Resummation at small x

Anna Stasto
Penn State University

INT Seattle, February 14, 2017

Question: *what is the range of applicability of standard collinear formalism with DGLAP evolution and the calculations with low x effects (including saturation)?*

Question: *what is the range of applicability of standard collinear formalism with DGLAP evolution and the calculations with low x effects (including saturation)?*

One possible answer: *it depends on the process*

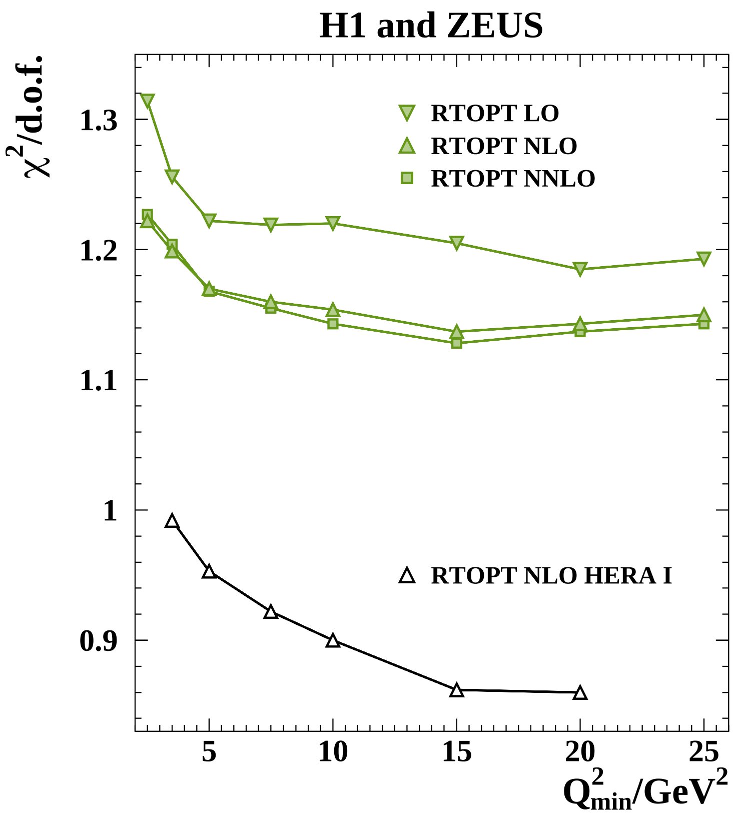
Question: *what is the range of applicability of standard collinear formalism with DGLAP evolution and the calculations with low x effects (including saturation)?*

One possible answer: *it depends on the process*

Another possible answer: *it depends on the accuracy of calculation in both cases. More specifically, it is possible to extend the region of validity of any of these approaches through the resummation.*

Are F_2 data compatible with DGLAP evolution?

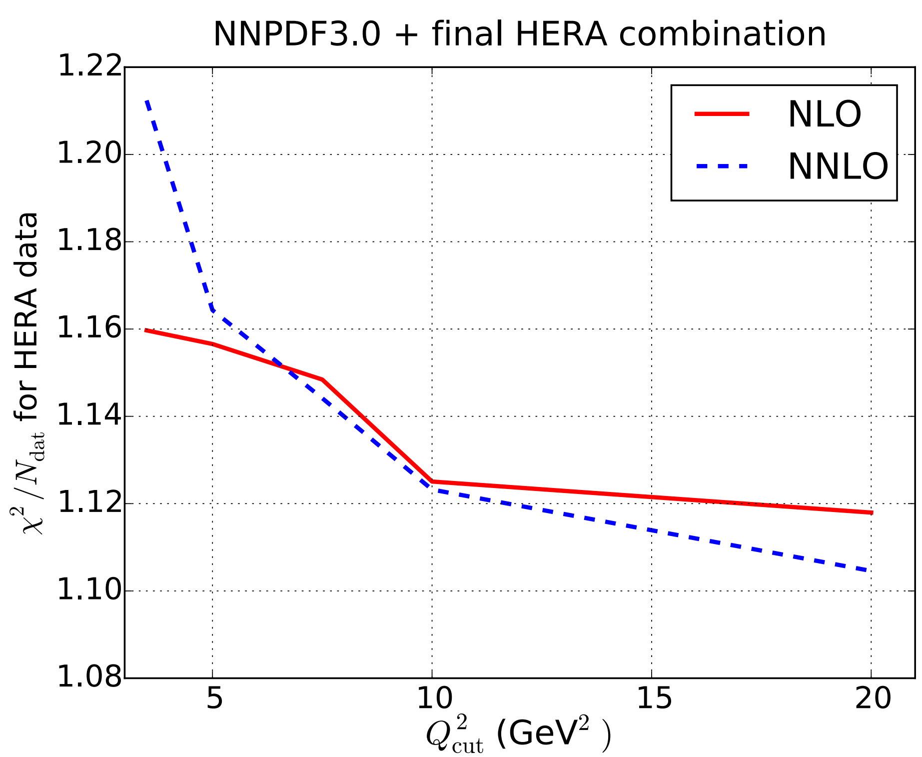
HERAPDF



Varying the cutoff Q^2_{min}

HERAPDF	Q^2_{min}[GeV^2]	χ^2	d.o.f.	χ^2/d.o.f
2.0 NLO	3.5	1357	1131	1.200
2.0HiQ2 NLO	10.0	1156	1002	1.154
2.0 NNLO	3.5	1363	1131	1.205
2.0HiQ2 NNLO	10.0	1146	1002	1.144
2.0 AG NLO	3.5	1359	1132	1.201
2.0HiQ2 AG NLO	10.0	1161	1003	1.158
2.0 AG NNLO	3.5	1385	1132	1.223
2.0HiQ2 AG NNLO	10.0	1175	1003	1.171
2.0 NLO FF3A	3.5	1351	1131	1.195
2.0 NLO FF3B	3.5	1315	1131	1.163
2.0Jets $\alpha_s(M_Z^2)$ fixed	3.5	1568	1340	1.170
2.0Jets $\alpha_s(M_Z^2)$ free	3.5	1568	1339	1.171

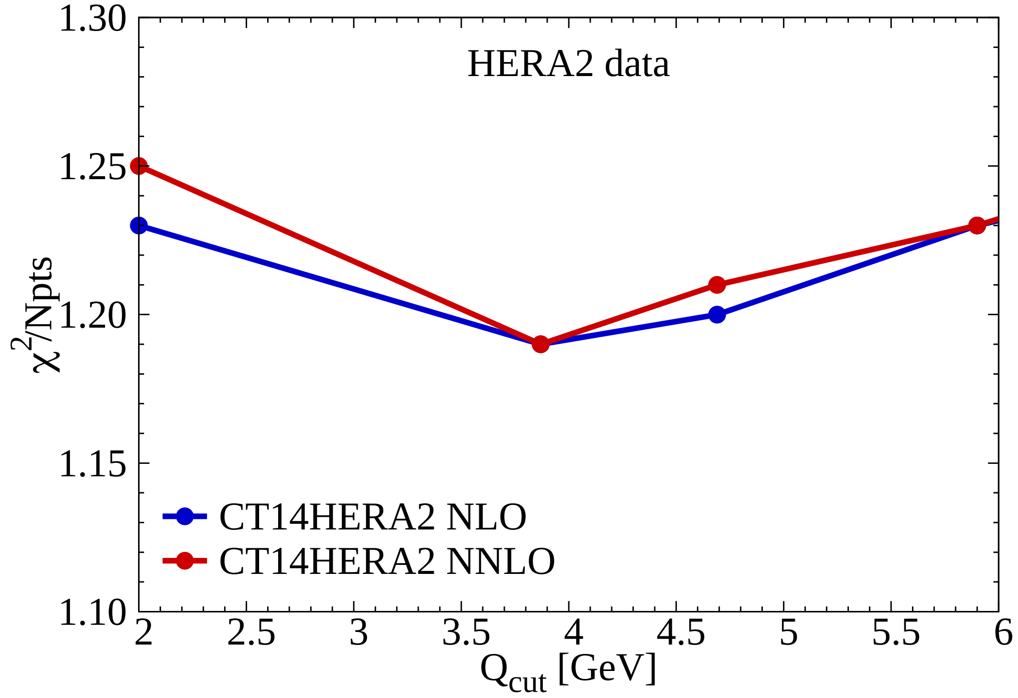
NNPDF

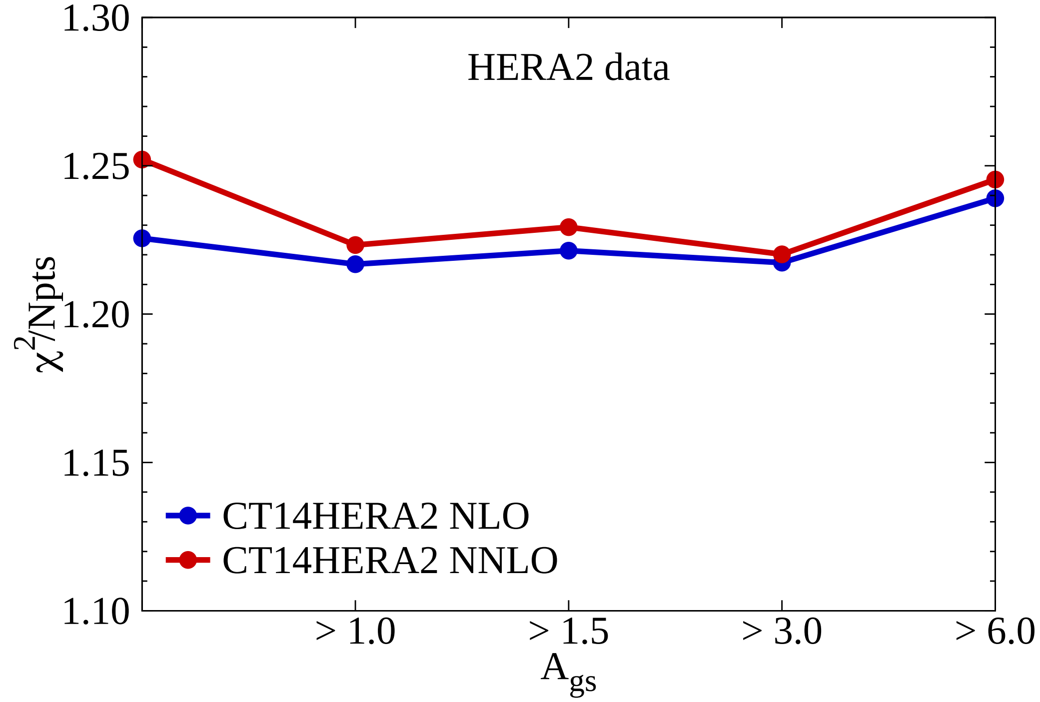


Both HERAPDF and NNPDF find some dependence on the cutoff.
Small x resummation? Or some other effects?

Are F_2 data compatible with DGLAP evolution?

CTEQ-TEA





CTEQ finds no dependence of χ^2 on the different cuts. DGLAP seems to work fine in the entire region down to very low x and Q.

Excluding data below the value set by the geometric scaling variable

$$A_{gs} = x^{\lambda} Q^2$$

Cannot conclude definitely about the tension between DGLAP and data in the low Q region.

High energy limit

$$\sqrt{s} \to \infty, x \to 0$$

Energy much larger than any other scale in the process

At small x there are large logs:

$$xP_{gg}(x) \sim \alpha_S^n \ln^{n-1}(1/x), \quad xP_{qg}(x) \sim \alpha_S^n \ln^{n-2}(1/x) \quad \text{and} \quad xC_{L,g}(x) \sim \alpha_S^n \ln^{n-2}(1/x).$$

High energy limit

$$\sqrt{s} \to \infty, x \to 0$$

Energy much larger than any other scale in the process

At small x there are large logs:

$$xP_{gg}(x) \sim \alpha_S^n \ln^{n-1}(1/x), \quad xP_{qg}(x) \sim \alpha_S^n \ln^{n-2}(1/x) \quad \text{and} \quad xC_{L,g}(x) \sim \alpha_S^n \ln^{n-2}(1/x).$$

At high energy, or small x we can have:

$$\alpha_S \ln 1/x \sim 1$$

High energy limit

$$\sqrt{s} \to \infty, x \to 0$$

Energy much larger than any other scale in the process

At small x there are large logs:

$$xP_{gg}(x) \sim \alpha_S^n \ln^{n-1}(1/x), \quad xP_{qg}(x) \sim \alpha_S^n \ln^{n-2}(1/x) \quad \text{and} \quad xC_{L,g}(x) \sim \alpha_S^n \ln^{n-2}(1/x).$$

At high energy, or small x we can have:

$$\alpha_S \ln 1/x \sim 1$$

Need to resum them as well to all orders:

$$(\alpha_S \ln 1/x)^n$$

High energy limit

$$\sqrt{s} \to \infty, x \to 0$$

Energy much larger than any other scale in the process

At small x there are large logs:

$$xP_{gg}(x) \sim \alpha_S^n \ln^{n-1}(1/x), \quad xP_{qg}(x) \sim \alpha_S^n \ln^{n-2}(1/x) \quad \text{and} \quad xC_{L,g}(x) \sim \alpha_S^n \ln^{n-2}(1/x).$$

At high energy, or small x we can have:

$$\alpha_S \ln 1/x \sim 1$$

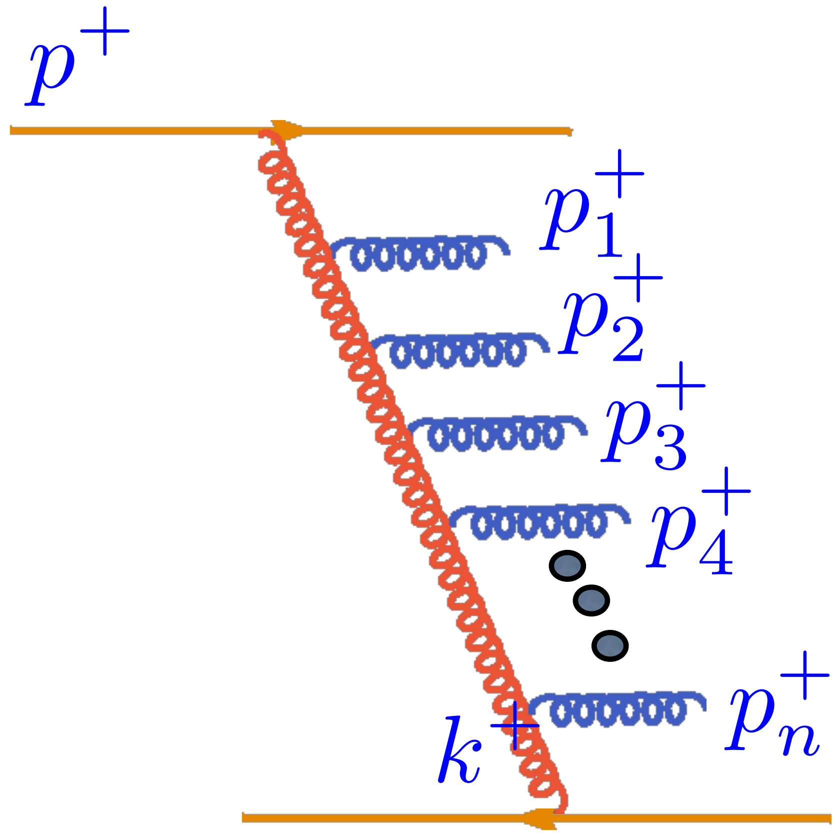
Need to resum them as well to all orders:

$$(\alpha_S \ln 1/x)^n$$

Any fixed order here would not be sufficient, potentially very large corrections.

Many soft gluon emissions in small x limit

Cascade of the n soft gluons



Strong ordering (in longitudinal momenta)

$$p^+ \gg p_1^+ \gg p_2^+ \gg \cdots \gg p_n^+ \gg k^+$$

Note: transverse momenta are not ordered

$$k^+ = xp^+$$

$$\frac{\alpha_s N_c}{\pi} \int_{k^+}^{p^+} \frac{dp_1^+}{p_1^+} = \frac{\alpha_s N_c}{\pi} \ln \frac{1}{x}$$

Large logarithm

Nested logarithmic integrals

$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{1}{x} \right)^n$$

Resummation of the gluon emissions performed by the equation

$$\frac{df_g(x, k_T^2)}{d \ln 1/x} = \frac{\alpha_s N_c}{\pi} \int d^2 k_T' \mathcal{K}(k_T, k_T') \; f_g(x, k_T')$$

integral over transverse momenta

kernel describing branching of gluons

gluon density

I.Balitsky, V.Fadin, E.Kuraev, L.Lipatov

Evolution equation in longitudinal momenta

$$\frac{df_g(x,k_T^2)}{d\ln 1/x} = \frac{\alpha_s N_c}{\pi} \int d^2k'_T \mathcal{K}(k_T,k'_T)\, f_g(x,k'_T)$$

Solution:

$$f_g(x,k_T) \sim x^{-\omega_P}$$

$$\omega_P = j-1 = \frac{\alpha_s N_c}{\pi} 4\ln 2$$

Leading exponent(spin)

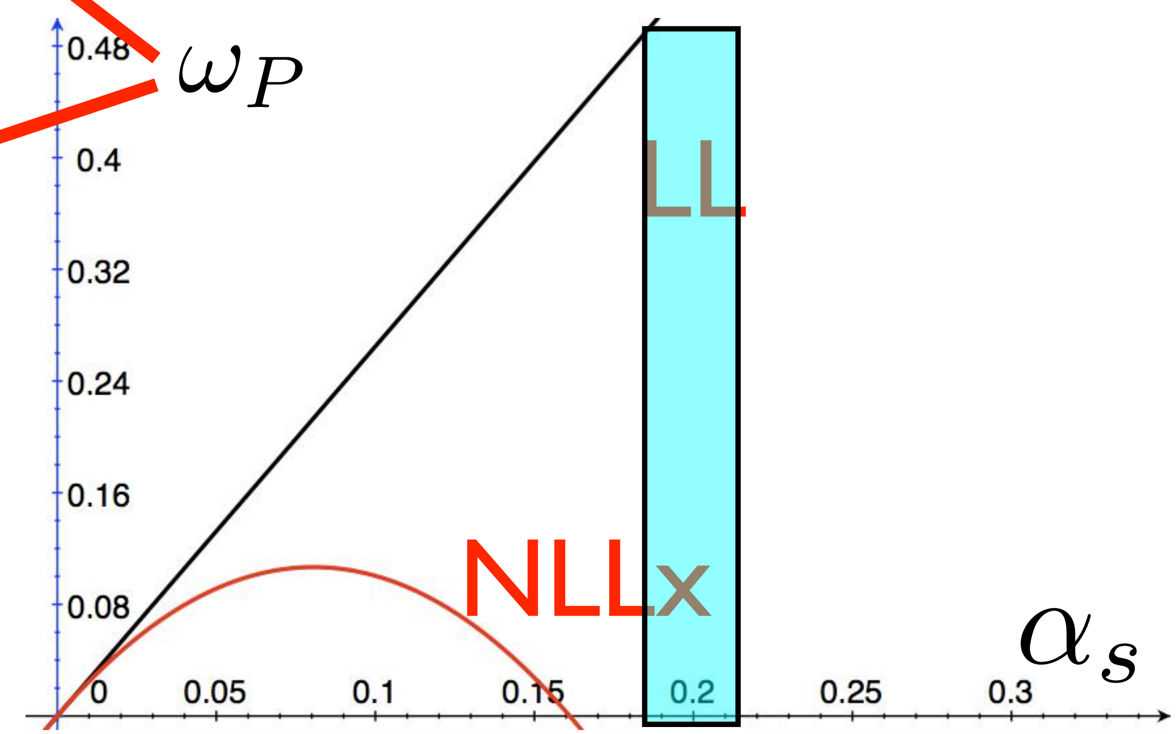
$$\sigma^{DIS}_{\gamma^* p} \sim s^{\omega_P}$$

Rise too strong for the data!

Take higher order corrections.

$$\alpha_s \mathcal{K}_0 + \alpha_s^2 \mathcal{K}_1 + \dots$$

V.Fadin,L.Lipatov, G.Camici,M.Ciafaloni



relevant values of α_s

$$\omega_P \simeq \bar{\alpha}_s 4\ln 2(1-6.5\bar{\alpha}_s)$$

Very large next-to-leading correction! Problems with convergence.

Why NLLx is so large in BFKL?

- Strong coupling constant is **not** a naturally small parameter in the Regge limit: $s \gg |t|, \Lambda^2_{QCD}$ but $\alpha_s(\mu^2),\ \mu^2 \neq s$

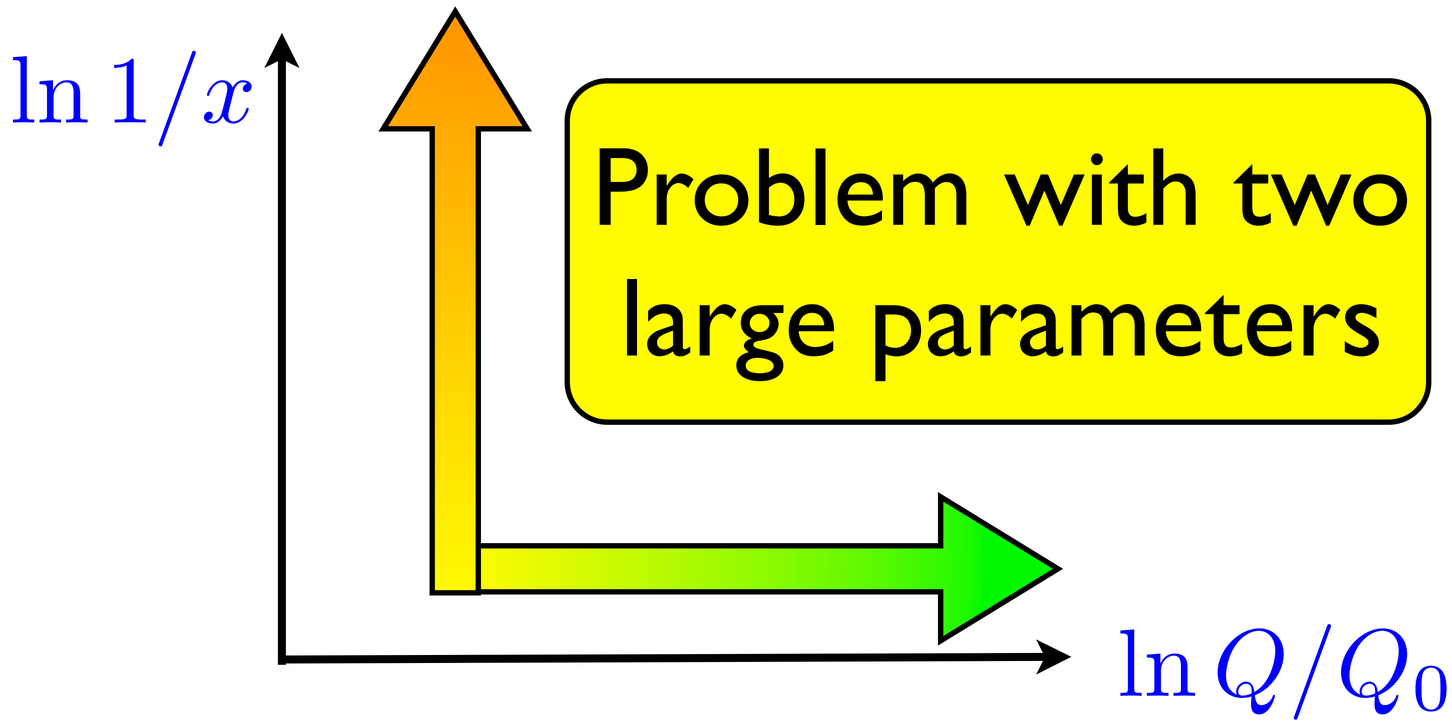
- Regge limit is inherently nonperturbative.

- Compare DGLAP (collinear approach): $Q^2 \gg \Lambda^2$ and $\alpha_s(Q^2) \ll 1$

- No momentum sum rule, since the evolution is local in x. In DGLAP: momentum sum rule satisfied at each order due to the initial assumption of the collinearity of the partons and the non-locality of the evolution in x.

- Approximations in the phase space (multi-Regge kinematics, quasi multi-Regge kinematics, etc..) cannot be recovered by the (fixed number of) the higher orders of expansion in the coupling constant.

Resummation



$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{1}{x}\right)^n$$

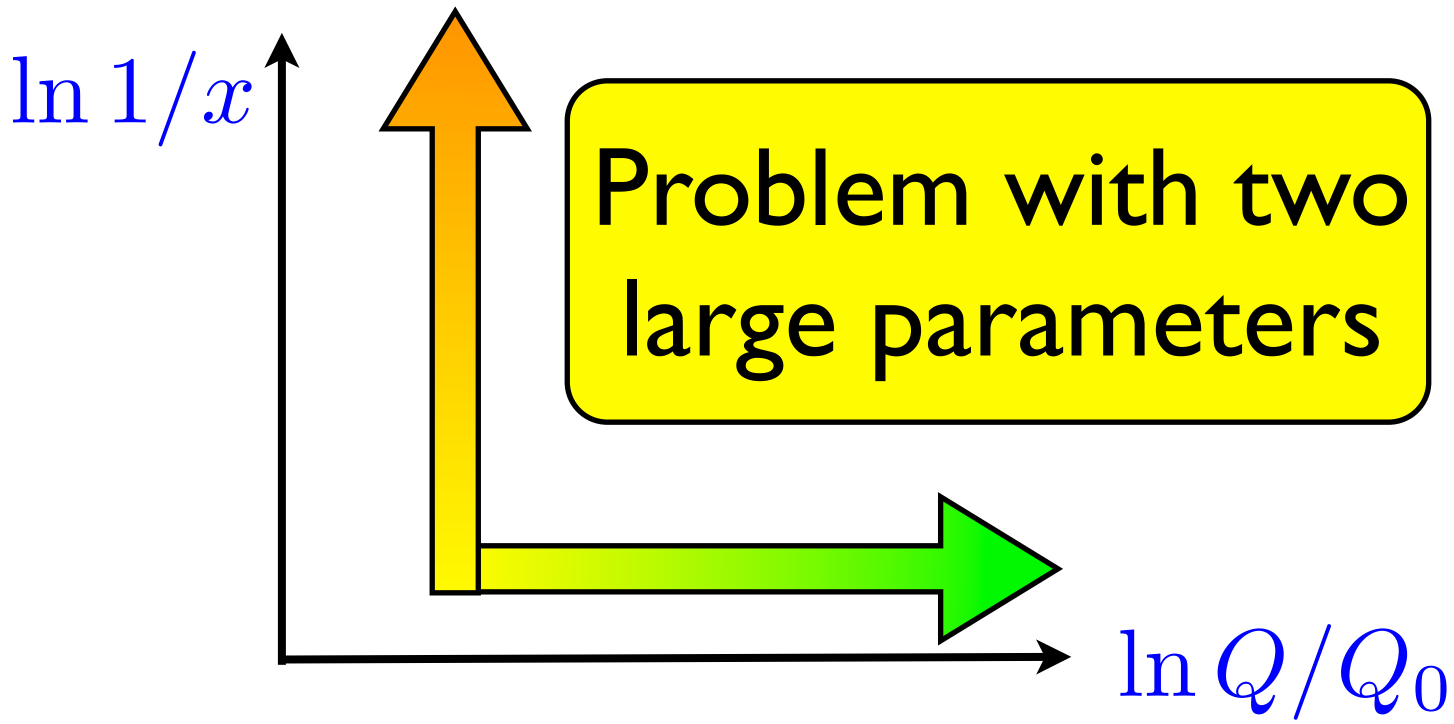
$$\left(\frac{\alpha_s N_c}{\pi} \ln \frac{Q}{Q_0}\right)^n$$

Mellin variables: $\gamma \leftrightarrow \ln k_T^2$ $\qquad \omega \leftrightarrow \ln 1/x$

Kernel in Mellin space $\qquad \chi(\gamma) = \int \frac{dk'^2}{k^2} K(k^2, k'^2) \left(\frac{k'^2}{k^2}\right)^\gamma$

Anomalous dimension $\qquad \gamma(\omega) = \int dz P(z) z^{-\omega}$

Resummation



$$\left(\frac{\alpha_s N_c}{\pi}\ln\frac{1}{x}\right)^n$$ energy

$$\left(\frac{\alpha_s N_c}{\pi}\ln\frac{Q}{Q_0}\right)^n$$ scale (related to transverse momentum)

Mellin variables: $\gamma \leftrightarrow \ln k_T^2$ $\omega \leftrightarrow \ln 1/x$

Kernel in Mellin space $$\chi(\gamma) = \int \frac{dk'^2}{k^2} K(k^2, k'^2)\left(\frac{k'^2}{k^2}\right)^\gamma$$

Anomalous dimension $$\gamma(\omega) = \int dz P(z) z^{-\omega}$$

Resummation

linear case

Anderson,Gustafson,Kharraziha,Samuelson Z.Phys. C71(1996) 613

Kwiecinski, Martin, Sutton Z.Phys. C71(1996) 585; Kwiecinski,Martin,AS Phys.Rev. D56 (1997) 3991

Salam JHEP 9807 (1998) 19; Ciafaloni,Colferai,Salam,AS Phys.Rev. D68(2003) 114003

Altarelli,Ball,Forte Nucl.Phys. B575(2000) 313; Bonvini,Marzani,Perano Eur. Phys. J C76(2016) 597.

Thorne Phys. Rev. D64 (2001) 074005

Sabio-Vera Nucl. Phys. B722 (2005) 65.

Brodsky,Fadin,Kim,Lipatov,Pivovarov JETP Lett. 70 (1999) 155.

nonlinear case

Motyka,AS Phys. Rev.D79(2009) 085016; Beuf Phys.Rev.D89(2014) 074039

Iancu,Madrigal,Mueller,Soyez; Phys.Lett. B744 (2015) 293;Lappi,Mantysaari Phys.Rev.D93(2016) 094004

General setup

- Kinematical constraint.
- DGLAP splitting function at LO and NLO.
- NLLx BFKL with suitable subtraction of terms included above.
- Momentum sum rule.
- Running coupling.
- Calculations done in momentum space, even though we use Mellin space as a guidance.

LLx + NLLx

Representation of the kernel $\mathcal{K} = \sum_{n=0}^{\infty} \bar{\alpha}_s^{n+1} \mathcal{K}_n$ $\bar{\alpha}_s \equiv \frac{N_c \alpha_s}{\pi}$

Mellin variables: $\gamma \leftrightarrow \ln k_T^2$ $\omega \leftrightarrow \ln 1/x$

LLx kernel in Mellin space

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

LLx + NLLx

Representation of the kernel $\mathcal{K} = \sum_{n=0}^{\infty} \bar{\alpha}_s^{n+1} \mathcal{K}_n$ $\bar{\alpha}_s \equiv \frac{N_c \alpha_s}{\pi}$

Mellin variables: $\gamma \leftrightarrow \ln k_T^2$ $\omega \leftrightarrow \ln 1/x$

LLx kernel in Mellin space

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

running coupling

triple poles

double poles

NLLx kernel in Mellin space

$$\chi_1(\gamma) = -\frac{b}{2}[\chi_0^2(\gamma) + \chi_0'(\gamma)] - \frac{1}{4}\chi_0''(\gamma) - \frac{1}{4}\left(\frac{\pi}{\sin \pi\gamma}\right)^2 \frac{\cos \pi\gamma}{3(1-2\gamma)}\left(11 + \frac{\gamma(1-\gamma)}{(1+2\gamma)(3-2\gamma)}\right)$$
$$+ \left(\frac{67}{36} - \frac{\pi^2}{12}\right)\chi_0(\gamma) + \frac{3}{2}\zeta(3) + \frac{\pi^3}{4\sin \pi\gamma}$$
$$- \sum_{n=0}^{\infty}(-1)^n \left[\frac{\psi(n+1+\gamma) - \psi(1)}{(n+\gamma)^2} + \frac{\psi(n+2-\gamma) - \psi(1)}{(n+1-\gamma)^2}\right]$$

Strictly speaking at NLLx this is not an eigenvalue. Still, one can consider Mellin transform of the kernel.

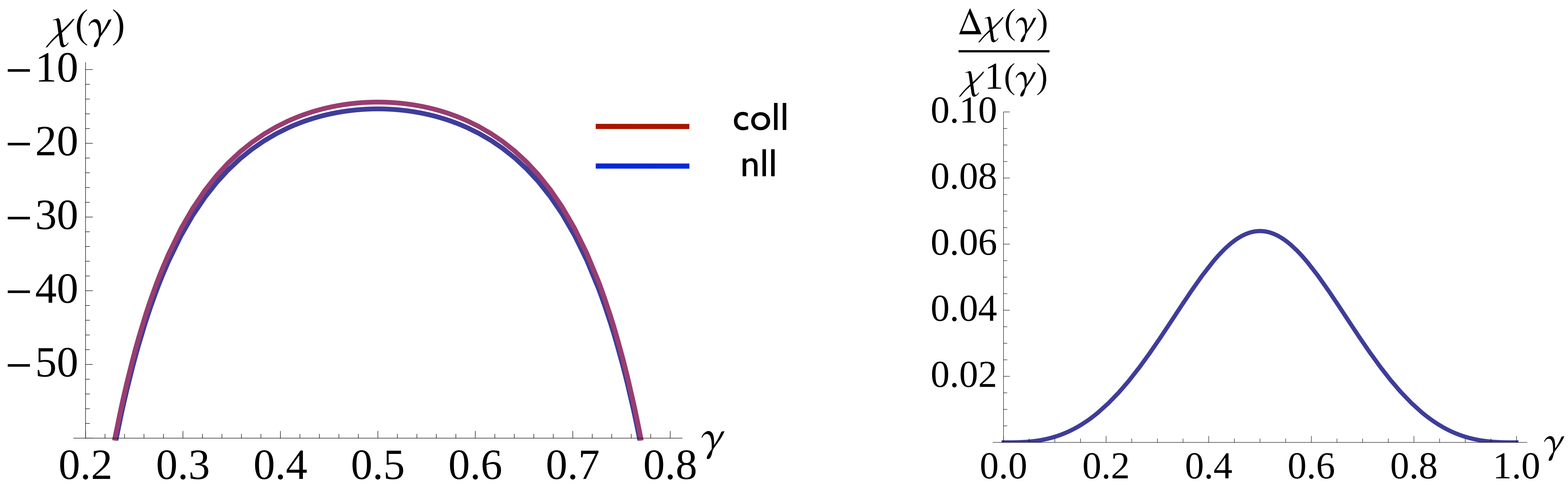
Collinear poles

$$\chi_1^{\rm coll}(\gamma) = -\frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3} + \frac{A_1(0)}{\gamma^2} + \frac{A_1(0)-b}{(1-\gamma)^2}$$

double and triple poles of the NLL part

LO DGLAP anomalous dimension $\gamma_{gg}^{(0)}(\omega) = \frac{\bar{\alpha}_s}{\omega} + \bar{\alpha}_s A_1(\omega)$ $\quad A_1(\omega) = -\frac{11}{12} + \mathcal{O}(\omega)$

Difference of about 7% at most



b=0

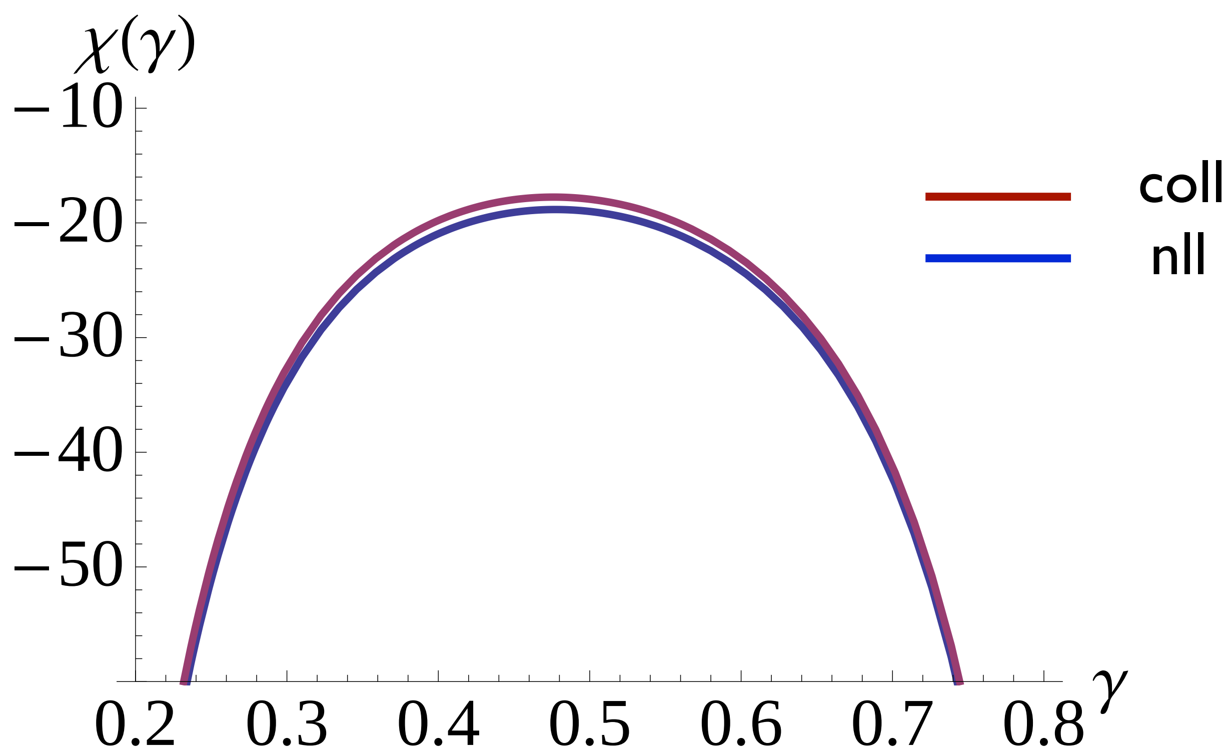
Collinear poles

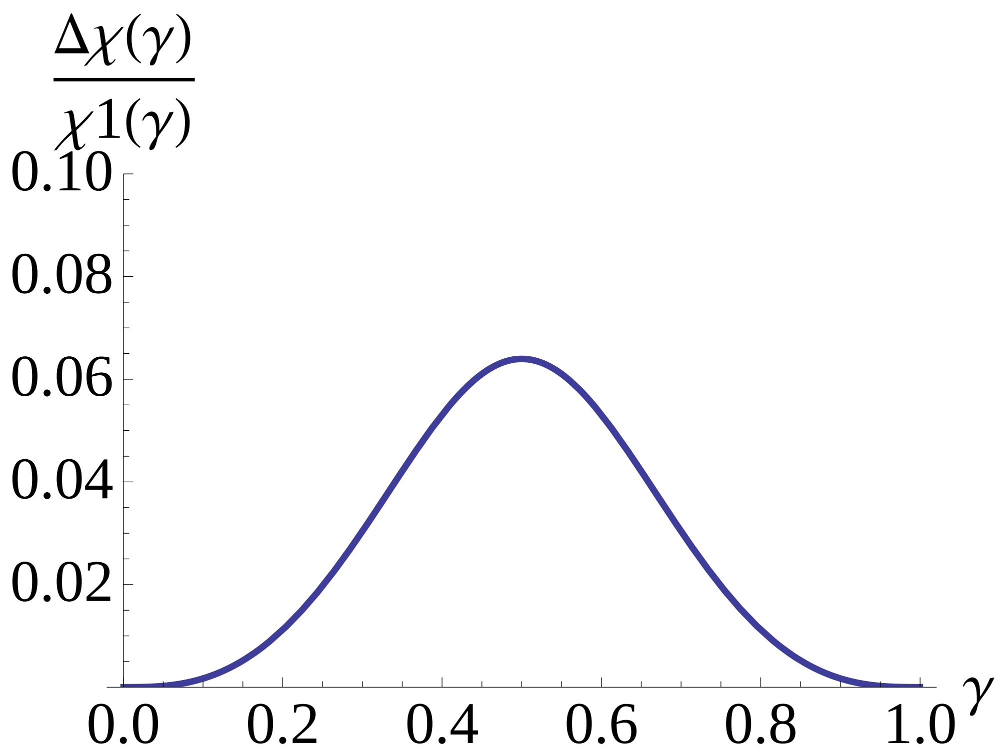
$$\chi_1^{\rm coll}(\gamma) = -\frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3} + \frac{A_1(0)}{\gamma^2} + \frac{A_1(0)-b}{(1-\gamma)^2}$$

double and triple poles of the NLL part

LO DGLAP anomalous dimension $\gamma_{gg}^{(0)}(\omega) = \frac{\bar{\alpha}_s}{\omega} + \bar{\alpha}_s A_1(\omega)$ $\quad A_1(\omega) = -\frac{11}{12} + \mathcal{O}(\omega)$

Difference of about 7% at most





b=0

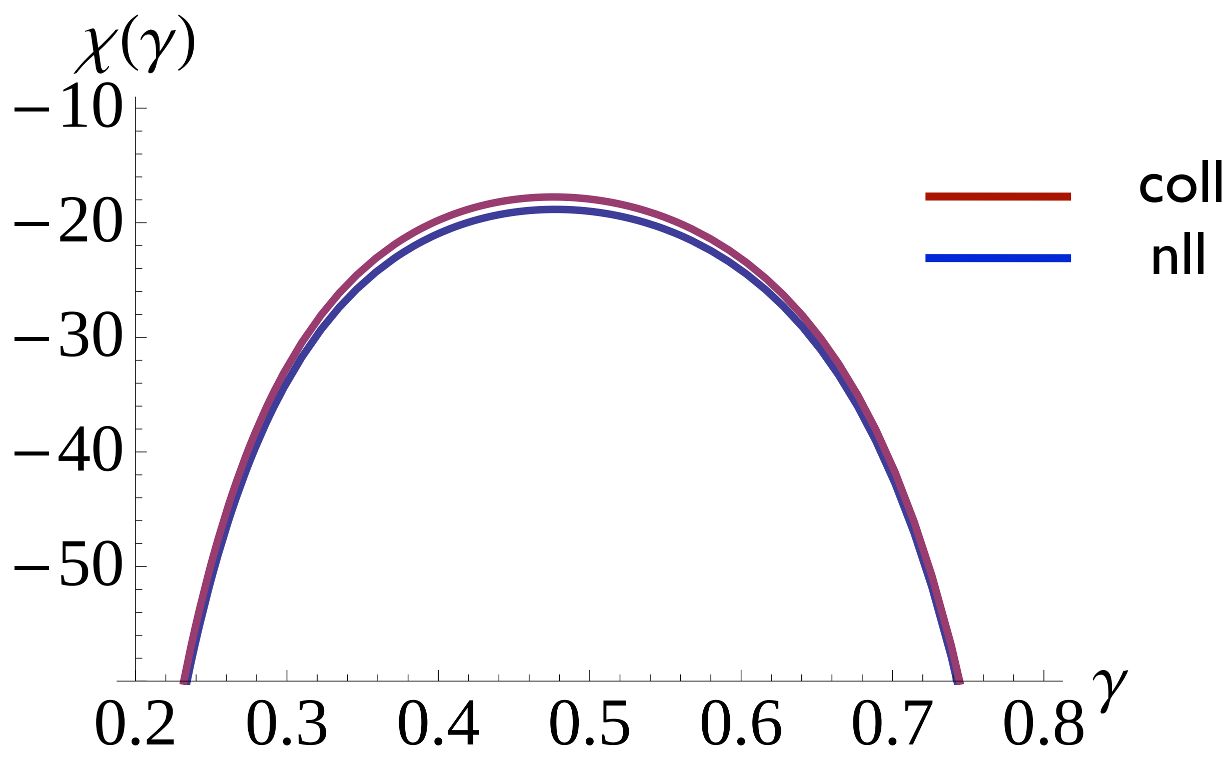
Collinear poles

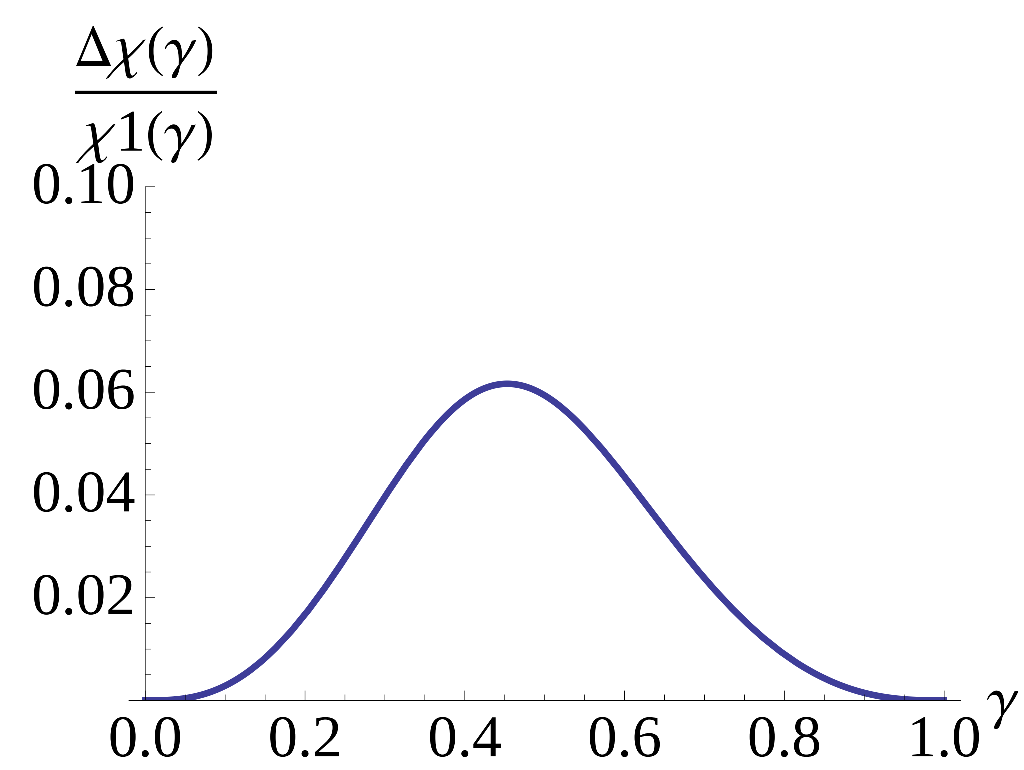
$$\chi_1^{\rm coll}(\gamma) = -\frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3} + \frac{A_1(0)}{\gamma^2} + \frac{A_1(0)-b}{(1-\gamma)^2}$$

double and triple poles of the NLL part

LO DGLAP anomalous dimension $\gamma_{gg}^{(0)}(\omega) = \frac{\bar{\alpha}_s}{\omega} + \bar{\alpha}_s A_1(\omega)$ $\quad A_1(\omega) = -\frac{11}{12} + \mathcal{O}(\omega)$

Difference of about 7% at most





b=0

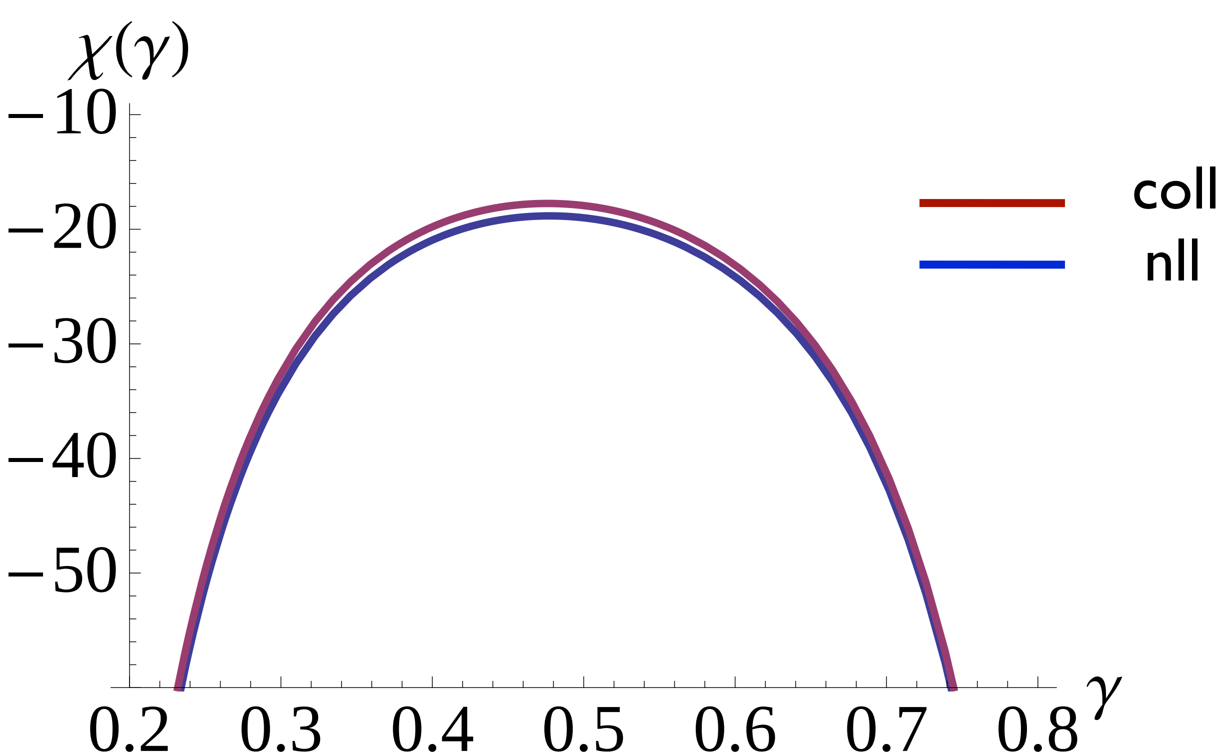
Collinear poles

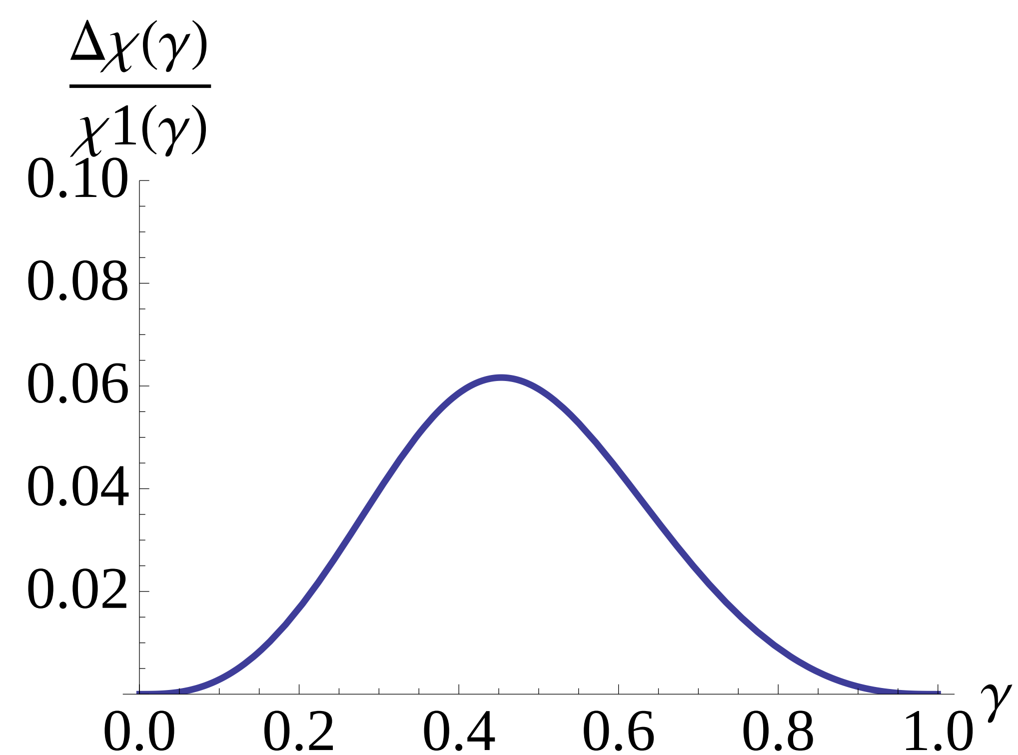
$$\chi_1^{\text{coll}}(\gamma) = -\frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3} + \frac{A_1(0)}{\gamma^2} + \frac{A_1(0) - b}{(1-\gamma)^2}$$

double and triple poles of the NLL part

LO DGLAP anomalous dimension $\gamma_{gg}^{(0)}(\omega) = \frac{\bar{\alpha}_s}{\omega} + \bar{\alpha}_s A_1(\omega)$ $\quad A_1(\omega) = -\frac{11}{12} + \mathcal{O}(\omega)$

Difference of about 7% at most





b=11/12

Scale choices

HE factorization for the cross section

$$\sigma_{AB}(s;Q,Q_0) = \int \frac{d\omega}{2\pi i}\,\frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2}\,\frac{d^2\boldsymbol{k}_0}{\boldsymbol{k}_0^2}\left(\frac{s}{QQ_0}\right)^{\omega} h_\omega^A(Q,\boldsymbol{k})\;\mathcal{G}_\omega(\boldsymbol{k},\boldsymbol{k}_0)\;h_\omega^B(Q_0,\boldsymbol{k}_0)$$

BFKL equation for the gluon Green's function

$$\omega\mathcal{G}_\omega(\boldsymbol{k},\boldsymbol{k}_0) = \delta^2(\boldsymbol{k}-\boldsymbol{k}_0) + \int \frac{d^2\boldsymbol{k}'}{\pi}\,\mathcal{K}_\omega(\boldsymbol{k},\boldsymbol{k}')\;\mathcal{G}_\omega(\boldsymbol{k}',\boldsymbol{k}_0)$$

Scale choices

HE factorization for the cross section

$$\sigma_{AB}(s;Q,Q_0) = \int \frac{d\omega}{2\pi i} \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2} \frac{d^2\boldsymbol{k}_0}{\boldsymbol{k}_0^2} \left(\frac{s}{QQ_0}\right)^{\omega} h_\omega^A(Q,\boldsymbol{k})\, \mathcal{G}_\omega(\boldsymbol{k},\boldsymbol{k}_0)\, h_\omega^B(Q_0,\boldsymbol{k}_0)$$

BFKL equation for the gluon Green's function

$$\omega \mathcal{G}_\omega(\boldsymbol{k},\boldsymbol{k}_0) = \delta^2(\boldsymbol{k}-\boldsymbol{k}_0) + \int \frac{d^2\boldsymbol{k}'}{\pi}\, \mathcal{K}_\omega(\boldsymbol{k},\boldsymbol{k}')\, \mathcal{G}_\omega(\boldsymbol{k}',\boldsymbol{k}_0)$$

Different possible scale choices:

symmetric (ex. two jets) $\nu_0 = k k_0$ $\quad k \sim k_0$

DIS type configuration

$\nu_0 = k^2$ $\quad k \gg k_0$

$\nu_0 = k_0^2$ $\quad k \ll k_0$

Scale choices

HE factorization for the cross section

$$\sigma_{AB}(s;Q,Q_0) = \int \frac{d\omega}{2\pi i}\,\frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2}\,\frac{d^2\boldsymbol{k}_0}{\boldsymbol{k}_0^2}\left(\frac{s}{QQ_0}\right)^{\omega} h_\omega^A(Q,\boldsymbol{k})\,\mathcal{G}_\omega(\boldsymbol{k},\boldsymbol{k}_0)\,h_\omega^B(Q_0,\boldsymbol{k}_0)$$

BFKL equation for the gluon Green's function

$$\omega\mathcal{G}_\omega(\boldsymbol{k},\boldsymbol{k}_0) = \delta^2(\boldsymbol{k}-\boldsymbol{k}_0) + \int \frac{d^2\boldsymbol{k}'}{\pi}\,\mathcal{K}_\omega(\boldsymbol{k},\boldsymbol{k}')\,\mathcal{G}_\omega(\boldsymbol{k}',\boldsymbol{k}_0)$$

Different possible scale choices:

symmetric (ex. two jets) $\qquad \nu_0 = kk_0 \qquad k \sim k_0$

DIS type configuration

$\nu_0 = k^2 \qquad k \gg k_0$

$\nu_0 = k_0^2 \qquad k \ll k_0$

Similarity transformation

$$\mathcal{G}_\omega \longrightarrow \left(\frac{k_>}{k_<}\right)^{\omega}\mathcal{G}_\omega$$

$$\mathcal{K}_\omega(k,k') \;\to\; \mathcal{K}_\omega^u(k,k') \;=\; \mathcal{K}_\omega(k,k')\left(\frac{k}{k'}\right)^{\omega}, \qquad \nu_0 = k^2\,,$$

$$\mathcal{K}_\omega(k,k') \;\to\; \mathcal{K}_\omega^l(k,k') \;=\; \mathcal{K}_\omega(k,k')\left(\frac{k'}{k}\right)^{\omega}, \qquad \nu_0 = k'^2\,,$$

Shift of poles

Shift of poles (symmetric case)

$$\chi_n^\omega(\gamma) = \chi_{nL}^\omega(\gamma + \tfrac{\omega}{2}) + \chi_{nR}^\omega(1-\gamma+\tfrac{\omega}{2})$$

LL case with shifts

$$\chi_0^\omega = 2\psi(1) - \psi(\gamma+\tfrac{\omega}{2}) - \psi(1-\gamma+\tfrac{\omega}{2})$$

Shifts are equivalent to the kinematical constraints imposed on the transverse momenta in the ladder

Expansion reproduces higher order poles:

$$\chi_0^\omega \simeq \chi_0^0 - \frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3} + \ldots$$

symmetric scale choice

Resummed kernel

$$\tilde{\mathcal{K}}_\omega = \bar{\alpha}_s(\boldsymbol{q}^2) K_0^\omega + \omega \bar{\alpha}_s(k_>^2) K_c^\omega + \bar{\alpha}_s^2(k_>^2) \tilde{K}_1^\omega$$

LL with shifts

non-singular DGLAP

NLL with subtractions

All the calculations are actually done in momentum space

Resummed kernel

$$\tilde{\mathcal{K}}_\omega = \bar{\alpha}_s(\boldsymbol{q}^2)K_0^\omega + \omega\bar{\alpha}_s(k_>^2)K_c^\omega + \bar{\alpha}_s^2(k_>^2)\tilde{K}_1^\omega$$

LL with shifts

non-singular DGLAP

NLL with subtractions

$$\chi_0^\omega = 2\psi(1) - \psi(\gamma + \tfrac{\omega}{2}) - \psi(1 - \gamma + \tfrac{\omega}{2})$$

$$\chi_c^\omega(\gamma) = \frac{A_1(\omega)}{\gamma + \frac{\omega}{2}} + \frac{A_1(\omega)}{1 - \gamma + \frac{\omega}{2}} \ ,$$

$$\begin{aligned}\tilde{\chi}_1(\gamma) &= \chi_1(\gamma) - \chi_0^0(\gamma)[\chi_0^1(\gamma) + \chi_c^0(\gamma)] - \chi_0^{\mathrm{run}}(\gamma) \\ &= \chi_1(\gamma) + \frac{1}{2}\chi_0(\gamma)\frac{\pi^2}{\sin^2(\pi\gamma)} - \chi_0(\gamma)\frac{A_1(0)}{\gamma(1-\gamma)} + \frac{b}{2}(\chi_0' + \chi_0^2)\end{aligned}$$

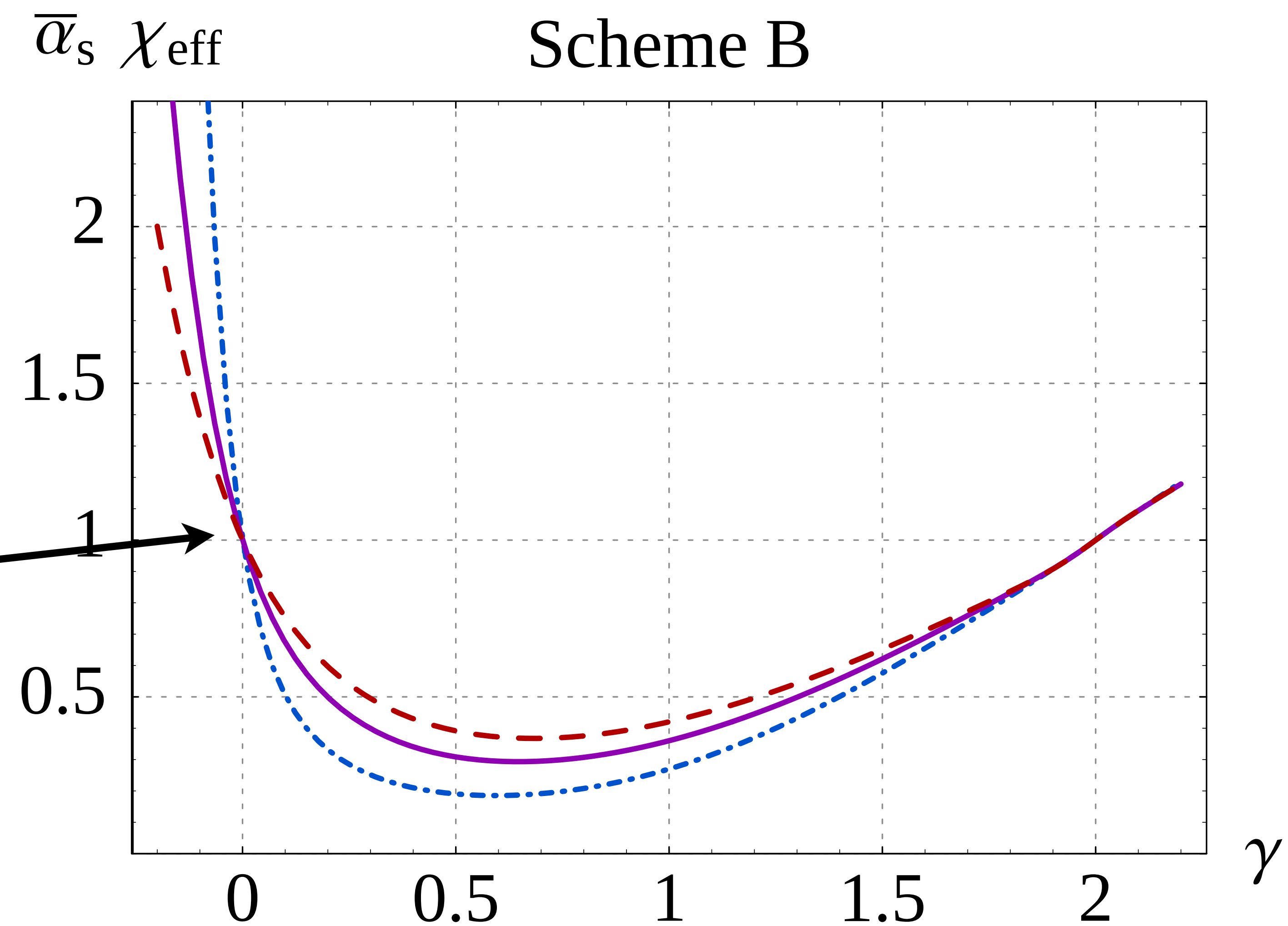
Additional subtraction needed to satisfy the momentum sum rule.

All the calculations are actually done in momentum space

Frozen coupling features

$$\bar{\alpha}_s \chi_\omega(\gamma, \bar{\alpha}_s) = \bar{\alpha}_s(\chi_0^\omega + \omega \chi_c^\omega) + \bar{\alpha}_s^2 \tilde{\chi}_1^\omega$$

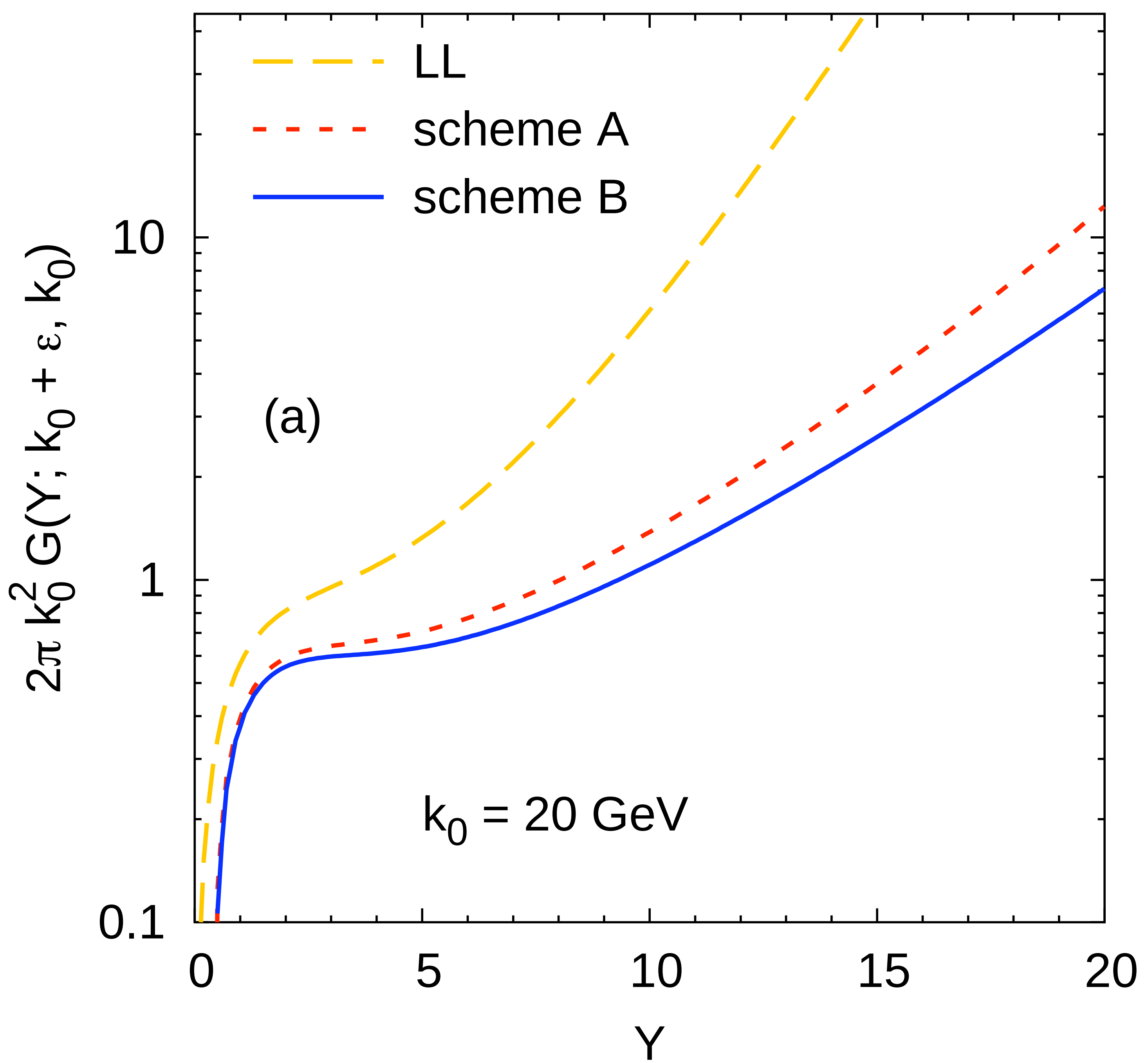
Effective characteristic function: $\omega = \bar{\alpha}_s \chi_{\text{eff}}^{(0)}(\gamma, \bar{\alpha}_s)$

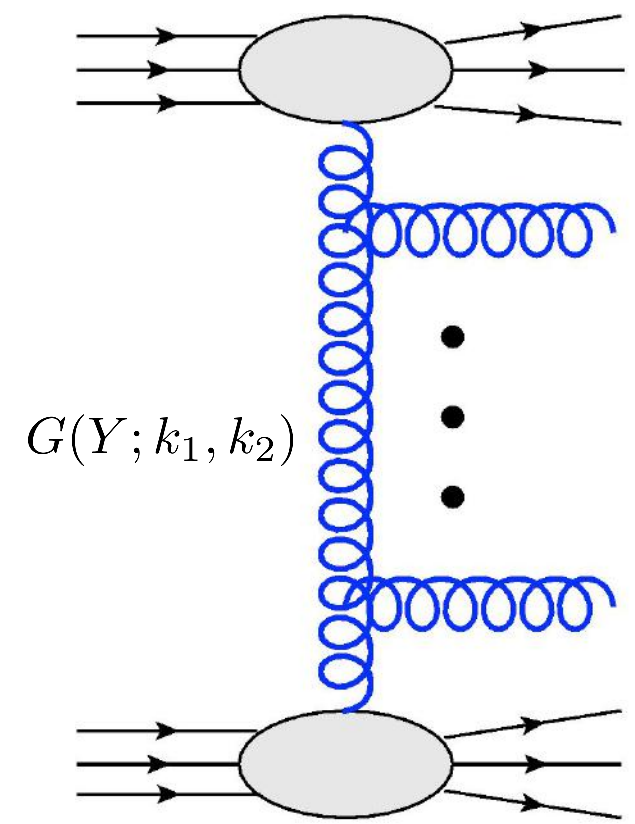


Fixed point (independent of the coupling).
Energy momentum conservation

Gluon Green's function

Solution to the BFKL equation (gluon Green's function)
Single channel: gluons only.

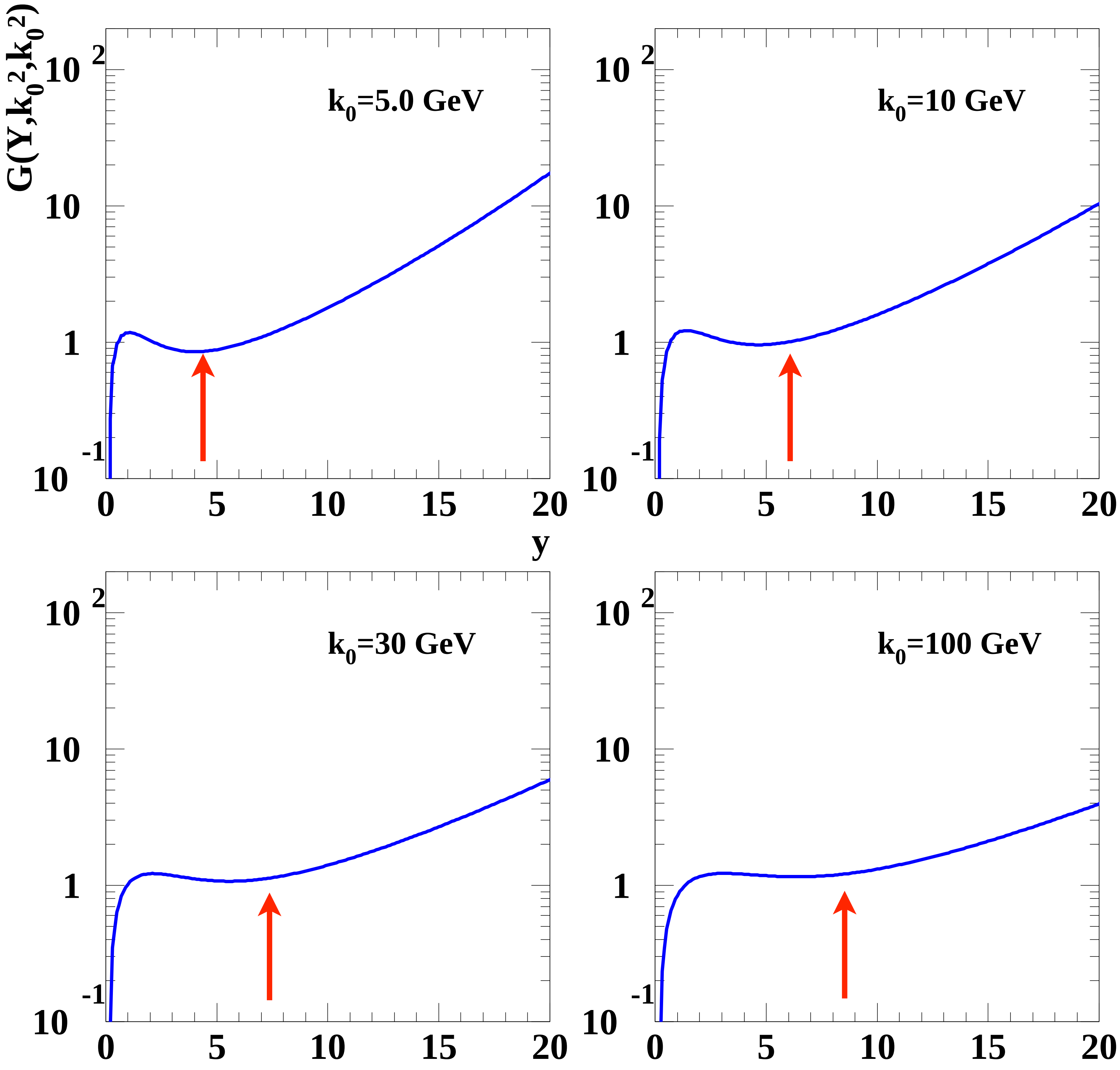




Large suppression as compared to LLx.

Two schemes, small differences.

Gluon Green's function



Effects of resummation:

Lowering effective power

Onset of small x rise delayed

Splitting function

- Deconvolution of the integral equation.
- Calculate the integrated density: $xg(x,Q^2) = \int^{Q^2} dk_T^2\, G^{(s_0=k_T^2)}(x;k_T,k_{0T})$
- Solve numerically for the splitting function:

$$\frac{dg(x,Q^2)}{d\log Q^2} = \int \frac{dz}{z} P_{\rm eff}(z,Q^2)\, g(\frac{x}{z},Q^2)$$

At large values of Q^2 the results should be independent of the regularization of the coupling and the choice of k_0 .

Factorization in Q^2 of the non-perturbative and perturbative contributions.

Splitting function

Gluon-gluon splitting function has logarithmic enhancements at small x

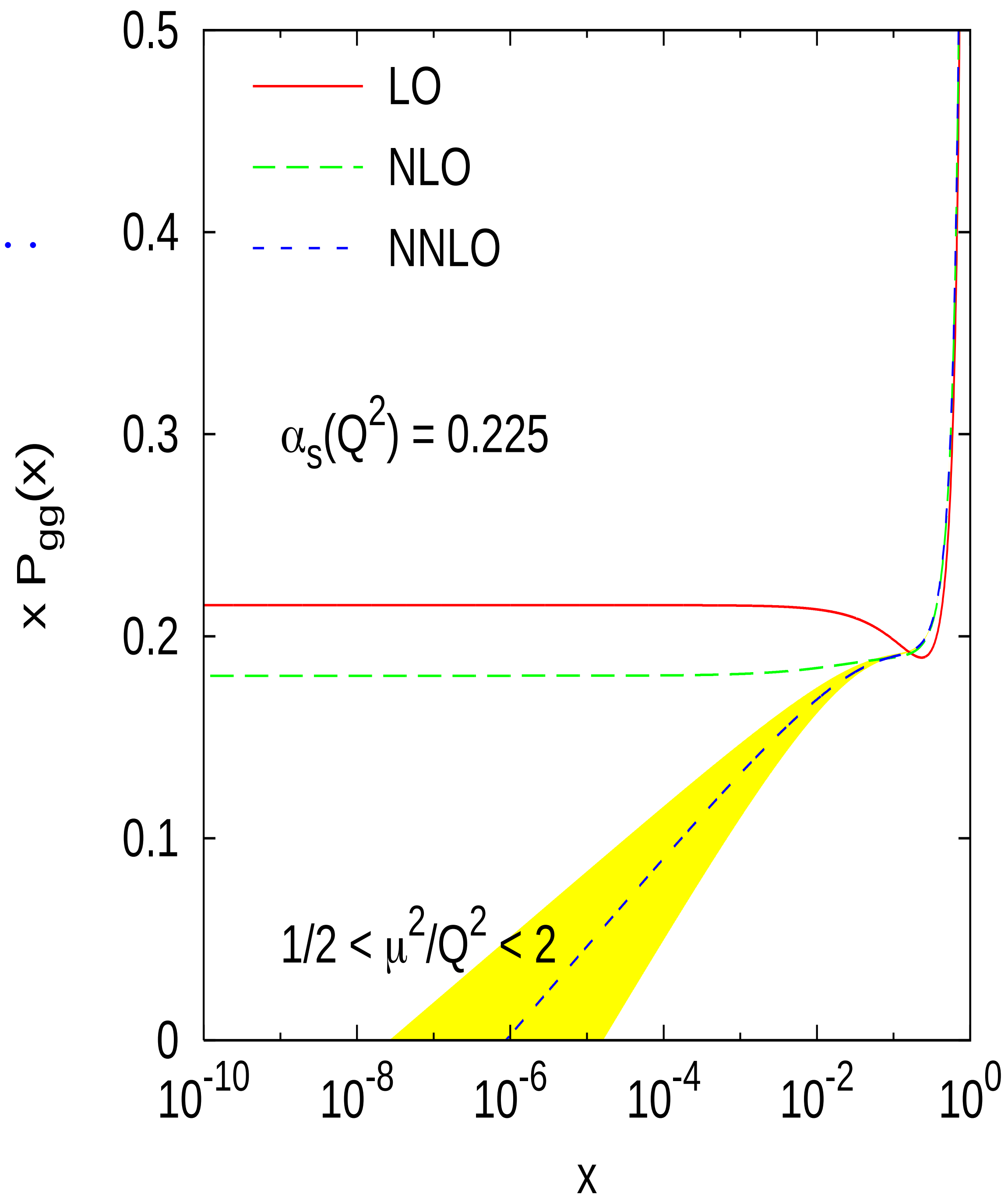
$$xP_{gg}(x) = \sum_{n=1} a_n \alpha_s^n \ln^{n-1}\frac{1}{x} + \sum_{n=2} b_n \alpha_s^n \ln^{n-2}\frac{1}{x} + \dots$$

LLx NLLx

First small x logarithmic term which belongs to NLLx hierarchy recovered at NNLO

$$-1.54\bar{\alpha}_s^3 \ln 1/x$$

Resummation at small x is inevitable.



Splitting function

Gluon-gluon splitting function has logarithmic enhancements at small x

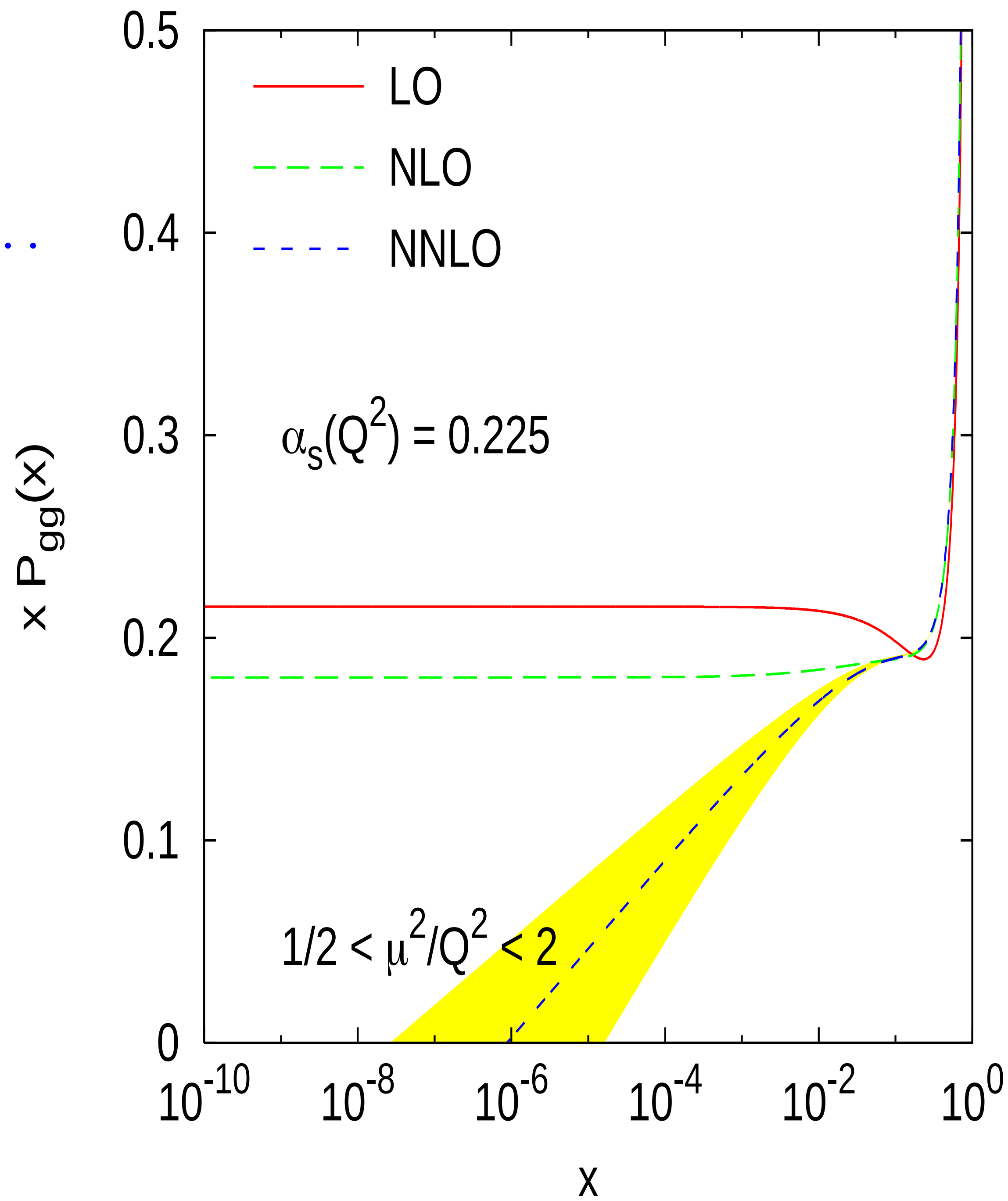
$$xP_{gg}(x) = \sum_{n=1} a_n \alpha_s^n \ln^{n-1}\frac{1}{x} + \sum_{n=2} b_n \alpha_s^n \ln^{n-2}\frac{1}{x} + \dots$$

LLx NLLx

First small x logarithmic term which belongs to NLLx hierarchy recovered at NNLO

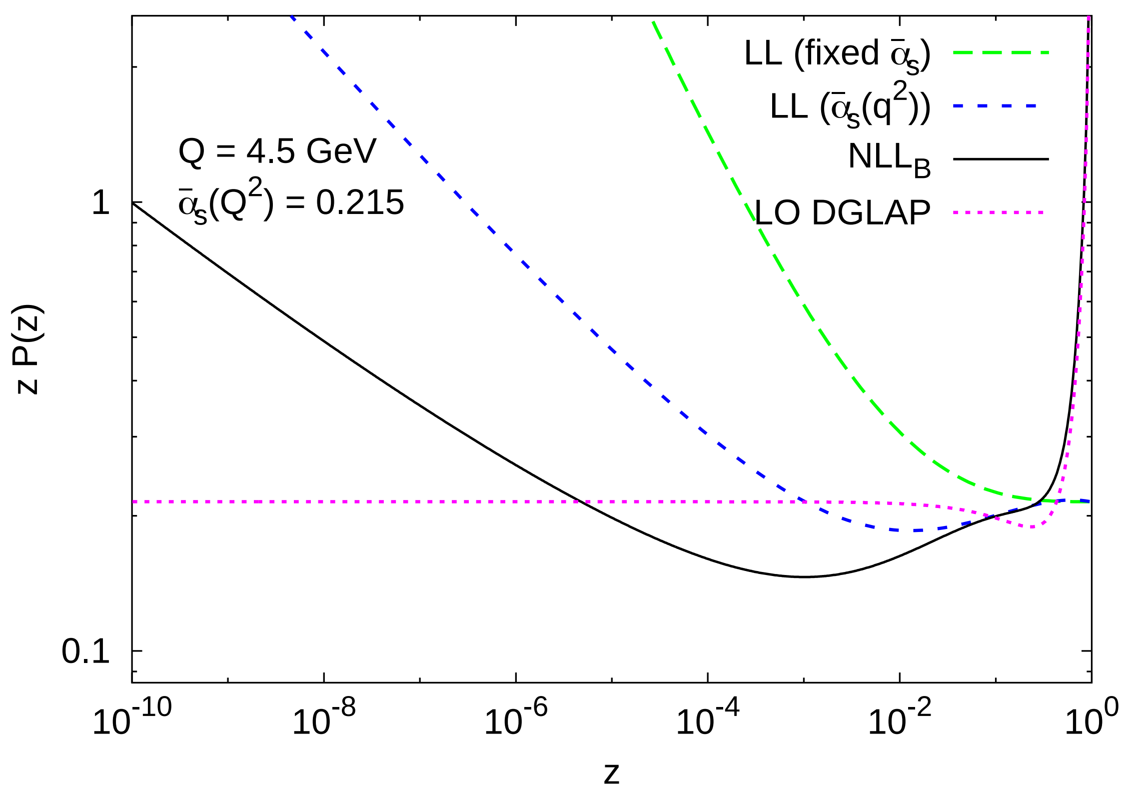
$$-1.54\bar{\alpha}_s^3 \ln 1/x$$

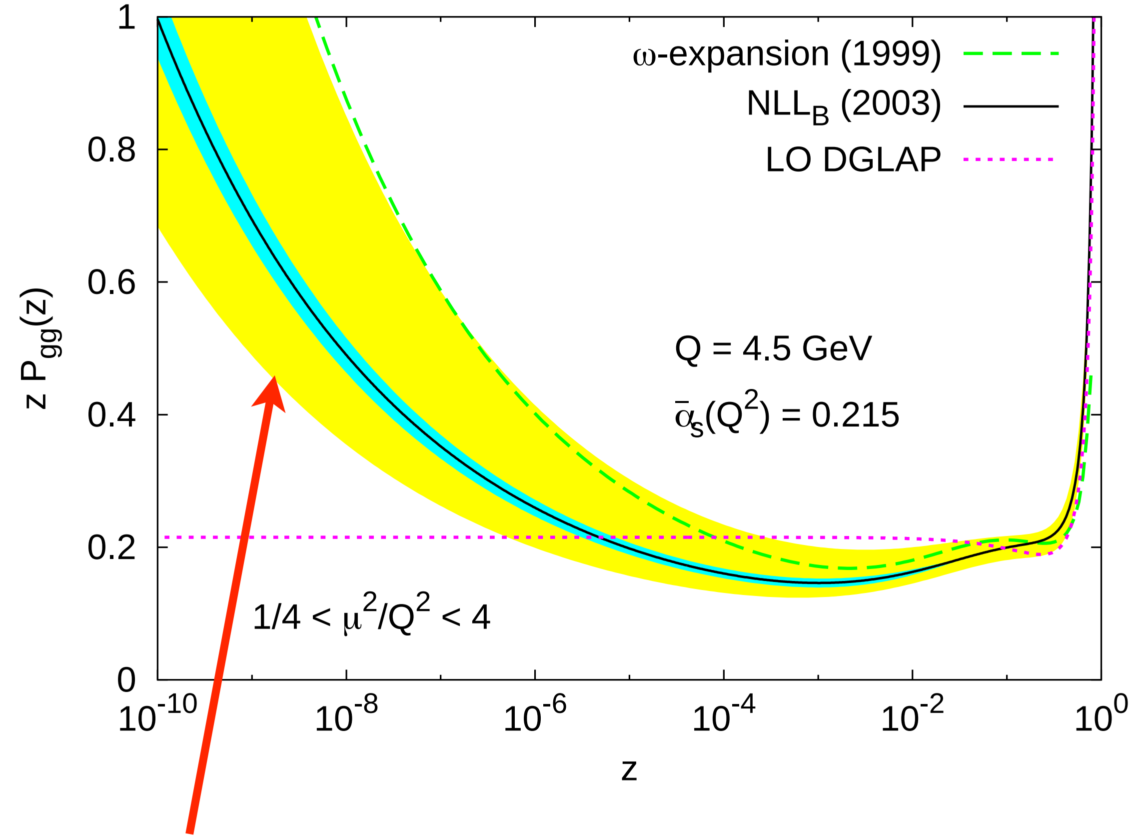
Resummation at small x is inevitable.



Moch, Vermaseren, Vogt

Resummed splitting function

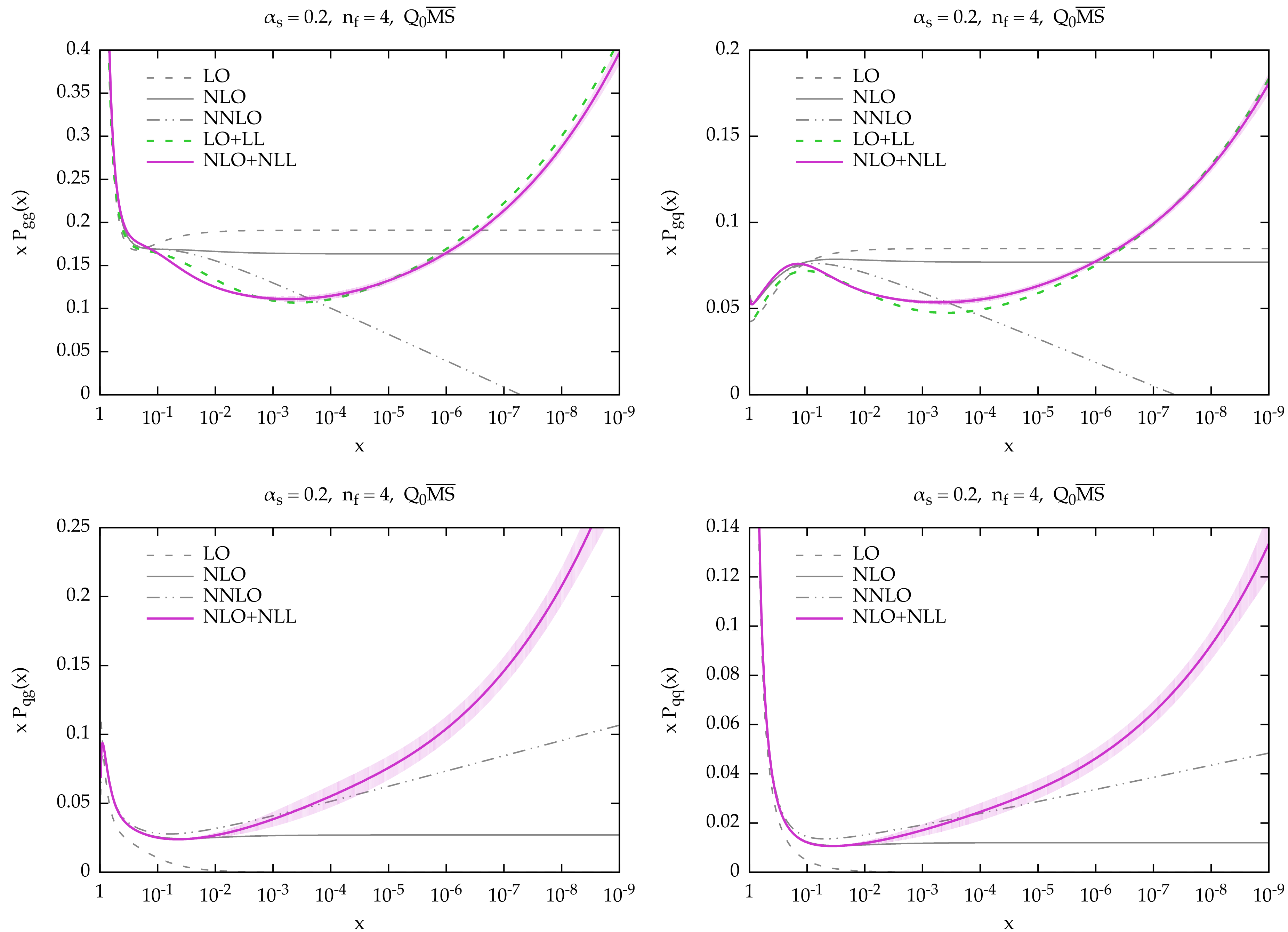




Dependence on the renormalization scale

- Small x growth delayed to much smaller values of x (beyond HERA)
- Interesting feature: a dip seen at around $x \simeq 10^{-3}$
- Is this universal feature ?
- Need to understand the origin of the dip.

Resummed splitting function



The same feature visible in other schemes of resummation

Bonvini,Marzani,Peraro based on Altarelli,Ball,Forte scheme

Understanding the structure of the splitting function

Perturbative terms in the splitting function

	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
$\vdots$				

const.

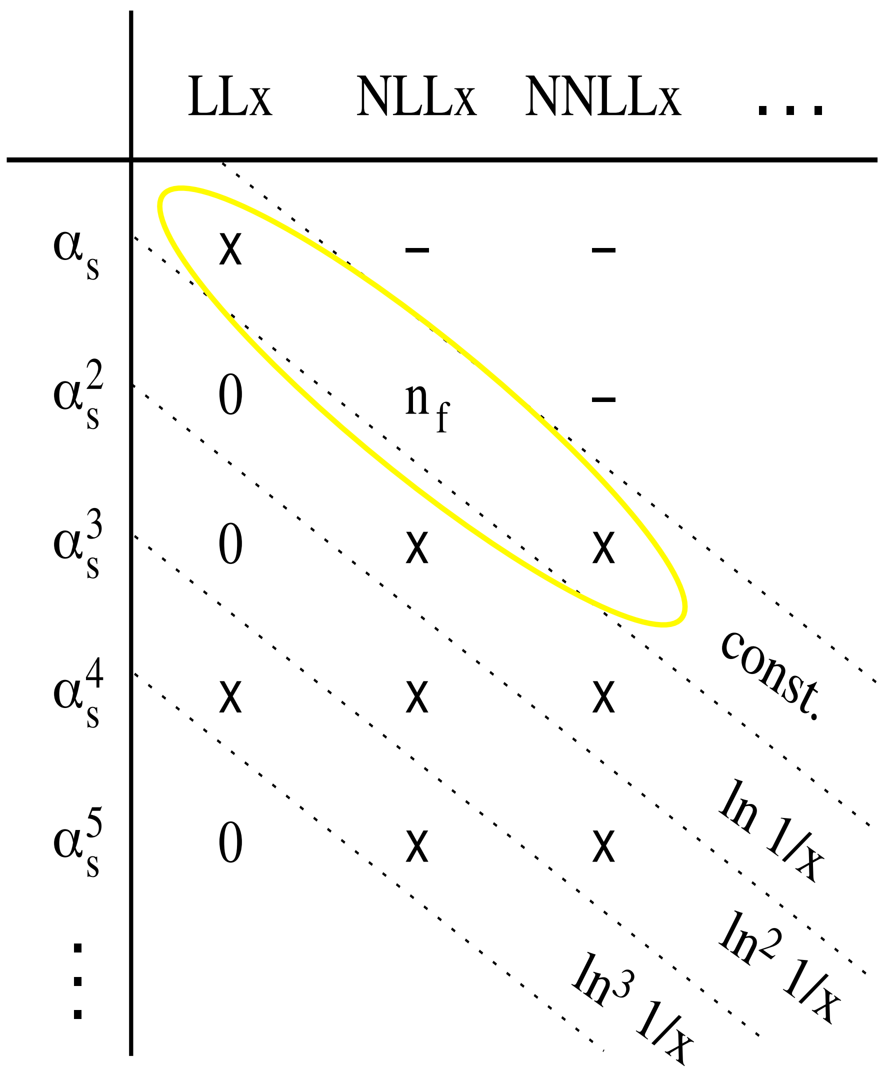
$\ln 1/x$

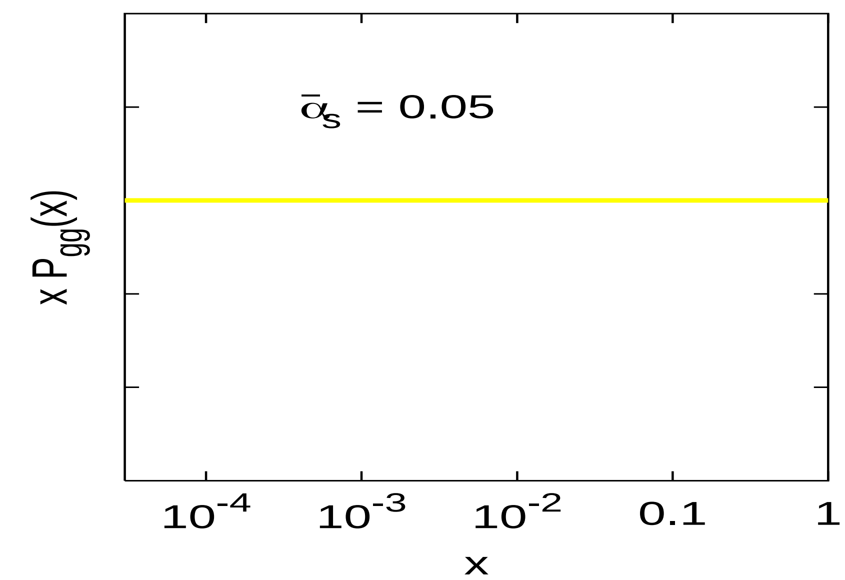
$\ln^2 1/x$

$\ln^3 1/x$

Understanding the structure of the splitting function

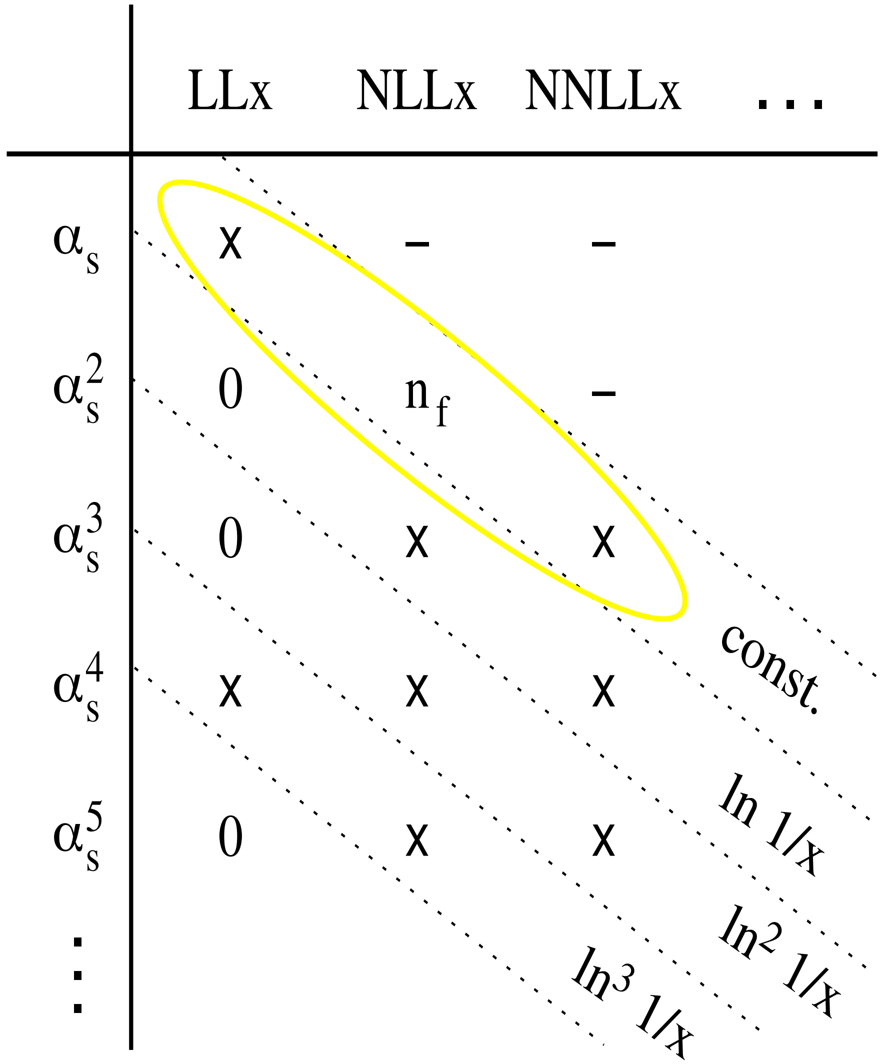
Perturbative terms in the splitting function





Understanding the structure of the splitting function

Perturbative terms in the splitting function



Understanding the structure of the splitting function

Perturbative terms in the splitting function

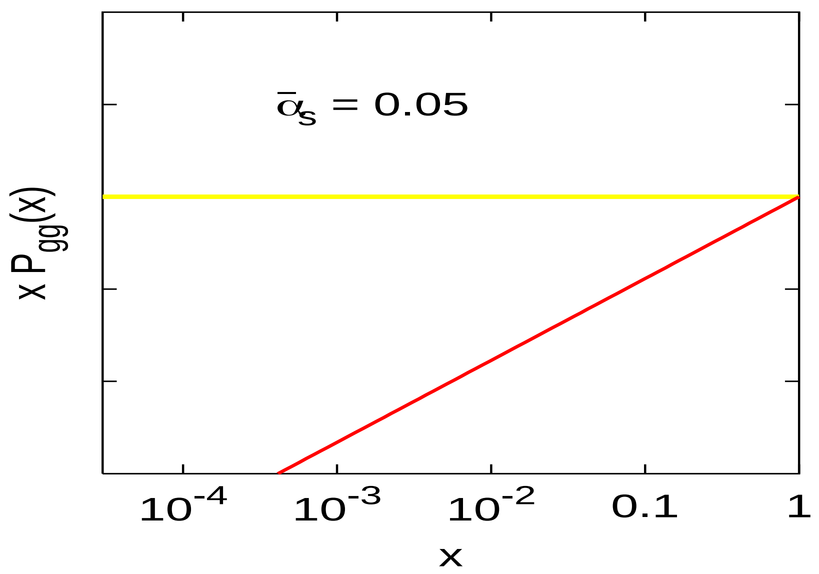
	LLx	NLLx	NNLLx	...
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
⋮				

const.

ln 1/x

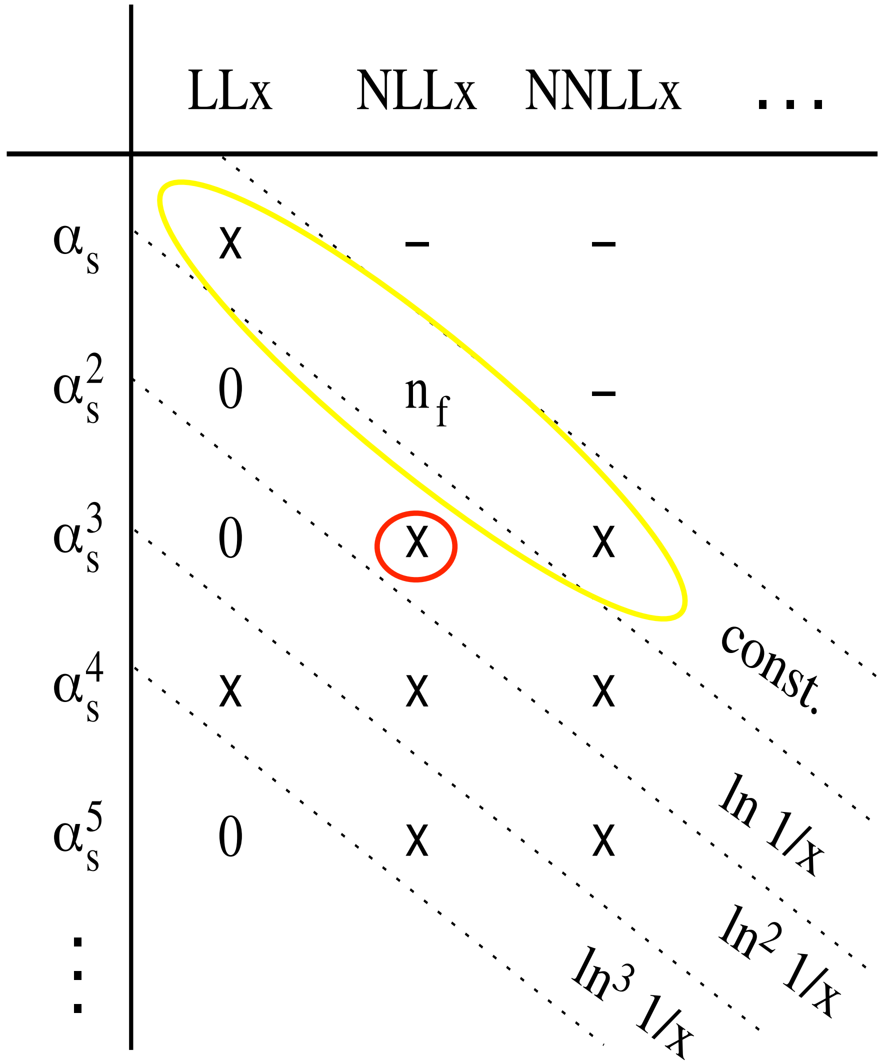
$\ln^2 1/x$

$\ln^3 1/x$



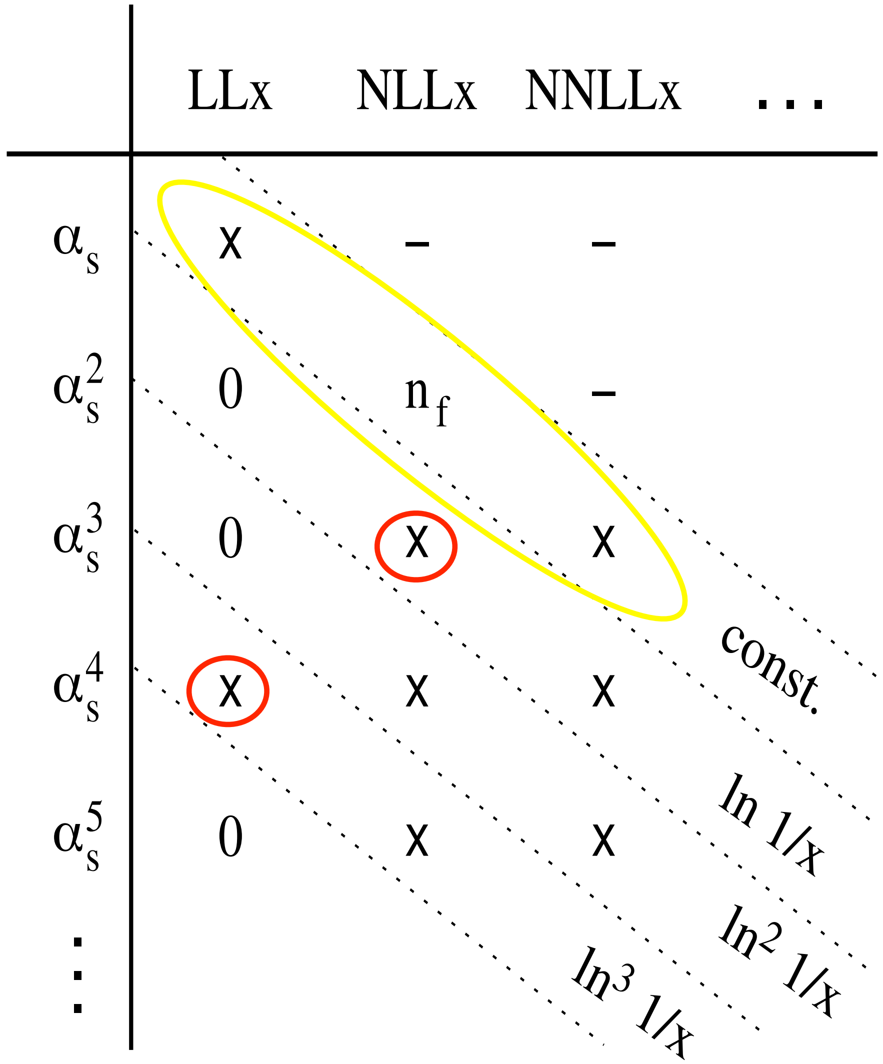
Understanding the structure of the splitting function

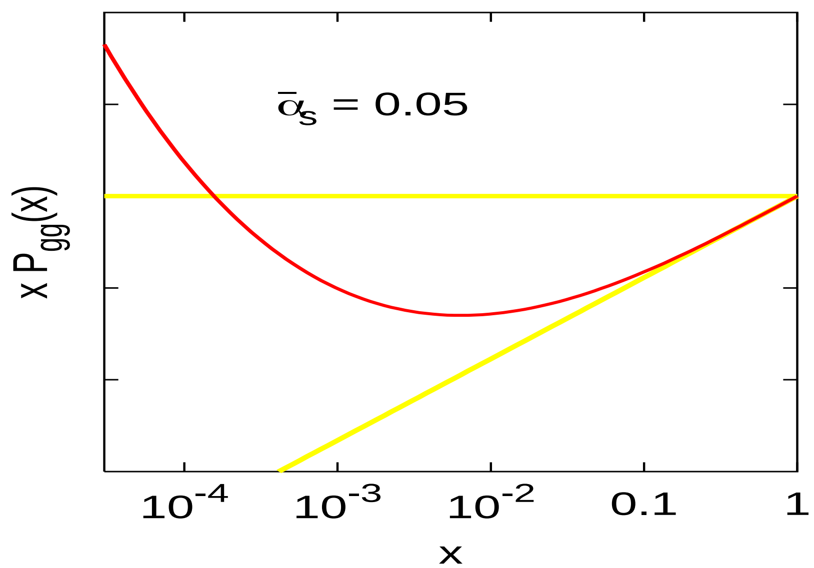
Perturbative terms in the splitting function



Understanding the structure of the splitting function

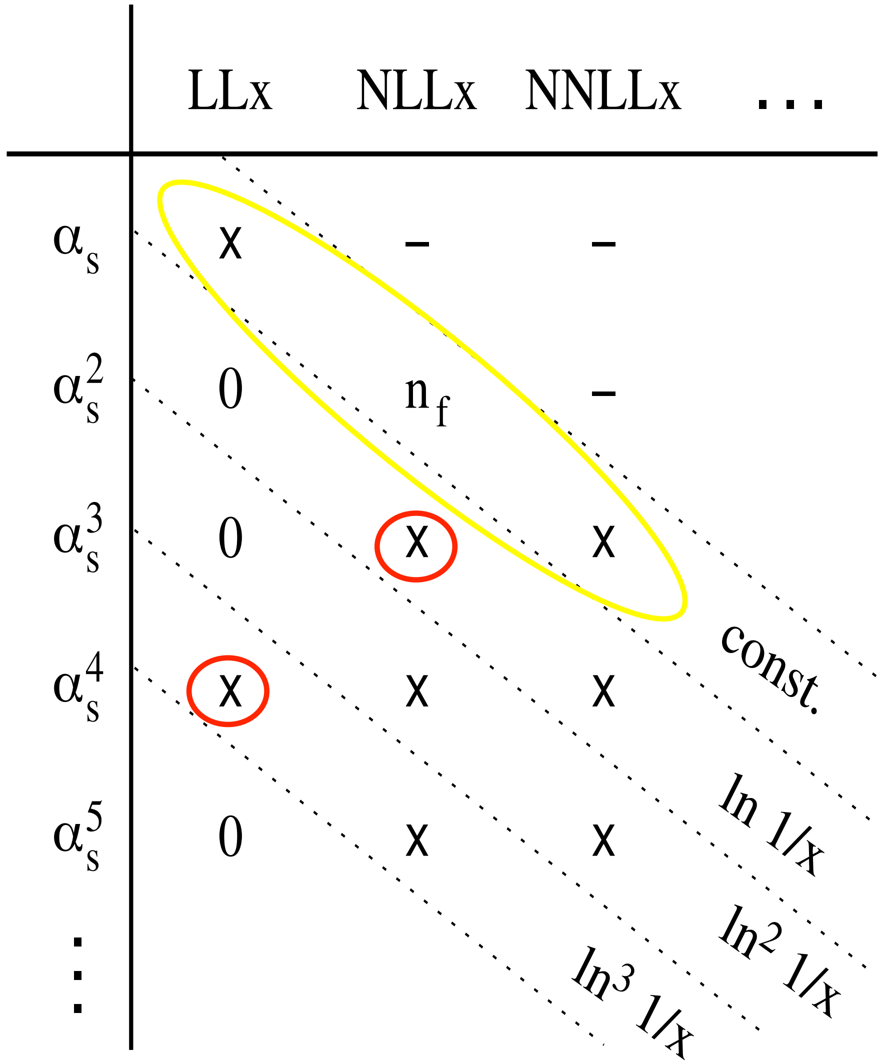
Perturbative terms in the splitting function

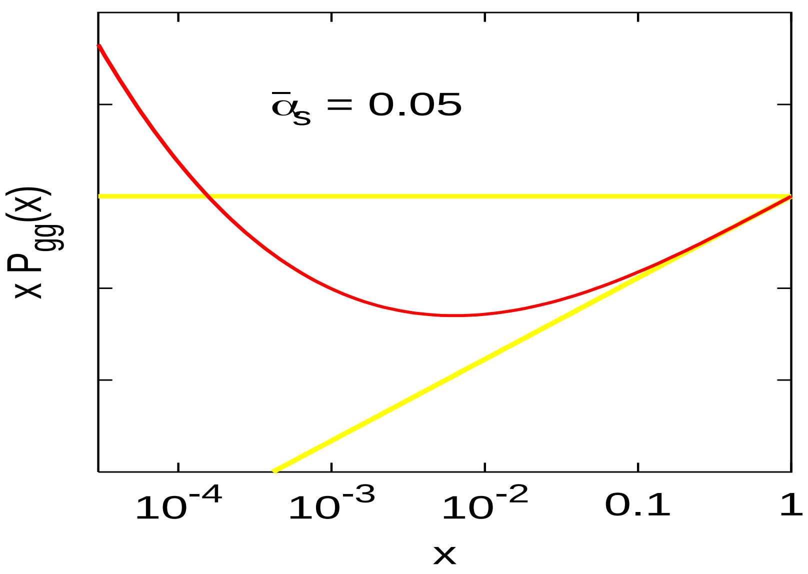




Understanding the structure of the splitting function

Perturbative terms in the splitting function

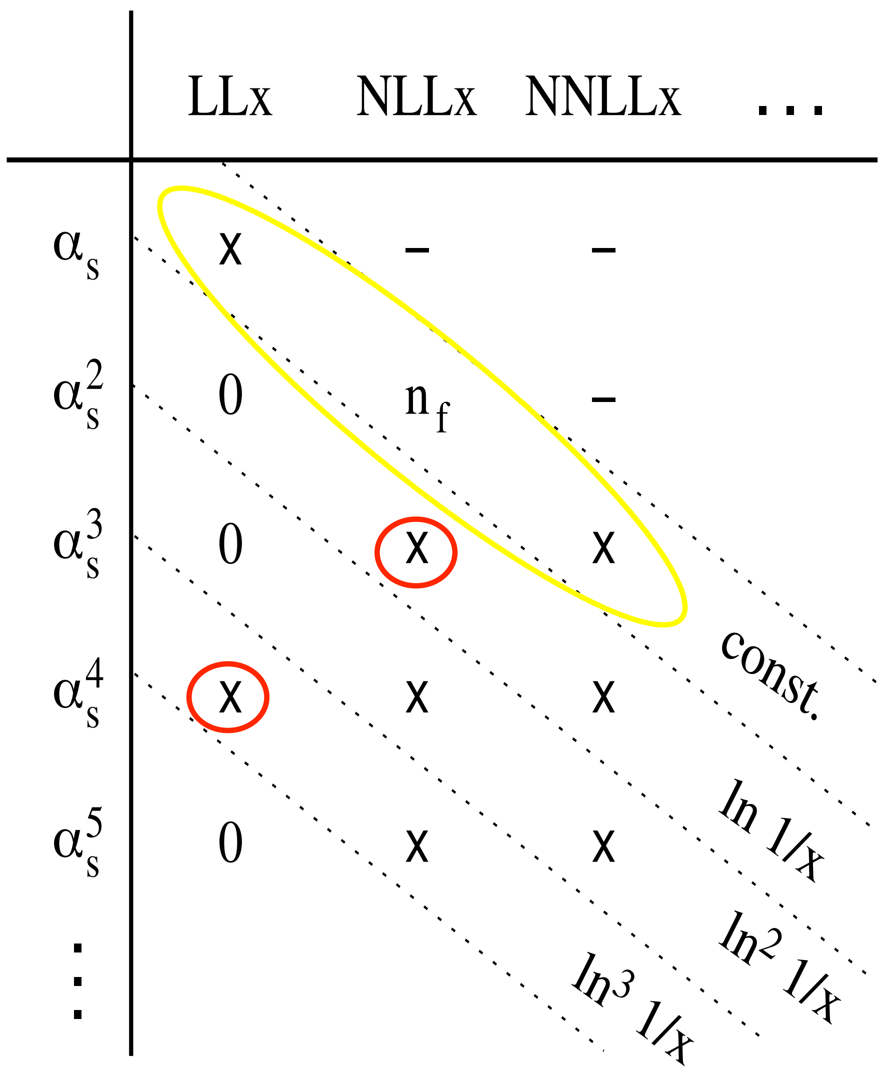


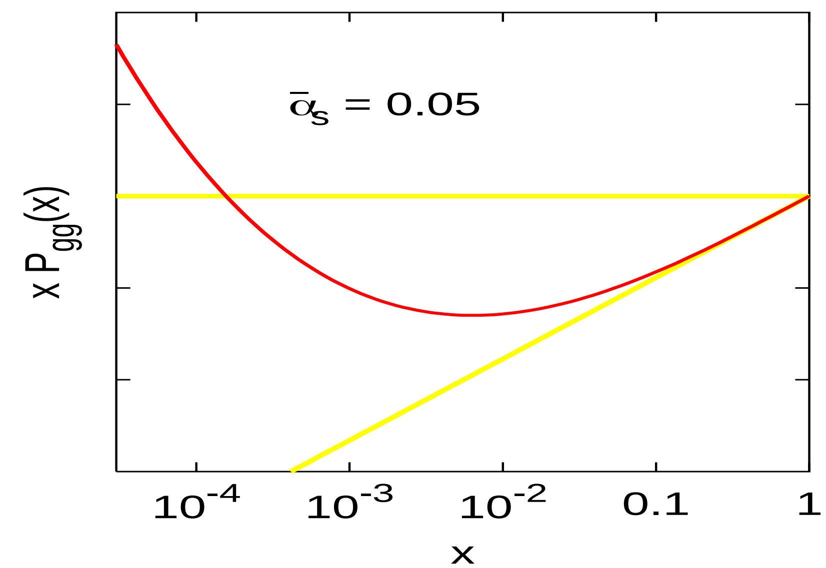


$$-1.54\bar{\alpha}_s^3 \ln 1/x + 0.401\bar{\alpha}_s^4 \ln^3 1/x$$

Understanding the structure of the splitting function

Perturbative terms in the splitting function





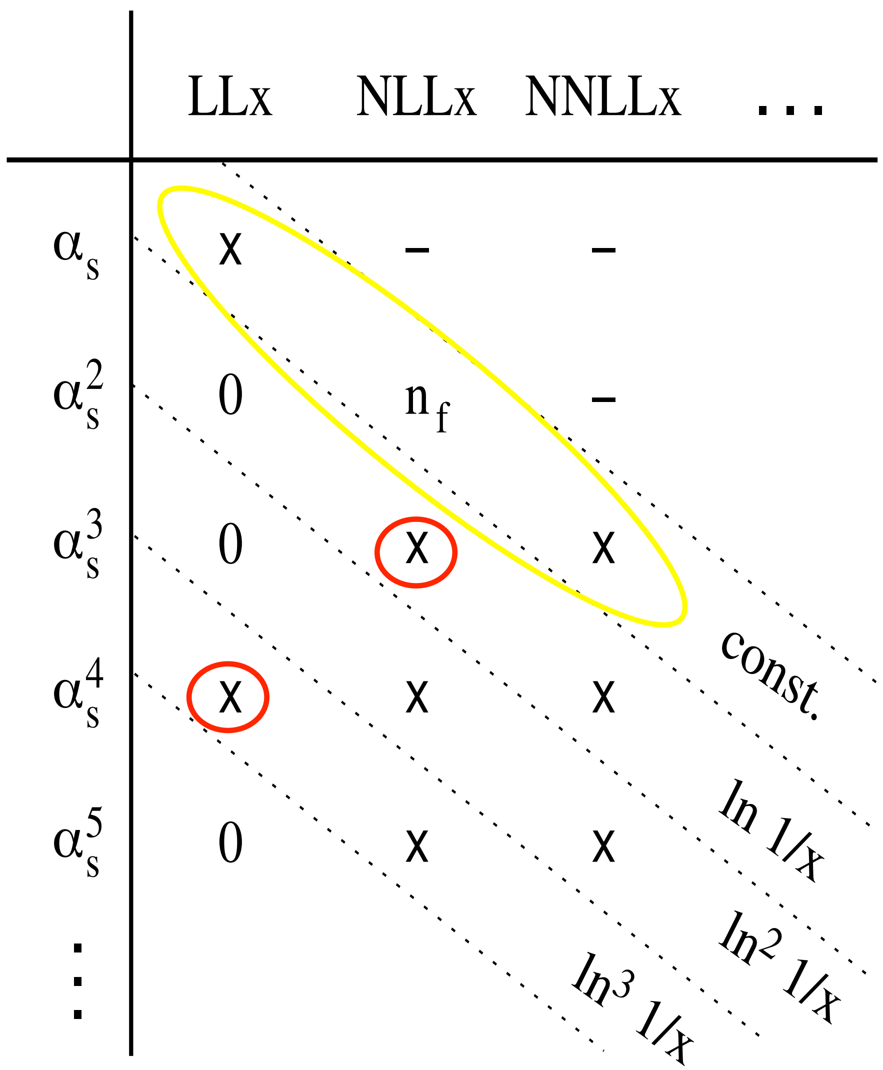
$$-1.54\bar{\alpha}_s^3 \ln 1/x + 0.401\bar{\alpha}_s^4 \ln^3 1/x$$

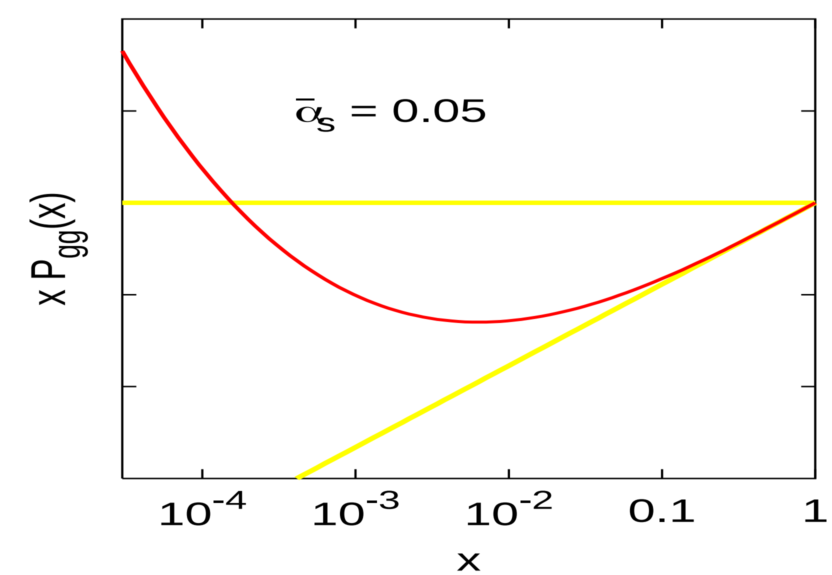
There is a minimum when

$$\alpha_s \ln^2 \frac{1}{x} \sim 1 \longrightarrow \ln \frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

Understanding the structure of the splitting function

Perturbative terms in the splitting function





$$-1.54\bar{\alpha}_s^3 \ln 1/x \,+\, 0.401\bar{\alpha}_s^4 \ln^3 1/x$$

There is a minimum when

$$\alpha_s \ln^2 \frac{1}{x} \sim 1 \;\longrightarrow\; \ln \frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$

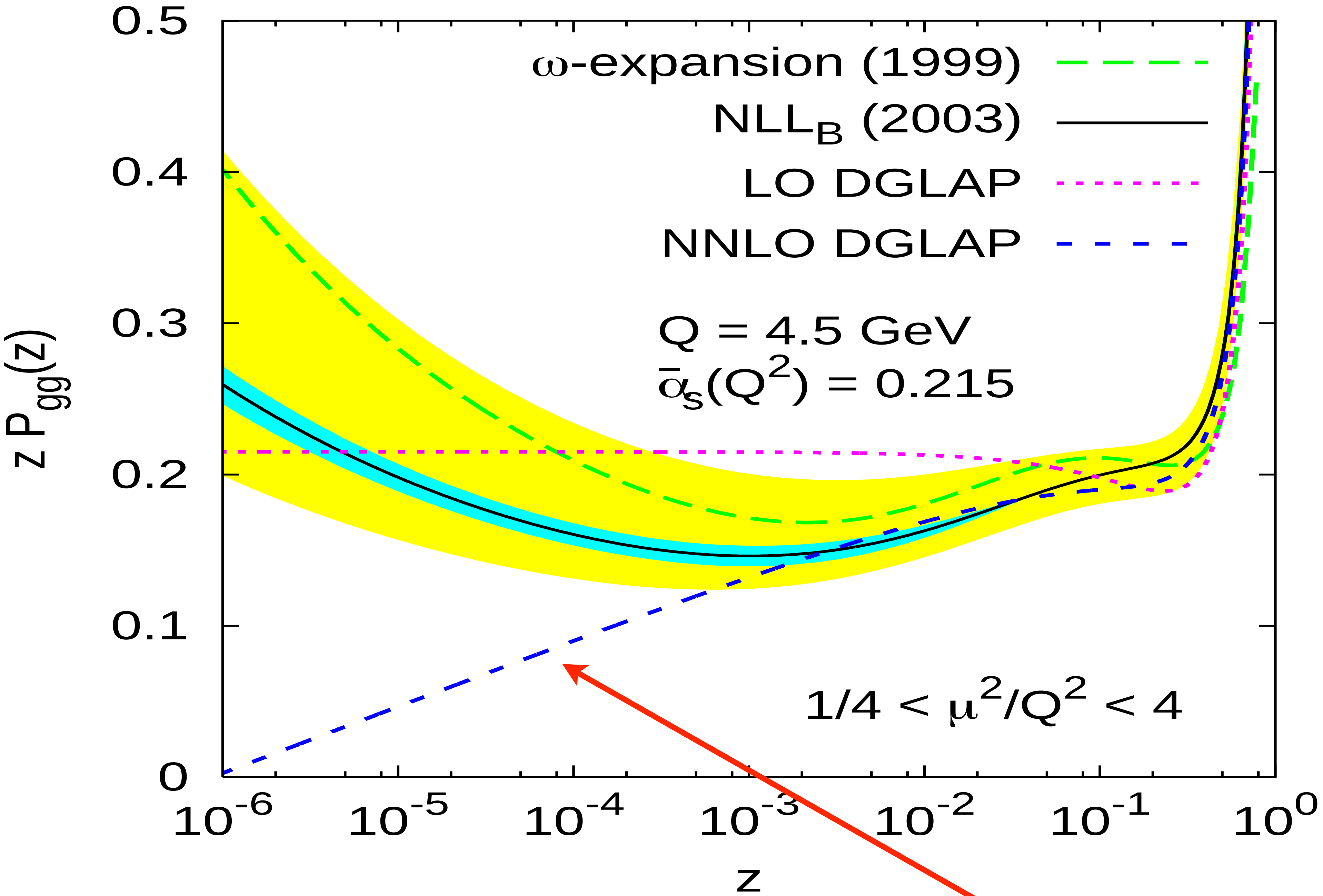
In general: dip comes from the interplay between NNLO and the resummation.

Summary and outlook

- Resummation schemes at low x based on collinear improvements: kinematical effects, matching to DGLAP
- Stability of the results demonstrated for scale changes and model changes.
- Characteristic features: reduced Pomeron intercept and small x growth delayed by several units of rapidity. Dip of the splitting function and dip in the Green's function.
- Impact on saturation: lowering the saturation scale.
- EIC : kinematic range where strong preasymptotic effects present. Still, increased luminosity and possibility of F_L measurement can help.
- Need NLO impact factors, and possibly resummation thereof to increase accuracy of theoretical predictions.

Backup

Where the dip comes from?



$$-1.54\bar{\alpha}_s^3 \ln 1/x$$

Initial decrease seems to be consistent with the small x NNLO term.

Resummed kernel in x,k$_T$

$$\int_x^1 \frac{dz}{z} \int dk'^2\, \tilde{\mathcal{K}}(z;k,k') f(\frac{x}{z},k')$$
$$= \int_x^1 \frac{dz}{z} \int dk'^2 \left[\bar{\alpha}_s(\boldsymbol{q}^2) K_0^{\mathrm{kc}}(z;\boldsymbol{k},\boldsymbol{k}') + \bar{\alpha}_s(k_>^2) K_{\mathrm{c}}^{\mathrm{kc}}(z;k,k') + \bar{\alpha}_s^2(k_>^2)\tilde{K}_1(k,k')\right] f(\frac{x}{z},k')$$

LL BFKL with consistency constraint

$$\int_x^1 \frac{dz}{z} \int dk'^2 \ \left[\bar{\alpha}_s(\boldsymbol{q}^2) K_0^{\mathrm{kc}}(z;\boldsymbol{k},\boldsymbol{k}')\right] f(\frac{x}{z},k')$$
$$= \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{q}}{\pi \boldsymbol{q}^2}\, \bar{\alpha}_s(\boldsymbol{q}^2) \left[f(\frac{x}{z},|\boldsymbol{k}+\boldsymbol{q}|)\Theta(\frac{k}{z}-k')\Theta(k'-kz) - \Theta(k-q) f(\frac{x}{z},k)\right]$$

non-singular DGLAP with consistency constraint

$$\int_x^1 \frac{dz}{z} \int dk'^2\, \bar{\alpha}_s(k_>^2) K_{\mathrm{c}}^{\mathrm{kc}}(z;k,k') f(\frac{x}{z},k')$$
$$= \int_x^1 \frac{dz}{z} \int_{(kz)^2}^{k^2} \frac{dk'^2}{k^2}\, \bar{\alpha}_s(k^2) z \frac{k}{k'} \tilde{P}_{gg}(z\frac{k}{k'}) f(\frac{x}{z},k')$$
$$+ \int_x^1 \frac{dz}{z} \int_{k^2}^{(k/z)^2} \frac{dk'^2}{k'^2}\, \bar{\alpha}_s(k'^2) z \frac{k'}{k} \tilde{P}_{gg}(z\frac{k'}{k}) f(\frac{x}{z},k')\ ,$$

Resummed kernel in x,k_T

NLL BFKL with subtractions

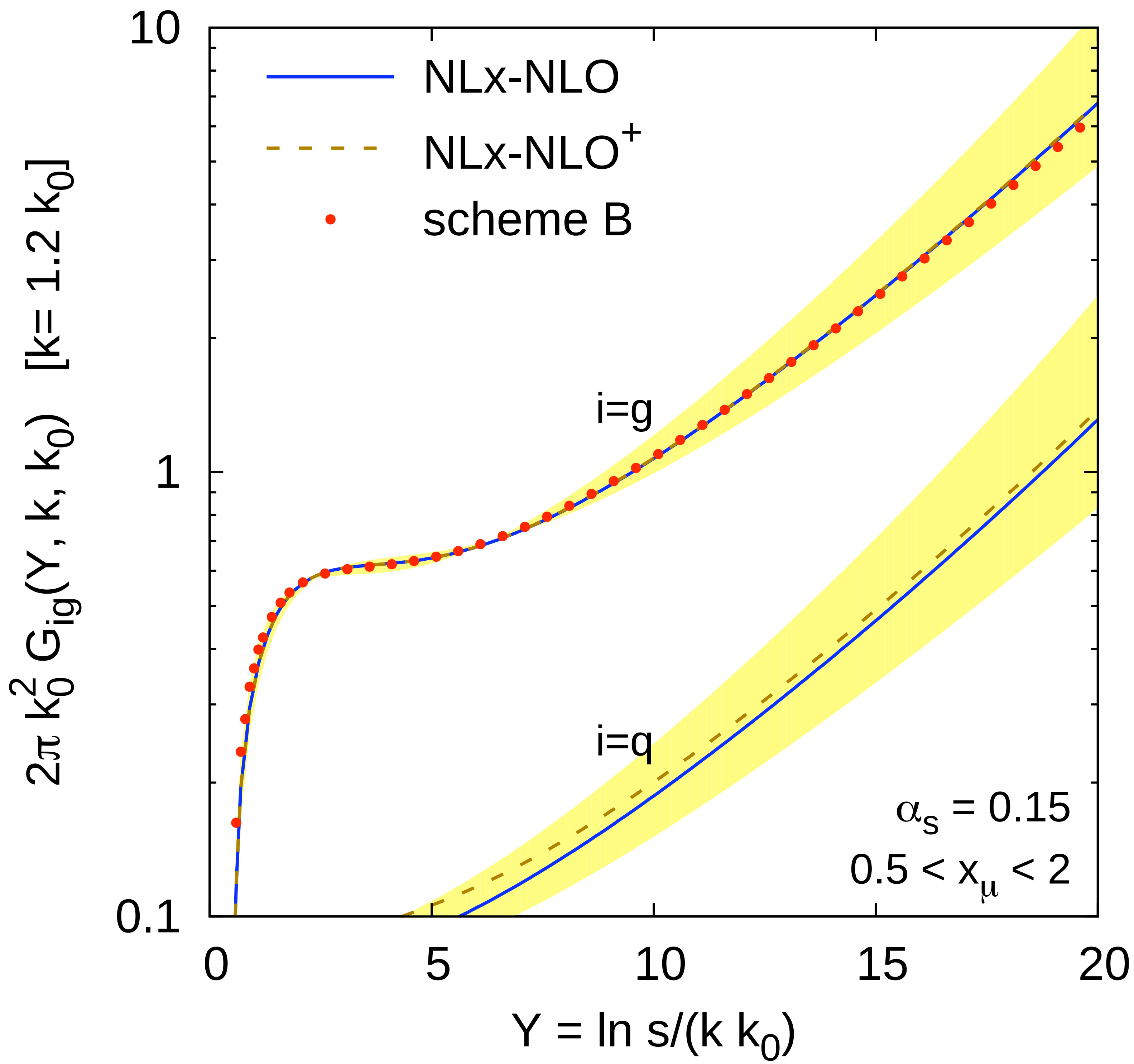
$$
\begin{aligned}
&\int_x^1 \frac{dz}{z} \int dk'^2\, \bar\alpha_s^2(k_>^2)\tilde K_1(k,k') f(\frac{x}{z},k') \\
&= \frac{1}{4}\int_x^1 \frac{dz}{z}\int dk'^2\, \bar\alpha_s^2(k_>^2)\Bigg\{ \\
&\left(\frac{67}{9}-\frac{\pi^2}{3}\right)\frac{1}{|k'^2-k^2|}\left[f(\frac{x}{z},k'^2)-\frac{2k_<^2}{(k'^2+k^2)}f(\frac{x}{z},k^2)\right]+ \\
&\Bigg[-\frac{1}{32}\left(\frac{2}{k'^2}+\frac{2}{k^2}+\left(\frac{1}{k'^2}-\frac{1}{k^2}\right)\log\left(\frac{k^2}{k'^2}\right)\right)+\frac{4\mathrm{Li}_2(1-k_<^2/k_>^2)}{|k'^2-k^2|} \\
&-4A_1(0)\mathrm{sgn}(k^2-k'^2)\left(\frac{1}{k^2}\log\frac{|k'^2-k^2|}{k'^2}-\frac{1}{k'^2}\log\frac{|k'^2-k^2|}{k^2}\right) \\
&-\left(3+\left(\frac{3}{4}-\frac{(k'^2+k^2)^2}{32k'^2k^2}\right)\right)\int_0^\infty \frac{dy}{k^2+y^2k'^2}\log|\frac{1+y}{1-y}| \\
&+\frac{1}{k'^2+k^2}\left(\frac{\pi^2}{3}+4\mathrm{Li}_2(\frac{k_<^2}{k_>^2})\right)\Bigg]f(\frac{x}{z},k')\Bigg\} \\
&+\frac{1}{4}6\zeta(3)\int_x^1\frac{dz}{z}\,\bar\alpha_s^2(k^2)f(\frac{x}{z},k)\,.
\end{aligned}
$$

Gluon Green's function

- Resummation identical to the single channel case in gg part.

- qg channel suppressed by factor of the coupling. Quarks are generated radiatively therefore growth in Y follows gg channel.

- Small difference between two resummation schemes:NLx-NLO and NLx-NLO. $_+$

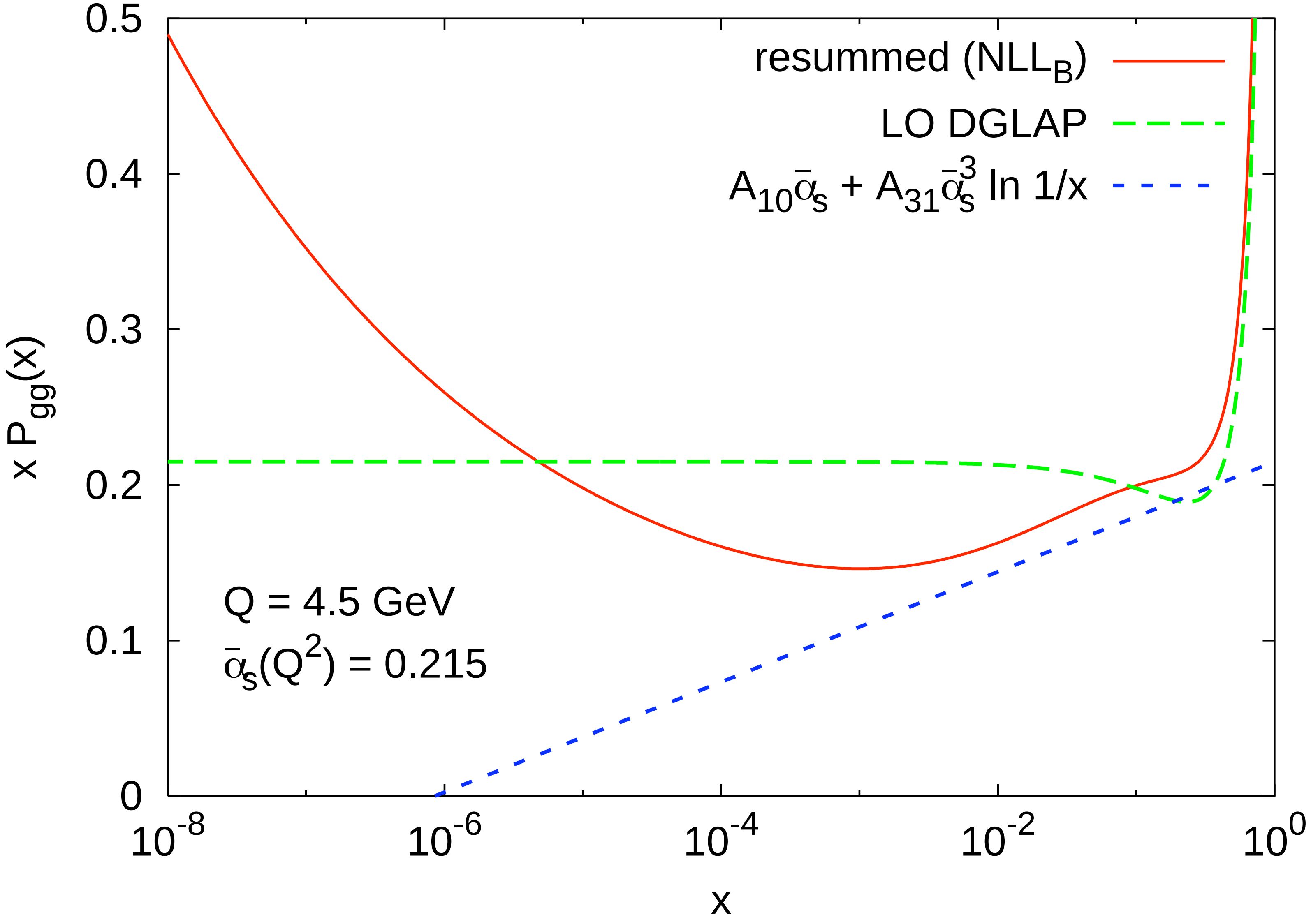


Momentum sum rule

		NLx-NLO		NLx-NLO$^+$	
α_s	Q [GeV]	$\sum_j \Gamma_{jq}(1)$	$\sum_j \Gamma_{jg}(1)$	$\sum_j \Gamma_{jq}(1)$	$\sum_j \Gamma_{jg}(1)$
0.20	6	0.0079	-0.0059	0.0074	-0.0055
0.15	20	0.0021	-0.0015	0.0018	-0.0012
0.10	220	0.00012	-0.00003	0.00006	0.00002

Momentum sum rule satisfied to very good accuracy.
Residual Q dependence (higher twist, non-perturbative regularization?)

Results on splitting function



The onset of small x rise delayed to x<0.0001.
Characteristic dip at around x=0.001.
Universal feature of the resummed approaches.

Results on splitting function