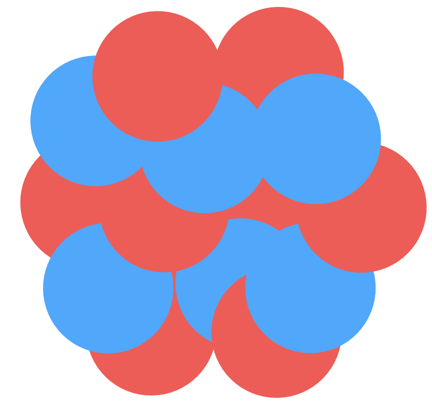
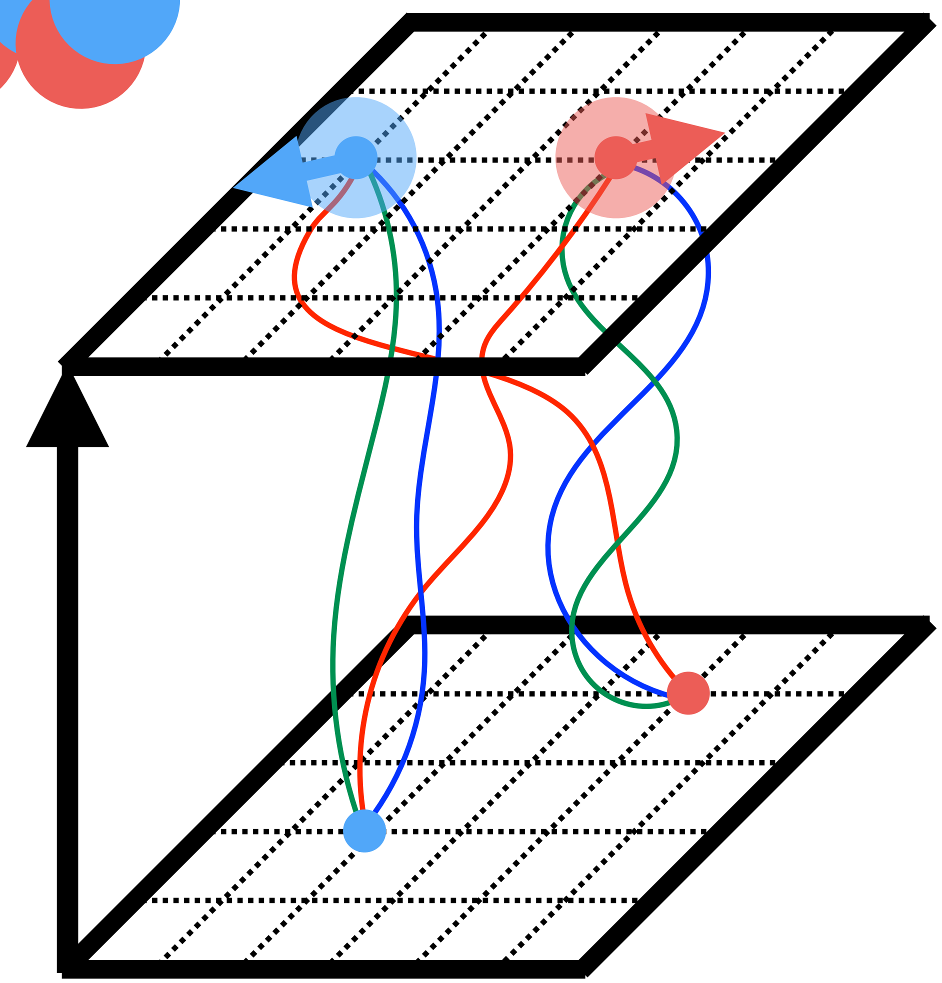
Developments in 2-baryon LQCD calculations





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Forschungszentrum Jülich

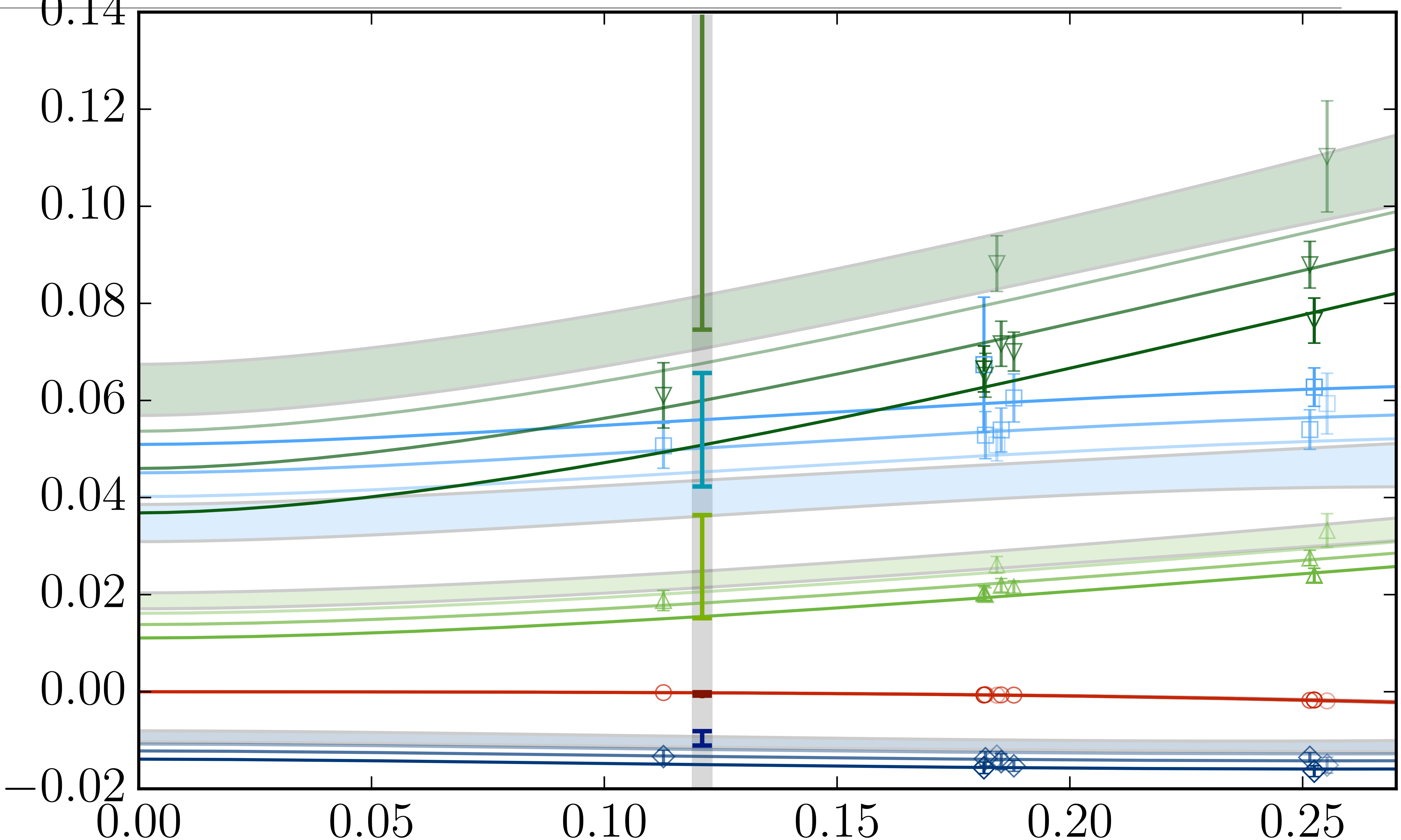
08 July 2017

0νββ / INT-17-67w

Seattle, Washington

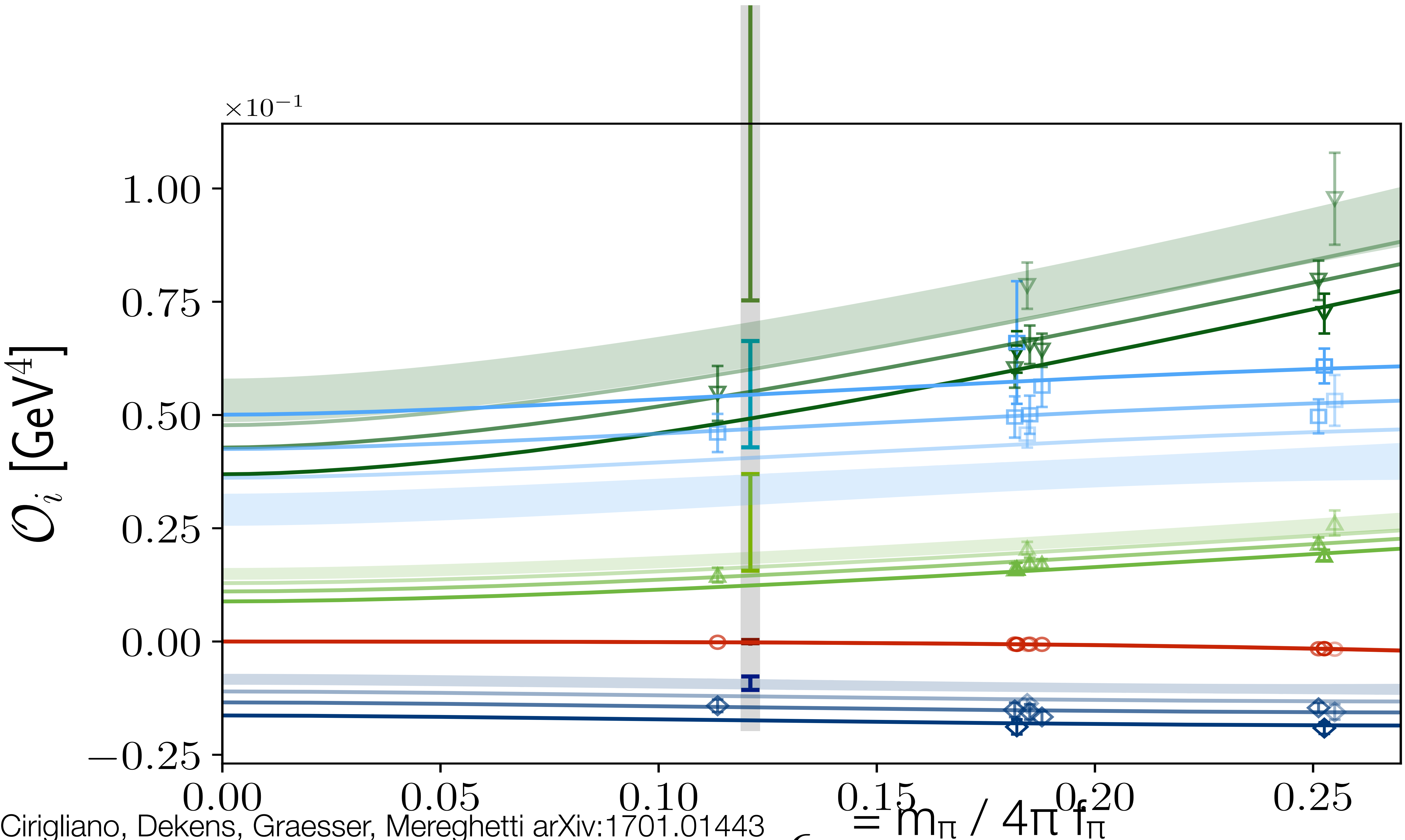
BUT FIRST

Preliminary Short-distance 0vββ w/o Renormalization

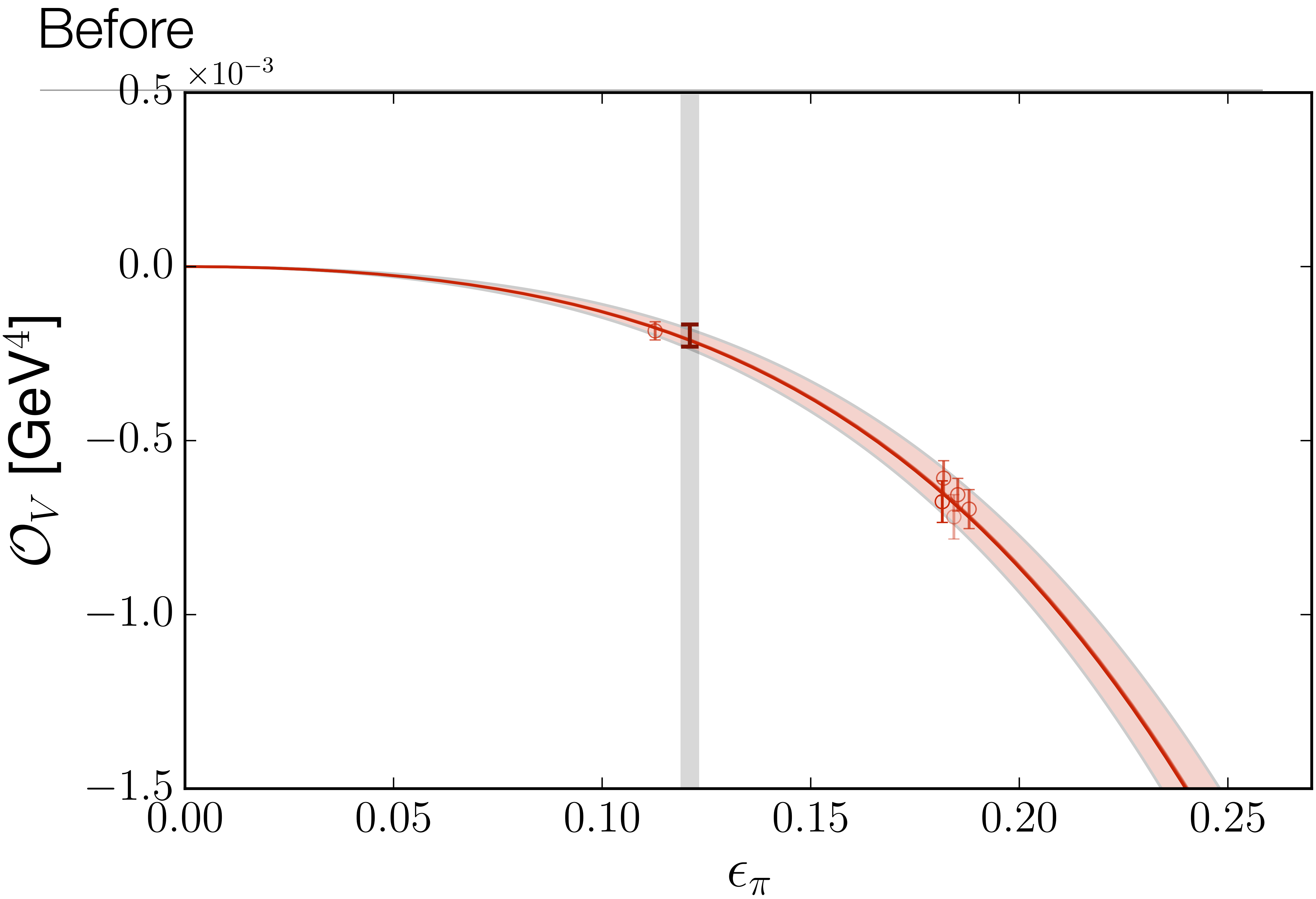


Cirigliano, Dekens, Graesser, Mereghetti arXiv:1701.01443 $\epsilon_\pi = m_\pi / 4\pi f_\pi$

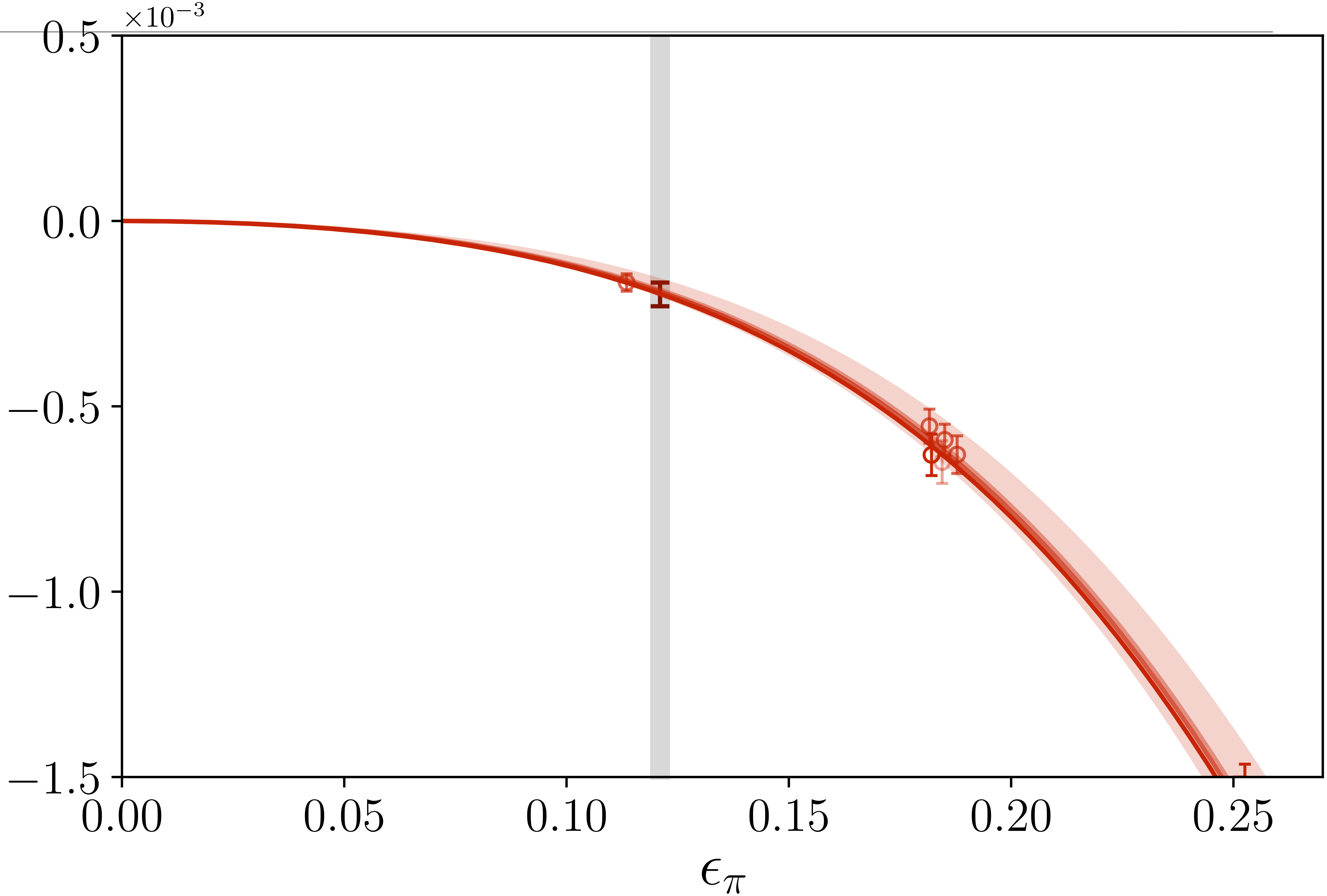
Preliminary Short-distance 0νββ w Renormalization



Cirigliano, Dekens, Graesser, Mereghetti arXiv:1701.01443


Before
×10^−3
0.5
0.0
−0.5
−1.0
−1.5
O_V [GeV^4]
0.00
0.05
0.10
0.15
0.20
0.25
ε_π

After

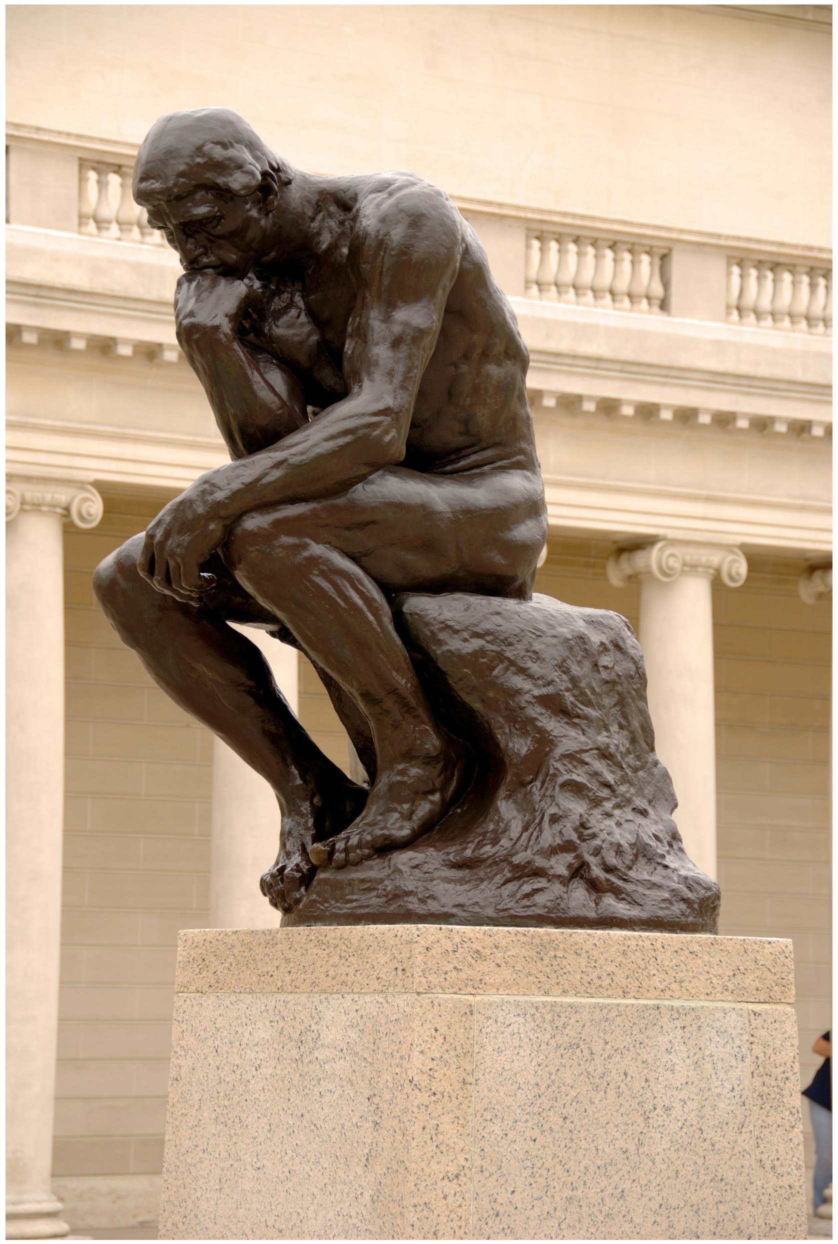


$\times 10^{-3}$
0.5
0.0
−0.5
−1.0
−1.5
$\mathcal{O}_V$ [GeV^4]
0.00
0.05
0.10
0.15
0.20
0.25
ϵ_π

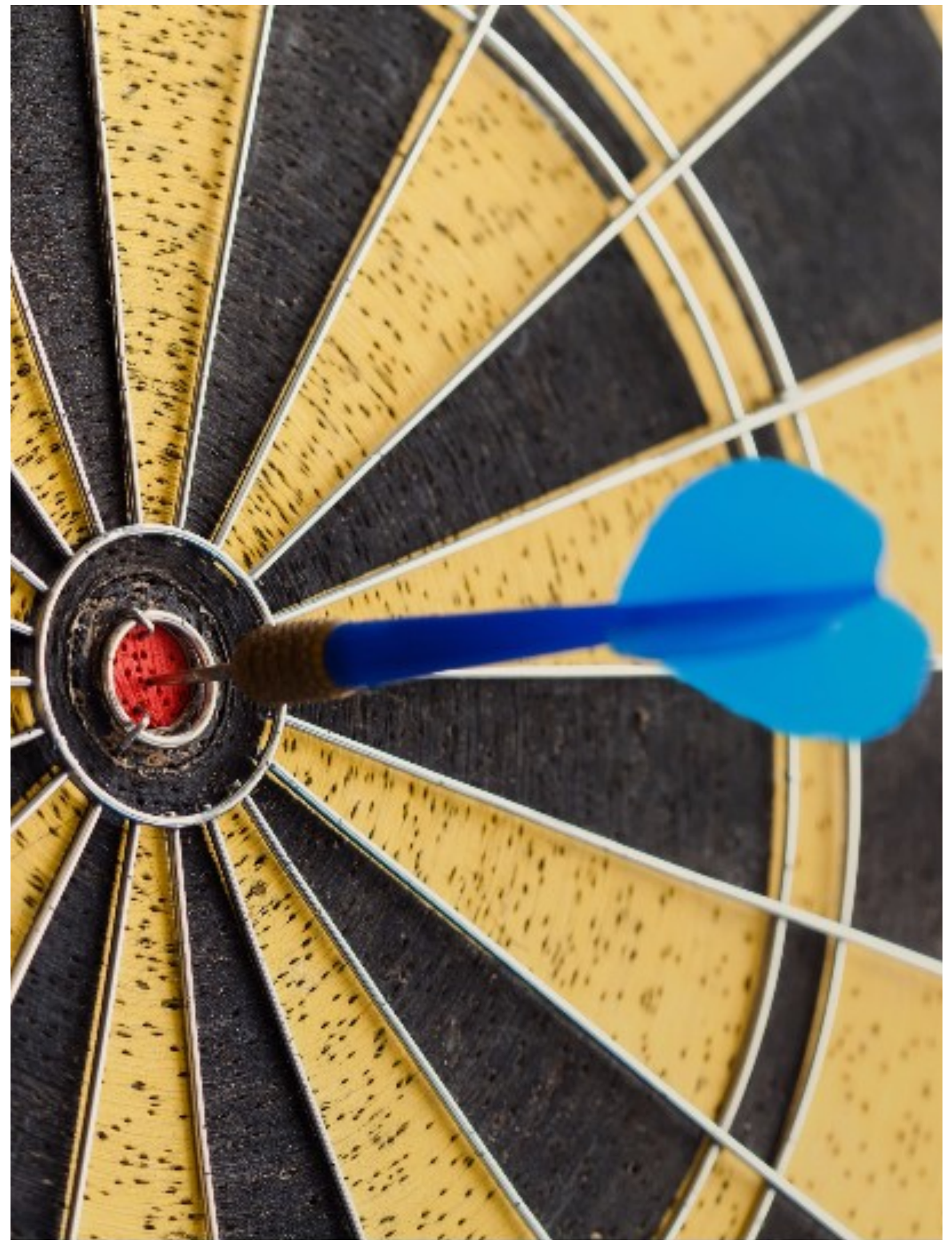
Outline

- Cubic Irreps / Considerations for cubic volume
    - Sinks and Sources: computational reuse
- Single nucleon improvement in NN calculations

New Methods



Generally Applicable



Improved Systematics

Computationally Affordable

Lüscher Formalism

$$\det\left[(\mathcal{M}^{\infty})^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

Infinite volume scattering amplitudes

finite volume spectrum + boundary conditions

Lattice calculation

Subduction

HadSpec 1004.4930

Isospin 0		Isospin 1	
Partial wave	Irreps	Partial wave	Irreps
${}^1P_1$	T_1^-	${}^1S_0$	A_1^+
${}^3S_1, {}^3D_1$	T_1^+	${}^3P_0$	A_1^-
${}^3D_2$	$E^+ \oplus T_2^+$	${}^3P_1$	T_1^-
${}^3D_3$	$A_2^+ \oplus T_1^+ \oplus T_2^+$	${}^3P_2, {}^3F_2$	$E^- \oplus T_2^-$
${}^1F_3$	$A_2^- \oplus T_1^- \oplus T_2^-$	${}^1D_2$	$E^+ \oplus T_2^+$
		${}^3F_3$	$A_2^- \oplus T_1^- \oplus T_2^-$
		${}^3F_4$	$A_1^- \oplus E^- \oplus T_1^- \oplus T_2^-$

Some states only couple to particular sources.

unphysical mixing

Two-Nucleon Spectrum

- Spectrum given by effective mass of (schematic) NN correlator:

$$\left\langle \Omega \middle| \mathcal{O}^{[J'\ell'S']}_{\Lambda'\mu',Im_I}(t)\,\mathcal{O}^{[J\ \ell\ S\ ]}_{\Lambda\mu,Im_I}(0) \middle| \Omega \right\rangle \rightarrow \delta\mathcal{G}^V$$

- Sink

$$\mathcal{O}_{Jm_JIm_I;S\ell}(t,|\boldsymbol{k}|) = \sum \text{Clebsh-Gordans} \sum_{R\in\mathcal{O}_h} Y_{\ell m_\ell}\left(\widehat{R\boldsymbol{k}}\right) N^{m_{I_1}}_{m_{s_1}}(t,R\boldsymbol{k})\, N^{m_{I_2}}_{m_{s_2}}(t,-R\boldsymbol{k})$$

- Source

$$\mathcal{O}_{Jm_JIm_I;S\ell}(t,\boldsymbol{x},\boldsymbol{\Delta x}) = \sum \text{Clebsch-Gordans} \sum_{R\in\mathcal{O}_h} Y_{\ell m_\ell}\left(\widehat{R\boldsymbol{\Delta x}}\right) N^{m_{I_1}}_{m_{s_1}}(t,\boldsymbol{x})\, N^{m_{I_2}}_{m_{s_2}}(t,x+R\boldsymbol{\Delta x})$$

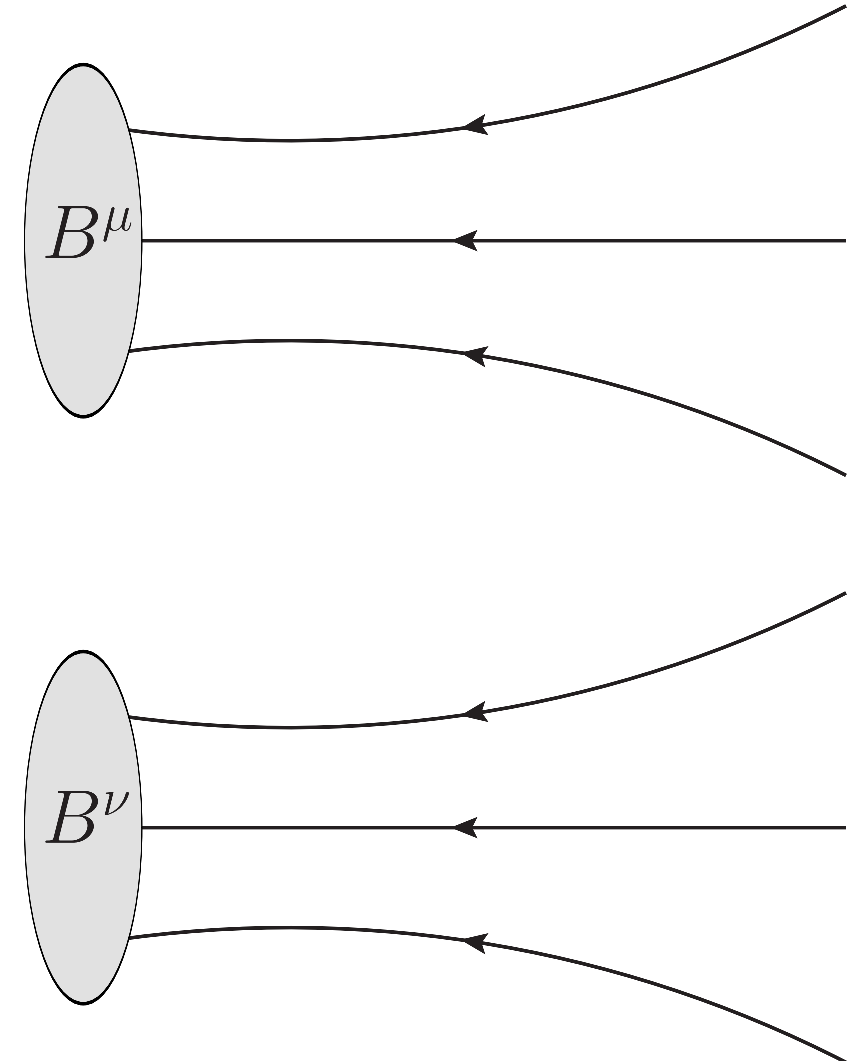
- Box breaks rotational symmetry → spectrum falls into irreps of O_h, not SO(3).
  - Subduction

$$\mathcal{O}^{[J\ell S]}_{\Lambda\mu,Im_I}(t,|\boldsymbol{k}|) = \sum_{m_J} [\mathrm{CG}^J_\Lambda]_{\mu,m_J}\, \mathcal{O}_{Jm_JIm_I;S\ell}(t,|\boldsymbol{k}|)$$

HPC

Doi & Endres 1205.0585, Detmold & Orginos 1207.1452

- Use baryon blocks
- Use sparse tensor contraction to take advantage of sparsity of L
- For each source displacement RΔx, store (sink-side) full-volume correlator for each $S'm'_S\, Sm_S\, Im_I$



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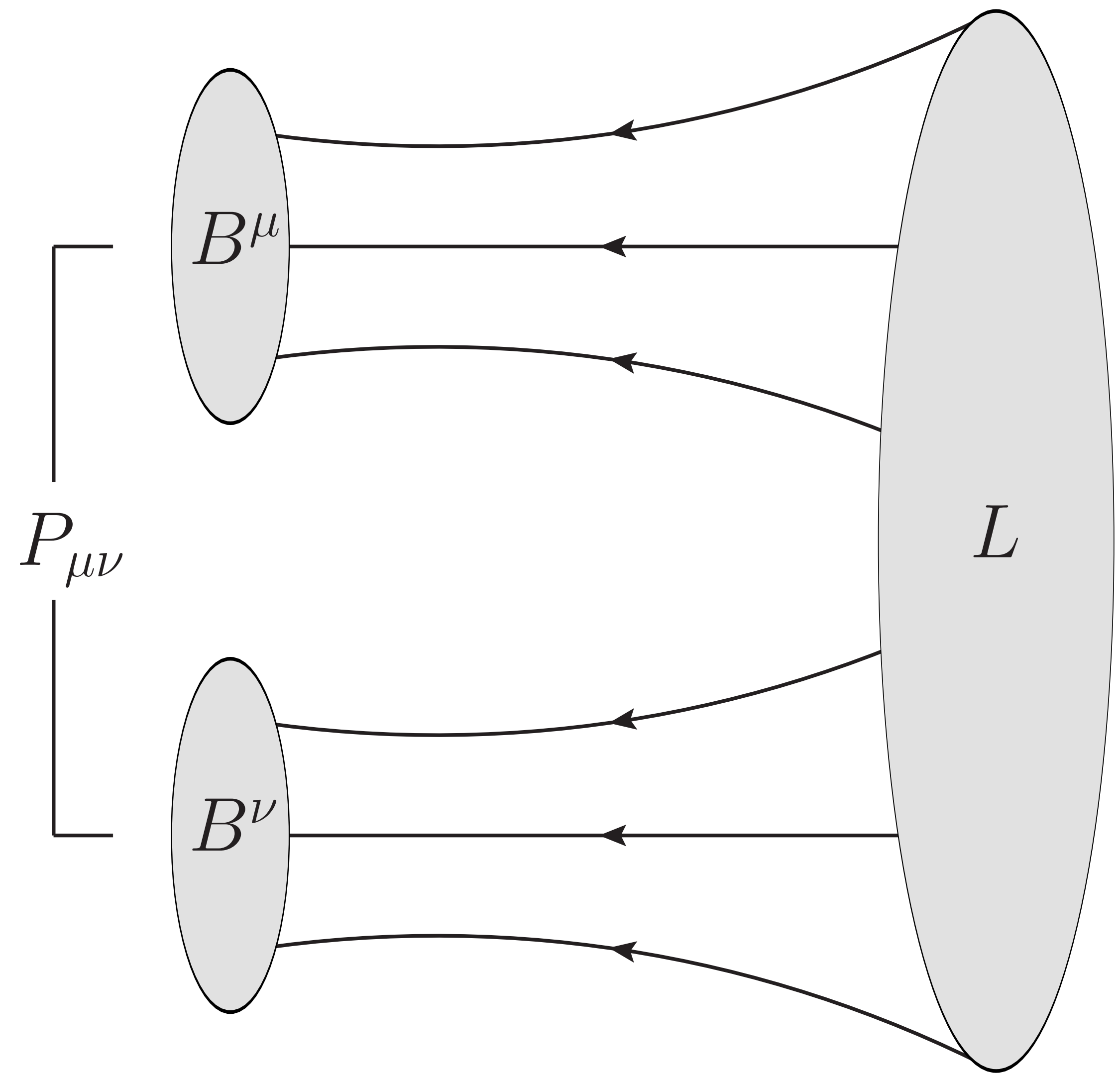
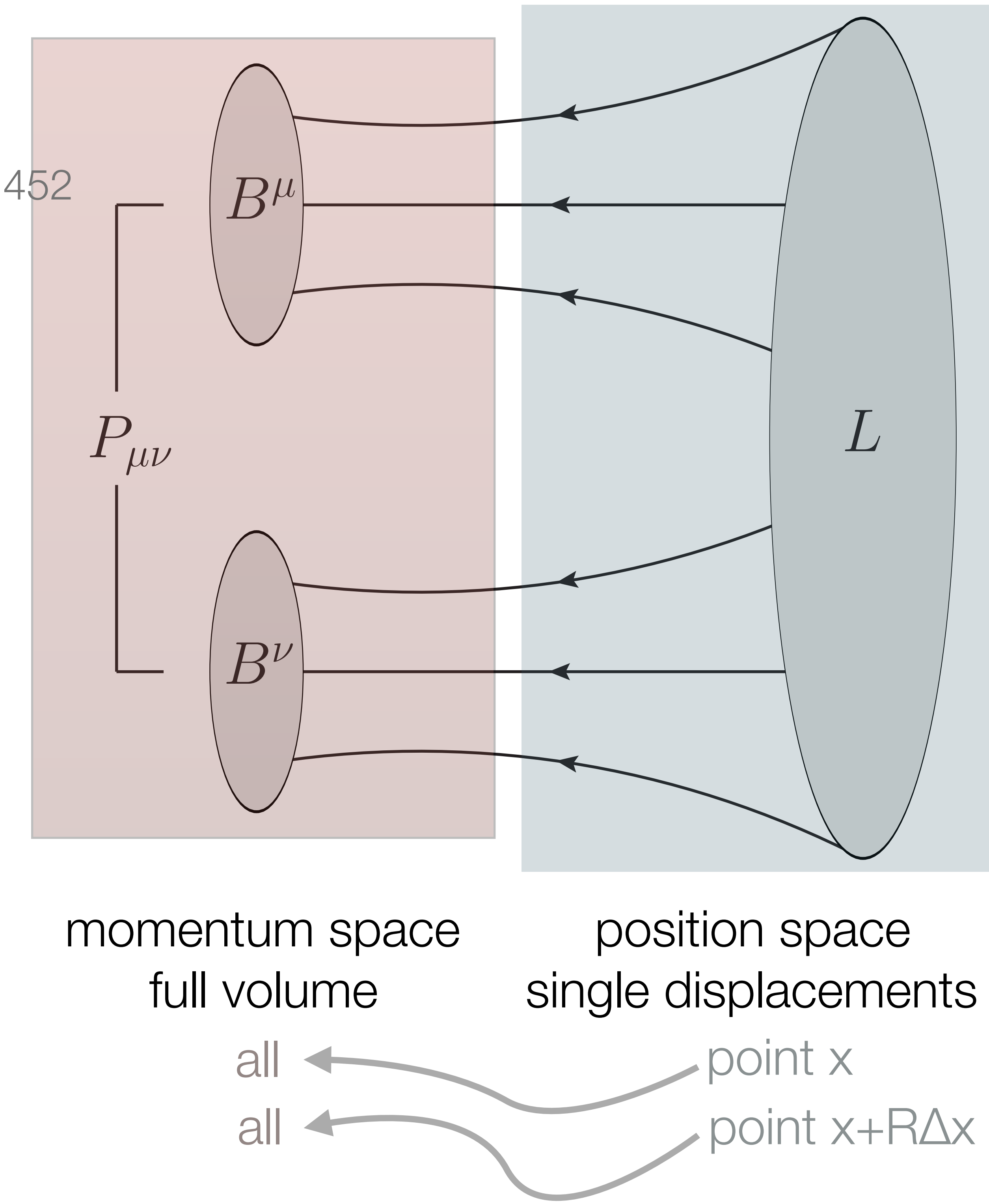


HPC

Doi & Endres 1205.0585, Detmold & Orginos 1207.1452

- Use baryon blocks
- Use sparse tensor contraction to take advantage of sparsity of L
- For each source displacement $R\Delta x$, store (sink-side) full-volume correlator for each $S' m'_S \, S m_S \, I m_I$



$$C_{Im_I}^{S'm'_S S m_S}(\boldsymbol{k}', t'-t, R\boldsymbol{\Delta x}) = \sum_{\boldsymbol{x}} \left\langle \Omega \right| \left(N_{i'}^{\mu'}(t',\boldsymbol{k}') P_{\mu'\nu'}^{S'm'_S} T_{Im_I}^{i'j'} N_{j'}^{\nu'}(t',-\boldsymbol{k}') \right) \left(\bar{N}_i^{\mu}(t,\boldsymbol{x}) P_{\mu\nu}^{Sm_S} T_{Im_I}^{ij} \bar{N}_j^{\nu}(t,\boldsymbol{x}+R\boldsymbol{\Delta x}) \right) \left| \Omega \right\rangle$$

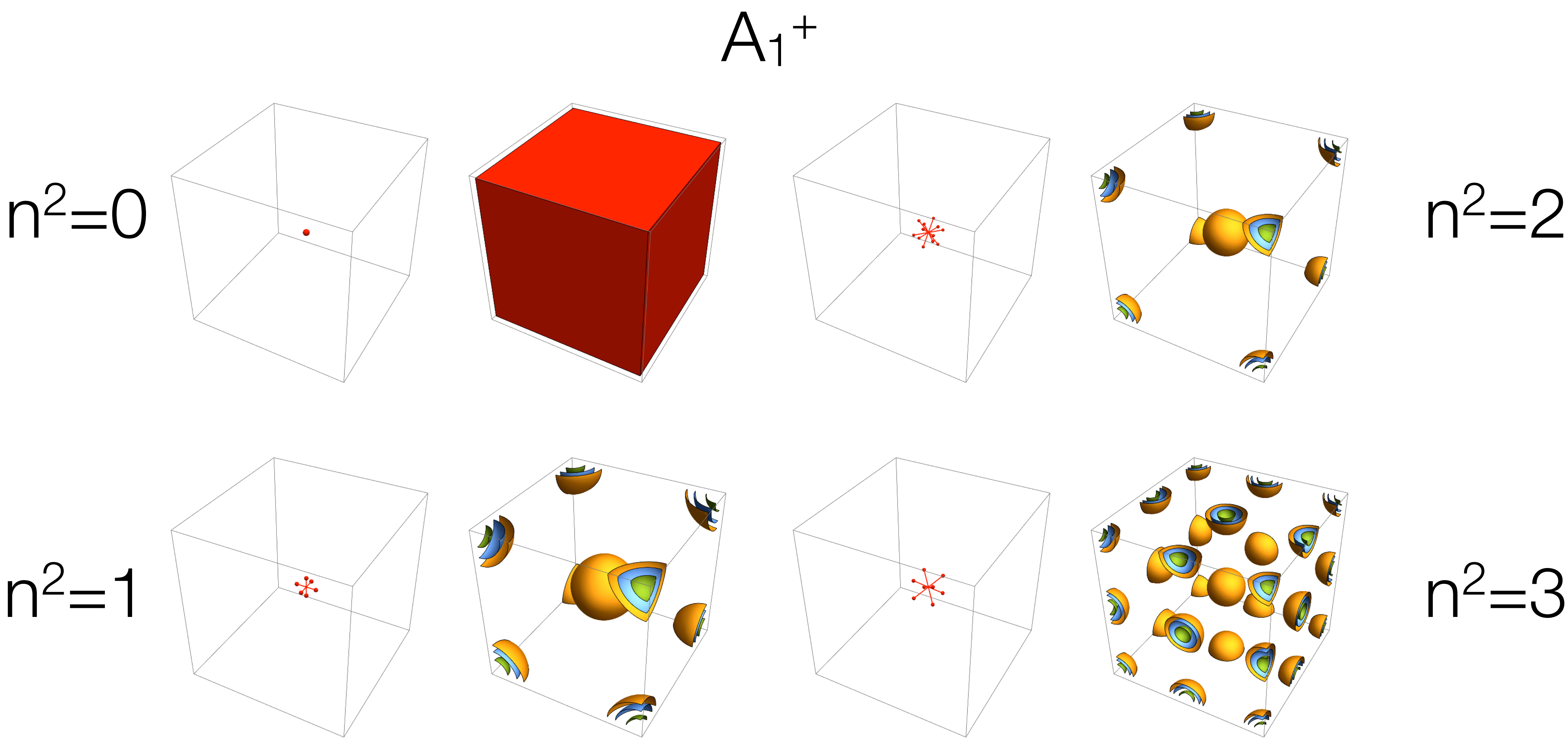
Sample with Sobol sequence

Projectors

Sinks

Luu & Savage 1101.3347

- Project to eigenstates of a noninteracting theory in a box.
- Full volume information → exactly project to any desired irrep



Sinks

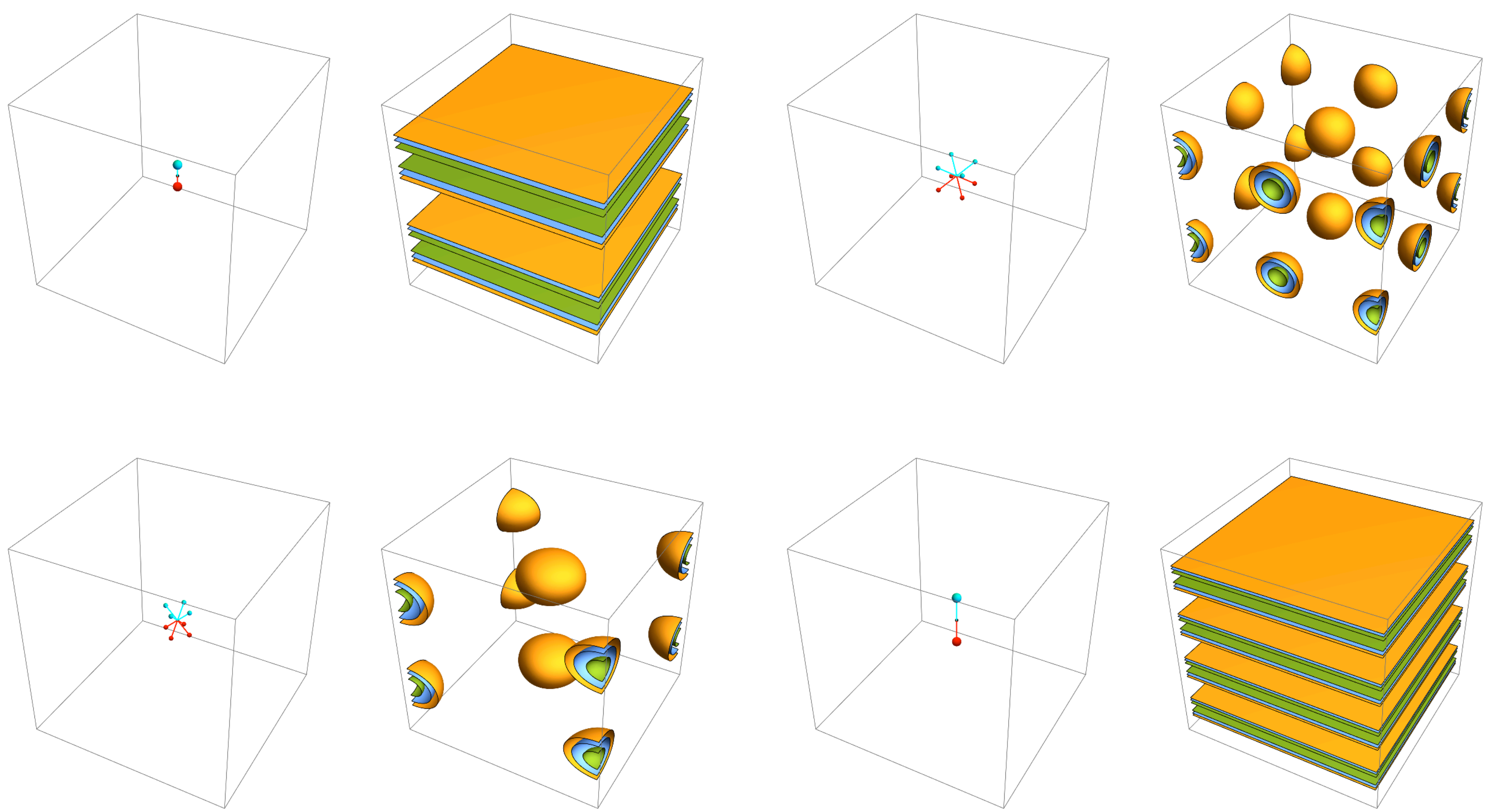
Luu & Savage 1101.3347

- Project to eigenstates of a noninteracting theory in a box.
- Full volume information → exactly project to any desired irrep

T_1^-

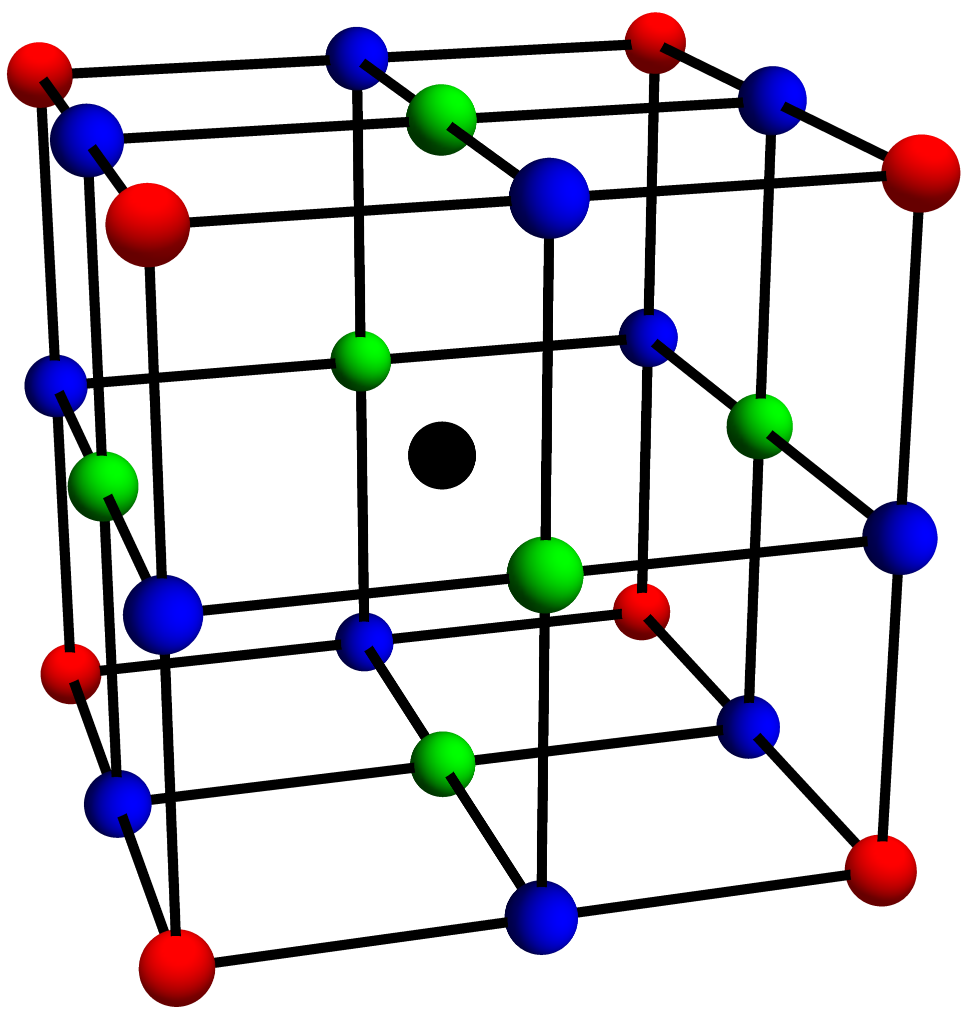
$n^2=1$ $n^2=3$

$n^2=2$ $n^2=4$



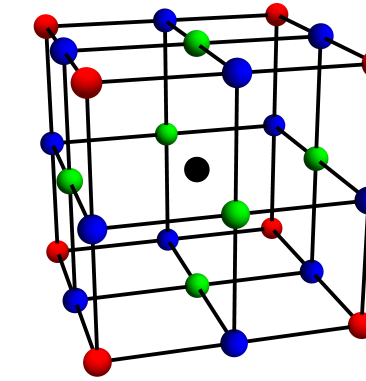
Sources

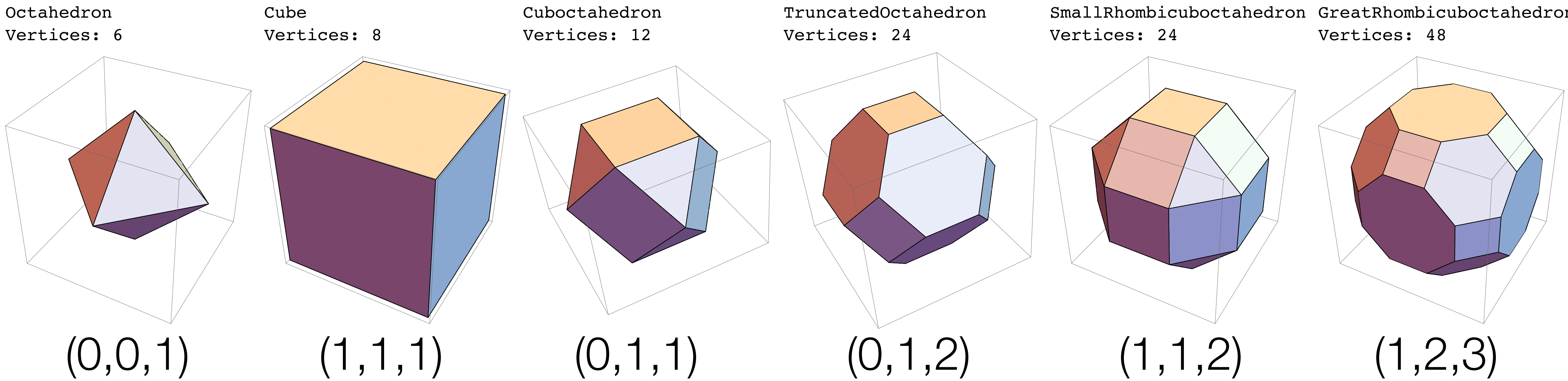
- Exact projection source-side requires spatial-volume-to-all propagators.
- Pick displacements



description	$\Delta x \propto$	count
local	(0,0,0)	1
face	(0,0,1)	6
edge	(0,1,1)	12
corner	(1,1,1)	8

Sources

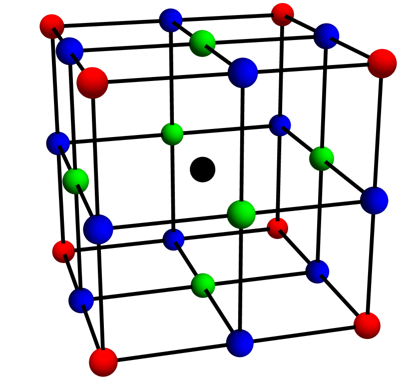




Solids generated by O_h ↔ Irreps of O_h

face	octahedron	6
edge	cuboctahedron	12
corner	cube	8
knight's move	trucated octahedron	24
more complicated	small rhombicutobctahedron	24
more complicated	great rhombicuboctahedron	48

Too expensive.



Source Overlap

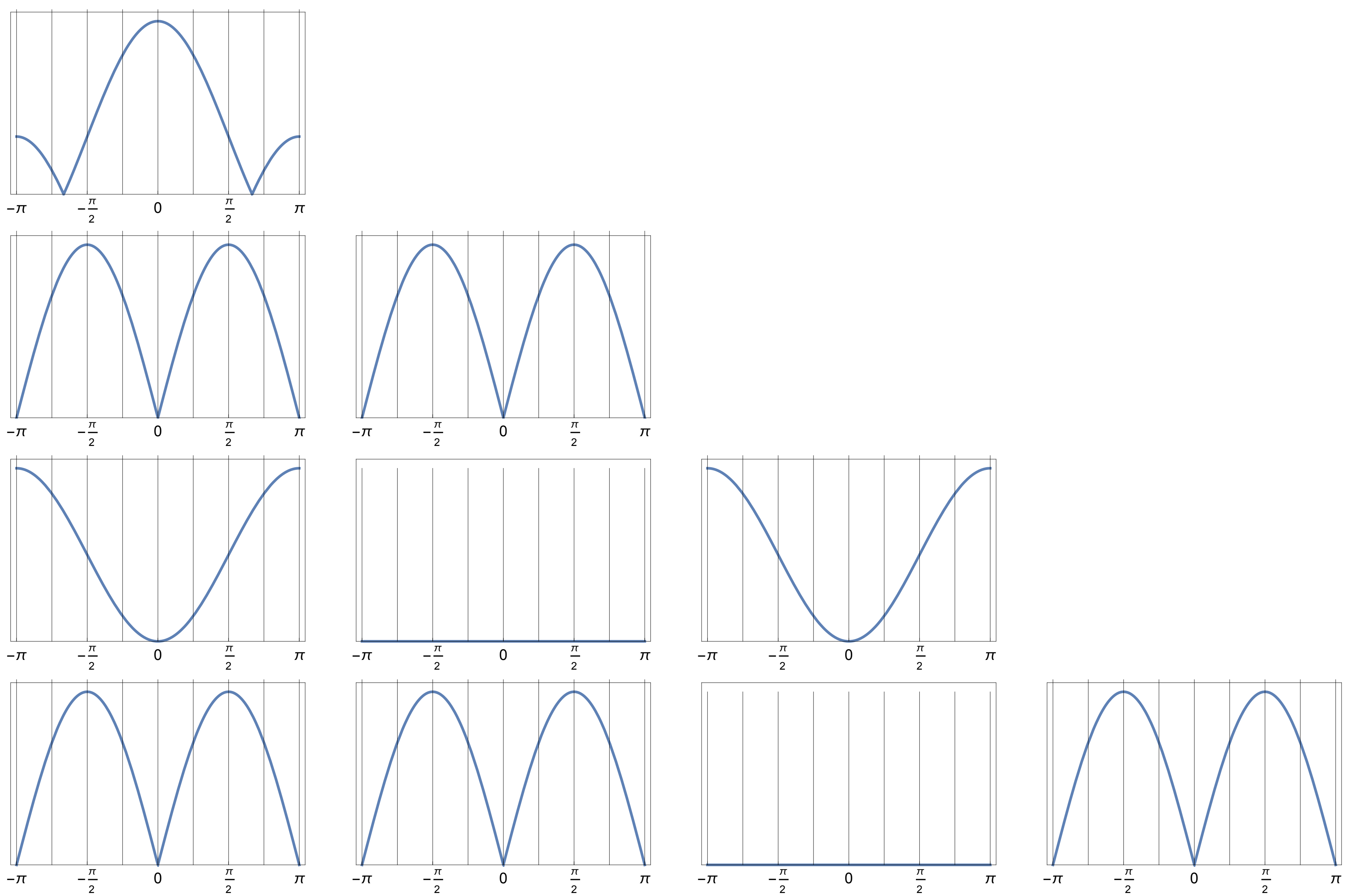
Project Luu & Savage momentum sources to faces as a function of $\pi\Delta x/L$

S

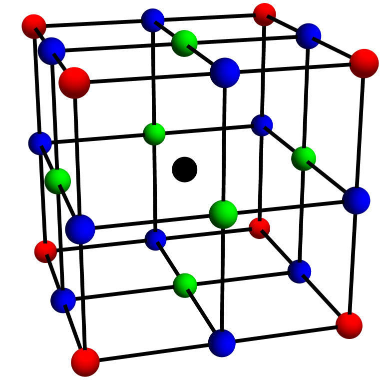
P

D

F

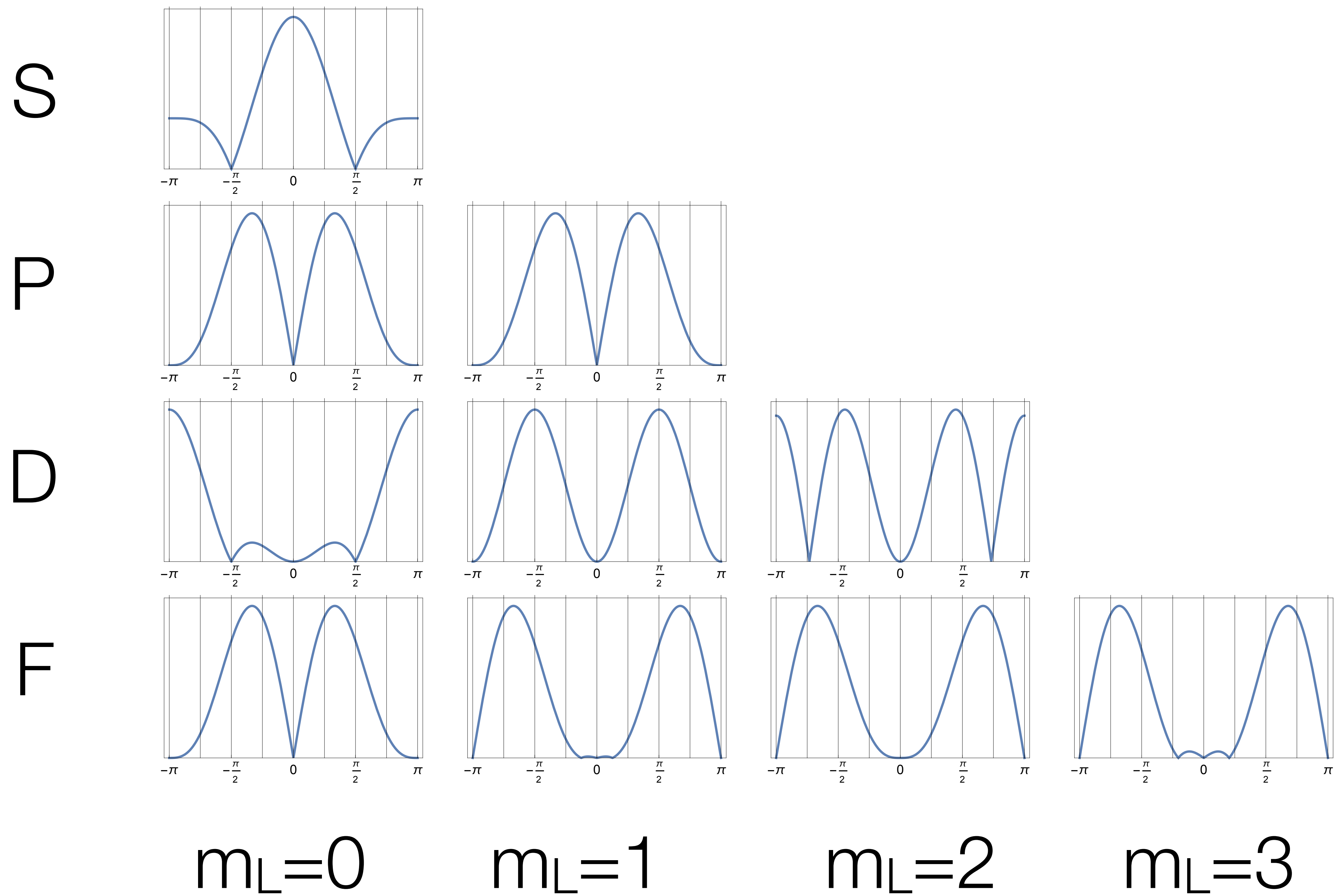


$m_L=0$ $m_L=1$ $m_L=2$ $m_L=3$

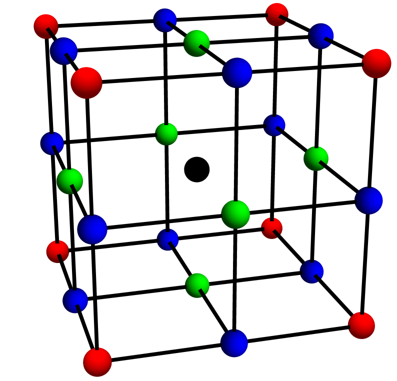


Source Overlap

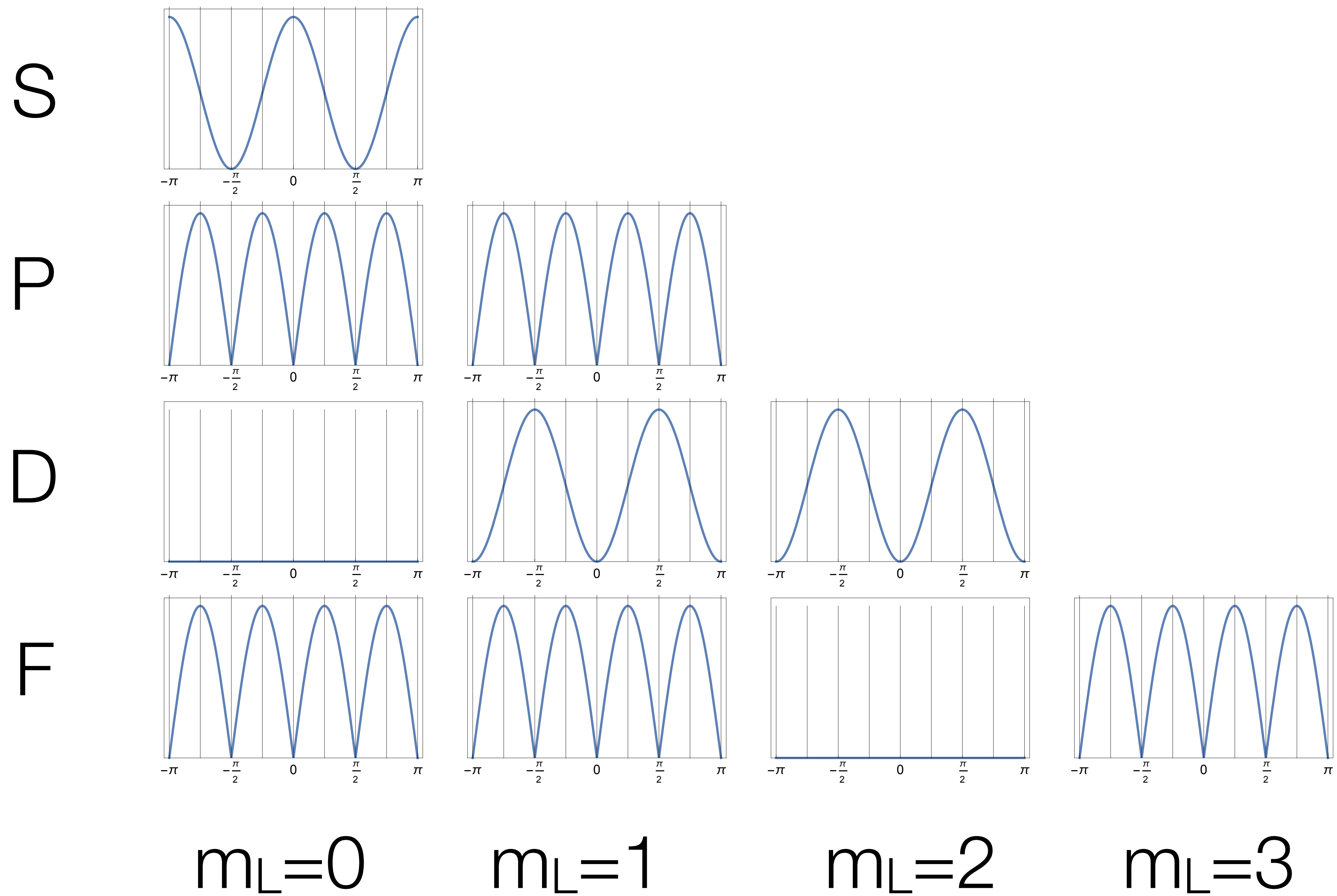
Project Luu & Savage momentum sources to edges as a function of $\pi\Delta x/L$

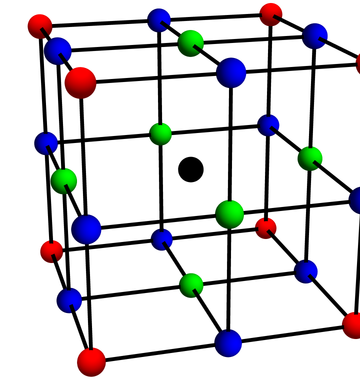


Source Overlap



Project Luu & Savage momentum sources to corner as a function of πΔx/L



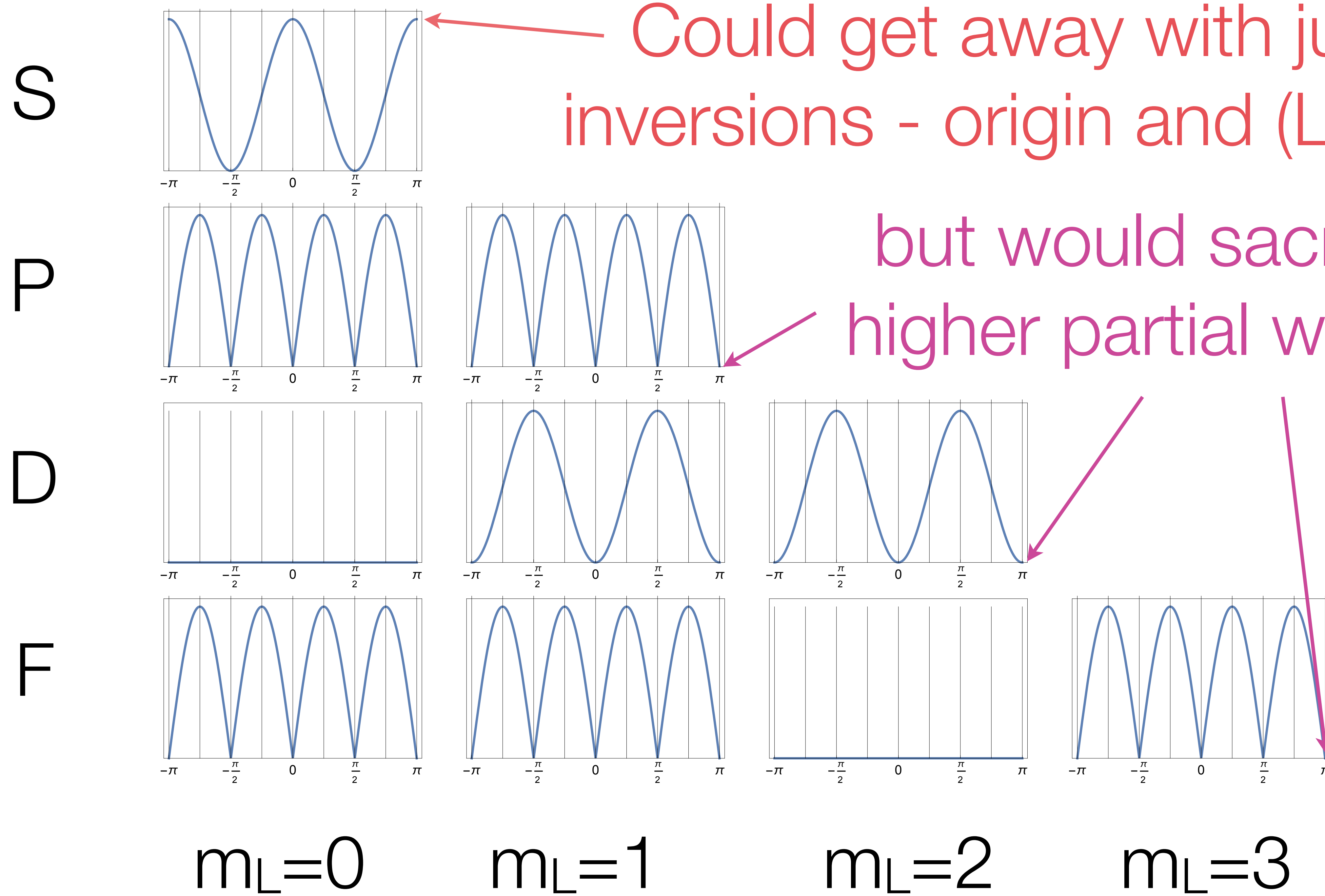


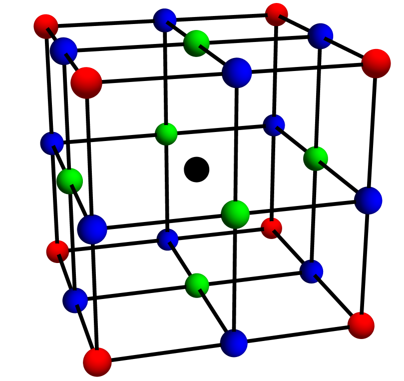
Source Overlap

Project Luu & Savage momentum sources to corner as a function of πΔx/L

Could get away with just 2 inversions - origin and (L,L,L)/2

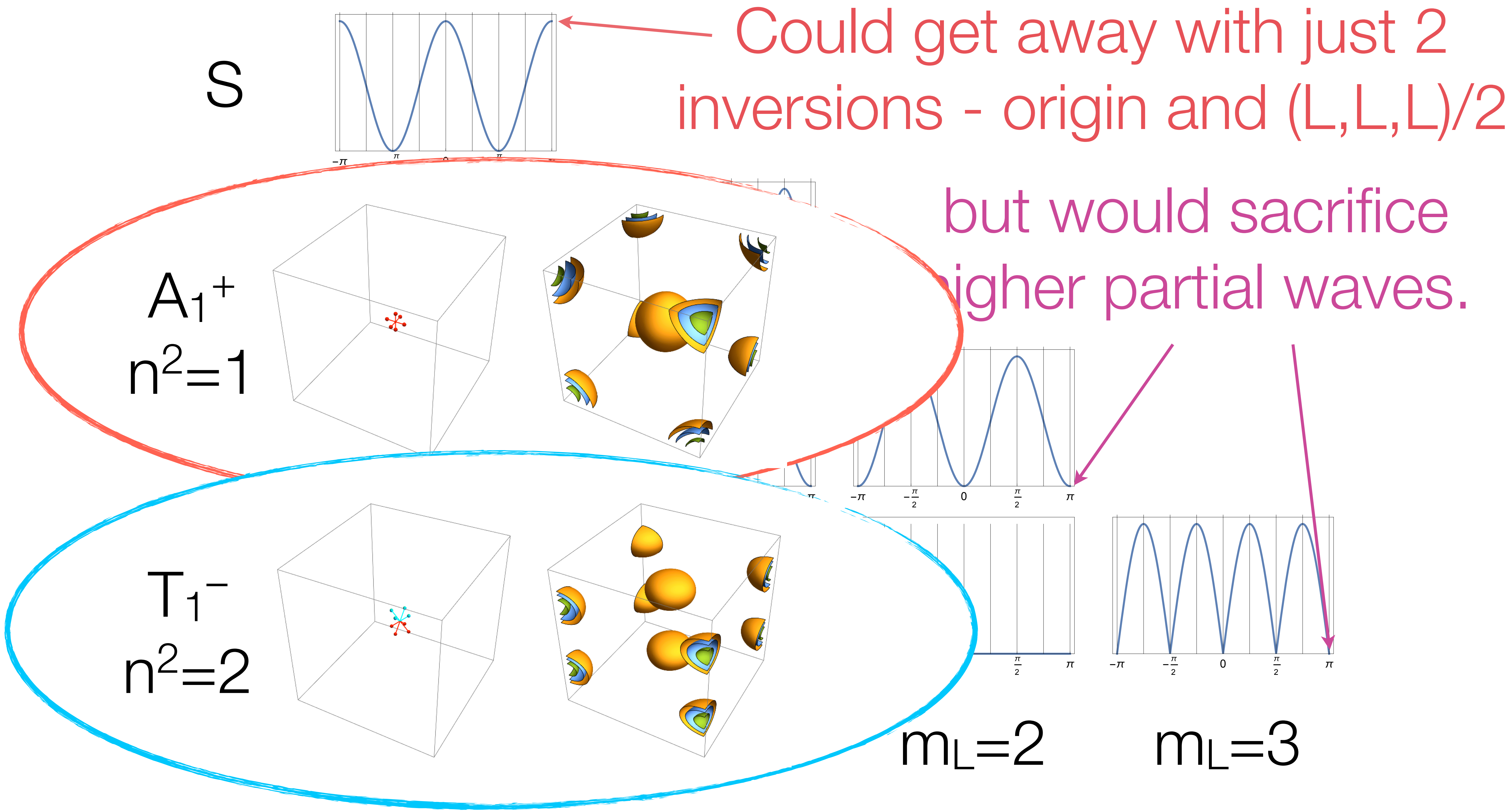
but would sacrifice higher partial waves.





Source Overlap

Project Luu & Savage momentum sources to corner as a function of πΔx/L



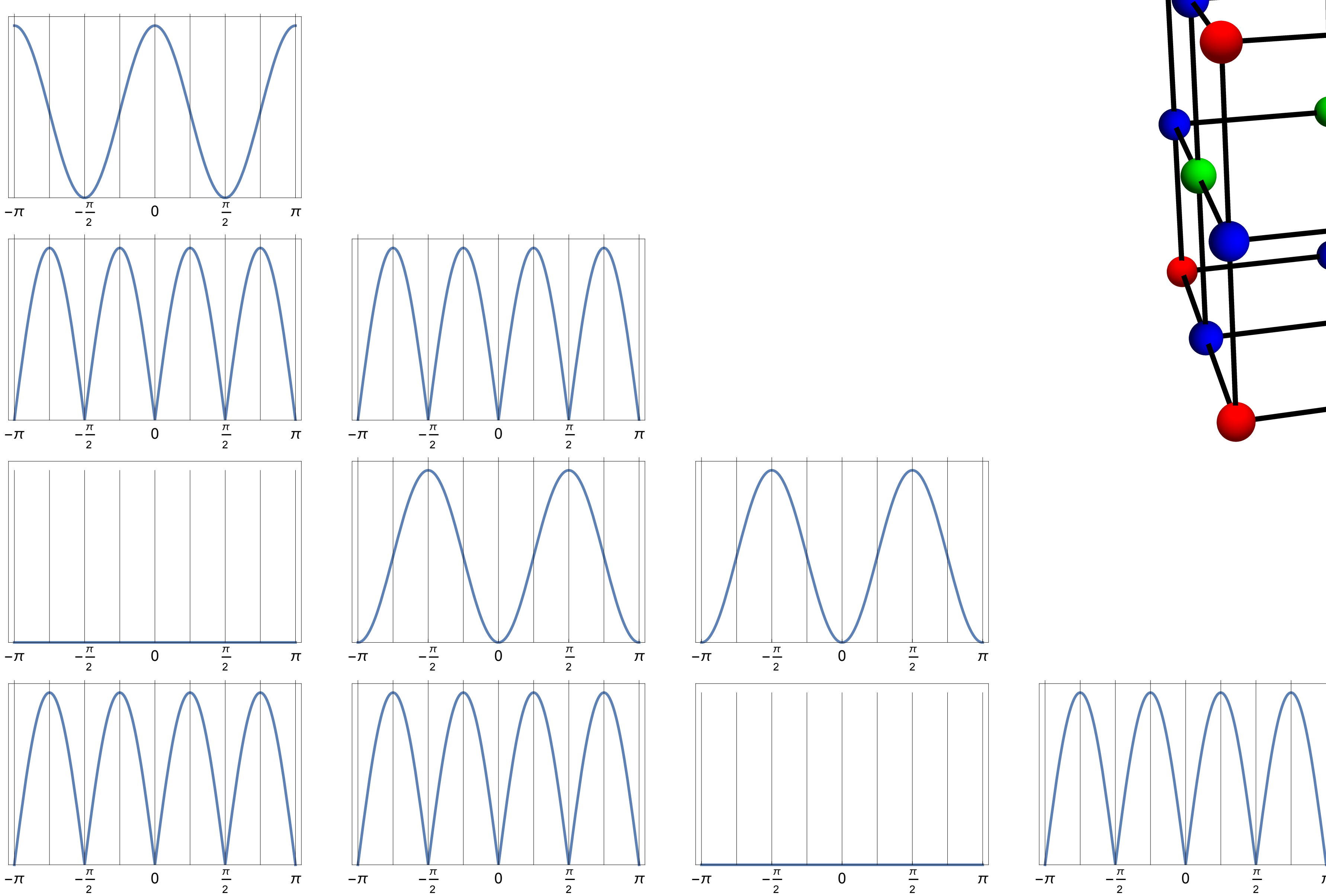
Propagator Reuse

S

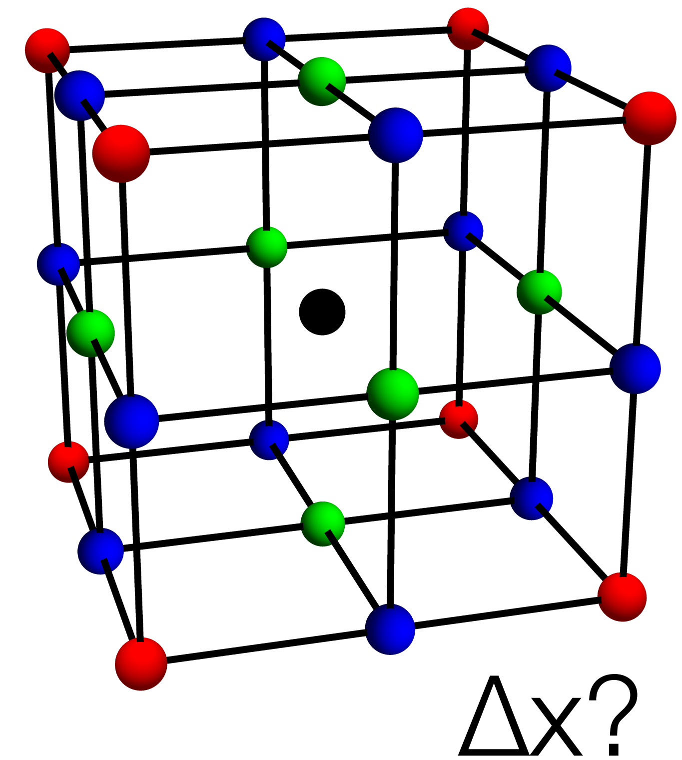
P

D

F



m_L=0 m_L=1 m_L=2 m_L=3



Propagator Reuse

$0 = (0,0,0)$

$A = (L,L,L)/2$

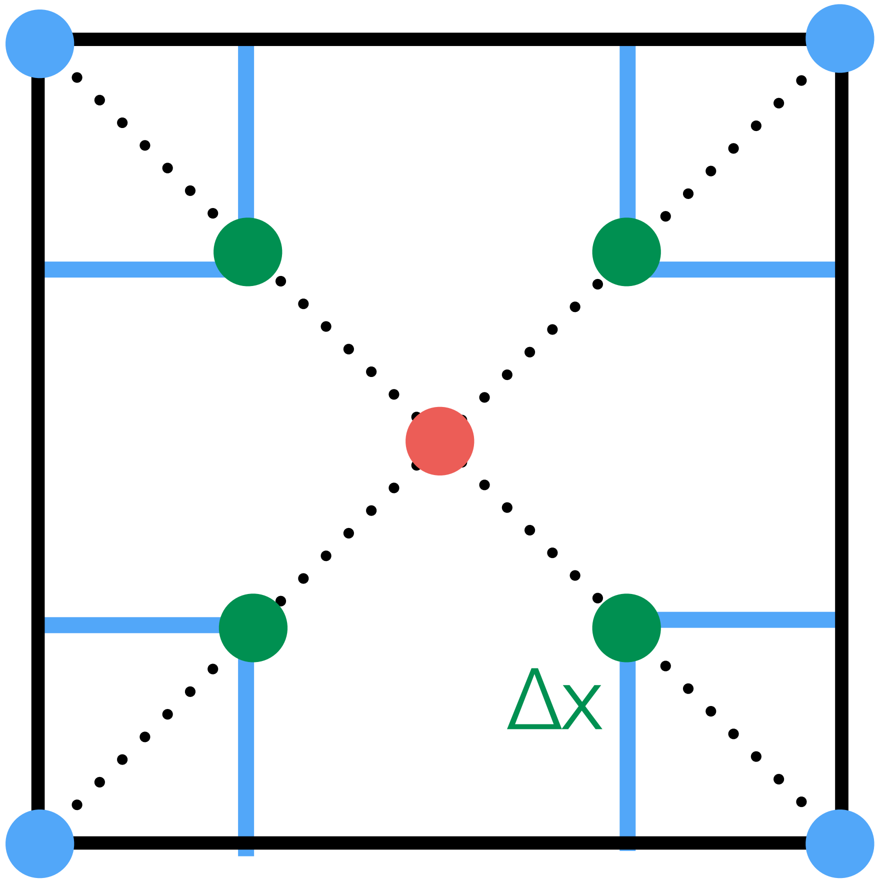
$C = (\pm 1,\pm 1,\pm 1)\ \Delta x$

10 local sources

+1 maximally displaced

+1 corner(Δx) around 0

+1 corner($L/2-\Delta x$) around A



Propagator Reuse

$0 = (0,0,0)$

$A = (L,L,L)/2$

$C = (\pm 1,\pm 1,\pm 1)\,\Delta x$

10 local sources

+1 maximally displaced

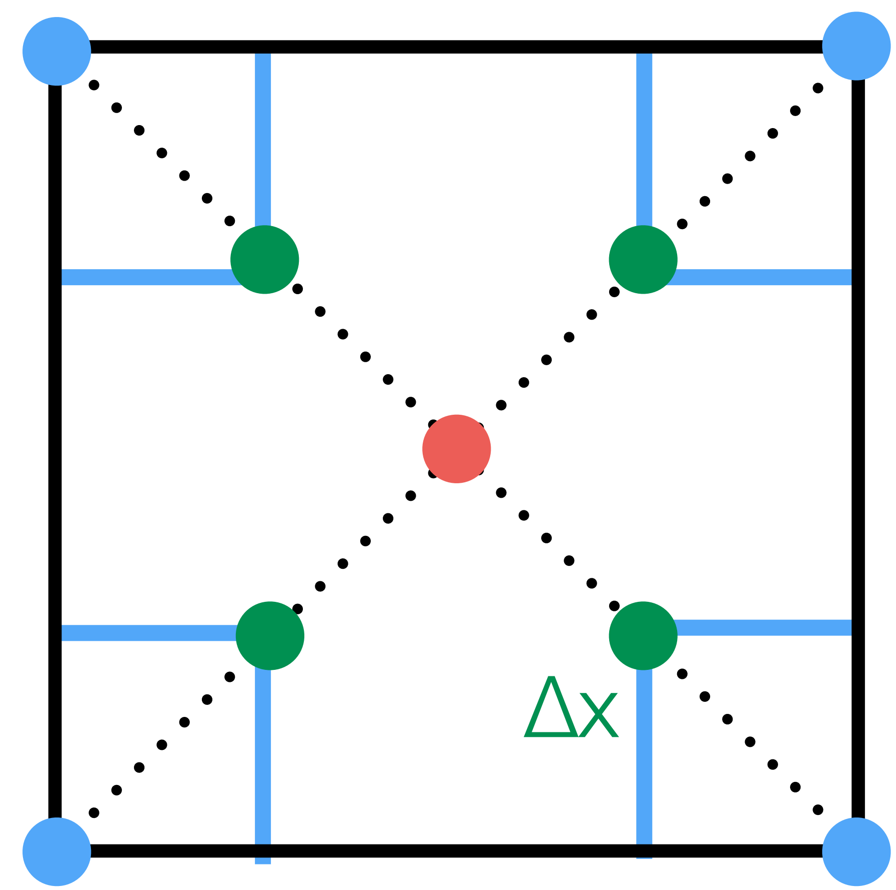
+1 corner(Δx) around 0

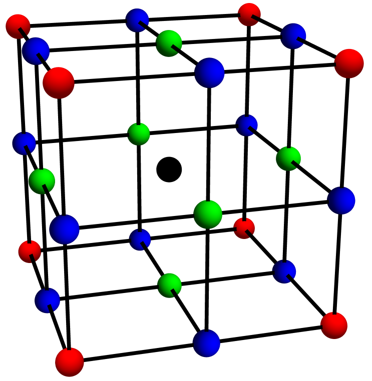
+1 corner(L/2-Δx) around A

+1/2 corner(2Δx) from C

+2 faces(2Δx) from C

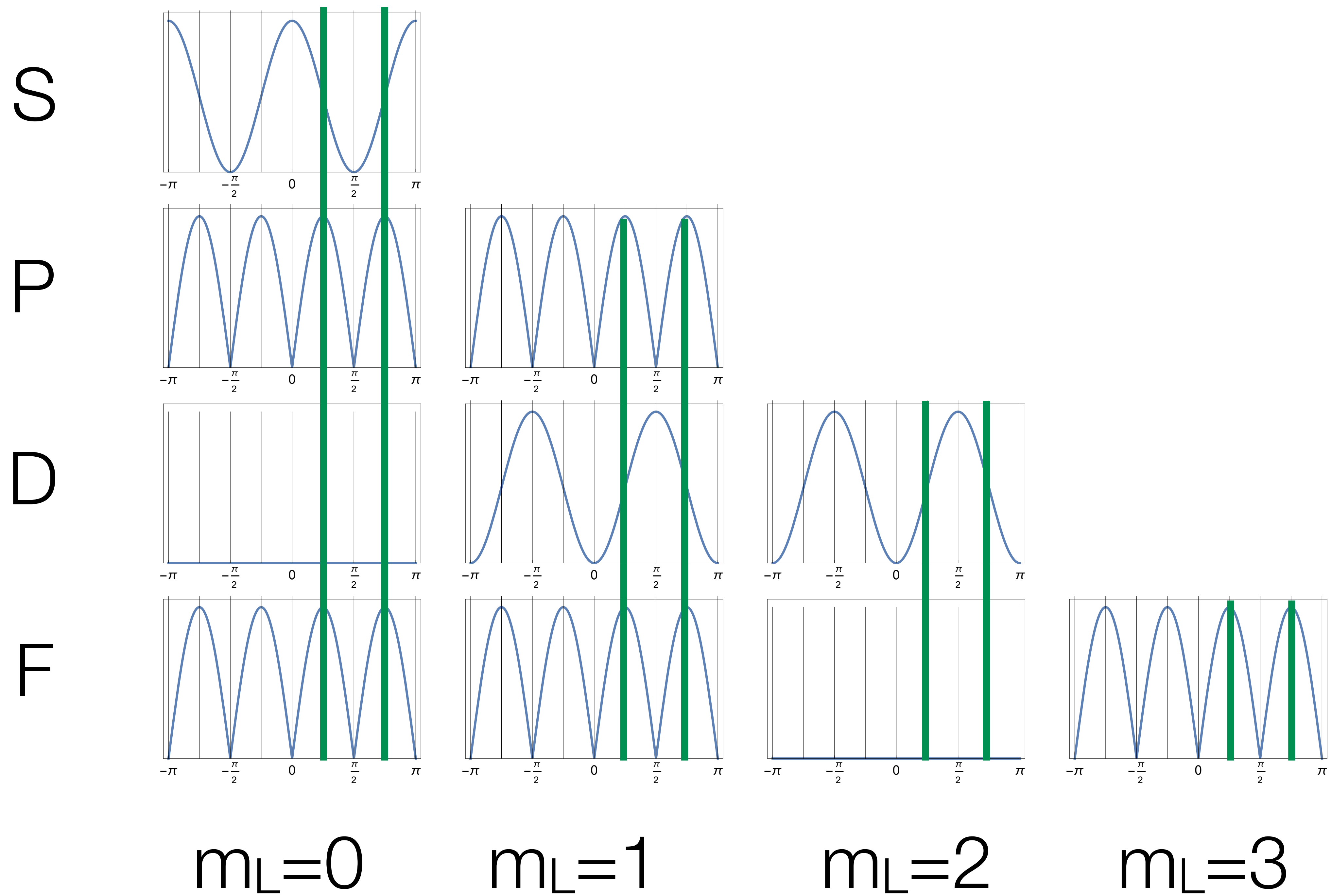
+1 edges(2Δx) from C



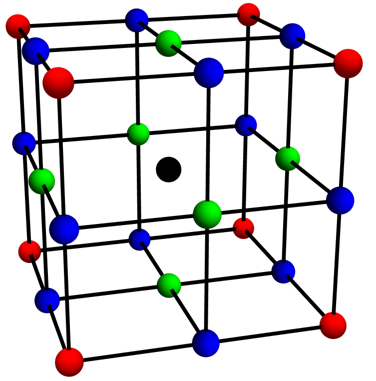


Magic Choice: Δx = L/8 (≡3L/8)

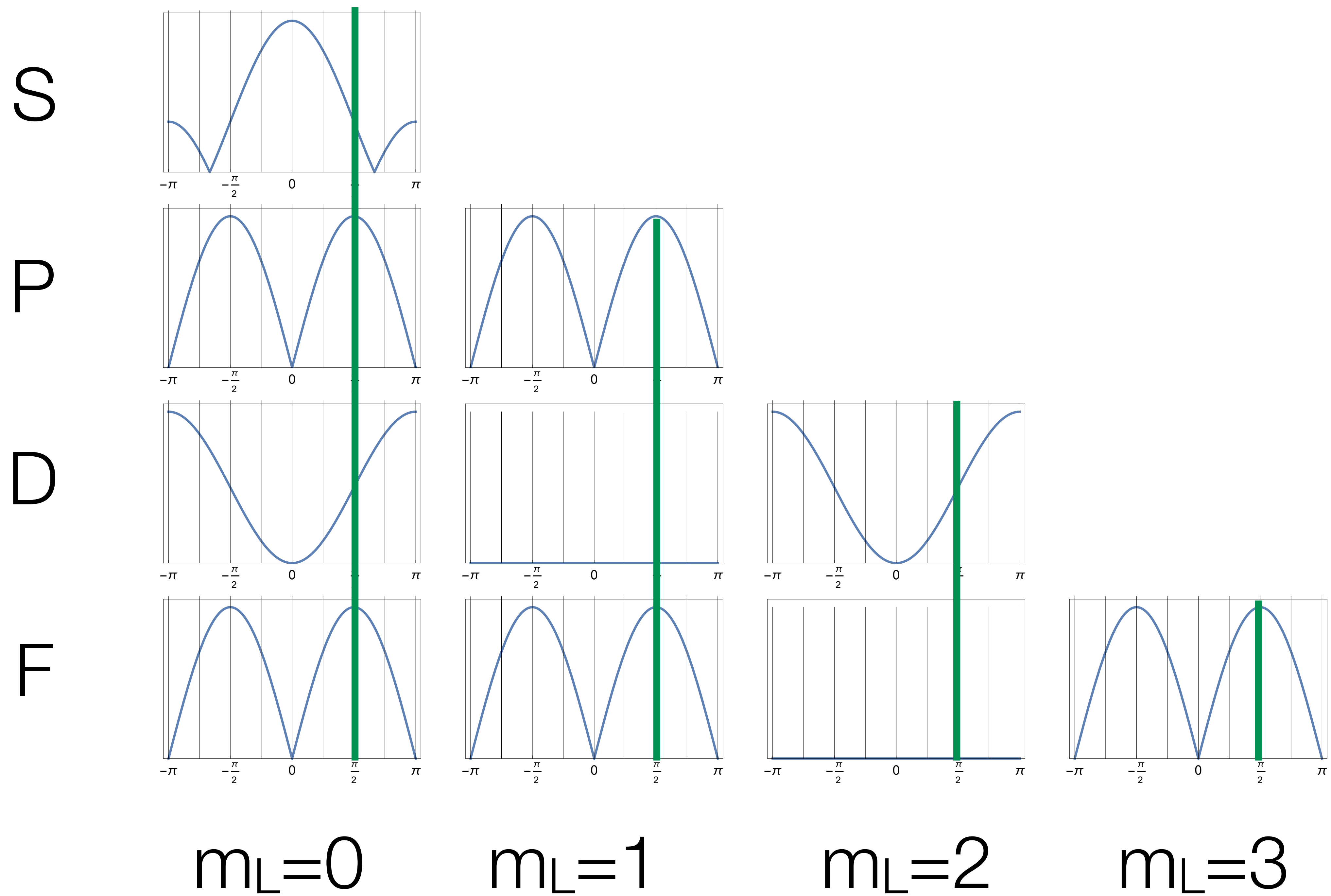
corner



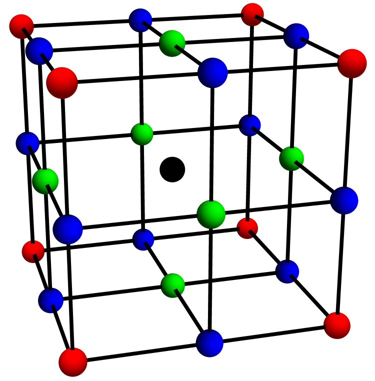
Magic Choice: $\Delta x = L/8$ ($\equiv 3L/8$)



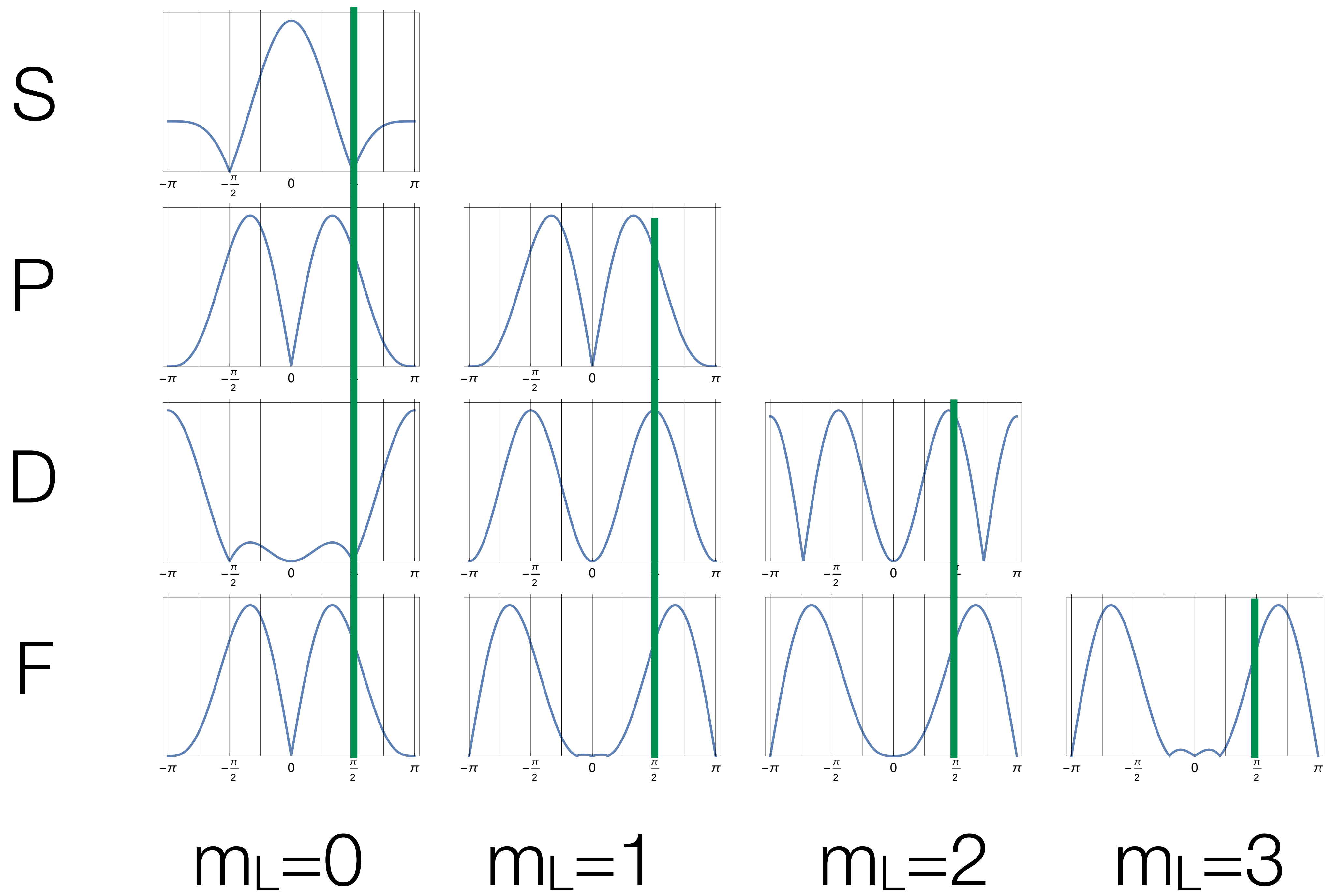
faces



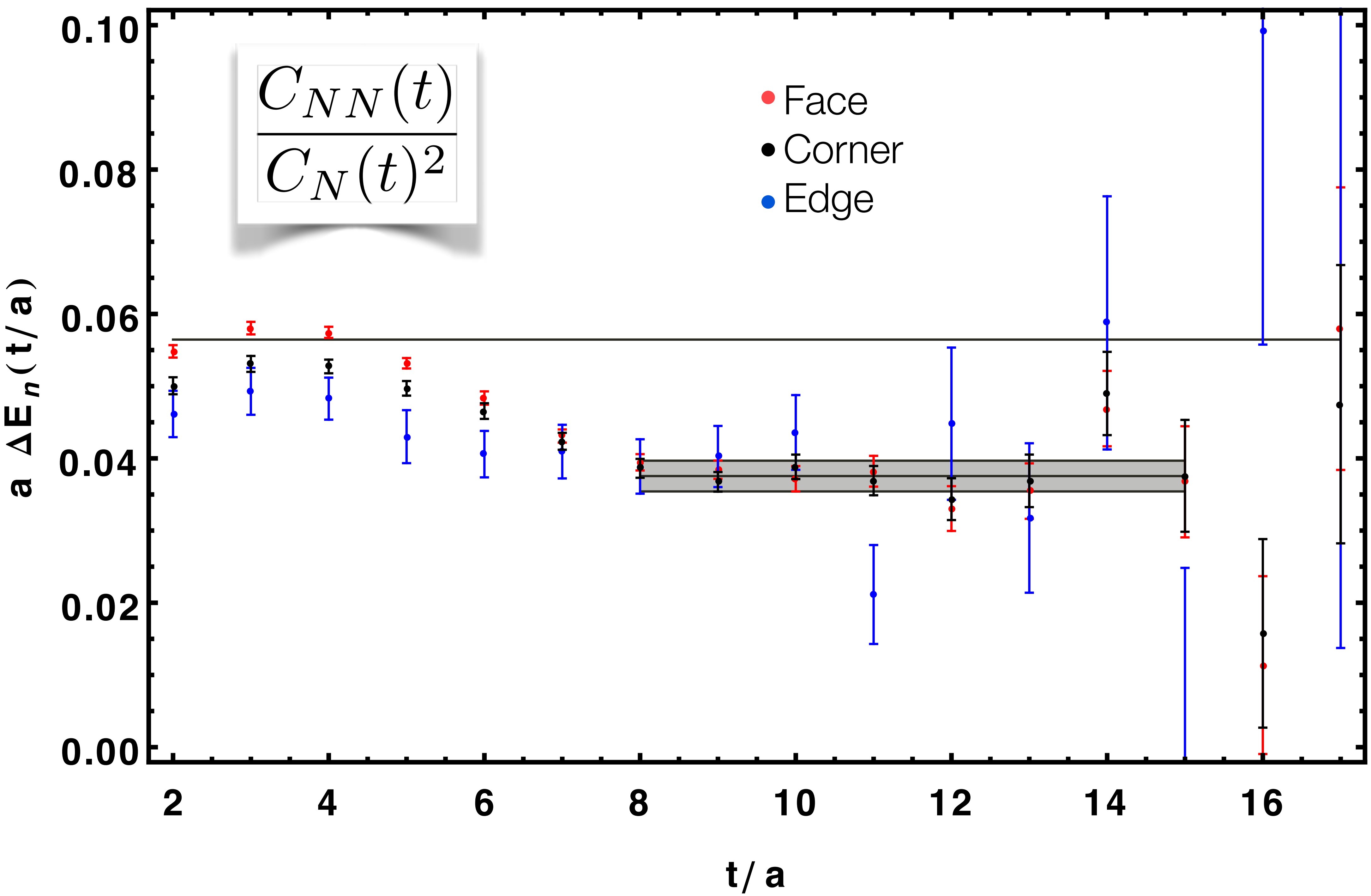
Magic Choice: $\Delta x = L/8$ ($\equiv 3L/8$)



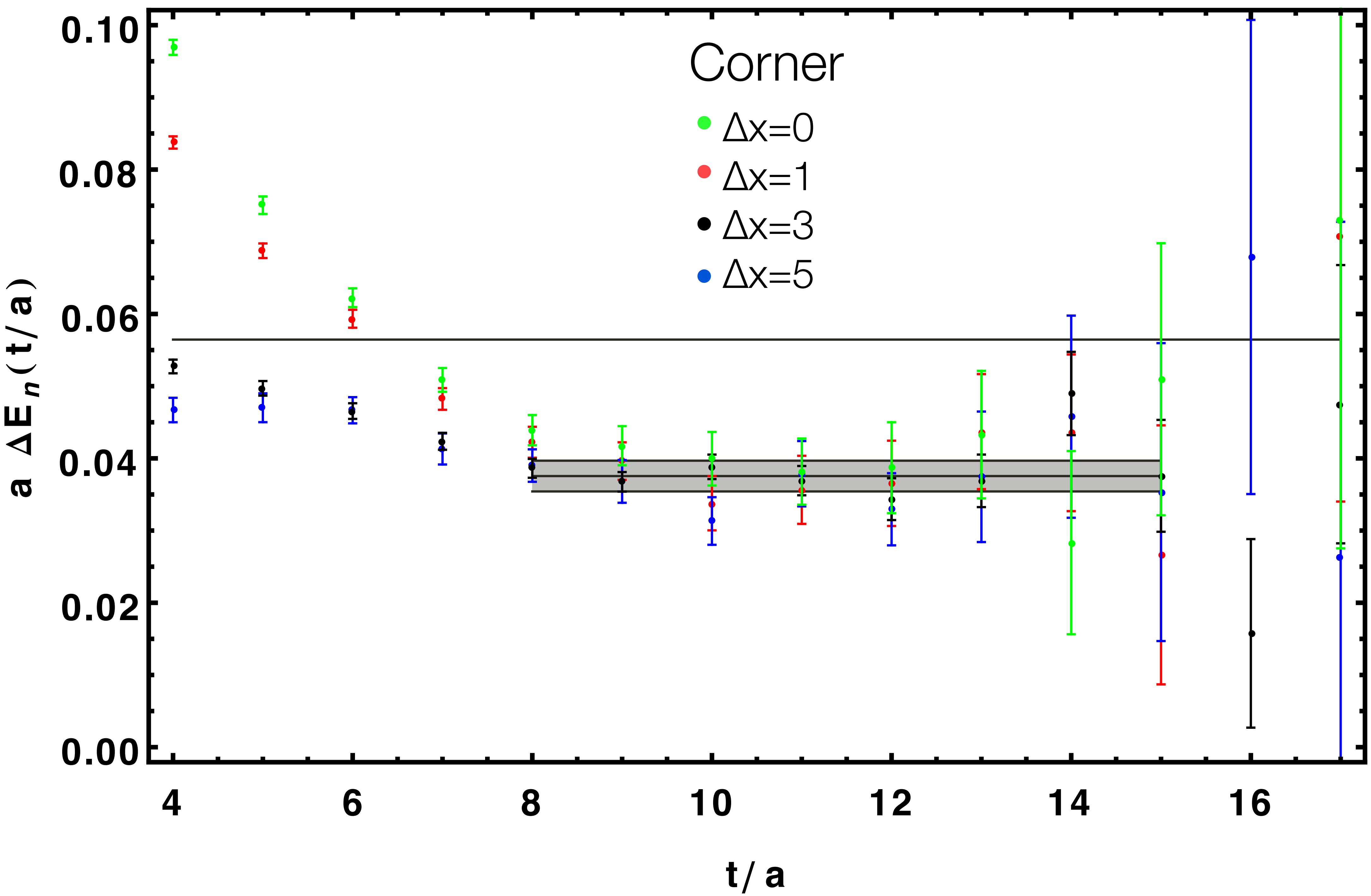
edges



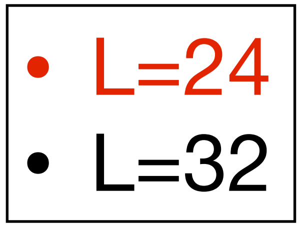
Different sources give same plateau A_1^+, $\Delta x=3$

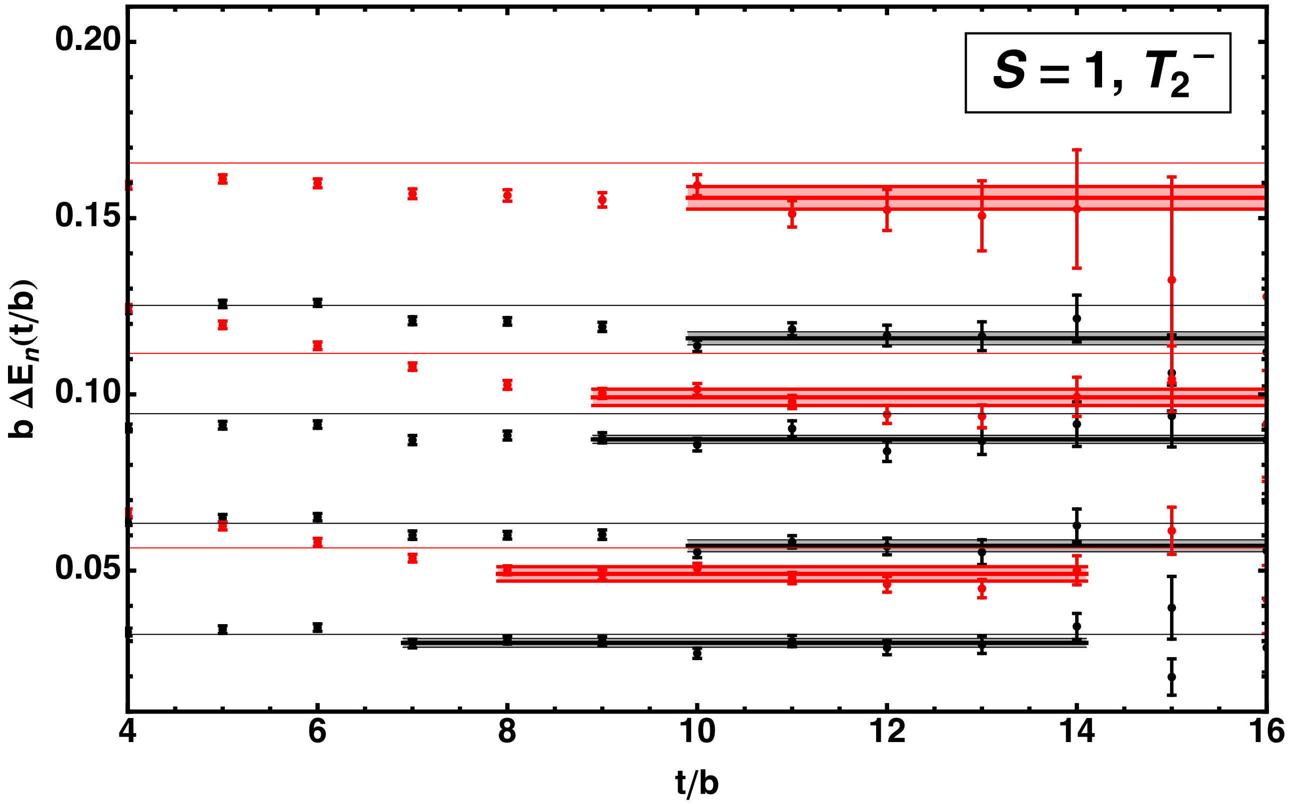


Different displacements give same plateau A_1^+



Clean separation of momentum shells

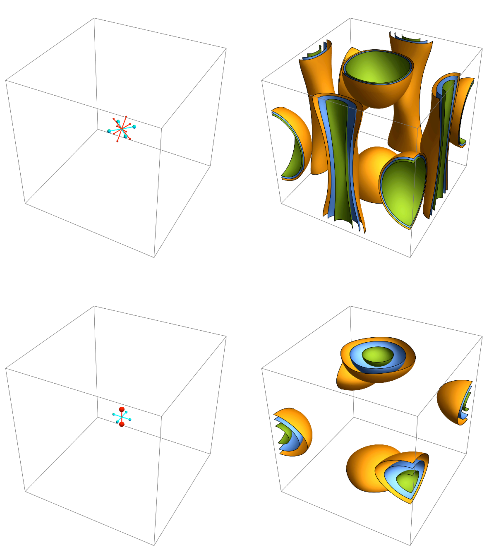


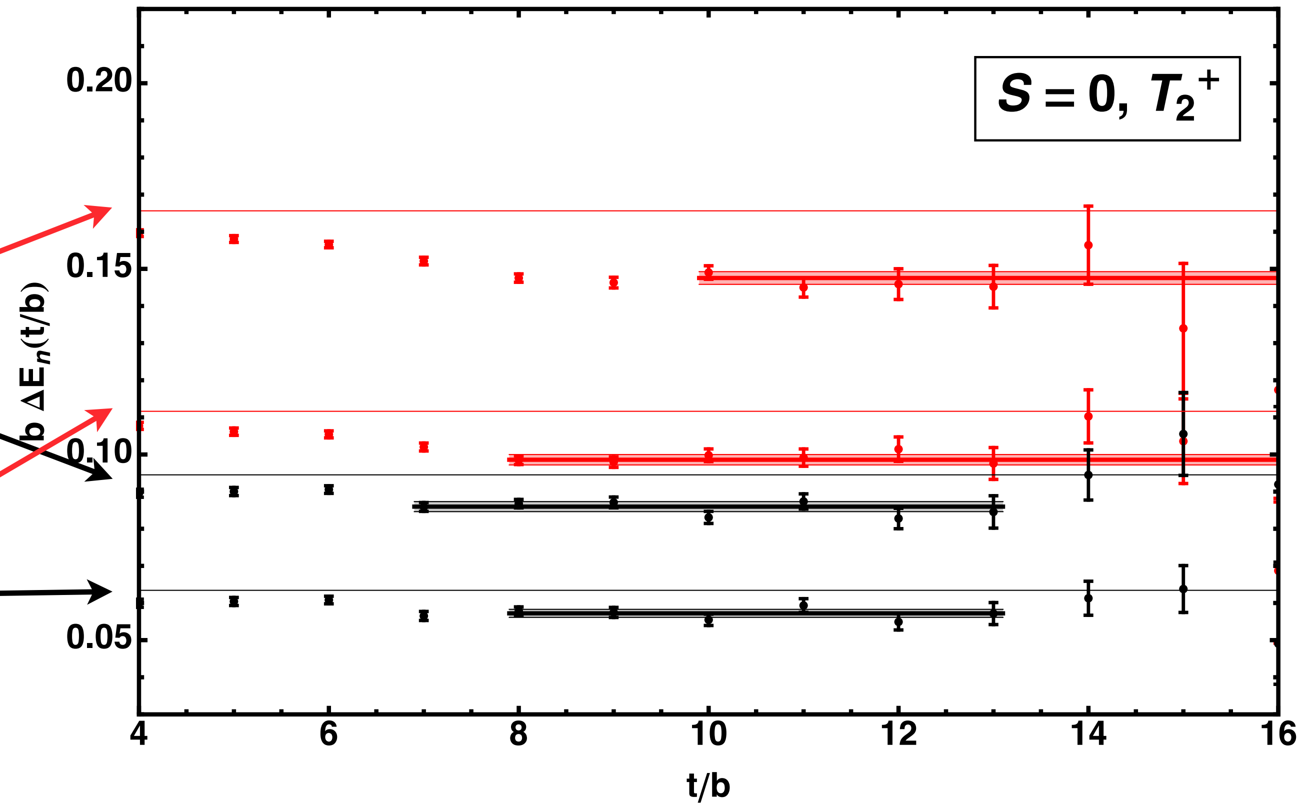


$\Delta E \rightarrow$ Lüscher

$n^2=2$

$n^2=1$



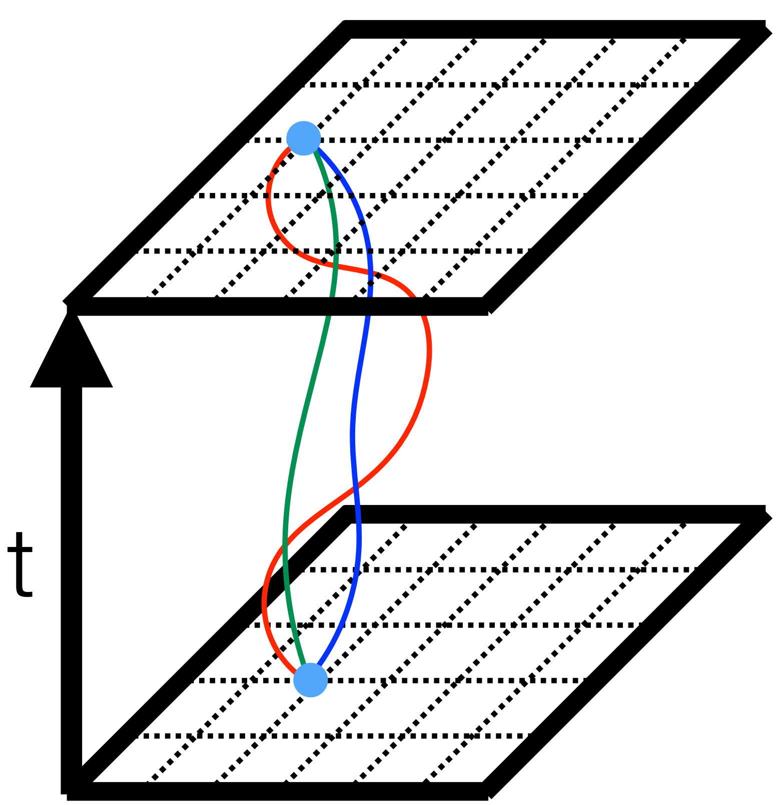


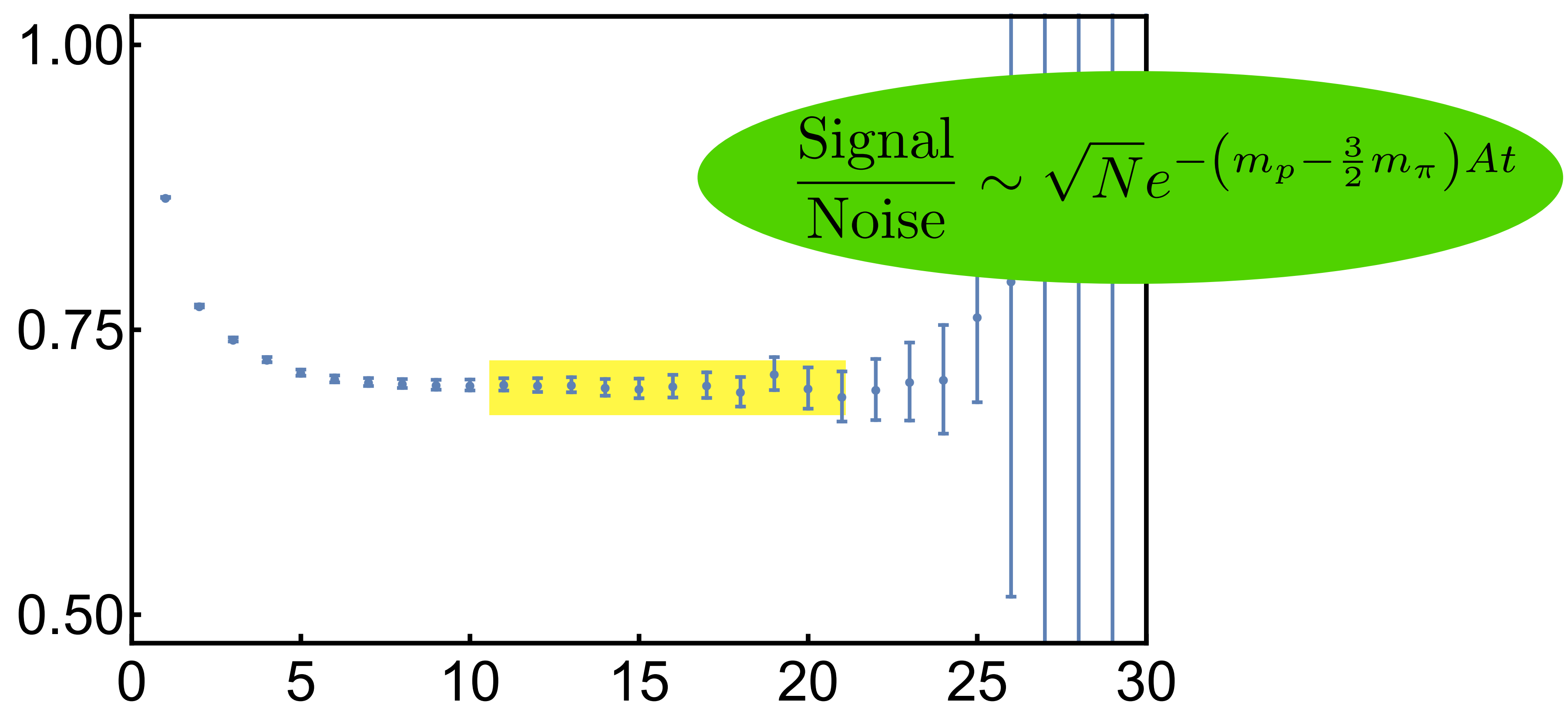
Correlation Functions and Effective Masses

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}U\ \mathcal{O}(t)\mathcal{O}^\dagger(0) e^{-S[\bar{\psi},\psi,U]}$$

$$= \sum_k \langle \Omega \mid \mathcal{O} \mid k \rangle \langle k \mid \mathcal{O}^\dagger \mid \Omega \rangle\, e^{-E_k t}$$

Effective mass $\quad E_0 = \lim_{t\to\infty} -\partial_t \log C(t)$





Correlation Functions and Effective Masses

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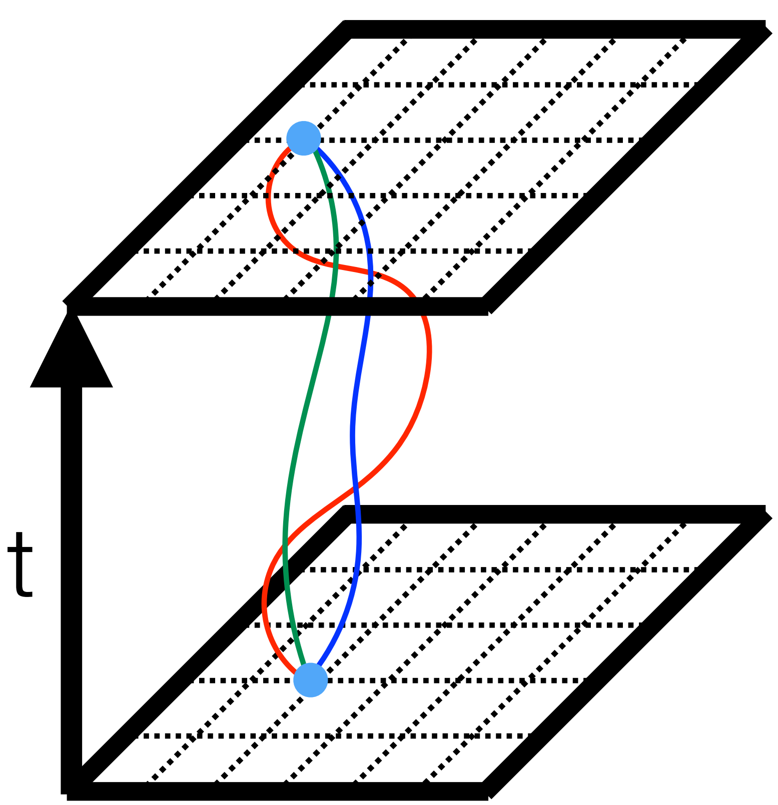
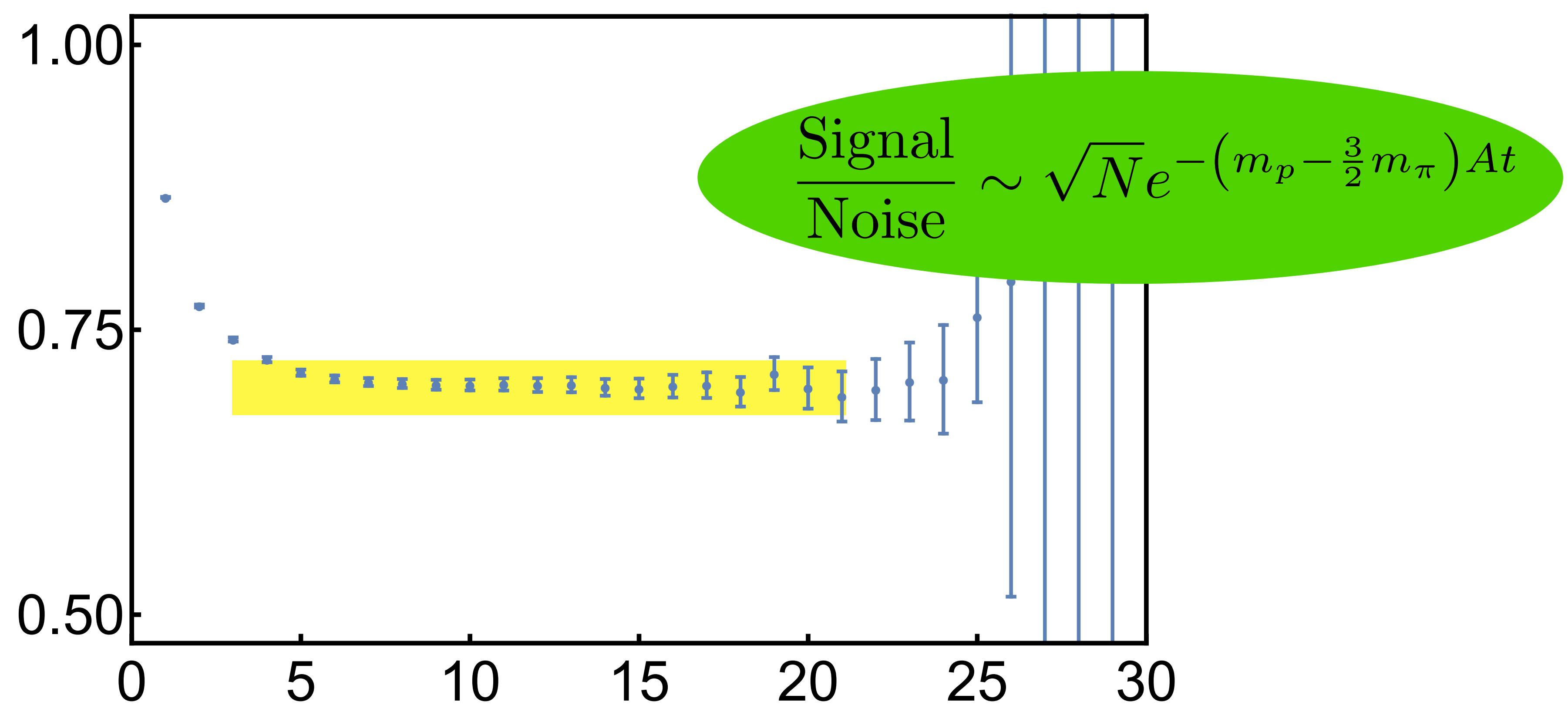
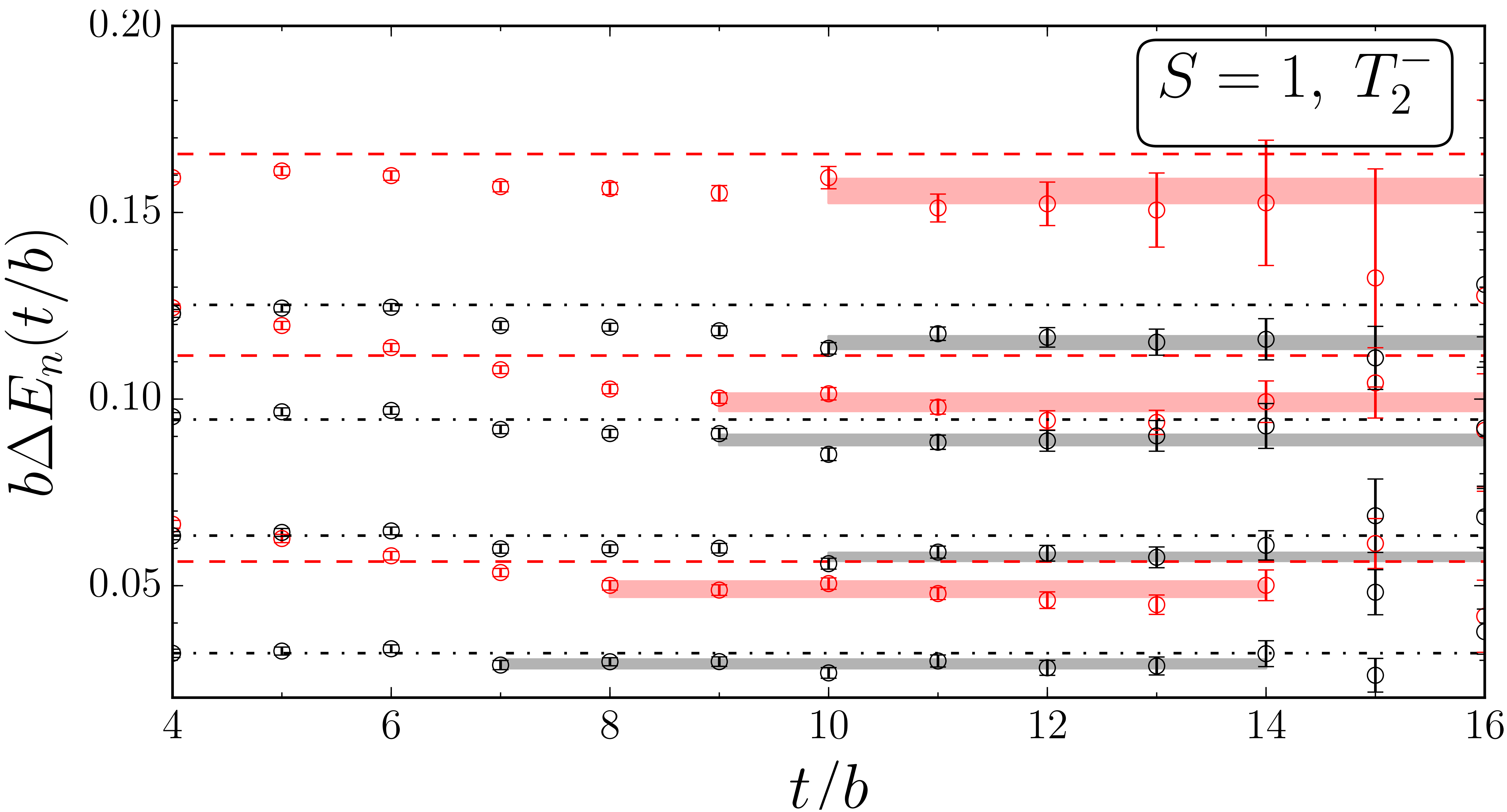




Fitting the Ratio

CalLat 1508.00886 Phys.Lett. B765 (2017) 285-292

$$E_{\text{interaction}} = \lim_{t\to\infty} \frac{C_{NN}(t)}{C_N(t)^2}$$



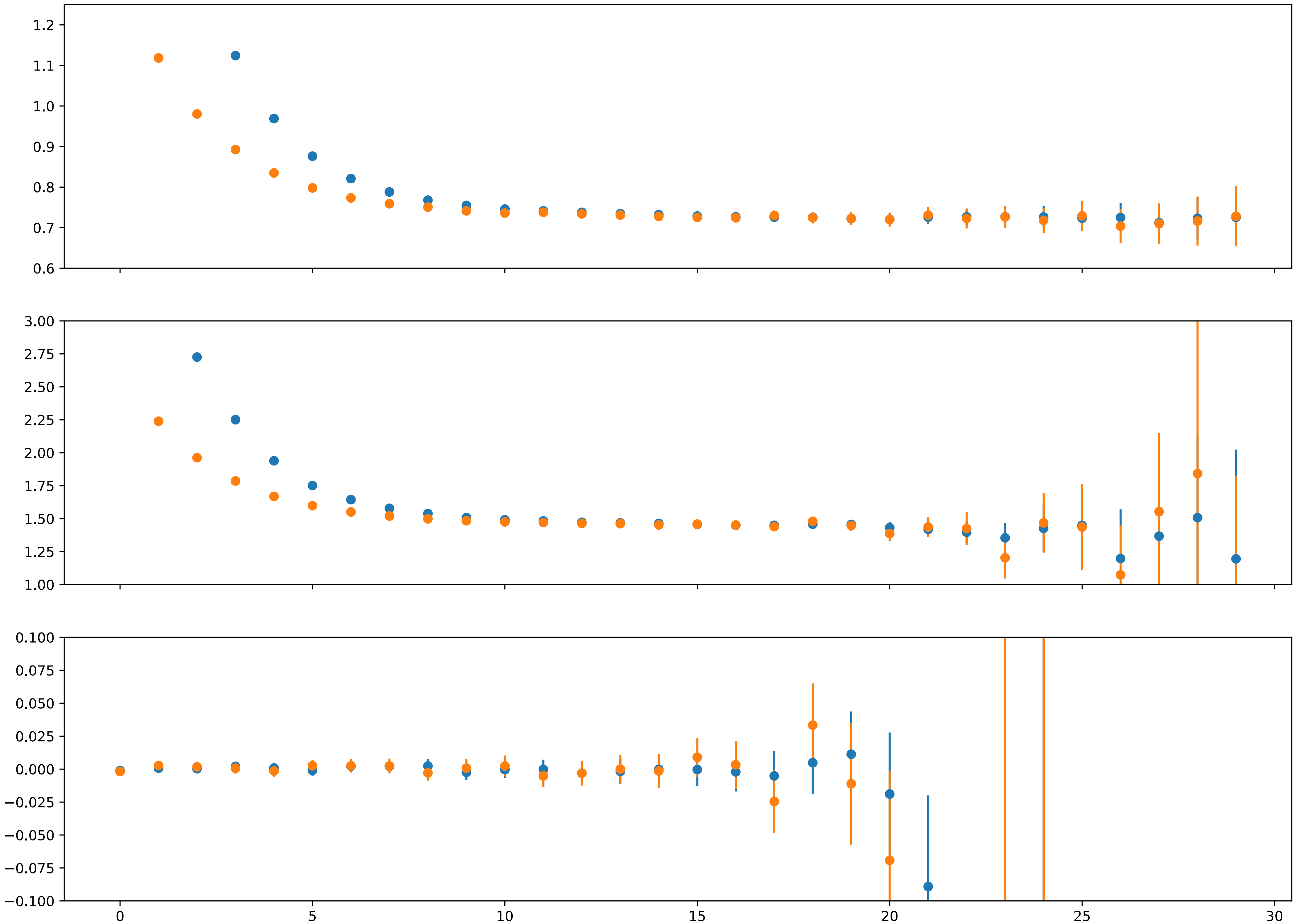
Individual correlators

mπ~700 MeV
gauss source

N

NN

ratio



point sink
gauss sink

Individual correlators

mπ~700 MeV
gauss source

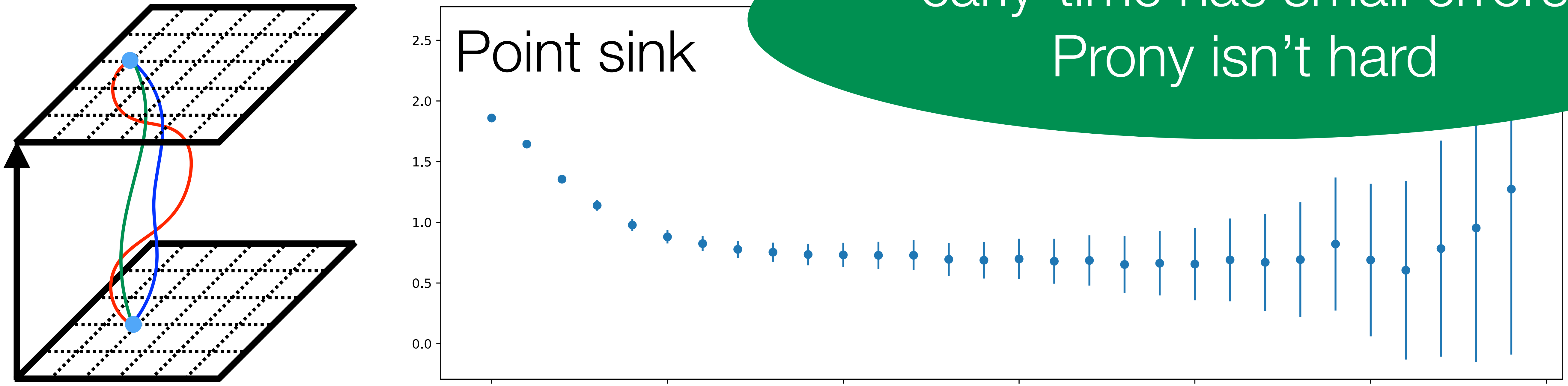
Suspicious coincidence

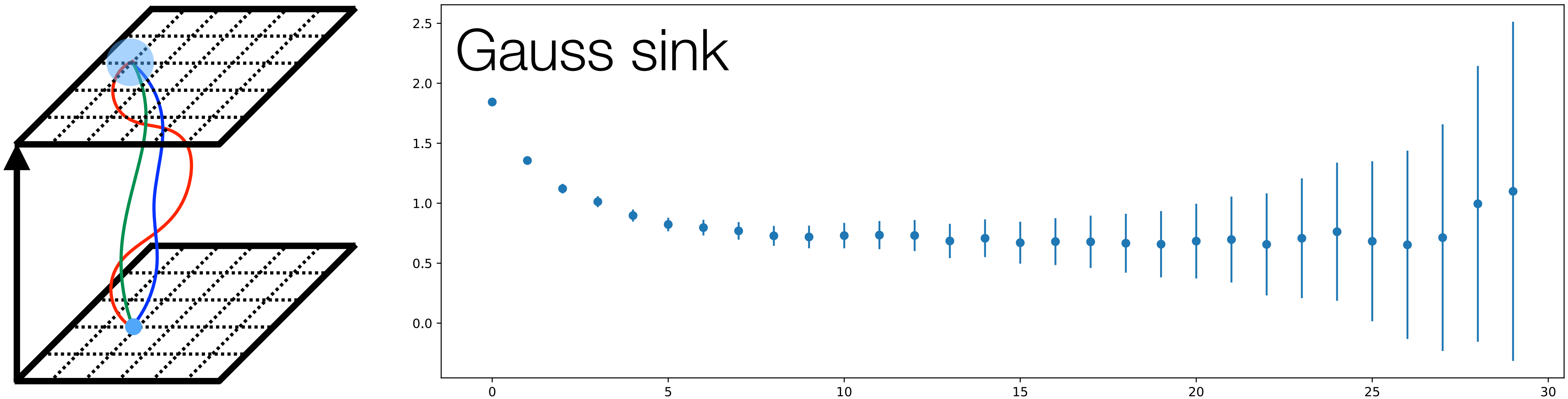
- Ratio does better than is really justified
- Ratio plateau starts way earlier than N or NN plateaus
- Matrix Prony on multiple NN signals doesn't help much

NN excited states are mostly single-nucleon excitations?

Single Nucleon Operator

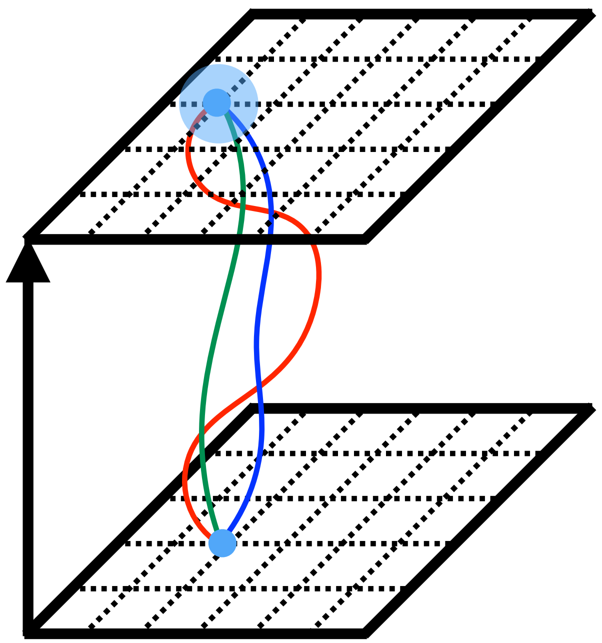
early-time has small errors
Prony isn't hard

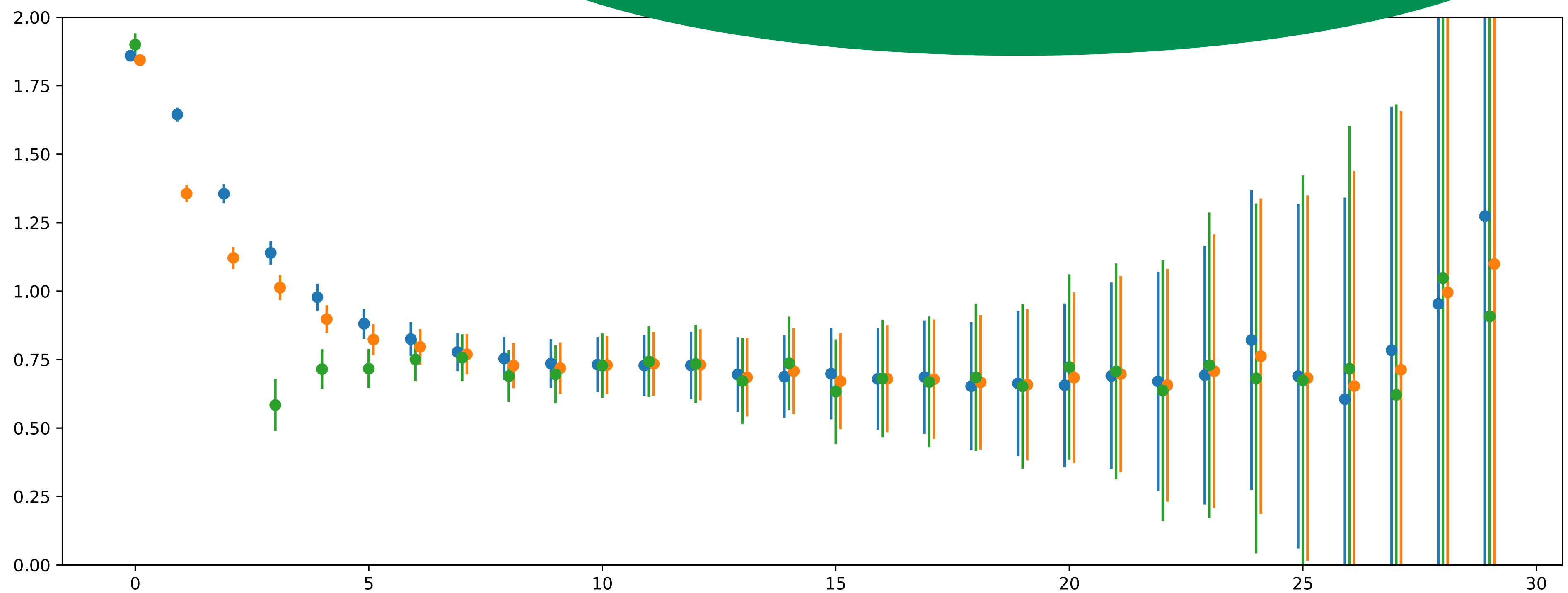




Single Nucleon Operator

LC lengthens N plateau
from t~10 to t~5

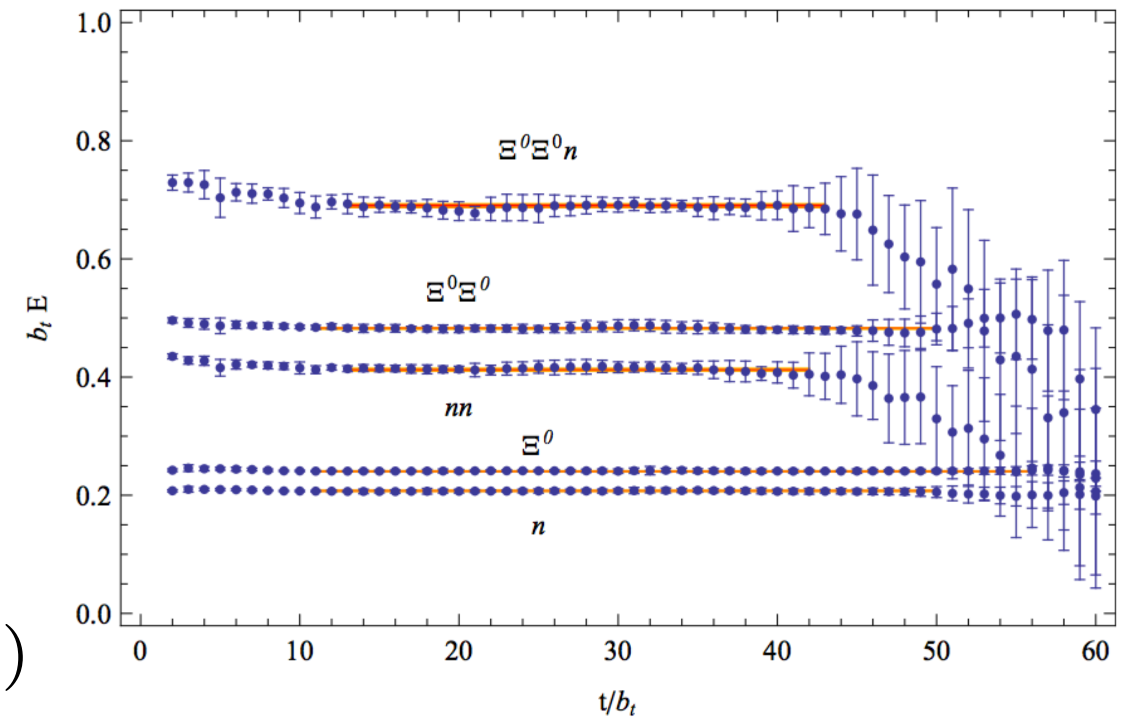




- Determining linear combination doesn't require large statistics
- No additional inversions.
- One (or more) smearing + single-nucleon contraction

This is NOT the same as linear combinations of NN

Beane et al. (NPLQCD) 0905.0466



$$NN^{(pt)} = N^{(pt)} N^{(pt)}$$

$$NN^{(smr)} = N^{(smr)} N^{(smr)}$$

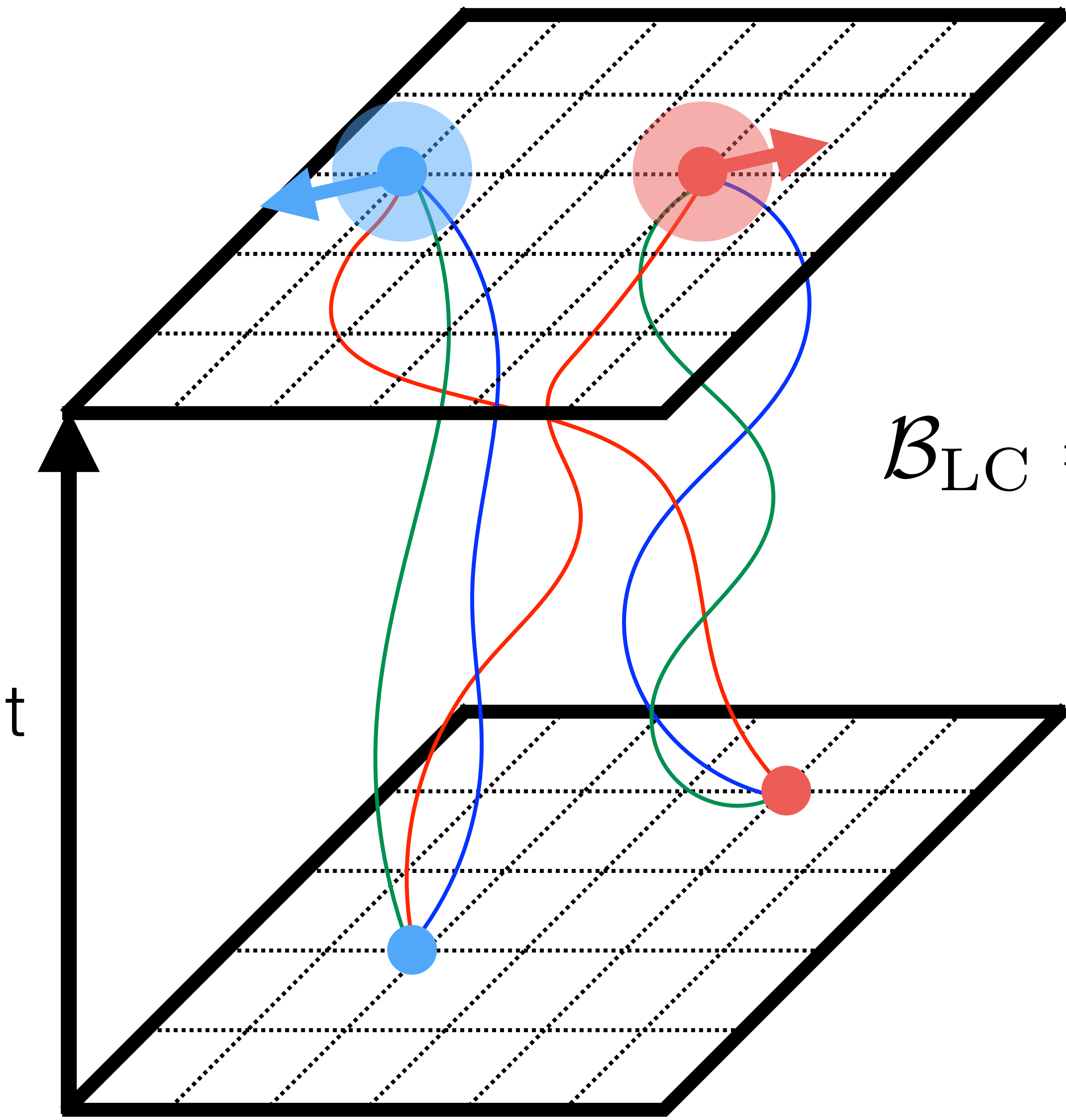
$$\text{Prony}(NN) = \alpha NN^{(pt)} + \beta NN^{(smr)}$$
$$= \alpha N^{(pt)} N^{(pt)} + \beta N^{(smr)} N^{(smr)}$$

$$NN^{(LC)} = N^{(LC)} N^{(LC)}$$
$$= (\gamma N^{(pt)} + \delta N^{(smr)})(\gamma N^{(pt)} + \delta N^{(smr)})$$

difference is cross terms

Easy to Incorporate into Baryon Blocks

Doi & Endres 1205.0585, Detmold & Orginos 1207.1452



multiple calls to baryon block construction routine

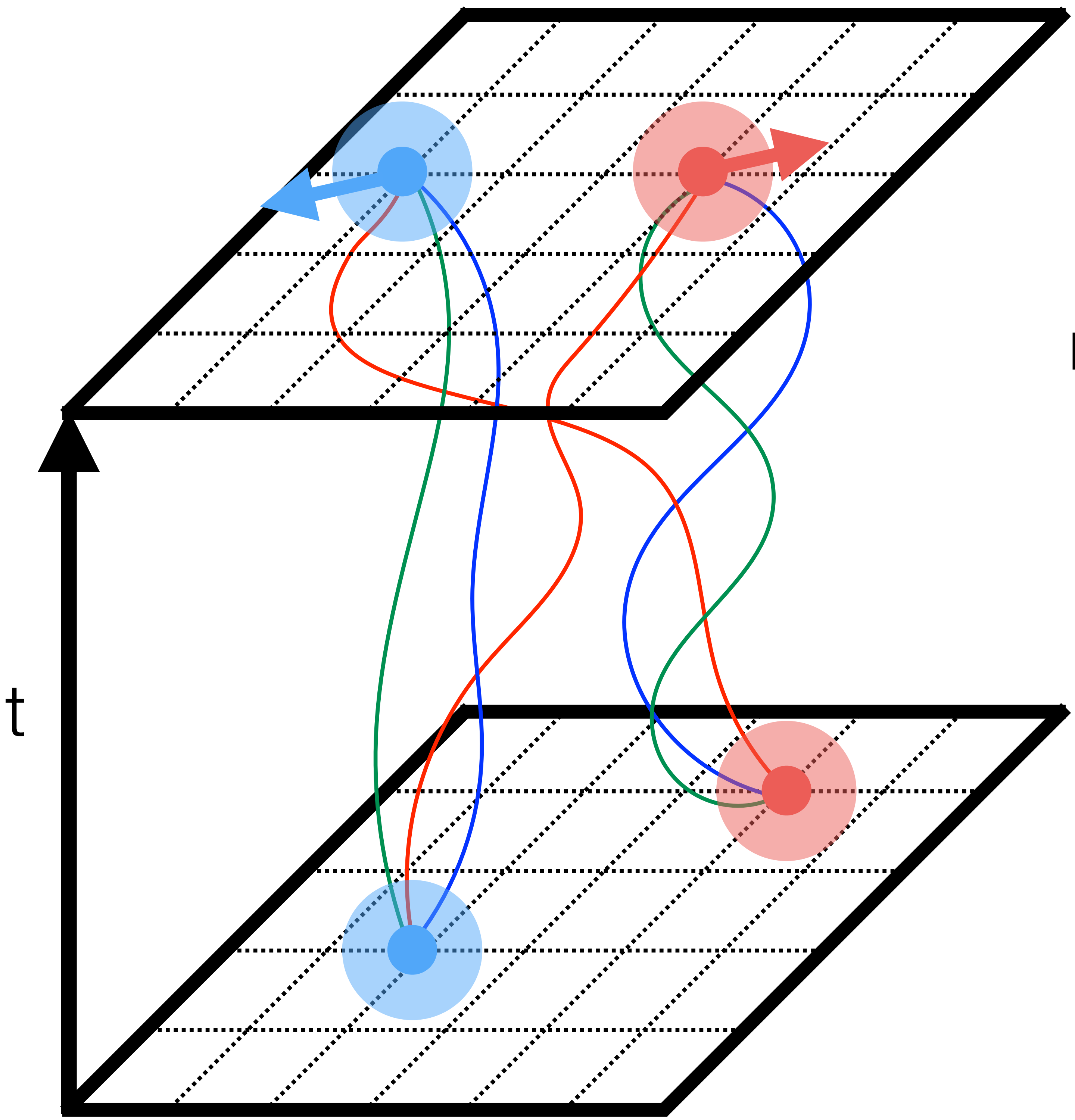
$$\mathcal{B}_{\mathrm{LC}} = \sum_j c_j \mathcal{B}(\mathrm{smearing}_j(S))$$

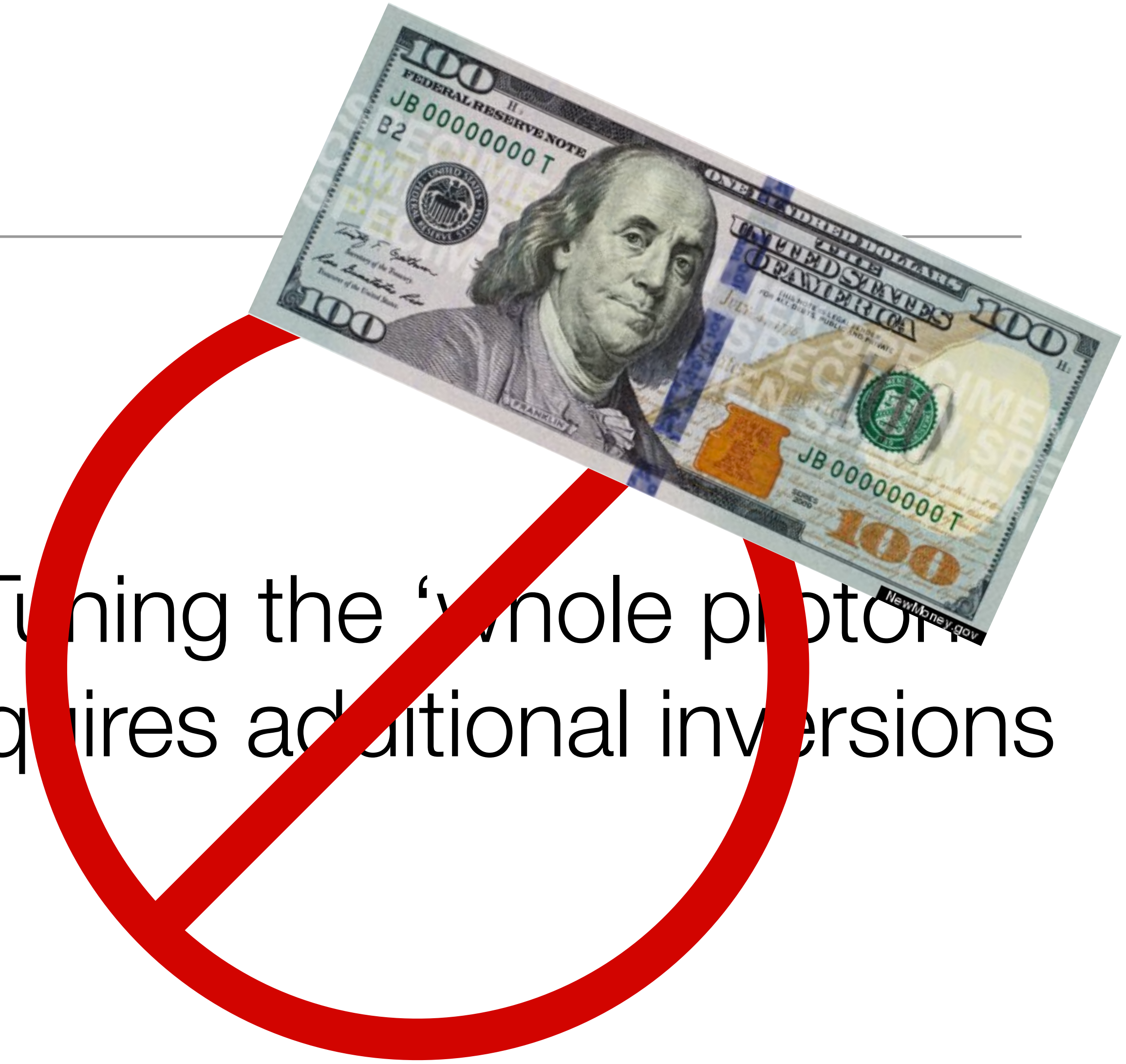
can be other baryon operators, too
e.g. HadSpec quark-displaced ops

same calls to tensor contraction routine

$$C_{\mathrm{LC}} = C(\mathcal{B}_{LC})$$

Source Improvement?





Tuning the ‘whole proton’ requires additional inversions

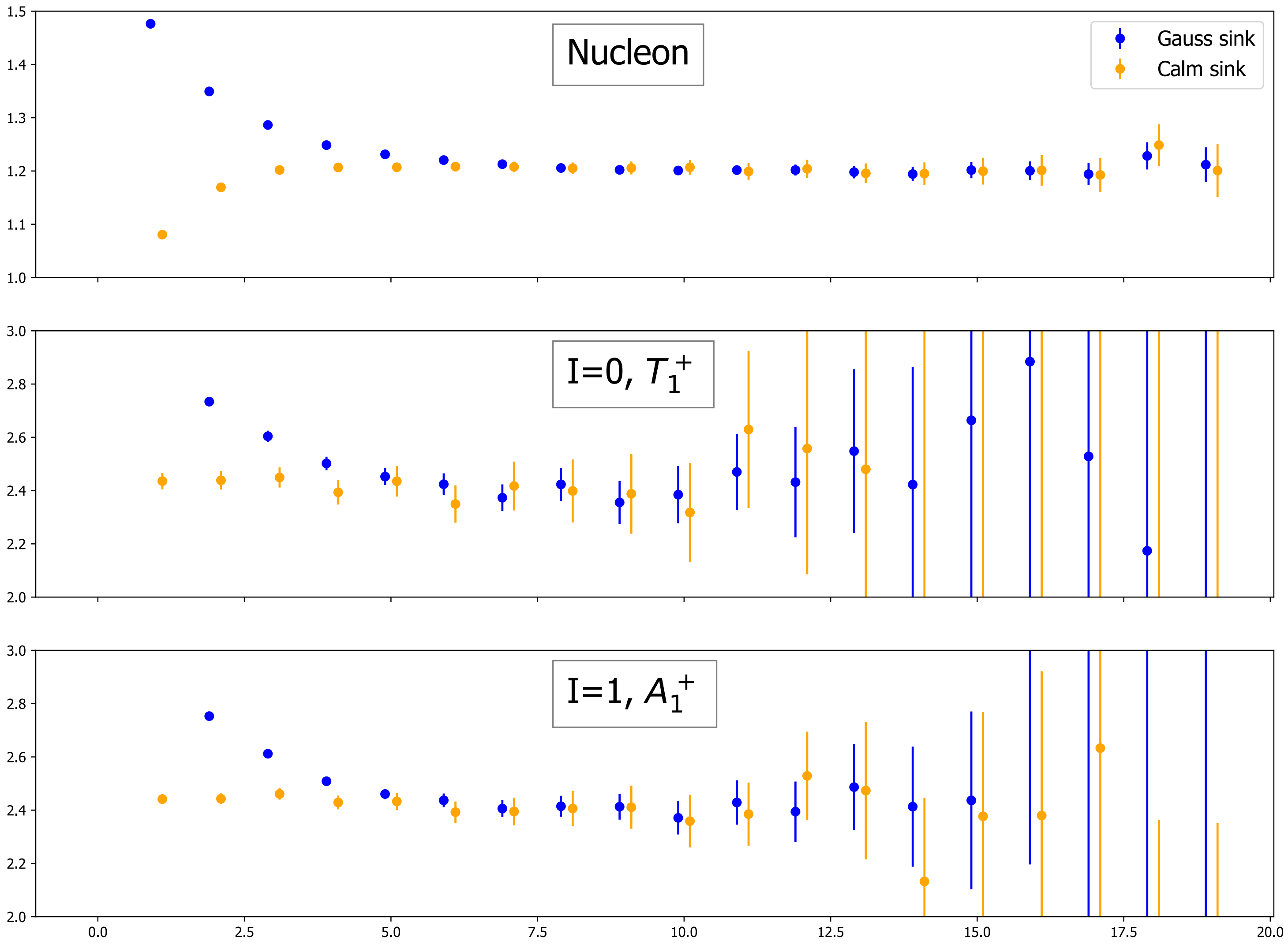
Adjusting quark sources is feasible but nonlinear

so far, unneeded

Defenestrated multi-nucleon Operators?

In preparation

- Easy to code
- Easy to tune
- Marginal additional cost (no extra inversions)
- Works even better on spatially displaced NN sources
  1508.00886
- Don't fight the noise
  But you still could!
  eg. Wagman & Savage 1704.07356
- Substantially longer plateaus for not much more work.



Defenestrated multi-nucleon Operators?

In preparation

- Ex-post-facto justification for fitting ratio?
- Plateau crisis? NN Prony or excited-state analysis should settle the issue
- Important single-N inelastic excited states in NN signal
- Not baryon (or QCD) specific, quite generally applicable
  1706.06494 / C. Körber 22 June 16:20
- Can be used in matrix element calculations etc.