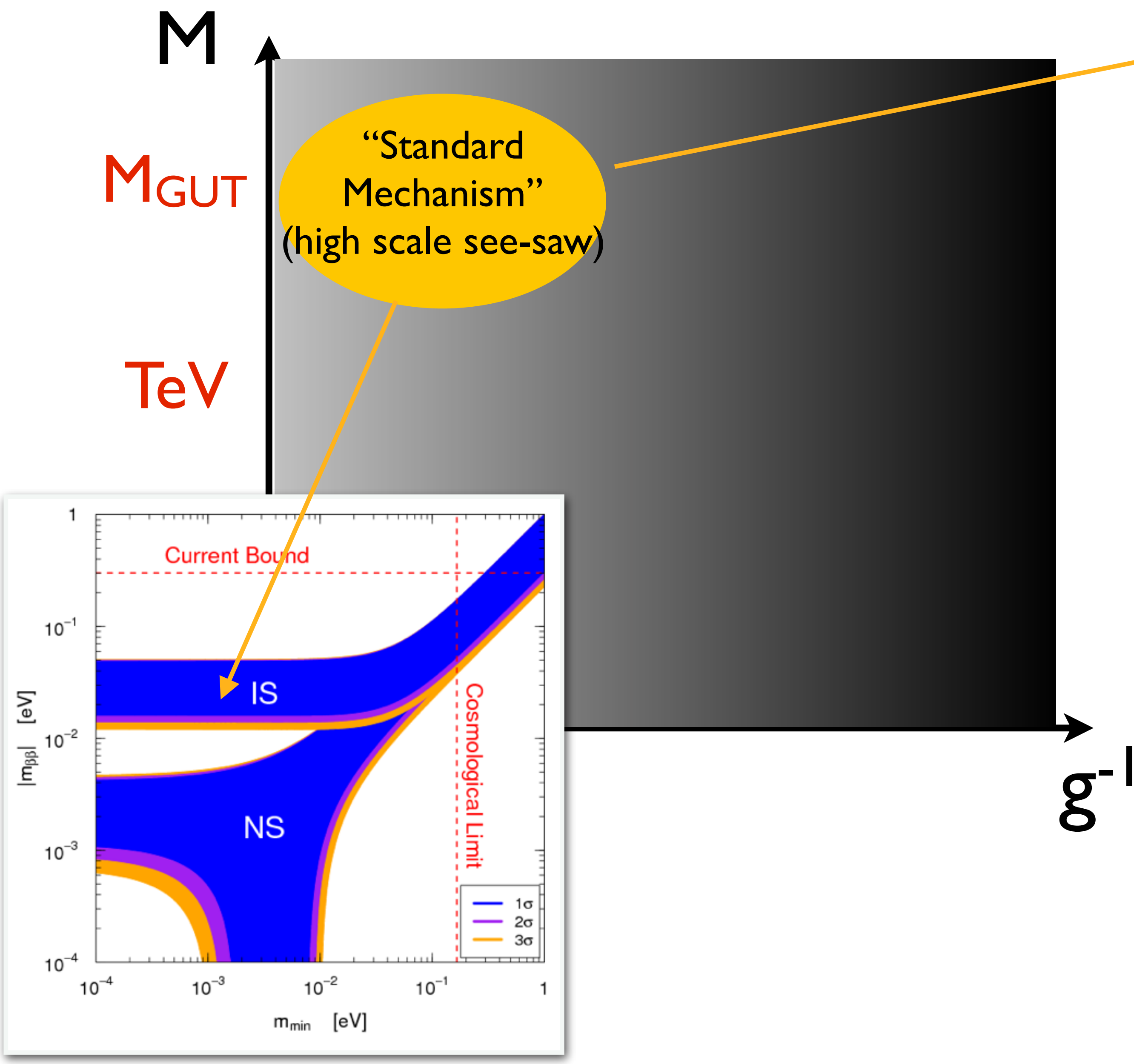
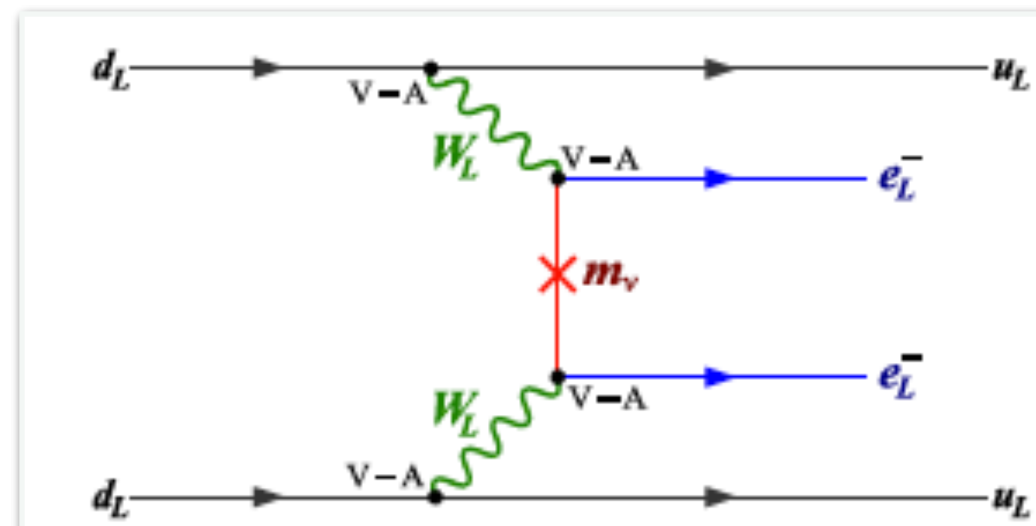


LNV dynamics at M >> TeV: leaves as the only low-energy footprint light Majorana neutrino (dim 5 op.)

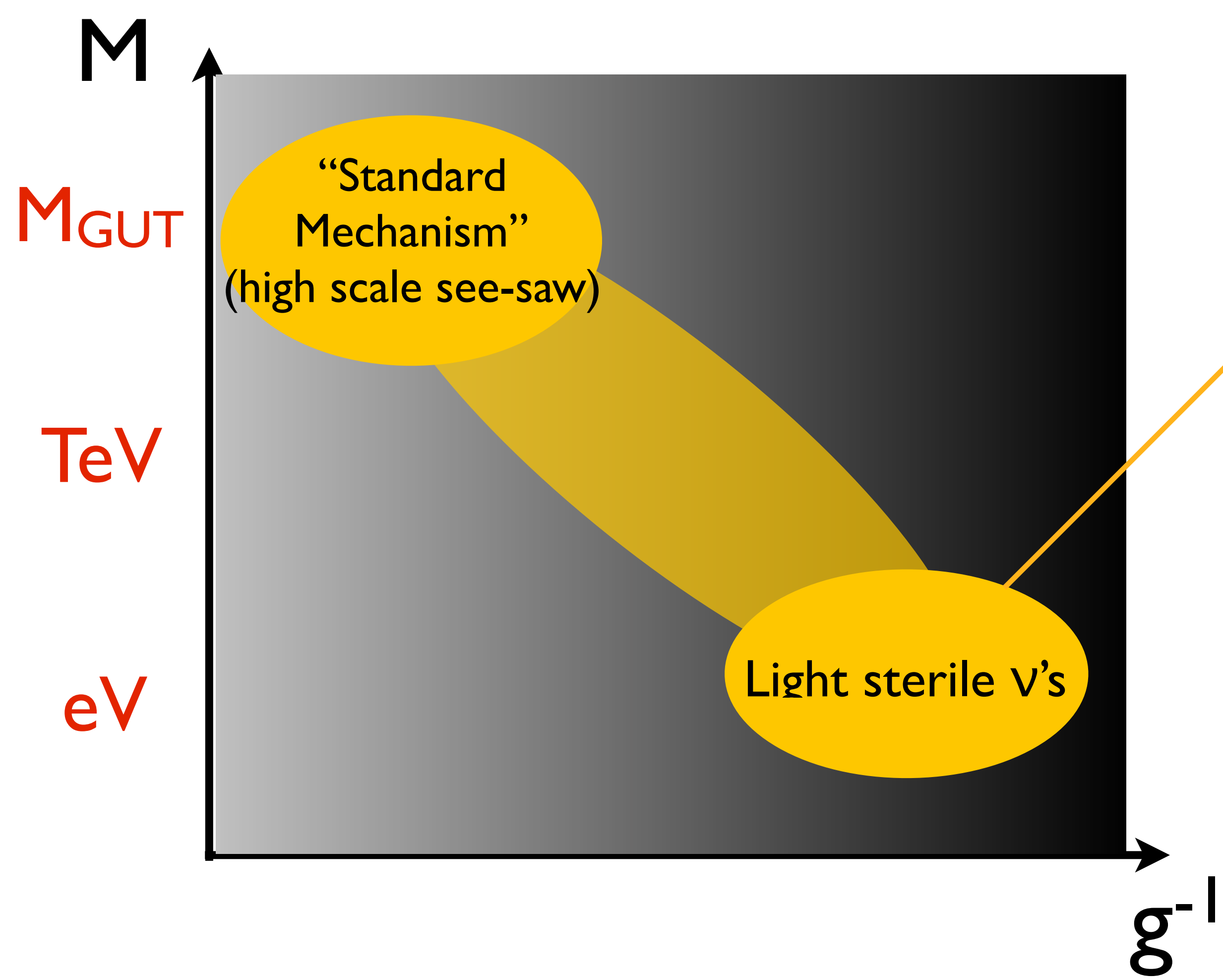


Clear interpretation framework and sensitivity goals ("inverted hierarchy"). Requires difficult nuclear matrix elements.

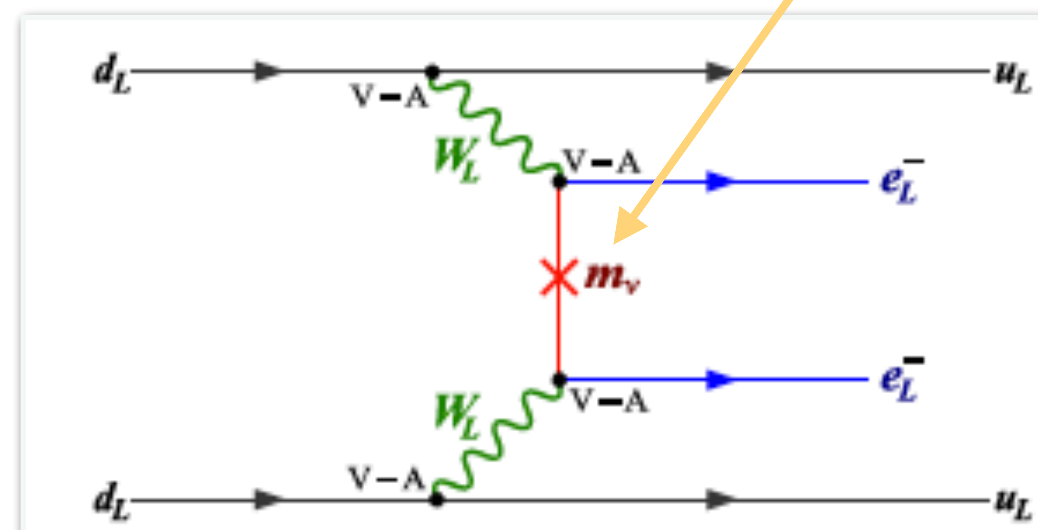
But only limited class of models!

0νββ and Lepton Number Violation

- Ton-scale 0νββ searches ($T_{1/2}$ >10^{27-28} yr) sensitive to LNV from a variety of mechanisms

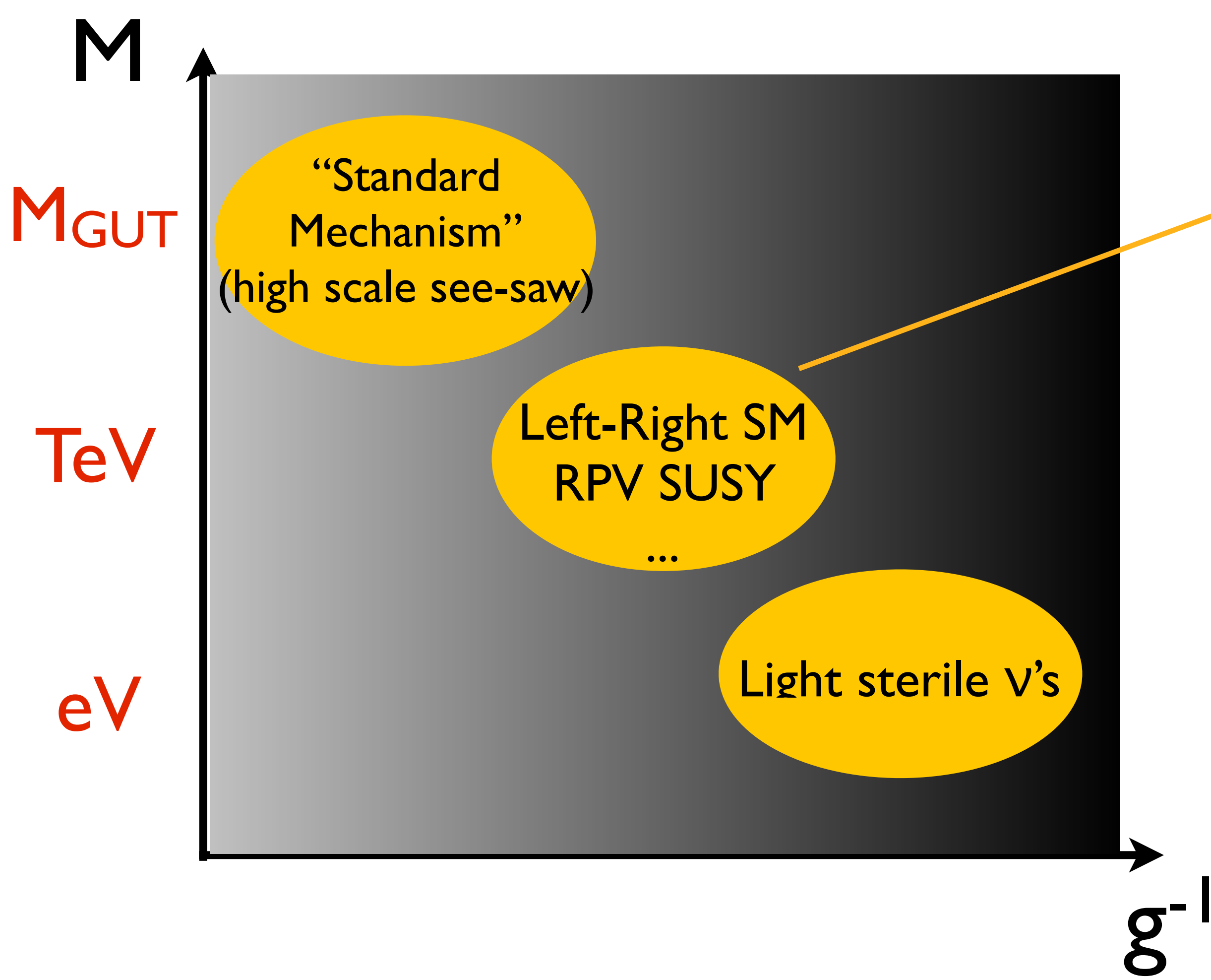


LNV dynamics at M_R ~ eV → GeV: additional light Majorana states

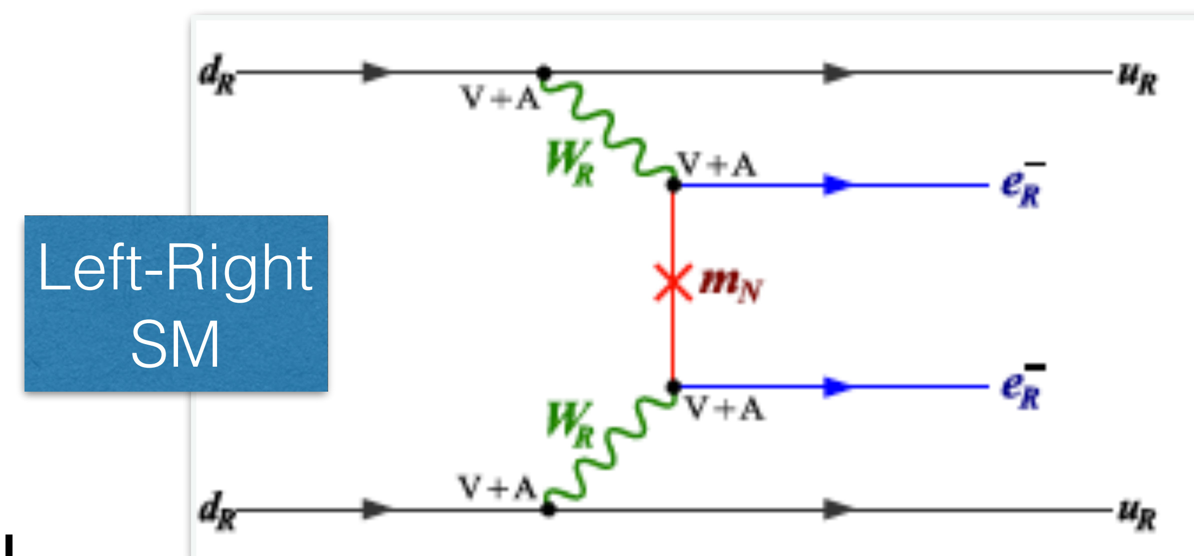


0νββ and Lepton Number Violation

- Ton-scale 0νββ searches ($T_{1/2} > 10^{27\text{-}28}$ yr) sensitive to LNV from a variety of mechanisms



LNV dynamics at M ~ 1-100 TeV
(dim 7 & 9 operators relevant):
1) new contribution to 0νββ not directly related to light neutrino mass;
2) pp → eejj at the LHC (or FCC)

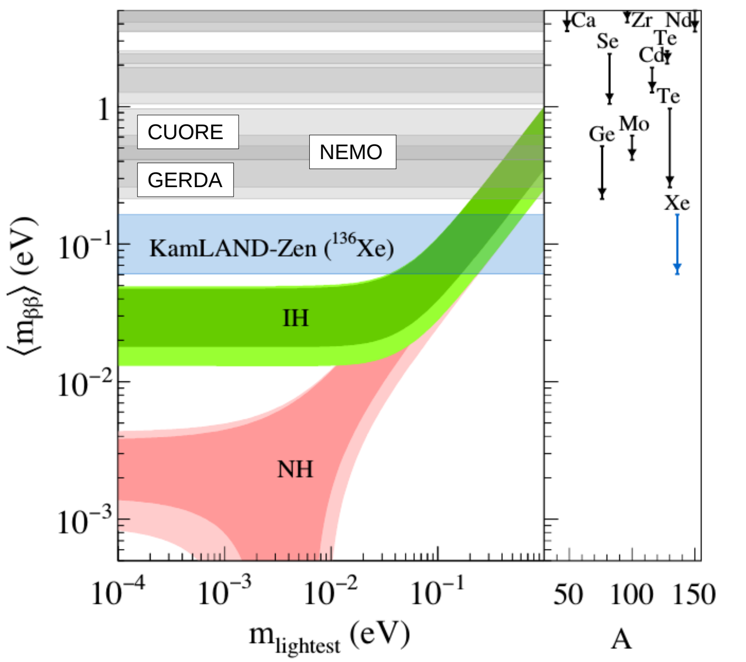


Discovery potential and interpretation of null results depend on a different set of (equally difficult) hadronic and nuclear matrix elements

See for example
Deppisch et al: 1208.0727
(and references therein)

EFT approach to LNV and 0νββ

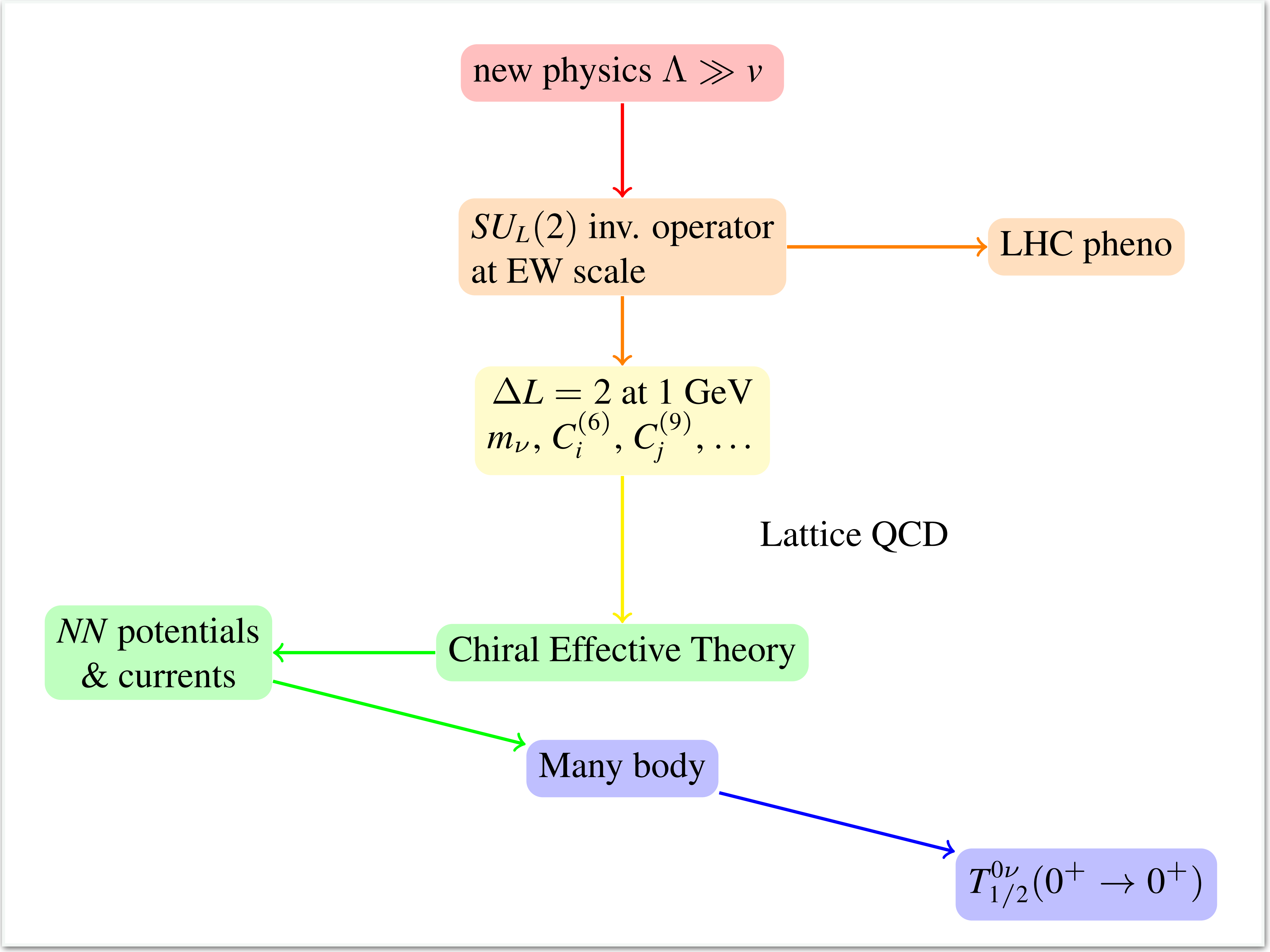
- To go beyond the "standard picture" in a systematic and general way use EFT



KamLAND-Zen coll., '16

- EFT techniques allow one to
  - parameterize sources of LNV at high energy with SU(3)×SU(2)×U(1) invariant operators (assumption: no light ν_R)
  - connect high-scale new physics to hadronic and nuclear scales
  - organize contributions to hadronic and nuclear matrix elements

The big picture



Courtesy of E. Mereghetti

LNV in the "Standard Model EFT"

- High scale ΔL=2 operators appear at dim = 5, 7, 9, …

$\Lambda \leftrightarrow M_{\rm BSM}$ $\qquad$ $C_i \; [g_{\rm BSM}, \; M_a/M_b]$

$$\mathcal{L}_{\rm eff}^{\Delta L=2} \;=\; \frac{C^{(5)}}{\Lambda}\, O^{(5)} \;+\; \sum_i \frac{C_i^{(7)}}{\Lambda^3}\, O_i^{(7)} \;+\; \sum_i \frac{C_i^{(9)}}{\Lambda^5}\, O_i^{(9)} \;+\; \ldots$$

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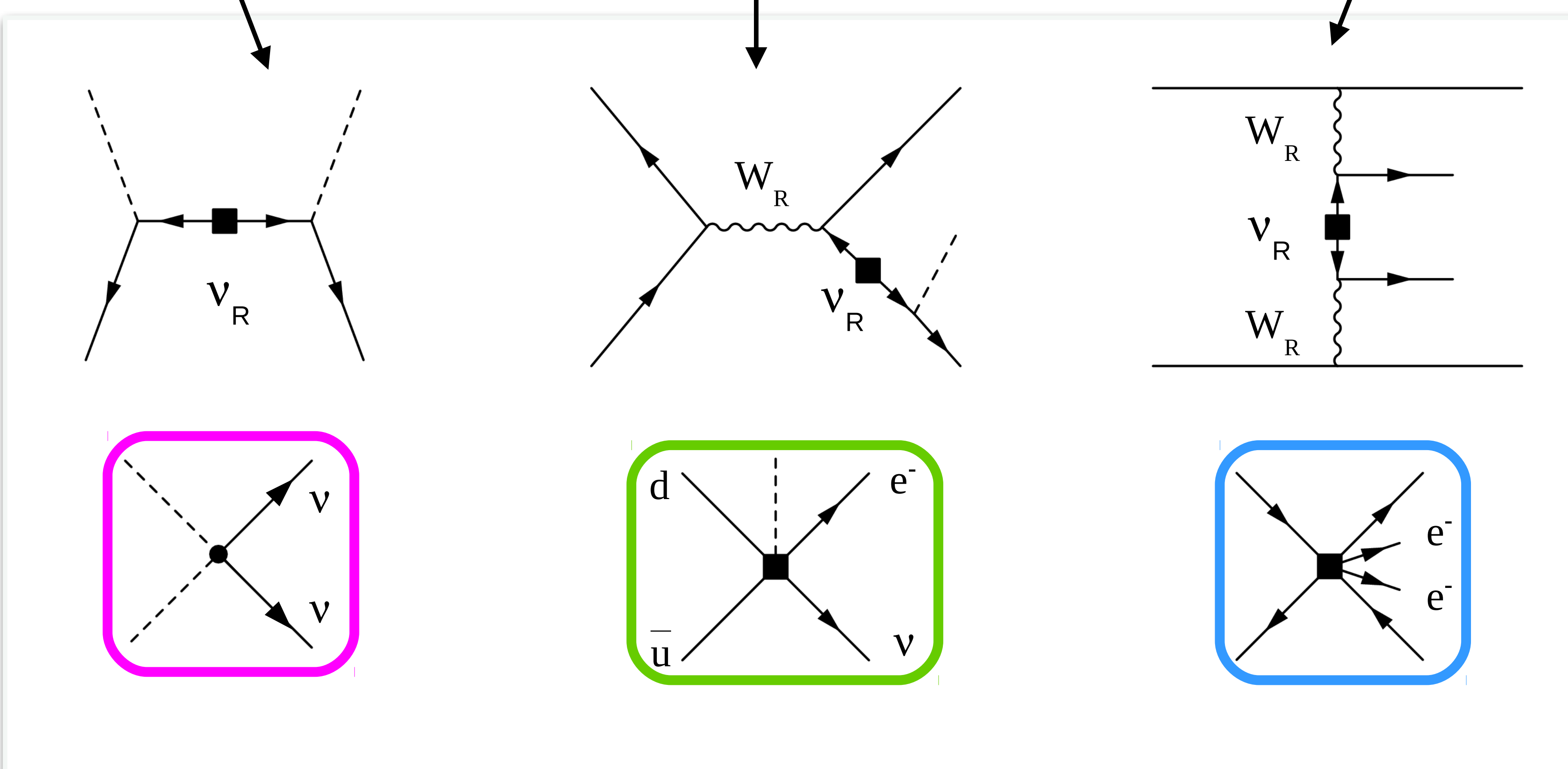
Simple model realization (Left-Right SM)



Systematic "unpacking"

Babu-Leung hep-ph/0106054

Bonnet et al 1212.3045

Helo et al 1602.03362

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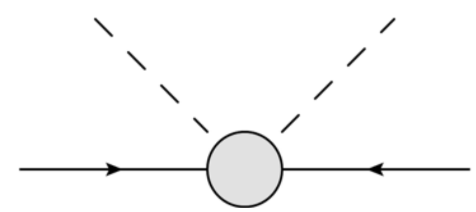
Weinberg 1979



$$O^{(5)} = \ell^T C \epsilon \varphi \; \varphi^T \epsilon \ell \qquad\qquad C = i\gamma_2\gamma_0$$

LNV in the "Standard Model EFT"

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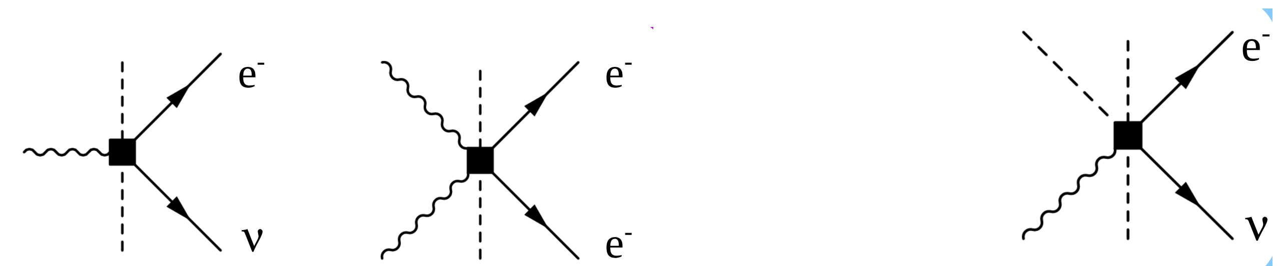
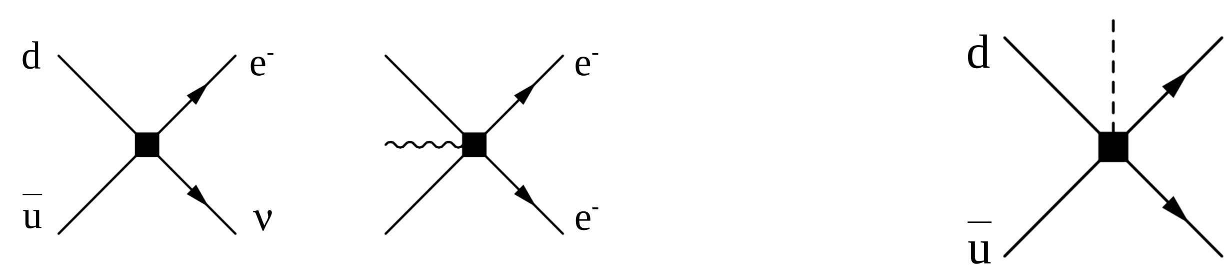
Lehman 1410.4193

- Twelve operators relevant to 0νββ
- W couplings & semileptonic 4-fermi with wrong-helicity neutrino
- WWee couplings



$\varepsilon_{ij}\varepsilon_{mn}\, L_i^T C (D_\mu L)_j\, H_m (D^\mu H)_n$ $\qquad$ $\varepsilon_{ij}\varepsilon_{mn}\, L_i^T C\gamma_\mu e\, H_j H_m (D^\mu H)_n$



$\varepsilon_{ij}\, \bar{d}\gamma_\mu u\, L_i^T C (D^\mu L)_j$ $\qquad$ $\varepsilon_{ij}\varepsilon_{mn} \bar{d} L_i\, Q_j^T C L_m\, H_n$

LNV in the "Standard Model EFT"

- High scale ΔL=2 operators appear at dim = 5, 7, 9, …

$\Lambda \leftrightarrow M_{\mathrm{BSM}}$ $\qquad$ $C_i\ [g_{\mathrm{BSM}},\ M_a/M_b]$

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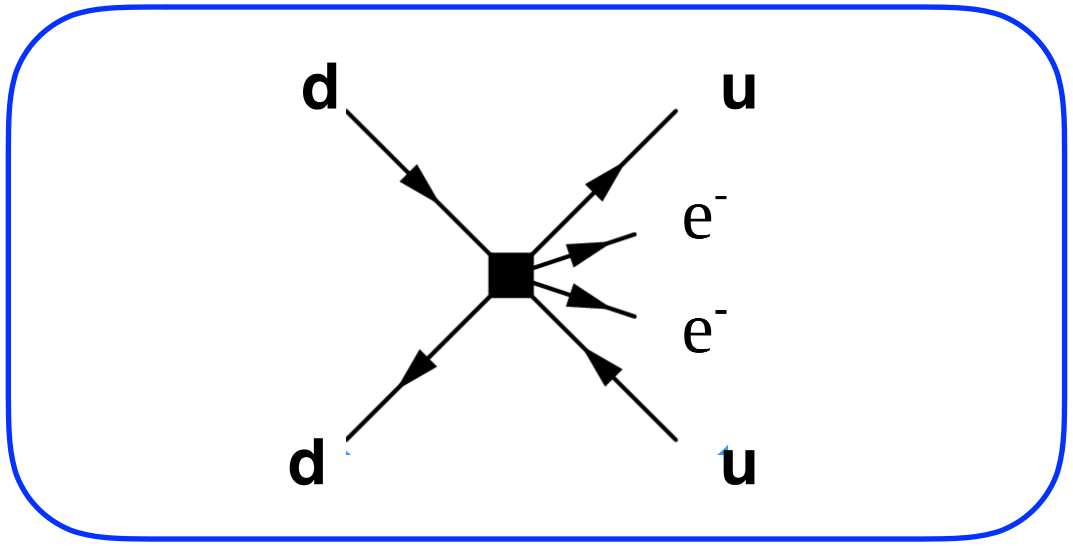
Graesser 1606.04549



- Eleven 6-fermion structures
  - 7 Lorentz scalar 4-quark
  - 4 Lorentz vector 4-quark

LNV in the "Standard Model EFT"

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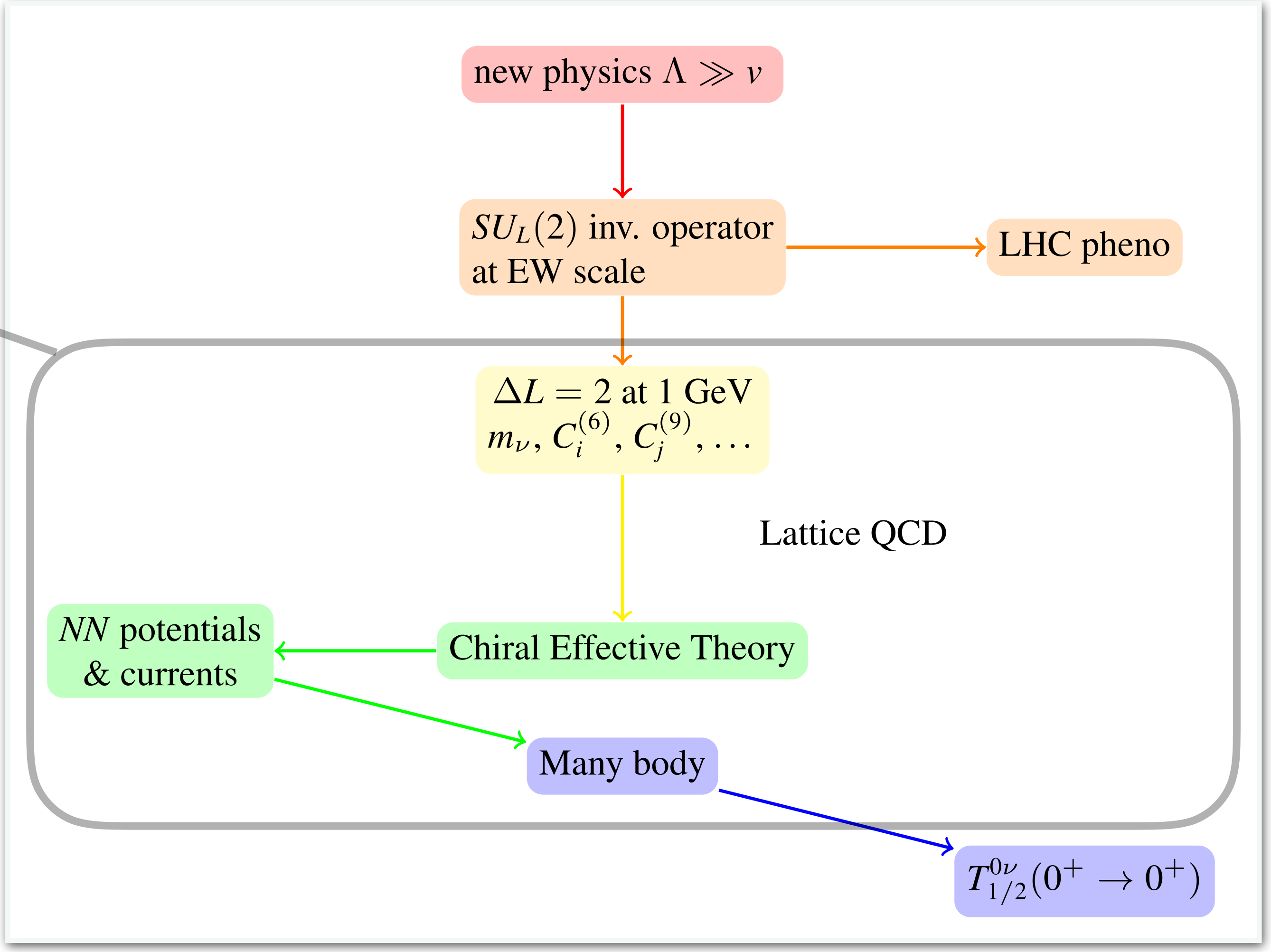
- Next, discuss low-energy manifestations of LNV from
  1. Dim 5 operator
  2. Dim 7 & Dim 9 operators



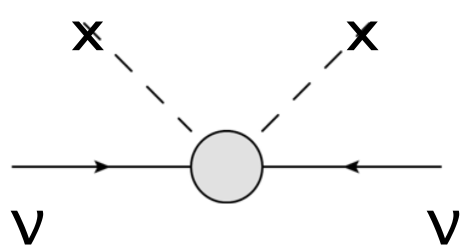
LNV from dimension-5 operator

Based on ongoing collaboration / discussions with:

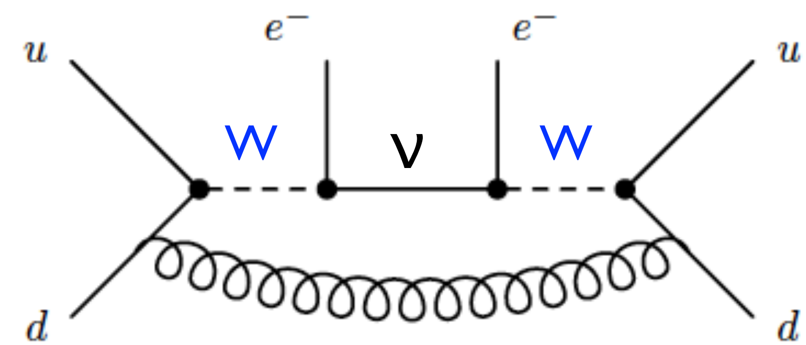
W. Dekens, E. Mereghetti, A. Walker-Loud

GeV-scale effective Lagrangian

- $\Delta L=2$ operators appear at dim 3 & 9



$$m_{\beta\beta} \sim \frac{v^2}{\Lambda}$$



$$\mathcal{L}^{(5)}_{\Delta L=2} = -\frac{1}{2} m_{\beta\beta}\, \nu^T_{eL} C \nu_{eL} \;+\; 8 V_{ud}^2 \frac{G_F^2 m_{\beta\beta}}{M_W^2} \frac{\alpha_s}{4\pi}\; \bar{e}_L e^c_L\; \bar{u}_L \gamma_\mu d_L\; \bar{u}_L \gamma^\mu d_L$$

- $m_{\beta\beta}$ + CC weak interaction → usual "neutrino potential"

- Seems negligible: $(k_F / M_W)^2$ relative suppression
- In a Wilsonian scheme $M_W \to \Lambda_\chi \sim$ GeV: there's some "short distance" physics even in the light-neutrino mechanism
- It shows up as a LEC in a mass-indep. scheme

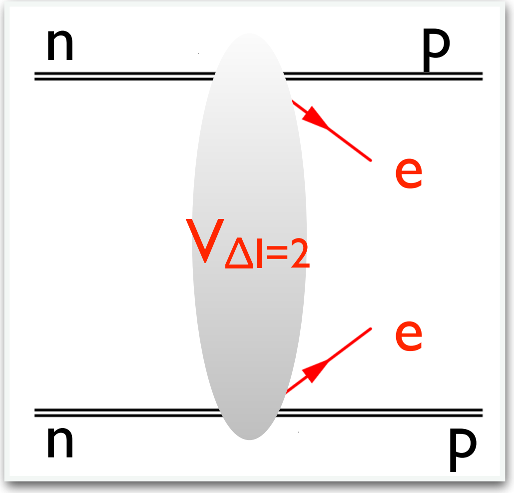
Chiral effective field theory

- At $E \sim \Lambda_\chi \sim$ GeV

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{Fermi}} + \mathcal{L}^{(5)}_{\Delta L=2}$$

- Map onto pion and nucleon operators; expand in Q/Λ_χ ($Q \sim k_F \sim m_\pi$)
- Identify "$0\nu\beta\beta$ potential" $V_{\Delta I=2}$ mediating $nn \rightarrow pp$ to a given order in Q/Λ_χ (generated by ν's and π's with 3-momenta $\sim Q$)

In the process, low-energy effective couplings appear, to be fixed through data or LQCD calculations



Chiral effective field theory

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$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm Fermi} + \mathcal{L}^{(5)}_{\Delta L=2}$$

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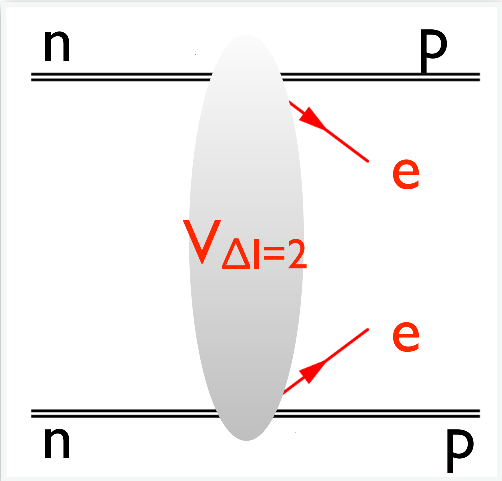

Use $V_{\Delta I=2}$ (or some further evolved/massaged version) in nuclear many-body calculations, as a perturbation to $H_{\rm strong}$

Note: "ultra-soft" neutrinos (ν_{US}) are still present in the low-energy nuclear effective Hamiltonian

Chiral effective field theory

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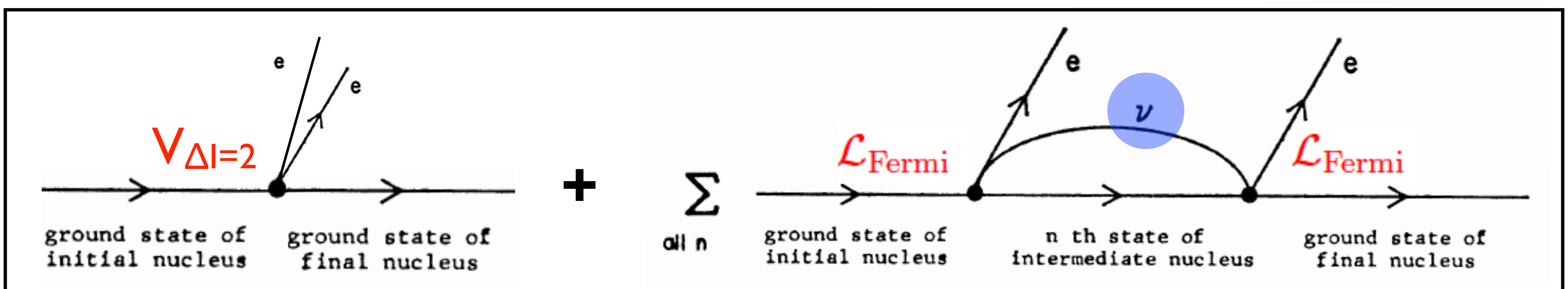
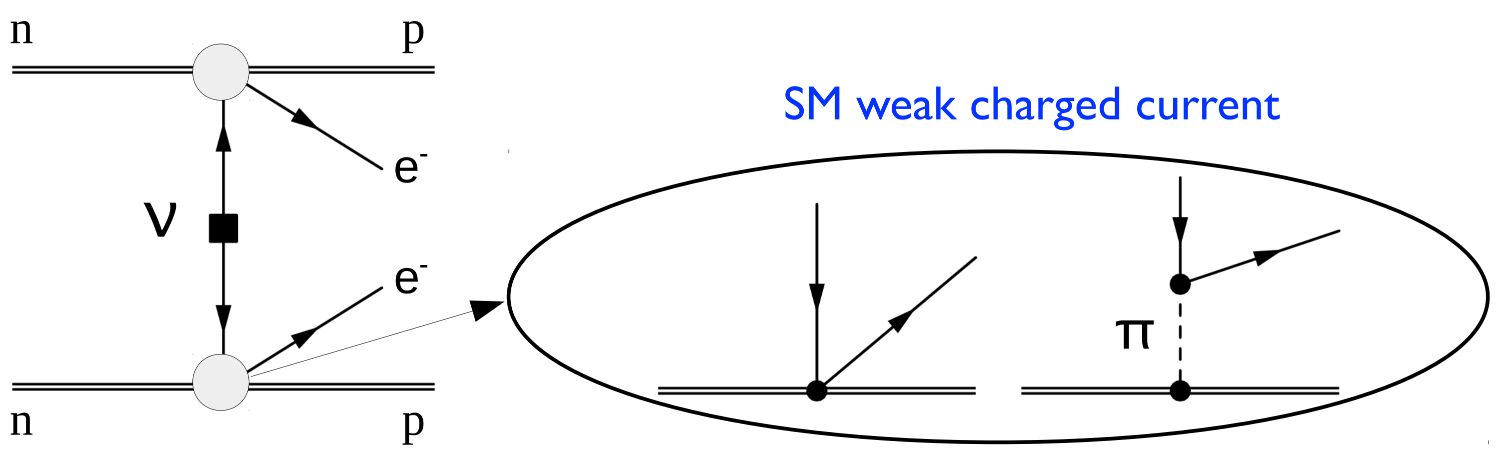


Figure adapted from Primakoff-Rosen 1969

ν_{US} contribution down by $(E_n - E_i)/k_F$
⇒ "Closure" approximation is OK at LO

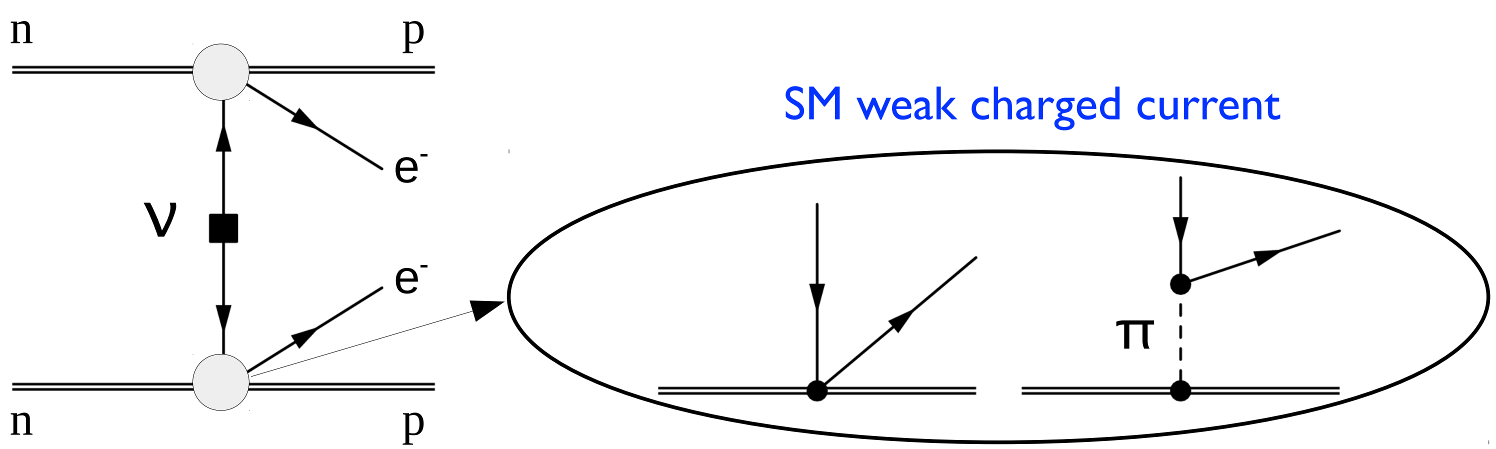
"0νββ potential" from $\mathcal{L}^{(5)}_{\Delta L=2}$



- Leading Order

$$\begin{aligned} J_V^\mu &= (g_V, \mathbf{0}) & g_V &= 1 \\ J_A^\mu &= -g_A \left(0, \boldsymbol{\sigma} - \frac{\mathbf{q}}{\mathbf{q}^2 + m_\pi^2}\, \boldsymbol{\sigma}\cdot\mathbf{q}\right) & g_A &= 1.27 \end{aligned}$$

"0νββ potential" from $\mathcal{L}^{(5)}_{\Delta L=2}$



- Leading Order

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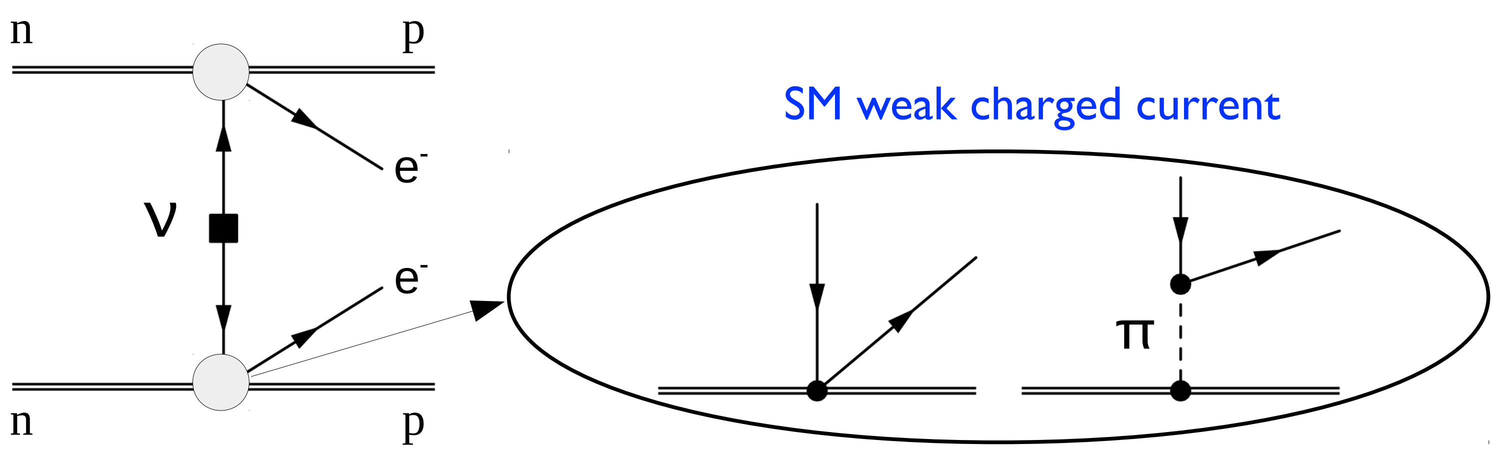
$$V_{\Delta I=2} = \mathcal{A}\frac{m_{\beta\beta}}{\mathbf{q}^2}\Big\{\mathbf{1}\times\mathbf{1} - g_A^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\left(1 - \frac{2}{3}\frac{\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2} + \frac{1}{3}\frac{(\mathbf{q}^2)^2}{(\mathbf{q}^2+m_\pi^2)^2}\right) - \frac{g_A^2}{3}S^{12}\left(-\frac{2\mathbf{q}^2}{\mathbf{q}^2+m_\pi^2} + \frac{(\mathbf{q}^2)^2}{(\mathbf{q}^2+m_\pi^2)^2}\right)\Big\}.$$

$$\mathcal{A} = e_L^T C e_L\, 8 G_F^2 V_{ud}^2 \qquad S^{(12)} = -\left(3\,\boldsymbol{\sigma}^{(1)}\cdot\hat{\mathbf{q}}\,\boldsymbol{\sigma}^{(2)}\cdot\hat{\mathbf{q}} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right)$$

"0νββ potential" from $\mathcal{L}^{(5)}_{\Delta L=2}$



- N²LO: $\mathcal{O}(Q/\Lambda_\chi)^2$

$$J_V^\mu = \left(g_V(\mathbf{q}^2), \frac{\mathbf{P}}{2m_N} - \frac{i(1+\kappa_1)}{2m_N}\boldsymbol{\sigma}\times\mathbf{q} \right) \qquad \kappa_1 = 3.7$$

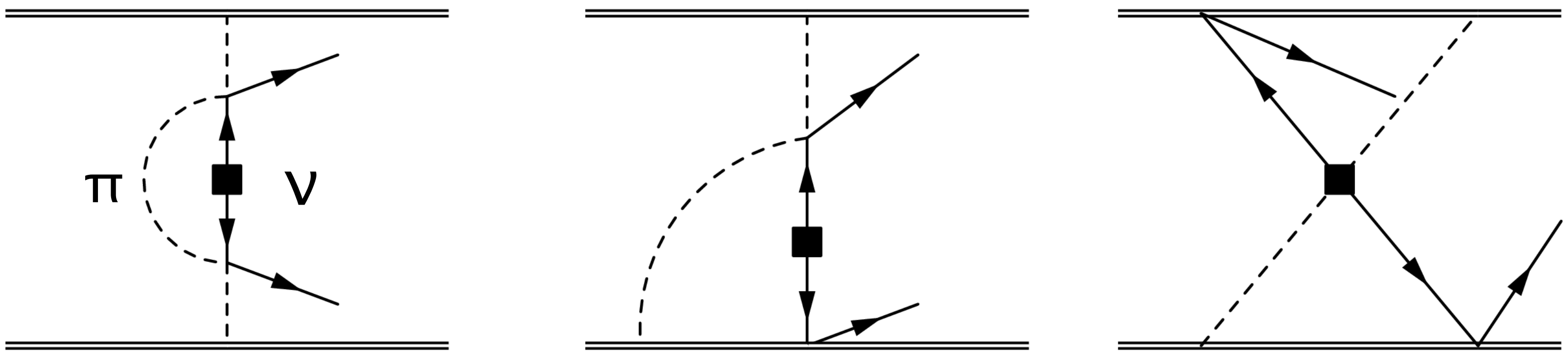
$$J_A^\mu = -g_A(\mathbf{q}^2)\left(\frac{\boldsymbol{\sigma}\cdot\mathbf{P}}{2m_N}, \boldsymbol{\sigma} - \frac{\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\boldsymbol{\sigma}\cdot\mathbf{q} \right)$$

1. Corrections to 1-body currents (radii, magnetic moment,…)
2. 2-body V & A currents
3. Short range effects

LQCD can help with all, but unique input for # 3 (in my opinion)

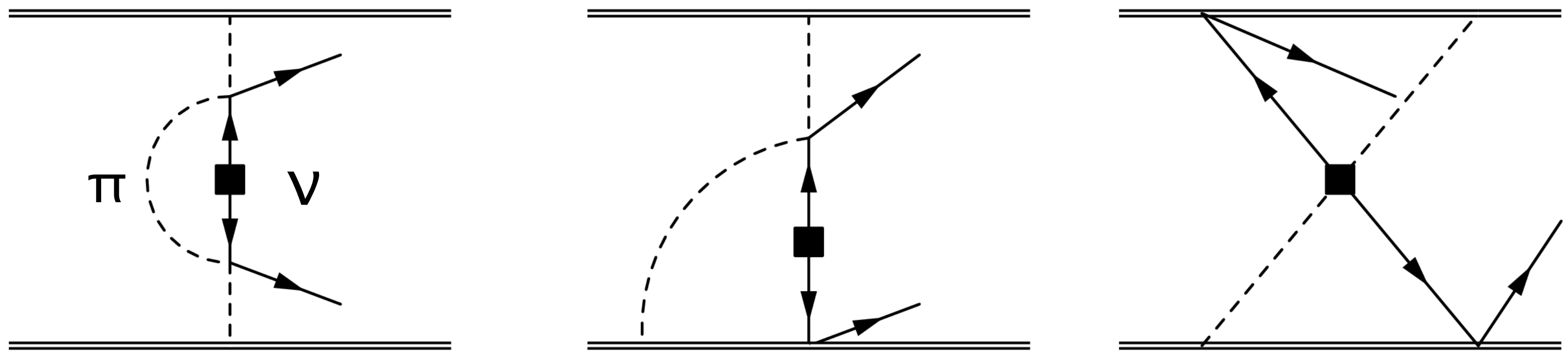
Short range effects from $\mathcal{L}^{(5)}_{\Delta L=2}$

Representative diagrams



Short range effects from $\mathcal{L}^{(5)}_{\Delta L=2}$

Representative diagrams



- Example: VV insertions

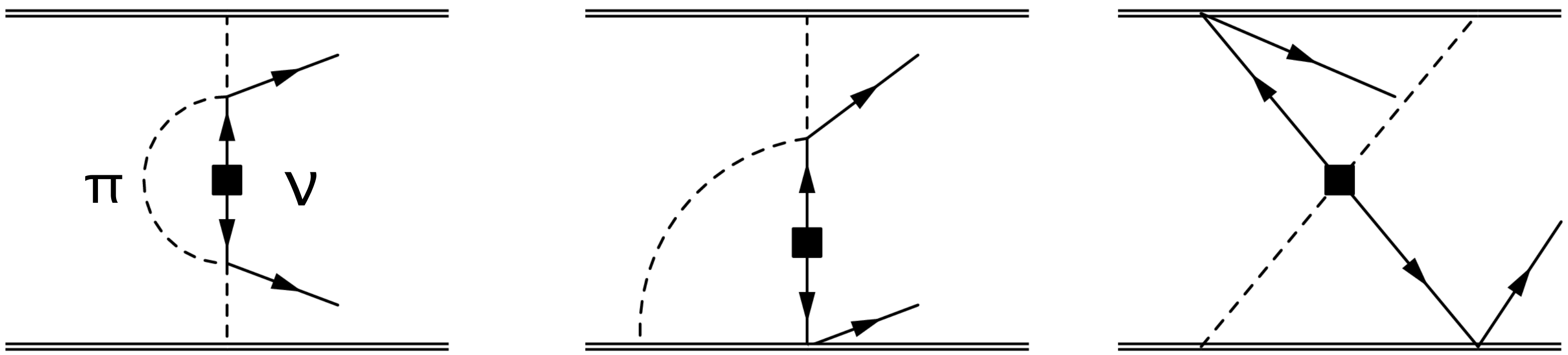
$L_\pi = \log \frac{m_\pi^2}{\mu^2}$

$$V_{\Delta I=2} = \mathcal{A}\, m_{\beta\beta} \frac{g_A^2}{3} (\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} - S^{12}) \frac{1}{(4\pi F_\pi)^2} \left\{ L_\pi \left(\frac{\mathbf{q}^2}{\mathbf{q}^2 + m_\pi^2} - 3 \frac{(\mathbf{q}^2)^2}{(\mathbf{q}^2 + m_\pi^2)^2} \right) + f\left(\frac{\mathbf{q}^2}{m_\pi^2} \right) \right\}$$

- UV divergence: need LECs that encode physics at E ≳ GeV

Short range effects from $\mathcal{L}^{(5)}_{\Delta L=2}$

Representative diagrams



- Example: VV insertions

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$$V_{\Delta I=2} = \mathcal{A}\, m_{\beta\beta} \frac{g_A^2}{3} (\boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} - S^{12}) \frac{1}{(4\pi F_\pi)^2} \left\{ L_\pi \left(\frac{\mathbf{q}^2}{\mathbf{q}^2 + m_\pi^2} - 3 \frac{(\mathbf{q}^2)^2}{(\mathbf{q}^2 + m_\pi^2)^2} \right) + f\left(\frac{\mathbf{q}^2}{m_\pi^2} \right) \right\}$$

- UV divergence: need LECs that encode physics at E ≳ GeV
- LQCD input: matrix elements of non-local effective action

$$S_{\rm NL} = \int dx\, dy\; S_\nu(x-y)\; T\left(J_\alpha^+(x)\, J_\beta^+(y)\right)\; g^{\alpha\beta} \qquad J_\alpha^+ = \bar{u}_L \gamma_\alpha d_L$$

$\langle \pi^+ | S_{NL} | \pi^- \rangle$ $\langle p\pi^+ | S_{NL} | n \rangle$ $\langle pp | S_{NL} | nn \rangle$

LNV from operators of dimension 7 & 9

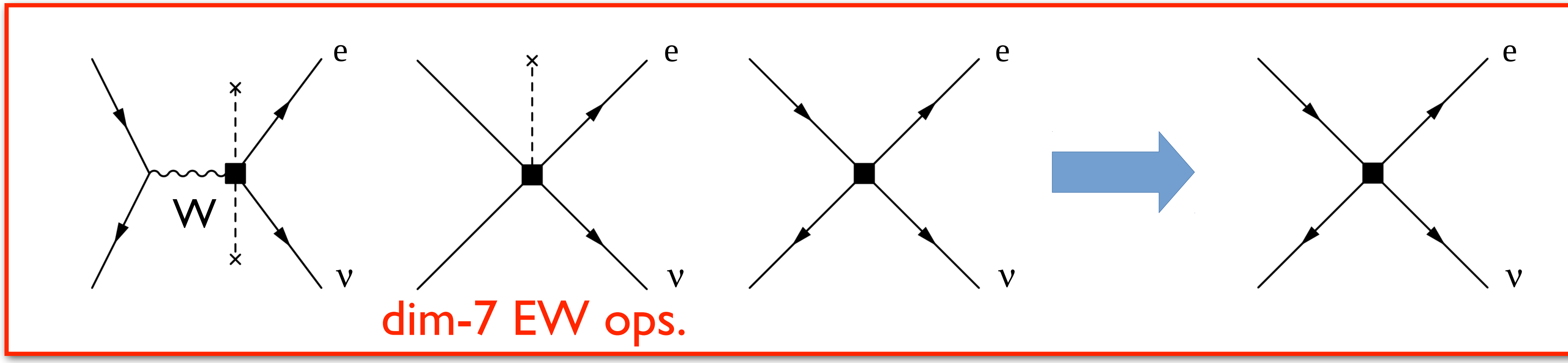
Based on ongoing collaborations / discussions with:

W. Dekens, J. de Vries, M. Graesser, E. Mereghetti
S. Pastore, J. Carlson, R. Wiringa

and on VC-Dekens-deVries-Mereghetti 1701.01443

GeV-scale effective Lagrangian

- ΔL=2 operators appear at dim = 6, 7 & 9



- β decay with "wrong" neutrino (five Lorentz structures) + two dim-7

$$\mathcal{L}^{(6)}_{\Delta L=2} = \frac{2G_F}{\sqrt{2}} \Big\{ C^{(6)}_{\rm VL}\, \bar{d}_L \gamma^\mu u_L\, \nu_L^T\, C\gamma_\mu e_R + C^{(6)}_{\rm VR}\, \bar{d}_R \gamma^\mu u_R\, \nu_L^T\, C\gamma_\mu e_R$$

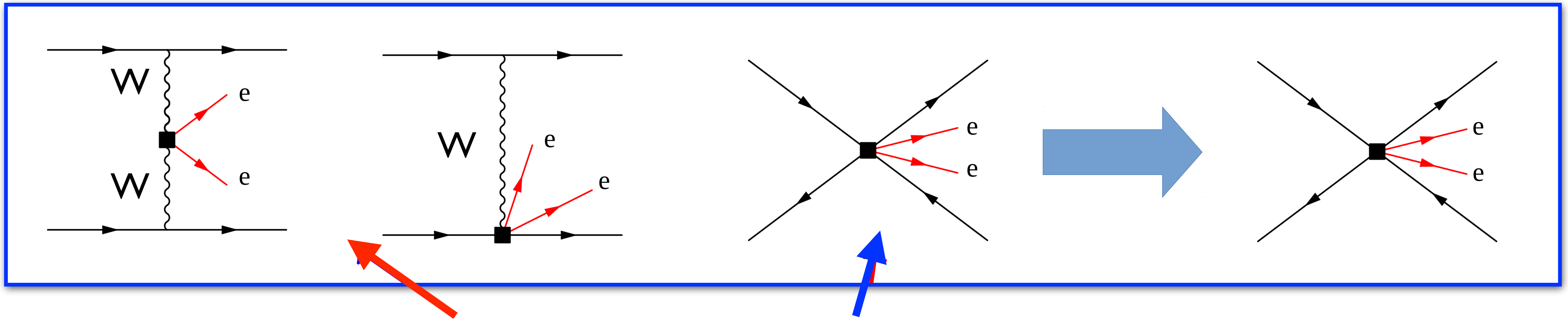
$$+ C^{(6)}_{\rm SL}\, \bar{d}_R u_L\, \nu_L^T\, C e_L + C^{(6)}_{\rm SR}\, \bar{d}_L u_R\, \nu_L^T\, C e_L + C^{(6)}_{\rm T}\, \bar{d}_R \sigma^{\mu\nu} u_L\, \nu_L^T\, C\sigma_{\mu\nu} e_L \Big\}$$

$$C_i^{(6),(7)} \sim \frac{v^3}{\Lambda^3}$$

$$\mathcal{L}^{(7)}_{\Delta L=2} = \frac{2G_F}{\sqrt{2}v} \Big\{ C^{(7)}_{\rm VL}\, \bar{d}_L \gamma^\mu u_L\, \nu_L^T\, C\, i \overleftrightarrow{\partial}_\mu e_L + C^{(7)}_{\rm VR}\, \bar{d}_R \gamma^\mu u_R\, \nu_L^T\, C i \overleftrightarrow{\partial}_\mu e_L \Big\}$$

GeV-scale effective Lagrangian

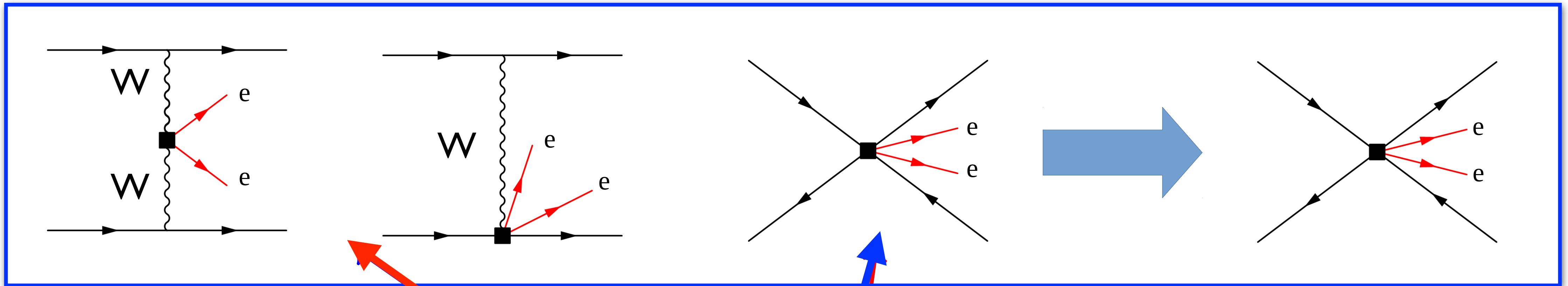
- ΔL=2 operators appear at dim = 6, 7 & 9



dim-7 EW ops. dim-9 EW ops.

GeV-scale effective Lagrangian

- ΔL=2 operators appear at dim = 6, 7 & 9



dim-7 EW ops. dim-9 EW ops.

$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{2G_F^2}{v}\left[\sum_{i=\text{scalar}} \left(C_i^{(9)}\,\bar{e}_L C\,\bar{e}_L^T + C_i^{(9)\prime}\,\bar{e}_R C\,\bar{e}_R^T\right) O_i \;+\; \bar{e}_R\gamma_\mu C\,\bar{e}_L^T \sum_{i=\text{vector}} C_{iV}^{(9)}\,O_i^\mu\right]$$

8 scalar 4-quark operators

8 vector 4-quark operators (from dim 9 EW ops.)

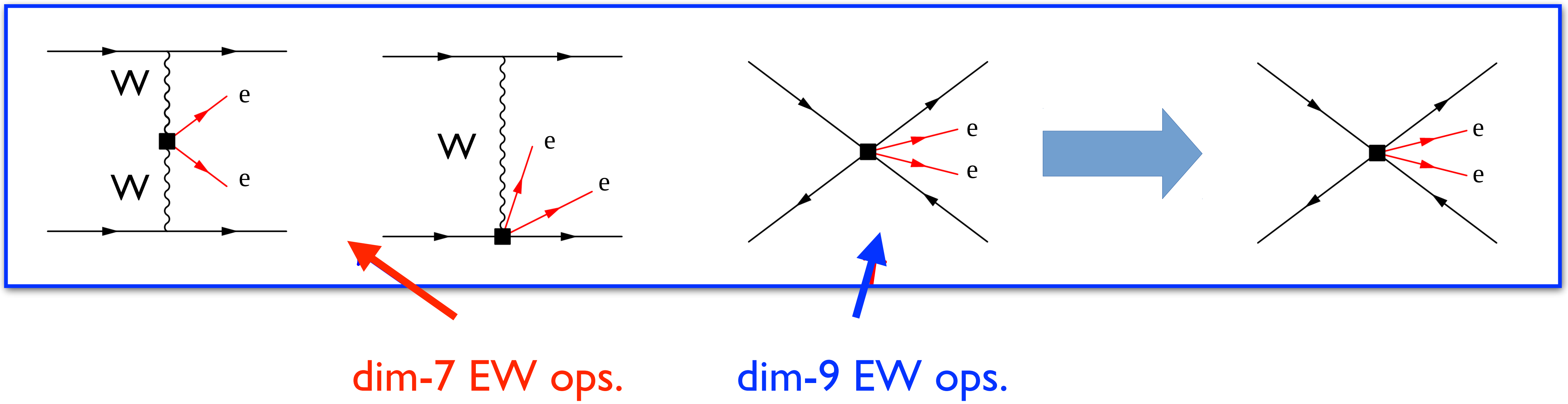
M. Graesser, 1606.04549

Prezeau, Ramsey-Musolf, Vogel hep-ph/0303205

Pas, Hirsch, Klapdor-Kleingrothaus, Kovalenko 1999

GeV-scale effective Lagrangian

- ΔL=2 operators appear at dim = 6, 7 & 9



$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{2G_F^2}{v}\left[\sum_{i=\text{scalar}}\left(C_i^{(9)}\,\bar{e}_L C\,\bar{e}_L^T + C_i^{(9)\prime}\,\bar{e}_R C\,\bar{e}_R^T\right) O_i \;+\; \bar{e}_R\gamma_\mu C\,\bar{e}_L^T \sum_{i=\text{vector}} C_{iV}^{(9)}\, O_i^\mu\right]$$

$$\mathcal{O}_1 = \bar{u}_L\gamma^\mu d_L\,\bar{u}_L\,\gamma_\mu d_L$$

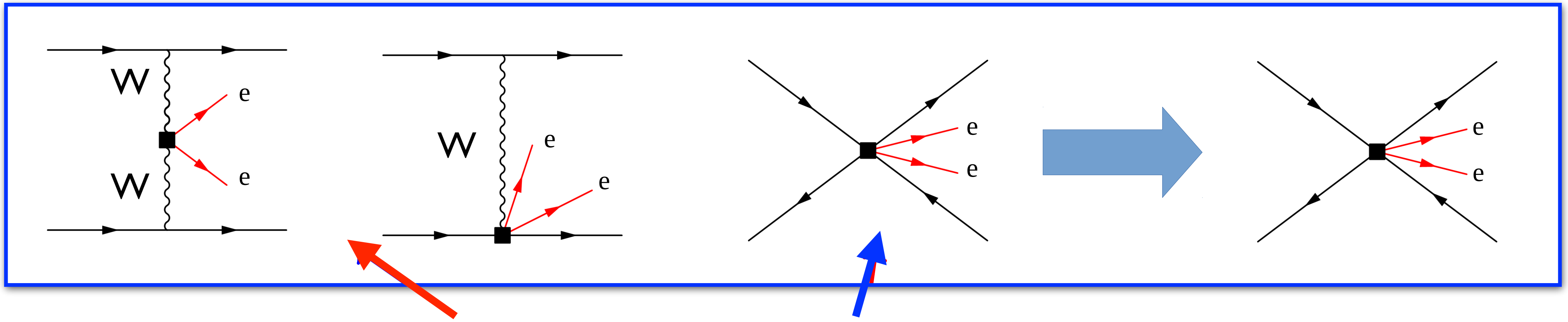
$\tilde{O}_{1,2,3}$ = $O_{1,2,3}$ with L ↔ R

$$\mathcal{O}_2 = \bar{u}_L d_R\,\bar{u}_L\,d_R, \qquad \mathcal{O}_3 = \bar{u}_L^\alpha d_R^\beta\,\bar{u}_L^\beta\,d_R^\alpha$$

$$\mathcal{O}_4 = \bar{u}_L\gamma^\mu d_L\,\bar{u}_R\,\gamma_\mu d_R, \qquad \mathcal{O}_5 = \bar{u}_L^\alpha\gamma^\mu d_L^\beta\,\bar{u}_R^\beta\,\gamma_\mu d_R^\alpha$$

GeV-scale effective Lagrangian

- ΔL=2 operators appear at dim = 6, 7 & 9



dim-7 EW ops.   dim-9 EW ops.

$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{2G_F^2}{v}\left[\sum_{i=\text{scalar}}\left(C_i^{(9)}\,\bar{e}_L C\,\bar{e}_L^T + C_i^{(9)\prime}\,\bar{e}_R C\,\bar{e}_R^T\right) O_i \;+\; \bar{e}_R\gamma_\mu C\,\bar{e}_L^T \sum_{i=\text{vector}} C_{iV}^{(9)}\,O_i^\mu\right]$$

$$C_1^{(9)}, C_{4,5}^{(9)} \sim \mathcal{O}\left(\frac{v^3}{\Lambda^3}\right)$$

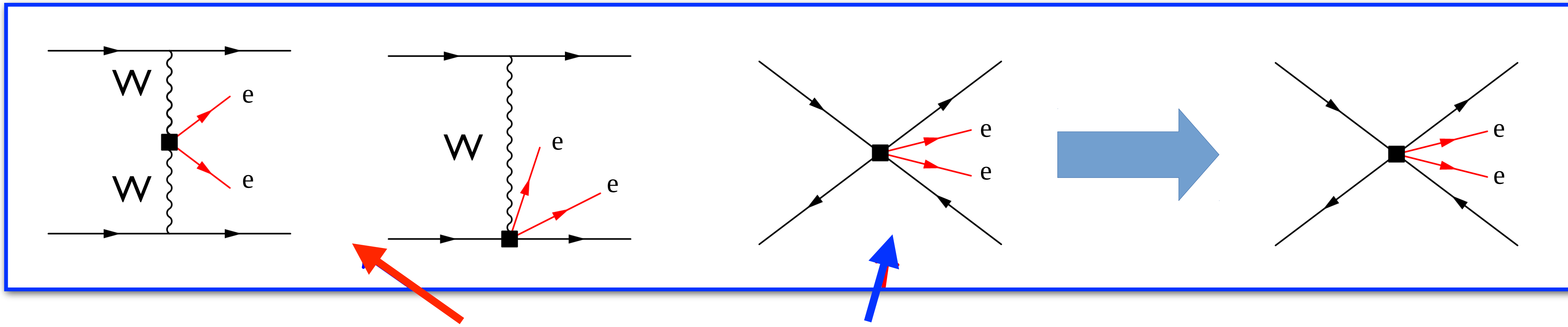
$$\tilde{C}_1^{(9)\prime}, C_i^{(9)} \sim \mathcal{O}\left(\frac{v^5}{\Lambda^5}\right)$$

$i = 2, 3, \tilde{2}, \tilde{3}, 4, 5$

8 leading scalar structures

GeV-scale effective Lagrangian

- ΔL=2 operators appear at dim = 6, 7 & 9



dim-7 EW ops. dim-9 EW ops.

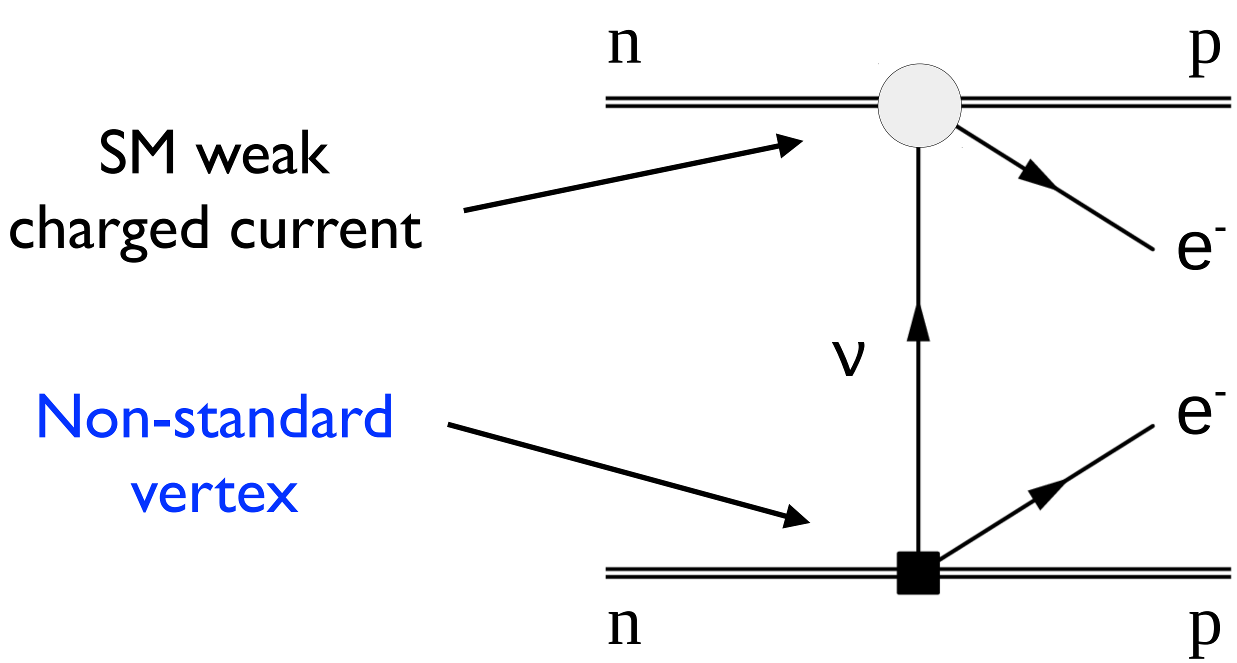
$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{2G_F^2}{v}\left[\sum_{i=\text{scalar}}\left(C_i^{(9)}\,\bar{e}_L C\,\bar{e}_L^T + C_i^{(9)\prime}\,\bar{e}_R C\,\bar{e}_R^T\right) O_i \;+\; \bar{e}_R \gamma_\mu C\,\bar{e}_L^T \sum_{i=\text{vector}} C_{iV}^{(9)}\, O_i^\mu\right]$$

$$O_1^\mu = \bar{u}_R \gamma^\mu d_R\ \bar{u}_L d_R \qquad O_3^\mu = \bar{u}_R \gamma^\mu d_R\ \bar{u}_R d_L$$

$$O_2^\mu = \bar{u}_R^\alpha \gamma^\mu d_R^\beta\ \bar{u}_L^\beta d_R^\alpha \qquad O_4^\mu = \bar{u}_R^\alpha \gamma^\mu d_R^\beta\ \bar{u}_R^\beta d_L^\alpha$$

4 leading vector structures

"$0\nu\beta\beta$ potential" from $\mathcal{L}^{(6,7)}_{\Delta L=2}$



Doi, Kotani, Takasugi 1985

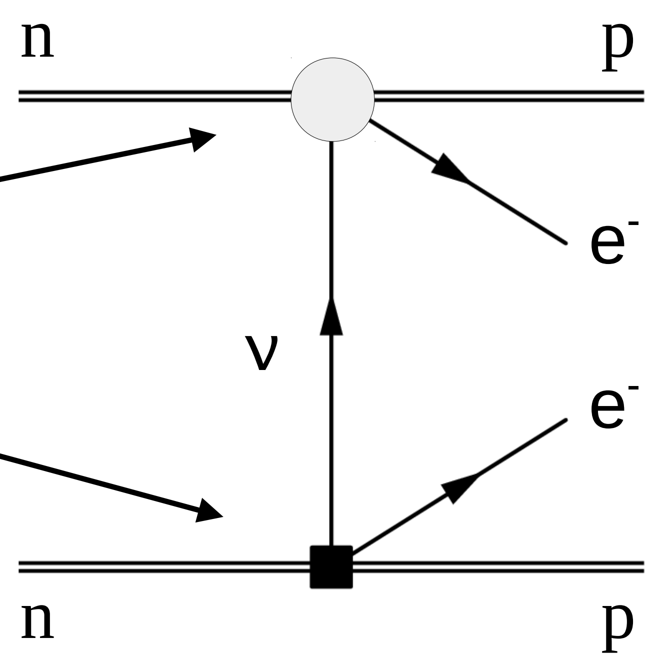
Pas, Hirsch, Klapdor-Kleingrothaus, Kovalenko 1999

- Long range neutrino exchange (but no mass insertion)
- Hadronic input: isovector nucleon V, A, S, P, T form factors

"0νββ potential" from $\mathcal{L}^{(6,7)}_{\Delta L=2}$

SM weak charged current

Non-standard vertex



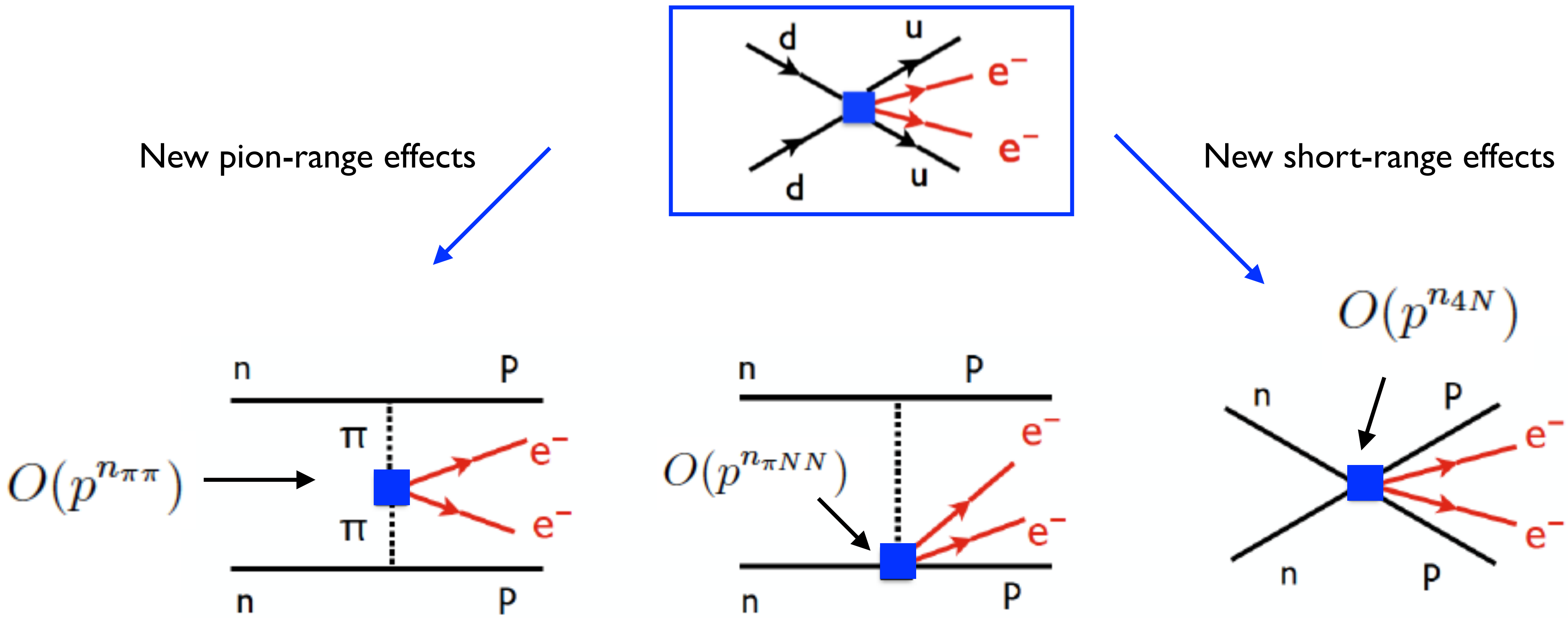
Doi, Kotani, Takasugi 1985

Pas, Hirsch, Klapdor-Kleingrothaus, Kovalenko 1999

- Long range neutrino exchange (but no mass insertion)
- Hadronic input: isovector nucleon V, A, S, P, T form factors
- Missing input from LQCD: $g_T^{(1)}$ (now taken from quark model)

$$\langle p(p_p)|\, \bar{u}\, \sigma_{\mu\nu}\, d\, |\, n(p_n)\rangle = \bar{u}_p(p_p) \Big[g_T(q^2)\, \sigma_{\mu\nu} + g_T^{(1)}(q^2)\, (q_\mu \gamma_\nu - q_\nu \gamma_\mu) + g_T^{(2)}(q^2)\, (q_\mu P_\nu - q_\nu P_\mu) \Big]\, u_n(p_n)$$

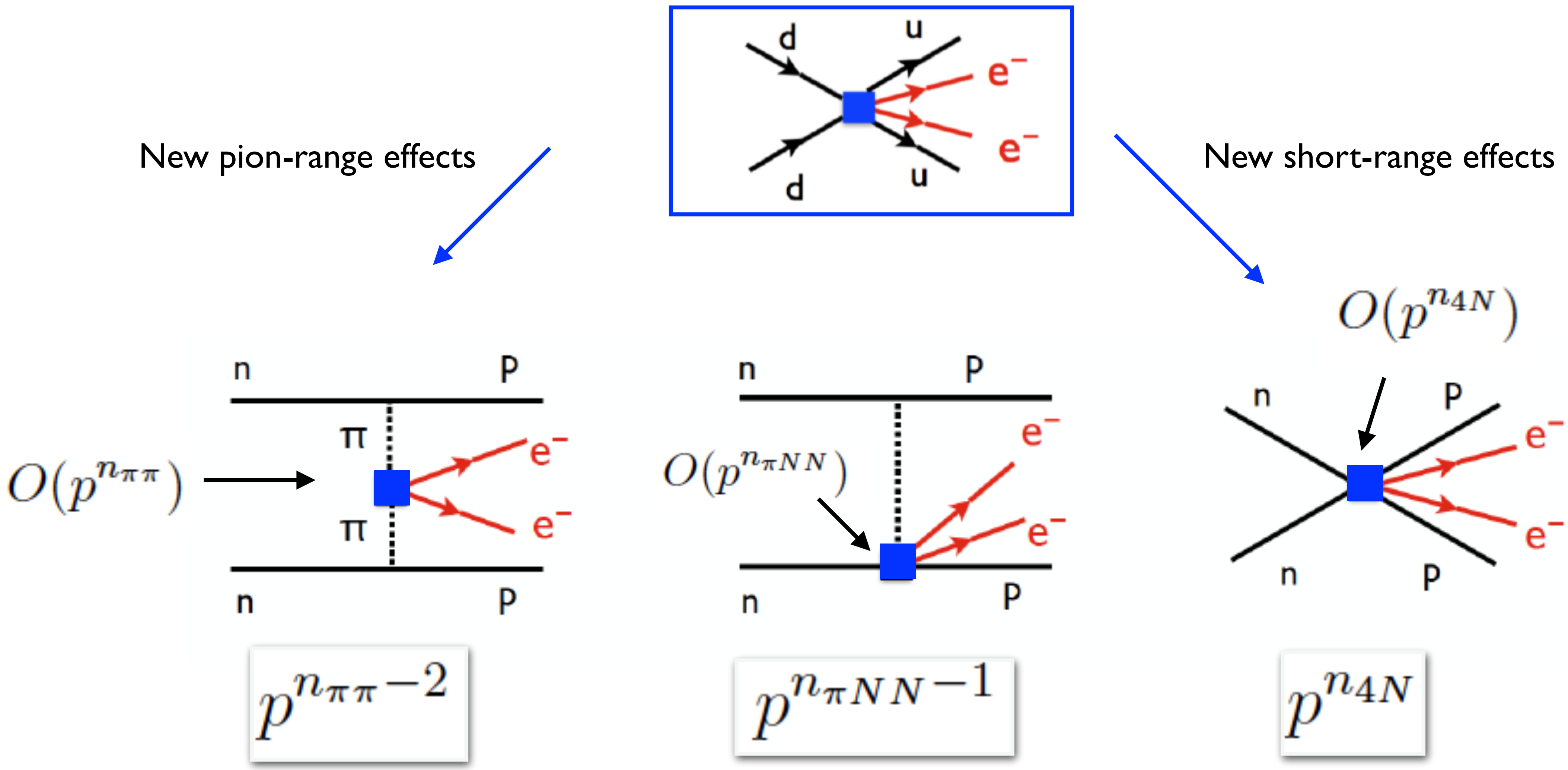
"0νββ potential" from $\mathcal{L}^{(9)}_{\Delta L=2}$



$p^{n_{\pi\pi}-2}$ $\quad$ $p^{n_{\pi NN}-1}$ $\quad$ $p^{n_{4N}}$

$p \sim Q/\Lambda_\chi$

"0νββ potential" from $\mathcal{L}^{(9)}_{\Delta L=2}$



$p \sim Q/\Lambda_\chi$

- Relative importance depends on O_i's chiral properties: in Weinberg's counting, 2-pion exchange dominates only if $n_{\pi\pi}=0$
- Needed input from LQCD: $\langle\pi^+|O_i|\pi^-\rangle$, $\langle p\pi^+|O_i|n\rangle$, $\langle pp|O_i|nn\rangle$

Scalar operators in $\mathcal{L}^{(9)}_{\Delta L=2}$

OPERATOR	$SU(3)_L \times SU(3)_R$ IRREP	LEADING CHIRAL REALIZATION
$\mathcal{O}_1 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_L \, \gamma_\mu d_L$	$(\mathbf{27_L}, \mathbf{1_R})$	$\frac{5}{3} g_{27\times 1} F_0^4 L_{\mu 12} L^\mu_{12}$
$\mathcal{O}_{2,3} = \bar{u}_L d_R \, \bar{u}_L \, d_R$	$(\bar{\mathbf{6}}_\mathbf{L}, \mathbf{6_R})$	$g_{6\times\bar{6}} \frac{F_0^4}{4} \mathrm{Tr}(t^a U \, t^b U)$
$\mathcal{O}_{4,5} = \bar{u}_L \gamma^\mu d_L \, \bar{u}_R \, \gamma_\mu d_R$	$(\mathbf{8_L}, \mathbf{8_R})$	$g_{8\times 8} \frac{F_0^4}{4} \mathrm{Tr}(t^a U \, t^b U^\dagger)$

$U = \exp\left(\frac{\sqrt{2} i \pi}{F_0}\right)$

$L_\mu = i U^\dagger \partial_\mu U$

Scalar operators in $\mathcal{L}^{(9)}_{\Delta L=2}$

OPERATOR	$SU(3)_L \times SU(3)_R$ IRREP	LEADING CHIRAL REALIZATION
$\mathcal{O}_1 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_L \gamma_\mu d_L$	$(\mathbf{27_L}, \mathbf{1_R})$	$\frac{5}{3} g_{27\times 1} F_0^4 L_{\mu 12} L^\mu_{12}$
$\mathcal{O}_{2,3} = \bar{u}_L d_R \, \bar{u}_L d_R$	$(\bar{\mathbf{6}}_\mathbf{L}, \mathbf{6_R})$	$g_{6\times\bar{6}} \frac{F_0^4}{4} \mathrm{Tr}(t^a U t^b U)$
$\mathcal{O}_{4,5} = \bar{u}_L \gamma^\mu d_L \, \bar{u}_R \gamma_\mu d_R$	$(\mathbf{8_L}, \mathbf{8_R})$	$g_{8\times 8} \frac{F_0^4}{4} \mathrm{Tr}(t^a U t^b U^\dagger)$

$U = \exp\left(\frac{\sqrt{2} i \pi}{F_0}\right)$

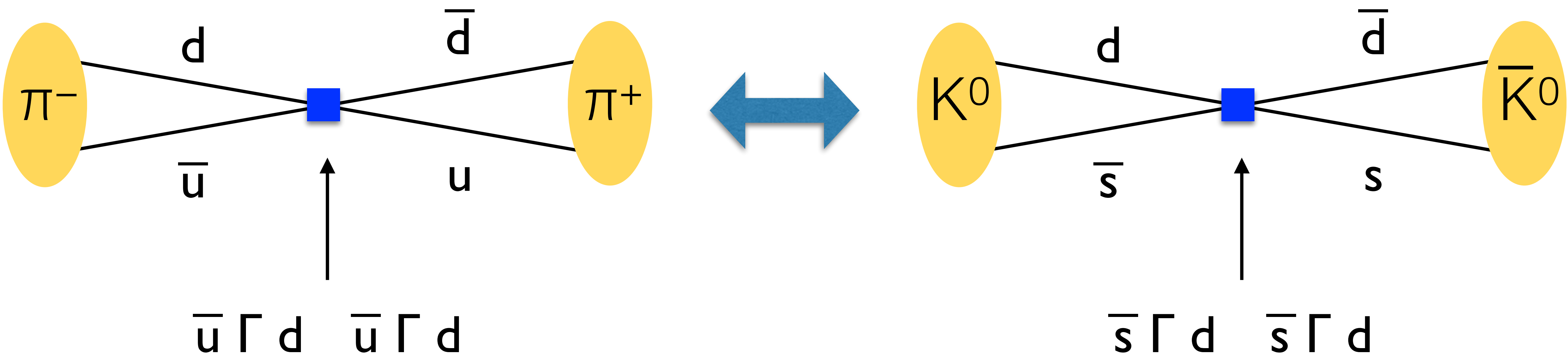
$L_\mu = i U^\dagger \partial_\mu U$

- Note: to determine $V_{\Delta I=2}[O_1]$ at LO, need all three $\langle\pi^+|O_1|\pi^-\rangle$, $\langle p\pi^+|O_1|n\rangle$, $\langle pp|O_1|nn\rangle$
- For all $\langle\pi^+|O_i|\pi^-\rangle$
  - Direct LQCD calculation (CalLat) Nicholson et al., 1608.04793
  - Indirect LQCD calculation: K-$\bar{K}$ + chiral SU(3)

VC-Dekens-Graesser-Mereghetti 1701.01443

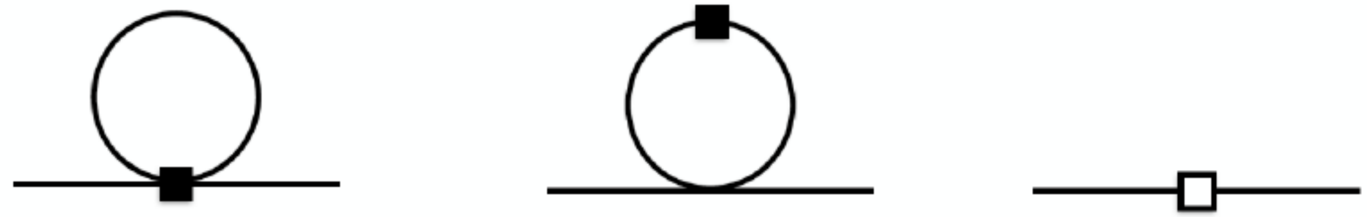
$\langle \pi^+|O_i|\pi^- \rangle$ from Kaon physics

- Chiral SU(3) relates $<\pi^+|O_i|\pi^->$ to $<K^0|O_i^{(X)}|K^0>$ (equal at LO!)



$\langle \pi^+ | O_i | \pi^- \rangle$ from Kaon physics

- Chiral SU(3) relates $<\pi^+|O_i|\pi^->$ to $<K^0|O_i^{(X)}|K^0>$
- Chiral corrections



$$\mathcal{M}^{\pi\pi}_{8\times 8} = \mathcal{M}^{K\bar{K}}_{8\times 8} \times \frac{F_\pi^2}{F_K^2} \times (1 + \Delta_{8\times 8})$$

$$\mathcal{M}^{\pi\pi}_{6\times \bar{6}} = \mathcal{M}^{K\bar{K}}_{6\times \bar{6}} \times \frac{F_\pi^2}{F_K^2} \times (1 + \Delta_{6\times \bar{6}})$$

$$\Delta_{8\times 8} = \frac{1}{(4\pi F_0)^2}\left[\frac{m_\pi^2}{4}(-4 + 5L_\pi) - m_K^2(-1 + 2L_K) + \frac{3}{4}m_\eta^2 L_\eta - a_{8\times 8}\,(m_K^2 - m_\pi^2)\right]$$

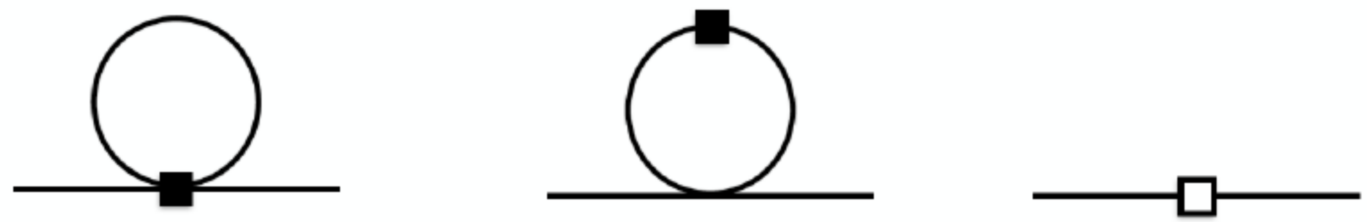
$$\Delta_{6\times \bar{6}} = \frac{1}{(4\pi F_0)^2}\left[-\frac{m_\pi^2}{4}(4 - 3L_\pi) - m_K^2(-1 + 2L_K) + \frac{5}{4}m_\eta^2 L_\eta - a_{6\times \bar{6}}\,(m_K^2 - m_\pi^2)\right]$$

$$L_{\pi,K,\eta} \equiv \log \mu_\chi^2 / m_{\pi,K,\eta}^2$$

LEC can be determined in principle by studying $m_{u,d}$ and m_s dependence of K-$\bar{K}$ matrix element

$\langle \pi^+ | O_i | \pi^- \rangle$ from Kaon physics

- Chiral SU(3) relates $<\pi^+|O_i|\pi^->$ to $<K^0|O_i^{(\chi)}|K^0>$
- Chiral corrections



$$\mathcal{M}^{\pi\pi}_{8\times 8} = \mathcal{M}^{K\bar{K}}_{8\times 8} \times \frac{F_\pi^2}{F_K^2} \times (1+\Delta_{8\times 8})$$
$$\mathcal{M}^{\pi\pi}_{6\times \bar{6}} = \mathcal{M}^{K\bar{K}}_{6\times \bar{6}} \times \frac{F_\pi^2}{F_K^2} \times (1+\Delta_{6\times \bar{6}})$$

$$\Delta_{8\times 8} = 0.02(30)$$
$$\Delta_{6\times \bar{6}} = 0.07(20)$$

$$\Delta_{8\times 8} = \frac{1}{(4\pi F_0)^2}\left[\frac{m_\pi^2}{4}(-4+5L_\pi) - m_K^2(-1+2L_K) + \frac{3}{4}m_\eta^2 L_\eta - a_{8\times 8}\,(m_K^2 - m_\pi^2)\right]$$
$$\Delta_{6\times \bar{6}} = \frac{1}{(4\pi F_0)^2}\left[-\frac{m_\pi^2}{4}(4-3L_\pi) - m_K^2(-1+2L_K) + \frac{5}{4}m_\eta^2 L_\eta - a_{6\times \bar{6}}\,(m_K^2 - m_\pi^2)\right]$$

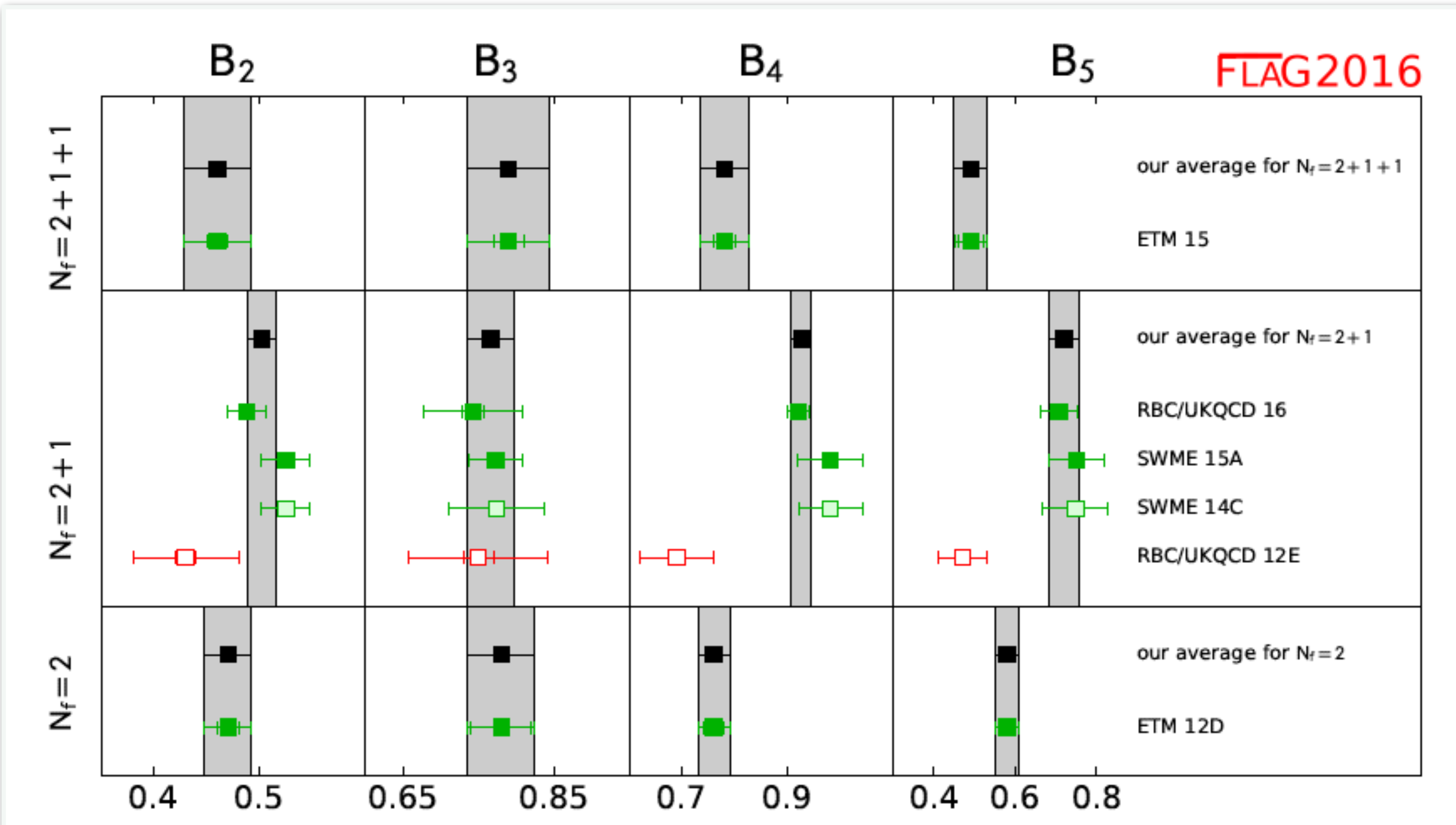
$$L_{\pi,K,\eta} \equiv \log \mu_\chi^2 / m^2_{\pi,K,\eta}$$

In practice set these to zero at $\mu_\chi = m_\rho$
and take as error the maximum between NDA and
$$\Delta_n^{(ct)} = \pm|d\Delta_n^{(loops)}/d(\log\mu_\chi)|$$

Results for $\langle \pi^+ | O_{2,3,4,5} | \pi^- \rangle$

- Input: K-$\bar{K}$ matrix elements ($M^{KK} \propto B_i$) at μ = 3 GeV in $\overline{MS}$ scheme
- Use conservative range from FLAG 2016 review

Aoki et al., 1607.00299



$6_L \times \bar{6}_R$ $\quad$ $8_L \times 8_R$

Results for $\langle \pi^+ | O_{2,3,4,5} | \pi^- \rangle$

- Input: K-$\bar{K}$ matrix elements ($M^{KK} \propto B_i$) at μ = 3 GeV in $\overline{MS}$ scheme
- Use conservative range from FLAG 2016 review

Aoki et al., 1607.00299

$$\langle \pi^+ | O_2 | \pi^- \rangle = -(2.7 \pm 0.3 \pm 0.5) \times 10^{-2}\ \mathrm{GeV}^4$$
$$\langle \pi^+ | O_3 | \pi^- \rangle = (0.9 \pm 0.1 \pm 0.2) \times 10^{-2}\ \mathrm{GeV}^4$$
$$\langle \pi^+ | O_4 | \pi^- \rangle = -(2.6 \pm 0.8 \pm 0.8) \times 10^{-2}\ \mathrm{GeV}^4$$
$$\langle \pi^+ | O_5 | \pi^- \rangle = -(11 \pm 2 \pm 3) \times 10^{-2}\ \mathrm{GeV}^4$$

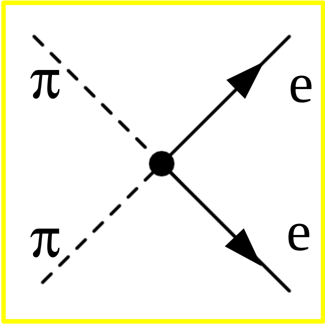
First error: lattice QCD input

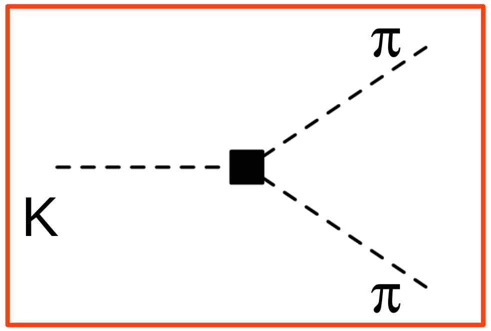
Second error: chiral corrections

- First controlled input for phenomenology
- Benchmark for CalLat calculation

$\langle\pi^+|O_1|\pi^-\rangle$ from $K\to\pi\pi$

$$\mathcal{O}_1 = \bar{u}_L\gamma^\mu d_L\, \bar{u}_L\, \gamma_\mu d_L \qquad (\mathbf{27_L}, \mathbf{1_R})$$





M Savage
nucl/th-9811087

- Chiral corrections to K-K mixing are large, use $K \to \pi\pi$
- RBC-UKQCD input on $\langle\pi^+\pi^0|Q_2|K^+\rangle$ + chiral corrections

Blum et al, 1502.00263 VC-Ecker-Neufeld-Pich hep-ph/0310351

$$\langle\pi^+|O_1|\pi^-\rangle \quad = \quad (1.0 \pm 0.1 \pm 0.2) \times 10^{-4}\ \mathrm{GeV}^4$$

Lattice QCD input Chiral corrections

15% larger than M. Savage 1998, who used experimental input on $g_{27\times1}$

Vector operators in $\mathcal{L}^{(9)}_{\Delta L=2}$

$$O_1^\mu = \bar{u}_R \gamma^\mu d_R \, \bar{u}_L d_R \qquad O_3^\mu = \bar{u}_R \gamma^\mu d_R \, \bar{u}_R d_L$$

$$O_2^\mu = \bar{u}_R^\alpha \gamma^\mu d_R^\beta \, \bar{u}_L^\beta d_R^\alpha \qquad O_4^\mu = \bar{u}_R^\alpha \gamma^\mu d_R^\beta \, \bar{u}_R^\beta d_L^\alpha$$

- For $L_\mu O_i^\mu$, $\pi\pi$ matrix elements proportional to m_e

Prezeau, Ramsey-Musolf, Vogel hep-ph/0303205

- $<p\pi^+|O_i^\mu|n>$, $<pp|O_i^\mu|nn>$ do not suffer from this suppression: need these from LQCD

Machinery at work

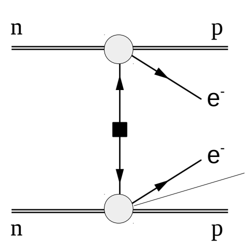
- For generic LNV physics one has

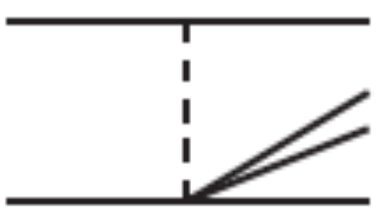
$$V_{\Delta I=2} \sim \mathcal{A}_{\rm lept}\,[m_{\beta\beta}\,V_\nu \;+\; C_\pi g_\pi\,V_\pi \;+\; C_{\pi\pi}g_{\pi\pi}\,V_{\pi\pi} \;+\; C_{CT}g_{\rm CT}\,V_{\rm CT}]$$

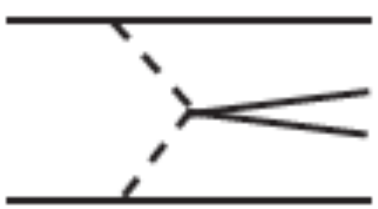
Wilson coefficient (model dependent)

Chiral LEC, determined by hadronic matrix element (e.g. ππ)

- Define rescaled potentials (that all behave as Q^{-3})





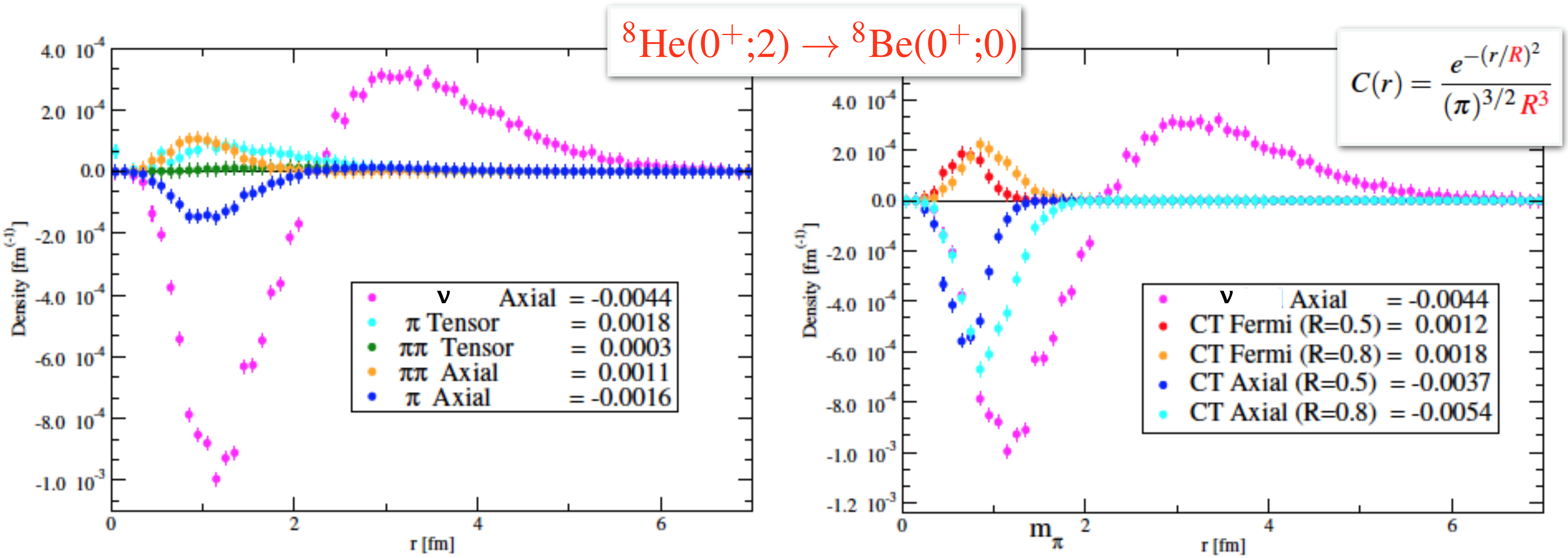


$$V_\nu = \frac{1}{m_\pi\,\mathbf{q}^2} \times O(\sigma,\tau) \qquad V_\pi = \frac{\mathbf{q}^2}{m_\pi^3\,(\mathbf{q}^2+m_\pi^2)} \times O(\sigma,\tau) \qquad V_{\pi\pi} = \frac{\mathbf{q}^2}{m_\pi\,(\mathbf{q}^2+m_\pi^2)^2} \times O(\sigma,\tau) \qquad V_{\rm CT} = \frac{1}{m_\pi^3} \times O(\sigma,\tau)$$

- Test power counting in light nuclei: relative size of $\langle V_n \rangle$

Matrix elements in light nuclei

Pastore, Mereghetti, Wiringa et al: PRELIMINARY



$$\text{Axial} \propto \tau_1^+ \tau_2^+ \sigma_1 \cdot \sigma_2$$
$$\text{Tensor} \propto \tau_1^+ \tau_2^+ S_{12}$$

- Power counting seems to work reasonably well in light nuclei
- Benchmark for other many-body methods

Courtesy of Saori Pastore

Summary: desirable LQCD input

- LNV from dim 5: matrix elements of non-local effective action

$$S_{\rm NL} = \int dx\, dy\; S_\nu(x-y)\; T\left(J_\alpha^+(x)\, J_\beta^+(y)\right)\; g^{\alpha\beta} \qquad J_\alpha^+ = \bar{u}_L \gamma_\alpha d_L$$

$\langle \pi^+|S_{NL}|\pi^-\rangle$ $\quad$ $\langle p\pi^+|S_{NL}|n\rangle^*$ $\quad$ $\langle pp|S_{NL}|nn\rangle^{*,**}$

- LNV from dim 7,9
  - "Long distance" neutrino exchange: tensor form factors $g_T^{(1)}$
  - "Short distance":
    - $\langle \pi^+|O_i|\pi^-\rangle$, $\langle p\pi^+|O_i|n\rangle^*$, $\langle pp|O_i|nn\rangle^{*,**}$ for five 4-quark scalars
    - $\langle p\pi^+|O_i^\mu|n\rangle^*$, $\langle pp|O_i^\mu|nn\rangle^{*,**}$ for four 4-quark vectors

* Need to be matched to non-perturbative EFT calculation

** Calculation at different m_q could give a handle on all needed LECs

Backup

Getting LECs from Kaon mixing

- Counterterms for matrix the two elements

$$\delta^{K\bar{K}}_{8\times 8} = a_{8\times 8}\, m_K^2 + b_{8\times 8}\left(m_K^2 + \frac{1}{2}m_\pi^2\right)$$

$$\delta^{\pi\pi}_{8\times 8} = a_{8\times 8}\, m_\pi^2 + b_{8\times 8}\left(m_K^2 + \frac{1}{2}m_\pi^2\right)$$

- "a" controls the ratio ("b" drops)
- To extract "a" need the m_s and m_q dependence of the matrix element

Dim-7 operators in the SM-EFT

Lehman 1410.4193

$1 : \psi^2 H^4 + \text{h.c.}$	
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n (H^\dagger H)$

$2 : \psi^2 H^2 D^2 + \text{h.c.}$	
$\mathcal{O}^{(1)}_{LHD}$	$\epsilon_{ij}\epsilon_{mn} L^i C (D^\mu L^j) H^m (D_\mu H^n)$
$\mathcal{O}^{(2)}_{LHD}$	$\epsilon_{im}\epsilon_{jn} L^i C (D^\mu L^j) H^m (D_\mu H^n)$

$3 : \psi^2 H^3 D + \text{h.c.}$	
$\mathcal{O}_{LHDe}$	$\epsilon_{ij}\epsilon_{mn}\left(L^i C \gamma_\mu e\right) H^j H^m D^\mu H^n$

$4 : \psi^2 H^2 X + \text{h.c.}$	
$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}\left(L^i C \sigma_{\mu\nu} L^m\right) H^j H^n B^{\mu\nu}$
$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\tau^I \epsilon)_{mn}\left(L^i C \sigma_{\mu\nu} L^m\right) H^j H^n W^{I\mu\nu}$

$5 : \psi^4 D + \text{h.c.}$	
$\mathcal{O}^{(1)}_{LL\bar{d}uD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C D^\mu L^j)$
$\mathcal{O}^{(2)}_{LL\bar{d}uD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C \sigma^{\mu\nu} D_\nu L^j)$
$\mathcal{O}^{(1)}_{\bar{L}QddD}$	$(Q C \gamma_\mu d)(\bar{L} D^\mu d)$
$\mathcal{O}^{(2)}_{\bar{L}QddD}$	$(\bar{L}\gamma_\mu Q)(d C D^\mu d)$
$\mathcal{O}_{ddd\bar{e}D}$	$(\bar{e}\gamma_\mu d)(d C D^\mu d)$

$6 : \psi^4 H + \text{h.c.}$	
$\mathcal{O}_{LLL\bar{e}H}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m)H^n$
$\mathcal{O}^{(1)}_{LLQ\bar{d}H}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m)H^n$
$\mathcal{O}^{(2)}_{LLQ\bar{d}H}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m)H^n$
$\mathcal{O}_{LL\bar{Q}uH}$	$\epsilon_{ij}(\bar{Q}_m u)(L^m C L^i)H^j$
$\mathcal{O}_{\bar{L}QQdH}$	$\epsilon_{ij}(\bar{L}_m d)(Q^m C Q^i)\widetilde{H}^j$
$\mathcal{O}_{\bar{L}dddH}$	$(dCd)(\bar{L}d)H$
$\mathcal{O}_{\bar{L}uddH}$	$(\bar{L}d)(uCd)\widetilde{H}$
$\mathcal{O}_{Leu\bar{d}H}$	$\epsilon_{ij}(L^i C \gamma_\mu e)(\bar{d}\gamma^\mu u)H^j$
$\mathcal{O}_{\bar{e}QddH}$	$\epsilon_{ij}(\bar{e}Q^i)(dCd)\widetilde{H}^j$

Dim-9 operators in the SM-EFT

Graesser 1601.04549

$$\begin{aligned}
\mathrm{LM1} &= i\sigma^{(2)}_{ab}(\overline{Q}_a\gamma^\mu Q_c)(\overline{u}_R\gamma_\mu d_R)(\overline{\ell}_b\ell^C_c) \\
\mathrm{LM2} &= i\sigma^{(2)}_{ab}(\overline{Q}_a\gamma^\mu\lambda^A Q_c)(\overline{u}_R\gamma_\mu\lambda^A d_R)(\overline{\ell}_b\ell^C_c) \\
\mathrm{LM3} &= (\overline{u}_R Q_a)(\overline{u}_R Q_b)(\overline{\ell}_a\ell^C_b) \\
\mathrm{LM4} &= (\overline{u}_R\lambda^A Q_a)(\overline{u}_R\lambda^A Q_b)(\overline{\ell}_a\ell^C_b) \\
\mathrm{LM5} &= i\sigma^{(2)}_{ab}i\sigma^{(2)}_{cd}(\overline{Q}_a d_R)(\overline{Q}_c d_R)(\overline{\ell}_b\ell^C_d) \\
\mathrm{LM6} &= i\sigma^{(2)}_{ab}i\sigma^{(2)}_{cd}(\overline{Q}_a\lambda^A d_R)(\overline{Q}_c\lambda^A d_R)(\overline{\ell}_b\ell^C_d) \\
\mathrm{LM7} &= (\overline{u}_R\gamma^\mu d_R)(\overline{u}_R\gamma_\mu d_R)(\overline{e}_R e^C_R) \\
\mathrm{LM8} &= (\overline{u}_R\gamma^\mu d_R)i\sigma^{(2)}_{ab}(\overline{Q}_a d_R)(\overline{\ell}_b\gamma_\mu e^C_R) \\
\mathrm{LM9} &= (\overline{u}_R\gamma^\mu\lambda^A d_R)i\sigma^{(2)}_{ab}(\overline{Q}_a\lambda^A d_R)(\overline{\ell}_b\gamma_\mu e^C_R) \\
\mathrm{LM10} &= (\overline{u}_R\gamma^\mu d_R)(\overline{u}_R Q_a)(\overline{\ell}_a\gamma_\mu e^C_R) \\
\mathrm{LM11} &= (\overline{u}_R\gamma^\mu\lambda^A d_R)(\overline{u}_R\lambda^A Q_a)(\overline{\ell}_a\gamma_\mu e^C_R)
\end{aligned}$$