Polarized parton production in DIS at small x

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in collaboration with

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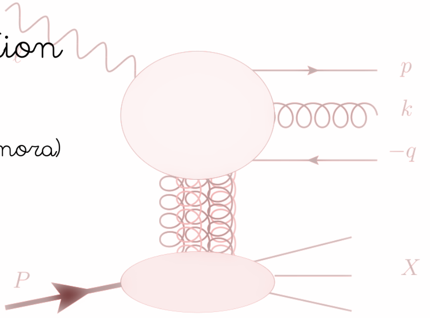


INSTITUTE for NUCLEAR THEORY

Probing QCD in γ-A Interactions at RHIC and LHC: the Path to EIC

INT Workshop INT-17-65W, February 13 - 17, 2017

Intro

- Effective action approach to gluon saturation which describes a high energy hadron or nucleus as a CGC: allows to apply semi-classical methods to particle production in high energy hadronic collisions L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994), Phys. Rev. D **49**, 3352 (1994)

- CGC: a state of high occupancy number which is a weakly-coupled yet non-pert system of gluons characterized by a semi-hard scale Q_s *saturation scale*

- importance of saturation physics in the small x regime of high energy collisions is well established (e.g. review by F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, Ann. Rev. Nucl. Part. Sci. **60**, 463 (2010))

- pheno studies are needed to constrain the parameters of the approach and to test its kinematic limits

Intro

- common $e + p$, $e + A$ DIS studies: inclusive obs. focus on F_2 and F_L, e.g. (G. Beuf, Phys. Rev. D **85**, 034039 (2012); I. Balitsky and G. A. Chirilli, Phys. Rev. D **83**, 031502 (2011), Phys. Rev. D **87**, no. 1, 014013 (2013); R. Boussarie, A. V. Grabovsky, L. Szymanowski and S. Wallon JHEP **1409**, 026 (2014); A. Kovner and U. A. Wiedemann, Phys. Rev. D **64**, 114002 (2001); J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D **70**, 114017 (2004))
- gives access 2-pt correlations of Wilson lines in the target (encoded in the extracted color dipole factors)
- motivation for studies of gluon saturation dynamics is far richer: CGC observables in terms of multi-point correlators of Wilson lines
- famous one *quadrupole*: 4-pt correlator of Wilson lines which appears in multi-parton production processes in DIS and pA collisions (A. Kovner and U. A. Wiedemann, Phys. Rev. D **64**, 114002 (2001); J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D **70**, 114017 (2004); A. Kovner and M. Lublinsky, JHEP **0611**, 083 (2006); F. Dominguez, C. Marquet, B. -W. Xiao and F. Yuan, Phys. Rev. D **83**, 105005 (2011))

Intro

- *quadrupoles* have more info than *dipoles*, but more difficult to constrain with experiment
- Use DIS clean experimental environment, observe multi-particle final states
- di-hadron azimuthal angular correlations: access to quadrupoles and key process in saturation searches at future Electron Ion Colliders (see *e.g.* L. Zheng, E. C. Aschenauer, J. H. Lee and B. W. Xiao, Phys. Rev. D **89**, no. 7, 074037 (2014).)
- di-hadron azimuthal angular correlations involve only one relative angle

Our proposal

We propose to use azimuthal angular correlations of 3 partons in inclusive DIS to explore the dynamics of saturated partonic matter

Reported in:

- *Polarized 3 parton production in inclusive DIS at small x*; A. Ayala, M. Hentschinski, J. Jalilian-Marian, M.E.T-Y; Phys.Lett. B761 (2016) 229-233, arXiv:1604.08526 [hep-ph]
- *Spinor helicity methods in high-energy factorization: efficient momentum-space calculations in the Color Glass Condensate formalism*; A. Ayala, M. Hentschinski, J. Jalilian-Marian, M.E.T-Y (2017), arXiv:1701.07143 [hep-ph]

√ Has an additional handle compared to di-hadron azimuthal angular correlations: 2 relative angles between 3 produced partons

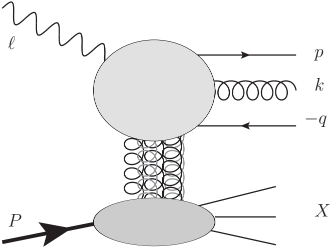
√ unlike 2 parton xsec, 3 parton xsec depends non-linearly on both quadrupoles and dipoles

The process: 3 partons in DIS

Study

$$\gamma^*(\ell) + \text{target}(P) \to q(p) + \bar{q}(q) + g(k) + X$$

in the HE limit: $\sqrt{s} \to \infty$ with $s = (\ell + P)^2$



- $Q^2 = -\ell^2$ photon virtuality
- model target (HE hadron/nucleus) with strong background color field
- *shock wave* $A_\mu \sim 1/g$
- light-cone gauge $A \cdot n = 0$ so $A^-(x^+, x_t) = \delta(x^+)\alpha(x_t)$ and $A_t = 0$

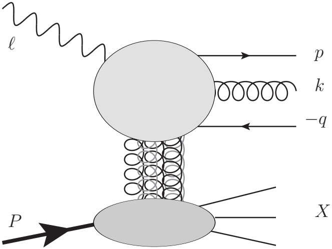
Use light-cone vectors $n, \bar{n}$

Any four vector v given by

$$v_\mu = v^+ \bar{n}_\mu + v^- n_\mu + v_t \,, \quad \text{where} \quad n \cdot \bar{n} = 1, \qquad n^2 = 0 = \bar{n}^2 \,,$$
$$v^+ = n \cdot v, \quad v^- = \bar{n} \cdot v \,, \qquad \text{and} \qquad v_t^2 = -\boldsymbol{v}^2$$

Momenta for target and virtual photon

$$P_\mu = P^- n_\mu \,, \qquad \ell_\mu = \ell^+ \bar{n}_\mu - \frac{Q^2}{\ell^+} n_\mu$$



Background field mom-space q and g propagators

L. D. McLerran and R. Venugopalan, Phys. Rev. D **50**, 2225 (1994); A. J. Baltz, F. Gelis, L. D. McLerran and A. Peshier, Nucl. Phys. A **695**, 395 (2001); F. Gelis and A. Peshier, Nucl. Phys. A **697**, 879 (2002); I. I. Balitsky and A. V. Belitsky, Nucl. Phys. B **629**, 290 (2002)

$$S_{F,il}(p,q) \equiv S^{(0)}_{F,il}(p)(2\pi)^4\delta^{(4)}(p-q) + S^{(0)}_{F,ij}(p)\,\tau_{F,jk}(p,q)\,S^{(0)}_{F,kl}(q)$$

$$G^{ad}_{\mu\nu}(p,q) \equiv G^{(0),ab}_{\mu\nu}(p)(2\pi)^4\delta^{(4)}(p-q) + G^{(0),ab}_{\mu\lambda}(p)\,\tau^{bc}_{G}(p,q)\,G^{(0),cd,\lambda}_{\nu}(q)$$

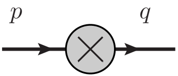
Free mom-space q and g propagators

$$S^{(0)}_{F,ij}(p) = \frac{i\delta_{ij}}{(\not p + i\epsilon)} \qquad \text{and} \qquad G^{(0),ab}_{\mu\nu} = \frac{i\delta^{ab} d_{\mu\nu}(k)}{(k^2+i\epsilon)}$$

where the polarization tensor in the light-cone gauge

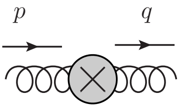
$$d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{n\cdot k}$$

Interaction with background field



$$= \tau_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\ \not{n}$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[V_{ij}(\boldsymbol{z}) - 1_{ij}\right] - \theta(-p^+)\left[V^\dagger_{ij}(\boldsymbol{z}) - 1_{ij}\right]\right\}$$



$$= \tau_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\ (-2p^+)$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[U^{ab}(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left(U^{ab}\right)^\dagger(\boldsymbol{z}) - 1\right]\right\}$$

where Wilson lines in fund (V) and adj (U) reps

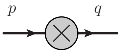
$$V(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+,\boldsymbol{z}) t^c \quad \text{and} \quad U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+,\boldsymbol{z}) T^c$$

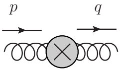
with $-iT^c_{ab} = f^{acb}$.

Amplitudes in presence of strong background field

Convenient to extend conventional QCD mom-space Feynman:

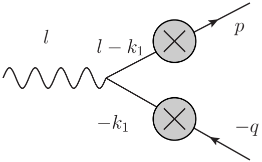
- adding the "vertices"



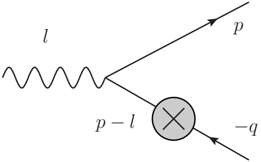


- *internal* momenta p are integrated over with measure $\int \frac{d^4p}{(2\pi)^4}$, as done for loop-momenta

For example:



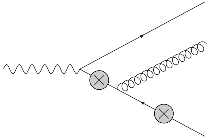
k_1 is integrated over with $\int d^4k_1/(2\pi)^4$



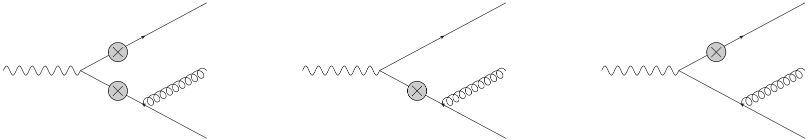
tree diagram with 1 insertion: all momenta are fixed by external momenta

Advantages

Cannot occur: diagrams with interaction not aligned along a vertical cut



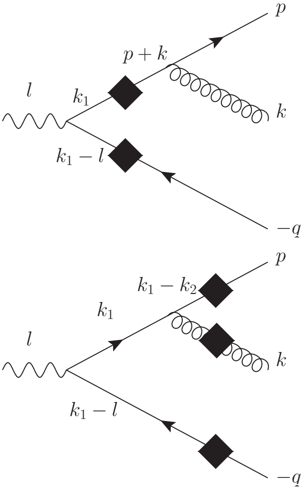
Can occur: diagrams with and without interaction which belong to the same cut



We end up with a minimal set of 4 diagrams

- √ Use spinor helicity method to construct polarized amplitudes
- √ Shown here the large N_c limit on Wilsons from amplitude squares

Minimal set of diagrams for $\gamma^*(l) + \text{target}(P) \to q(p) + \bar{q}(q) + g(k) + X$



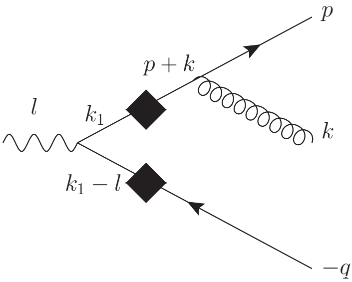
Minimal set of amplitudes for $\gamma^*(l) + \text{target}(P) \to q(p) + \bar{q}(q) + g(k) + X$

$$
\begin{aligned}
i\mathcal{A}_1 =& (ie)(ig)\int \frac{d^4k_1}{(2\pi)^4}\,\bar{u}(p)\gamma^\mu\, t^a\,\overline{S}_F(p+k,k_1)\gamma^\nu\overline{S}_F(k_1-l,-q) \\
&\cdot\left[S_F^{(0)}(-q)\right]^{-1} v(q)\epsilon_\nu(l)\,\epsilon^*_\mu(k)\,, \\
i\mathcal{A}_2 =& (ie)(ig)\int \frac{d^4k_1}{(2\pi)^4}\,\bar{u}(p)\left[S_F^{(0)}(p)\right]^{-1}\overline{S}_F(p,k_1)\,\gamma^\nu\overline{S}_F(k_1-l,-q-k) \\
&\cdot\gamma^\mu\, t^a\, v(q)\,\epsilon_\nu(l)\,\epsilon^*_\mu(k)\,, \\
i\mathcal{A}_3 =& (ie)(ig)\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p)\left[S_F^{(0)}(p)\right]^{-1}\overline{S}_F(p,k_1-k_2)\gamma^\lambda\, t^c\, S_F^{(0)}(k_1)\gamma^\nu \\
&\overline{S}_F(k_1-l,-q)\left[S_F^{(0)}(-q)\right]^{-1} v(q)\left[\overline{G}_\lambda{}^\delta\right]^{ca}(k_2,k)\left[G_\delta^{(0),\mu}(k)\right]^{-1}\epsilon_\nu(l)\epsilon^*_\mu(k)\,, \\
i\mathcal{A}_4 =& (ie)(ig)\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p)\left[S_F^{(0)}(p)\right]^{-1}\overline{S}_F(p,l-k_1)\,\gamma^\nu\, S_F^{(0)}(-k_1)\gamma^\lambda\, t^c \\
&\overline{S}_F(k_2-k_1,-q)\left[S_F^{(0)}(-q)\right]^{-1}\left[\overline{G}_\lambda{}^\delta\right]^{ca}(k_2,k)\left[G_\delta^{(0)\mu}(k)\right]^{-1}\epsilon_\nu(l)\,\epsilon^*_\mu(k)
\end{aligned}
$$

Focus on $\mathcal{A}_1$

$$i\mathcal{A}_1^a = eg \int d^2\boldsymbol{x}_1\, d^2\boldsymbol{x}_2 \int \frac{d^4 k_1}{(2\pi)^2}\, e^{i(\boldsymbol{k}_1-\boldsymbol{p}-\boldsymbol{k})\cdot\boldsymbol{x}_1}\, e^{-i(\boldsymbol{k}_1+\boldsymbol{q})\cdot\boldsymbol{x}_2}$$

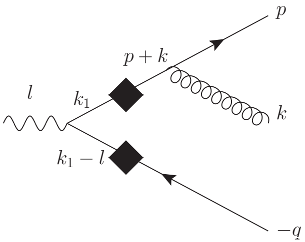
$$\delta(p^+ + k^+ - k_1^+)\delta(k_1^+ - l^+ + q^+)\, t^a V(x_{1,t}) V^\dagger(x_{2,t})\, \frac{-i\cdot N_1}{k_1^2\,(k_1-l)^2}$$



For $\mathcal{A}^a_{\#_1,ij}\mathcal{A}^{a,\dagger}_{\#_2,ij}$:

→ spinor helicity methods for N_1
→ traces of Wilsons in colour space
→ integrations: mom/config space

Spinor helicity methods for N_1



$$N_1^{\lambda_\gamma;\lambda_q\lambda_{\bar{q}}\lambda_g} = \bar{u}_{\lambda_q}(p)\left(\not\epsilon^{(\lambda_g)}\right)^*(k)\frac{(\not p + \not k)}{(p+k)^2}\not n \not k_1 \not\epsilon^{(\lambda_\gamma)}(l)(\not k_1 - \not l)\not n v_{\lambda_{\bar{q}}}(q)$$

$\lambda_\gamma = (L, T = \pm), \lambda_q = (\pm), \lambda_{\bar{q}} = (\pm), \lambda_g = (\pm)$ encode the possible helicity states of initial and final state particles

Spinor helicity methods

For massless fermions, helicity is a conserved, quantum number. On-shell helicity eigenstates (spinors) are defined as (M. L. Mangano and S. J. Parke, Phys. Rept. **200**, 301 (1991))

$$u_\pm(k) = \frac{1 \pm \gamma_5}{2} u(p) \qquad\qquad v_\mp(k) = \frac{1 \pm \gamma_5}{2} v(p)$$

Use short-hands $|i^\pm\rangle \equiv |k_i^\pm\rangle \equiv u_\pm(k_i) = v_\mp(k_i)$ for spinor products

$$\langle k_i k_j \rangle \equiv \langle k_i^- | k_j^+ \rangle = \bar{u}_-(k_i) u_+(k_j), \qquad [k_i k_j] \equiv \langle k_i^+ | k_j^- \rangle = \bar{u}_+(k_i) u_-(k_j)$$

And $\langle k_i^\pm | k_j^\pm \rangle = 0$ and $\langle k_i^\mp | k_i^\pm \rangle = 0$. γ matrices Dirac rep. to calculate the spinor products

$$\langle k_i k_j \rangle = (k_i^+ k_j^+)^{-\frac{1}{2}} \left(k_j^+ |\boldsymbol{k}_i| e^{i\phi_{k_i}} - k_i^+ |\boldsymbol{k}_j| e^{i\phi_{k_j}} \right)$$
$$[k_i k_j] = -(k_i^+ k_j^+)^{-\frac{1}{2}} \left(k_j^+ |\boldsymbol{k}_i| e^{-i\phi_{k_i}} - k_i^+ |\boldsymbol{k}_j| e^{-i\phi_{k_j}} \right)$$

with $e^{i\phi_k} \equiv \frac{k^1 + ik^2}{\sqrt{\boldsymbol{k}^2}} = \sqrt{2} \frac{\boldsymbol{k} \cdot \boldsymbol{\epsilon}}{\sqrt{\boldsymbol{k}^2}}$ and $\boldsymbol{\epsilon} = \frac{1}{\sqrt{2}}(1, i)$

Spinor helicity methods

Identities such as

$$\langle i^{\pm}|\gamma^{\mu}|j^{\pm}\rangle\langle k^{\pm}|\gamma_{\mu}|l^{\pm}\rangle = 2\langle i^{\pm}|k^{\mp}\rangle\langle l^{\mp}|j^{\pm}\rangle$$

together with

- $\not{p} = |p^{+}\rangle\langle p^{+}| + |p^{-}\rangle\langle p^{-}|$ for on-shell momentum $p^2 = 0$
- $k^{\mu}_{1,2} = \bar{k}^{\mu}_{1,2} + \frac{k^2_{1,2}}{2k^{+}_{1,2}} n^{\mu}$ for off-shell momenta (internal, integrated over), where $\bar{k}^2_{1,2} = 0$ is on-shell

to get relevant (polarization specific) amplitudes in a simple (factorized) way.

Going back to $\mathcal{A}_1$

$$N_1^{\lambda_\gamma;\lambda_q\lambda_{\bar{q}}\lambda_g} = \bar{u}_{\lambda_q}(p)\left(\not{\epsilon}^{(\lambda_g)}\right)^*(k)\frac{(\not{p}+\not{k})}{(p+k)^2}\not{n}\not{k}_1\not{\epsilon}^{(\lambda_\gamma)}(l)(\not{k}_1-\not{l})\not{n}v_{\lambda_{\bar{q}}}(q)$$

Can be written as

$$N_1^{\lambda_\gamma;\lambda_q\lambda_{\bar{q}}\lambda_g} = Q^{\lambda_\gamma\lambda_q}_{\gamma^*\to q\bar{q}}(\bar{k}_1)\cdot Q^{\lambda_q\lambda_g}_{q\to qg}(\bar{k}_1)$$

where

$$Q^{\lambda_\gamma\lambda_q}_{\gamma^*\to q\bar{q}}(\bar{k}_1) = \bar{u}_{\lambda_q}(\bar{k}_1)\not{\epsilon}^{(\lambda_\gamma)}(l)(\not{k}_1-\not{l})\not{n}v_{\lambda_{\bar{q}}}(q)\,,$$

$$Q^{\lambda_q\lambda_g}_{q\to qg}(\bar{k}_1) = \frac{1}{(p+k)^2}\cdot\bar{u}_{\lambda_q}(p)\left(\not{\epsilon}^{(\lambda_g)}\right)^*(k)(\not{p}+\not{k})\not{n}u_{\lambda_q}(\bar{k}_1)$$

$\mathcal{A}_1$ for longitudinal photon

$$N_1^{L;\lambda_q\lambda_{\bar{q}}\lambda_g} = Q^{L\lambda_q}_{\gamma^*\to q\bar{q}}(\bar{k}_1)\cdot Q^{\lambda_q\lambda_g}_{q\to qg}(\bar{k}_1)$$

$$Q^{L+}_{\gamma^*\to q\bar{q}} = \frac{Q}{l^+}[\bar{k}_1 n](\langle n\bar{k}_1\rangle[\bar{k}_1 n] - \langle n\bar{l}\rangle[\bar{l}n])\langle nq\rangle = -4\frac{Q}{l^+}(p^+ + k^+)^{\frac{1}{2}}(q^+)^{\frac{3}{2}}$$

$$Q^{++;1}_{q\to qg} = -\frac{\sqrt{2}[pn][pn]\langle n\bar{k}_1\rangle}{[nk][pk]} = \frac{2^{\frac{3}{2}}\cdot(p^+)^{\frac{3}{2}}(p^+ + k^+)^{\frac{1}{2}}}{p^+|\boldsymbol{k}|e^{-i\phi_k} - k^+|\boldsymbol{p}|e^{-i\phi_p}},$$

$$Q^{+-;1}_{q\to qg} = \frac{\sqrt{2}(\langle np\rangle[pn] + \langle nk\rangle[kn])\langle n\bar{k}_1\rangle}{\langle nk\rangle\langle kp\rangle} = \frac{2^{\frac{3}{2}}\cdot(p^+)^{\frac{1}{2}}(p^+ + k^+)^{\frac{3}{2}}}{p^+|\boldsymbol{k}|e^{i\phi_k} - k^+|\boldsymbol{p}|e^{i\phi_p}}$$

where we used $(p+k)^2 = [pk]\langle kp\rangle$ and id of collinear/soft sing

Amplitude squared and xsec

With flux factor $\mathcal{F} = 2l^+$, photon momentum fractions $\{z_1, z_2, z_3\} = \{p^+/l^+, q^+/l^+, k^+/l^+\}$, and the three-particle phase space,

$$d\Phi^{(3)} = \frac{1}{(2\pi)^8}\, d^4p\, d^4q\, d^4k\, \delta(p^2)\delta(q^2)\delta(k^2)\delta(l^+ - p^+ - k^+ - q^+)$$
$$= 2l^+ \frac{d^2\boldsymbol{p}\, d^2\boldsymbol{q}\, d^2\boldsymbol{k}}{(64\pi^4 l^+)^2} \frac{dz_1\, dz_2\, dz_3}{z_1\, z_2\, z_3} \delta(1 - \sum_{i=1}^{3} z_i)$$

the differential 3-parton production cross-section reads

$$d\sigma = \frac{1}{\mathcal{F}} \left\langle |\sum_{i=1}^{4} \mathcal{M}_i(A^+) - \mathcal{M}_i(0)|^2 \right\rangle_{A^+} d\Phi^{(3)}$$

where $\langle \ldots \rangle_{A_+}$ denotes the average over background field configurations

$$\mathcal{A}_i = 2\pi\delta(l^+ - p^+ - k^+ - q^+)\mathcal{M}_i \qquad i = 1, \ldots, 4$$

Wilson lines for diff xsec

$$\mathcal{A}^{a}_{\#_1,ij}\mathcal{A}^{a,\dagger}_{\#_2,ij} = 2\mathcal{A}^{a}_{\#_1,ij} t^a_{kl} t^b_{lk} \mathcal{A}^{b,\dagger}_{\#_2,ij}$$

contain

$$\begin{aligned}
N^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4) &\equiv 1 + S^{(4)}_{(x_1x_2x_3x_4)} - S^{(2)}_{(x_1x_2)} - S^{(2)}_{(x_3x_4)}\,,\\
N^{(22)}(\boldsymbol{x}_1, \boldsymbol{x}_2 | \boldsymbol{x}_3, \boldsymbol{x}_4) &\equiv \left[S^{(2)}_{(x_1x_2)} - 1\right]\left[S^{(2)}_{(x_3x_4)} - 1\right]\\
N^{(24)}(\boldsymbol{x}_1, \boldsymbol{x}_2 | \boldsymbol{x}_3, \boldsymbol{x}_4, \boldsymbol{x}_5, \boldsymbol{x}_6) &\equiv 1 + S^{(2)}_{(x_1x_2)} S^{(4)}_{(x_3x_4x_5x_6)} - S^{(2)}_{(x_1x_2)} S^{(2)}_{(x_3x_6)} - S^{(2)}_{(x_4x_5)}\,,\\
N^{(44)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4 | \boldsymbol{x}_5, \boldsymbol{x}_6, \boldsymbol{x}_7, \boldsymbol{x}_8) &\equiv 1 + S^{(4)}_{(x_1x_2x_3x_4)} S^{(4)}_{(x_5x_6x_7x_8)}\\
&\qquad - S^{(2)}_{(x_1x_4)} S^{(2)}_{(x_5x_8)} - S^{(2)}_{(x_2x_3)} S^{(2)}_{(x_6x_7)}
\end{aligned}$$

where dipole and quadrupole

$$S^{(2)}_{(x_1x_2)} \equiv \frac{1}{N_c}\mathrm{tr}\left[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)\right], \quad S^{(4)}_{(x_1x_2x_3x_4)} \equiv \frac{1}{N_c}\mathrm{tr}\left[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)V(\boldsymbol{x}_3)V^\dagger(\boldsymbol{x}_4)\right]$$

sub-leading in N_c, operators $N^{(4)}, N^{(22)}, N^{(24)}, N^{(44)}$ need to be replaced by $1/N_c \cdot N^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2)$.

Wilson lines and large N_c limit for diff xsec in diffraction

Diffractive reactions: color exchange is at amplitude level restricted to the color singlet, so project $q\bar{q}g$ system onto color-singlet

$$\mathcal{A}^{a}_{\#_1,ij} P^{ab}_{ji;i'j'} \mathcal{A}^{a,\dagger}_{\#_2,i'j'} = 2\mathcal{A}^{a}_{\#_1,ij} t^{a}_{ji} t^{b}_{i'j'} \mathcal{A}^{b,\dagger}_{\#_2,i'j'}$$

$$\begin{aligned}
N^{(4)}_{\text{diff.}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4) &\equiv \left[1 - S^{(2)}_{(x_1 x_2)}\right]\left[1 - S^{(2)}_{(x_3 x_4)}\right] , \\
N^{(22)}_{\text{diff.}}(\boldsymbol{x}_1, \boldsymbol{x}_2 | \boldsymbol{x}_3, \boldsymbol{x}_4) &\equiv \left[S^{(2)}_{(x_1 x_2)} - 1\right]\left[S^{(2)}_{(x_3 x_4)} - 1\right] , \\
N^{(24)}_{\text{diff.}}(\boldsymbol{x}_1, \boldsymbol{x}_2 | \boldsymbol{x}_3, \boldsymbol{x}_4, \boldsymbol{x}_5, \boldsymbol{x}_6) &\equiv \left[1 - S^{(2)}_{(x_1 x_2)} S^{(2)}_{(x_3 x_6)}\right]\left[1 - S^{(2)}_{(x_4 x_5)}\right] , \\
N^{(44)}_{\text{diff.}}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4 | \boldsymbol{x}_5, \boldsymbol{x}_6, \boldsymbol{x}_7, \boldsymbol{x}_8) &\equiv \left[1 - S^{(2)}_{(x_1 x_4)} S^{(2)}_{(x_5 x_8)}\right]\left[1 - S^{(2)}_{(x_2 x_3)} S^{(2)}_{(x_6 x_7)}\right]
\end{aligned}$$

sub-leading terms in N_c, operators $N^{(4)}_{\text{diff}}$, $N^{(22)}_{\text{diff}}$, $N^{(24)}_{\text{diff}}$, $N^{(44)}_{\text{diff}}$ need to be replaced by $1/N_c \cdot N^{(4)}_{\text{diff}}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2)$.

Polarized diff xsec to study azimuthal correlations

Leading N_c result for polarized diff xsec for 3 partons in DIS

$$\frac{d\sigma^{T,L}}{d^2\boldsymbol{p}\, d^2\boldsymbol{k}\, d^2\boldsymbol{q}\, dz_1 dz_2} = \frac{\alpha_s \alpha_{em} e_f^2 N_c^2}{z_1 z_2 z_3 \cdot 2} \prod_{i,j=1}^{3} \int \frac{d^2\boldsymbol{x}_i}{(2\pi)^2} \int \frac{d^2\boldsymbol{x}'_j}{(2\pi)^2} e^{i\boldsymbol{p}(\boldsymbol{x}_1-\boldsymbol{x}'_1)+i\boldsymbol{q}(\boldsymbol{x}_2-\boldsymbol{x}'_2)+i\boldsymbol{k}(\boldsymbol{x}_3-\boldsymbol{x}'_3)}$$

$$\Big\langle (2\pi)^4 \Big[\Big(\delta^{(2)}(\boldsymbol{x}_{13})\delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Big) N^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}'_1, \boldsymbol{x}'_2, \boldsymbol{x}_2)$$

$$+ \Big(\delta^{(2)}(\boldsymbol{x}_{23})\delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{2;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Big) N^{(22)}(\boldsymbol{x}_1, \boldsymbol{x}'_1 | \boldsymbol{x}'_2, \boldsymbol{x}_2) \Big]$$

$$+ (2\pi)^2 \Big[\delta^{(2)}(\boldsymbol{x}_{13}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(24)}(\boldsymbol{x}_{3'}, \boldsymbol{x}_{1'} | \boldsymbol{x}_{2'}, \boldsymbol{x}_2, \boldsymbol{x}_1, \boldsymbol{x}_{3'}) + \{1\} \leftrightarrow \{2\}$$

$$+ \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) N^{(24)}(\boldsymbol{x}_1, \boldsymbol{x}_3 | \boldsymbol{x}_{2'}, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_{1'}) + \{1'\} \leftrightarrow \{2'\} \Big]$$

$$+ \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(44)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{3'}, \boldsymbol{x}_3 | \boldsymbol{x}_3, \boldsymbol{x}_{3'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2) \Big\rangle_{A^+}$$

where $\psi_{i'} \equiv \psi_i$, $i = 1, \ldots, 3$ and $z_3 = 1 - z_1 - z_2$, while $c_L = 1$, $c_T = 1/2$. ϕ_{ij} the azimuthal angle of $\boldsymbol{x}_{ij}$, $i, j = 1 \ldots, 3$ and $X_j^2 = \boldsymbol{x}_{12}^2 (z_j + z_3)\left(1 - z_j - z_3\right)$ and $X_3^2 = z_1 z_2 \boldsymbol{x}_{12}^2 + z_1 z_3 \boldsymbol{x}_{13}^2 + z_2 z_3 \boldsymbol{x}_{23}^2$

$$\psi^{L}_{j,hg} = -2\sqrt{2} Q K_0\left(QX_j\right) \cdot a^{(L)}_{j,hg}, \qquad j = 1,2$$

$$\psi^{T}_{j,hg} = 2ie^{\mp i\phi_{x_{12}}} \sqrt{(1 - z_3 - z_j)(z_j + z_3)} Q K_1\left(QX_j\right) \cdot a^{\pm}_{j,hg} \qquad j = 1,2$$

$$\psi^{L}_{3,hg} = 4\pi i Q \sqrt{2 z_1 z_2} K_0\left(QX_3\right)\left(a^{(L)}_{3,hg} + a^{(L)}_{4,hg}\right), \qquad \psi^{T}_{3,hg} = -4\pi Q \sqrt{z_1 z_2} \frac{K_1\left(QX_3\right)}{X_3}\left(a^{\pm}_{3,hg} + a^{\pm}_{4,hg}\right)$$

Numerical specs for physics scenario

$$S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} = \int d^2\boldsymbol{l}\, e^{-i\boldsymbol{l}\cdot\boldsymbol{x}_{12}}\ \Phi(\boldsymbol{l}^2) = 2\left(\frac{Q_0|\boldsymbol{x}_{12}|}{2}\right)^{\rho-1}\frac{K_{\rho-1}(Q_0|\boldsymbol{x}_{12}|)}{\Gamma(\rho-1)},$$

where $$\Phi(\boldsymbol{l}^2) = \frac{\rho-1}{Q_0^2\pi}\left(\frac{Q_0^2}{Q_0^2+\boldsymbol{l}^2}\right)^{\rho}$$

and Q_0 is a scale proportional to the saturation scale.

$\rho = 2.3$ and $Q_0^{\text{proton}} = 0.69$ GeV motivated from inclusive DIS fits of the dipole dist $x = 0.2\times 10^{-3}$ and $Q_0^{\text{Au}} = A^{1/6}\cdot Q_0^{\text{proton}} = 1.67$ GeV

Quadrupoles: write them in terms of dipole in the large N_c and Gaussian approx (A. Kovner and U. A. Wiedemann, Phys. Rev. D **64**, 114002 (2001); J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D **70**, 114017 (2004); A. Kovner and M. Lublinsky, JHEP **0611**, 083 (2006); F. Dominguez, et al, Phys. Rev. D **83**, 105005 (2011))

Dipole profile motivated by a fit to the solution of rcBK equation (J. L. Albacete, et al, Eur. Phys. J. C **71**, 1705 (2011); A. H. Rezaeian, et al, Phys. Rev. D **87**, no. 3, 034002 (2013); A. H. Rezaeian and I. Schmidt, Phys. Rev. D **88**, 074016 (2013); E. Iancu, et al, Phys. Lett. B **750**, 643 (2015))

Numerical specs for physics scenario

- dilute limit: xsec at large photon virtuality $Q^2 = 9\ \text{GeV}^2$
- expand diff xsec to quadratic order in $N^{(2)} = 1 - S^{(2)}$
- $|\boldsymbol{p}| = |\boldsymbol{k}| = |\boldsymbol{q}|$ in analogy to the back-to-back configuration in di-parton production
- the 'collinear'limit $\boldsymbol{p} + \boldsymbol{k} + \boldsymbol{q} = 0$ (vanishing transverse momentum transfer between projectile and target) configuration

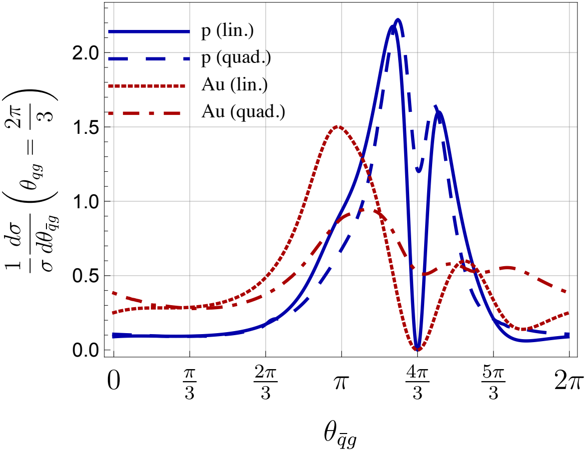
$$\{\Delta\theta_{qg}, \Delta\theta_{\bar{q}g}\} = \{2\pi/3, 4\pi/3\}$$
$$\{\Delta\theta_{qg}, \Delta\theta_{\bar{q}g}\} = \{4\pi/3, 2\pi/3\}$$

- expect strong peaks of the angular distribution at these points

Azimuthal correlations between 3 partons in DIS

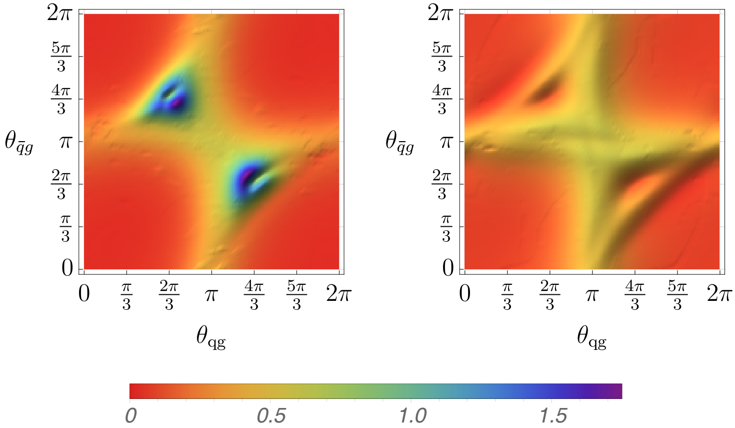
$z_1 = z_2 = 0.2$, $|\boldsymbol{p}| = |\boldsymbol{k}| = |\boldsymbol{q}| = 2$ GeV and $Q = 3$ GeV

Normalized cross-section against $\Delta\theta_{\bar{q}g}$ with $\Delta_{qg} = 2\pi/3$ for proton and gold up to linear and quadratic order in $N^{(2)}$



Azimuthal correlations between 3 partons in DIS

$z_1 = z_2 = 0.2$, $|\boldsymbol{p}| = |\boldsymbol{k}| = |\boldsymbol{q}| = 2$ GeV and $Q = 3$ GeV
Combined $\Delta\theta_{qg}$ and $\Delta\theta_{\bar{q}g}$ dependence of the normalized cross-section for proton and gold at quadratic order



Azimuthal correlations between 3 partons in DIS

- partonic xsec vanishes at 'collinear'configurations + strong double peak
- behavior is also observed in studies of quark-gluon, photon-quark and dilepton-quark angular correlations
- double peak goes away at the hadronic level and/or when adding quadratic corrections in $N^{(2)}$
- we include sub-leading corrections in the dilute expansion: small for p, sizeable for Au

Outlook: use $\gamma^* T \to q\bar{q}gX$ for

- real photons ($Q^2 \to 0$): UPCs in AA (inclusive 3-jet production, NLO inclusive di-jet production)

$$\begin{array}{lcl} qT & \to & qg\gamma^* X \\ \bar{q}T & \to & \bar{q}g\gamma^* X \\ gT & \to & q\bar{q}\gamma^* X \end{array}$$

for pA colisions: di-jet and photoproduction $pA \to h_1 h_2 \gamma^* X$

- MPIs: double/triple parton scattering

$$\begin{array}{c} q\bar{q}T \to g\gamma^* X \\ g\bar{q}T \to \bar{q}\gamma^* X \\ qgT \to q\gamma^* X \end{array} \qquad q\bar{q}gT \to \gamma^* X$$

for pA colisions: 2GPDs $pA \to h_1 \gamma^* X$

Conclusions

- We calculated the triple differential cross section for inclusive production of three partons in DIS
- We keep the full helicity dependence of initial and final state particles
- Potential use of the 3 parton production process in providing experimental evidence for saturation effects and in particular for exploring the 4-point correlator $S^{(4)}$
- Our analytic expressions can be used to compute the real contributions to the Next to Leading Order (NLO) corrections to inclusive di-hadron production in DIS

EXTRAS

Is this efficient?

- method is efficient for small number of final state partons

  (J. Jalilian-Marian and Y. V. Kovchegov, Phys. Rev. D **70**, 114017 (2004); F. Gelis and J. Jalilian-Marian, Phys. Rev. D **67**, 074019 (2003))

- as usual, final states with large multiplicities: large number of Feynman diagrams
  $\gamma^* +$ target $\rightarrow q + \bar{q}$ with 3 diagrams
  $\gamma^* +$ target $\rightarrow q + \bar{q} + g$ with 16 diagrams
- large redundancy: calcs based on configuration space propagators $\rightsquigarrow$ large cancellations among different diagrams

$\surd$ There is a more efficient way to use the emerging simplicity of the configuration space amplitudes for calculations in momentum space.

Key ideas to be more efficient

(1) Study cut diagrams before and after interaction $x^+ = 0$

(2) Fourier transform into config-space the complete propagators (non-interaction and interaction terms)

(3) Combine ideas, add a "zero "contribution and Fourier transform back into mom-space

(1) Exploit $A^{-}(x^{+}, x_t) = \delta(x^{+})\alpha(x_t)$

$A^{-}(x^{+}, x_t) \sim \delta(x^{+})$ gives restriction on projectile-target intn to the light-cone time-slice $x^{+} = 0$ so:

- propagators get a separate minus-momentum variable and at each (standard QCD) vertex (where minus-momenta are conserved), introduce a delta function as

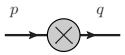
$$\delta(\{p_{\rm in}^{-}\} - \{p_{\rm out}^{-}\}) = \int \frac{dx^{+}}{2\pi} e^{-ix^{+}\cdot(\{p_{\rm in}^{-}\}-\{p_{\rm out}^{-}\})}$$

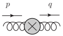
  $\{p_{\rm in\ (out)}\}$ full set of incoming (outgoing) momenta of the vertex

- get an extra integral over x_i^{+}, $i = 1, \ldots N$ with N the total number of (standard QCD)vertices
- vertices connected by mom-space propagators which are Fourier transformed w.r.t. their plus momentum

(2) Fourier transform into config-space the non-intn + intn propagators

Minus momentum is not conserved $\rightarrow$ no integration over minus coordinates in vertices





$$\tilde{S}_{F,kl}(x_{ij}^{+};p^{+},\boldsymbol{p}) = \int \frac{dp^{-}}{2\pi} e^{-ip^{-}x_{ij}^{+}} S_{F,kl}(p) =$$

$$= \delta_{kl} \frac{e^{-ip^{-}x_{ij}^{+}}}{2p^{+}} \left[\left(\theta(p^{+})\theta(x_{ij}^{+}) + \theta(-p^{+})\theta(-x_{ij}^{+}) \right) (\not{p} + m) + \delta(x_{ij}^{+})\not{n} \right]_{p^{-}=\frac{\boldsymbol{p}^2+m^2}{2p^{+}}}$$

$$\tilde{G}_{\mu\nu}^{ab}(x_{ij}^{+};p^{+},\boldsymbol{p}) = \int \frac{dp^{-}}{2\pi} e^{-ip^{-}x_{ij}^{+}} G_{\mu\nu}^{ab}(p) =$$

$$= \delta_{ab} \frac{e^{-ip^{-}x_{ij}^{+}}}{2p^{+}} \left[\left(\theta(p^{+})\theta(x_{ij}^{+}) + \theta(-p^{+})\theta(-x_{ij}^{+}) \right) \cdot d_{\mu\nu}(p) + 2\delta(x_{ij}^{+}) \frac{n_{\mu}n_{\nu}}{p \cdot n} \right]_{p^{-}=\frac{\boldsymbol{p}^2+m^2}{2p^{+}}}$$

where $x_{ij}^{+} \equiv x_{i}^{+} - x_{j}^{+}$.

Well known because after integration over light-cone times x_i^{+} at each vertex: light-cone time ordered Feynman rules of light-front pert theory (G. Beuf, Phys. Rev. D **94**, no. 5, 054016 (2016))

(2) Fourier transform into config-space the non-intn + intn propagators

Organize QCD intn vertices into vertices before ($x_i^+ < 0$) and after intn ($x_i^+ > 0$) with target

In practise, we evaluate to zero configurations which are absent in the light-front formalism for momentum space diagrams

Potential contact terms are absent:

- quark prop ($\sim \delta(x_{ij}^+)\not{n}$) $\times$ intn vertex: $\not{n}\not{n} = 0$
- gluon prop ($\sim \delta(x_{ij}^+) n_\mu n_\nu$) $\times$ intn vertex: $d_{\mu\nu}(p) \cdot n^\nu = 0$ and $n^2 = 0$ (if the vertex connects to a virtual gluon) and $\epsilon^{(\lambda)}(p)_\nu \cdot n^\nu = 0$ (if the vertex connects to a real gluon)

(3) Skeleton diagrams with cuts before/after intn

Skeleton real corrections, $l^+, p^+, q^+, k^+ > 0$ and $l^+ = p^+ + k^+ + q^+$

Each skeleton diagram can be organized according to three possible "s-channel" cuts