APPLIED QUANTUM TOMOGRAPHY

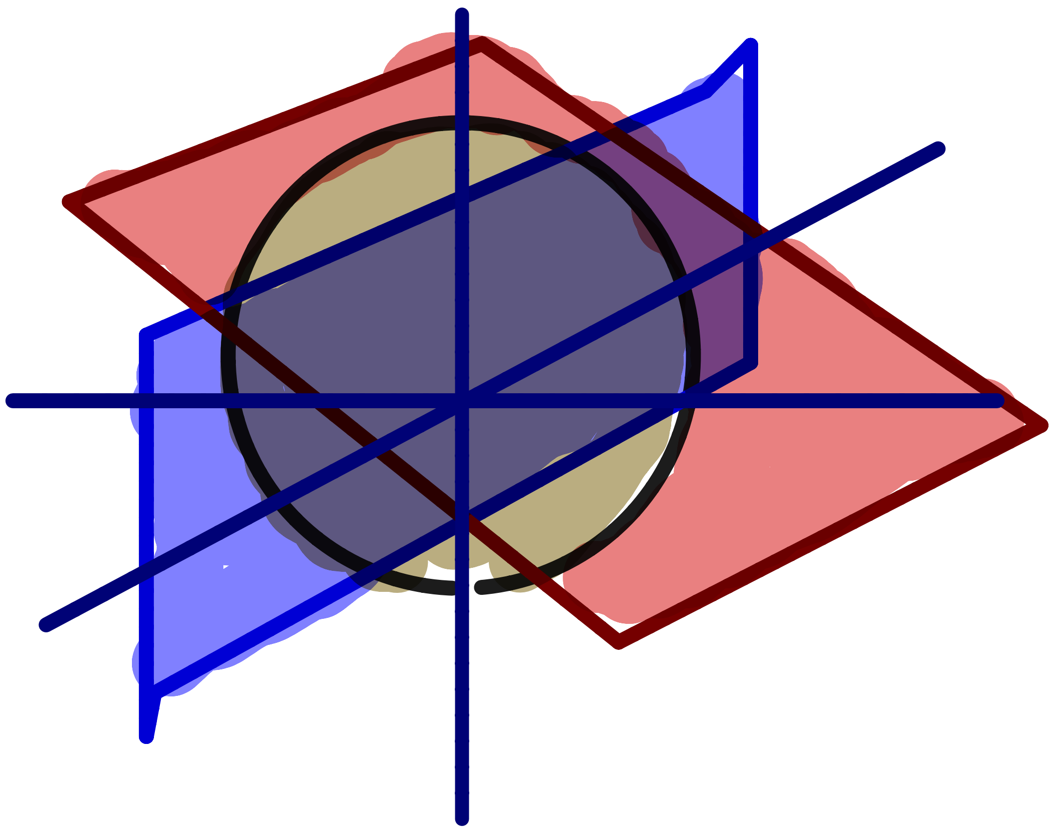
a new approach to what experiments measure

- "old topics" produce new results
- an explanation finally of Lam-Tung relⁿ!
- give us data! find new things!

John MARTENS
Daniel TAPIA TAKAKI
John Ralston

TOMOGRAPHY

reconstructs higher dimensional objects from lower dimensional projections



QUANTUM TOMOGRAPHY

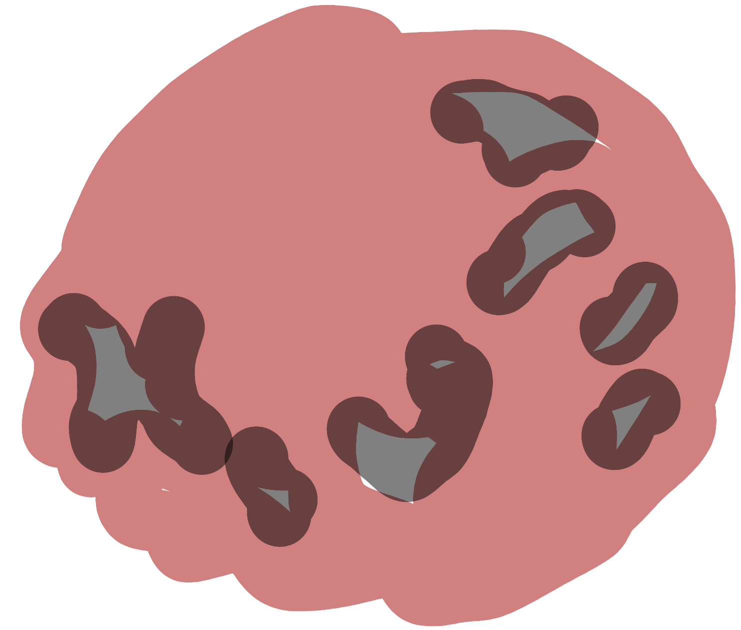
reconstructs density matrix or wave function from quantum observables

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle \longrightarrow \mathrm{tr}(\rho \hat{A})$$

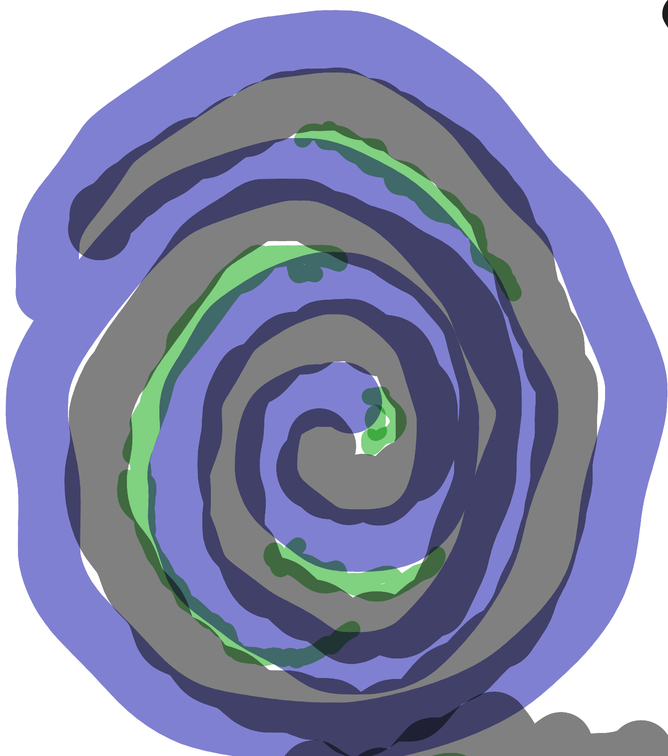
$\rho > 0$ positive evals

$\longrightarrow \rho = \rho^{\dagger}$

WHEN DID YOU LAST SPEND QUALITY TIME WITH QUANTUM MECHANICS?

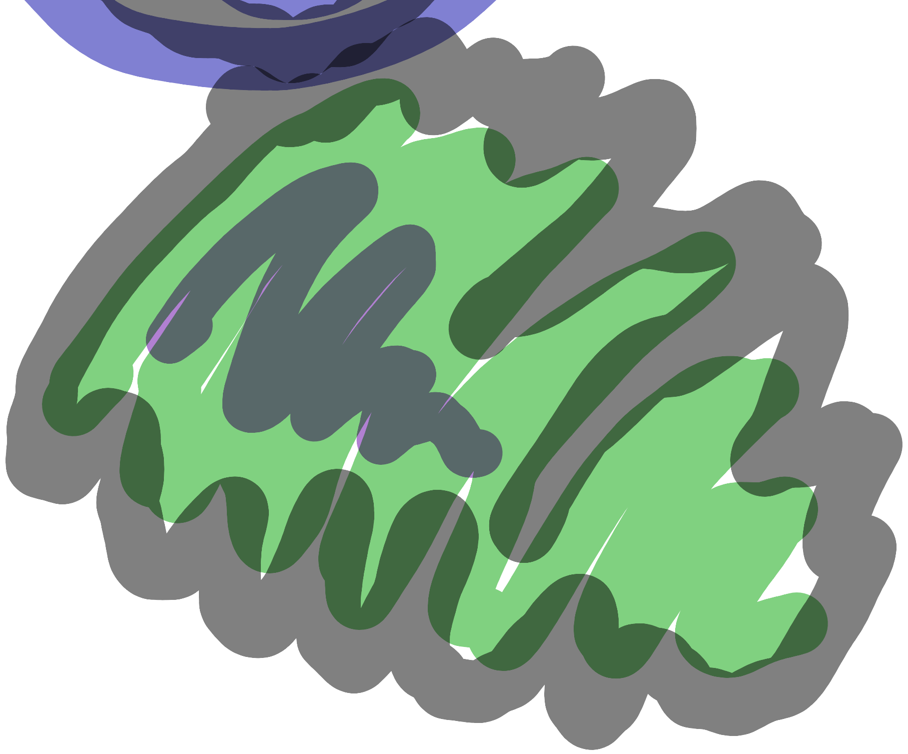


could you have done more?



HAVE YOU BEEN TAKING QUANTUM MECHANICS FOR GRANTED?

how does that make you feel?



IN YOUR ABSENSCE, WHO MANAGES YOUR RELATIONSHIP WITH QUANTUM MECHANICS?

do you regret your decisions?

QM "pure states" $i|\dot{\psi}\rangle = H|\psi\rangle$

$\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle = \mathrm{tr}(\hat{A}\,|\psi\rangle\langle\psi|)$

$\rho = |\psi\rangle\langle\psi|$ iff $\mathrm{rank}(\rho) = 1$

else $\rho = \sum p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha| \rightarrow \langle\hat{A}\rangle = \mathrm{tr}(\hat{A}\rho)$

observe

$\langle\hat{A}|\hat{B}\rangle = \mathrm{tr}(\hat{A}^\dagger\hat{B})$

Hilbert Schmidt inner product on operator space

$\mathrm{tr}(\hat{A}\rho) = \langle\hat{A}|\rho\rangle$

$\hat{A} = \hat{A}^\dagger$

$\uparrow$ dim N^2 if dim $|\psi\rangle = N$

OBSERVABLES ARE PROJECTIVE MAPS OF RAY REPRESENTATIONS

QM "pure states" $i|\dot{\psi}\rangle = H|\psi\rangle$

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Hilbert Schmidt inner product on operator space

$tr(\hat{A}\rho) = \langle\hat{A}|\rho\rangle$

$\hat{A} = \hat{A}^\dagger$ $\uparrow$ dim N^2 if dim $|\psi\rangle = N$

operator basis G_ℓ

$G_\ell = G_\ell^\dagger$ dim N^2

$$tr(G_\ell G_m) = \langle G_\ell|G_m\rangle = \delta_{\ell m}$$

generators of $u(N)$

COMPLETENESS

$$\rho = \sum_\ell |G_\ell\rangle\langle G_\ell|\rho\rangle$$

$$\rho = \sum_\ell G_\ell\, tr(G_\ell \rho)$$

OBSERVABLE

OBSERVABLE

PURE STATE:

If $\rho = |\psi\rangle\langle\psi|$

then $\rho|\psi\rangle = |\psi\rangle$

find $|\psi\rangle = \lambda|\psi\rangle$

$\lambda \in \mathbb{C}$

REDUCTION

the key to quantum probability

$$A^{i'a'\alpha'\ldots}_{ia\alpha\ldots} = A^{i'}_{i}\,\delta_{aa'}\,\delta_{\alpha\alpha'}\ldots$$

$$\rho^{i'a'\alpha'\ldots}_{ia\alpha\ldots} \rightarrow \rho^{i'}_{i} = \sum_{\substack{a\alpha\ldots \\ a'\alpha'\ldots}} \rho^{i'a'\alpha'\ldots}_{ia\alpha\ldots}\,\delta_{aa'}\,\delta_{\alpha\alpha'}\ldots$$

trace out in advance what will not be observed

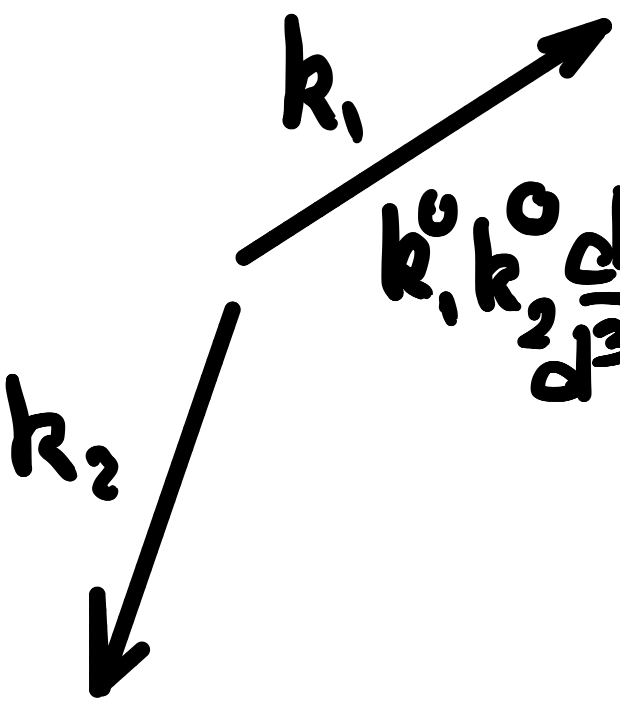
PURE STATES... EXPONENTIAL COMPLICATION... DELAY REDUCTION TO LAST STEP ← perturbative QFT

DENSITY MATRIX... ... PROBE DEFINED... REDUCTION IMMEDIATELY ← tomography

ALSO: FOCUS ON INVARIANTS

GIVE US DATA: WE'LL DO TOMOGRAPHY

bypassing all the unobservable crap of QFT



pair of any momenta k_1^μ, k_2^μ

$$k_1^0 k_2^0 \frac{d\sigma}{d^3k_1\, d^3k_2} = \frac{d\sigma}{d^4q\, d^2\hat{\ell}}$$

$d\Omega$ in frame

$$q^\mu = k_1^\mu + k_2^\mu \to (Q, \vec{0})$$

$$\ell^\mu = k_1^\mu - k_2^\mu \to (0, \hat{\ell})$$

NEVER BOOST TO A FRAME
DEFINE ALL THINGS COVARIANTLY

$$(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta) = (\ell_\mu X^\mu, \ell_\mu Y^\mu, \ell_\mu Z^\mu)$$

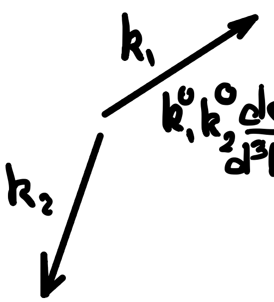
$$P(\hat{\ell}, q) = P(\hat{\ell}\,|\,q)\, P(q)$$

partons, kinematic

spin & dynamics

GIVE US DATA: WE'LL DO TOMOGRAPHY

bypassing all the unobservable crap of QFT



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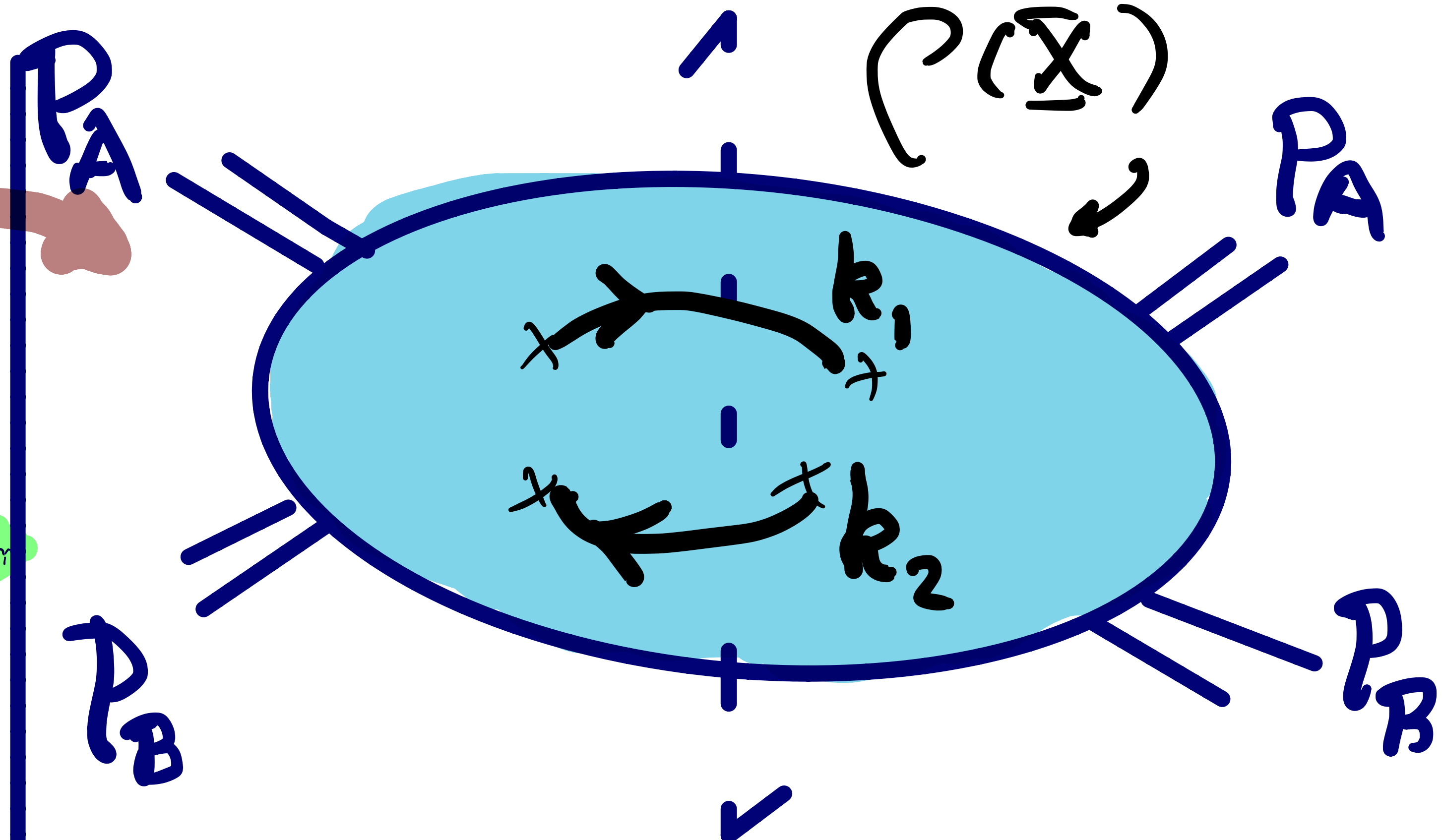

PROBE

SYSTEM

2 fermions:

$$P(\hat{\ell}\,|\,q) = \mathrm{tr}\big(\rho(\ell)\,\rho(\underline{X})\big)$$

$$\rho(\ell)_{\alpha\alpha'}^{\beta\beta'} = (\not{k}_1)_{\alpha\alpha'}(\not{k}_2)_{\beta\beta'} = k_{1\mu} k_{2\nu}\, \gamma^\mu_{\alpha\alpha'}\gamma^\nu_{\beta\beta'}$$

EXPAND PROBE IN COMPLETE SET.
NOTHING ELSE WILL BE OBSERVABLE

GIVE US DATA : WE'LL DO TOMOGRAPHY

bypassing all the unobservable crap of QFT

pair of any momenta k_1^μ, k_2^μ

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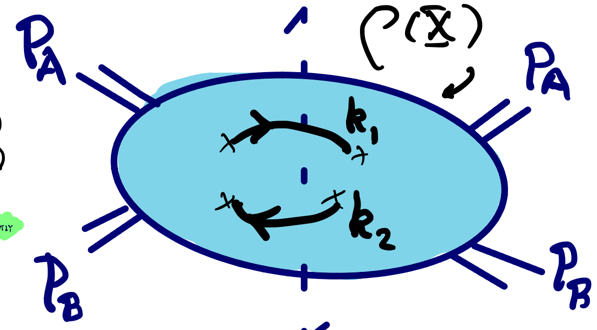
NEVER BOOST TO A FRAME
DEFINE ALL THINGS COVARIANTLY

$$P(\hat{\ell}, q) = P(\hat{\ell}|q)P(q)$$

partons, kinematic

spin & dynamics



PROBE

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EXPAND PROBE IN COMPLETE SET.
NOTHING ELSE WILL BE OBSERVABLE

PROBE : bilinear, 2x chirally even

$$\rho_{\mu\nu} = \rho^*_{\nu\mu}$$

$$\rho_{\mu\nu}(\ell) = \alpha\, k_1 \cdot k_2\, \eta_{\mu\nu} + \beta(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}) + i\gamma(k_{1\mu}k_{2\nu} - k_{1\nu}k_{2\mu}) + i\delta\, \varepsilon_{\mu\nu\alpha\beta}\, k_1^\alpha k_2^\beta$$

$$\rho_{ij} = \frac{1}{3}\delta_{ij} + a\, J^p_{ij}\,\hat{\ell}_p - b\, \mathcal{U}_{ij}$$

imaginary, antisym.
spin - 1

real, symmetric,
traceless spin 2

$$\mathcal{U}_{ij}(\hat{\ell}) = \hat{\ell}_i\hat{\ell}_j - \frac{1}{3}\delta_{ij}$$

THIS IS the EXPANSION the Probe CAN OBSERVE

QFT & STANDARD MODEL
ADD NOTHING BUT $a = C_A C_V;\ b = \frac{1}{2}$

$$\rho_{ij} = \frac{1}{3}\delta_{ij} + a J^{p}_{ij}\hat{l}_p - b U_{ij}$$

imaginary, antisym. spin - 1

real, symmetric, traceless spin 2
$U_{ij}(\hat{l}) = \hat{l}_i\hat{l}_j - \frac{1}{3}\delta_{ij}$

QFT & STANDARD MODEL
ADD NOTHING BUT $a = C_A C_V$; $b = \frac{1}{2}$

3 numbers from $\rho(\underline{X})$

5 numbers from $\rho(\underline{X})$

THE MIRROR TRICK

SYSTEM MIRRORS PROBE

PROBE		SYSTEM
$\rho_{ij} = \frac{1}{3}\delta_{ij}$	$\leftrightarrow$	$\frac{1}{3}\delta_{ij} = \rho_{ij}(\underline{X})$
$+ a J^{p}_{ij}\hat{l}_p$	$\leftrightarrow$	$+ J^{p}_{ij} S_p$
$+ b U_{ij}(\hat{l})$	$\leftrightarrow$	$+ U_{ij}(\underline{X})$

measurement = projection of system onto probe

NO MODEL of the SYSTEM

a description of what can be observed

MODEL INDEPENDENT CLASSIFICATIONS

$$P(\vartheta,\varphi\,|\,q) = \frac{1}{4\pi} + \frac{3}{4\pi} S_x \sin\vartheta\cos\varphi + \frac{3}{4\pi} S_y \sin\vartheta\sin\varphi$$
$$+ \frac{3}{4\pi} S_z \cos\vartheta + c\rho_0 \left(\frac{1}{\sqrt{3}} - \sqrt{3}\cos^2\vartheta\right)$$
$$- c\rho_1 \sin 2\vartheta\cos\varphi + c\rho_2 \sin^2\vartheta\cos 2\varphi$$
$$+ c\rho_3 \sin^2\vartheta\sin 2\varphi - c\rho_4 \sin 2\vartheta\sin\varphi$$

$$c = \frac{3}{8\sqrt{2\pi}}$$

item	$d\sigma/d\Omega$	C_ℓ	P	T	$C_\ell P$
ℓ	$\cdot$	$-$	$-$	$-$	$+$
X	$\cdot$		$-$	$-$	$-$
Y	$\cdot$		$+$	$+$	$+$
Z	$\cdot$		$-$	$-$	$-$
$X\ell$	$\sin\vartheta\cos\varphi$	$-$	$+$	$+$	$-$
$Y\ell$	$\sin\vartheta\sin\varphi$	$-$	$-$	$-$	$+$
$Z\ell$	$\cos\vartheta$	$-$	$+$	$+$	$-$
$XX\ell\ell$	$\sin^2\vartheta\cos 2\varphi$	$+$	$+$	$+$	$+$
$XY\ell\ell$	$\sin^2\vartheta\sin 2\varphi$	$+$	$-$	$-$	$-$
$XZ\ell\ell$	$\sin 2\vartheta\cos\varphi$	$+$	$+$	$+$	$+$
$YZ\ell\ell$	$\sin 2\vartheta\sin\varphi$	$+$	$-$	$-$	$-$
$ZZ\ell\ell$	$1/\sqrt{3} - \sqrt{3}\cos^2\vartheta$	$+$	$+$	$+$	$+$

C_ℓ = lepton charge conjugation

"T-odd:
Im parts, loops"

"C_ℓ-odd"
charge asymmetry

"P-odd"
parity asymmetry

SO FAR, HAVE NOT EVEN ASSUMED ONE BOSON EXCHANGE

GIVE US DATA: WE'LL DO TOMOGRAPHY

bypassing all the unobservable crap of QFT

EXAMPLE: ATLAS $pp \to Z^0 + \bar{X} \to \mu^+\mu^- + \bar{X}$

1606.00689

"mature theory"

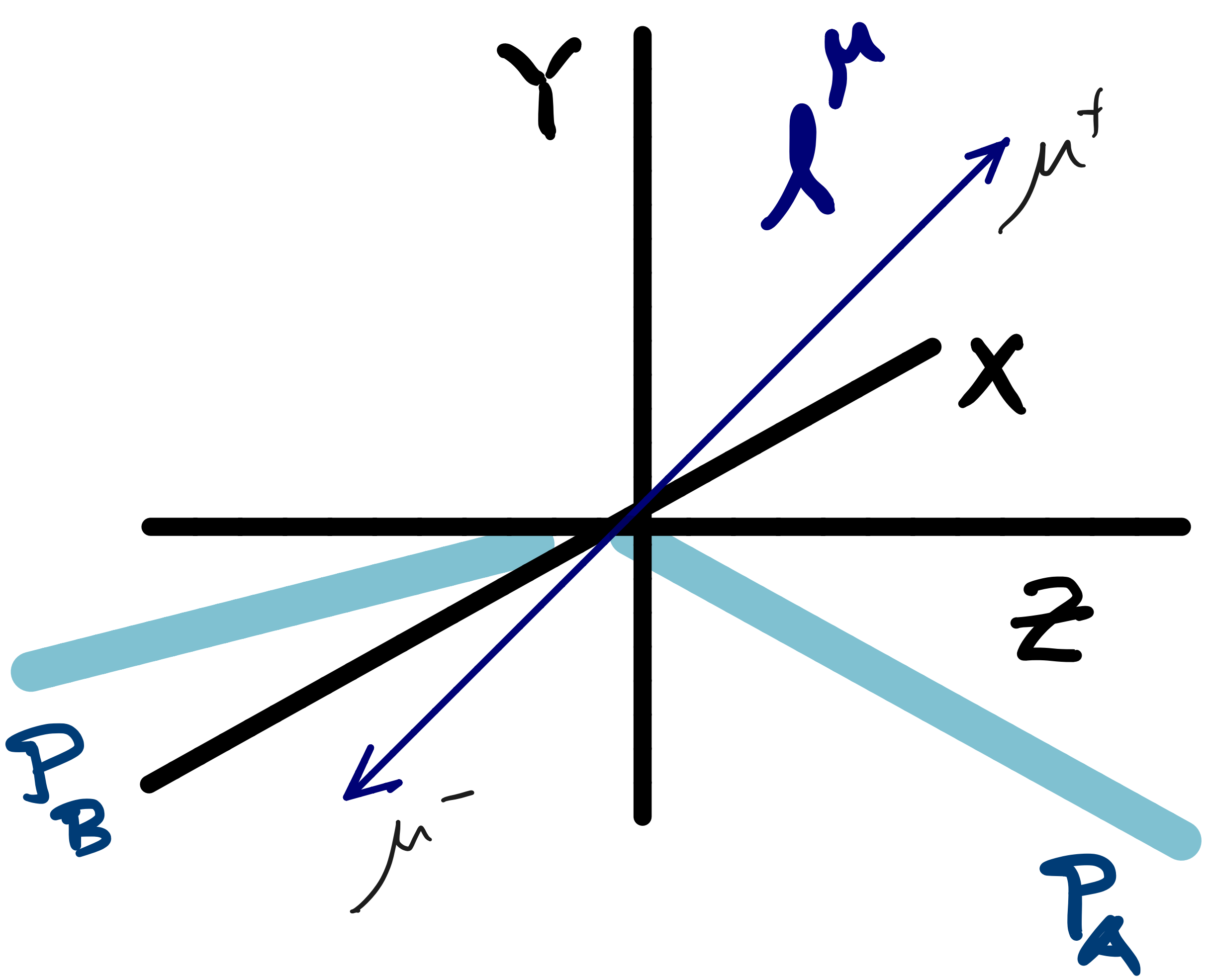
3 or more frame currents

COLLINS SOPER FRAME

$$Z^\mu = P_A^\mu Q\cdot P_B - P_B^\mu Q\cdot P_A$$

$$X^\mu = Q^\mu \text{ with } Q\cdot X = Z\cdot X = 0$$

$$Y^\mu = (Z \times X)^\mu$$

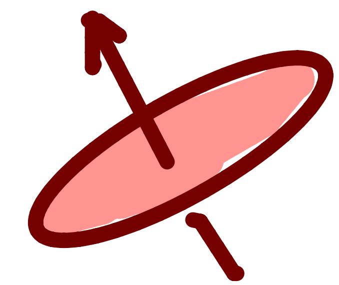


WHAT ARE THE INVARIANTS?

LEPTON HELICITY CONSERVATION

$$\rho_{lep} = \begin{array}{c|ccc} & + & - & 0 \\ \hline + & & & \\ - & & & \\ 0 & & & \end{array} \quad \begin{array}{c} \\ + \\ - \\ 0 \end{array}\begin{pmatrix} 1/2 & & \\ & 1/2 & \\ & & 0 \end{pmatrix}$$

eigenvalues



leptons

HADRON SPIN-2

$$\rho_X = \begin{array}{c} \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{pmatrix} \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark \end{pmatrix} \end{array} \xrightarrow{LO} \begin{pmatrix} \checkmark & 0 & \checkmark \\ 0 & \rho_0/\sqrt{6} - \rho_2/\sqrt{2} & 0 \\ \checkmark & 0 & \checkmark \end{pmatrix}$$

LEADING ORDER

$$\rho_0/\sqrt{6} - \rho_2/\sqrt{2} = \frac{1}{2}$$

- LAM-TUNG RELATION
- Eigenvector $Y \ldots Y_i Y_j$ always if T symmetry

we prove LAM-TUNG EXACT all orders TREE APPX

plus it's an invariant @ " ".

$$\rho_X = \begin{array}{c|ccc} & X & Y & Z \\ \hline X & \checkmark & \checkmark & \checkmark \\ Y & \checkmark & \checkmark & \checkmark \\ Z & \checkmark & \checkmark & \checkmark \end{array} \xrightarrow{LO} \begin{pmatrix} \checkmark & 0 & \checkmark \\ 0 & \rho_0/\sqrt{6} - \rho_2/\sqrt{2} & 0 \\ \checkmark & 0 & \checkmark \end{pmatrix}$$

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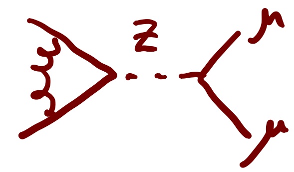
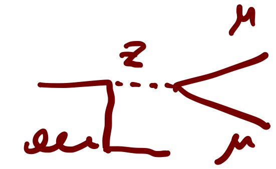
NLO: $Q_T \neq 0$ no loops matter





NNLO $Q_T \neq 0$ + loop

LAM-TUNG FAILS......

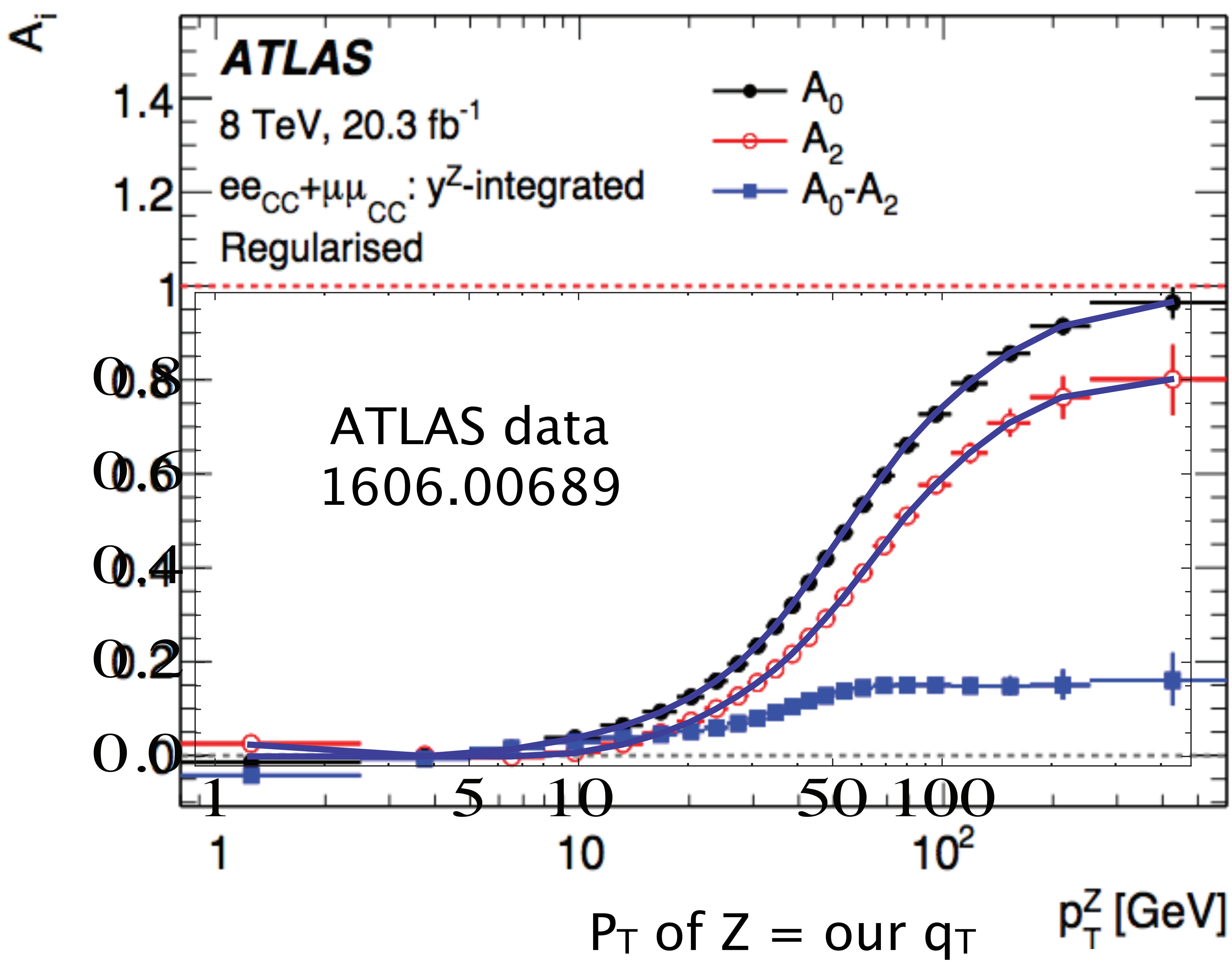
FINALLY EXPLAINED WHY

WHAT ARE THE INVARIANTS?

Faccioli et all – attempted invariants rotating about Y

Positivity means $\rho > 0$, not $d\sigma > 0$: Botched!

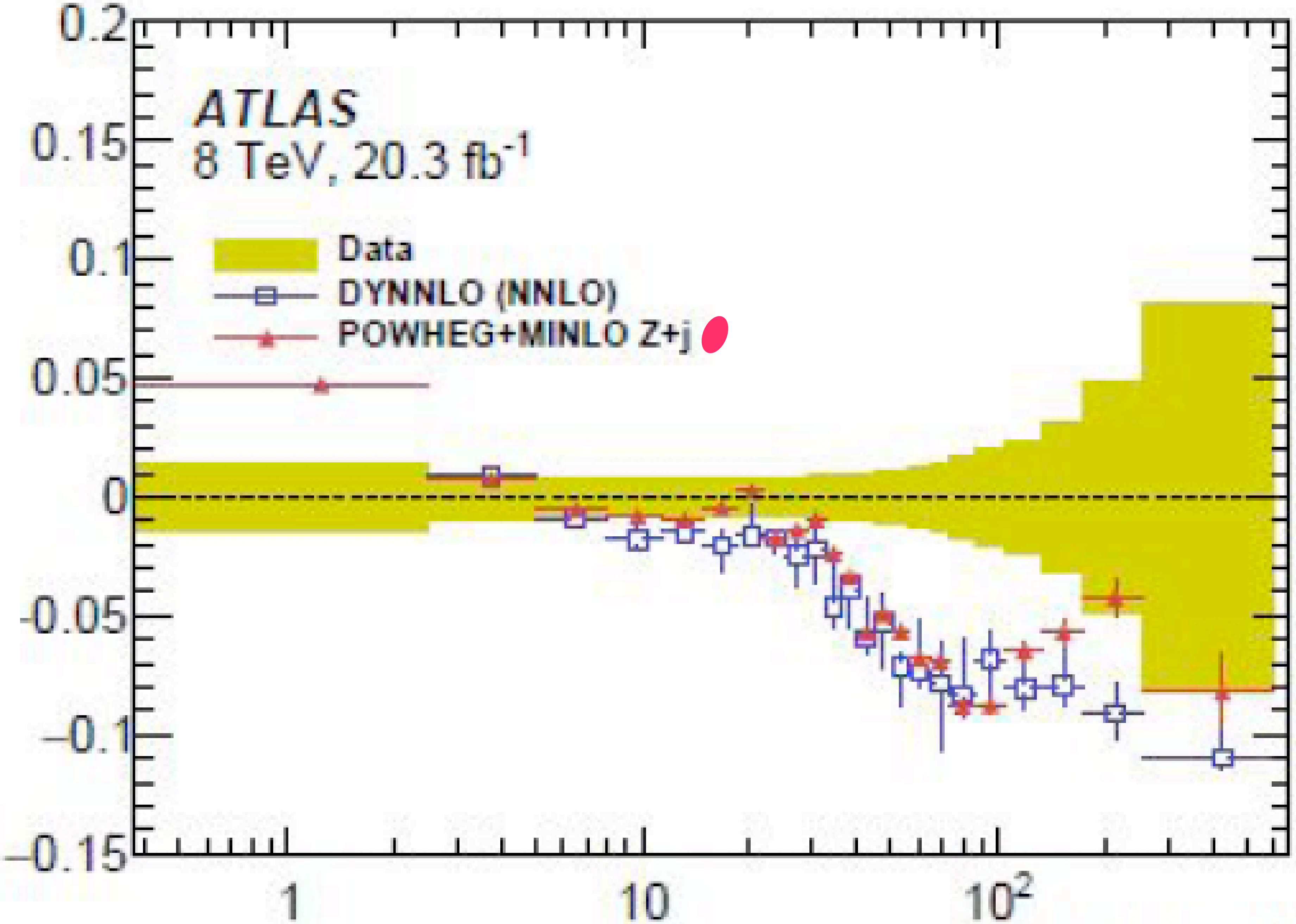
THE INARIANTS ARE THE EXPERIMENTALLY DERIVED EIGENVALUES



ATLAS
8 TeV, 20.3 fb^{-1}
ee$_{CC}$+μμ$_{CC}$: y^Z-integrated
Regularised
A_0
A_2
A_0-A_2
ATLAS data
1606.00689
A_i
p_T^Z [GeV]
P_T of Z = our q_T

ATLAS data
1606.00689

$(A_0 - A_2)$



$p_T\ [GeV]$

P_T of Z = our q_T

Our data analysis

* download data. Reproduce curves and claims of ATLAS

* transcribe arbitrary $A_{k \text{ convention}}$ to spin-1 and spin-2 density matrix elements

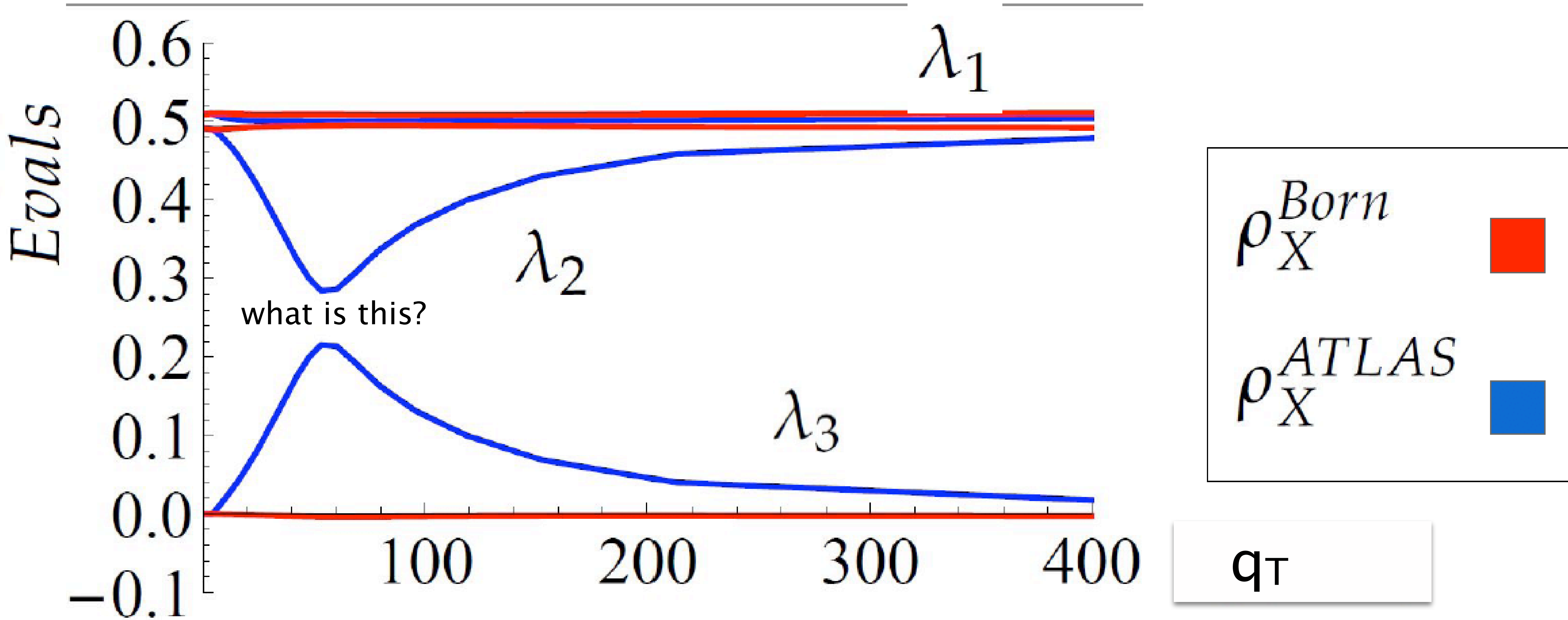
* compute invariants, entanglement entropy, and covariants like \vec S

* for this talk, ignore experimental error bars, if small compared to data values

hadronic eigenvalues versus p_T, same as q_T

y-integrated data. **arXiv: 1606.00689**

Evals λ_j of ρ_X^{ATLAS} (blue) are very different from evals from Born-level physics (red).



new: no precedent

the entanglement entropy

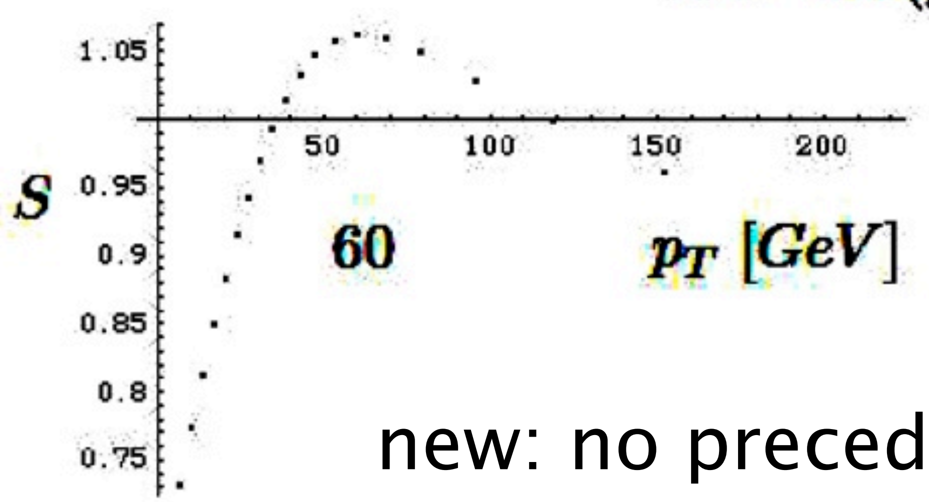
$$\mathcal{S} = -tr(\rho)log(\rho); \qquad 0 < \mathcal{S} < log(N)$$

pure　　　unpolz

What's the bump at pT = 60 GeV about?

We plot the entropy S and the degree of polarization d

$$S = -tr(\rho_X log \rho_X) \qquad d = \sqrt{(3tr(\rho_X) - 1)/2}$$



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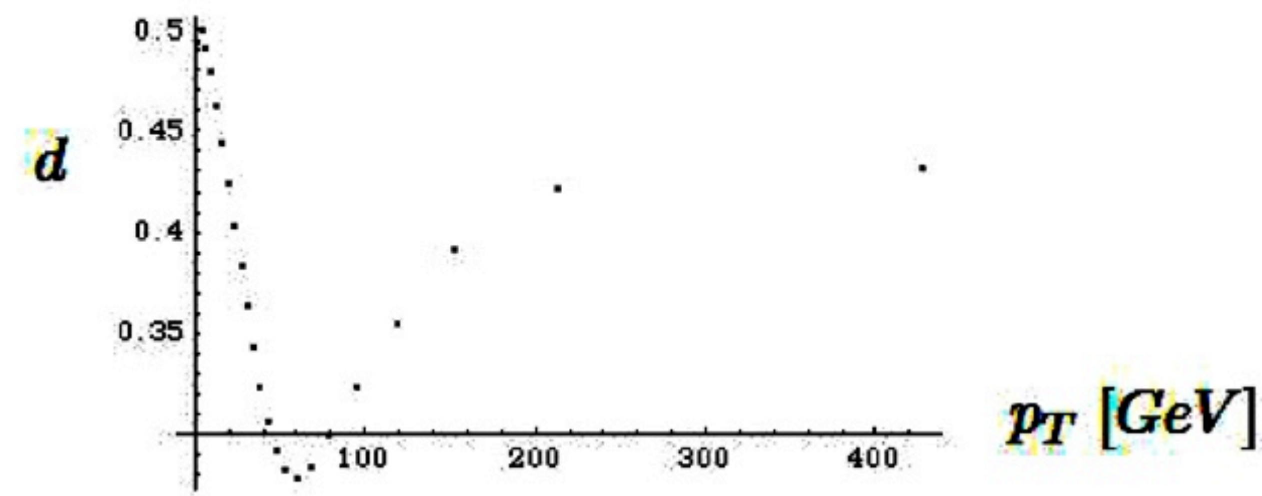
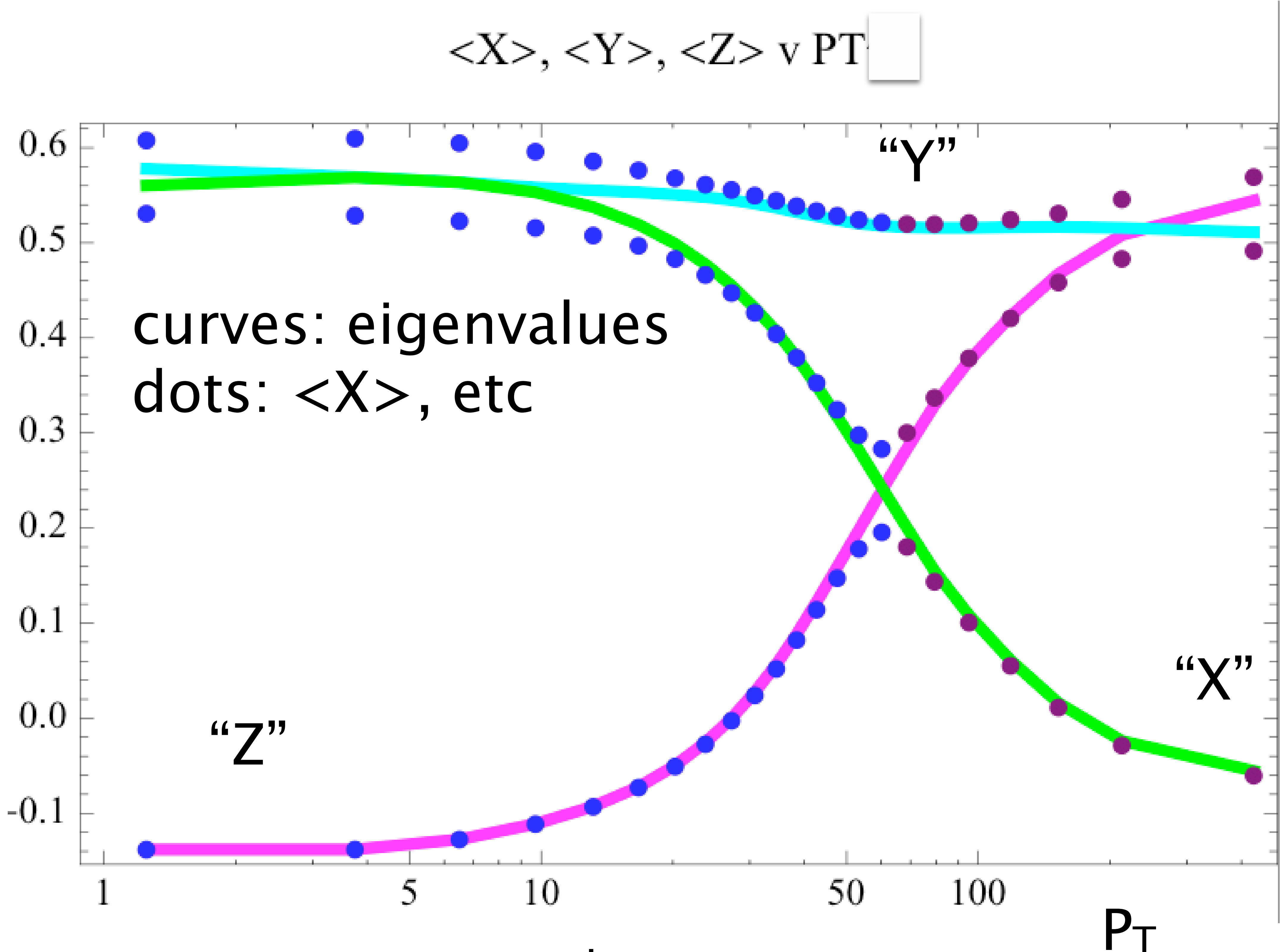


60

P_T of Z = our q_T

Bump has the accidents if not the substance of a phase transition..

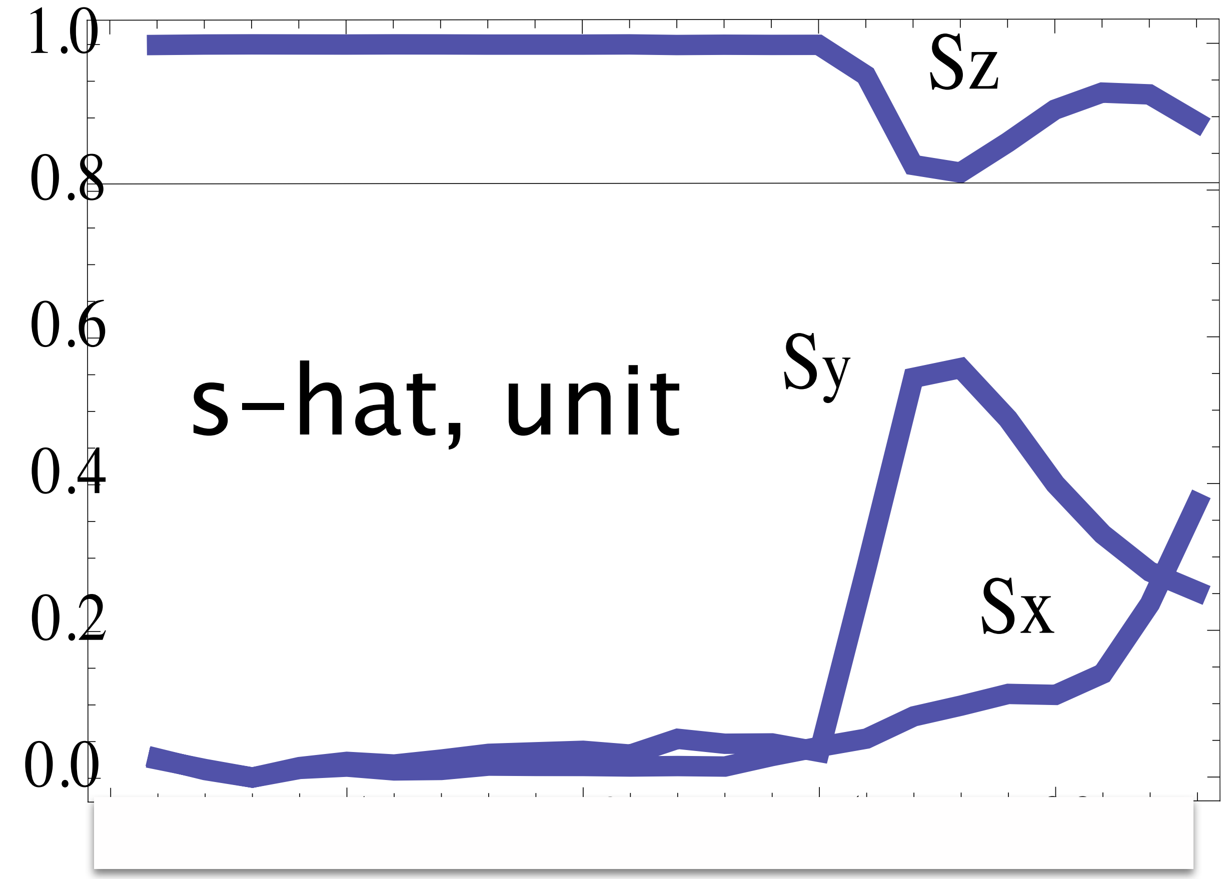
Avoided level crossing.
Eigenvectors swap



new: no precedent

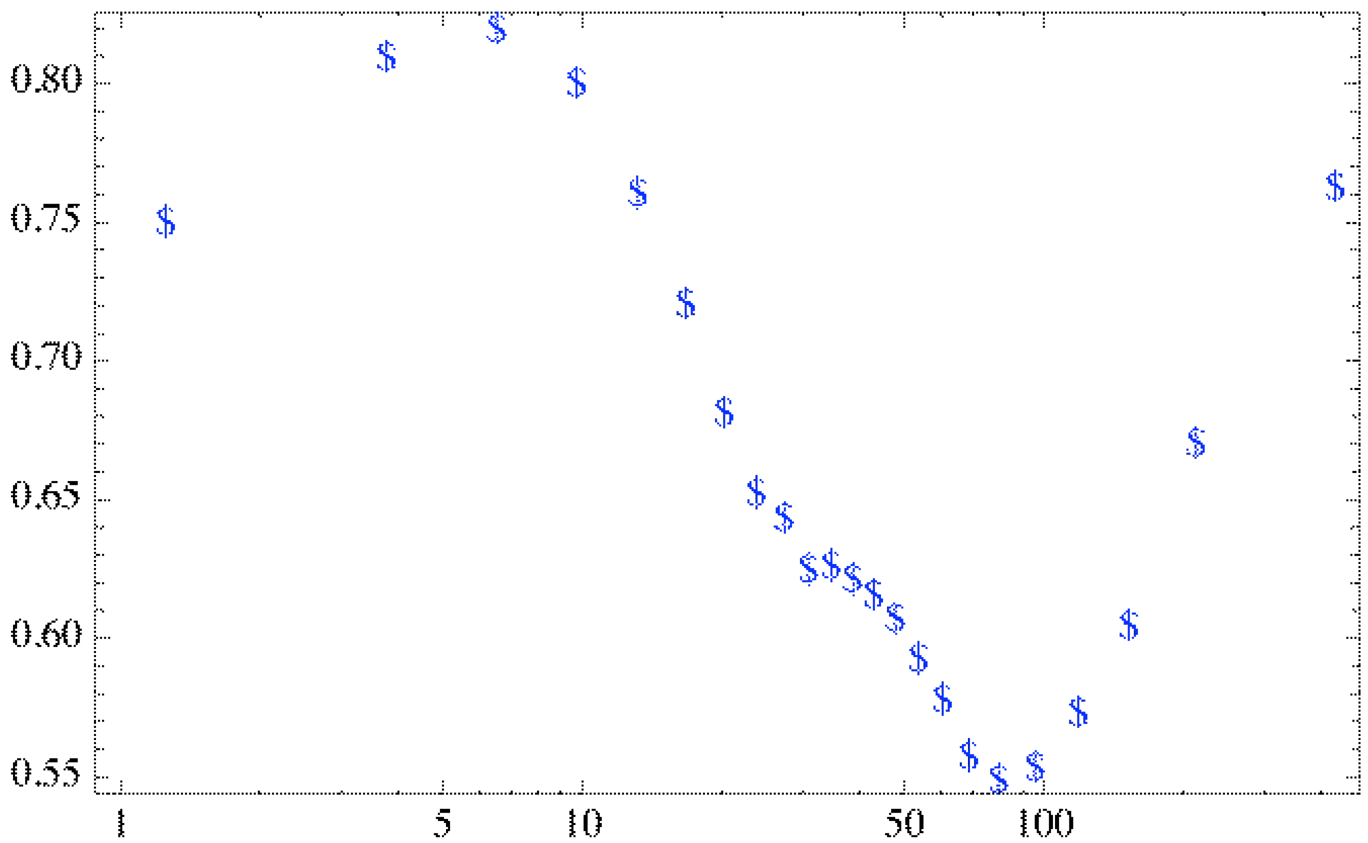
P_T of Z = our q_T

strange spin magnitude and directions



P_T

10 x |S|



P_T

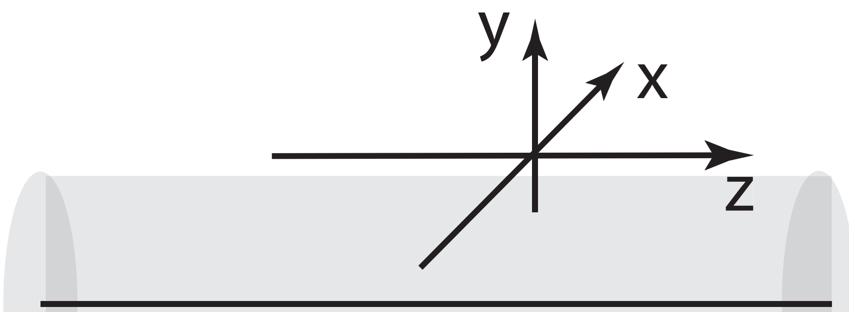
P_T of Z = our q_T

unexpected structure
in
spin parameters of Z

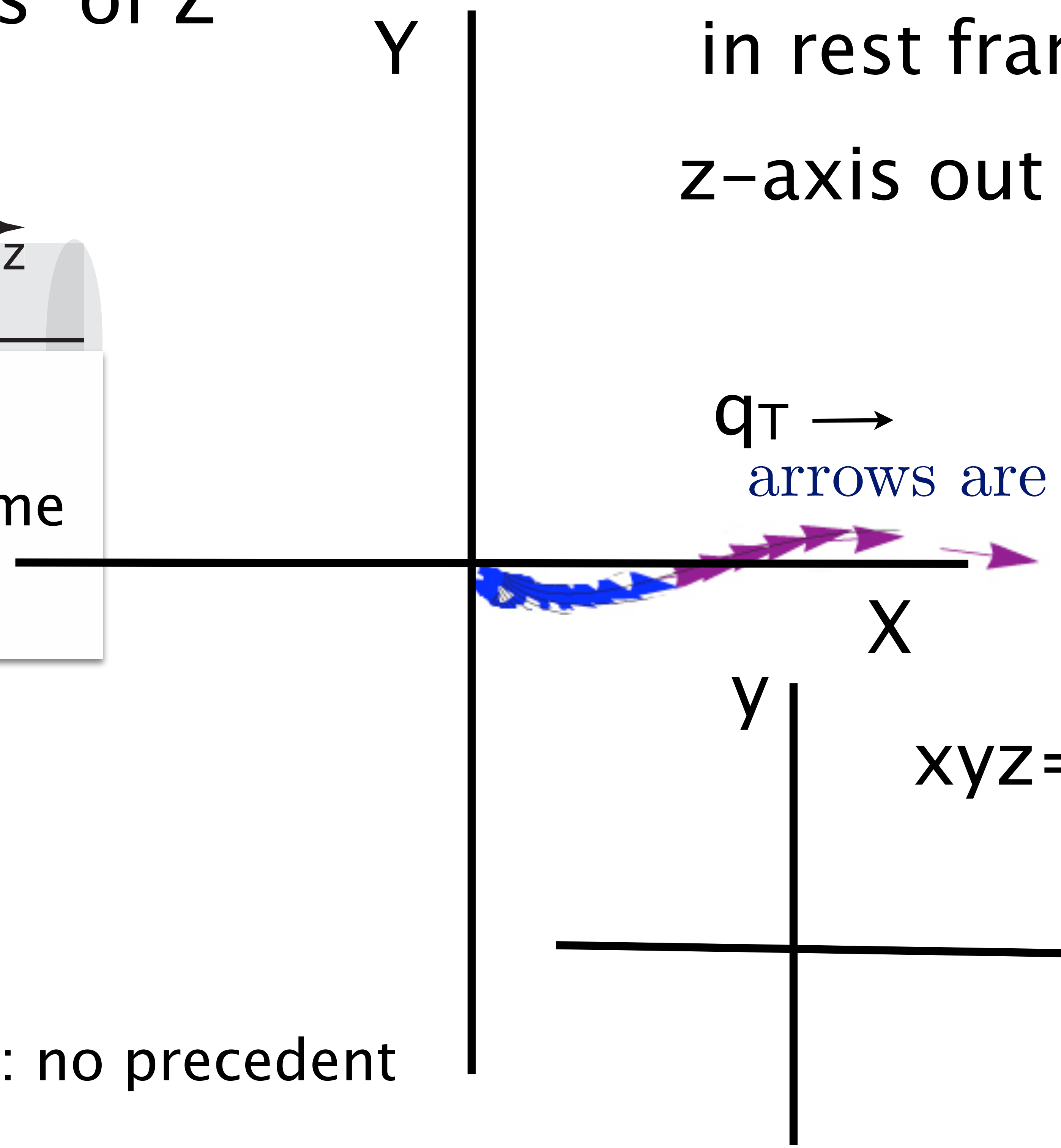
XYZ= collins soper

in rest frame of Z

z-axis out of page



sideways view of frame

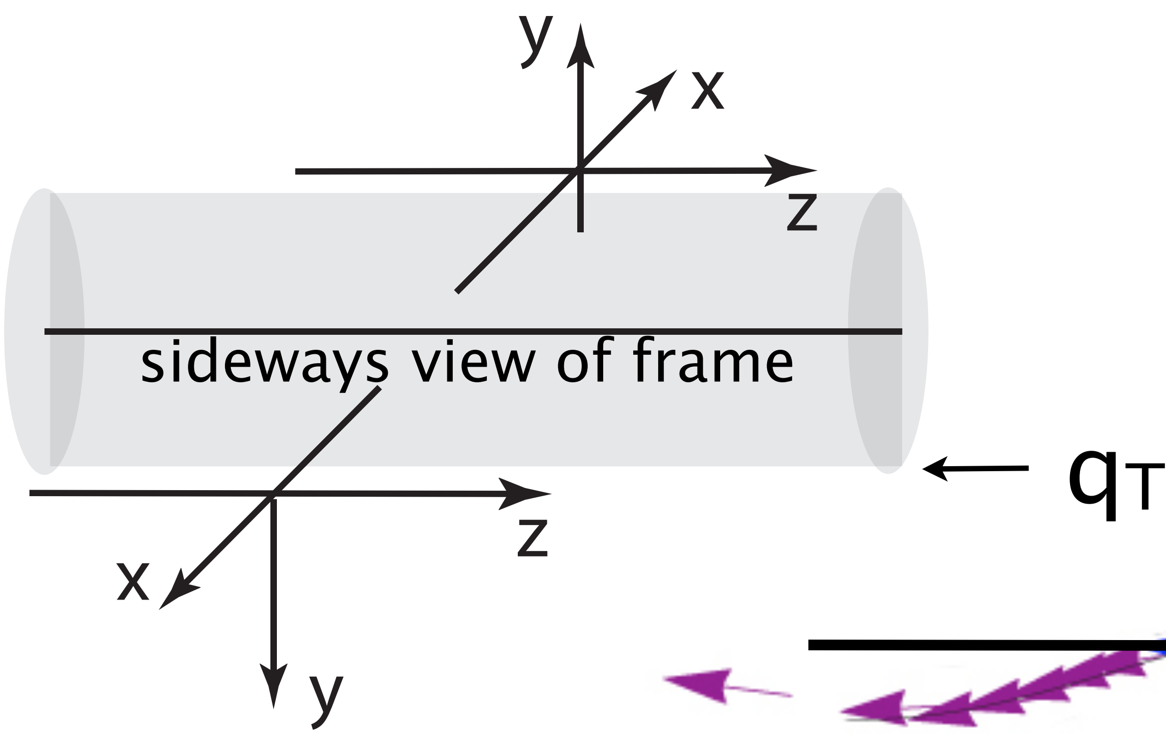


xyz= lab

z \times x=y

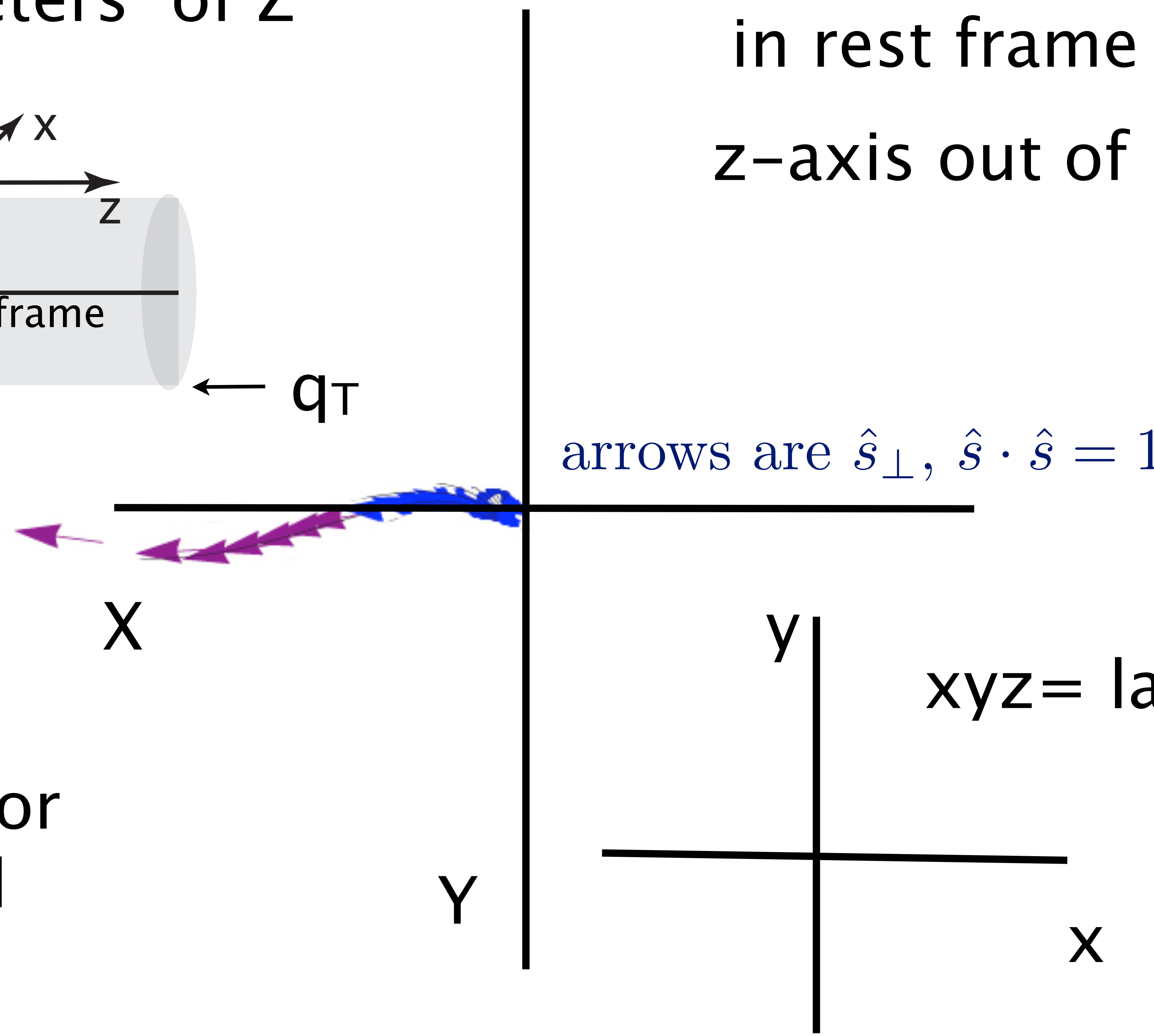
new: no precedent

unexpected structure in spin parameters of Z



in rest frame of Z

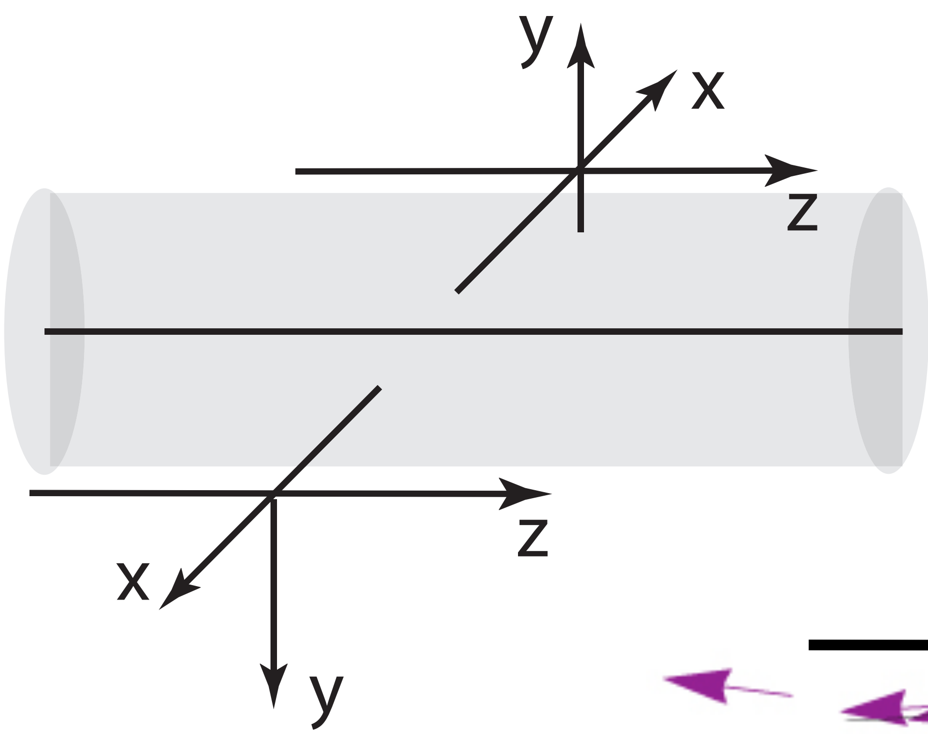
z−axis out of page

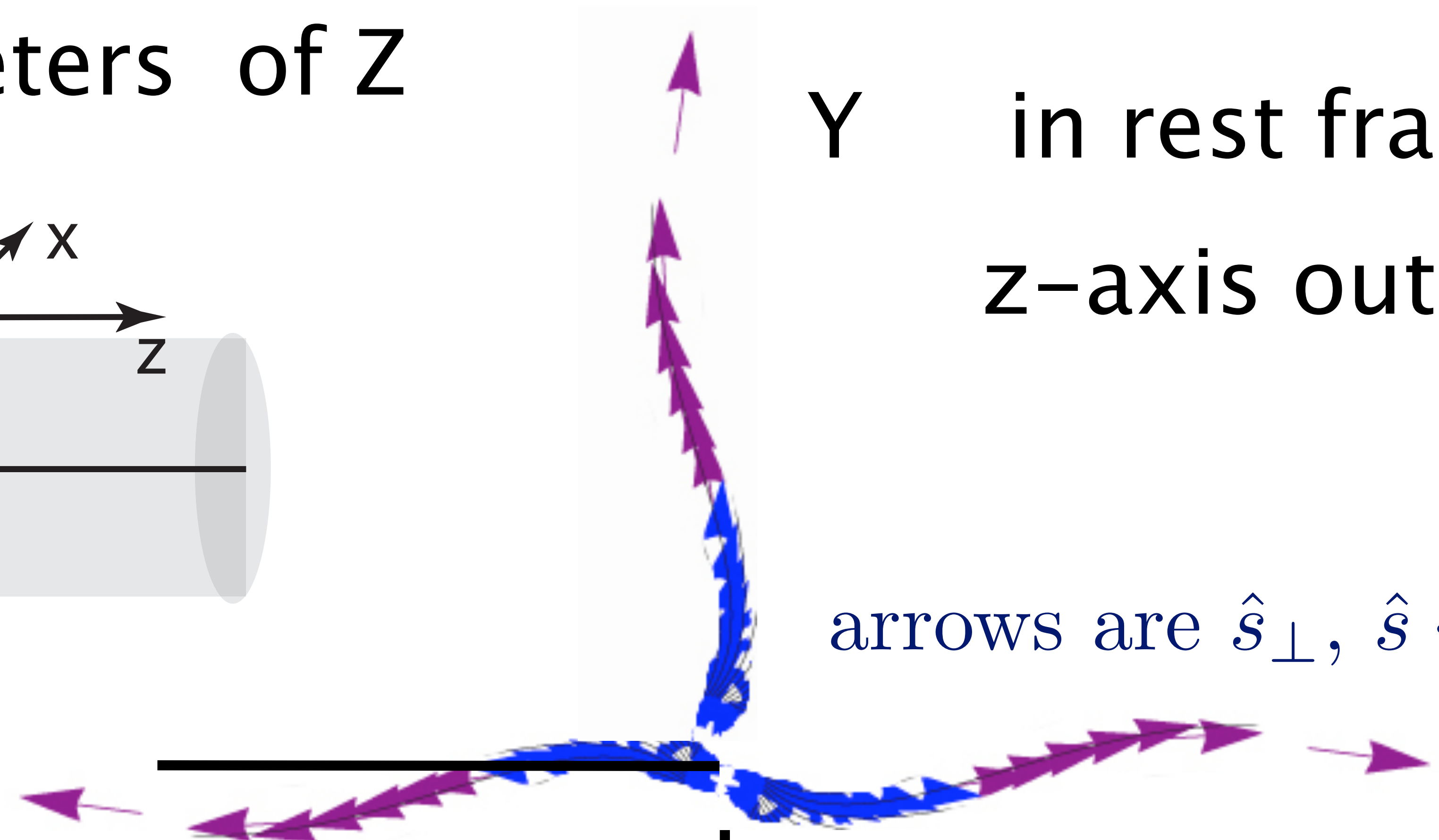


xyz= lab

Y is pseudovector and T odd

unexpected structure
in
spin parameters of Z

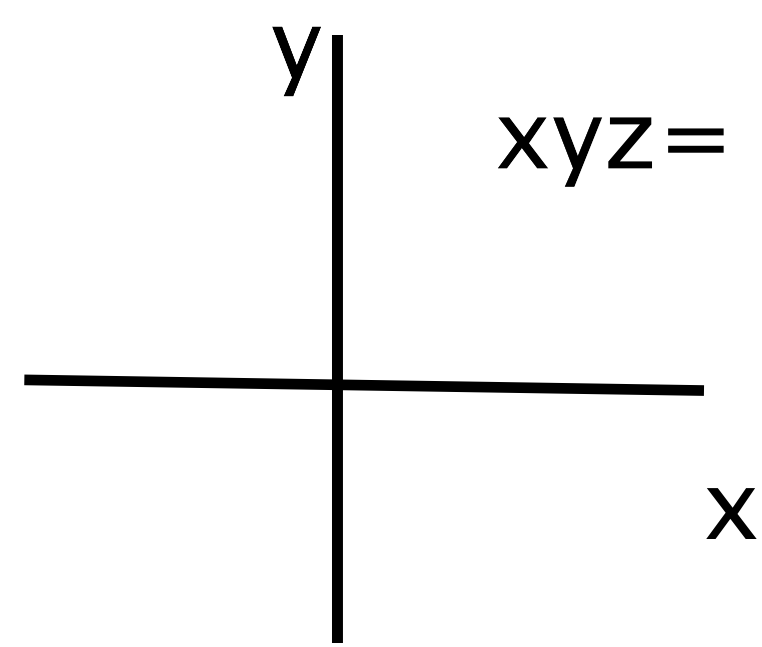




in rest frame of Z

z-axis out of page

arrows are $\hat{s}_\perp$, $\hat{s} \cdot \hat{s} = 1$

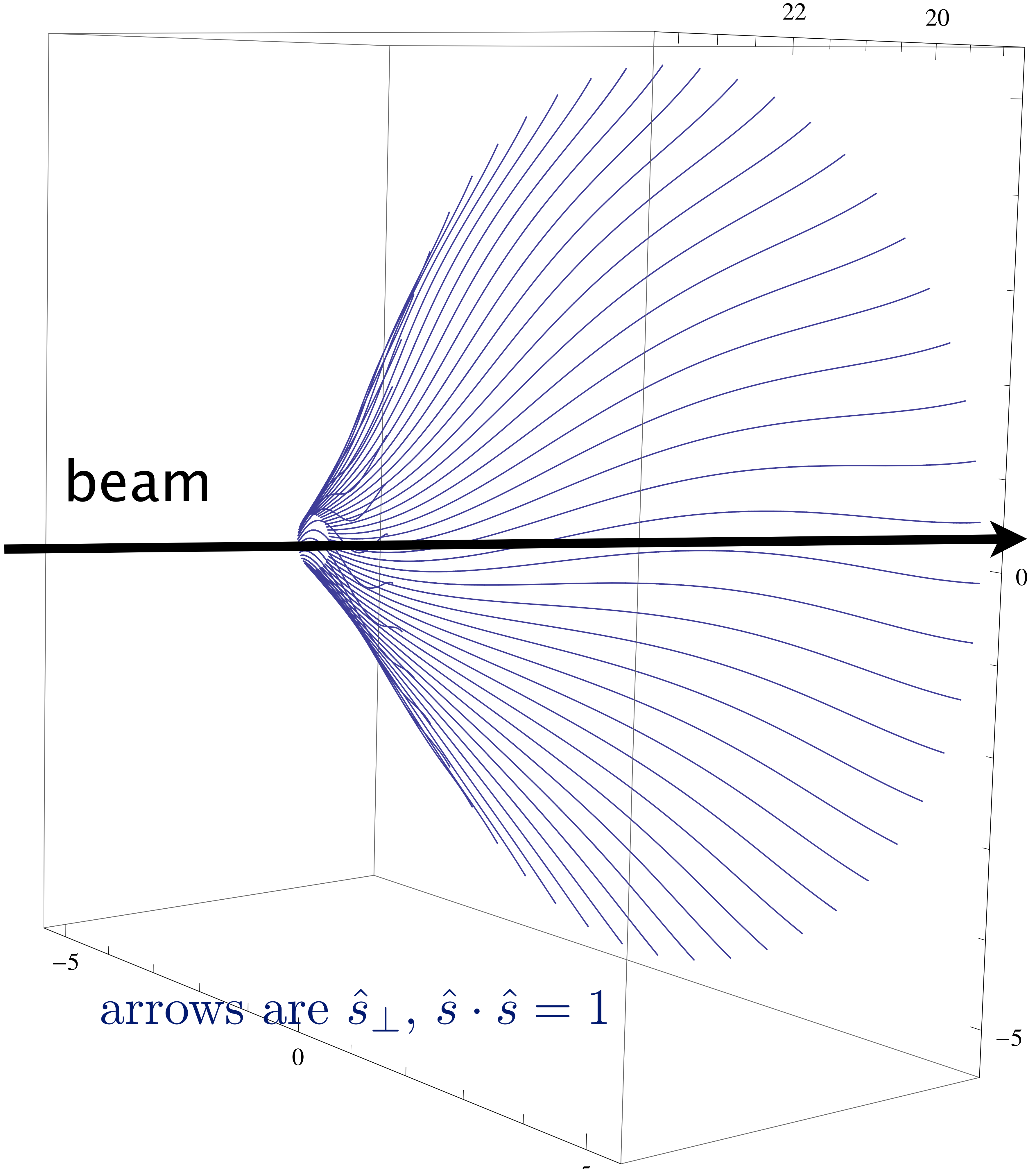


xyz= lab

rotational symmetry:
q_T itself is distributed
isotropically about the beam

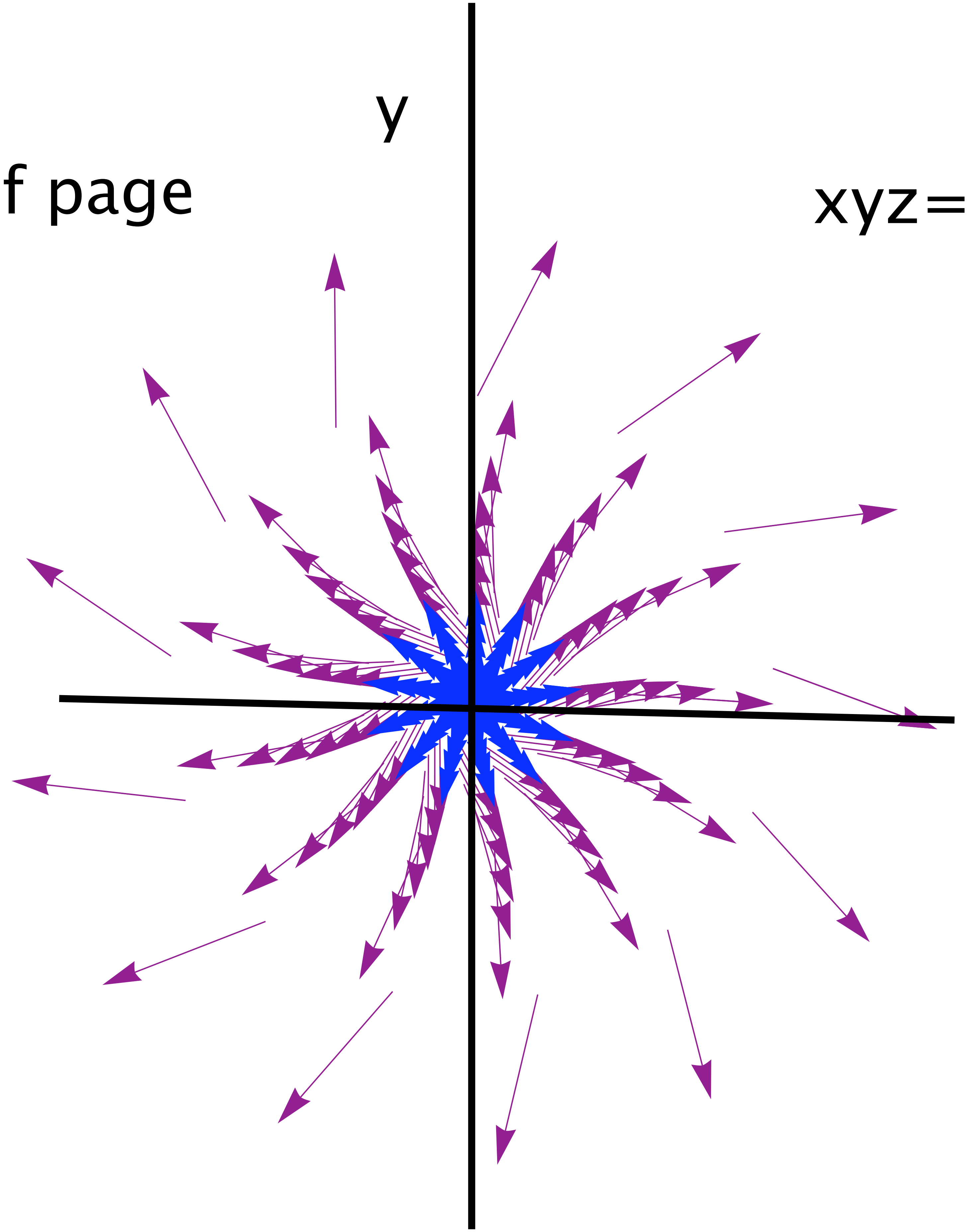
3D holography of the Z spin, lab frame

(q_x, q_y, q_z)



2% of Z's are polarized pure state spinning as shown beam−axis out of page

xyz= lab



2% of
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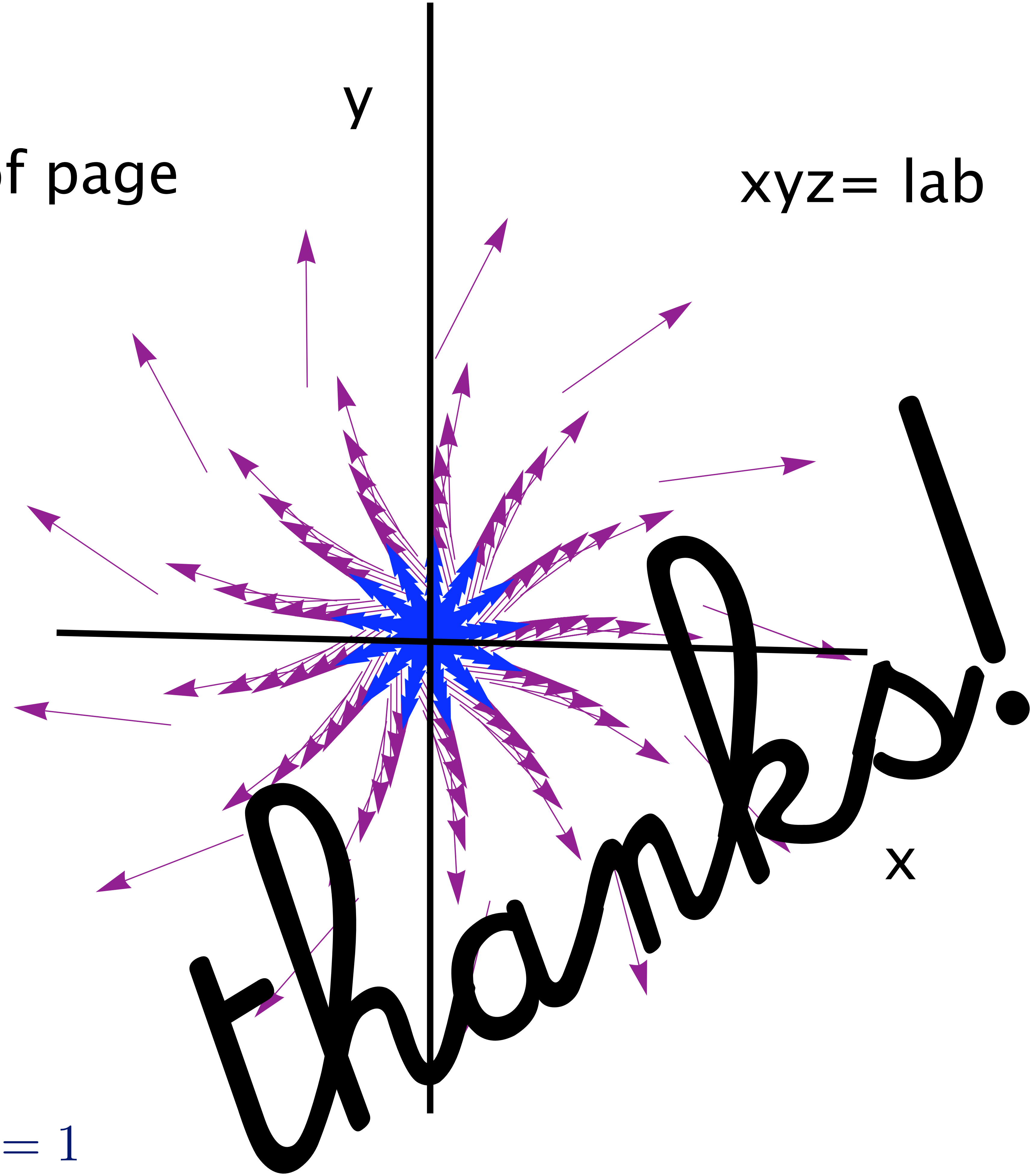
spinning
as shown

arrows are $\hat{s}_\perp$, $\hat{s} \cdot \hat{s} = 1$

beam−axis out of page

xyz= lab



2% of
Z's are
polarized
pure
state
spinning
as shown

arrows are $\hat{s}_{\perp}$, $\hat{s} \cdot \hat{s} = 1$

entanglement entropy

$$S = -\mathrm{tr}(\rho_x \log \rho_x)$$

$S = 0 \Rightarrow \rho = |\psi\rangle\langle\psi|$

pure state

$S = \log(N) \Rightarrow \rho = \frac{1}{N}{}_{N\times N}$

$U(N)$ invariant

unique extensive $S(\rho_{AB}) \ni S(\rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B)$