

# Realistic calculations of GPDs of light nuclei



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# Outline

**The nucleus:** *“a Lab for QCD fundamental studies”*

**Realistic calculations:** use of few-body wave functions, exact solutions of the Schrödinger equation, with realistic  $NN$  potentials (Av18, Nijmegen, CD Bonn) and 3-body forces

**GPDs of light nuclei** (deuteron aside):

 **1 - GPDs for  $^3\text{He}$ :**

A complete impulse approximation realistic study is reviewed  
(SS PRC 2004, PRC 2009; M. Rinaldi and S.S., PRC 2012, PRC 2013)  
No data; proposals? Prospects at JLAB-12 and EIC;

 **2 - DVCS off  $^4\text{He}$ :**

data available from JLab at 6 GeV; new data expected at 12 GeV;  
our calculation: planned, in progress; not yet realistic

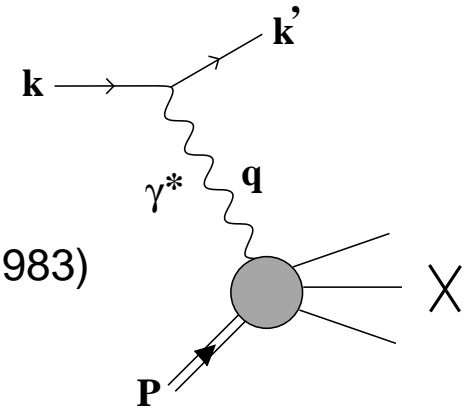
**My point:** *I do not know if realistic calculations will describe the data. I think they are necessary to distinguish effects due to “conventional” or to “exotic” nuclear structure*



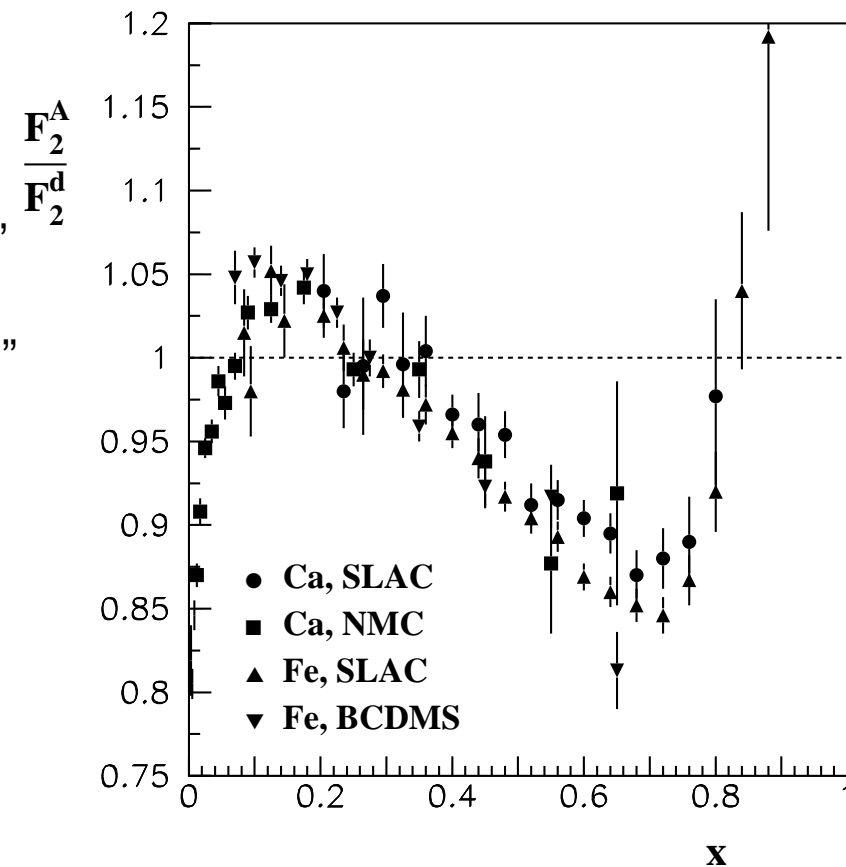
# EMC effect in A-DIS

Measured in  $A(e, e')X$ , ratio of  $A$  to  $d$  SFs  $F_2$  (EMC Coll., 1983)

One has  $0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$



- $x \leq 0.1$  "Shadowing region"
- $0.1 \leq x \leq 0.2$  "Enhancement region"
- $0.2 \leq x \leq 0.8$  "EMC (binding) region"
- $0.8 \leq x \leq 1$  "Fermi motion region"
- $x \geq 1$  "TERRA INCOGNITA"



# EMC effect: explanations?

In general, with a few parameters any model explains the data:

EMC effect = “Everyone’s Model is Cool” (G. Miller)

Situation: basically not understood. Very unsatisfactory. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:

- the knowledge of nuclear parton distributions is crucial for the data analysis of heavy ions collisions;
- the partonic structure of the neutron is measured with nuclear targets and several QCD sum rules involve the neutron information (Bjorken SR, for example): importance of Nuclear Physics for QCD

## Inclusive measurements cannot distinguish between models

One has to go beyond

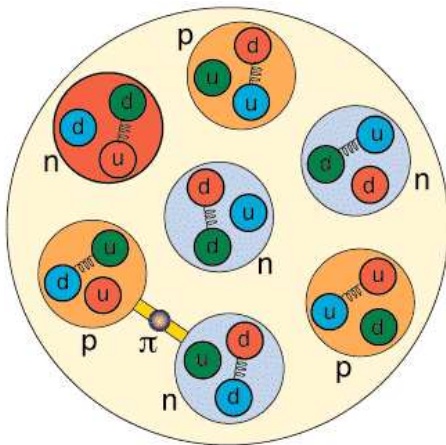
(R. Dupré and S.S., EPJA 52 (2016) 159)

- **SIDIS (TMDs)** - not treated here
- **Hard Exclusive Processes (GPDs)**



# EMC effect: way out?

**Question:** Which of these transverse sections is more similar to that of a nucleus?



To answer, we should perform a *tomography...*

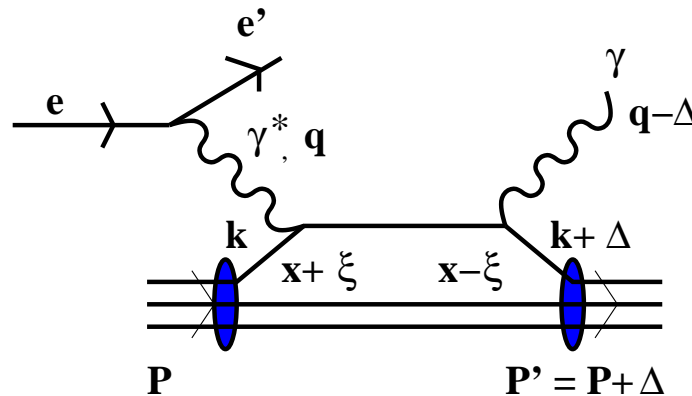
**We can!** M. Burkardt, PRD 62 (2000) 07153

**Answer:** Deeply Virtual Compton Scattering  
& Generalized Parton Distributions (GPDs)



# GPDs: Definition (X. Ji PRL 78 (97) 610)

For a  $J = \frac{1}{2}$  target,  
in a hard-exclusive process,  
(handbag approximation)  
such as (coherent) DVCS:



the GPDs  $H_q(x, \xi, \Delta^2)$  and  $E_q(x, \xi, \Delta^2)$  are introduced:

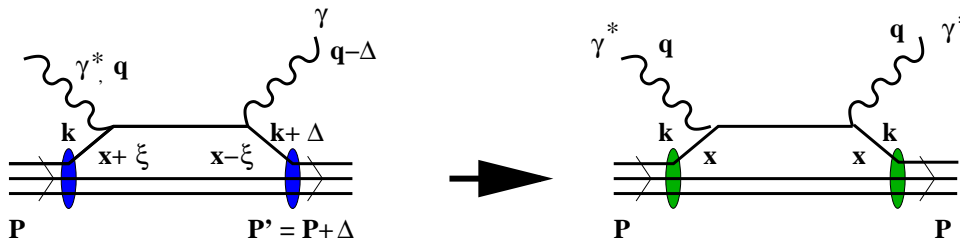
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma^\mu \psi_q(\lambda n/2) | P \rangle = H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P) + E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \dots$$

- $\Delta = P' - P$ ,  $q^\mu = (q_0, \vec{q})$ , and  $\bar{P} = (P + P')^\mu / 2$
- $x = k^+ / P^+$ ;  $\xi = \text{"skewness"} = -\Delta^+ / (2\bar{P}^+)$
- $x \leq -\xi \longrightarrow$  GPDs describe *antiquarks*;  
 $-\xi \leq x \leq \xi \longrightarrow$  GPDs describe  *$q\bar{q}$  pairs*;  $x \geq \xi \longrightarrow$  GPDs describe *quarks*



# GPDs: constraints

- when  $P' = P$ , i.e.,  $\Delta^2 = \xi = 0$ , one recovers the usual PDFs:



$$H_q(x, \xi, \Delta^2) \implies H_q(x, 0, 0) = q(x); \quad E_q(x, 0, 0) \text{ unknown}$$

- the  $x$ -integration yields the  $q$ -contribution to the Form Factors (ffs)

$$\int dx \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma^\mu \psi_q(\lambda n/2) | P \rangle =$$

$$\int dx H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P) + \int dx E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \dots$$

$$\implies \int dx H_q(x, \xi, \Delta^2) = F_1^q(\Delta^2) \quad \int dx E_q(x, \xi, \Delta^2) = F_2^q(\Delta^2)$$

$$\implies \text{Defining } \boxed{\tilde{G}_M^q = H_q + E_q} \quad \text{one has } \int dx \tilde{G}_M^q(x, \xi, \Delta^2) = G_M^q(\Delta^2)$$

# GPDs: a unique tool...

- not only 3D structure, at **parton level**; many other aspects, e.g., contribution to the solution to the “**Spin Crisis**” (J.Ashman et al., EMC collaboration, PLB 206, 364 (1988)), yielding parton total angular momentum...

... but also an experimental challenge:

- Hard exclusive process  $\rightarrow$  small  $\sigma$ ;

- Difficult extraction:

$$T_{\text{DVCS}} \propto CFF \propto \int_{-1}^1 dx \frac{H_q(x, \xi, \Delta^2)}{x - \xi + i\epsilon} + \dots$$

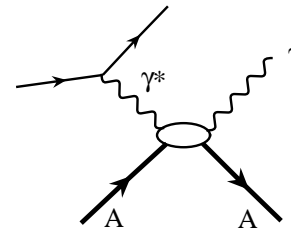
- Competition with the **BH** process! ( $\sigma$  asymmetries measured).

$$d\sigma \propto |T_{\text{DVCS}}|^2 + |T_{\text{BH}}|^2 + 2 \Re\{T_{\text{DVCS}}T_{\text{BH}}^*\}$$

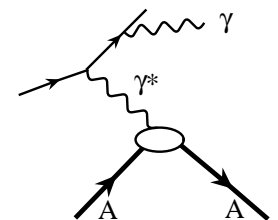
Nevertheless, for the proton, we have results:

(Guidal et al., Rep. Prog. Phys. 2013...

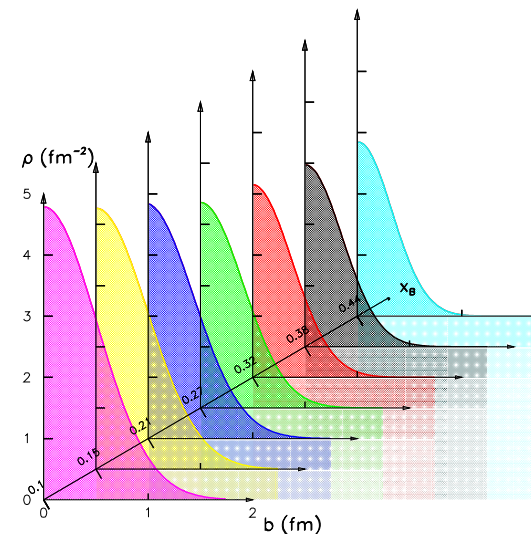
Dupré, Guidal, Niccolai, Vanderhaeghen arXiv:1704.07330 [hep-ph])



DVCS



BH

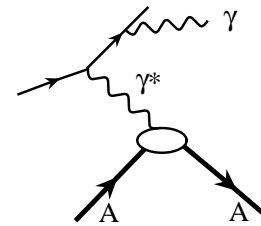
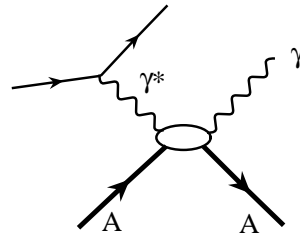
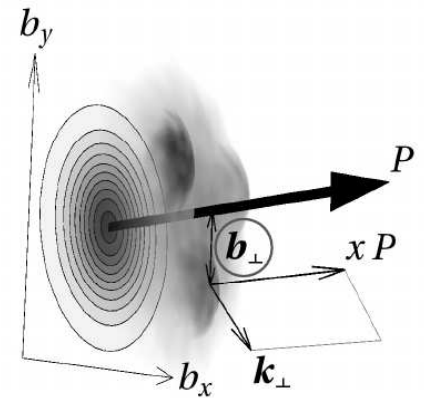




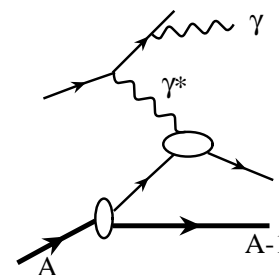
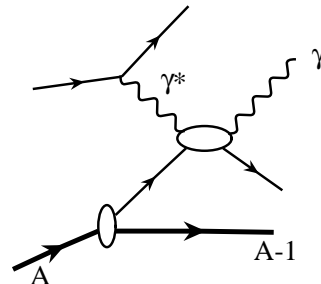
# Nuclei and DVCS tomography

In impact parameter space, GPDs are *densities*:

$$\rho_q(x, \vec{b}_\perp) = \int \frac{d\vec{\Delta}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H^q(x, 0, \Delta^2)$$



Coherent DVCS: nuclear tomography

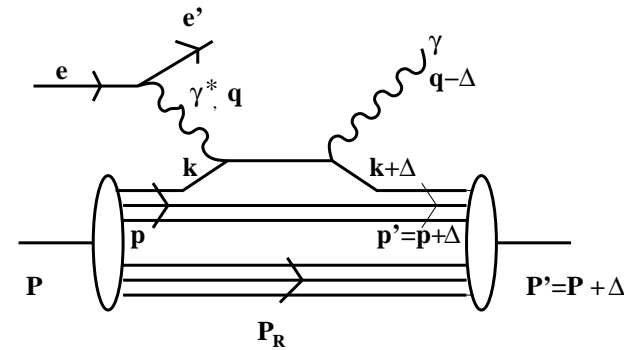


Incoherent DVCS: tomography of bound nucleons: realization of the EMC effect



# Nuclei: why? - not only tomography

ONE of the reasons is understood by studying coherent DVCS in the I.A. to the handbag contribution:



In a symmetric frame ( $\bar{p} = (p + p')/2$ ) :

$$k^+ = (x + \xi)\bar{P}^+ = (x' + \xi')\bar{p}^+ ,$$

$$(k + \Delta)^+ = (x - \xi)\bar{P}^+ = (x' - \xi')\bar{p}^+ ,$$

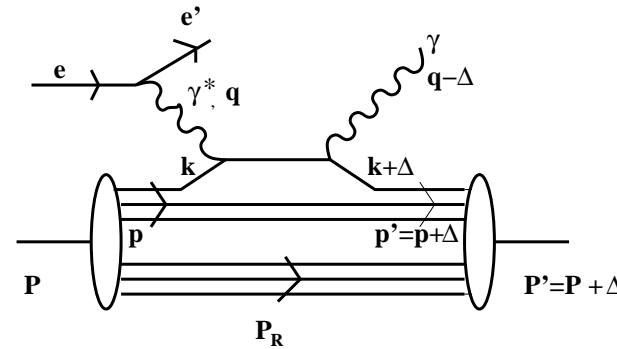
one has, for a given GPD

$$GPD_q(x, \xi, \Delta^2) \simeq \int \frac{dz^-}{4\pi} e^{ix\bar{P}^+z^-} {}_A \langle P' S' | \hat{O}_q^\mu | P S \rangle_A |_{z^+=0, z_\perp=0} .$$



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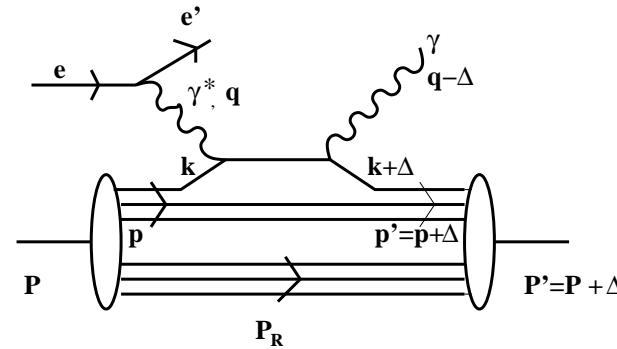
$$GPD_q(x, \xi, \Delta^2) \simeq \int \frac{dz^-}{4\pi} e^{ix\bar{P}^+z^-} {}_A\langle P' S' | \hat{O}_q^\mu | P S \rangle_A |_{z^+=0, z_\perp=0} .$$

By properly inserting complete sets of states for the interacting nucleon and the recoiling system :



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In a symmetric frame ( $\bar{p} = (p + p')/2$ ):

$$\begin{aligned} k^+ &= (x + \xi)\bar{P}^+ = (x' + \xi')\bar{p}^+ , \\ (k + \Delta)^+ &= (x - \xi)\bar{P}^+ = (x' - \xi')\bar{p}^+ , \end{aligned}$$

one has, for a given GPD

$$\begin{aligned} GPD_q(x, \xi, \Delta^2) &= \int \frac{dz^-}{4\pi} e^{ix'\bar{p}^+z^-} \langle P'S' | \sum_{\vec{P}'_R, S'_R, \vec{p}', s'} \{ |P'_R S'_R\rangle |p' s'\} \} \langle P'_R S'_R | \\ &\langle p' s' | \hat{O}_q^\mu \sum_{\vec{P}_R, S_R, \vec{p}, s} \{ |P_R S_R\rangle |ps\rangle \} \{ \langle P_R S_R | \langle ps | \} |PS\rangle , \end{aligned}$$

and, since  $\{ \langle P_R S_R | \langle ps | \} |PS\rangle = \langle P_R S_R, ps | PS\rangle (2\pi)^3 \delta^3(\vec{P} - \vec{P}_R - \vec{p}) \delta_{S, S_R s}$ ,



# Why nuclei?

a convolution formula can be obtained (S.S. PRC 70, 015205 (2004)):

$$H_q^A(x, \xi, \Delta^2) \simeq \sum_N \int \frac{d\bar{z}}{\bar{z}} h_N^A(\bar{z}, \xi, \Delta^2) H_q^N\left(\frac{x}{\bar{z}}, \frac{\xi}{\bar{z}}, \Delta^2\right)$$

in terms of  $H_q^N(x', \xi', \Delta^2)$ , the GPD of the free nucleon  $N$ , and of the light-cone off-diagonal momentum distribution:

$$h_N^A(z, \xi, \Delta^2) = \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta\left(\bar{z} - \frac{\bar{p}^+}{\bar{P}^+}\right)$$

where  $P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E)$ , is the one-body off-diagonal spectral function for the nucleon  $N$  in the nucleus,

$$\begin{aligned} P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) &= \frac{1}{(2\pi)^3} \frac{1}{2} \sum_M \sum_{R,s} \langle \vec{P}' M | (\vec{P} - \vec{p}) S_R, (\vec{p} + \vec{\Delta}) s \rangle \\ &\times \langle (\vec{P} - \vec{p}) S_R, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_R^*). \end{aligned}$$



# Why nuclei?

The obtained expressions have the correct **limits**:

- the **x-integral** gives the f.f.  $F_q^A(\Delta^2)$  in **I.A.**:

$$\int dx H_q^A(x, \xi, \Delta^2) = F_q^N(\Delta^2) \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) = F_q^A(\Delta^2)$$

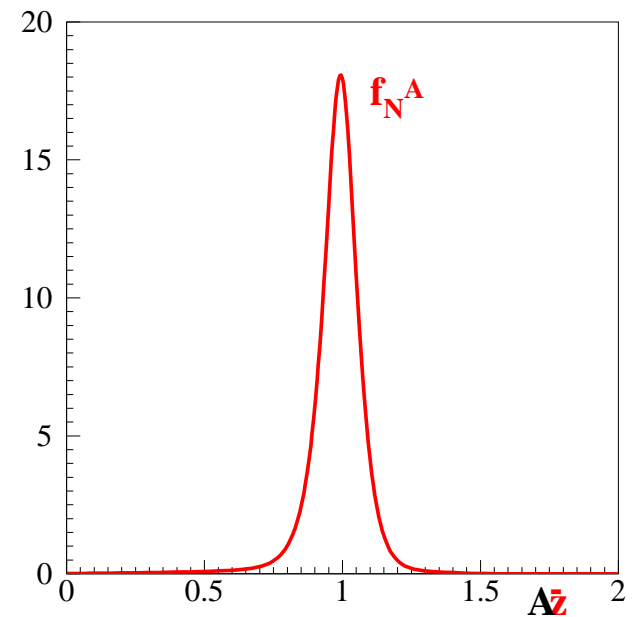
- **forward limit** (standard DIS):

$$q^A(x) \simeq \sum_N \int_x^1 \frac{d\tilde{z}}{\tilde{z}} f_N^A(\tilde{z}) q^N\left(\frac{x}{\tilde{z}}\right)$$

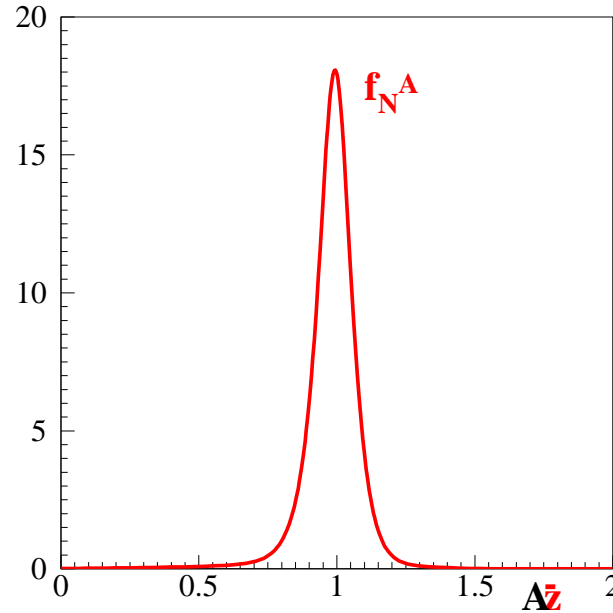
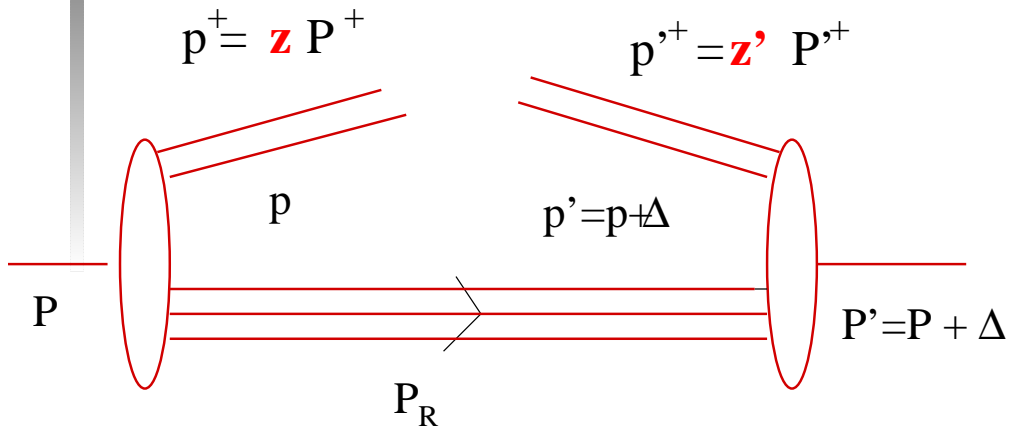
with the **light-cone momentum distribution**:

$$f_N^A(\tilde{z}) = \int dE d\vec{p} P_N^A(\vec{p}, E) \delta\left(\tilde{z} - \frac{p^+}{P^+}\right),$$

which is strongly peaked around  $A\tilde{z} = 1$ :



# Why nuclei?



Since  $z - z' = -x_B(1 - z)/(1 - x_B)$ ,  $\xi \simeq x_B/(2 - x_B)$  can be tuned to have  $z - z'$  larger than the width of the narrow nuclear light-cone momentum distribution  $f_N^A(\bar{z} = (z + z')/2)$ : in this case IA predicts a *vanishing* GPD, *at small  $x_B$* .

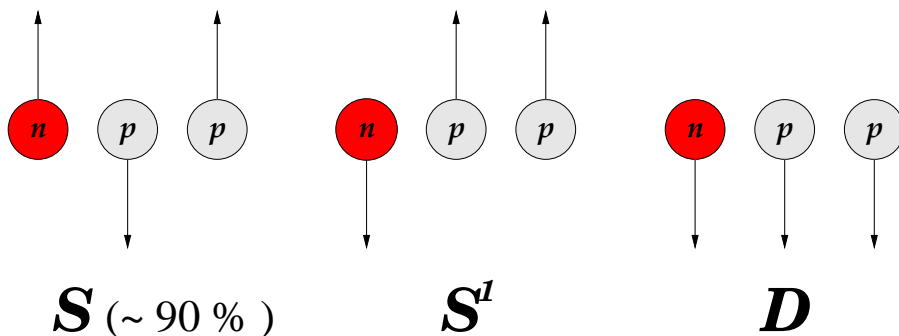
If DVCS were observed at this kinematics, *exotic* effects beyond IA, *non-nucleonic degrees of freedom*, would be pointed out (Berger, Cano, Diehl and Pire, PRL 87 (2001) 142302)

Similar effect predicted in DIS at  $x_B > 1$ , where DIS data are not accurate enough.



# GPDs for ${}^3\text{He}$ : why?

- ${}^3\text{He}$  is theoretically well known. Even a relativistic treatment may be implemented.
- ${}^3\text{He}$  has been used extensively as an effective neutron target, especially to unveil the spin content of the free neutron, due to its peculiar spin structure:



In  $S$ -wave  
 ${}^3\vec{H}e = \vec{n}$  !

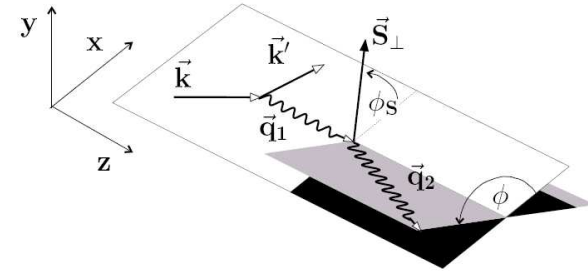
- ${}^3\text{He}$  always promising when the neutron angular momentum properties have to be studied. To what extent for OAM?
- ${}^3\text{He}$  is a unique target for GPDs studies. Examples:
  - \* access to the neutron information in coherent processes
  - \* heavier targets do not allow refined theoretical treatments. Test of the theory
  - \* Between  ${}^2\text{H}$  (“not a nucleus”) and  ${}^4\text{He}$  (a true one). Not isoscalar!





# Extracting GPDs: ${}^3\text{He} \simeq p$

One measures asymmetries:  $A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$



- Polarized beam, unpolarized target:

$$\Delta\sigma_{LU} \simeq \sin\phi \left[ F_1 \mathcal{H} + \xi(F_1 + F_2) \tilde{\mathcal{H}} + (\Delta^2 F_2 / M^2) \mathcal{E} / 4 \right] d\phi \quad \Rightarrow \quad H$$

- Unpolarized beam, longitudinally polarized target:

$$\Delta\sigma_{UL} \simeq \sin\phi \left\{ F_1 \tilde{\mathcal{H}} + \xi(F_1 + F_2) [\mathcal{H} + \xi / (1 + \xi) \mathcal{E}] \right\} d\phi \quad \Rightarrow \quad \tilde{H}$$

- Unpolarized beam, transversely polarized target:

$$\Delta\sigma_{UT} \simeq \cos\phi \sin(\phi_S - \phi) \left[ \Delta^2 (F_2 \mathcal{H} - F_1 \mathcal{E}) / M^2 \right] d\phi \quad \Rightarrow \quad E$$

To evaluate cross sections, e.g. for experiments planning, one needs  $H, \tilde{H}, E$

This is what we have calculated for  ${}^3\text{He}$ .  $H$  alone, already very interesting.



# GPDs of $^3\text{He}$ in IA

$H_q^A$  can be obtained in terms of  $H_q^N$  (S.S. PRC 70, 015205 (2004), PRC 79, 025207 (2009)):

$$H_q^A(x, \xi, \Delta^2) = \sum_N \int dE \int d\vec{p} \sum_S \sum_s P_{SS,ss}^N(\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} H_q^N(x', \Delta^2, \xi'),$$

and  $\tilde{G}_M^{3,q}$  in terms of  $\tilde{G}_M^{N,q}$  (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

$$\tilde{G}_M^{3,q}(x, \Delta^2, \xi) = \sum_N \int dE \int d\vec{p} \left[ P_{+-,+ -}^N - P_{+-,- +}^N \right] (\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} \tilde{G}_M^{N,q}(x', \Delta^2, \xi'),$$

where  $P_{SS',ss'}^N(\vec{p}, \vec{p}', E)$  is the one-body, spin-dependent, off-diagonal spectral function for the nucleon  $N$  in the nucleus,

$$P_{SS',ss'}^N(\vec{p}, \vec{p}', E) = \frac{1}{(2\pi)^6} \frac{M\sqrt{ME}}{2} \int d\Omega_t \sum_{s_t} \langle \vec{P}' S' | \vec{p}' s', \vec{t}_{s_t} \rangle_N \langle \vec{p} s, \vec{t}_{s_t} | \vec{P} S \rangle_N,$$

evaluated by means of a **realistic** treatment based on **Av18 wave functions**

(“CHH” method in A. Kievsky *et al* NPA 577, 511 (1994); Av18 + UIX overlaps in E. Pace *et. al*, PRC 64, 055203 (2001)).

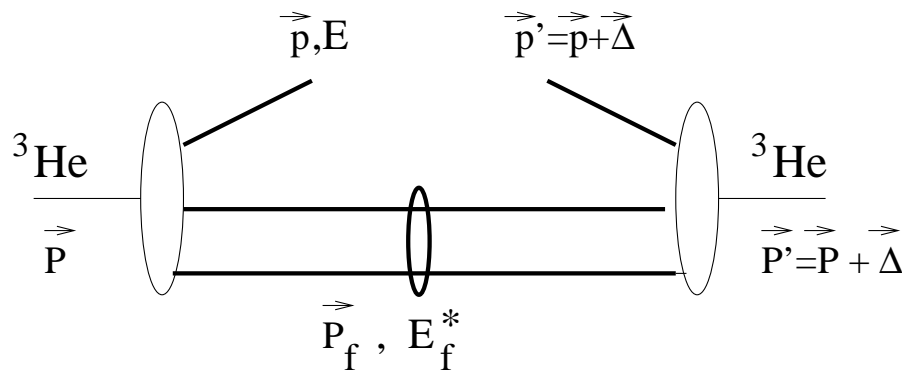
**Nucleon GPDs** given by an old version of the VGG model

(VGG 1999,  $x$ – and  $\Delta^2$ – dependencies factorized)



# A few words about $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$ :

$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \frac{1}{(2\pi)^3} \frac{1}{2} \sum_M \sum_{f,s} \langle \vec{P}' M | (\vec{P} - \vec{p}) S_f, (\vec{p} + \vec{\Delta}) s \rangle \\ \times \langle (\vec{P} - \vec{p}) S_f, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_f^*).$$



- the two-body recoiling system can be either the deuteron or a scattering state;
- when a deeply bound nucleon, with high removal energy  $E = E_{min} + E_f^*$ , leaves the nucleus, the recoiling system is left with high excitation energy  $E_f^*$ ;
- the three-body bound state and the two-body bound or scattering state are evaluated within the same (Av18) interaction: the extension of the treatment to heavier nuclei is extremely difficult
- correlations of any kind naturally present

# The calculation has the correct limits:

1 - Forward limit: the ratio:

$$R_q(x, 0, 0) = \frac{H_q^3(x, 0, 0)}{2H_q^p(x, 0, 0) + H_q^n(x, 0, 0)}$$

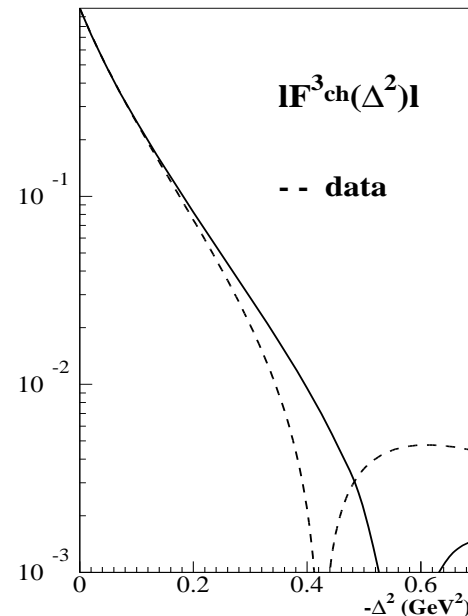
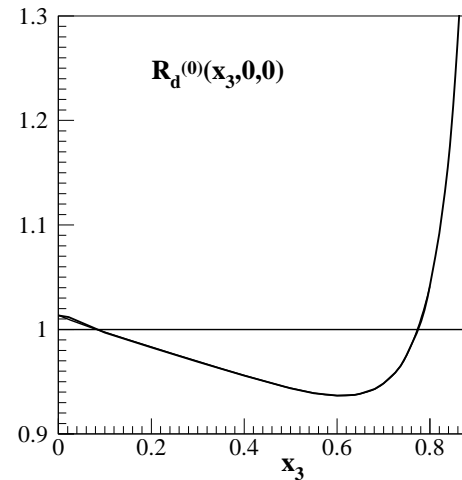
$$= \frac{q^3(x)}{2q^p(x) + q^n(x)}$$

shows an EMC-like behavior;

2 - Charge F.F.:

$$\sum_q \int dx H_q^3(x, \xi, \Delta^2) = F^3(\Delta^2)$$

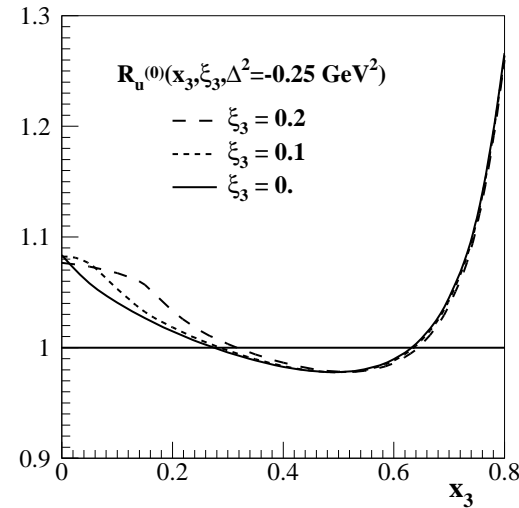
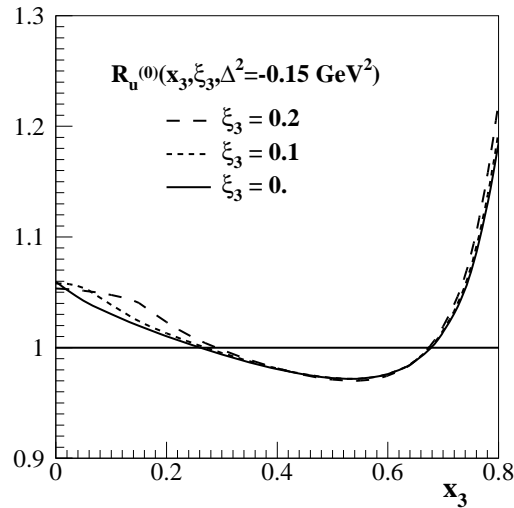
in good agreement with data in the region relevant to the coherent process,  $\Delta^2 \ll 0.25 \text{ GeV}^2$ .



# Nuclear effects - general features



Nuclear effects grow with  $\xi$  at fixed  $\Delta^2$ , and with  $\Delta^2$  at fixed  $\xi$ :



$$R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2)}$$

$$H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi) F_q^3(\Delta^2)$$

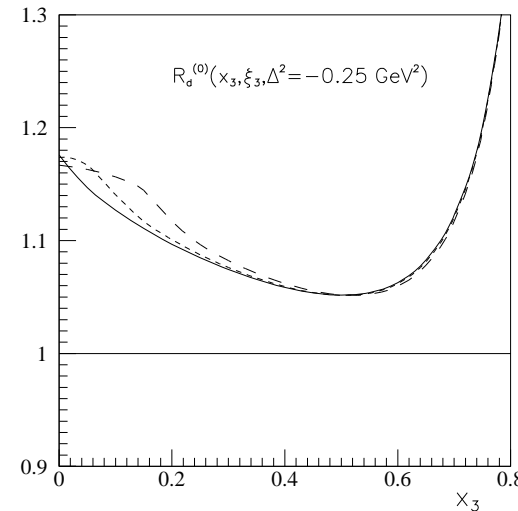
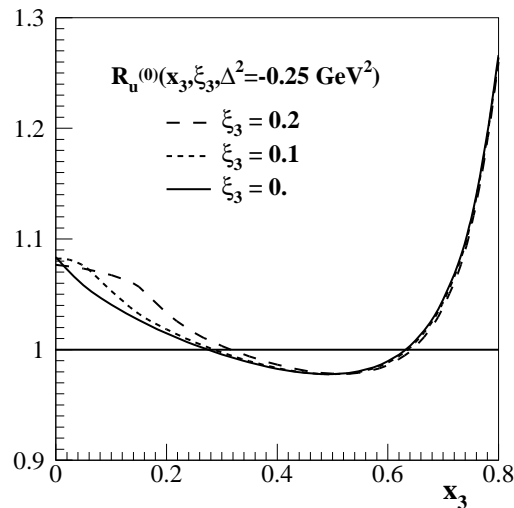
$R_q^{(0)}(x, \xi, \Delta^2)$  would be one if there were no nuclear effects;  
as it is found also for the deuteron, there is **no factorization** into terms  
dependent separately on  $\Delta^2$  and  $x, \xi$  (the factorization hypotheses has been  
used to estimate nuclear **GPDs**), even if the nucleonic model is factorized



# Nuclear effects - flavor dependence



Nuclear effects are bigger for the  $d$  flavor rather than for the  $u$  flavor:



$$R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2)}$$

$$H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi) F_q^3(\Delta^2)$$

$R_q^{(0)}(x, \xi, \Delta^2)$  would be one if there were no nuclear effects;

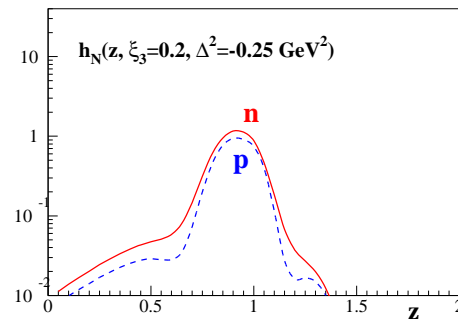
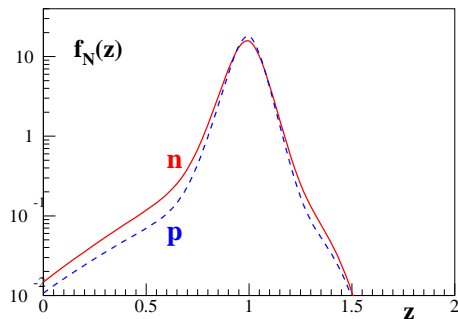
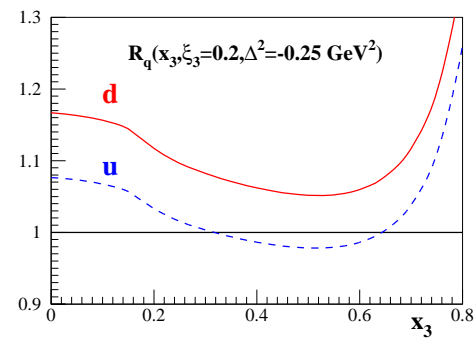
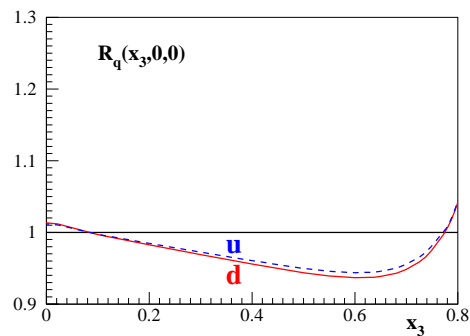


This is a typical **conventional, IA** effect (spectral functions are different for  $p$  and  $n$  in  ${}^3\text{He}$ , not isoscalar!); if (not) found, clear indication on the reaction mechanism of **DIS off nuclei**. Not seen in  ${}^2\text{H}$ ,  ${}^4\text{He}$



# Nuclear effects - flavor dependence

- The **d** and **u** distributions follow the pattern of the **neutron** and **proton** light-cone momentum distributions, respectively:

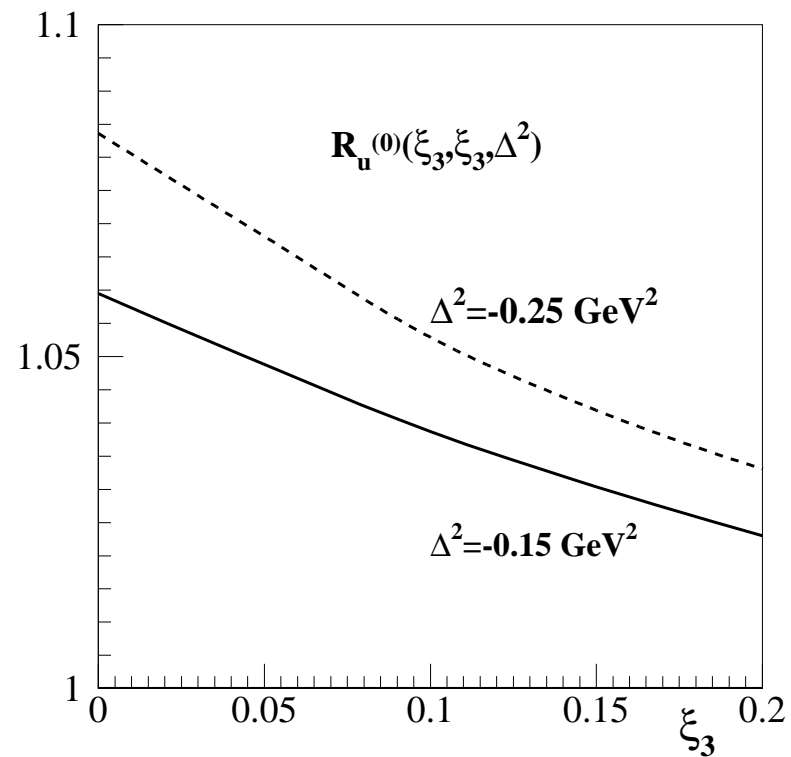


- How to perform a flavor separation? Take **the triton  ${}^3\text{H}$**  !  
Possible (see MARATHON@JLab). Possible for DVCS (ALERT).  
Studied in **S.S. Phys. Rev. C 79 (2009) 025207**

$$H_t, H_H \rightarrow H_u^H \simeq H_d^t, H_d^H \simeq H_u^t \text{ in the valence region...}$$

# Nuclear effects @ $x = \xi$

- Nuclear effects are large also in the important region  $x = \xi$ :





# Nuclear effects - the binding

General IA formula:  $H_q^A(x, \xi, \Delta^2) \simeq \sum_N \int_x^1 \frac{dz}{z} h_N^A(z, \xi, \Delta^2) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right)$

where

$$h_N^A(z, \xi, \Delta^2) = \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta\left(z + \xi - \frac{p^+}{P^+}\right)$$

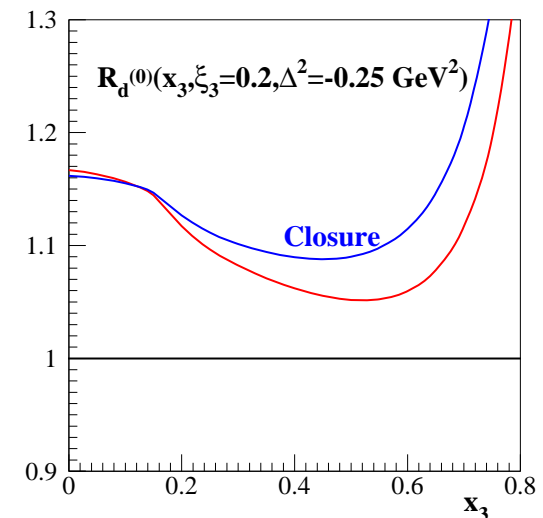
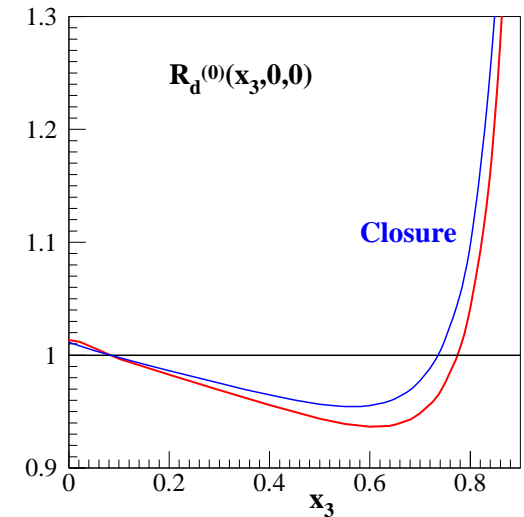
$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \bar{\sum}_M \sum_{s,f} \langle \vec{P}' M | \vec{P}_f, (\vec{p} + \vec{\Delta}) s \rangle \\ \times \langle \vec{P}_f, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_f^*)$$

using the Closure Approximation,  $E_f^* = \bar{E}$ :

$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) \simeq \bar{\sum}_M \sum_s \langle \vec{P}' M | a_{\vec{p}+\vec{\Delta},s} a_{\vec{p},s}^\dagger | \vec{P} M \rangle$$

$$\delta(E - E_{min} - \bar{E}) = \\ = n(\vec{p}, \vec{p} + \vec{\Delta}) \delta(E - E_{min} - \bar{E}),$$

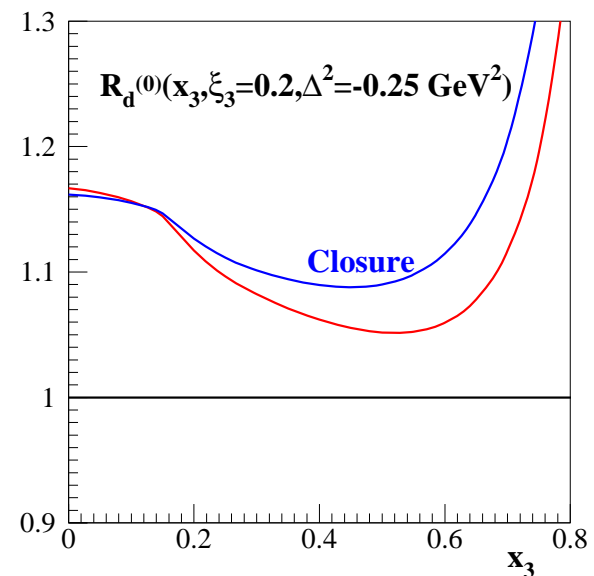
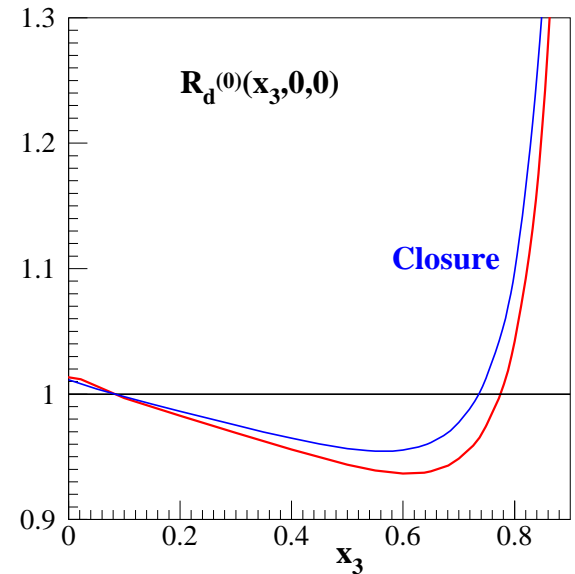
Spectral function substituted by a Momentum distribution  
(forward case in C. Ciofi, S. Liuti PRC 41 (1990) 1100)



# Nuclear effects - the binding

Nuclear effects are bigger than in the forward case: dependence on the binding

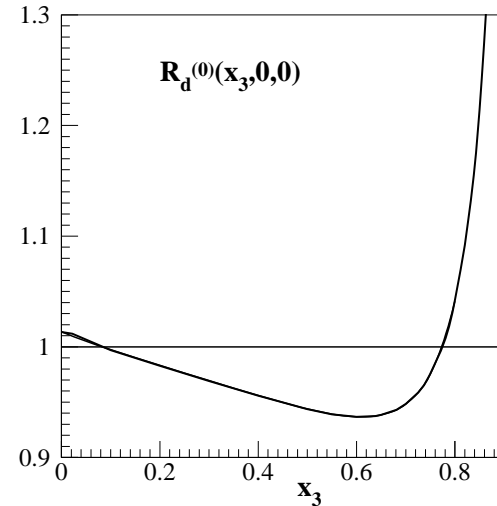
- In calculations using  $n(\vec{p}, \vec{p} + \vec{\Delta})$  instead of  $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$ , in addition to the IA, also the Closure approximation has been assumed;
- 5 % to 10 % binding effect between  $x = 0.4$  and  $0.7$  - much bigger than in the forward case;
- for  $A > 3$ , the evaluation of  $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$  is difficult - such an effect is not under control: Conventional nuclear effects can be mistaken for exotic ones;
- for  ${}^3\text{He}$  it is possible : this makes it a unique target, even among the Few-Body systems.



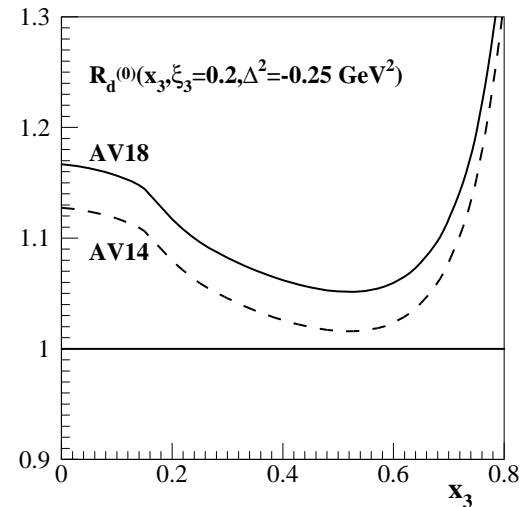
# Dependence on the NN interaction

Nuclear effects are bigger than in the forward case: dependence on the potential

● **Forward case:** Calculations using the **AV14** or **AV18** interactions are **indistinguishable**



● **Non-forward case:** Calculations using the **AV14** and **AV18** interactions **do differ:**



# $\tilde{G}_M^{3,q}$ calculation: correct limits

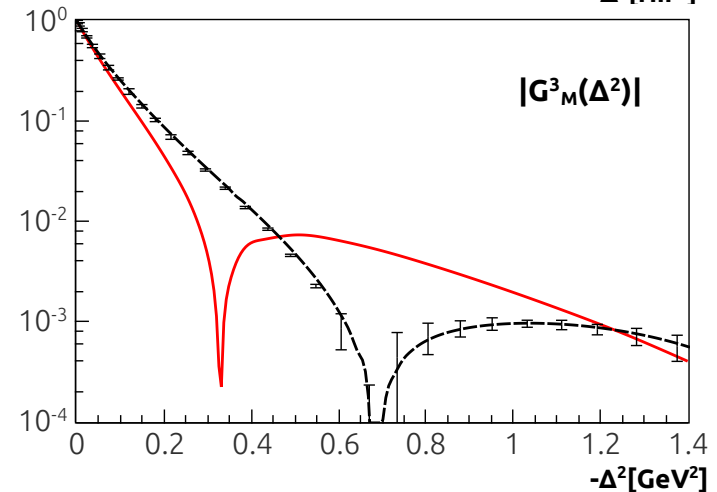
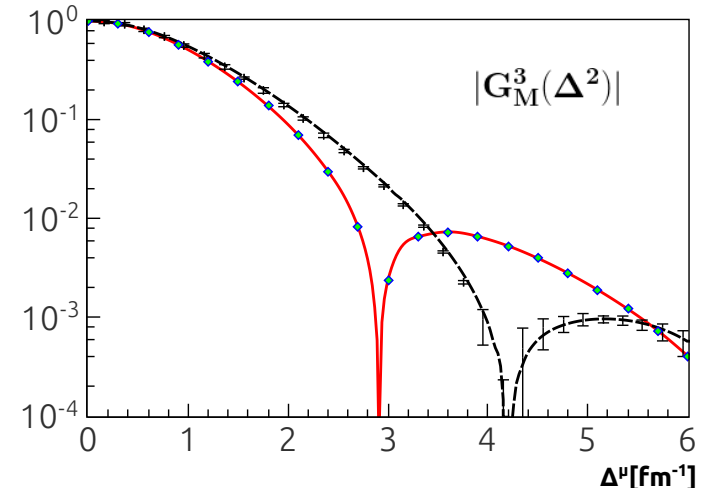
For  $\tilde{G}_M^3$  (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013) ):

1 - Forward limit: no control on  $E_q^3(x, 0, 0)$   
no possible check;

2 - Magnetic F.F.:

$$\sum_q \int dx \tilde{G}_M^{3,q}(x, \xi, \Delta^2) = G_M^3(\Delta^2)$$

- in perfect agreement with previous IA, Av18 calculations ( L.E. Marcucci et al. PRC 58 (1998))
- in good agreement with data in the region relevant to the coherent process,  $-\Delta^2 \ll 0.15 \text{ GeV}^2$
- To have agreement at higher  $\Delta^2$ , effects beyond IA are necessary: not important for the coherent channel!



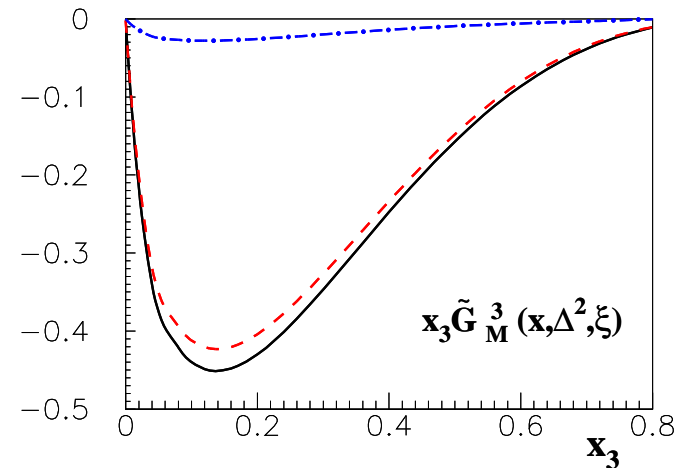
# $\tilde{G}_M^{3,q}$ : proton and neutron contributions

1 - Forward limit,  $\Delta^2 = 0$ ,  $\xi = 0$ :

As we hoped, the **neutron** contribution to  ${}^3\text{He}$  largely dominates!

( $x_3 = (M_A/M)x \simeq 3x$ ):

The **proton** contribution to  ${}^3\text{He}$  is almost negligible!

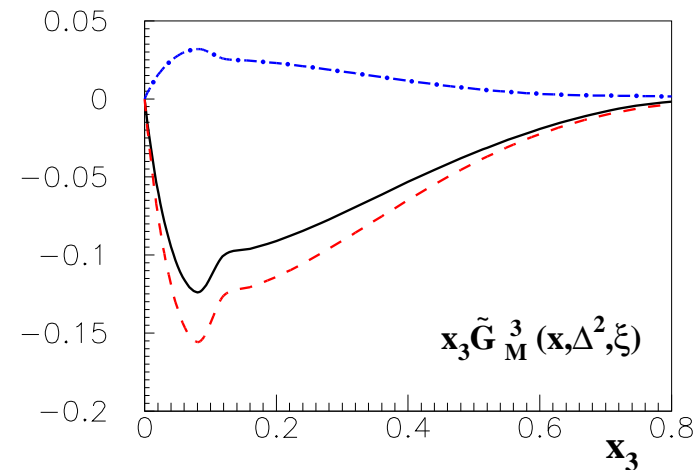


2 - Non-forward,  $\Delta^2 = -0.1 \text{ GeV}^2$ ,  $\xi = 0.1$ :

The **neutron** contribution to  ${}^3\text{He}$  still dominates

The **proton** contribution to  ${}^3\text{He}$  gets sizable

How to get the **neutron** information?



# $\tilde{G}_M^{3,q}$ : Flavor separation

For the  $u$  flavor, the neutron contribution (dashed) to  ${}^3\text{He}$  (full) is less important than for the  $d$  flavor:

Understandable, sketching the formula:

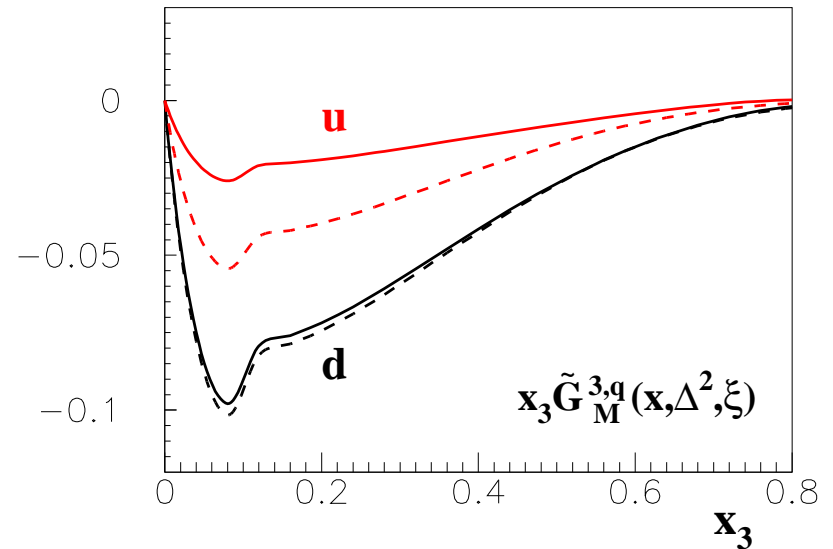
$$\tilde{G}_M^{3,q} \approx P_p^3 \otimes \tilde{G}_M^{p,q} + P_n^3 \otimes \tilde{G}_M^{n,q},$$

where  $P_{p(n)}^3$  describes the proton (neutron) dynamics in  ${}^3\text{He}$ .

As already explained, due to the spin structure of  ${}^3\text{He}$ ,  $P_n^3 \gg P_p^3 \rightarrow$  neutron dominates in the forward limit.

With increasing  $\Delta^2$ , for the  $u$  flavor,  $\tilde{G}_M^{p,u} \gg \tilde{G}_M^{n,u} \rightarrow$  the proton contribution grows. Not for  $d$ !

Besides, 1/2 of the  $d$  content of  ${}^3\text{He}$  comes from the neutron, only 1/5 of the  $u$  one comes from it.



# Extracting the neutron - I:

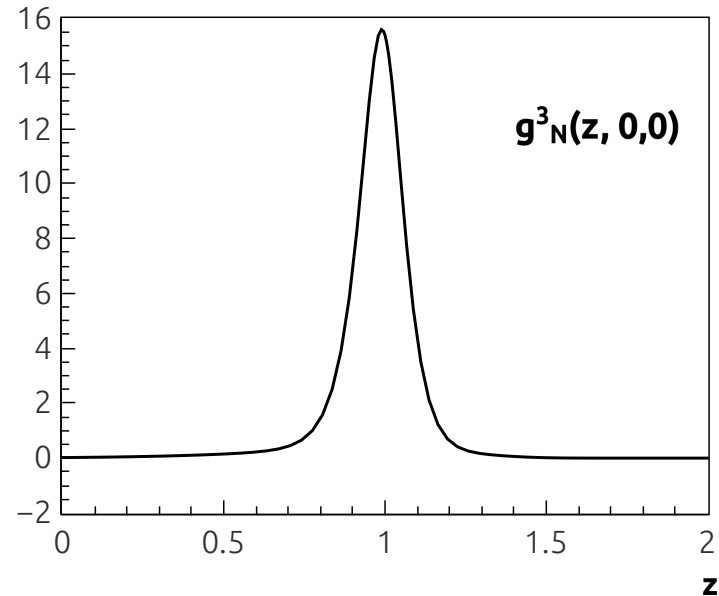
The convolution formula can be written as

$$\tilde{G}_M^{3,q}(x_3, \Delta^2, \xi) = \sum_N \int_{x_3}^{\frac{M_A}{M}} \frac{dz}{z} g_N^3(z, \Delta^2, \xi) \tilde{G}_M^{N,q}\left(\frac{x_3}{z}, \Delta^2, \frac{\xi}{z}\right),$$

where  $g_N^3(z, \Delta^2, \xi)$  is a “light cone off-forward momentum distribution” and, since close to the forward limit it is strongly peaked around  $z = 1$

$$g_N^3(z, \Delta^2, \xi) = \int dE \int d\vec{p} \tilde{P}_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$$

$$\delta\left(z + \xi - \frac{M_A}{M} \frac{p^+}{P^+}\right)$$



# Extracting the neutron - I:

The convolution formula can be written as

$$\tilde{G}_M^{3,q}(x_3, \Delta^2, \xi) = \sum_N \int_{x_3}^{\frac{M_A}{M}} \frac{dz}{z} g_N^3(z, \Delta^2, \xi) \tilde{G}_M^{N,q} \left( \frac{x_3}{z}, \Delta^2, \frac{\xi}{z} \right),$$

where  $g_N^3(z, \Delta^2, \xi)$  is a “light cone off-forward momentum distribution” and, since close to the forward limit it is strongly peaked around  $z = 1$

$$\begin{aligned} \tilde{G}_M^{3,q}(x_3, \Delta^2, \xi) &\simeq \text{low } \Delta^2 \simeq \sum_N \tilde{G}_M^{N,q}(x_3, \Delta^2, \xi) \int_0^{\frac{M_A}{M}} dz g_N^3(z, \Delta^2, \xi) \\ &= G_M^{3,p,point}(\Delta^2) \tilde{G}_M^p(x_3, \Delta^2, \xi) + G_M^{3,n,point}(\Delta^2) \tilde{G}_M^n(x_3, \Delta^2, \xi). \end{aligned}$$

where, at  $x_3 < 0.7$ , the **magnetic point like ff** has been introduced

$$G_M^{3,N,point}(\Delta^2) = \int dE \int d\vec{p} \tilde{P}_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \int_0^{\frac{M_A}{M}} dz g_N^3(z, \Delta^2, \xi).$$



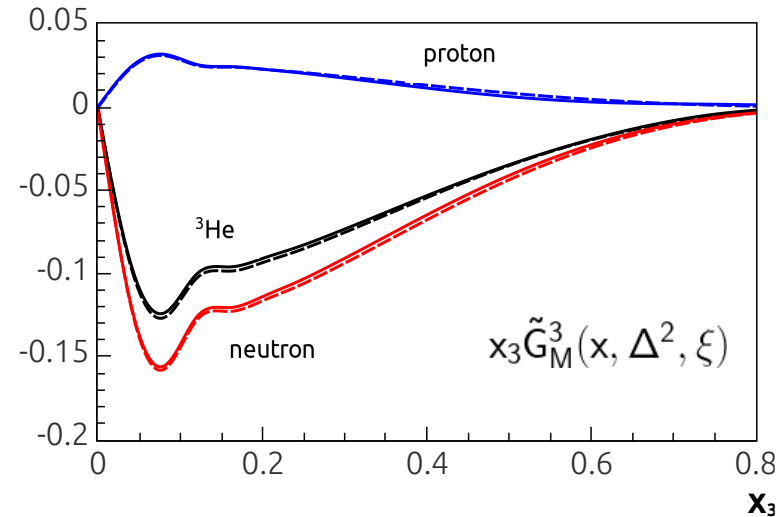


## Extracting the neutron - II:

Validity of the approximated formula:

full: IA calculation,  $\tilde{G}_M^3(x, \Delta^2, \xi)$  and  
**proton** and **neutron** contributions to it,  
 at  $\Delta^2 = -0.1 \text{ GeV}^2$ ,  $\xi = 0.1$ ;

dashed: same quantities, with the  
 approximated formula:



$$\begin{aligned} \tilde{G}_M^{3,q}(x, \Delta^2, \xi) &\simeq G_M^{3,p,point}(\Delta^2) \tilde{G}_M^p(x, \Delta^2, \xi) \\ &+ G_M^{3,n,point}(\Delta^2) \tilde{G}_M^n(x, \Delta^2, \xi) \end{aligned}$$

Impressive agreement! The **only Nuclear Physics ingredient** in the approximated formula is the **magnetic point like ff**, which is under good theoretical control:

$\Delta^2$ [GeV <sup>2</sup> ]	$G_M^{3,p,point}$ Av18	$G_M^{3,p,point}$ Av14	$G_M^{3,n,point}$ Av18	$G_M^{3,n,point}$ Av14
0	-0.044	-0.049	0.879	0.874
-0.1	0.040	0.038	0.305	0.297
-0.2	0.036	0.035	0.125	0.119

# Extracting the neutron - III:

The approximated relation can now be solved to extract the neutron contribution:

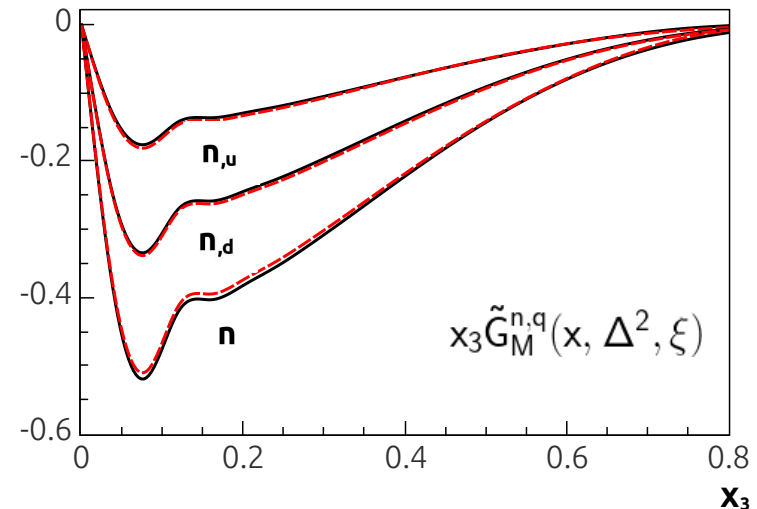
$$\tilde{G}_M^{n,extr}(x, \Delta^2, \xi) \simeq \frac{1}{G_M^{3,n,point}(\Delta^2)} \left\{ \tilde{G}_M^3(x, \Delta^2, \xi) - G_M^{3,p,point}(\Delta^2) \tilde{G}_M^p(x, \Delta^2, \xi) \right\},$$

from data for  $\tilde{G}_M^3(x, \Delta^2, \xi)$  and  $\tilde{G}_M^p(x, \Delta^2, \xi)$ , using as theoretical ingredients the **magnetic point like ffs** only.

The procedure works nicely!

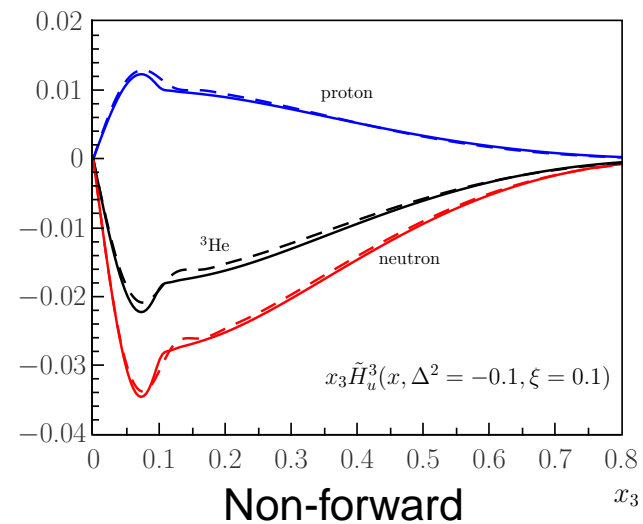
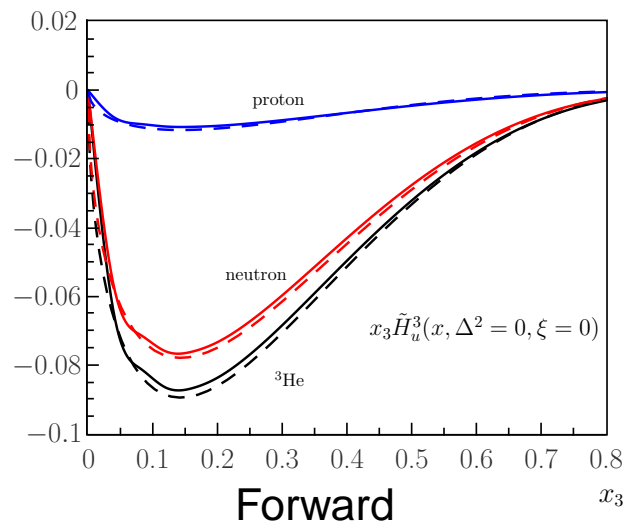
full : the neutron model for  $\tilde{G}_M^n(x, \Delta^2, \xi)$  and the different flavor contributions to it used in the IA calculation, at  $\Delta^2 = -0.1 \text{ GeV}^2$ ,  $\xi = 0.1$ ;

**dashed: the neutron extracted** using the IA calculation for  $\tilde{G}_M^3(x, \Delta^2, \xi)$  and the model used in it for  $\tilde{G}_M^p(x, \Delta^2, \xi)$  together with the **magnetic point like ffs**.



# The GPD $\tilde{H}$ : M. Rinaldi, S.S, Few-Body Systems 55, 861 (2014)

$\tilde{H}^{3,u}(x, \Delta^2, \xi)$  and **proton** and **(dominant!) neutron** contributions to it:



full: IA calculation;      dashed: approximated formula:

$$\tilde{H}^{3,u}(x, \Delta^2, \xi) \simeq g_A^{3,p,point}(\Delta^2) \tilde{H}^{p,u}(x, \Delta^2, \xi) + g_A^{3,n,point}(\Delta^2) \tilde{H}^{n,u}(x, \Delta^2, \xi)$$

Good agreement! The **only Nuclear Physics ingredient** in the approximated formula is the **axial point like ff**, which is under good theoretical control.

One has  $g_A^{3,N,point}(\Delta^2 = 0) = p_N$ , nucleon effective polarizations (within AV18,  $p_n = 0.878$ ,  $p_p = -0.024$ ), used in DIS for extracting the neutron information from  ${}^3\text{He}$  (C. Ciofi, S.S., E. Pace and G. Salmè, PRC 48 R968 (1993)). **Forward limit recovered!**

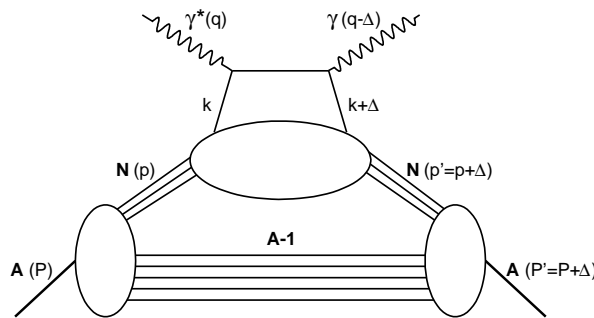
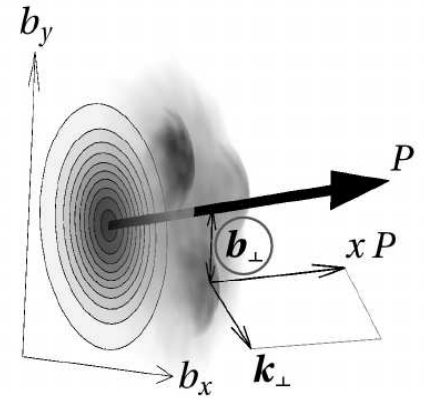
## $^3\text{He}$ calculations: summary

- Our results, for  $^3\text{He}$ : (S.S. PRC 2004, 2009; M. Rinaldi and S.S., PRC 2012, 2013)
  - \* I.A. calculation of  $H_3, E_3, \tilde{H}_3$ , within AV18;
  - \* Interesting predictions: strong sensitivity to details of nuclear dynamics:
  - \* extraction procedure of the neutron information, able to take into account all the nuclear effects encoded in an IA analysis;
- Coherent DVCS off  $^3\text{He}$  would be:
  - \* a test of IA; relevance of non-nucleonic degrees of freedom;
  - \* a test of the  $A$ -dependence of nuclear effects;
  - \* complementary to incoherent DVCS off the deuteron in extracting the neutron information (with polarized targets).
- No data; no proposals at JLAB... difficult to detect slow recoils using a polarized target... But even unpolarized,  $^3\text{He}$  would be interesting!  
Together with  $^3\text{H}$ , nice possibilities (flavor separation of nuclear effects, test of IA)
- at the EIC, beams of polarized light nuclei will operate.  $^3\vec{H}e$  can be used.
- Our codes available to interested colleagues.

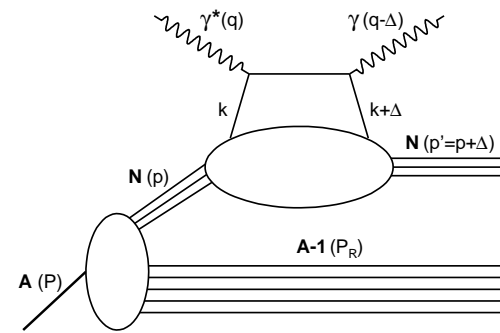
# Data on nuclear DVCS

In impact parameter space, GPDs are *densities*:

$$\rho_q(x, \vec{b}_\perp) = \int \frac{d\vec{\Delta}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H^q(x, 0, \Delta^2)$$



Coherent DVCS (in IA):  
nuclear tomography;



Incoherent DVCS (in IA):  
tomography of bound nucleons:  
realization of the EMC effect

- **Very difficult to distinguish coherent and incoherent channels** (for example, in Hermes data, Airapetian et al., PRC 2011).
- Large energy gap between the photons and the slow-recoiling systems: very different detection systems required at the same time... **Very difficult...**



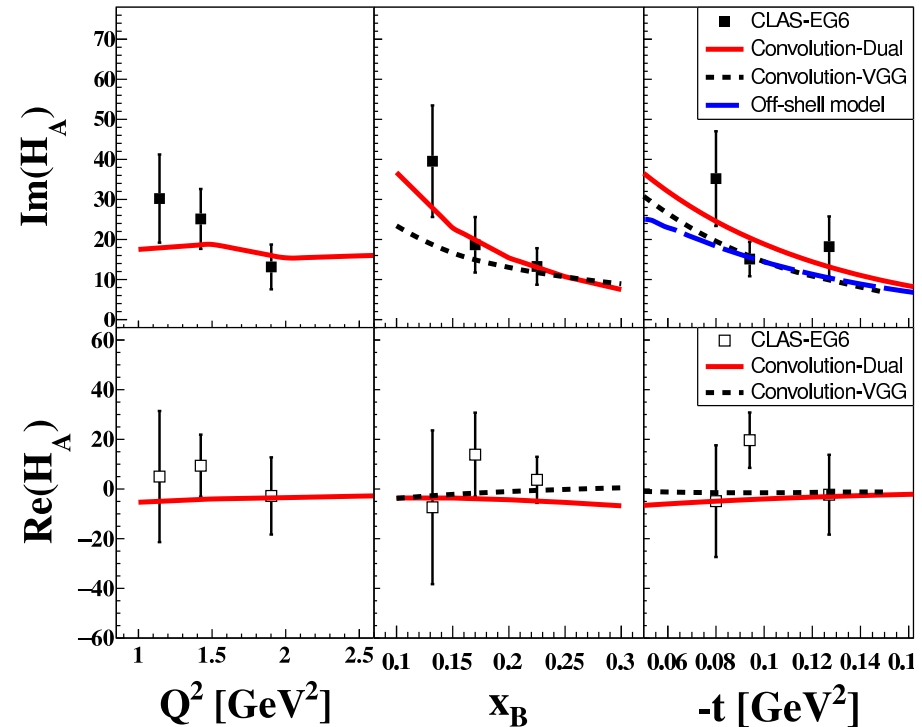
# ... But possible! Just released from CLAS!

( M. Hattawy et al. arXiv:1707.03361v1 [nucl-ex] )

Coherent data (incoherent will follow) of DVCS off  $^4\text{He}$ :

off-shell model by  
Gonzalez, Liuti, Goldstein, Kathuria  
(blue dashed)  
(PRC 88, 065206 (2013))

IA calculation, Guzey  
(full, dashed, different GPD models)  
(PRC 78, 025211 (2008))

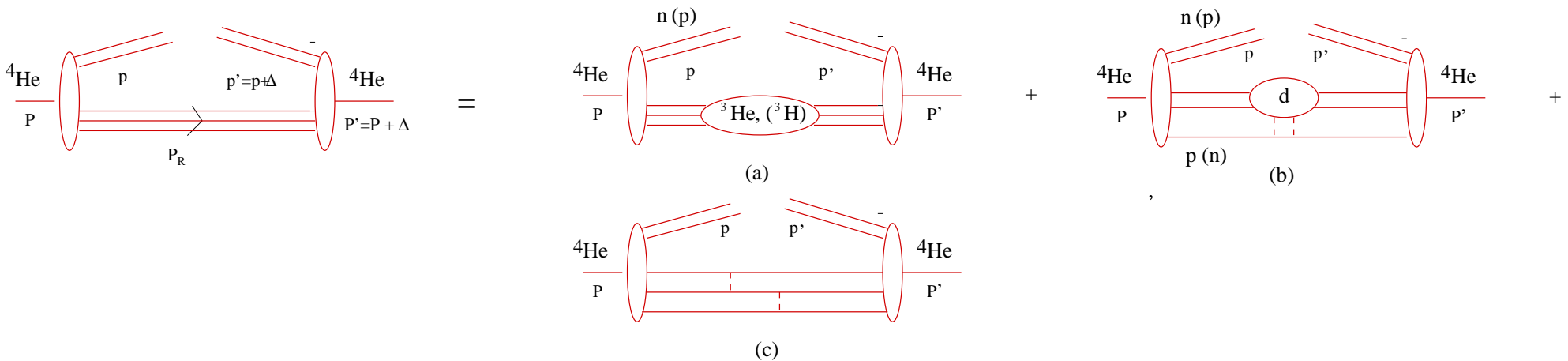
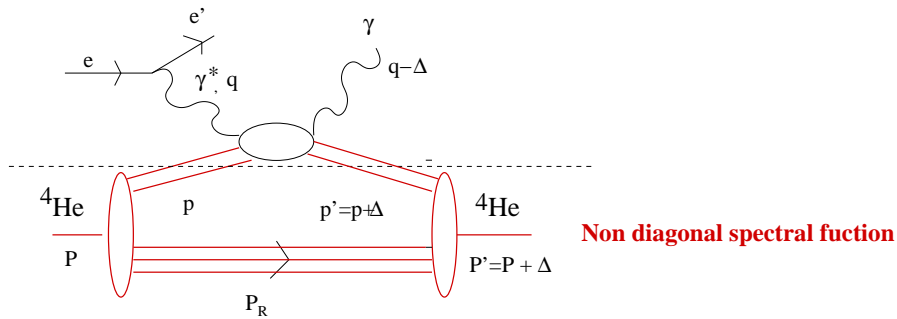


- $^4\text{He}$ :  $J = 0, I = 0$ , easy *formal* description (1 chiral-even twist-2 GPD); but a true nucleus (deeply bound, dense...)
- Next generation of experiments (ALERT run-group), just approved (A-rate), will distinguish models: precisely what is needed to understand nuclei at parton level!
- Good prospects for the EIC at low  $x_B$ , easy recoil detection...

# DVCS off $^4\text{He}$

- CLAS data demonstrate that measurements are possible, separating coherent and incoherent channels;
- Realistic microscopic calculations are necessary. A collaboration has started with Sara Fucini (Perugia, graduating student), Michele Viviani (INFN Pisa).

## Coherent channel in IA:

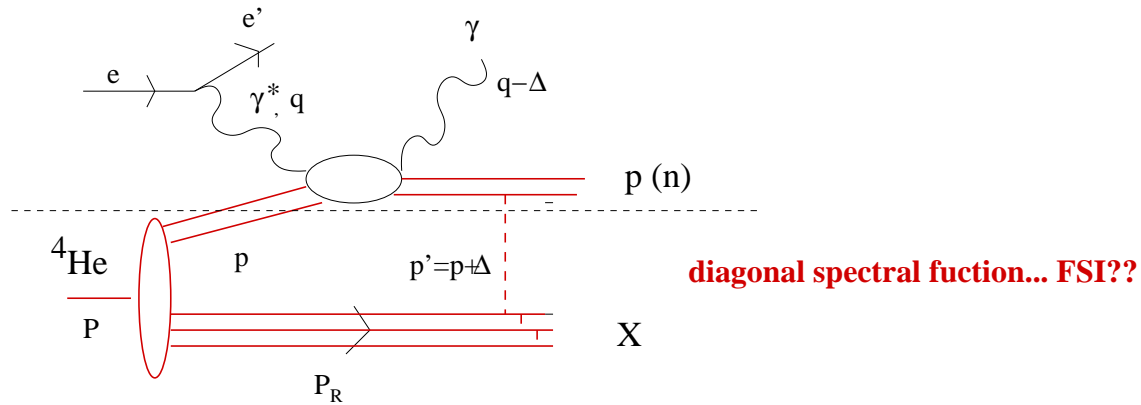


we are working on a); b) is feasible; c) is really challenging

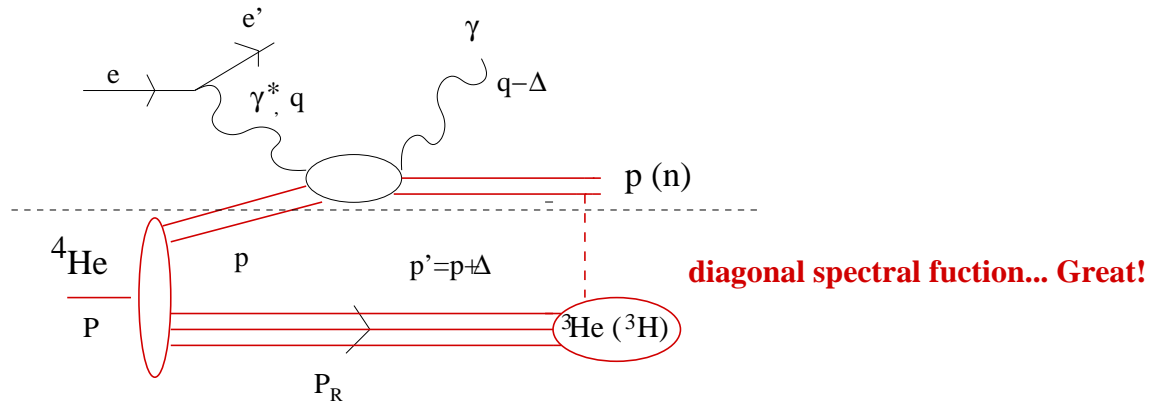


# Incoherent DVCS off $^4\text{He}$ in IA

**$^4\text{He}(e, e'\gamma p(n))X$**



**Tagged! e.g.,  $^4\text{He}(e, e'\gamma p)^3\text{H}$  ( arXiv:1708.00835 [nucl-ex]  $\rightarrow$  Armstrong)**

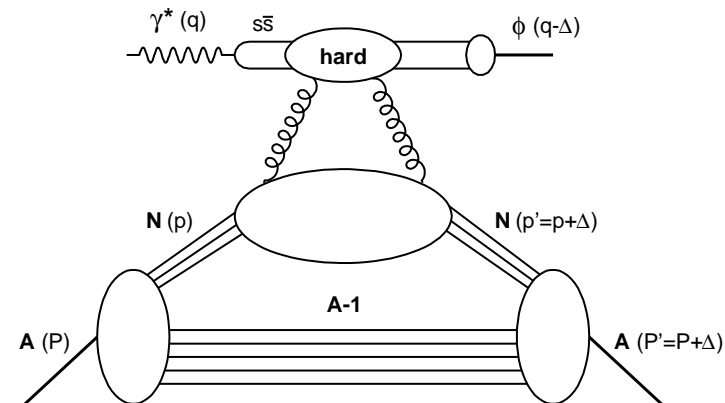




# Many other issues...

- $x$ -moments of GPDs (ffs of energy momentum tensor): information on spatial distribution of energy, momentum and forces experienced by the partons. Predicted an  $A$  dependence stronger than in IA (not seen at HERMES); M. Polyakov, PLB 555, 57 (2003); H.C. Kim et al. PLB 718, 625 (2012)...

- Gluon GPDs in nuclei



For GPDs, shadowing (low  $x_B$ ) stronger than for PDFs

A. Freund and M. Strikman, PRC 69, 015203 (2004)...

Exclusive  $\phi$ - electroproduction, unique source of information, studied by ALERT, waiting for EIC...

- Deuteron: an issue aside.

Extraction of the neutron information; access to a new class of distribution ( $J = 1$ )

Studied by different collaborations (by ALERT too, coherent and incoherent DVCS)

theory: Cano and Pire EPJA 19,423 (2004); Taneja et al. PRD 86,036008 (2012)...



# The quest for covariance

- Mandatory to achieve polynomiality for GPDs, and sum rules in DIS: number of particle and momentum sum rule not fulfilled at the same time in not covariant IA calculations
- Numerically not very relevant for forward Physics. It becomes relevant for non-diagonal observables at high momentum transfer. Example: form factors (well known since a long time, see, i.e., **Cardarelli et al., PLB 357 (1995) 267**)
- I do not expect big problems in the coherent case at low  $t$ ;  
Crucial for incoherent at higher  $t$
- Certainly it has to be studied.  
For  $^3\text{He}$ , formal developments available in a Light-Front framework (**A. Del Dotto, E. Pace, S.S., G. Salmè, PRC 95 (2017) 014001** ).  
Talk here by Gianni in two weeks.  
Calculations in progress, starting from a diagonal, spin-independent spectral function.  
 $^4\text{He}$ ... Later (very cumbersome).



# Conclusions

- **Exciting time thanks to new data and accepted next-generation experiments at JLab...**
- **... a prelude to “Great expectations” for the E-Ion-C**
- **“Ion” structure effects: not only relevant. Essential**
- **Easy to predict a growing interest and an important contribution from (low-energy) nuclear theorists**

