Extraction of unpolarized TMD PDFs at NNLO: analysis and result

Alexey A. Vladimirov

Institut für Theoretische Physik
Universität Regensburg

in collaboration with I.Scimemi
based on [1706.01473]**(ver 2)**

Spatial and Momentum Tomography of Hadrons and Nuclei
Seattle
Sep.2017

Motivation

The theory of TMD made huge progress in recent years.

General theory

- Proof of collinear part of TMD factorization [Collins,84→11],[Becher,Neubert,Stewart...]
- Proof of rapidity divergences factorization [AV,1707.07606]
- W to Y matching [Collins,*at al*,1605.00671]
- Leading order factorization of Y-term [Fiedge,*et al*,1703.03411],[Balitsky&Tarasov, 1706.01415]

Perturbation theory

- TMD evolution kernels (anomalous dimensions)
  - UV evolution: 3 loop [Moch,*et al*,0505039]
  - rapidity evolution: 3 loop [Li & Zhu,1604.01404; AV,1610.05791]
- Hard coefficient functions: 3 loop [Moch,*et al*,0505039]
- Matching coefficients to PDFs [many]
- Structure of power suppressed terms of small-b OPE [Scimemi & AV,1609.06047]

Not widely used in TMD phenomenology!

Perturbation theory is important!

- Follow the world!

  All PDF extraction and α_s extraction are made at NNLO and higher. To use it in TMD phenomenology, one should use equivalent perturbative inputs.
- Mixing effects ⇔ induced flavour dependence

  TMD hard part (and TMD evolution) is flavour-diagonal. All mixing effects comes from the matching, and they are large. (mixing with gluon at NLO, with see at NNLO)
- Quantitative effects

  They could be very large. E.g. DY-normalization: 0.85(LO)→0.95(NLO)→0.99(NNLO)
  Ultimately important for high-energy data, e.g. LHC
- Theoretical (perturbative) uncertainty

  It decreases from order to order. Very significant at low energies, due to large α_s.

But requires a lot of work to include

- Involved coding

  TMD cross-section requires MANY different integrations. E.g. to analyse LHC
  $6(+2)(+2)(x_1, x_2, b, y, Q, p_T(+\text{evolution})(+\text{fid.}))$
- Requires some extra theory studies
- Reanalysis of data

  We cannot study $f_{1T}^{\perp}$ without d_1, and d_1 without f_1

That's what we do!

That's what we do!

But we just start.

Status

Theory state	
Universal	
Hard part	N^{3}LO
Evolution	N^{3}LO
Matching	
f_1	NNLO
g_1	NLO
h_1	**NNLO**
$h_{1T}^{\perp}$	**NNLO** ($\neq 0$!)
$f_{1T}^{\perp}$	NLO - ?
$h_1^{\perp}$	–
$h_{1L}^{\perp}$	–
g_{1T}	–

- `arTeMiDe` package for evaluation of TMDs and related cross-sections.
  Ver.1.1 includes f_1
  (https://teorica.fis.ucm.es/artemide)
- We have extracted f_1 from DY and Z-boson production. (Presented here)

That's what we do!

But we just start.

Status

Extracted from
jet evolution
AV,PRL**118**(2017)

Universal	
Hard part	N^3LO
Evolution	N^3LO
Matching	
f_1	NNLO
g_1	NLO
h_1	**NNLO**
$h_{1T}^{\perp}$	**NNLO** ($\neq 0$!)
$f_{1T}^{\perp}$	NLO - ?
$h_1^{\perp}$	–
$h_{1L}^{\perp}$	–
g_{1T}	–

- `arTeMiDe` package for evaluation of TMDs and related cross-sections. Ver.1.1 includes f_1 (https://teorica.fis.ucm.es/artemide)
- We have extracted f_1 from DY and Z-boson production. (Presented here)

unpolarized Drell-Yan⇒ unpolarized TMDPDF

Theory input

$$\frac{d\sigma}{dQdy\,d^2q_T} = H(Q,\mu)\int\frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta)+Y$$

$$F(x,\mathbf{b};\mu,\zeta) = R[\mathbf{b};(\mu,\zeta)\to(\mu_{\text{low}},\zeta_\mu)]F^{\text{low}}(x;\mathbf{b})$$

$$F_k^{\text{low}}(x,\mathbf{b}) = \int_x^1\frac{dy}{y}C_{k\leftarrow l}(y,\mathbf{b};\mu)f_l\left(\frac{x}{y},\mu\right)f_{NP}(y;\mathbf{b})$$

unpolarized Drell-Yan⇒ unpolarized TMDPDF

Theory input

$$\frac{d\sigma}{dQdy\,d^2q_T} = H(Q,\mu)\int \frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta)+Y$$

hard c.
LO
NLO
NNLO

$$F(x,\mathbf{b};\mu,\zeta) = R[\mathbf{b};(\mu,\zeta)\to(\mu_{\text{low}},\zeta_\mu)]F^{\text{low}}(x;\mathbf{b})$$

evolution kernel

cusp	ADs
NLO	LO
NNLO	NLO
N^{3}LO	**NNLO**

$$F_k^{\text{low}}(x,\mathbf{b}) = \int_x^1 \frac{dy}{y}C_{k\leftarrow l}(y,\mathbf{b};\mu)f_l\left(\frac{x}{y},\mu\right)f_{NP}(y;\mathbf{b})$$

small-b mach.
LO
NLO
NNLO

MHHT2014

We can define four successive orders

Name	$\lvert C_V\rvert^2$	$C_{f\leftarrow f'}$	Γ	γ_V	$\mathcal{D}$	PDF set	a_s(run)	ζ_μ
NLL	a_s^0	a_s^0	a_s^2	a_s^1	a_s^1	nlo	nlo	NLL
NLO	a_s^1	a_s^1	a_s^2	a_s^1	a_s^1	nlo	nlo	NLO
NNLL	a_s^1	a_s^1	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLL
NNLO	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

unpolarized Drell-Yan⇒ unpolarized TMDPDF

Theory input

$$\frac{d\sigma}{dQdy\,d^2q_T} = H(Q,\mu)\int \frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta)+\cancel{Y}$$

hard c.
LO
NLO
NNLO

pure TMD factorization
⇒ small q_T

$$F(x,\mathbf{b};\mu,\zeta)=R[\mathbf{b};(\mu,\zeta)\rightarrow(\mu_{\text{low}},\zeta_\mu)]F^{\text{low}}(x;\mathbf{b})$$

evolution kernel

cusp	ADs
NLO	LO
NNLO	NLO
N^{3}LO	**NNLO**

$$F_k^{\text{low}}(x,\mathbf{b})=\int_x^1\frac{dy}{y}C_{k\leftarrow l}(y,\mathbf{b};\mu)f_l\left(\frac{x}{y},\mu\right)\underline{f_{NP}(y;\mathbf{b})}$$

small-b mach.
LO
NLO
NNLO

To fit
minory restricted

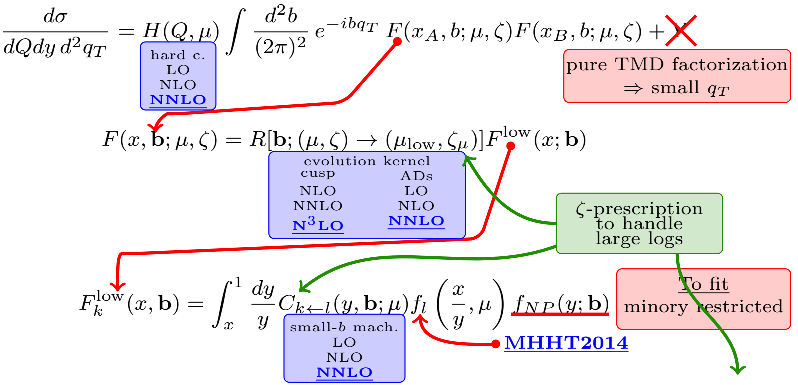
MHHT2014

We can define four successive orders

Name	$\|C_V\|^2$	$C_{f\leftarrow f'}$	Γ	γ_V	$\mathcal{D}$	PDF set	a_s(run)	ζ_μ
NLL	a_s^0	a_s^0	a_s^2	a_s^1	a_s^1	nlo	nlo	NLL
NLO	a_s^1	a_s^1	a_s^2	a_s^1	a_s^1	nlo	nlo	NLO
NNLL	a_s^1	a_s^1	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLL
NNLO	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

unpolarized Drell-Yan⇒ unpolarized TMDPDF

Theory input



$$\frac{d\sigma}{dQdy\,d^2q_T}=H(Q,\mu)\int\frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta)+\cancel{Y}$$

hard c.
LO
NLO
NNLO

pure TMD factorization
⇒ small q_T

$$F(x,\mathbf{b};\mu,\zeta)=R[\mathbf{b};(\mu,\zeta)\to(\mu_{\text{low}},\zeta_\mu)]F^{\text{low}}(x;\mathbf{b})$$

evolution kernel

cusp	ADs
NLO	LO
NNLO	NLO
N^{3}LO	**NNLO**

ζ-prescription
to handle
large logs

$$F_k^{\text{low}}(x,\mathbf{b})=\int_x^1\frac{dy}{y}C_{k\leftarrow l}(y,\mathbf{b};\mu)f_l\left(\frac{x}{y},\mu\right)\underline{f_{NP}(y;\mathbf{b})}$$

small-b mach.
LO
NLO
NNLO

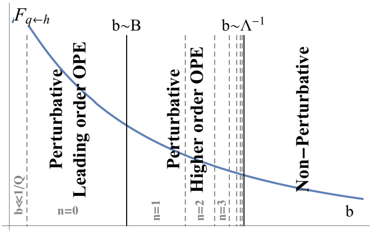
MHHT2014

To fit
minory restricted

We can define four successive orders

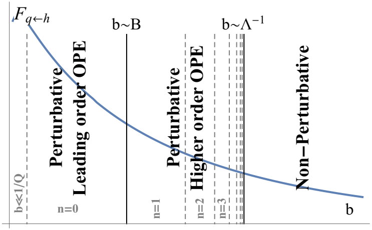
Name	$\lvert C_V\rvert^2$	$C_{f\leftarrow f'}$	Γ	γ_V	$\mathcal{D}$	PDF set	a_s(run)	ζ_μ
NLL	a_s^0	a_s^0	a_s^2	a_s^1	a_s^1	nlo	nlo	NLL
NLO	a_s^1	a_s^1	a_s^2	a_s^1	a_s^1	nlo	nlo	NLO
NNLL	a_s^1	a_s^1	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLL
NNLO	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

Non-perturbative input



$b \ll Q^{-1}$	Perturbative	Not observable, deeply in Y-term dominated region
$b \ll B$	Perturbative	Leading twist contribution $F(x,b) \sim C(x,b) \otimes f(x)$
$b \sim B$	Perturbative	Higher twist $F(x,b) \sim \sum_n \left(\frac{xb^2}{B^2}\right)^n C_n(x,b) \otimes f_n(x)$
	but not calculable	the main scale parameter is xb^2 [I.Scimemi,AV, 1609.06047]
		$n = 1$ term can be estimated.
$b > \Lambda^{-1}$	Non-perturbative	Nothing is know. Exponential? Gaussian?

Non-perturbative input



$b \ll Q^{-1}$	Perturbative	Not observable, deeply in Y-term dominated region
$b \ll B$	Perturbative	Leading twist contribution $F(x,b) \sim C(x,b) \otimes f(x)$
$b \sim B$	Perturbative but not calculable	Higher twist $F(x,b) \sim \sum_n \left(\frac{xb^2}{B^2}\right)^n C_n(x,b) \otimes f_n(x)$ the main scale parameter is xb^2 [I.Scimemi,AV, 1609.06047] $n = 1$ term can be estimated.
$b > \Lambda^{-1}$	Non-perturbative	Nothing is know. Exponential? Gaussian?

Additionally, there can be non-perturbative contribution to the rapidity evolution

only even powers can appear

$$\mathcal{D}(b) = \mathcal{D}^{\text{perp}}(b) + g_K b^2 + \ldots$$

Theory prediction: very small or zero $g_K = 0.01 \pm 0.03\text{GeV}^2$ [I.Scimemi,AV, 1609.06047]

ζ-prescription

$$\ln(\mu^2 \mathbf{b}^2), \qquad \ln(\zeta\, \mathbf{b}^2)$$

- There are (potentially large) logs of **b**. Some prescription is needed to handle it.
- Typically, b^*-prescription used ⇒ induces power corrections and new parameters
- ζ-prescription does not introduce any artificial dependence

ζ-prescription uses the freedom
(granted to us by factorization theorem)
to choose the scale in any convenient way.

ζ-prescription

$$\ln(\mu^2 \mathbf{b}^2), \qquad \ln(\zeta\, \mathbf{b}^2)$$

- There are (potentially large) logs of **b**. Some prescription is needed to handle it.
- Typically, b^*-prescription used ⇒ induces power corrections and new parameters
- ζ-prescription does not introduce any artificial dependence

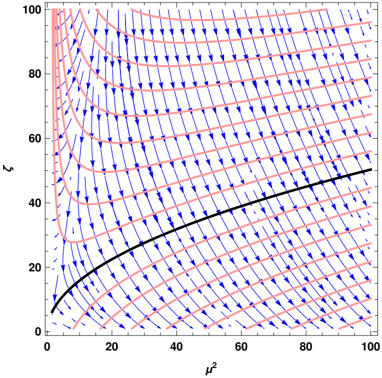
ζ-prescription uses the freedom
(granted to us by factorization theorem)
to choose the scale in any convenient way.

This freedom has not been used yet.

TMD evolution is multi-scale evolution

$$\begin{aligned}\mu^2\frac{dF(x,\mathbf{b};\mu,\zeta)}{d\mu^2} &= \frac{1}{2}\gamma_K(\mu,\zeta)F(x,\mathbf{b};\mu,\zeta)\\ \zeta\frac{dF(x,\mathbf{b};\mu,\zeta)}{d\zeta} &= -\mathcal{D}(\mu,\mathbf{b})F(x,\mathbf{b};\mu,\zeta)\end{aligned}$$

The (μ,ζ)-plane has a rich structure

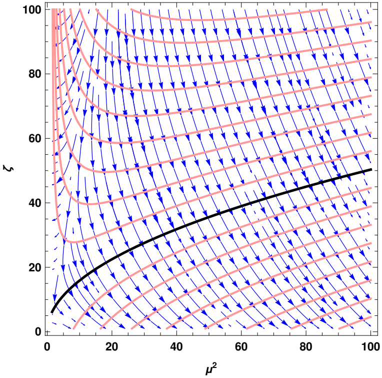


TMD evolution is multi-scale evolution

$$\mu^2 \frac{dF(x,\mathbf{b};\mu,\zeta)}{d\mu^2} = \frac{1}{2}\gamma_K(\mu,\zeta)F(x,\mathbf{b};\mu,\zeta)$$

$$\zeta \frac{dF(x,\mathbf{b};\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,\mathbf{b})F(x,\mathbf{b};\mu,\zeta)$$

The (μ,ζ)-plane has a rich structure



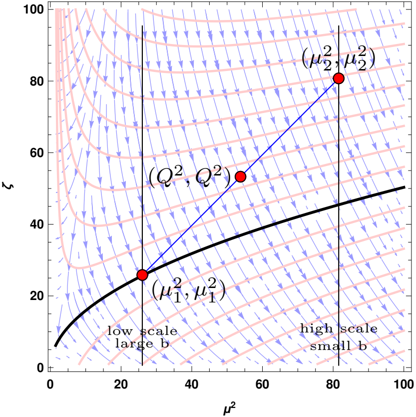
There are equi-evolution lines in the (μ,ζ)-plane

$$\mu^2 \frac{dF(x,\mathbf{b};\mu,\zeta(\mu))}{d\mu^2} = 0.$$

$$F(x,\mathbf{b};Q,Q^2) = R[(Q,Q^2) \to (\mu_i,\zeta_i)]F(x,\mathbf{b};\mu_i,\zeta_i)$$

Typical choice

$$\zeta_i = \mu_i^2$$

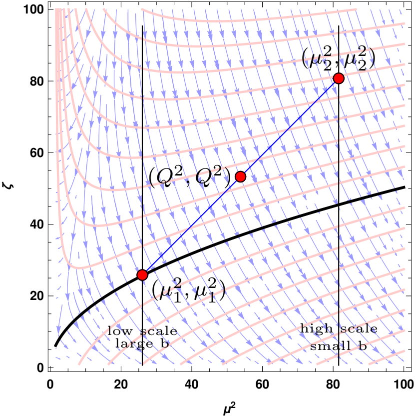


WARNING: Picture is not entirely correct, because scales depend on b (i.e. it should be 3D)

$$F(x,\mathbf{b};Q,Q^2)=R[(Q,Q^2)\to(\mu_i,\zeta_i)]F(x,\mathbf{b};\mu_i,\zeta_i)$$

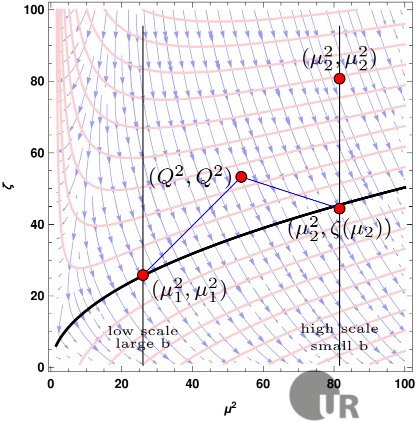
Typical choice

$$\zeta_i=\mu_i^2$$



ζ-prescription

$$\zeta=\zeta(\mu)\text{ equi-evolution line}$$

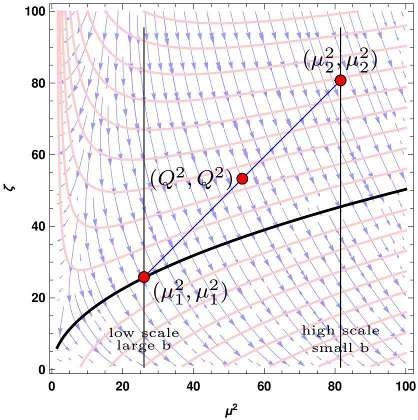


WARNING: Picture is not entirely correct, because scales depend on b (i.e. it should be 3D)

$$F(x, \mathbf{b}; Q, Q^2) = R[(Q, Q^2) \to (\mu_i, \zeta_i)]F(x, \mathbf{b}; \mu_i, \zeta_i)$$

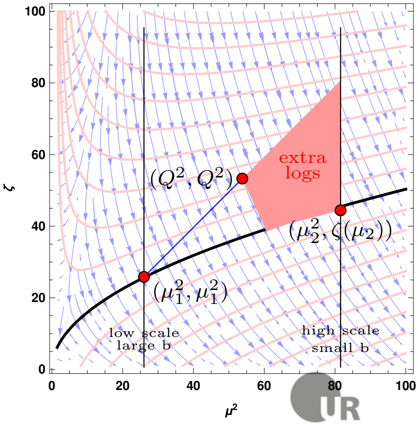
Typical choice

$$\zeta_i = \mu_i^2$$



ζ-prescription

$$\zeta = \zeta(\mu) \text{ equi-evolution line}$$



WARNING: Picture is not entirely correct, because scales depend on b (i.e. it should be 3D)

ζ-prescription in practice

In the ζ-prescription a TMD is EVOLUTIONless

$$\mu^2 \frac{d}{d\mu^2} F(x, \mathbf{b}; \mu, \zeta_\mu) = 0 \qquad \Leftrightarrow \qquad \zeta_\mu = \frac{2\mu}{|\mathbf{b}|} e^{-\gamma_E} \overbrace{e^{3/2+\dots}}^{\text{PT-calculable here LO}} .$$

ζ-prescription eliminates large logs from the expressions. Good example is the coefficient function:

$$F(x, \mathbf{b}; \mu, \zeta) = C(x, \mathbf{b}; \mu, \zeta) \otimes f(x, \mu)$$

ζ-prescription in practice

In the ζ-prescription a TMD is EVOLUTIONless

$$\mu^2 \frac{d}{d\mu^2} F(x,\mathbf{b};\mu,\zeta_\mu) = 0 \qquad \Leftrightarrow \qquad \zeta_\mu = \frac{2\mu}{|\mathbf{b}|} e^{-\gamma_E} \overbrace{e^{3/2+\dots}}^{\text{PT-calculable here LO}} .$$

ζ-prescription eliminates large logs from the expressions. Good example is the coefficient function:

$$F(x,\mathbf{b};\mu,\zeta) = C(x,\mathbf{b};\mu,\zeta) \otimes f(x,\mu)$$

$$C = \delta(\bar{x}) + a_s C_F \Big[-2 \underbrace{\mathbf{L}_\mu p(x)}_{\substack{\text{never large}\\ \text{thanks to}\\ \text{charge}\\ \text{conservation}}} + 2\bar{x} + \delta(\bar{x}) \Big(\overbrace{-\mathbf{L}_\mu^2 + \mathbf{L}_\mu \mathbf{l}_\zeta + 3\mathbf{L}_\mu}^{\text{usually large}} - \zeta_2 \Big) \Big]$$

ζ-prescription in practice

In the ζ-prescription a TMD is EVOLUTIONless

$$\mu^2 \frac{d}{d\mu^2} F(x,\mathbf{b};\mu,\zeta_\mu) = 0 \qquad \Leftrightarrow \qquad \zeta_\mu = \frac{2\mu}{|\mathbf{b}|} e^{-\gamma_E} \overbrace{e^{3/2+\dots}}^{\substack{\text{PT-calculable}\\ \text{here LO}}} .$$

ζ-prescription eliminates large logs from the expressions. Good example is the coefficient function:

$$F(x,\mathbf{b};\mu,\zeta) = C(x,\mathbf{b};\mu,\zeta) \otimes f(x,\mu)$$

$$C = \delta(\bar{x}) + a_s C_F \Big[-2 \underbrace{\mathbf{L}_\mu p(x)}_{\substack{\text{never large}\\ \text{thanks to}\\ \text{charge}\\ \text{conservation}}} + 2\bar{x} + \delta(\bar{x}) \Big(\overbrace{\underbrace{-\mathbf{L}_\mu^2 + \mathbf{L}_\mu \mathbf{l}_\zeta + 3\mathbf{L}_\mu}_{\substack{=0\\ \text{at } \zeta_\mu}}}^{\text{usually large}} - \zeta_2 \Big) \Big]$$

charge conservation: $\int_0^1 dx C(x,b) \otimes f(x) = \text{const}$

ζ-prescription in practice

In the ζ-prescription a TMD is EVOLUTIONless

$$\mu^2 \frac{d}{d\mu^2} F(x,\mathbf{b};\mu,\zeta_\mu)=0 \qquad \Leftrightarrow \qquad \zeta_\mu = \frac{2\mu}{|\mathbf{b}|} e^{-\gamma_E} \overbrace{e^{3/2+\dots}}^{\substack{\text{PT-calculable}\\ \text{here LO}}} .$$

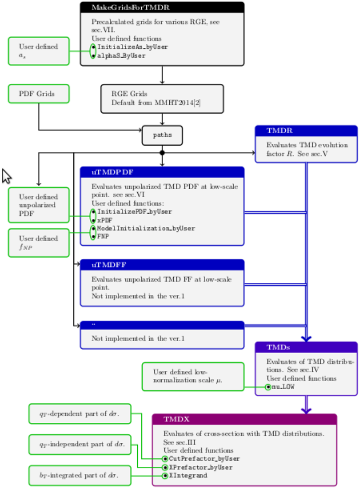
ζ-prescription eliminates large logs from the expressions. Good example is the coefficient function:

$$F(x,\mathbf{b};\mu,\zeta)=C(x,\mathbf{b};\mu,\zeta)\otimes f(x,\mu)$$

No need for b^*, since there are no large logs.
But b^* can be used, (we are not).
Our choose the simplest function: $\mu_{\text{low}}=\mu_{\text{OPE}}=C_0\left(\frac{1}{b}+2\right)$

charge conservation: $\int_0^1 dx C(x,b)\otimes f(x) = \text{const}$

arTeMiDe



MakeGridsForTMDR
Precalculated grids for various RGE, see sec.VII.
User defined functions
`InitializeAs_byUser`
`alphaS_ByUser`

User defined a_s

PDF Grids

RGE Grids
Default from MMHT2014[2]

`paths`

TMDR
Evaluates TMD evolution factor R. See sec.V

uTMDPDF
Evaluates unpolarized TMD PDF at low-scale point. see sec.VI
User defined functions:
`InitializePDF_byUser`
`xPDF`
`ModelInitialization_byUser`
`FNP`

User defined unpolarized PDF

User defined f_{NP}

uTMDFF
Evaluates unpolarized TMD FF at low-scale point.
Not implemented in the ver.1

..
Not implemented in the ver.1

TMDs
Evaluates of TMD distributions. See sec.IV
User defined functions
`mu_LOW`

User defined low-normalization scale μ.

TMDX
Evaluates of cross-section with TMD distributions. See sec.III
User defined functions
`CutPrefactor_byUser`
`XPrefactor_byUser`
`XIntegrand`

q_T-dependent part of $d\sigma$.

q_T-independent part of $d\sigma$.

b_T-integrated part of $d\sigma$.

- FORTRAN 90 code
- Module structure
- Convolutions, evolution (LO,NLO,NNLO)
- Fourier to q_T-space, integrations over phase space
- Scale-variation (ζ-prescription)
- User defined PDFs, scales, f_{NP}
- Efficient code ($\sim 10^9$ TMDs $\sim$ 6. min at NNLO)

Currently ver 1. (soon performance update to ver.1.1)

Available at: https://teorica.fis.ucm.es/artemide

Future plans: add modules for fragmentations, and polarized TMDs

High- & low-energy data are used

- High-energy ⇒ precise fixation of asymptotic
- Low-energy ⇒ better access to NP structure
- To start with we considered only "well-established" data
- In the final fit $309 = \underbrace{163}_{\text{high}} + \underbrace{146}_{\text{low}}$ points used.

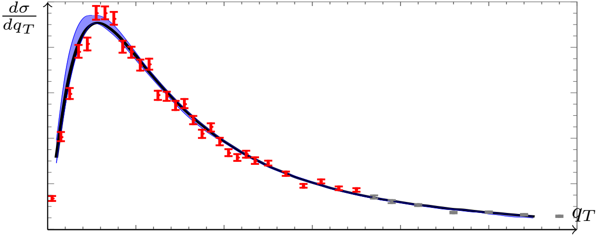
Included data (at $q_T < 0.2Q$)

	reaction	$\sqrt{s}$	Q	comment	points
E288	$p + Cu \to \gamma^* \to \mu\mu$	19.4 GeV	4-9 GeV	norm=0.8	35
E288	$p + Cu \to \gamma^* \to \mu\mu$	23.8 GeV	4-9 GeV	norm=0.8	45
E288	$p + Cu \to \gamma^* \to \mu\mu$	27.4 GeV	4-9 & 11-14 GeV	norm=0.8	66
CDF+D0	$p + \bar{p} \to Z \to ee$	1.8 TeV	66-116 GeV	tiny errors!	44
CDF+D0	$p + \bar{p} \to Z \to ee$	1.96 TeV	66-116 GeV		43
ATLAS	$p + p \to Z \to \mu\mu$	7 & 8 TeV	66-116 GeV		18
CMS	$p + p \to Z \to \mu\mu$	7 & 8 TeV	60-120 GeV		14
LHCb	$p + p \to Z \to \mu\mu$	7 & 8 & 13 TeV	60-120 GeV		30
ATLAS	$p + p \to Z/\gamma^* \to \mu\mu$	8 TeV	46-66 GeV		5
ATLAS	$p + p \to Z/\gamma^* \to \mu\mu$	8 TeV	116-150 GeV		9
				Total	309

Limits of application of TMD factorizaiton ↔ size of Y-term

$$\frac{d\sigma}{dQdy\,d^2q_T}=H(Q,\mu)\int\frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta)+Y$$

TMD factorization derived at small q_T
the leading correction $\sim q_T^2/Q^2$



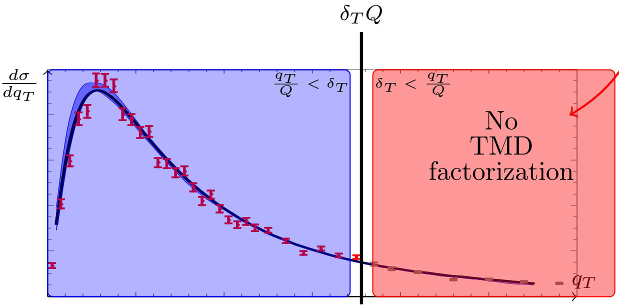
Limits of application of TMD factorizaiton ↔ size of Y-term

$$\frac{d\sigma}{dQdy\,d^2q_T}=\boxed{H(Q,\mu)\int\frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta)}\boxed{+Y}$$

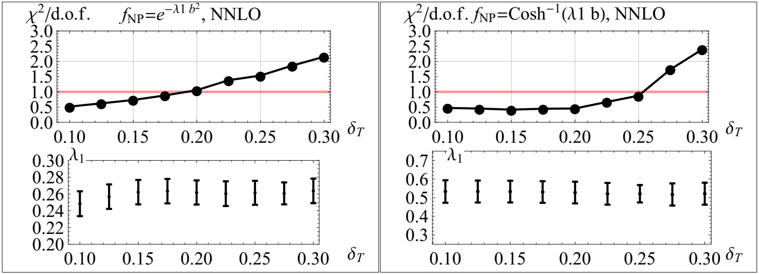
We include all points with $q_T<\delta_T Q$

To find the value of δ_T, we check **the stability of the fit**

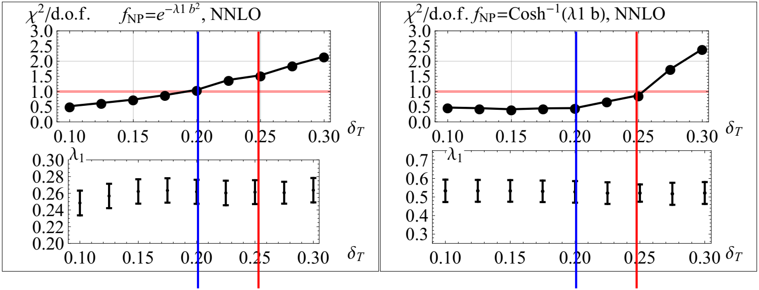
- Make fits with increasing δ_T (0.1→0.3)(165→399 points)
- The value of $\chi^2/d.o.f.$ **blows up** for δ_T **outside allowed region**



Scans of δ_T (E288 not included)



Scans of δ_T (E288 not included)



- $\delta_T < 0.2$ save region,
- $\delta_T < 0.25$ un-save region,
- $\delta_T > 0.25$ TMD factorization does not work.

To be on the save side we used $\delta_T = 0.2$
There are 309 data points

Selection of f_{NP}

We have tested many models with different behaviour.

Lessons

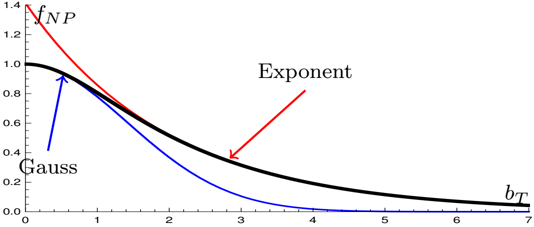
- Test at NNLO. Since at NLO (or NLL) all models are equally good/bad.
- High-energy experiments favour Gaussian-like
- Low-energy experiment favour exponent-like
- Need at least 2 parameters (to control b^2 correction and the tail)

Selection of f_{NP}

We have tested many models with different behaviour.

Lessons

- Test at NNLO. Since at NLO (or NLL) all models are equally good/bad.
- High-energy experiments favour Gaussian-like
- Low-energy experiment favour exponent-like
- Need at least 2 parameters (to control b^2 correction and the tail)



Best models (2 parameters)+g_K

$$f_{NP} = \frac{\cosh(\lambda_2 b)}{\cosh(\lambda_1 b)}$$

$$f_{NP} = \exp\left[-\frac{z\lambda_2 b^2}{\sqrt{1+\left(zb\frac{\lambda_2}{\lambda_1}\right)^2}}\right]$$

both

$$\frac{\chi^2}{dof} \simeq 1.2$$

Just as expected from theory!

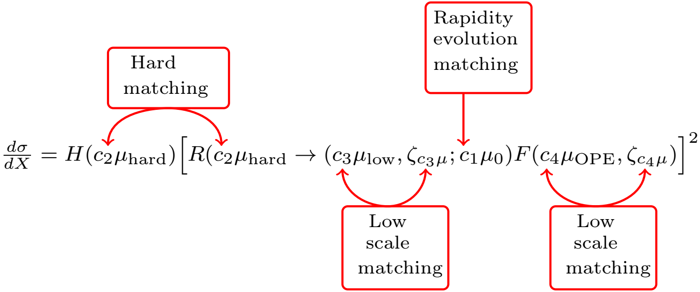
Perturbative uncertainties with in TMD cross-section

There are four perturbative scale entries ⇒ four constants to vary $\{c_1, c_2, c_3, c_4\}$.

$$\frac{d\sigma}{dX} = H(c_2\mu_{\text{hard}})\Big[R(c_2\mu_{\text{hard}} \to (c_3\mu_{\text{low}}, \zeta_{c_3\mu}; c_1\mu_0)F(c_4\mu_{\text{OPE}}, \zeta_{c_4\mu})\Big]^2$$

Perturbative uncertainties with in TMD cross-section

There are four perturbative scale entries ⇒ four constants to vary $\{c_1, c_2, c_3, c_4\}$.



Hard matching

Rapidity evolution matching

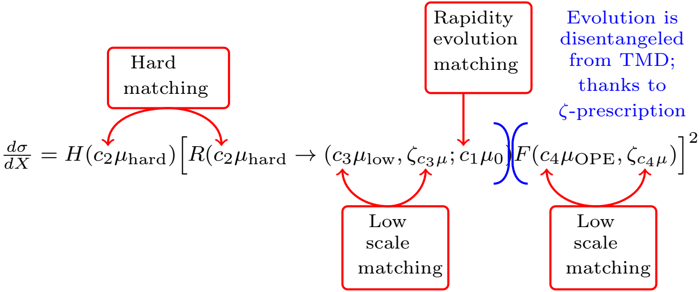
$$\frac{d\sigma}{dX} = H(c_2\mu_{\text{hard}})\Big[R(c_2\mu_{\text{hard}} \to (c_3\mu_{\text{low}}, \zeta_{c_3\mu}; c_1\mu_0)F(c_4\mu_{\text{OPE}}, \zeta_{c_4\mu})\Big]^2$$

Low scale matching

Low scale matching

Perturbative uncertainties with in TMD cross-section

There are four perturbative scale entries ⇒ four constants to vary $\{c_1, c_2, c_3, c_4\}$.



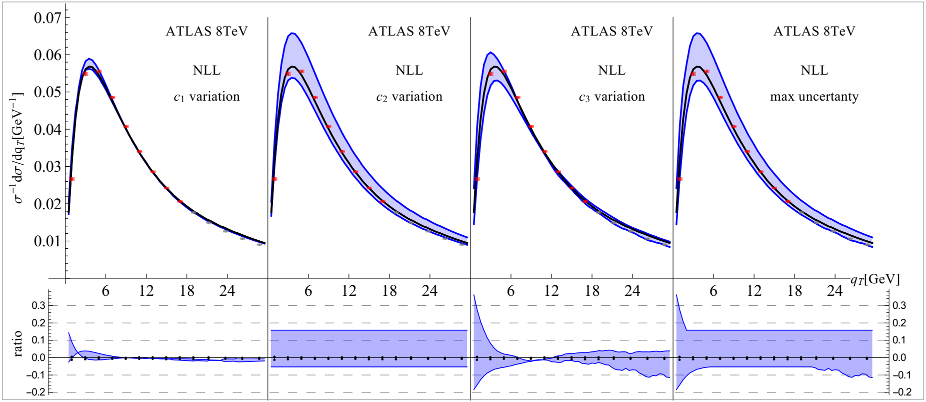
$$\frac{d\sigma}{dX} = H(c_2\mu_{\text{hard}})\Big[R(c_2\mu_{\text{hard}} \to (c_3\mu_{\text{low}}, \zeta_{c_3\mu}; c_1\mu_0)\Big)\Big(F(c_4\mu_{\text{OPE}}, \zeta_{c_4\mu})\Big]^2$$

Evolution is disentangeled from TMD; thanks to ζ-prescription

Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition	$c_2 \to$ uncertainty of hard matching	$c_3 \to$ uncertainty of small-b matching	Total uncertainty is the maximum of three
$\int_{c_1\mu_0}^{\mu}\Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

High-energy example: ATLAS 8 TeV (best precision)



Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition

$$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$$

$c_2 \to$ uncertainty of hard matching

$$H(c_2\mu)F(c_2\mu)F(c_2\mu)$$

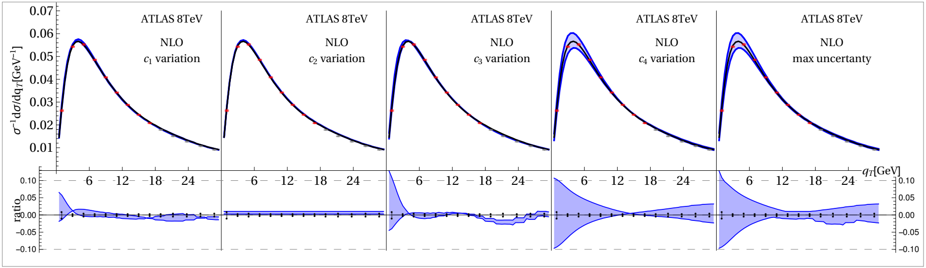
$c_3 \to$ uncertainty of small-b matching

$$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$$

Total uncertainty is the maximum of three

$$c_i \in (0.5, 2)$$

High-energy example: ATLAS 8 TeV (best precision)



Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition

$$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$$

$c_2 \to$ uncertainty of hard matching

$$H(c_2\mu)F(c_2\mu)F(c_2\mu)$$

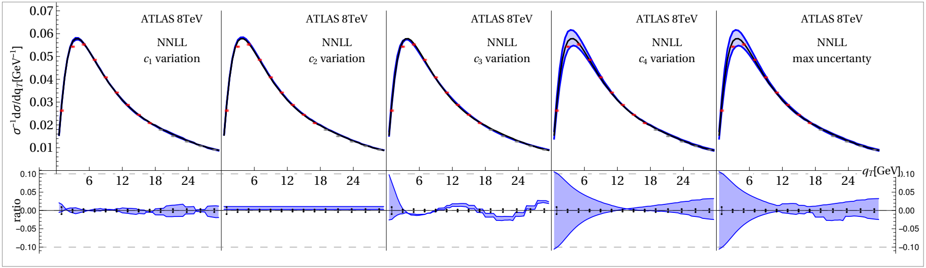
$c_3 \to$ uncertainty of small-b matching

$$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$$

Total uncertainty is the maximum of three

$$c_i \in (0.5, 2)$$

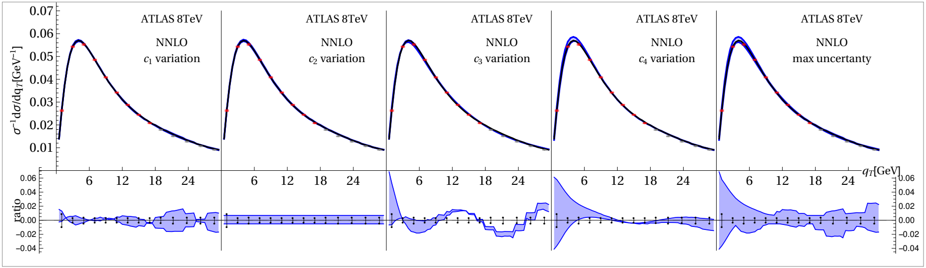
High-energy example: ATLAS 8 TeV (best precision)



Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition	$c_2 \to$ uncertainty of hard matching	$c_3 \to$ uncertainty of small-b matching	Total uncertainty is the maximum of three
$\int_{c_1\mu_0}^{\mu}\Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

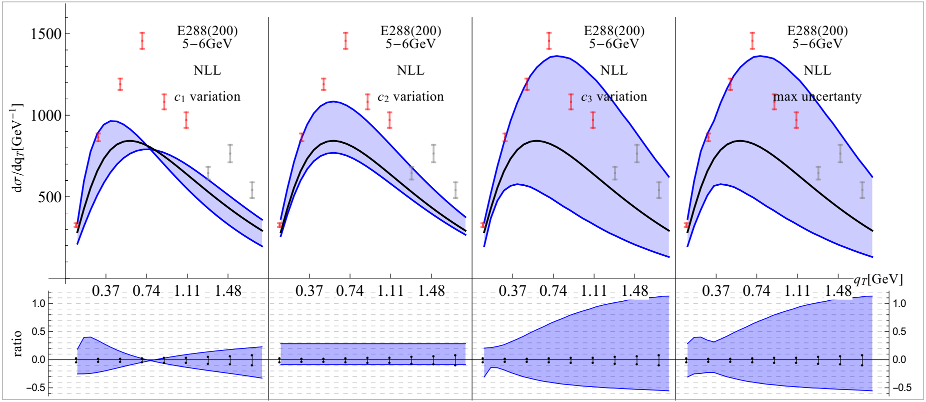
High-energy example: ATLAS 8 TeV (best precision)



Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition	$c_2 \to$ uncertainty of hard matching	$c_3 \to$ uncertainty of small-b matching	Total uncertainty is the maximum of three
$\int_{c_1\mu_0}^{\mu}\Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

Low-energy example: E288 $\sqrt{s} = 19.4$ GeV, $Q = 4 - 5$ GeV



Perturbative uncertainties

$c_1 \rightarrow$ uncertainty of RAD definition

$$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$$

$c_2 \rightarrow$ uncertainty of hard matching

$$H(c_2\mu)F(c_2\mu)F(c_2\mu)$$

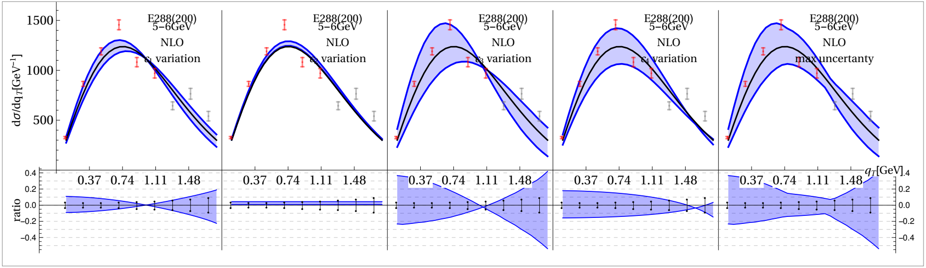
$c_3 \rightarrow$ uncertainty of small-b matching

$$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$$

Total uncertainty is the maximum of three

$$c_i \in (0.5, 2)$$

Low-energy example: E288 $\sqrt{s} = 19.4$ GeV, $Q = 4 - 5$ GeV



Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition

$$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$$

$c_2 \to$ uncertainty of hard matching

$$H(c_2\mu)F(c_2\mu)F(c_2\mu)$$

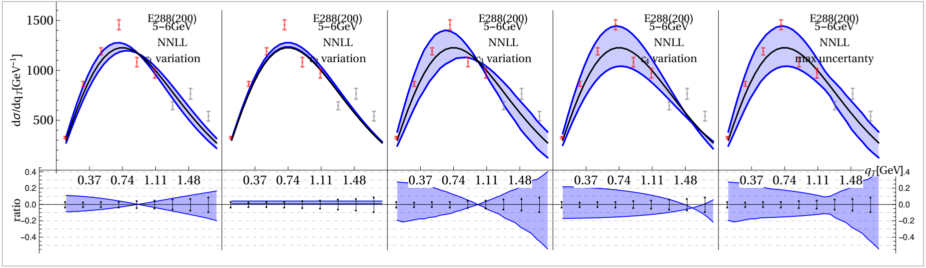
$c_3 \to$ uncertainty of small-b matching

$$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$$

Total uncertainty is the maximum of three

$$c_i \in (0.5, 2)$$

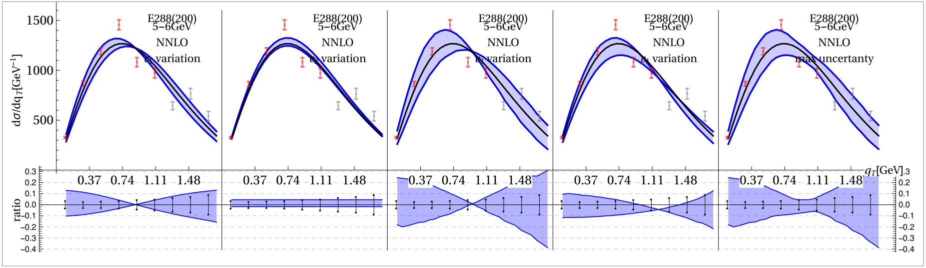
Low-energy example: E288 $\sqrt{s} = 19.4$ GeV, $Q = 4 - 5$ GeV



Perturbative uncertainties

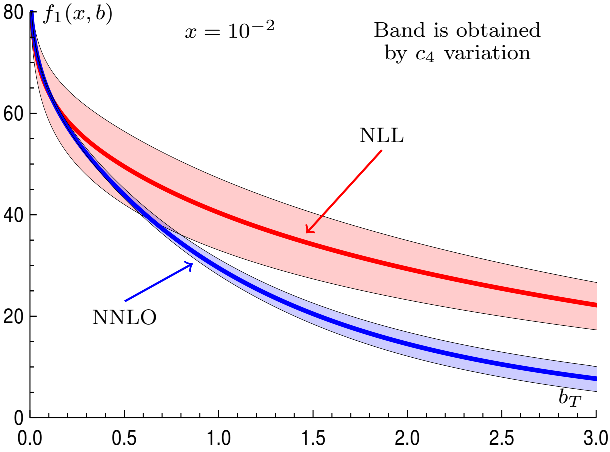
$c_1 \to$ uncertainty of RAD definition	$c_2 \to$ uncertainty of hard matching	$c_3 \to$ uncertainty of small-b matching	Total uncertainty is the maximum of three
$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

Low-energy example: E288 $\sqrt{s} = 19.4$ GeV, $Q = 4 - 5$ GeV

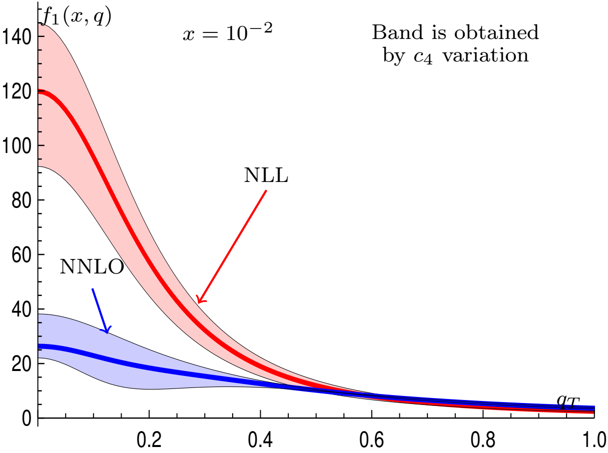


Universität Regensburg

Uncertainties in TMD



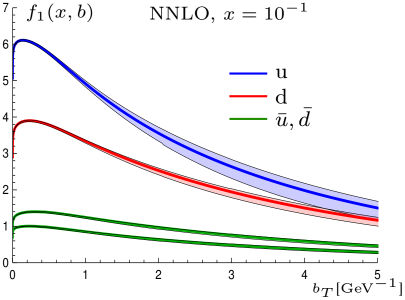
Uncertainties in TMD

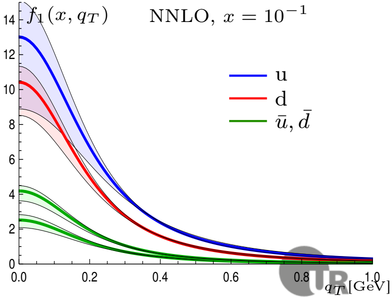


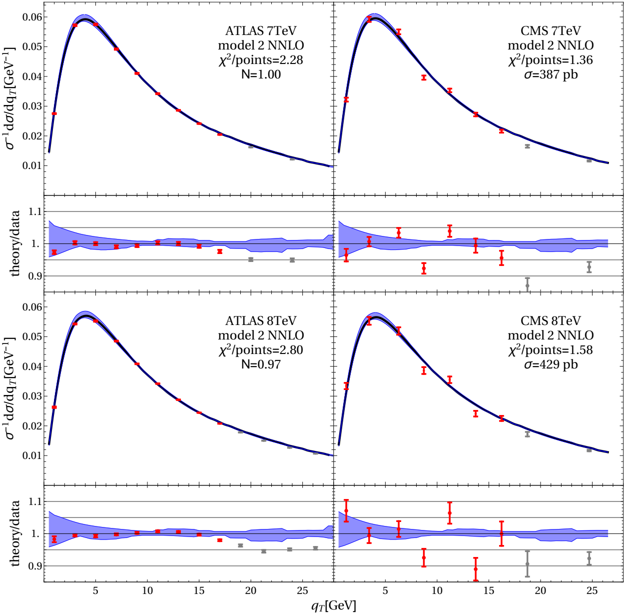
Results

Results of fit

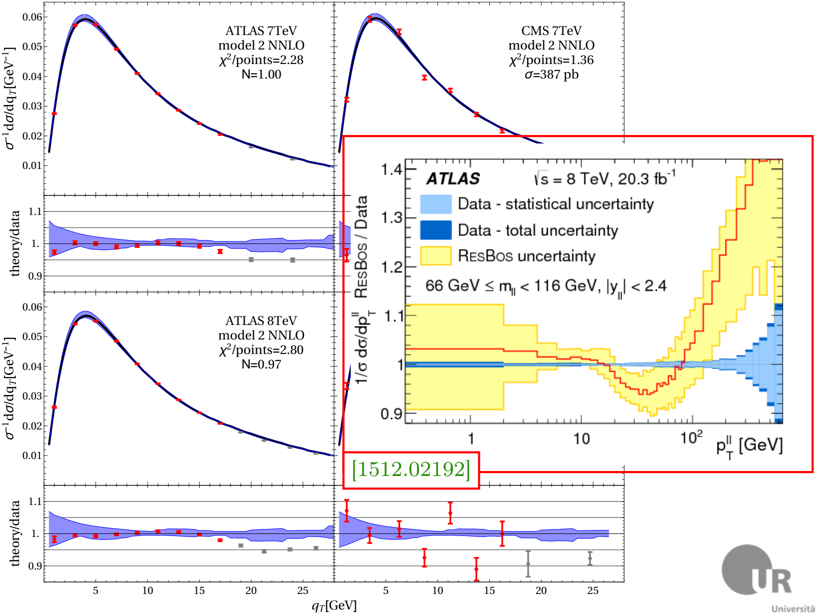
	χ^2/dof	λ_1	λ_2	g_K	norm
NLL	-	-	-	-	~ 0.83
NLO	1.18	0.20	0.43	0.021	~ 0.94
NNLL	1.30	0.17	1.30	0.012	~ 0.97
NNLO	1.23	0.244	0.307	0.006	~ 0.99



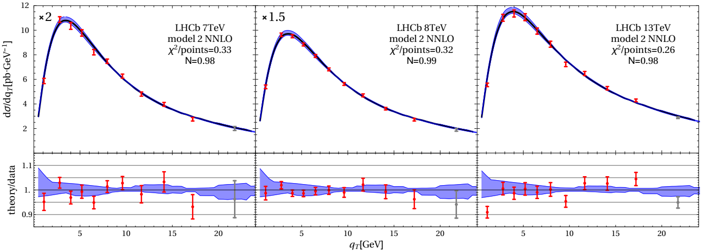


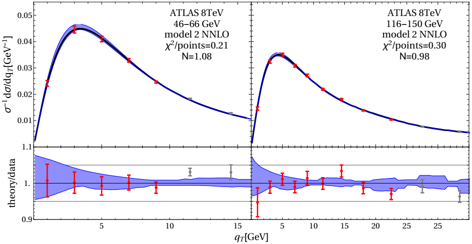


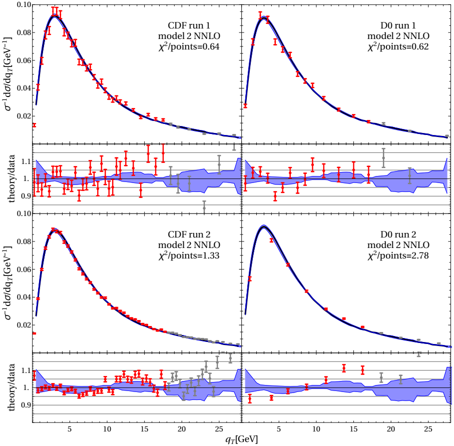
Universität Regensburg

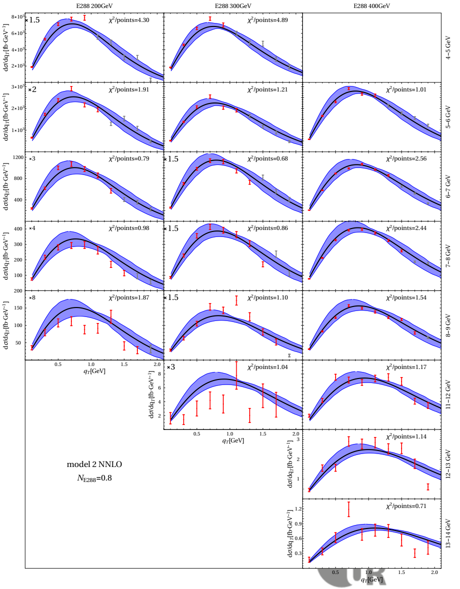

ATLAS 7TeV
model 2 NNLO
χ^2/points=2.28
N=1.00
CMS 7TeV
model 2 NNLO
χ^2/points=1.36
σ=387 pb
ATLAS 8TeV
model 2 NNLO
χ^2/points=2.80
N=0.97
$\sigma^{-1}d\sigma/dq_T$[GeV^{-1}]
theory/data
q_T[GeV]
ATLAS
$\sqrt{s}$ = 8 TeV, 20.3 fb^{-1}
Data - statistical uncertainty
Data - total uncertainty
ResBos uncertainty
66 GeV ≤ m_{ll} < 116 GeV, $|y_{ll}|$ < 2.4
$1/\sigma\ d\sigma/dp_T^{ll}$ ResBos / Data
p_T^{ll} [GeV]

[1512.02192]

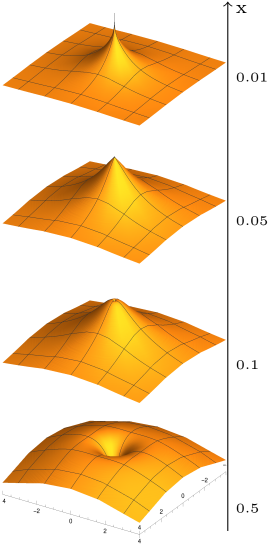

×2
LHCb 7TeV
model 2 NNLO
χ^2/points=0.33
N=0.98
×1.5
LHCb 8TeV
model 2 NNLO
χ^2/points=0.32
N=0.99
LHCb 13TeV
model 2 NNLO
χ^2/points=0.26
N=0.98
$d\sigma/dq_T$[pb·GeV^{-1}]
12
10
8
6
4
2
theory/data
1.1
1.
0.9
5
10
15
20
q_T[GeV]


ATLAS 8TeV
46–66 GeV
model 2 NNLO
χ^2/points=0.21
N=1.08
ATLAS 8TeV
116–150 GeV
model 2 NNLO
χ^2/points=0.30
N=0.98
$\sigma^{-1}d\sigma/dq_T$[GeV^{-1}]
0.05
0.04
0.03
0.02
0.01
theory/data
1.1
1.
0.9
5
10
15
5
10
15
20
25
25
q_T[GeV]


CDF run 1
model 2 NNLO
χ^2/points=0.64
D0 run 1
model 2 NNLO
χ^2/points=0.62
CDF run 2
model 2 NNLO
χ^2/points=1.33
D0 run 2
model 2 NNLO
χ^2/points=2.78
$\sigma^{-1}d\sigma/dq_T$[GeV^{-1}]
theory/data
0.10
0.08
0.06
0.04
0.02
1.1
1.
0.9
5
10
15
20
25
q_T[GeV]


E288 200GeV
E288 300GeV
E288 400GeV
4–5 GeV
5–6 GeV
6–7 GeV
7–8 GeV
8–9 GeV
11–12 GeV
12–13 GeV
13–14 GeV
×1.5
χ^2/points=4.30
χ^2/points=4.89
×2
χ^2/points=1.91
χ^2/points=1.21
χ^2/points=1.01
×3
χ^2/points=0.79
×1.5
χ^2/points=0.68
χ^2/points=2.56
×4
χ^2/points=0.98
×1.5
χ^2/points=0.86
χ^2/points=2.44
×8
χ^2/points=1.87
×1.5
χ^2/points=1.10
χ^2/points=1.54
×3
χ^2/points=1.04
χ^2/points=1.17
χ^2/points=1.14
χ^2/points=0.71
$d\sigma/dq_T$[fb·GeV^{-1}]
q_T[GeV]
0.5
1.0
1.5
2.0
model 2 NNLO
N_{E288}=0.8

Conclusion



- Perturbative input significantly affect extraction of non-perturbative part.
- At least NLO is needed (better go NNLO).
- ζ-prescription (as a clever distribution of logs between parts of factorization theorem) help to reduce the theory uncertainty.

Ongoing work/Future plans

- Include SIDIS
- `arTeMiDe` updates
- Polarized TMDs.