Issues of TMD Extraction from SIDIS Data

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09/13/2017, INT Workshop

Suggested questions for discussion:

(same of the questions were raised in the Jlab SIDIS working group)

Question#1: How to define the current fragmentation region?

Question#2: Issues in current fragmentation region?

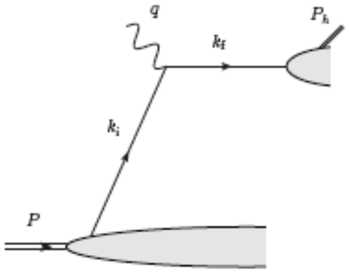
Question#3: How to separate TMDs from Fragmentation Functions?

Question#4: Treatment of Evolution, High Twist, Radiation and others Effects in SIDIS data?

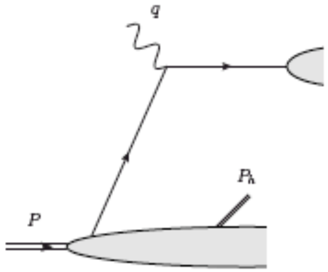
Other questions related to TMD extraction from SIDIS data (Drell-Yan as well) that we should ask here and think about?

Question#1: How to define the current fragmentation region?

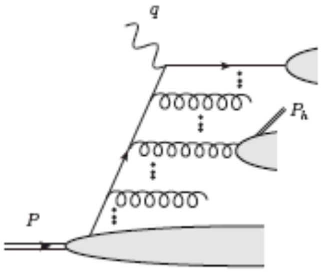
- Three kinematic regions can contribute to the SIDIS reaction



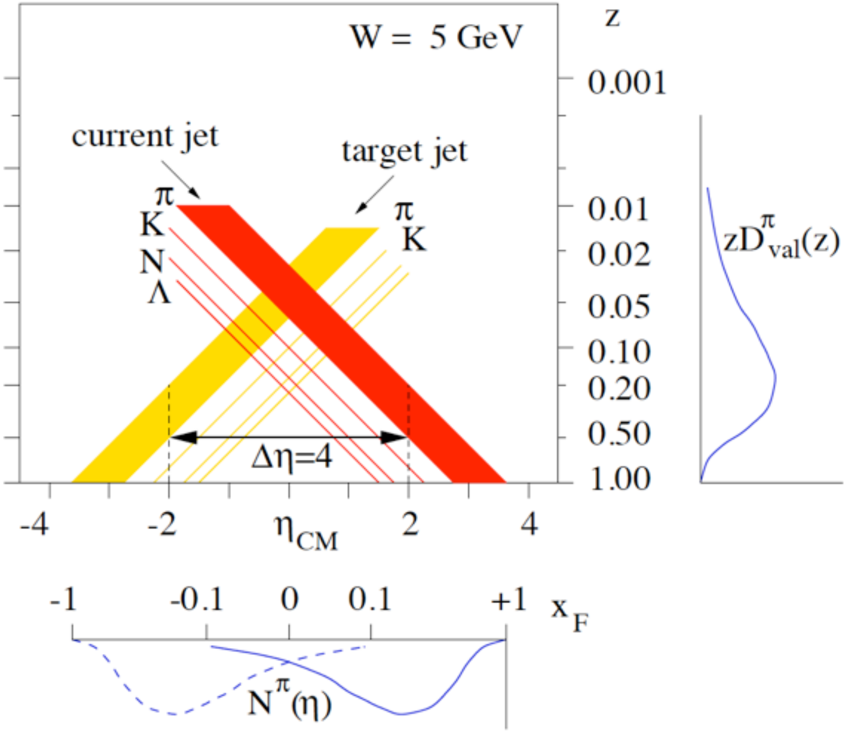
TMDs and Fragmentation Functions



Fracture Functions



No theoretical development



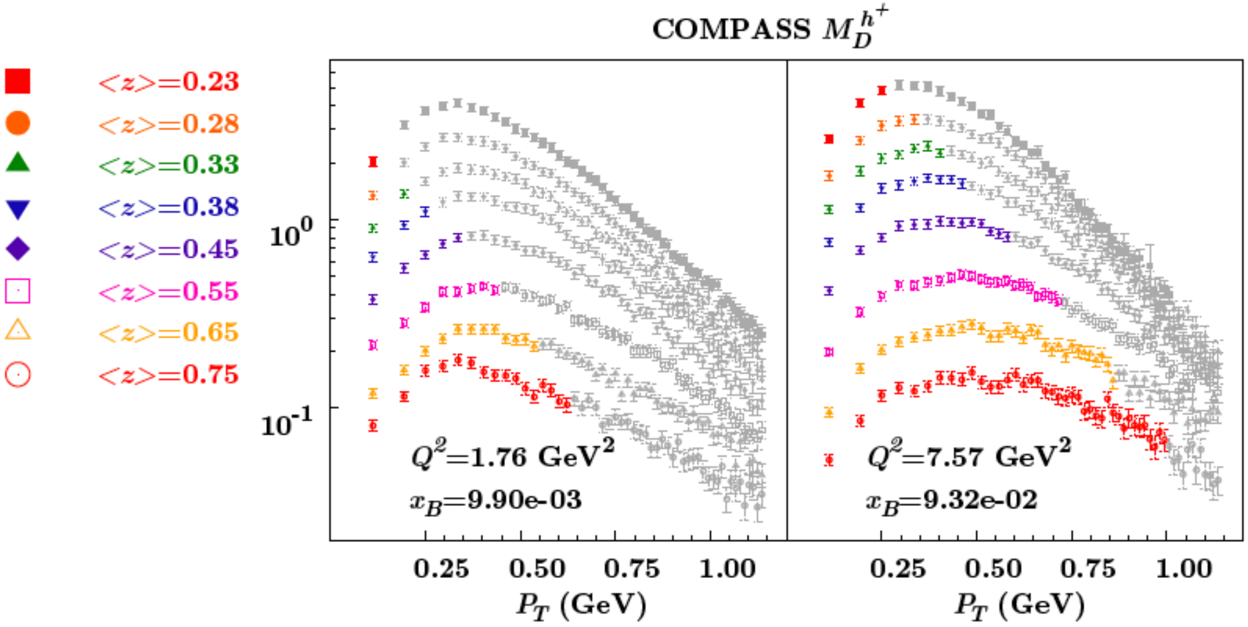
Berger Criterion: Phase space should be large enough to distinguish current/target regions.

Qualitatively, a rapidity separation of $\Delta y > 4$ can be considered as the current fragmentation region

Rapidity: $y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$

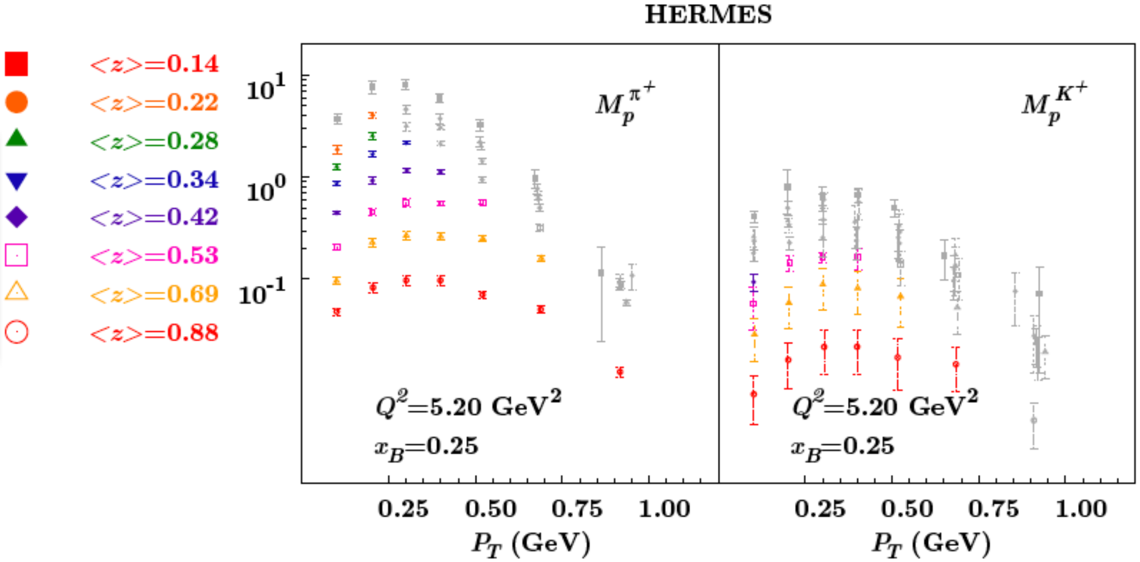
Question#1: How to define the current fragmentation region?

How to quantitatively determine the current fragmentation region?



M. Boglione, J. Osvaldo Gonzalez Hernandez, L. Gamberg, T. Rogers, N. Sato (2017)

Color: within the current fragmentation region based on new criteria
Grey: outside the current fragmentation region

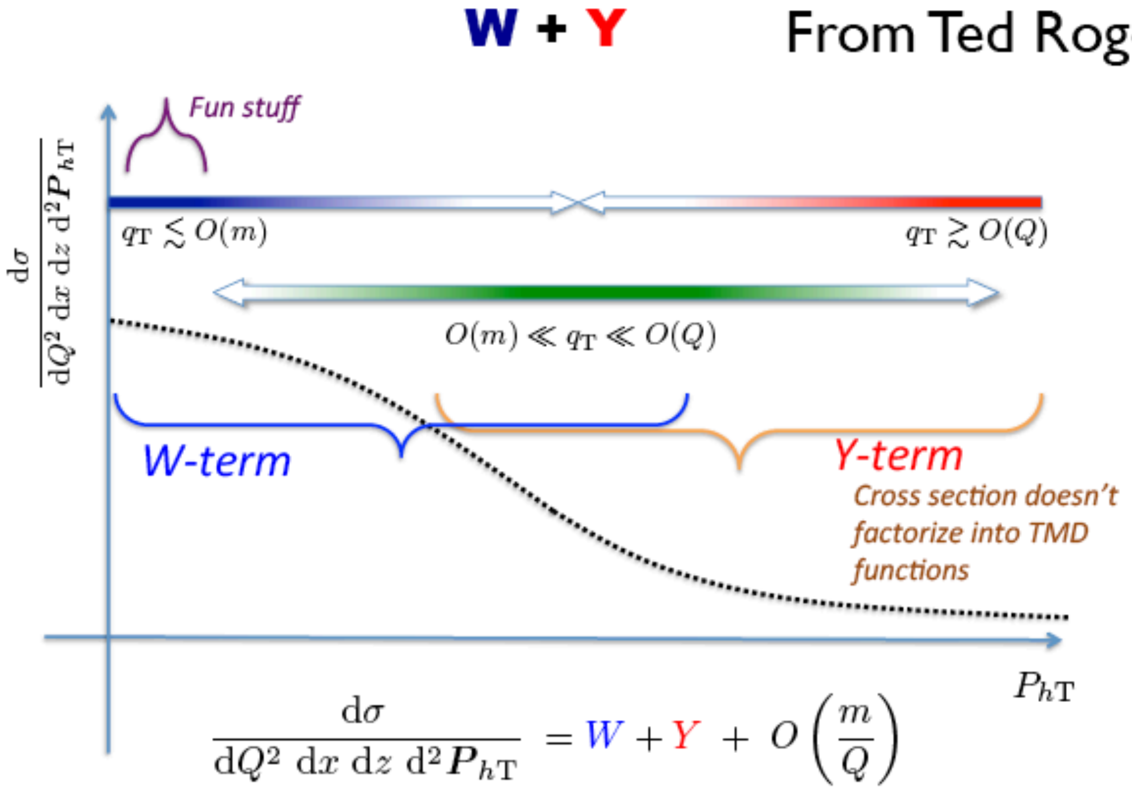


Question#2: Issues in current fragmentation region?

e.g, the Y-Term:

W + Y construction
(Collins-Soper-Sterman)

- W →describes the TMD region
- Y→ pQCD corrections at larger $q_T = P_T/z_h \sim Q$



$$\frac{d\sigma}{dQ^2\,dx\,dz\,d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

note $P_{hT} = zq_T$

Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

Question#3: How to separate TMDs from Fragmentation Functions?

How can we do better than Gaussian Ansatz?

- At leading order, the unpolarized cross section:

$$\frac{d\sigma^h}{dx\,dy\,dz\,d^2\mathbf{P_T}} = \frac{4\pi\alpha^2 s}{Q^4}\left(1 - y + \frac{y^2}{2}\right)\sum_q e_q^2 [f_1^q \otimes D_{1q}^h],$$

$$[\ldots \otimes \ldots] = \int d^2 p_T d^2 k_T \delta^{(2)}\left(\mathbf{p_T} - \frac{\mathbf{P_T}}{z} - \mathbf{k_T}\right)[\ldots].$$

- With Gaussian Approximation: *PRD 71 074006 (2005)*

$$f_q(x, k_\perp) = f_q(x)\frac{1}{\pi\langle k_\perp^2\rangle} e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\langle k_\perp^2\rangle = 0.25\,(\mathrm{GeV}/c)^2 \qquad \langle p_\perp^2\rangle = 0.20\,(\mathrm{GeV}/c)^2$$

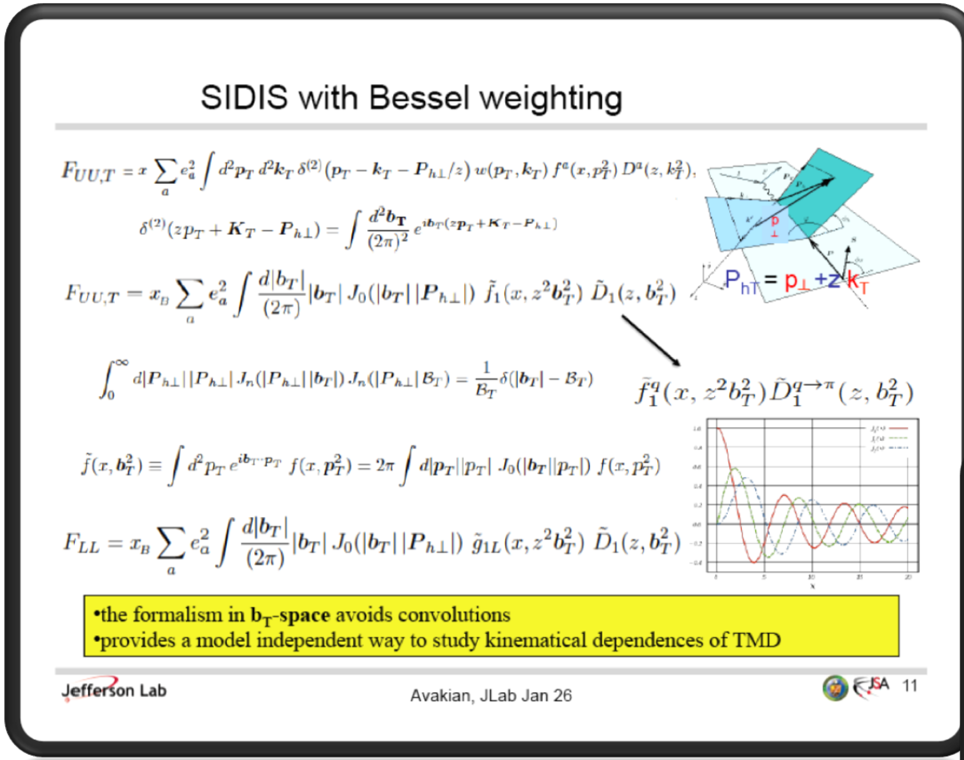
$$D_q^h(z, p_\perp) = D_q^h(z)\frac{1}{\pi\langle p_\perp^2\rangle} e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle \qquad \boldsymbol{p}_\perp = \boldsymbol{P}_T - z_h \boldsymbol{k}_\perp + \mathcal{O}\left(\frac{k_\perp^2}{Q^2}\right)$$

Question#3: How to separate TMDs from Fragmentation Functions?

Bessel Weighting?

Boer, Gamberg, Musch &Prokudin arXiv:1107.5294



SIDIS with Bessel weighting

$$F_{UU,T} = x \sum_a e_a^2 \int d^2 p_T \, d^2 k_T \, \delta^{(2)}(p_T - k_T - P_{h\perp}/z) \, w(p_T, k_T) \, f^a(x, p_T^2) \, D^a(z, k_T^2),$$

$$\delta^{(2)}(z p_T + K_T - P_{h\perp}) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i b_T (z p_T + K_T - P_{h\perp})}$$

$$F_{UU,T} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T| \, J_0(|b_T|\,|P_{h\perp}|) \, \tilde{f}_1(x, z^2 b_T^2) \, \tilde{D}_1(z, b_T^2)$$

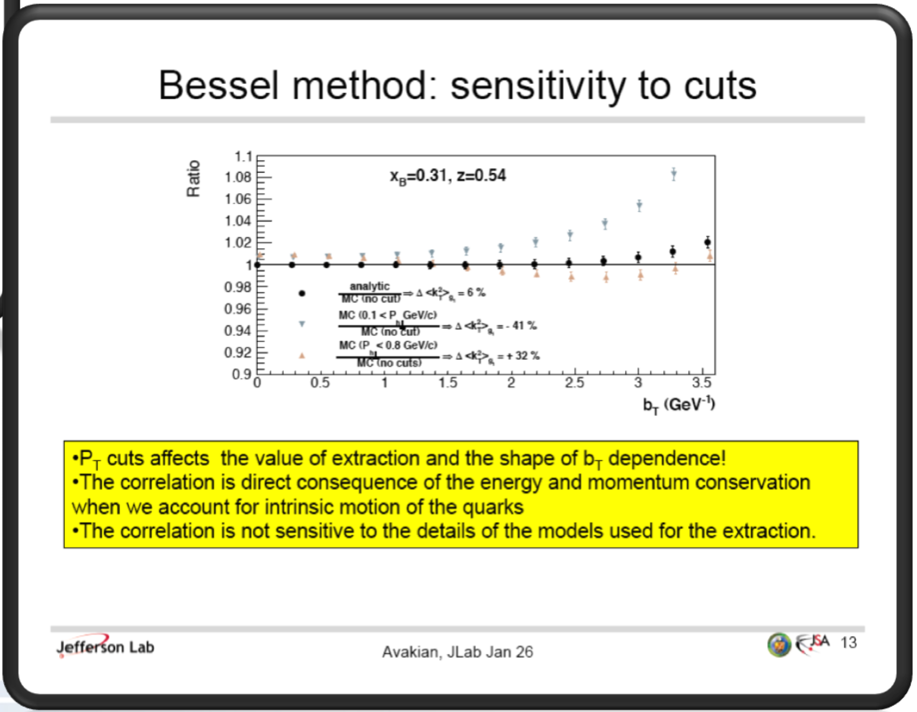
$$\int_0^\infty d|P_{h\perp}|\,|P_{h\perp}|\, J_n(|P_{h\perp}|\,|b_T|)\, J_n(|P_{h\perp}|\,\mathcal{B}_T) = \frac{1}{\mathcal{B}_T}\delta(|b_T| - \mathcal{B}_T)$$

$$\tilde{f}_1^q(x, z^2 b_T^2) \tilde{D}_1^{q\to\pi}(z, b_T^2)$$

$$\tilde{f}(x, b_T^2) \equiv \int d^2 p_T \, e^{i b_T \cdot p_T} \, f(x, p_T^2) = 2\pi \int d|p_T|\,|p_T|\, J_0(|b_T|\,|p_T|)\, f(x, p_T^2)$$

$$F_{LL} = x_B \sum_a e_a^2 \int \frac{d|b_T|}{(2\pi)} |b_T| \, J_0(|b_T|\,|P_{h\perp}|) \, \tilde{g}_{1L}(x, z^2 b_T^2) \, \tilde{D}_1(z, b_T^2)$$

- the formalism in $\mathbf{b_T}$**-space** avoids convolutions
- provides a model independent way to study kinematical dependences of TMD



Bessel method: sensitivity to cuts

- P_T cuts affects the value of extraction and the shape of b_T dependence!
- The correlation is direct consequence of the energy and momentum conservation when we account for intrinsic motion of the quarks
- The correlation is not sensitive to the details of the models used for the extraction.

Question#4:Treatment of Evolution, High Twist, Radiation and others Effects in SIDIS data?

(Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, Phys. Rev. D 93, 014009)

How to deal with Evolution?

e.g, treatment for Collins Asymmetry:

$$A_{UT}^{\sin(\phi_h+\phi_S)}(x_B,y,z_h,P_{h\perp})=\frac{2(1-y)}{1+(1-y)^2}\frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}}$$

$$F_{UU} = \frac{1}{z_h^2}\int\frac{db\,b}{2\pi}J_0\left(\frac{P_{h\perp}b}{z_h}\right)e^{-S_{\rm PT}(Q,b_*)-S_{\rm NP}^{\rm (SIDIS)}(Q,b)}$$
$$\times\ C_{q\leftarrow i}\otimes f_1^i(x_B,\mu_b)\ \hat{C}^{\rm (SIDIS)}_{j\leftarrow q}\otimes\hat{D}_{h/j}(z_h,\mu_b),$$

Unpolarized PDF

Unpolarized FF

$$F_{UT}^{\sin(\phi_h+\phi_s)} = -\frac{1}{2z_h^3}\int\frac{db\,b^2}{2\pi}J_1\left(\frac{P_{h\perp}b}{z_h}\right)e^{-S_{\rm PT}(Q,b_*)-S_{\rm NP\,coll}^{\rm (SIDIS)}(Q,b)}$$
$$\times\ \delta C_{q\leftarrow i}\otimes h_1^i(x_B,\mu_b)\ \delta\hat{C}^{\rm (SIDIS)}_{j\leftarrow q}\otimes\hat{H}^{(3)}_{h/j}(z_h,\mu_b),$$

Collins FF

Transversity

Non-perturbative factors containing the initial conditions of evolution

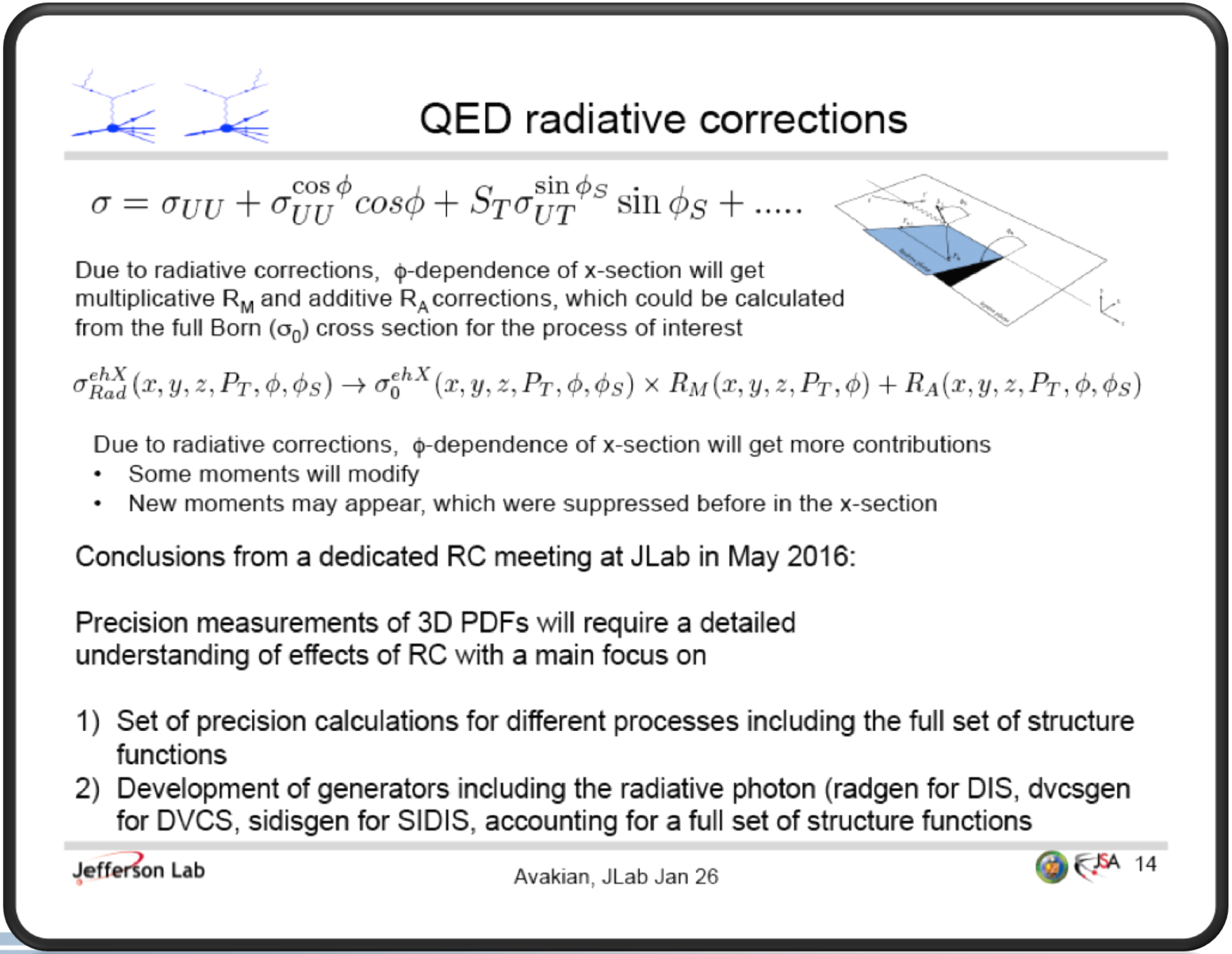
$$S_{\rm NP}^{\rm (SIDIS)} = g_2\ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right)+\left(g_q+\frac{g_h}{z_h^2}\right)b^2\ ,$$

$$S_{\rm NP\,collins}^{\rm SIDIS} = g_2\ln\left(\frac{b}{b_*}\right)\ln\left(\frac{Q}{Q_0}\right)+\left(g_q+\frac{g_h-g_c}{z_h^2}\right)b^2\ ,$$

$$S_{\rm PT}(Q,b_*)=\int_{\mu_b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[A\ln\frac{Q^2}{\mu^2}+B\right]\ ,$$

A free parameter in the global fit to allow Collins FF to change its shape w.r.t unpolarized FF.

Question#4:Treatment of Evolution, High Twist, Radiation and others Effects in SIDIS data?



QED radiative corrections

$$\sigma = \sigma_{UU} + \sigma_{UU}^{\cos\phi} cos\phi + S_T \sigma_{UT}^{\sin\phi_S} \sin\phi_S + ...$$

Due to radiative corrections, ϕ-dependence of x-section will get multiplicative R_M and additive R_A corrections, which could be calculated from the full Born (σ_0) cross section for the process of interest

$$\sigma^{ehX}_{Rad}(x,y,z,P_T,\phi,\phi_S) \rightarrow \sigma_0^{ehX}(x,y,z,P_T,\phi,\phi_S) \times R_M(x,y,z,P_T,\phi) + R_A(x,y,z,P_T,\phi,\phi_S)$$

Due to radiative corrections, ϕ-dependence of x-section will get more contributions

- Some moments will modify
- New moments may appear, which were suppressed before in the x-section

Conclusions from a dedicated RC meeting at JLab in May 2016:

Precision measurements of 3D PDFs will require a detailed understanding of effects of RC with a main focus on

1) Set of precision calculations for different processes including the full set of structure functions
2) Development of generators including the radiative photon (radgen for DIS, dvcsgen for DVCS, sidisgen for SIDIS, accounting for a full set of structure functions

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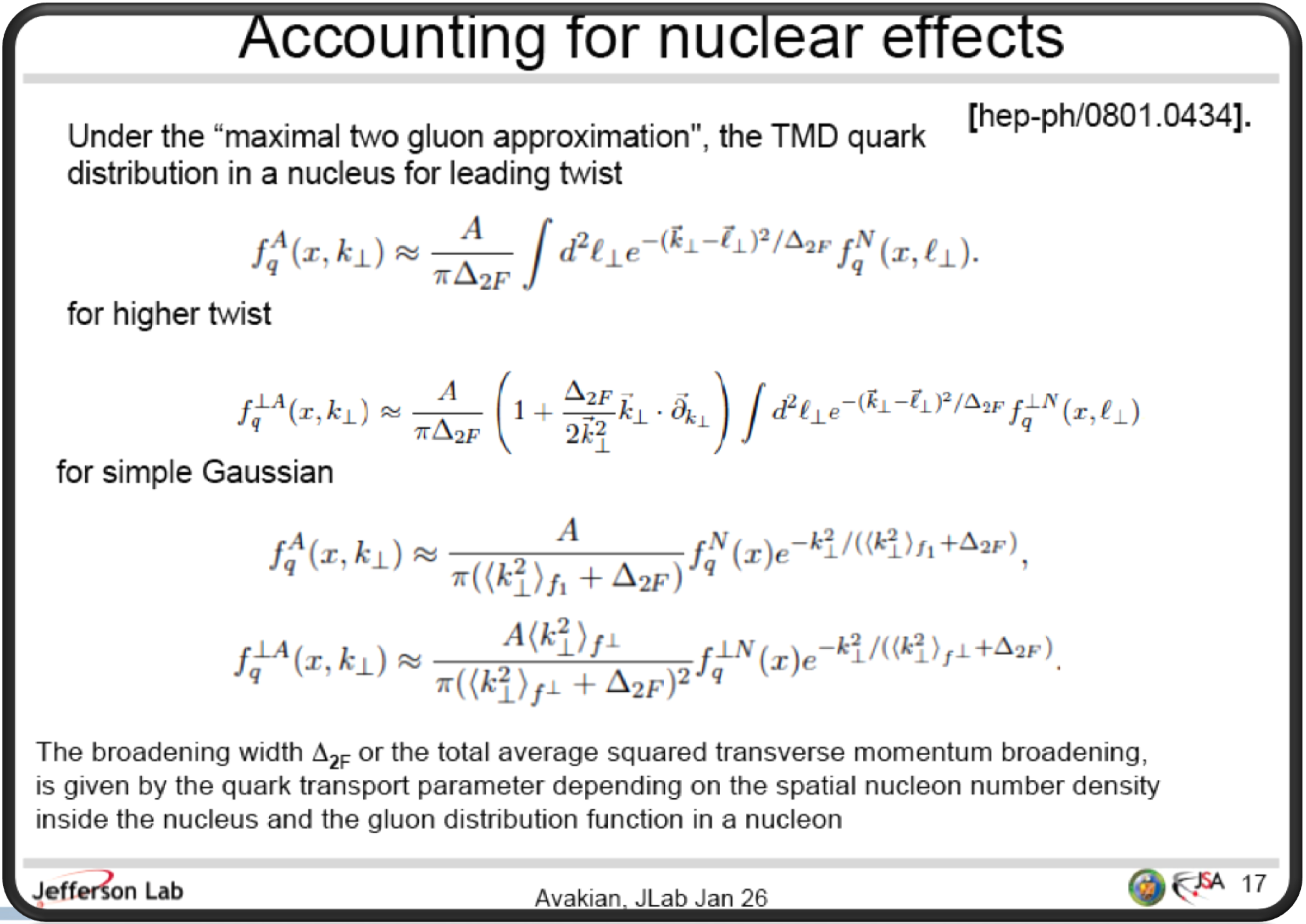
Question#4:Treatment of Evolution, High Twist, Radiation and others Effects in SIDIS data?



Accounting for nuclear effects

[hep-ph/0801.0434].

Under the “maximal two gluon approximation", the TMD quark distribution in a nucleus for leading twist

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}} f_q^N(x,\ell_\perp).$$

for higher twist

$$f_q^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\left(1+\frac{\Delta_{2F}}{2\vec{k}_\perp^2}\vec{k}_\perp\cdot\vec{\partial}_{k_\perp}\right)\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}} f_q^{\perp N}(x,\ell_\perp)$$

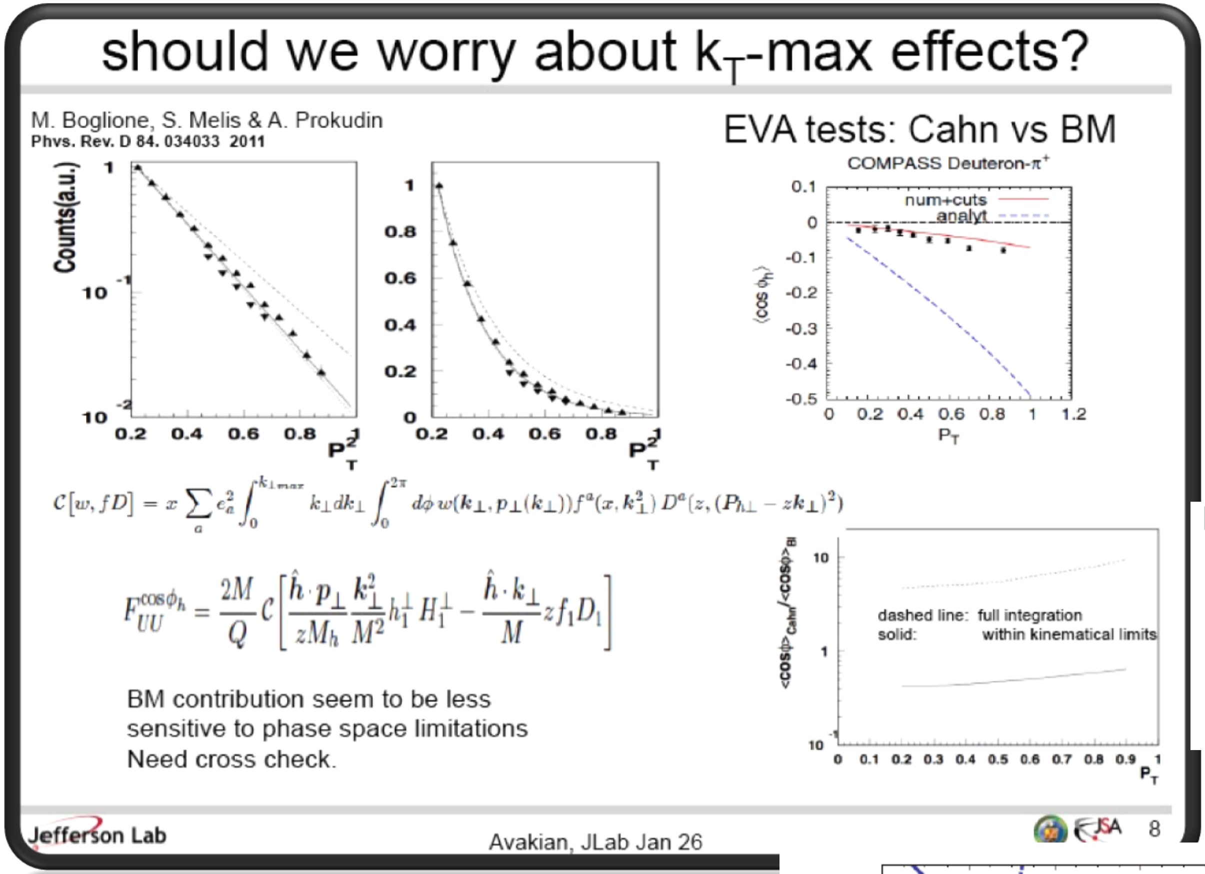
for simple Gaussian

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi(\langle k_\perp^2\rangle_{f_1}+\Delta_{2F})} f_q^N(x) e^{-k_\perp^2/(\langle k_\perp^2\rangle_{f_1}+\Delta_{2F})},$$

$$f_q^{\perp A}(x,k_\perp) \approx \frac{A\langle k_\perp^2\rangle_{f^\perp}}{\pi(\langle k_\perp^2\rangle_{f^\perp}+\Delta_{2F})^2} f_q^{\perp N}(x) e^{-k_\perp^2/(\langle k_\perp^2\rangle_{f^\perp}+\Delta_{2F})}.$$

The broadening width Δ_{2F} or the total average squared transverse momentum broadening, is given by the quark transport parameter depending on the spatial nucleon number density inside the nucleus and the gluon distribution function in a nucleon

Question#4:Treatment of Evolution, High Twist, Radiation and others Effects in SIDIS data?



should we worry about k_T-max effects?

M. Boglione, S. Melis & A. Prokudin
Phys. Rev. D 84, 034033 2011

EVA tests: Cahn vs BM

$$\mathcal{C}[w, fD] = x\sum_a e_a^2 \int_0^{k_{\perp max}} k_\perp dk_\perp \int_0^{2\pi} d\phi\, w(k_\perp, p_\perp(k_\perp)) f^a(x,k_\perp^2)\, D^a(z,(P_{h\perp}-zk_\perp)^2)$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\frac{\hat{h}\cdot p_\perp}{zM_h}\frac{k_\perp^2}{M^2} h_1^\perp H_1^\perp - \frac{\hat{h}\cdot k_\perp}{M} z f_1 D_1\right]$$

dashed line: full integration
solid: within kinematical limits

BM contribution seem to be less sensitive to phase space limitations
Need cross check.

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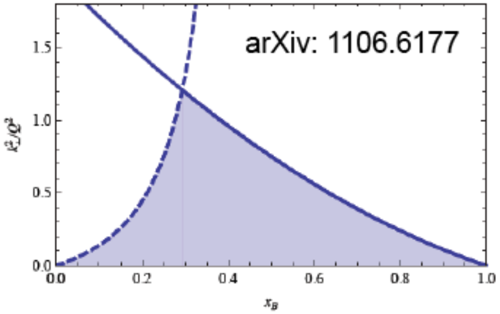
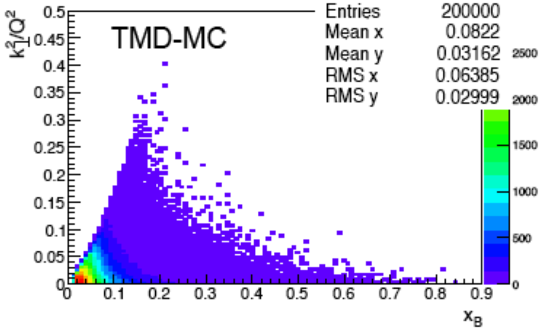
From energy/momentum conservation

$$xP_0 + \frac{k_\perp^2}{4xP_0} \le P_0 \;\Rightarrow\; k_\perp^2 \le 4x(1-x)P_0^2$$

$$\Rightarrow\; k_\perp^2 \le \frac{x(1-x)}{x_B(1-x_B)}Q^2$$



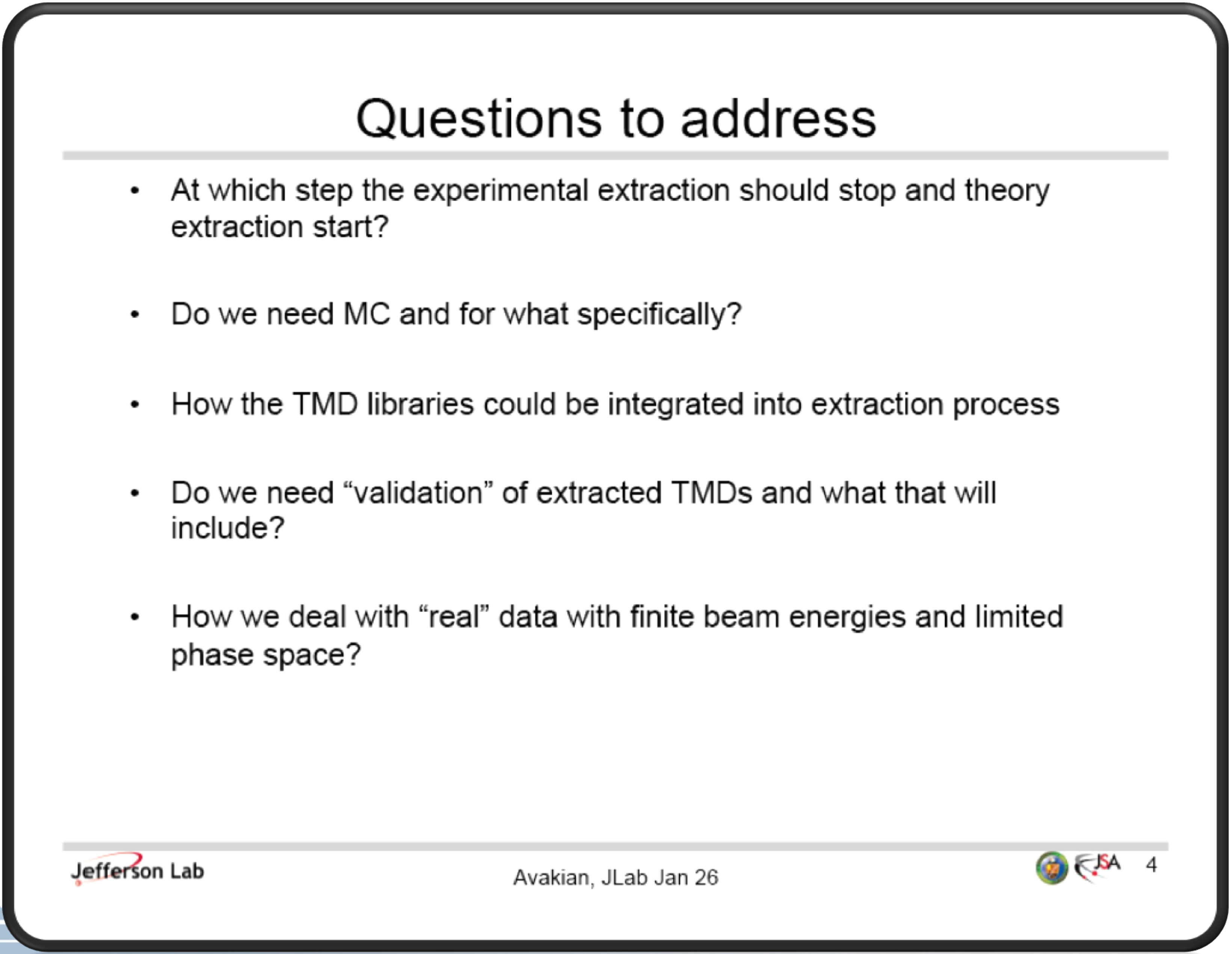
arXiv: 1106.6177



TMD-MC

x and k_T are not independent at low Q^2 even in factorized Gaussian approach!

Questions from Harut Avakian et al. (EVA SIDIS working group)



Questions to address

- At which step the experimental extraction should stop and theory extraction start?
- Do we need MC and for what specifically?
- How the TMD libraries could be integrated into extraction process
- Do we need "validation" of extracted TMDs and what that will include?
- How we deal with "real" data with finite beam energies and limited phase space?

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Gunar’s Discussion

- 1. Pythia has constant width of <pT> and <kT> = 0.44 GeV/c. Need to tune the data integrated over whole kinematic range to get a better Gaussian shape (0.44 → 0.38)
- Tuning needs multiple (5D) dimensions.
- Gunar’s biggest concern is the radiative corrections (most of experts are out of jobs).
- Tuning Jetset (J. Rubin, Ph.D. Thesis, UIUC)