

Large x Physics in UPCs and EIC

Misak Sargsian
Florida International University, Miami



Probing QCD in Photon-Nucleus Interactions at RHIC and the LHC -> EIC
Feb, 13-17, 2017, INT, UW, Seattle

(I) Probing Nuclear Dynamics at Very Short Distances

- $r < 0.5 \text{ fm}$ ($> 1\text{GeV}$)

- Physics of Nuclear Core

- We know why nuclei are bound
- We don't know why they are stable

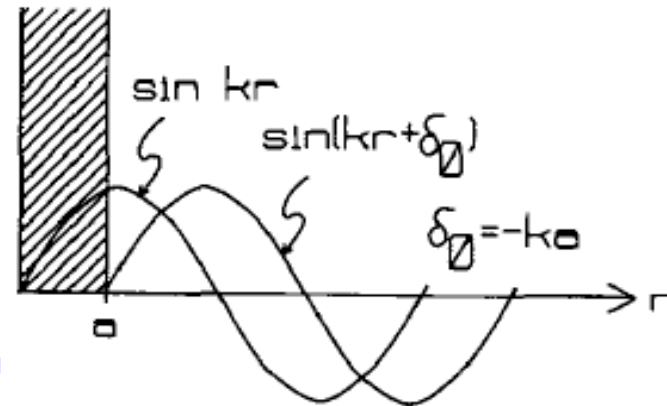
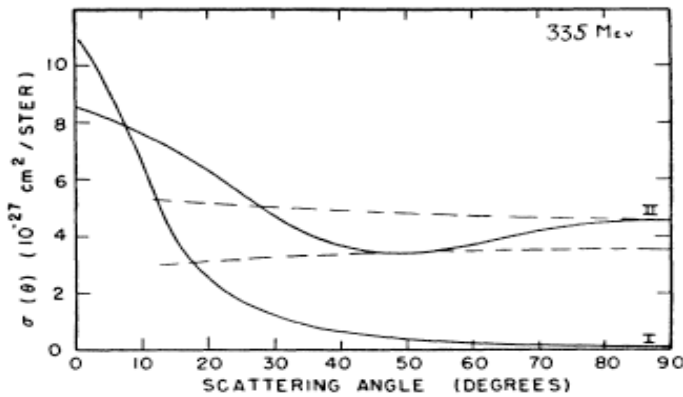
- Hadron- quark/gluon transition
- Color non-singlet states
- Gluonic content of NN core
- Delta-Delta NN* components

(II) Probing Nuclear Partonic Distributions at large x

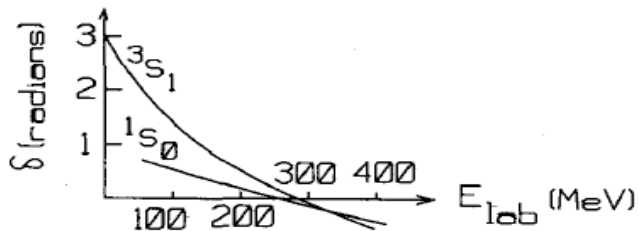
- Physics of the EMC Effect
- Modification of parton distributions in the bound nucleon
 - Dependence of modification on local densities/momenta
 - x and Q^2 dependence of parton modifications

(I) Probing Nuclear Dynamics at very short distances: NN repulsive core

Jastrow 1951 assumed the existence of the hard core to explain the angular distribution of pp cross section at 340 MeV ($r_0=0.6\text{fm}$)



$r_0 = 0.4\text{fm}$



Stability Theorem: Nuclei will Collapse without Repulsive interaction 1950s Weisskopf, Blatt

Modern NN Potentials

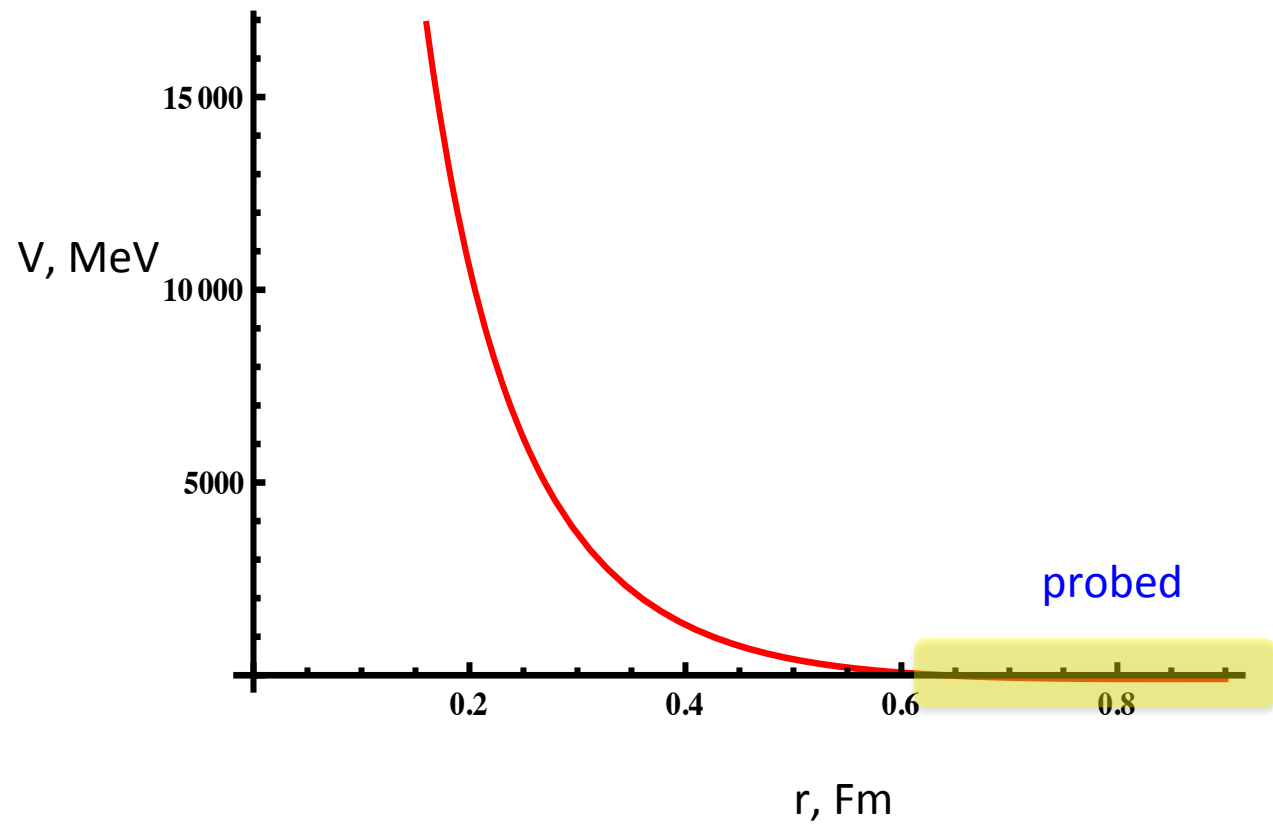
$$V^{2N} = V_{EM}^{2N} + V_{\pi}^{2N} + V_R^{2N}$$

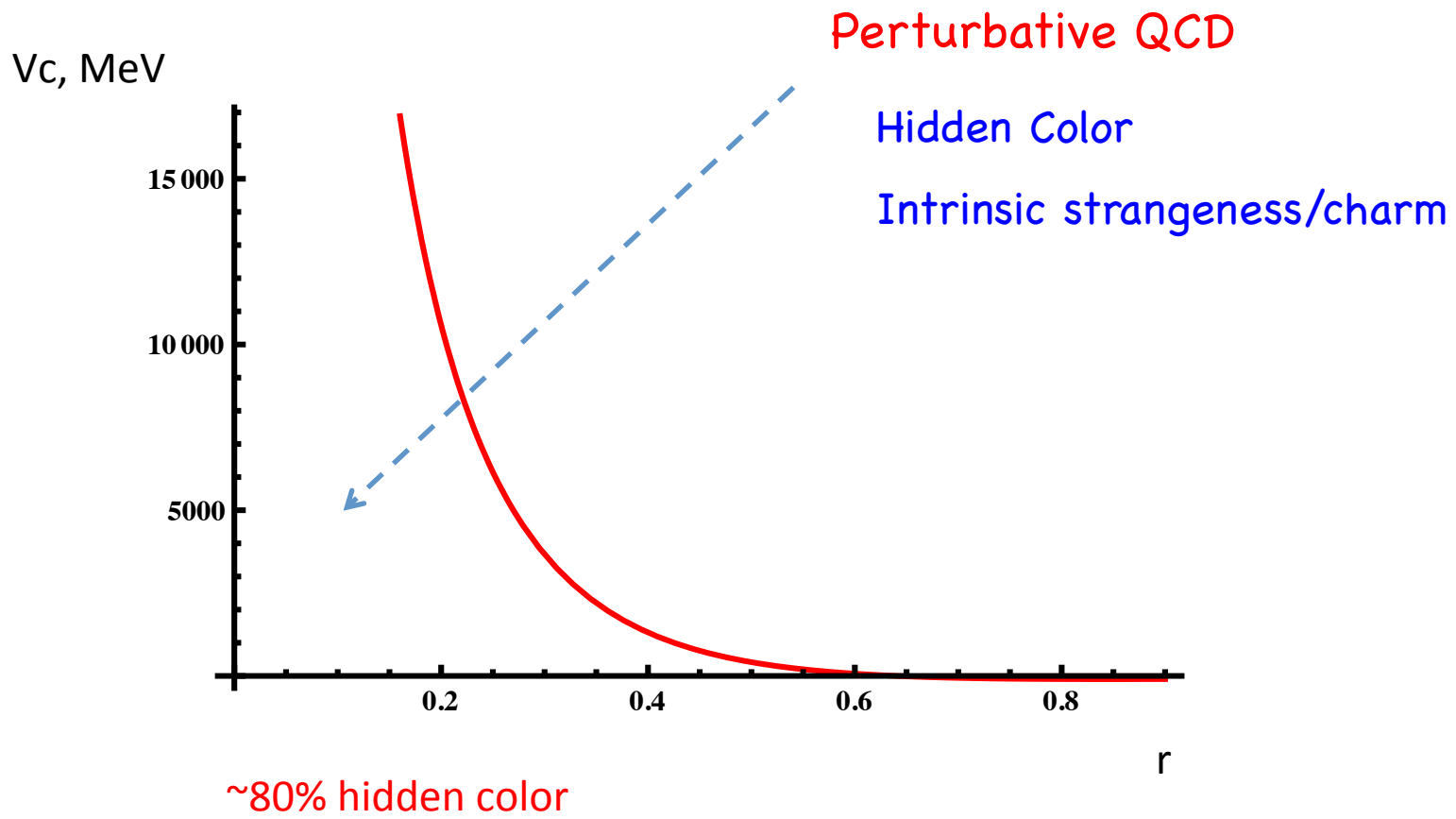
$$V_R^{2N} = V^c + V^{l^2} L^2 + V^t S_{12} + V^{ls} L \cdot S + v^{ls^2} (L \cdot S)^2$$

$$V^i = V_{int,R} + V_{core}$$

$$V_{core} = \left[1 + e^{\frac{r-r_0}{a}} \right]^{-1}$$

60's





(II) Nuclear Medium Modifications of PDFs (EMC Effect)

$$R_{EMC} = \frac{2\sigma_A}{A\sigma_D} \cdot f_{iso}$$

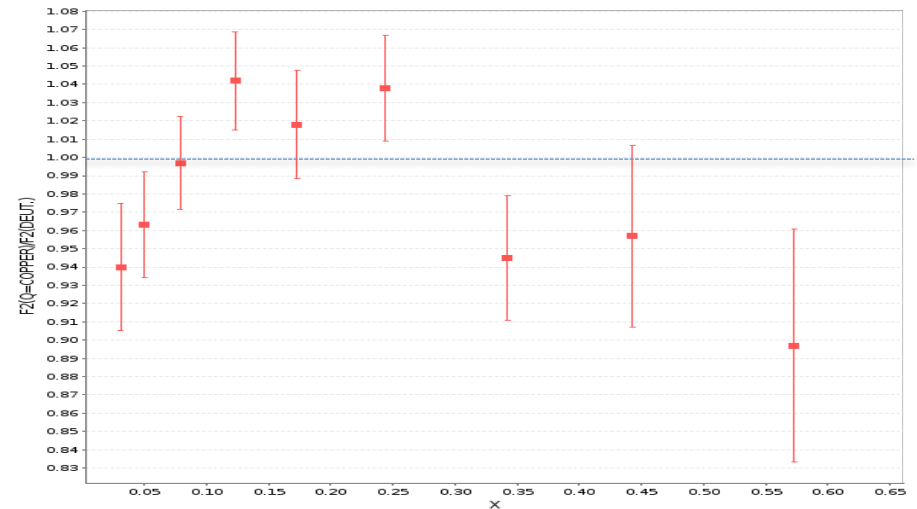
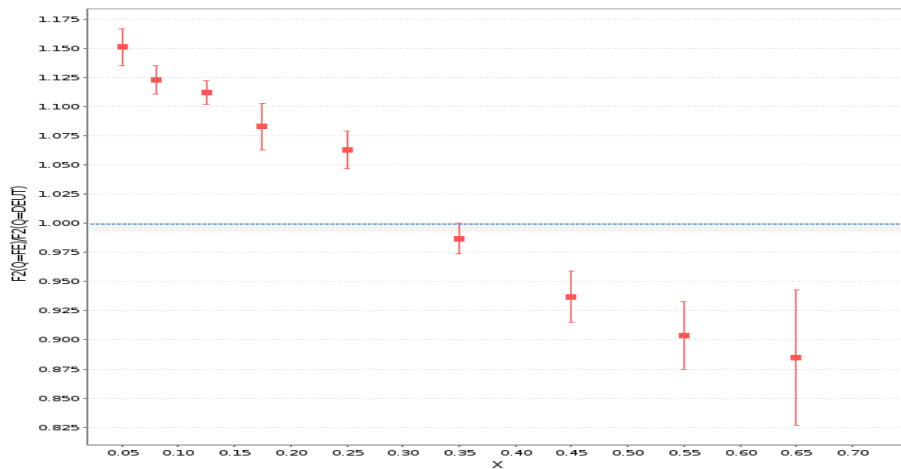
$$\sigma_{eA} = \frac{d\sigma}{d\Omega dE'}$$

and

$$f_{iso} = \frac{\sigma_p + \sigma_n}{\frac{2Z}{A}\sigma_p + \frac{2(A-Z)}{A}\sigma_n}$$

Expected $R = 1$ in DIS region

CERN 1983-1988



$$R_{EMC} = \frac{2\sigma_A}{A\sigma_D} \cdot f_{iso}$$

SLAC 1994: Gomez et al

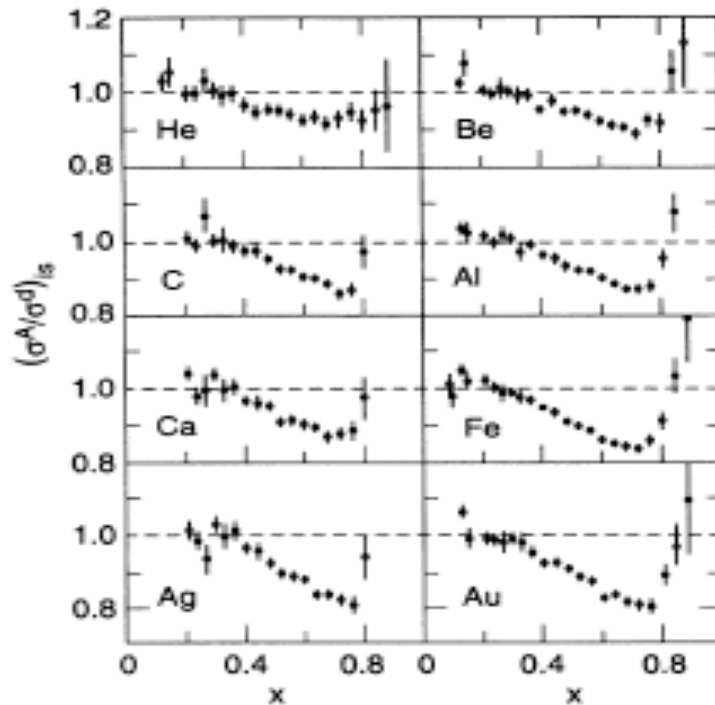


FIG. 15. Q^2 -averaged $(\sigma^A/\sigma^d)_{is}$ ratios for isoscalar nuclei as a function of x . The data have been binned in fine x bins. Errors are the same as in Fig. 14.

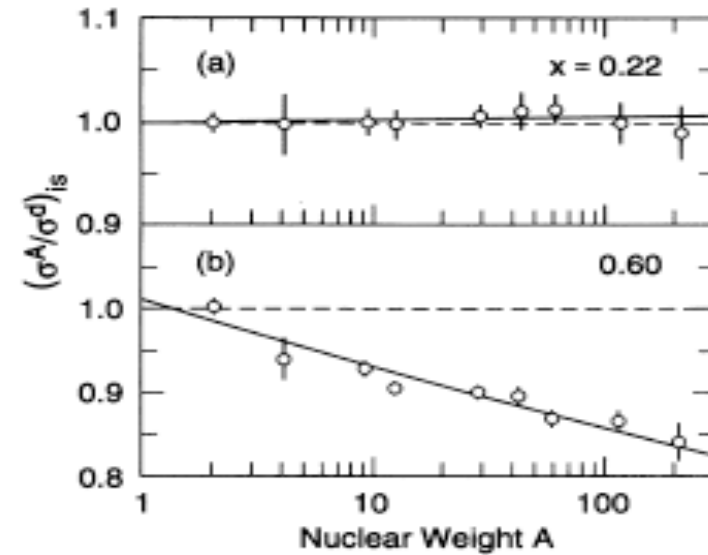


FIG. 18. Ratios $(\sigma^A/\sigma^d)_{is}$ versus atomic weight A at (a) $x=0.220$ and (b) $x=0.600$. The solid lines are a parametrization of the data in terms of $(\sigma^A/\sigma^d)_{is} = C(x)A^{\alpha(x)}$. The errors shown include statistical, point-to-point systematic, and target-to-target errors. The overall uncertainty due to the deuterium target is included only at the $A=2$ point.

$$R_{EMC} = \frac{2\sigma_A}{A\sigma_D} \cdot f_{iso}$$

SLAC 1994: Gomez et al

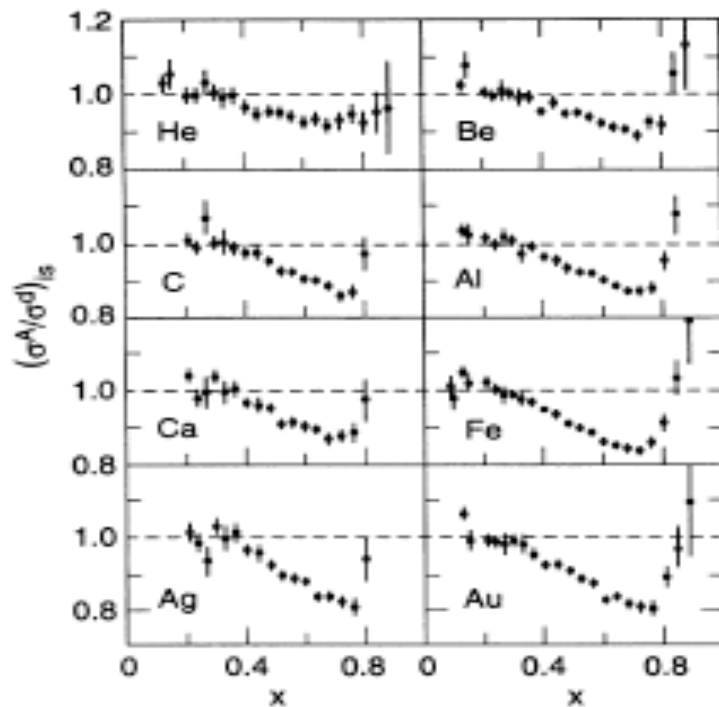


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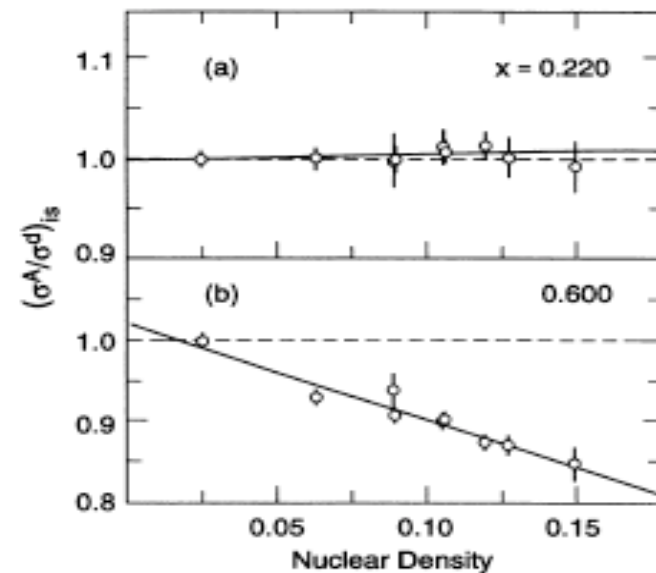
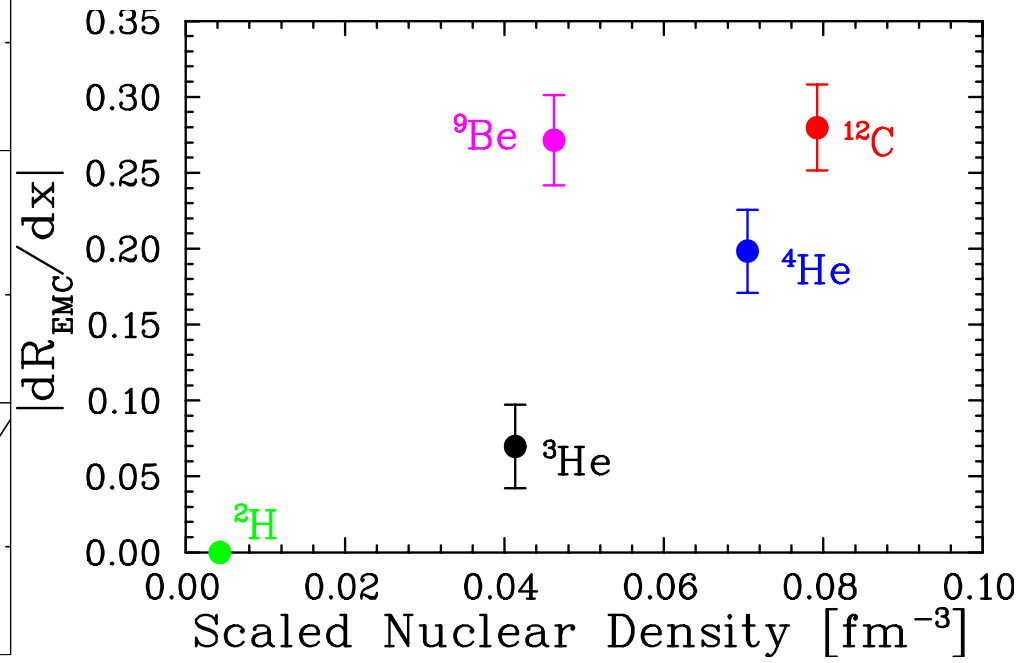
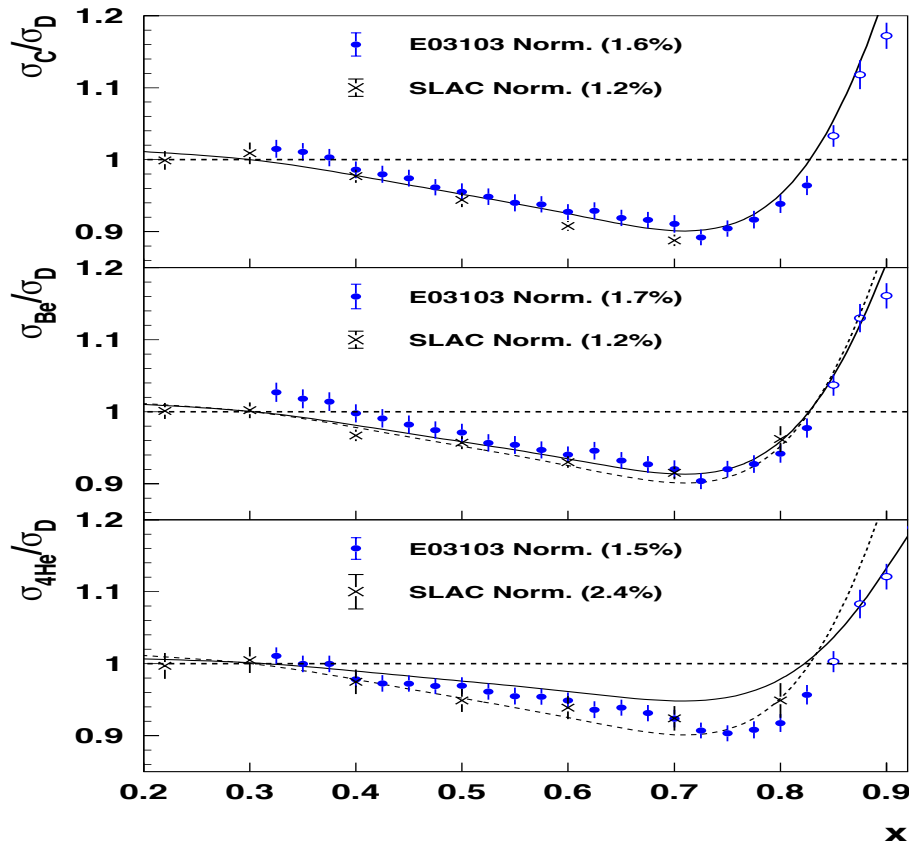


FIG. 20. Ratios $(\sigma^A/\sigma^d)_{is}$ versus nuclear density at (a) $x=0.220$ and (b) $x=0.600$. The solid lines represent the parametrization $(\sigma^A/\sigma^d)_{is} = d(x)[1 + \beta(x)\rho(A)]$. The errors shown include statistical, point-to-point systematic, and target-to-target errors. The overall uncertainty due to the deuterium target is included only at the $A=2$ point.

EMC Effect

JLAB 2009: Silly et al PRL09

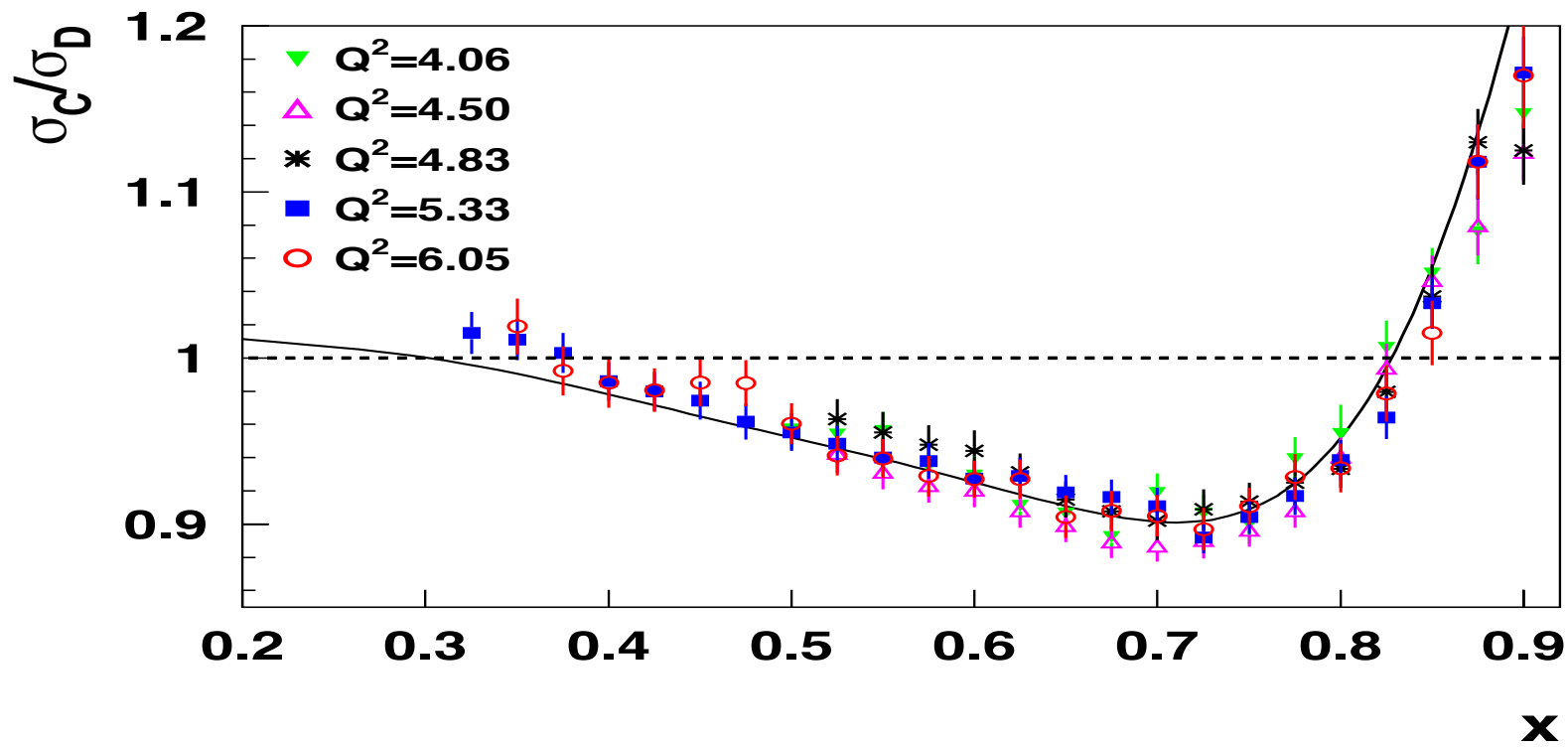
$$R_{EMC} = \frac{2\sigma_A}{A\sigma_D} \cdot f_{iso}$$



EMC Effect

JLAB 2009

$$R_{EMC} = \frac{2\sigma_A}{A\sigma_D} \cdot f_{iso}$$



EMC - SRC - Correlation

SLAC 1993:Day Frankfurt, MS, Strikman PRC93

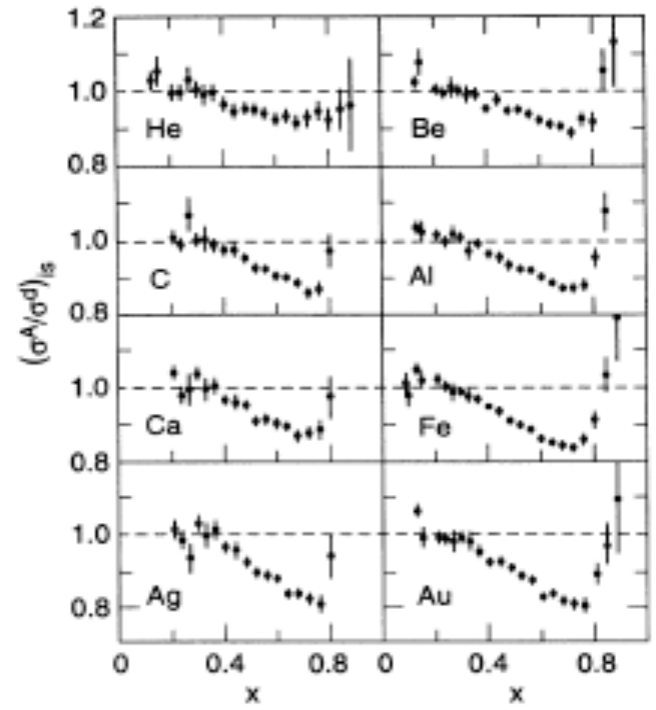
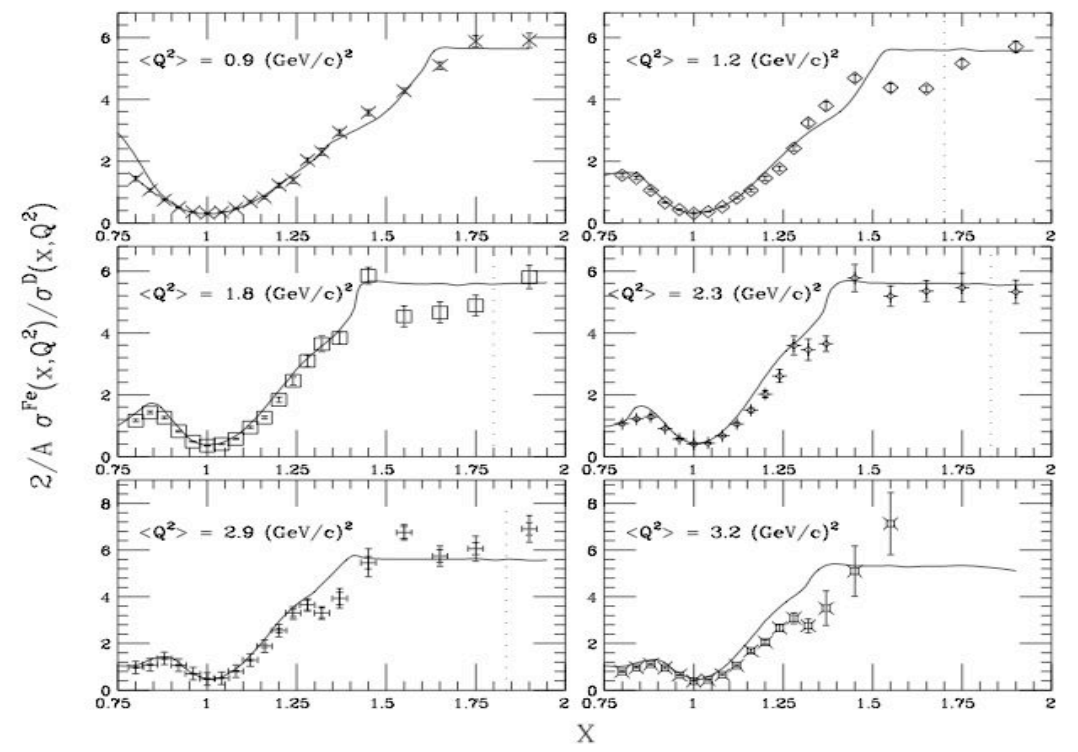
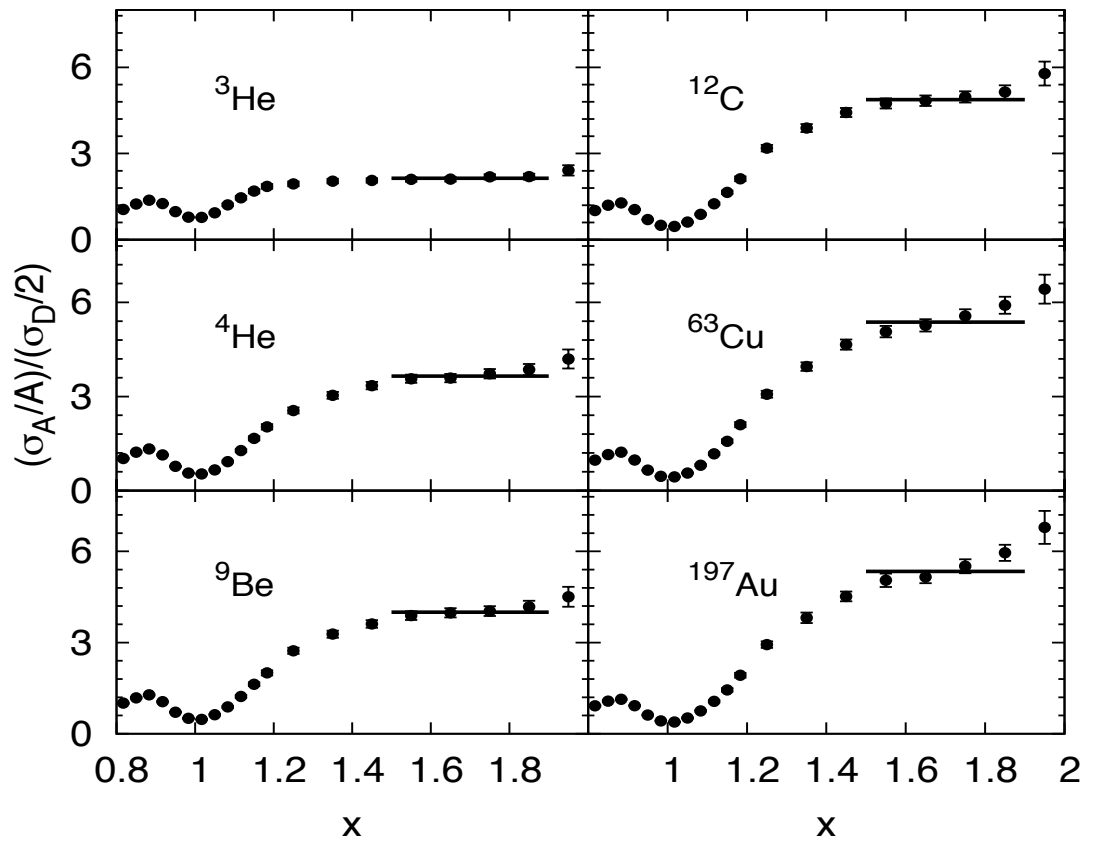
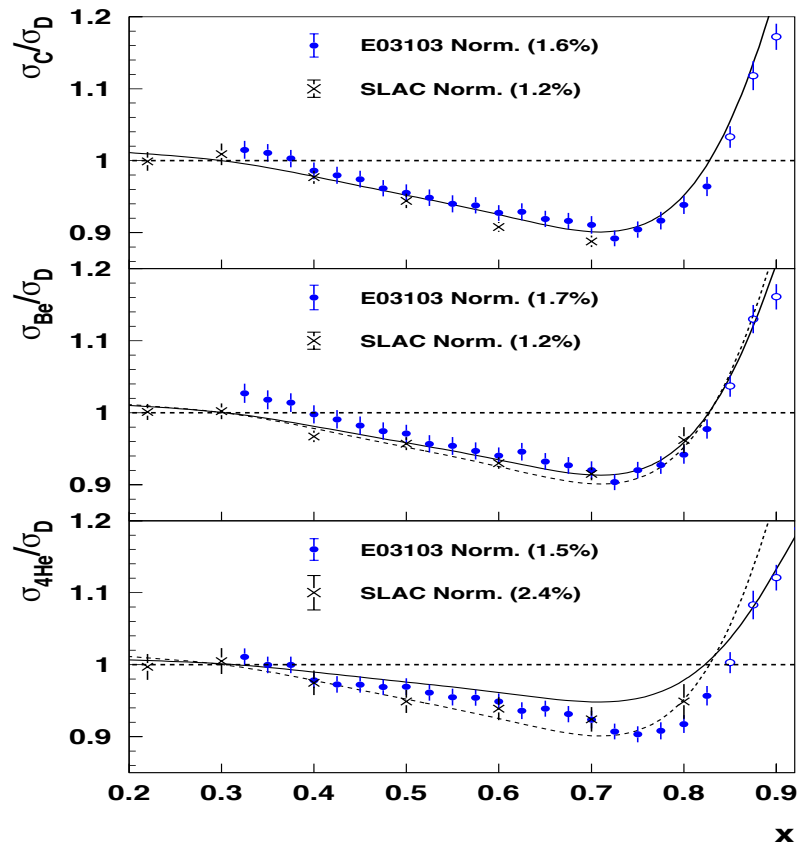


FIG. 15. Q^2 -averaged $(\sigma^A/\sigma^d)_{is}$ ratios for isoscalar nuclei as a function of x . The data have been binned in fine x bins. Errors are the same as in Fig. 14.

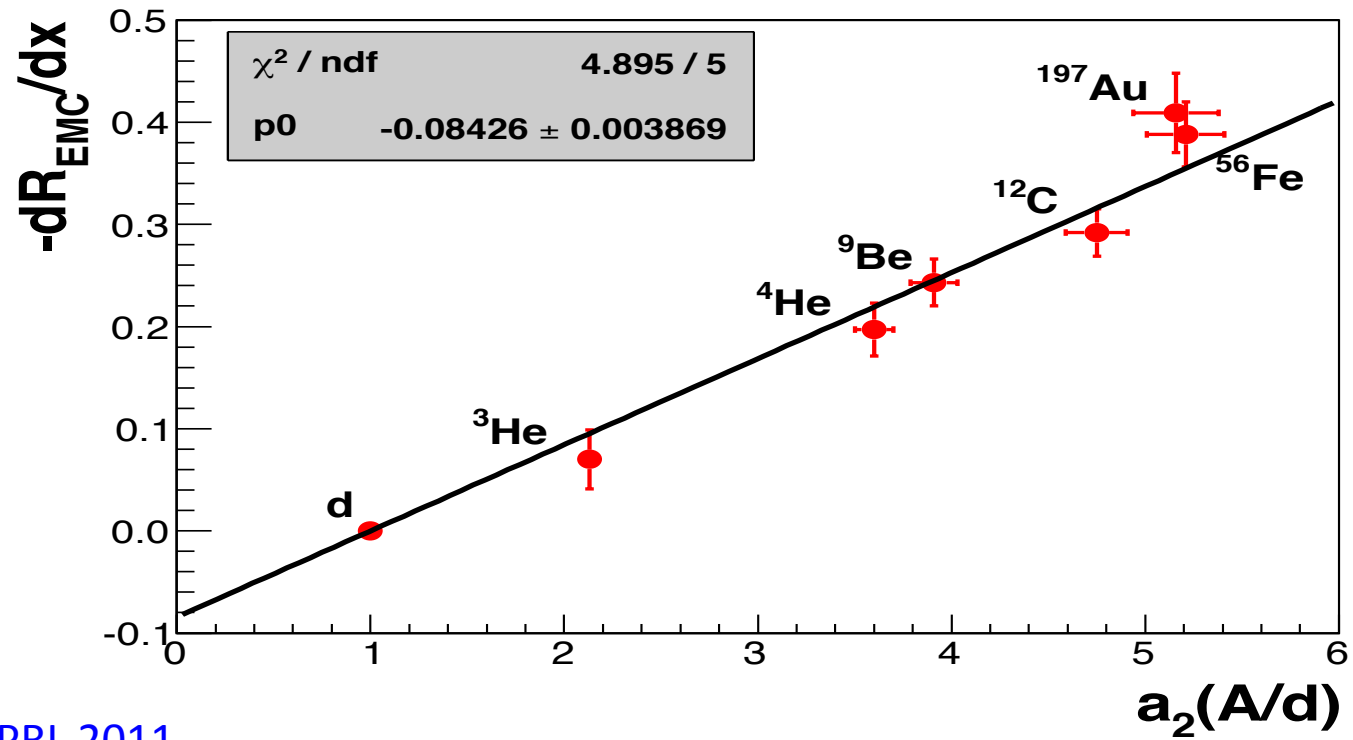


EMC - SRC - Correlation

Jlab 2012
Fomin et al, PRL 2012



EMC-SRC Correlations



-Weinstein et al PRL 2011,
arxiv 2012

Conceptually: How to probe nuclei
at short nucleon separations

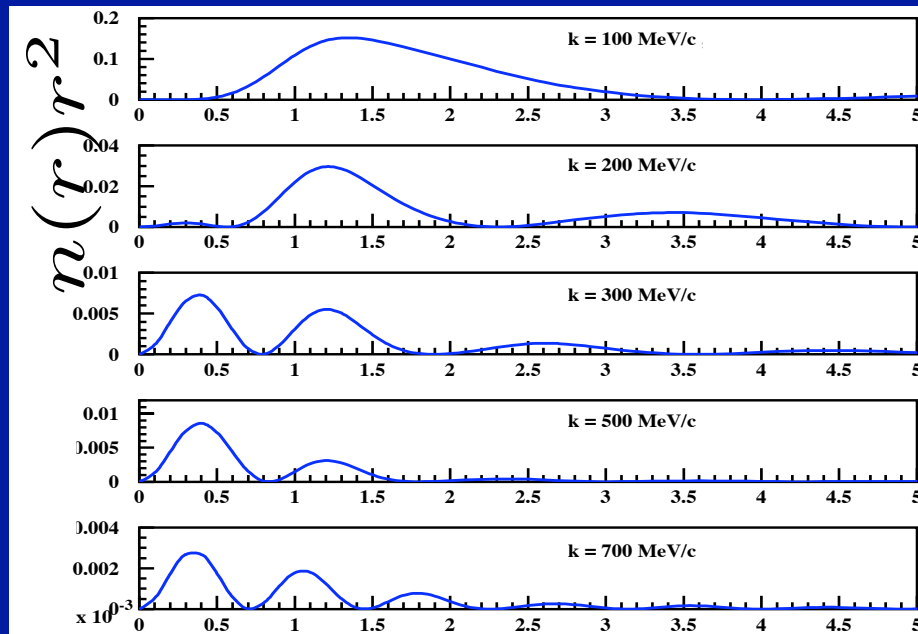
□ How to probe small distances in nuclei for NN-system (Deuteron)

1. Probe nucleon with large virtuality

$$\frac{1}{k^2 - m^2 + i\epsilon} \sim \frac{1}{m(\frac{k^2}{2m} + |\epsilon_B| - i\epsilon)}$$

2. Probe large relative momenta

$$E_m \sim \frac{k^2}{2m} \gtrsim 100 \text{ MeV}$$



$$k \gtrsim 300 \text{ MeV/c}$$

High Density Fluctuations

Emergence of Short-Range Correlations

- start with A-body Schroedinger equation interacting by two body potential only

$$\left[- \sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2} \sum_{i,j} V(x_i - x_j) \right] \psi(x_1, \dots, x_A) = E \psi(x_1, \dots, x_A)$$

- Introducing

$$\psi(x_1, \dots, x_A) = \int \Phi(k_1, \dots, k_A) e^{i \sum_i k_i x_i} \prod_i \frac{d^3 k_i}{(2\pi)^{3/2}}$$

$$V(x_i - x_j) = \int U(q) e^{iq(x_i - x_j)} d^3 q$$

$$\left(\sum_i \frac{k_i^2}{2m} - E_b \right) \Phi(k_1, \dots, k_A) = -\frac{1}{2} \sum_{i,j} \int U(q) \Phi(k_1, \dots, k_i - q, \dots, k_j + q, \dots, k_A) d^3 q$$

- Assume: system is dilute

$$\left(\sum_i \frac{k_i^2}{2m} - E_b \right) \Phi(k_1, \dots, k_A) = -\frac{1}{2} \sum_{i,j} \int U(q) \Phi(k_1, \dots, k_i - q, \dots, k_j + q, \dots, k_A) d^3q$$

- then the k dependence of the wave function for $k^2/2m_N \gg |E_B|$ Amado, 1976

$$\Phi^{(1)}(k_1, \dots, k_c, \dots, -k_c, \dots, k_A) \approx \frac{U_{NN}(k_c)}{k_c^2} F_A(k_1, \dots, \dots, \dots, k_A)$$

- Assume: $U_{NN}(q) \sim \frac{1}{q^n}$ with $n > 1$

$$\Phi^{(2)}(\dots k_c, \dots) \sim \frac{1}{k_c^{2+n}} \int \frac{1}{q^n} dq \sim \frac{U_{NN}(k_c)}{k_c^2} \int_{q_{min}} \frac{1}{q^n} dq$$

- For large k_c $\Phi^{(2)}(k_c) \ll \Phi^{(1)}(k_c)$ Frankfurt, Strikman 1981
Frankfurt, MS, Strikman 2008

- 3N SRCs are parametrically smaller than 2N SRC

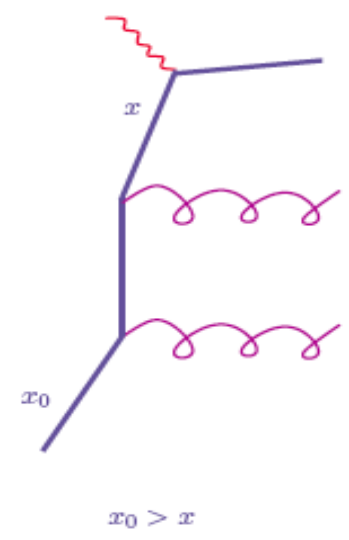
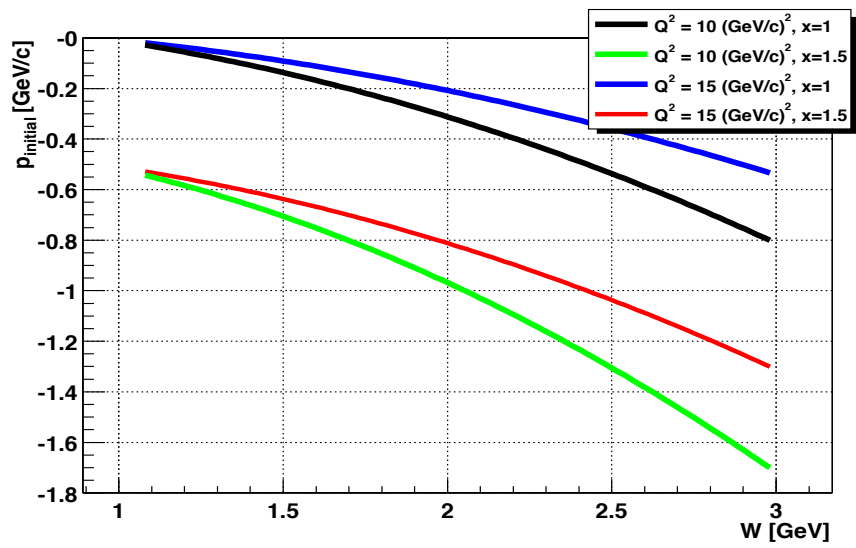
SuperFast quarks – short distance probes in nuclei

$$x = \frac{Q^2}{2m_N q_0} > 1$$

Two factors driving nucleons close together

Kinematic $p_{min} \equiv p_z = m_N \left(1 - x - x \left[\frac{W_N^2 - m_N^2}{Q^2} \right] \right)$

Dynamical: QCD evolution



Existing Experiments:

1. BCDMS Collaboration 1994 (CERN): $52 \leq Q^2 \leq 200 \text{ GeV}^2$
2. CCFR Collaboration 2000 (FermiLab): $Q^2 = 120 \text{ GeV}^2$
3. E02-019 Experiment 2010 (JLab) $Q_{AV}^2 = 7.4 \text{ GeV}^2$
4. Approved Experiments at JLab12
5. Alternative Studies at LHC $p+A \rightarrow 2 \text{ jets} + X$
6. Ultra Peripheral Collision: $\gamma + A \rightarrow 2 \text{ jets} + X ?$
7. Electron Ion Collider: $e + A \rightarrow e' + (N) + X ?$

1. BCDMS Collaboration 1994 (CERN): Z.Phys C63 1994

Structure function of Carbon in deep-inelastic scattering of 200GeV muons

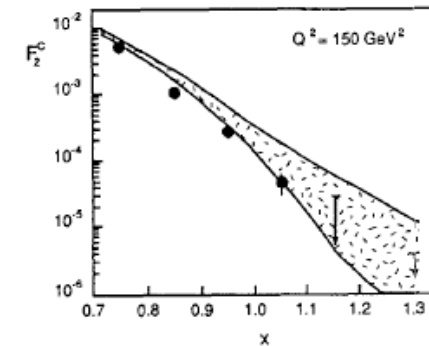
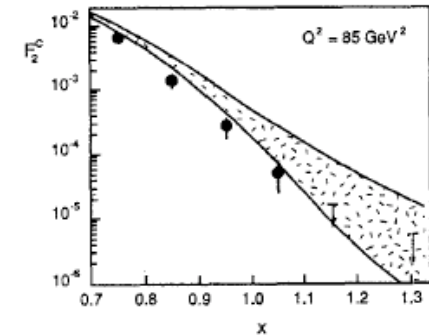
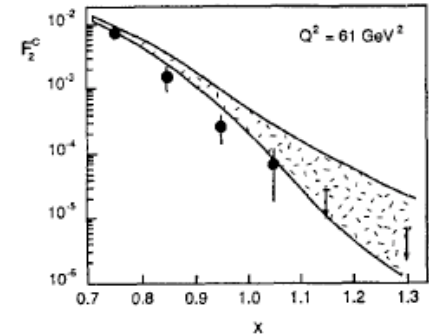
$$Q^2 = 61, 85 \text{ and } 150 \text{ GeV}^2$$

$$x = 0.85, 0.95, 1.05, 1.15 \text{ and } 1.3$$

$$F_{2A}(x, Q^2) = F_{2A}(x_0 = 0.75, Q^2) e^{-s(x-0.75)}$$

$$s = 16.5 \pm 0.6$$

More than Fermi Gas but very marginal high momentum component



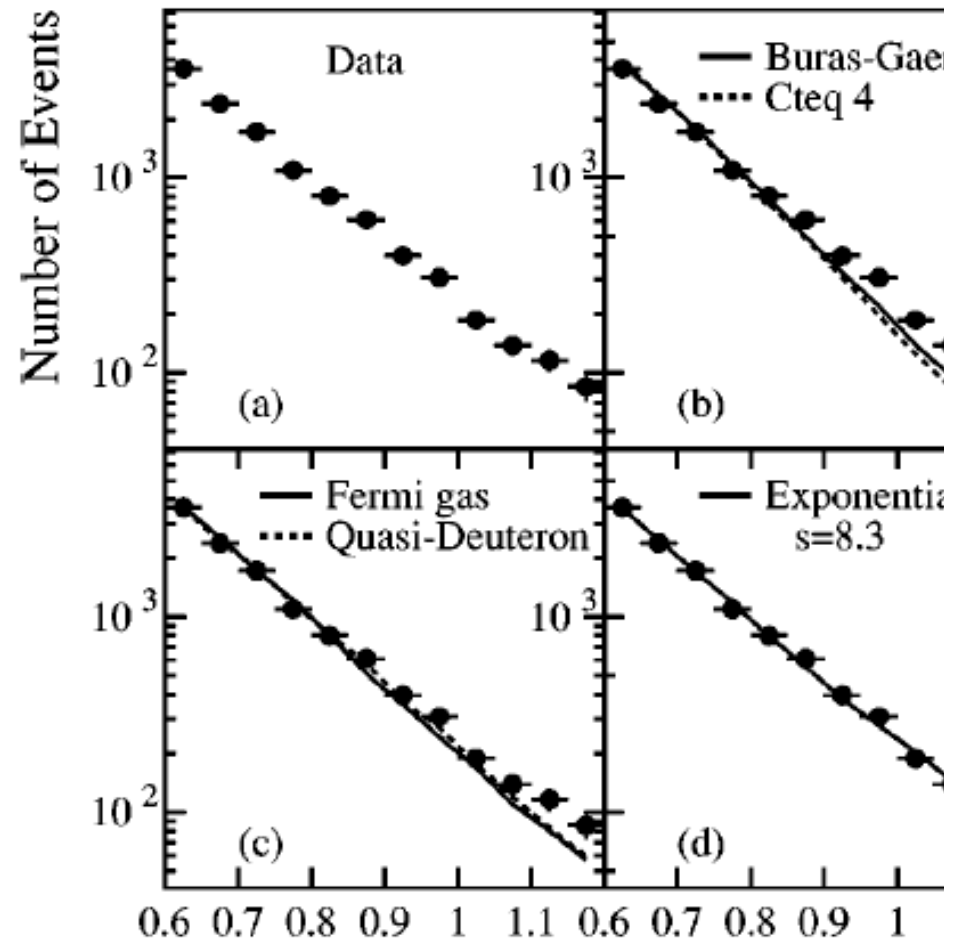
2. CCFR Collaboration 2000 (FermiLab):
 Phys. Rev. D61 2000

Using the neutrino and antineutrino beams in which structure function Iron was measured in the charged current sector for average

$$Q^2 = 120 \text{ GeV}^2 \text{ and } 0.6 \leq x \leq 1.2.$$

$$F_{2A} \sim e^{-s(x-x_0)}$$

$$s = 8.3 \pm 0.7(\text{stat}) \pm 0.7(\text{sys})$$



3. E02-019 Experiment 2010 (JLab)

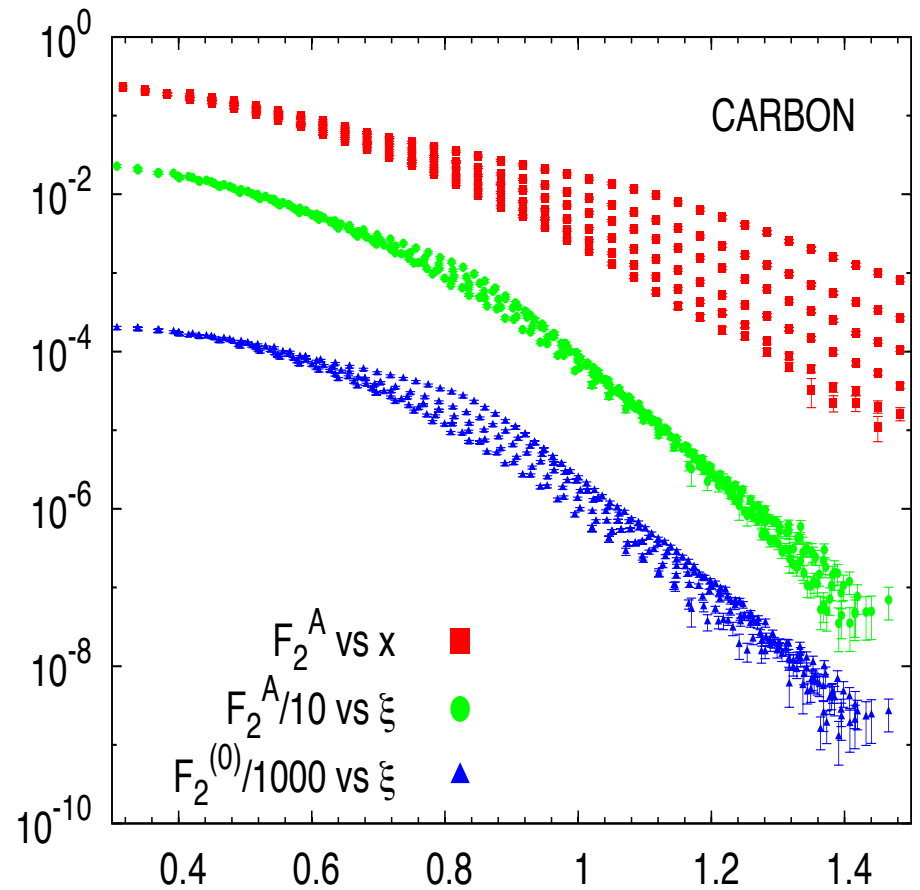
Phys.Rev.Lett 204 2010

(ee') scattering of

^2H , ^3He , ^4He , ^9Be , ^{12}C , ^{64}Cu and ^{197}Au

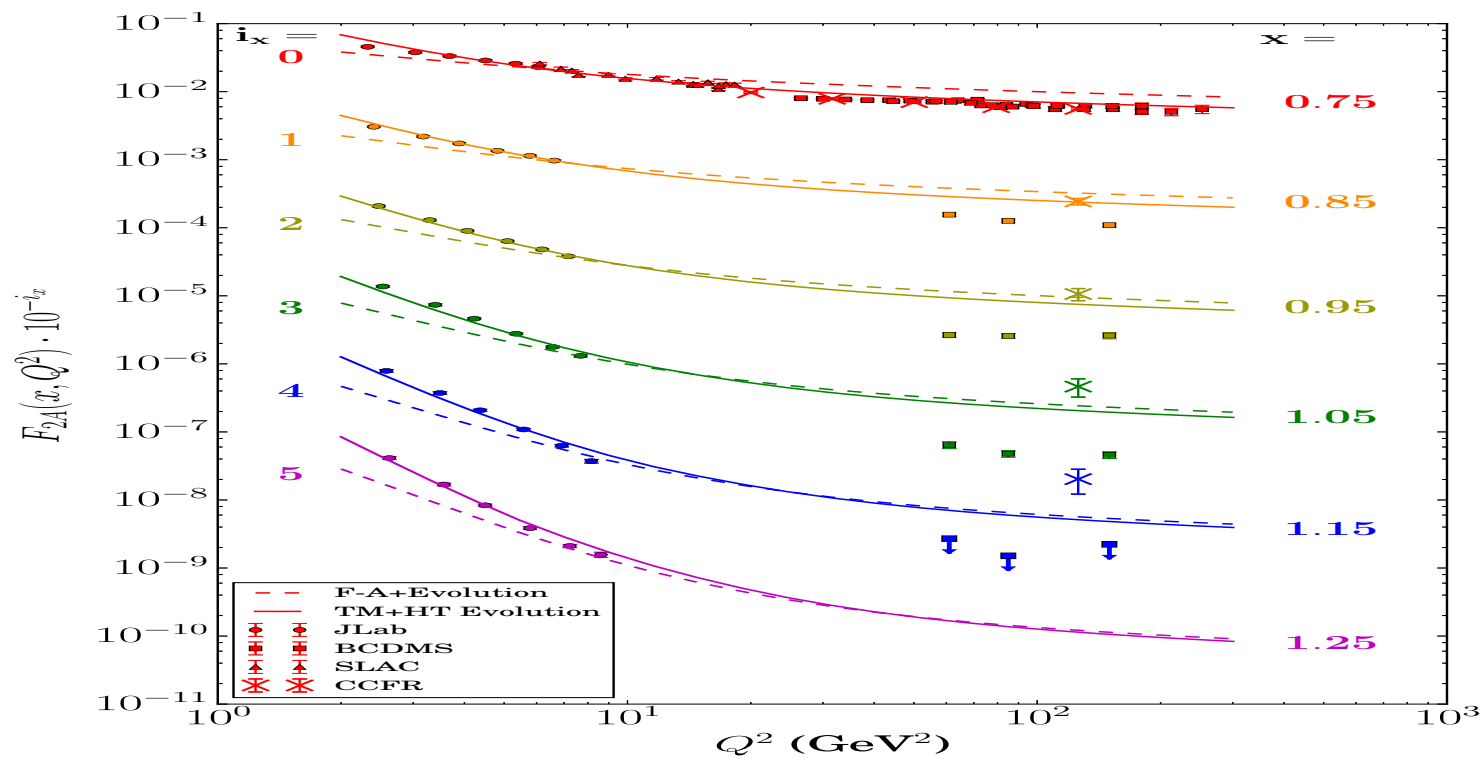
$$6 < Q^2 < 9 \text{ GeV}^2$$

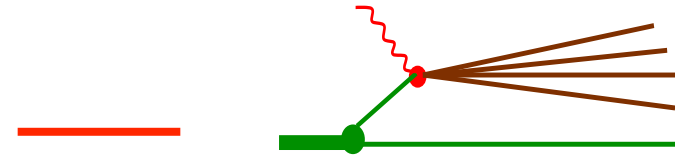
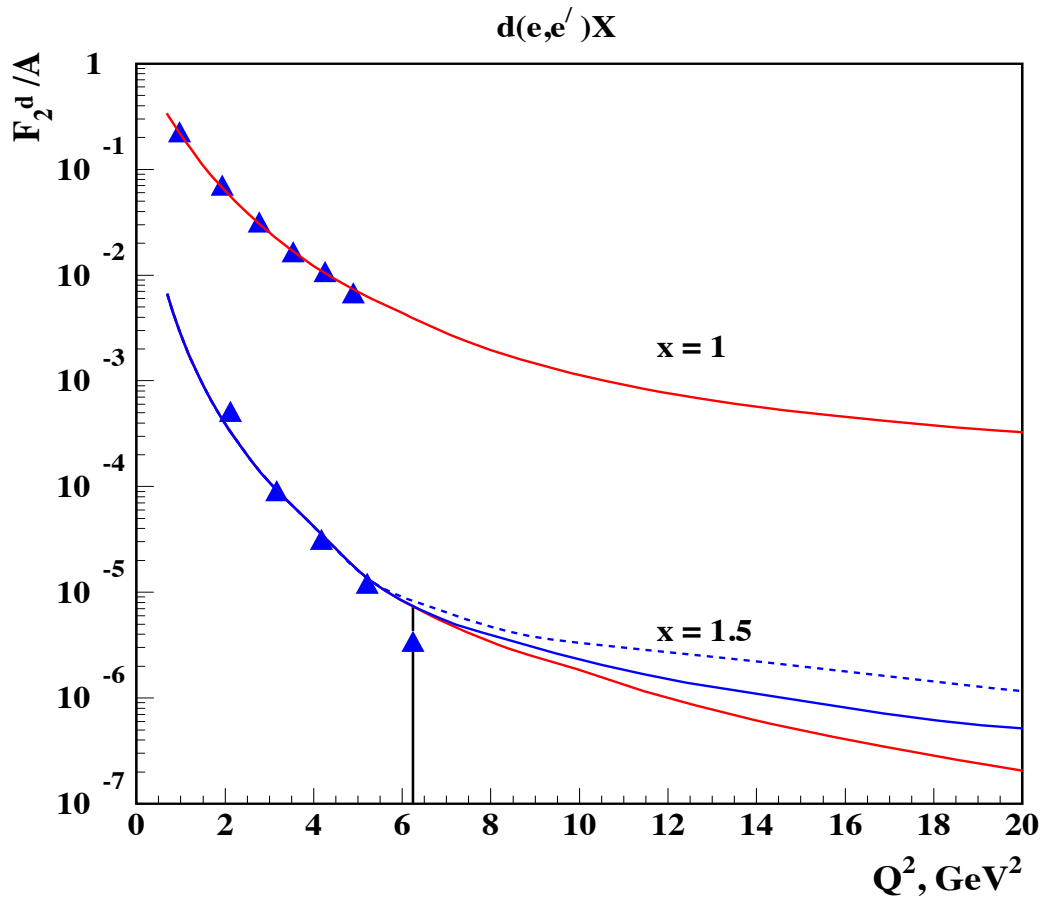
$$\xi = \frac{2x}{(1+r)} \text{ where } r = \sqrt{1 + \frac{4M_N^2 x^2}{Q^2}}$$



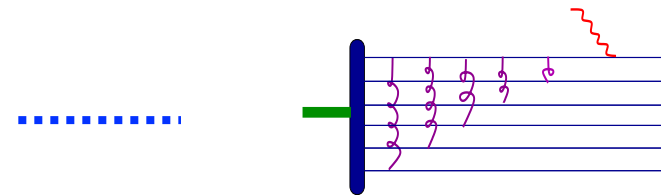
$$\frac{dF_{2A}(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left\{ 2 \left(1 + \frac{4}{3} \log \left(1 - \frac{x}{A} \right) \right) F_{2,A}(x, Q^2) + \frac{4}{3} \int \frac{dz}{1-z} \left(\frac{1+z^2}{z} F_{2A} \left(\frac{x}{z}, Q^2 \right) - 2F_{2A}(x, Q^2) \right) \right\}.$$

A. Freese & M.S
ArXiv 2016

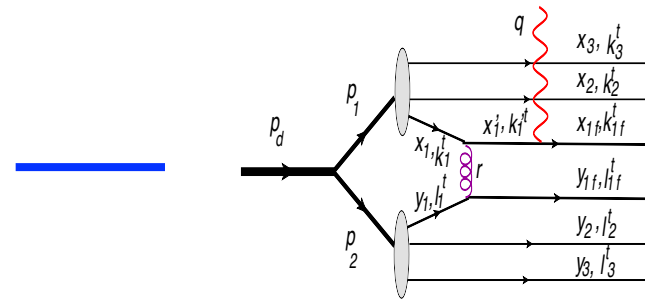




$$F_{2d} = \int_x^2 \rho_d^N(\alpha, p_t) F_{2N}\left(\frac{x}{\alpha}, Q^2\right) \frac{d^2\alpha}{\alpha} d^2p_t$$



$$F_{2D} = F_{2,(6q)} \sim \left(1 - \frac{x}{2}\right)^{10}$$



$$F_{2d}(x_{Bj}, Q^2) \sim \sum_{i,j} x_{Bj} e_i^2 \int dx_1 dy_1 f_i(x_1, Q^2) f_j(y_1, l_{1f,t}^2) \frac{1}{y_1^2} \left[1 - \frac{x_{Bj}}{x_1 + y_1}\right]^2 \Theta(x_1 + y_1 - x_{Bj})$$

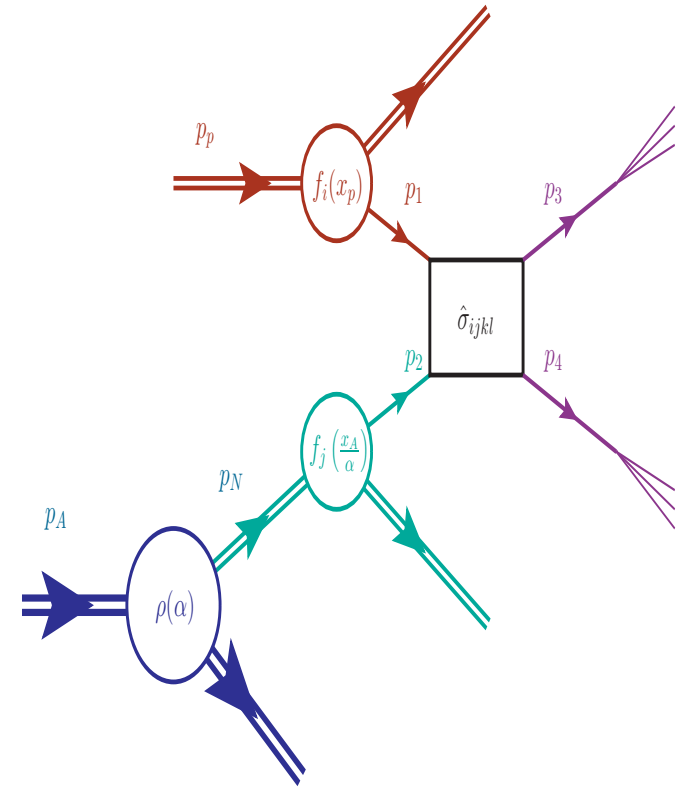
MS, in progress

5. Probing Superfast quarks in $p+A \rightarrow 2 \text{ jets} + X$ reaction

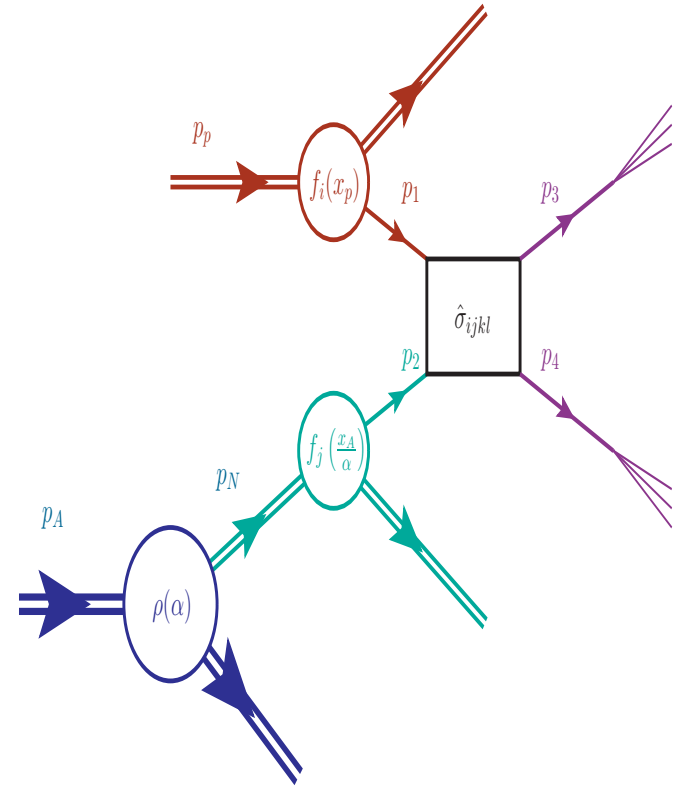
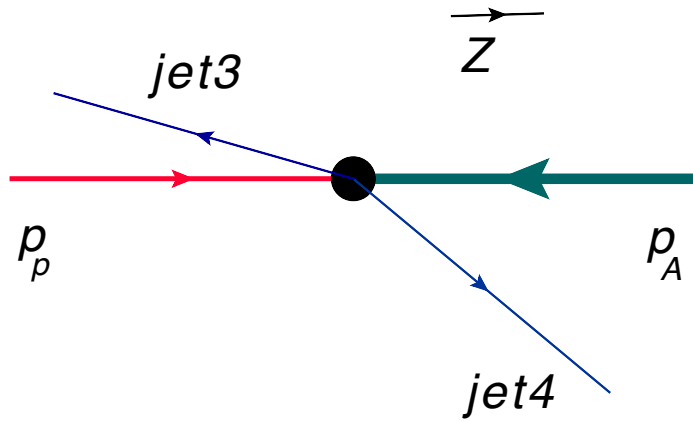
A.Freese, M.S.
M.Strikman, EPJ 2015

$$p + A \rightarrow \text{dijet} + X$$

- Reaction is treated in Leading Twist Approximation
- Jets are produced in two-body parton-parton scattering
- one parton from the probe – other from the nucleus
- nuclear parton originated from the bound nucleon



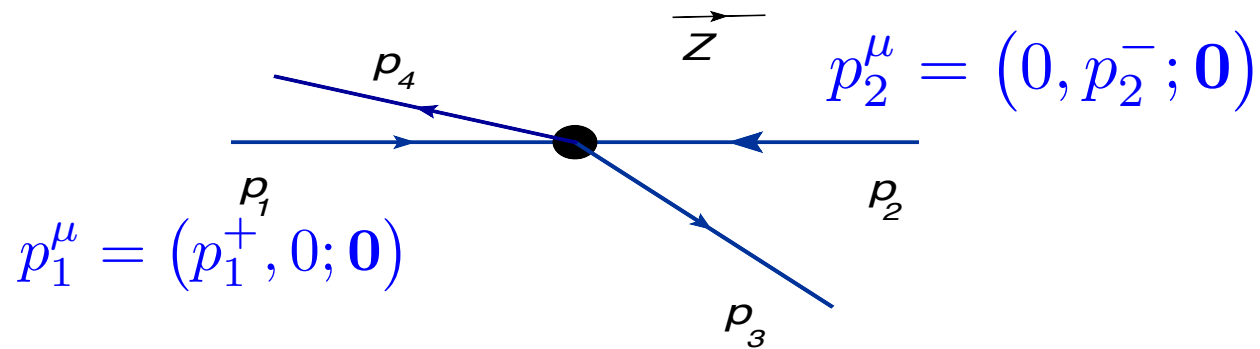
Jet - kinematics



$$p_p^\mu = \left(p_p^+, \frac{m_p^2}{p_p^+}, \mathbf{0}_T \right) = (2E_0, 0, \mathbf{0}_T) = \left(\sqrt{\frac{As_{NN}^{avg.}}{Z}}, 0, \mathbf{0}_T \right)$$

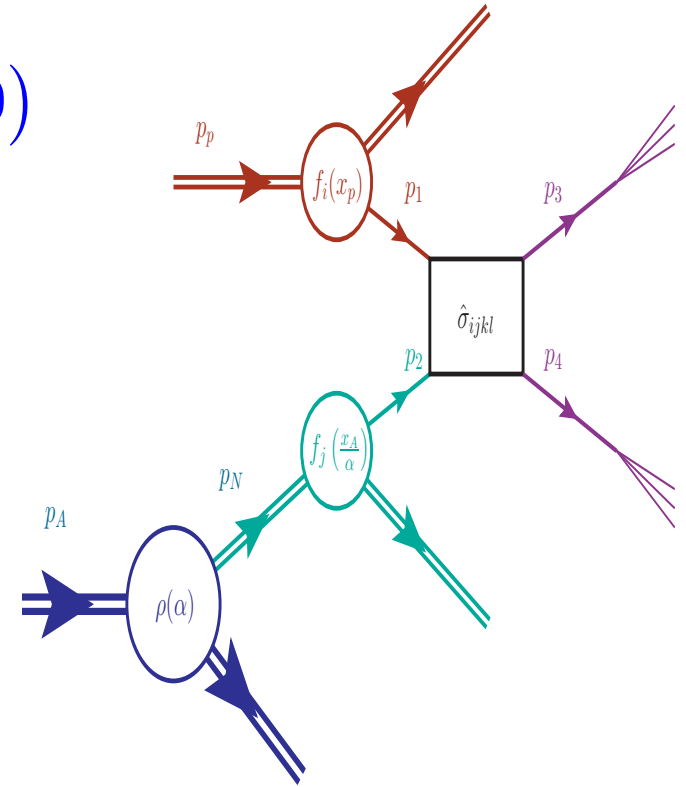
$$p_A^\mu = \left(\frac{M_A^2}{p_A^-}, p_A^-, \mathbf{0}_T \right) = (0, 2ZE_0, \mathbf{0}_T) = \left(0, \sqrt{AZs_{NN}^{avg.}}, \mathbf{0}_T \right)$$

Parton - kinematics



$$x_p = \frac{p_1^+}{p_p^+} = \sqrt{\frac{Z}{A}} \frac{p_1^+}{\sqrt{s_{NN}^{\text{avg.}}}}$$

$$x_A = A \frac{p_2^-}{p_A^-} = \sqrt{\frac{A}{Z}} \frac{p_2^-}{\sqrt{s_{NN}^{\text{avg.}}}}$$



$$p_1^\mu = (p_1^+, 0; \mathbf{0}) \quad x_p = \frac{p_1^+}{p_p^+}$$

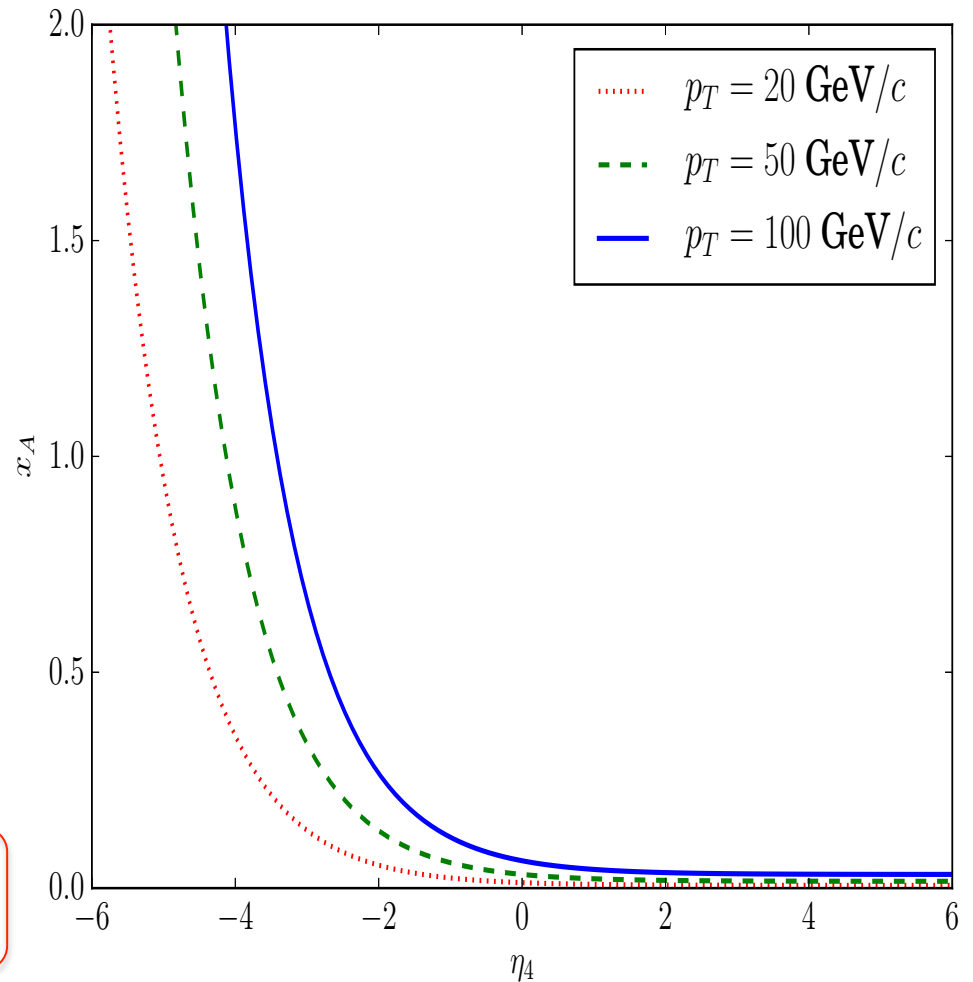
$$p_2^\mu = (0, p_2^-; \mathbf{0}) \quad x_A = A \frac{p_2^-}{p_A^-}$$

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$

$$\eta = \frac{1}{2} \log \left(\frac{p^+}{p^-} \right)$$

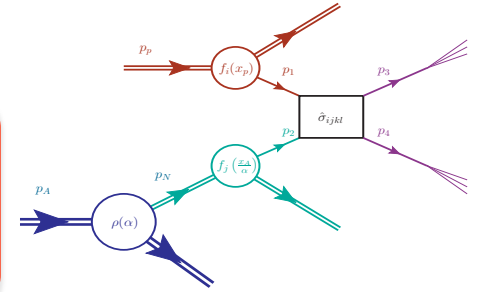
$$x_p = \sqrt{\frac{Z}{A}} \frac{p_T}{\sqrt{s_{NN}^{\text{avg.}}}} (e^{\eta_3} + e^{\eta_4})$$

$$x_A = \sqrt{\frac{A}{Z}} \frac{p_T}{\sqrt{s_{NN}^{\text{avg.}}}} (e^{-\eta_3} + e^{-\eta_4})$$



Differential Cross Section of the Reaction

$$\frac{d^3\sigma}{d\eta_3 d\eta_4 dp_T^2} = \sum_{ijkl} \frac{1}{16\pi (s_{NN}^{\text{avg.}})^2} \frac{f_{i/p}(x_p, Q^2)}{x_p} \frac{f_{j/A}(x_A, Q^2)}{x_A} \frac{|\overline{\mathcal{M}}_{ij \rightarrow kl}|^2}{1 + \delta_{kl}}$$



$$s_{NN}^{\text{avg}} = \frac{p_p^+ p_A^-}{A}$$

$$Q^2 = -(p_1 - p_3)^2 \approx p_T^2$$

$$f_{i/p}(x_p, Q^2)$$

$$f_{j/A}(x_A, Q^2)$$

Subprocess	$\frac{ \overline{\mathcal{M}} ^2}{g_s^4}$
$q_j + q_k \rightarrow q_j + q_k$	$\frac{4}{9} \frac{s^2 + u^2}{t^2}$
$q_j + q_j \rightarrow q_j + q_j$	$\frac{4}{9} \left(\frac{s^2 + u^2}{t^2} + \frac{s^2 + t^2}{u^2} \right) - \frac{8}{27} \frac{s^2}{ut}$
$q_j + \bar{q}_j \rightarrow q_k + \bar{q}_k$	$\frac{4}{9} \frac{t^2 + u^2}{s^2}$
$q_j + \bar{q}_j \rightarrow q_j + \bar{q}_j$	$\frac{4}{9} \left(\frac{s^2 + u^2}{t^2} + \frac{t^2 + u^2}{s^2} \right) - \frac{8}{27} \frac{u^2}{st}$
$q_j + \bar{q}_j \rightarrow g + g$	$\frac{32}{27} \frac{u^2 + t^2}{ut} - \frac{8}{3} \frac{u^2 + t^2}{s^2}$
$g + g \rightarrow q_j + \bar{q}_j$	$\frac{1}{6} \frac{u^2 + t^2}{ut} - \frac{3}{8} \frac{u^2 + t^2}{s^2}$
$q_j + g \rightarrow q_j + g$	$-\frac{4}{9} \frac{u^2 + s^2}{us} + \frac{8}{3} \frac{u^2 + s^2}{t^2}$
$g + g \rightarrow g + g$	$\frac{9}{2} \left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2} \right)$

Nuclear Partonic Distributions

$$f_{i/A}(x_A, Q^2) = \sum_N \int_{x_A}^A \frac{d\alpha}{\alpha} \int d^2\mathbf{p}_T f_{N/A}(\alpha, \mathbf{p}_T) f_{i/N}^{(b)}\left(\frac{x_A}{\alpha}, \alpha, \mathbf{p}_T, Q^2\right)$$

$f_{N/A}(\alpha, \mathbf{p}_T)$ Light-Front fractional distribution of nucleon in the nucleus

$f_{i/N}^{(b)}\left(\frac{x_A}{\alpha_N}, \alpha_N, \mathbf{p}_{N,T}, Q^2\right)$ i-parton distribution in the bound nucleon N

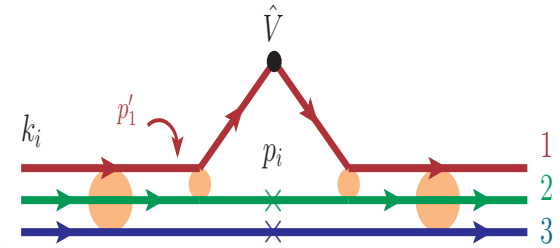
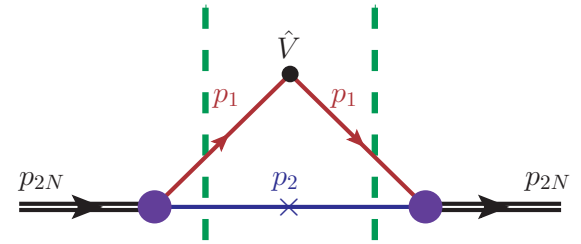
Light-Front Distribution of Nucleon in the Nucleus

$$f_{N/A}(\alpha, \mathbf{p}_T) = f_{N/A}^{(MF)}(\alpha, \mathbf{p}_T) + f_{N/A}^{(2)}(\alpha, \mathbf{p}_{NT}) + f_{N/A}^{(3)}(\alpha, \mathbf{p}_{NT}) \cdots$$

$$f_{N/A}^{(MF)}(\alpha, \mathbf{p}_T) = \frac{m_A}{A} \left| \Psi_{MF}^{(N)}(p) \right|^2$$

$$f_{N/A}^{(2)}(\alpha, \mathbf{p}_T) = \frac{a_2(A)}{2\chi_N} \frac{|\psi_d(k)|^2}{\alpha(2-\alpha)} \Theta(k - k_F)$$

$$f_{N/A}^{(3)}(\alpha, \mathbf{p}_T) = \frac{\{a_2(A)\}^2}{|\psi_d(k_{12})|^2} \frac{1}{\alpha} \int \frac{d\alpha_3 d^2\mathbf{p}_{3T}}{\alpha_3(3-\alpha-\alpha_3)} \left\{ \frac{3-\alpha_3}{2(2-\alpha_3)} \right\}^2 \frac{|\psi_d(k_{23})|^2}{\Theta(k_{23} - k_F)} \Theta(k_{12} - k_F)$$



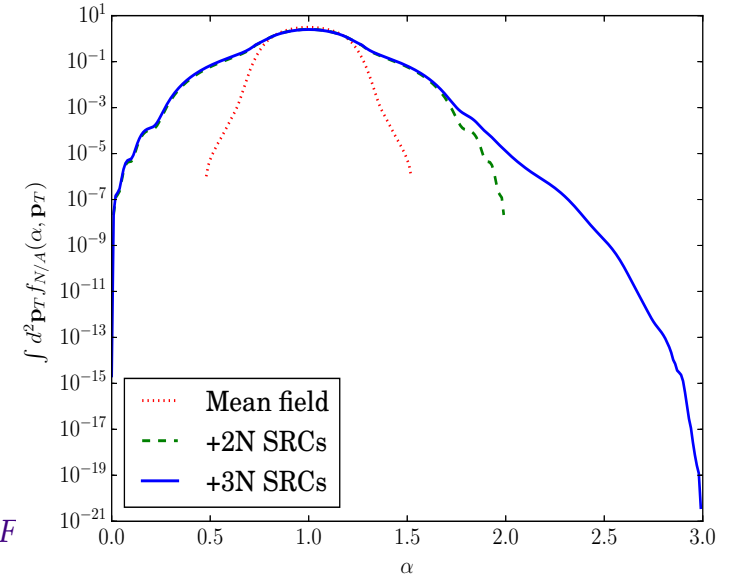
Light-Front Distribution of Nucleon in the Nucleus

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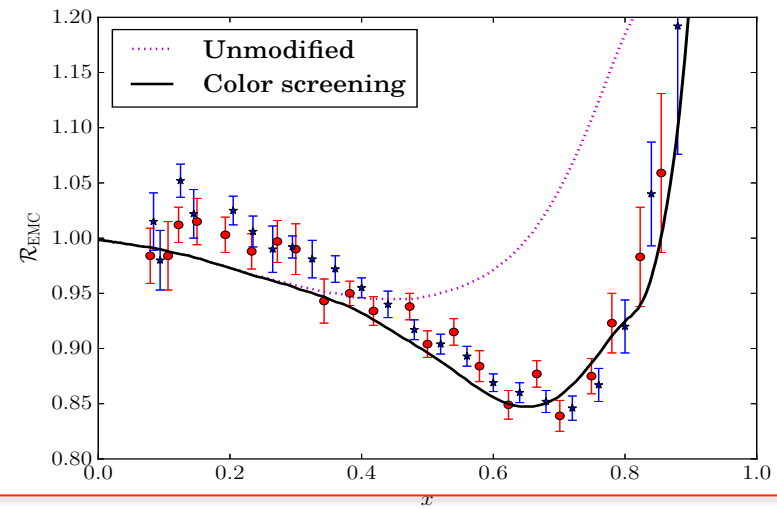
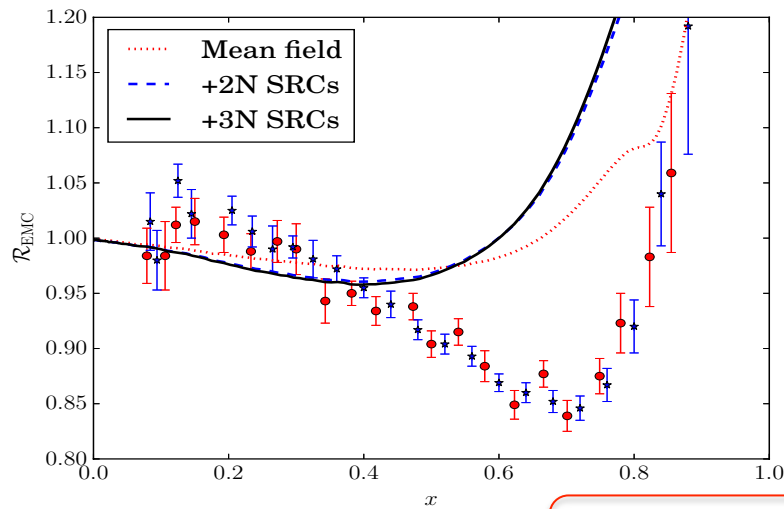
$$f_{N/A}^{(3)}(\alpha, \mathbf{p}_T) = \frac{\{a_2(A)\}^2}{|\psi_d(k_{12})|^2} \frac{1}{\alpha} \int \frac{d\alpha_3 d^2\mathbf{p}_{3T}}{\alpha_3(3-\alpha-\alpha_3)} \left\{ \frac{3-\alpha_3}{2(2-\alpha_3)} \right\}^2 \Theta(k_{12} - k_F) \frac{|\psi_d(k_{23})|^2}{\Theta(k_{23} - k_F)}$$



Partonic distribution in the bound nucleon: Medium Modification effects

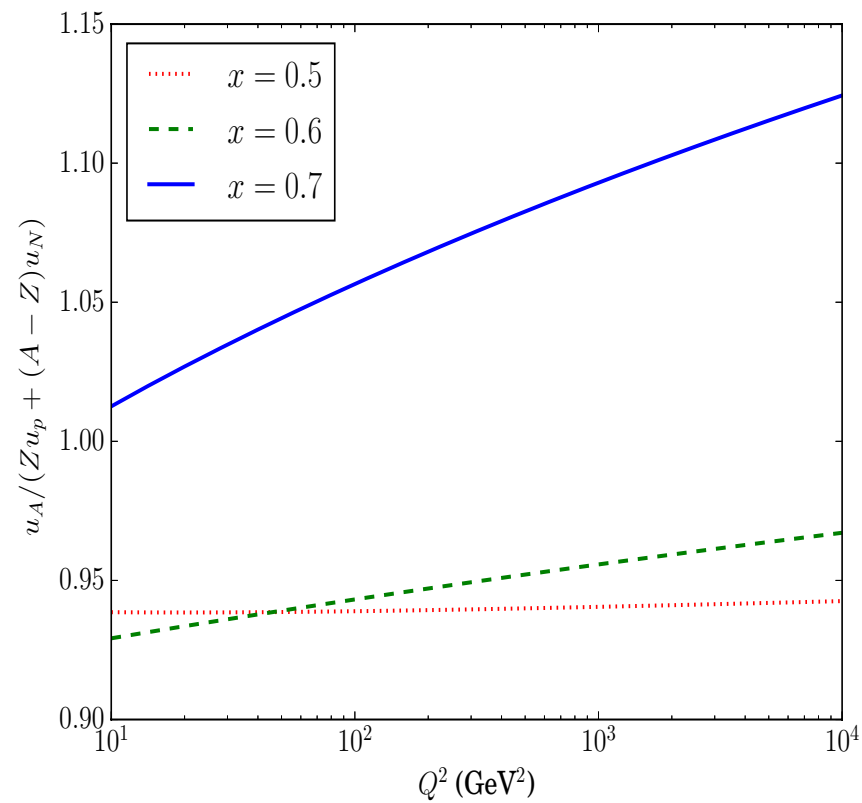
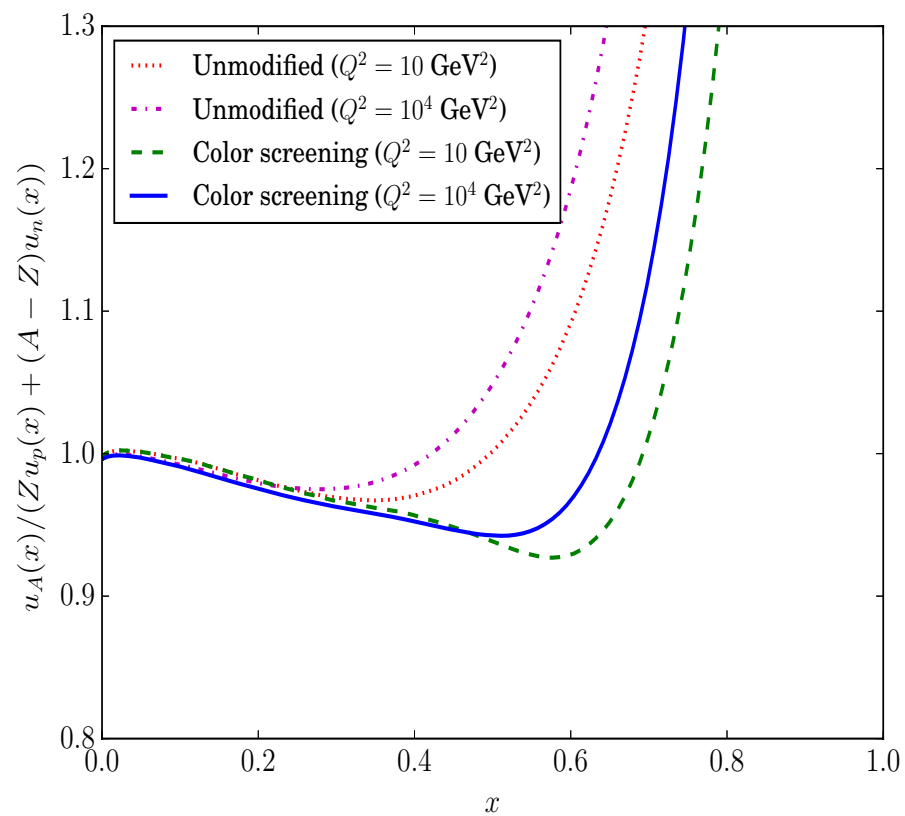
$$F_2^{(A)}(x, Q^2) = \sum_N \int_x^A d\alpha \int d^2\mathbf{p}_T f_{N/A}(\alpha, \mathbf{p}_T) F_2^{(N,b)}\left(\frac{x}{\alpha}, \alpha, \mathbf{p}_T, Q^2\right)$$

$$\mathcal{R}_{\text{EMC}}(x, Q^2) = \frac{2}{A} \frac{\sigma_{eA}}{\sigma_{ed}} f_{\text{iso}} \approx \frac{2}{A} \frac{F_2^{(A)}(x, Q^2)}{F_2^{(d)}(x, Q^2)} f_{\text{iso}} \quad Q^2 = 10 \text{ GeV}^2$$



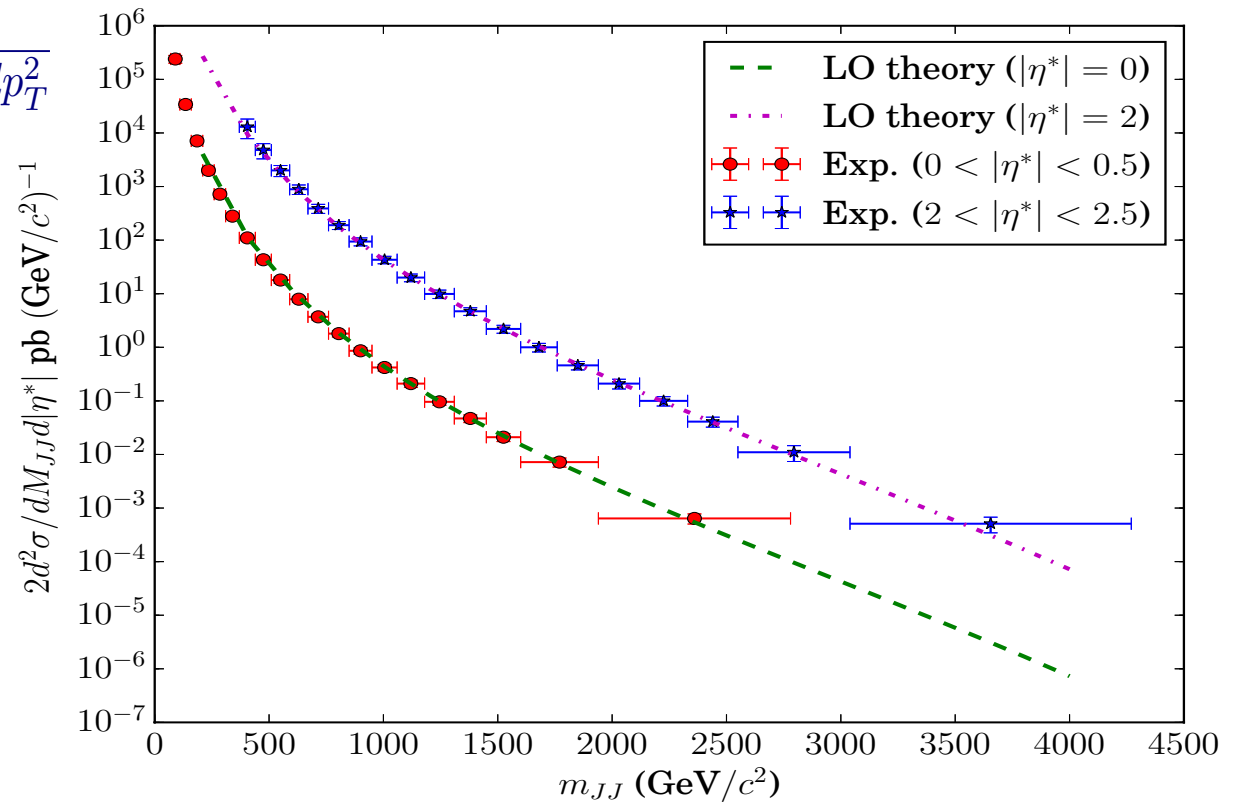
Color Screening Model \rightarrow
$$F_2^{(N,b)}\left(\frac{x}{\alpha}, \alpha, \mathbf{p}_T, Q^2\right) = F_2^{(N)}\left(\frac{x}{\alpha}, Q^2\right) \delta\left(k^2(\alpha, \mathbf{p}_T), \frac{x}{\alpha}\right)$$

QCD Evolution of Medium Modifications

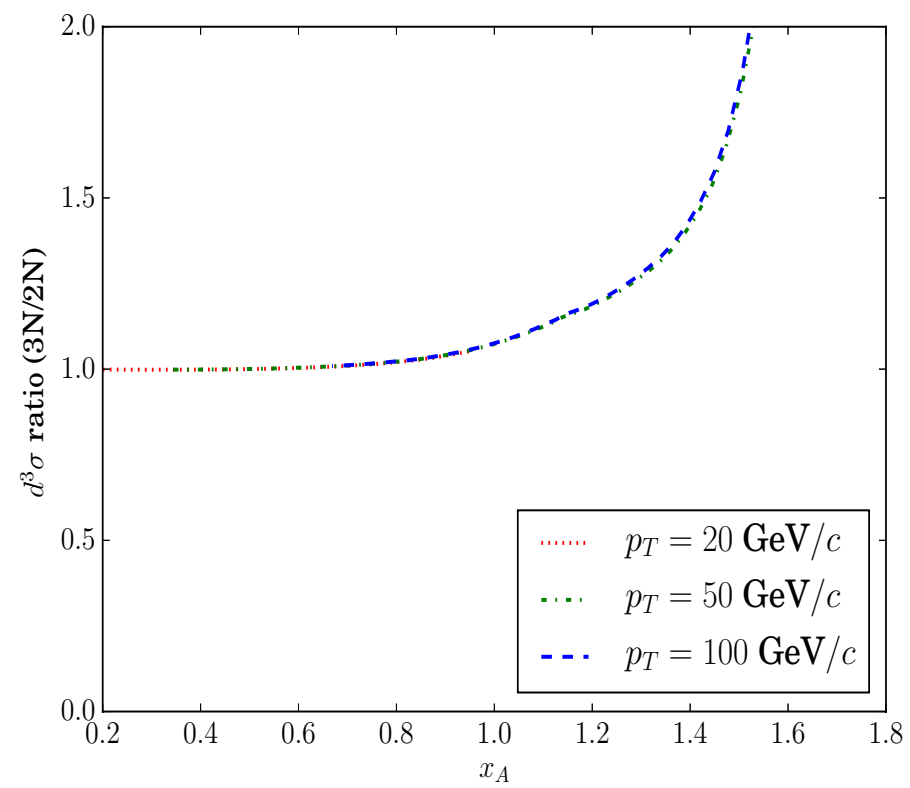
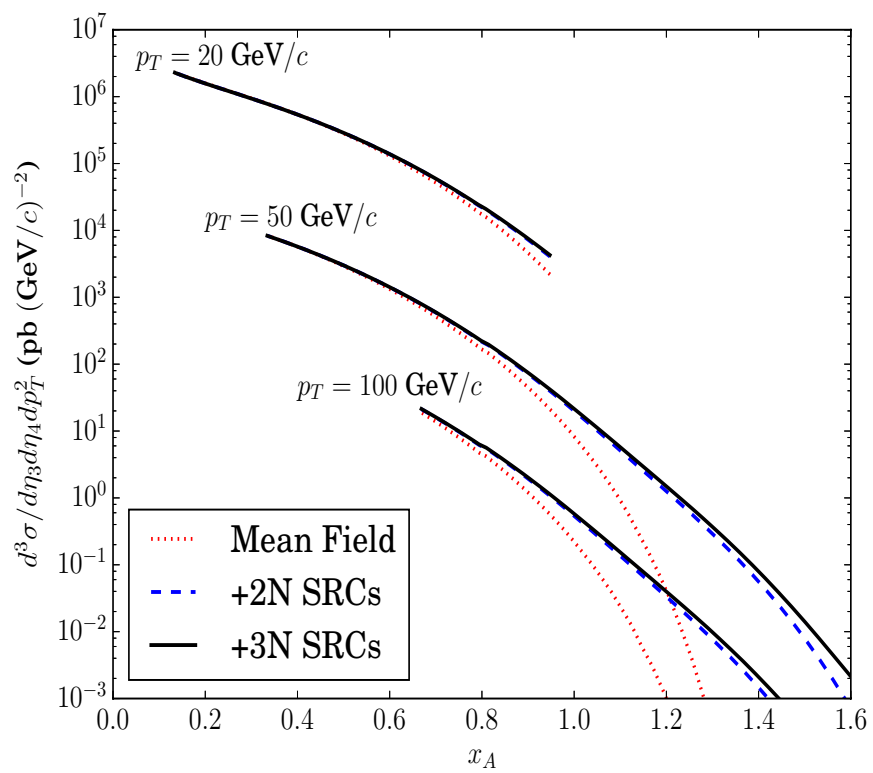


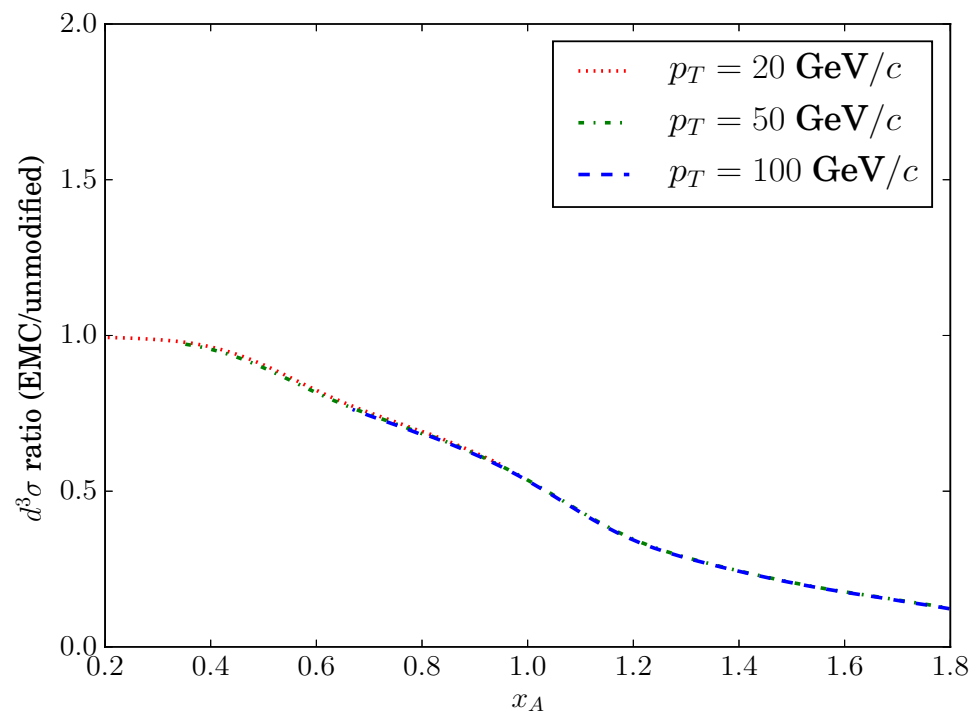
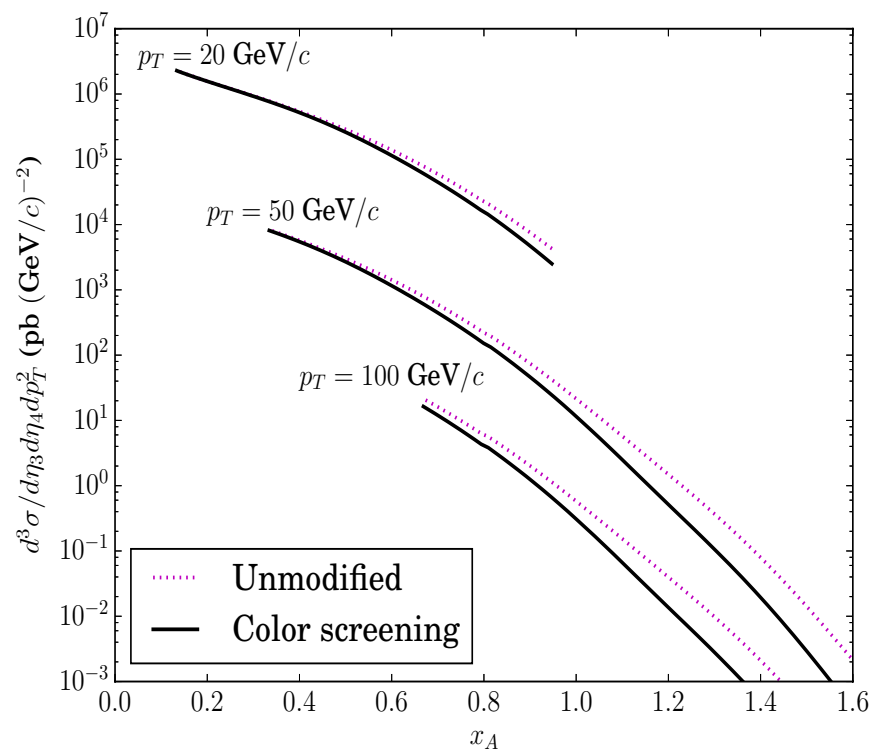
Checking Calculation for "Conventional" kinematics

$$\frac{2d^2\sigma}{dm_{JJ}d\eta^*} = \frac{4p_T}{\cosh(\eta^*)} \int d\bar{\eta} \frac{d^3\sigma}{d\eta_3 d\eta_4 dp_T^2}$$

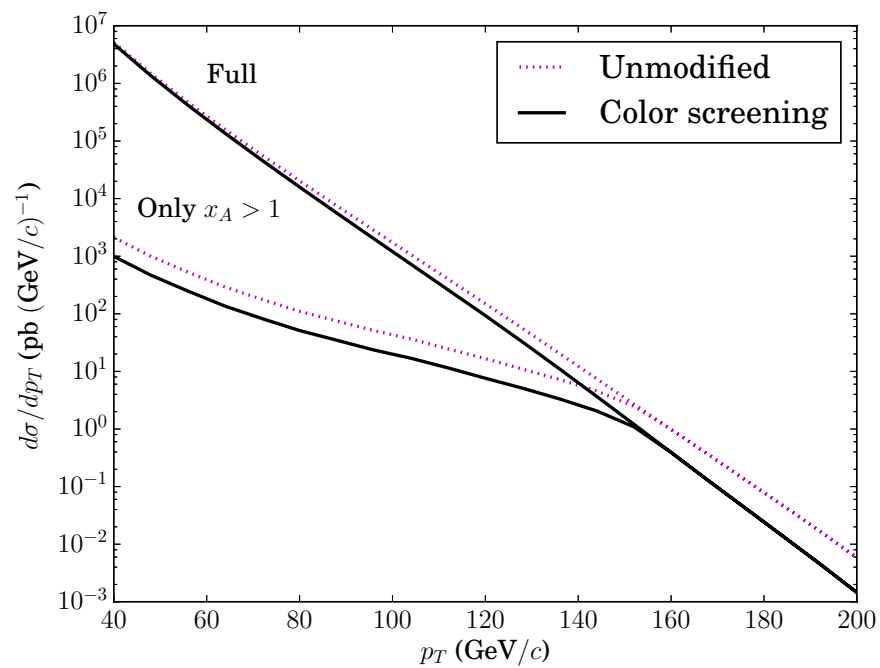
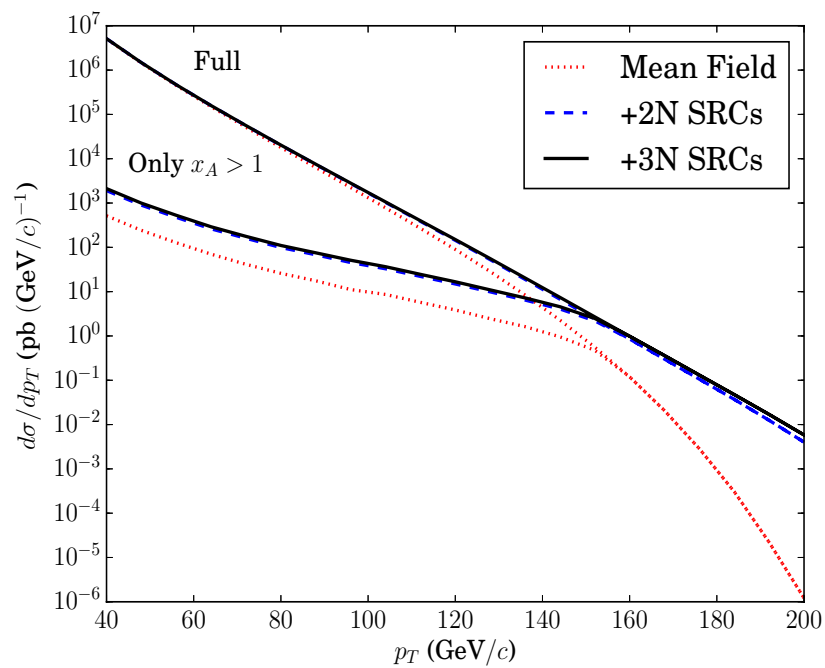


G. Aad et al. (ATLAS Collaboration), Phys. Rev. D 86, 014022(2012).





$$\frac{d\sigma(x_A > 1)}{dp_T} = \int_{-2.5}^{2.5} d\eta_3 \int_{-5}^{-3} d\eta_4 \frac{2p_T d^3\sigma}{d\eta_3 d\eta_4 dp_T^2} \Theta(x_A - 1)$$



Integrated cross section
at 7TeV per proton

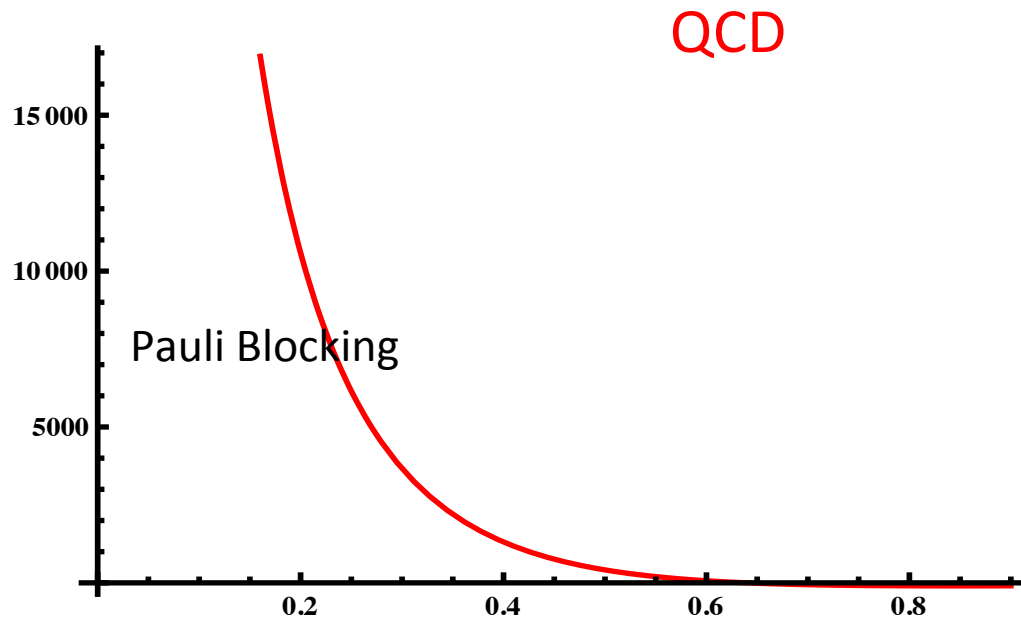
$$\frac{d\sigma(x_{max} > x_A > x_{min})}{dp_T} = \int_{50\text{GeV}/c} dp_T \int_{-2.5}^{2.5} d\eta_3 \int_{-5}^{-3} d\eta_4 \frac{2p_T d^3\sigma}{d\eta_3 d\eta_4 dp_T^2} \Theta(x_A - x_{min}) \Theta(x_{max} - x_A)$$

	Unmodified (SRCs)	Modified (no SRCs)	Modified (SRCs)
All x_A	58 μb	55 μb	55 μb
$0.6 < x_A < 0.7$	1.7 μb	1.2 μb	1.3 μb
$0.7 < x_A < 0.8$	0.60 μb	0.37 μb	0.43 μb
$0.8 < x_A < 0.9$	0.20 μb	0.11 μb	0.13 μb
$0.9 < x_A < 1$	59 nb	20 nb	33 nb
$1 < x_A$	21 nb	3.0 nb	9.3 nb

The expected yield for $x_A > 1$ events at the LHC is **326 events for a month of** run time based on previously achieved **luminosity of 35.5/nb**.

Summary & Outlook

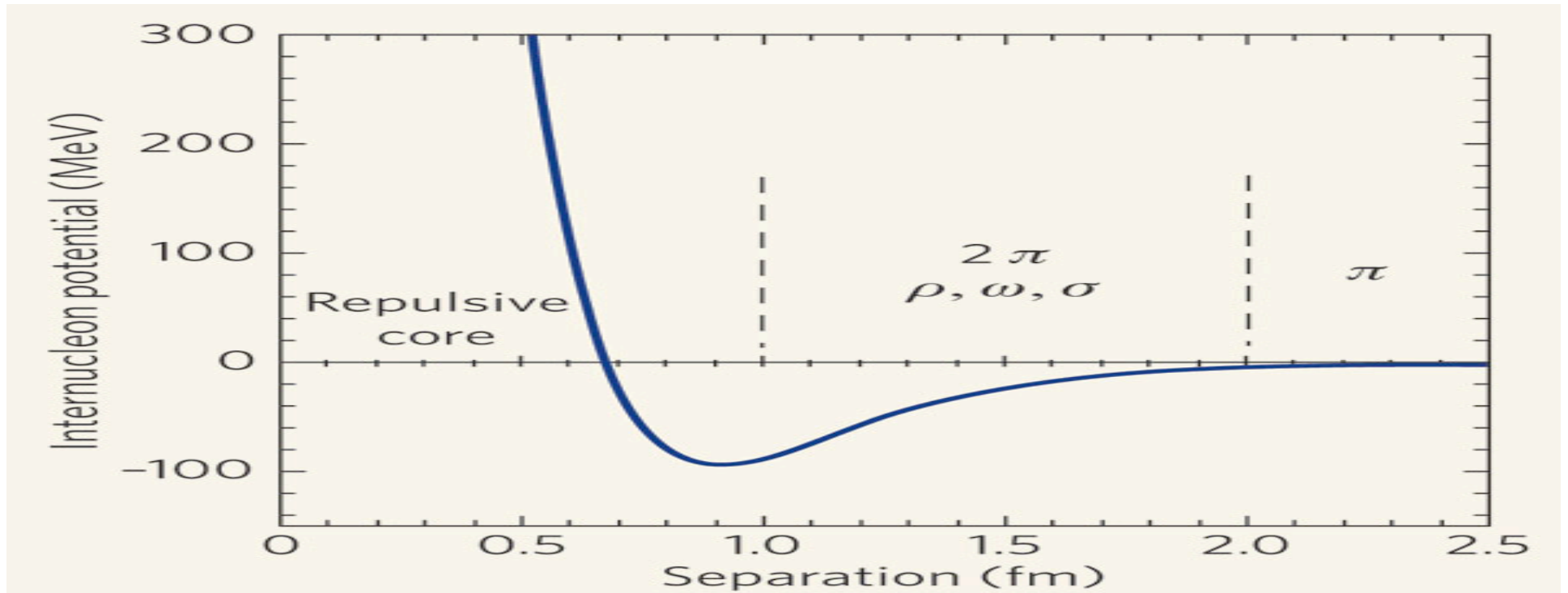
- p+A → 2jets+X at large p_T allow to reach practically unexplored $x > 1$ region
- Cross section in these kinematics is sensitive to the nuclear structure at very short distances
- It is sensitive also to the medium modification effects of PDFs allowing to study its x and Q^2 dependences
- Special care should be given in separating above to phenomena

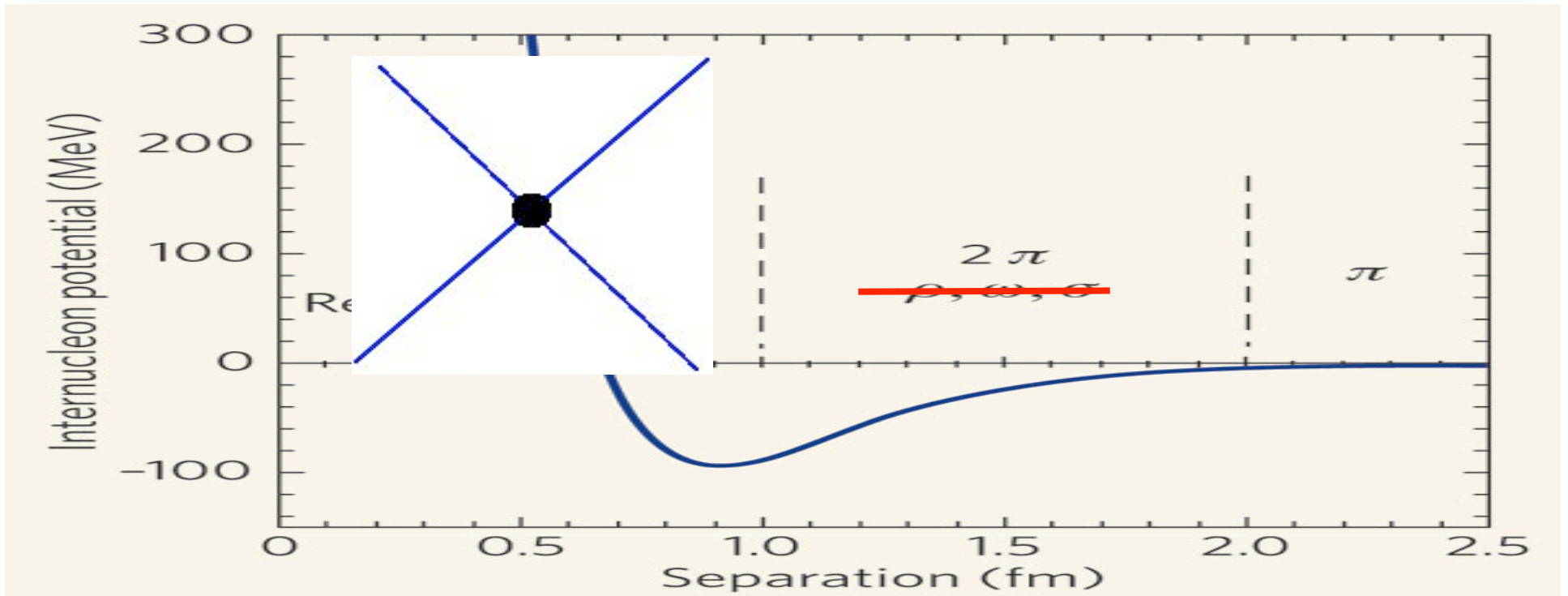
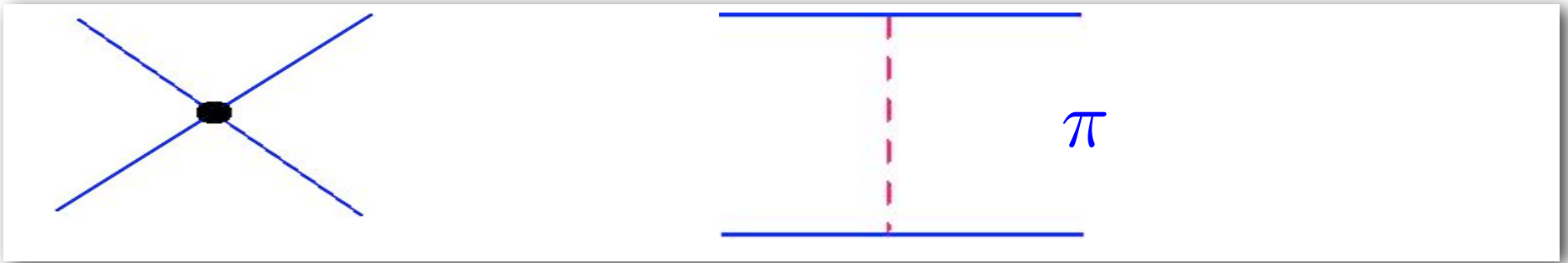


Contradicts Neutron Star Observations:
will predict masses not more than 0.1 - 0.6 Solar mass

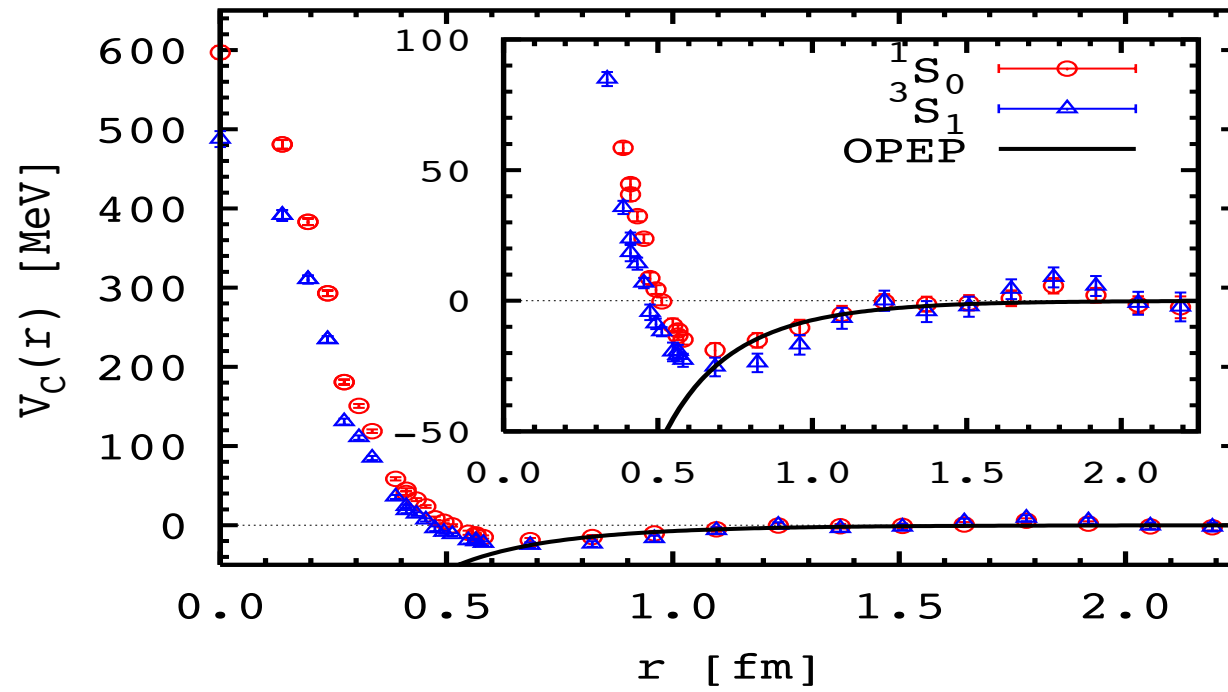


$\sigma, \pi, \rho, \omega, \dots$





Lattice Calculations



□ How to probe high density fluctuations in nuclei *for A > 2*

Probe large pre-existing relative momenta of nucleons in the nucleus

$$\begin{aligned} (E_B - \frac{k^2}{2m} - \sum_{i=2,\dots,A} T_i) \psi_A &= \sum_{i=2,\dots,A} \int V(k - k'_i) \psi_A(k, k'_i, \dots, k_j, \dots, k_A) \frac{d^3 k'_i}{(2\pi)^3} \\ &+ \sum_{i=2,\dots,A} \int V(k_i - k'_i) \psi_A(k, k'_i, \dots, k_j, \dots, k_A) \frac{d^3 k'_i}{(2\pi)^3}, \end{aligned}$$

If at large k, $V_{NN}(k) \sim \frac{1}{k^n}$ and $n > 1$

Frankfurt M.S.
Strikman IJMP 08

in $k^2/2m_N \gg |E_B|$ limit

$$\psi_A \sim \frac{V_{NN}(k)}{k^2} f(k_3, \dots, k_A),$$

where $f(k_3, \dots, k_A)$ is a smooth function of spectator nucleon's momenta with $k_2 \sim -k$.