NUCLEON STRUCTURE FROM LATTICE QCD

James Zanotti
The University of Adelaide

QCDSF Collaboration

Hadron imaging at Jefferson Lab and at a future EIC
September 25-29, 2017,
Seattle, USA

CSSM/QCDSF/UKQCD Collaborations

- **J. Bickerton** (Adelaide)
- **A. Chambers** (Adelaide)
- R. Horsley (Edinburgh)
- Y. Nakamura (RIKEN, Kobe)
- H. Perlt (Leipzig)
- P. Rakow (Liverpool)
- G. Schierholz (DESY)
- A. Schiller (Leipzig)
- **K. Somfleth** (Adelaide)
- H. Stüben (Hamburg)
- R. Young (Adelaide)

Proton Imaging

Lattice QCD can play important role

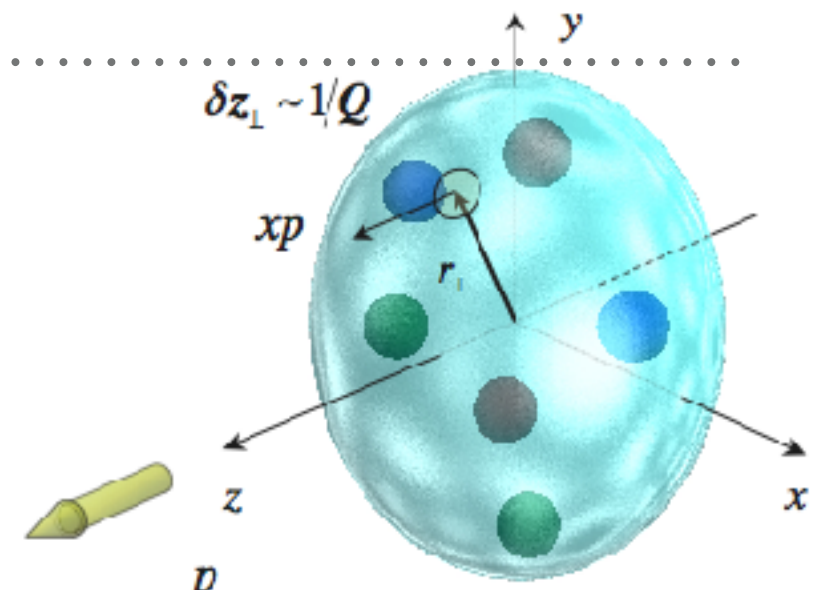
- Obtain a "3-D picture" of the proton
  - electromagnetic form factors
    - small Q^2 - proton size (radius)
    - large Q^2 - charge/magnetisation distributions
  - (generalised) parton distribution functions
    - decomposition of spin

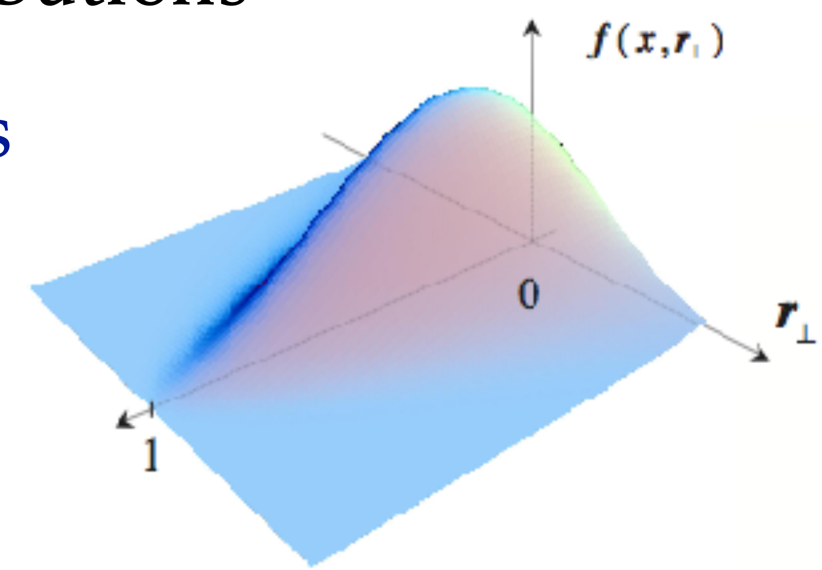
$$\frac{1}{2} = \sum_q J_q(\mu^2) + J_g(\mu^2)$$

$$J_q = \frac{1}{2}\Delta\Sigma_q + L_q$$

    - decomposition of momentum

$$1 = \sum_q \langle x \rangle_q + \langle x \rangle_g$$

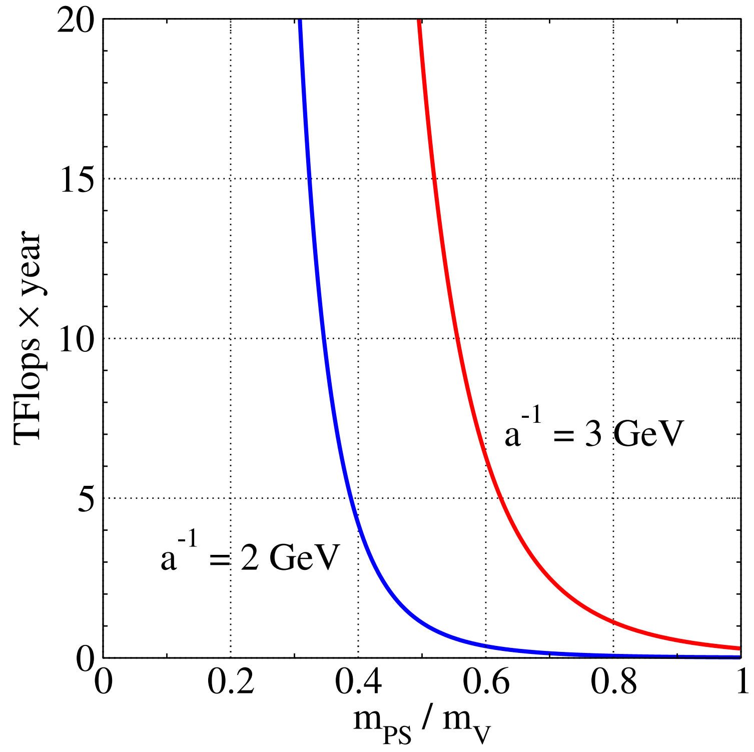




Speed of a Lattice Calculation

1000 configurations with L=2fm

[Ukawa (Berlin, 2001)]

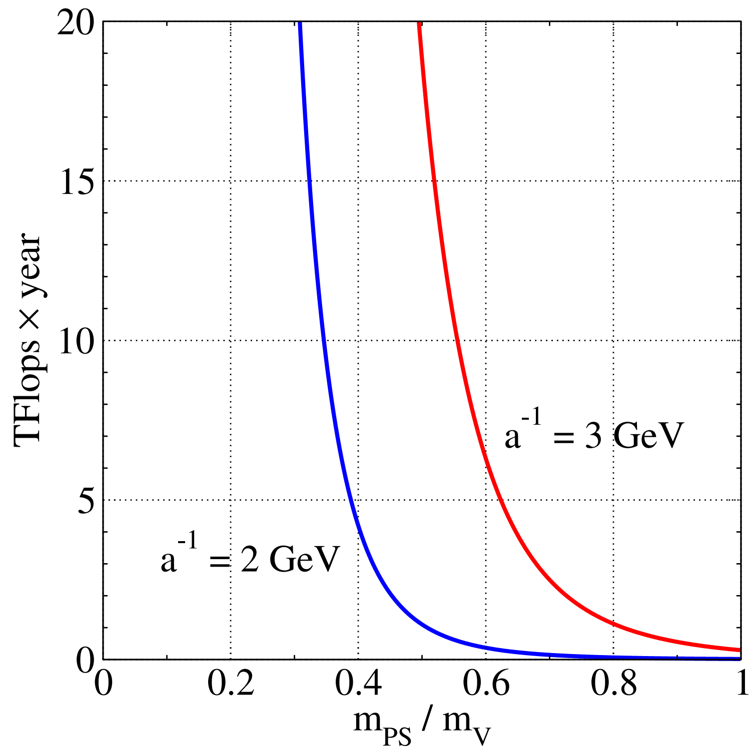


June 2007: BlueGene/L (DOE), 280 TFlops

Speed of a Lattice Calculation

1000 configurations with L=2fm

[Ukawa (Berlin, 2001)]



Algorithmic improvements

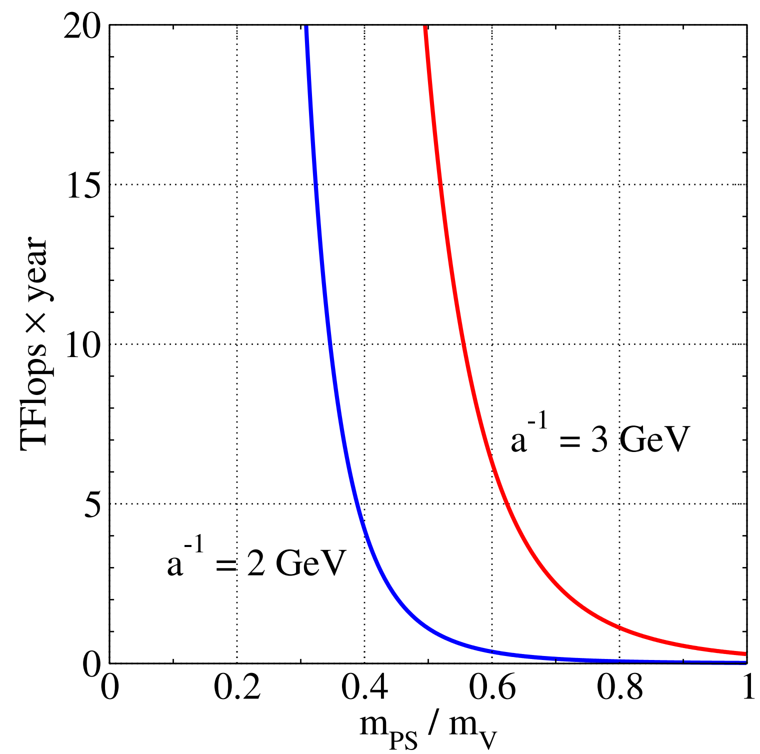
Faster supercomputers

June 2007: BlueGene/L (DOE), 280 TFlops

Speed of a Lattice Calculation

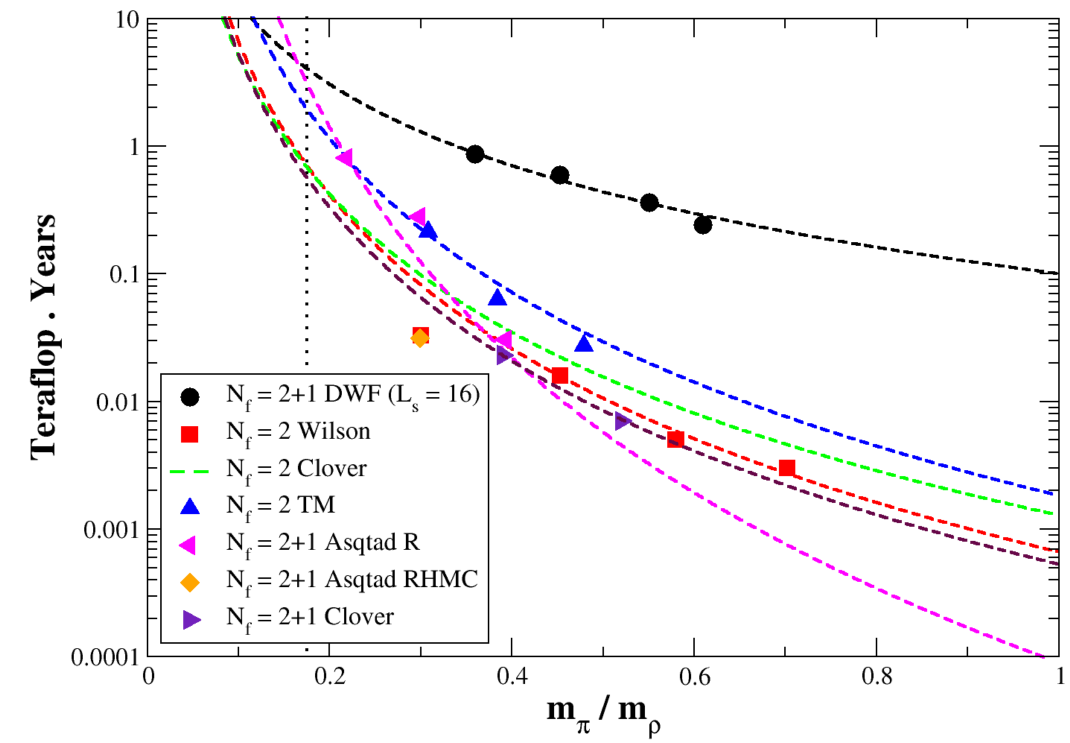
1000 configurations with L=2fm

[Ukawa (Berlin, 2001)]



Algorithmic improvements

[Clark (Tucson, 2006)]



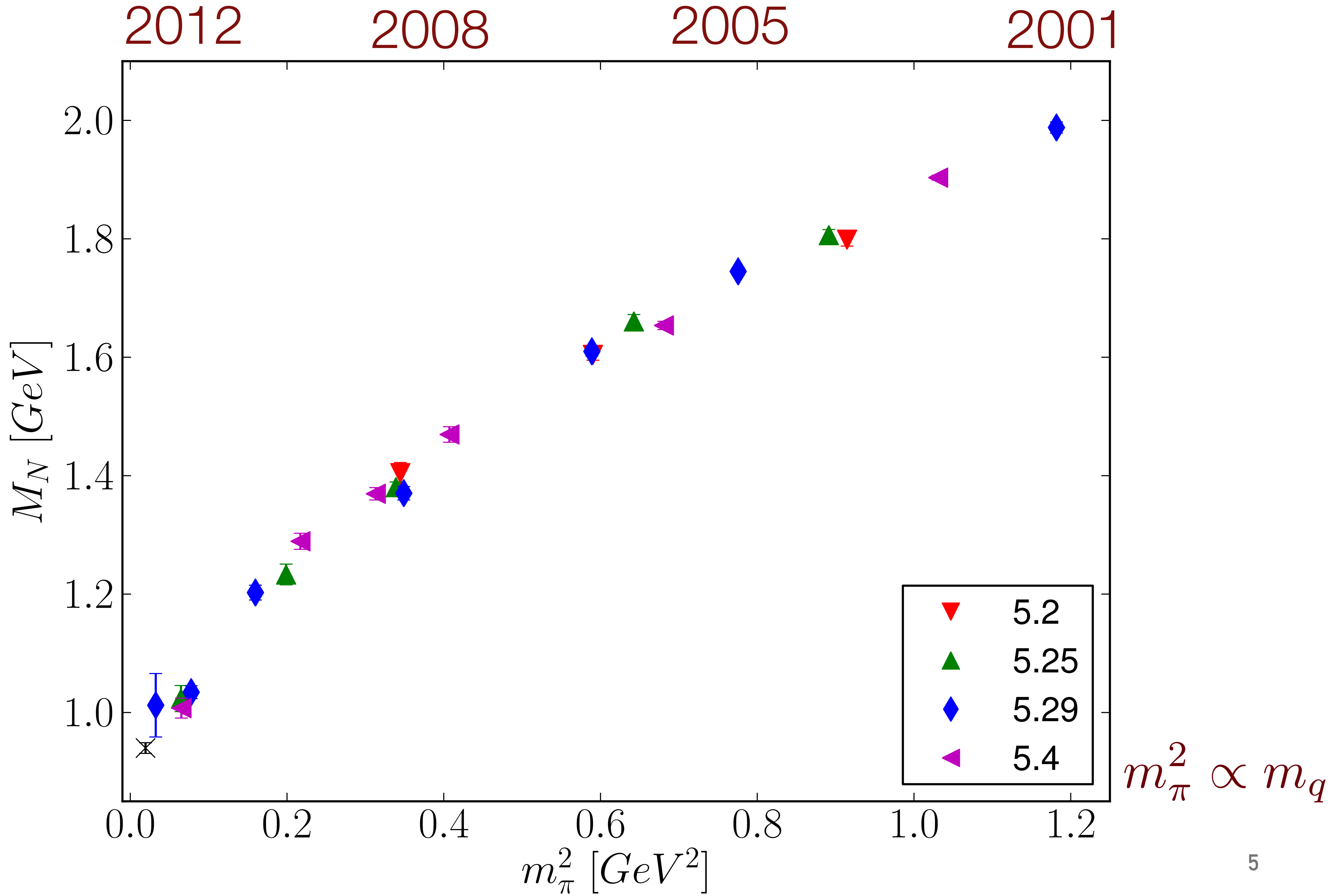
Faster supercomputers

June 2017: Sunway TaihuLight, 93 PFlops

June 2007: BlueGene/L (DOE), 280 TFlops

Real-Time Evolution of Lattice Results

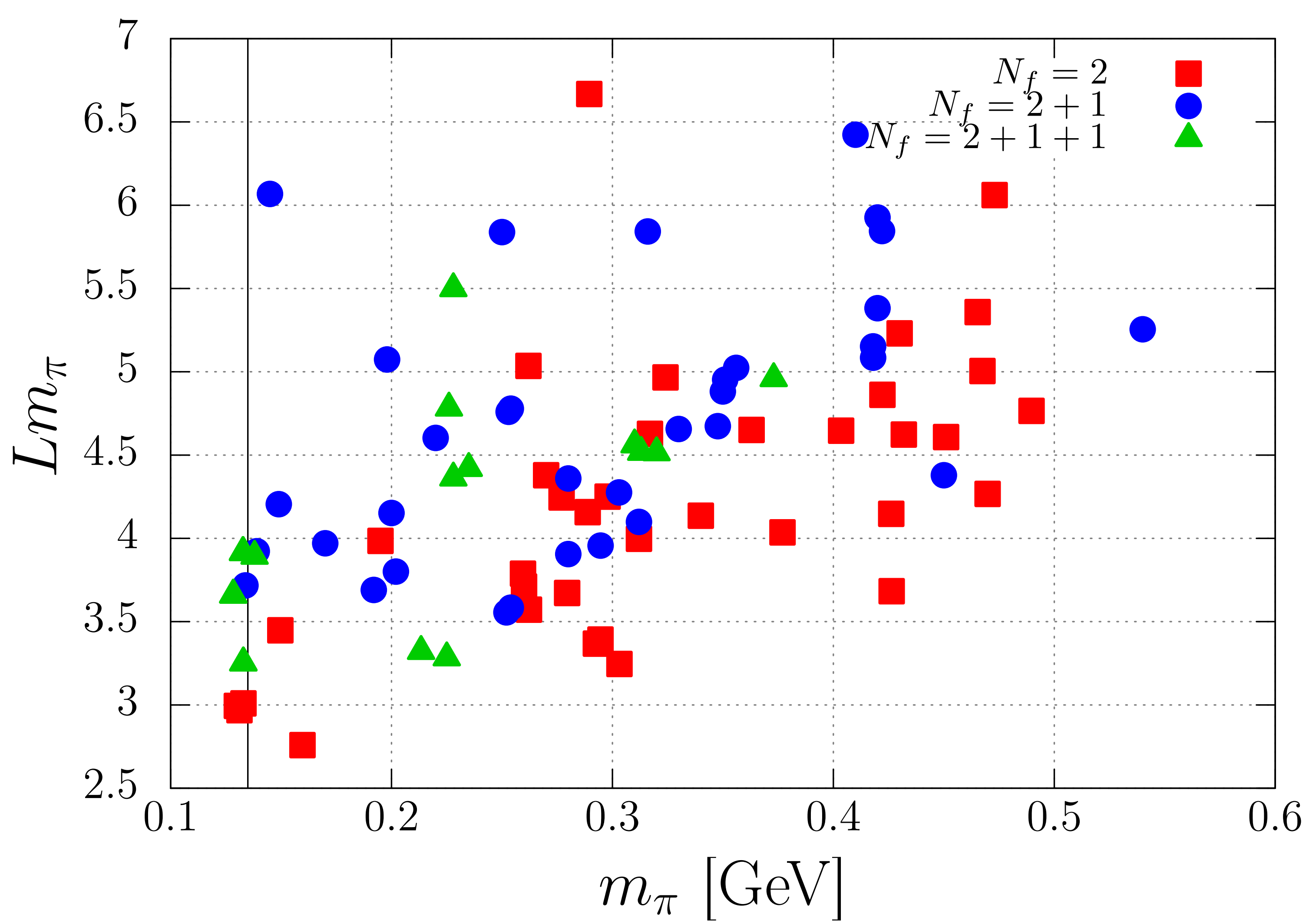
Nucleon Mass



The Lattice Landscape - Hadron Structure

- **Leading sources of error:**
  - Unphysically large quark masses
  - Finite Volume
- Several Collaborations now consider a large range of lattice parameters

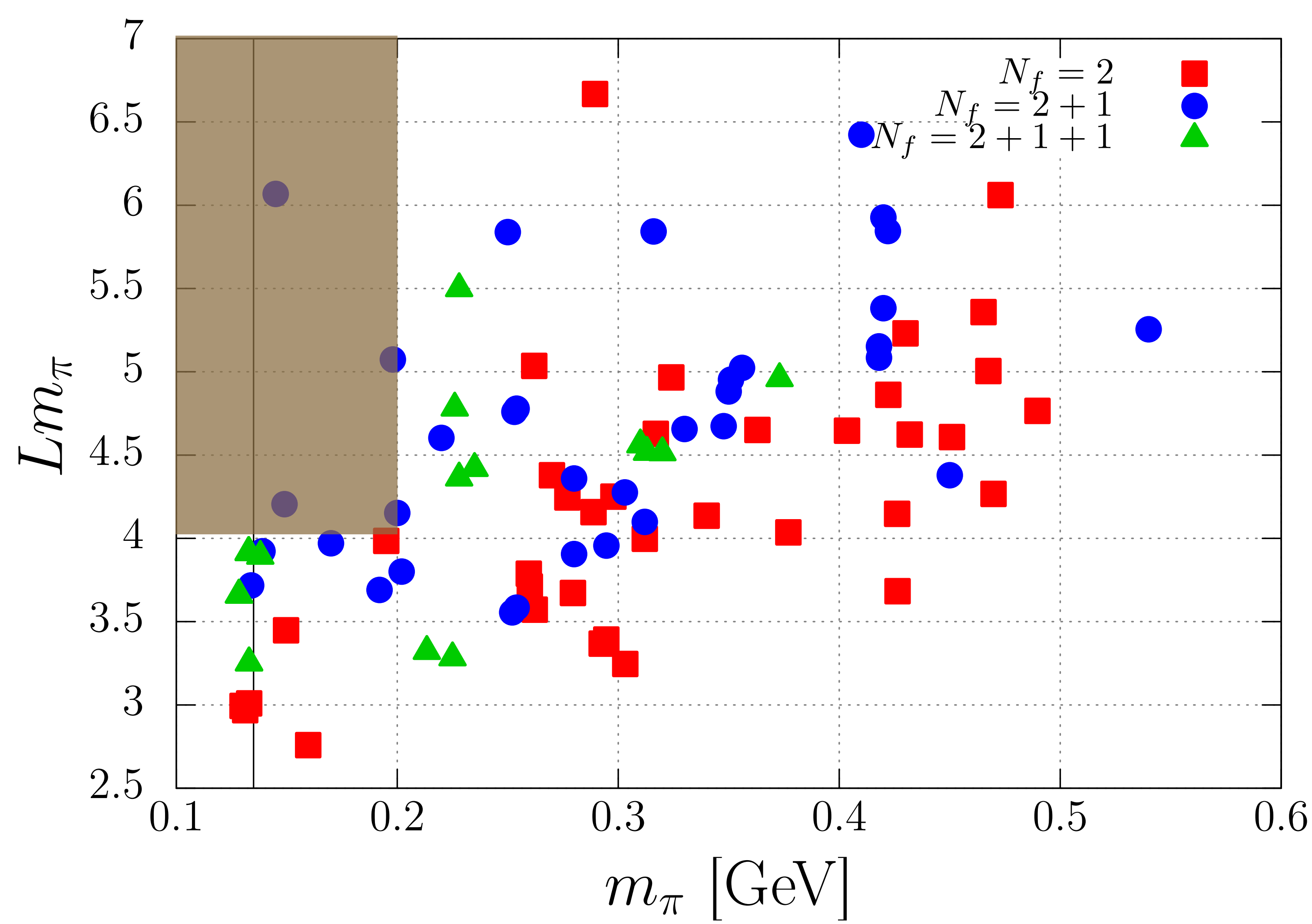
[Plot from S. Collins, Lattice 2016]



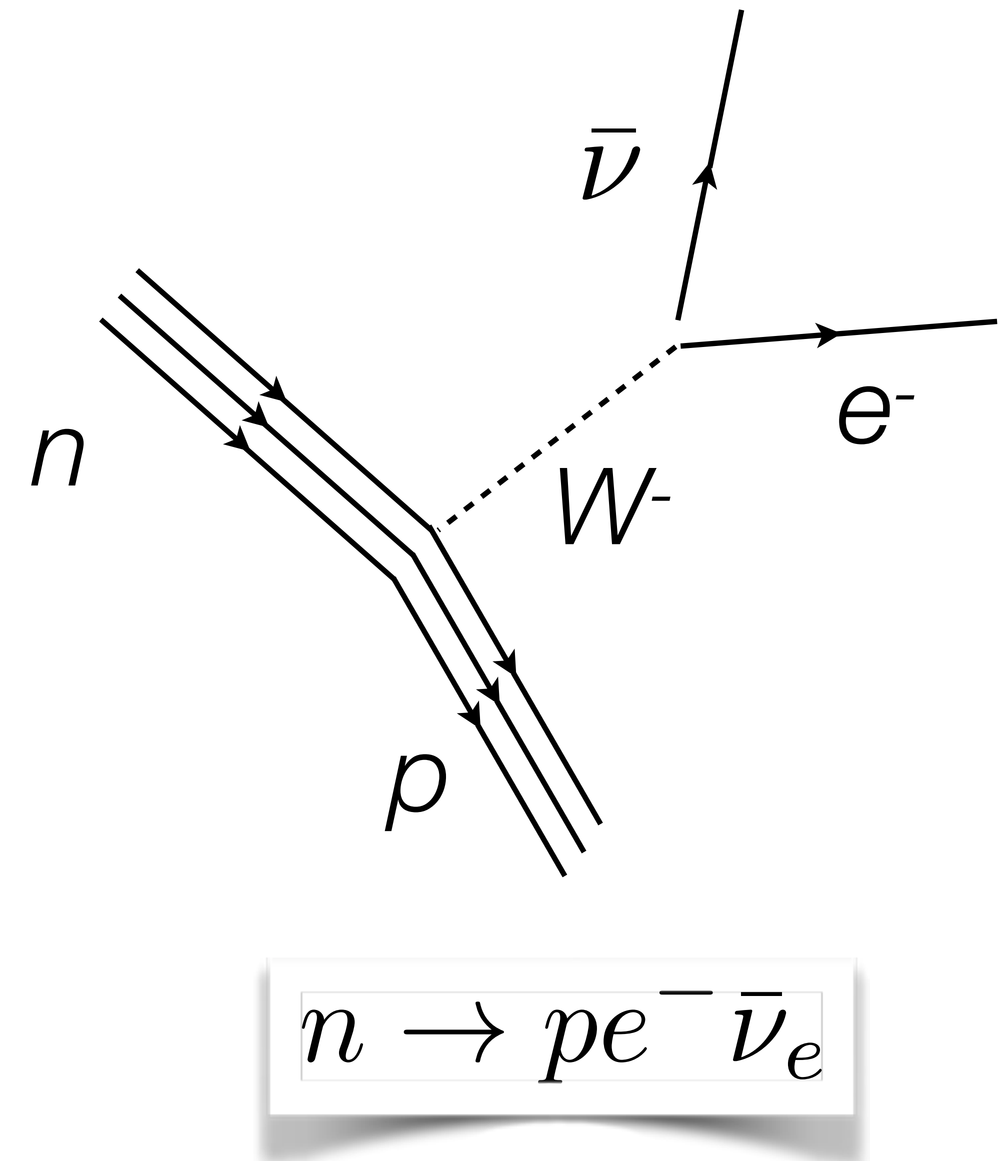
The Lattice Landscape - Hadron Structure

- **Leading sources of error:**
  - Unphysically large quark masses
  - Finite Volume
- Several Collaborations now consider a large range of lattice parameters

[Plot from S. Collins, Lattice 2016]



NUCLEON AXIAL CHARGE



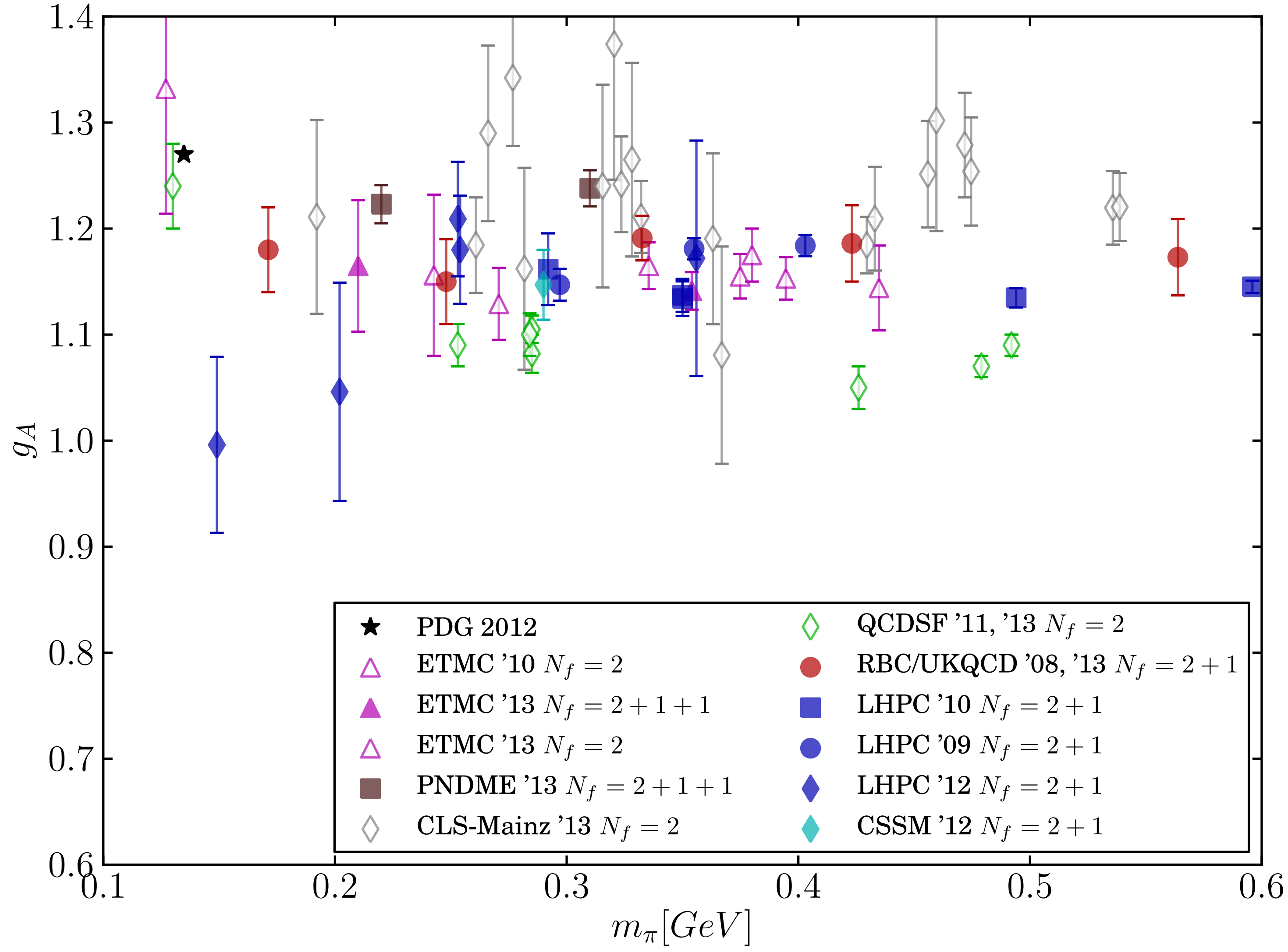
Relatively simple to compute on the lattice

Good benchmark for hadron structure (understanding systematic errors)

g_A from the Lattice

- Some scatter in the results
- Underestimating g_A

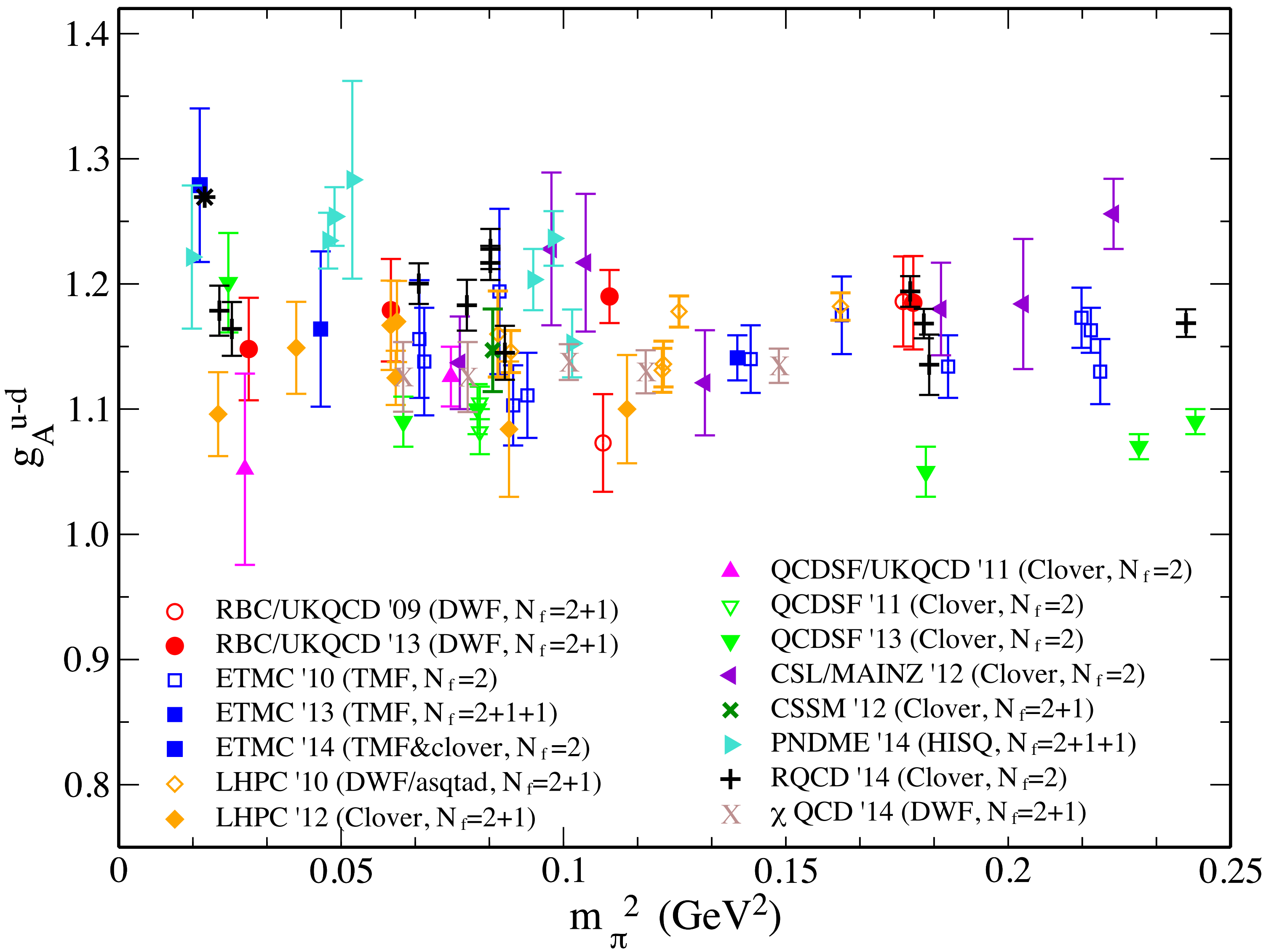
[Plot from S. Syritsyn, Lattice 2013]



g_A from the Lattice

- Some scatter in the results
- Underestimating g_A

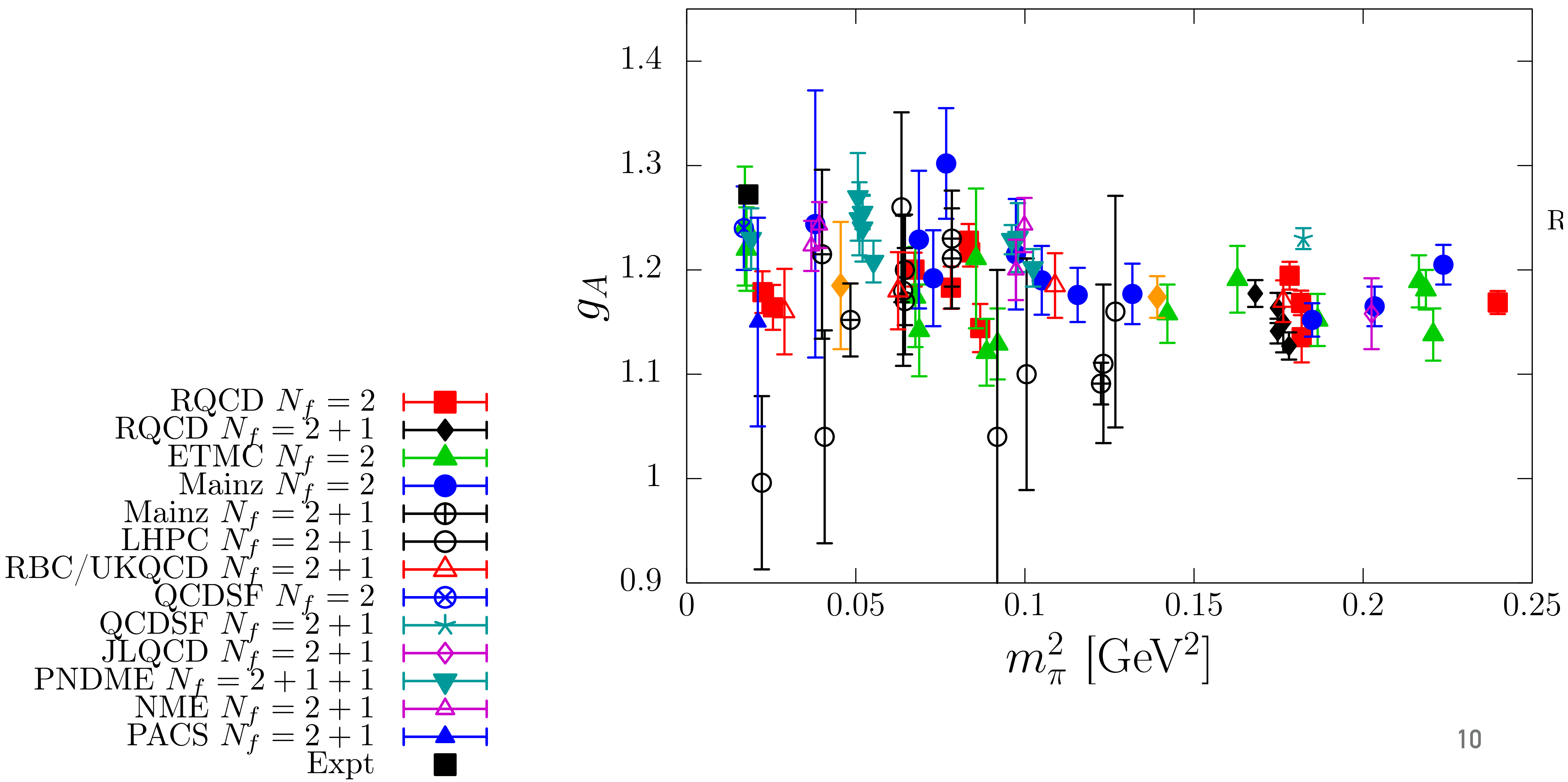
[Plot from M. Constaninou, Lattice 2014]



g_A from the Lattice

- Some scatter in the results
- Underestimating g_A

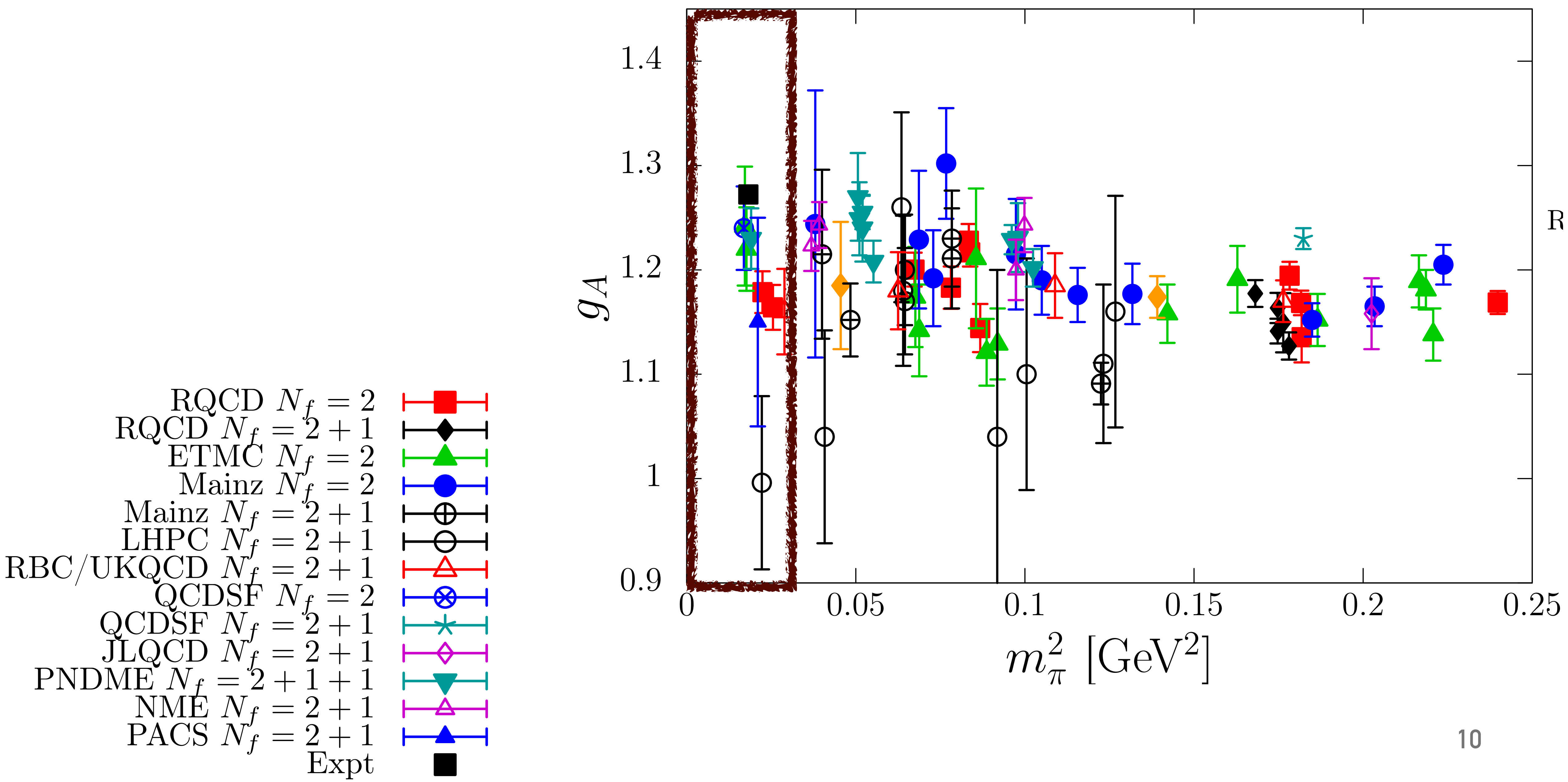
[Plot from S. Collins, Lattice 2016]



g_A from the Lattice

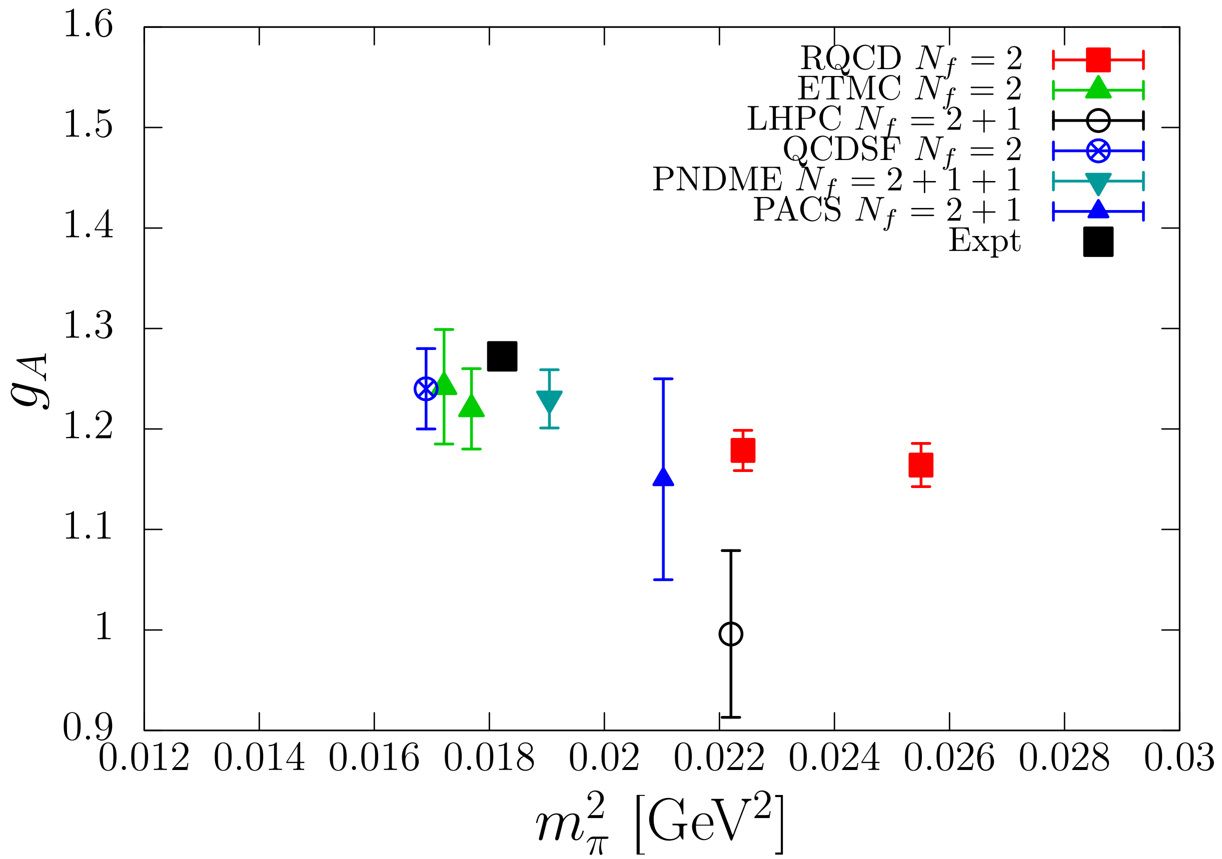
- Some scatter in the results
- Underestimating g_A

[Plot from S. Collins, Lattice 2016]

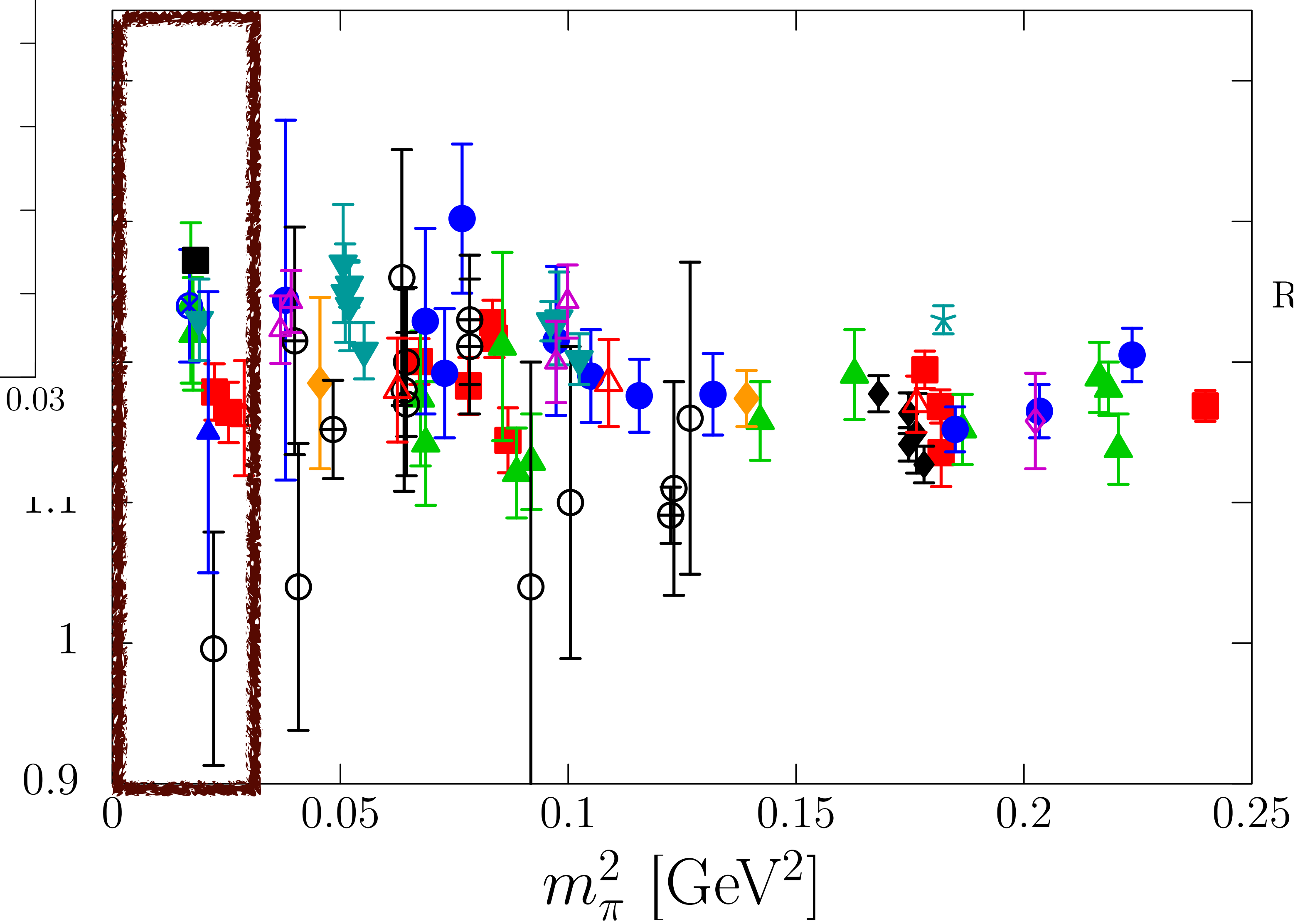


g_A from the Lattice

- Some scatter in the results

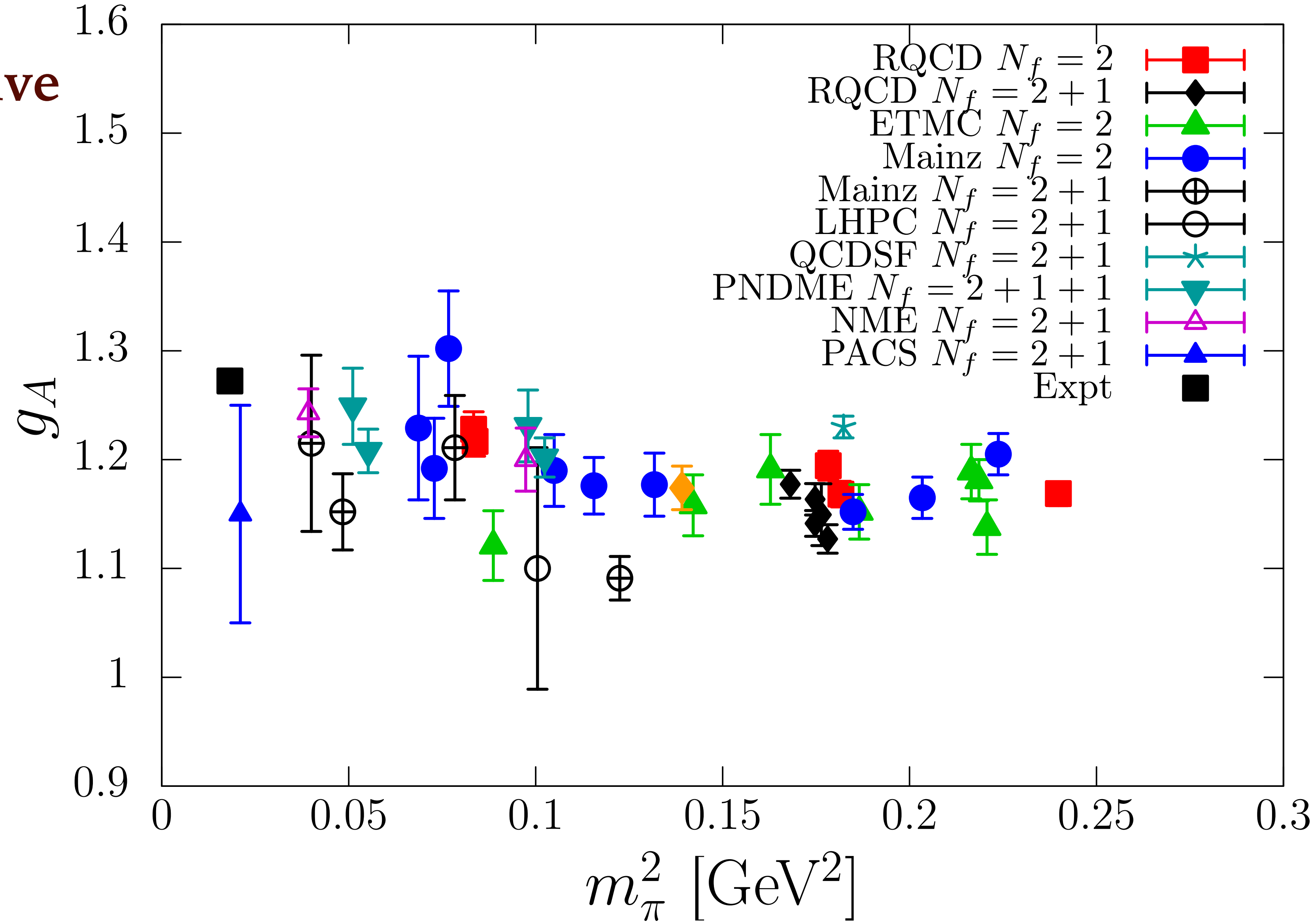


[Plot from S. Collins, Lattice 2016]



g_A from the Lattice

- Some scatter in the results
- Impose "filter": $m_\pi L > 4, \;\; a < 0.1\,\mathrm{fm}$ [Plot from S. Collins, Lattice 2016]
- Converging on physical result
- **g_A appears to be very sensitive to Lattice systematics**
  - e.g. Contamination from excited states
- Lots of effort in reducing systematic errors

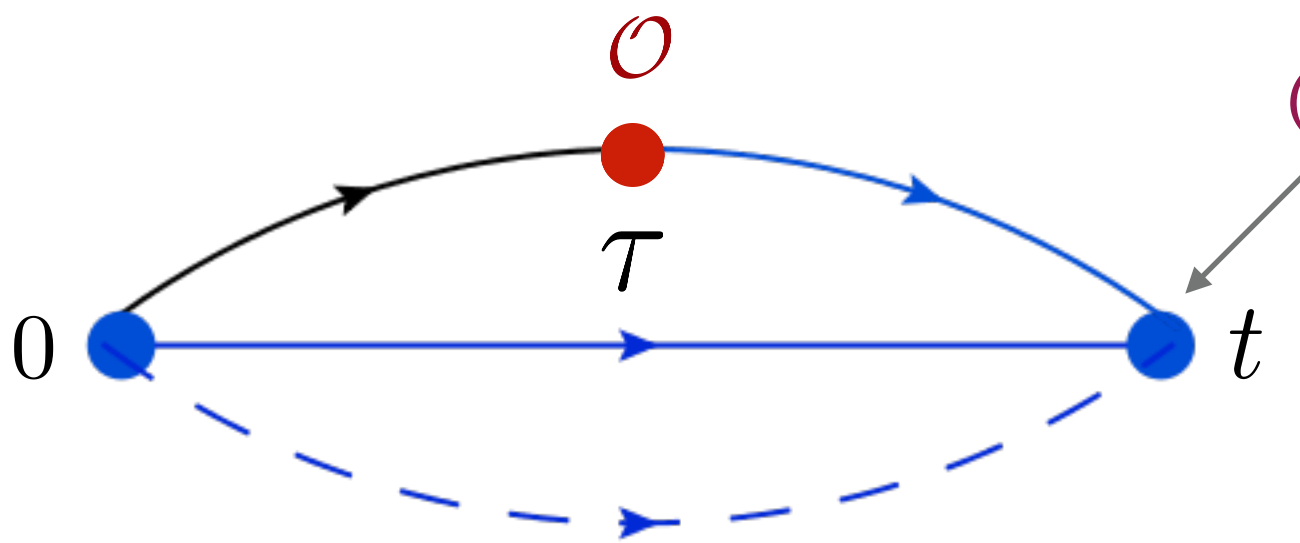


➡ **flow on for other quantities**

Lattice 3pt Functions

Most common method for determining matrix elements relevant for hadron structure calculations - 3pt function

(fix hadron, flavour, p')



$$G(t,\tau,\vec{p},\vec{p}\,') = \sum_{s,s'} e^{-E_{\vec{p}\,'}(t-\tau)} e^{-E_{\vec{p}}\tau} \Gamma_{\beta\alpha} \langle \Omega | \chi_\alpha(0) | N(p',s') \rangle \langle N(p',s') | \mathcal{O}(\vec{q}) | N(p,s) \rangle \langle Np, s) | \overline{\chi}_\beta(0) | \Omega \rangle$$

For large times $1 \ll \tau \quad 1 \ll t - \tau$

(remove excited states: control 2 time windows)

→ **Extract matrix element**

→ **Determine form factors, charges, moments, …**

Feynman-Hellmann Theorem

Also talk by R.Young (Tuesday)

➤ Provides an alternative method for determining hadronic matrix elements $\langle H|\mathcal{O}|H\rangle$ from energy shifts

1. Modify Lagrangian by

$$\mathcal{L} \to \mathcal{L} + \lambda\mathcal{O}$$

simple excited state removal

2. Measure hadron energy while changing λ

$$G(\lambda;\vec{p};t) = \int dx\, e^{-\vec{p}\cdot\vec{x}} \langle \chi'(x)\chi(0)\rangle \overset{\text{large } t}{\propto} e^{-E_H(\lambda,\vec{p})t}$$

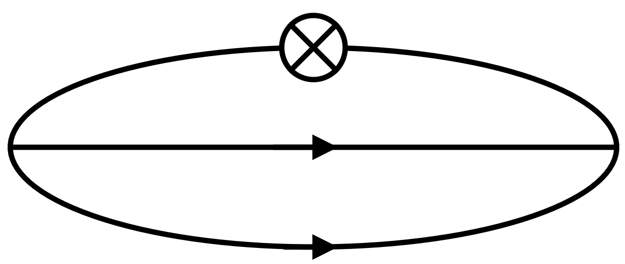
3. Calculate matrix element from energy shifts

$$\left.\frac{\partial E_H(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p})}\langle H(\vec{p})|\mathcal{O}(0)|H(\vec{p})\rangle$$

Calculation of matrix elements $\equiv$ hadron spectroscopy

Feynman-Hellmann Theorem

- Can modify fermion action in 2 places:
  - quark propagators
  - fermion determinant

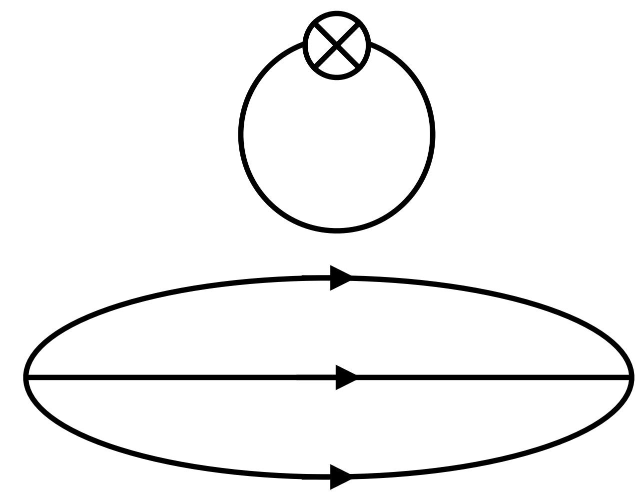


Connected

g_A, $\Delta\Sigma$ [PRD90 (2014)]

G_E,G_M [1702.01513]

OPE [PRL118 (2017)]



Disconnected

(Requires new gauge configurations)

Δs [PRD92 (2015)]

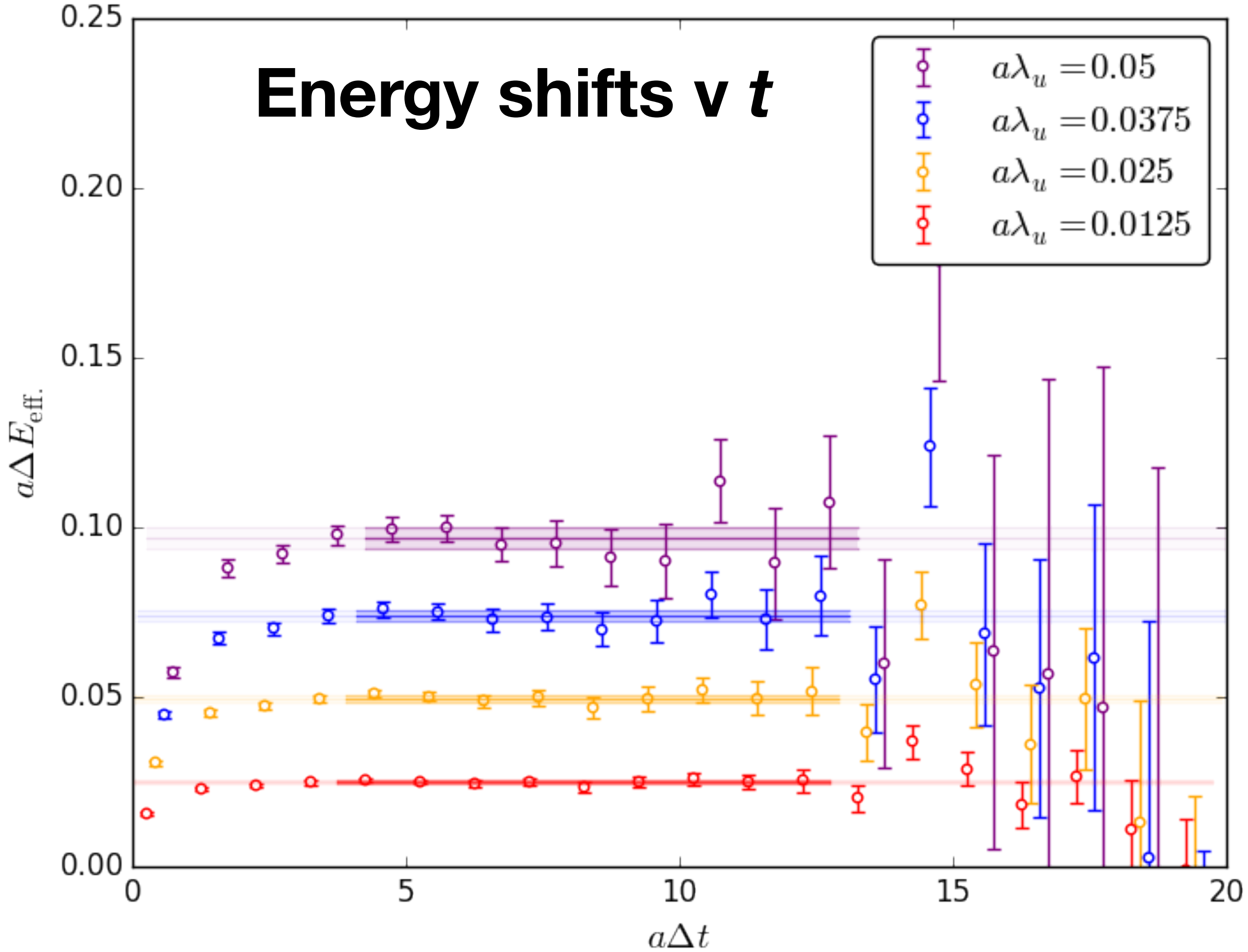
Talk by R.Young (Tuesday)

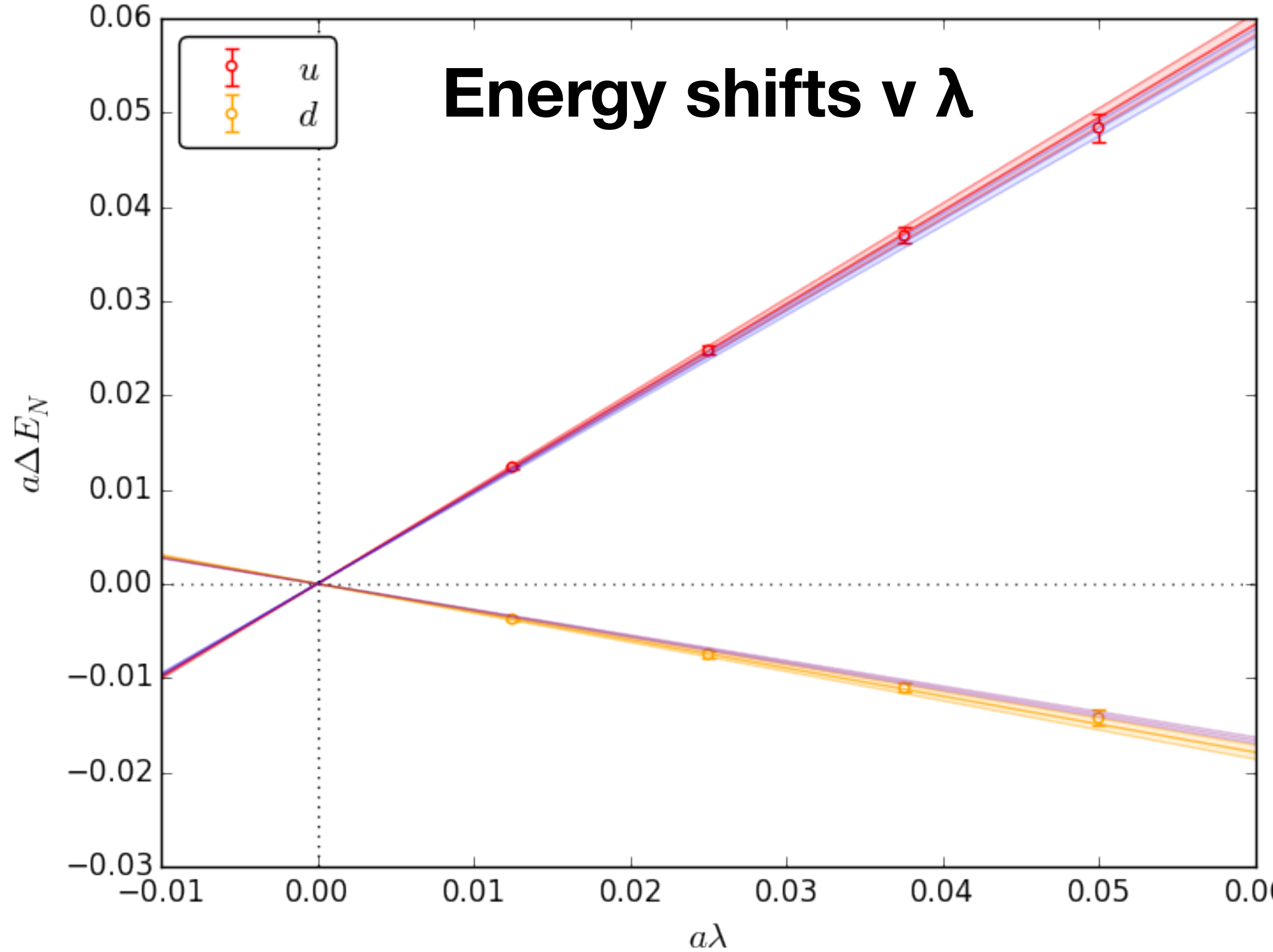
Demonstration: Axial Charges

(Connected only)

- Want $\langle N_s(\vec{p})|\bar{q}(0)\gamma_\mu\gamma_5 q(0)|N_s(\vec{p})\rangle = 2is_\mu \Delta q \qquad q \in (u,d)$

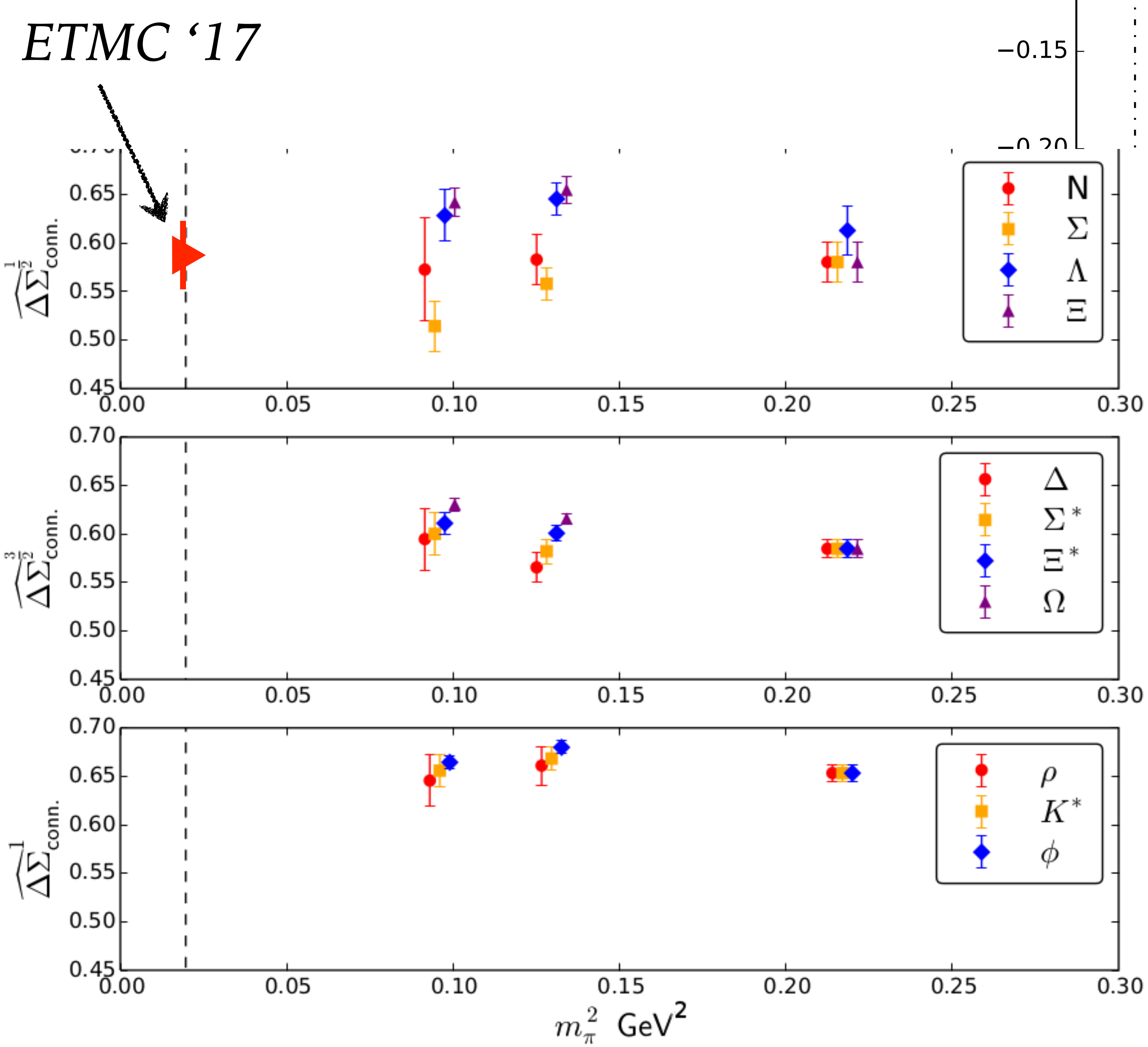
- Employ $\mathcal{L} \to \mathcal{L} + \lambda\bar{q}(-i\gamma_3\gamma_5)q \implies \left.\frac{\partial E_N(\lambda)}{\partial\lambda}\right|_{\lambda=0}^{\Gamma_\pm} = \pm\Delta q_{\text{conn.}}$

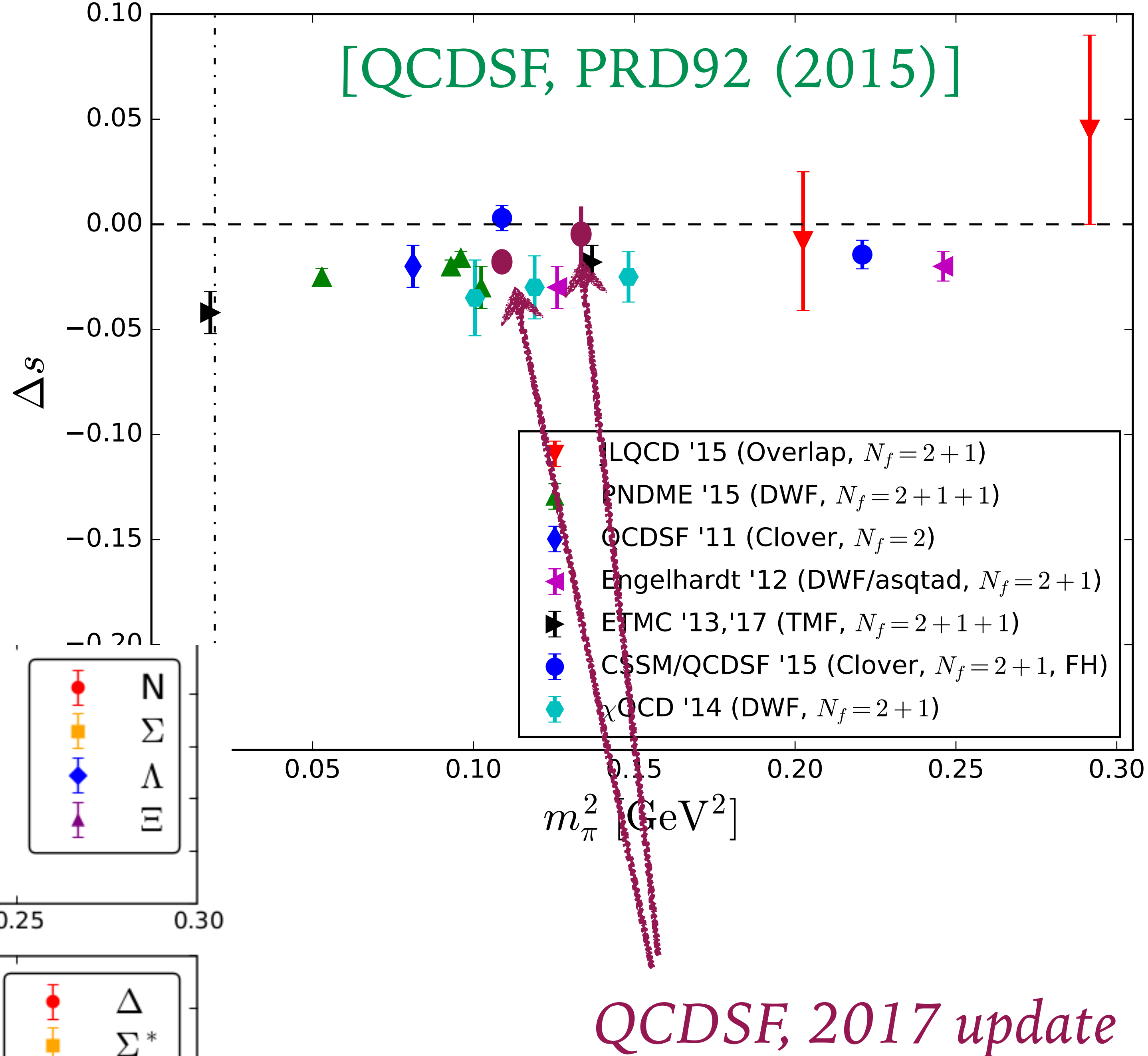




$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$

Quark Axial Charges



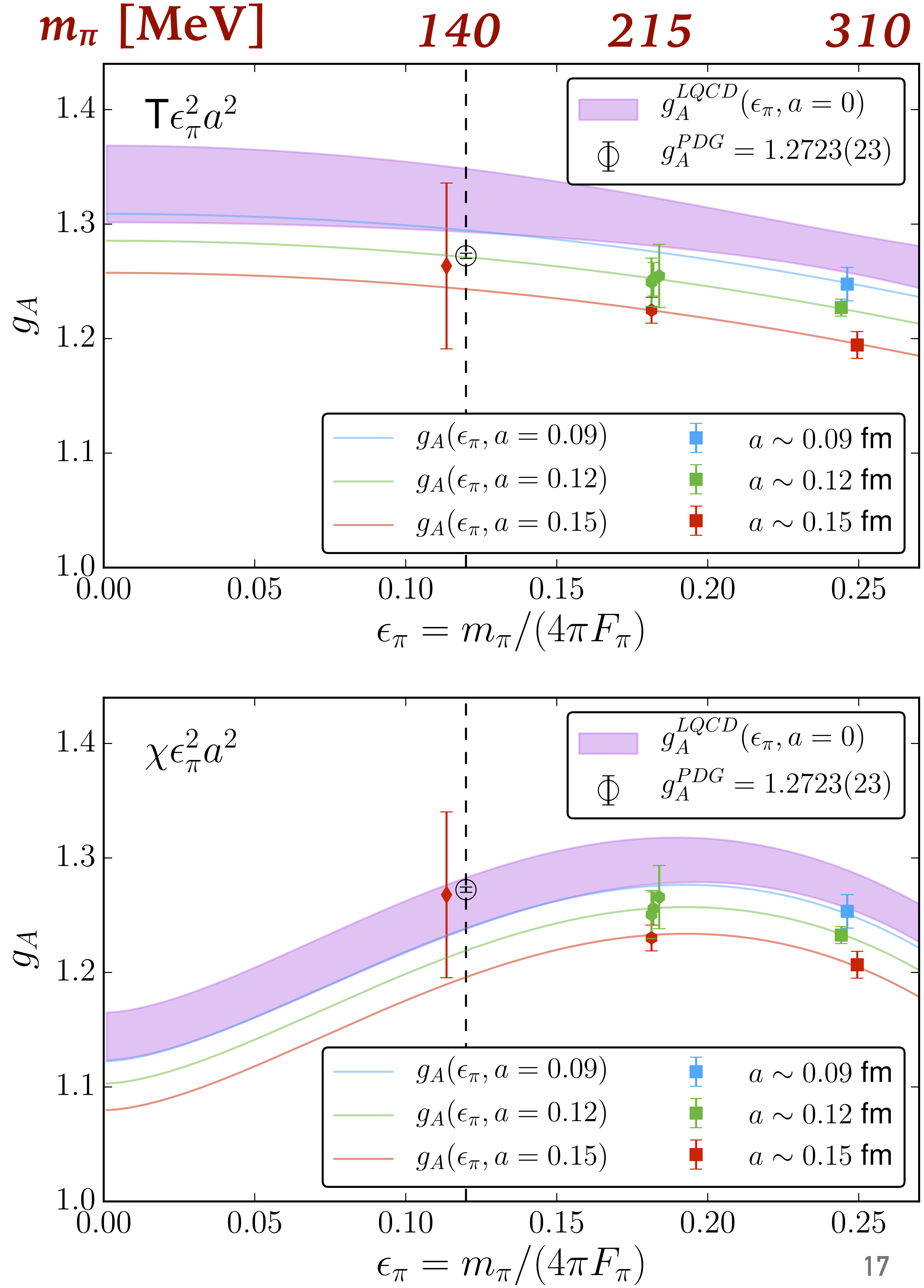


Quark spin suppression appears to be a universal feature of all hadrons

Recent g_A Calculation

[CalLat, 1704.01114]

- Feynman-Hellman-inspired
  - High-statistics
  - Multiple lattice spacings
  - Physical quark masses



Electromagnetic Form Factors

$$\langle p',\, s'|J^\mu(\vec{q})|p,\, s\rangle \;= \bar{u}(p',\, s')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(p,\, s)$$

$$F_1(0) = Q$$
$$F_1(0) + F_2(0) = \mu$$

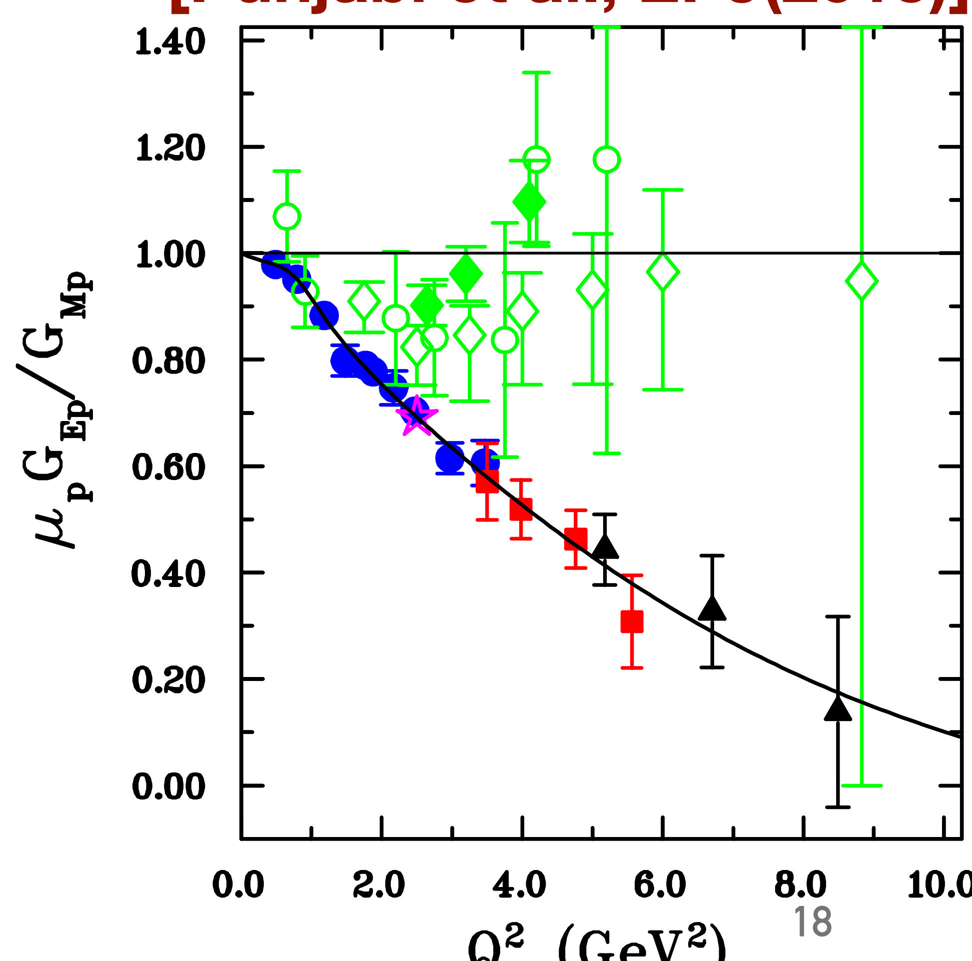
$$Q_p = 1,\; Q^n = 0$$
$$\mu_p = 2.79\mu_N,\; \mu_n = -1.91\mu_N$$

Radii: $r_i^2 = -6\frac{dF_i(q^2)}{dq^2}\Big|_{q^2=0}$

$q^2 > 0$: "Look inside" hadron

Zero crossing in G_E?

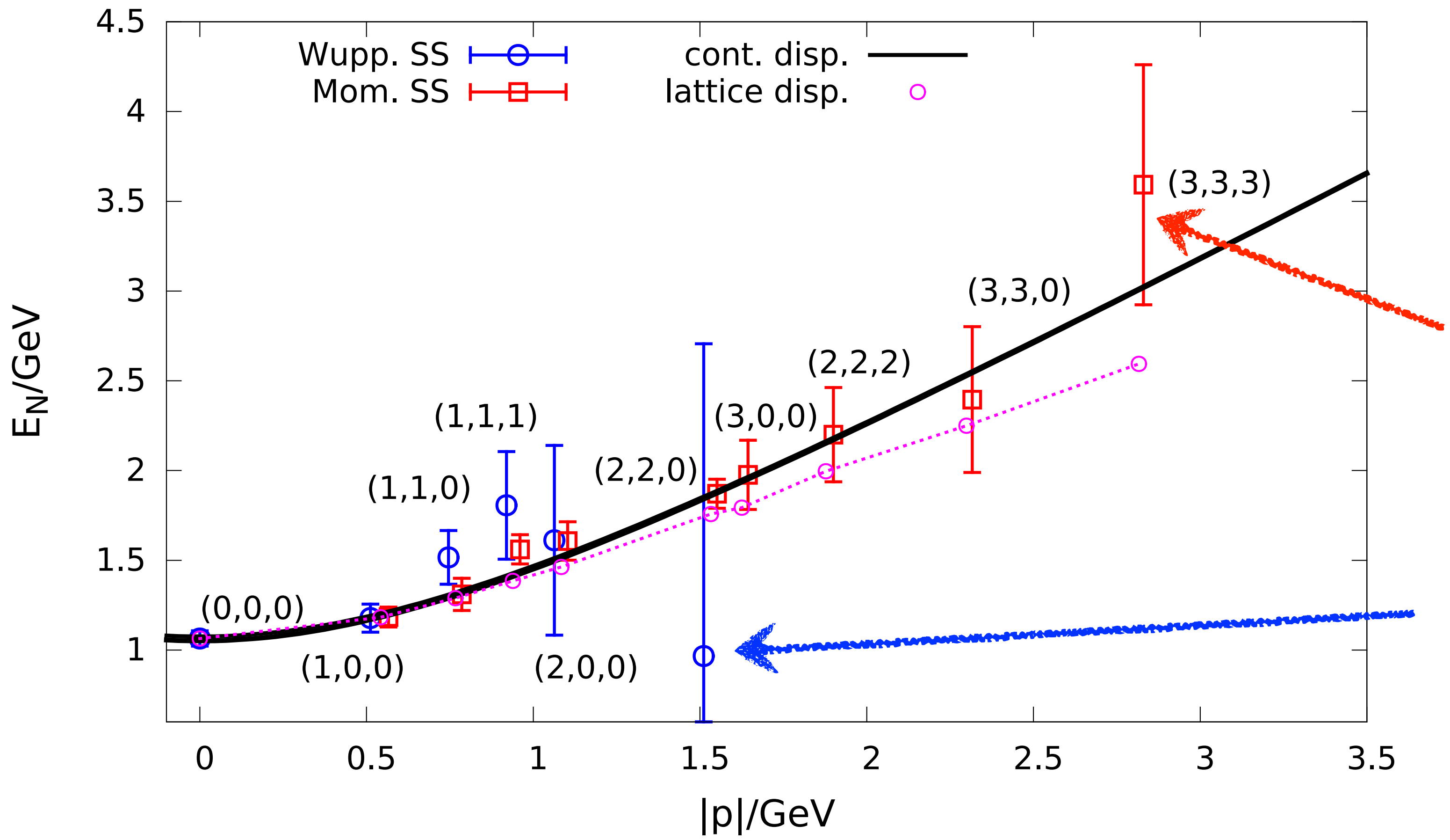
[Punjabi et al., EPJ(2015)]



Large Q²

- Large momentum transfer region a challenge on Lattice
- Recent developments:
  - Feynman-Hellmann
  - Boosted quark operators (momentum smearing)

(can be used together - next year)



[Bali, PRD 93 (2016) 094515]

Boosted operators

Traditional operators

Feynman-Hellmann Theorem (Non-Forward Case)

- Form factors ➡ extend to non-forward FH

1. Modify Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda\left(e^{i\vec{q}\cdot\vec{x}} + e^{-i\vec{q}\cdot\vec{x}}\right)\mathcal{O}(x)$$

2. Measure hadron energy while changing λ

$$G(\lambda;\vec{p}\,';t) \overset{\text{large } t}{\propto} e^{-E_H(\lambda,\vec{p}\,')t}$$

Requires Breit frame

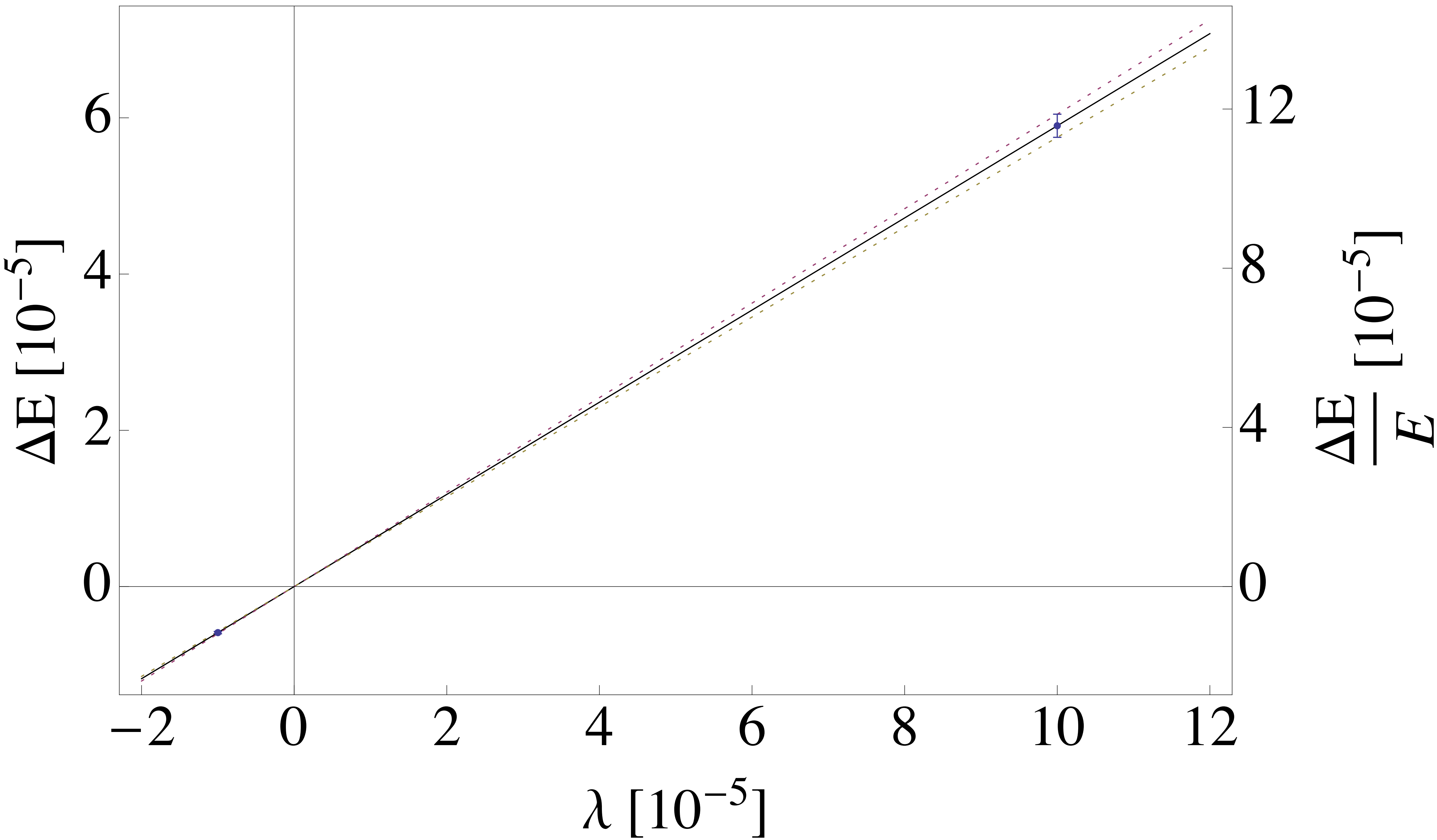
3. Calculate matrix element from energy shifts

$$\left.\frac{\partial E_H(\lambda,\vec{p}\,')}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p}\,')}\langle H(\vec{p}\,')|\mathcal{O}(0)|H(\vec{p})\rangle$$

Form factor extraction simplified for $\vec{p}\,' = -\vec{p}$

Energy Shifts

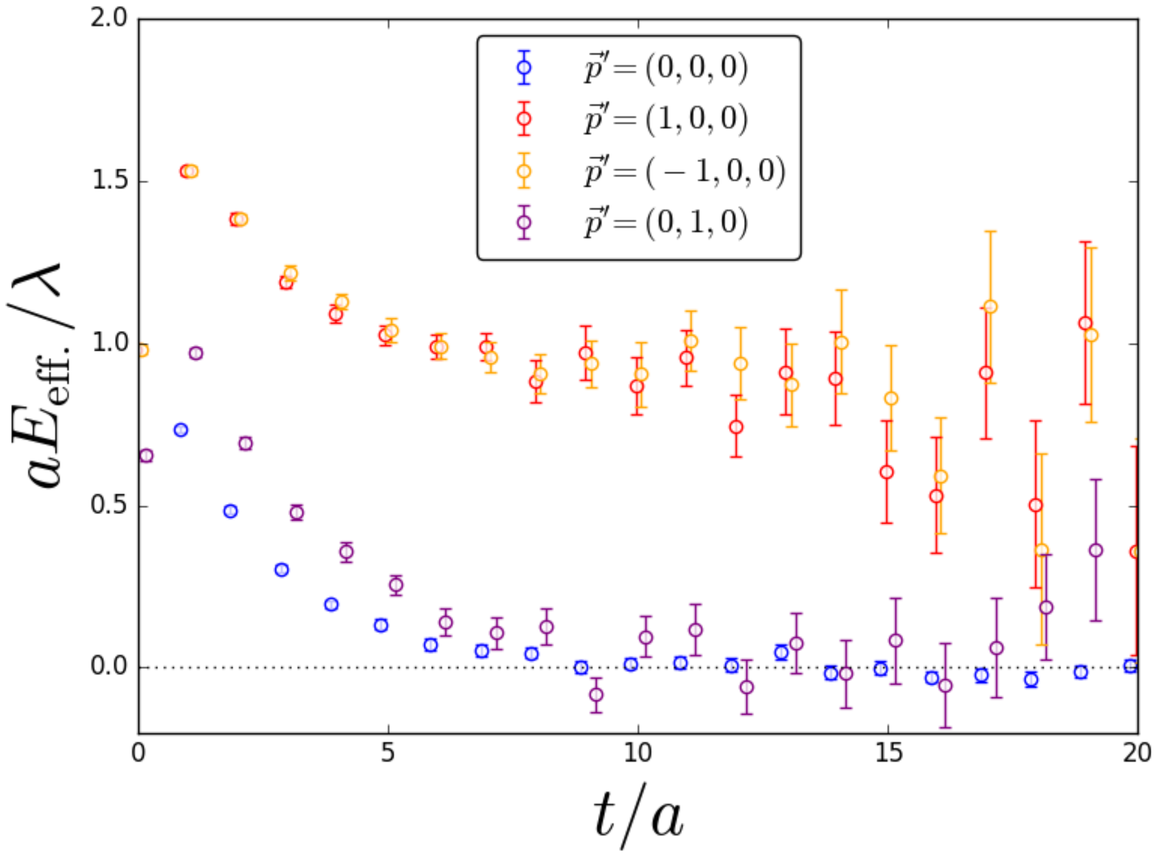
➤ Choose small λ (~10^{-4} - 10^{-5}) ➡ minimise quadratic effects



Pion Form Factor

$$\langle \pi(\vec{p}\,')|\bar{q}(0)\gamma_\mu q(0)|\pi(\vec{p})\rangle = (p'_\mu + p_\mu)F^q_\pi(Q^2)$$

$m_\pi \approx 470$ MeV $\quad N_{\text{conf}} = 750 \quad 32^3 \times 64 \quad \mathbf{q} = (2,0,0)$



Require Breit frame kinematics

$$\mathbf{q} = (2,0,0) \implies \mathbf{p}' = (\pm 1, 0, 0)$$

Otherwise no signal at $\mathcal{O}(\lambda)$

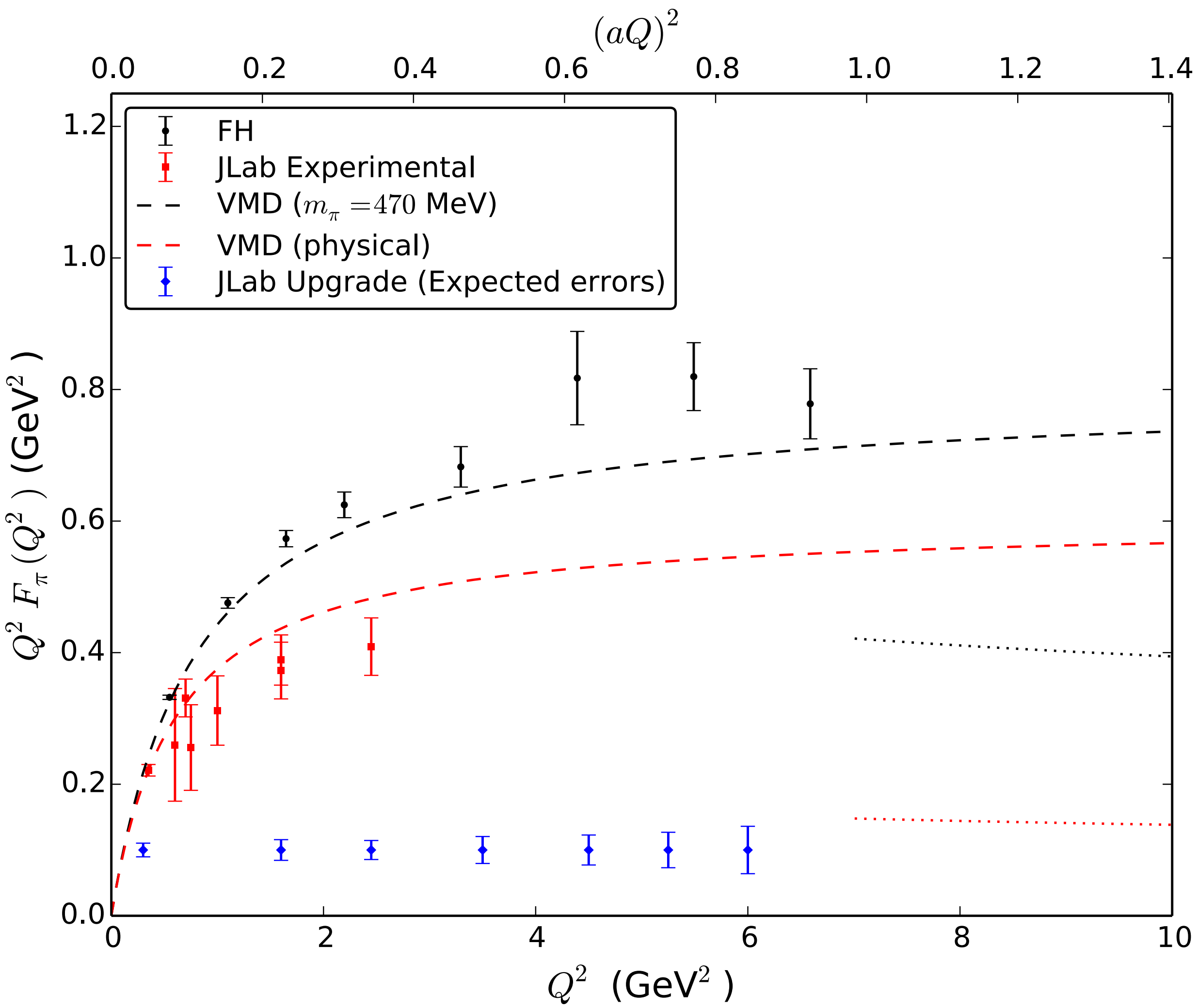
- Selecting ***p* = -*p'***
  - maximises q^2 while minimising hadron momentum
  - ➡ minimises statistical noise

Pion Form Factor

$m_\pi \approx 470$ MeV

1000-1500 measurements

Breit frame



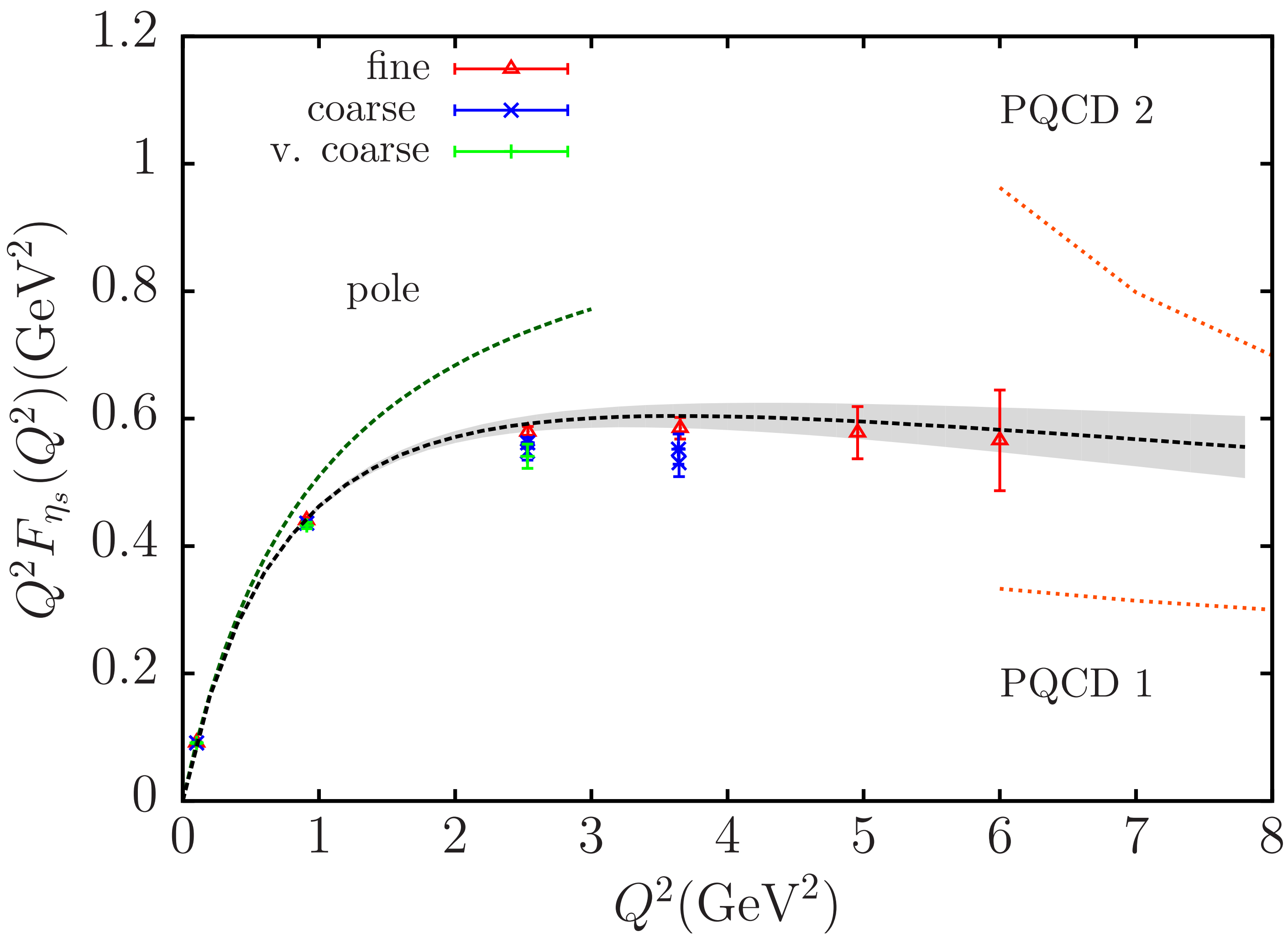
[Chambers, et al. 1702.01513]

Feynman-Hellmann

$m_\pi \approx 690$ MeV

48,000 - 192,000 measurements

Breit frame



[Koponen et al., PRD96 (2017)]

3pt functions

[Chambers, et al. 1702.01513]

Proton Form Factors

$m_\pi \approx 470$ MeV *1000-1500 measurements*

Breit frame

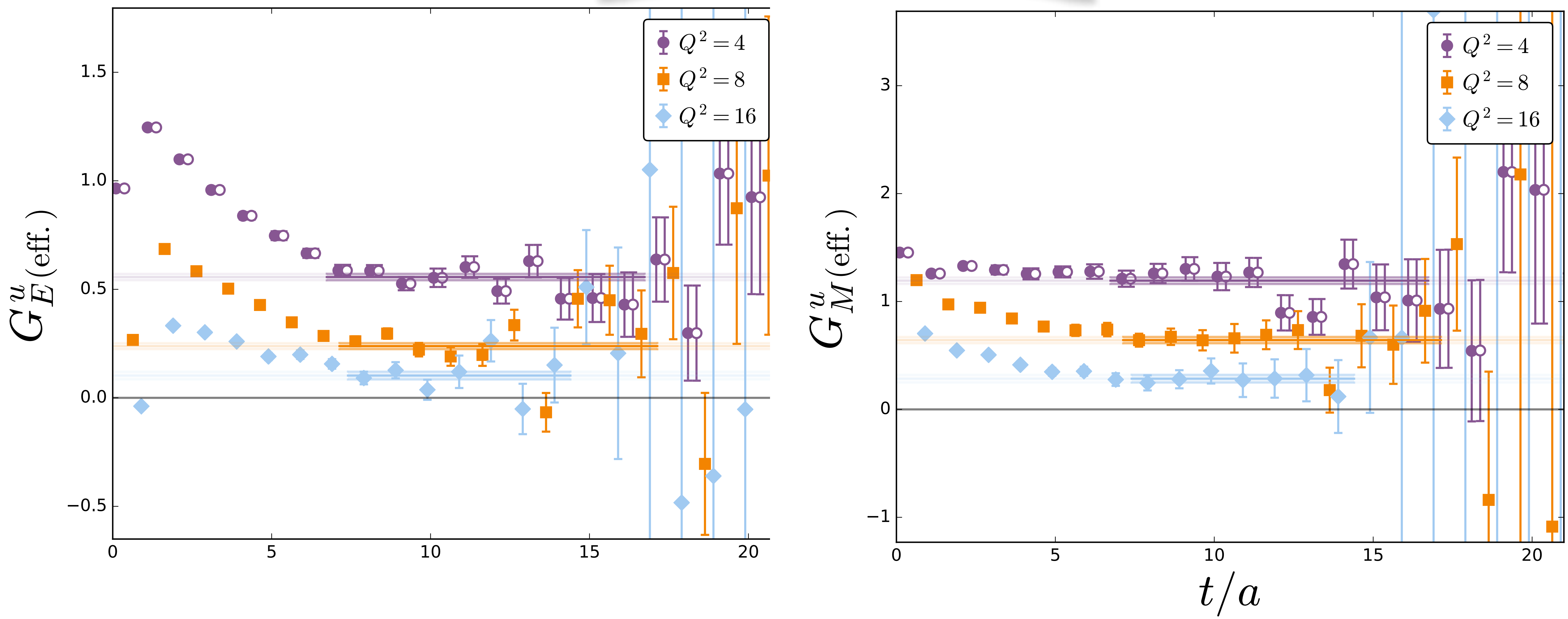
Temporal current

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0}^{\gamma_{\rm unpol}} \overset{\vec{p}'=-\vec{p}}{=} \frac{M}{E} G_E^q(Q^2)$$

Spatial current

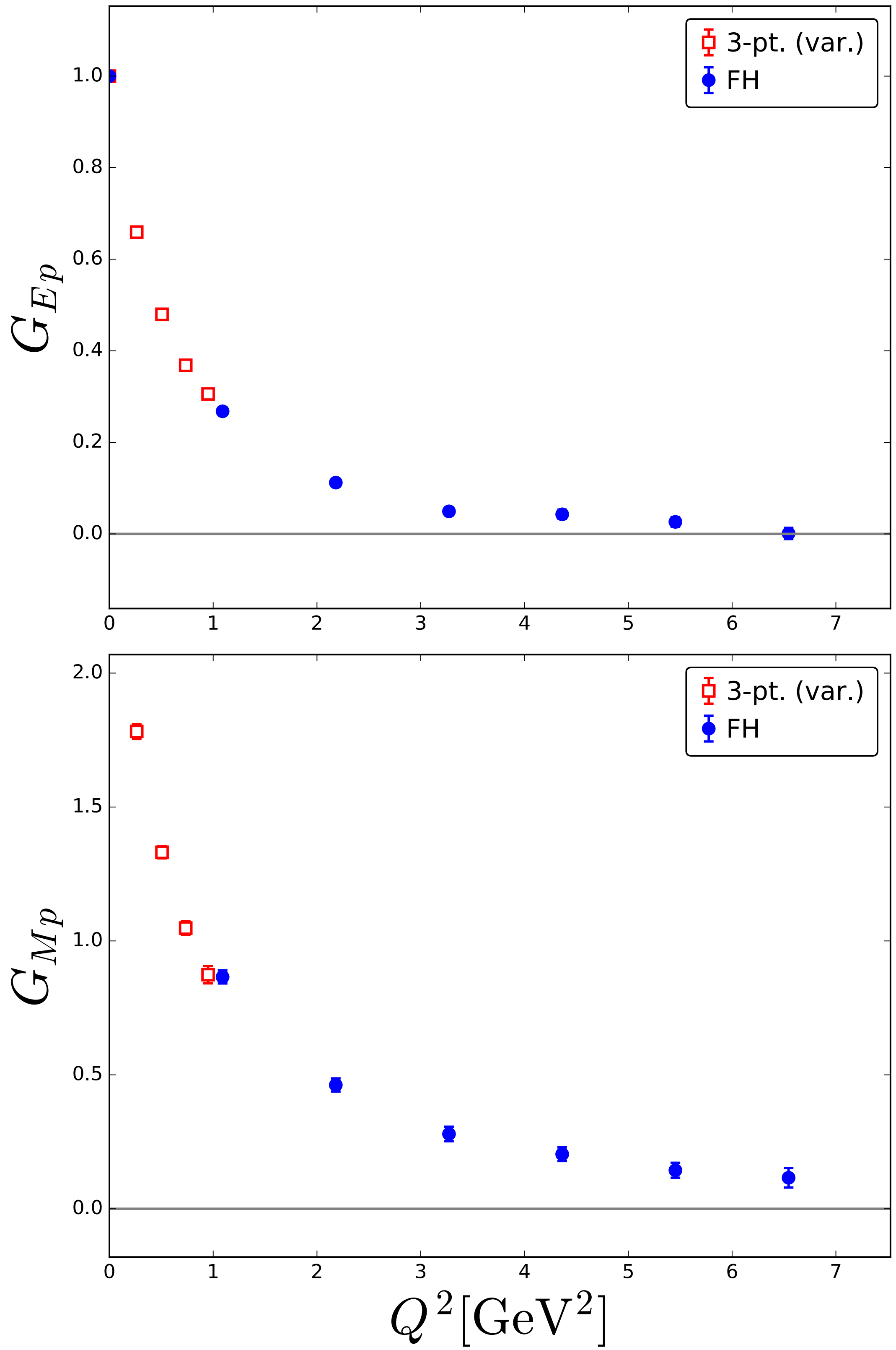
$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0}^{\gamma_{\pm}} \overset{\vec{p}'=-\vec{p}}{=} \pm\frac{\vec{q}\times\hat{e}}{2E} G_M^q(Q^2)$$

$$G^q_{E/M(\text{eff.})} = G^q_{E/M}/\lambda$$



Proton Form Factors

$m_\pi \approx 470$ MeV
Breit frame

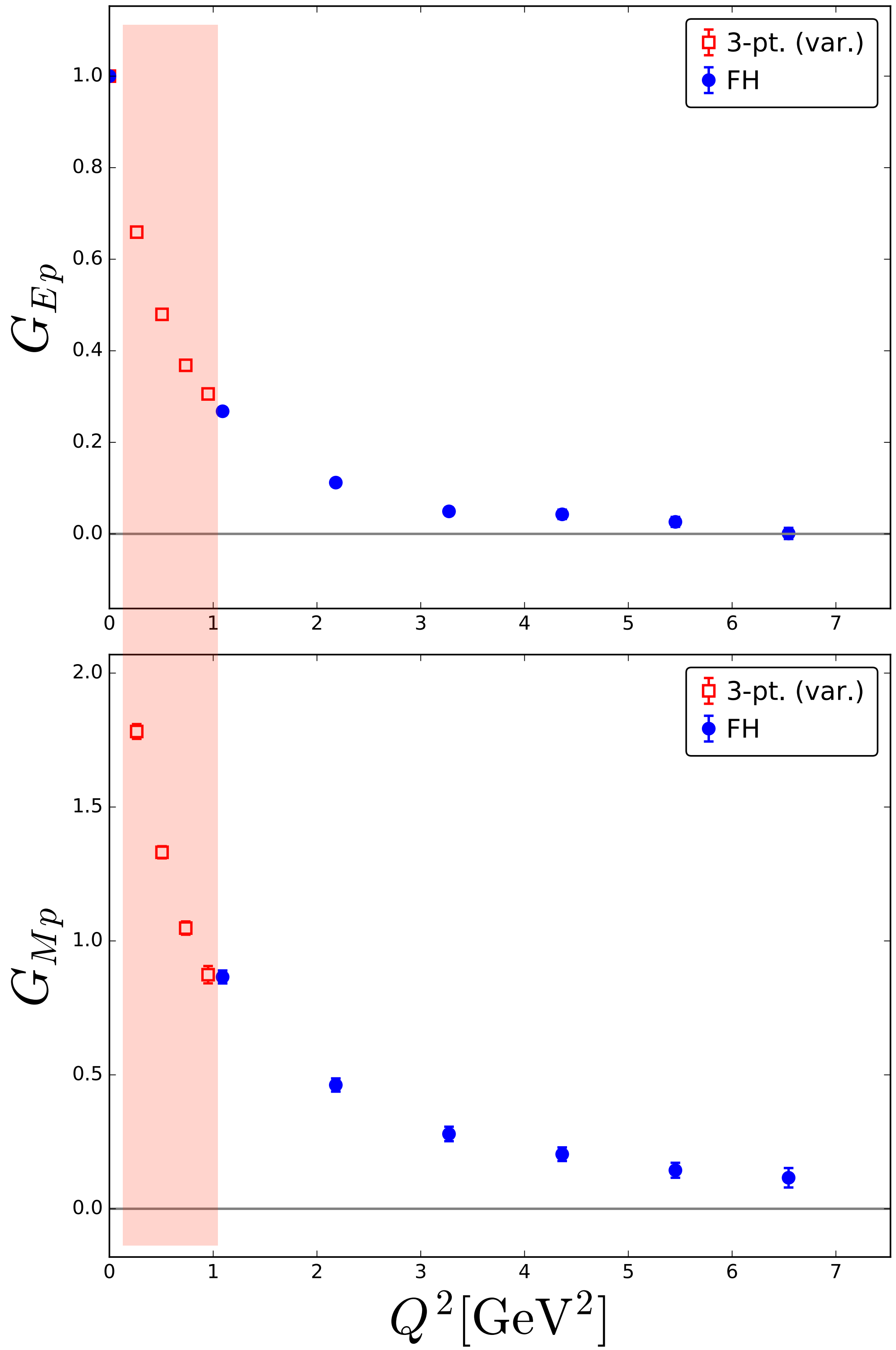


[Chambers *et al*. arXiv:1702.01513]

Proton Form Factors

3-pt functions

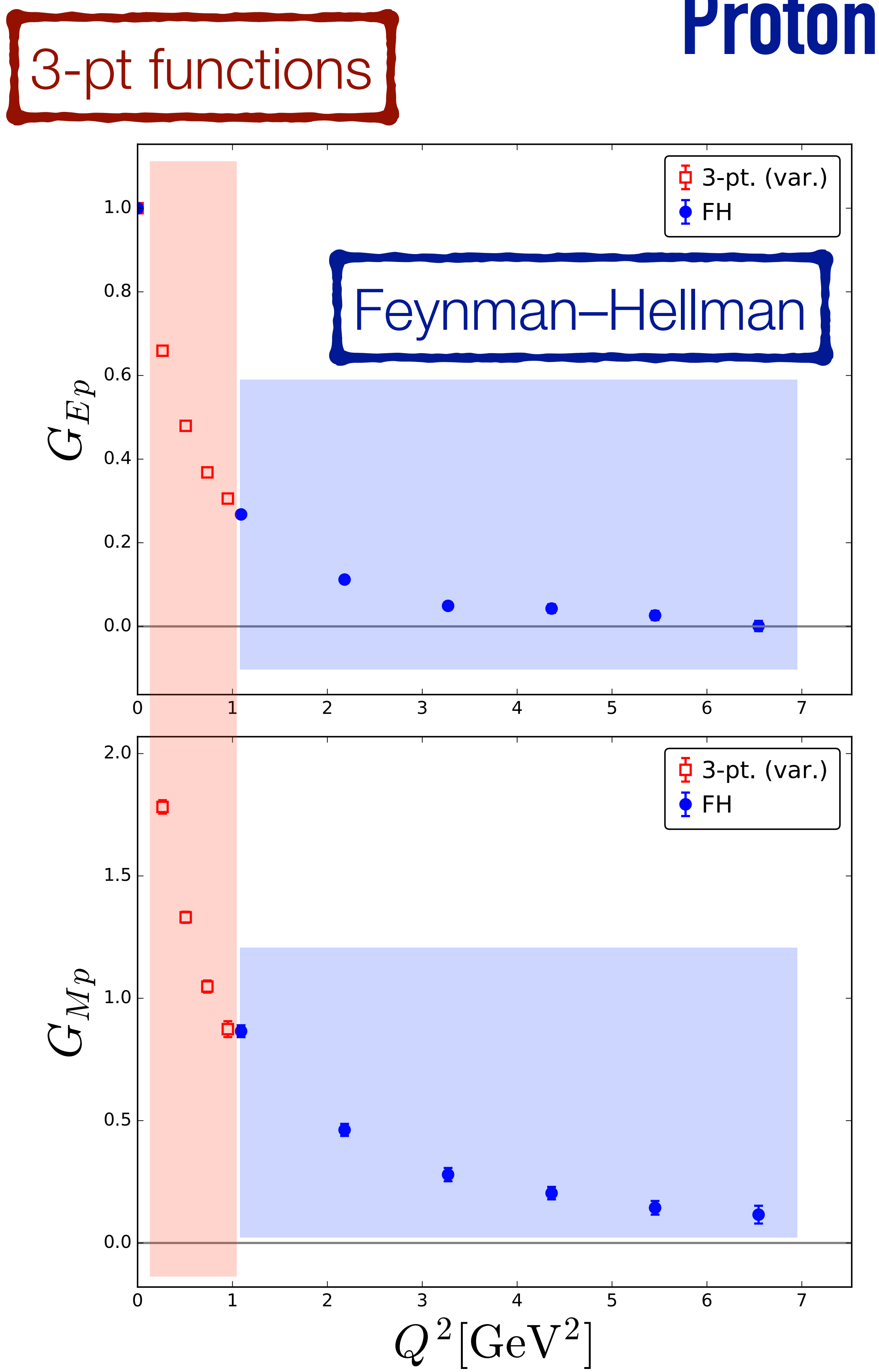
$m_\pi \approx 470$ MeV
Breit frame



[Chambers *et al*. arXiv:1702.01513]

Proton Form Factors

$m_\pi \approx 470$ MeV
Breit frame

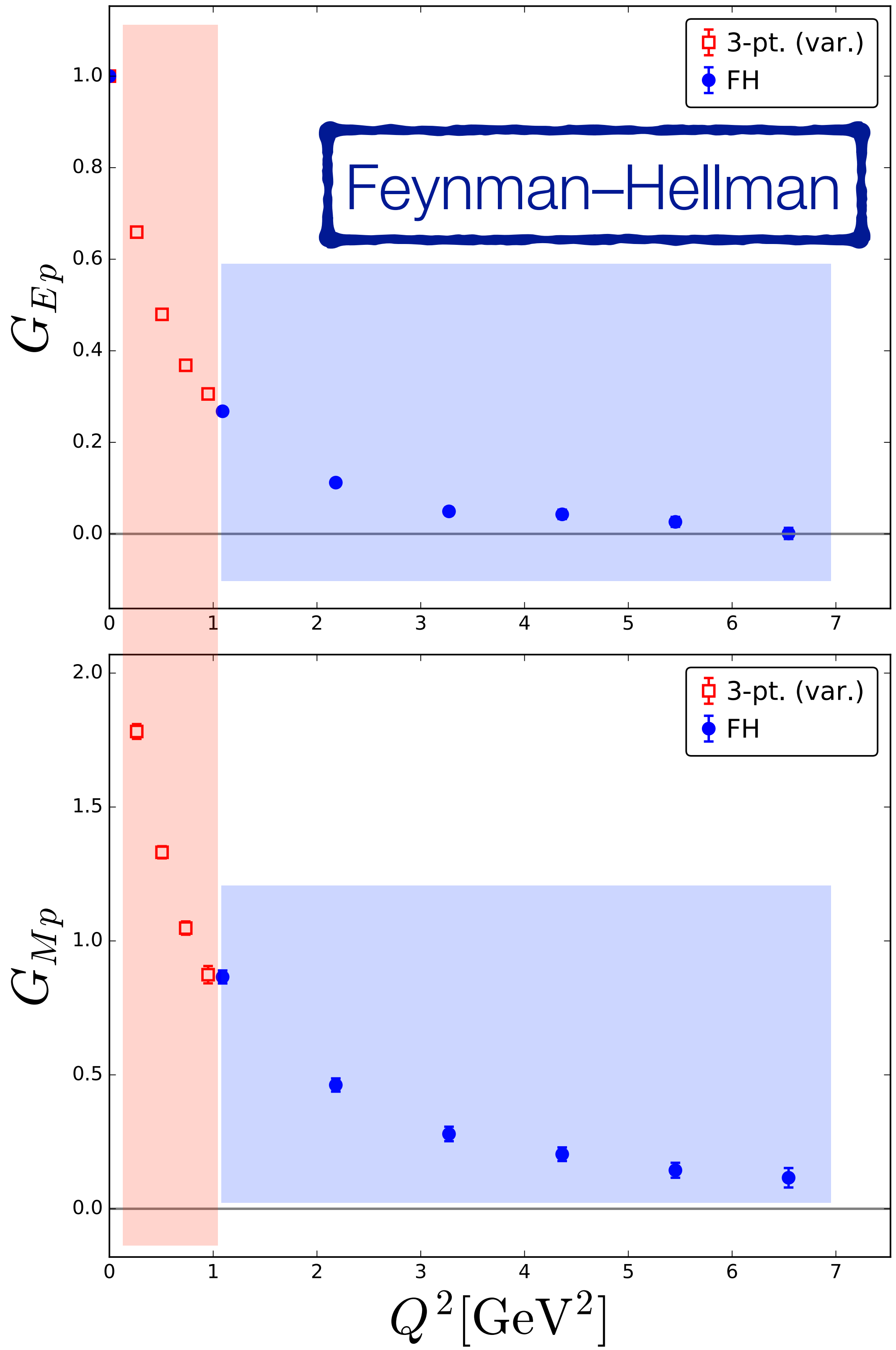


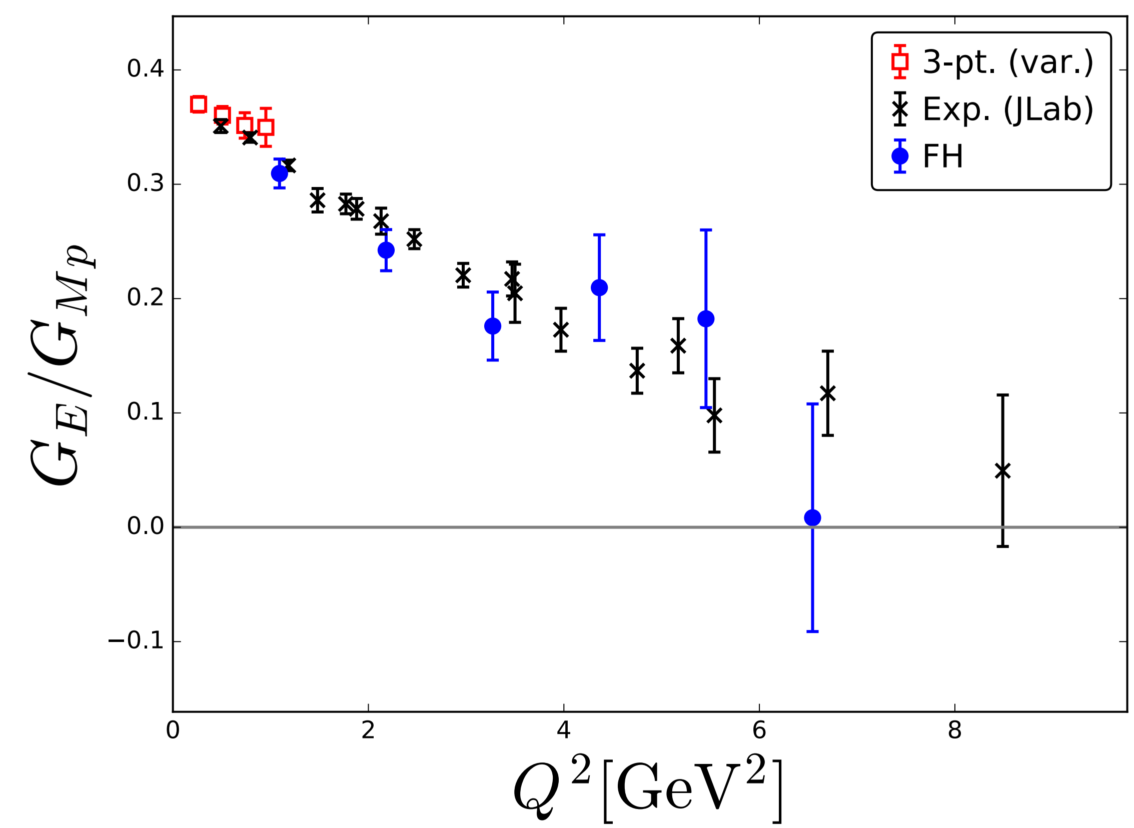
[Chambers *et al*. arXiv:1702.01513]

Proton Form Factors

$m_\pi \approx 470$ MeV
Breit frame

3-pt functions



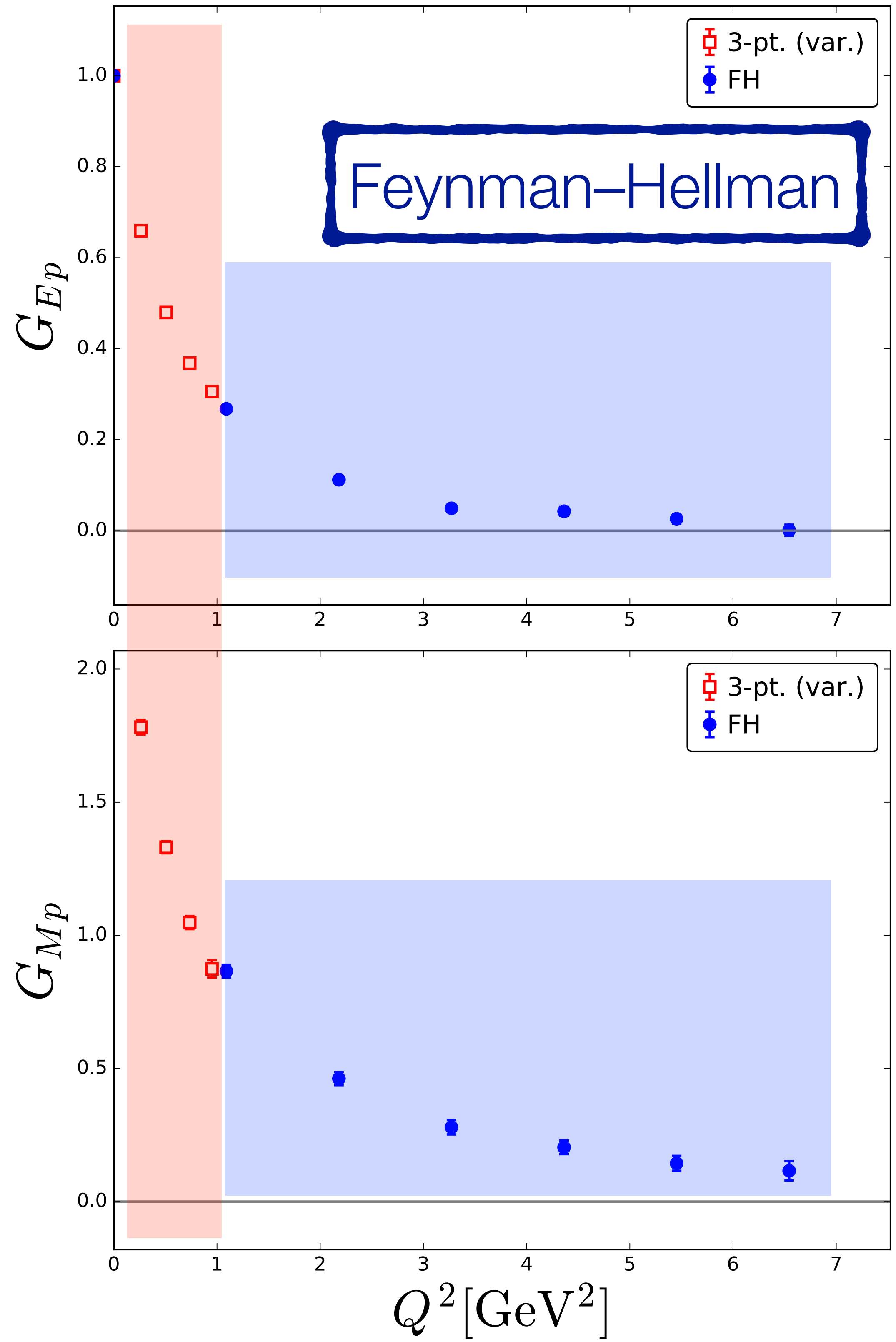


[Chambers *et al*. arXiv:1702.01513]

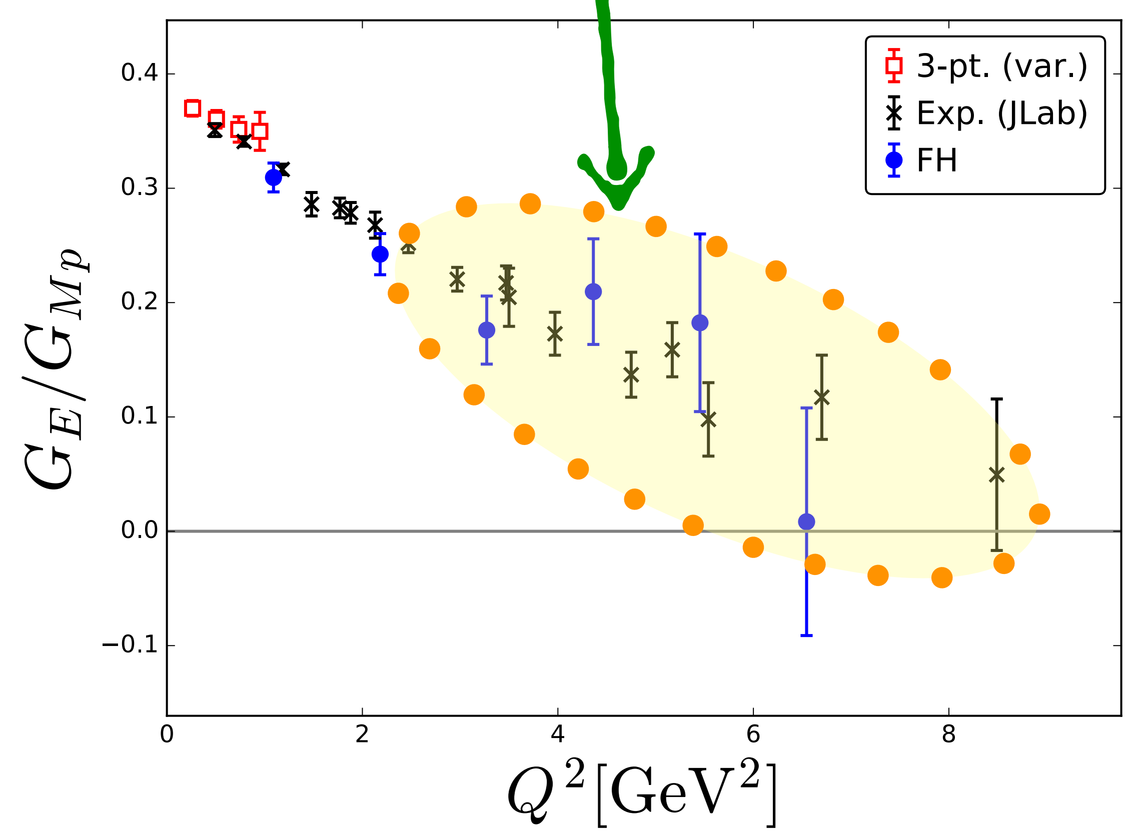
Proton Form Factors

$m_\pi \approx 470$ MeV
Breit frame

3-pt functions



Phenomenologically-interesting region.
Domain dominated by model calculations…
previously prohibitive to study in lattice QCD.

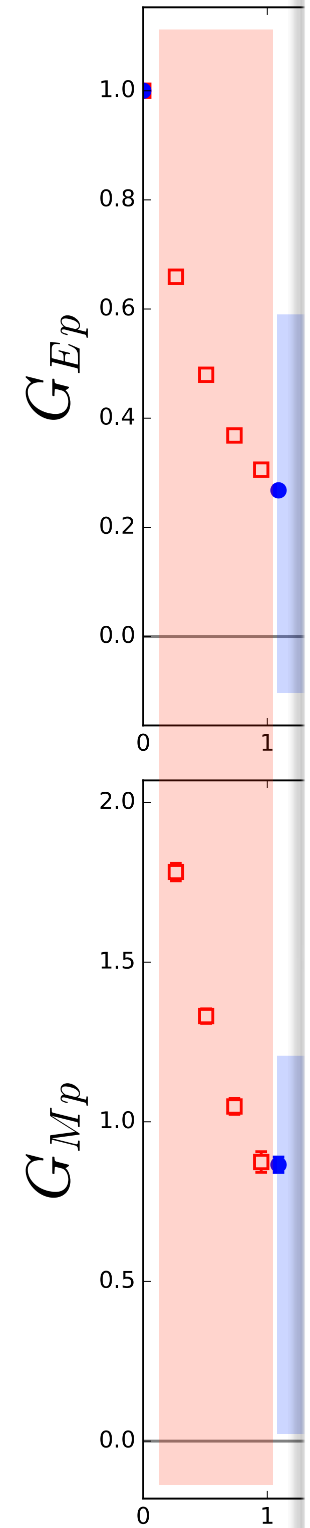


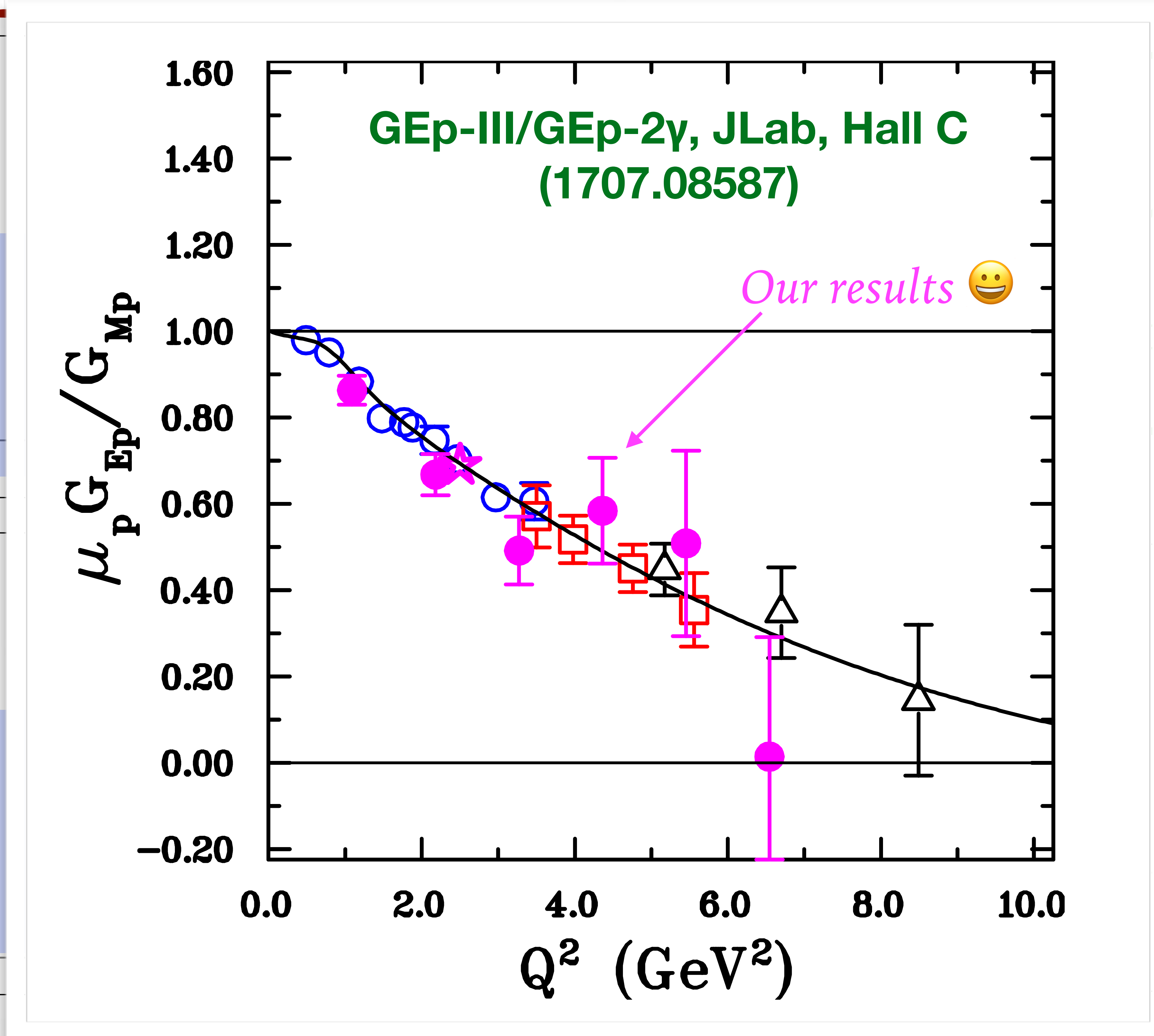
[Chambers *et al*. arXiv:1702.01513]

Proton Form Factors

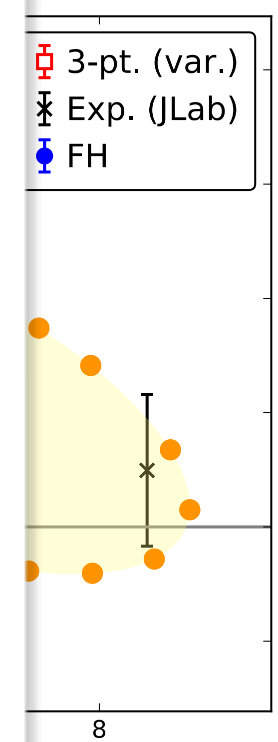
$m_\pi \approx 470$ MeV
Breit frame

3-pt functions





model



[Chambers *et al*. arXiv:1702.01513]

Flavour Form Factors

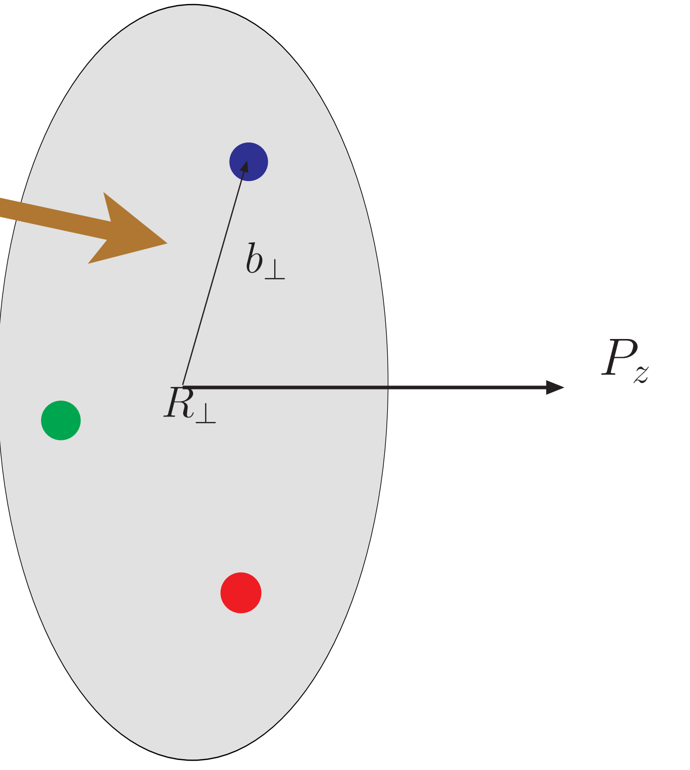
- Separate form factors into individual quark contributions

➡ **Quark densities in transverse plane**

$$q(b_\perp^2) = \int d^2 q_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot q_\perp} F_1(q^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon

Provide information on the size and internal charge densities



Flavour Form Factors

- Separate form factors into individual quark contributions
- Obtain via:
  - Assume charge symmetry

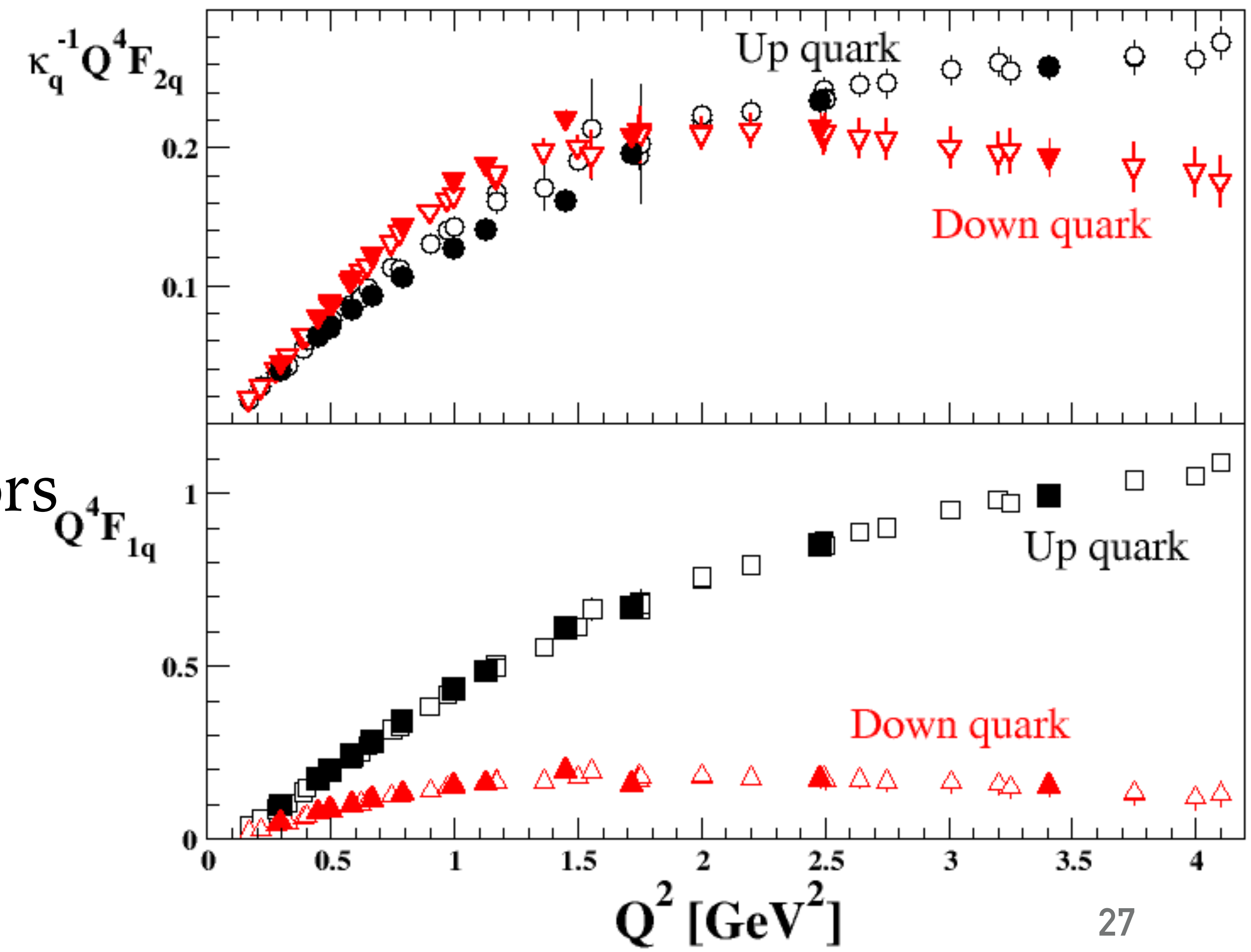
$$G^{p,u}_{E/M} = G^{n,d}_{E/M}$$
$$G^{p,d}_{E/M} = G^{n,u}_{E/M}$$

  - Assume strange FF=0
  - Decompose p and n FFs

$$G^{p}_{E/M} = \frac{2}{3}G^{u}_{E/M} - \frac{1}{3}G^{d}_{E/M}$$
$$G^{n}_{E/M} = \frac{2}{3}G^{d}_{E/M} - \frac{1}{3}G^{u}_{E/M}$$

  - Solve for u and d form factors

[Punjabi et al., EPJ(2015)]



Flavour Form Factors

- Separate form factors into individual quark contributions
- On the Lattice:
    - Work with individual quark sectors
    - Construct hadron form factors

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}\,',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}\,'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\,\Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\;C\gamma_5\;d^b(x)\right)u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

u-quark (connected - 4 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\;C\gamma_5\;d^b(x_2)\right)u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha} \langle\Omega| T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\; C\gamma_5\; d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

u-quark (connected - 4 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\; C\gamma_5\; d^b(x_2)\right) u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\ C\gamma_5\ d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

u-quark (connected - 4 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\ C\gamma_5\ d^b(x_2)\right) u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha} \langle\Omega| T\left[\chi_\alpha(t,\vec{x}_2)\, \mathcal{O}(\tau,\vec{x}_1)\, \overline{\chi}_\beta(0)\right] |\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc} \left(u^{Ta}(x)\ C\gamma_5\ d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

u-quark (connected - 4 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'} \left(u^{Ta}(x_2)\ C\gamma_5\ d^b(x_2)\right) u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0) \left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\; C\gamma_5\; d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

d-quark (connected - 2 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\; C\gamma_5\; d^b(x_2)\right) u^c_\alpha(x_2)\bar{d}(x_1)\mathcal{O}d(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}^{\,\prime},\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}^{\,\prime}\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha} \langle \Omega | T \left[\chi_\alpha(t,\vec{x}_2)\, \mathcal{O}(\tau,\vec{x}_1)\, \overline{\chi}_\beta(0) \right] | \Omega \rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc} \left(u^{Ta}(x)\ C\gamma_5\ d^b(x) \right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

d-quark (connected - 2 terms)

$$\epsilon^{abc}\epsilon^{a'b'c'} \left(u^{Ta}(x_2)\ C\gamma_5\ d^b(x_2) \right) u^c_\alpha(x_2)\bar{d}(x_1)\mathcal{O}d(x_1)\bar{u}^{c'}(0) \left(\bar{d}^{b'}(0) C\gamma_5 \bar{u}^{Ta'}(0) \right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha} \langle\Omega| T\left[\chi_\alpha(t,\vec{x}_2)\, \mathcal{O}(\tau,\vec{x}_1)\, \overline{\chi}_\beta(0)\right] |\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\; C\gamma_5\; d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

u-quark (disconnected)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\; C\gamma_5\; d^b(x_2)\right) u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

proton

$$G_\Gamma(t,\tau;\vec{p}',\vec{p}) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}'\cdot(\vec{x}_2-\vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1}\, \Gamma_{\beta\alpha}\langle\Omega|T\left[\chi_\alpha(t,\vec{x}_2)\,\mathcal{O}(\tau,\vec{x}_1)\,\overline{\chi}_\beta(0)\right]|\Omega\rangle$$

- Use the following interpolating operator to create a proton

$$\chi_\alpha(x) = \epsilon^{abc}\left(u^{Ta}(x)\ C\gamma_5\ d^b(x)\right) u^c_\alpha(x)$$

- And insert the local operator (quark bi-linear) $\bar{q}(x)\mathcal{O}q(x)$

$\mathcal{O}$: Combination of γ matrices and derivatives

- Perform all possible Wick contractions

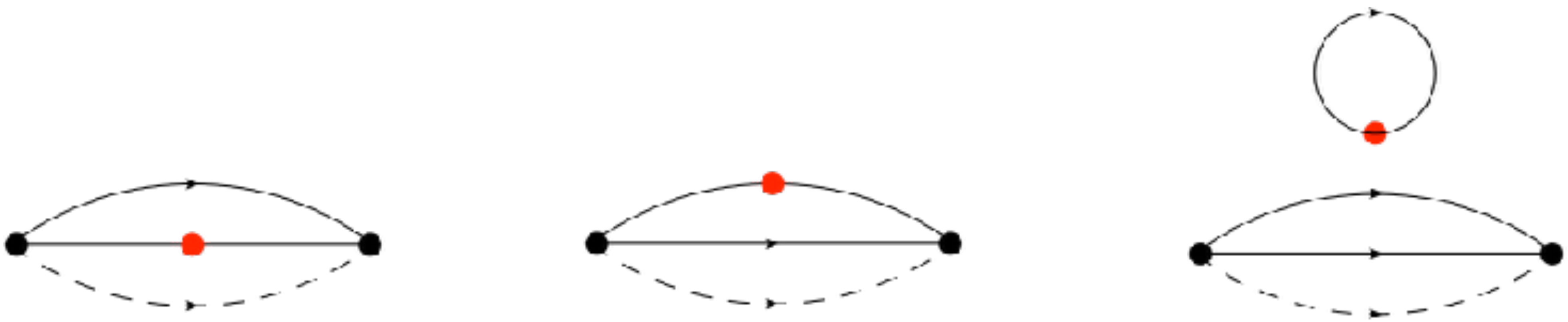
u-quark (disconnected)

$$\epsilon^{abc}\epsilon^{a'b'c'}\left(u^{Ta}(x_2)\ C\gamma_5\ d^b(x_2)\right) u^c_\alpha(x_2)\bar{u}(x_1)\mathcal{O}u(x_1)\bar{u}^{c'}(0)\left(\bar{d}^{b'}(0)C\gamma_5\bar{u}^{Ta'}(0)\right)$$

Lattice 3pt Functions

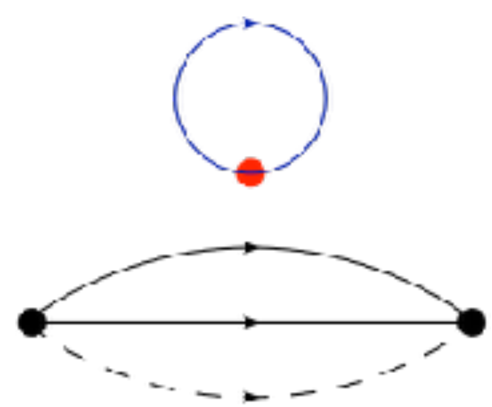
proton

- Pictorially:
  - u-quark



  - d-quark

  - s-quark

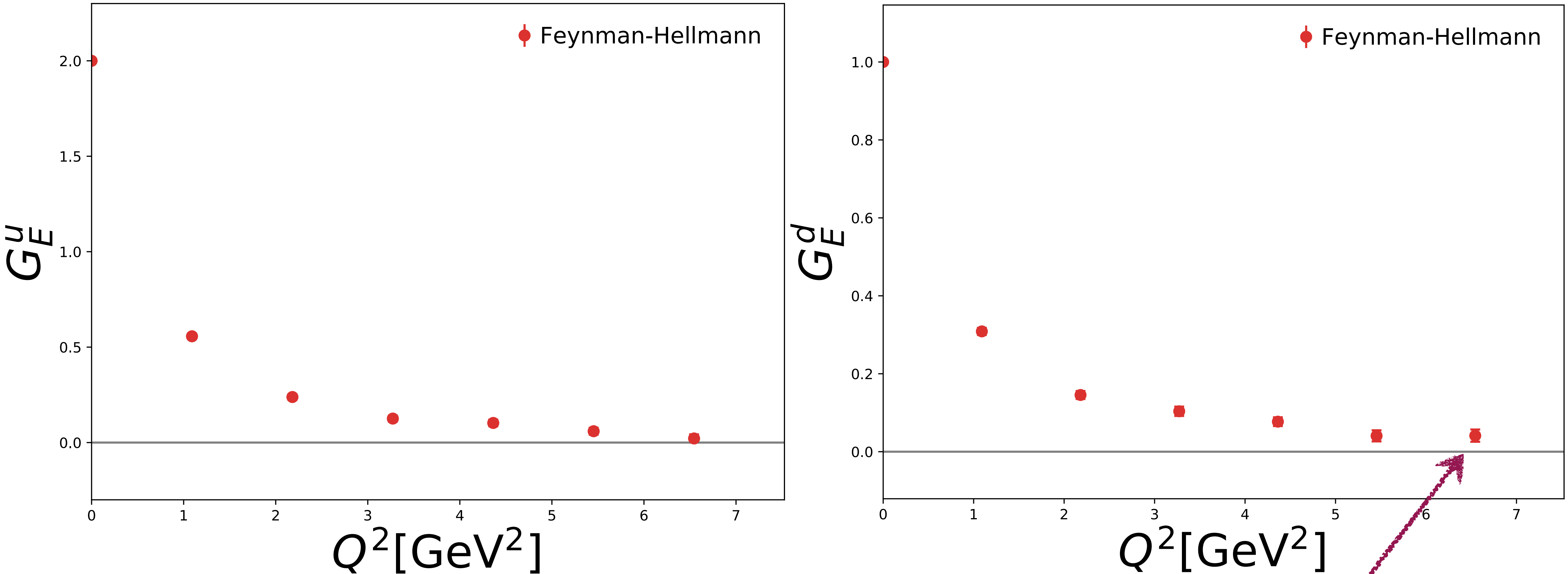


Flavour Form Factors (Connected)

Preliminary

$m_\pi \approx 470$ MeV

➤ (Unit charged) u and d contributions to G_E

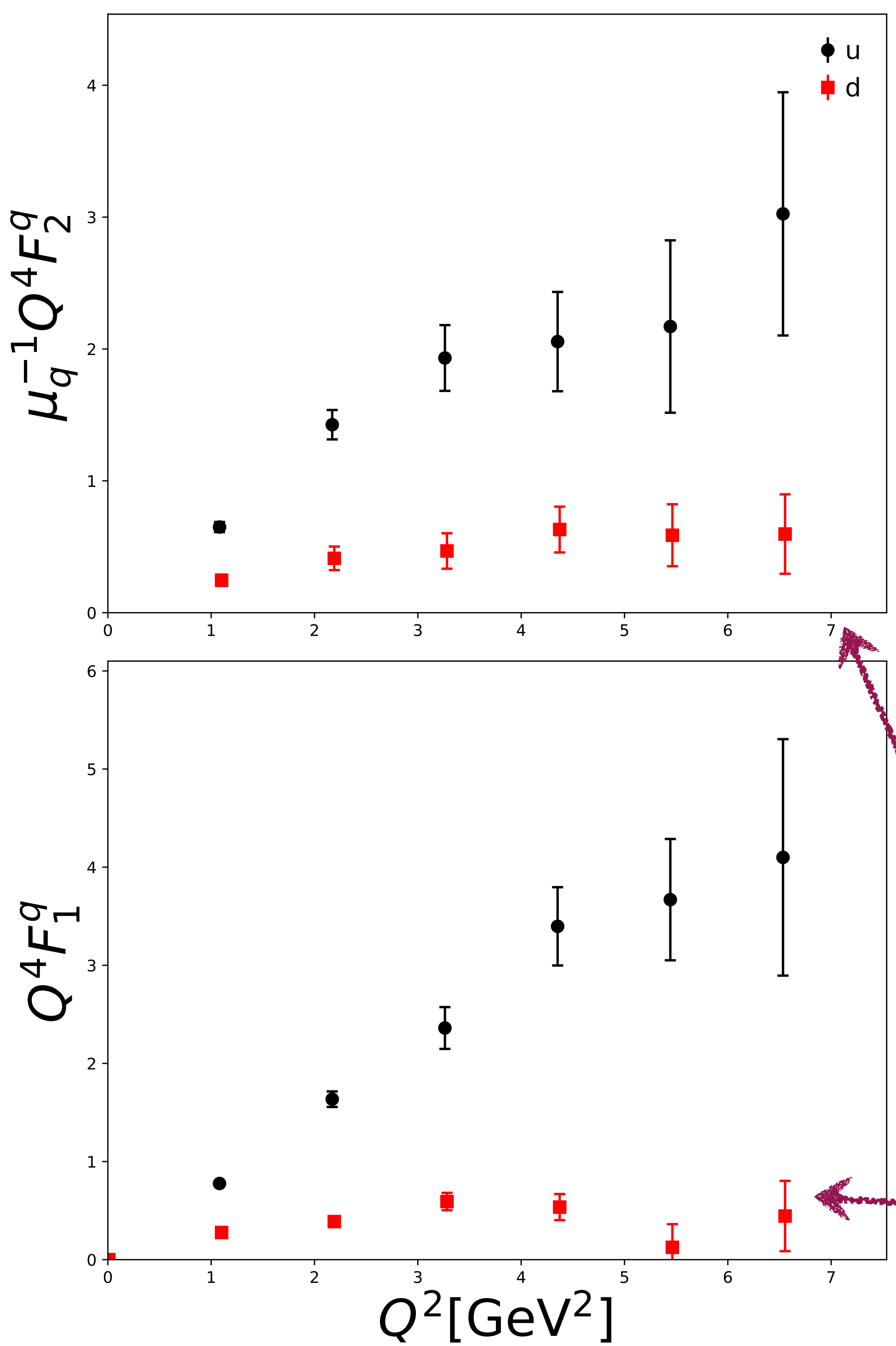


Clear non-zero signal observed

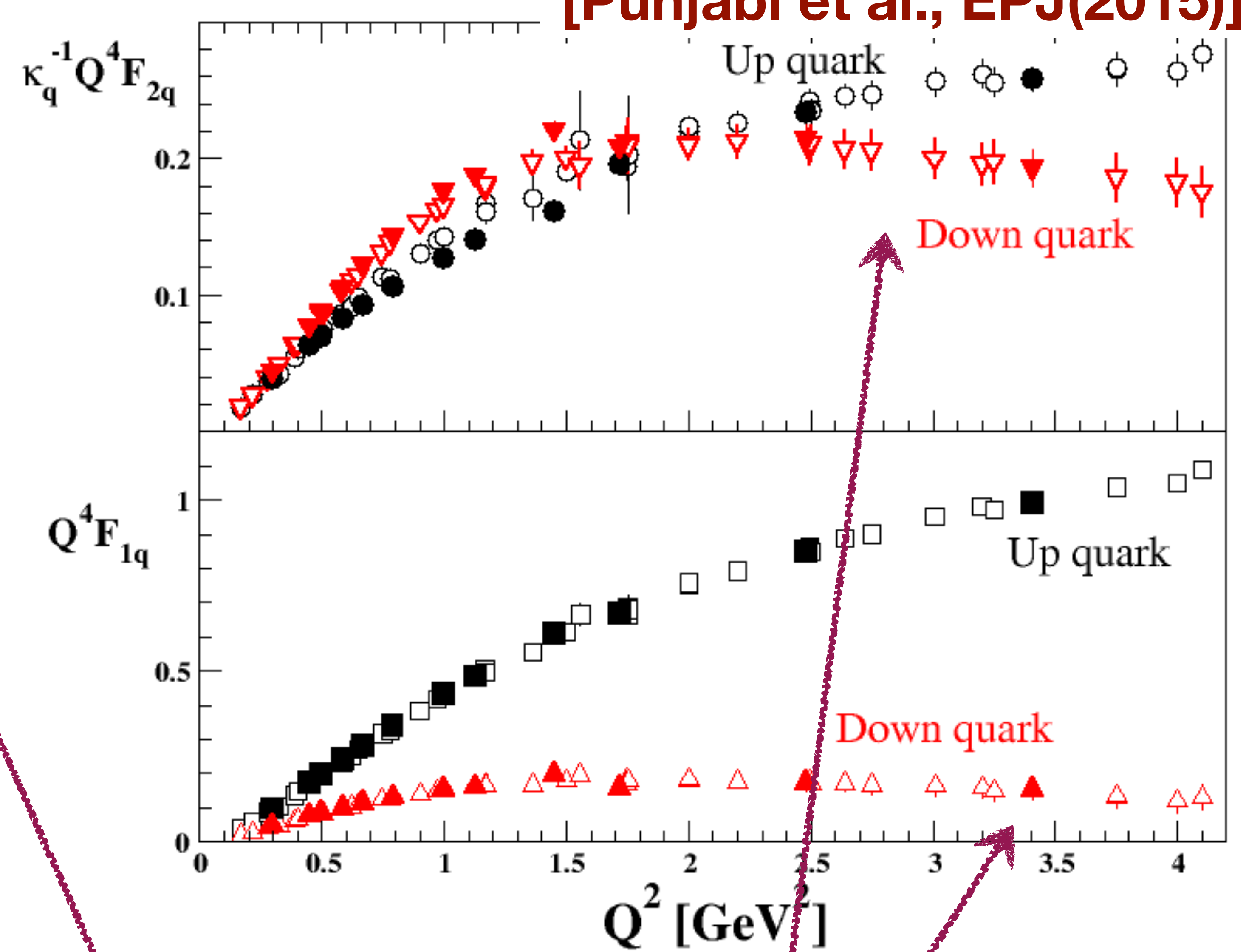
Preliminary

Flavour Form Factors (Connected)

$m_\pi \approx 470$ MeV



[Punjabi et al., EPJ(2015)]



d quark plateau observed

Impact Parameter Gpds

[Burkardt, (2000)]

Quark densities in the transverse plane

No momentum transfer in longitunidal direction

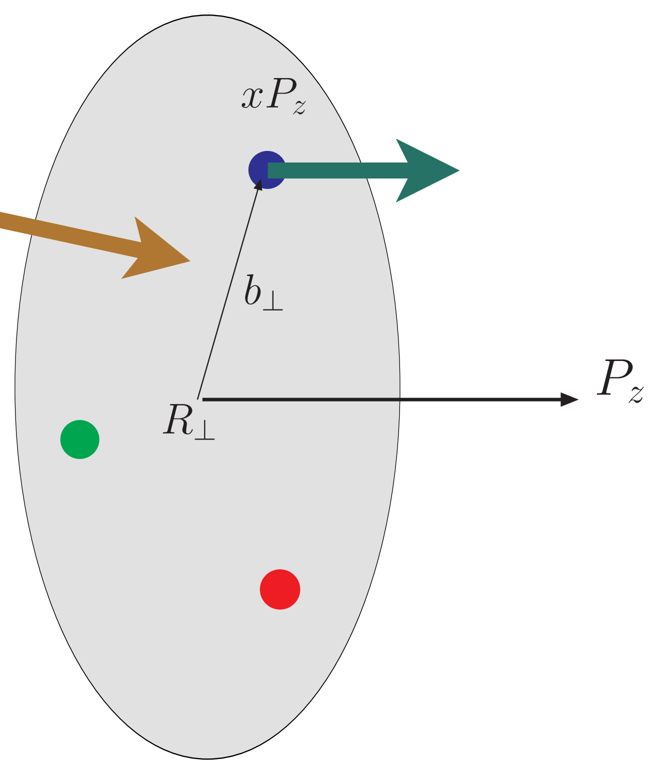
- Quark (charge) distribution in the transverse plane

$$q(b_\perp^2) = \int d^2 q_\perp \, e^{-i\vec{b}_\perp \cdot q_\perp} F_1(q^2)$$

- Probabilistic interpretation of GPDs, e.g. $H(x,\xi,q^2)$ at $\xi = 0$

$$q(x, b_\perp^2) = \int d^2 q_\perp \, e^{-i\vec{b}_\perp \cdot \vec{q}_\perp} H(x, 0, q_\perp^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon



Decompose into contributions from individual quarks with momentum fraction, x

Spin Dependence?

Diehl & Hägler, EPJ C44 (2005) 87-101 [hep-ph/0504175]

- What about nucleon/quark spin?
- How do they affect these quark distributions?
- Consider transverse nucleon $\vec{S}_\perp$ and/or quark $\vec{s}_\perp$ polarisations
- Probability density for finding quark at impact parameter $\vec{b}_\perp$ is then

$$F(b_\perp^2) = \int d^2 q_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \vec{q}_\perp} \, F(q_\perp^2)$$

$$\begin{aligned}
\rho^n(b_\perp, s_\perp, S_\perp) &= \int_{-1}^{1} dx \, x^{n-1} \, \rho(x, b_\perp, s_\perp, S_\perp) \\
&= \frac{1}{2}\Big\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T\,n0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{T\,n0}(b_\perp^2) \right) \\
&+ \frac{b_\perp^i \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{T\,n0}(b_\perp^2) \right) + s_\perp^i \left(2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij} \right) S_\perp^j \frac{1}{m^2} \tilde{A}''_{T\,n0}(b_\perp^2) \Big\}
\end{aligned}$$

- These *A*, *B*, …, functions define the moments w.r.t *x* of GPDs: "generalised form factors"
- This talk: only $n=1$ (F_1, F_2, g_T, …)

Spin Dependence?

- What about nucleon/quark spin?
- How do they affect these quark distributions?
- Consider transverse nucleon $\vec{S}_\perp$ and/or quark $\vec{s}_\perp$ polarisations
- Probability density for finding quark at impact parameter $\vec{b}_\perp$ is then

Diehl & Hägler, EPJ C44 (2005) 87-101 [hep-ph/0504175]

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1}\, \rho(x, b_\perp, s_\perp, S_\perp)$$

$$= \frac{1}{2}\Big\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \Big(A_{T\,n0}(b_\perp^2) - \frac{1}{4m^2}\nabla_{b_\perp}\tilde{A}_{T\,n0}(b_\perp^2)\Big)$$

$$+ \frac{b_\perp^i \epsilon^{ji}}{m}\big(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{T\,n0}(b_\perp^2)\big) + s_\perp^i\big(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\big)S_\perp^j \frac{1}{m^2}\tilde{A}''_{T\,n0}(b_\perp^2)\Big\}$$

Unpolarised

$$F(b_\perp^2) = \int d^2 q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\vec{q}_\perp}\, F(q_\perp^2)$$

Spin Dependence?

- What about nucleon/quark spin?
- How do they affect these quark distributions?
- Consider transverse nucleon $\vec{S}_\perp$ and/or quark $\vec{s}_\perp$ polarisations
- Probability density for finding quark at impact parameter $\vec{b}_\perp$ is then

Diehl & Hägler, EPJ C44 (2005) 87-101 [hep-ph/0504175]

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1}\, \rho(x, b_\perp, s_\perp, S_\perp)$$

$$= \frac{1}{2}\Big\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \Big(A_{T\,n0}(b_\perp^2) - \frac{1}{4m^2}\nabla_{b_\perp}\tilde{A}_{T\,n0}(b_\perp^2)\Big)$$

$$+ \frac{b_\perp^i \epsilon^{ji}}{m}\big(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{T\,n0}(b_\perp^2)\big) + s_\perp^i \big(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\big) S_\perp^j \frac{1}{m^2}\tilde{A}''_{T\,n0}(b_\perp^2)\Big\}$$

Polarised Nucleon

$$F(b_\perp^2) = \int d^2 q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\vec{q}_\perp}\, F(q_\perp^2)$$

Spin Dependence?

- What about nucleon/quark spin?
- How do they affect these quark distributions?
- Consider transverse nucleon $\vec{S}_\perp$ and/or quark $\vec{s}_\perp$ polarisations
- Probability density for finding quark at impact parameter $\vec{b}_\perp$ is then

Diehl & Hägler, EPJ C44 (2005) 87-101 [hep-ph/0504175]

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1}\, \rho(x, b_\perp, s_\perp, S_\perp)$$
$$= \frac{1}{2}\Big\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \Big(A_{T\,n0}(b_\perp^2) - \frac{1}{4m^2}\nabla_{b_\perp}\tilde{A}_{T\,n0}(b_\perp^2)\Big)$$
$$+ \frac{b_\perp^i \epsilon^{ji}}{m}\big(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{T\,n0}(b_\perp^2)\big) + s_\perp^i \big(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\big) S_\perp^j \frac{1}{m^2}\tilde{A}''_{T\,n0}(b_\perp^2)\Big\}$$

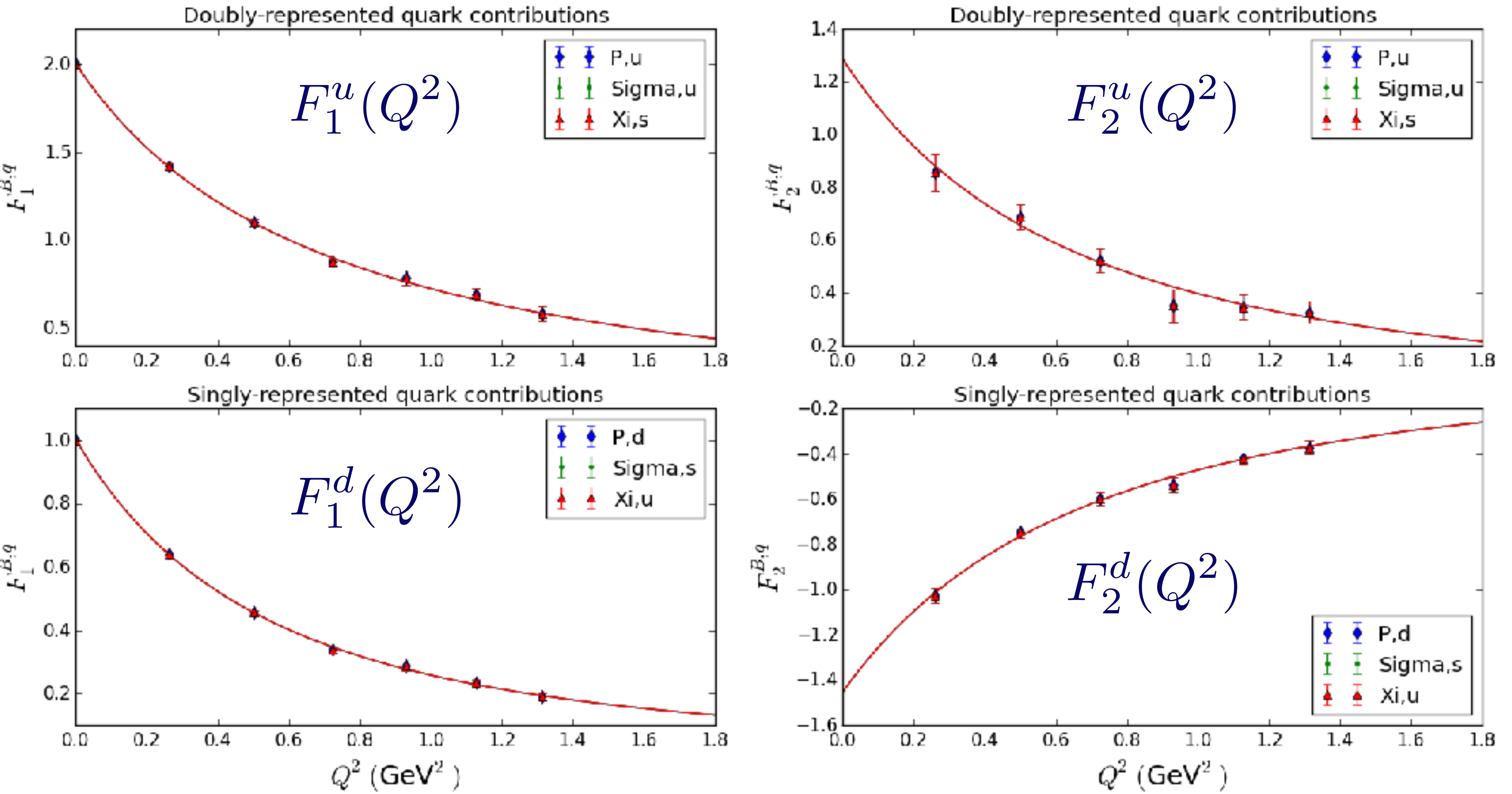
Polarised quark

$$F(b_\perp^2) = \int d^2 q_\perp\, e^{-i\vec{b}_\perp\cdot\vec{q}_\perp}\, F(q_\perp^2)$$

Vector Form Factors

SU(3) symmetric

$$F_1(Q^2) = \frac{F(0)}{(1 + Q^2/M^2)^2}$$

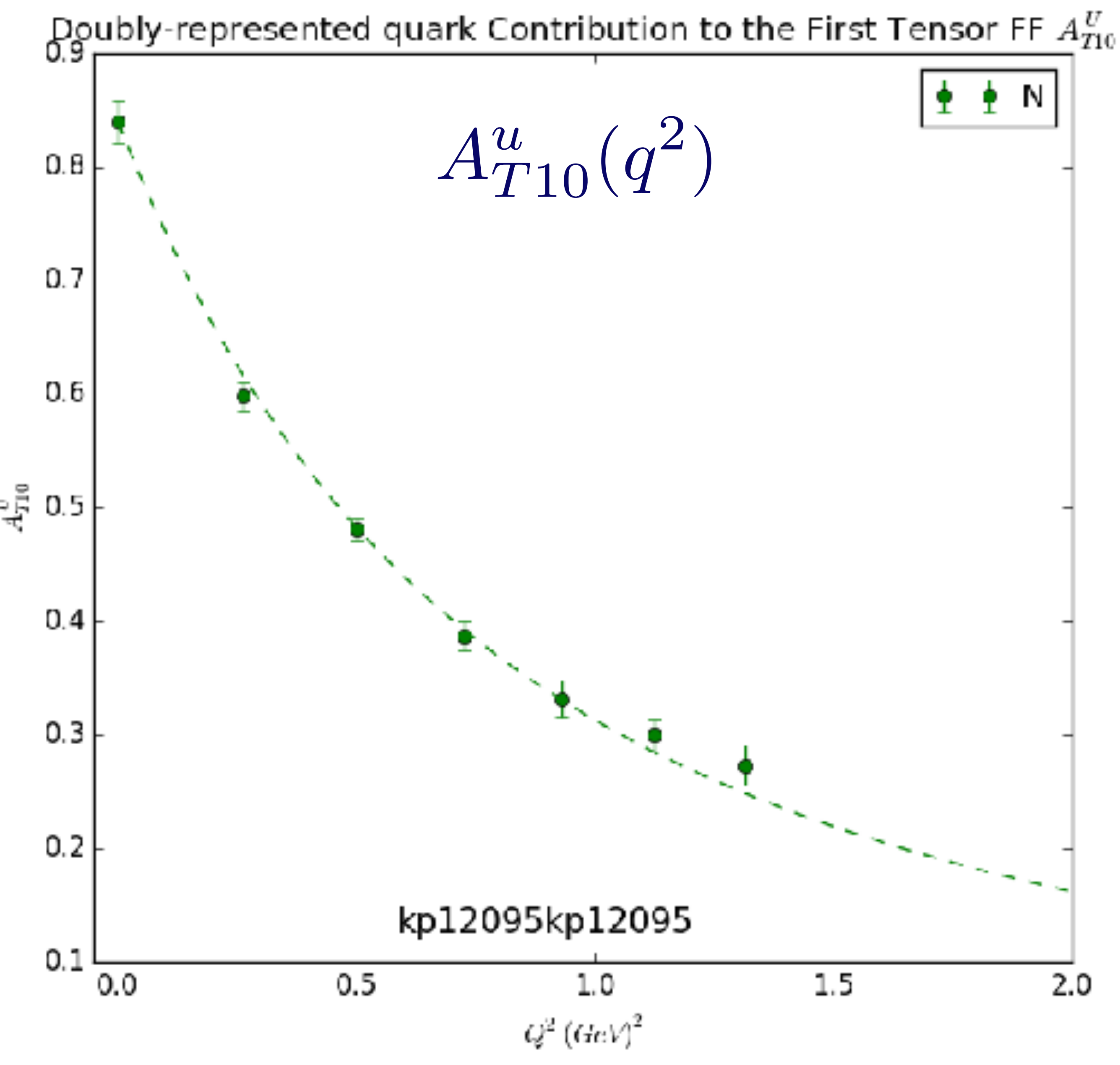


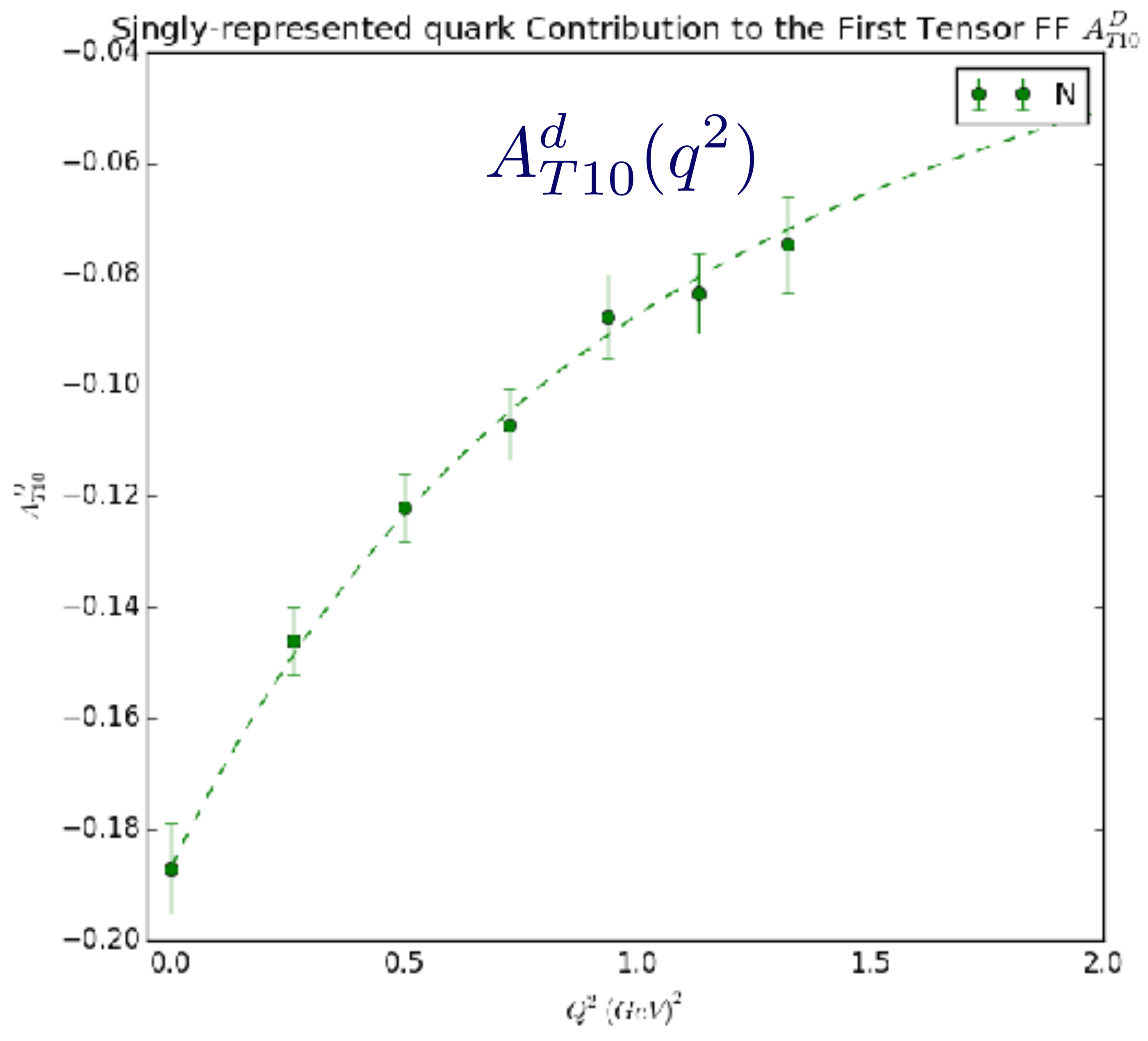
Tensor Form Factors

➤ Tensor form factors are obtained from the matrix elements

$$\bar{P}^{\mu} = \frac{1}{2}(p' + p)^{\mu}$$

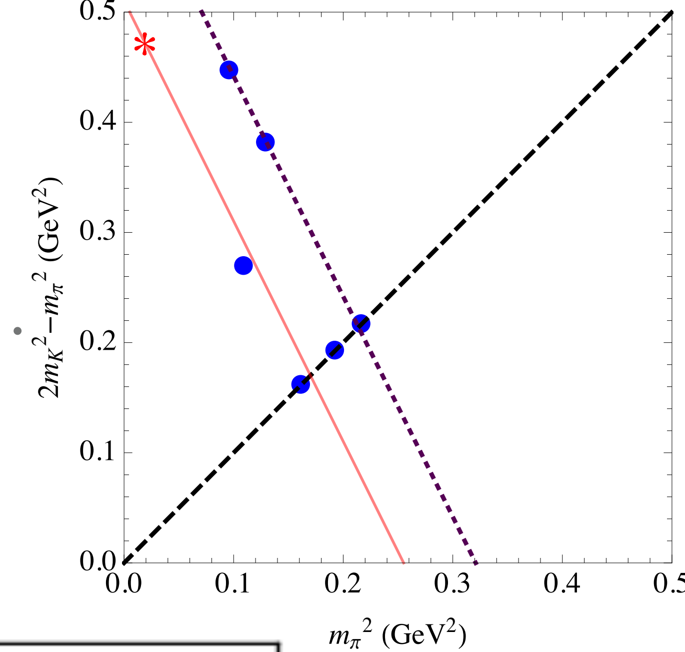
$$\langle p',\, s'|\bar{\psi}(0) i\sigma^{\mu\nu}\psi(0)|p,\, s\rangle = \bar{u}(p',\, s')\{i\sigma^{\mu\nu} A_{T10}(q^2) + \frac{\overline{P}^{[\mu}q^{\nu]}}{m^2}\tilde{A}_{T10}(q^2) + \frac{\gamma^{[\mu}q^{\nu]}}{2m} B_{T10}(q^2)\}u(p,\, s)$$



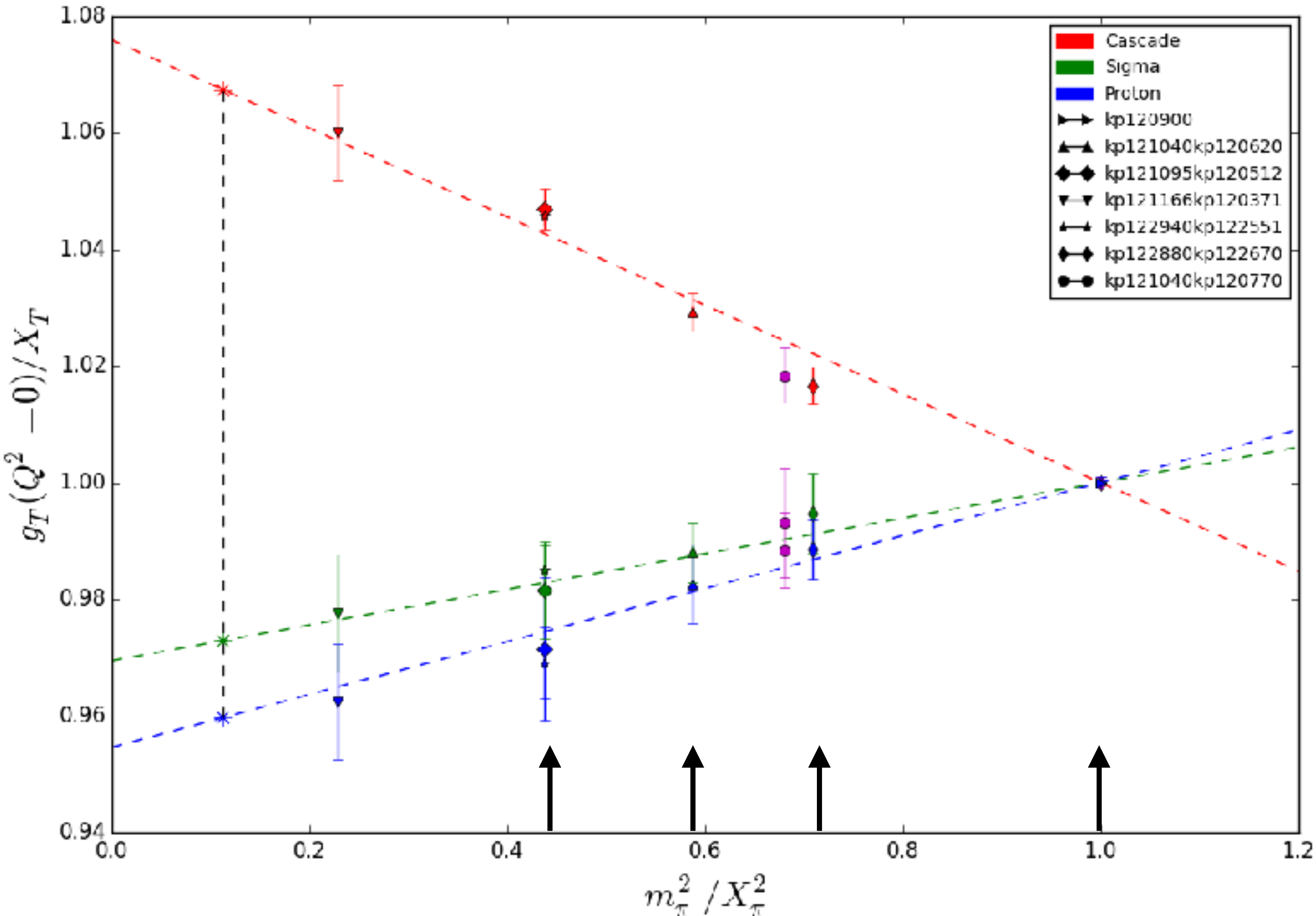


u-Quark Fan

$$Q^2 = 0$$



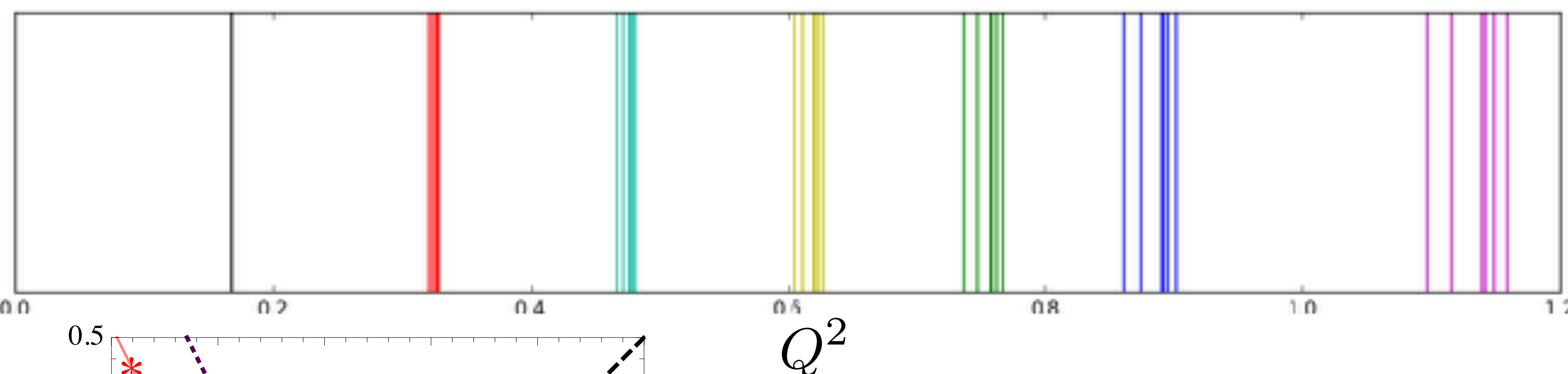
- Tensor charge (doubly-represented quark contribution)
- Ratio "fans out" from SU(3)-symmetric point

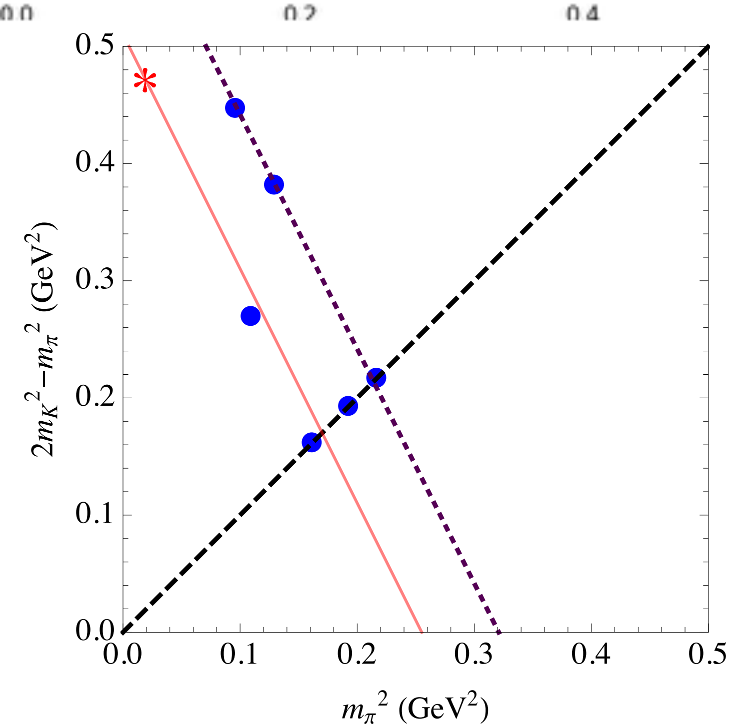


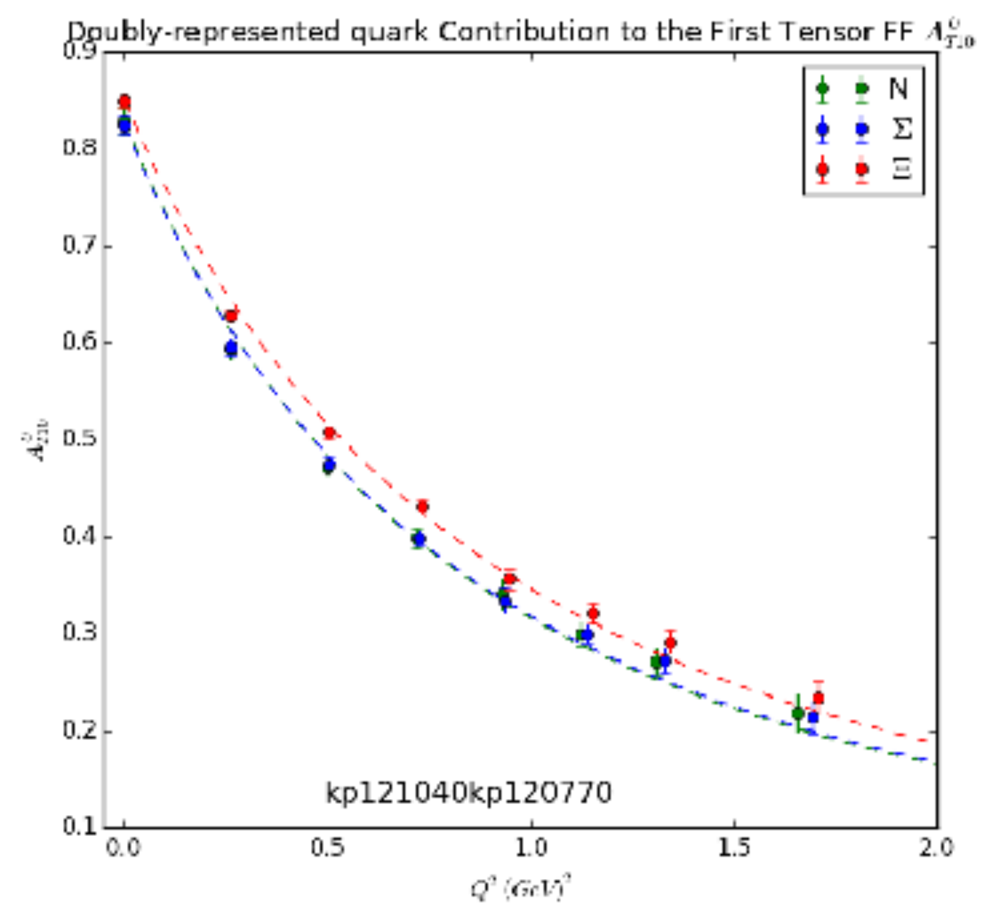
used in fit

Form Factors at $Q^2 \neq 0$

- SU(3) breaking expansion valid at a fixed Q^2
- All ensembles have $L=32$
- Small difference in Q^2 comes from difference in baryon masses across ensembles
- Bin results in Q^2 and shift form factors to centre of bin using dipole form

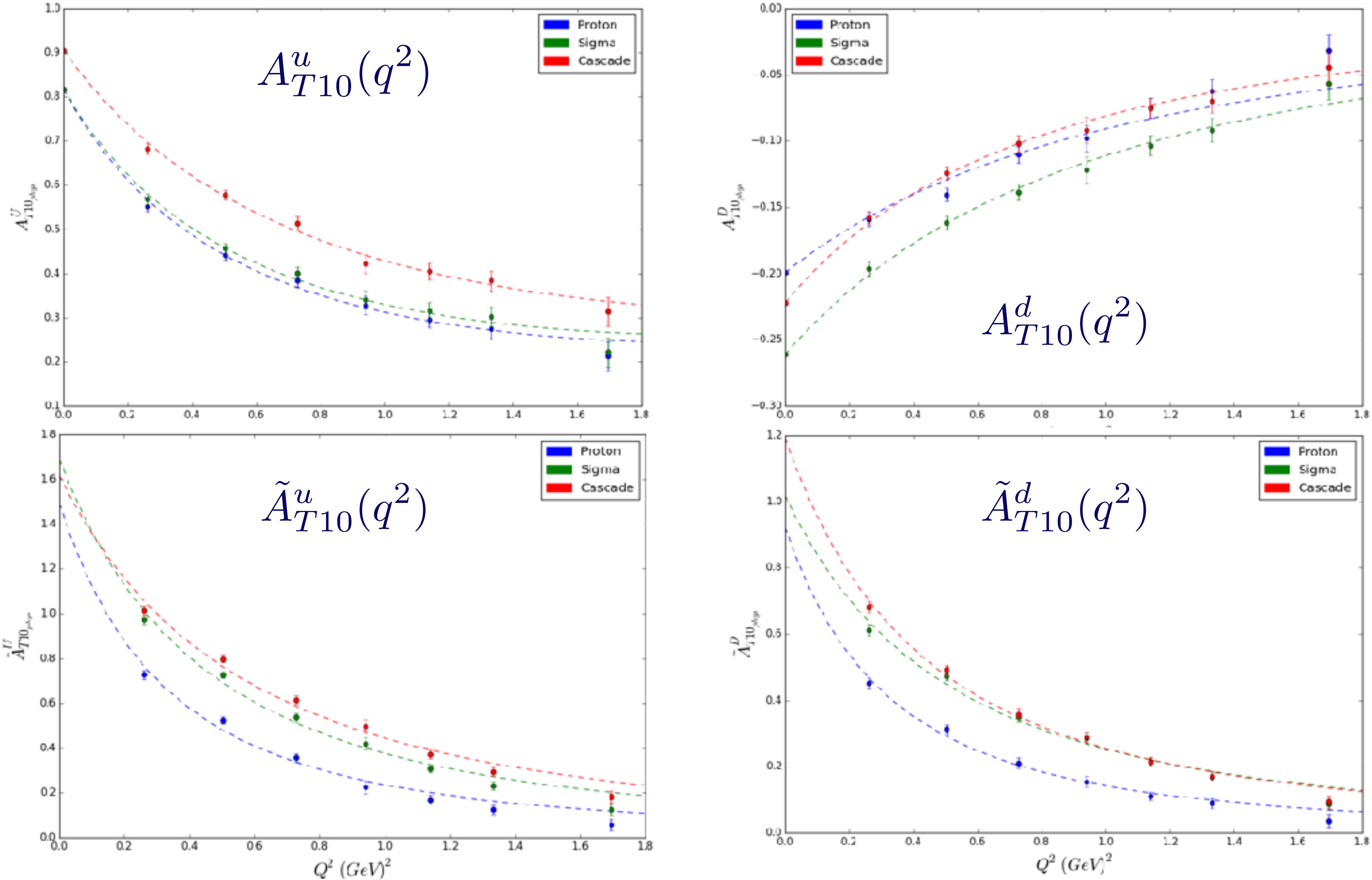






Physical Mass Form Factors

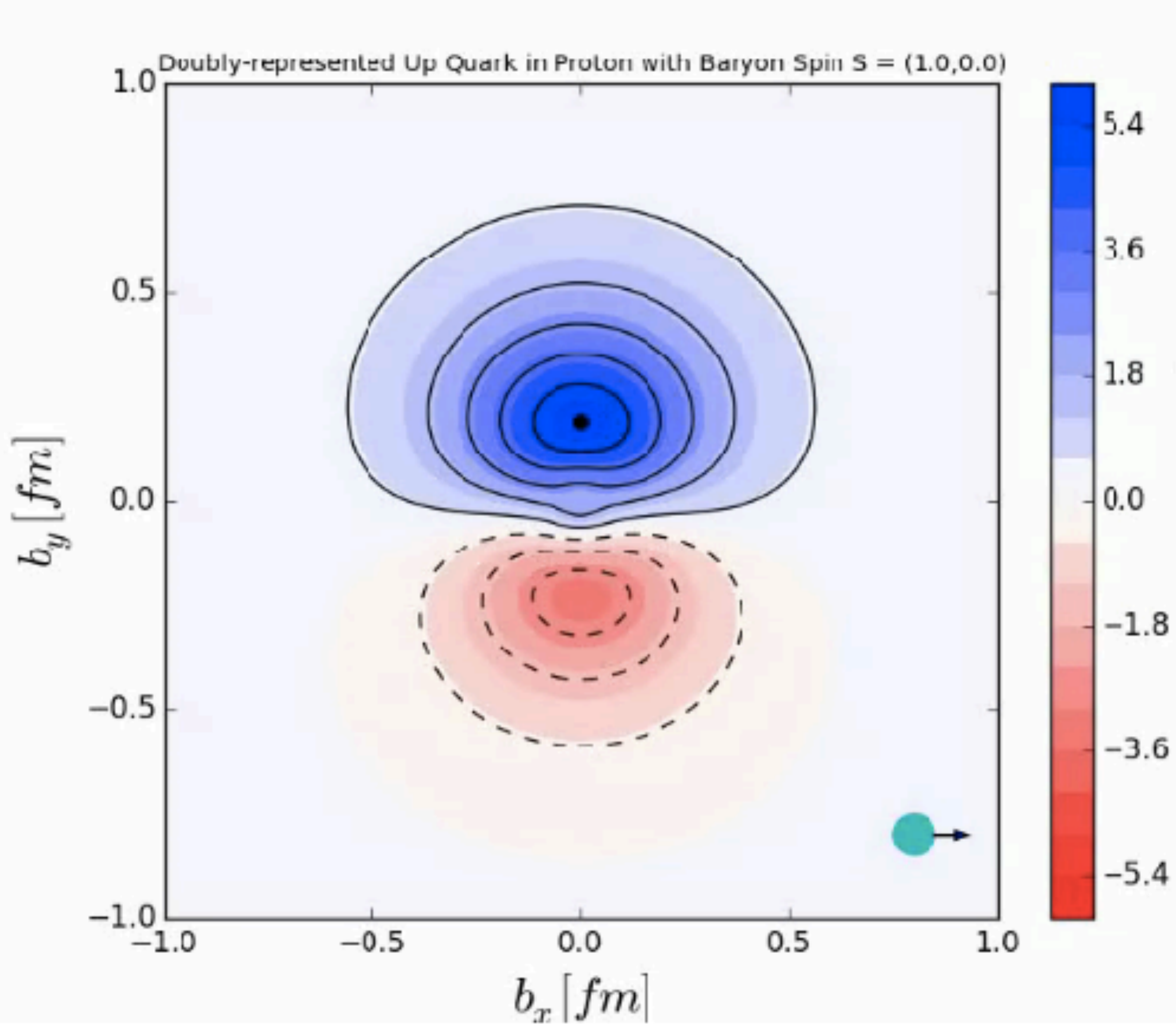
Preliminary



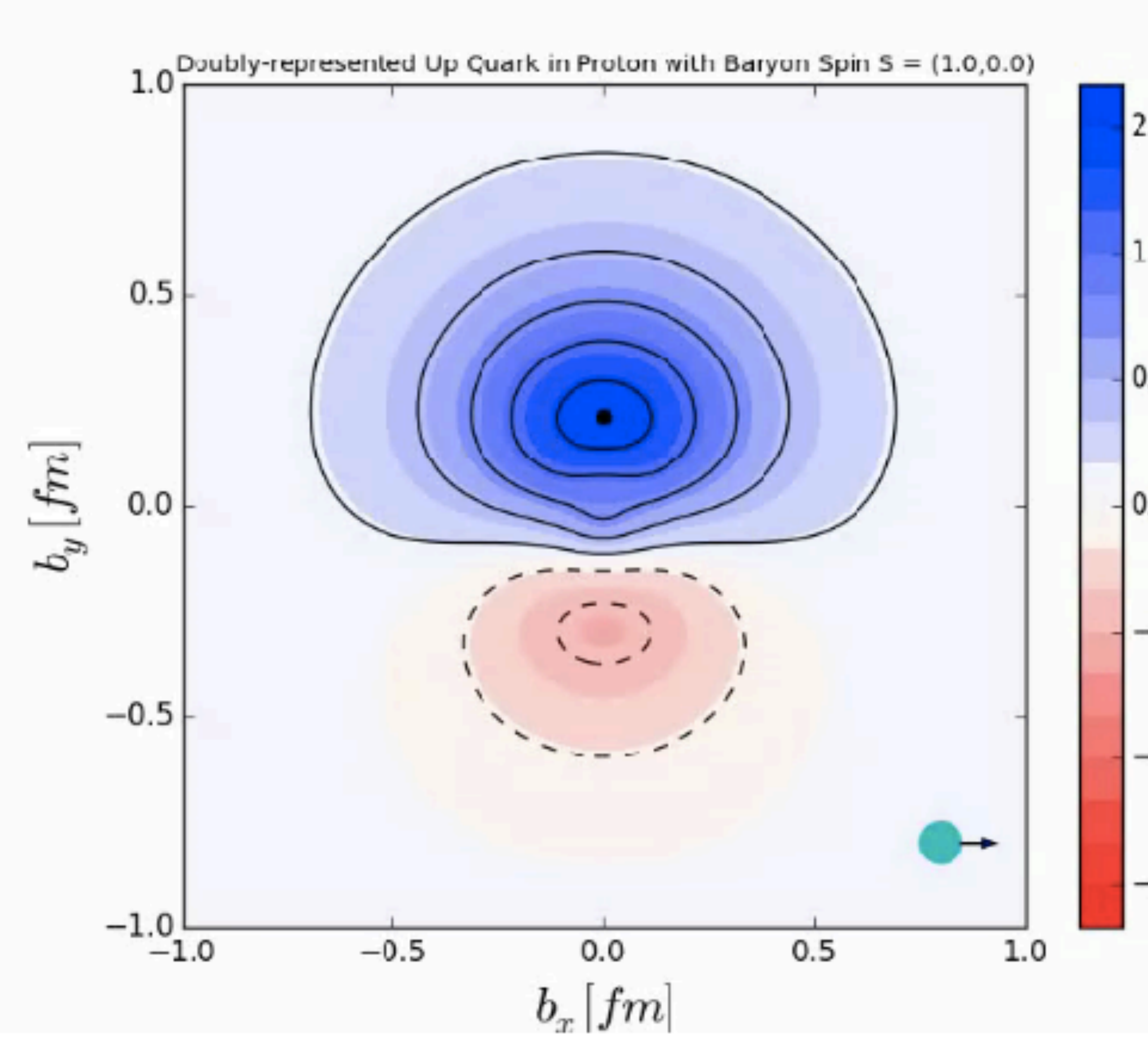
u-Quark

Hyperon spin

Proton



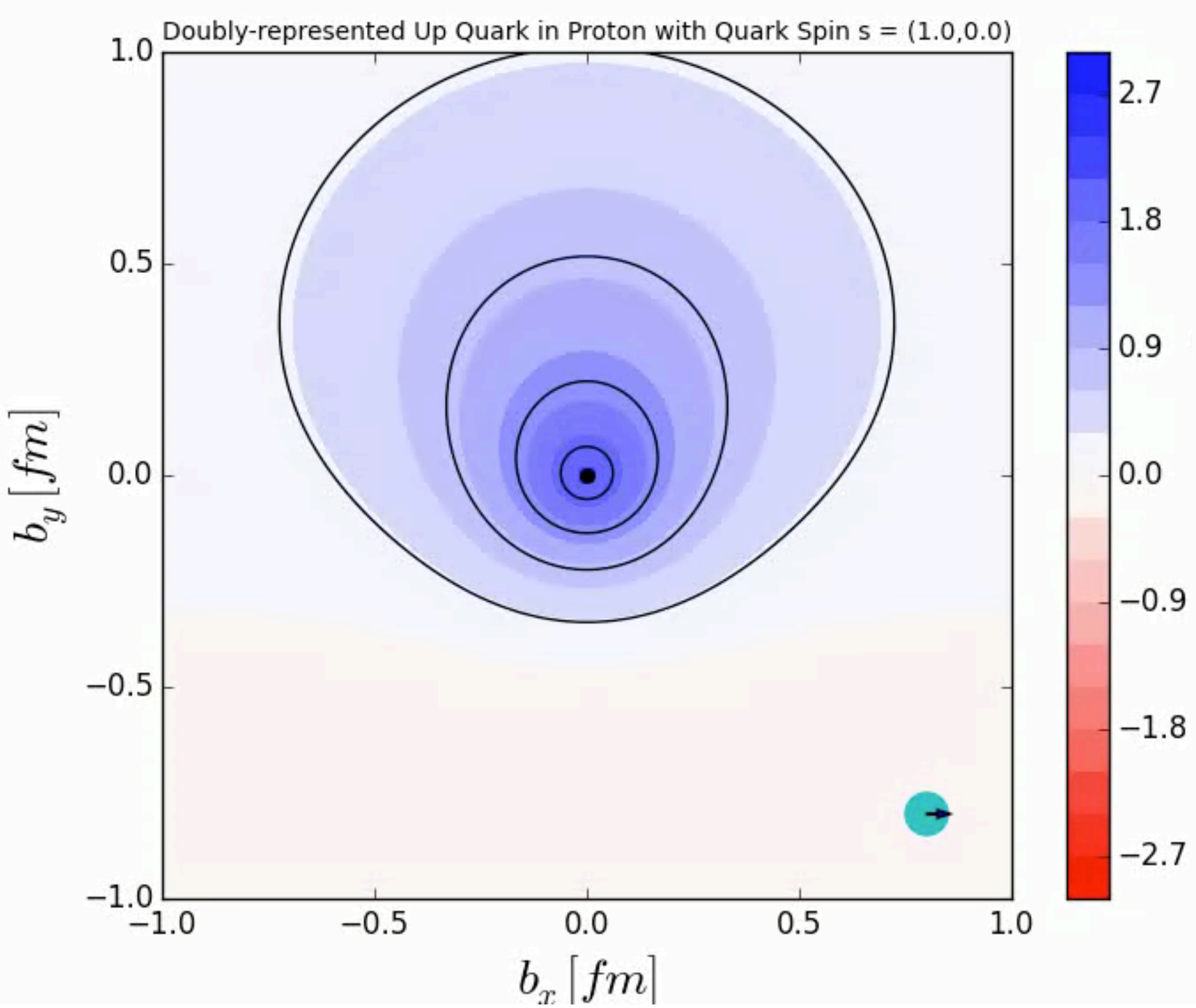
Sigma



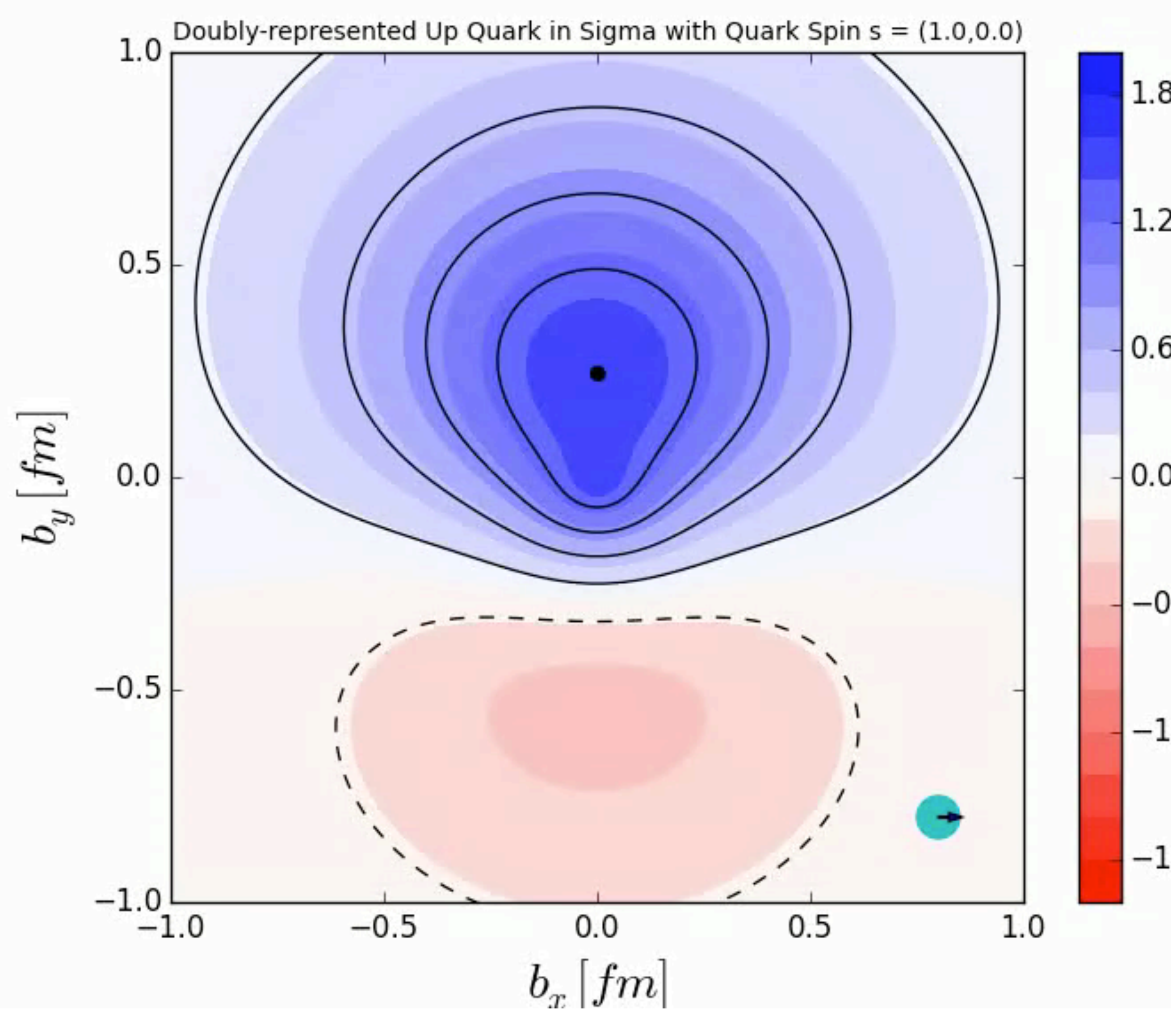
u-Quark

Quark spin

Proton

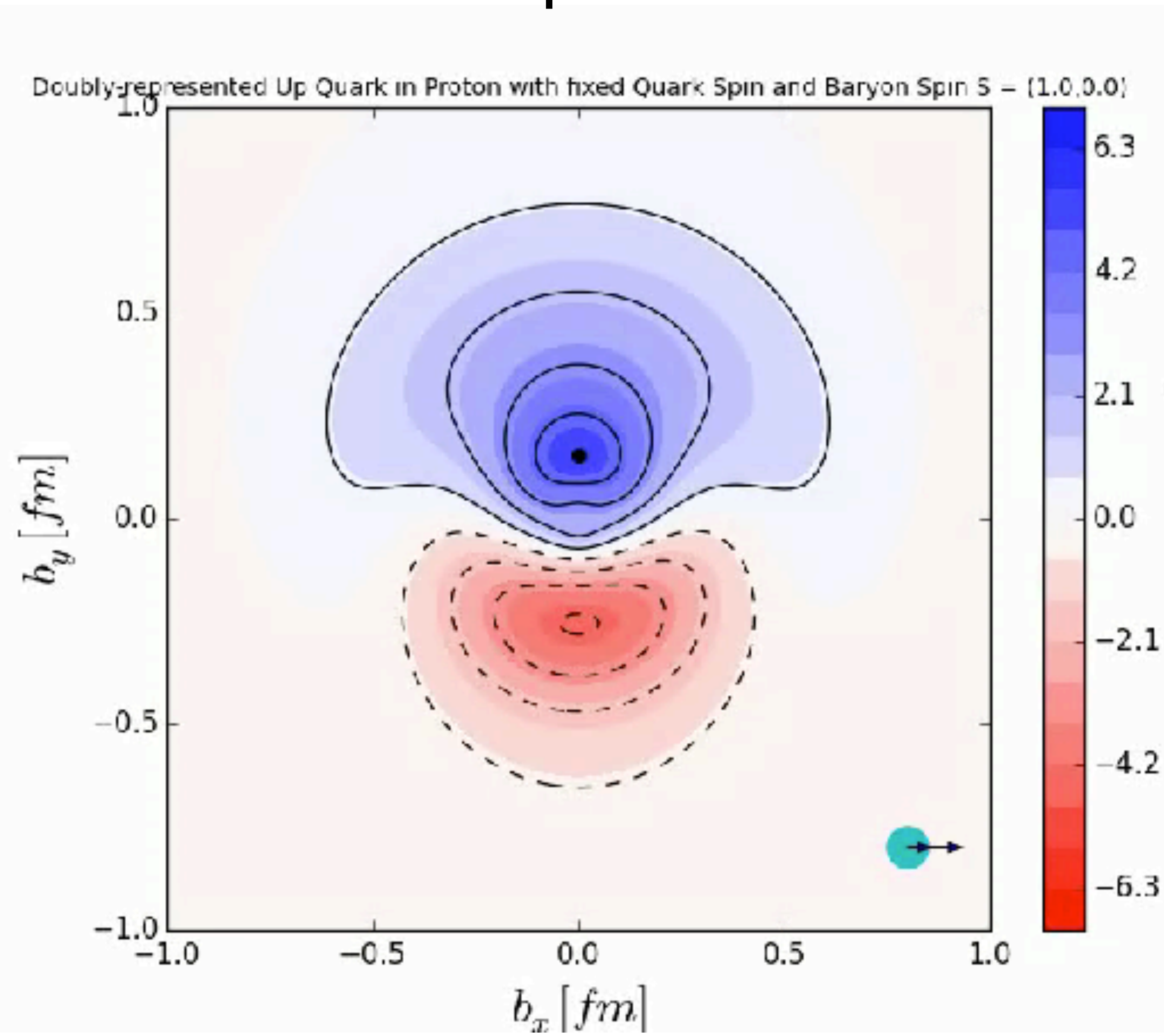


Sigma

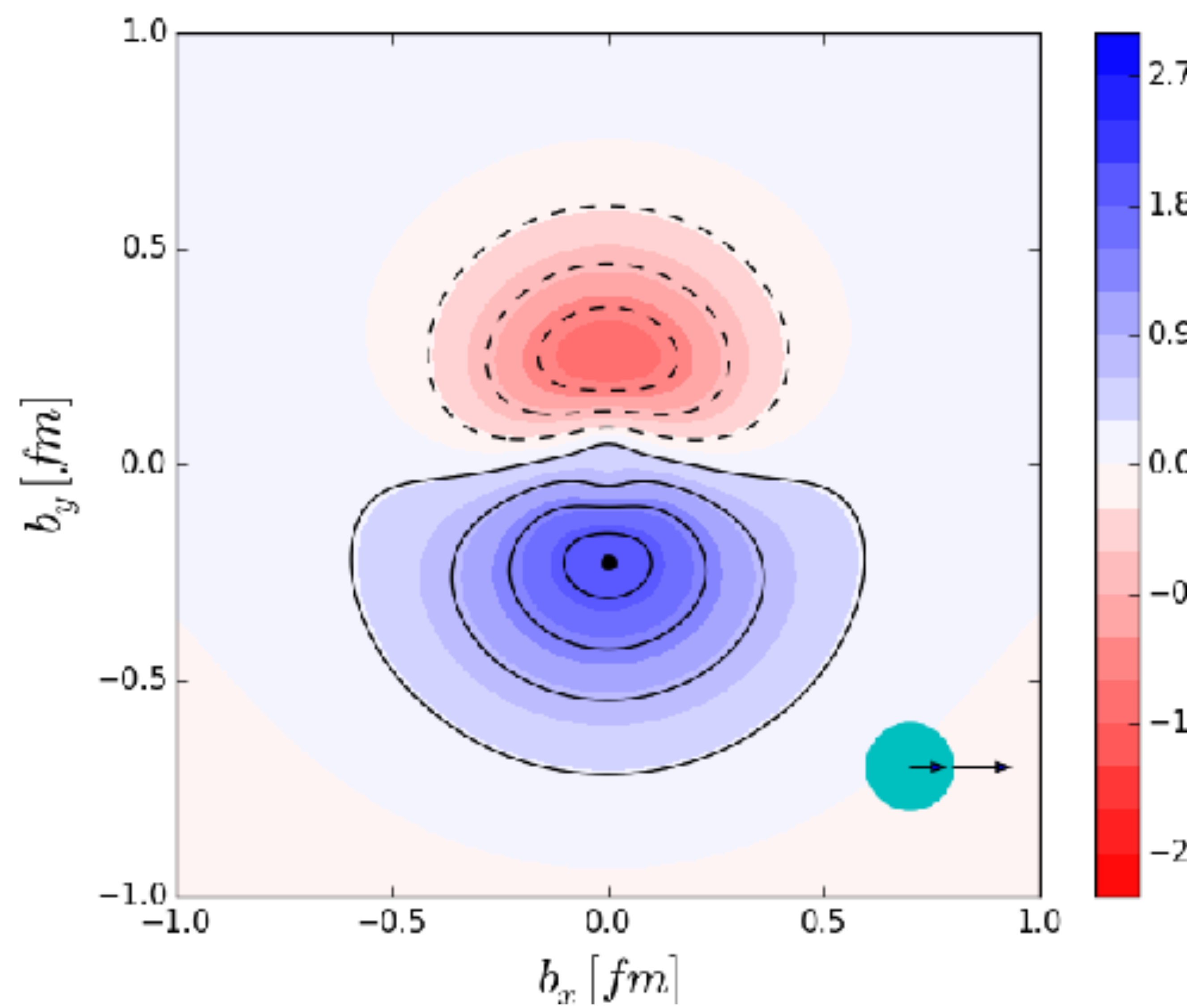


Spin Densities - Proton

u-quark

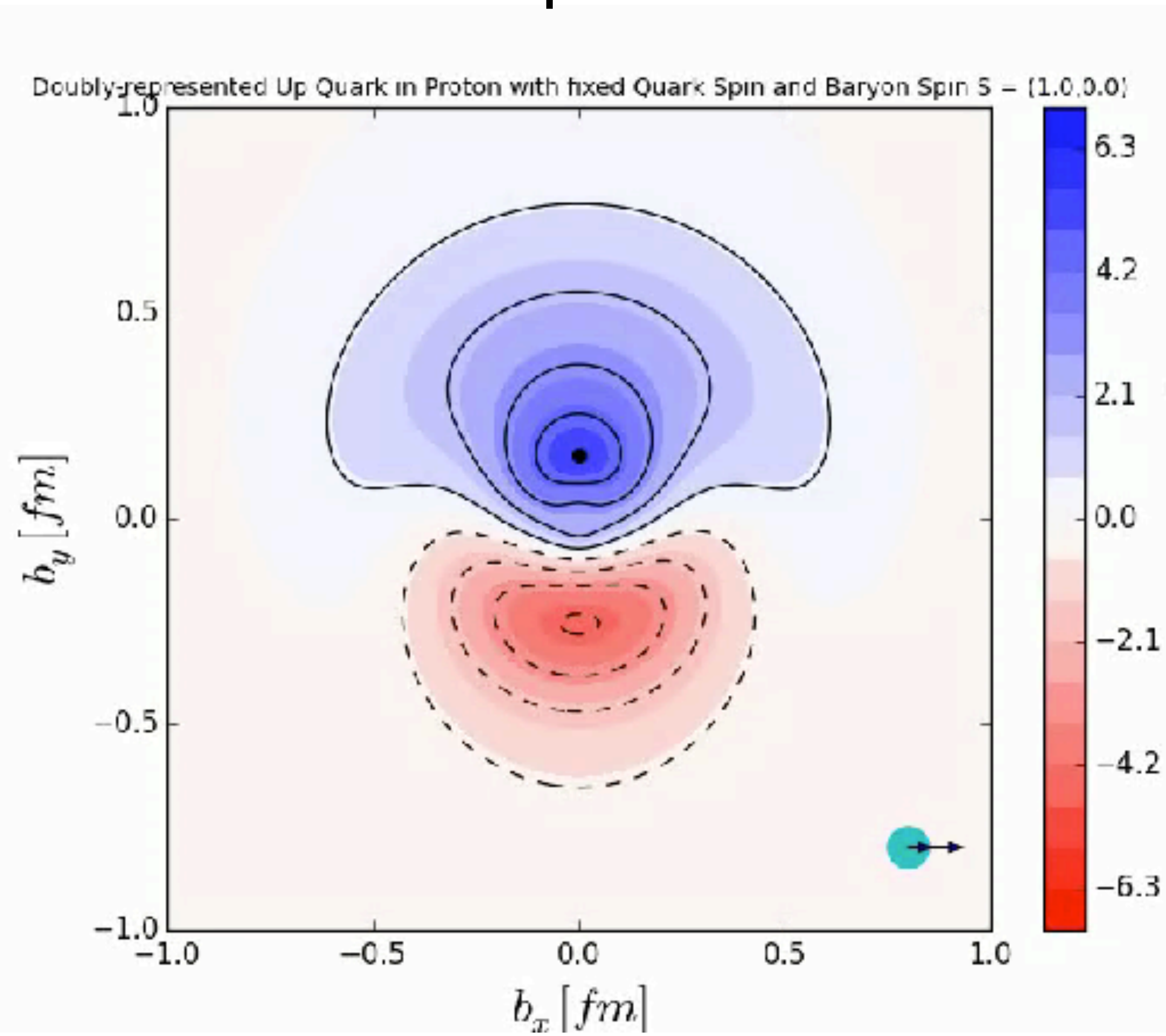


d-quark

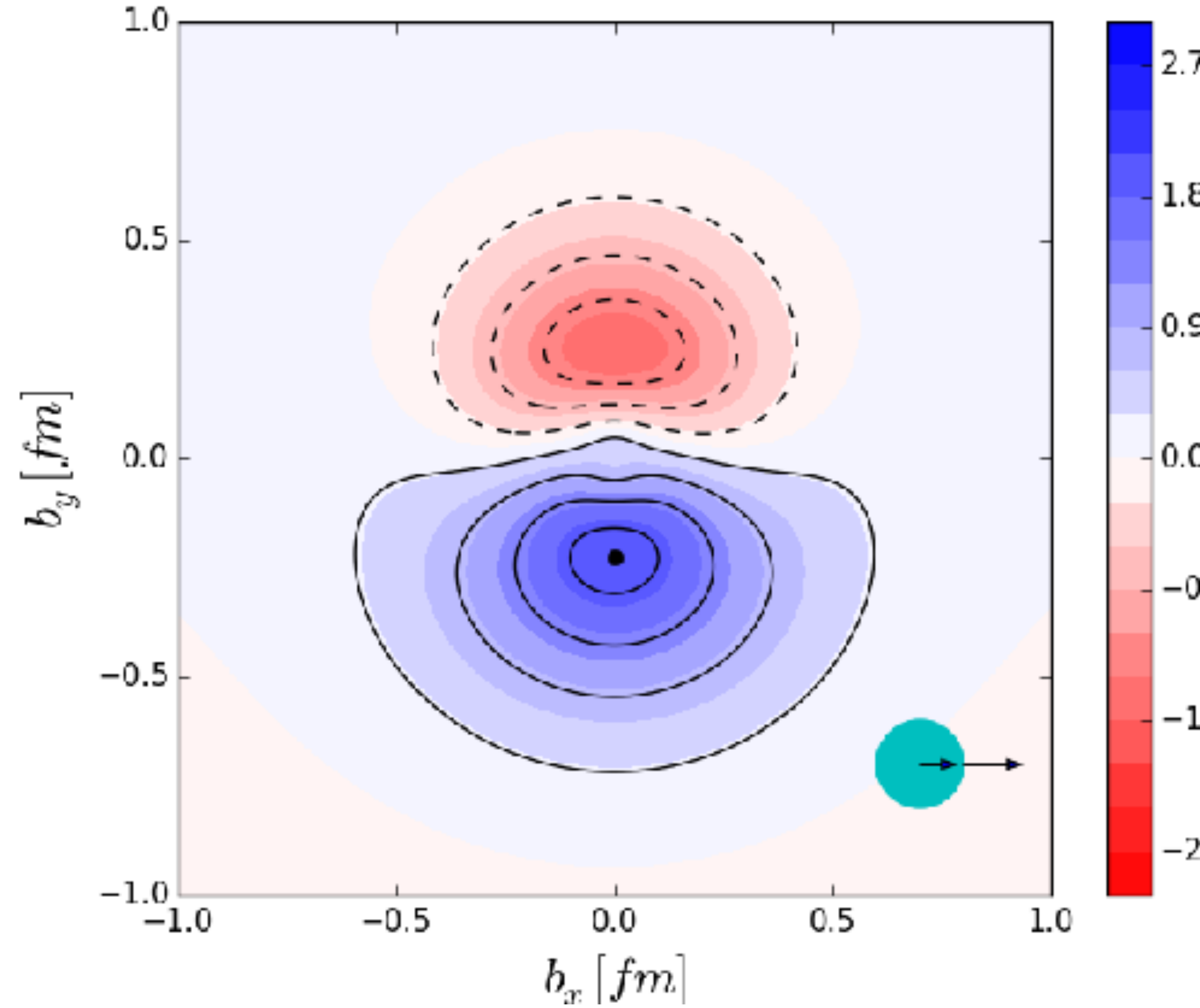


Spin Densities - Proton

u-quark

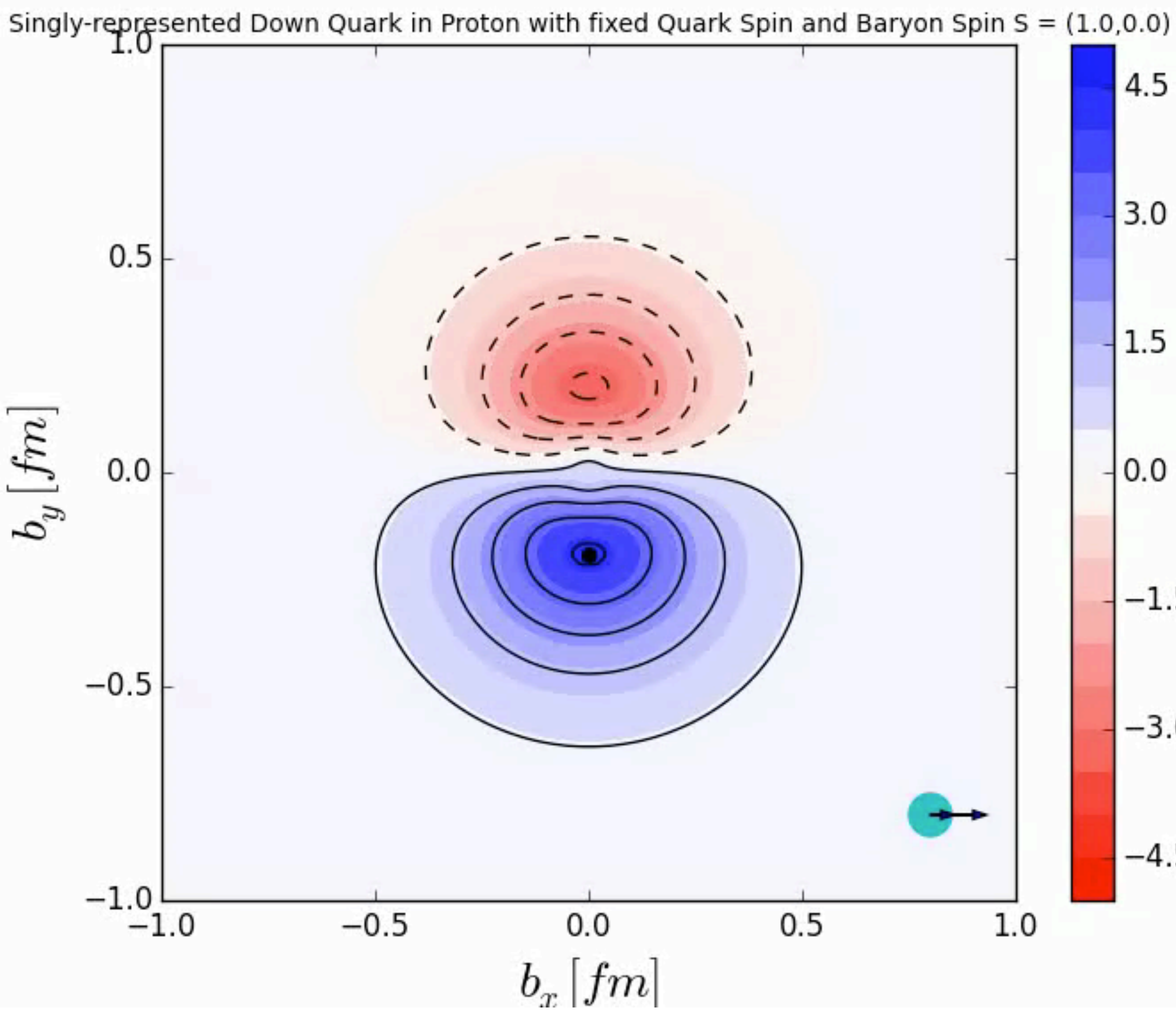


d-quark

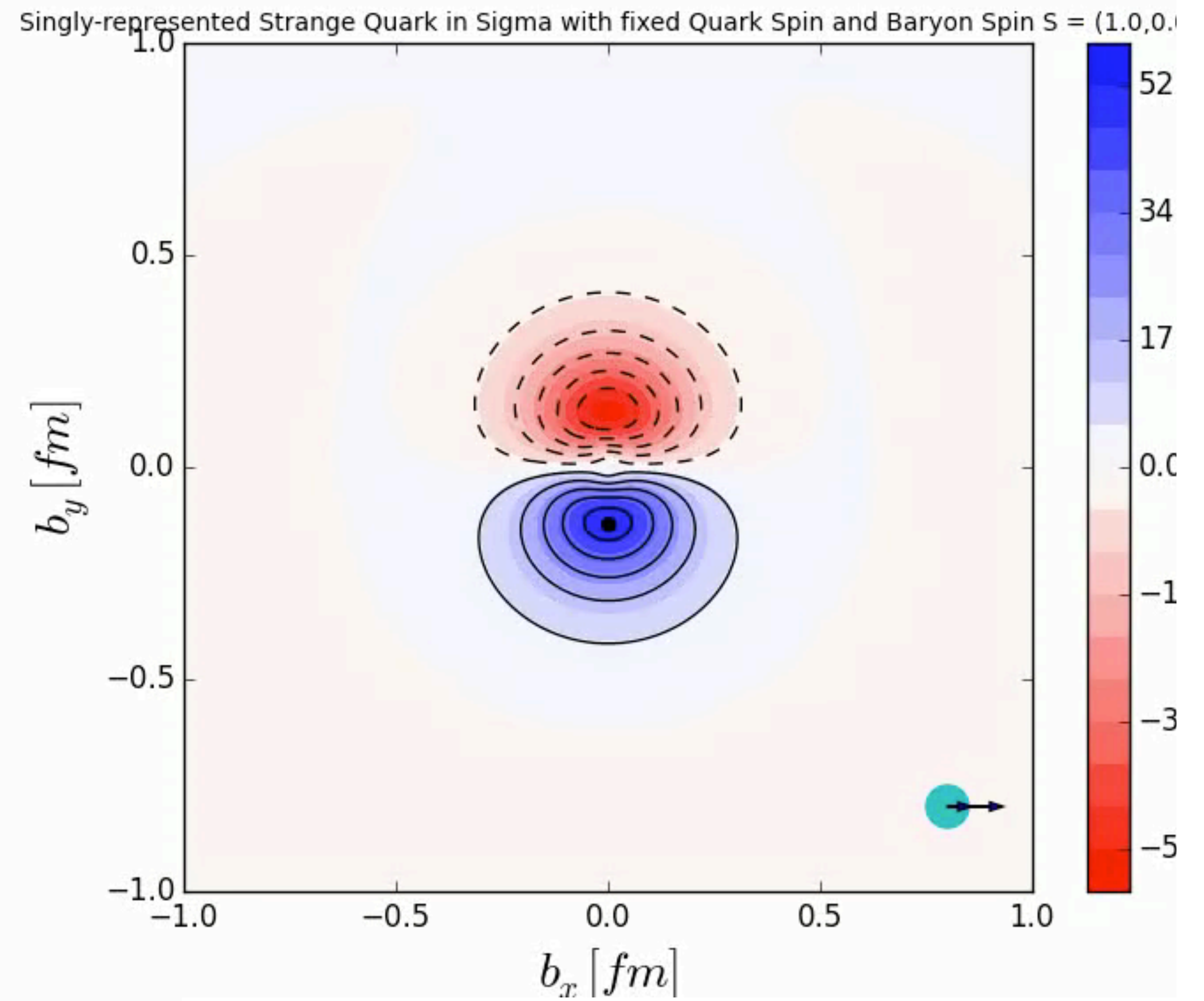


“d”-Quark

Proton



Sigma



Summary

- Results for standard observables (g_A, g_T, $\langle x \rangle$, *FFs at low* Q^2) now approaching the physical point
- Lots of effort in understanding systematic errors
- Lots of progress in more exotic quantities
  - Disconnected quark and glue
  - Quasi-PDFs
  - Compton amplitude
  - FFs at large Q^2
  - Angular momentum
  - …