Analysis of generalized distribution amplitudes

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S. Kumano, Qin-Tao Song and O. Teryaev, KEK-TH-1959, J-PARC-TH-0086, research in progress.

Outline

Generalized distribution amplitude (GDA) **of pion**

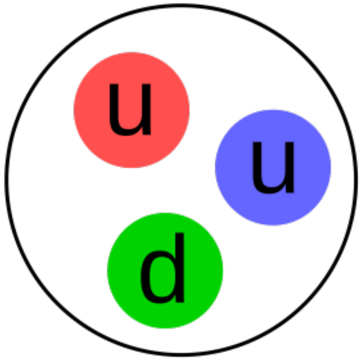
- Motivation
- GDA in two-photon process
- GDA analysis for Belle data

Structure of hadrons: 3D structure

In quark model,
quarks are in S wave

$$\frac{1}{2}(\Delta u_v + \Delta d_v) = \frac{1}{2}$$

$$\Delta L = 0$$



Proton

In patron picture, proton
spin is composed as

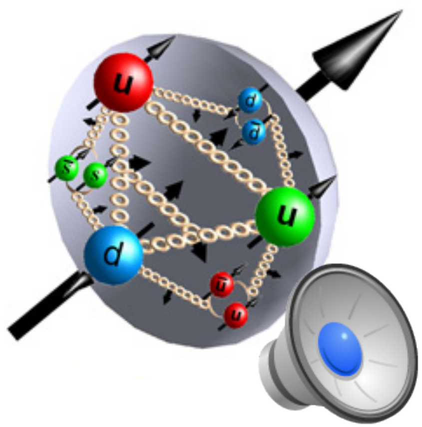
$$\frac{1}{2}(\Delta u^+ + \Delta d^+ + \Delta s^+) + \Delta g + \Delta L = \frac{1}{2}$$

$$q^+ = q + \bar{q}$$

However, recent research
shows

$$\Delta u^+ + \Delta d^+ + \Delta s^+ \approx 0.3$$

$$\Delta g + \Delta L \neq 0$$



Generalized Parton Distributions (GPDs) provide information on ΔL
to solve the proton puzzle!

Generalized Distribution Amplitudes (GDAs) <––> s-t crossing of GPDs
Pion GDAs are investigated.

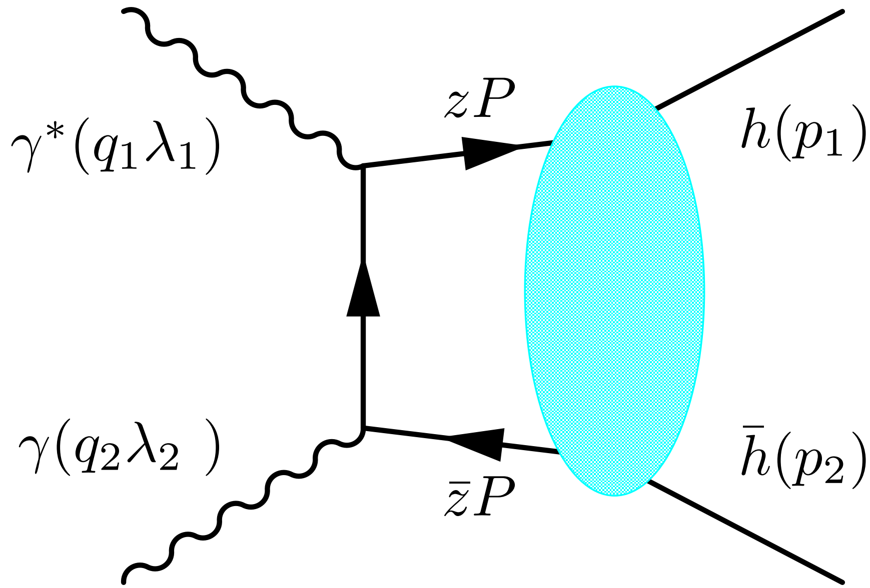
GDA carry many important physical quantities of the hadron, such as distribution amplitudes (DAs) and timelike form factors.

Three-dimensional structure functions are investigated in this talk. The generalized parton amplitudes (GDAs) are one type of three-dimensional structure functions, and they are related to the generalized parton distributions (GPDs) by the s-t crossing. Here, s and t are Mandelstam variables. We explain the GDA studies on the pion by using the two-photon process $\gamma^*\gamma \to \pi^0\pi^0$ at KEKB.

Generalized distribution amplitude for pion

Introduction to GDA

In the process **γγ***→h bar{h}, an hard part describing the process **γγ***→q bar{q} with produced collinear and on-shell quark, and a soft part describing the production of the hadron h pair from a q bar{q}.This soft part is called Generalized Distribution Amplitude (GDA).



The process $\gamma^*\gamma \rightarrow$h bar{h}

GDA is an important quantity of hadron, it is defined as

$$\Phi^q\left(z,\xi,W^2\right)=\int\frac{dx^-}{2\pi}e^{-izP^+x}\left\langle h(p)\bar{h}(p')|\bar{q}(x^-)\gamma^+q(0)|0\right\rangle$$

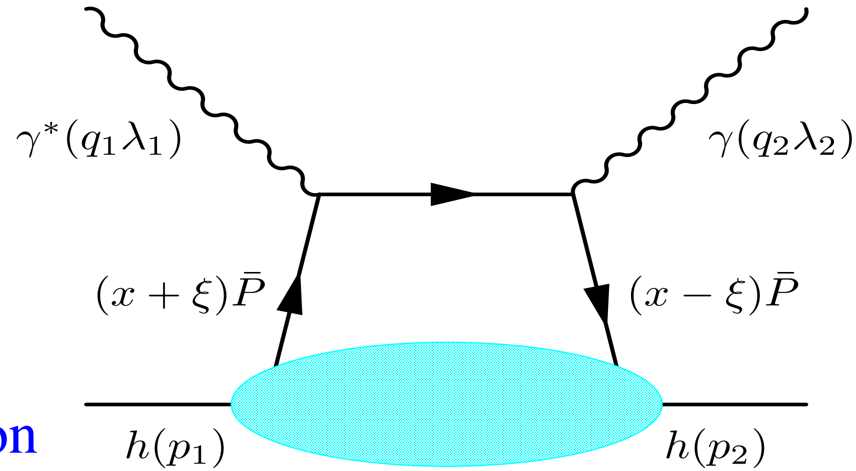
$$z=\frac{k^+}{P^+},\ \xi=\frac{p^+}{P^+},\ \mathrm{s=W}^2=(p+p')^2=P^2$$

M. Diehl, Phys. Rep. 388 (2003), 41.
M. Diehl and P. Kroll, EPJC 73, 2397 (2013).

GDA is closely related to generalized parton distribution (GPD) by the s-t crossing, so GDA could provide another way to obtain GPD information.

$$\Phi^q\left(z,\xi,W^2\right) \leftrightarrow H^q\left(x=\frac{1-2z}{1-2\xi},\varsigma=\frac{1}{1-2\xi},t=W^2\right)$$

GDA GPD



$\gamma^* h \rightarrow \gamma\ h$

GPD can be used to study the proton spin puzzle!

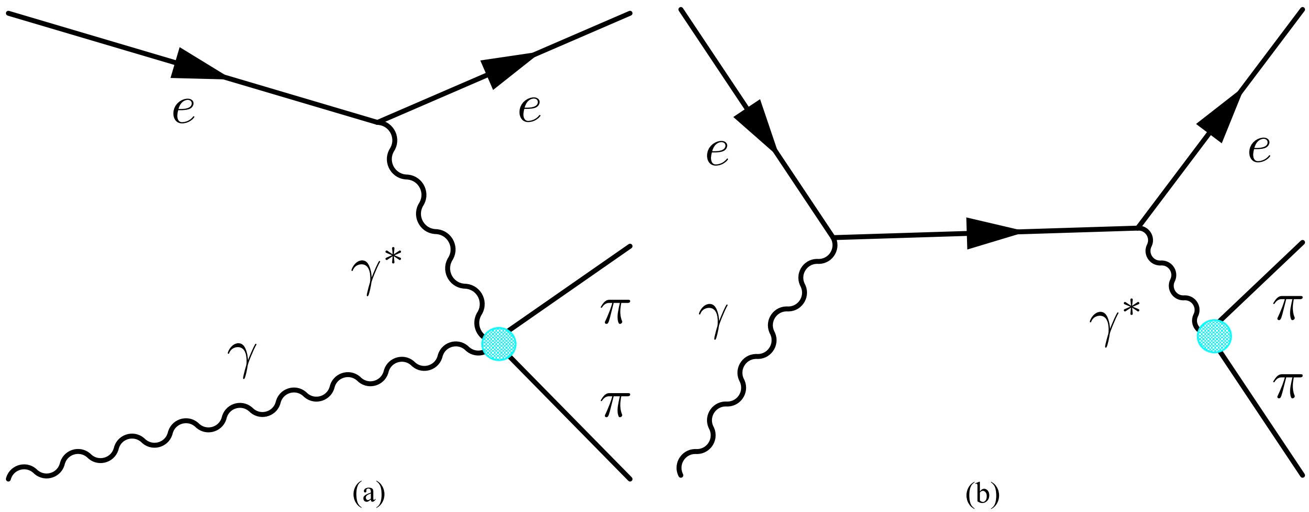
$$\int \frac{dx^-}{2\pi} e^{-iz(\bar{P}^+ x)} \left\langle h(p_2)\,|\,\bar{q}(x^-)\gamma^+ q(0)\,|\,h(p_1)\right\rangle$$

$$= \frac{1}{2\bar{P}^+}\left[H^q(x,\xi,t)\bar{u}(p_2)\gamma^+ u(p_1) + E^q(x,\xi,t)\bar{u}(p_2)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_1)\right]$$

$$\bar{P} = (p_1+p_2)/2,\ \Delta = p_2 - p_1,\ x = \frac{-q_1^2}{2p_1 q_1},\ \xi = \frac{\Delta^+}{p_1^+ + p_2^+}$$

M. Diehl, Phys. Rep. 388 (2003), 41.
H. Kawamura and S. Kumano, PRD 89 (2014), 054007.

GDA in the two-photon process at KEKB



$\gamma\gamma^*$ scattering

Bremsstrahlung

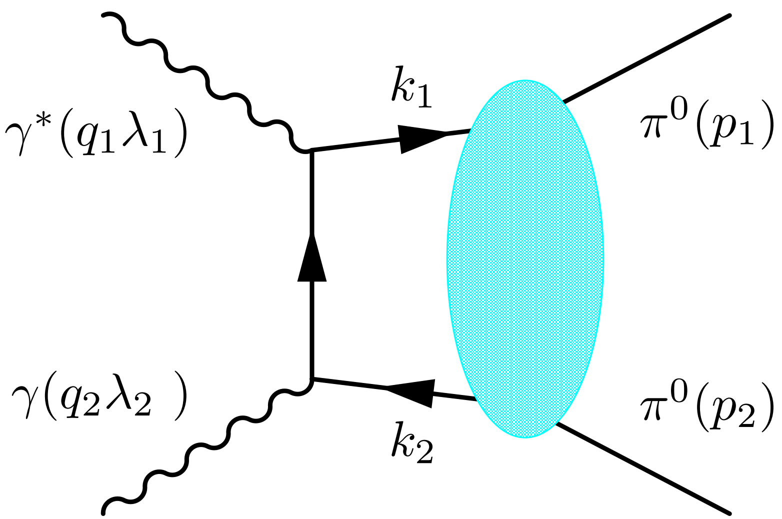
There are two subprocesses for the reaction $e\boldsymbol{\gamma}\to e\pi\pi$. The $\pi\pi$ pair must have C = + for the charge conjugation in the $\boldsymbol{\gamma\gamma}^*$ scattering process, and the GDA determine the amplitude. In the Bremsstrahlung process, only $\pi^+\pi^-$ can be produced due to the negative C parity, the amplitude is expressed by distribution amplitude (DA) or electromagnetic form factor.

DA definition: $\phi(z)=\frac{i}{f_\pi}\int\frac{dx^-}{2\pi}e^{-iz(P^+x^-)}\langle\pi(p_1)|\bar{q}(x^-)\gamma^+\gamma^5 q(0)|0\rangle$

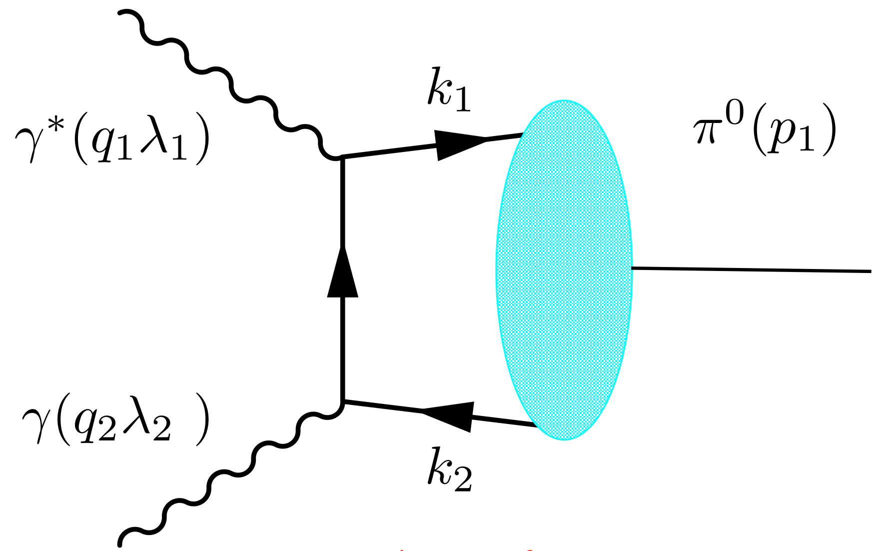
M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301
Belle Collaboration,PRD 93, 032003 (2016)

GDA in the two-photon process

Compare $\gamma^*\gamma \to \pi^0\pi^0$ with $\gamma^*\gamma \to \pi^0$



$\gamma^*\gamma\to\pi^0\pi^0$



$\gamma^*\gamma\to\pi^0$

The hard part of $\gamma^*\gamma\to\pi^0\pi^0$ is the same with that of $\gamma^*\gamma\to\pi^0$. The soft part of of $\gamma^*\gamma\to\pi^0\pi^0$ involves GDA by the vector current. However, the soft part of latter one is the distribution amplitude (DA) of pion by axial vector current. This difference comes from the parity invariance. In the process $\gamma^*\gamma\to\pi^0$, the amplitude is also called the transition form factor $F_{\gamma\gamma\to\pi}(Q^2)$, which can be expressed by the pion DA at high energy.

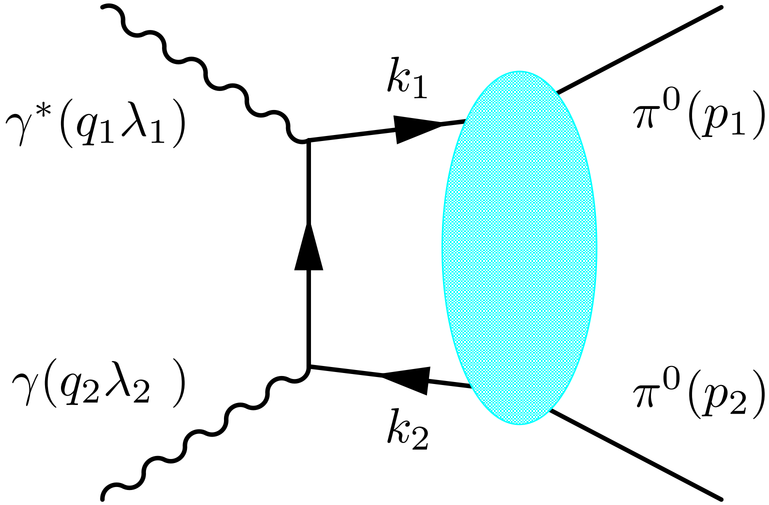
DA definition:

$$\phi(z)=\frac{i}{f_\pi}\int\frac{dx^-}{2\pi}e^{-iz(P^+x^-)}\langle\pi(p_1)|\bar{q}(x^-)\gamma^+\gamma^5q(0)|0\rangle$$

The cross section of process $\gamma^*\gamma \to \pi^0\pi^0$

$$d\sigma = \frac{1}{4}\frac{1}{4\sqrt{(q_1q_2)^2 - q_1^2q_2^2}}\sum_{\lambda_1\lambda_2}|-iT_{\mu\nu}\varepsilon^{\mu}(q_1)\varepsilon^{\nu}(q_2)|^2 d\Phi_2$$

$$d\sigma = \frac{\pi\alpha^2\sqrt{1-\frac{4m^2}{s}}}{4(Q^2+s)}|A_{++}|^2 \sin\theta d\theta$$



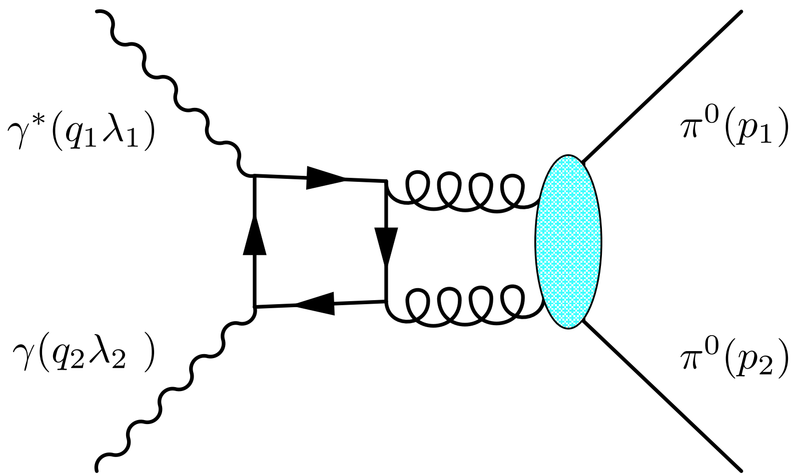
$A_{\lambda1\lambda2}$ is the helicity amplitude, and there are 3 independent helicity amplitudes, they are A_{++} ,A_{0+} and A_{+-}. The leading-twist amplitude A_{++} has a close relation with the generalized distribution amplitude (GDA) $\Phi^q(z, \xi, W^2)$.

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\varepsilon^{\mu}(\lambda_1)\varepsilon^{\nu}(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi^q\left(z,\xi,W^2\right)$$

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL 81 (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.

Higher-twist contribution A_{0+} requires a helicity flip along the fermion line, and it decreases as 1/Q. Higher-order contribution A_{+-} contributes with the GDA of gluon, since A_{+-} indicates the angular momentum $L_z = 2$ .Therefore A_{+-} is suppressed by running coupling constant $\alpha_{s.}$



Gluon GDA

M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.
N. Kivel, L. Mankiewicz and M.V. Polyakov PLB 467 (1999) 263.

GDA expression

Generalized distribution amplitude (GDA) follows the ERBL (Efremov-Radyushkin-Brodsky-Lepage) evolution as the distribution amplitude (DA). We can get the general expression of GDA from the ERBL evolution equation.

$$\sum_q \Phi_q^+\left(z,\xi,W^2;Q^2\right) = z(1-z)\sum_{odd\ \mathrm{n}}[6n_f \sum_{even\ \mathrm{l}}^{n+1} B_{nl}(W;Q^2)P_l(2\xi-1)]C_n^{3/2}(2z-1)$$

$$B_{nl}(W;Q^2) = B_{nl}^+(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)})^{k_n^+} + (\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)})^{k_n^-}$$

where P_l is the Legendre polynomial and $C_n{}^a$ is the Gegenbauer polynomial.

At very high energy Q^2, only the terms of n=1 in the above expression will survive.

$$\sum_q \Phi_q^+\left(z,\xi,W^2\right) = 18n_f z(1-z)(2z-1)[B_{10}(W)+B_{12}(W)P_2(2\xi-1)]$$

$$= 18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

This is the asymptotic form of the GDA.

In the two-photon process, only C even GDAs Φ^+ are needed, and there is a sum rule for Φ^+. In this case, GDAs are related to the energy-momentum form factor in the timelike region.

$$\int dz(2z-1)\Phi_q^+\left(z,\xi,W^2\right)=\frac{1}{2(P^+)^2}\left\langle\pi^+(p_1)\pi^-(p_2)\right|T_q^{++}(0)\left|0\right\rangle$$

$$=[A(W^2)(\xi-\frac{1}{2})^2-B(W^2)]$$

where the energy-momentum form factor for quarks is defined as

$$\left\langle\pi^+(p_1)\pi^-(p_2)\right|T_q^{\mu\nu}(0)\left|0\right\rangle=2[A(W^2)\frac{q^\mu q^\nu}{4}+B(W^2)(W^2 g^{\mu\nu}-P^\mu P^\nu)]$$

$P=p_1+p_2$, $q=p_1-p_2$

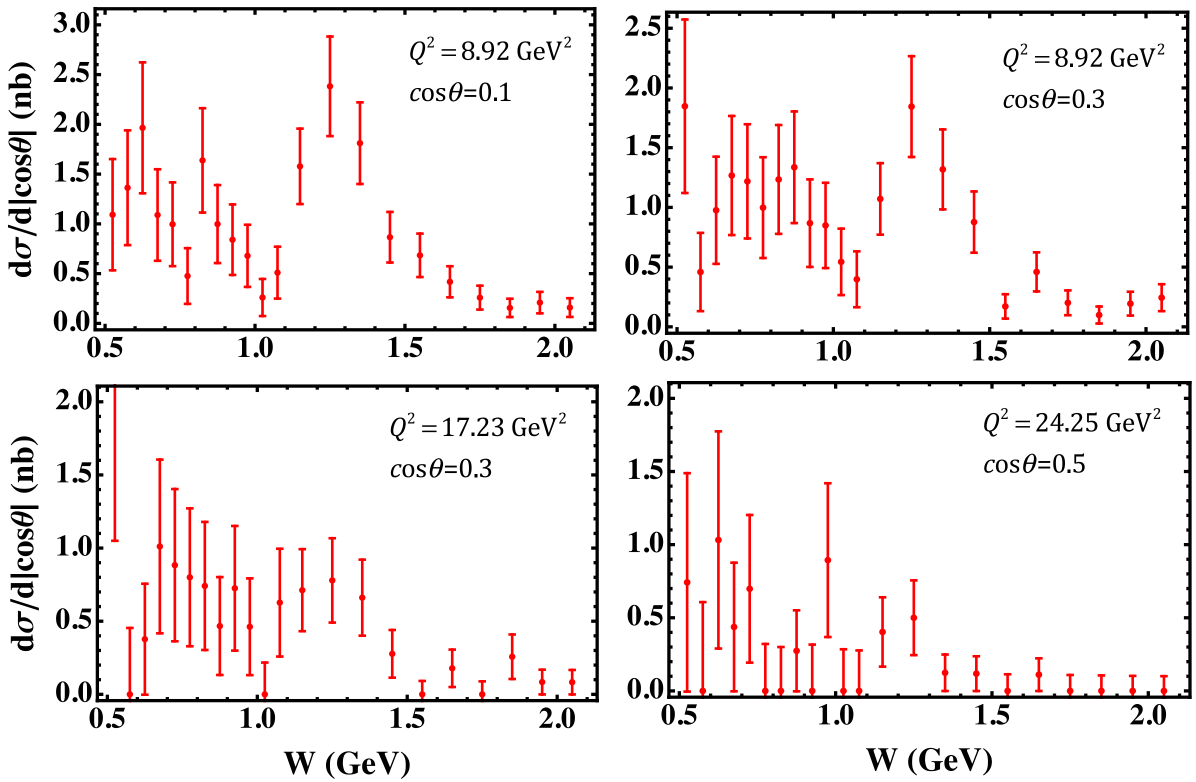
By using this sum rule we can obtain

$$B_{12}(0)=\frac{5R_\pi}{9}$$

where R_π is the momentum fraction carried by quarks in the pion.

M. V. Polyakov, NPB **555** (1999) 231.
M. V. Polyakov and C. Weiss PRD 60 (1999) 114017.

In 2016, the Belle Collaboration released the measurement of differential cross section for $\gamma^{*}\gamma \to \pi^0\pi^0$. The GDAs can be obtained by analyzing the Belle data.



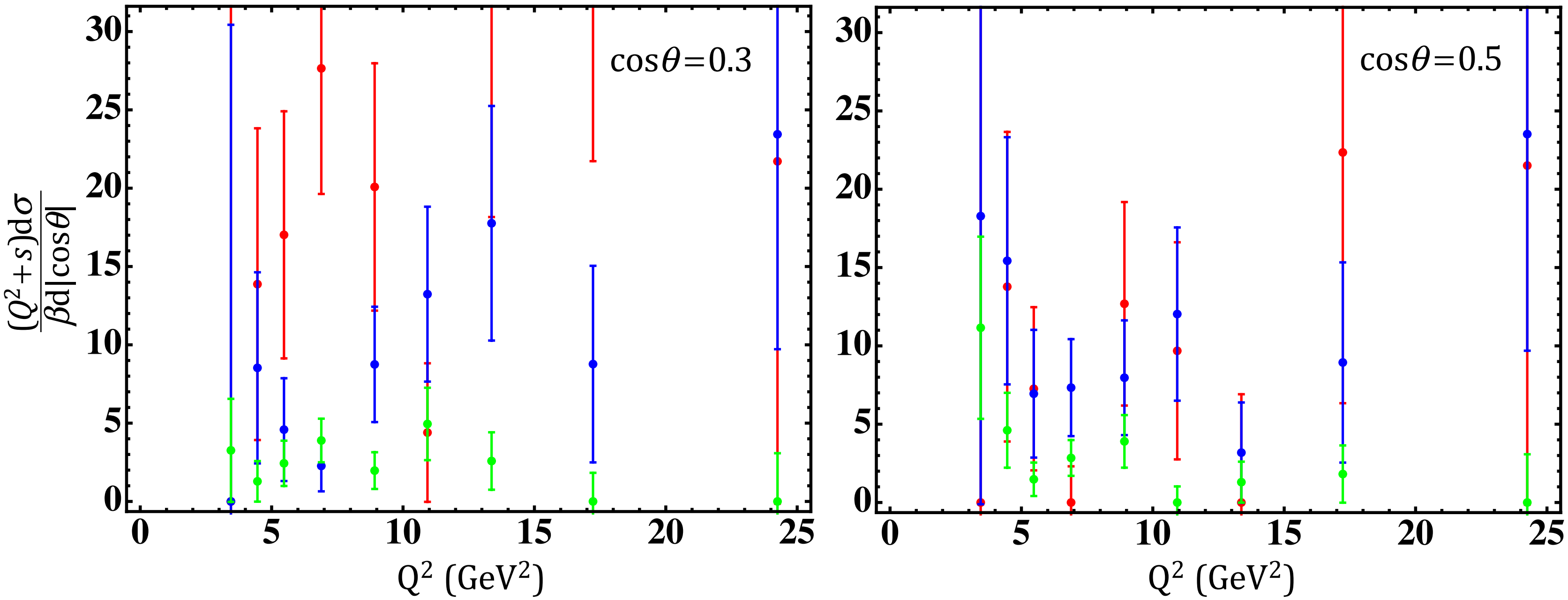
Differential cross section for $\gamma^{*}\gamma \to \pi^0\pi^0$

In these figures, the resonance $f_2(1270)$ is clearly seen around W = 1.25 GeV, however, other resonances are not clearly seen due to the large errors.

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.

Scale violation of GDA based on Belle data

$$\frac{(Q^2+s)d\sigma}{\beta d|\cos\theta|} \propto \left|\Phi^{\pi^0\pi^0}(z,\,\cos\theta,\,W,\,Q)\right|^2$$



The scale dependence of the Belle data. We have red color for W = 0. 525 GeV, blue color for W = 0. 975 GeV, and green color for W = 1. 55 GeV.

The scaling violation of the GDAs is not so obvious in the Belle data on account of the large errors, so that the Q^2-independent GDAs could be used in analyzing the Belle data.

Q^2-independent (asymptotic form) GDAs

$$\sum_q \Phi_q^+\left(z,\xi,W^2\right)=18n_f z(1-z)(2z-1)[B_{10}(W)+B_{12}(W)P_2(2\xi-1)]$$
$$=18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W)=\overline{B}_{10}(W)e^{i\delta_0},\tilde{B}_{12}(W)=\overline{B}_{12}(W)e^{i\delta_2}$$

We will try to find a reasonable expression for GDAs to analyze the Belle data, and there are some initial conditions for $B_{12}(0)$ and $B_{10}(0)$ we need to consider.

$$B_{12}(0)=-B_{10}(0)=\frac{5R_\pi}{9}$$

where R_π=0.5 is the momentum fraction carried by quarks in the pion.

M. V. Polyakov, NPB **555** (1999) 231.

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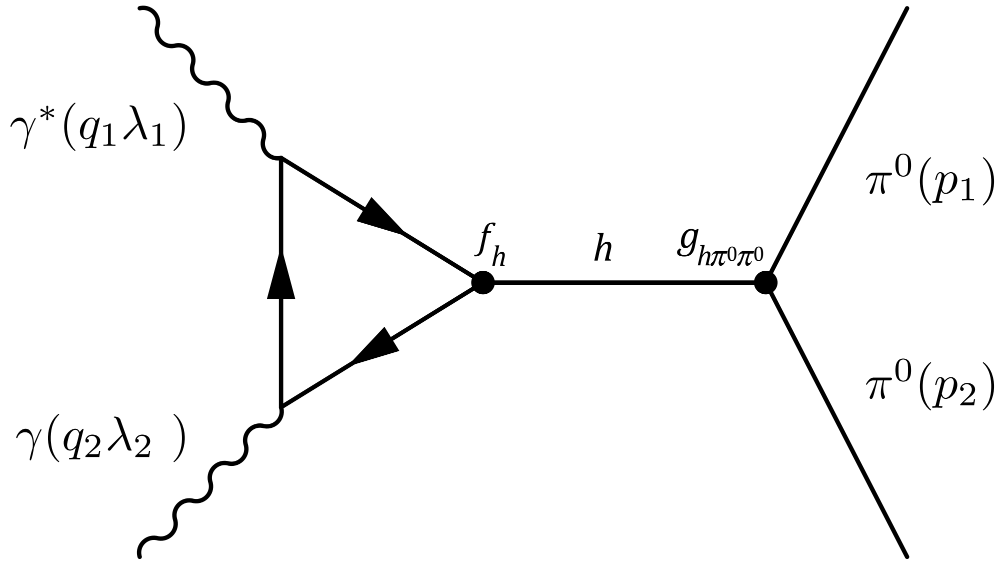
Resonance effects

In the process $\boldsymbol{\gamma}^{*}\boldsymbol{\gamma}\rightarrow\pi^0\pi^0$, the $\pi^0\pi^0$ can be produced through intermediate meson state h. The q bar{q}→h amplitude should be proportional to the decay constant f_h or the distribution amplitude (DA), and the h→$\pi^0\pi^0$ amplitude can be expressed by the coupling constant $g_{h\pi\pi}$. These resonance contributions read

$$\bar{B}_{12}(W)=\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M^3_{f_2}\Gamma_{f_2}}{9\sqrt{(M^2_{f_2}-W^2)^2-\Gamma^2_{f_2}M^2_{f_2}}}$$

$$\bar{B}_{10}(W)=\frac{5g_{f_2\pi\pi}f_{f_2}M^3_{f_2}\Gamma_{f_2}}{3\sqrt{(M^2_{f_0}-W^2)^2-\Gamma^2_{f_0}M^2_{f_0}}}$$



The resonance effects play an important role in the resonance regions.

Phase shifts δ_0 and δ_2

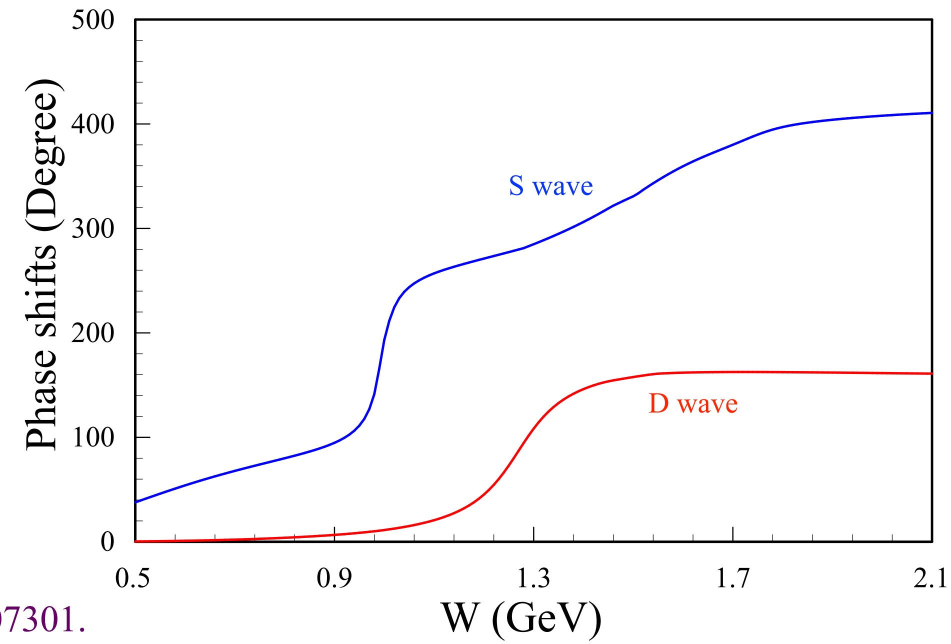
$$\sum_q \Phi_q^+\left(z,\xi,W^2\right)=18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W)=\bar{B}_{10}(W)e^{i\delta_0},\tilde{B}_{12}(W)=\bar{B}_{12}(W)e^{i\delta_2}$$

In the above equation δ_0 and δ_2 and are the ππ elastic scattering phase shifts in the isospin=0 channel (see the figure). Above the KK threshold, we parameterize the relative phase δ_0-δ_2 as

$$\delta_0(W)-\delta_2(W)=\delta_0(2m_K)-\delta_2(2m_K)+a(W-2m_K)^b \qquad W>2m_K$$

The S wave and D-wave ππ scattering phase shifts.



M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.
P. Bydzovsky, R. Kamiski and V. Nazari, PRD 90 (2014) , 116005; PRD 94 (2016), 116013.

We adopt a simple expression of GDA to analyze Belle data, here resonance effects of $f_0(500)$ and $f_2(1270)$ are introduced.

$$\Phi_q^+\left(z,\xi,W^2\right)=\frac{-3}{20}N_h z^\alpha(1-z)^\alpha(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

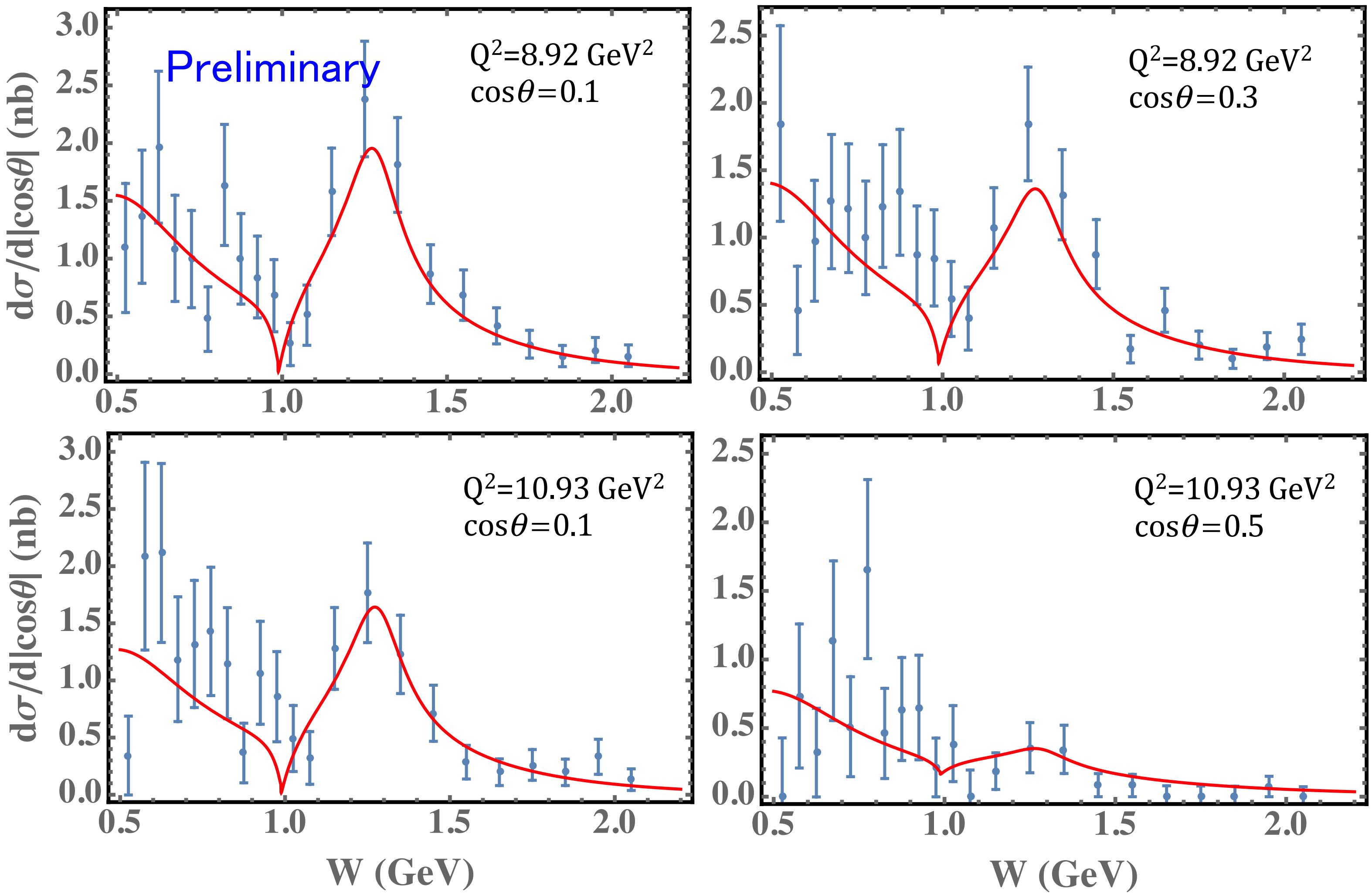
$$\tilde{B}_{10}(W)=[\frac{-3+\beta^2}{2}\frac{5R_\pi}{9}F_h(W^2)+\frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{(M_{f_0}^2-W^2)^2-\Gamma_{f_0}^2M_{f_0}^2}}]e^{i\delta_0}$$

$$\tilde{B}_{12}(W)=[\beta^2\frac{5R_\pi}{9}F_h(W^2)+\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^2}{9\sqrt{(M_{f_2}^2-W^2)^2-\Gamma_{f_2}^2M_{f_2}^2}}]e^{i\delta_2}$$

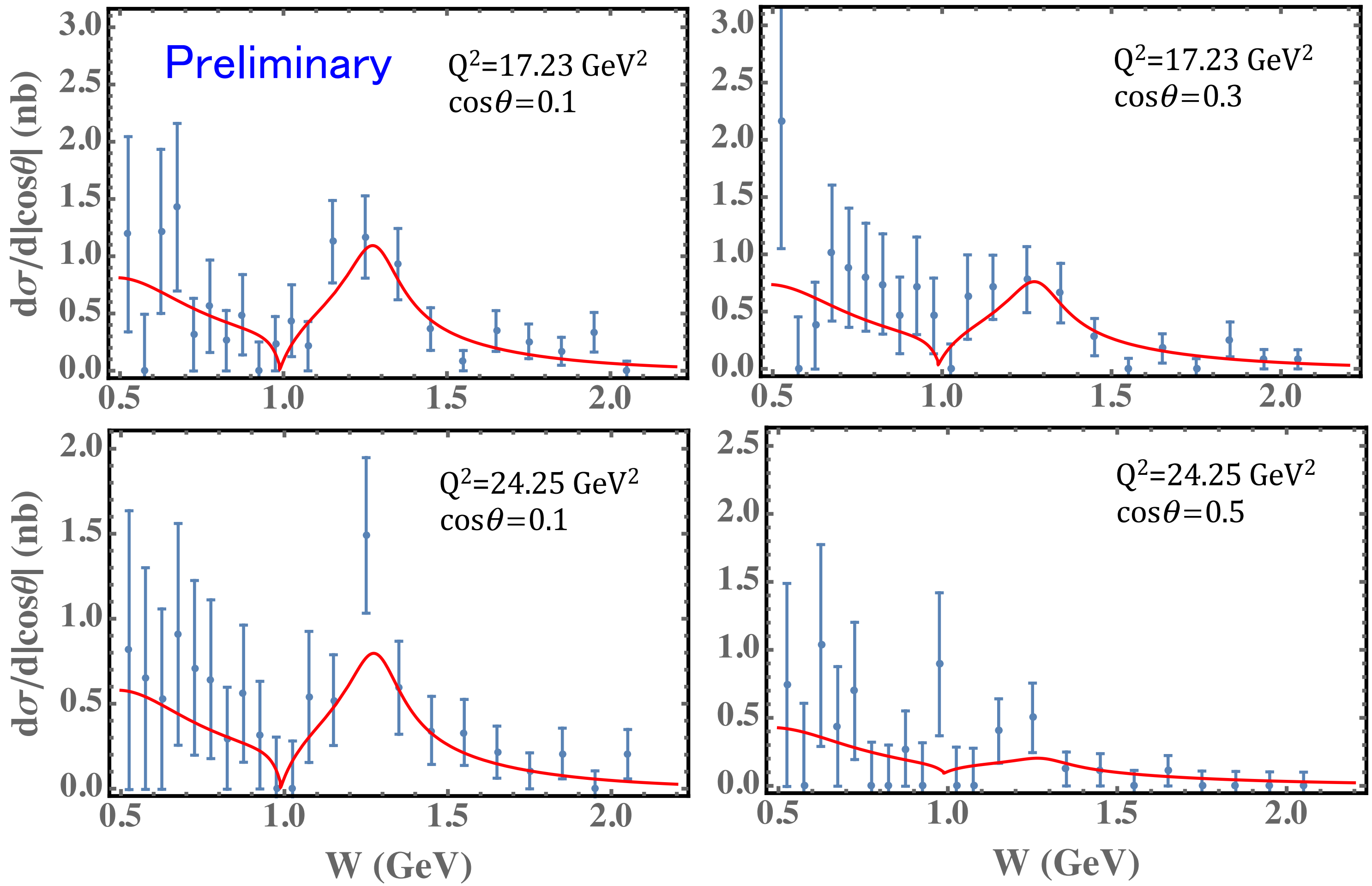
$$F_h(W^2)=\frac{1}{\left[1+\frac{W^2-4m_\pi^2}{\Lambda^2}\right]^{n-1}}$$

The function $F_h(W^2)$ is the form factor of the quark part of the energy-momentum tensor, and the parameter Λ is the momentum cutoff in the form factor. The parameter n is predicted as $n = 2$ at very high energy, because we have $d\sigma/d|\cos\theta|$/~ $1/W^6$ by the counting rule. In the asymptotic limit, $\boldsymbol{\alpha} = 1$.

By analyzing the Belle data, the values of parameters are obtained. $\frac{\chi^2}{NOF} = 0.997$



The W dependence of the differential cross section (in units of nb), and in comparison with Belle data.



The W dependence of the differential cross section (in units of nb), and in comparison with Belle data.

Summary

In this talk, we showed the basic formalism of the GDAs for the cross section of $\boldsymbol{\gamma}^{*}\boldsymbol{\gamma}\rightarrow\pi^0\pi^0$. By analyzing the Belle data the pion GDAs are obtained, and the obtained GDAs can also give a good description of experimental data. For the next step, we try to obtain some important quantities of pion by using GDA.

Thank you very much