

# Multi parton interactions in UP collisions

B.Blok (Technion)

*University of Washington at Seattle February 2017*



# Introduction

The conventional dijet production in high energy process:

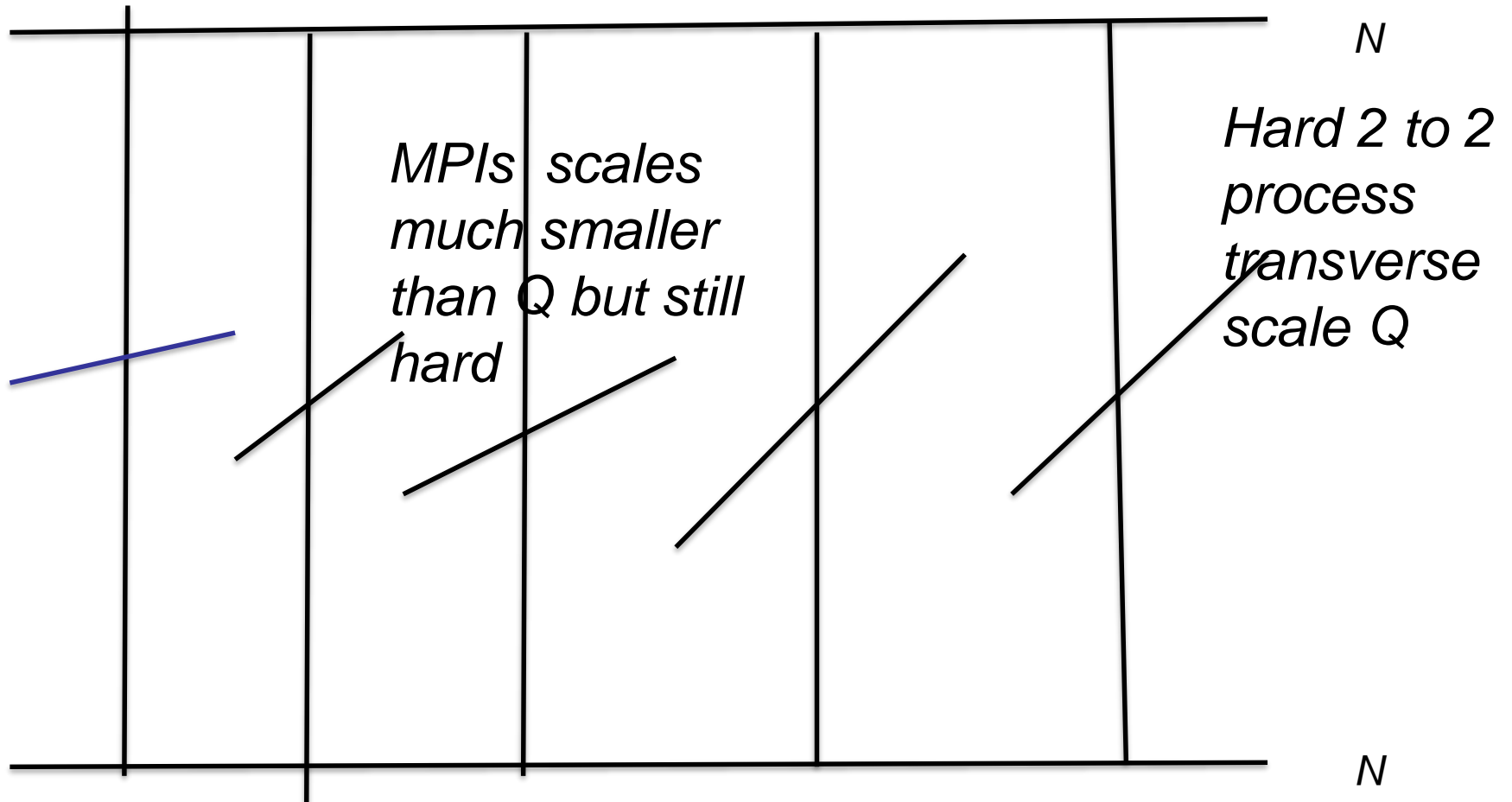
$$\frac{d\sigma^{(2\rightarrow 2)}}{d\hat{t}} \propto \frac{\alpha_s^2}{Q^4}.$$



2

*Realistic process however-more than 1 hard interactions-MPI. Can occur either accompanying hard process –form 1-2 (Minimal Bias) to 2-3 (Underlying way (UE)-to large number-high multiplicity events.-or on its own-Double Parton Scattering (DPS)-two hard processes both scales are hard-currently 40-20 GeV.*

# *MPI picture of UE in hard processes at high energies*



# *DPS-Double parton scattering-4 jet production*

*Basic evidence for MPI: a) UE b) DPS c) close on direct observation  $W+W+d$  heavy flavor production in forward kinematics (LHCb)*

*The description of MPI demands new theoretical concepts beyond 2 to 2 processes: Generalised parton distributions-GPD, ladder splitting. MPI are important for precision measurements at hadronic colliders at high energies and can be a source of interesting correlations at high energies.*

## Why we need to study MPI in pA and UPC?

1. There are more MPI in pA than in pp-A  $1/3$  since all nucleons at a given impact parameter interact with a hard probe together.

2. The full description of MPI includes 4 to 4 and 3 to 4 processes. The pp Collisions combine these two processes in a particular way (see below)

UPC with direct photons can permit us to disentangle 3 to 4 and 4 to 4 and see 4 to 4 directly. MPI in pA give us cross section in terms of 4 to 4 mechanism.

So combining pp,pA and UPC we can disentangle different aspects of pQCD in MPI  
And look separately on different processes in central kinematics.

3. Most of multiplicity may come from MPI

Let us briefly review MPI in pp collisions.

The most interesting property of MPI-they can be seen in the back-to-back kinematics.

Indeed, **they are not the leading twist process.**

The **2 to 4** processes give a contribution to cross section

$$\frac{d\sigma^{(2\rightarrow 4)}}{d\hat{t}_1 d\hat{t}_2} \propto \frac{\alpha_s^4}{Q^6}$$

On the other hand

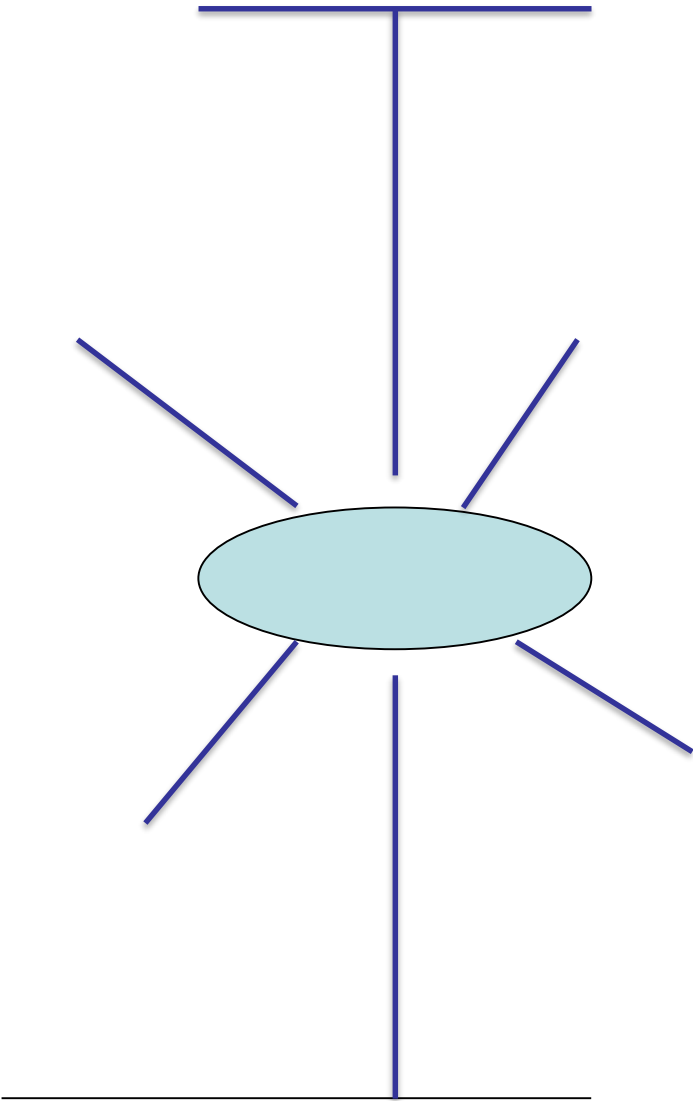
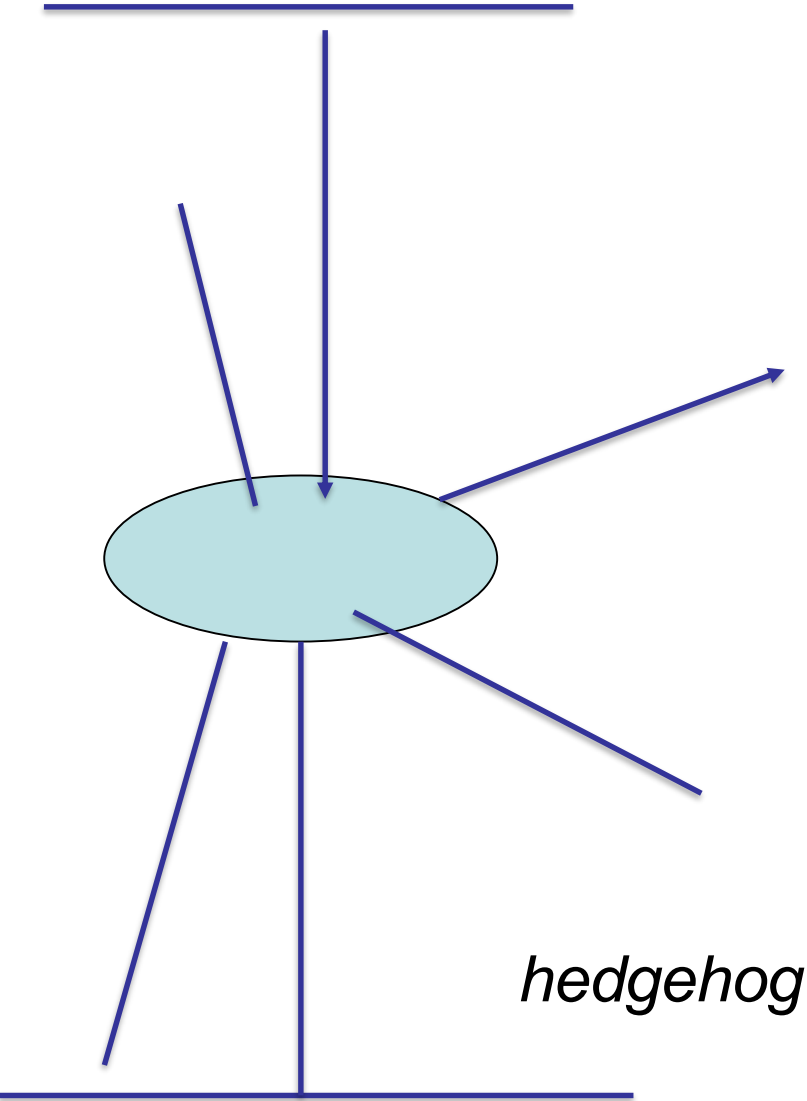
$$\frac{d\sigma^{(4\rightarrow 4)}}{d\hat{t}_1 d\hat{t}_2} \propto R^{-2} \cdot \left( \frac{\alpha_s^2}{Q^4} \right)^2 \propto \frac{\alpha_s^4}{R^2 Q^8}$$

i.e. they can be seen as a **higher twist process.**

the scale R is given by

$R^2 = 1/\langle \Delta^2 \rangle$  the characteristic distance between  
the two partons in the hadron wave function.

*Back to back*





*Recently new pQCD formalism for MPI was developed:*

*B.Blok, Yu.Dokshitzer, L. Frankfurt ,M. Strikman, Phys.Rev. D83 (2011) 071501  
Eur.Phys.J. C72 (2012) 1963, Eur.Phys.J. C74 (2014) 2926*

*Numerical implementation:B.  
Blok, P. Gunnellini Eur.Phys.J. C75 (2015) no.6, 282Eur.Phys.J. C76 (2016) no.4, 202*

*B.Blok, M. Strikman Eur.Phys.J. C76 (2016) no.12, 694 soft correlations*

*There are now experimental measurements by ATLAS CMS LHCb and D0 at Tevatron  
.at different regions of the phase space.*

*We shall talk here: mean field approach, impact parameter distributions, 3 to 4 versus 4 to 4 mechanism,  
MPI in pA collisions, MPI in direct photon-nuclei collisions, first steps in numerical implementation.*

*New basic ideas: 1) new universal objects 2 parton GPD  $D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta})$  :*

*2) DGLAP/parton ladder splitting mechanism-3 to 4, numerically gives  
the same order contribution as parton model 4 to 4 mechanism.*

*3) Double collinearly enhanced classification*

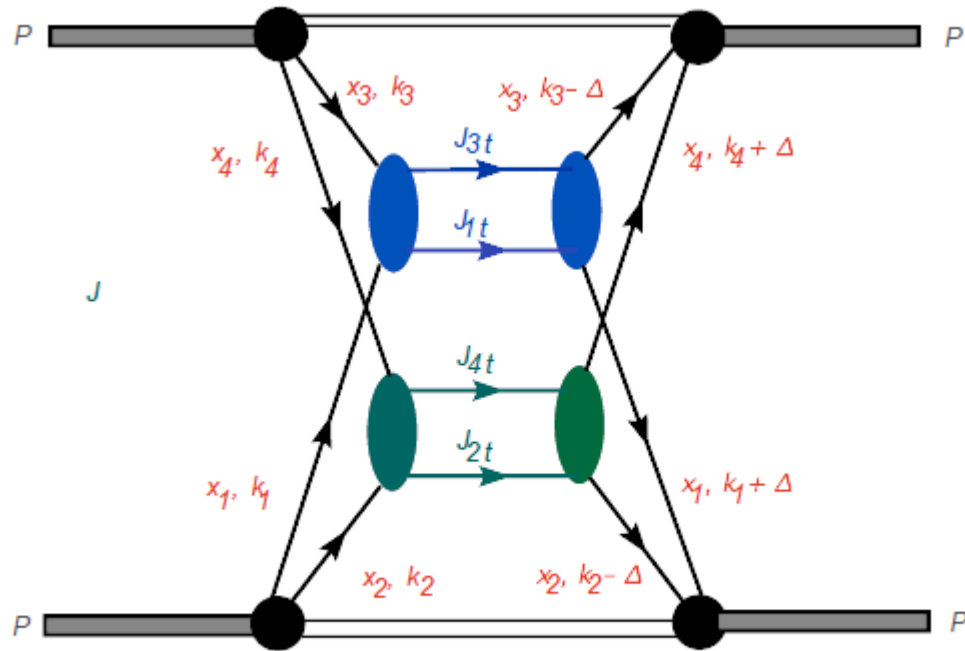
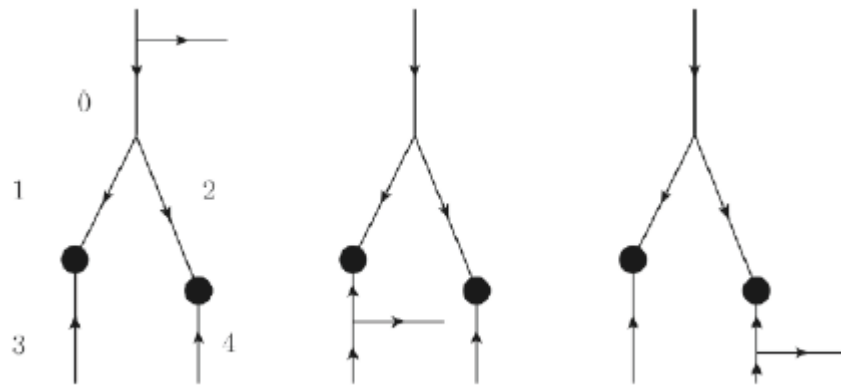


Fig. 1: Kinematics of double hard collision - momenta of t.

$$d\sigma^{(4\rightarrow 4)} \propto \frac{\alpha_s^2}{\delta_{13}^2 \delta_{24}^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma,$$

$$\delta_{13}^2 \ll Q^2, \quad \delta_{24}^2 \ll Q^2;$$

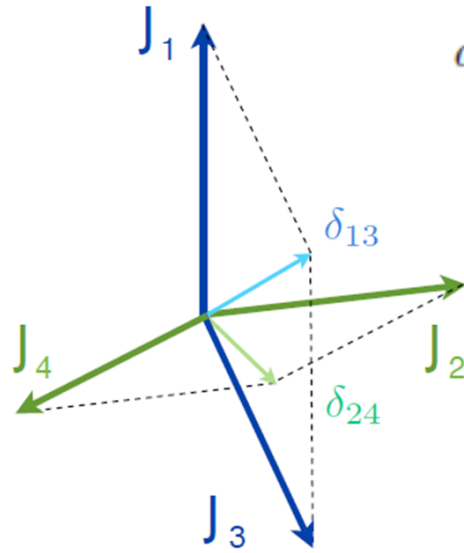
$$d\sigma^{(3\rightarrow 4)} \propto \frac{\alpha_s^2}{\delta'^2 \delta^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma,$$

$$\delta'^2 \ll \delta^2 \ll Q^2, \quad \delta^2 = \delta_{13}^2 \simeq \delta_{24}^2.$$

*In back-to back kinematics they are double collinear enhanced (and 2 to 4 are not)*

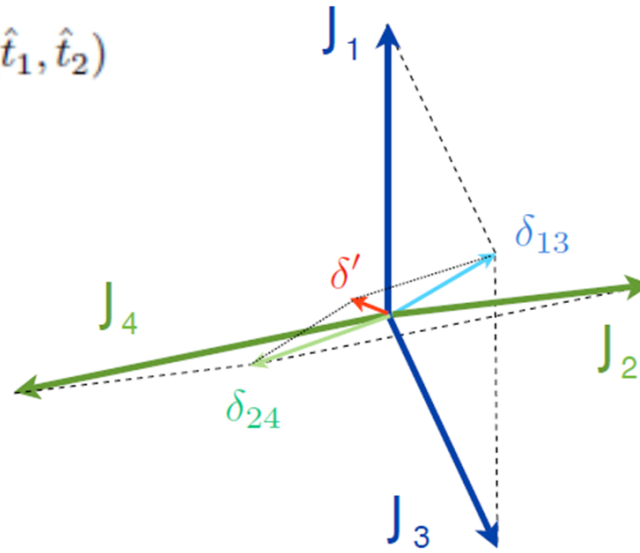
back-to-back kinematics

$$\delta_{13}^2, \delta_{24}^2 \ll J_{i\perp}^2$$



$$d\sigma^{(4\rightarrow 4)} \propto \frac{\alpha_s^2}{\delta_{13}^2 \delta_{24}^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

$$\delta'^2 \ll \delta_{13}^2 \simeq \delta_{24}^2 \ll J_{i\perp}^2$$



$$d\sigma^{(3\rightarrow 4)} \propto \frac{\alpha_s^2}{\delta'^2 \delta^2} d^2 j_{3\perp} d^2 j_{4\perp} \cdot d\Sigma$$

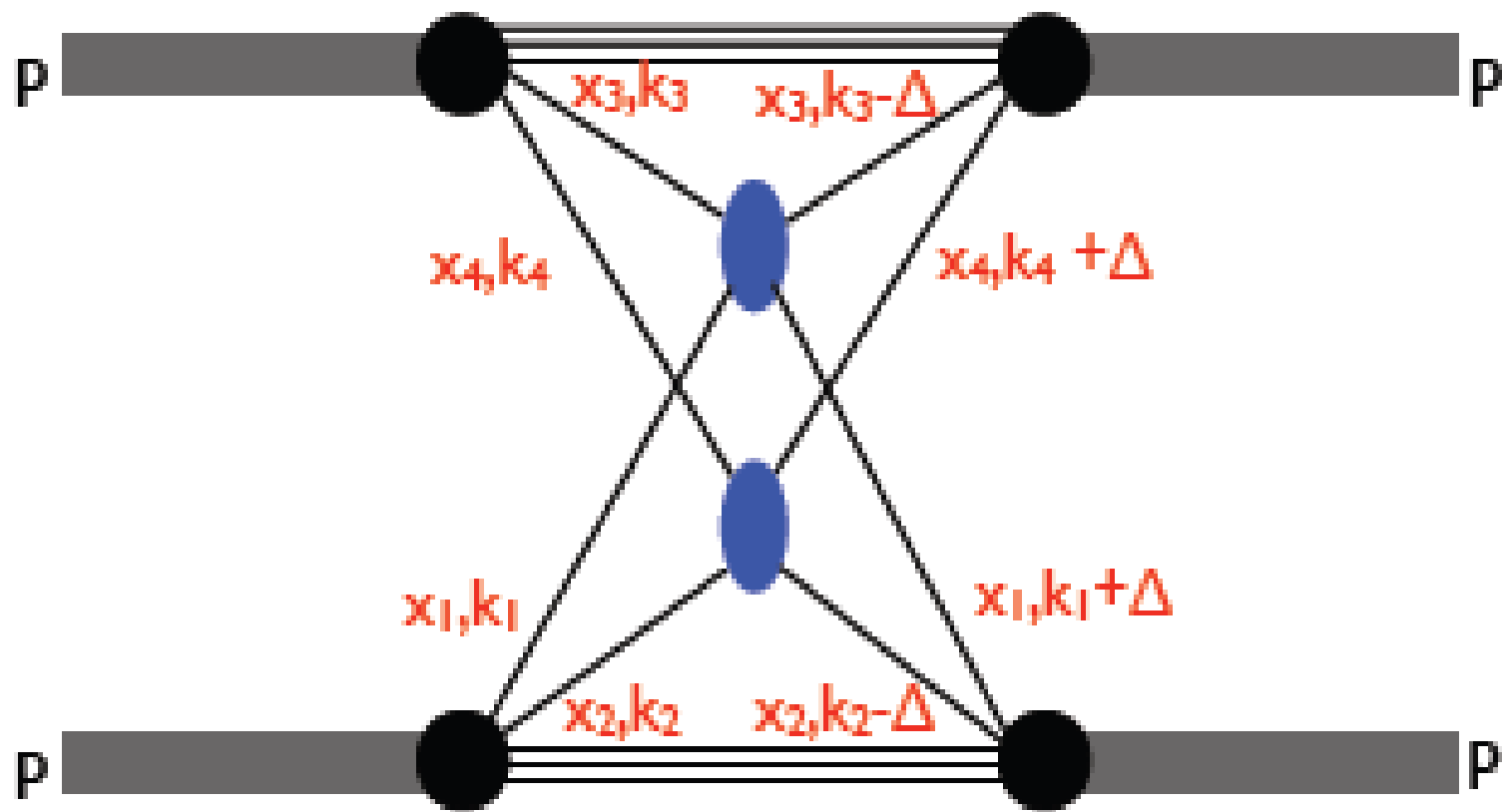
# The four jet cross-section in the parton model.

The four jet cross-section can be directly calculated in **momentum space** and is given by the formula:

$$\begin{aligned} \sigma_4(x_1, x_2, x_3, x_4) &= \int \frac{d^2\vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) \times D_b(x_3, x_4, p_1^2, p_2^2, -\vec{\Delta}) \\ &\times \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} d\hat{t}_1 d\hat{t}_2. \end{aligned} \quad (2)$$

*Experimentalists often denote:*

$$\sigma_4 = \sigma_1 \sigma_2 / \pi R_{\text{int}}^2,$$



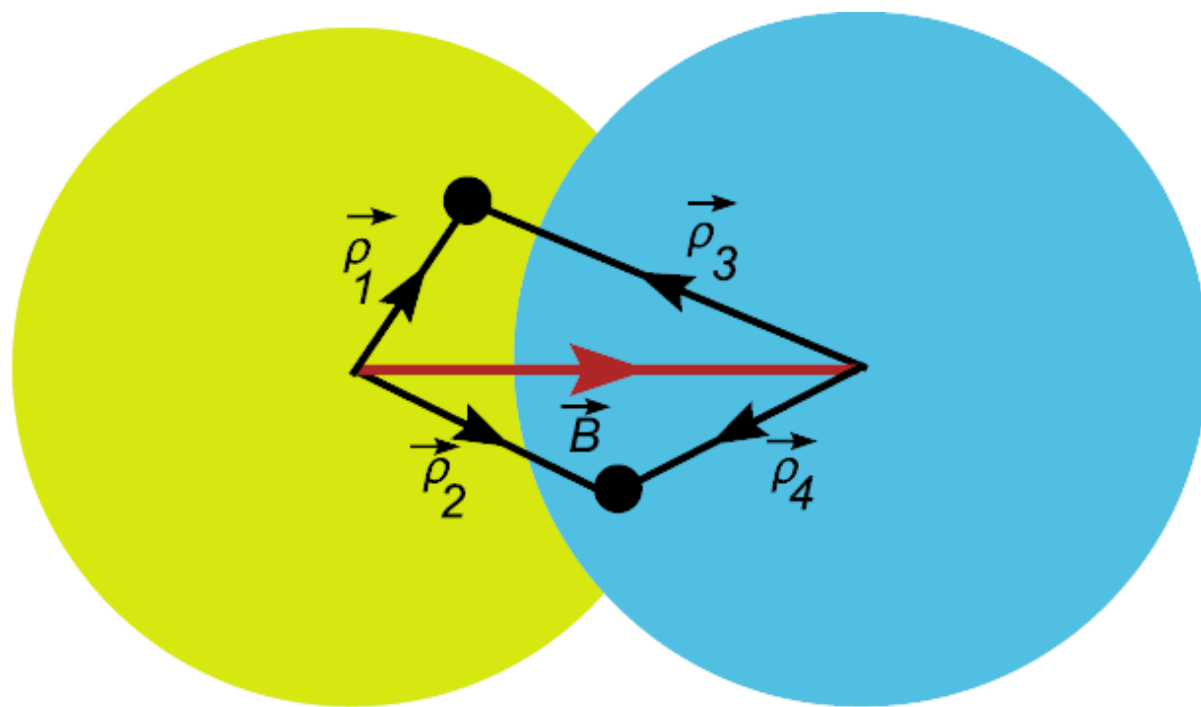


FIG. 2 (color online). Geometry of two hard collisions in impact parameter picture.

It follows from the discussion above that the area can be written explicitly in terms of these new **two particle GPDs** as

$$\frac{1}{\pi R_{\text{int}}^2} = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} \frac{D(x_1, x_2, -\vec{\Delta}) D(x_3, x_4, \vec{\Delta})}{D(x_1) D(x_2) D(x_3) D(x_4)},$$

*This formula is valid for inclusive dijet production. When the momentum fraction are different, the exclusive production DDT formula can be easily obtained. This formula expresses the interaction area in the model independent way as the **single integral over the transverse momenta**.*



The new **GPDs** can be explicitly expressed through the **light cone wave functions** of the hadron as

$$\begin{aligned}
 D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) &= \sum_{n=3}^{\infty} \int \frac{d^2 k_1}{(2\pi)^2} \frac{d^2 k_2}{(2\pi)^2} \theta(p_1^2 - k_1^2) \\
 &\times \theta(p_2^2 - k_2^2) \int \prod_{i \neq 1,2} \frac{d^2 k_i}{(2\pi)^2} \int_0^1 \prod_{i \neq 1,2} dx_i \\
 &\times \psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, \dots, \vec{k}_i, x_i \dots) \\
 &\times \psi_n^+(x_1, \vec{k}_1 + \vec{\Delta}, x_2, \vec{k}_2 - \vec{\Delta}, x_3, \vec{k}_3, \dots) \\
 &\times (2\pi)^3 \delta\left(\sum_{i=1}^{i=n} x_i - 1\right) \delta\left(\sum_{i=1}^{i=n} \vec{k}_i\right). \quad (!)
 \end{aligned}$$

*Here psi are the light cone wave functions of the nucleon in the initial and final states.*

## The approximation of independent particles.

Suppose the multiparton wave function factorise, i.e. we neglect possible interparton correlations and recoil effects. Then it's straightforward to see that the two particle GPDs **factorise** and acquire a form:

$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta})G(x_2, p_2^2, \vec{\Delta}),$$

The one-particle GPD-s  $G$  are conventionally written in the dipole form:

$$G_N(x, Q^2, \vec{\Delta}) = G_N(x, Q^2)F_{2g}(\Delta)$$

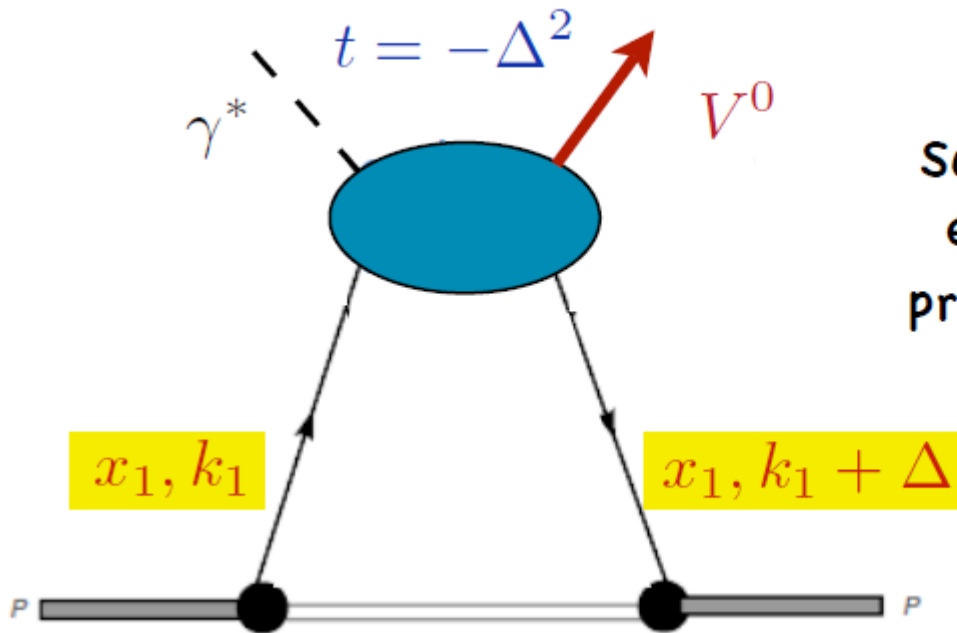
$G$  - the usual 1-parton distribution (determining DIS structure functions)

$F$  - the two-gluon form factor of the nucleon

the dipole fit :

$$F_{2g}(\Delta) \simeq \frac{1}{(1 + \Delta^2/m_g^2)^2} \quad m_g^2(x \sim 0.03, Q^2 \sim 3\text{GeV}^2) \simeq 1.1\text{GeV}^2$$

# G P D



Such an amplitude describes  
exclusive photo-(/electro-)  
production of **vector mesons**  
at HERA !

$$\frac{1}{\pi R_{\text{int}}^2} = \int \frac{d^2 \Delta}{(2\pi)^2} F_{2g}^4(\Delta) = \frac{m_g^2}{28\pi}.$$

$$R_{\text{int}}^2 = 7/2 r_g^2, \quad r_g^2/4 = dF_{2g}(t)/dt_{t=0}.$$

Let us note that this result coincides with the one obtained in a geometric picture (Frankfurt, Strikman and Weiss 2003) However the latter computation involved a **complicated 6 dimensional integral** that potentially could lead to large numerical uncertainties

The dependence of  $r_g^2$  on  $Q^2$  and  $x$  is given by the approximate formula that takes into account the **DGLAP evolution**:

$$\langle \rho^2 \rangle(x, Q^2) = \langle \rho^2 \rangle(x, Q_0^2) \left( 1 + A \ln \frac{Q^2}{Q_0^2} \right)^{-a},$$

where

$$\langle \rho^2 \rangle = \frac{8}{m_g^2}.$$

$$Q_0^2 = 3 \text{ GeV}^2, \quad A = 1.5, \quad a = 0.0090 \ln \frac{1}{x}.$$

The similar analysis for quark sea leads to slightly bigger transverse area (Strikman and Weiss 2009). Recoil may be important for large  $x_i$  but also leads to smaller total cross section, i.e. to larger  $R_{int}$

Then we **see the problem: the approximation of independent particles leads to the cross section two times smaller than the experimental one** (Frankfurt, Strikman and Weiss 2004),

The experimental result is **15 mb**, while the use of the electromagnetic radius of the nucleon leads to this area being 60 mb while we obtain in independent particle approximation **34 mb**

Even more naïve **way-take**  $\sigma_{eff} = 1/\pi R^2$

(most MC generators do)

# Perturbative QCD and differential cross sections

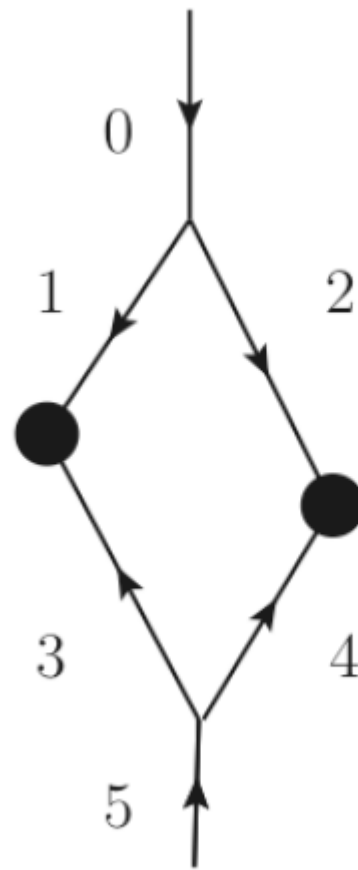
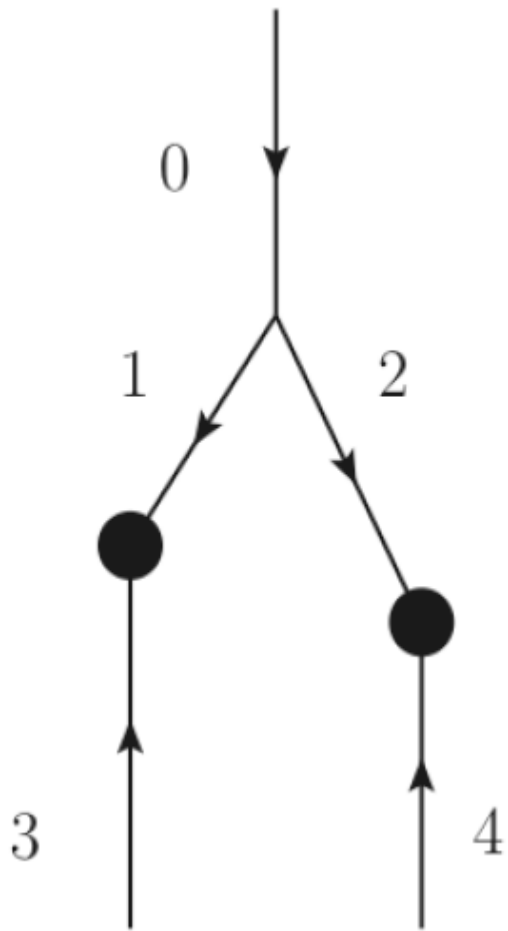
Two basic ideas (relative to conventional one dijet processes-2 to2 in our notations):

1. Double collinear enhancement in total cross sections-i.e. double pole enhancement in differential two dijet cross sections.
2. new topologies-in addition to conventional pQCD bremsstrahlung-parton/ladder splitting .

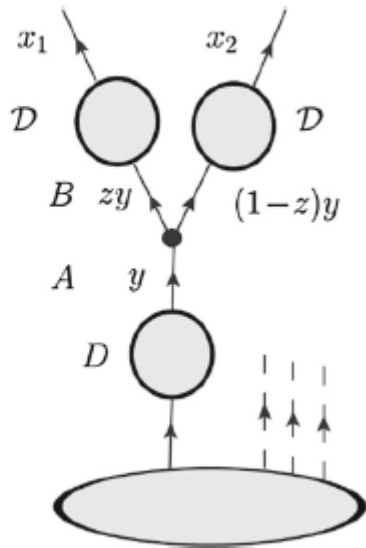
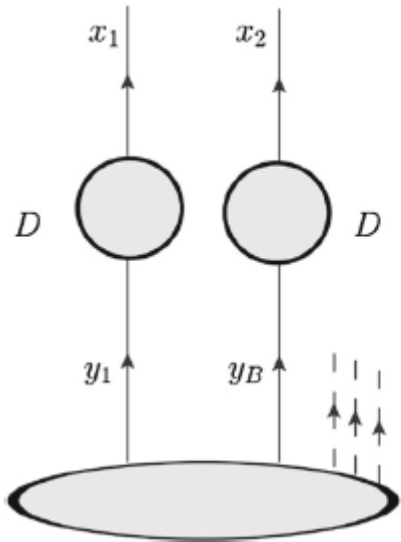
a) *4 to 4*

b) *3 to 4*

*But no 2 to 4*



$$\begin{aligned}
& [1]D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) \\
&= \sum_{a', b', c'} \int_{Q_{\min}^2}^{\min(q_1^2, q_2^2)} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \\
&\quad \times \int \frac{dy}{y^2} G_a^{a'}(y; k^2, Q_0^2) \\
&\quad \times \int \frac{dz}{z(1-z)} P_{a'}^{b'[c']}(z) G_{b'}^b\left(\frac{x_1}{zy}; q_1^2, k^2\right) \\
&\quad \times G_{c'}^c\left(\frac{x_2}{(1-z)y}; q_2^2, k^2\right).
\end{aligned}$$



$$\begin{aligned}
& [2]D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) \\
&= S_b(q_1^2, Q_{\min}^2) S_c(q_2^2, Q_{\min}^2) [2]D_a^{b,c}(x_1, x_2; Q_0^2, Q_0^2; \vec{\Delta}) \\
&\quad + \sum_{b'} \int_{Q_{\min}^2}^{q_1^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} S_b(q_1^2, k^2) \\
&\quad \times \int \frac{dz}{z} P_{b'}^b(z) [2]D_a^{b',c}\left(\frac{x_1}{z}, x_2; k^2, q_2^2; \vec{\Delta}\right) \\
&\quad + \sum_{c'} \int_{Q_{\min}^2}^{q_2^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} S_c(q_2^2, k^2) \\
&\quad \times \int \frac{dz}{z} P_{c'}^c(z) [2]D_a^{b,c'}\left(x_1, \frac{x_2}{z}; q_1^2, k^2; \vec{\Delta}\right). \quad (16)
\end{aligned}$$

$$\begin{aligned}
D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) &= [2]D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) \\
&\quad + [1]D_a^{b,c}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta})
\end{aligned}$$



$$\frac{1}{\sigma_{eff}} \equiv \int \frac{d^2\vec{\Delta}}{(2\pi)^2} [ {}_{[2]}G_2(x_1, x_3, Q_1^2, Q_2^2; \vec{\Delta}) {}_{[2]}G_2(x_2, x_4, Q_1^2, Q_2^2; -\vec{\Delta})$$

$$+ {}_{[1]}G_2(x_1, x_3, Q_1^2, Q_2^2; \vec{\Delta}) {}_{[2]}G_2(x_2, x_4, Q_1^2, Q_2^2; -\vec{\Delta})$$

$$+ {}_{[1]}G_2(x_2, x_4, Q_1^2, Q_2^2; \vec{\Delta}) {}_{[2]}G_2(x_1, x_3, Q_1^2, Q_2^2; -\vec{\Delta}) ].$$

*2G2 and 1G2 are two parts of GPD ,calculated in two different ways. 2G2-in mean field approach, using GPD1 from charmonium photoproduction at HERA*

$${}_{[2]}GPD_2(x_1, x_3, Q_1^2, Q_2^2, \Delta) = D_q(x_1, Q_1)D_g(x_3, Q_2)F_{2q}(\Delta, x_1)F_{2g}(\Delta, x_3),$$

$$GPD_{q,g}(x, Q^2, \Delta) = D_{q,g}(x, Q)F_{2g,2q}(\Delta, x).$$

*We use parametrisation due to Frankfurt,Strikman,Weiss (2011)*

*1G2 is calculated solving evolution equation for GPD*

*The final answer for effective cross section is convenient to represent as*

$$\sigma_{eff} = \frac{\sigma_{eff}^{(0)}}{1 + R},$$

*Here  $\sigma_{eff}^{(0)}$  is the 4 to 4 cross section in mean field approximation while the function R corresponds to contribution due to 3 to 4 mechanism, and is calculated analytically.*

*Note: only one unknown paramter-Q0, separating soft and hard scales, so approach is practically model independent.*

Consequently, in the differential distributions we have **3** •  
 terms, corresponding to **4 to 4** and **3 to 4** (long and  
 short):

$$\pi^2 \frac{d\sigma^{(4 \rightarrow 4)}}{d^2\delta_{13} d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1 d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \left\{ [2]D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \times [2]D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \right. \\ \left. \times S_1(Q^2, \delta_{13}^2) S_3(Q^2, \delta_{13}^2) \times S_2(Q^2, \delta_{24}^2) S_4(Q^2, \delta_{24}^2) \right\}.$$

$$\pi^2 \frac{d\sigma_1^{(3 \rightarrow 4)}}{d^2\delta_{13} d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{d\hat{t}_1 d\hat{t}_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \left\{ [1]D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \cdot [2]D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \right. \\ \left. \times S_1(Q^2, \delta_{13}^2) S_3(Q^2, \delta_{13}^2) \cdot S_2(Q^2, \delta_{24}^2) S_4(Q^2, \delta_{24}^2) \right\}.$$

here S are the corresponding **Sudakov formfactors** . We •  
 see that **4 to 4** and long split **3 to 4** are expressed  
 through convolution of 2GPD of two colliding hadrons –  
 the expressions look  
 quite similar to DDT formula •

# The total cross sections

$$\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1 d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} \times \left\{ \frac{1}{S_4} + \frac{1}{S_3} \right\}.$$

$$\frac{1}{S_4} = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} [2]D_{h_1}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) [2]D_{h_2}(x_3, x_4; q_1^2, q_2^2; -\vec{\Delta}).$$

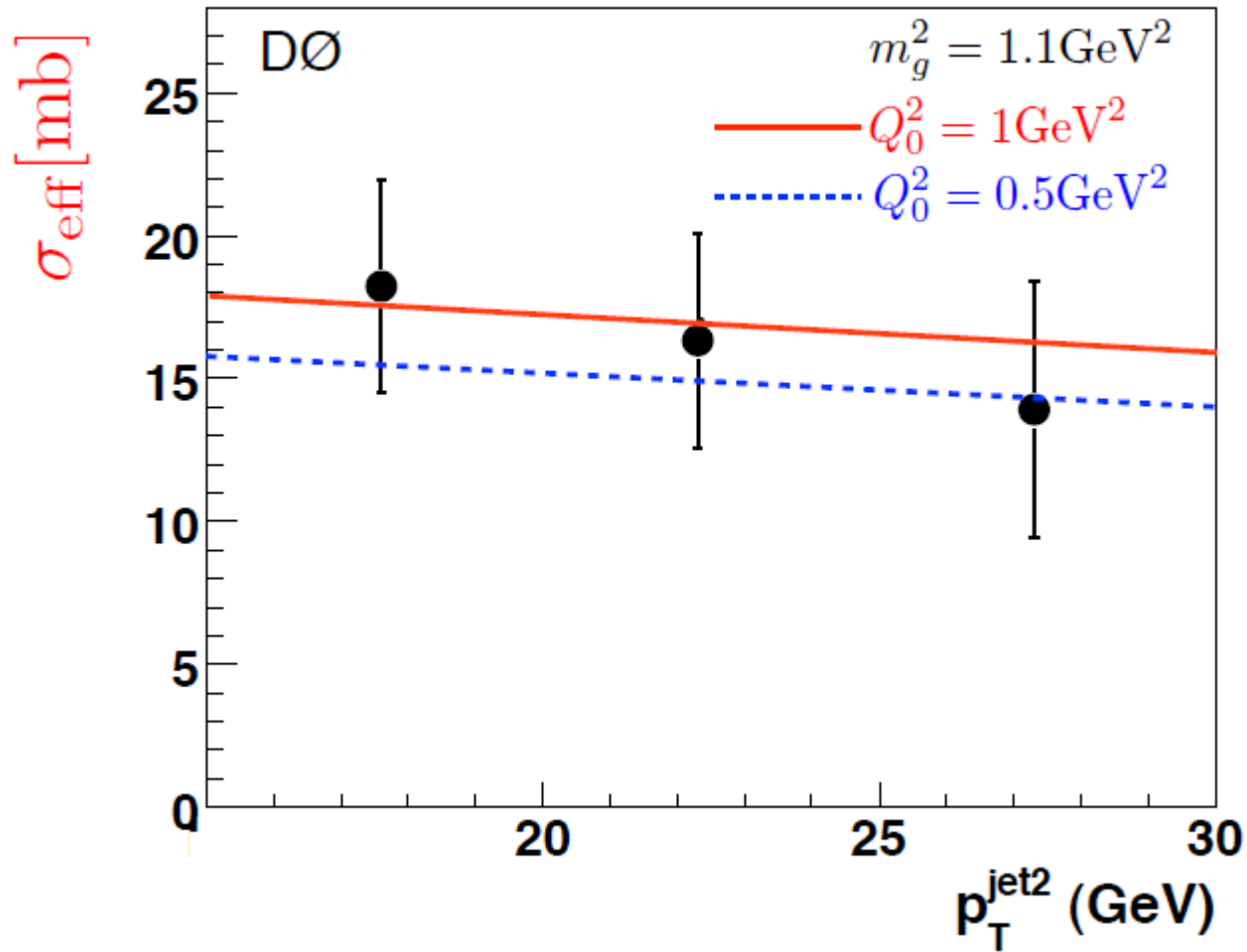
$$\frac{1}{S_3} = \int \frac{d^2\vec{\Delta}}{(2\pi)^2} \left[ [2]D_{h_1}(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) [1]D_{h_2}(x_3, x_4; q_1^2, q_2^2) + [1]D_{h_1}(x_1, x_2; q_1^2, q_2^2) [2]D_{h_2}(x_3, x_4; q_1^2, q_2^2; \vec{\Delta}) \right].$$

## *Analytical estimate*

$$\frac{D^{bc}(x_1, x_2; 0)}{G^b(x_1)G^c(x_2)} - 1 \simeq \frac{N_c}{2(n_q C_F + n_g N_c)}.$$

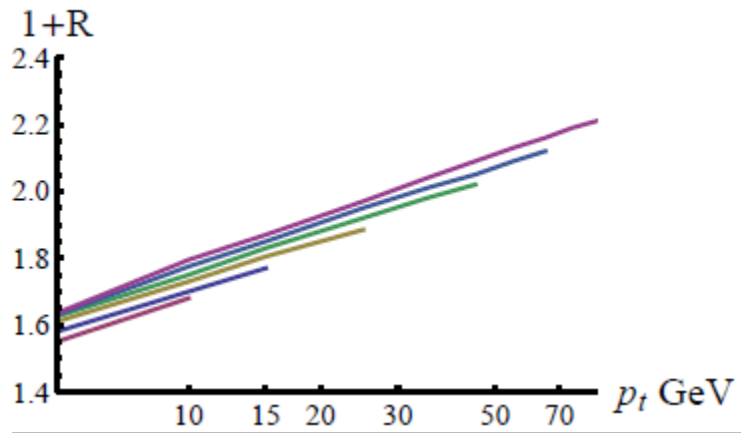
*Decreases with  $x_i$ , of order 2 increase of cross sections*

*D0 physics (slightly larger energies )*

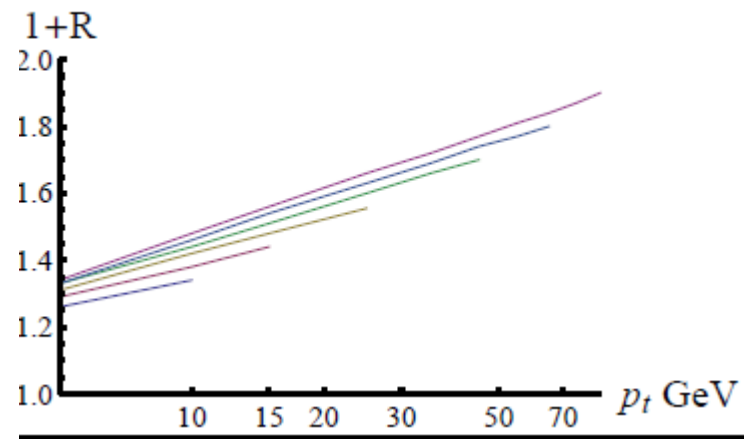


*The dependence on transverse momentum of a gluonic dijet for different Photon momenta.*

$Q_0^2 = 0.5 \text{ GeV}^2$



$Q_0^2 = 1 \text{ GeV}^2$



*Good agreement with Tevatron, soft and NLO to be included*

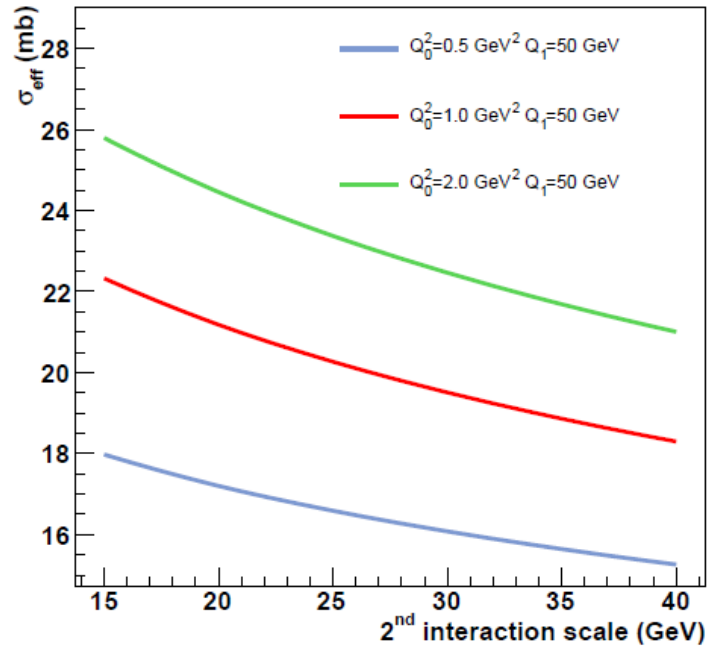


FIG. 10: Values of  $\sigma_{eff}$  as a function of the scale of the 2<sup>nd</sup> interaction for different scales of the first interaction,  $Q_1$ , and different choices of  $Q_0^2$ . The values of the longitudinal momentum fractions correspond to the maximal transverse momentum exchange.

Detailed study of DPS observables-in B. Blok, P. Gunnellini, EJPC 2015,2016  
improves agreement with data both for 4 jet and Wjj processes

*Additional contributions in pp collisions –soft correlations  
Due to nongactorizable initial conditions –B. Blok, M.  
Strikman, Eur. J. of Physics, C76 (2016) 694+  
B. Blok, M. Strikman, hep-ph/1611.03649, submitted to  
PLB*

[Open charm prod](#)



*Shadowing-for heavy nuclei factorization, for pD-B. Blok,  
M. Strikman, Eur. Phys. J. C74(2014) 3038*

*Can one see pQCD 3 to 4 directly? direct photons  
in ultraperipheral collisions*

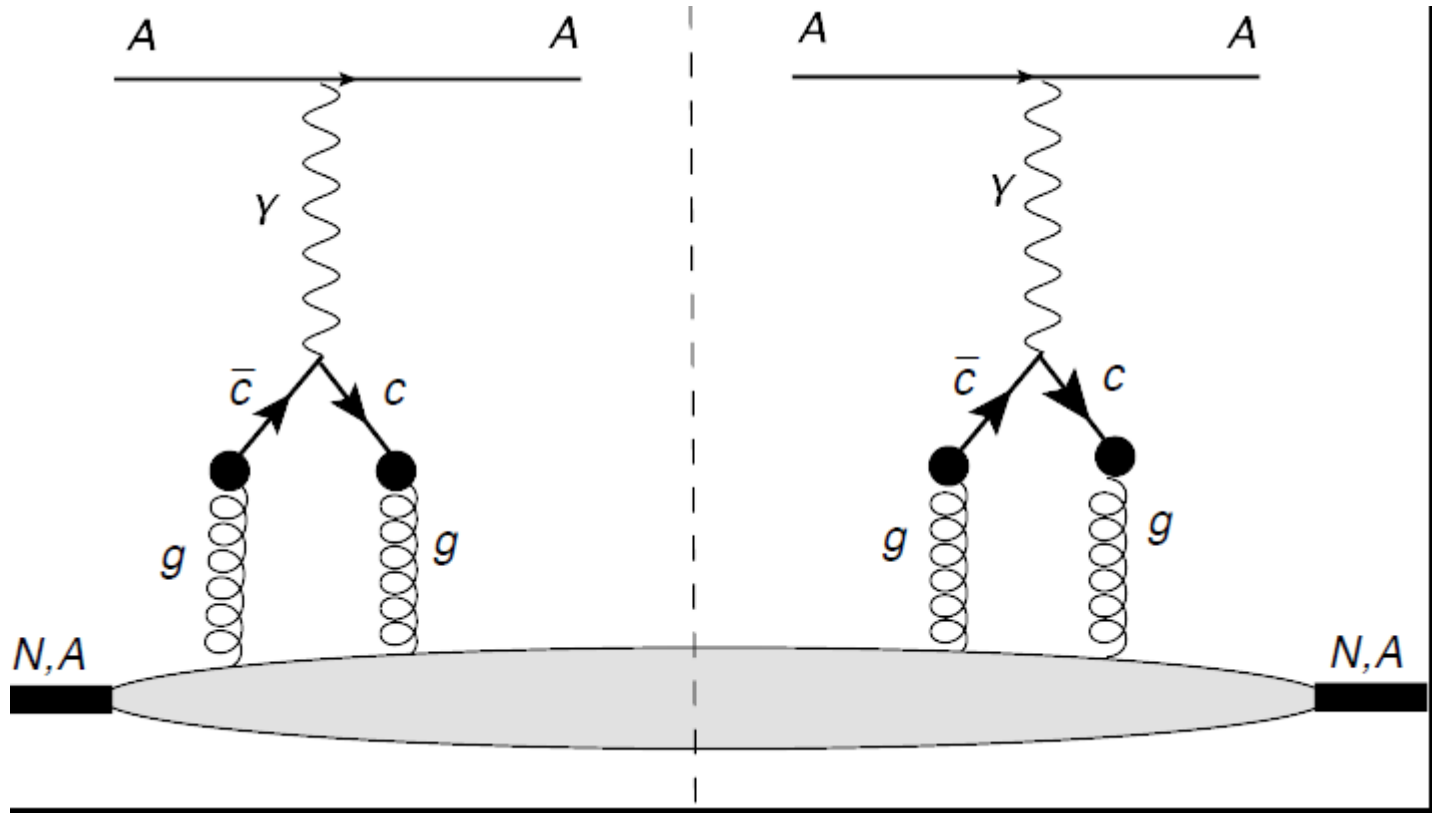
[B. Blok](#) , [M. Strikman](#)

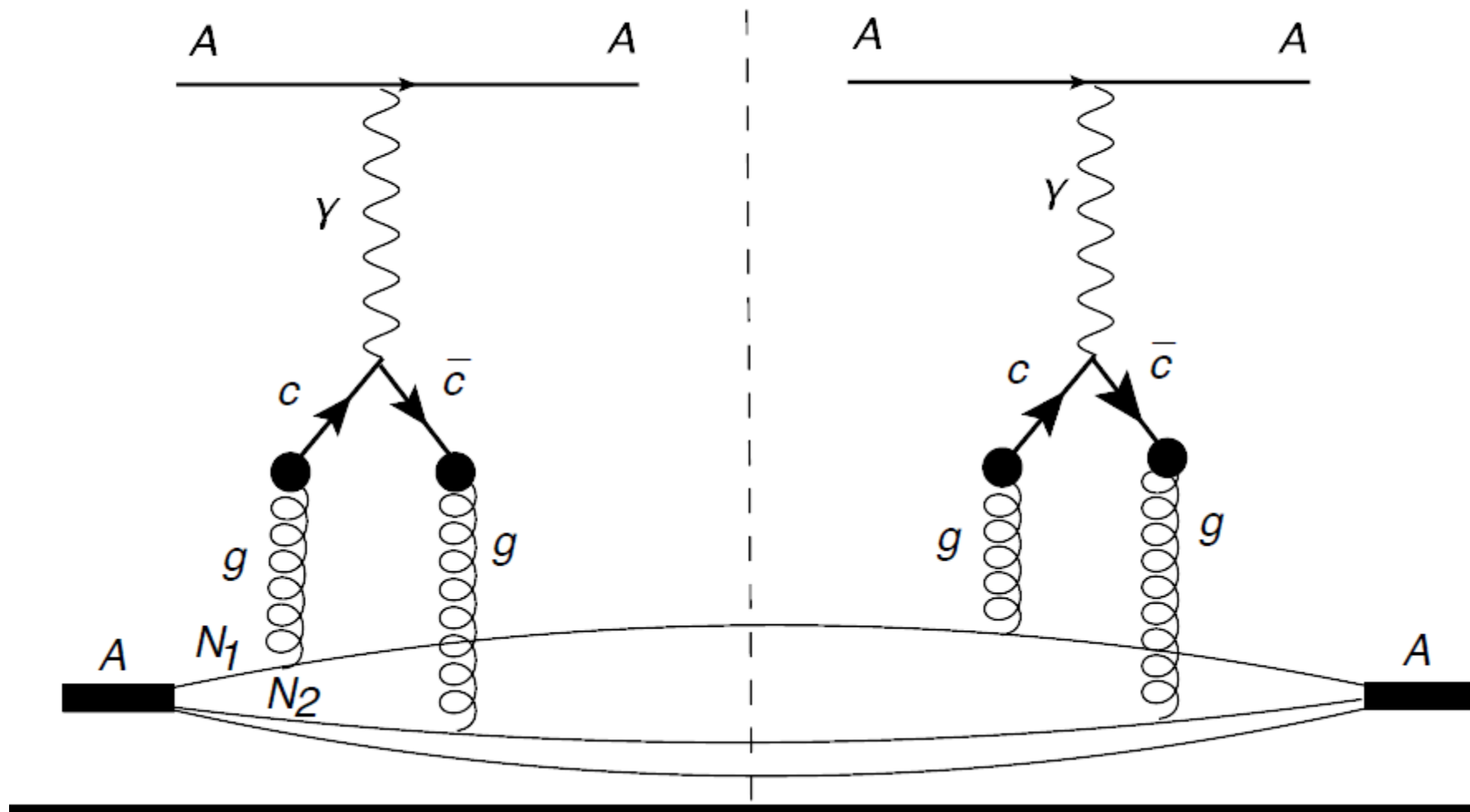
*Published in **Eur.Phys.J. C74 (2014) no.12, 3214***

UPC photon proton-give possibility of clean-free of soft QCD study of MPI.  
Kinematics:  $x_1, x_2$  for charmed jets are large, i.e. we consider processes

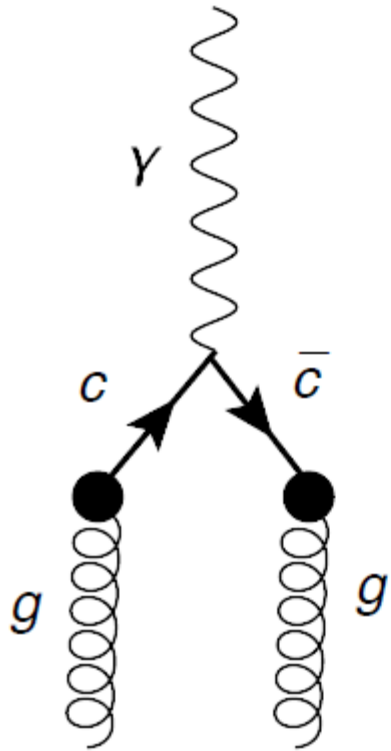
With **direct** photons, and large pseudorapidity gaps in each pair of dijets, 3—4, with  $p_t$  7-15 GeV (can be done due to photons).

This is very different kinematics from the processes with **resolved** photons MPI considered at HERA/LHC by Butterworth, Forshaw, Seymour (JIMMY) which are very similar to  $pp$  and contain all the related uncertainties)

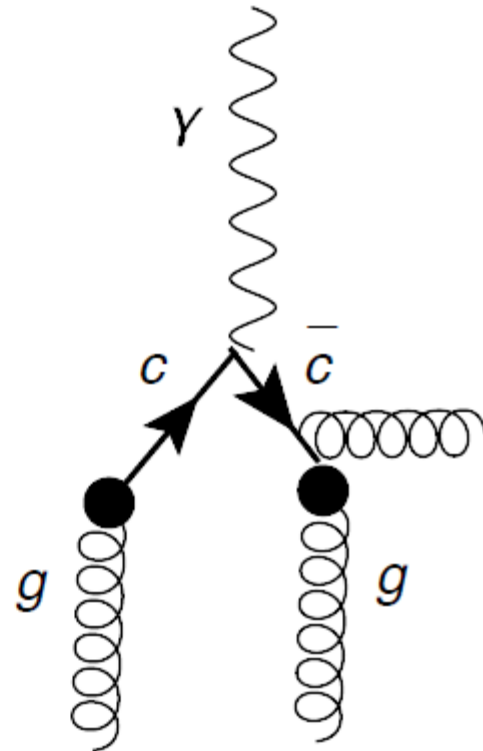




*parton model*



*QCD*



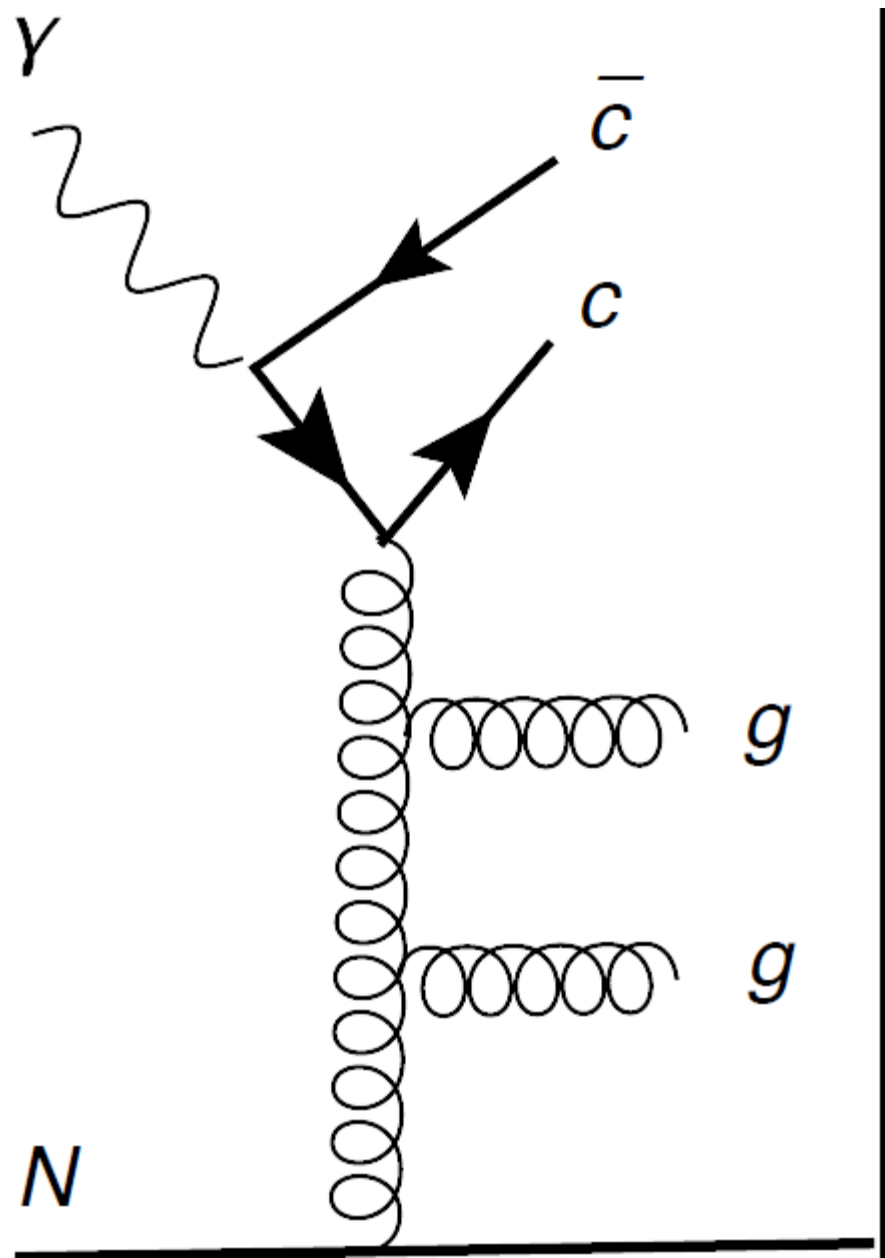
$$\pi^2 \frac{d\sigma_1^{(3 \rightarrow 4)}}{d^2\delta_{13} d^2\delta_{24}} = \frac{d\sigma_{\text{part}}}{dt_1 dt_2} \cdot \frac{\partial}{\partial\delta_{13}^2} \frac{\partial}{\partial\delta_{24}^2} \left\{ [1]D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \cdot [2]D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \right. \\ \left. \times S_1(Q^2, \delta_{13}^2) S_3(Q^2, \delta_{13}^2) \cdot S_2(Q^2, \delta_{24}^2) S_4(Q^2, \delta_{24}^2) \right\}.$$

$$[1]D(x_1, x_2; q_1^2, q_2^2; \vec{\Delta}) = \int_{m_c^2 + \Delta^2}^{\min(q_1^2, q_2^2)} \frac{dk^2}{k^2} \frac{\alpha_{em}}{4\pi} \times \int \frac{dz}{z(1-z)} R(z) G_{q'}^q\left(\frac{x_1}{z}; q_1^2, k^2\right) G_{q'}^q\left(\frac{x_2}{(1-z)}; q_2^2, k^2\right).$$

$$R(z) = z^2 + (1-z)^2$$

$$\sigma_{\gamma p \rightarrow 4j+X} = \int dp_{1t}^2 \int dp_{2t}^2 \int dk \int \frac{dx_1 dx_2 dx_3 dx_4}{x_1 x_2 x_3 x_4} \frac{dN}{dk} D(x_1, x_2, p_{1t}, p_{2t})$$

$$\frac{d\sigma_1}{dt_1} \frac{d\sigma_2}{dt_2} \times \frac{m_g^2}{12\pi^2} G(x_3, p_{1t}^2) G(x_4, p_{2t}^2)$$



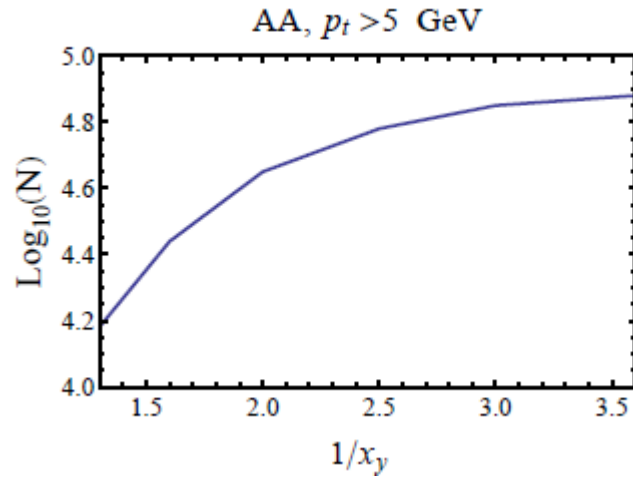
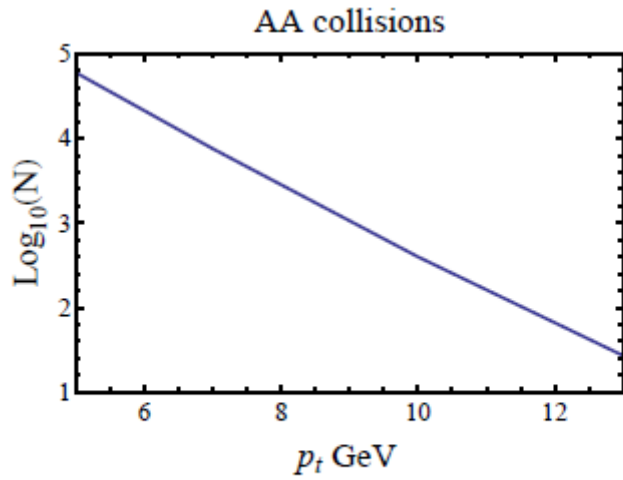
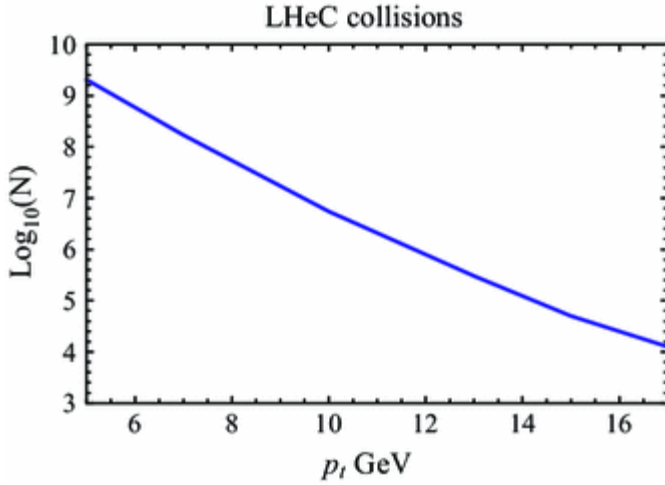


FIG. 5: event rate for MPI in AA collisions

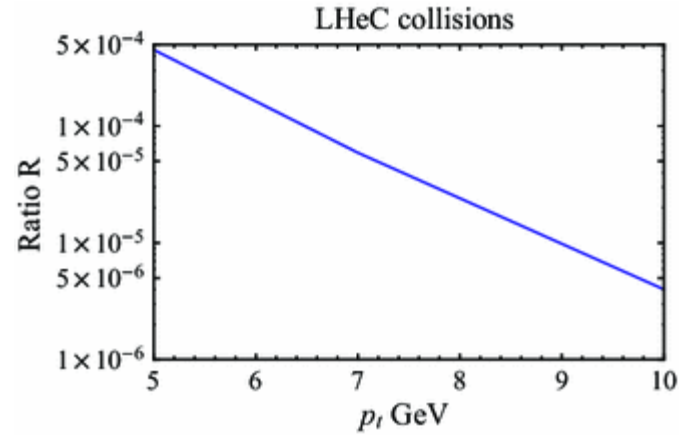
For AA collisions we use: (i) luminosity  $10^{27} \text{cm}^{-2} \text{s}^{-1}$ , (ii) running time  $10^6$  s, and  $\sqrt{s} = 5.6$  TeV. The radius of the nuclei Pb is 6.5 Fermi. The factor gamma is nucleon energy divided by  $2m_p$  equal to  $1.4 \cdot 10^3$ . The exponentially decreasing Macdonald function cuts off the contribution of high photon energy. The total number of events for the  $p_t$  cut 5 GeV is  $5 \cdot 10^4$ , while the ratio of MPI events to the total number of dijet events is rather high—0.037%.



To estimate the MPI event rate at LHeC at  $\sqrt{s} = 1.3$  TeV we used luminosity  $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  and the running time of 1 year ( $10^7$  s). The number of events and their ratio to the total number of dijet event are presented in Figs. 5 and 6. For the cutoff  $p_t > 5$  GeV we get  $2 \times 10^9$  events for the running time  $10^7$  s, corresponding to a year of work of

## *LHeC collider.*

the LHeC collider. The ratio to the number of dijet events with the same cutoffs on  $x$  and  $p_t$  is 0.045 %. Overall it is clear that high statistics studies would be possible at least for the 17 GeV cutoff, where we expect approximately  $10^4$  events.



We also considered the MPI event rate in the similar kinematics at HERA. To estimate the MPI event rate at HERA we use the total integrated luminosity accumulated at HERA of  $1 \text{ fb}^{-1}$ , at the  $\sqrt{s} = 300$  GeV. For the cutoff  $p_t > 5$  GeV we get  $1.2 \times 10^5$  events. The ratio to the number of dijet events with the same cutoffs on  $x$  and  $p_t$  is 0.0125 %. It would be very interesting to reanalyze the HERA data in the direct photon kinematics to look for the MPI charm dijets [45,56].



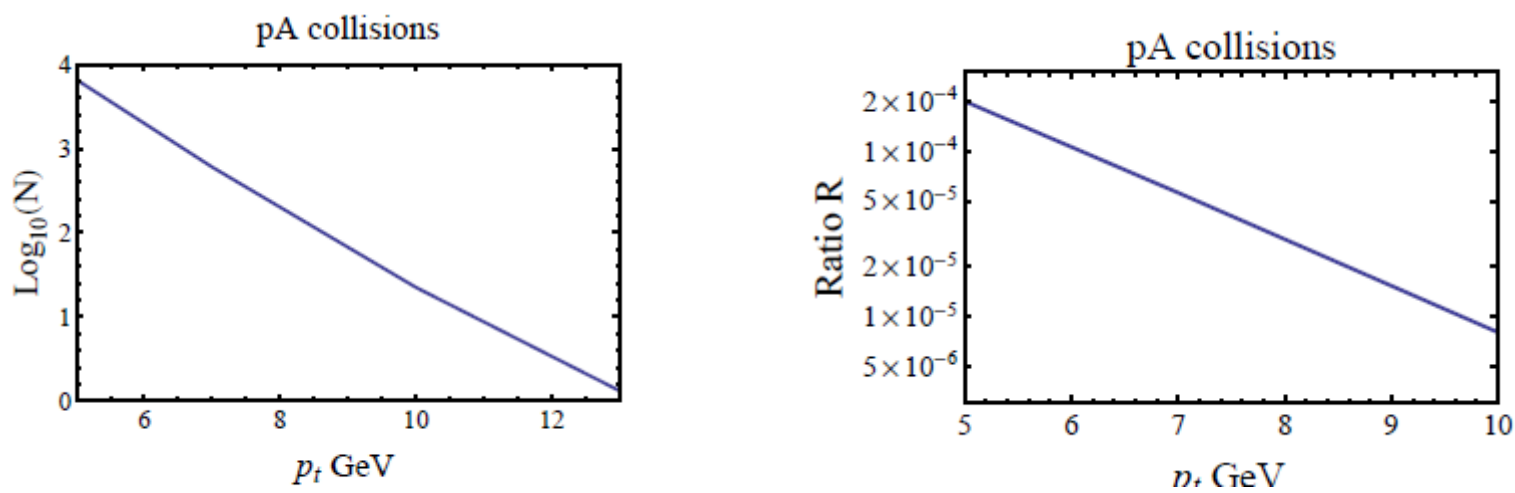


FIG. 8: The event rate for MPI in pA collisions

For pA collisions we use luminosity  $10^{29} \text{cm}^{-2} \text{s}^{-1}$ , running time  $10^6$  s, and the energies  $\sqrt{s} = 10$  TeV. The number of events for cut  $5 \text{GeV}$  is of the order  $6.6 \cdot 10^3$ , and ratio is of order 0.02%, rapidly decreasing as in AA case with increase of  $p_t$ . We have seen above that there is geometric enhancement of a ratio of AA events to dijets, relative to pA case by a factor of the order 2.5, for the same c.m.s. energy per nucleon. The remaining factor 2 difference is connected with difference in energies.

*Conclusions: measuring MPI in UPC will lead to new insights*

- 1. in the nature of Underlying event in pp pA and AA*
- 2. in the nature of pQCD mechanisms for MPI .*

*Resolved photons can be studied along the same line as in pp, pA-can improve qfewwmwnt between experiment and theory as in Hera (Butterworth, Forshaw and Seymour 1996*