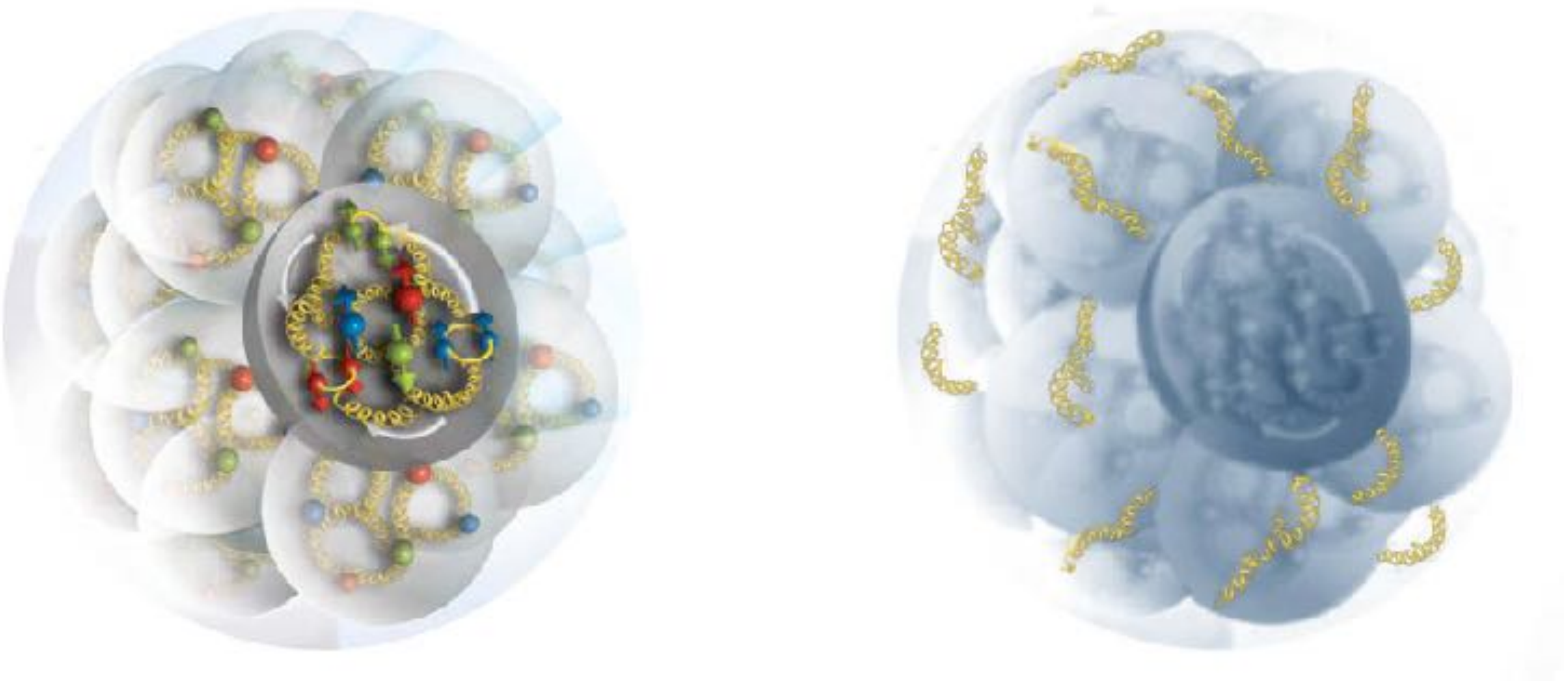
Gluon Structure of Hadrons and Nuclei



Phiala Shanahan
William and Mary
Thomas Jefferson National Accelerator Facility

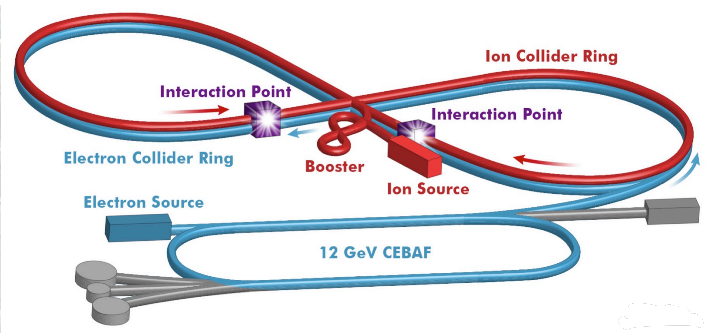
Gluon Structure

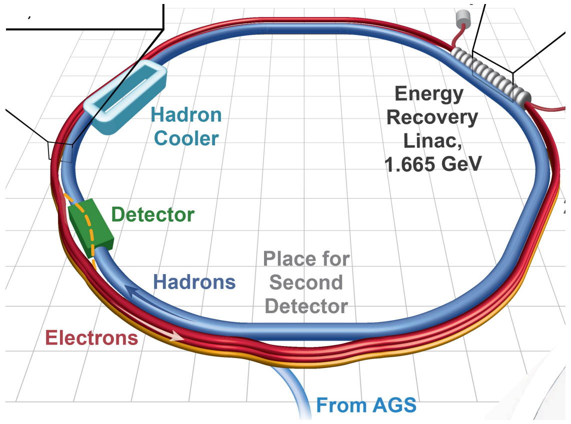
- Past 60+ years: detailed view of quark structure of nucleons
- Gluonic structure (beyond gluon density) relatively unexplored
- Electron-Ion Collider
  - Priority in 2015 nuclear physics long range plan
  - "Understanding the glue that binds us all"
- Insights from Lattice QCD?


Electron Ion Collider:
The Next QCD Frontier

Understanding the glue
that binds us all

Cover image from EIC whitepaper arXiv::1212.1701





Gluon Structure from LQCD

1

How much do gluons contribute to the proton's

- Momentum
- Mass
- Spin

2

What is the 3D gluon distribution of a proton

- PDFs
- TMDs
- GPDs
- 'Gluon radius'

3

How is the gluon structure of a proton modified in a nucleus

- Gluonic 'EMC' effect
- 'Exotic' glue

Nucleon momentum decomposition

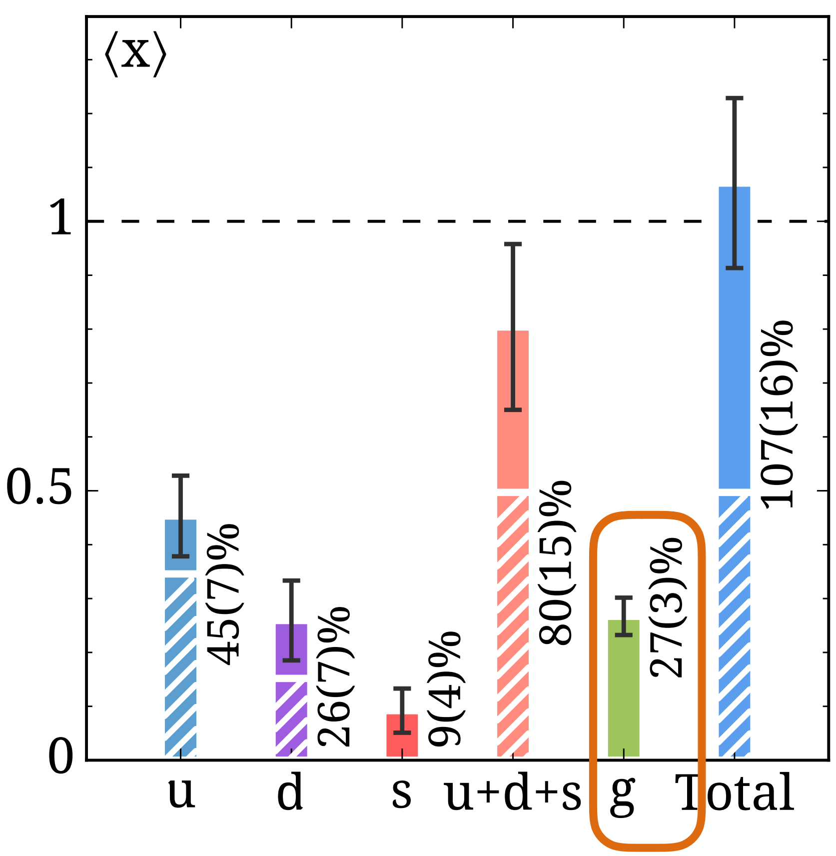
Gluon Momentum fraction

- Two direct calculations at the physical point since last year

C. Alexandrou et al., arXiv:1706.02973

Y.-B. Yang et al., χQCD, in preparation

$\overline{\text{MS}}$-scheme at 2 GeV



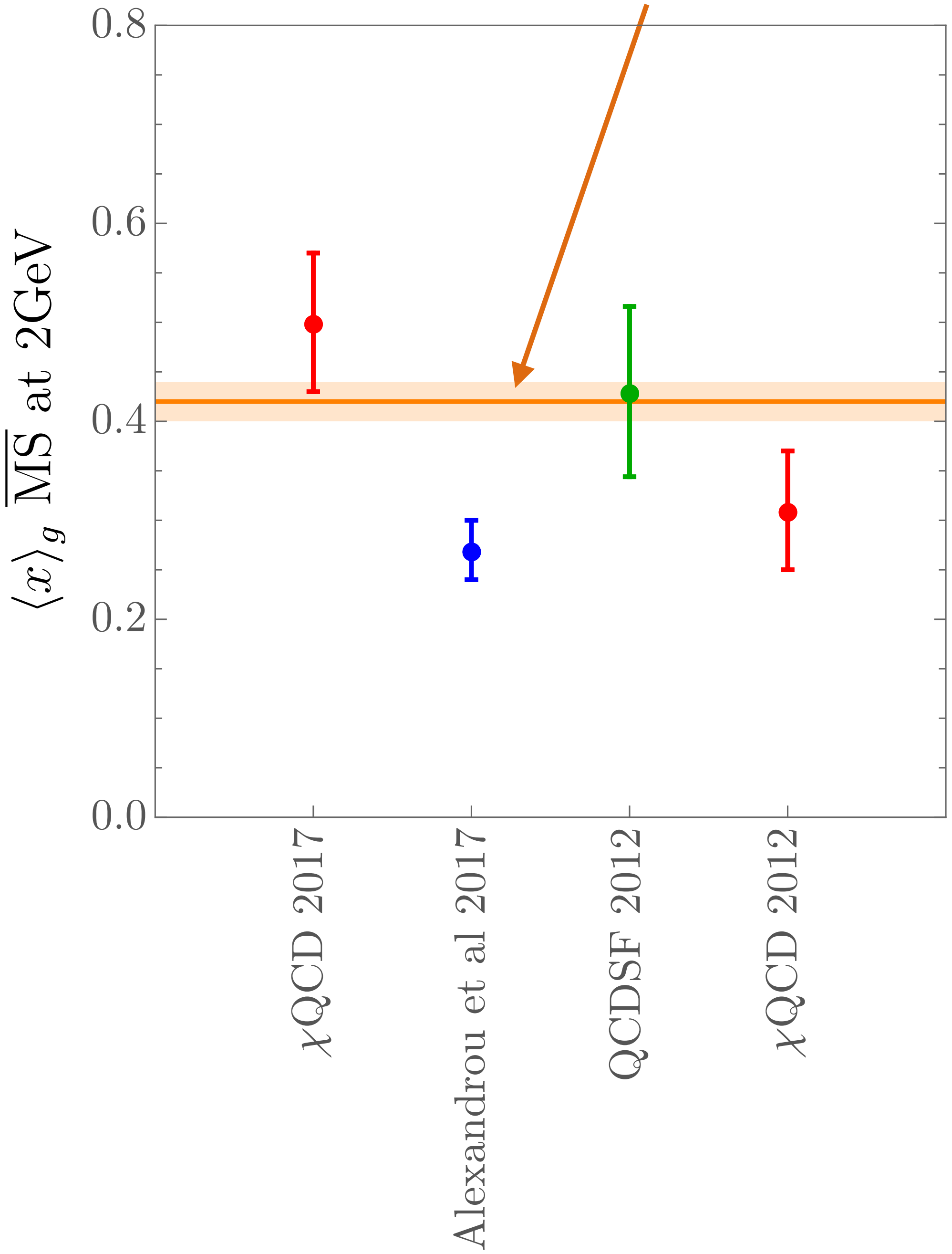
Solid: disconnected

Hashed: connected

gluon momentum fraction

CT14NNLO

S. Dulat et al, PRD 93, 033006 (2016)



Nucleon spin decomposition

Two decompositions of the proton spin:

- **Ji (1996)**

quark orbital
angular momentum

$$J_N = \sum_{q=u,d,s,c\ldots} \left(\frac{1}{2} \Delta\Sigma_q + L_q \right) + \boxed{J_g}$$

gluon spin

quark helicity

- **Jaffe-Manohar (1990)**

$$J_N = \sum_{q=u,d,s,c\ldots} \left(\frac{1}{2} \Delta\Sigma_q + \mathcal{L}_q \right) + \boxed{\Delta g} + \mathcal{L}_g$$

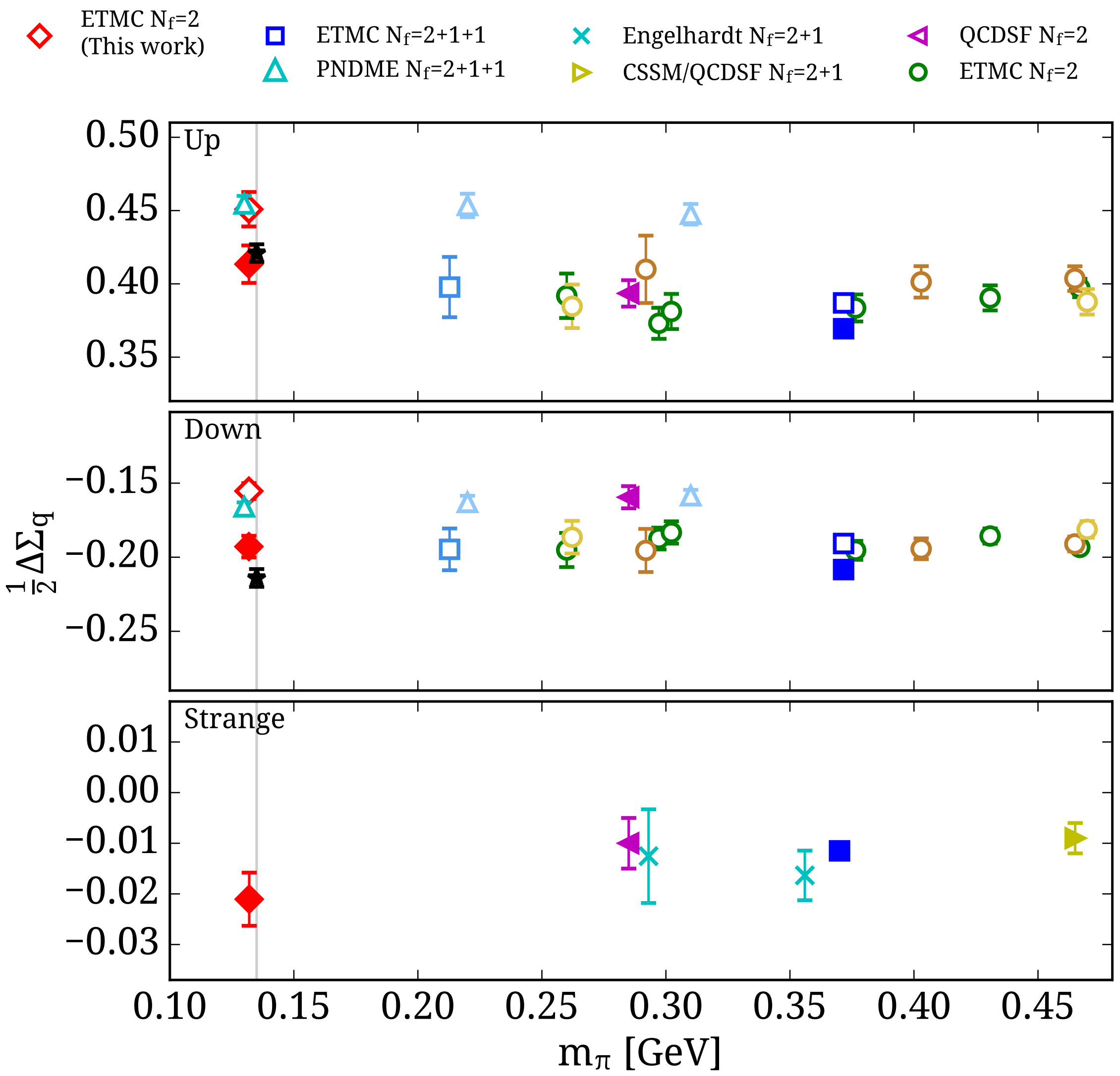
gluon helicity

Interpolation between decompositions: M. Engelhardt, PRD 95 094505 (2017)

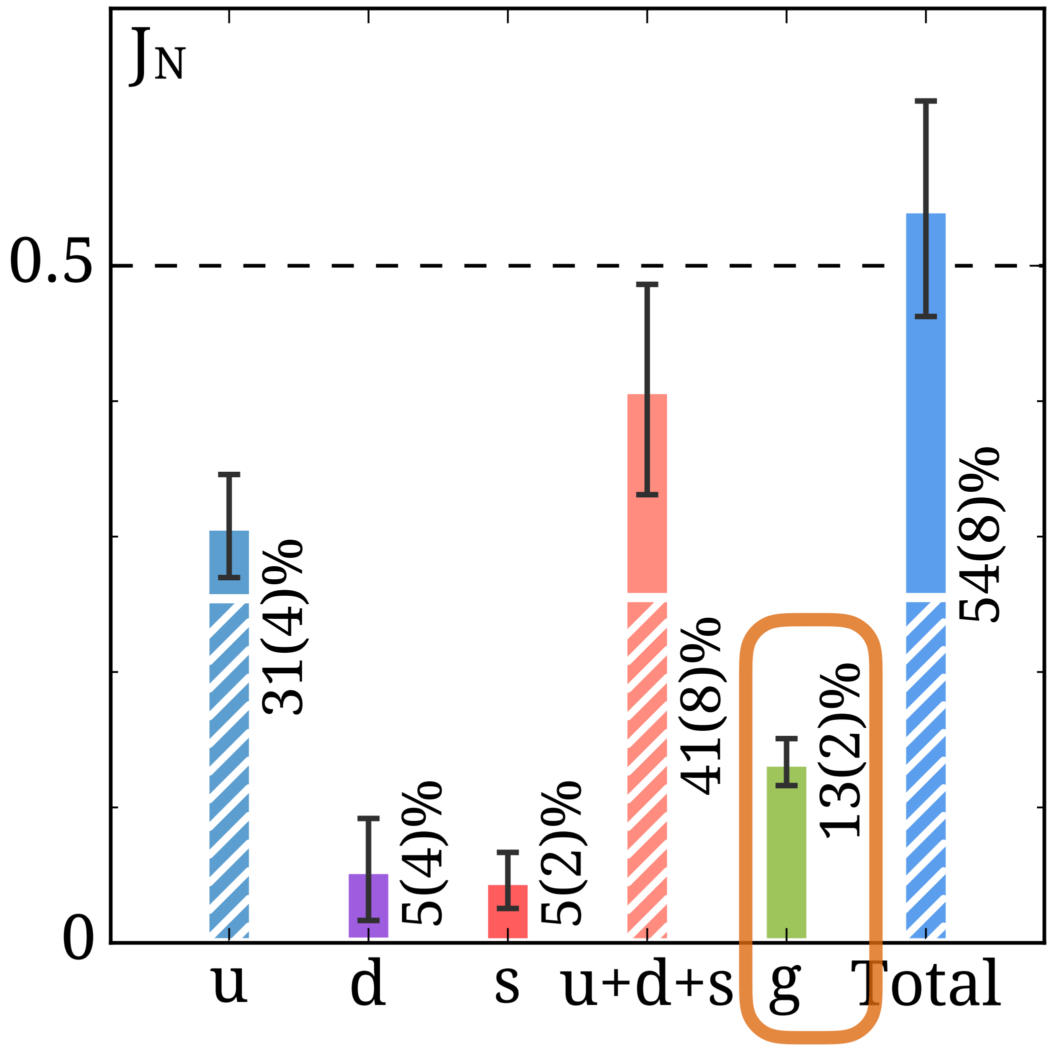
Ji spin decomposition

C. Alexandrou et al., arXiv:1706.02973

- Physical pion mass
- All terms calculated directly



$\overline{\text{MS}}$-scheme at 2 GeV



gluon spin

Hashed: connected
Solid: disconnected

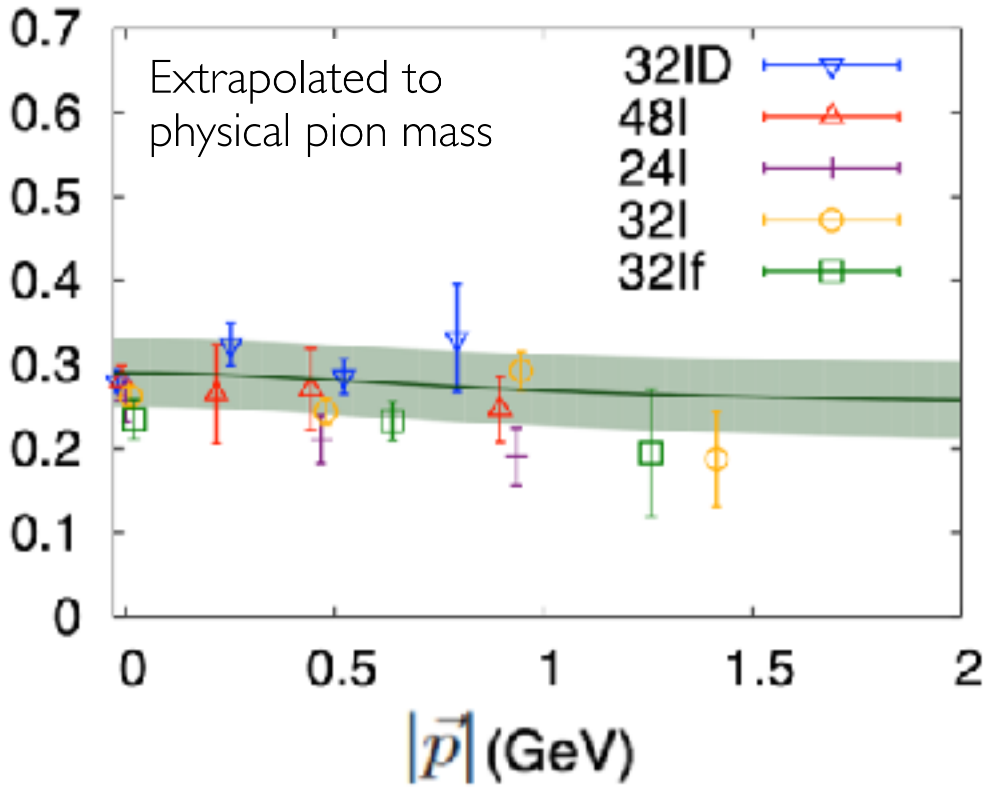
J-M spin decomposition

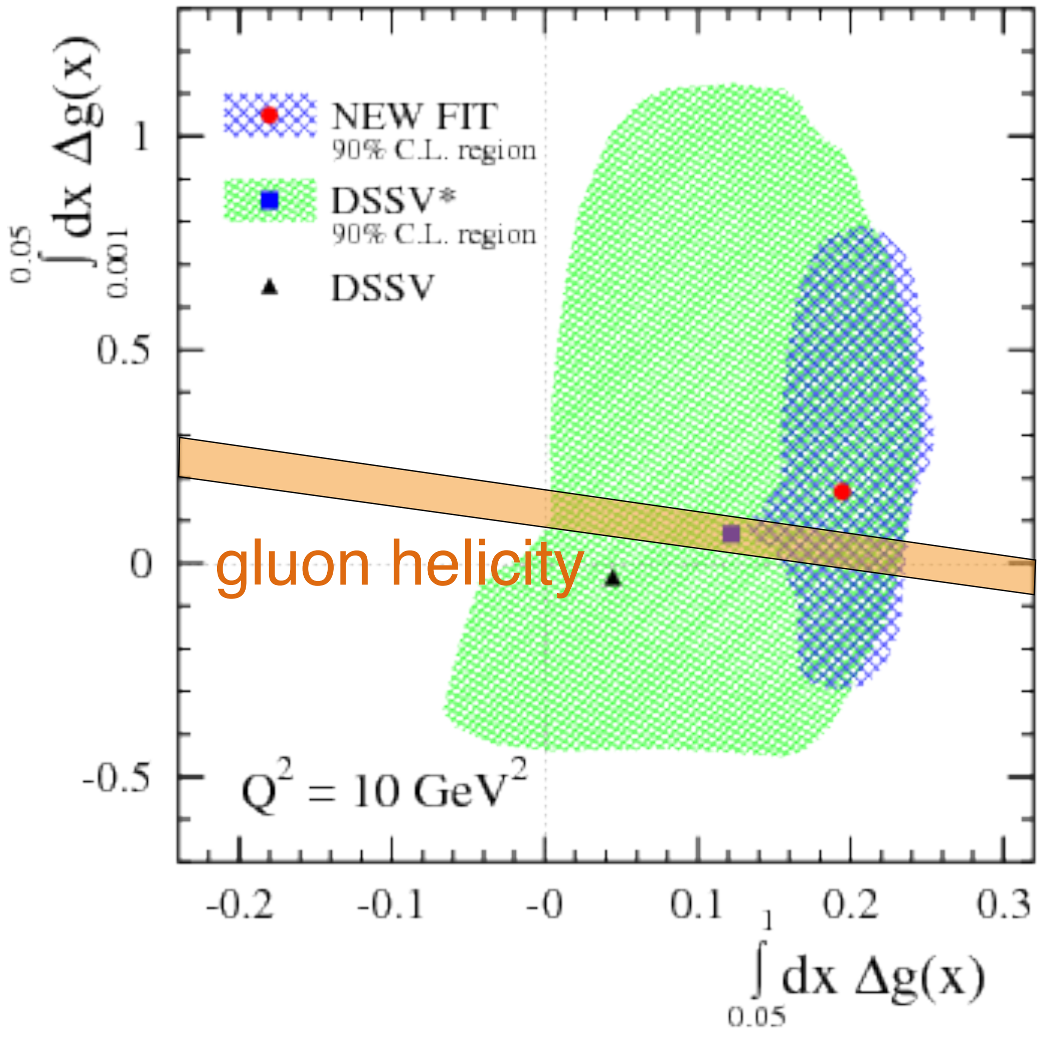
Y.-B. Yang et al., PRL 118, 102001 (2017)

Gluon Helicity

- Can't be calculated directly
- Match to calculable ME in infinite momentum frame limit using large momentum effective theory

LaMET: X. Ji et al., PRL 111 112002 (2013)





de Florian et. al, Phys.Rev.Lett. 113, 012001 (2014)

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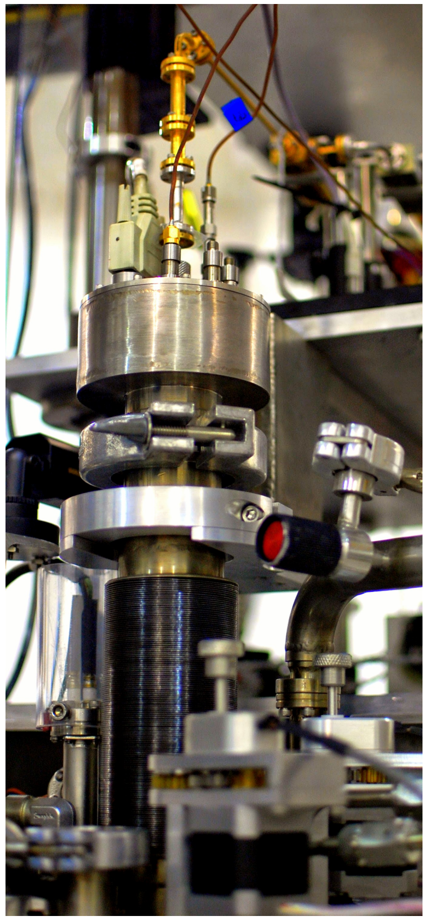
Gluonic Transversity

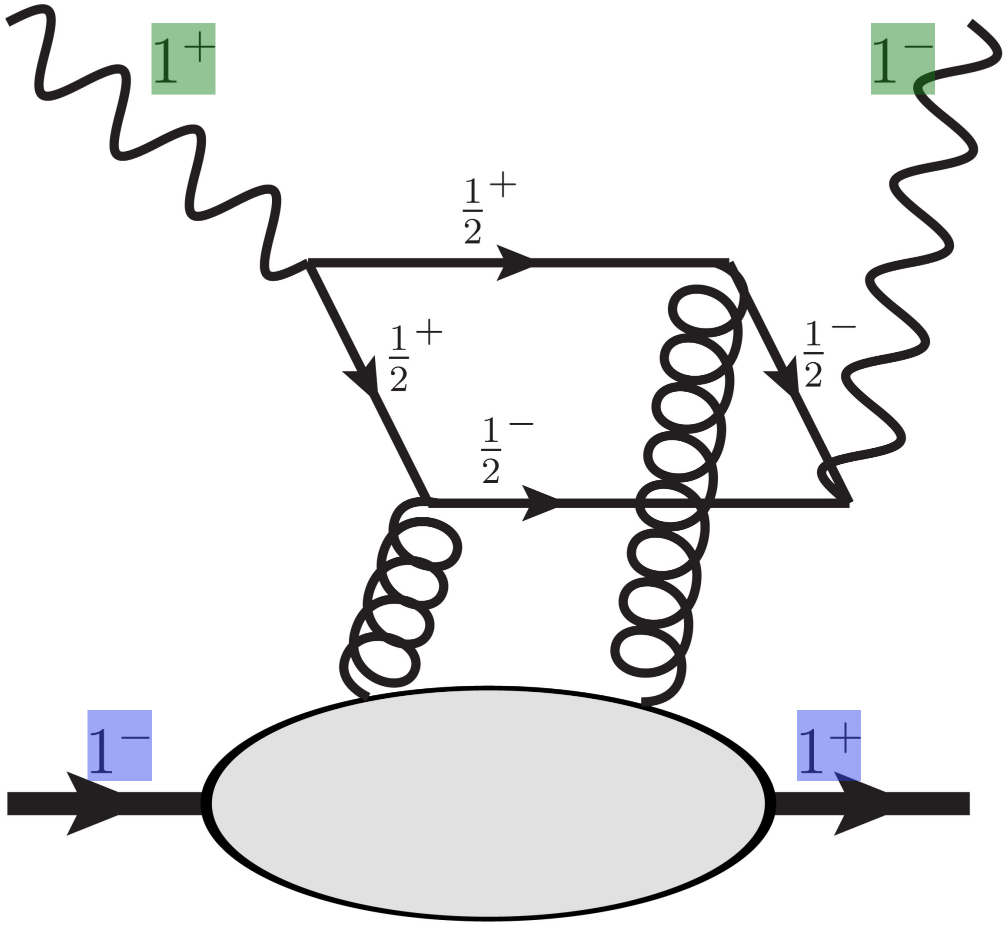
Targets with J≥1 have leading twist gluon parton distribution $\Delta(x,Q^2)$: double helicity flip [Jaffe & Manohar 1989]

- **Unambiguously gluonic:** no analogous quark PDF at twist-2
- Non-vanishing in forward limit for targets with spin≥1
- **Experimentally measurable** in unpolarised electron DIS on polarised target
  - Nitrogen target: JLab LoI 2015
  - Polarised nuclei at EIC
- **Moments calculable in LQCD**



Gluonic Transversity

Double helicity flip structure function Δ(x,Q²)



Changes both photon and target helicity by 2 units

Gluonic Transversity

Double helicity flip structure function Δ(x,Q²)

- **Hadrons:** Gluonic Transversity (parton model interpretation)

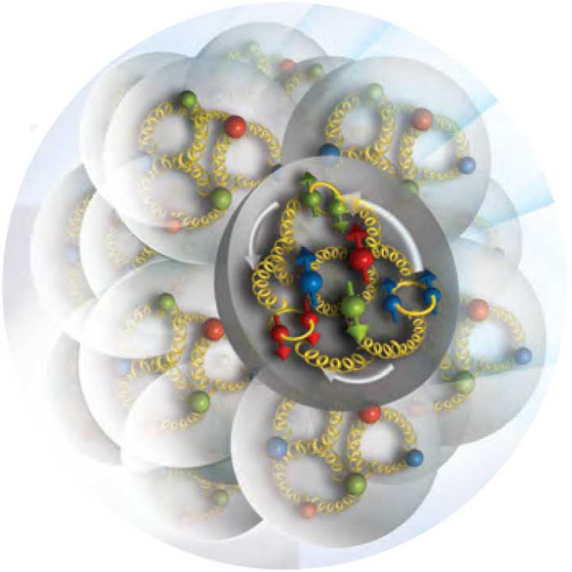
$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathcal{Q}^2\,x^2\int_x^1\frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2)-g_{\hat{y}}(x,Q^2)\right]$$

$g_{\hat{x},\hat{y}}(y,Q^2)$: probability of finding a gluon with momentum fraction y linearly polarised in $\hat{x}$, $\hat{y}$ direction

- **Nuclei:** Exotic Glue

gluons not associated with individual nucleons in nucleus

$$\langle p|\mathcal{O}|p\rangle = 0$$
$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$



Gluonic Transversity

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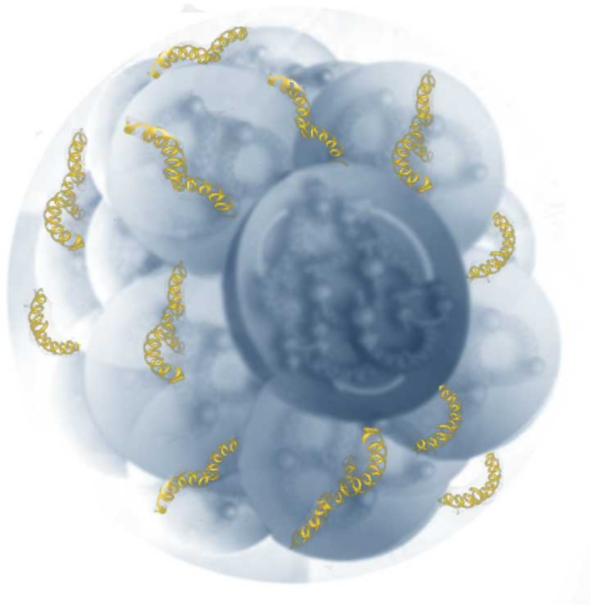


Gluonic Transversity

Moments of Δ(x,Q^2) are calculable in LQCD

Moment of Structure Function

Reduced Matrix Element

$$\boxed{\int_0^1 dx\, x^{n-1}\Delta(x,Q^2)} = \frac{\alpha_s(Q^2)}{3\pi}\frac{\boxed{A_n(Q^2)}}{n+2}, \qquad n = 2,4,6\ldots,$$

Determined by matrix elements of local gluonic operators

$$\langle pE'|\underline{S}\left[G_{\mu\mu_1}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n}G_{\nu\mu_2}\right]|pE\rangle$$

Symmetrise in $\mu_1,\ldots,\mu_n$, trace subtract in all free indices

$$= (-2i)^{n-2}\underline{S}\left[(p_\mu E'^*_{\mu_1} - p_{\mu_1}E'^*_{\mu})(p_\nu E_{\mu_2} - p_{\mu_2}E_\nu) + (\mu\leftrightarrow\nu)\right]p_{\mu_3}\ldots p_{\mu_n}\boxed{A_n(Q^2)}\ldots,$$

Reduced Matrix Element

LQCD Calculation

Gluon transversity of the ϕ meson

- First moment in ϕ meson (simplest spin-1 system → nuclei)
- Lattice details: clover fermions, Lüscher-Weisz gauge action

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- Many systematics not addressed (yet)
  - Quark mass effects
  - Volume effects
  - Discretisation
  - Renormalisation

Alexandrou et al. arXiv:1611.06901

W. Detmold, PES, PRD 94 (2016), 014507 + W. Detmold, D. Pefkou, PES PRD 95 (2017), 114515

Doing lattice QCD

- Correlation decays exponentially with distance in time:

$$C(t) = \sum_{n} Z_n \exp(-E_n t)$$

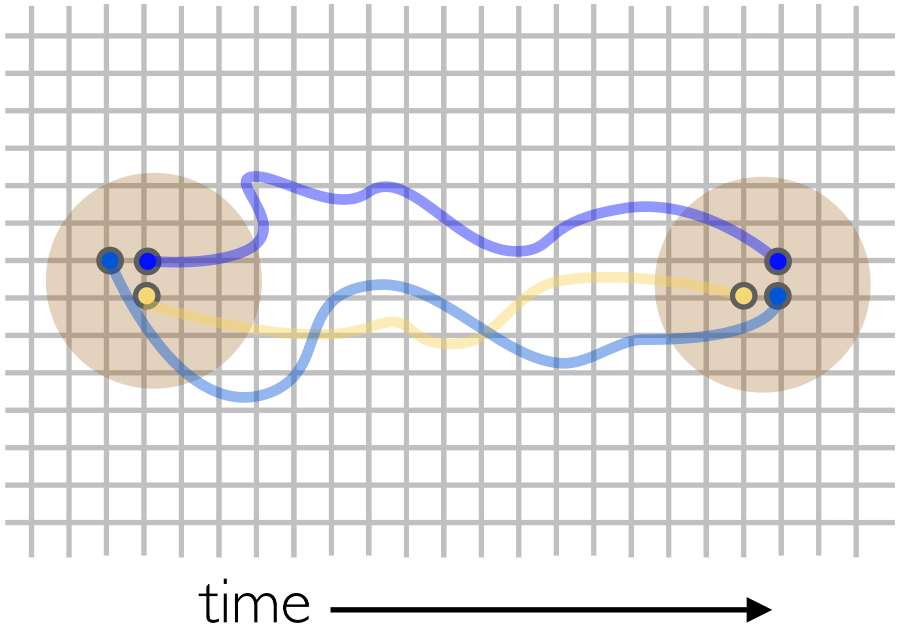
$n \leftarrow$ all eigenstates with q#'s of proton

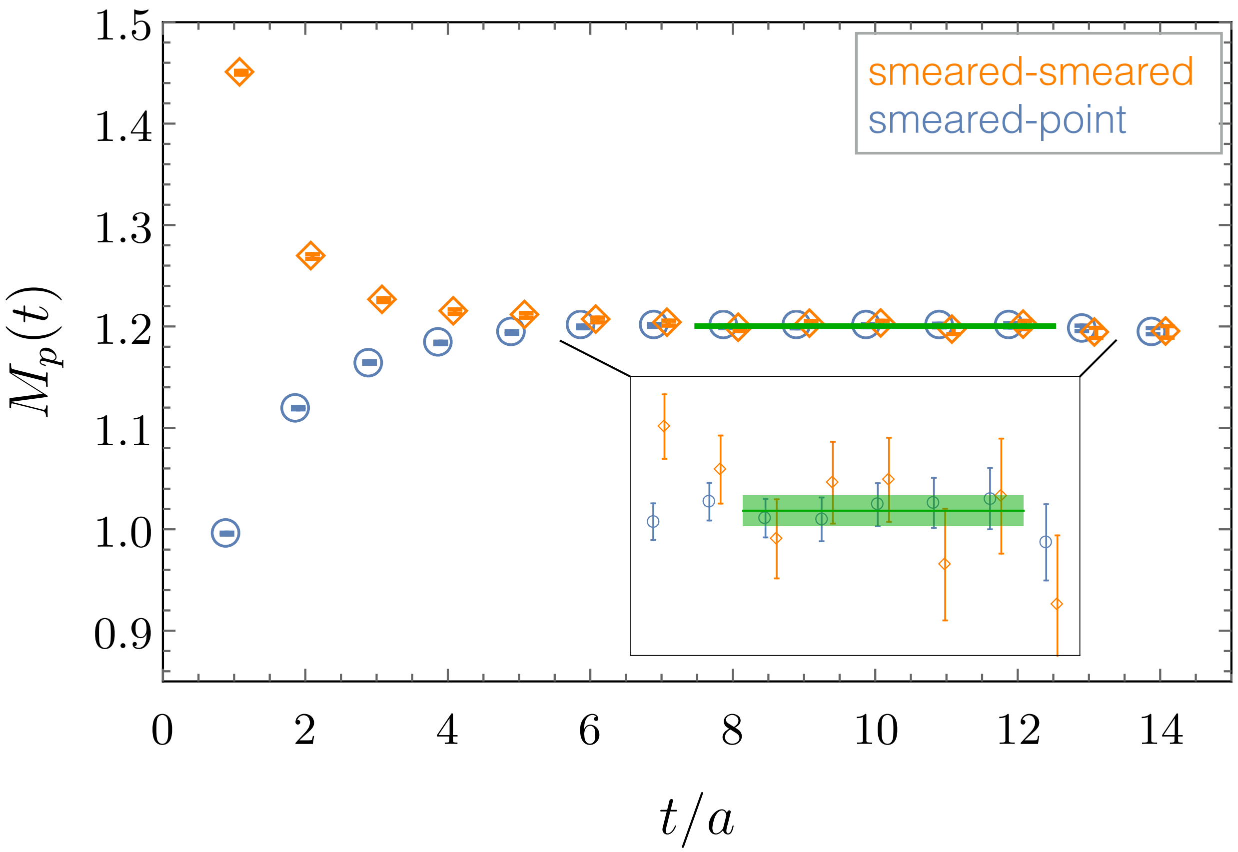
At late times:

$$\rightarrow Z_0 \exp(-E_0 t)$$

- Ground state mass revealed through "effective mass plot"

$$M(t) = \ln\left[\frac{C(t)}{C(t+1)}\right] \xrightarrow{t\rightarrow\infty} E_0$$

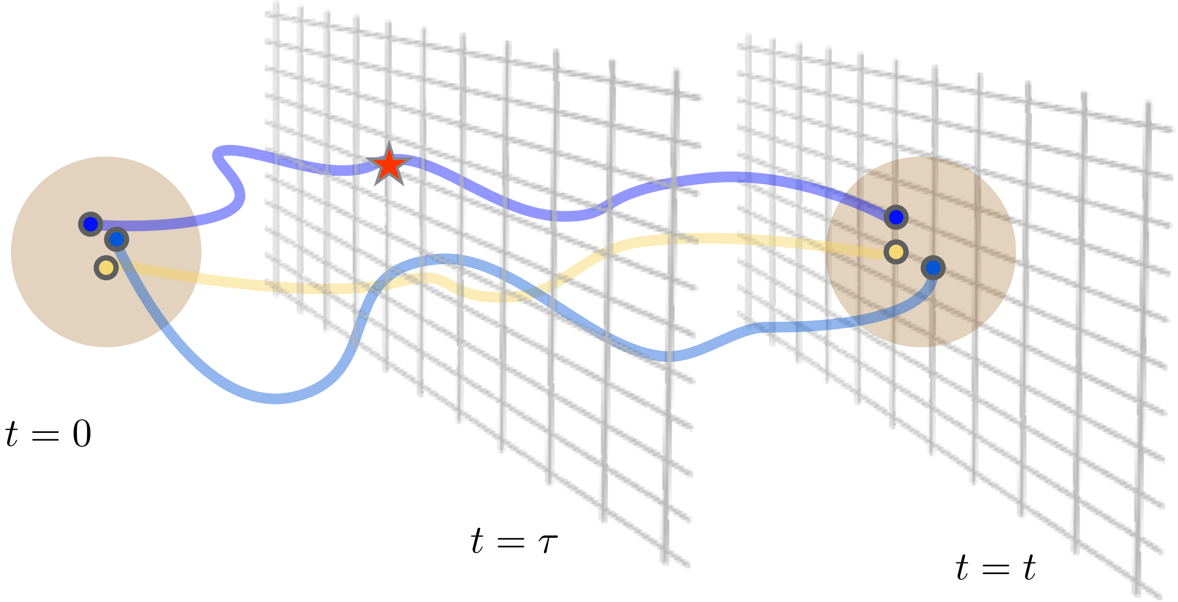




LQCD matrix elements

How do we calculate matrix elements?

- Create three quarks (correct quantum numbers) at a source and annihilate the three quarks at sink far from source
- Insert operator at intermediate timeslice



- Remove time-dependence by dividing out with two-point correlators: $\frac{C_3(t,\tau,\vec{p'},\vec{q})}{C_2(t-\tau,p')C_2(\tau,p)} \xrightarrow{t\to\infty} \langle N(p')|\mathcal{O}(q)|N(p)\rangle$

LQCD Calculation

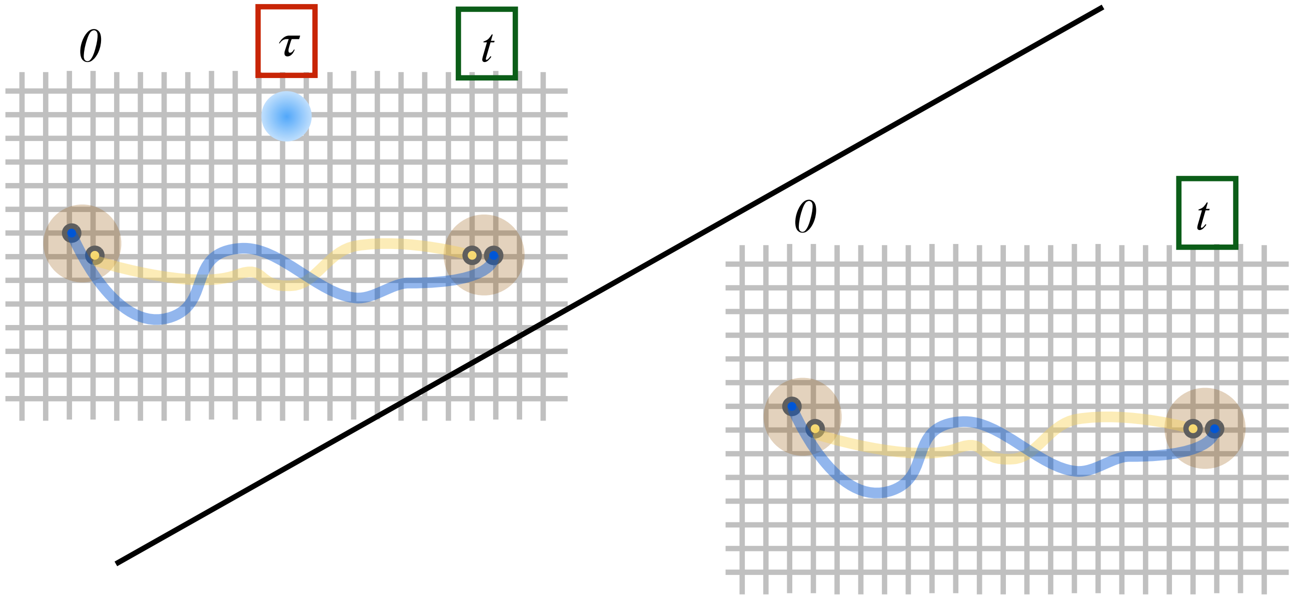
Calculate lowest moment of **Δ**(x,Q²):

Moment of Structure Function

Reduced Matrix Element

$$\boxed{\int_0^1 dx x^{n-1}\Delta(x,Q^2)} = \frac{\alpha_s(Q^2)}{3\pi}\frac{\boxed{A_n(Q^2)}}{n+2}, \quad n = 2, 4, 6\ldots,$$

Ratio of LQCD correlators $R_{jk}(t,\tau,\vec{p})$:



$$\propto A_2, \qquad 0 \ll \boxed{\tau} \ll \boxed{t}$$

operator insertion time

sink time

LQCD Calculation

- Discrete lattice: rotational symmetry → hypercubic symmetry
- Take linear combinations of operators that transform irreducibly under hypercubic group: safe from mixing

  e.g., for $\mathcal{O}^{(E)}_{\mu\nu\mu_1\mu_2} = G^{(E)}_{\mu\mu_1} G^{(E)}_{\nu\mu_2}$ use $\mathcal{O}^{(E)}_{1,1} = \frac{1}{8\sqrt{3}}\left(-2\mathcal{O}^{(E)}_{1122} + \mathcal{O}^{(E)}_{1133} + \mathcal{O}^{(E)}_{1144} + \mathcal{O}^{(E)}_{2233} + \mathcal{O}^{(E)}_{2244} - 2\mathcal{O}^{(E)}_{3344}\right)$

$$C^{2\text{pt}}_{jk}(t,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \eta_j(t,\vec{x}) \eta^\dagger_k(0,\vec{0})\rangle$$
$$= Z_\phi \left(e^{-Et} + e^{-E(T-t)}\right) \sum_{\lambda} \epsilon^{(E)}_j(\vec{p},\lambda)\epsilon^{(E)*}_k(\vec{p},\lambda)$$

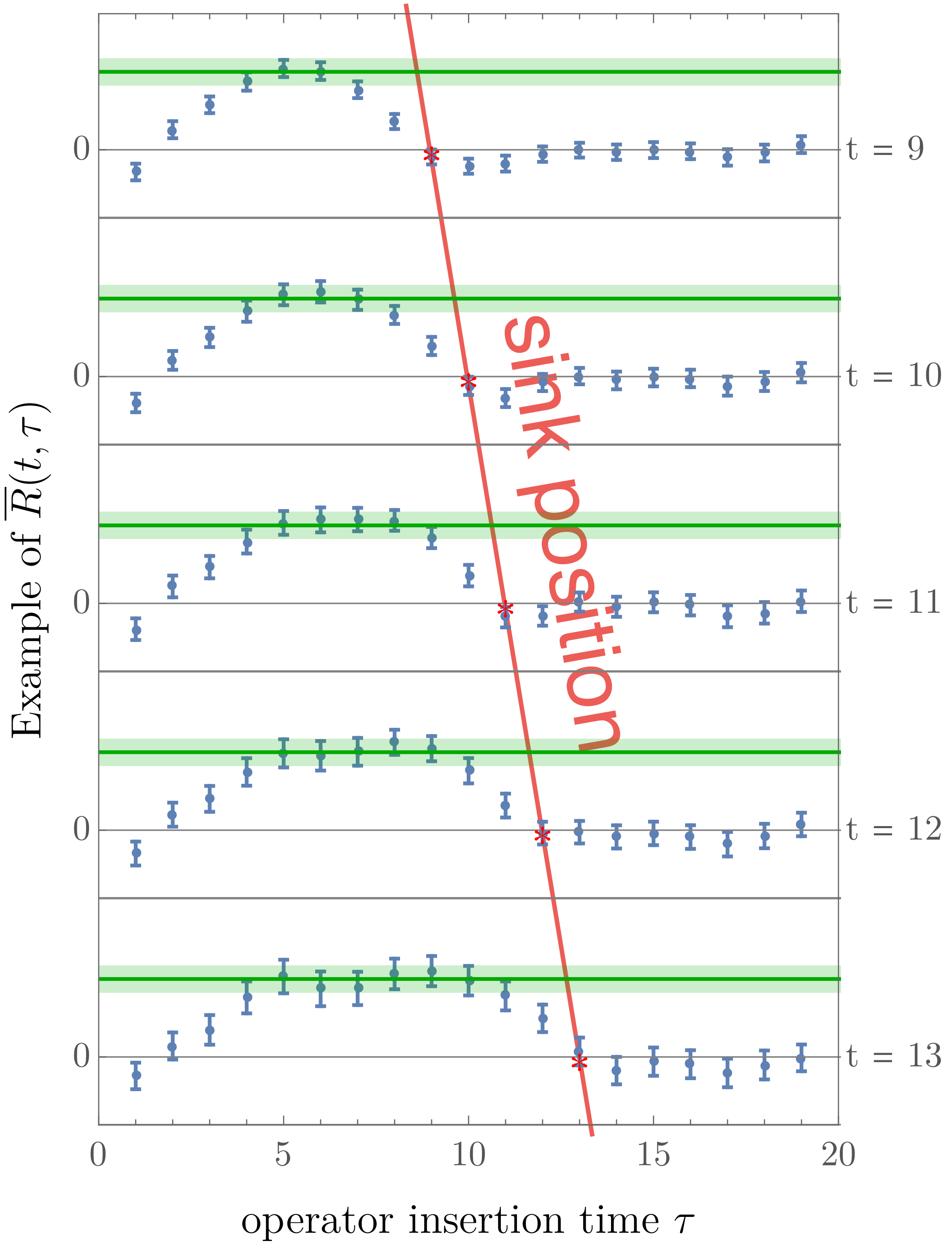
$$C^{3\text{pt}}_{jk}(t,\tau,\vec{p}) = \sum_{\vec{x}}\sum_{\vec{y}} e^{i\vec{p}\cdot\vec{x}} \langle \eta_j(t,\vec{p})\ \mathcal{O}(\tau,\vec{y})\ \eta^\dagger_k(0,\vec{0})\rangle$$
$$= Z_\phi e^{-Et} \sum_{\lambda\lambda'} \epsilon^{(E)}_j(\vec{p},\lambda)\epsilon^{(E)*}_k(\vec{p},\lambda')\langle \vec{p},\lambda|\mathcal{O}|\vec{p},\lambda'\rangle$$

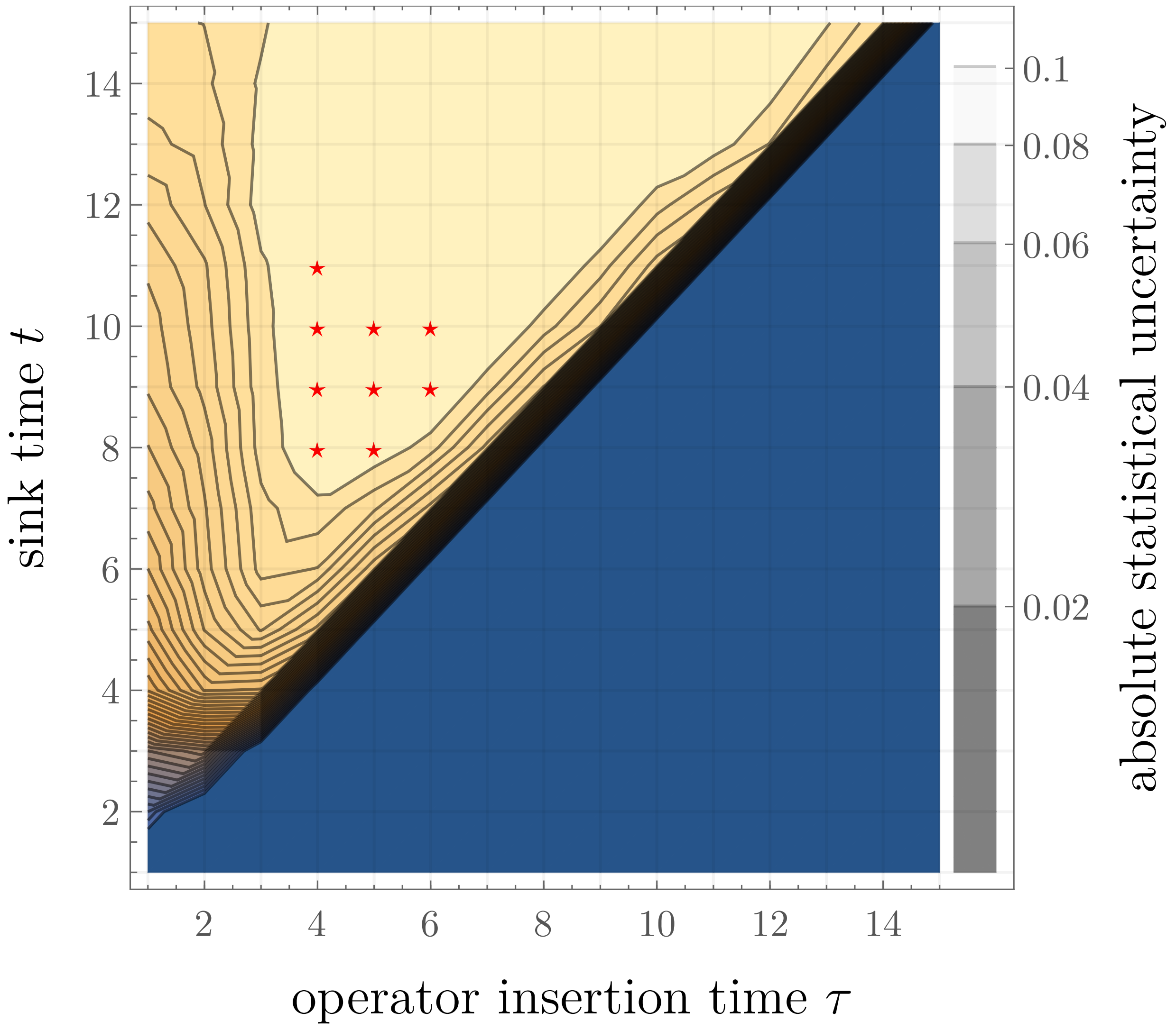
$$R_{jk}(t,\tau,\vec{p}) = \frac{C^{3\text{pt}}_{jk}(t,\tau,\vec{p}) + C^{3\text{pt}}_{jk}(T-t,T-\tau,\vec{p})}{C^{2\text{pt}}_{jk}(t,\vec{p})}$$

ratio depends on polarisations, momentum, operator

- All polarisation combinations (j,k)
- Boost momenta up to (1,1,1)
- Examine all elements of each hypercubic irrep.

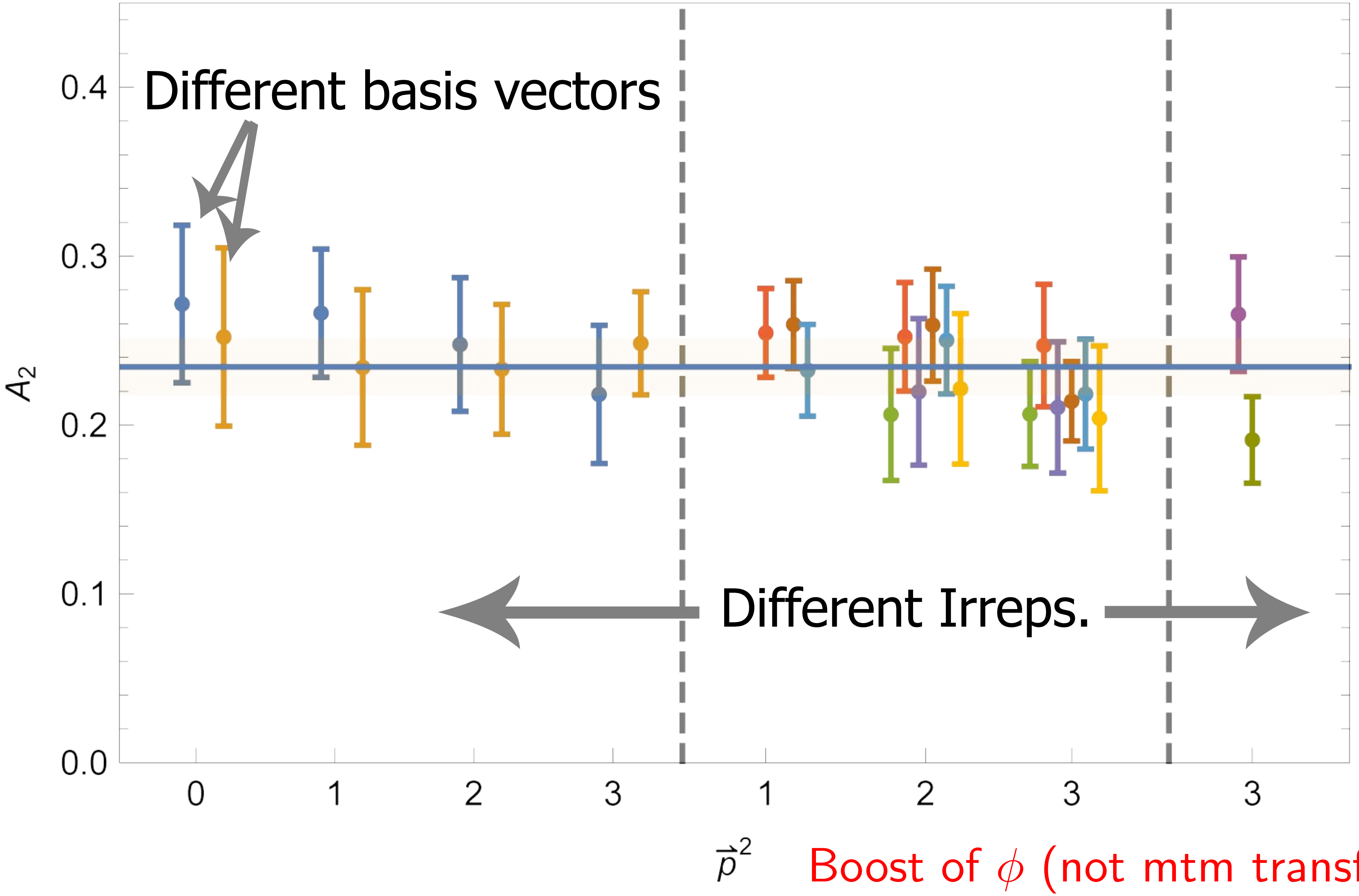
LQCD Calculation





LQCD Calculation

W. Detmold, PES, PRD 94 (2016), 014507



Boost of ϕ (not mtm transfer)

Soffer-type Bounds

Constraint relating **transversity**, **spin-indep.** and **spin-dep.** distributions

For quark distributions in spin 1/2 state:

Transversity

Spin-independent

$$|\delta q(x)| \leq \frac{1}{2}\left(q(x) + \Delta q(x)\right)$$

Spin-dependent

Analogue for first moments of gluon distributions?

- Need to calculate moments of spin independent gluon distribution (first moment of spin-dependent gluon distribution vanishes by operator symmetries)

Spin-indep. gluon structure

W. Detmold, PES, PRD 94 (2016), 014507

Spin-independent gluon operator:

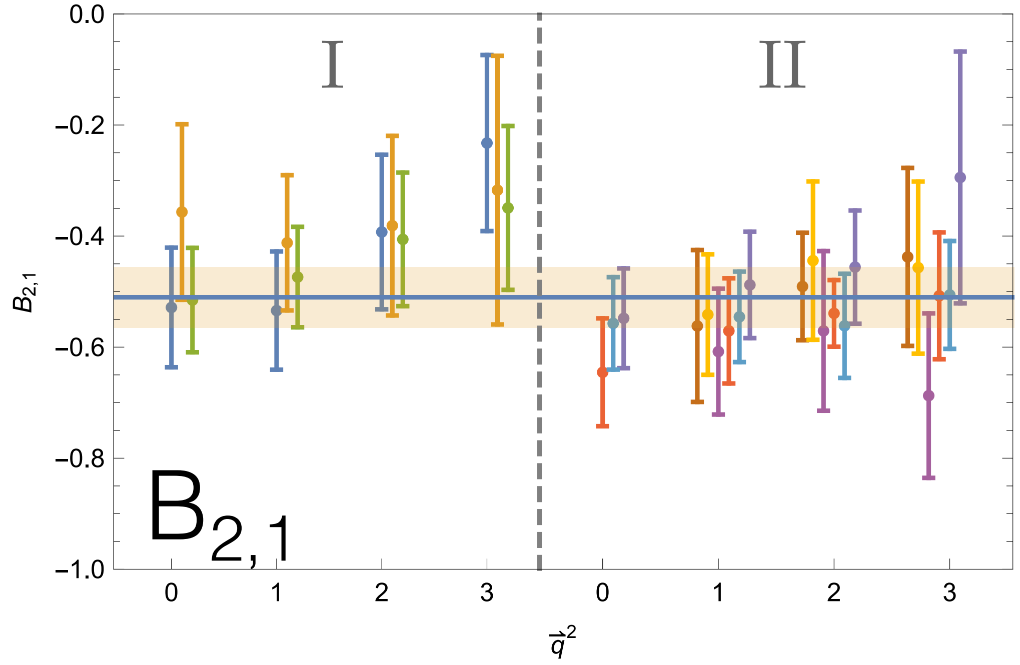
$$\overline{\mathcal{O}}_{\mu_1\ldots\mu_n} = S\left[G_{\mu_1\alpha}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n}G_{\mu_2}{}^{\alpha}\right]$$

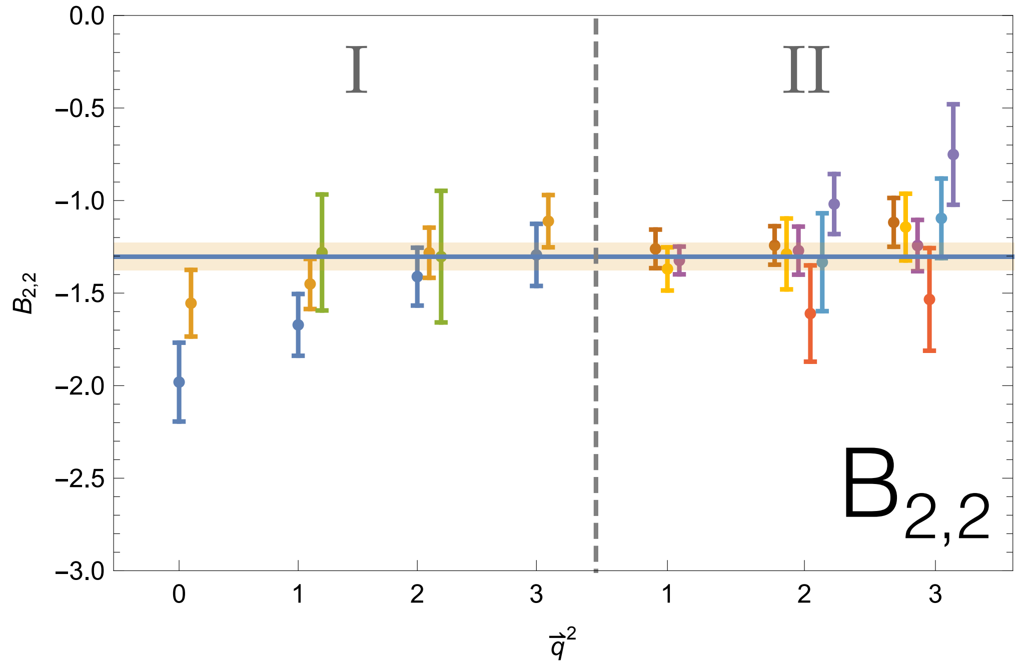
Matrix elements at n=2 define lowest moment of structure functions

$$\begin{aligned}\langle pE'|\overline{\mathcal{O}}_{\mu_1\mu_2}|pE\rangle \\ &= S\left[M^2 E'^*_{\mu_1}E_{\mu_2}\right]\boxed{B_{2,1}(\mu^2)} \\ &\quad + S\left[(E\cdot E'^*)p_{\mu_1}p_{\mu_2}\right]\boxed{B_{2,2}(\mu^2)}\end{aligned}$$

Two reduced matrix elements

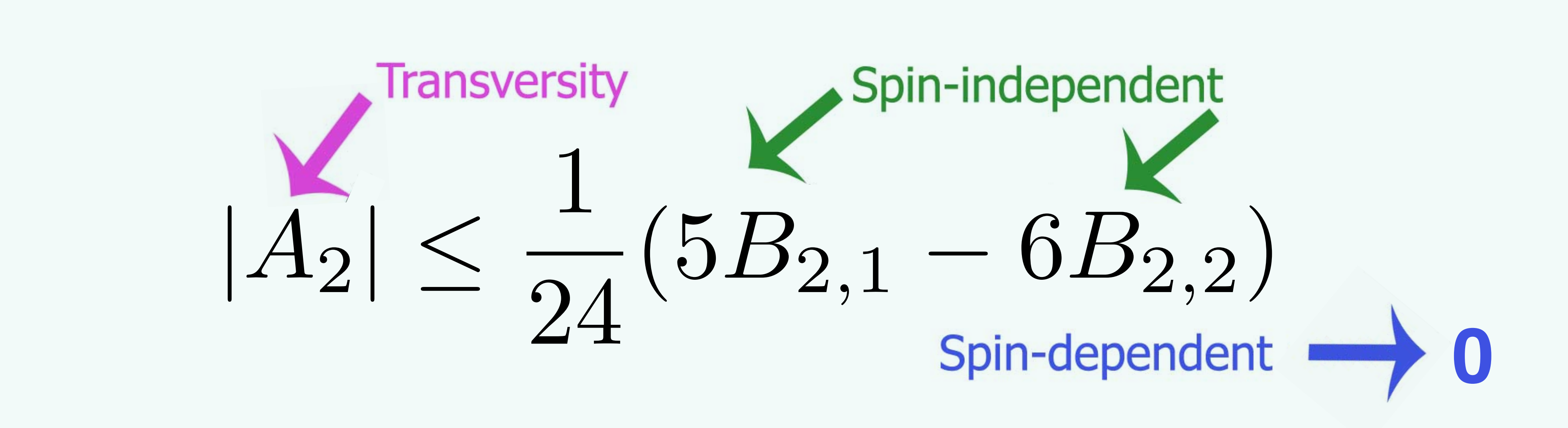
- Analysis as in transversity case
- Mixing with quark ops. neglected, pQCD calcs. shown that it is small: Alexandrou 1611.06901





Soffer-type Bounds

Soffer-type bound for leading moments of gluon distributions (spin-1 state):



Transversity

Spin-independent

$$|A_2| \leq \frac{1}{24}(5B_{2,1} - 6B_{2,2})$$

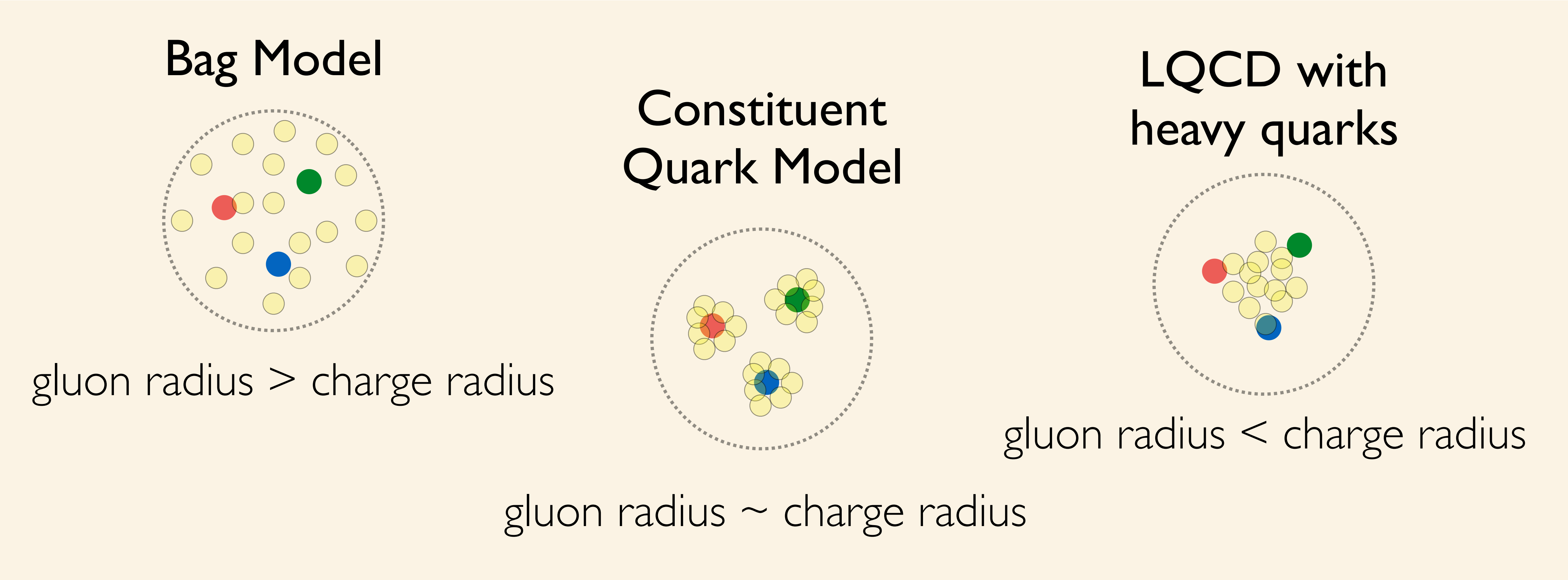
Spin-dependent → 0

$$|0.24| \leq \frac{1}{24}\left[5(-0.5) - 6(-1.4)\right] = 0.24$$

Soffer-like bound approximately saturated

Gluon Radii

How does the gluon radius of a proton compare to the quark/charge radius?



Bag Model

gluon radius > charge radius

Constituent Quark Model

gluon radius ~ charge radius

LQCD with heavy quarks

gluon radius < charge radius

Or is the picture more complicated?

Gluon Generalised FFs

Matrix elements of the spin-independent gluon structure function

- Off-forward matrix elements are complicated:

$$
\begin{aligned}
&\left\langle p'E' \left| S\left[G_{\mu\alpha} i\overleftrightarrow{D}_{\mu_1}\dots i\overleftrightarrow{D}_{\mu_n} G_\nu{}^{\alpha}\right]\right| pE\right\rangle \\
&\quad = \sum_{\substack{m\ \text{even}\\ m=0}}^{n} \Bigg\{ B_{1,m}^{(n+2)}(\Delta^2) M^2 S\left[E_\mu E'^*_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{2,m}^{(n+2)}(\Delta^2) S\left[(E\cdot E'^*) P_\mu P_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{3,m}^{(n+2)}(\Delta^2) S\left[(E\cdot E'^*) \Delta_\mu \Delta_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{4,m}^{(n+2)}(\Delta^2) S\left[\left((E'^*\cdot P) E_\mu P_\nu + (E\cdot P) E'^*_\mu P_\nu\right) \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{5,m}^{(n+2)}(\Delta^2) S\left[\left((E'^*\cdot P) E_\mu \Delta_\nu - (E\cdot P) E'^*_\mu \Delta_\nu\right) \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + \frac{B_{6,m}^{(n+2)}(\Delta^2)}{M^2} S\left[(E\cdot P)(E'^*\cdot P) P_\mu P_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + \frac{B_{7,m}^{(n+2)}(\Delta^2)}{M^2} S\left[(E\cdot P)(E'^*\cdot P) \Delta_\mu \Delta_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \Bigg\} .
\end{aligned}
$$

Gluon Generalised FFs

Matrix elements of the spin-independent gluon structure function

- Off-forward matrix elements are complicated:

$$\left\langle p'E' \left| S\left[G_{\mu\alpha} i\overleftrightarrow{D}_{\mu_1}\dots i\overleftrightarrow{D}_{\mu_n} G_\nu{}^\alpha\right]\right| pE\right\rangle$$

$$= \sum_{\substack{m \text{ even} \\ m=0}}^{n} \left\{ \boxed{B_{1,m}^{(n+2)}(\Delta^2)} M^2 S\left[E_\mu E'^*_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \right.$$

$$+ \boxed{B_{2,m}^{(n+2)}(\Delta^2)} S\left[(E \cdot E\right.$$

$$+ \boxed{B_{3,m}^{(n+2)}(\Delta^2)} S\left[(E \cdot E\right.$$

$$+ \boxed{B_{4,m}^{(n+2)}(\Delta^2)} S\left[((E'^* \quad \dots P_{\mu_n}\right]$$

$$+ \boxed{B_{5,m}^{(n+2)}(\Delta^2)} S\left[((E \quad \dots P_{\mu_n}\right]$$

$$+ \frac{\boxed{B_{6,m}^{(n+2)}(\Delta^2)}}{M^2} S\left[(E \cdot P)(E'^* \cdot P) P_\mu P_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right]$$

$$\left. + \frac{\boxed{B_{7,m}^{(n+2)}(\Delta^2)}}{M^2} S\left[(E \cdot P)(E'^* \cdot P) \Delta_\mu \Delta_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \right\}.$$

Many gluonic radii:

Defined by slope of each form factor at $Q^2=t=0$

Gluon Generalised FFs

Matrix elements of the gluon transversity structure function

- Similarly complicated:

$$\begin{aligned}
&\left\langle p'E' \middle| S\left[G_{\mu\mu_1}\overset{\leftrightarrow}{D}_{\mu_3}\dots\overset{\leftrightarrow}{D}_{\mu_n}G_{\nu\mu_2}\right]\middle| pE\right\rangle \\
&= \sum_{\substack{m\ \mathrm{odd}\\ m=3}}^{n}\Bigg\{\boxed{A^{(n)}_{1,m-3}(t,\mu^2)}\,S\left[(P_\mu E_{\mu_1}-E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\boxed{A^{(n)}_{2,m-3}(t,\mu^2)}\,S\left[(\Delta_\mu E_{\mu_1}-E_\mu\Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2}-E'^*_\nu\Delta_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\boxed{A^{(n)}_{3,m-3}(t,\mu^2)}\,S\Big[\big((\Delta_\mu E_{\mu_1}-E_\mu\Delta_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2})-(\Delta_\mu E'^*_{\mu_1}-E'^*_\mu\Delta_{\mu_1})(P_\nu E_{\mu_2}-E_\nu P_{\mu_2})\big) \\
&\qquad\qquad\times\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\Big] \\
&\quad+\boxed{A^{(n)}_{4,m-3}(t,\mu^2)}\,S\left[(E_\mu E'^*_{\mu_1}-E_{\mu_1}E'^*_\mu)(P_\nu\Delta_{\mu_2}-P_{\mu_2}\Delta_\nu)\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\frac{\boxed{A^{(n)}_{5,m-3}(t,\mu^2)}}{M^2}\,S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(\Delta_\nu E'^*_{\mu_2}-E'^*_\nu\Delta_{\mu_2}) \\
&\qquad\qquad+\ (E'^*\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(\Delta_\nu E_{\mu_2}-E_\nu\Delta_{\mu_2})\big)\,\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\Big] \\
&\quad+\frac{\boxed{A^{(n)}_{6,m-3}(t,\mu^2)}}{M^2}\,S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2}) \\
&\qquad\qquad-\ (E'^*\cdot P)\ (P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2}-E_\nu P_{\mu_2})\big)\,\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\Big] \\
&\quad+\frac{\boxed{A^{(n)}_{7,m-3}(t,\mu^2)}}{M^2}(E'^*\cdot E)\,S\left[(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2}-\Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\frac{\boxed{A^{(n)}_{8,m-3}(t,\mu^2)}}{M^4}(E\cdot P)(E'^*\cdot P)\,S\left[(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2}-\Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right]\Bigg\}
\end{aligned}$$

Gluon Generalised FFs

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- Some GFFs suppressed by orders of magnitude
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Simplest example:
Transversity GFFs
One basis (2 vectors)
Mtm 1 (lattice units)

$$\begin{pmatrix}
0.604 & 0.0424 & 0 & 0 & 0 & 0 & 0.0588 & 0 \\
0.592 & -2.45\times10^{-3} & 0.0785 & -0.0785 & 6.58\times10^{-3} & -0.0992 & -0.103 & -4.15\times10^{-3} \\
0.485 & 0.0429 & 0 & 0 & 0 & 0 & 0.0379 & 0 \\
0.481 & 0.0431 & -3.02\times10^{-5} & 3.02\times10^{-5} & -2.53\times10^{-6} & -4.03\times10^{-7} & 0.0374 & -1.69\times10^{-8} \\
0.475 & -3.29\times10^{-3} & 0.0791 & -0.0791 & 6.59\times10^{-3} & -0.0791 & -0.0824 & -3.29\times10^{-3} \\
0.353 & -7.97\times10^{-4} & 0.0385 & -0.0385 & 3.28\times10^{-3} & -0.0598 & -0.0631 & -2.54\times10^{-3} \\
0.347 & -0.0382 & 0 & 0 & 0 & 0 & 0.0962 & 0 \\
0.258 & 0.0806 & 0 & 0 & 0 & 0 & -0.0374 & 0 \\
0.258 & 0.0808 & 0 & 0 & 0 & 0 & -0.0379 & 0 \\
0.253 & 0.101 & -8.60\times10^{-4} & 8.60\times10^{-4} & -7.20\times10^{-5} & 6.32\times10^{-7} & -0.0588 & 2.65\times10^{-8} \\
0.239 & -1.66\times10^{-3} & 0.0401 & -0.0401 & 3.29\times10^{-3} & -0.0393 & -0.0402 & -1.61\times10^{-3} \\
0.238 & -1.65\times10^{-3} & 0.0396 & -0.0396 & 3.29\times10^{-3} & -0.0396 & -0.0412 & -1.65\times10^{-3} \\
0.228 & -0.0581 & 8.30\times10^{-4} & -8.30\times10^{-4} & 6.94\times10^{-5} & -1.04\times10^{-6} & 0.0962 & -4.33\times10^{-8} \\
0.228 & -0.0379 & 0 & 0 & 0 & 0 & 0.0758 & 0 \\
0.0590 & -0.0109 & 0.139 & -0.139 & 0.0112 & -4.97\times10^{-3} & -3.94\times10^{-4} & -8.24\times10^{-6} \\
0.0578 & -2.56\times10^{-4} & 9.42\times10^{-3} & -9.42\times10^{-3} & 3.89\times10^{-4} & -4.65\times10^{-3} & 2.51\times10^{-4} & 5.25\times10^{-6} \\
0.0338 & 1.59\times10^{-3} & -0.128 & 0.128 & -0.0107 & 3.18\times10^{-4} & 0.0154 & 1.33\times10^{-5} \\
0.0183 & 6.36\times10^{-3} & -1.29\times10^{-4} & 1.29\times10^{-4} & 3.84\times10^{-4} & 4.84\times10^{-3} & 5.99\times10^{-3} & 5.18\times10^{-6} \\
0.0155 & -4.78\times10^{-3} & -0.128 & 0.128 & -0.0111 & -4.52\times10^{-3} & 9.41\times10^{-3} & 8.14\times10^{-6} \\
1.19\times10^{-3} & -0.0106 & 0.129 & -0.129 & 0.0108 & -3.22\times10^{-4} & -6.45\times10^{-4} & -1.35\times10^{-5} \\
0.549 & 2.44\times10^{-3} & 0 & 0 & 0 & 0 & 0.0895 & 0 \\
0.546 & -1.88\times10^{-3} & 0.0676 & -0.0676 & 5.69\times10^{-3} & -0.0918 & -0.0960 & -3.86\times10^{-3} \\
0.498 & 0.0710 & 0 & 0 & 0 & 0 & 0.0123 & 0 \\
0.480 & -2.37\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0799 & -0.0828 & -3.33\times10^{-3} \\
0.429 & 0.0714 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.424 & 0.0834 & -5.14\times10^{-4} & 5.14\times10^{-4} & -4.30\times10^{-5} & 1.33\times10^{-7} & -0.0123 & 5.55\times10^{-9} \\
0.412 & 2.85\times10^{-3} & 0 & 0 & 0 & 0 & 0.0657 & 0 \\
0.412 & -2.85\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0685 & -0.0714 & -2.85\times10^{-3} \\
0.409 & -8.65\times10^{-3} & 4.61\times10^{-4} & -4.61\times10^{-4} & 3.86\times10^{-5} & -8.30\times10^{-7} & 0.0771 & -3.47\times10^{-8} \\
0.0674 & -6.43\times10^{-3} & 0.0856 & -0.0856 & 6.70\times10^{-3} & -5.55\times10^{-3} & -8.26\times10^{-5} & -1.73\times10^{-6} \\
0.0656 & 4.96\times10^{-4} & -9.21\times10^{-4} & 9.21\times10^{-4} & -6.37\times10^{-6} & -0.0119 & -0.0132 & -5.32\times10^{-4} \\
0.0514 & -0.0685 & 0 & 0 & 0 & 0 & 0.0771 & 0 \\
0.0347 & -0.0124 & 0.155 & -0.155 & 0.0127 & -3.05\times10^{-3} & -6.00\times10^{-4} & -1.26\times10^{-5} \\
0.0327 & 5.99\times10^{-3} & -0.0692 & 0.0692 & -6.03\times10^{-3} & -2.50\times10^{-3} & 5.17\times10^{-4} & 1.08\times10^{-5} \\
0.0301 & 4.59\times10^{-3} & -0.0738 & 0.0738 & -5.95\times10^{-3} & 2.98\times10^{-3} & 0.0123 & 1.07\times10^{-5} \\
0.0285 & -1.84\times10^{-3} & -0.147 & 0.147 & -0.0126 & -2.43\times10^{-3} & 0.0143 & 1.24\times10^{-5} \\
0.0171 & 0.0685 & 0 & 0 & 0 & 0 & -0.0657 & 0 \\
0.0146 & 0.0920 & -9.75\times10^{-4} & 9.75\times10^{-4} & -8.17\times10^{-5} & 9.63\times10^{-7} & -0.0895 & 4.03\times10^{-8} \\
1.59\times10^{-3} & 6.43\times10^{-3} & 0.0736 & -0.0736 & 6.61\times10^{-3} & 5.40\times10^{-3} & -1.97\times10^{-3} & -1.71\times10^{-6}
\end{pmatrix}
\begin{pmatrix}
A^{(2)}_{1,0}(1) \\
A^{(2)}_{2,0}(1) \\
A^{(2)}_{3,0}(1) \\
A^{(2)}_{4,0}(1) \\
A^{(2)}_{5,0}(1) \\
A^{(2)}_{6,0}(1) \\
A^{(2)}_{7,0}(1) \\
A^{(2)}_{8,0}(1)
\end{pmatrix}
=
\begin{pmatrix}
0.179(36) \\
0.150(38) \\
0.152(30) \\
0.154(37) \\
0.129(32) \\
0.056(31) \\
0.067(41) \\
0.056(35) \\
0.069(21) \\
0.093(36) \\
0.028(32) \\
0.041(27) \\
0.012(33) \\
0.029(30) \\
0.024(11) \\
-0.005(21) \\
-0.0056(96) \\
-0.002(11) \\
0.009(16) \\
0.0162(91) \\
0.086(26) \\
0.131(31) \\
0.155(33) \\
0.086(33) \\
0.098(16) \\
0.094(17) \\
0.088(27) \\
0.114(25) \\
0.075(27) \\
0.034(25) \\
-0.006(22) \\
-0.001(31) \\
0.022(11) \\
0.014(16) \\
0.0010(16) \\
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0.018(23) \\
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0.005(18)
\end{pmatrix}$$

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0.258	0.0806	0	0	0	0	−0.0374	0
0.258	0.0808	0	0	0	0	−0.0379	0
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0.228	−0.0379	0	0	0	0	0.0758	0
0.0590	−0.0109	0.139	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.0578	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.0338	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.0183	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.0155	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.19×10^{-3}	−0.0106	0.129	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.549	2.44×10^{-3}	0	0	0	0	0.0895	0
0.546	-1.88×10^{-3}	0.0676	−0.0676	5.69×10^{-3}	−0.0918	−0.0960	-3.86×10^{-3}
0.498	0.0710	0	0	0	0	0.0123	0
0.480	-2.37×10^{-3}	0.0685	−0.0685	5.70×10^{-3}	−0.0799	−0.0828	-3.33×10^{-3}
0.429	0.0714	0	0	0	0	0	0
0.424	0.0834	-5.14×10^{-4}	5.14×10^{-4}	-4.30×10^{-5}	1.33×10^{-7}	−0.0123	5.55×10^{-9}
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0.0514	−0.0685	0	0	0	0	0.0771	0
0.0347	−0.0124	0.155	−0.155	0.0127	-3.05×10^{-3}	-6.00×10^{-4}	-1.26×10^{-5}
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0.0171	0.0685	0	0	0	0	−0.0657	0
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1.59×10^{-3}	6.43×10^{-3}	0.0736	−0.0736	6.61×10^{-3}	5.40×10^{-3}	-1.97×10^{-3}	-1.71×10^{-6}

$$\begin{pmatrix} [illegible] \\ \vdots \\ A^{(2)}_{5,0}(1) \\ A^{(2)}_{6,0}(1) \\ A^{(2)}_{7,0}(1) \\ A^{(2)}_{8,0}(1) \end{pmatrix}$$

$$\begin{pmatrix} 0.179(36) \\ 0.150(38) \\ 0.152(30) \\ 0.154(37) \\ 0.129(32) \\ 0.056(31) \\ 0.067(41) \\ 0.056(35) \\ 0.069(21) \\ 0.093(36) \\ 0.028(32) \\ 0.041(27) \\ 0.012(33) \\ [illegible] \\ 0.086(26) \\ 0.131(31) \\ 0.155(33) \\ 0.086(33) \\ 0.098(16) \\ 0.094(17) \\ 0.088(27) \\ 0.114(25) \\ 0.075(27) \\ 0.034(25) \\ -0.006(22) \\ -0.001(31) \\ 0.022(11) \\ 0.014(16) \\ 0.0010(16) \\ 0.0008(85) \\ 0.018(23) \\ 0.001(29) \\ 0.005(18) \end{pmatrix}$$

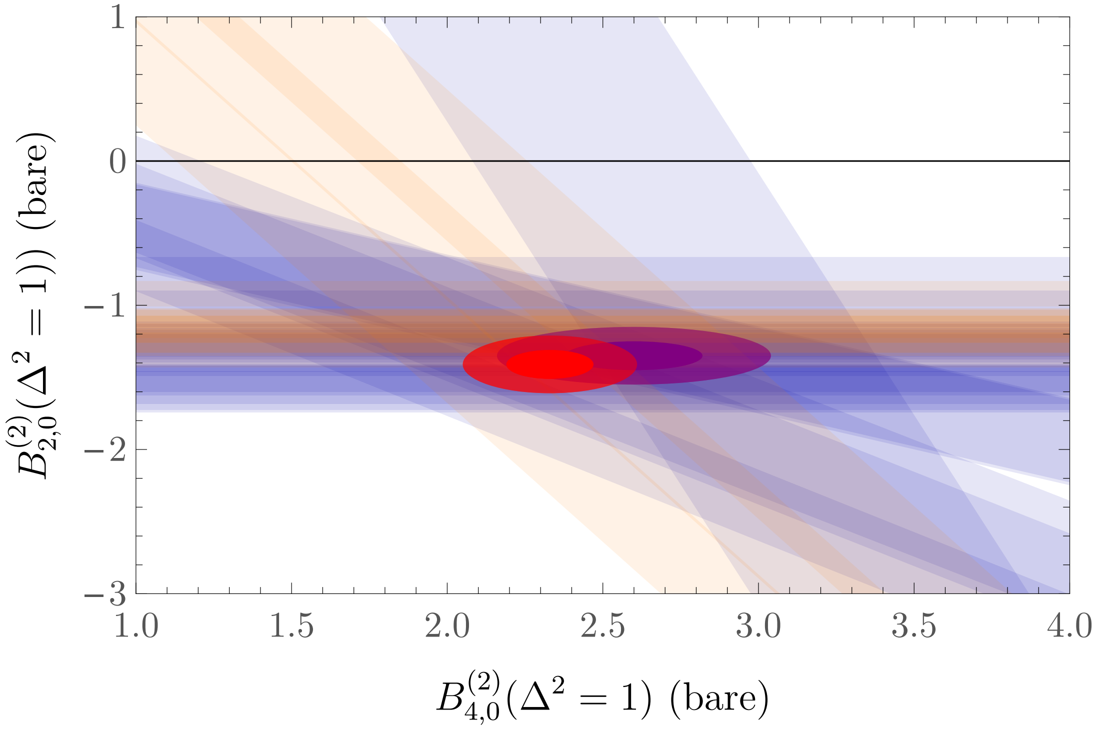
Target a subset of “dominant GFFs”

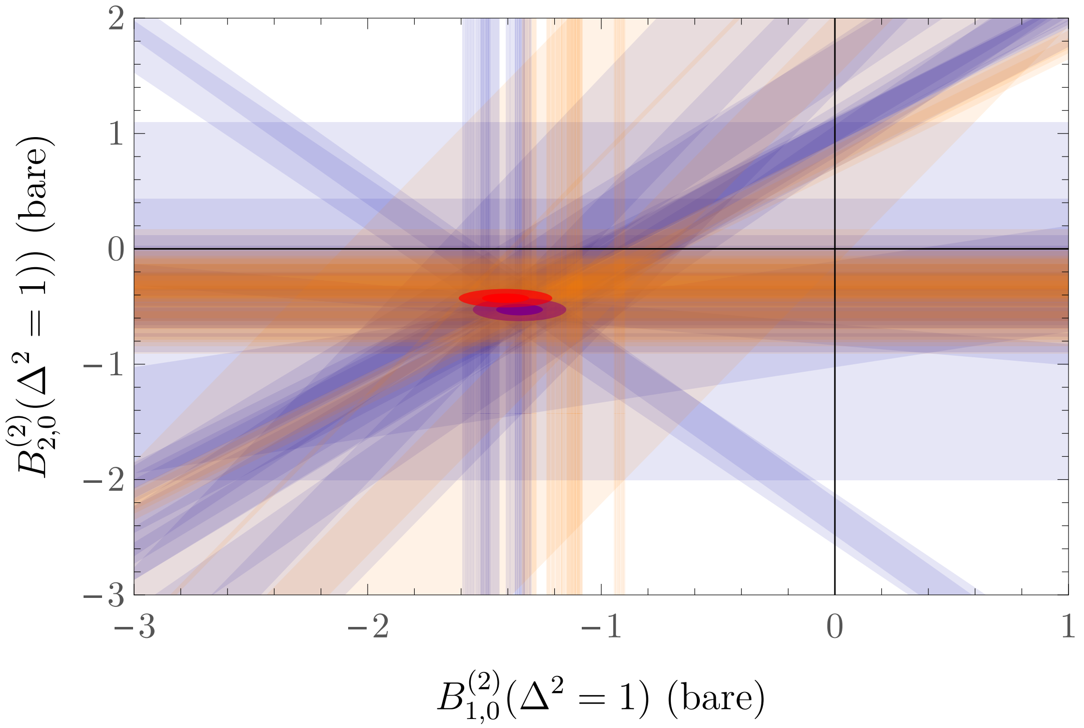
Gluon Generalised FFs

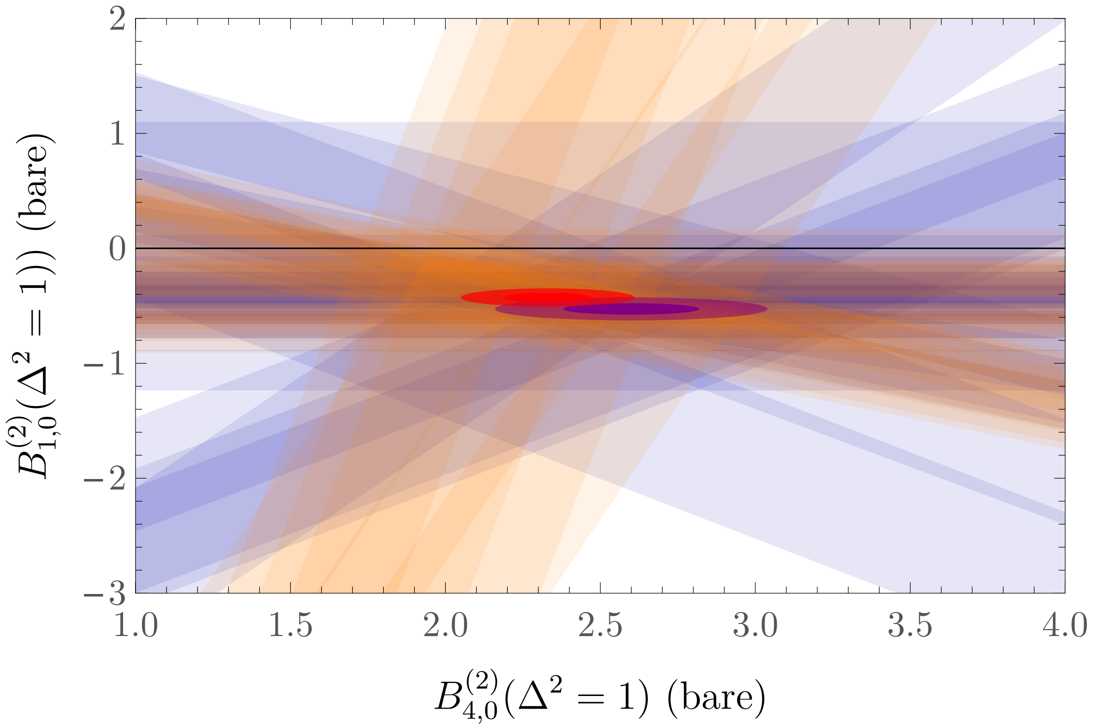
Example:

Spin-indep GFFs, lowest non-zero momentum transfer

- Projection into planes of dominant GFFs
- Others set to 0±10
- Only tightly-constrained bands shown in each projection.



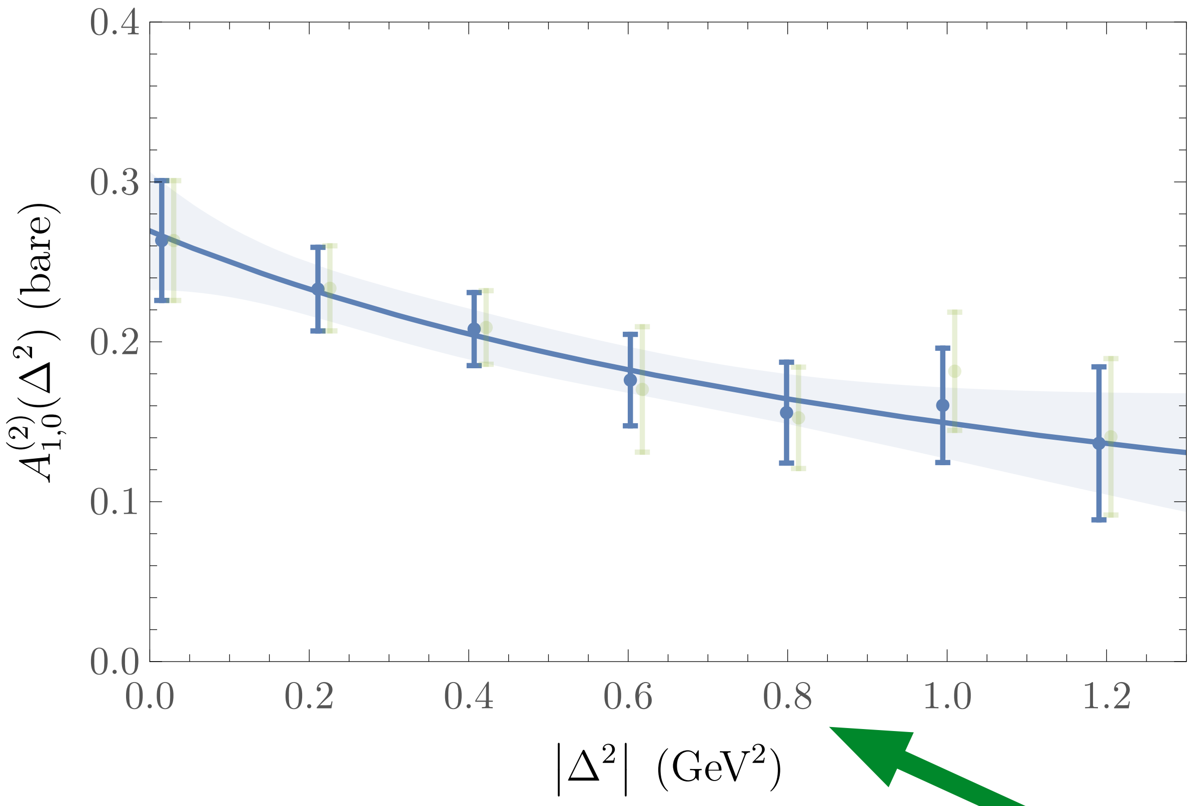


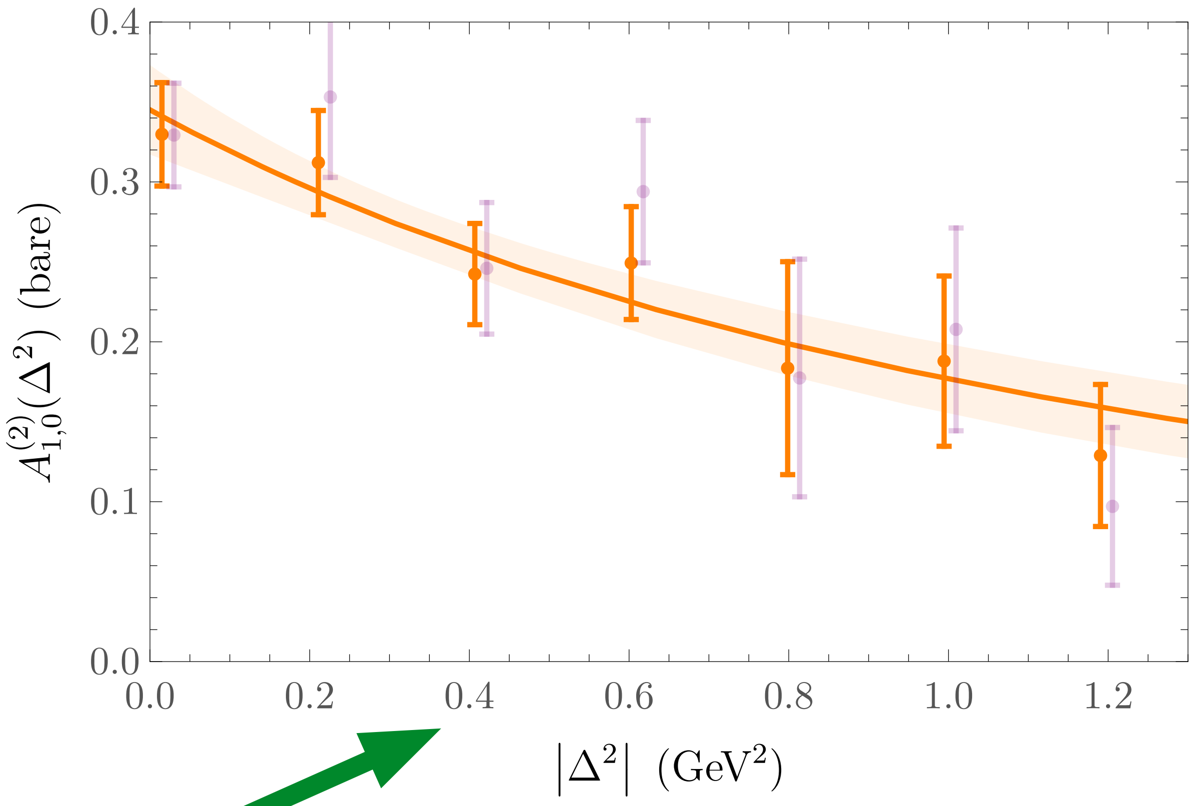


Gluon Transversity GFFs

W. Detmold, PES, PRD 94 (2016), 014507 + W. Detmold, D. Pefkou, PES PRD 95 (2017), 114515

One GFF can be resolved for all momenta



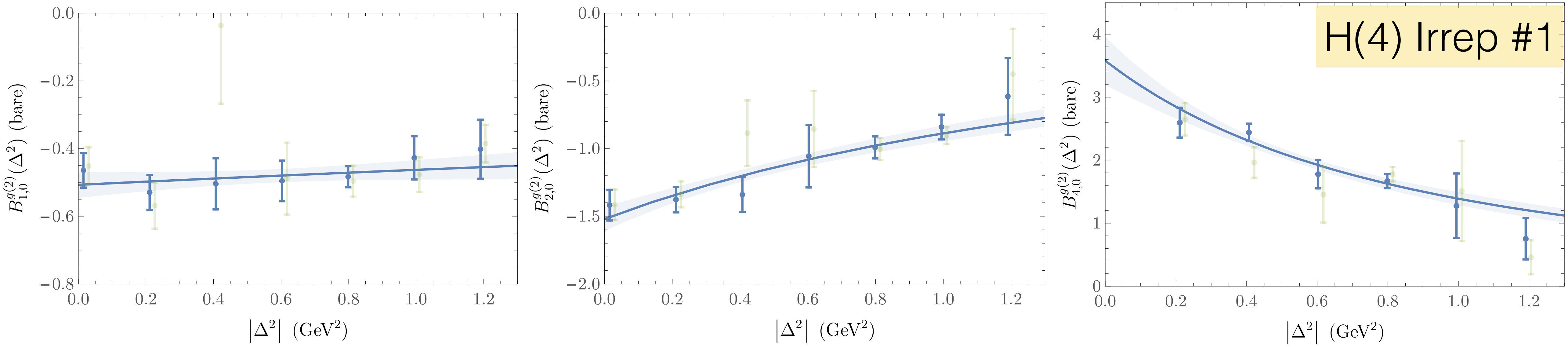


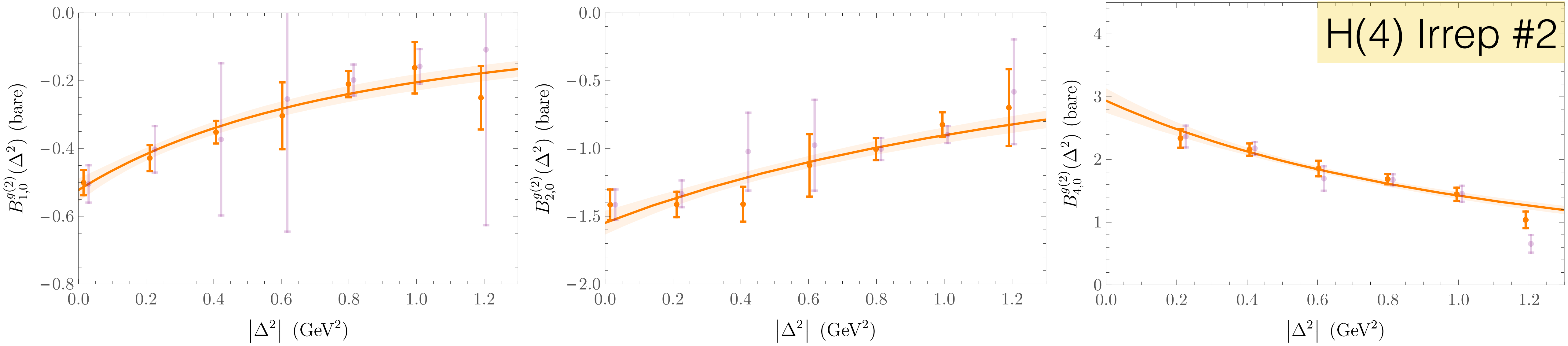
Different H(4) irreps

Spin-Indep. Gluon GFFs

W. Detmold, PES, PRD 94 (2016), 014507 + W. Detmold, D. Pefkou, PES PRD 95 (2017), 114515

Three GFFs can be resolved for all momenta





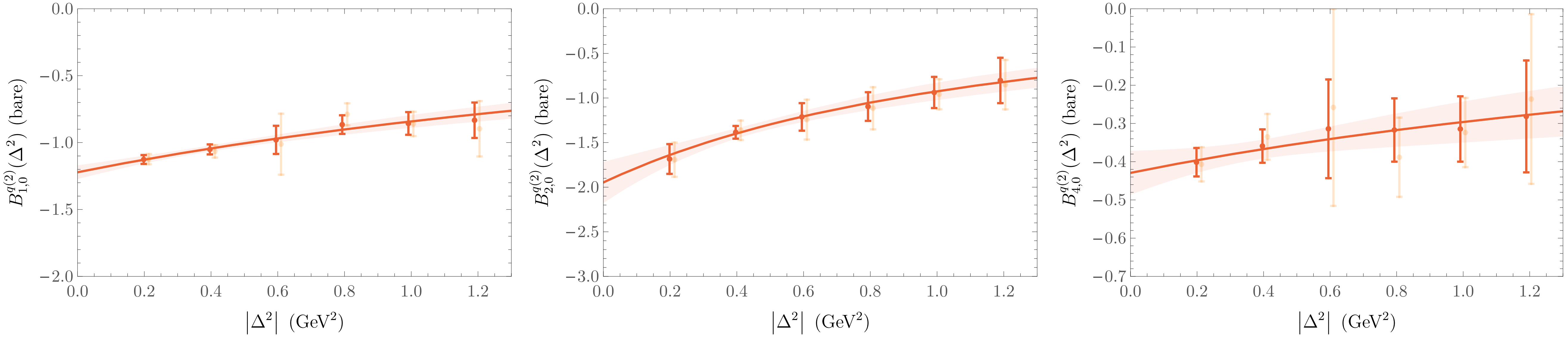
Spin-Indep. Quark GFFs

W. Detmold, PES, PRD 94 (2016), 014507 + W. Detmold, D. Pefkou, PES PRD 95 (2017), 114515

Three GFFs can be resolved for all momenta

GFF decomposition has precisely the same structure as in the spin-independent gluon case

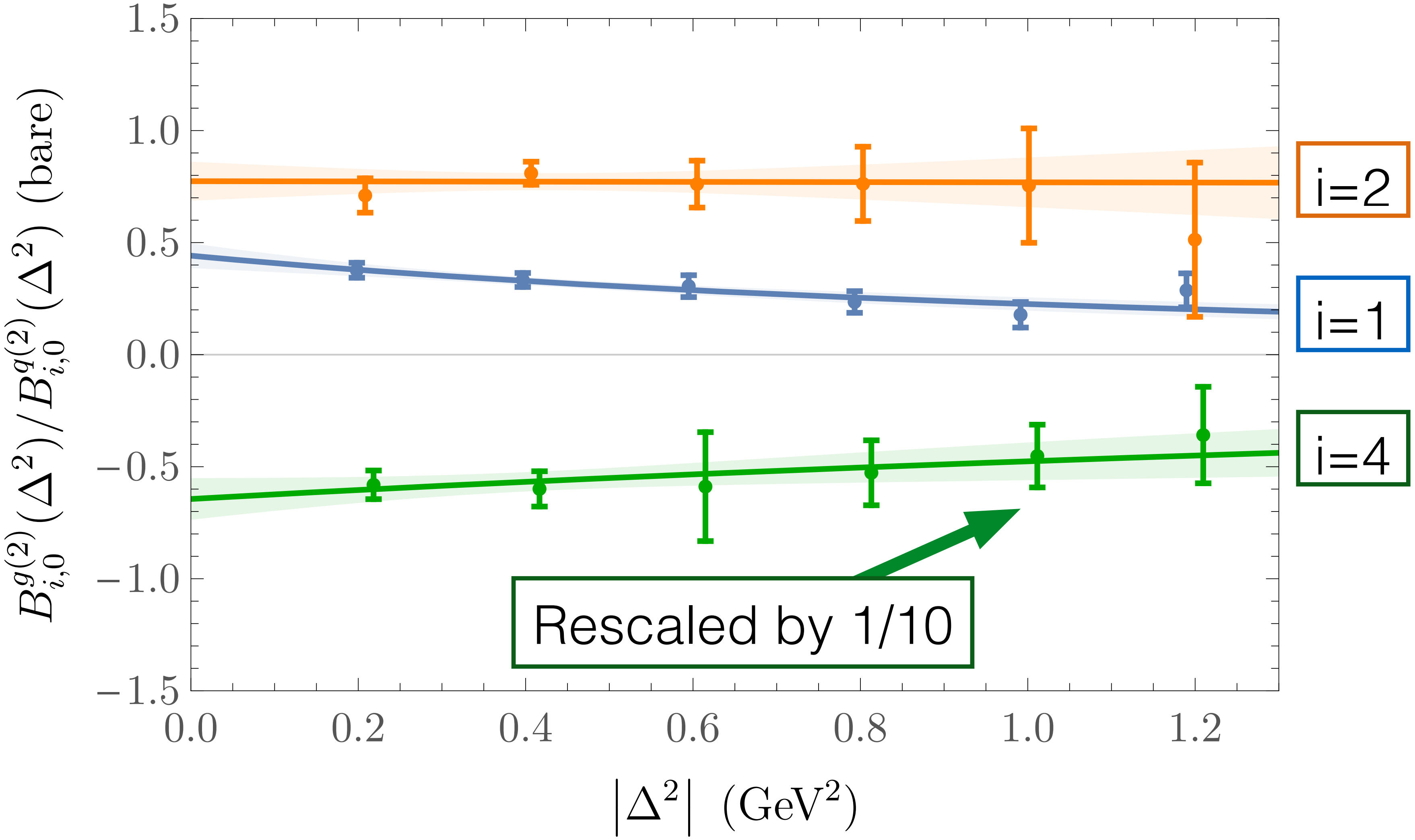
One H(4) irrep.



Same three GFFs that are resolved in the gluon case

Quark and Gluon GFFs

Ratio of gluon to quark unpolarised GFFs



Gluon vs quark radius is a non-trivial question

Much more complicated than intuitive pictures

Gluon Structure from LQCD

1

How much do gluons contribute to the proton's

- Momentum
- Mass
- Spin

2

What is the 3D gluon distribution of a proton

- PDFs
- TMDs
- GPDs
- 'Gluon radius'

3

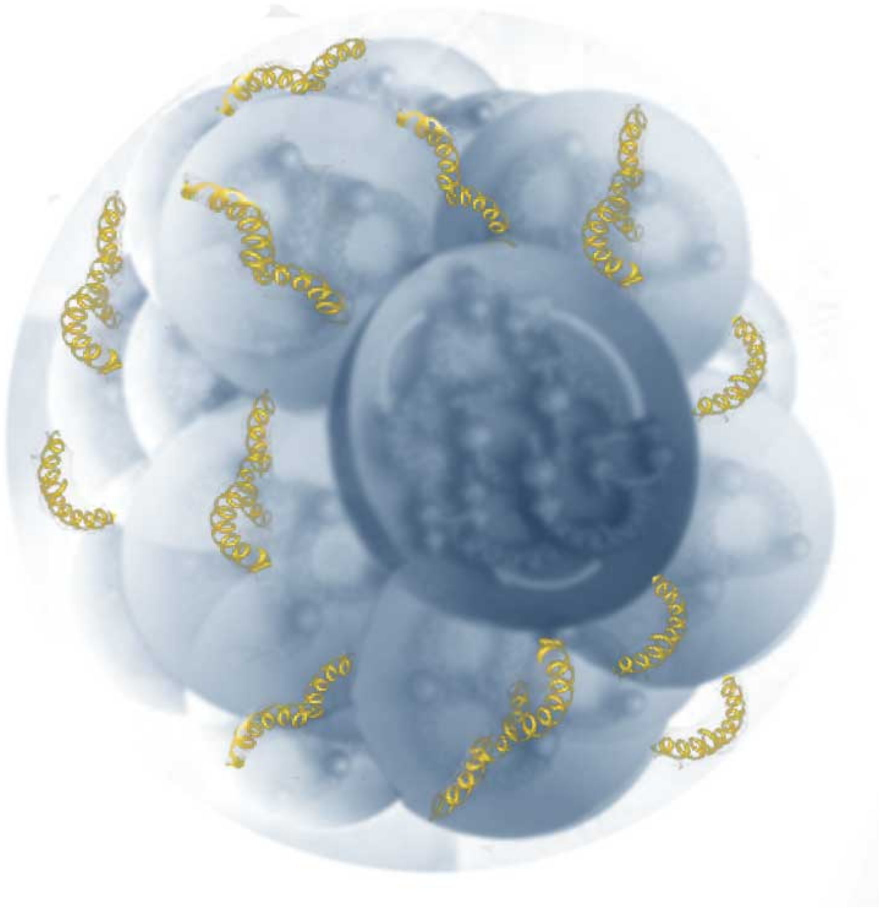
How is the gluon structure of a proton modified in a nucleus

- Gluonic 'EMC' effect
- 'Exotic' glue

Glue structure of nuclei

- **First investigations:**
  ϕ meson
  simplest spin-1 system (has fwd limit gluon transversity)

- **Phenomenologically relevant:**
  nucleon, nuclei



Gluon structure - nuclei

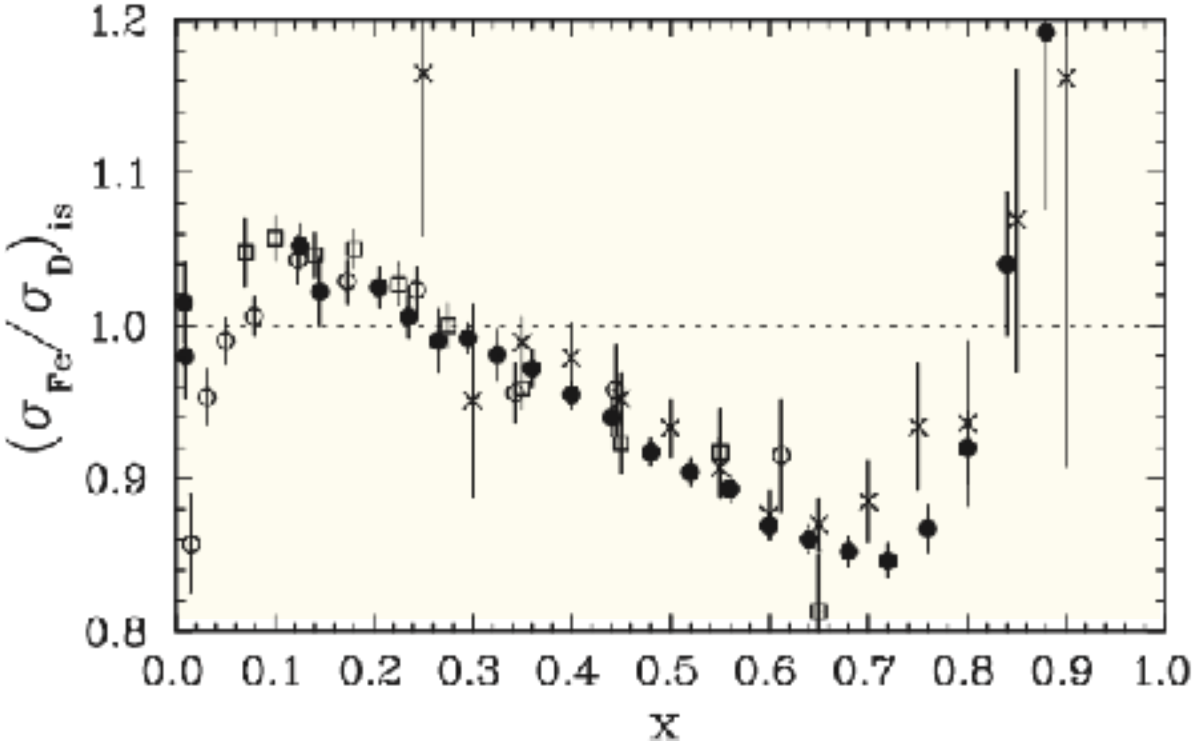
European Muon Collaboration (1983):

Modification of per-nucleon cross section of nucleons bound in nuclei

Precise understanding of nuclear targets essential for DUNE expt: extraction of neutrino mass hierarchy, mixing parameters

Ratio of structure function F_2 per nucleon for iron and deuterium

$$\mathrm{F}_2(x, Q^2) = \sum_{\mathrm{q=u,d,s..}} x z_{\mathrm{q}}^2 \left[\mathrm{q}(x, Q^2) + \bar{\mathrm{q}}(x, Q^2)\right]$$

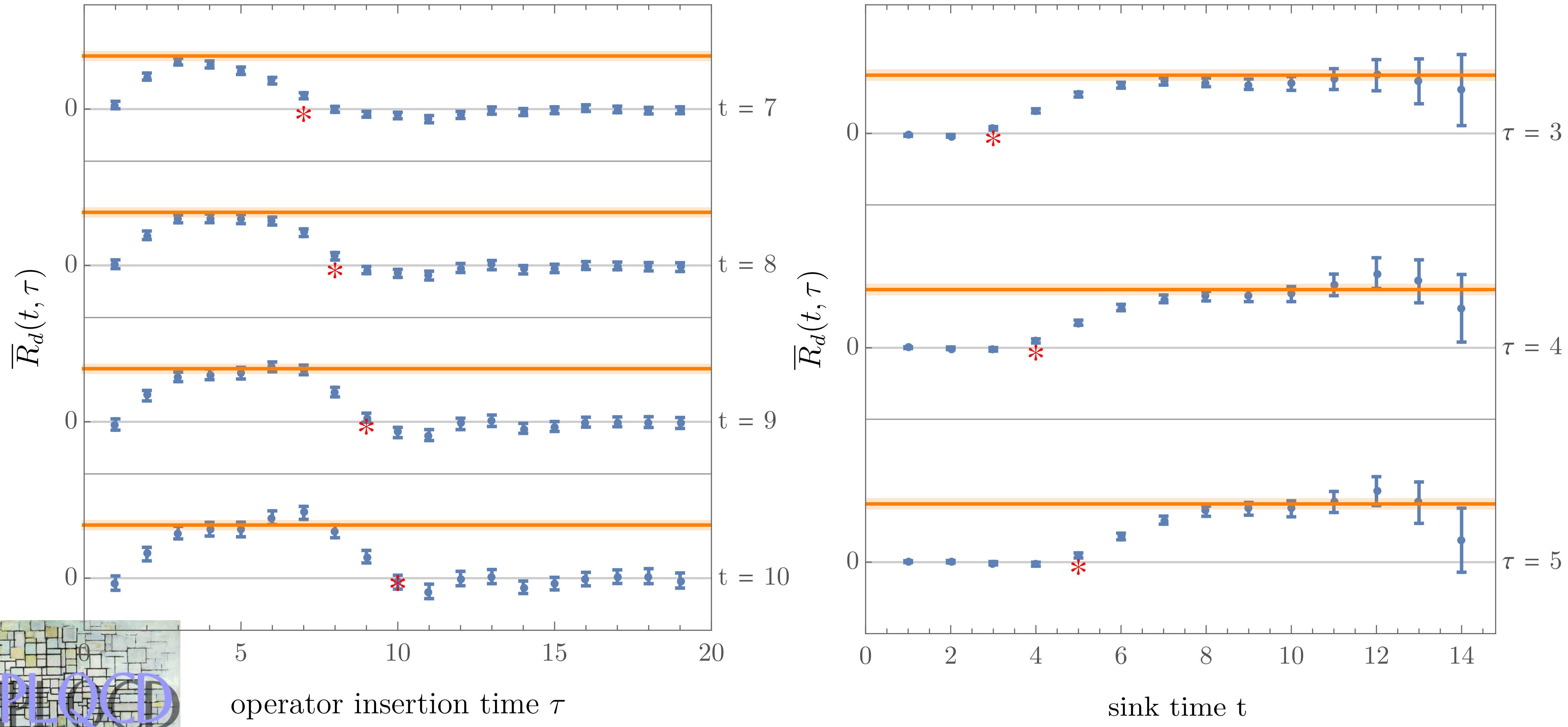


What is the gluonic analogue of the EMC effect?

Nuclear glue, m_π ~450 MeV

NPLQCD Collaboration, arXiv:1709.00395

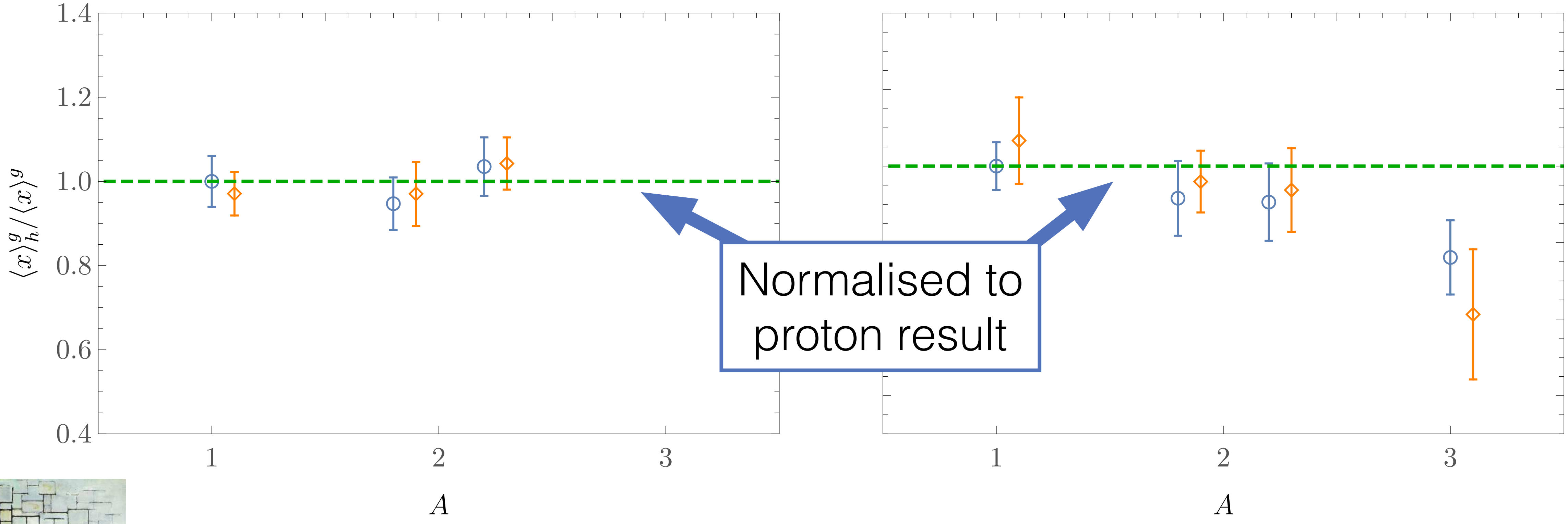
Signals for spin-independent gluon operator in deuteron



Gluon momentum fraction

NPLQCD Collaboration, arXiv:1709.00395

- Matrix elements of the **Spin-independent gluon operator** in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators



Normalised to proton result

m_π ~450 MeV

m_π ~800 MeV

Gluonic Transversity

Double helicity flip structure function Δ(x,Q^2)

Jaffe and Manohar, "Nuclear Gluonometry" Phys. Lett. B223 (1989) 218

- **Hadrons:** Gluonic Transversity (parton model interpretation)

$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}Q^2\,x^2\int_x^1\frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2)-g_{\hat{y}}(x,Q^2)\right]$$

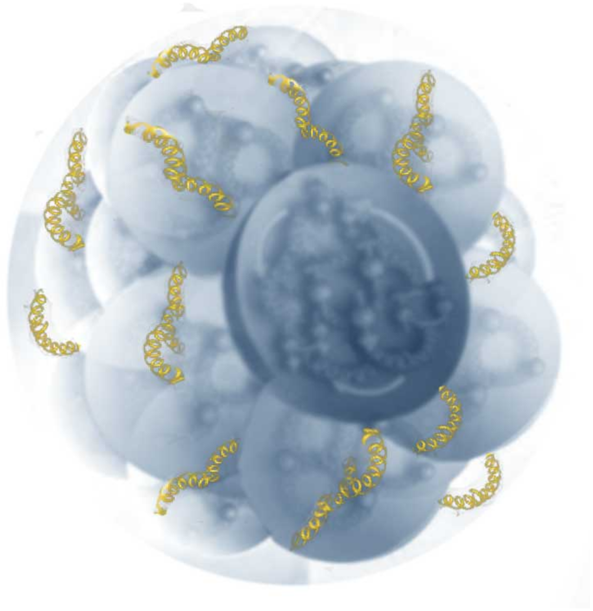
$g_{\hat{x},\hat{y}}(y,Q^2)$: probability of finding a gluon with momentum fraction y linearly polarised in $\hat{x}$, $\hat{y}$ direction

- **Nuclei:** Exotic Glue

gluons not associated with individual nucleons in nucleus

$$\langle p|\mathcal{O}|p\rangle = 0$$

$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$



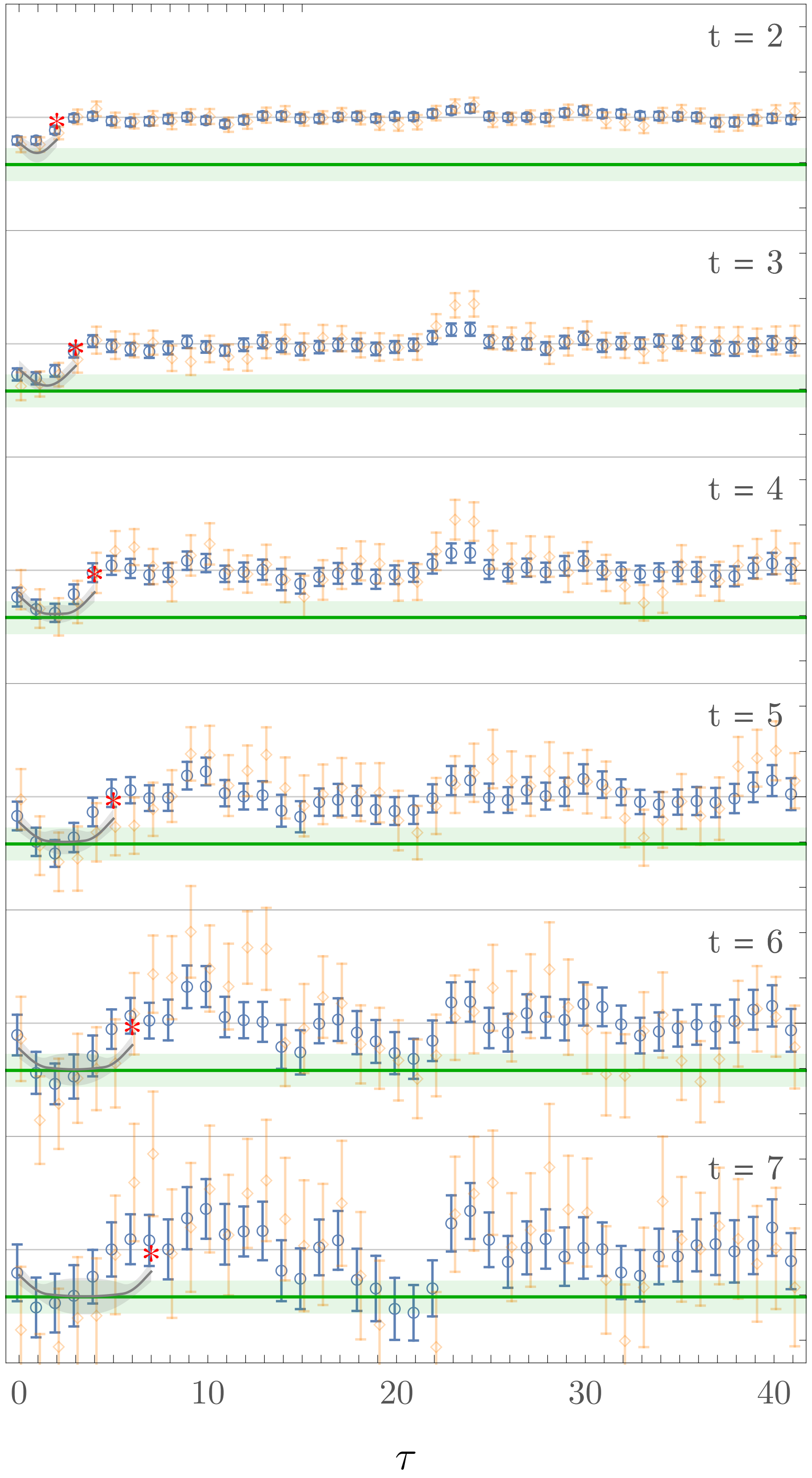
Non-nucleonic Glue in Deuteron

NPLQCD Collaboration, arXiv:1709.00395

First moment of gluon transversity distribution in the deuteron,
m_π ~800 MeV

- **First evidence for non-nucleonic gluon contributions to nuclear structure**
- Magnitude relative to momentum fraction as expected from large-Nc

Ratio of 3pt and 2pt functions

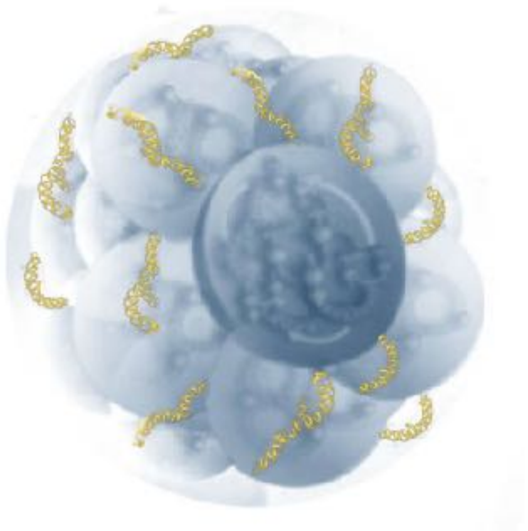


Gluon structure circa 2025

- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
  - Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)
  - $\Delta(x,Q^2)$ has an interesting role

    Purely gluonic

    Non-nucleonic: directly probe nuclear effects



  - Compare quark and gluon distributions in hadrons and nuclei
- Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature