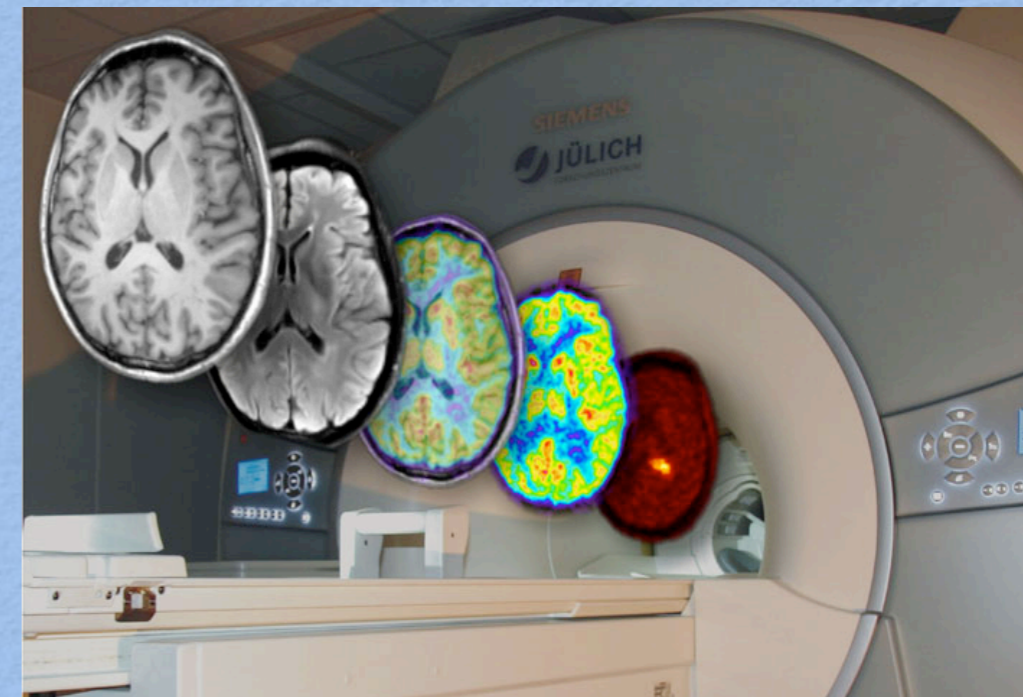




JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ



# Spatial tomography of the proton from present data



Marc Vanderhaeghen

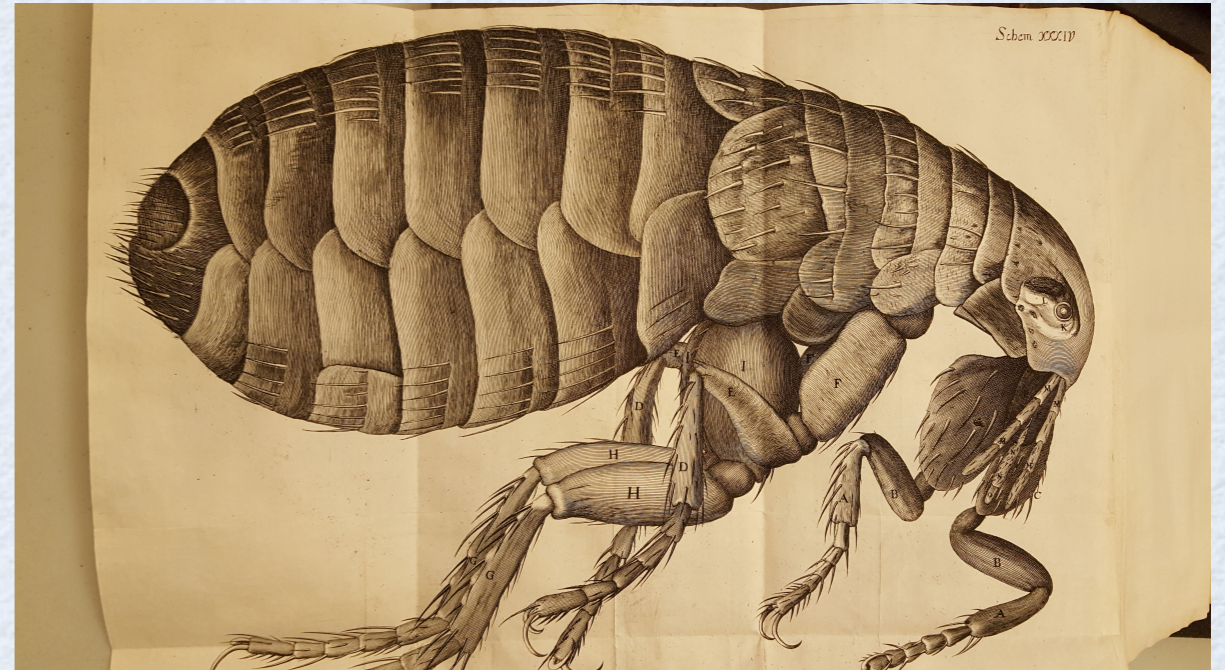
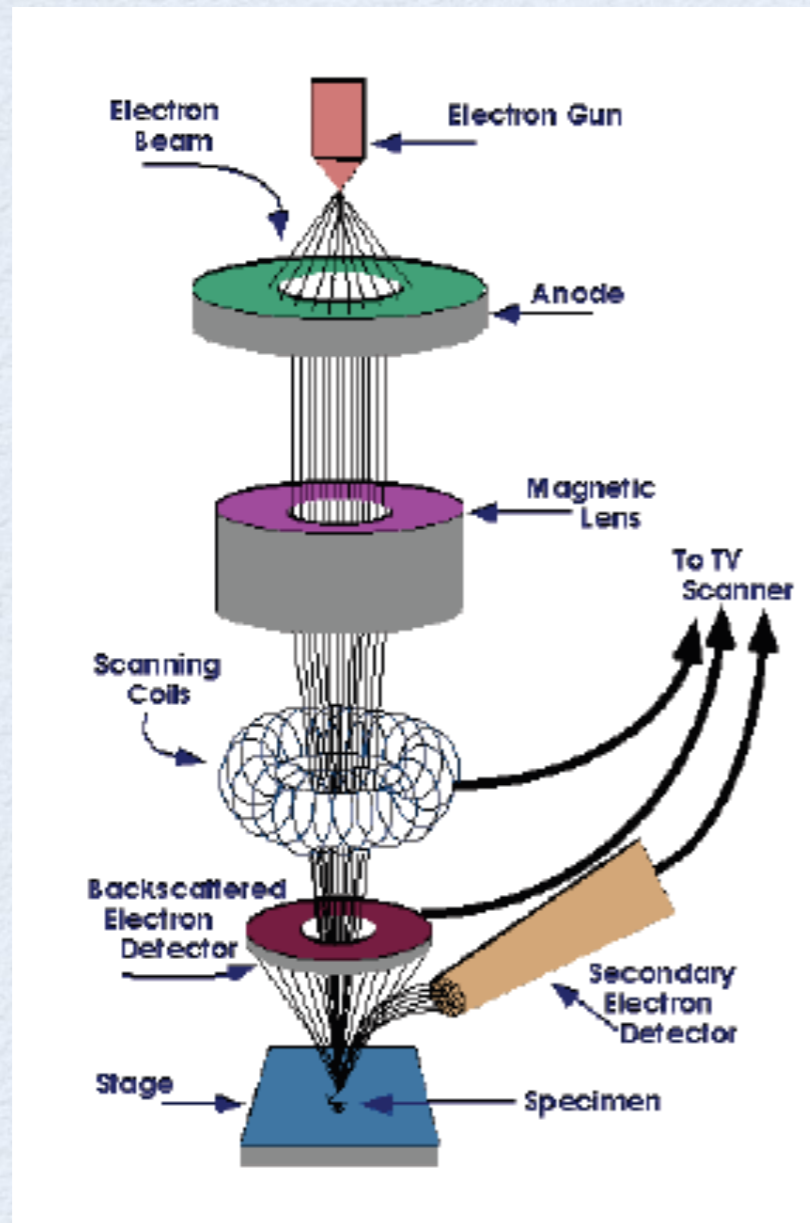
INT Workshop "Spatial and Momentum Tomography of Hadrons and Nuclei"

August 28 - September 29, 2017, Seattle, USA



# how to image a system

R.Hooke (Micrographia, 1665)



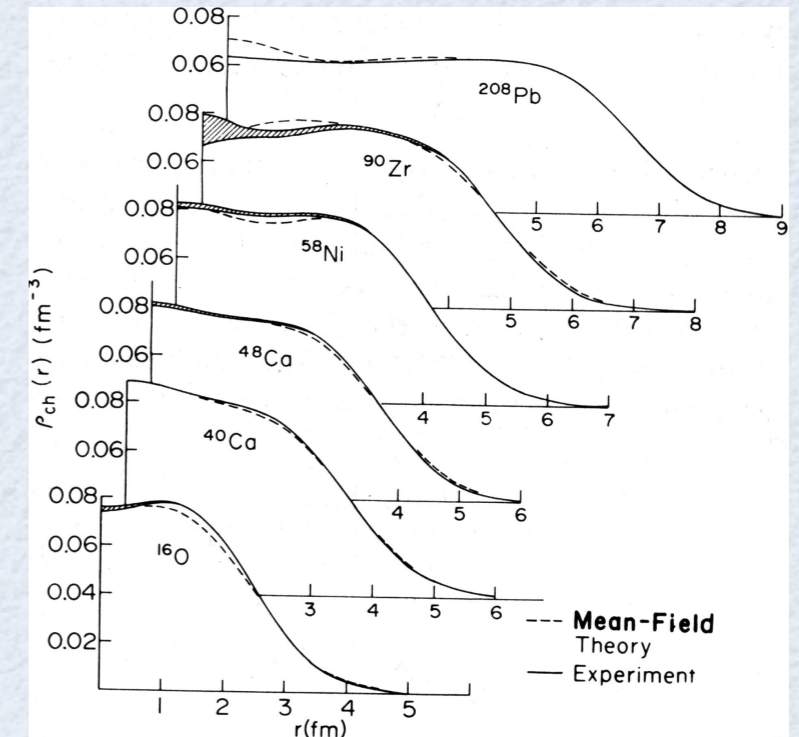
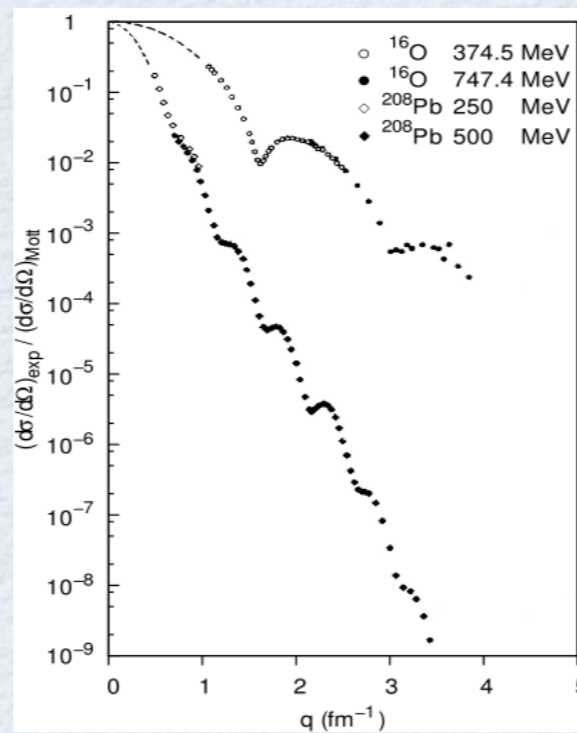
when target is static  
( $m_{\text{constituent}}, m_{\text{target}} \gg Q$ )

the 3D **Fourier transform** of **form factors**  
gives the distribution of electric charge and magnetization

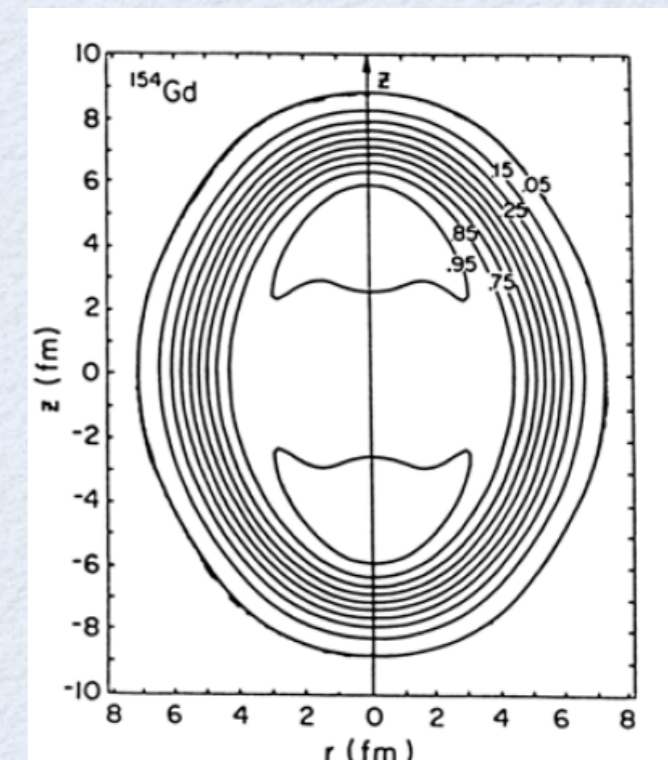
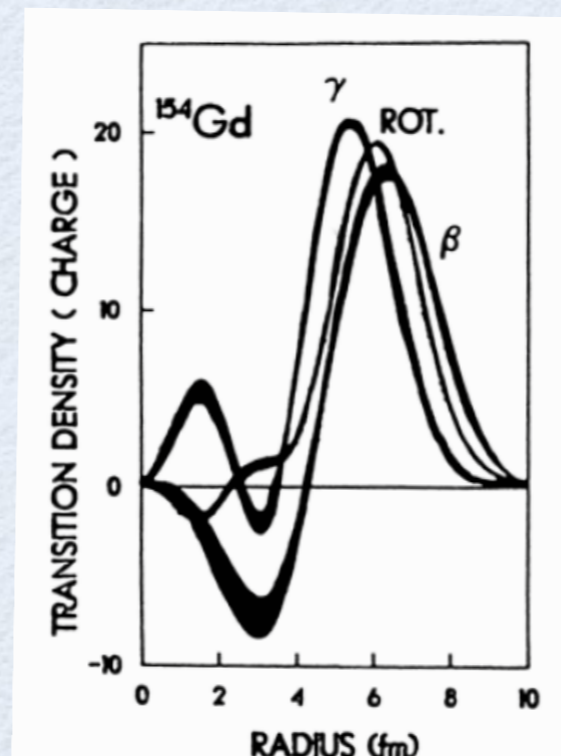


# what do we know about spatial distributions of charges in nuclei?

**sizes** of nuclei:  
as revealed through  
**elastic** electron scattering



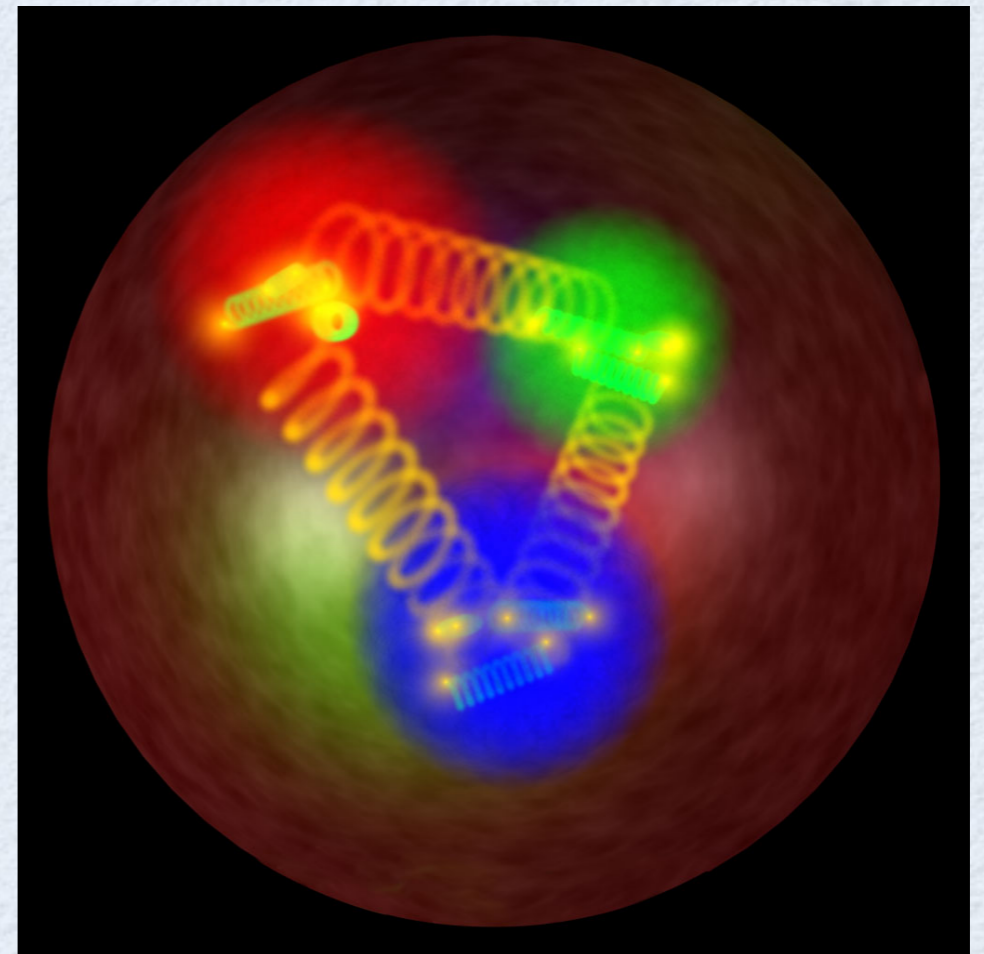
**shapes** of nuclei:  
as revealed through  
**inelastic** electron scattering





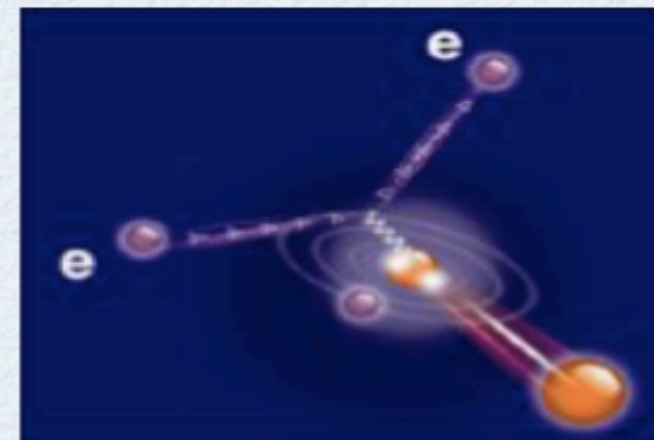
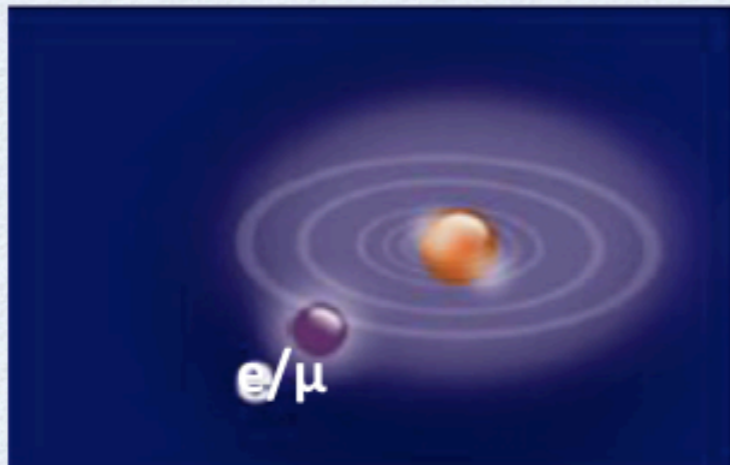
# what do we know about the proton size and its charge distributions?

- ➔ proton **size**: charge radius  $R_E$   
very low  $Q^2$  **elastic** electron scattering,  
**atomic spectroscopy** (Lamb shift)
- ➔ proton **spatial (charge) distributions**  
**elastic** electron scattering  
e.m. FFs:  $F_1(Q^2) \rightarrow \rho(\mathbf{b})$
- ➔ proton **3D transverse spatial/  
longitudinal momentum distributions**  
**deeply virtual Compton scattering**  
GPDs  $H(x, \xi, t) \rightarrow \rho(x, \mathbf{b})$  for  $\xi=0$



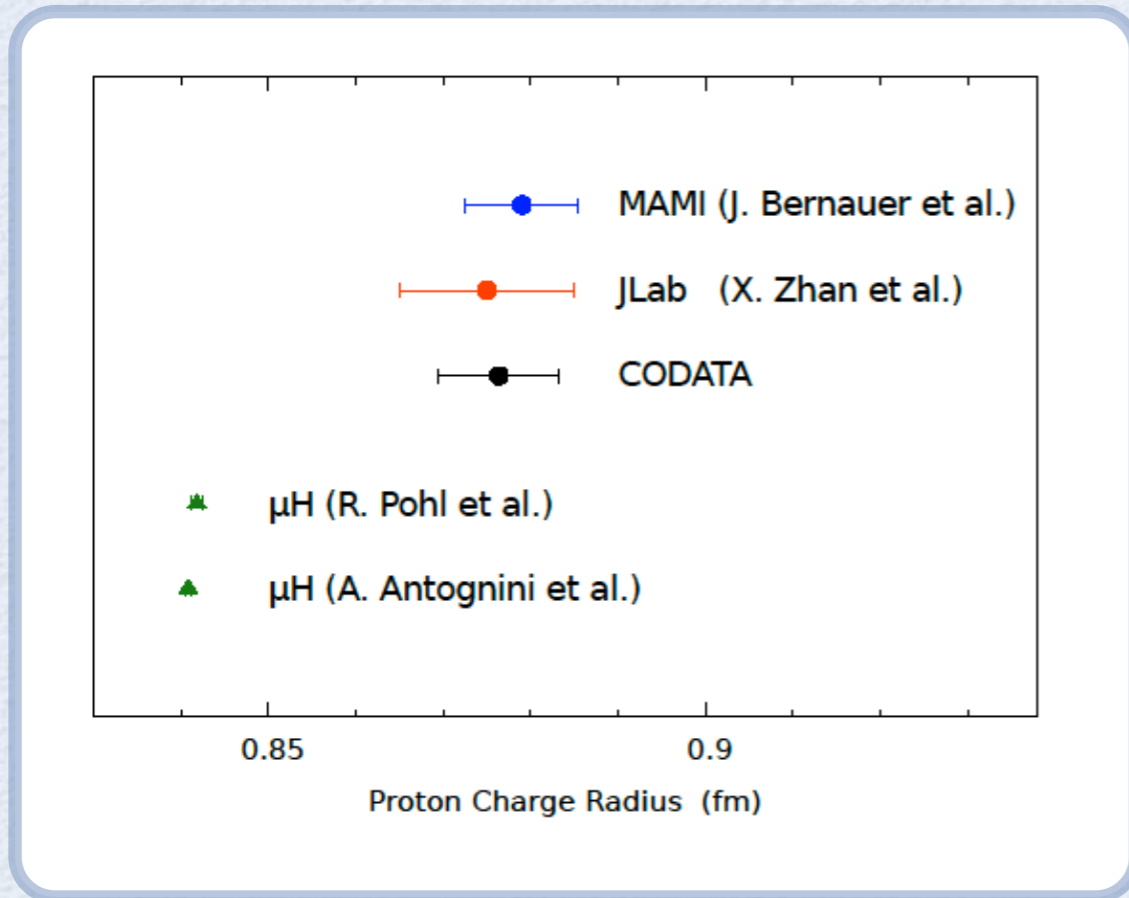


# proton size, proton radius puzzle





# Proton radius puzzle



**$\mu\text{H}$  data:**

$$R_E = 0.8409 \pm 0.0004 \text{ fm}$$

Pohl et al. (2010)

Antognini et al. (2013)



**$7 \sigma$  difference !?**

**ep data:**

$$R_E = 0.8775 \pm 0.0051 \text{ fm}$$

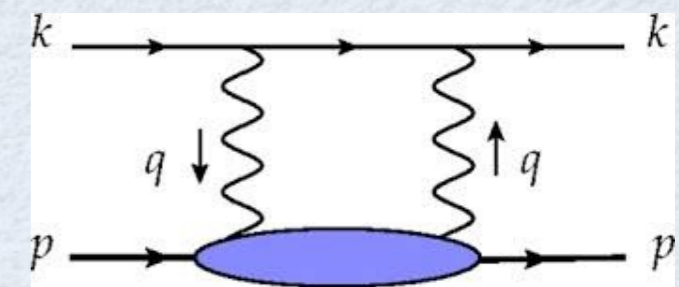
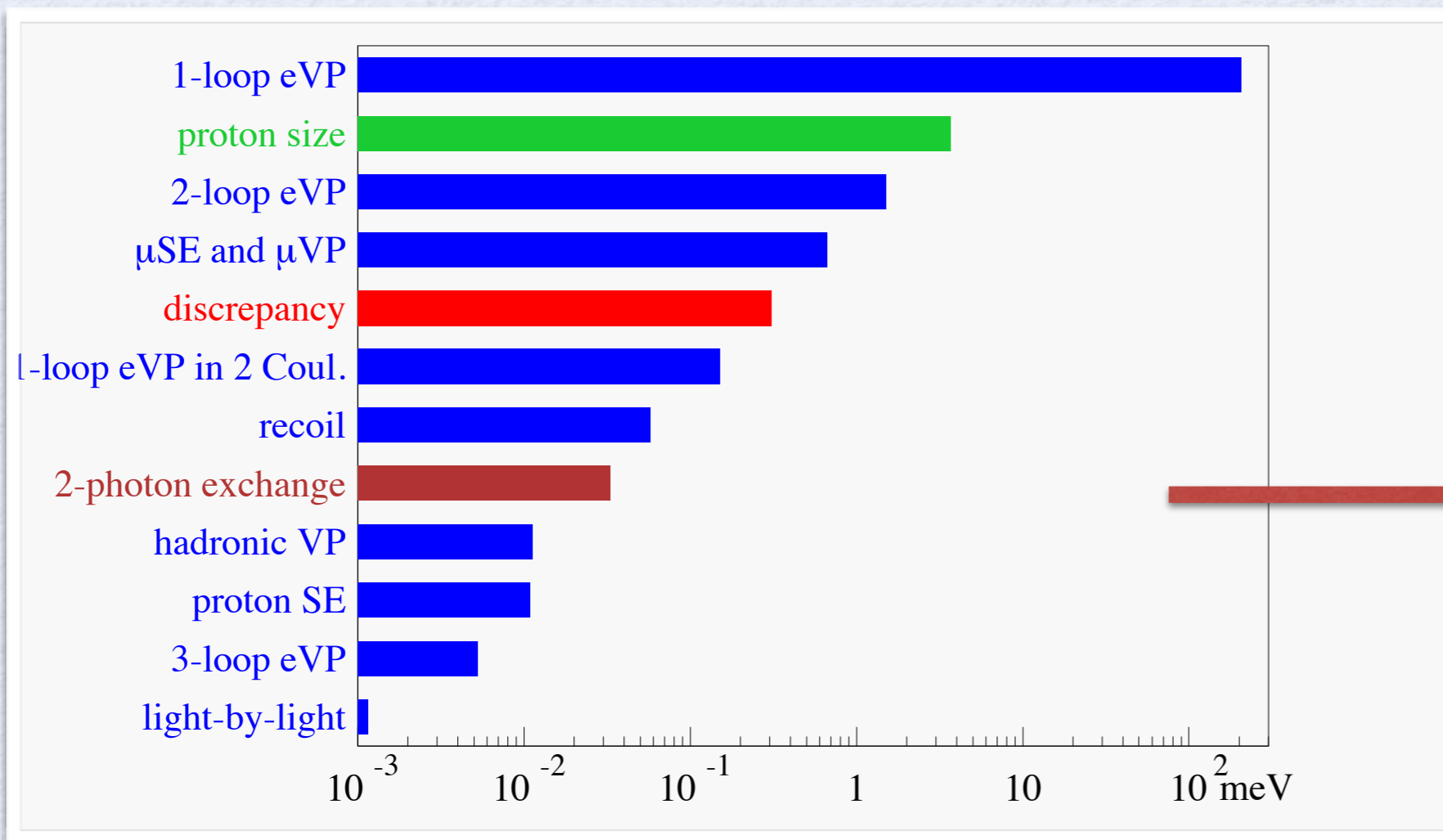
CODATA (2012)





# Lamb shift: status of known corrections

## $\mu\text{H}$ Lamb shift: summary of corrections



largest theoretical uncertainty

➔ elastic contribution on 2S level:  $\Delta E_{2S} = -23 \mu\text{eV}$

➔ inelastic contribution: Carlson, Vdh (2011) + Birse, McGovern (2012)

total hadronic correction on Lamb shift

$$\Delta E_{(2P - 2S)} = (33 \pm 2) \mu\text{eV}$$

...or about 10% of needed correction



# Proton radius puzzle: what's next ?

➔  $\mu$  atom Lamb shift:  $\mu$  D,  $\mu$   $^3\text{He}^+$ ,  $\mu$   $^4\text{He}^+$  have been performed

➔ electronic H Lamb shift: higher accuracy measurements

➔ electron scattering analysis:

- radius extraction fits (use fits with correct analytical behavior:  $2\pi$  cut)
  - radiative corrections, two-photon exchange corrections
- new fit  $R_E = 0.904$  (15) fm ( $4\sigma$  from  $\mu\text{H}$ )

➔ electron scattering experiments:

new  $G_{Ep}$  experiments down to  $Q^2 \approx 2 \times 10^{-4} \text{ GeV}^2$

- **MAMI/A1**: Initial State Radiation (2013/4)
- **JLab/Hall B**: HyCal, magnetic spectrometer-free experiment, norm to Møller (2016/7)
- **MESA**: low-energy, high resolution spectrometers

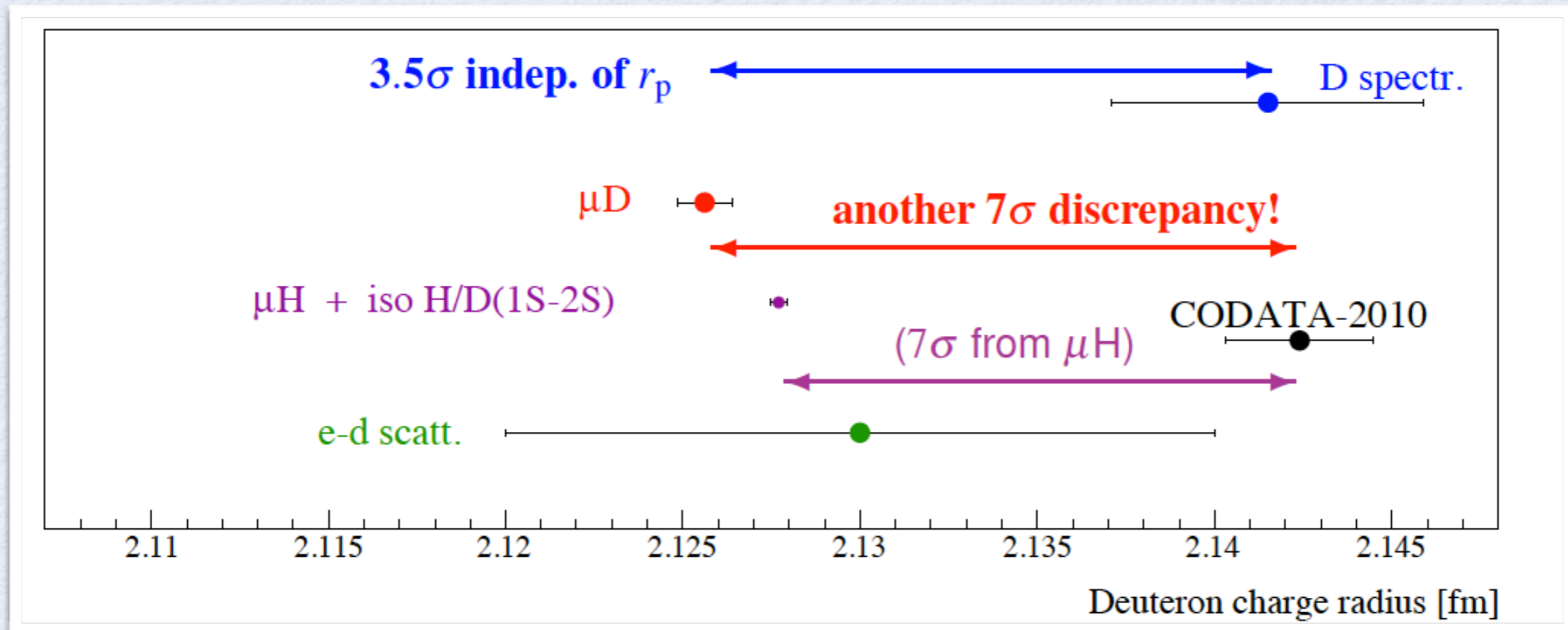
➔ muon scattering experiments: **MUSE@PSI** (2018/9)

➔  $e^-e^+$  versus  $\mu^-\mu^+$  photoproduction: lepton universality test



# $\mu\text{D}$ Lamb shift experiment

- H/D isotope shift (1S - 2S):  $r_d^2 - r_p^2 = 3.82007 (65) \text{ fm}^2$  Parthey et al. (2010)
- CODATA 2010:  $r_d = 2.14240 (210) \text{ fm}$
- $r_p$  from  $\mu\text{H}$  + isotope shift :  $r_d = 2.12771 (22) \text{ fm}$
- new  $\mu\text{D}$  Lamb shift @ PSI:  $r_d = 2.12562 (13)_{\text{theo}} (77)_{\text{theo}} \text{ fm}$  Pohl et al., Science 353,417 (2016)



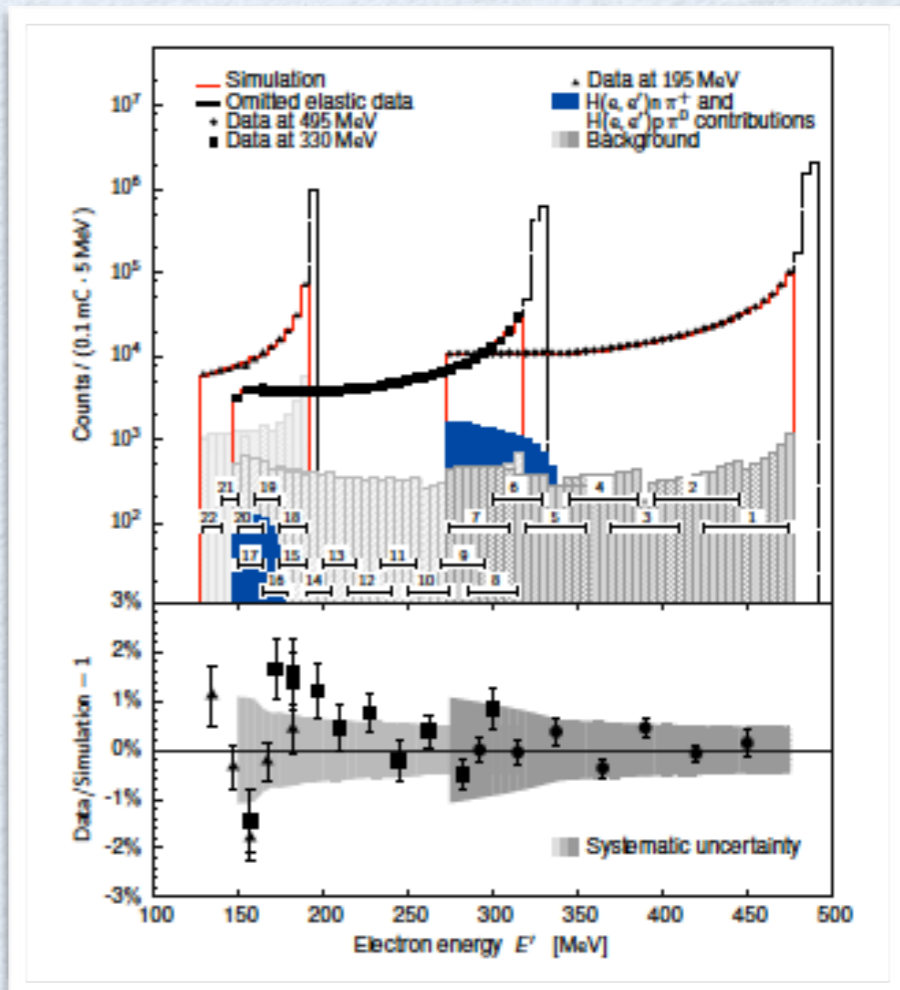
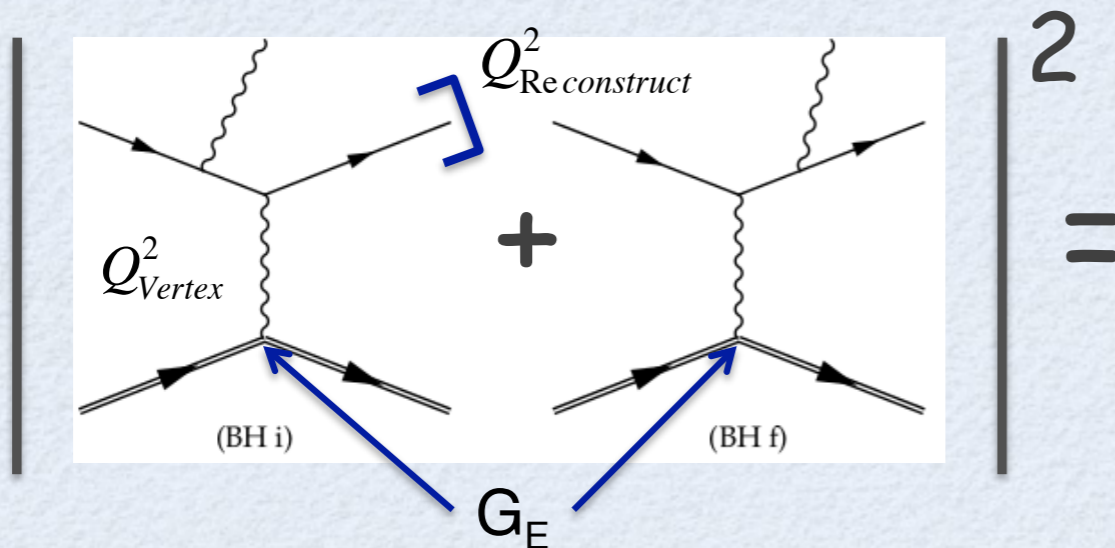
- electronic D ( $r_p$  indep.):  $r_d = 2.14150 (450) \text{ fm}$   $\leftarrow 3.5 \sigma$  Pohl et al. (2016)

- improved radius measurement from e-d scattering was performed @ MAMI (2014)

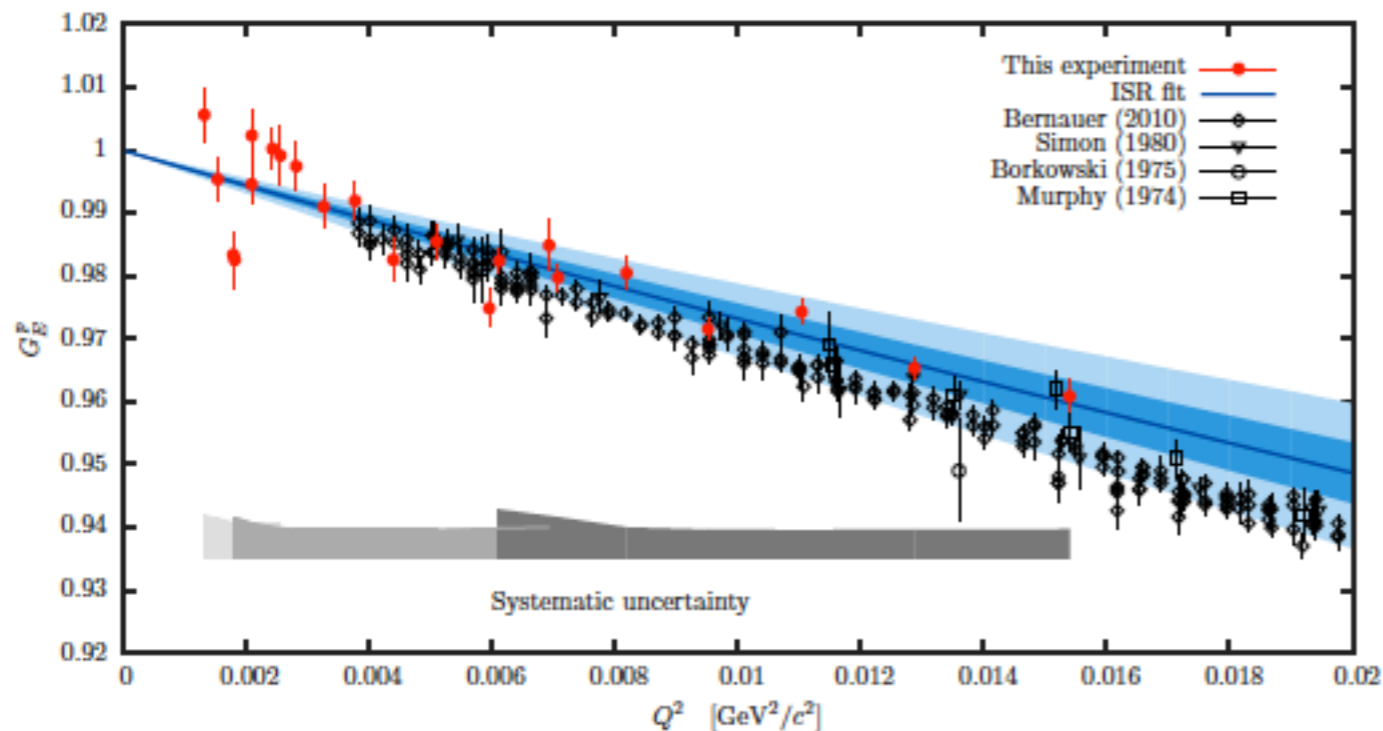


# ISR@MAMI experiment

- **Extracting FFs from the radiative tail.**
- Radiative tail dominated by coherent sum of two Bethe-Heitler diagrams.



Mihovilovic et al. (2016)

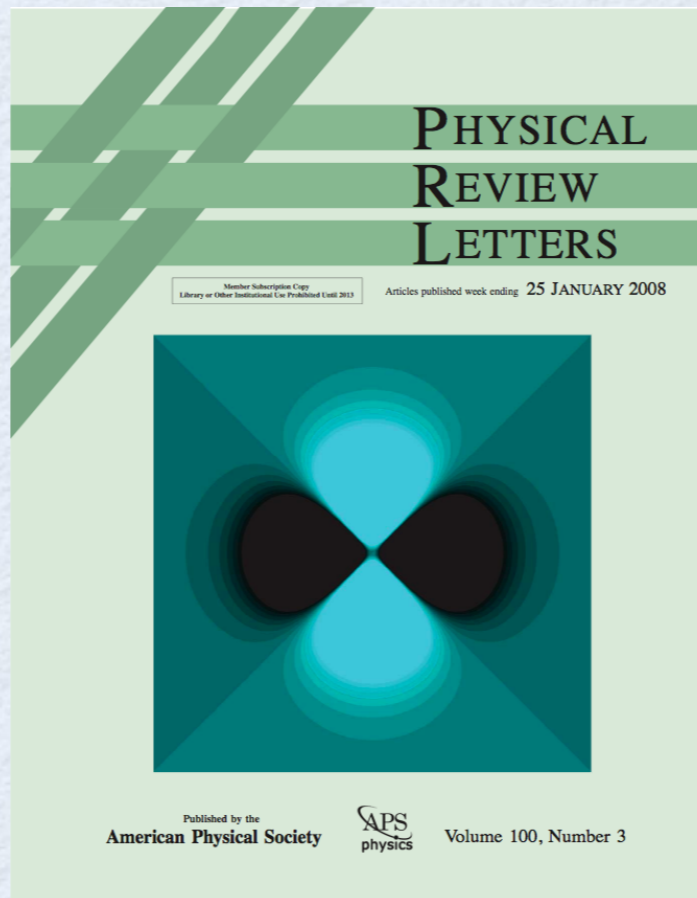


good understanding of radiative tail ( $\sim 1\%$ )

follow up experiment:  
down to  $Q^2 \approx 2 \times 10^{-4} \text{ GeV}^2$



# proton e.m. form factors, charge distributions



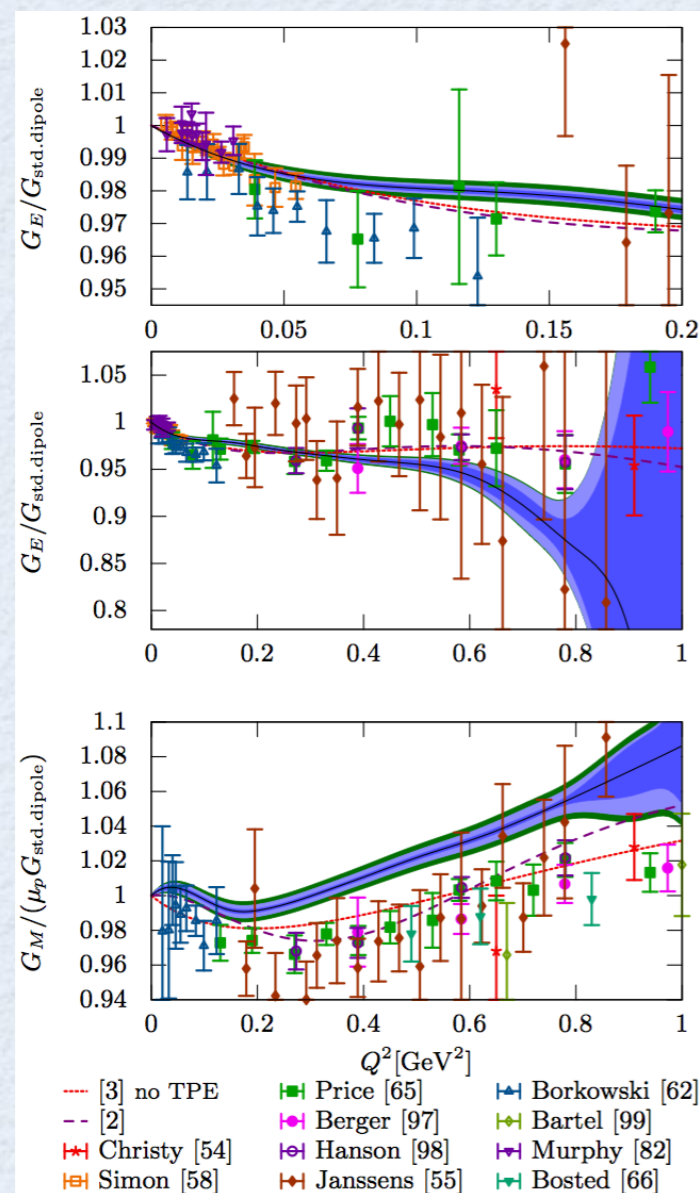


# $e^-$ scattering cross sections

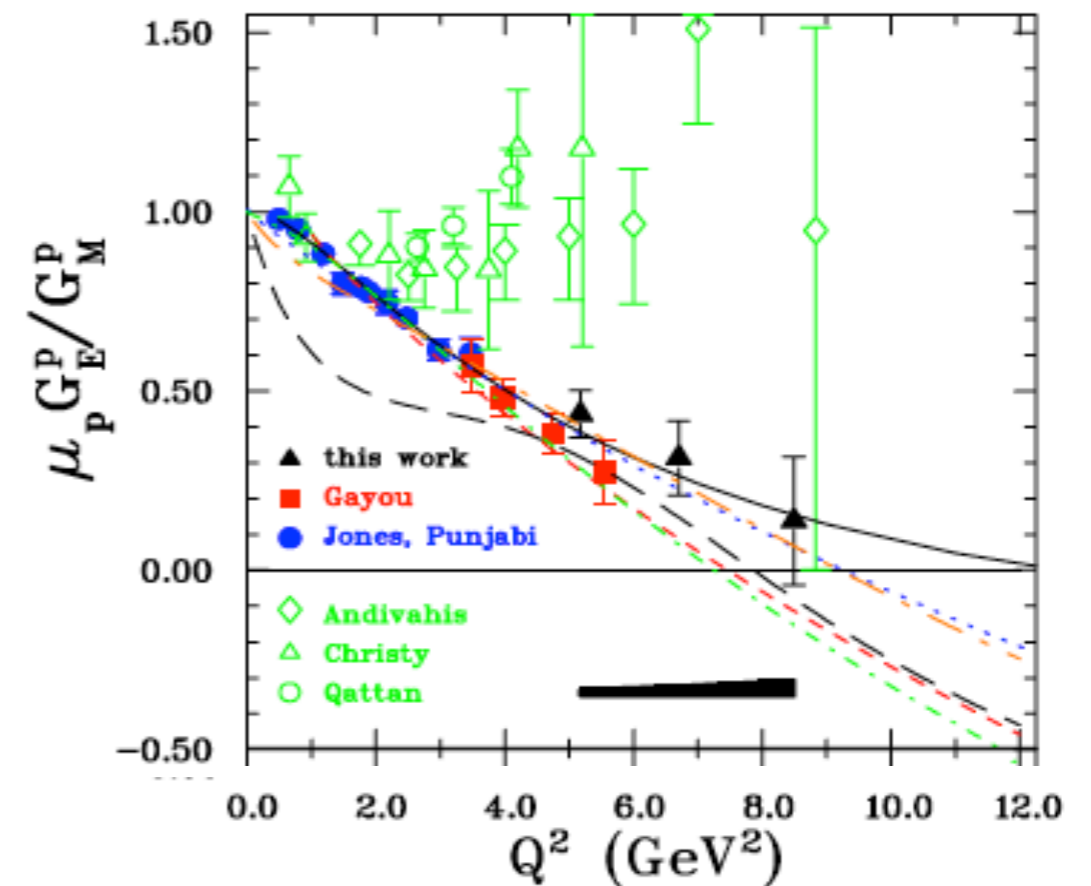
Electron scattering facilities JLab (12 GeV), MAMI (1.6 GeV):  
uniquely positioned to deliver high precision data

MAMI/A1 achieved  $< 1\%$  measurement  
of proton charge radius  $R_E$

JLab polarization transfer measurements:  
 $G_{Ep} / G_{Mp}$  difference with Rosenbluth



Bernaer et al. (2010, 2013)

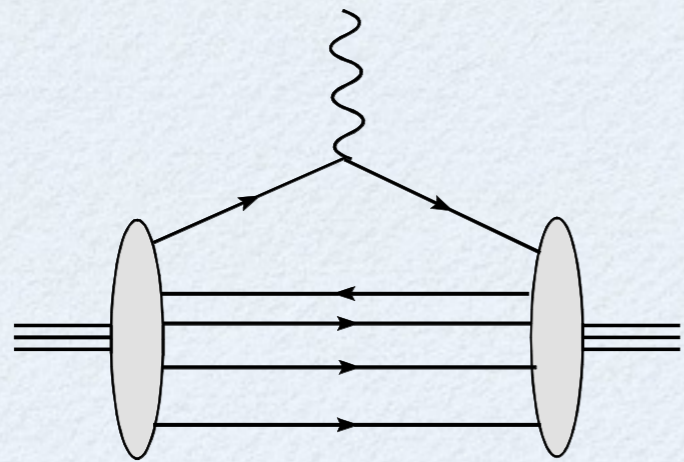
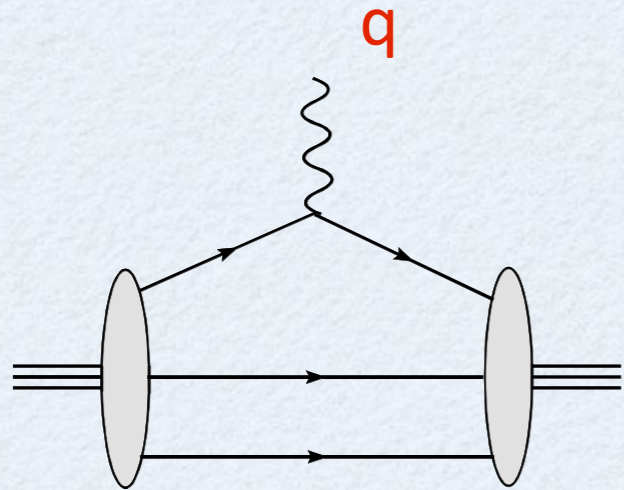


Jones et al. (2000) Punjabi et al. (2005)

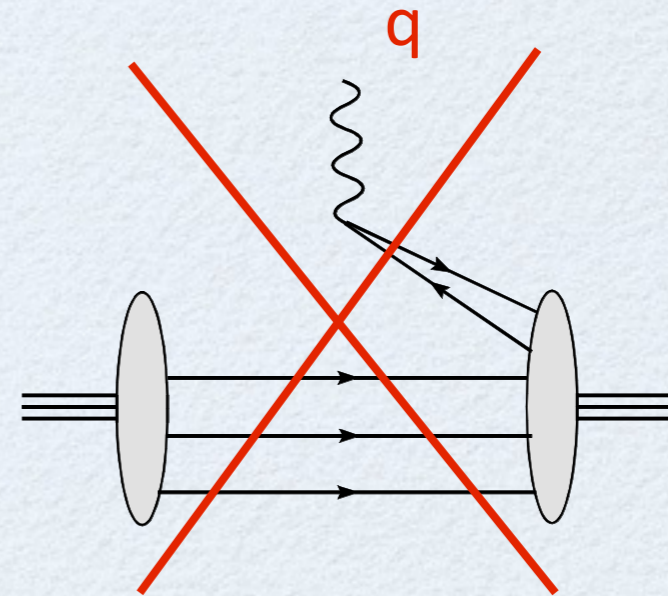
Gayou et al. (2002) Puckett et al. (2010)



# Interpretation of form factor as quark density



overlap of wave function  
Fock components  
with **same** number of quarks



overlap of wave function  
Fock components  
with **different** number of quarks  
**NO** probability / charge density  
interpretation

absent in a light-front frame!

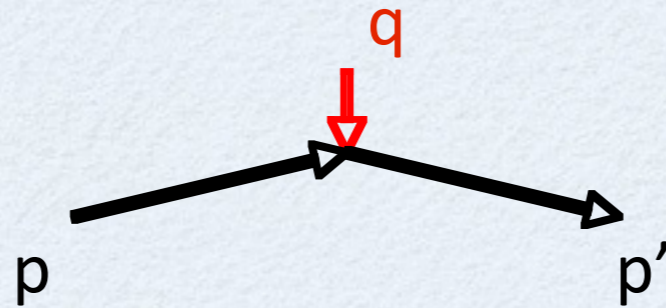
$$q^+ = q^0 + q^3 = 0$$



# quark transverse charge densities in nucleon (1)

→ light-front

$$q^+ = q^0 + q^3 = 0$$

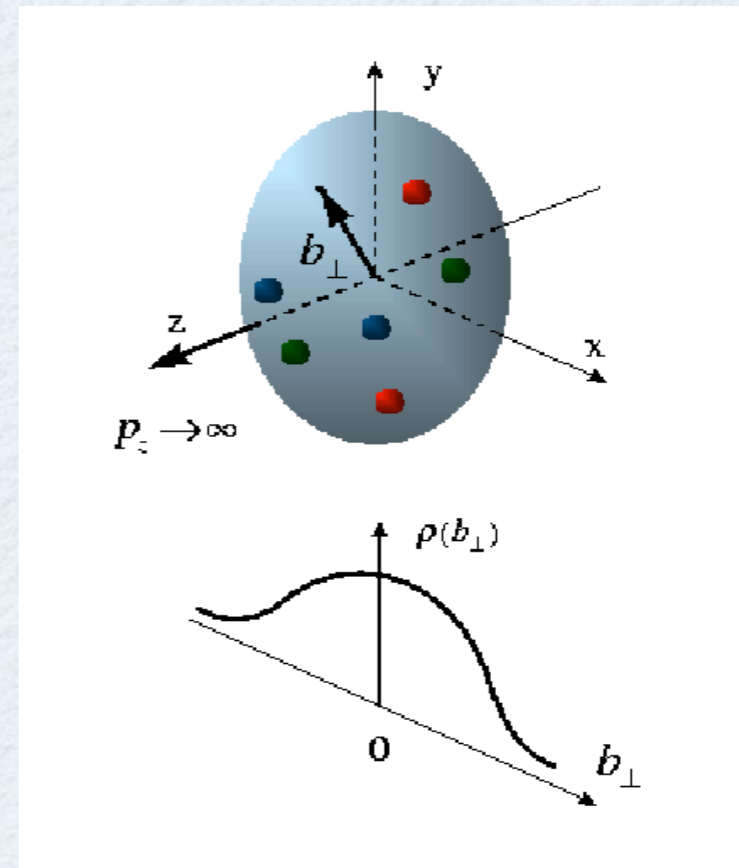


photon only couples to forward moving quarks

→ quark charge density operator

$$J^+ = J^0 + J^3 = \bar{q}\gamma^+q = 2q_+^\dagger q_+$$

with  $q_+ \equiv \frac{1}{4}\gamma^-\gamma^+q$



→ longitudinally polarized nucleon

$$\begin{aligned} \rho_0^N(\vec{b}) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle \\ &= \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1(Q^2) \end{aligned}$$

Soper (1997)

Burkardt (2000)

Miller (2007)



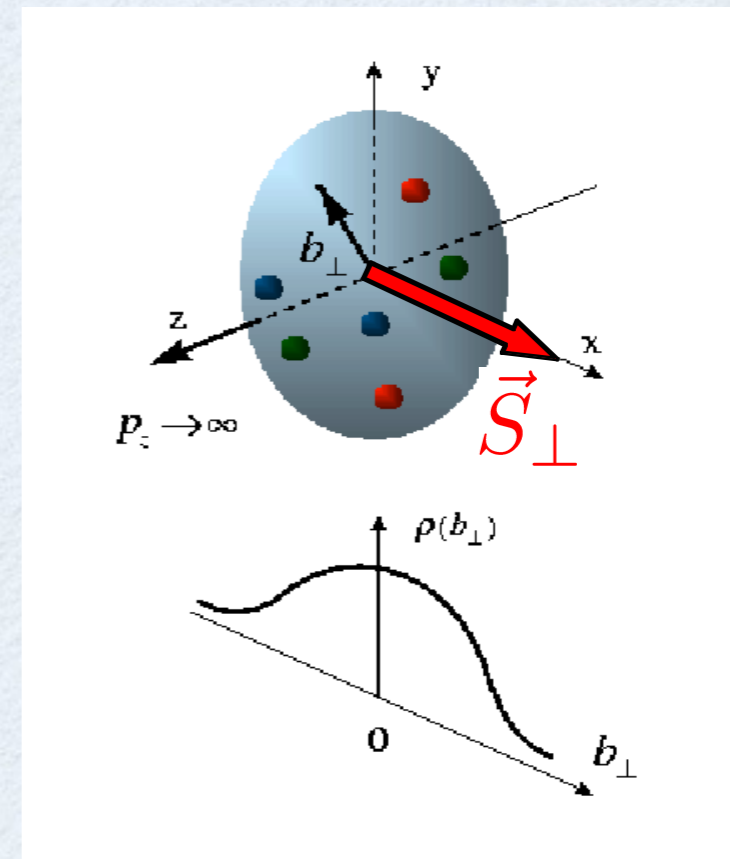
# quark transverse charge densities in nucleon (2)

→ transversely polarized nucleon

transverse spin  $\vec{S}_\perp = \cos \phi_S \hat{e}_x + \sin \phi_S \hat{e}_y$

e.g. along x-axis  $\phi_S = 0$

$$\vec{b} = b(\cos \phi_b \hat{e}_x + \sin \phi_b \hat{e}_y)$$



$$\begin{aligned} \rho_T^N(\vec{b}) &\equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \rangle \\ &= \rho_0^N(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(bQ) F_2(Q^2) \end{aligned}$$

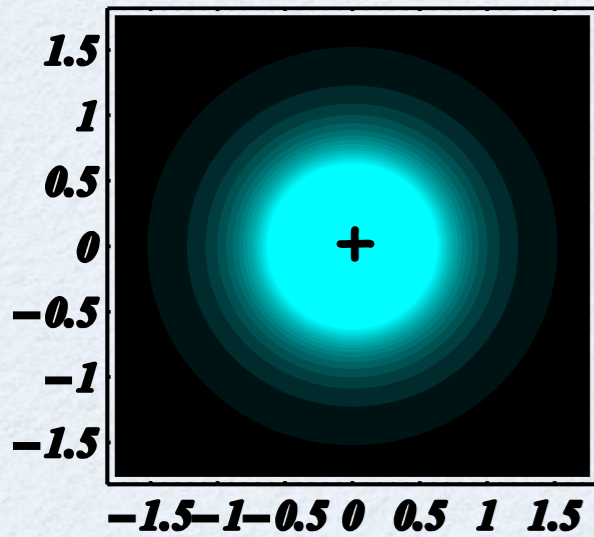
↑ dipole field pattern

Carlson, Vdh (2007)

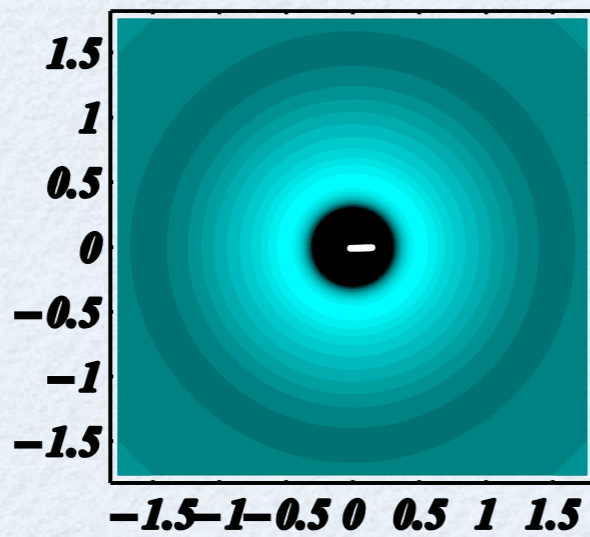


# spatial imaging of hadrons

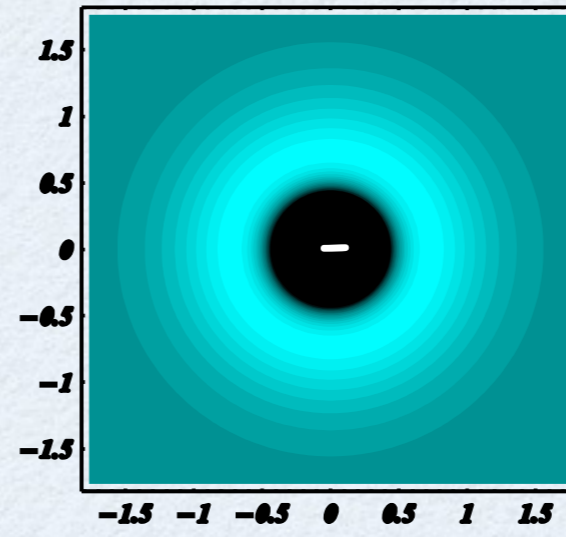
proton



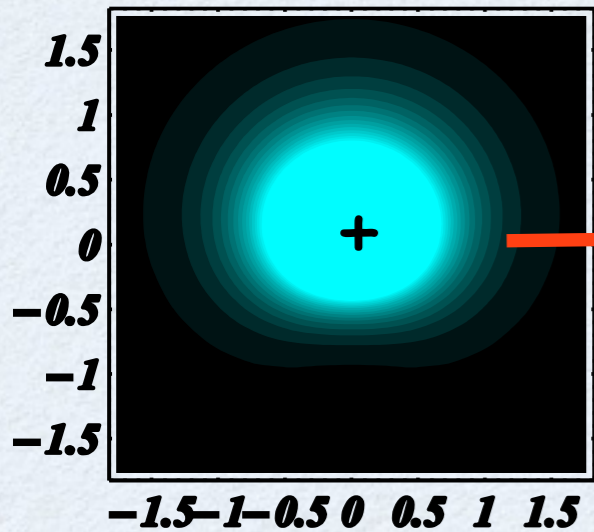
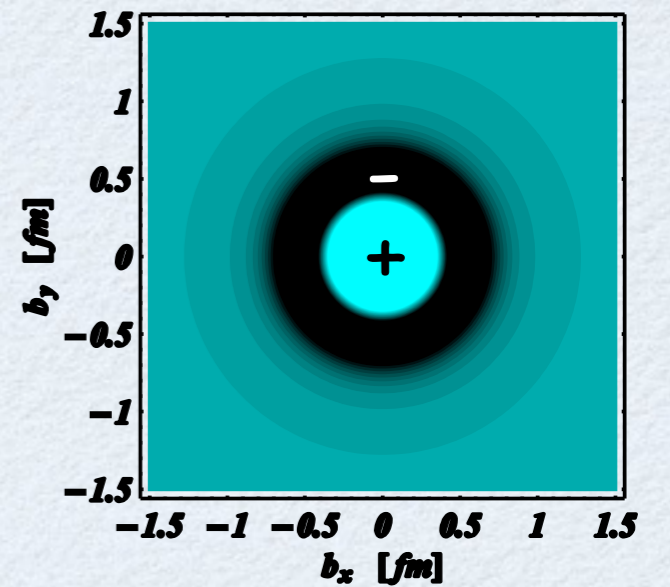
neutron



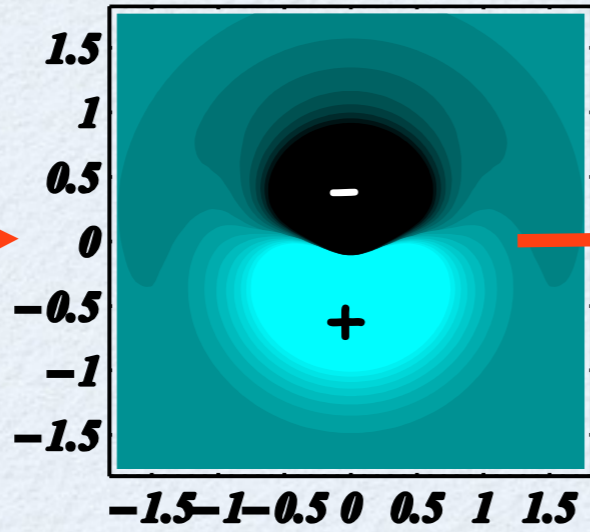
$p \rightarrow \Delta^+$



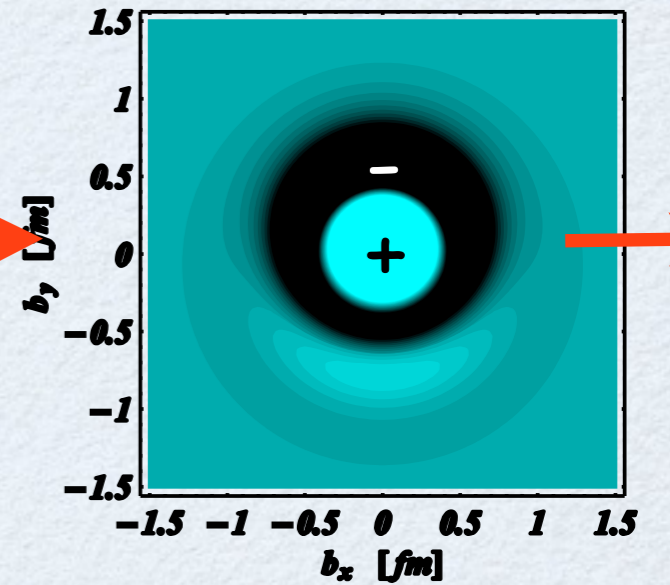
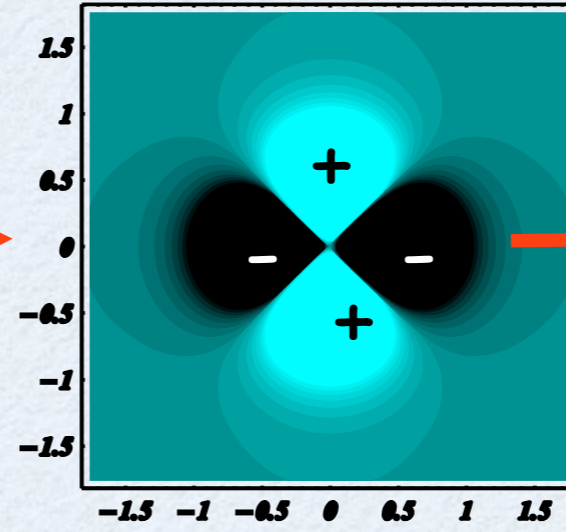
$p \rightarrow N^* (1440)$



Miller (2007)



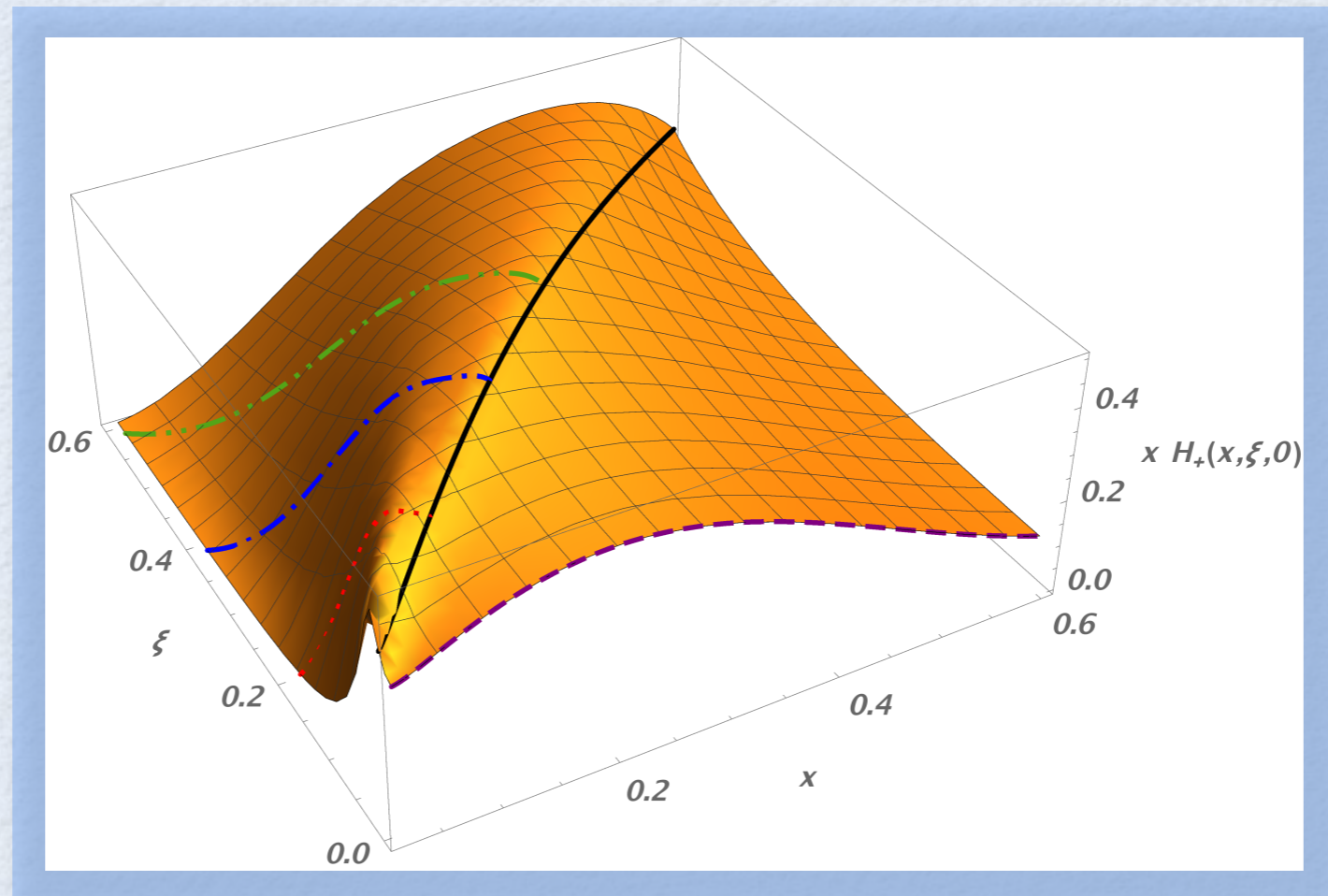
Carlson, Vdh (2007)



Tiator, Vdh (2007)



# Generalized Parton Distributions





# Correlations in transverse position/longitudinal momentum

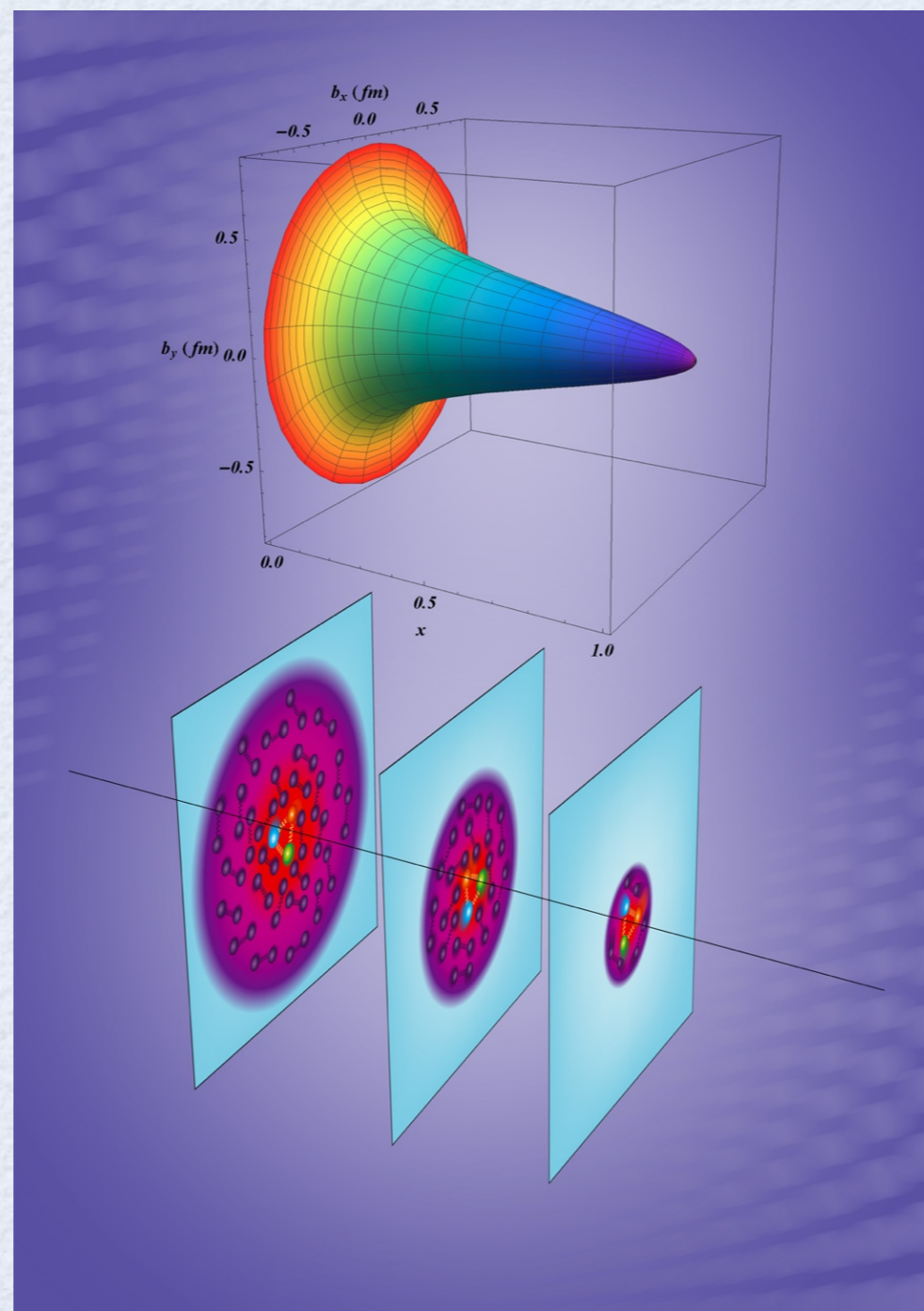
**elastic  
scattering**



**DIS**

quark  
distributions in  
transverse  
position space

quark  
distributions in  
longitudinal  
momentum



proton  
3D imaging

Burkardt (2000, 2003)

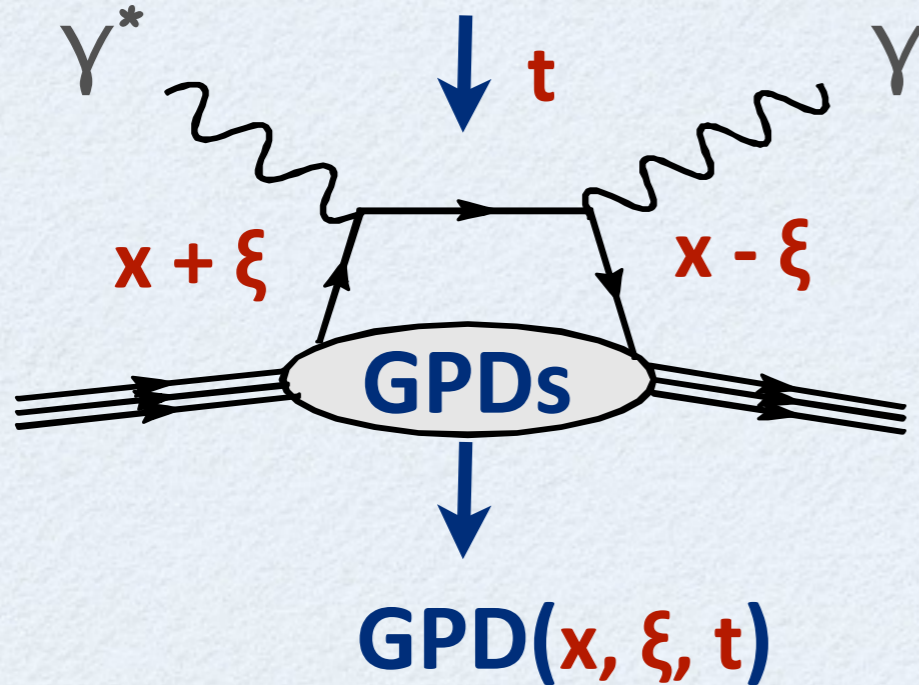
Belitsky, Ji, Yuan  
(2004)



# DVCS: tool to access GPDs

world data on proton  $F_2$

$Q^2 \gg 1 \text{ GeV}^2$



➔ at large  $Q^2$ : QCD factorization theorem

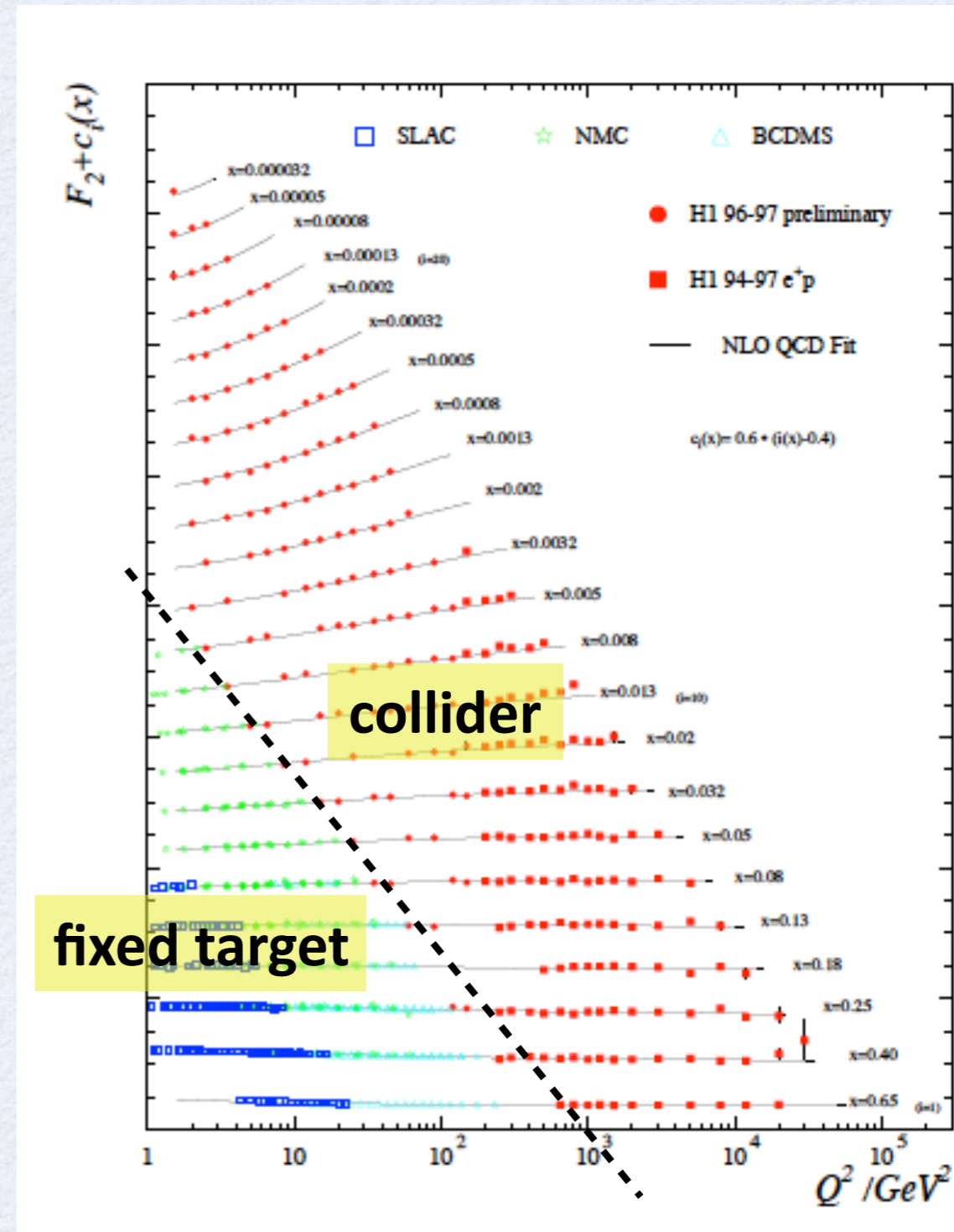
Müller et al (1994)

Ji (1995) Radyushkin (1996)

Collins, Frankfurt, Strikman (1996)

at twist-2: 4 quark helicity conserving GPDs

➔ key:  $Q^2$  leverage needed to test QCD scaling





# GPDs: known limits

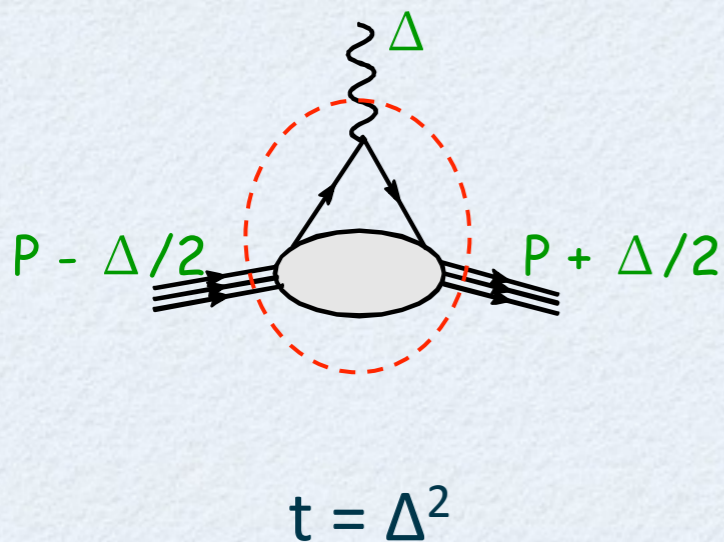
➔ in forward kinematics ( $\xi=0, t = 0$ ) : **PDF limit**

$$H^q(x, \xi = 0, t = 0) = q(x)$$

$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$$

$E, \tilde{E}^q$  do not appear in forward kinematics (DIS) ➔ **new information**

➔ first moments of GPDs : **elastic form factor limit**



$$\int_{-1}^{+1} dx H^q(x, \xi, t) = F_1^q(t)$$

➔ Dirac FF

$$\int_{-1}^{+1} dx E^q(x, \xi, t) = F_2^q(t)$$

➔ Pauli FF

$$\int_{-1}^{+1} dx \tilde{H}^q(x, \xi, t) = G_A^q(t)$$

➔ axial FF

$$\int_{-1}^{+1} dx \tilde{E}^q(x, \xi, t) = G_P^q(t)$$

➔ pseudoscalar FF



# GPDs: moments, total angular momentum

$$\int_{-1}^{+1} dx x H^q(x, \xi, t) = A(t) + \xi^2 C(t)$$

$$\int_{-1}^{+1} dx x E^q(x, \xi, t) = B(t) - \xi^2 C(t)$$

form factors of energy-momentum tensor

Polyakov, Weiss (1999)

Polyakov (2003)

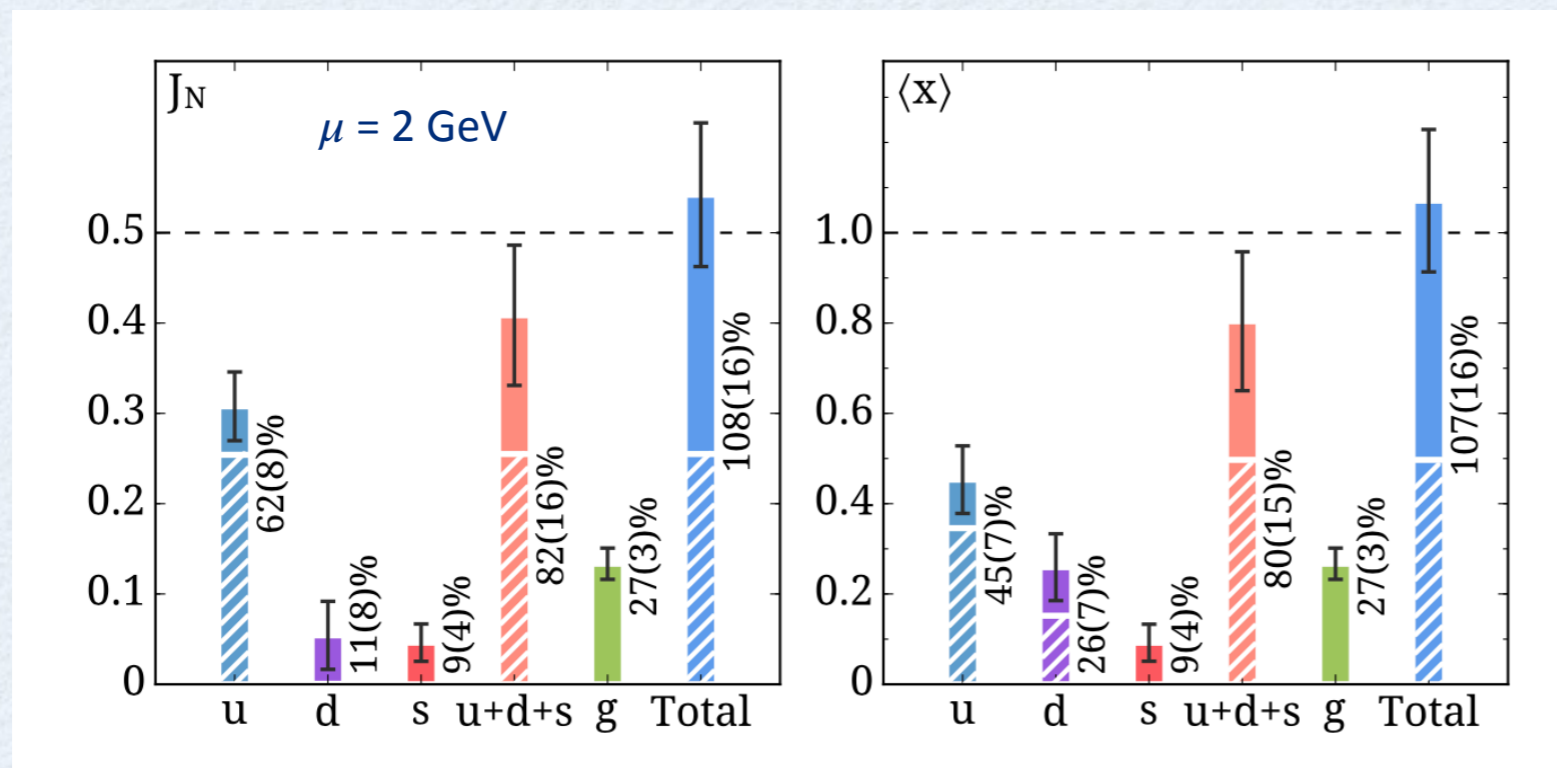
Goeke, Schweitzer et al. (2007)

Ji's angular momentum sum rule

$$\int_{-1}^{+1} dx x \{ H^q(x, \xi, 0) + E^q(x, \xi, 0) \} = A(0) + B(0) = 2J^q$$

lattice QCD calculations at the physical point

Alexandrou et al. (2017)



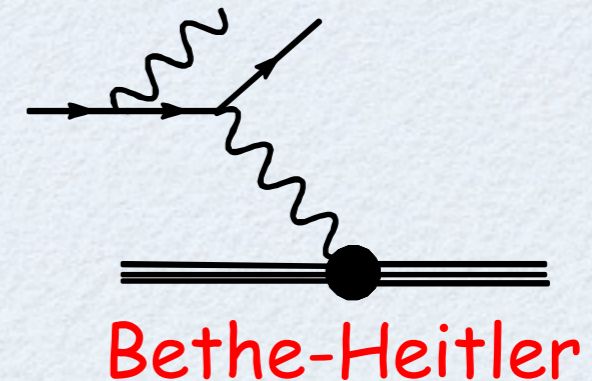
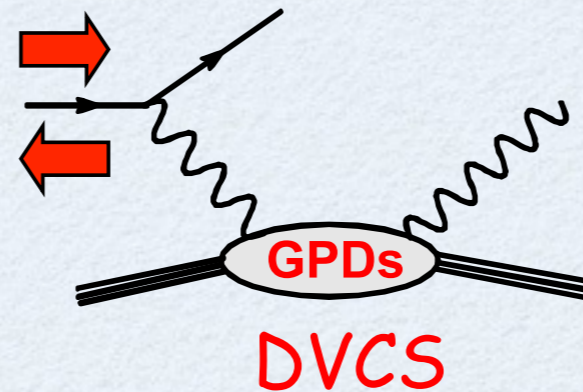
d, s-quarks carry very small total J in proton, u-quark carries around 60%, gluons around 30%

Sharing of momentum and total angular momentum between quarks and gluons identical in proton !



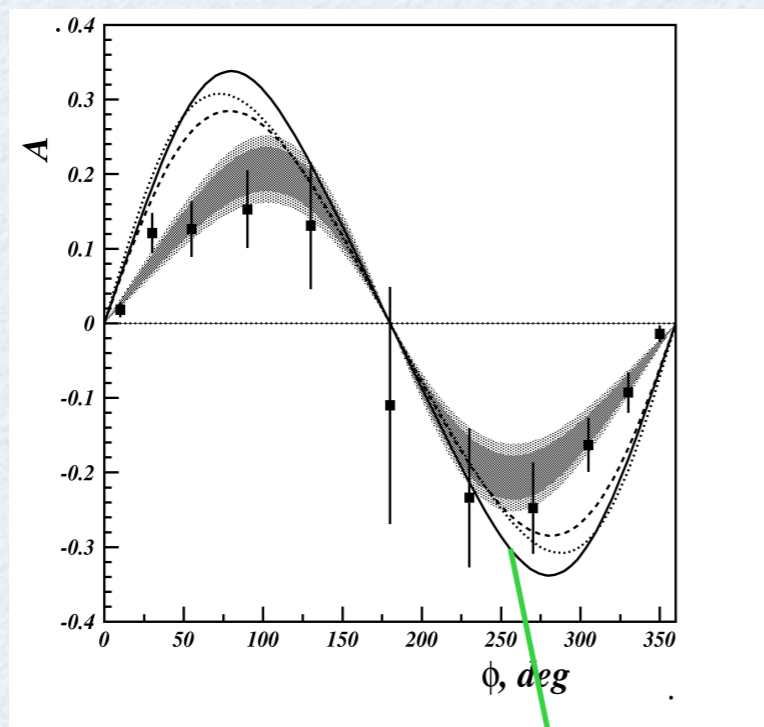
# DVCS beam spin asymmetries: first observations around 2000

$$A_{LU} = \frac{(BH) * \text{Im}(DVCS) * \sin \Phi}{(BH^2 + DVCS^2)}$$

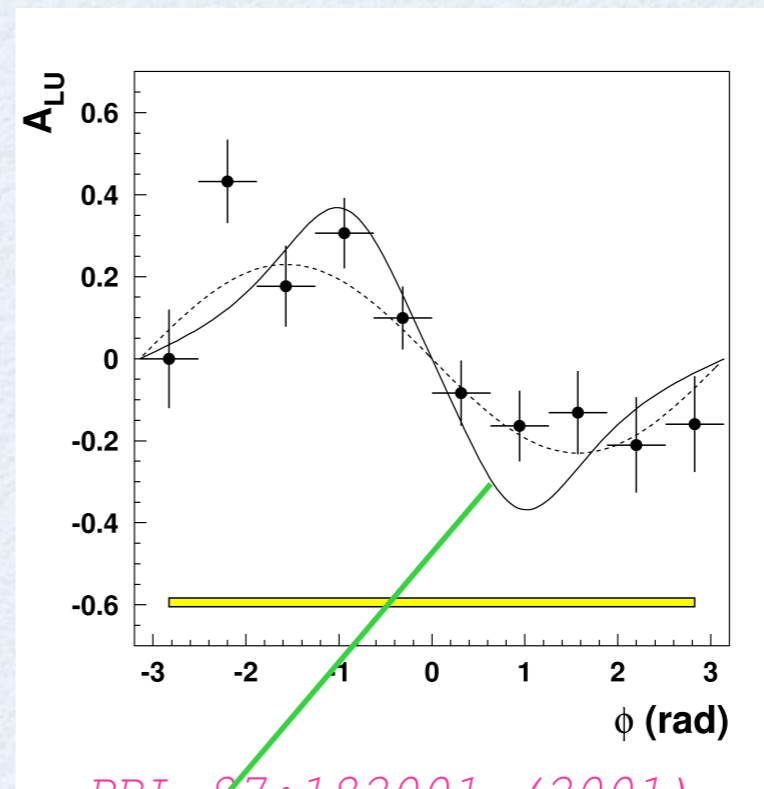


CLAS

$Q^2 = 1.25 \text{ GeV}^2$ ,  
 $x_B = 0.19$ ,  
 $-t = 0.19 \text{ GeV}^2$



PRL 87:182002 (2001)



PRL 87:182001 (2001)

HERMES

$Q^2 = 2.6 \text{ GeV}^2$ ,  
 $x_B = 0.11$ ,  
 $-t = 0.27 \text{ GeV}^2$

twist-2 + twist-3

Vdh, Guichon, Guidal (1999)  
Kivel, Polyakov, Vdh (2000)



# DVCS accesses Compton Form Factors: 8 CFFs at twist-2



$$\mathcal{H}_{Re}(\xi, t) \equiv \mathcal{P} \int_0^1 dx \left\{ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right\} H_+(x, \xi, t)$$

$$\mathcal{H}_{Im}(\xi, t) \equiv H_+(\xi, \xi, t)$$

$$\tilde{\mathcal{H}}_{Re}(\xi, t) \equiv \mathcal{P} \int_0^1 dx \left\{ \frac{1}{x - \xi} - \frac{1}{x + \xi} \right\} \tilde{H}_+(x, \xi, t)$$

$$\tilde{\mathcal{H}}_{Im}(\xi, t) \equiv \tilde{H}_+(\xi, \xi, t)$$

and analogous  
formulas for  
GPDs  $E, \tilde{E}^q$   
respectively

with singlet GPD combinations  
(quark + anti-quark):

$$H_+(x, \xi, t) \equiv H(x, \xi, t) - H(-x, \xi, t)$$

$$\tilde{H}_+(x, \xi, t) \equiv \tilde{H}(x, \xi, t) + \tilde{H}(-x, \xi, t)$$



CFF fit extractions from data:

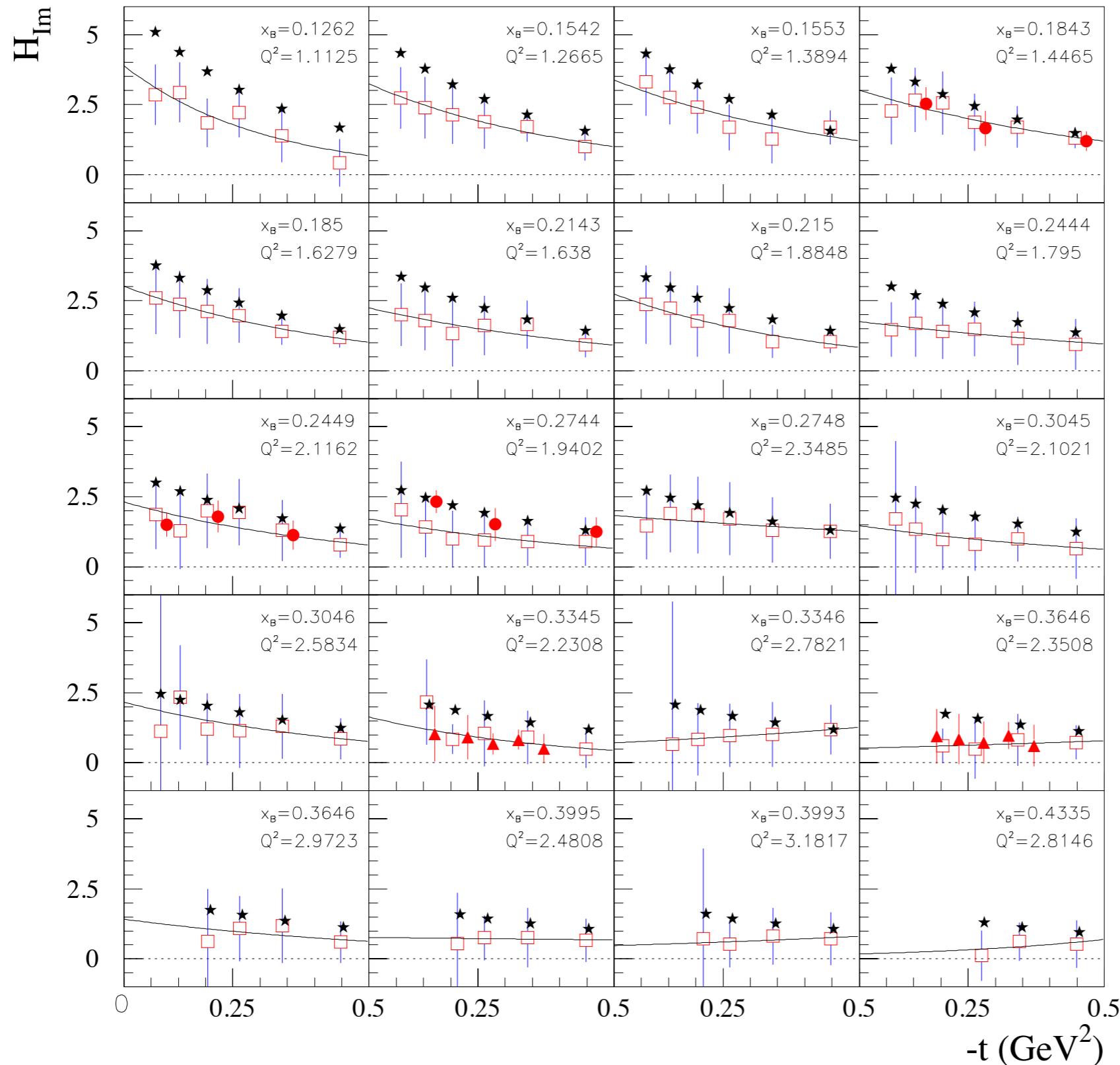
Guidal (2008, ...)

Guidal, Moutarde (2009, ...)

Kumericki, Mueller, Passek-Kumericki (2008, ...)



# global analysis of JLab 6 GeV data



$$\mathcal{H}_{Im}(\xi, t)$$

red solid circles:  
CLAS:  $\sigma$ ,  $A_{LU}$ ,  $A_{UL}$ ,  $A_{LL}$

red open squares:  
CLAS:  $\sigma$ ,  $A_{LU}$

red triangles:  
Hall A:  $\sigma$ ,  $A_{LU}$

black stars  
VGG model values

Dupré, Guidal,  
vdh (2017)



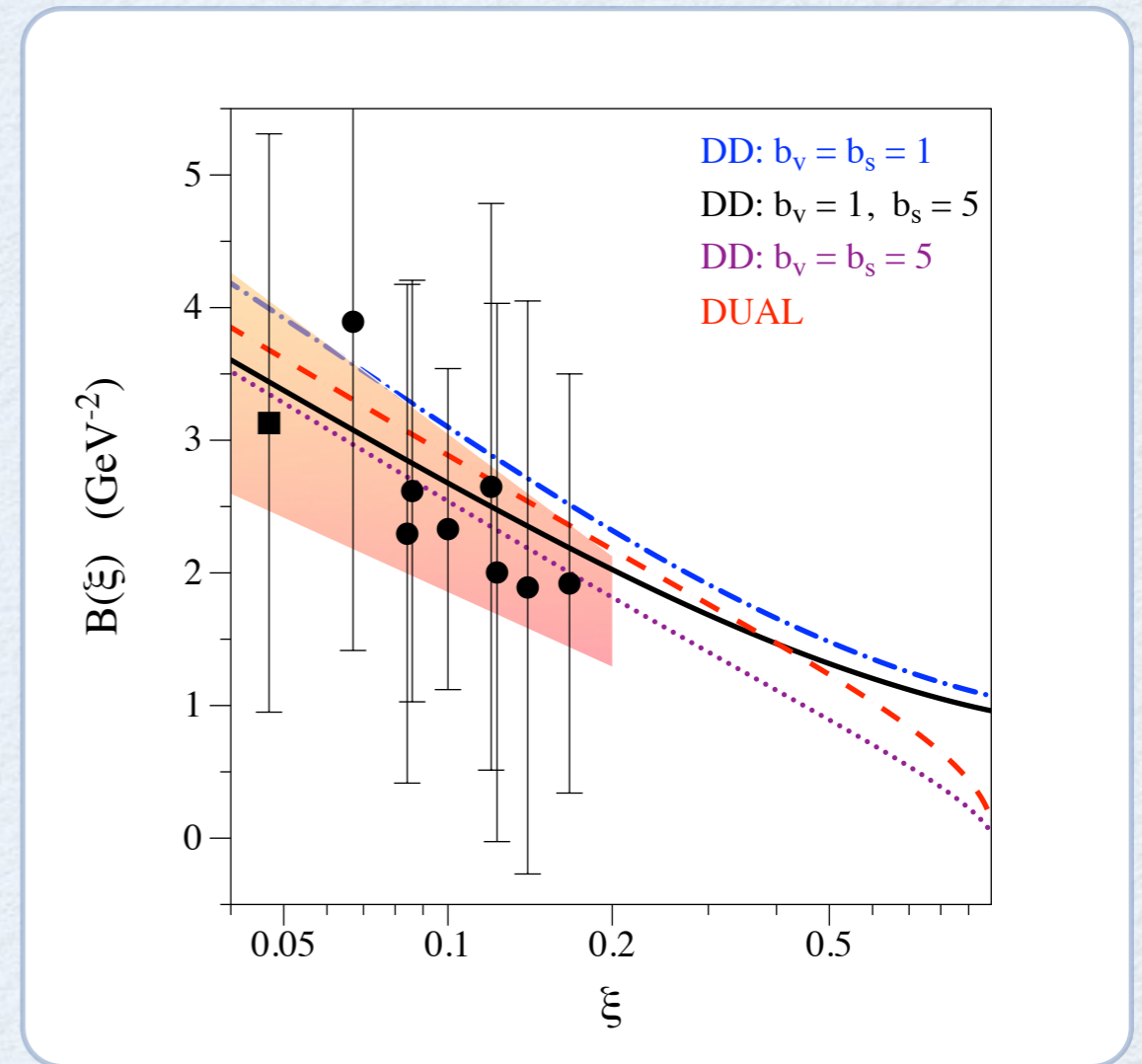
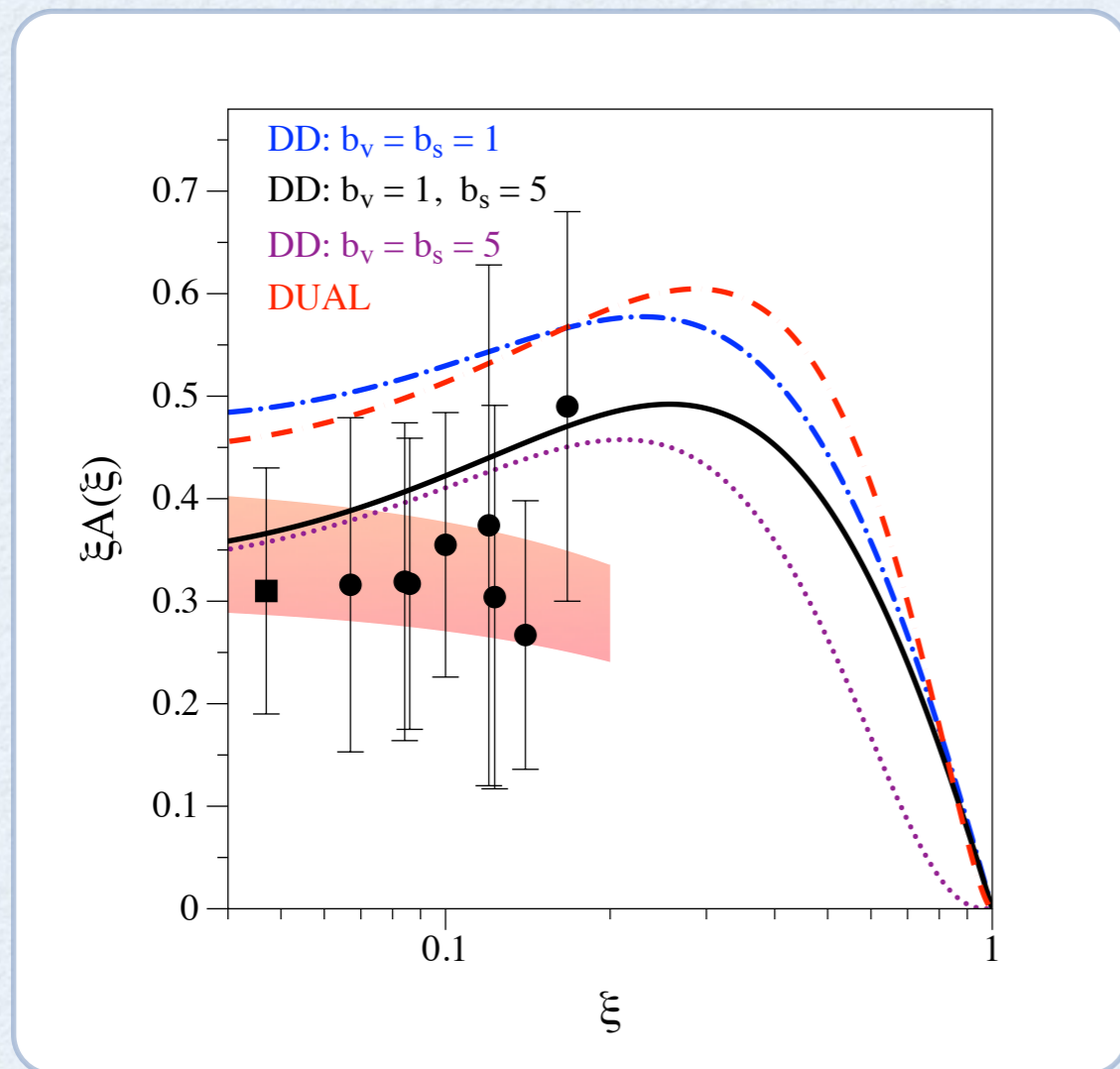
CFF  $\mathcal{H}_{Im}$ :  $\mathcal{H}_{Im}(\xi, t) = A(\xi)e^{B(\xi)t}$

black circles: CFF fit of JLab data

Dupré, Guidal, Vdh (2017)

black squares: CFF fit of HERMES data

Guidal, Moutarde (2009)



$A(\xi) = a_A(1 - \xi)/\xi$

red bands:  
1- parameter  
fits of data

$B(\xi) = a_B \ln(1/\xi)$



# 3D imaging

$$\rho^q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} H_-^q(x, \xi = 0, -\Delta_\perp^2)$$

Burkardt (2000)

number density of quarks (q) with longitudinal momentum x  
at a transverse distance  $\mathbf{b}_\perp$  in proton

non-singlet (valence quark) GPDs:  $H_-^q(x, 0, t) \equiv H^q(x, 0, t) + H^q(-x, 0, t)$

x-dependent  
radius

$$\langle b_\perp^2 \rangle^q(x) \equiv \frac{\int d^2 \mathbf{b}_\perp \mathbf{b}_\perp^2 \rho^q(x, \mathbf{b}_\perp)}{\int d^2 \mathbf{b}_\perp \rho^q(x, \mathbf{b}_\perp)} = -4 \frac{\partial}{\partial \Delta_\perp^2} \ln H_-^q(x, 0, -\Delta_\perp^2) \Big|_{\Delta_\perp=0}$$

$$H_-^q(x, 0, t) = q_v(x) e^{B_0(x)t} \longrightarrow \langle b_\perp^2 \rangle^q(x) = 4B_0(x)$$

x-independent  
radius

$$\langle b_\perp^2 \rangle^q = \frac{1}{N_q} \int_0^1 dx q_v(x) \langle b_\perp^2 \rangle^q(x)$$

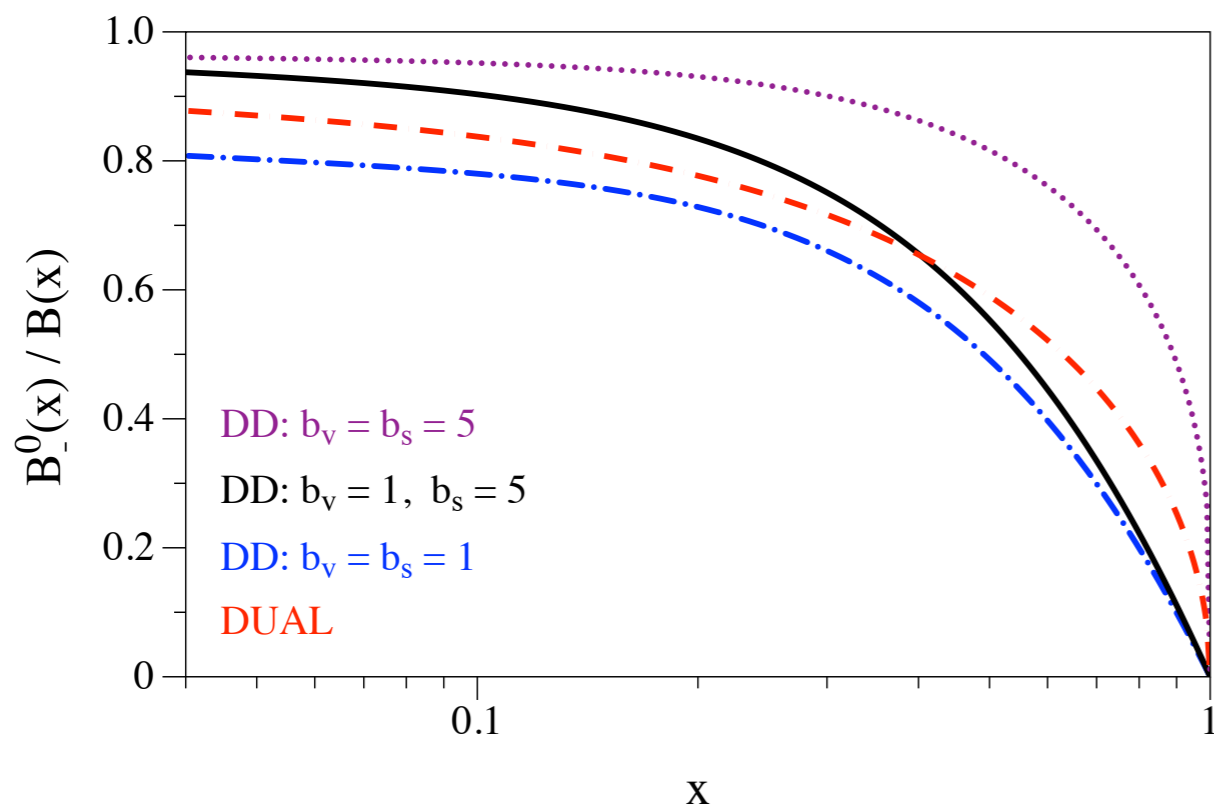
$N_u=2, N_d=1$

$$\langle b_\perp^2 \rangle = 2e_u \langle b_\perp^2 \rangle^u + e_d \langle b_\perp^2 \rangle^d = 2/3 \langle r_1^2 \rangle = 0.43 \pm 0.01 \text{ fm}^2$$

Bernauer (2014)



# x-dependent radius in proton

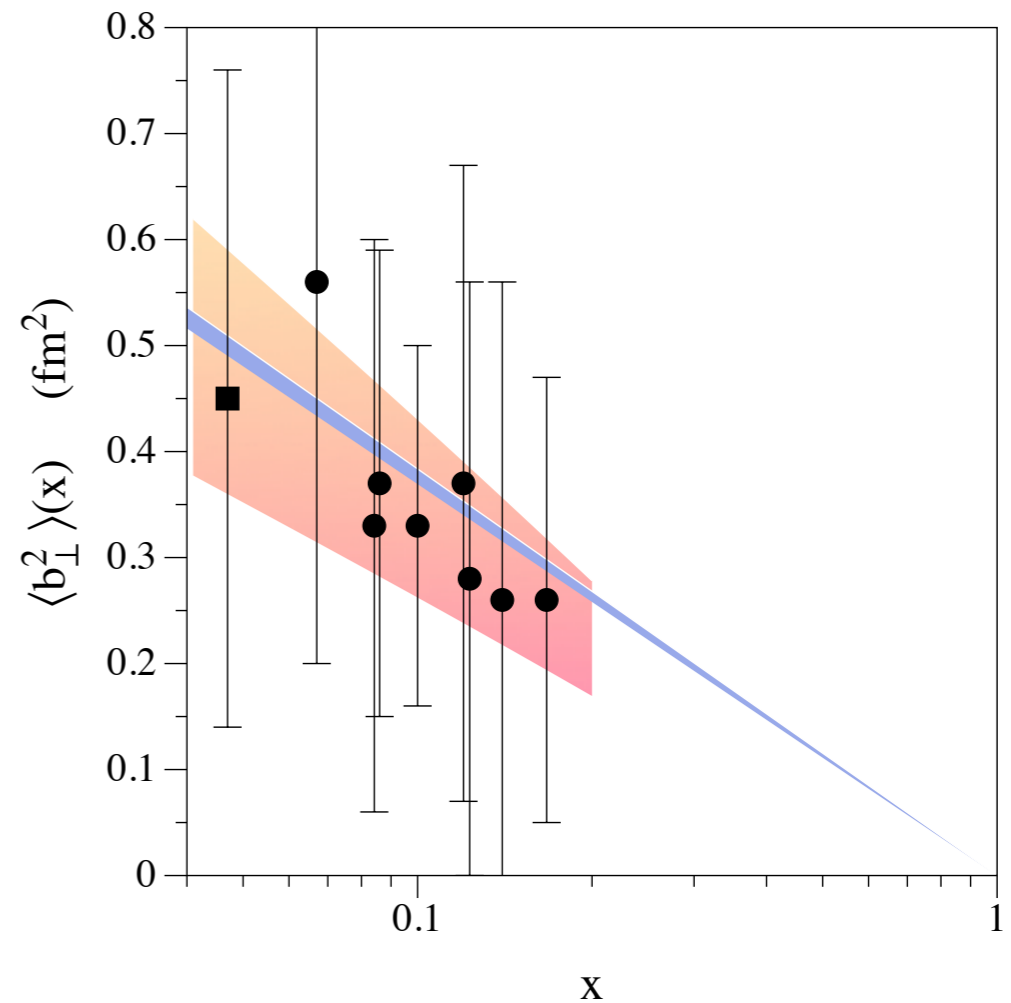


for  $x < 0.15$ :  $B_0/B > 0.9$

Dupré, Guidal, Niccolai, Vdh (2017)

black circles: CFF fit of JLab data

black squares: CFF fit of HERMES data

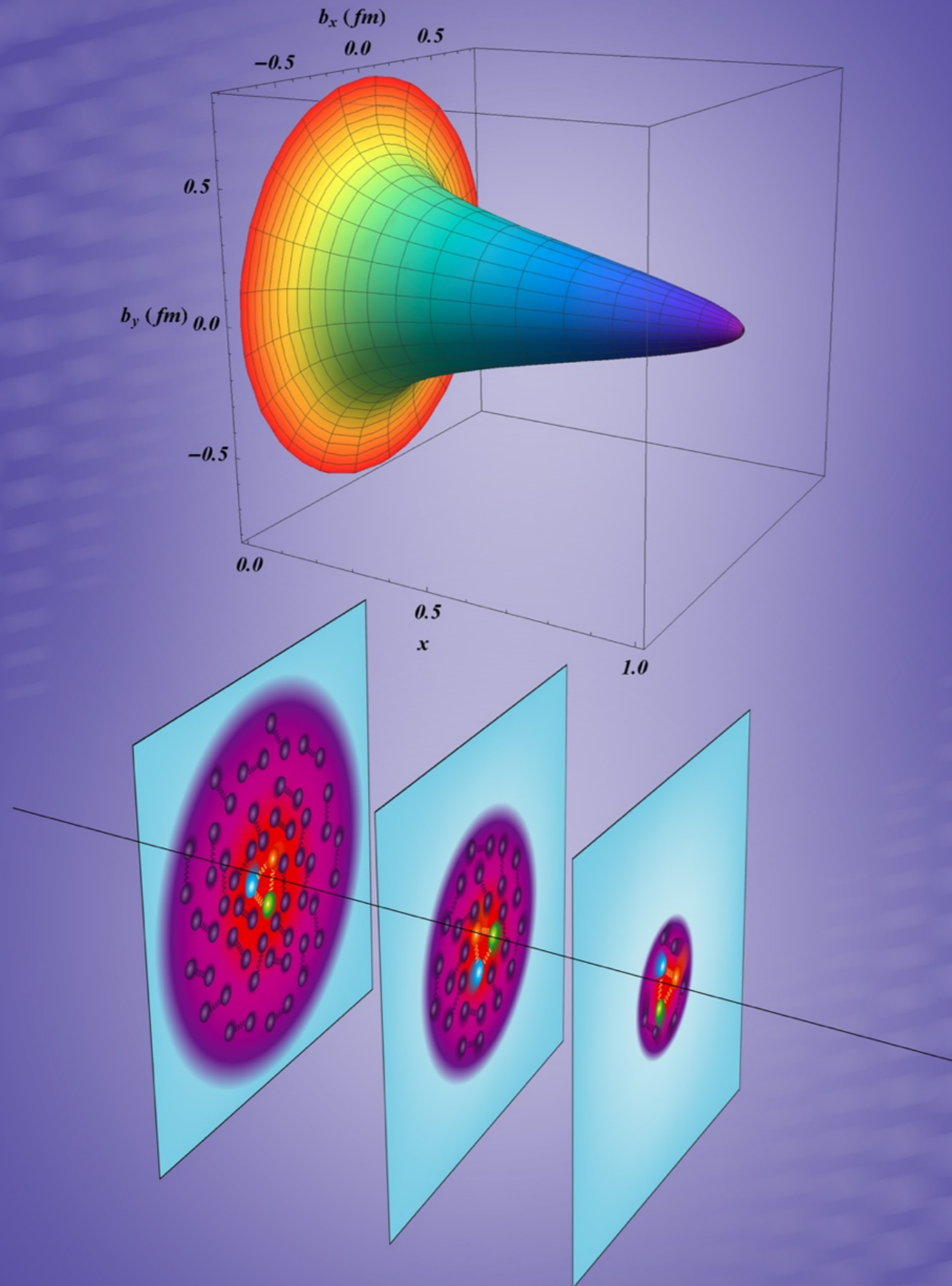


band: using  $B_0(x) = a_{B_0} \ln(1/x)$

$a_{B_0}$  fixed from elastic scattering



# 3D imaging of proton



Dupré, Guidal, Vdh (2017)



# CFF $\mathcal{H}_{Re}$ : dispersion relation formalism

Anikin, Teryaev (2007)

Diehl, Ivanov (2007)

Polyakov, Vdh (2008)

Kumericki-Passek, Mueller, Passek (2008)

Goldstein, Liuti (2009)

Guidal, Moutarde, Vdh (2013)

➔ once-subtracted fixed-t dispersion relation

$$\mathcal{H}_{Re}(\xi, t) = -\Delta(t) + \mathcal{P} \int_0^1 dx H_+(x, x, t) \left[ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

$\xi$ -independent  
subtraction function

known from CFF  
 $\mathcal{H}_{Im}(x, t)$

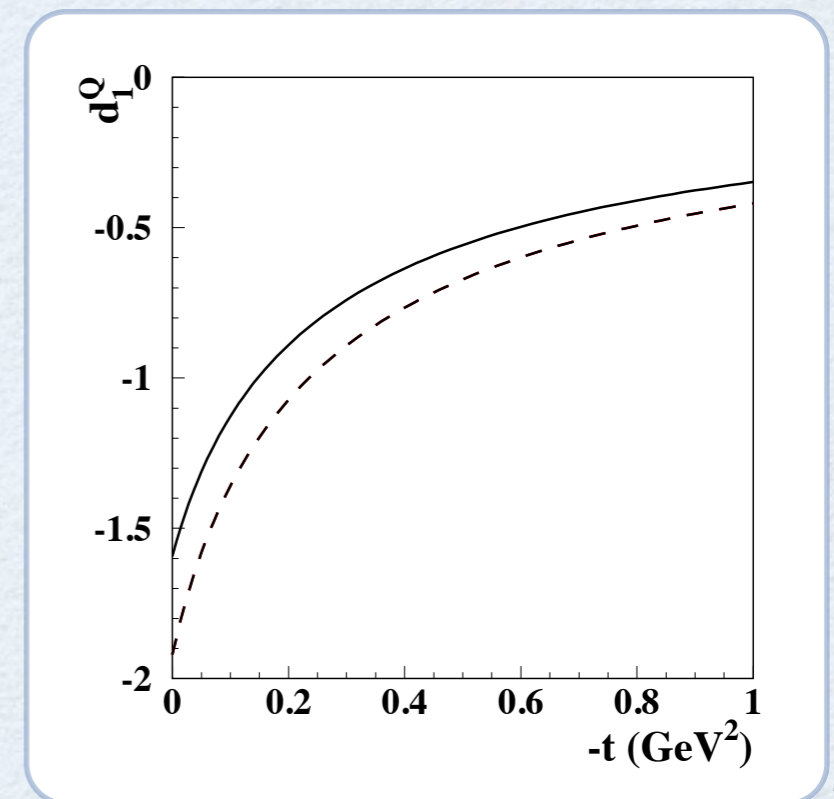
Pasquini, Polyakov, Vdh (2014)

$$\Delta(t) \equiv \frac{2}{N_f} \int_{-1}^1 dz \frac{D(z, t)}{1 - z}$$

D-term

$$D(z, t) = (1 - z^2) \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} d_n(t) C_n^{3/2}(z)$$

Polyakov, Weiss (1999)

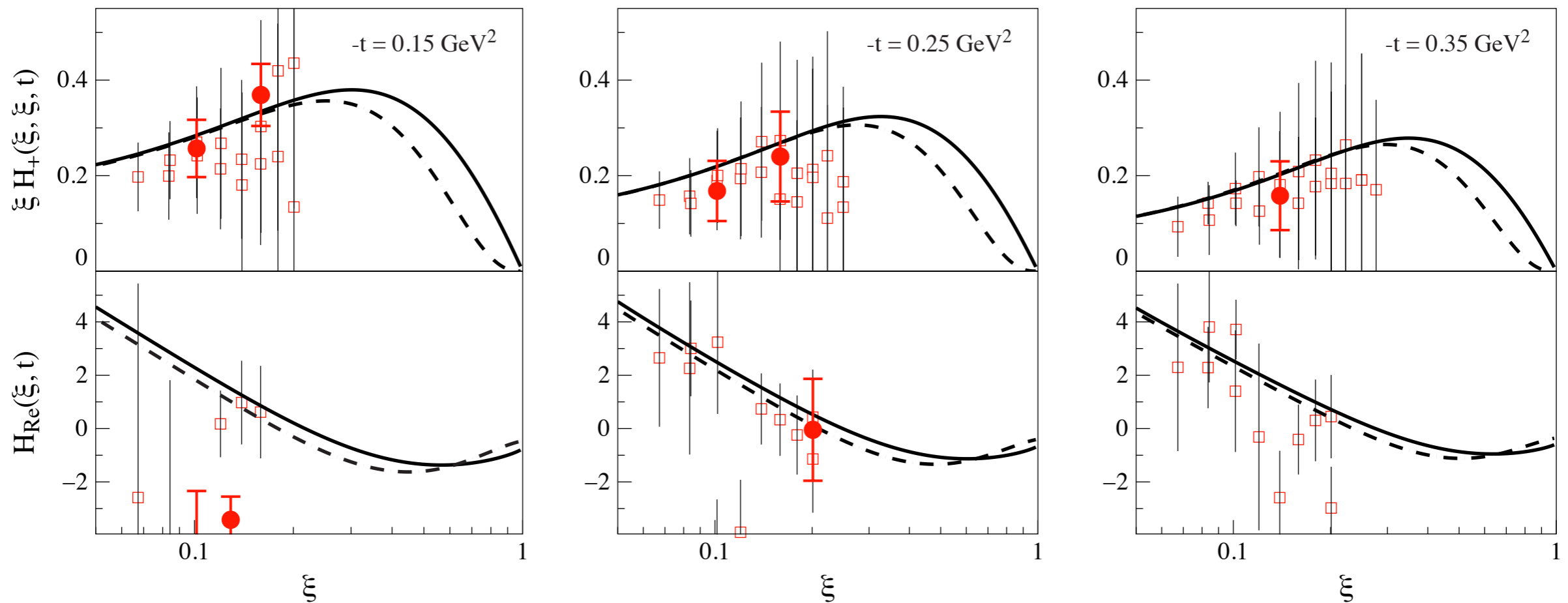




# experimental strategy for CFF $\mathcal{H}_{\text{Re}}$ : direct extraction vs dispersion formalism

red solid circles: CLAS:  $\sigma$ ,  $A_{\text{LU}}$ ,  $A_{\text{UL}}$ ,  $A_{\text{LL}}$

red open squares: CLAS:  $\sigma$ ,  $A_{\text{LU}}$



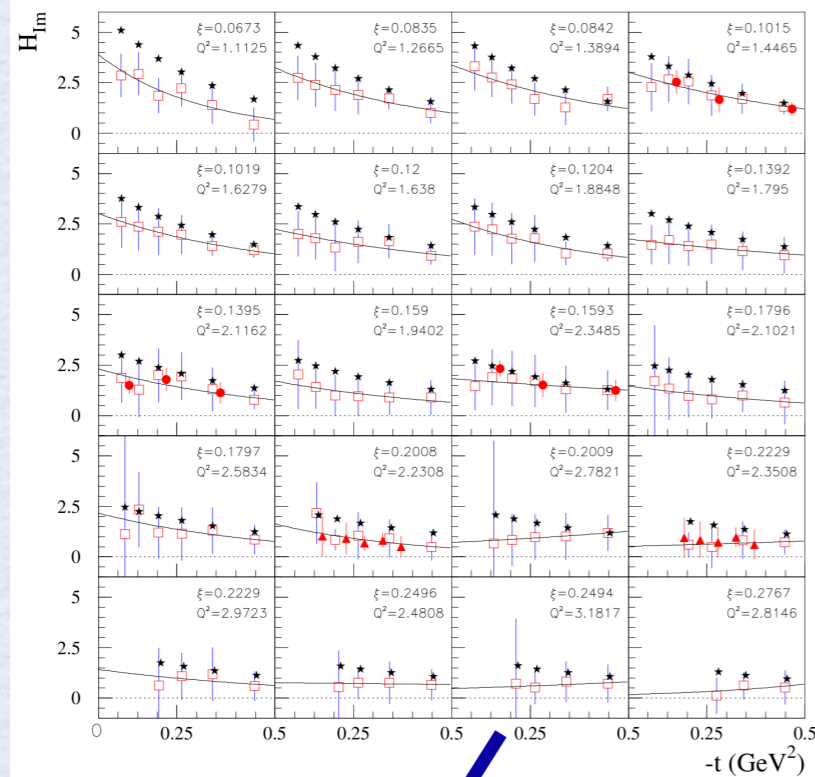
Curves for  $\Delta(t) = 0$ ;  $\Delta(t) < 0$  would shift  $H_{\text{Re}}$  curves up !

Dupré, Guidal, Niccolai, Vdh (2017)

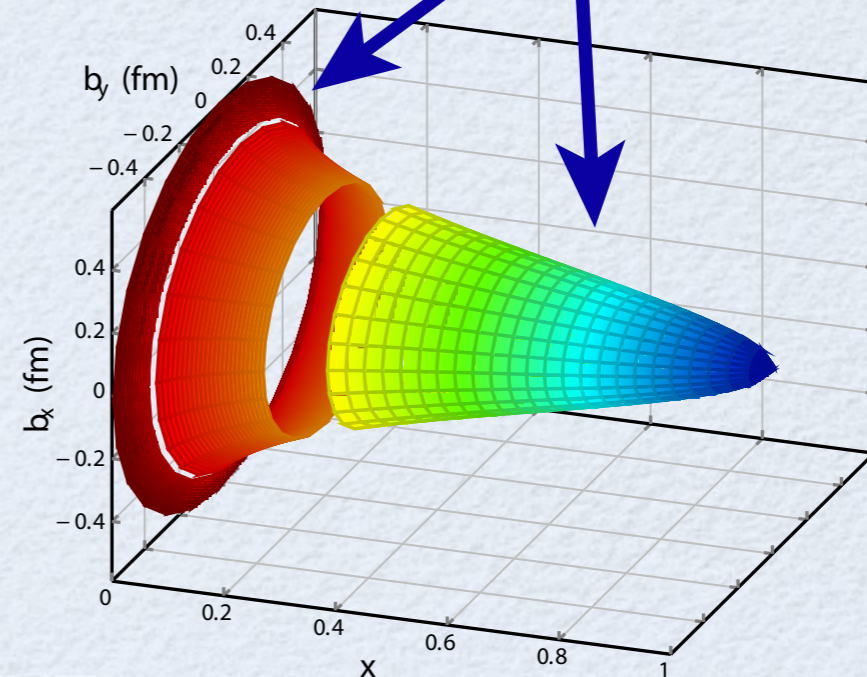
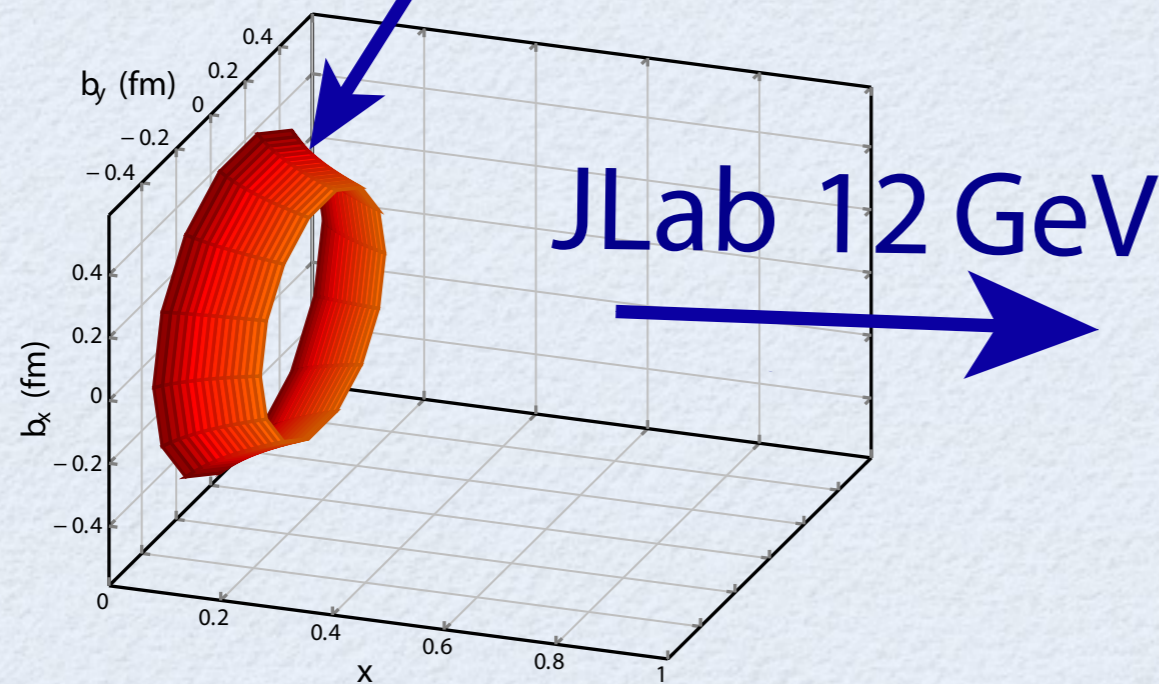
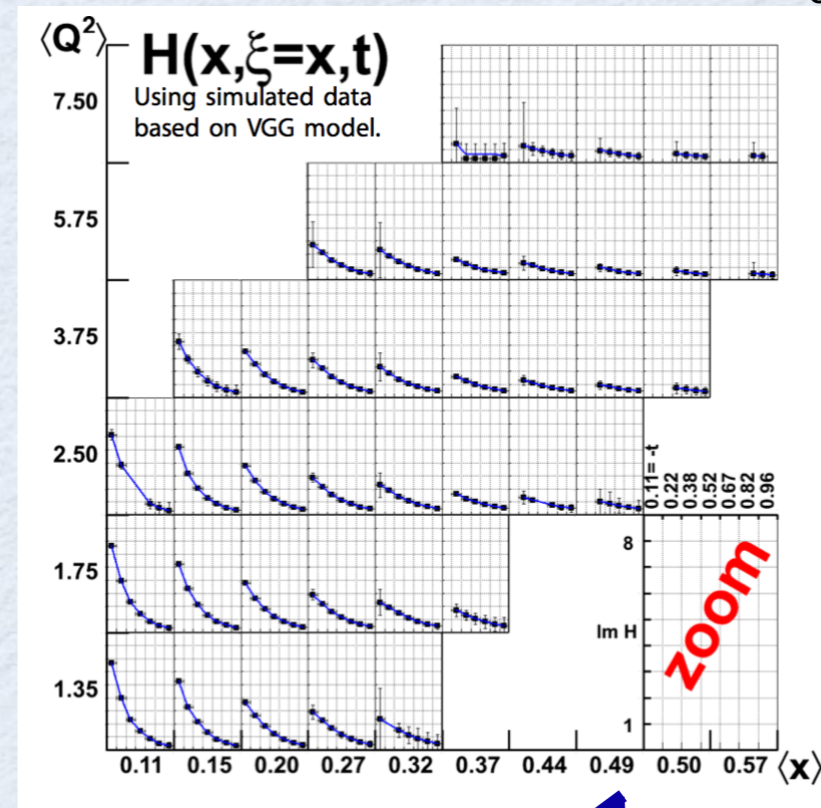


# Projections for CFFs at JLab 12 GeV

Düpré-Guidal-Vanderhaeghen-PRD **95** 011501 (R) (2017)



CLAS12 projections E12-06-119 with DVCS  $A_{UL}$  and  $A_{LU}$



JLab 12 GeV

courtesy of Z.E. Meziani



# Outlook

- ➔ elastic / transition FFs have allowed to get a first glimpse at the spatial distributions of quarks in nucleons
- ➔ GPDs allow for a proton imaging in longitudinal momentum and transverse position
- ➔ global analysis of JLab 6 GeV data have shown a proof of principle of such 3D imaging (tools available: fitters, dispersive analyses)
- ➔ systematic 3D imaging is possible now: COMPASS, JLab 12 GeV,...EIC

