Frontiers in Lattice PDF Study

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and

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The parton distribution function (PDF)

The original quark PDF defined in the light front frame is,

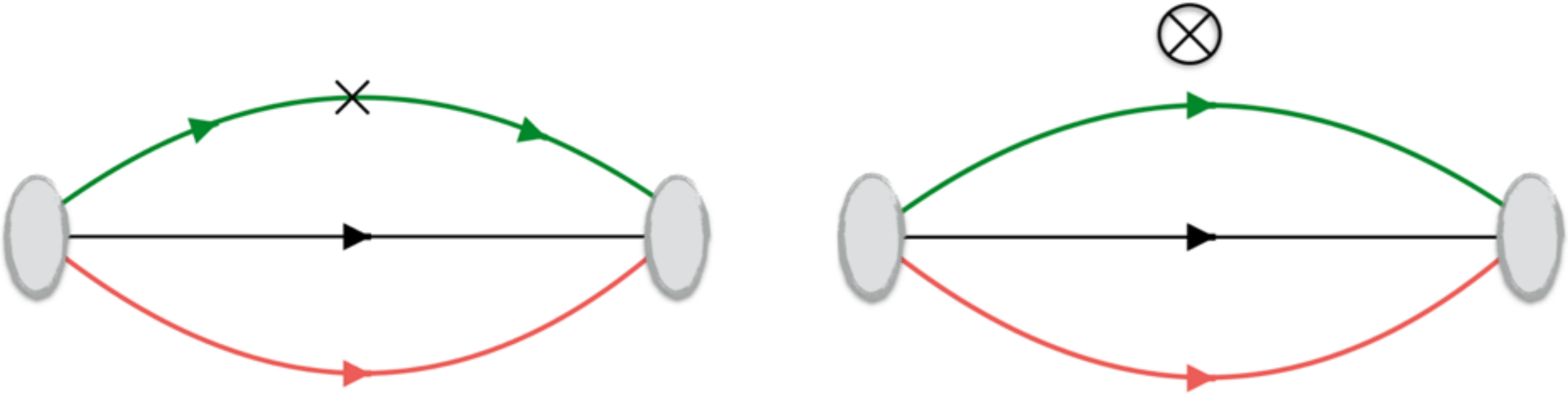
$$q(x, \mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \overline{\psi}(\xi^-) \gamma^+ \times \exp\left(-ig \int_0^{\xi^-} d\eta^- A^+(\eta^-) \right) \psi(0) | P \rangle$$

- PDF is a universal distribution in nucleon;
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …);
- Important inputs to discern new physics at LHC, and currently dominate errors in Higgs production.
- The real time dependence in the PDF definition makes the direct lattice simulation to be impossible.

The alternative solutions on the lattice:

- **The moments of the PDF which are the matrix elements of local operators:**
  - How to do the lattice calculation is clear;
  - The renormalization of the quark part is also clear, while the gluon ones and the mixing are unclear;
  - Limited to a few moments and most of the recent calculations concentrate on the zero/first moments.
- **Reconstruct the PDF from the lattice “cross section”:**
  - The lattice calculation is relatively non-trivial;
  - The renormalization is non-trivial for part of the approaches;
  - Can access the full Bjorken x dependence of PDF, while the result may depend on the way to do the reconstruction.

The calculations of the moments



The calculations of the moments

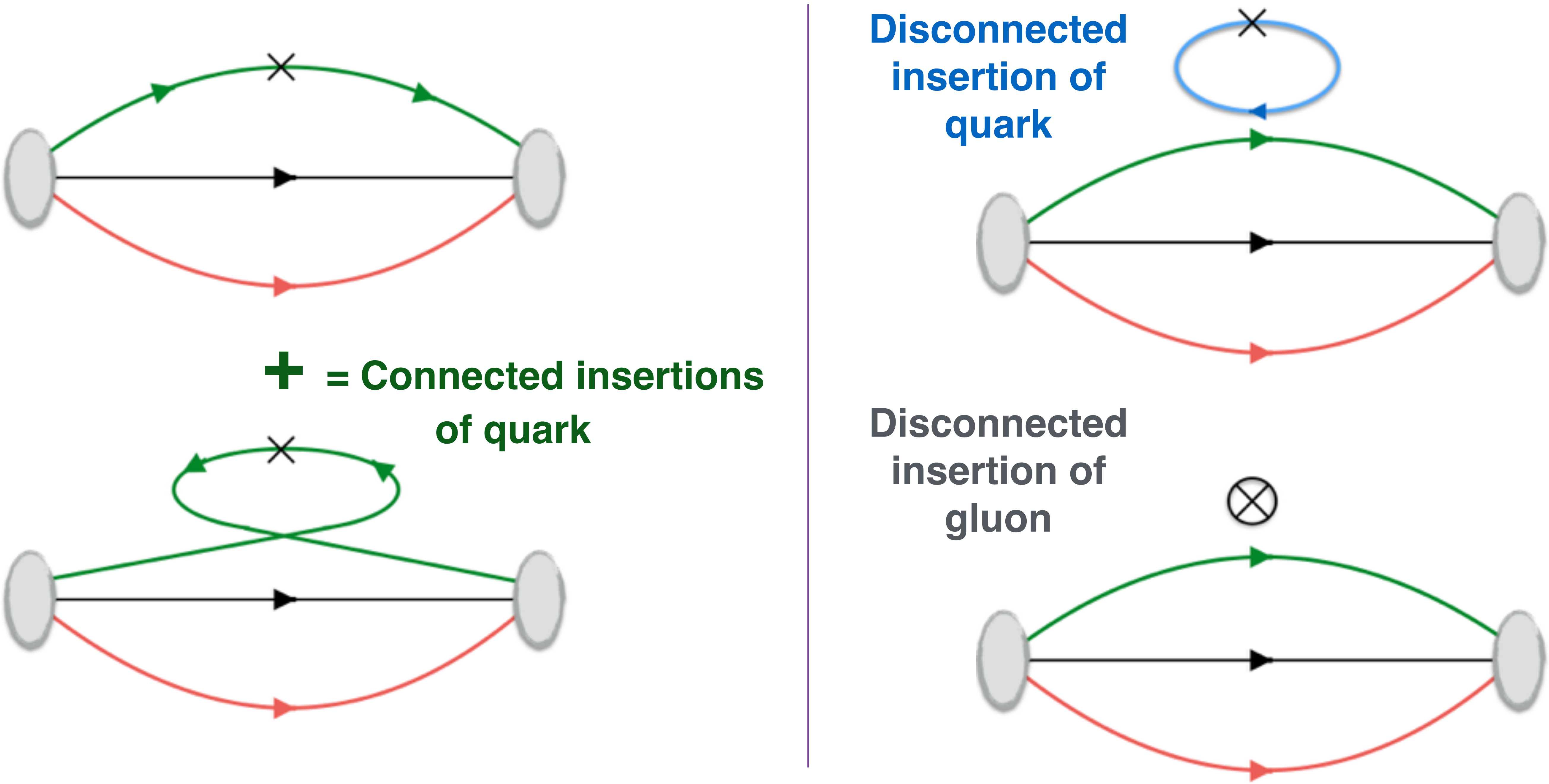
- The moments of the PDF which are the matrix elements of local light-cone operators: $O^n = \overline{\psi}\gamma^+ D^{+_2} \ldots D^{+_n}\psi$

  or,

$$O^{\mu_1\ldots\mu_n} = \overline{\psi}\gamma^{(\mu_1} iD^{\mu_2} \ldots iD^{\mu_n)}\psi - \mathrm{trace}$$

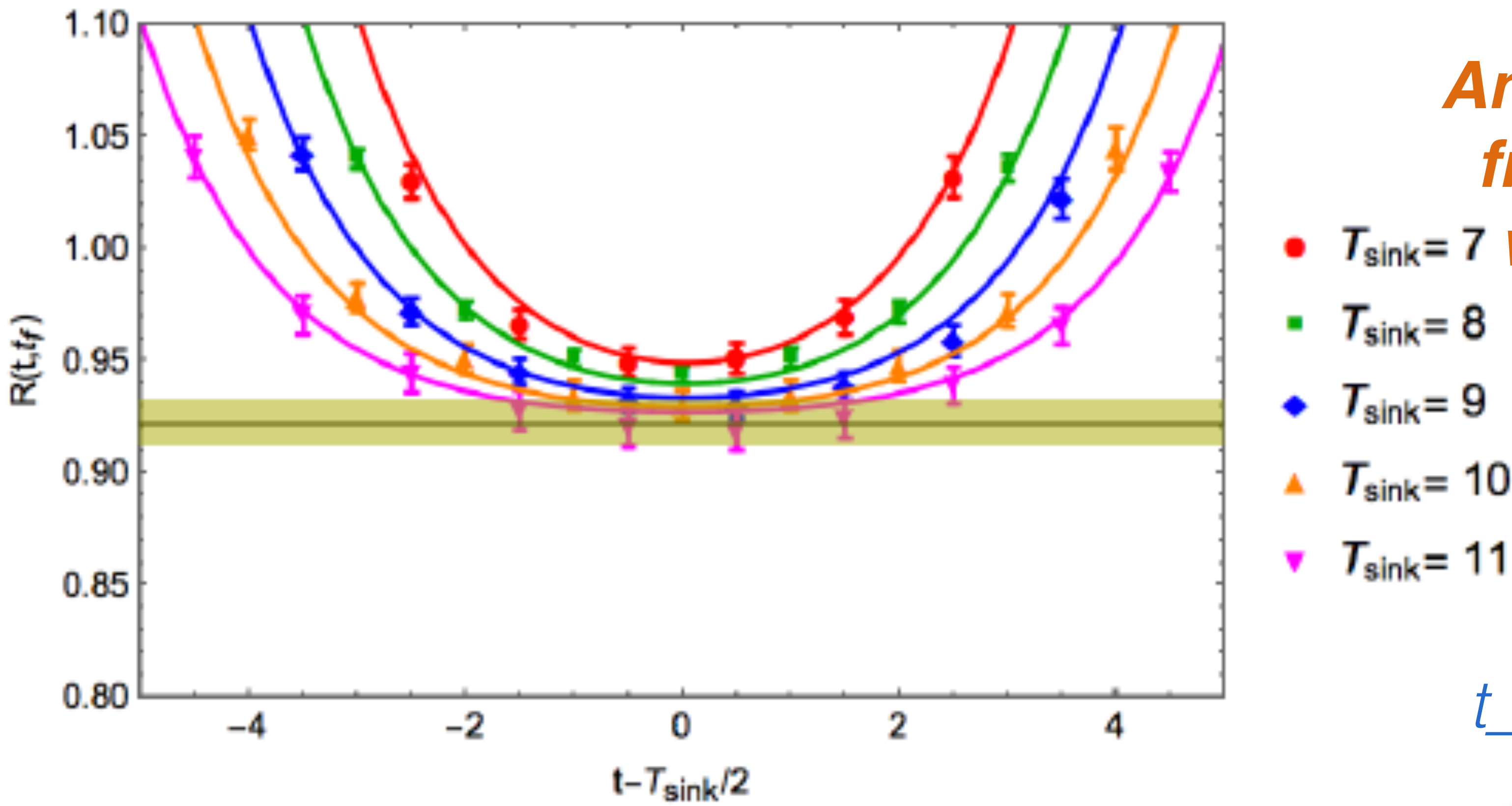
- How to do the lattice calculation is clear;
- The renormalization of the quark part is also clear, while the gluon ones and the mixing are unclear;
- Limited to a few moments and most of the recent calculations concentrate on the first/second moments.

Connected and disconnect insertions

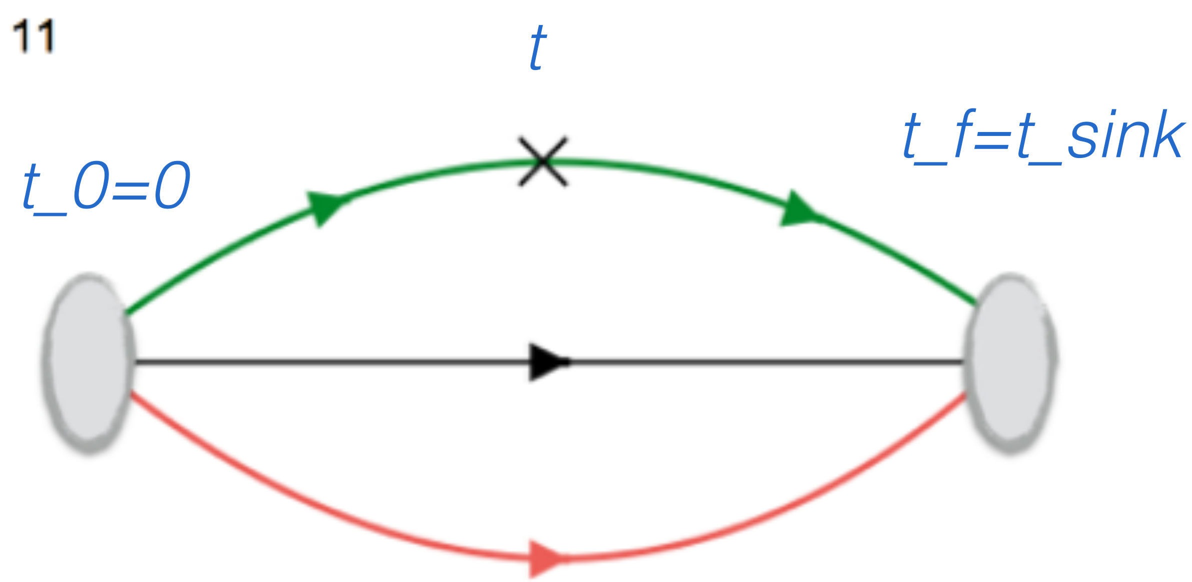


- Quark propagates in the gluon background

Two state fit
to remove the excited states contaminations



An example of the two-state fit from the tensor charge case which has obvious excited states contaminations.



$$R(t_f, t) = S_G + C_1 e^{-\Delta E(t_f - t)} + C_2 e^{-\Delta E t}$$

the difference between the proton mass and effective excited state mass

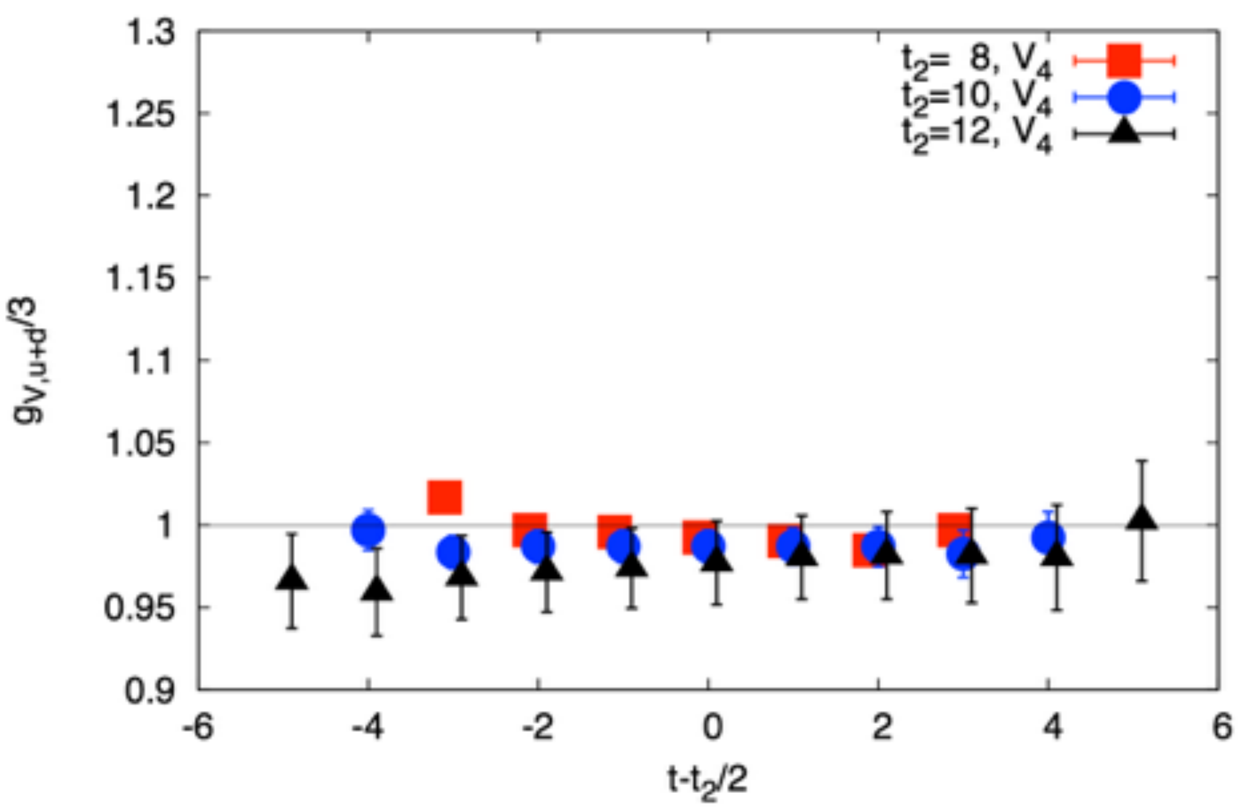
Integration of PDF

Connected insertion(CI) parts

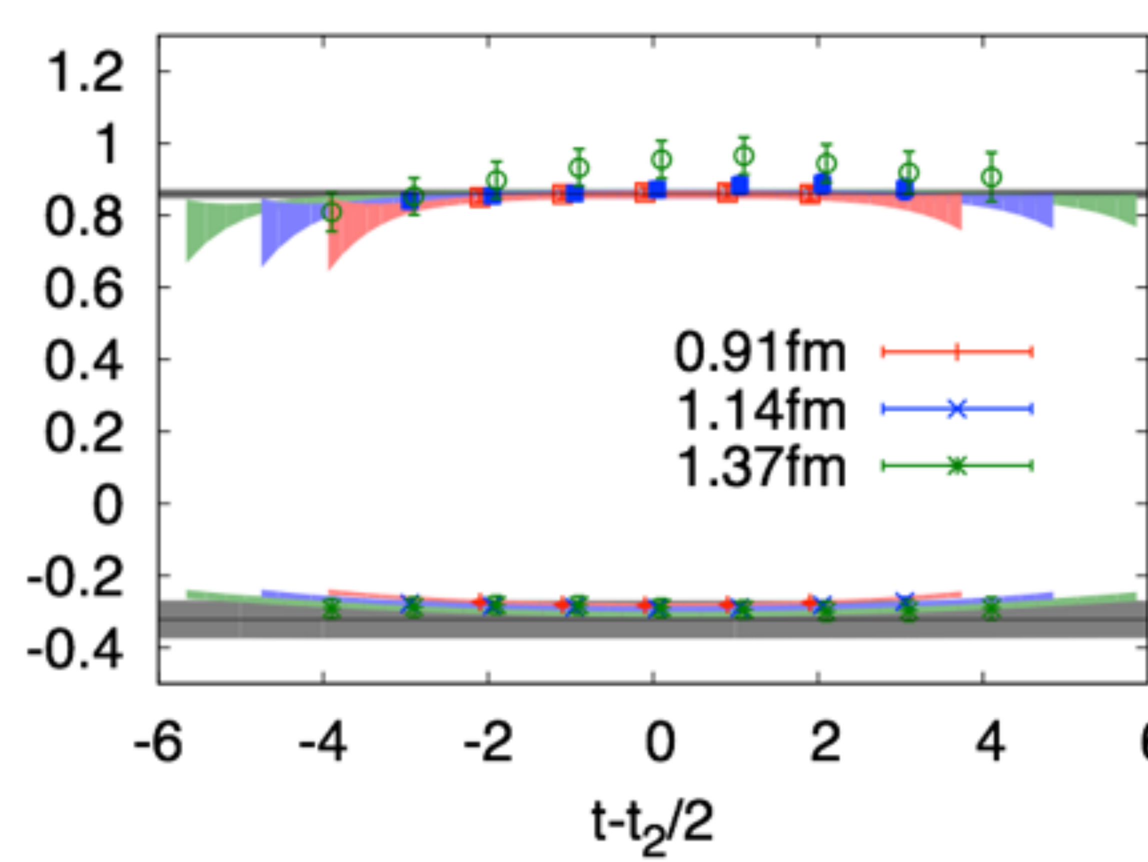
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

Replace γ^+ by $\gamma^+\gamma^5$ to get the polarized case.

YBY. et.al, χQCD, in preparation



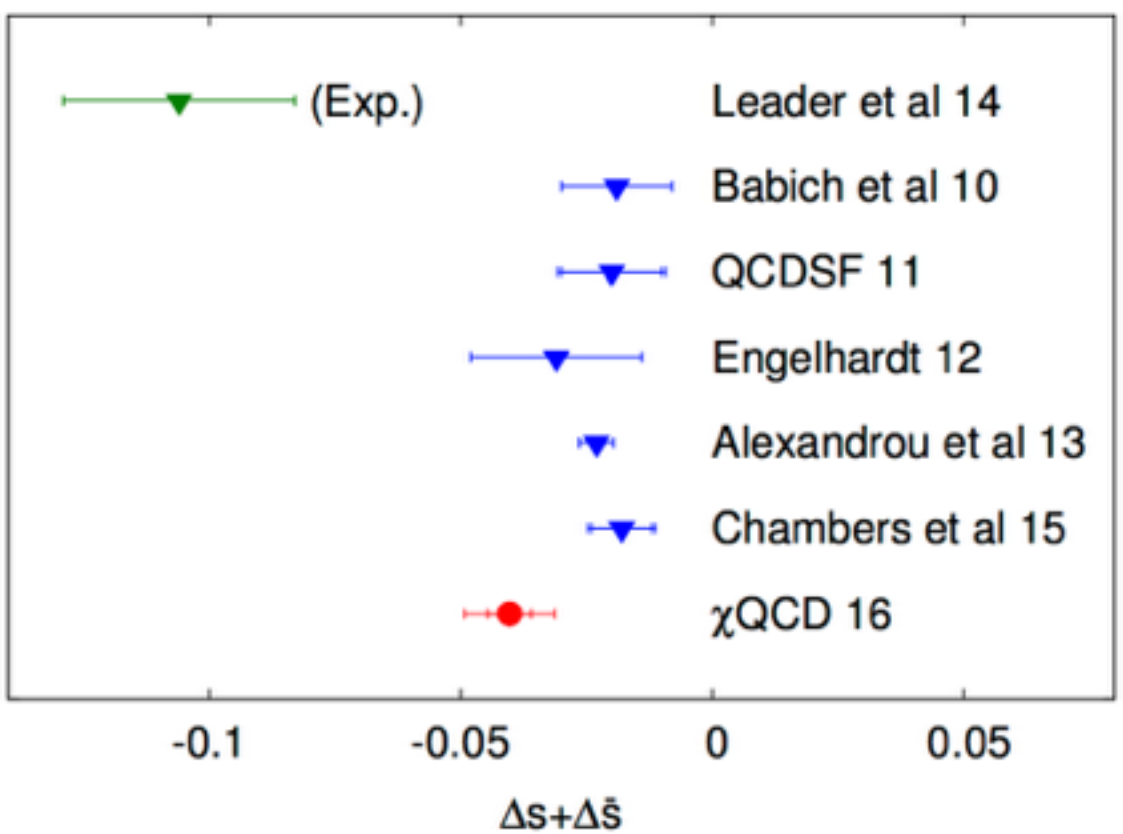
The averaged vector charge



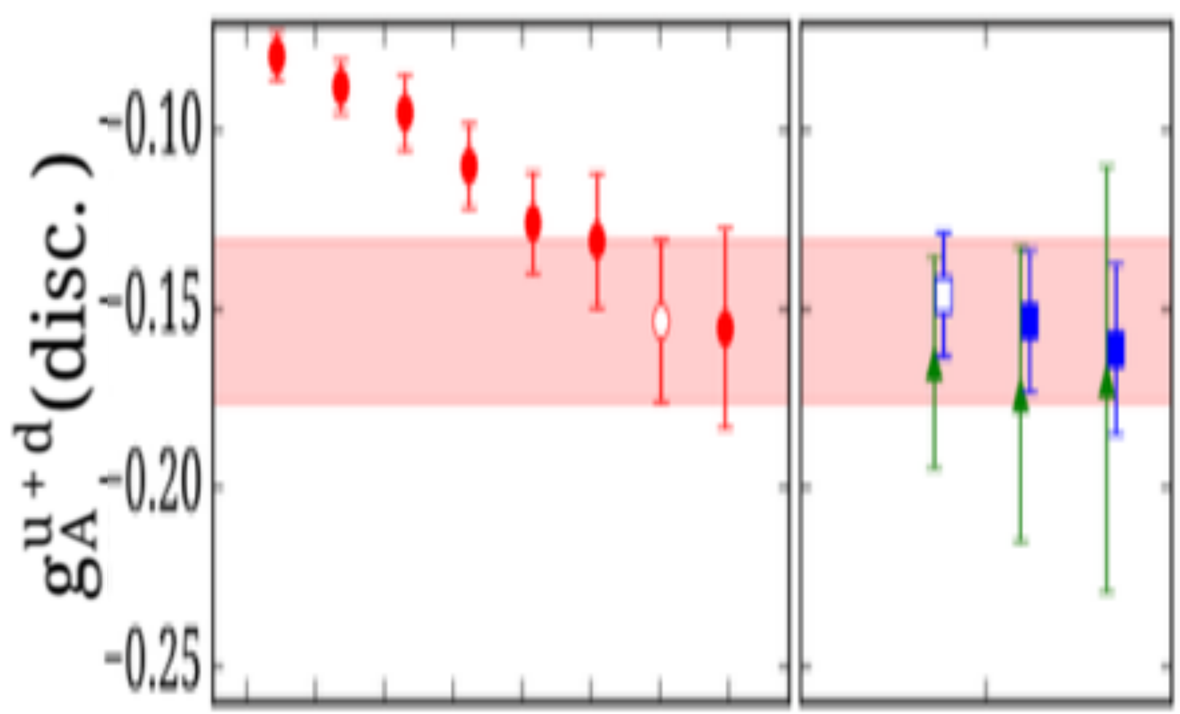
The connected insertion parts of the axial vector charge

Integration of PDF
quark disconnected insertion (DI) parts

- The DI parts of the vector charge should be zero due to the cancellation between the quarks and anti-quarks;
- But those of the axial vector charge can be large.



M. Gong, YBY. et.al, χQCD, Phys.Rev. D95 (2017), 114509, 1511.03671



C. Alexandrou et.al, ETMC 1706.02973

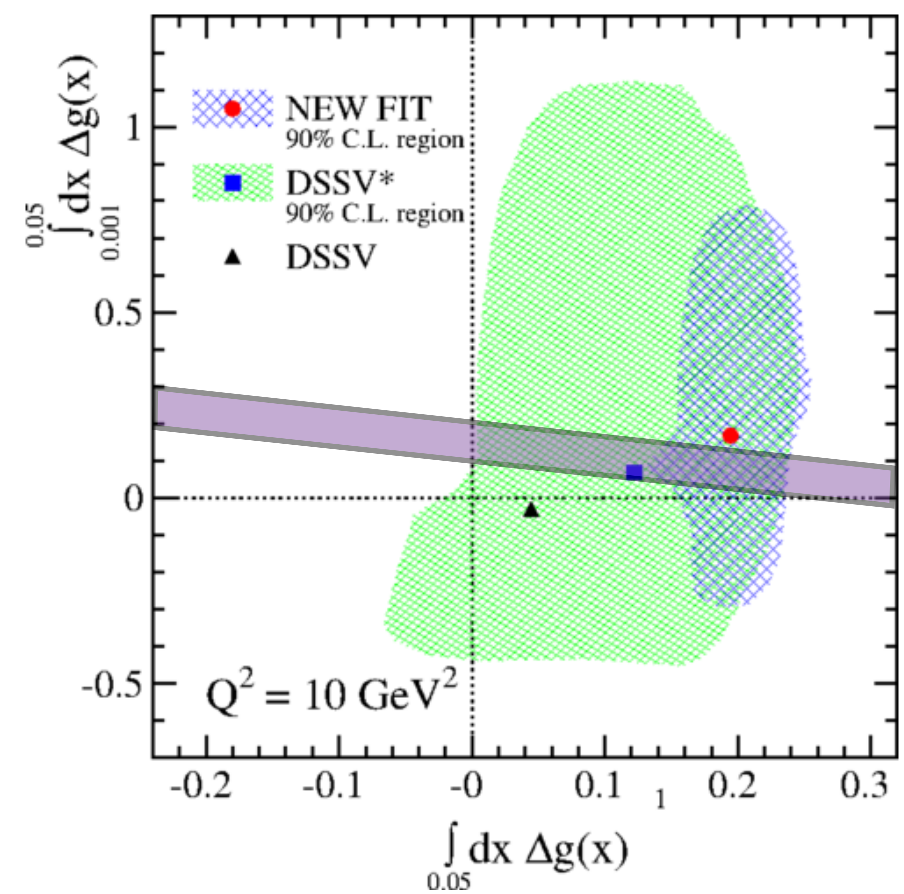
Integration of PDF

gluon parts

- The integration of the unpolarized gluon PDF is undefined.
- But the polarized one can be a finite value.

$$\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle$$

- *The gluon spin under the Coulomb gauge can be calculated on the lattice.*
- *It can be matched to ΔG based on the large momentum effective theory (LaMET).*
- *With renormalization at the 1-loop level, we predicts ΔG=0.25(6) when the matching effects can be ignored.*



LaMET: X. Ji, et. al, PRL 111(2013) 112002, 1304.6708
Exp. fit: Florian et. al, PRL 113 (2014), 012001, 1404.4293
Lattice: YBY, et al, χQCD, PRL 118 (2017), 102001, 1609.05937

Integration of GPD

The form factors

$$F_q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\; e^{ix\bar{P}^+z^-}\; \langle P'|\bar{\psi}(-z/2)\gamma^+\psi(z/2)|P\rangle \Big|_{z^+=z^1=z^2=0}$$

$$= \frac{1}{2\bar{P}^+}\left(H_q(x,\xi,t)\;\bar{u}(P')\gamma^+u(P) \;+\; E_q(x,\xi,t)\;\bar{u}(P')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(P)\right)$$

When we integrate F over x, we get the Dirac and Pauli form factors,

$$\int_{-1}^{1} dx H_q(x,\xi,t) = F_1^q(t) \qquad \int_{-1}^{1} dx E_q(x,\xi,t) = F_2^q(t)$$

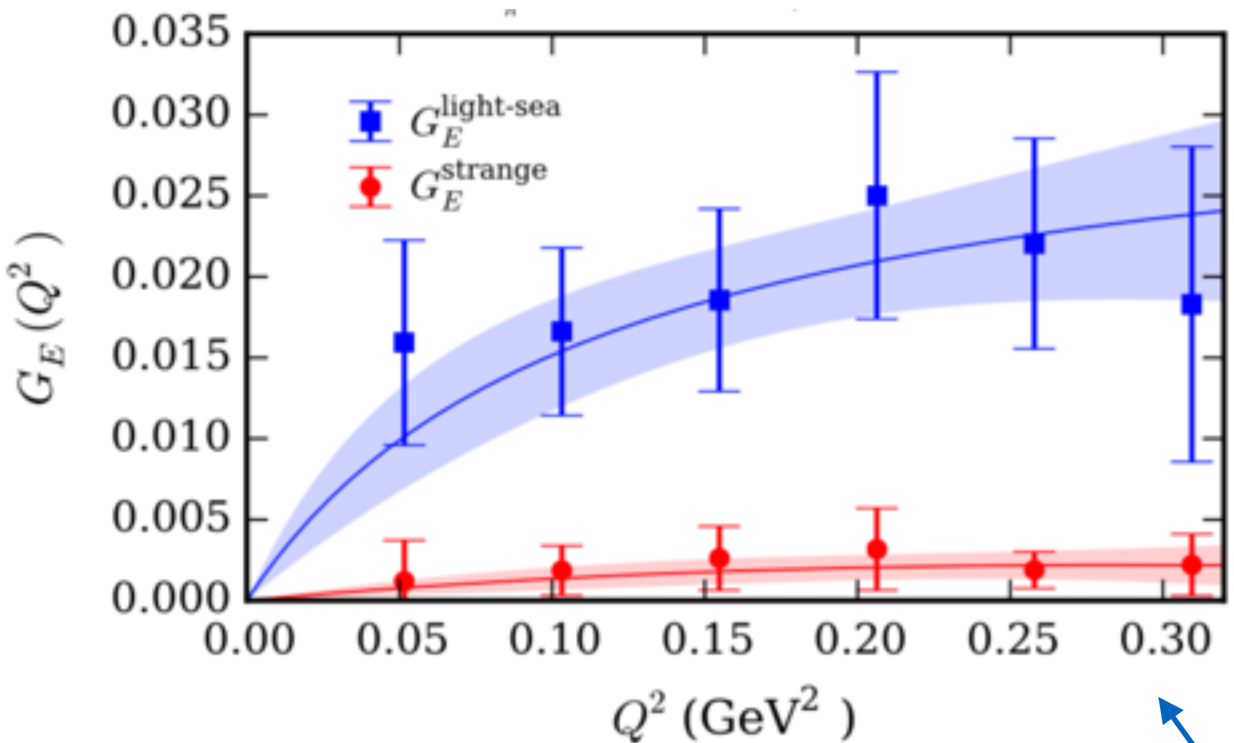
The lattice simulation of their combinations

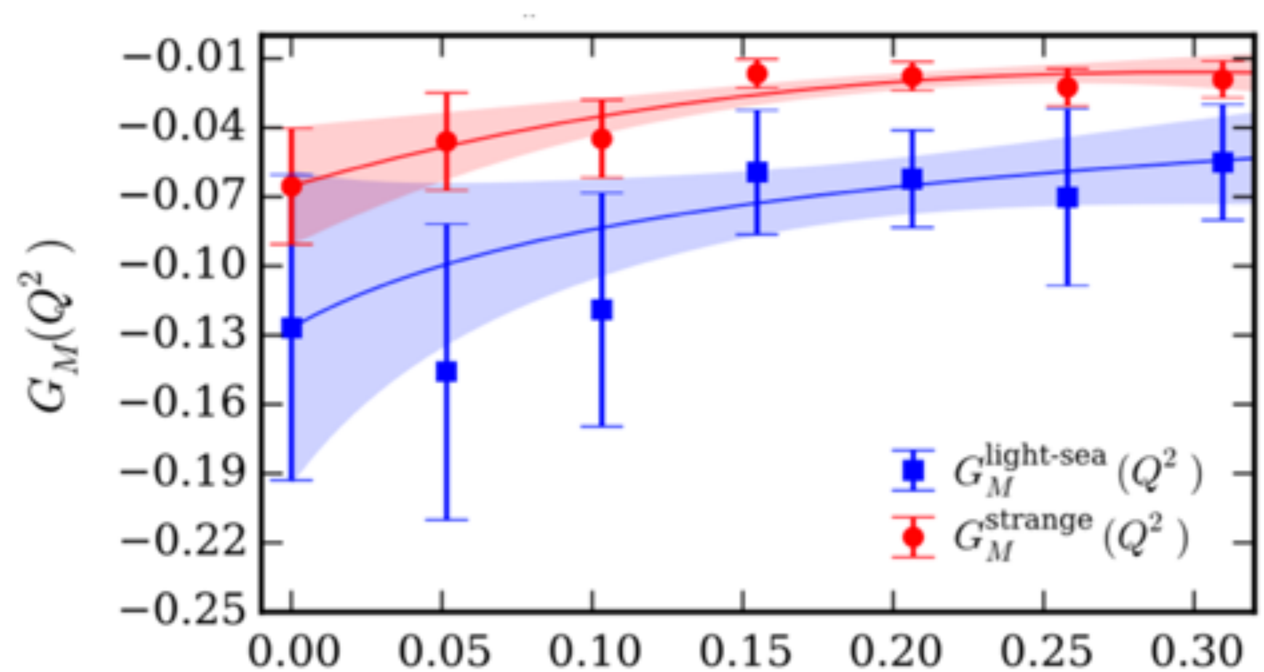
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

are more straightforward to calculate, as they cam be obtained by the off-forward vector current in the unpolarized and polarized proton.

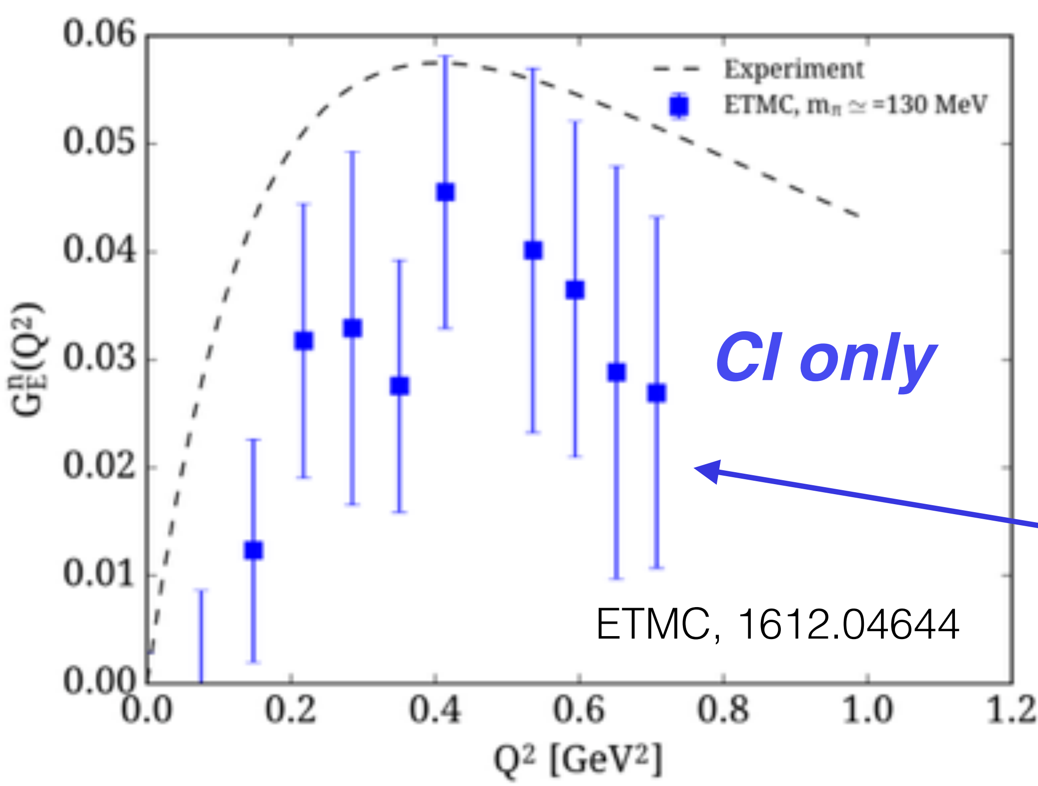
Integration of GPD

Example: *The DI form factors*





R. S. Sufian, YBY, et al, χQCD, PRL 118 (2017), 042001, 1606.07075
R. S. Sufian, YBY, et al, χQCD, 1705.05849



- *The sea quark contributions should be important to understand the tension between the ETMC valance quarks results and the experiment values.*

Moments of GPD

The first moment

$$F_q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\; e^{ix\bar{P}^+z^-}\; \langle P'|\bar{\psi}(-z/2)\gamma^+\psi(z/2)|P\rangle \Big|_{z^+=z^1=z^2=0}$$

$$= \frac{1}{2\bar{P}^+}\left(H_q(x,\xi,t)\;\bar{u}(P')\gamma^+u(P) \;+\; E_q(x,\xi,t)\;\bar{u}(P')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(P)\right)$$

For the first moment, we have the Ji sum rule:

$$\int_{-1}^{1} dx x[H_q(x,\xi,t) + E_q(x,\xi,t)] = A_q(t) + B_q(t)$$

for the quark GPD and also the gluon one,

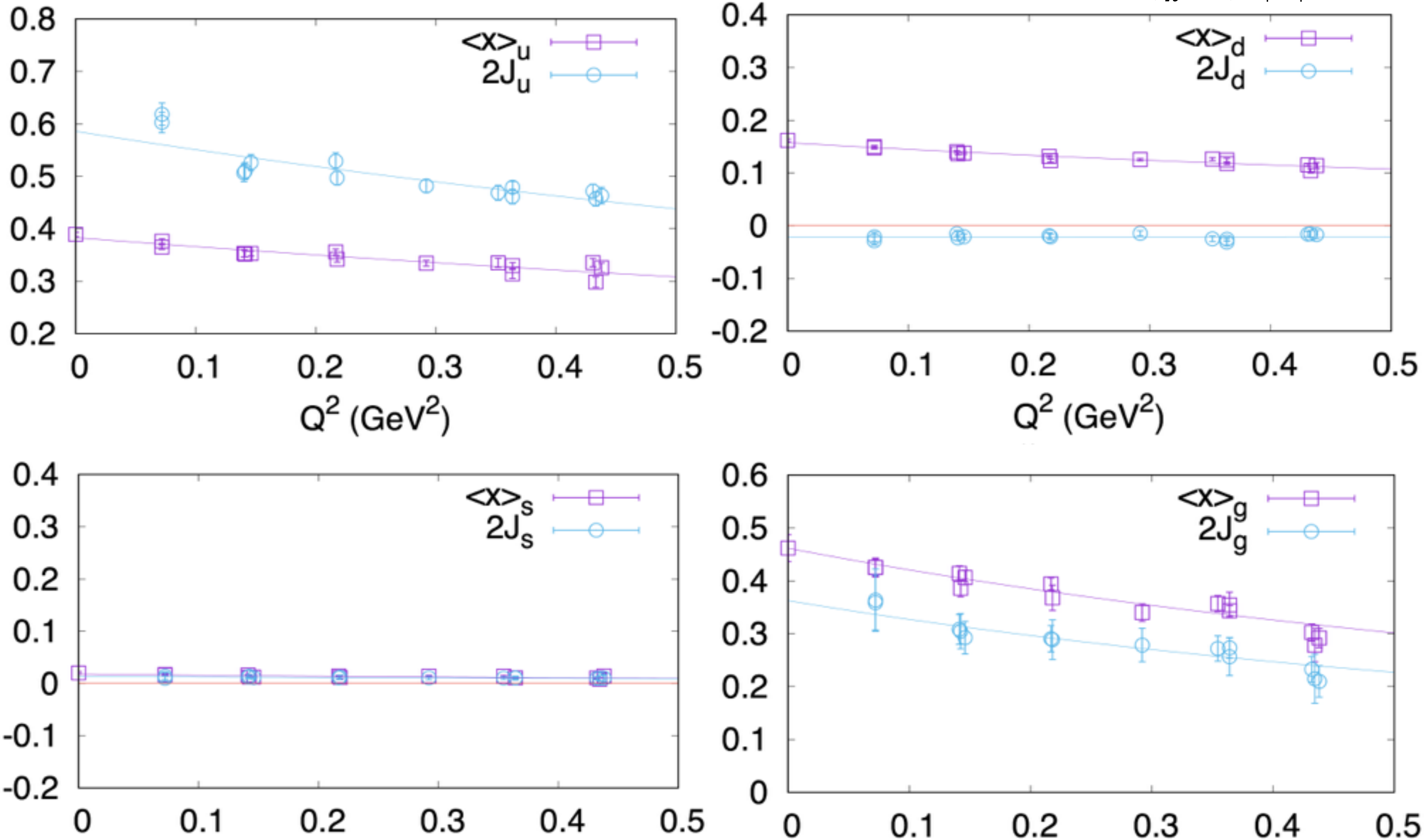
where the form factor can be obtained from the M.E. of the Energy-Momentum tensor,

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\Big[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(t)\Delta^{(\mu}\Delta^{\nu)}/M\Big]U(P)$$

Note that A(0) corresponds to the momentum fraction,
and A(0)+B(0) corresponds to the angular momentum fraction.

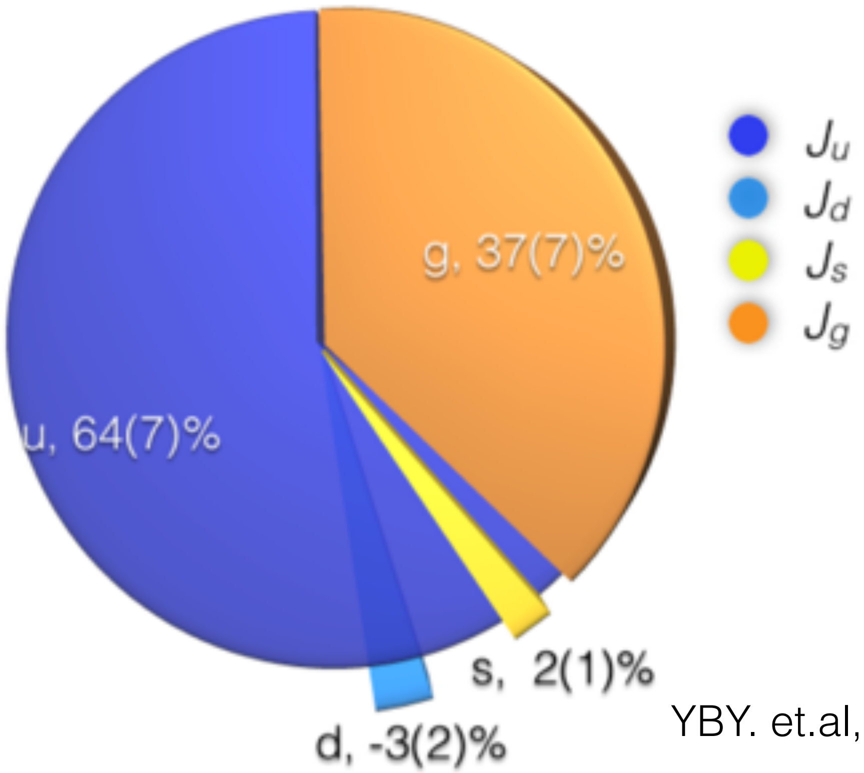
The lattice calculations

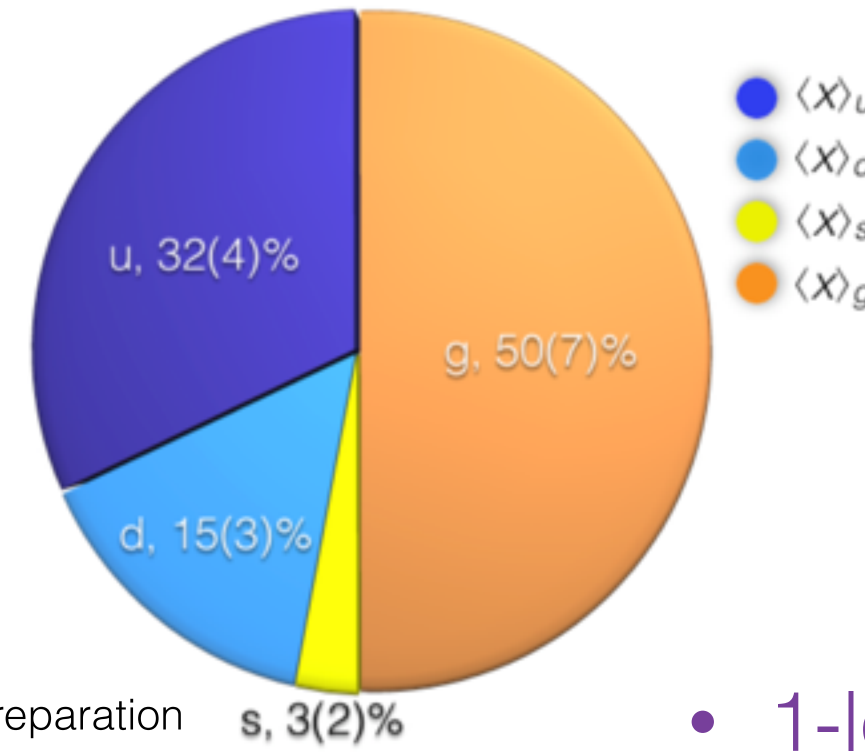
YBY. et.al, χQCD, in preparation



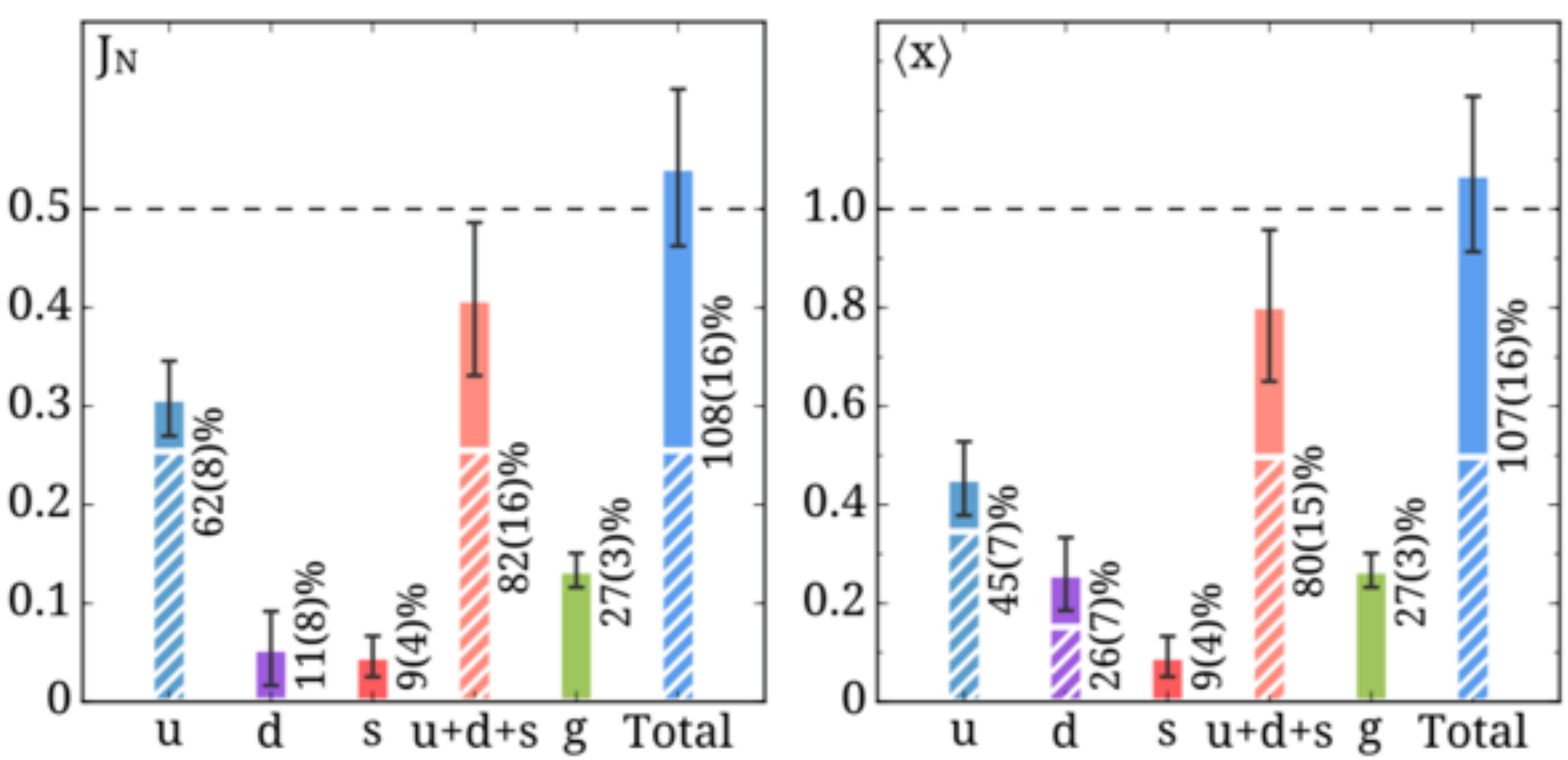
The first moments of GPD

Results





YBY. et.al, χQCD, in preparation

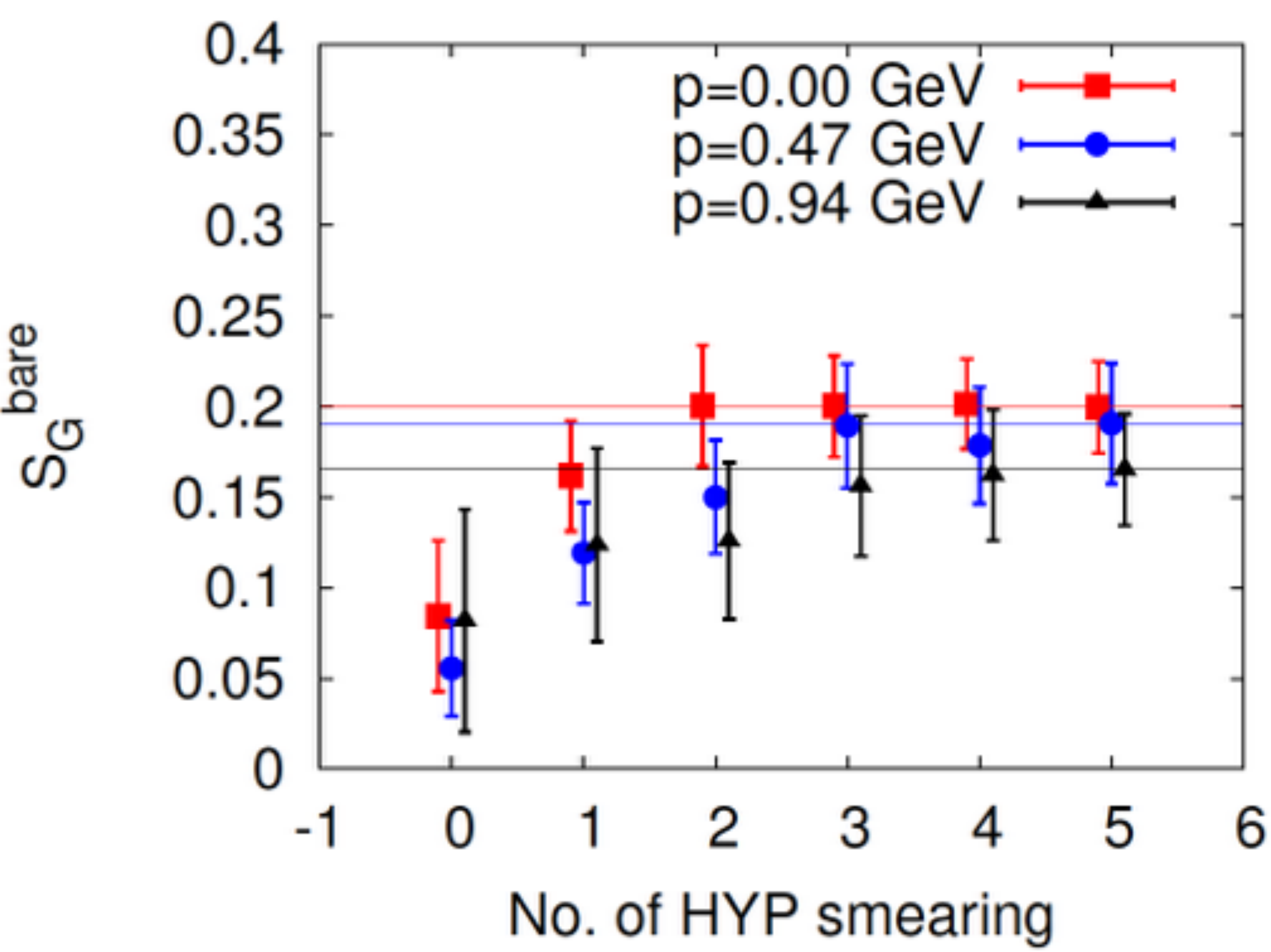


ETMC, 1706.02973

- 1-loop perturbative renormalization on the glue parts.
- Systematic uncertainties from finite lattice spacing, pion mass, and etc. are not fully included.

Non-perturbative renormalization
of the gluon parts?

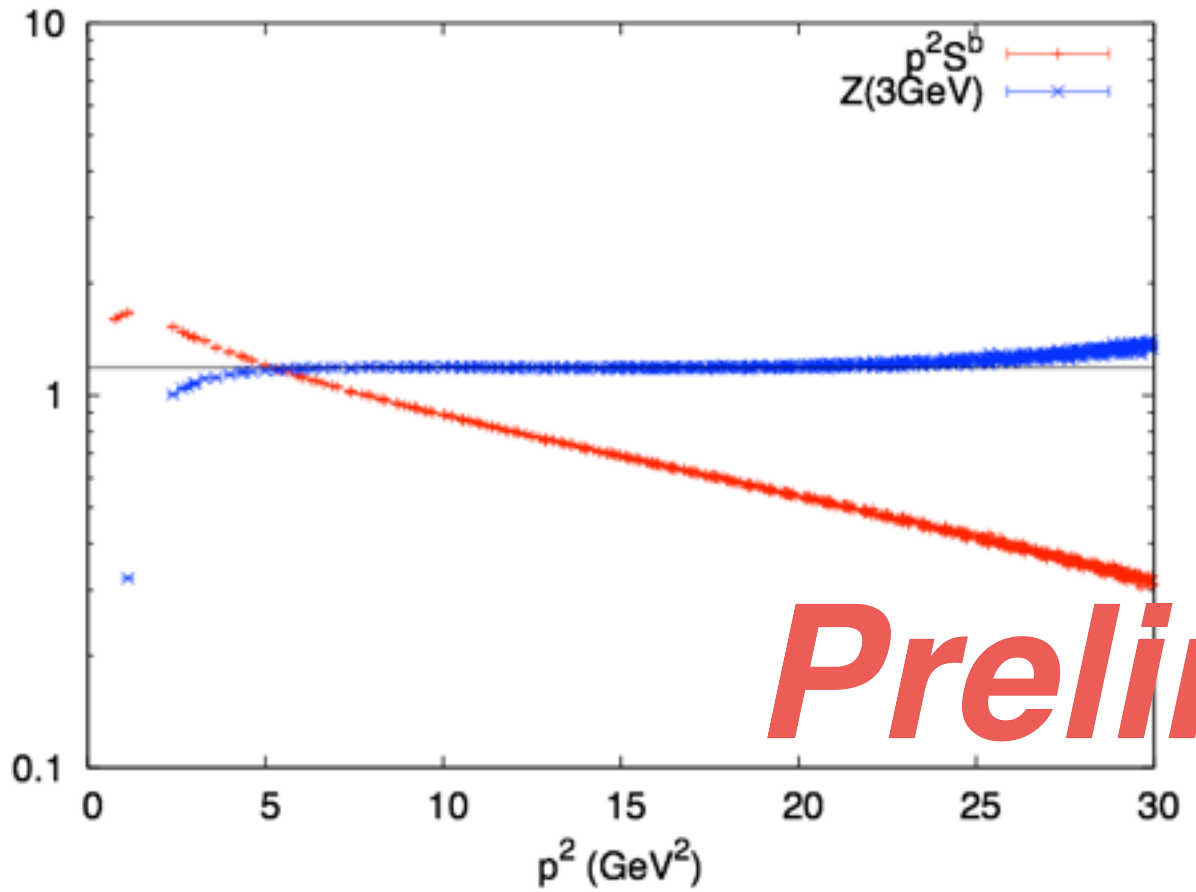
YBY, R. S. Sufian, et al, χQCD, PRL 118 (2017), 102001, 1609.05937



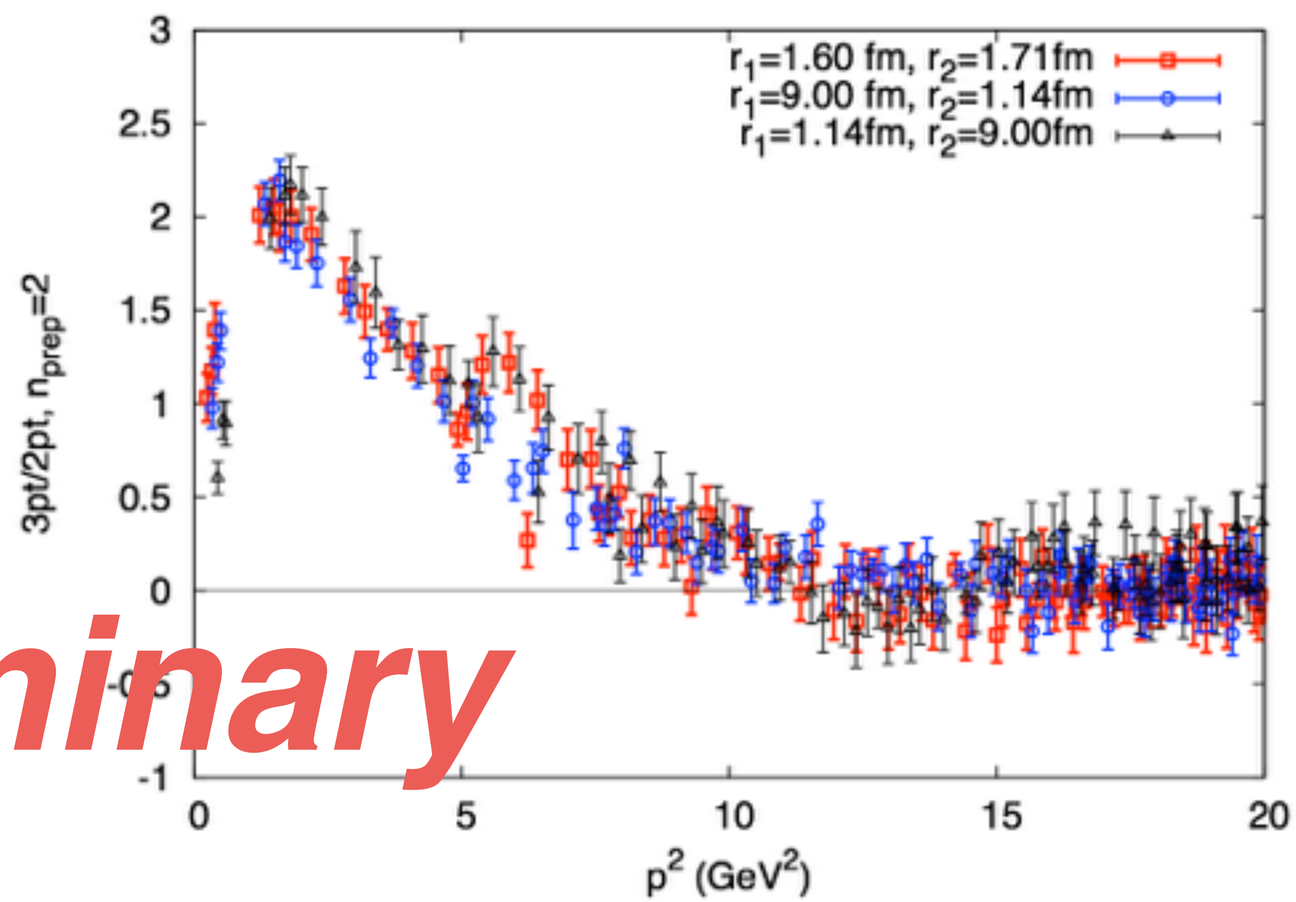
- The gluon M.E. with different steps HYP smearing can be quite different.
- The values converge after several steps, and the 1-loop renormalization effect there is reasonable in the case shown here.
- The non-perturbative renormalization will be necessary to confirm the final results are independent to the No. of HYP smearing steps.

Non-perturbative renormalization
of the gluon parts?

The renormalization factor of the gluon self energy



The renormalization factor of the gluon EMT operator



Preliminary

YBY. et.al, in preparation

Summary

of the moment calculations

- The lattice communities have many achievements recently on the first a few moments of PDF and also GPD, for both quark and gluon.

Some recents works didn't mentioned in this talk:

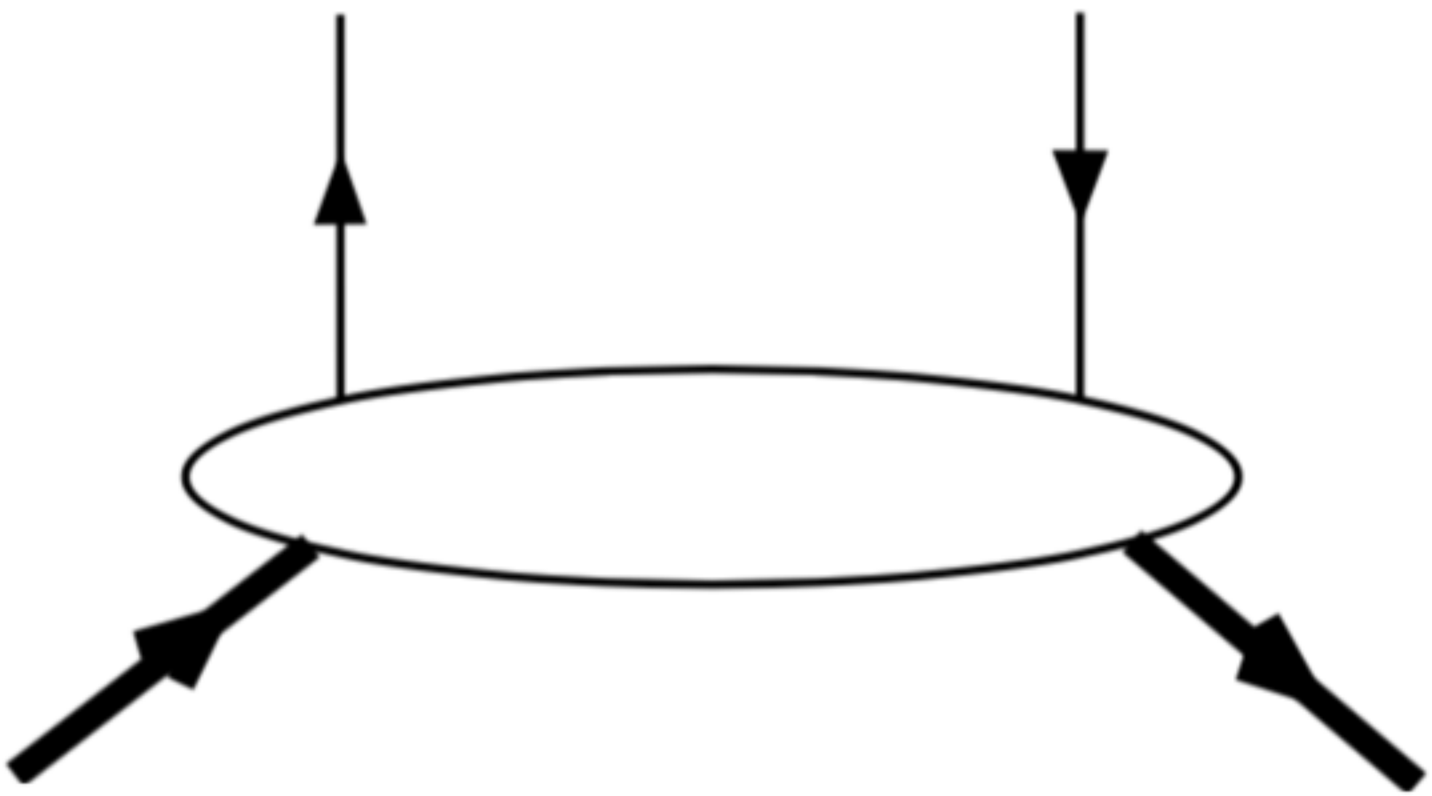
The integrations of the polarized quark GPD: 1705.03399
The moments of the gluon GPD in vector meson: 1703.08220

- The 3rd or lower moments can be obtained on the lattice without the mixing with the even lower moments, by choosing the pure off-diagonal parts; but the 4th or higher ones would be impossible.

$$O^{\mu_1 \ldots \mu_n} = \overline{\psi} \gamma^{(\mu_1} i D^{\mu_2} \ldots i D^{\mu_n)} \psi - \mathrm{trace}$$

- The non-perturbative renormalization of the gluon part is still in progress.

Reconstruct the PDF from the lattice "cross section"



Reconstruct the PDF from the lattice "cross section"

- Lattice cross section: the hadronic matrix elements of a finite Pz, which are calculable in lattice QCD and whose continuum limit can be perturbatively factorized in terms of PDFs. Y.Q. Ma and J.-W. Qiu, 1404.6860

- The quasi-PDF proposed by Ji. are good lattice "cross sections", with well-developed lattice techniques on the local operators likes the moments discussed in the previous section.

- The hadronic tensor are also good lattice "cross sections", while the calculation will be much more non-trivial;

- **Can access the full Bjorken x dependence of PDF**, while the result may depend on the way to do the reconstruction.

Definition *of the quasi-PDF*

The original quark PDF defined in the light front frame is,

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

The quasi-PDF is defined by

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^{z} dz' A^z(z')\right)\psi(0)|P\rangle + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) ,$$

X.D. Ji, Phys.Rev.Lett. 110 (2013) 262002

From the renormalized quasi-PDF

to the real PDF

X.D. Ji, Phys.Rev.Lett. 110 (2013) 262002, 1305.1539

1- loop matching from quasi-PDF to PDF

Tree level

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} [\delta(1-\frac{x}{y})\{1 + \frac{\alpha_s C_F}{2\pi} \int_{-\infty}^{+\infty} d\xi\, C^{OM}\left(\xi, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\} - \frac{\alpha_s C_F}{2\pi} C^{OM}\left(\frac{x}{y}, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)]$$
$$\int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iyP^z z} \langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_R$$
$$+ \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right) + \mathcal{O}\left((\frac{\alpha_s}{\pi})^2\right)$$

Renormalized Lattice quasi-PDF matrix elements with the mass correction which can be calculated analytically

Residual higher-twist contribution

Residual higher-loop matching

X. Xiong, X. Ji, J. Zhang, Y. Zhao, Phys. Rev. D 90 (2014) 014051, 1310.7471
LP3, Nucl.Phys. B911 (2016) 246-273, 1603.06664

From *the bare quasi-PDF*

to the real PDF

Y-Q. Ma, J-W. Qiu, 1404.6860
C. Alexandrou et. al., Phys. Rev. D92 014502
J.-W. Chen, X. Ji, J. Zhang, Nucl.Phys. B915 (2017) 1

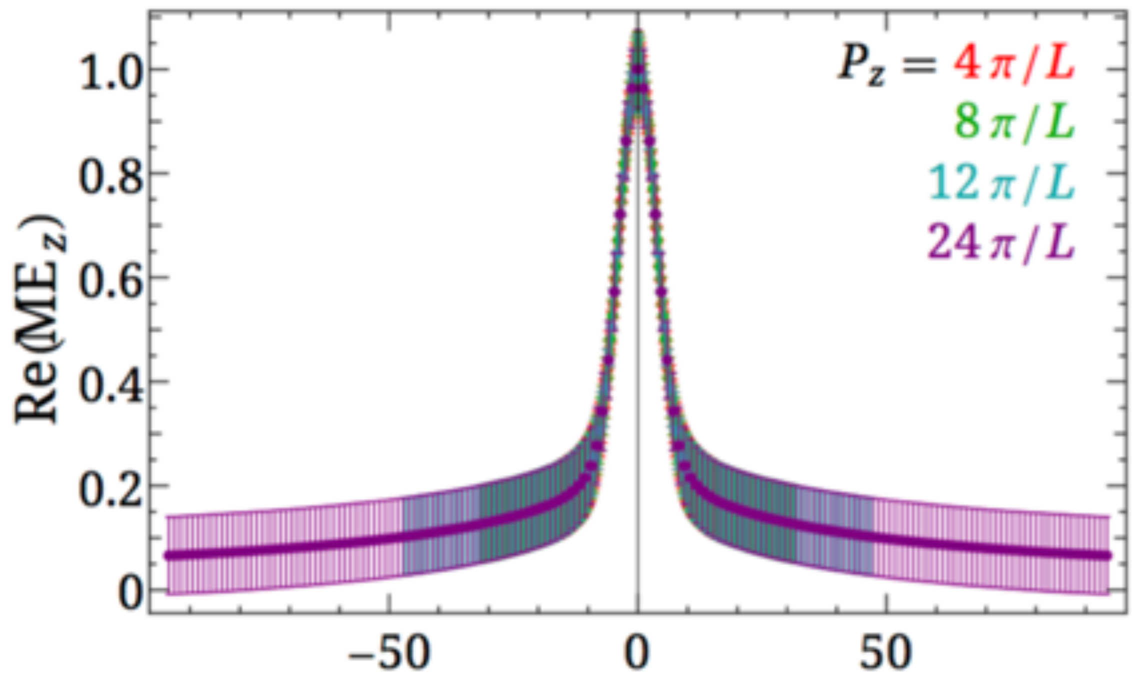
$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} \left[\delta(1-\frac{x}{y})\left\{1 + \frac{\alpha_s C_F}{2\pi}\int_{-\infty}^{+\infty} d\xi\, C^{OM}\left(\xi, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\right\} - \frac{\alpha_s C_F}{2\pi} C^{OM}\left(\frac{x}{y}, \frac{\mu_R}{P^z}, \frac{\mu}{P^z}\right)\right]$$
$$\int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iyP^z z} \langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_R$$
$$+\,\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}\right) + \mathcal{O}\left((\frac{\alpha_s}{\pi})^2\right)$$

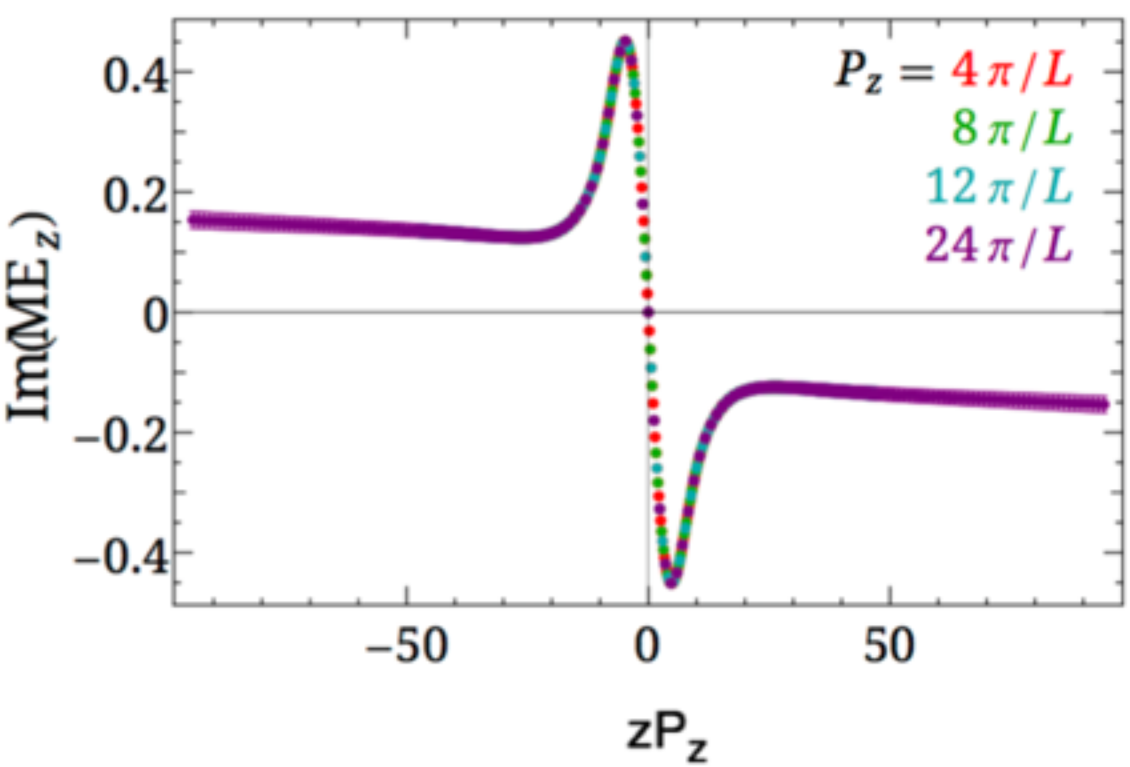
The linear divergence under the lattice regularization can break the convergence of the perturbative series!

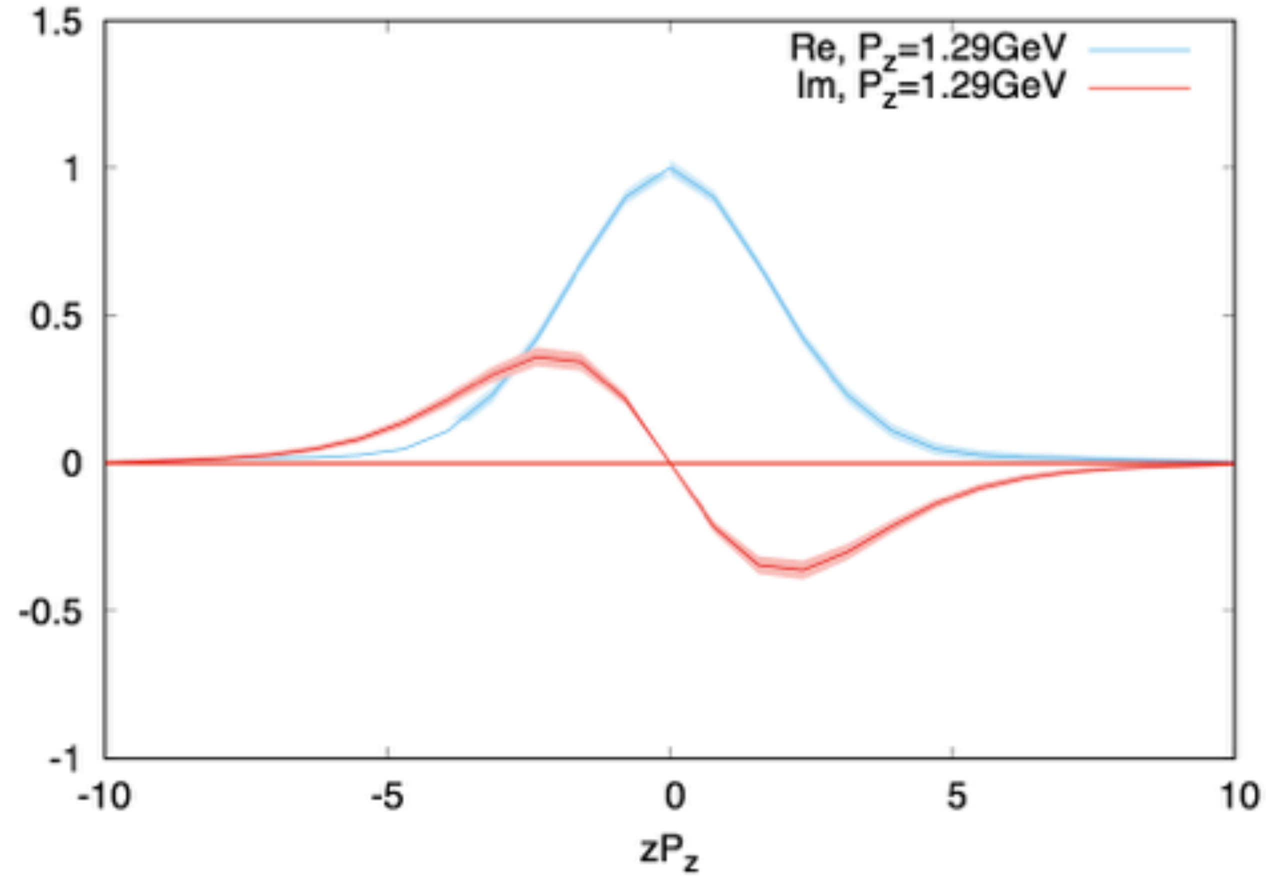
$$\langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_R = (1 + \frac{\alpha_S}{4\pi}(\frac{C}{a} + \text{Log}(p^2a^2) + ...) + \mathcal{O}((\frac{\alpha_S}{4\pi})^2))\langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_{bare}$$

Linear divergence

another hint

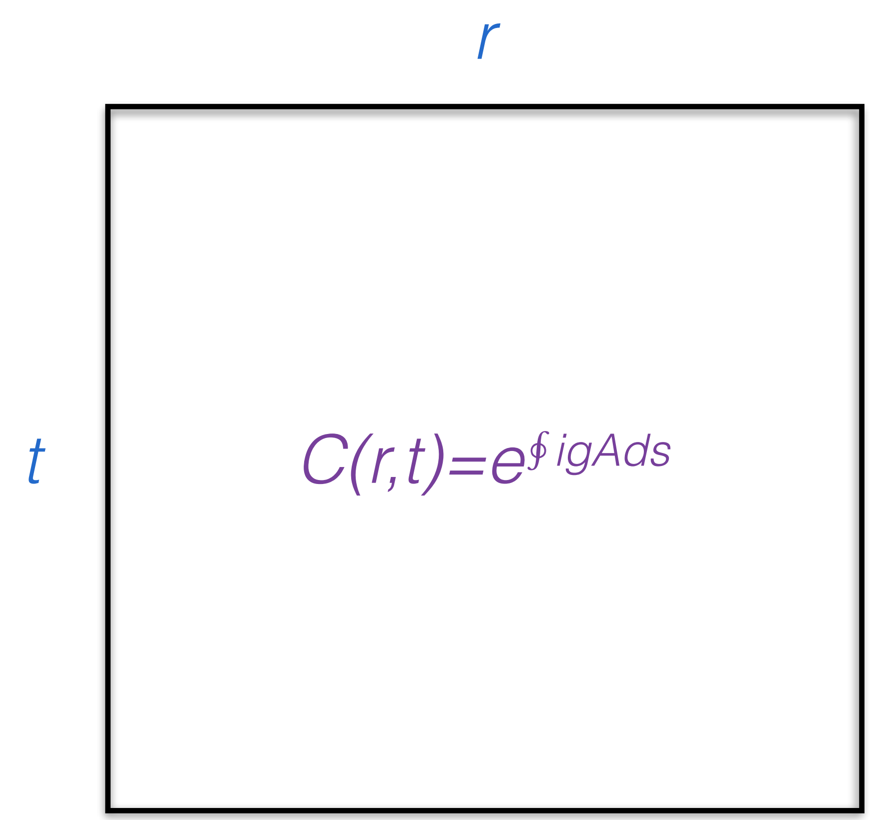




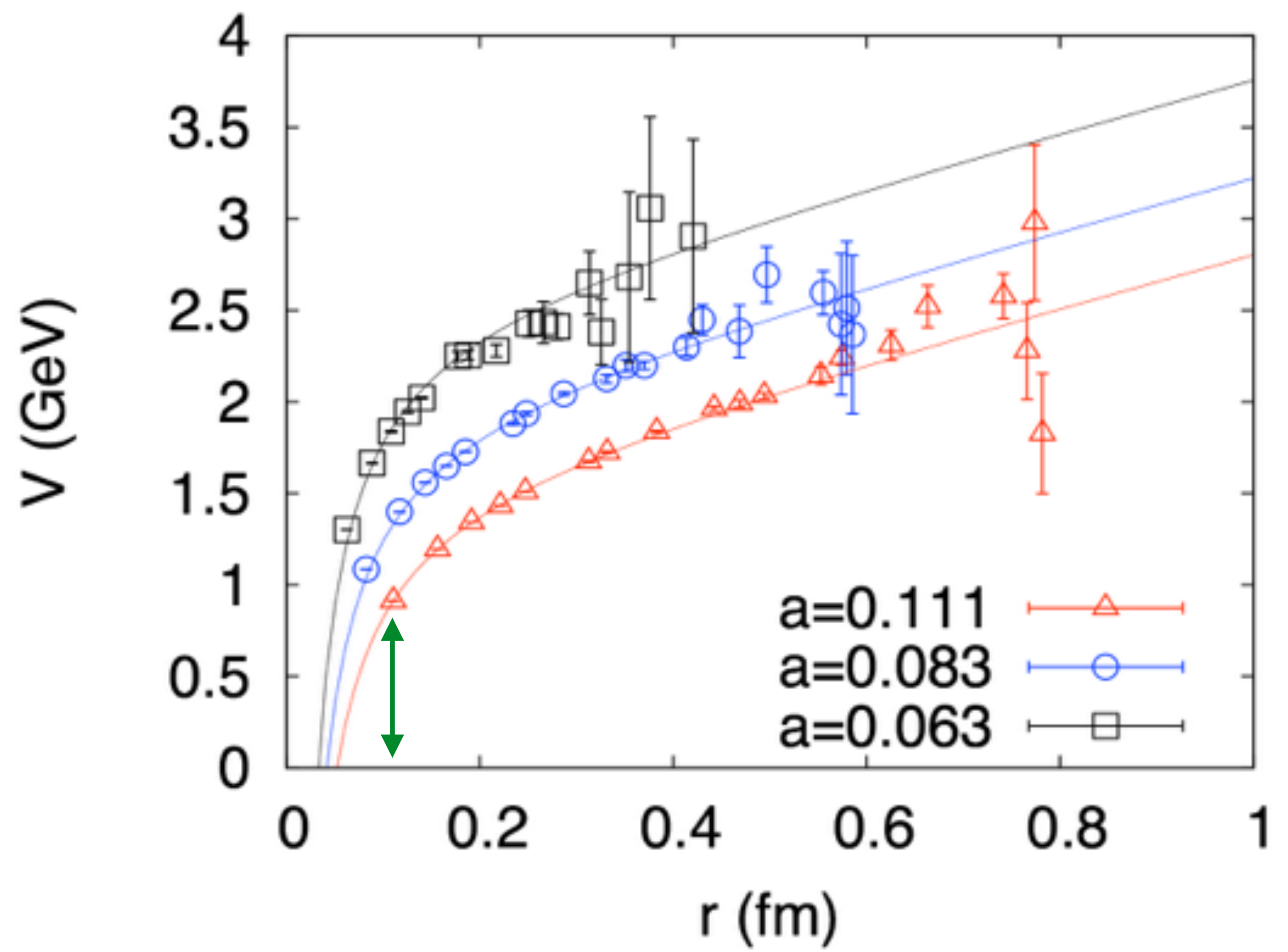


- Right: The bare quasi-PDF M.E. from the lattice simulation;
- The bare quasi-PDF M.E. decays much faster than the FT of the real PDF **due to the linear divergence.**

Linear divergence



in the wilson loop



The statical potential is defined by,

$$V(R)=Log[\langle C(r,t)\rangle/\langle C(r,t+1)\rangle]\,|_{t\to\infty,\ r\to\infty}$$

$$=\alpha/r+2\mathbf{A}+\mathbf{B}r,$$

with

$$\mathbf{A}\sim \Delta m\, \mathbf{a_0/a} + \mathbf{A_0}$$

The non-perturbative renormalization of the quasi-PDF operator

- The quasi-PDF operator $\langle P|\bar{\psi}(z)\gamma^z W_z(z,0)\psi(0)|P\rangle_{bare}$ with different z will NOT mix with each other.

T. Ishikawa, et.al, 1707.03107

- The linear divergence of the wilson link can be removed by multiply a factor $\sim e^{\Delta m z}$ *with* Δm from the wilson line/loop.

X.Ji. et.al, 1706.08962
J. Green, et.al, 1707.07152

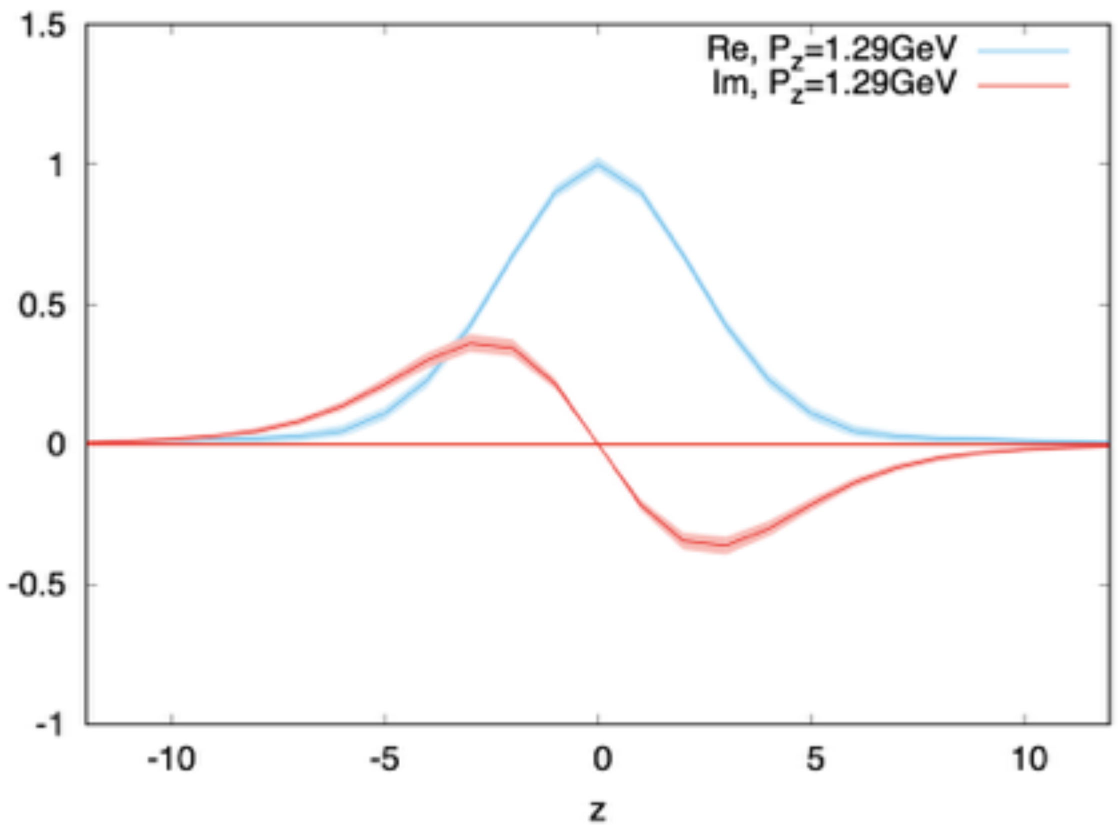
- **Or, both the linear and logarithmic UV divergence can be removed with non-perturbative renormalization (NPR) under RI/MOM scheme.**

ETMC, NPB923(2017) 394, 1706.00265
LP3, 1706.01295

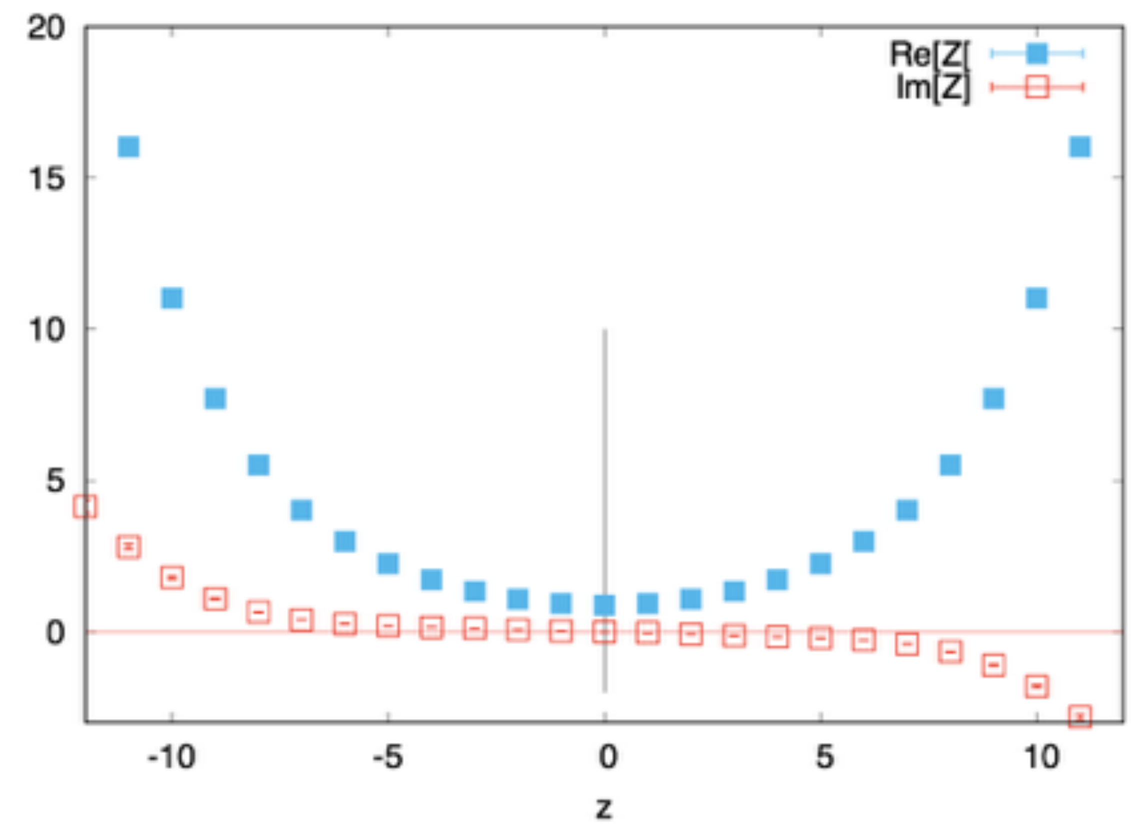
- **Or, the UV divergence can be removed by taking ratio of the hadron M.E. with the same operator and different momenta.**

A. Radyushkin, 1705.01488
K. Orginos, et.al, 1706.05373

The non-perturbative renormalization of the quasi-PDF operator



The bare matrix elements

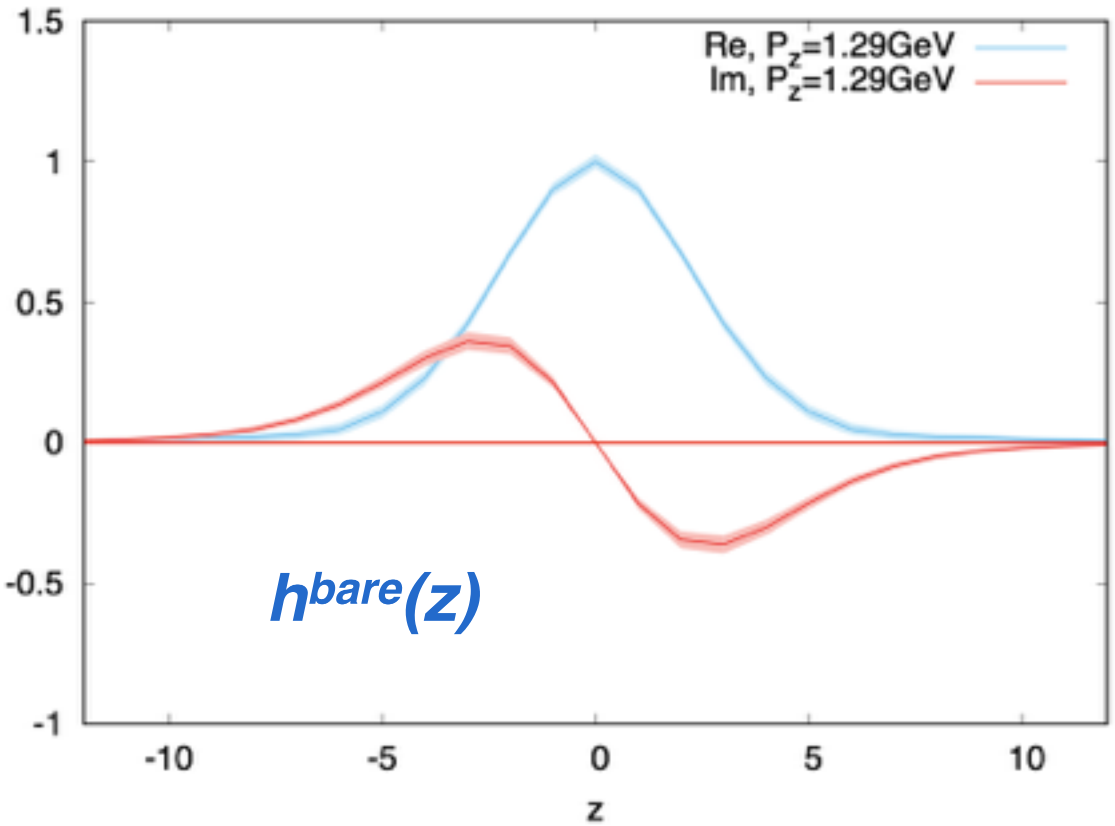


The renormalization factors including the factor $e^{\Delta m z}$

LP3, 1706.01295

The renormalized quasi-PDF

LP3, 1706.01295



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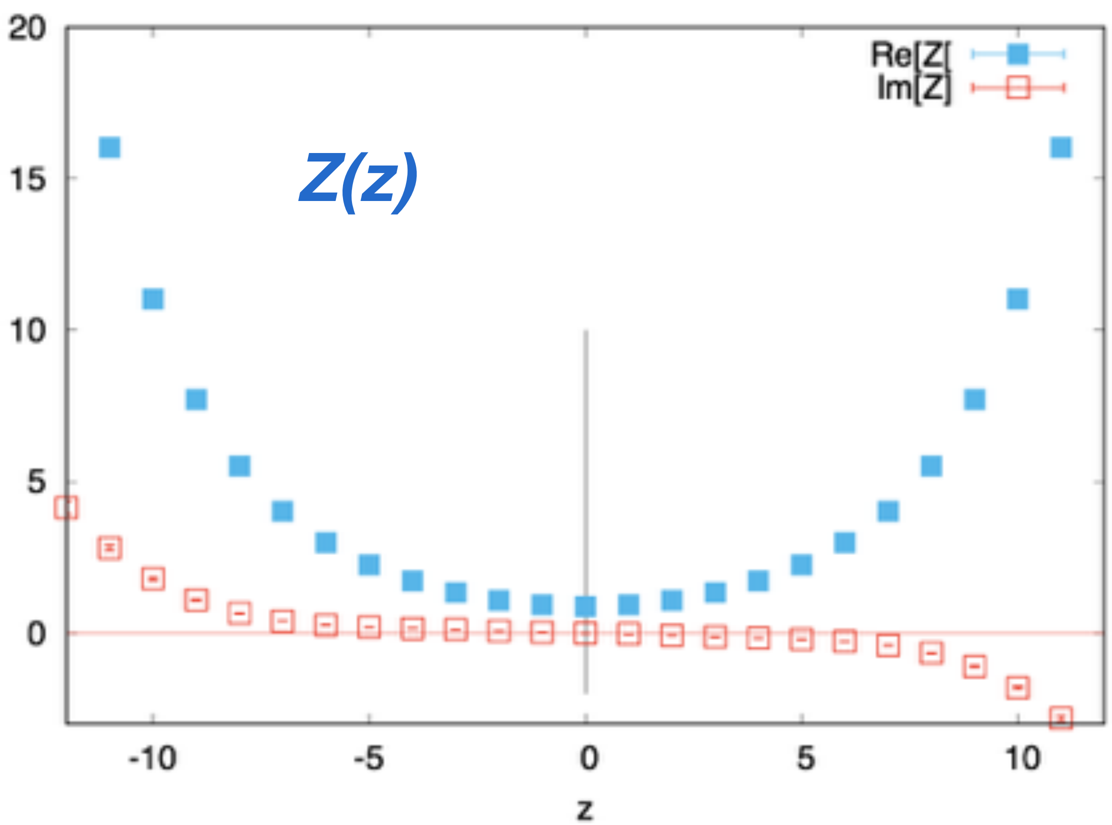

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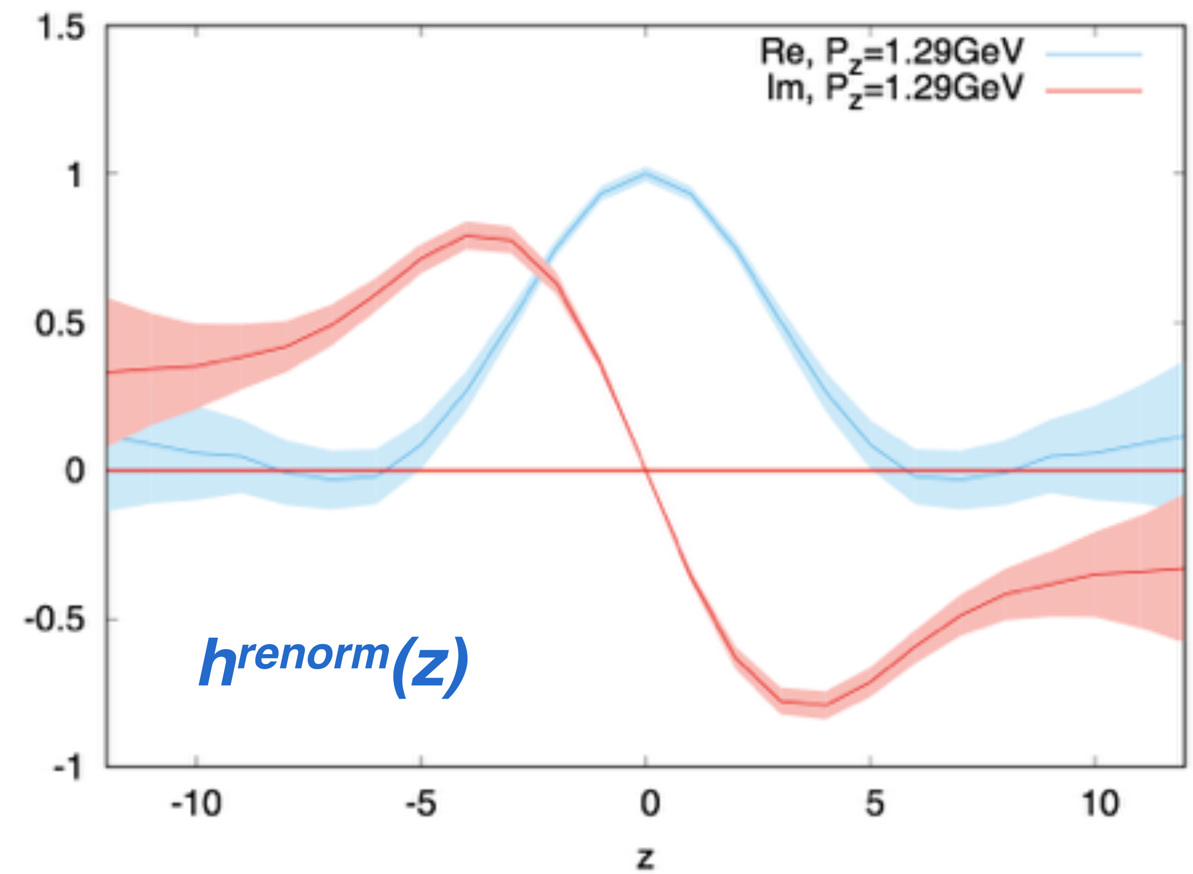
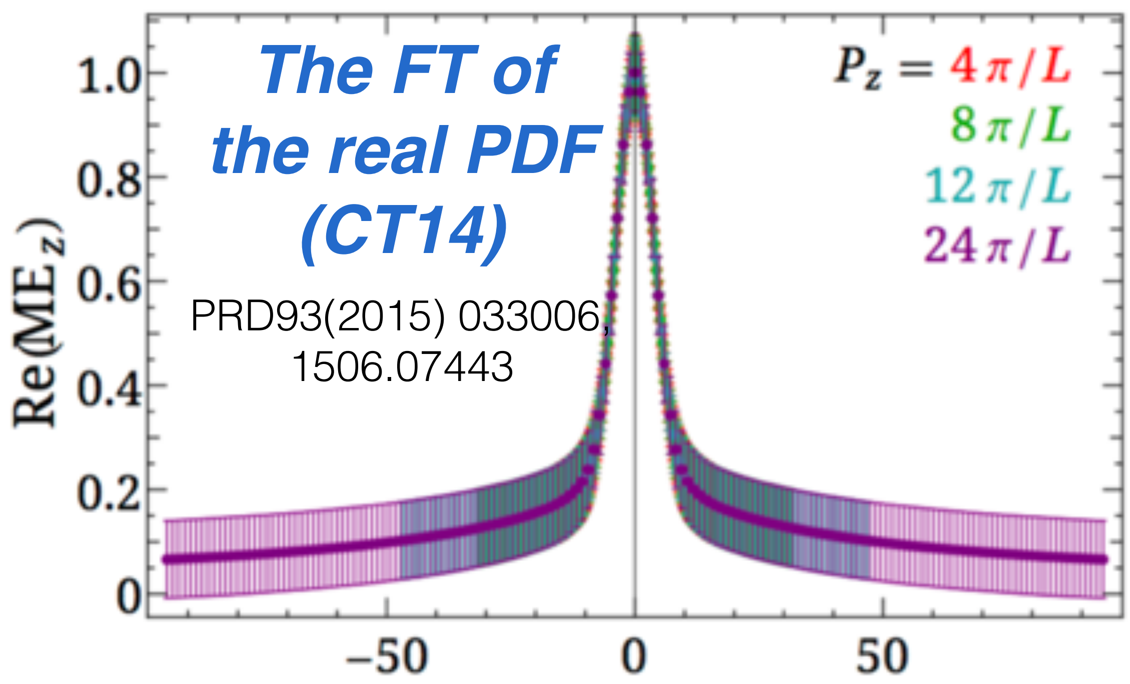
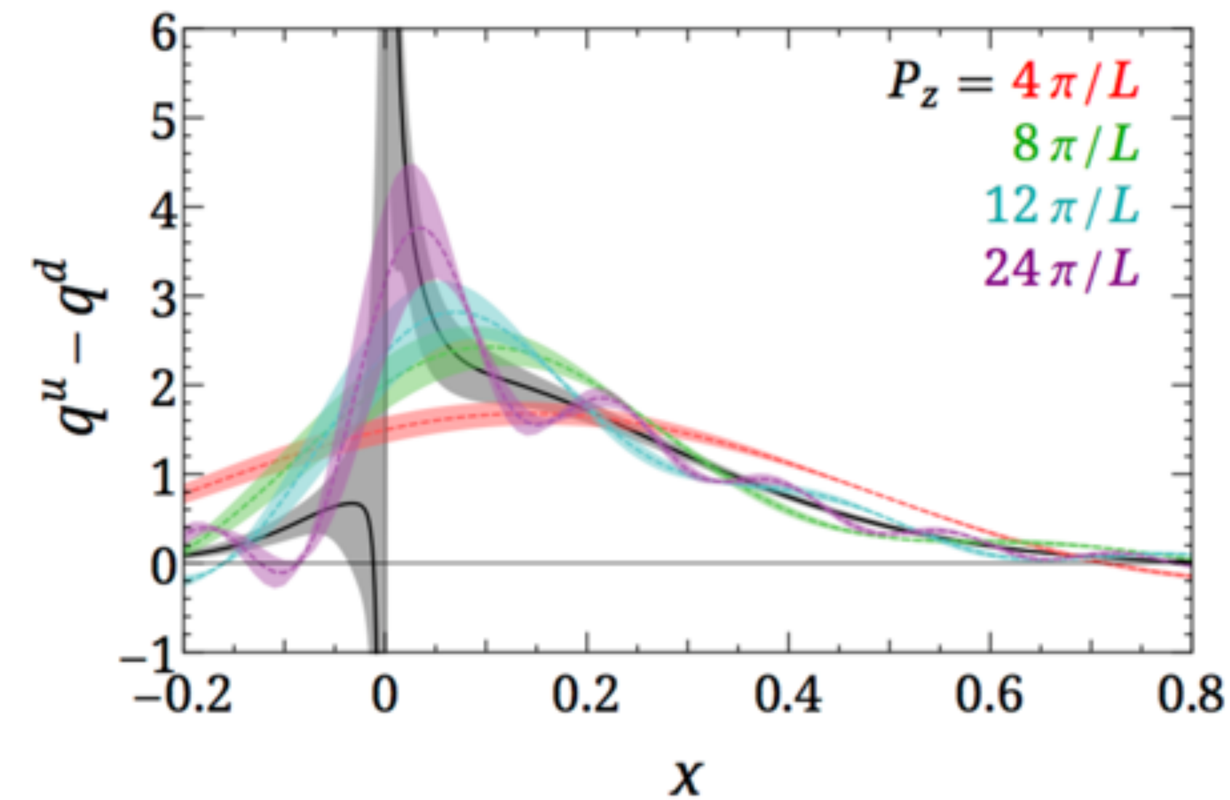


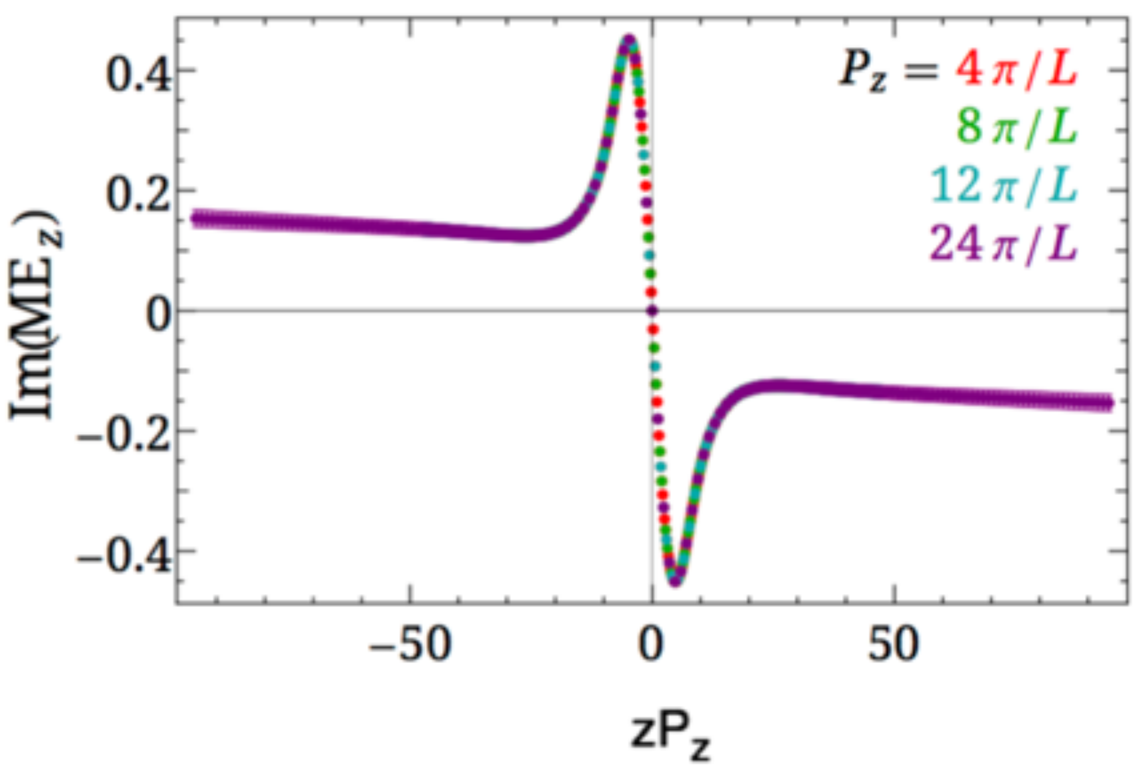
- The unphysical exponential decay in the bare M.E. has been cured with the NPR;
- The uncertainties at large z have been enhanced by the huge factors $e^{\Delta mz}$.

Reconstruct PDF

H.-W. Lin, J.-W. Chen, T. Ishikawa, J.-H. Zhang, 1708.05301



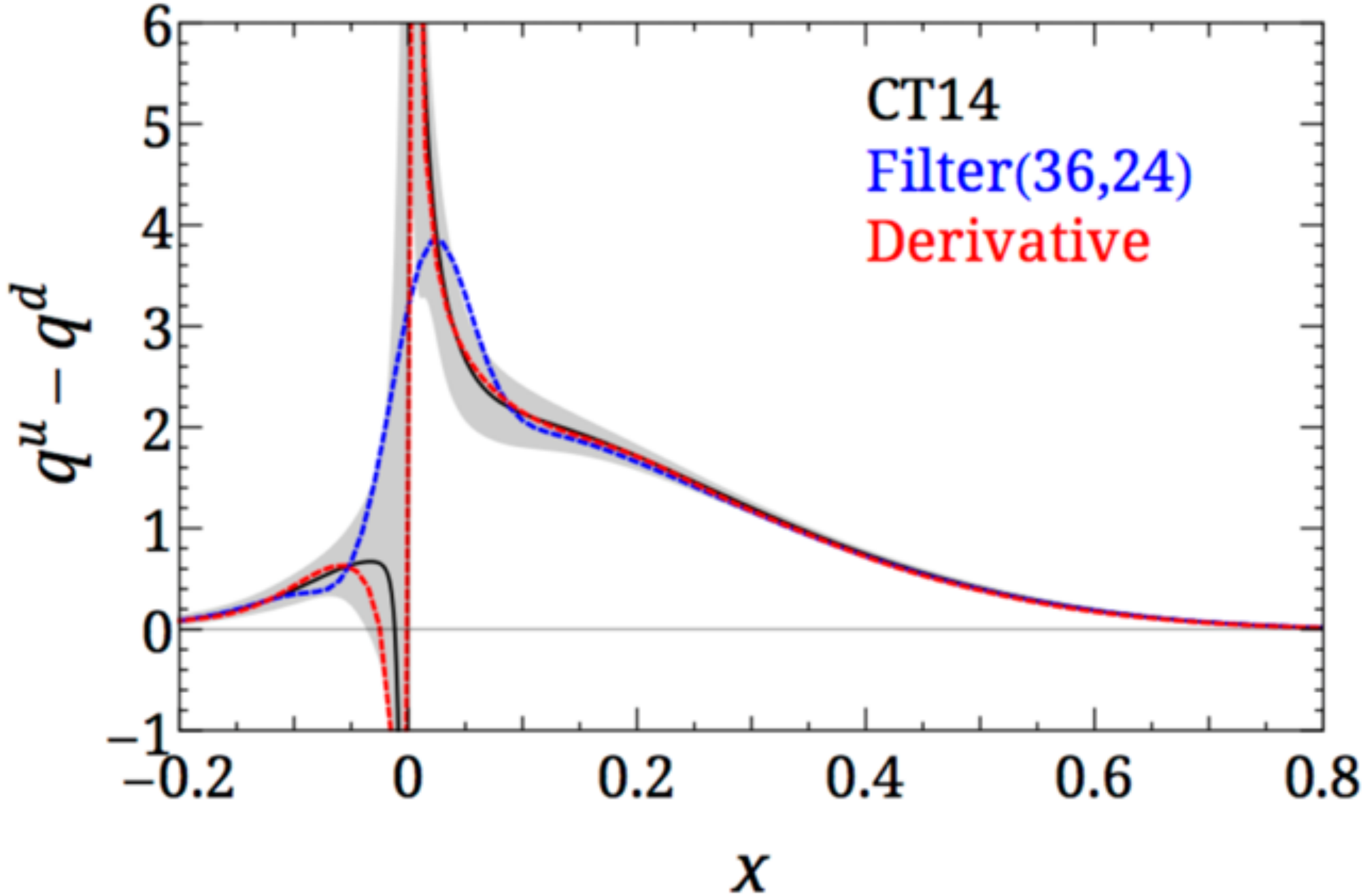




- *The data can be obtained on the lattice are limited by some z_{max}*
- *If we do FT back with limited z_{max}, strong oscillations will be observed.*

Reconstruct PDF with filters

H.-W. Lin, J.-W. Chen, T. Ishikawa, J.-H. Zhang, 1708.05301



*Derivative: Require the derivative of the original PDF to be **zero** when |z| >L/2=32*

Filter(36,24): Applying the filter

$$F(z_{\rm lim}, z_{\rm wid}) = \frac{1 + {\rm erf}\left(\frac{z + z_{\rm lim}}{z_{\rm wid}}\right)}{2} \frac{1 - {\rm erf}\left(\frac{z - z_{\rm lim}}{z_{\rm wid}}\right)}{2}$$

on the FT of the original PDF and FT back.

Both of the them can improve the quality of the reconstruction.

Hadronic tensor

and the PDF

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4z \, e^{iq\cdot z} \left\langle P,S \left| \left[J^\dagger_\mu(z), J_\nu(0) \right] \right| P,S \right\rangle$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F_2(x,Q^2)$$

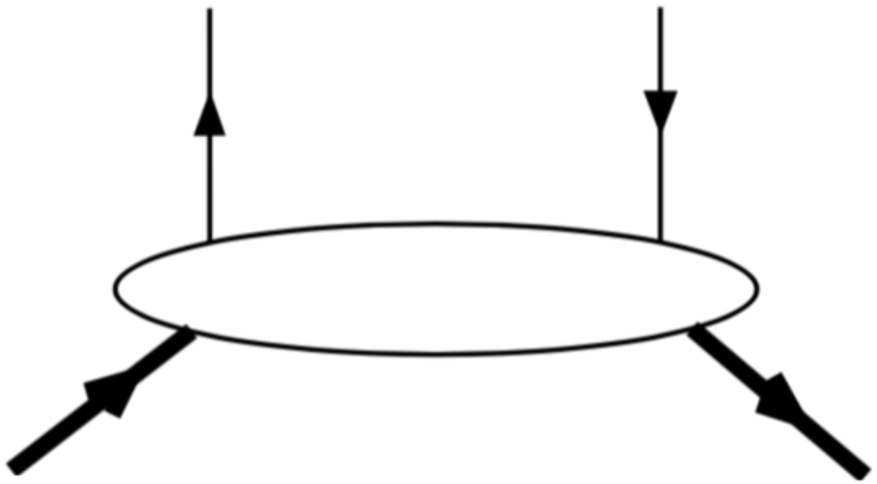
$$- i\varepsilon_{\mu\nu\alpha\beta} \frac{q^\alpha P^\beta}{2P\cdot q} F_3(x,Q^2)$$

$$+ i\varepsilon_{\mu\nu\alpha\beta} \frac{q^\alpha}{P\cdot q} \left[S^\beta g_1(x,Q^2) + \left(S^\beta - \frac{S\cdot q}{P\cdot q} P^\beta \right) g_2(x,Q^2) \right]$$

$$+ \frac{1}{P\cdot q} \left[\frac{1}{2} \left(\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\nu \right) - \frac{S\cdot q}{P\cdot q} \hat{P}_\mu \hat{P}_\nu \right] g_3(x,Q^2)$$

$$+ \frac{S\cdot q}{P\cdot q} \left[\frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} g_4(x,Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) g_5(x,Q^2) \right]$$

$$\hat{P}_\mu = P_\mu - \frac{P\cdot q}{q^2} q_\mu, \qquad \hat{S}_\mu = S_\mu - \frac{S\cdot q}{q^2} q_\mu \, .$$



- *The form factors can be reduced to the PDFs in the large Q^2 limit with reasonable assumptions.*
- *Thus one can reconstruct PDF with the "measurements" on the hadronic tensor.*

PDG review, Sec 19, Chin. Phys. C, 40(2016) 100001

Reconstruct the PDF from lattice operator product expansion

QCDSF, Phys.Rev.Lett. 118 (2017), 242001, 1703.01153

Given forward hadronic tensor

$$T_{\mu\nu}(p,q) = \rho_{\lambda\lambda'} \int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x} \langle p, \lambda' | T J_\mu(x) J_\nu(0) | p, \lambda \rangle$$

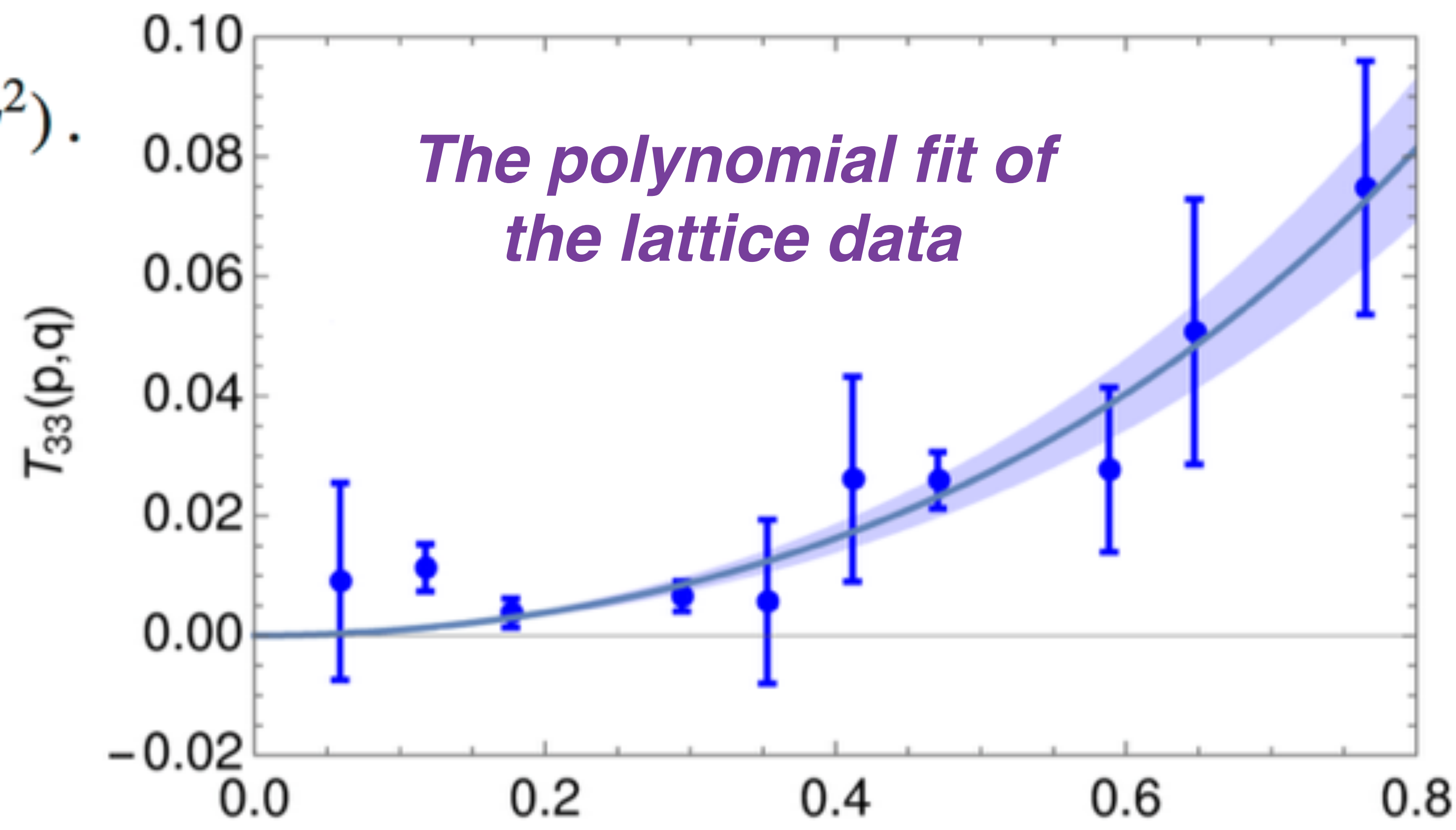
choose $\mu = \nu = 3$ *and* $p_3 = q_3 = q_4 = 0$

one has,

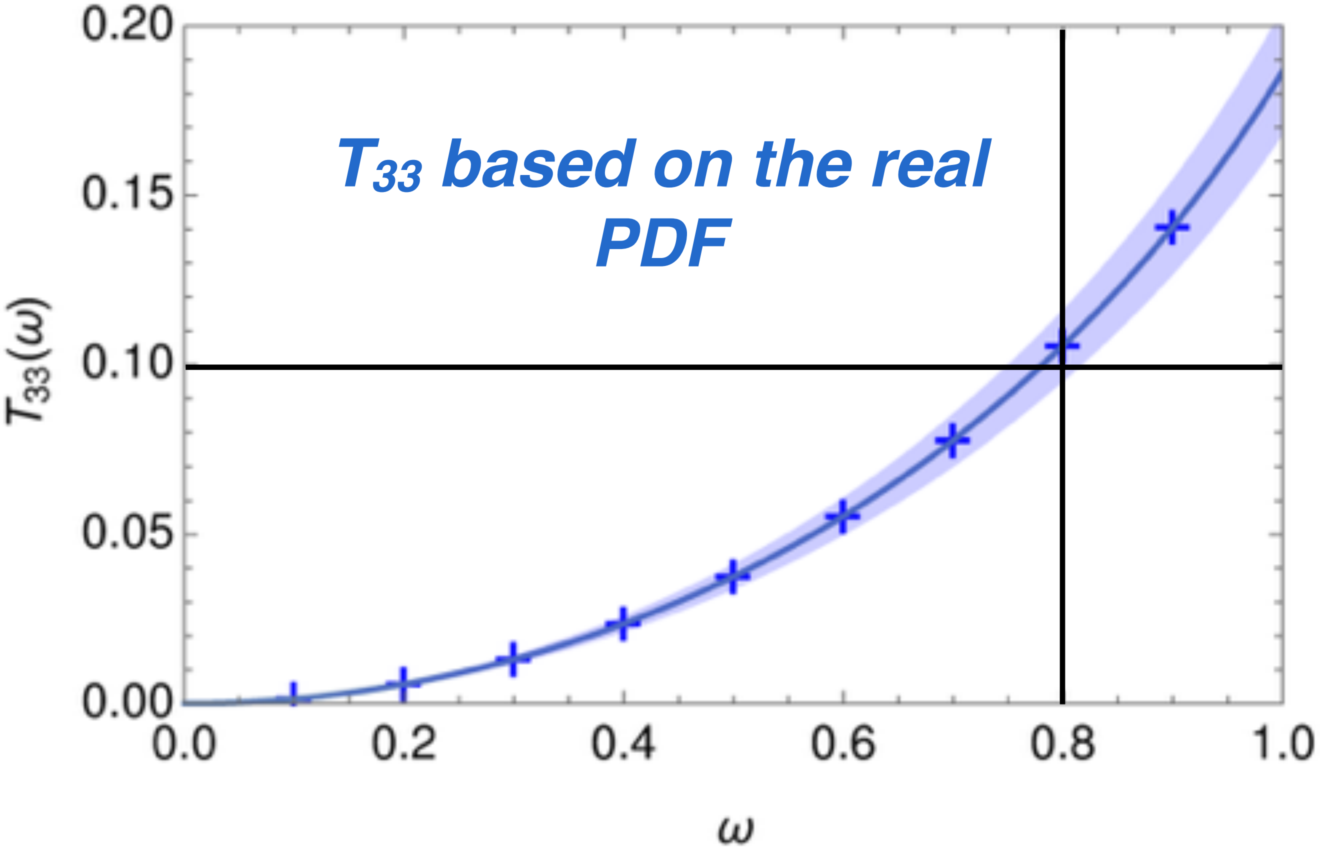
$$T_{33}(p,q) = 4\omega \int_0^1 dx \, \frac{\omega x}{1-(\omega x)^2} F_1(x, q^2) \, .$$

with $\omega = 2p.q/q^2$

Then one can reconstruct the PDF from the results of the hadronic tensor.



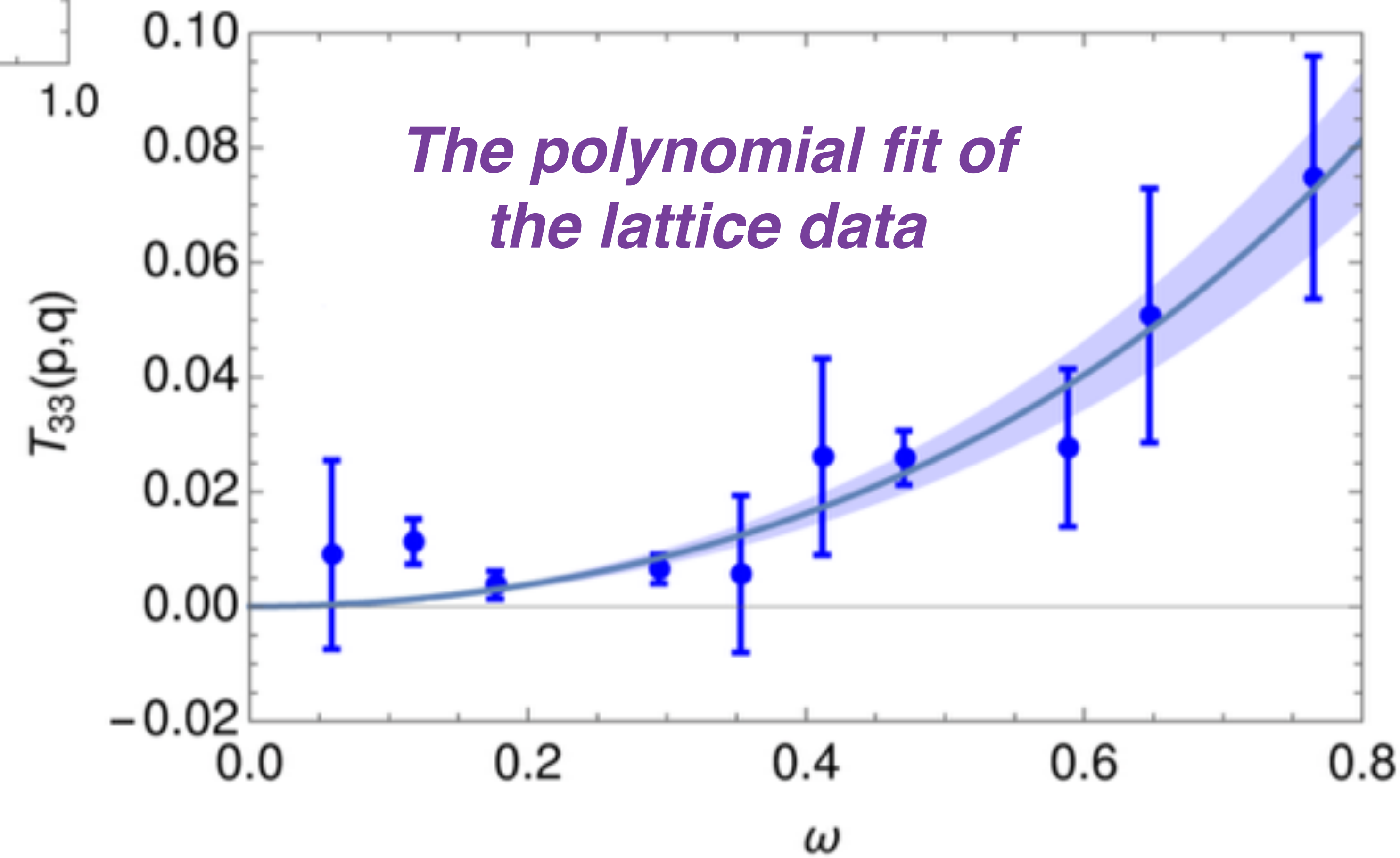
Reconstruct the PDF from lattice operator product expansion



QCDSF, Phys.Rev.Lett. 118 (2017), 242001, 1703.01153

The reconstruction strategy has been tested with the real PDF data;

and the lattice results on T_{33} seem to have a similar shape as that from the real PDF data.



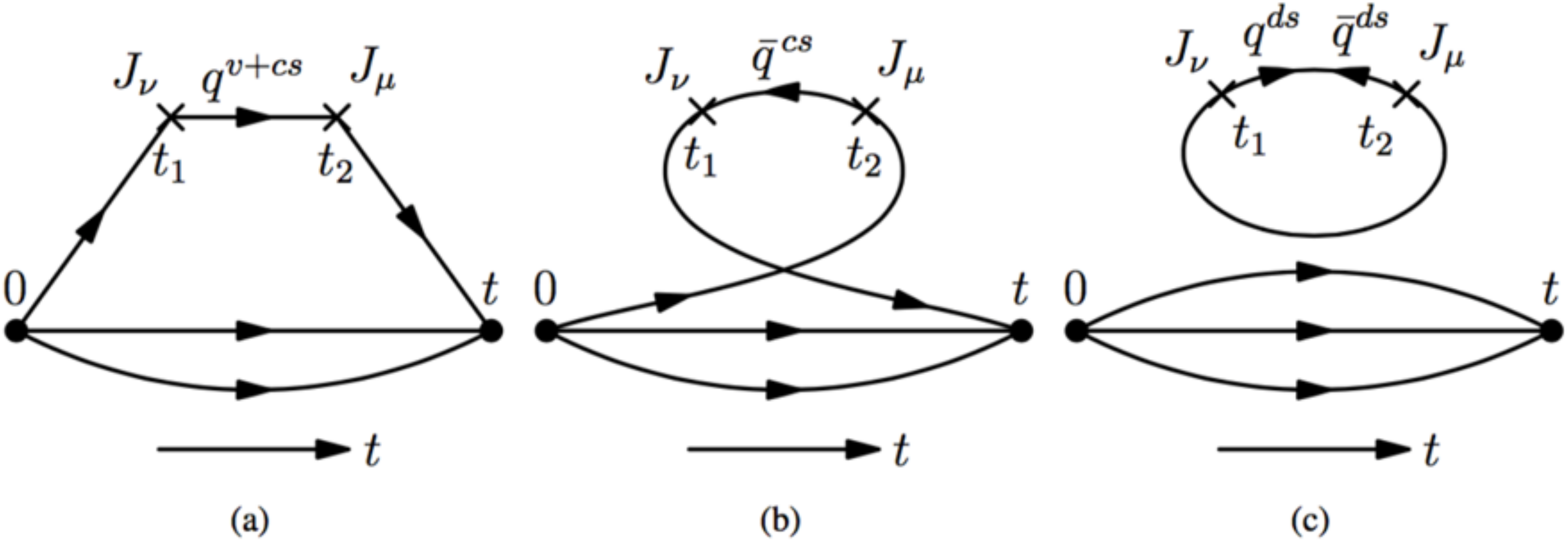
Access the connected sea

with a special currents setup

K.-F. Liu, 1606.07075,
K.-F. Liu, J. Liang, YBY. Lattice 2017 poster

$$\tilde{W}_{\mu\nu}(q^2,\tau) = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)} \langle \chi_N(\vec{x}_f,t_f) J_\mu(\vec{x}_2,t_2) J_\nu(\vec{x}_1,t_1) \bar{\chi}_N(\vec{0},t_0)\rangle,$$

$$\tau = t_2 - t_1.$$

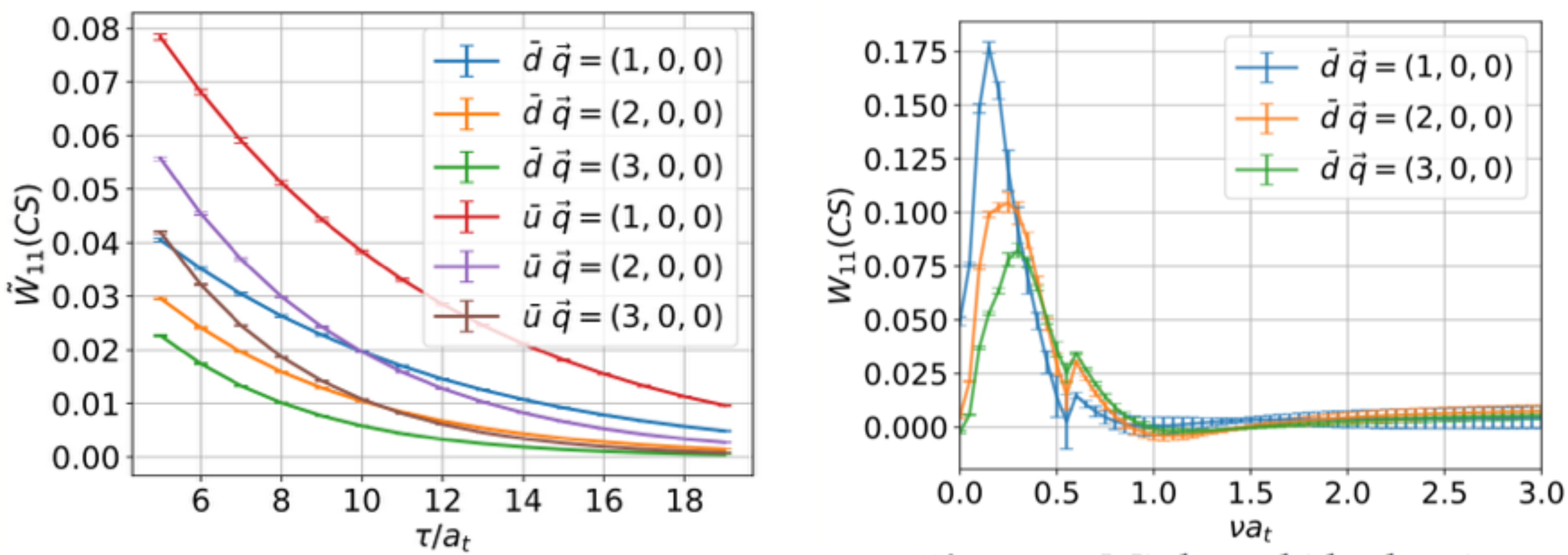


By using different time order of the two vector currents, the contribution of ***the anti-quark connected sea*** *can be obtained independently.*

Access the connected sea

with a special currents setup

K.-F. Liu, 1606.07075,
K.-F. Liu, J. Liang, YBY. Lattice 2017 poster



$$\tilde{W}_{\mu\nu}(q^2,\tau) = \int d\omega\, e^{-\nu\tau}\, W_{\mu\nu}(q^2,\nu)$$

The hadronic tensor of the connected sea part can be implemented by several methods, such as the Backus-Gilbert method, the improved maximum entropy method, the χ^2 fit with model functions and so on.

Backus-Gilbert Method: Maxwell T. Hansen, Harvey B. Meyer, Daniel Robaina,1704.08993
Improved Maximun entropy method: Yannis Burnier, Alexander Rothkopf PRL. 111 (2013) 182003, 1307.6106

Summary

The reconstruction of the entire PDF

- The linear divergence in the quasi-PDF can be safely removed by several strategies.

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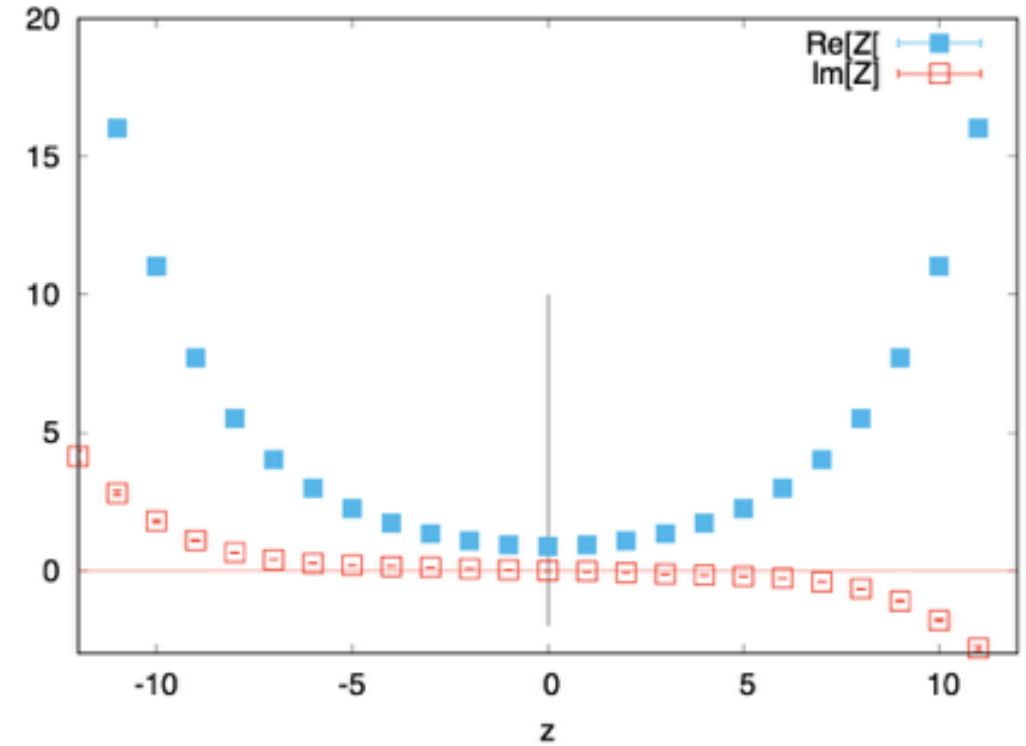


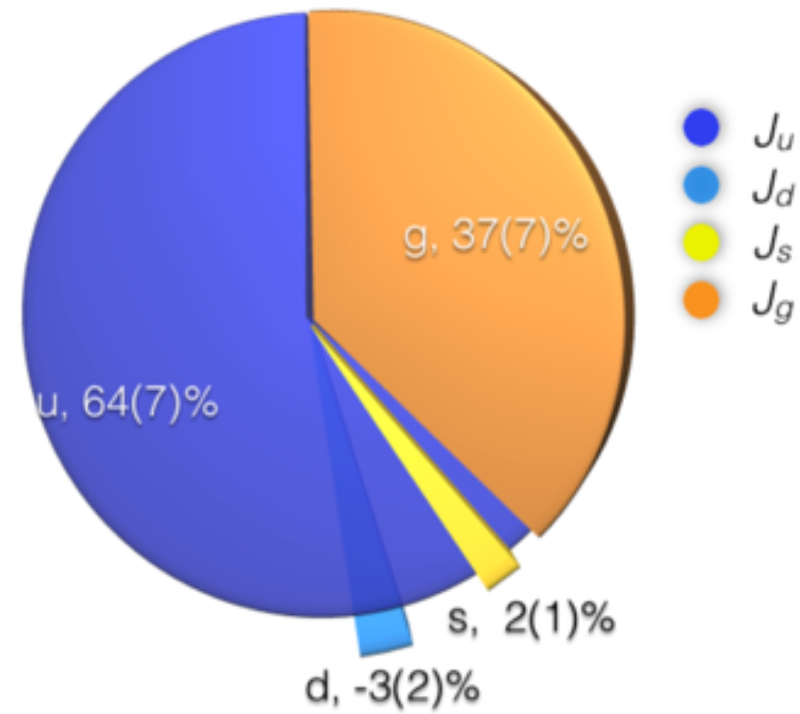
$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4z\, e^{iq\cdot z} \left\langle P,S \middle| \left[J^{\dagger}_{\mu}(z), J_{\nu}(0) \right] \middle| P,S \right\rangle$$

- Reconstructing PDF from the hadronic tensor are also in progress with several approaches.

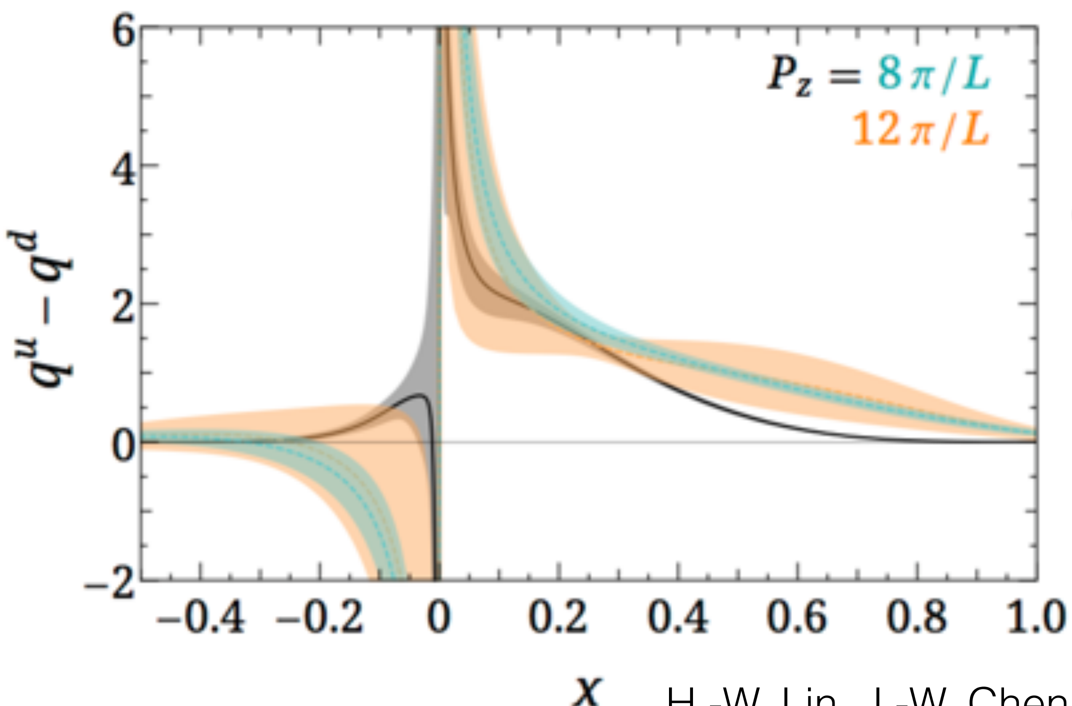
- The systematic uncertainties for both the directions are under investigation, and the statistical uncertainties will be under control with increasing computer resources.

Summary

walking on two legs



- The moments calculation can provide relatively precise informations on limited moments of PDF and GPD;



H.-W. Lin, J.-W. Chen, T. Ishikawa, J.-H. Zhang, 1708.05301

- The reconstruction approaches likes quasi-PDF can provide much more informations which may not be very precise in the present stage;

- We should combine two approaches to obtain a precise picture of the parton distributions.