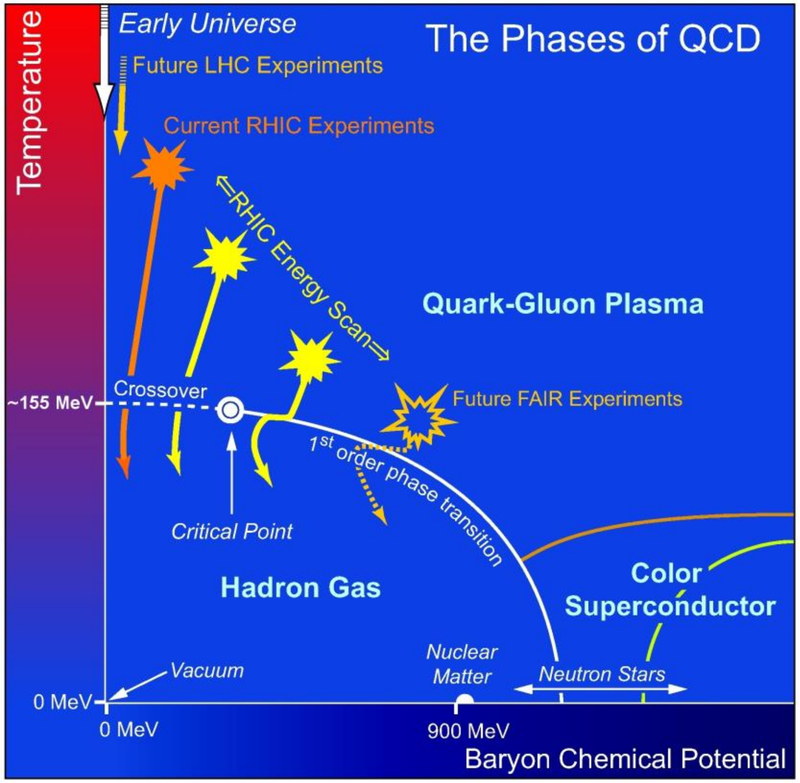
Hot QCD and Quantum Simulations

Péter Petreczky

BROOKHAVEN NATIONAL LABORATORY



The system created at RHIC behaves like perfect liquid (2005) How does the system thermalize ?

Is there is a critical point on the QCD phase diagram ? (2019-2021)

INT, Novemver 14-15, 2017

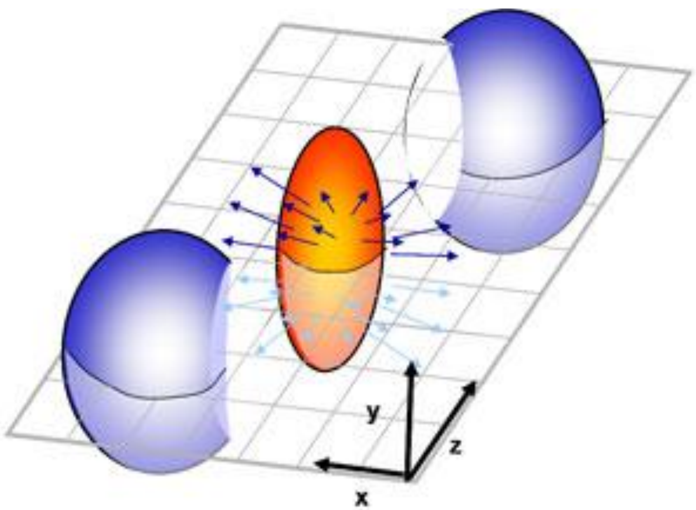
Viscous hydrodynamics and flow

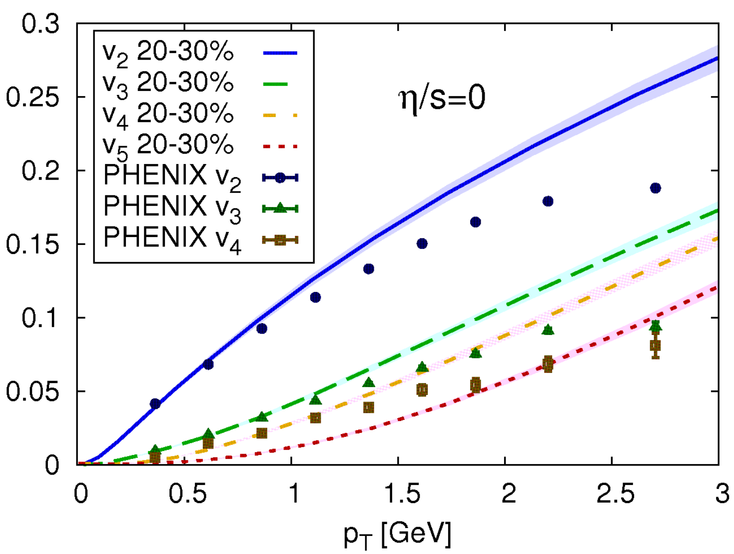
Assume that a thermal system is created shortly after the collisions that expands hydrodynamically.

To describe the experimental data very small shear viscosity to entropy ratio is needed

RHIC Scientists Serve Up "Perfect" Liquid, New state of matter more remarkable than predicted -raising many new questions
April 18, 2005

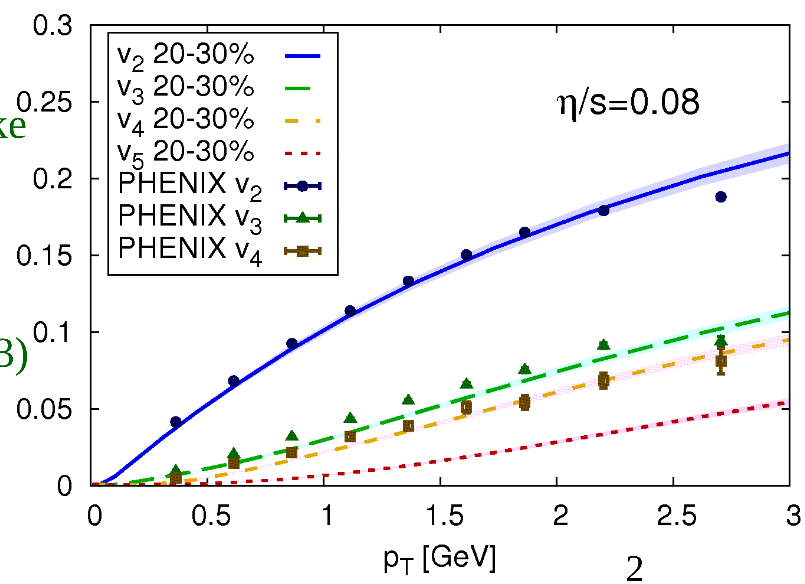
$$\frac{dN}{d\Phi} = v_0\left(1 + 2\,v_1\,\cos(\Phi) + 2\,v_2\,\cos(2\Phi)\right)$$





Bjoern Schenke
BNL

Gale et al,
PRL 110 (2013)
012302

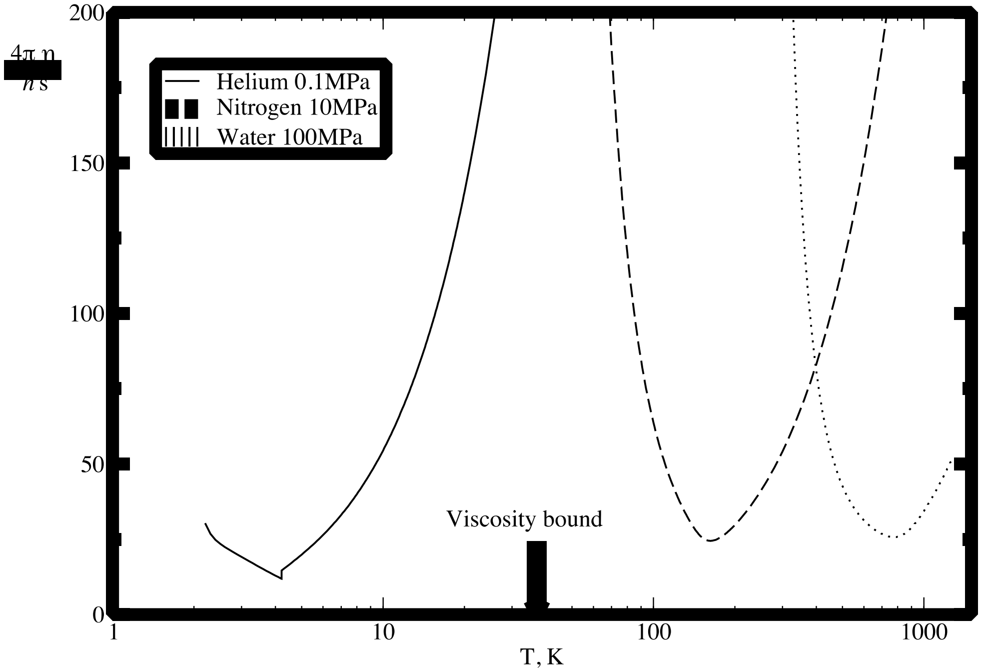


How small is the shear viscosity ?

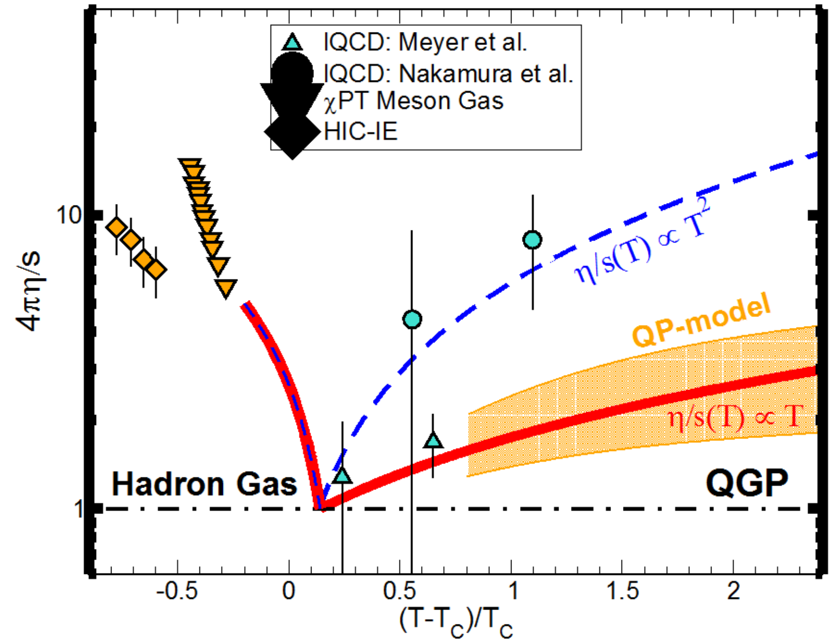
Validity of the hydrodynamics is governed by η/s
Hadron gas and QGP at very high temperature have large value η/s

Super-symmetric gauge theories at strong coupling have small η/s with lower bound dictated by quantum mechanics $\eta/s>1/(4\pi)$ (Kovtun, Son Starinets 2005)
⇒QGP near the transition temperature T_c has close to minimal η/s

Kovtun, Son Starinets, 2005



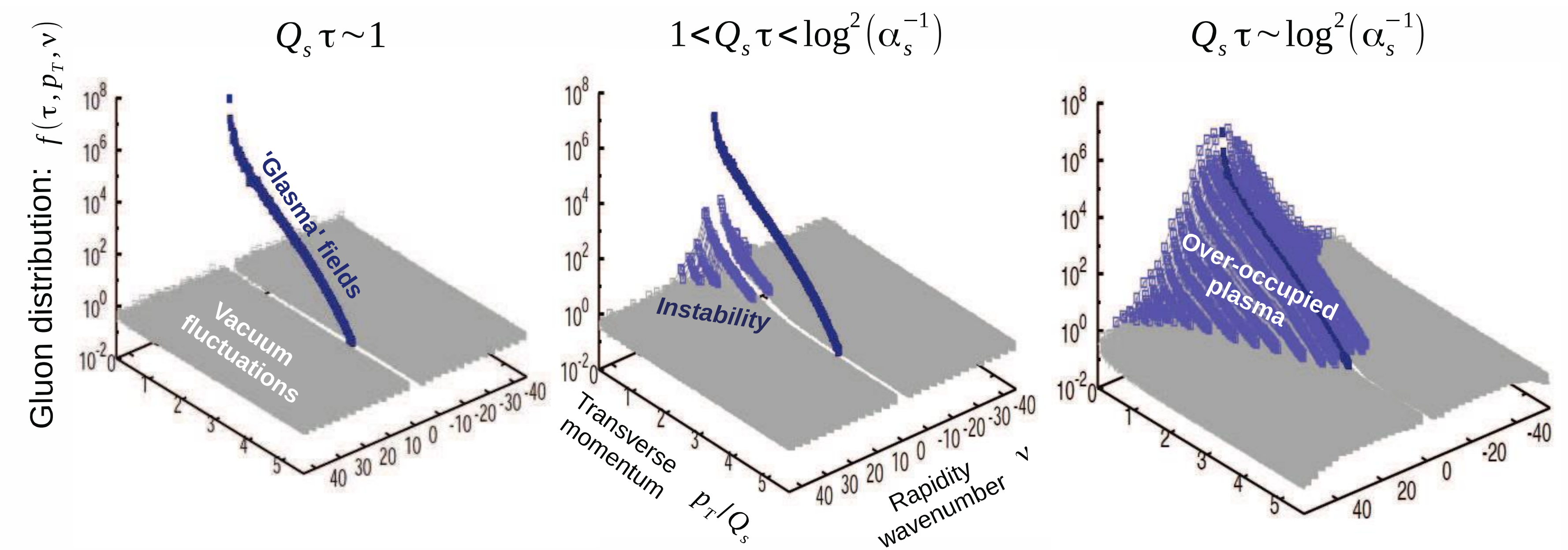
Csernai et al, 2013



Initial time dynamics and thermalization in heavy ion collisions

Classical-statistical calculations of gluon distribution at early times (large gluon occupation numbers)

Berges Schenke, Schlichting, Venugopalan, Nucl. Phys. A 931 (2014) 348



The gluon occupation number decreases at later times reaching *O(1)*, the system becomes quantum and strongly coupled

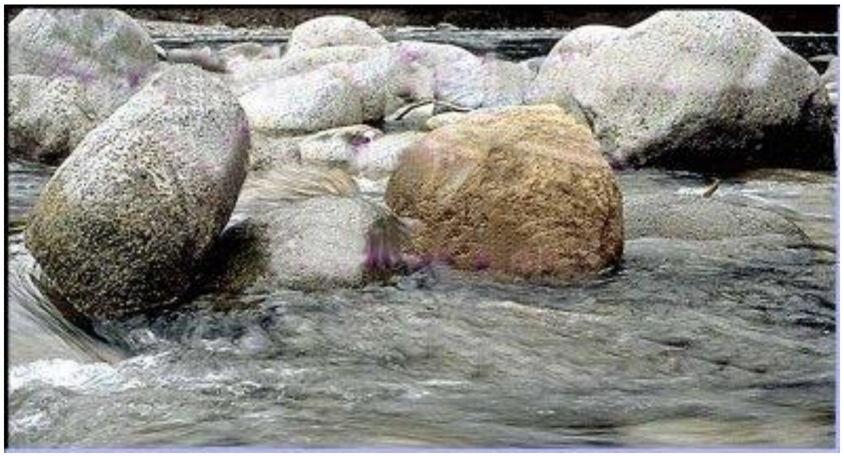
➡ quantum simulations are needed

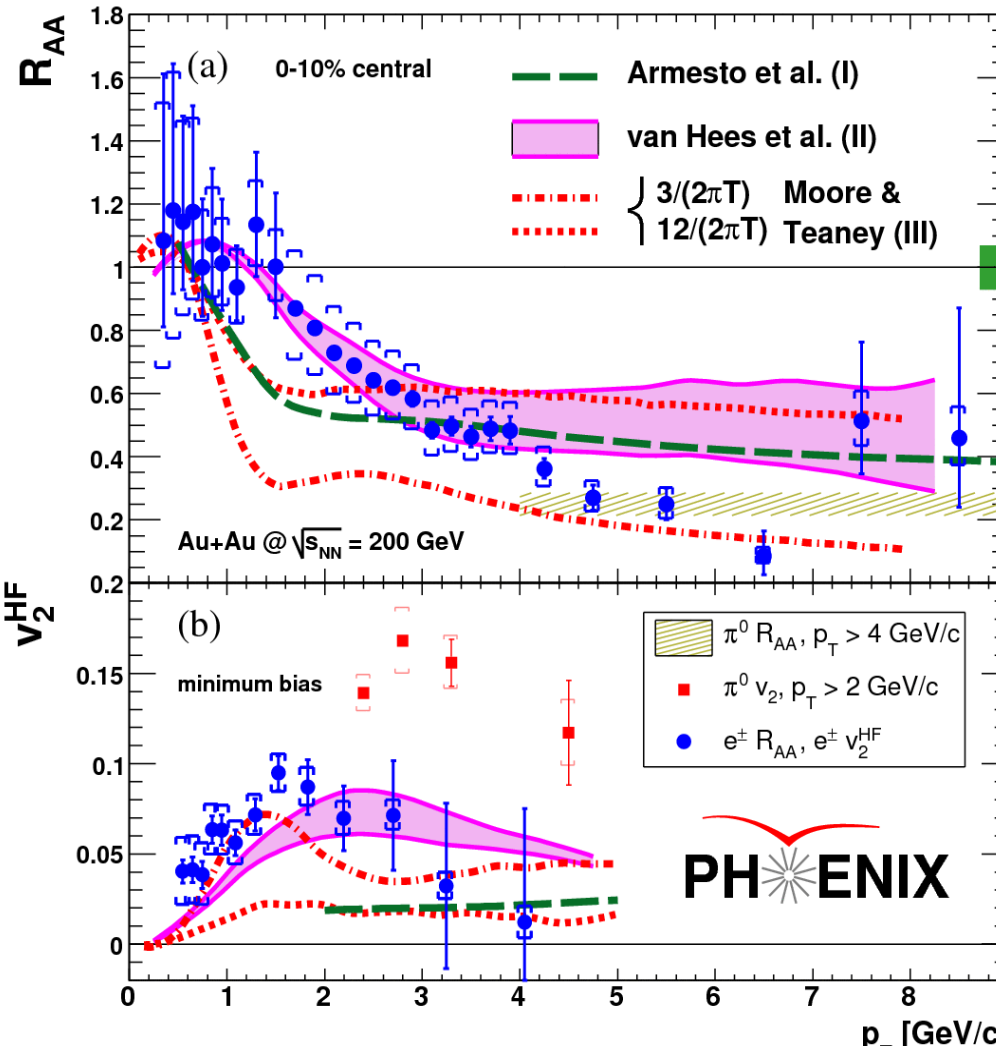
Early time dynamics is important event-by-event fluctuations in *AA*, and high multiplicity *pA* and *AA* collisions

Strongly coupled QGP and heavy quarks

Heavy quarks ($M_c \sim 1.5\ GeV$) flow in the strongly coupled QGP

Analogy from Jamie Nagle





$$t_{rel}^{heavy} \sim \frac{M_c}{T} t_{rel}^{light} \Rightarrow \text{Langevin dynamics:}$$

$$\frac{dx^i}{dt} = \frac{p^i}{M}, \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

$$\langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

$$\eta = \frac{\kappa}{2MT}, \quad D = \frac{T}{M\eta}$$

Finite Temperature QCD and its Lattice Formulation

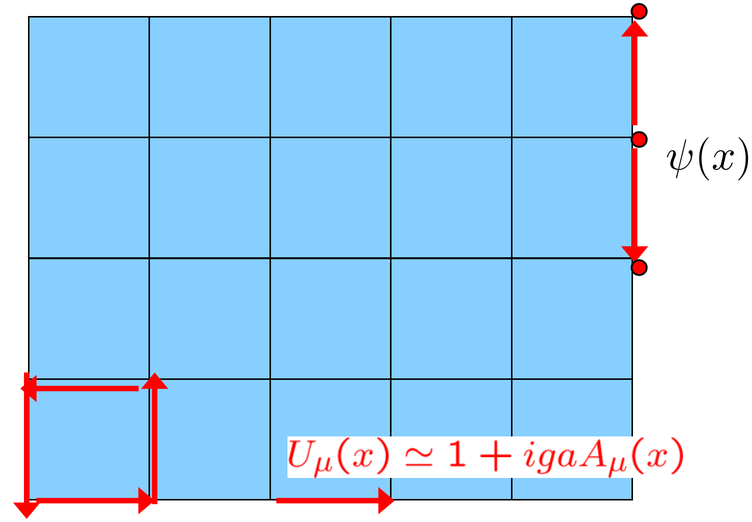
$$\langle O \rangle = \mathrm{Tr} O e^{-\beta H - \mu N} \qquad \beta = 1/T$$

$$\langle O \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}}$$

$$A_\mu(0,\mathbf{x}) = A_\mu(\beta,\mathbf{x}) \quad \psi(0,\mathbf{x}) = -\psi(\beta,\mathbf{x})$$

Lattice

integral with very large dimensions



$$\langle O \rangle = \int \prod_x dU_\mu(x) O(det D_q[U,m,\mu]) e^{-\sum_x S_G[U(x)]}, \; U_\mu(x) = e^{igaA_\mu(x)}$$

$\mu = 0$ → Monte-Carlo Methods

cost ~ $1/a^7$

$\mu \neq 0 : \; det D_q(U,m,\mu)$ complex → sign problem → Taylor expansion for not too large μ

$$\frac{p(T,\mu)}{T^4} = \sum_{n=1}^{\infty} \frac{1}{(2n)!} \chi_{2n}(T) \mu^{2n}$$

Calculable in LQCD but the computational difficulty increases with *n* !
(noise problem vs. sign problem)
Current calculations exist only to *n*=6.

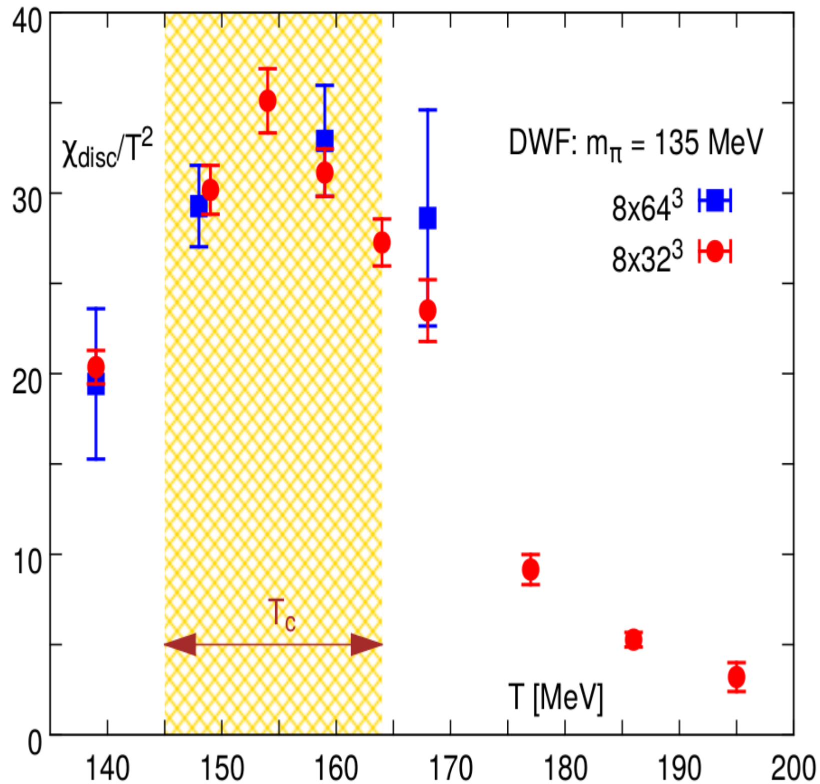
LQCD Accomplishments: The Transition in QCD

The QCD transition at zero net baryon density is a smooth crossover (not a phase transition) the cross-over temperature has been determined in the continuum limit

Fluctuations of the order parameter:

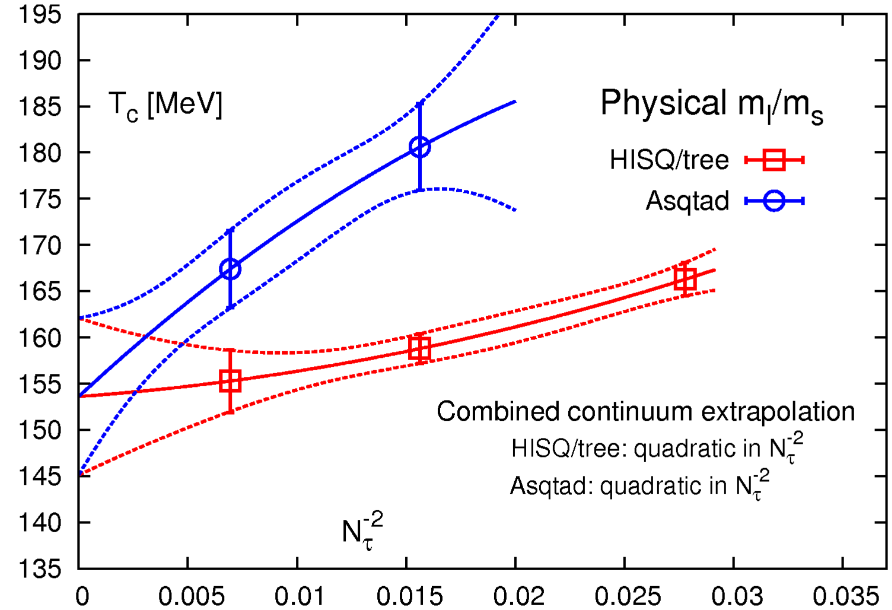
$$\chi_{disc} = VT^{-1}\left(\langle(\bar{\psi}\psi)^2\rangle - \langle\bar{\psi}\psi\rangle^2\right)$$

Bhattacharya et al (HotQCD), PRL 113 (2014)082001



Bazavov et al, PRD85 (2012) 054503

$T_c = (154 \pm 8 \pm 1(scale))$MeV



no increase with the volume

$\Rightarrow$ Crossover transition

Aoki et al, Nature 443 (2006) 675

Equation of state in the continuum limit

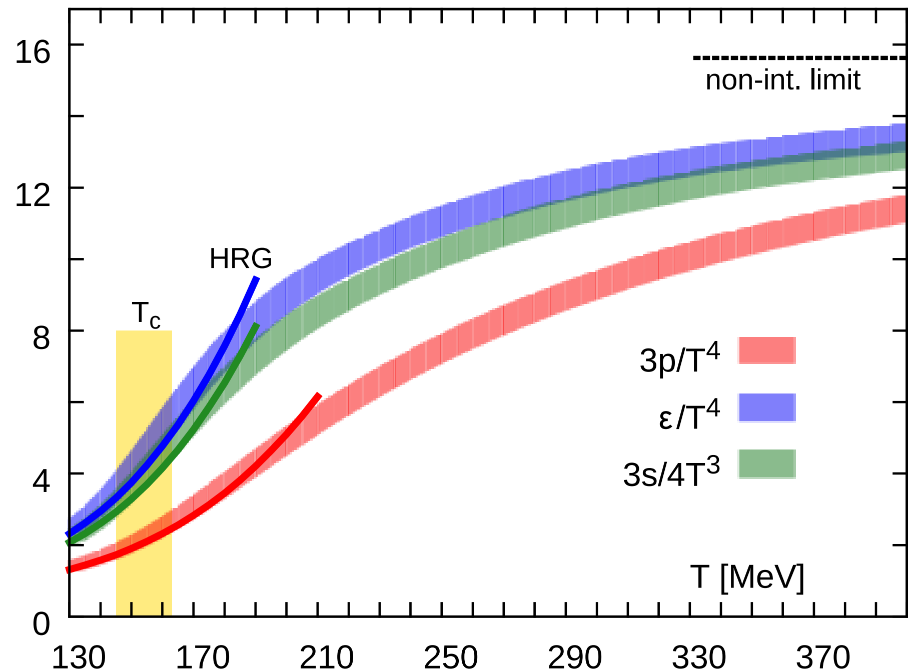
Equation of state has be calculated in the continuum limit up to *T=400* MeV

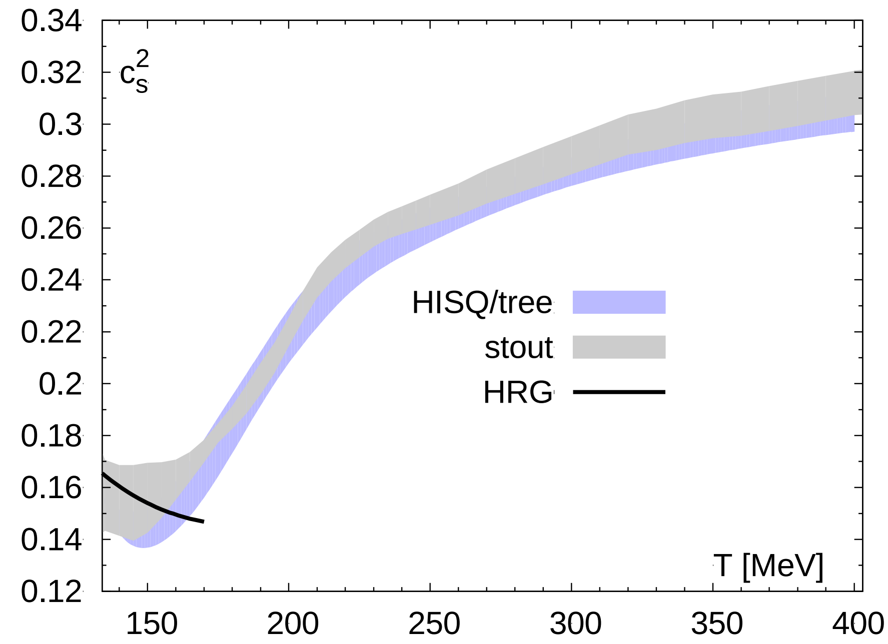
$$p \sim \rho T, \;\; \rho \sim T^3$$

in ultra-relativistic case

Calculations that use two different discretization schemes agree:

Bazavov et al, PRD 90 (2014) 094503





Hadron resonance gas (HRG):
Interacting gas of hadrons = non-interacting gas of hadrons and hadron resonances
(virial expansion, Prakash & Venugopalan)

HRG agrees with the lattice for *T< 145* MeV

$$T_c = (154 \pm 9)\text{MeV}$$

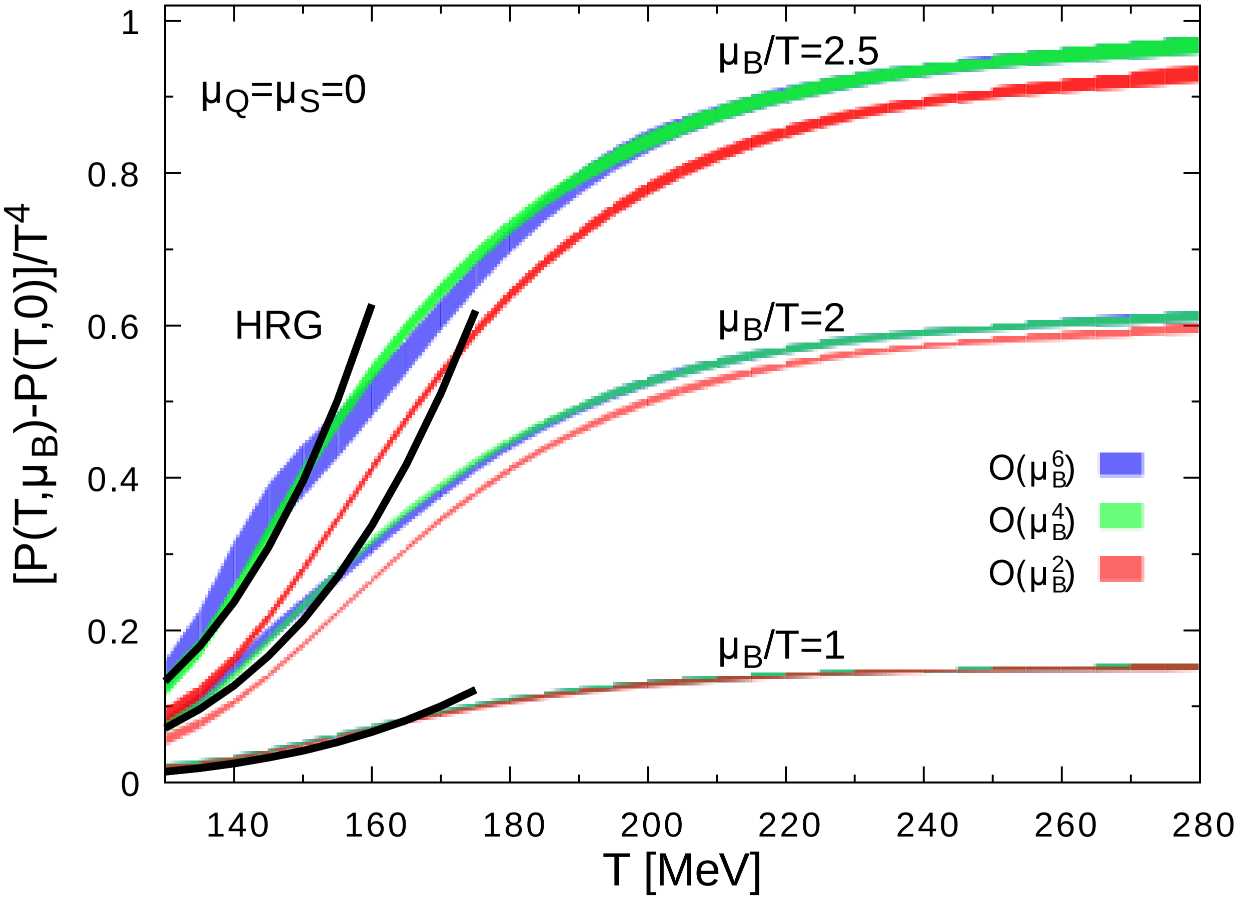
$$\epsilon_c \simeq 300\text{MeV}/\text{fm}^3$$

$$\epsilon_{low} \simeq 180\text{MeV}/\text{fm}^3 \leftrightarrow \epsilon_{nucl} \simeq 150\text{MeV}/\text{fm}^3$$

$$\epsilon_{high} \simeq 500\text{MeV}/\text{fm}^3 \leftrightarrow \epsilon_{proton} \simeq 450\text{MeV}/\text{fm}^3$$

Thermodynamics at non-zero net baryon density

6^{th} order Taylor expansion, Bazavov et al, PRD 95 (2017) 054504



Truncation errors of the 6^{th} order Taylor expansions are small for $\mu_B/T<2.5$

Critical point is strongly disfavored for $\mu_B/T<2.0$

Correlation functions and transport coeffocients

Transport coefficients are encoded in the spectral functions:

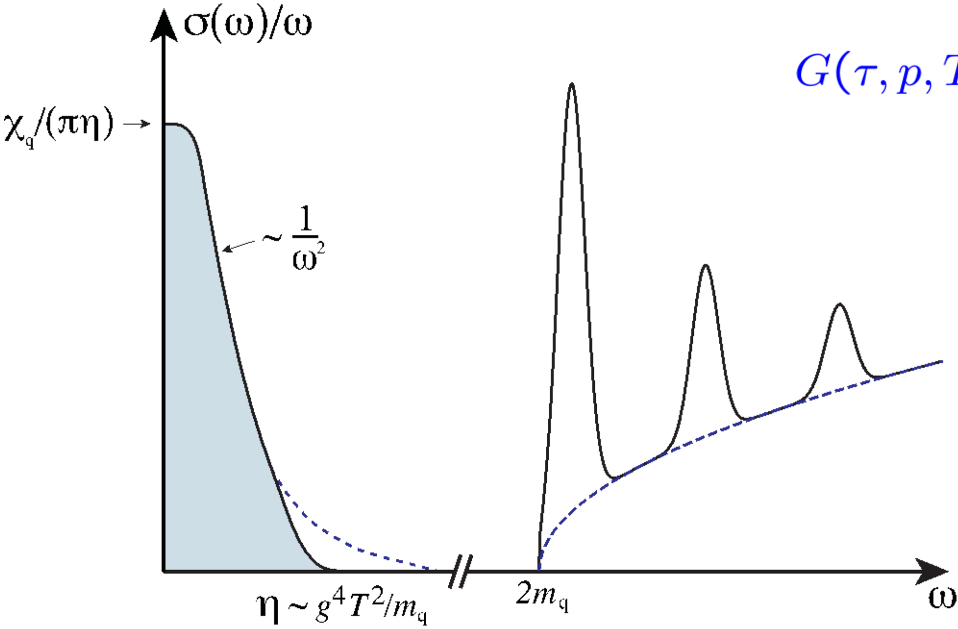
$$\sigma(\omega,p,T) = \frac{1}{2\pi}\,\mathrm{Im}\int_{-\infty}^{\infty} dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t),J(x,0)]\rangle_T$$

In LQCD one can calculated the Euclidean time

$$\text{transport coefficients} = \lim_{\omega\to 0}\frac{\sigma(\omega)}{\omega}$$

$$G(\tau,p,T) = \int d^3x e^{ipx}\langle J(x,-i\tau),J(x,0)\rangle_T$$

Due to analytic continuation $G(\tau,T) = D^{>}(-i\tau)$



$$G(\tau,p,T) = \int_0^{\infty} d\omega\,\sigma(\omega,p,T)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Challenge: resolve a potentially narrow transport peak at zero energy

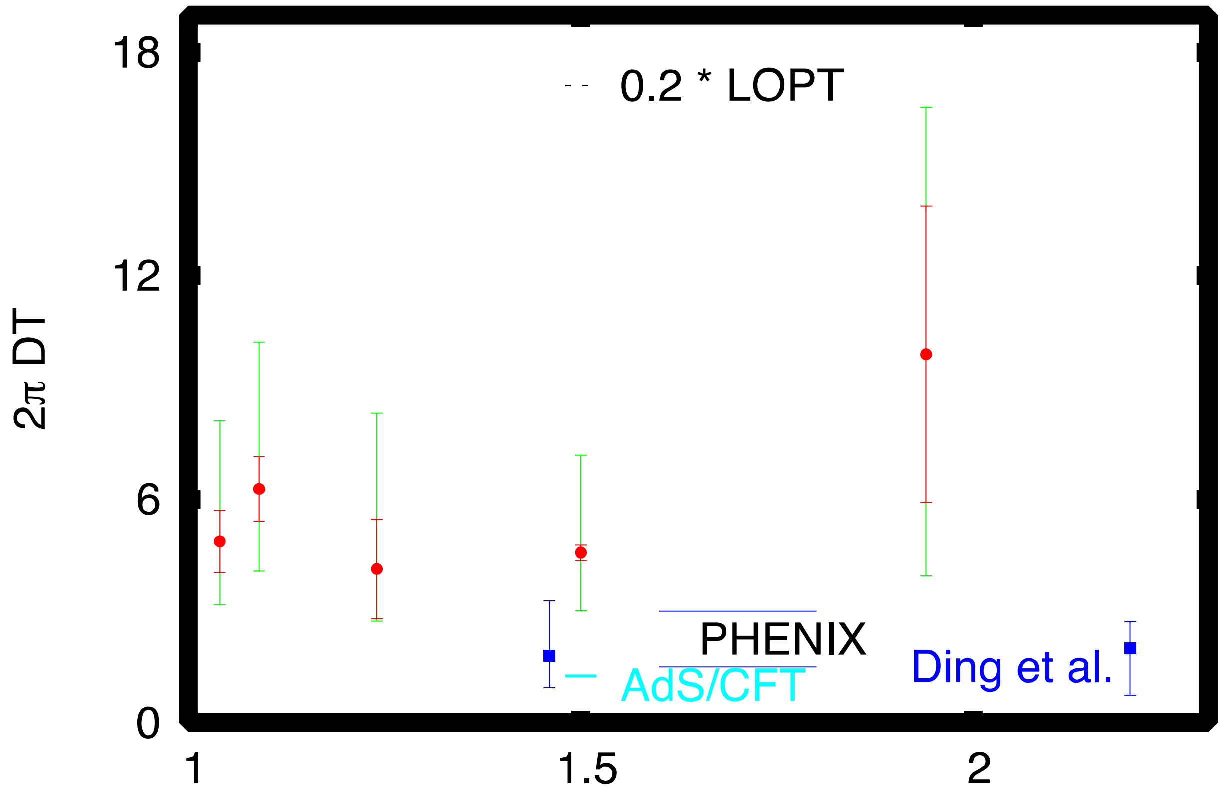
with temporal extent in Euclidean time that is limited by *1/T*

Heavy quark diffusion constant from quenched LQCD

Direct method: determine the width of the transport peak,
Ding et al, arXiv:1204:4954, quenched $128 \times N_\tau$ lattices, N_τ=24-48

Integrate out the heavy quark fields: $< J_i(\tau)\, J_i(0) > \Rightarrow <E_i^a(\tau)\, E_i^a(0)>$
Banarjee et al, arXiv:1109.5738, Kaczmarek et al, arXiv:1109:3941, N_τ=16-24

Lattice find values of D consistent with experiment and sQGP scenario



the width of
the transport peak is
potentially
overestimated

Open questions

- How does the system created at RHIC thermalizes at time scales < 1fm/c ?

What is the effect of the early time out of equilibrium stage on experimental observables ?

- What is the shear viscosity and heavy quark diffusion coefficient as function of temperature

- Where is the critical point of the QCD phase diagram ? Or is the QCD transition is always a crossover ?

⇒ Quantum Simulations of QCD !

Hamiltonian formulation of gauge theories

Consider continuous time and discretized space :

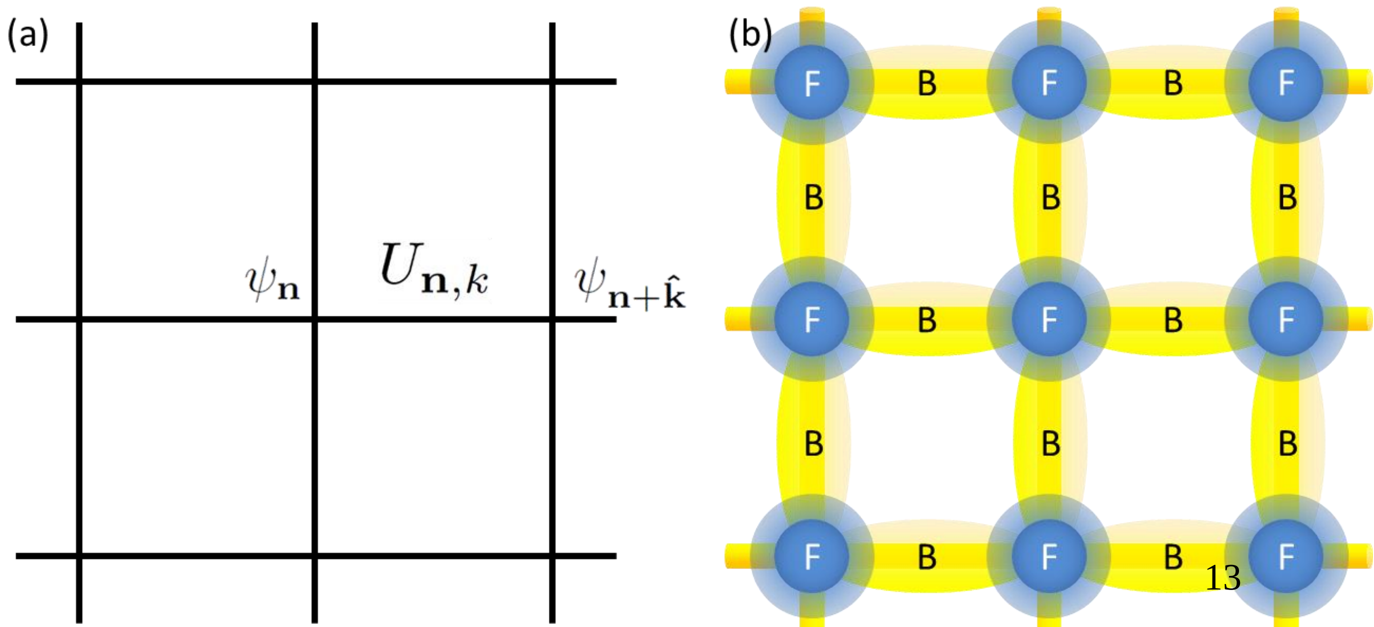
$$H = H^G + H^M$$

$$H^G = \frac{g^2}{2}\sum_{n,k} L_{n,k}^2 - \frac{1}{2g^2}\sum_{plaq} \mathrm{tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger + h.c.\right) \qquad L_{\mathbf{n},k} = -i\dot{U}_{\mathbf{n},k} U_{\mathbf{n},k}^\dagger$$

$$H^M = \sum_{n,k}(\psi_n^\dagger U_{n,k}\psi_{n+k} + h.c.) + m\sum_n (-1)^{\sum_k n_k}\psi_n^\dagger \psi_n^\dagger \psi_n$$

Simulate using interacting cold atom systems in optical lattices:

Zohar, Cirac, Reznik, Rep. Prog. Phys. 79 (2016) 014401



Realizations of gauge theories on optical lattices and challenges

Concrete proposal for quantum simulations:

- Z_N gauge theories in 2+1 dimensions: Zohar, Brunello, PRD 91 (2015) 054506, Zohar, Farace, Reznik, Cirac, PRL 118 (2017) 070501, PRA 95 (2017) 023604

- U(1) Higgs model in 2+1 dimensions: Gonzales-Cuadra, Zohar, Cirac, New. J. Phys. 19 (2017) 063038

Theoretical challenges:
Finite Hilbert space, continuum limit ?
Higher dimensions ?

Experimental:

1) Generating complex lattices and superlattice structures
2) Better control of scattering parameters
3) Longer lived lattices
4) Improved measurement techniques

Summary

- The are compelling questions in hot QCD that require quantum computations:

1) What is the QCD phase diagram at high baryon density ? Is there critical point ?
2) How does thermalization in ultra-relativistic heavy ion collisions happen ?
3) What are the QCD transport coefficients ?

- Quantum simulations using optical lattices may provide an avenue addressing these questions but many open challenges remain