Strong forces inside the nucleon and their applications

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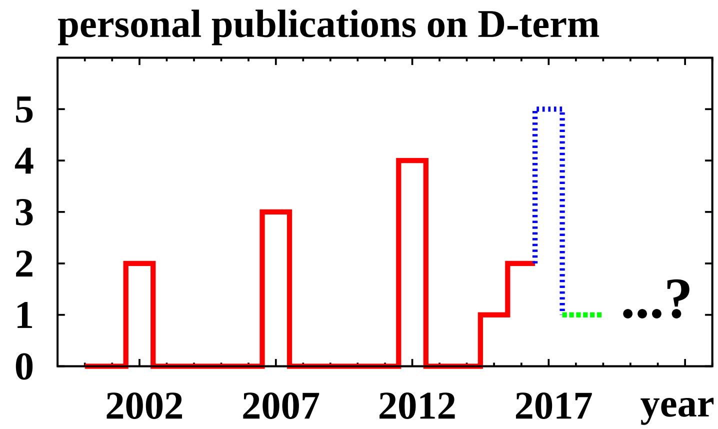
Outline

- **Introduction**
  hard-exclusive reactions → GPDs → so what?
  tomography, Ji sum rule, and ...
- **Energy-momentum tensor**
  form factors & D-term
  last unknown global property(!)
- **D-term**
  What do we know?
  theory & experiment
- **Physical interpretation**
  3D densities: limitations & uses
  stress tensor and stability
- **Applications**
  insights in mechanical stability
  from hard-exclusive reactions at JLab ...
  to $c\bar{c}$ pentaquark spectroscopy at LHCb
- **Outlook**



based on:
PS, Boffi, Radici, PRD**66**, 114004 (2002)
Goeke et al, PRD**75**, 094021; PRC**75**, 055207
Cebulla et al, Nucl. Phys. A**794**, 87 (2007)
Mai, PS, PRD**86**, 076001 & **86**, 096002 (2012)
Cantara, Mai, PS, Nucl. Phys. A**953**, 1 (2016)
Perevalova, Polyakov, PS, PRD**94**, 054024
Neubelt, Sampino, PS, in progress
Hudson, PS, forthcoming (2017)
supported by: NSF

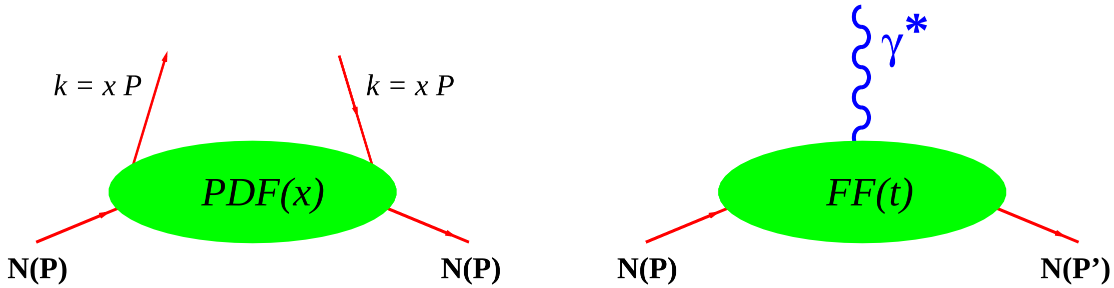
Introduction

- what can we learn from GPDs?

- GPDs generalize
  form factors, PDFs

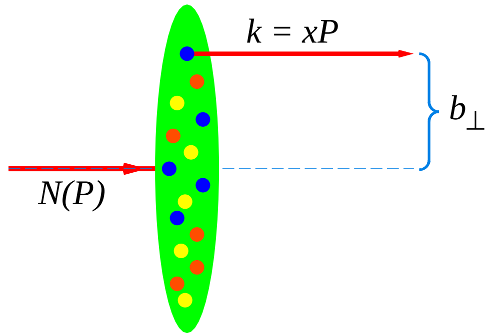
  $\int \mathrm{d}x\; H^q(x,\xi,t) = F_1^q(t)$

  $\lim_{\Delta\to 0} H^q(x,\xi,t) = f_1^q(x)$



- explore impact parameter space
  allow tomography (M. Burkardt, ...)

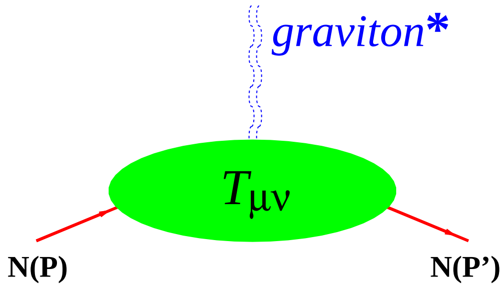
$$H^q(x,b_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\left[\lim_{\xi\to 0} H^q(x,\xi,t)\right] e^{i\boldsymbol{\Delta}_T \boldsymbol{b}_T}$$



- allow to access (polynomiality)
  gravitational form factors

  $\int \mathrm{d}x\; x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$

  $\int \mathrm{d}x\; x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$



- and **gravity** couples to
  energy momentum tensor
  probably most fundamental quantity

Energy-momentum tensor (EMT)

- instead of arguing how important EMT is, question:
  are you aware of introductory QFT text books
  which *do not* discuss EMT in first chapters?*

- if a theory can be solved (like free theory):
  construct $\boldsymbol{T_{\mu\nu}}$ and generators of Poincaré group
  learn what is $\underbrace{\textbf{mass,}}_{T_{00}}$ $\underbrace{\textbf{spin},}_{\varepsilon^{ijk}x_j T_{0k}}$ $\underbrace{\boldsymbol{D}\textbf{-term}}_{T_{ij}}$ particles

- even if theory cannot be solved, studies of EMT insightful
  prominent example: **Ji sum rule** PRL 78 (1997) 610
  $$\int \mathrm{d}x\, x \Big(H^q(x,\xi,t) + E^q(x,\xi,t)\Big) = A^q(t) + B^q(t) \xrightarrow{t\to 0} 2J^q(0)$$
  mass-decomp. (recent workshops) + more!

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* interestingly, advanced QFT books discuss EMT in later chapters: *trace anomaly*
$\hat{T}^{\mu}_{\mu} \equiv \frac{\beta}{2g} F^{\mu\nu}F_{\mu\nu} + (1+\gamma_m)\sum_q m_q \bar{\psi}_q \psi_q$ Adler, Collins, Duncan, PRD15 (1977) 1712;
Nielsen, NPB 120, 212 (1977); Collins, Duncan, Joglekar, PRD 16, 438 (1977)

nucleon EMT form factors

$$\langle p'|\hat{T}^{q,g}_{\mu\nu}|p\rangle = \bar{u}(p')\bigg[\; A^{q,g}(t)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^{q,g}(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M_N} + D^{q,g}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} \pm \bar{c}(t)g_{\mu\nu}\bigg]u(p)$$

- $\hat{T}^q_{\mu\nu}$, $\hat{T}^g_{\mu\nu}$ both gauge-invariant (not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$ is conserved $\partial_\mu \hat{T}^{\mu\nu} = 0$
- constraints: **mass** $\Leftrightarrow$ $A^q(0) + A^g(0) = 1$ (100 % of nucleon momentum carried by quarks + gluons)

  spin $\Leftrightarrow$ $B^q(0) + B^g(0) = 0$ (i.e. $J^q + J^g = \frac{1}{2}$ nucleon spin due to quarks + gluons)*
- property: **D-term** $\Leftrightarrow$ $D^q(0) + D^g(0) \equiv D$ $\rightarrow$ unconstrained! **Unknown! Last global unknown!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
$$t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M_2^q(t)$

* also expressed as: vanishing of total gravitomagnetic moment

last global unknown: How do we learn about hadrons?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\|J^\mu_{\rm em}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\|J^\mu_{\rm weak}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\rm grav} = 0$	$\langle N'\|T^{\mu\nu}_{\rm grav}\|N\rangle$	$\longrightarrow$	$M,\ J,\ D,\ \ldots$

global properties:

$$Q_{\rm prot} = 1.602176487(40) \times 10^{-19}\,\mathrm{C}$$
$$\mu_{\rm prot} = 2.792847356(23)\,\mu_N$$
$$g_A = 1.2694(28)$$
$$g_p = 8.06(0.55)$$
$$M = 938.272013(23)\ \mathrm{MeV}$$
$$J = \tfrac{1}{2}$$
$$\boldsymbol{D} = \ ??$$

and more:
t-dependence . . . . . .
parton structure, etc . . .

$\hookrightarrow$ $\boldsymbol{D}$ = *"last" global unknown*
which value does it have?
what does it mean?

D-term in theory

pions

- free Klein-Gordon field $D = -1$
  (Pagels 1965; Hudson, PS 50 years later)
- Goldstone bosons of chiral symmetry breaking $D = -1$ in soft pion limit
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)
- chiral perturbation theory for Goldstone bosons
  Donoghue, Leutwyler (1991)

$$
\begin{aligned}
D_\pi &= -1 + 16\,a\,\frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\,I_\pi - \frac{m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_K &= -1 + 16\,a\,\frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_\eta &= -1 + 16\,a\,\frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\,I_\pi + \frac{8m_K^2}{3F^2}\,I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4)
\end{aligned}
$$

where
$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}\,(\log\frac{\mu^2}{m_i^2} - 1)$
$i = \pi,\ K,\ \eta.$

$$
\begin{aligned}
D_\pi &= -0.97 \pm 0.01 \\
D_K &= -0.77 \pm 0.15 \\
D_\eta &= -0.69 \pm 0.19
\end{aligned}
$$

nuclei

- nuclei in liquid drop model $\boldsymbol{D = -0.2 \times A^{7/3}}$ $\rightarrow$ potential for DVCS wit nuclei!
  Maxim Polyakov (2002) (see below)

- nuclei in Walecka model
  Guzey, Siddikov (2006)

$$
\begin{aligned}
{}^{12}\text{C}: \quad & D = -3 \\
{}^{16}\text{O}: \quad & D = -58 \\
{}^{40}\text{Ca}: \quad & D = -610 \\
{}^{90}\text{Zr}: \quad & D = -3300 \\
{}^{208}\text{Pb}: \quad & D = -19700
\end{aligned}
$$

Q-balls

- Q-balls, non-topological solitons in strongly interacting theory: $90 \leq -D =\leq \infty$
  Mai, PS (2012)

- N^{th} excited Q-ball state (decay into ground states): $D = -\,\text{const}\,N^8$
  Mai, PS (2012)

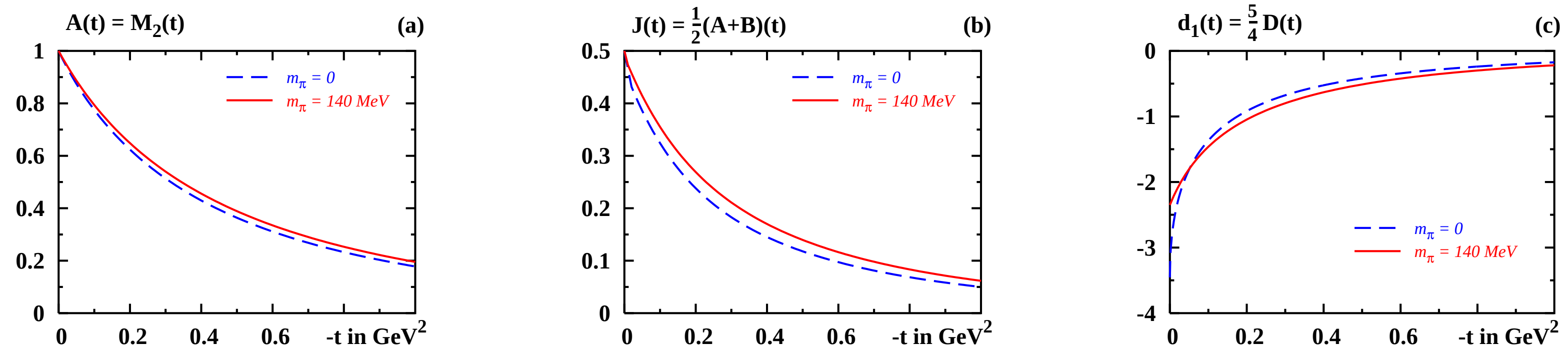
- Q-cloud limit, most extreme instability we could find: $D = -\,\text{const}/\varepsilon^2$ in the limit $\varepsilon \to 0$
  Cantara, Mai, PS (2016)

- Q-cloud excitations, even more extreme instability: $D < 0$ divergent and even more negative
  Bergabo, Cantara, PS (2017)

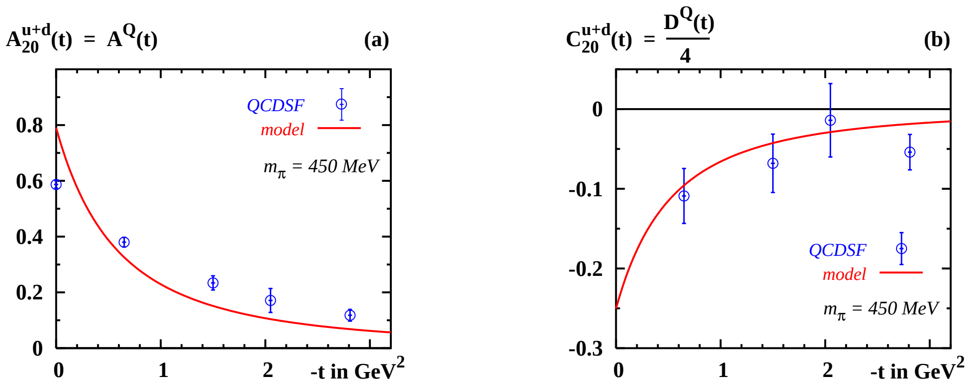
nucleon

- bag model (always good starting point!) $D < 0$ due to bag pressure!
  Ji, Melnitchouk, Song (1997); Neubelt, Sampino, PS (2017+)

- chiral quark soliton model
  Petrov et al 1998, Goeke et al, PRD75 (2007) 094021



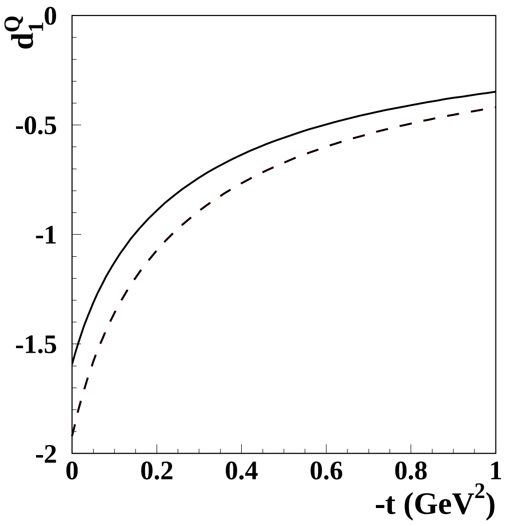
- lattice D^Q: QCDSF Collaboration, Göckeler et al, PRL92 (2004) 042002 & hep-ph/0312104



- χPT cannot predict D-term, but $d_1(m_\pi) = \mathring{d}_1 + \frac{5k\, g_A^2 M_N}{64\,\pi\, f_\pi^2}\, m_\pi + \ldots, \quad \mathring{d}_1'(0) = -\frac{k\, g_A^2\, M_N}{32\,\pi f_\pi^2\, m_\pi} + \ldots$
  $k = 1$ for finite N_c, and $k = 3$ for $N_c \to \infty$ Belitsky, Ji (2002), Diehl et al (2006)

nucleon (theory cont.)

- unsubtracted t-channel dispersion relations (sensitive to pion PDFs)
  Barbara Pasquini, Maxim Polyakov, Marc Vanderhaeghen (2014)



D-term in experiment (waiting, soon)

- HERMES Ellinghaus [HERMES Collaboration], NPA711, 171 (2002); PRD 75, 011103 (2007)
- JLab talk by V. Burkert at SPIN 2016 in Urbana-Champaign, September 25-30, 2016; PRL115 (2015); arXiv:1707.03361; PR12-16-010; talk by Daria yesterday
- COMPASS PoS DIS **2016**, 235 (2016).

interpretation in terms of 3D-densities

- Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$
- analog to electric form factor $G_E(\vec{\Delta}^2) = \int d^3\vec{r}\, \rho_E(\vec{r})\, e^{i\vec{\Delta}\vec{r}} \to$ charge distribution
  Sachs, PR126 (1962) 2256
  $\hookrightarrow Q = \int d^3\vec{r}\, \rho_E(\vec{r})$
- static EMT $T_{\mu\nu}(\vec{r},\vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{i\vec{\Delta}\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle \to$ mechanical properties of nucleon
  M.V.Polyakov, PLB 555 (2003) 57
  $\hookrightarrow M_N = \int d^3\vec{r}\, T_{00}(\vec{r})$, etc

limitations ($\exists$ in contrast to 2D Fourier transforms $\leftrightarrow$ tomography)

well known since earliest days (Sachs, 1962)
comprehensive studies, e.g. by
- Belitsky & Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2;
- X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect),
- G. Miller, PRC80 (2009) 045210 (toy model, dramatic effect)

No doubt: mathematical operation well-defined
The question: is the concept justified?
Answer: yes, modulo **corrections**!
How large are the corrections?

illustration in simplest framework

$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} m^2 \Phi^2$ free neutral elementary point-like scalar particle

("Higgs" modulo standard model corrections...)

evaluate EMT:

$$\langle \vec{p}\,'|\hat{T}^{\mu\nu}(x)|\vec{p}\,\rangle = e^{i(p'-p)x}\,\frac{1}{2}\left\{P^\mu P^\nu\, A(t) + \left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right)D(t)\right\}, \qquad A(t) = -D(t) = 1$$

compute **energy density**

$$T_{00}(\vec{r}) \overset{\text{gen.}}{=} m^2 \int \frac{d^3\Delta}{E(2\pi)^3}\, e^{i\vec{\Delta}\vec{r}} \left[A(t) - \frac{t}{4m^2}(A(t)+D(t))\right] \quad \text{in Breit frame } E = E' = \sqrt{m^2 + (\vec{\Delta}/2)^2}$$

$$\overset{\text{here}}{=} \frac{m}{\sqrt{1 - \vec{\nabla}^2/(4m^2)}}\,\delta^{(3)}(\vec{r})$$

reproduces correctly: $\int d^3r\, T_{00}(\vec{r}) \overset{!}{=} m$

but yields also $\langle r_E^2\rangle = \frac{1}{m}\int d^3r\, r^2 T_{00}(\vec{r}) \overset{!??}{=} \frac{3}{4m^2}$

corrections generate **mean square radius** $\neq 0$ for point-like particle???

- take **heavy mass limit** to recover "correct" intuitive 3D description

$$T_{00}(\vec{r}) \longrightarrow m\,\delta^{(3)}(\vec{r}) \quad \text{for } m \to \text{large} \ldots \qquad \text{large with respect to what?}$$

- let's give particle a **finite size R** (i.e. **"smear out"** δ-function)

$$T_{00}(\vec{r})_{\text{true}} \overset{\text{e.g.}}{=} m\,\frac{e^{-r^2/R^2}}{\pi^{3/2}\,R^3} \quad \text{"true energy density"}$$

$$\to \quad \langle r_E^2\rangle = \langle r_E^2\rangle_{\text{true}}\left(1+\delta_{\text{rel}}\right) \simeq \langle r_E^2\rangle_{\text{true}} \quad \textbf{if } \delta_{\text{rel}} = \frac{1}{2m^2R^2} \ll 1 \qquad \left(\text{it is } \langle r_E^2\rangle_{\text{true}} = \tfrac{3}{2}R^2 \text{ if Gaussian}\right)$$

- **for nuclei** $(M_A \sim M_N A,\ R_A \sim R_0\,A^{1/3}) \quad \to \quad \delta_{\text{rel}} \sim 0.16\,A^{-8/3} \lesssim 10^{-4}$ (for [4]He, less for heavier)

- **nucleon** $(M_N \sim 940\,\text{MeV},\ R_N \sim 1\,\text{fm}) \quad \to \quad \delta_{\text{rel}} \sim 3\,\% \overset{!?}{\ll} 1$ small enough!?

- **large-N_c nucleon** $(M_N \sim N_c,\ R_N \sim N_c^0) \quad \to \quad \delta_{\text{rel}} \sim \frac{1}{N_c^2} \overset{!!}{\ll} 1$ conceptually small enough!

remark: fair enough to use large-N_c limit!

$\frac{1}{N_c}$ *only (known) small parameter of QCD at* ***all*** *energies* (S. Coleman in "Aspects of Symmetry")

(... besides large-N: powerful theoretical method, much more general than QCD)

particle	J^π	mass [GeV]	size [fm]	$\delta_{\rm rel}$
pion	0^-	0.14	0.67	2.2
kaon	0^-	0.49	0.56	2.5×10^{-1}
η-meson	0^-	0.55	0.68	1.4×10^{-1}
η_c-meson	0^-	2.98	0.26	3.8×10^{-2}
proton	$\frac{1}{2}^+$	0.94	0.89	2.8×10^{-2}
deuteron	1^+	1.88	2.14	1.2×10^{-3}
^{4}He	0^+	3.73	1.68	5.0×10^{-4}
^{12}C	0^+	11.2	2.47	2.6×10^{-5}
^{20}Ne	0^+	18.6	3.01	6.2×10^{-6}
^{32}S	0^+	29.8	3.26	2.1×10^{-6}
^{56}Fe	0^+	52.1	3.74	5.1×10^{-7}
^{132}Xe	0^+	123	4.79	5.6×10^{-8}
^{208}Pb	0^+	194	5.50	1.7×10^{-8}
^{244}Pu	0^+	227	5.89	1.1×10^{-8}

from:
Hudson, PS
"D-terms of
spin-0 systems"
new preprint (2017)

- important distinction:

  2D densities = partonic probability densities (unitarity)

  must be exact! → M. Burkardt (2000)

  is exact √

  apply to any particle including pion

  vs

  3D densities = mechanical response functions

  correlation functions subject to corrections → M. Polyakov (2002)

  okay, if corrections acceptably small √

  apply to nucleon or heavier

- besides: no 2D interpretations exist for stress tensor and pressure
  inherently 3D concepts, have to pay the prize for (and deal with corrections)

- **interpretation as 3D-densities** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $T_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\vec{r}} \langle P'|\hat{T}_{\mu\nu}|P\rangle$

all formulae correct, interpretation in terms of 3D-densities has limitations (see above)

$$\int d^3r\, T_{00}(\vec{r}) = M_N \qquad \text{known}$$

$$\int d^3r\, \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5} M_N \int d^3r \left(r^i r^j - \frac{r^2}{3}\delta^{ij}\right) T_{ij}(\vec{r}) \equiv D \qquad \textbf{new!}$$

with: $$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij} \qquad \textbf{stress tensor}$$

$s(r)$ related to distribution of *shear forces*
$p(r)$ distribution of *pressure* inside hadron
$\Big\} \longrightarrow$ **"mechanical properties"**

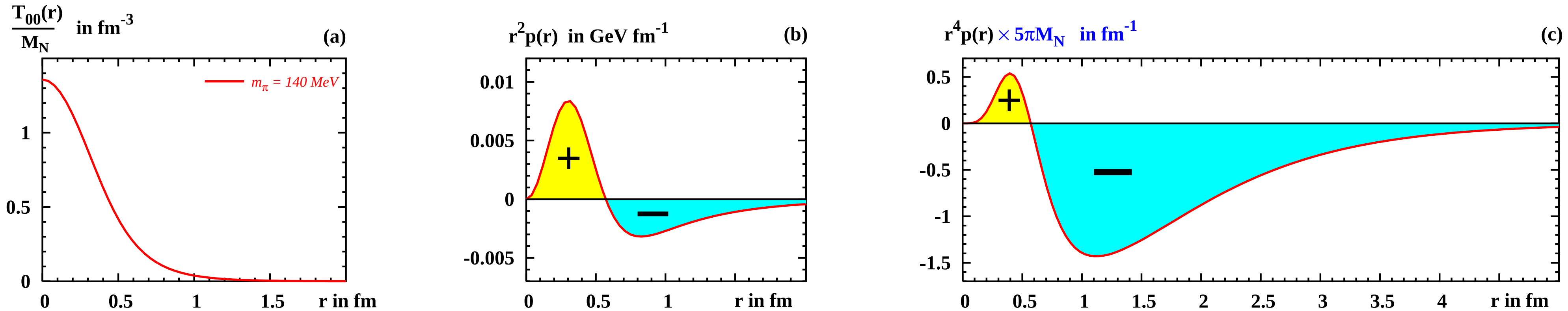
- **relation to stability**: EMT conservation $\Leftrightarrow \quad \partial^\mu \hat{T}_{\mu\nu} = 0 \quad \Leftrightarrow \quad \nabla^i T_{ij}(\vec{r}) = 0$

$\hookrightarrow$ necessary condition for stability $\int_0^\infty dr\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

$$\boldsymbol{D} = -\frac{16\pi}{15} M_N \int_0^\infty dr\, r^4 s(r) = 4\pi M_N \int_0^\infty dr\, \boldsymbol{r^4\, p(r)}$$

$\hookrightarrow$ shows how internal forces balance

- lessons from model



energy density — pressure: $r^2 p(r)$ — $r^4 p(r) \rightsquigarrow$ sign of d_1 negative

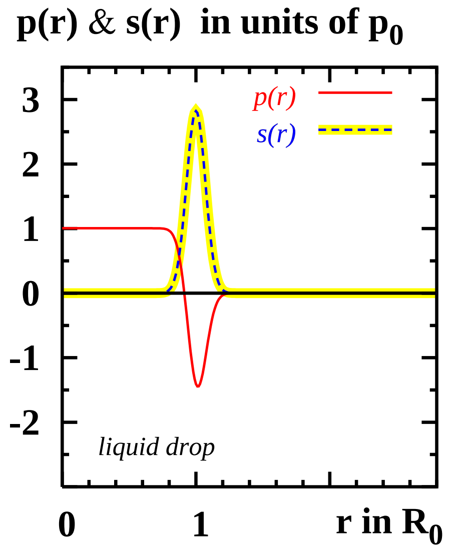
$T_{00}(0) = 1.70\,\mathrm{GeV/fm^3} \approx 3 \times 10^{15}\, \rho(H_2O) \approx 13 \times$ (nuclear density)

$p(0) = 0.23\,\mathrm{GeV/fm^3} \approx 4 \times 10^{34}\,\mathrm{N/m^2} \gtrsim 100\times$(pressure in center of neutron star)

in chiral quark soliton model (Goeke et al, PRD75 (2007) 094021)

... how does it look like in QCD? We do not know. Wouldn't it be fascinating to know??

• **intuition on shear forces and pressure**



liquid drop

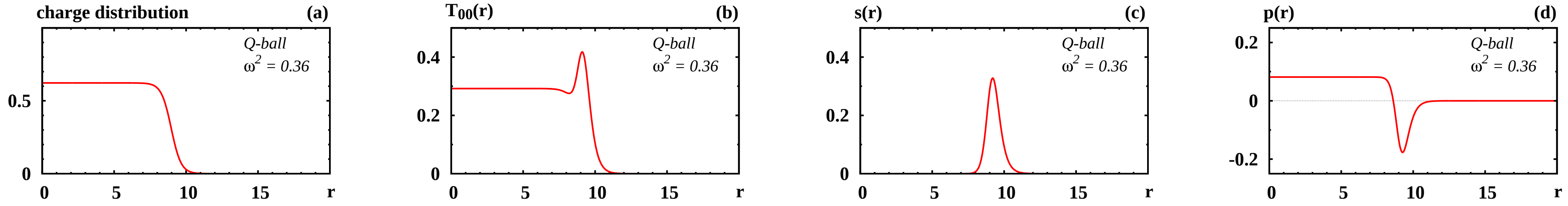
radius R_0
inside pressure p_0
surface tension $\gamma = \frac{1}{2} p_0 R_0$

$$s(r) = \gamma\, \delta(r - R_0)$$

$$p(r) = p_0\, \Theta(R_0 - r) - \frac{1}{3} p_0 R_0\, \delta(r - R_0)$$

application: liquid drop model of nucleus
(M.V.Polyakov, PLB 555 (2003) 57)

realized in field theoretical Q-ball system



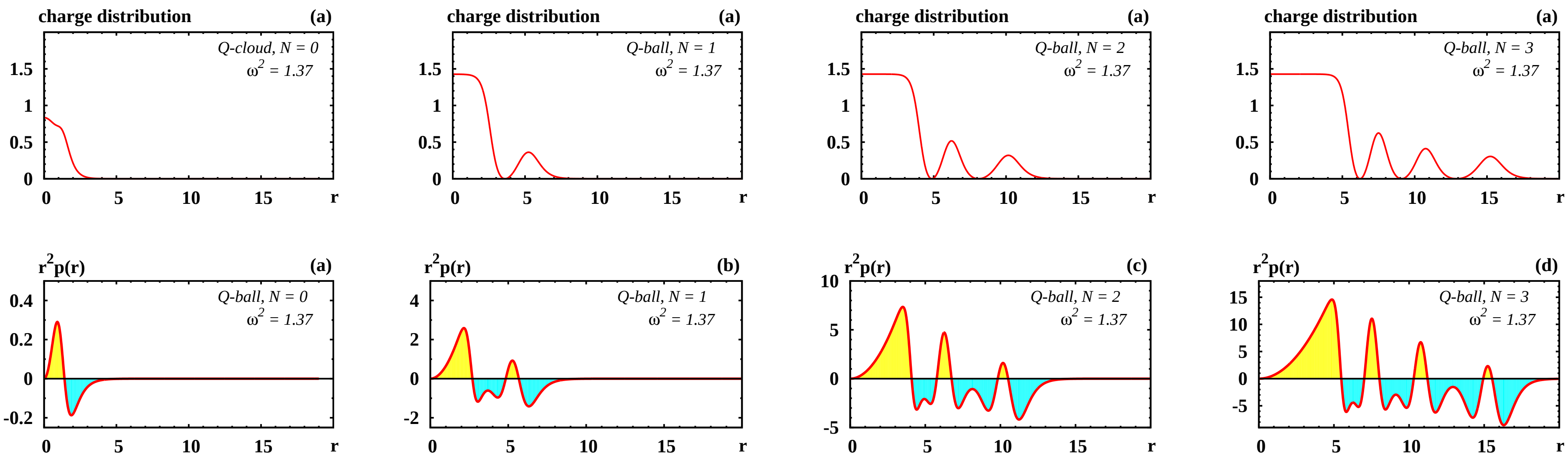
$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$ with U(1) global symm., $V = A\,(\Phi^*\Phi) - B\,(\Phi^*\Phi)^2 + C\,(\Phi^*\Phi)^3$, $\Phi(t, \vec{r}) = e^{i\omega t}\phi(r)$
S. R. Coleman, NPB262 (1985) 263, 269 (1986) 744E; M. Mai, PS PRD86 (2012) 076001

to satisfy $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0 \;\rightarrow\; p(r)$ **must have a zero! Could it have more zeros?**

N^{th} radial excitations of Q-balls

$N = 0$ ground state,
$N = 1$ first excited state, etc

Mai, PS PRD86 (2012) 096002
charge density exhibits N shells
$p(r)$ exhibits $(2N+1)$ zeros



$N > 0$ radial excitations all unstable
$\rightarrow$ decay to ground state Q-balls of smaller total energy and same total charge

nevertheless $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0$ always valid
$\rightarrow$ necessary (not sufficient) condition $\Leftrightarrow$ (local extremum of action, not global)

D-term always negative!
$\rightarrow$ is it a theorem? $\rightarrow$ for Q-balls formulated Mai, PS PRD86 (2012) 076001
$\rightarrow$ rigorous proof that $d_1 < 0$ for hadrons in QCD and other particles still awaiting

so far **all D-terms** negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q-balls, Q-clouds*

(Q-**cloud:** most extreme instability(!); parametric limit where Q-balls dissociate in free quanta; still $D < 0$)

Application I: investigating forces

question: how do strong forces balance inside the nucleon?

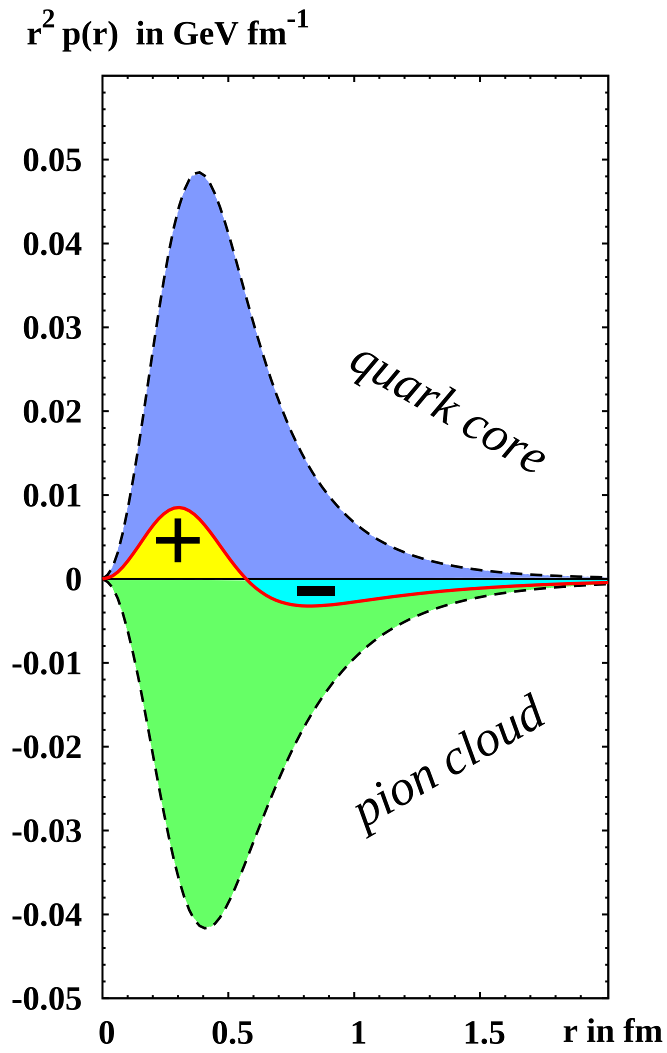
- answer in **model:** strong cancellation of repulsive forces due to quark core, and attractive forces from pion cloud
  cf. $V_{\text{conf}}(r) \approx k\,r$ with $k \approx 1\,\text{GeV/fm}$

- answer in **QCD:** we do not know nice pictures, attractive insights

  be aware:
  proton $\tau_{\text{prot}} > 2.1 \times 10^{29}$ years
  neutron $\tau_{\text{neut}} = 14\,\text{min}\;40\,\text{sec}$
  Δ-resonance $\tau_{\Delta} \sim 10^{-23}\,\text{sec}$
  $\rightarrow$ necessary condition!

- support for GPD program

- practical application!

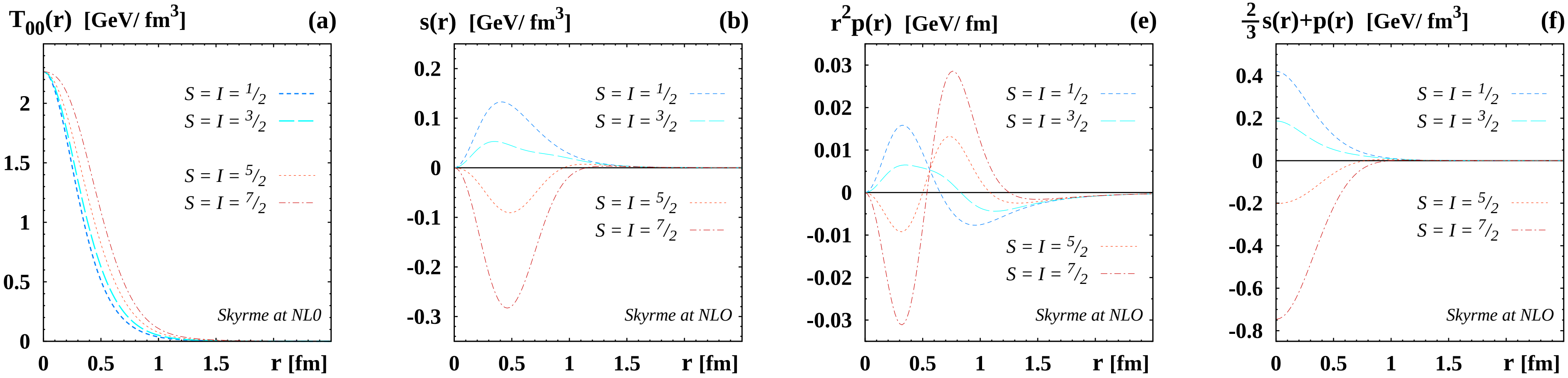


in chiral quark soliton model
chiral symmtry breaking ✓
realization of QCD in large-N_c ✓
built on instanton vacuum calculus ✓
not bad, but after all a model ...
Goeke et al, PRD75 (2007)

Application II: nucleon, Δ, large-N_c artifacts

Witten 1979

in large N_c baryons = rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \ldots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$$

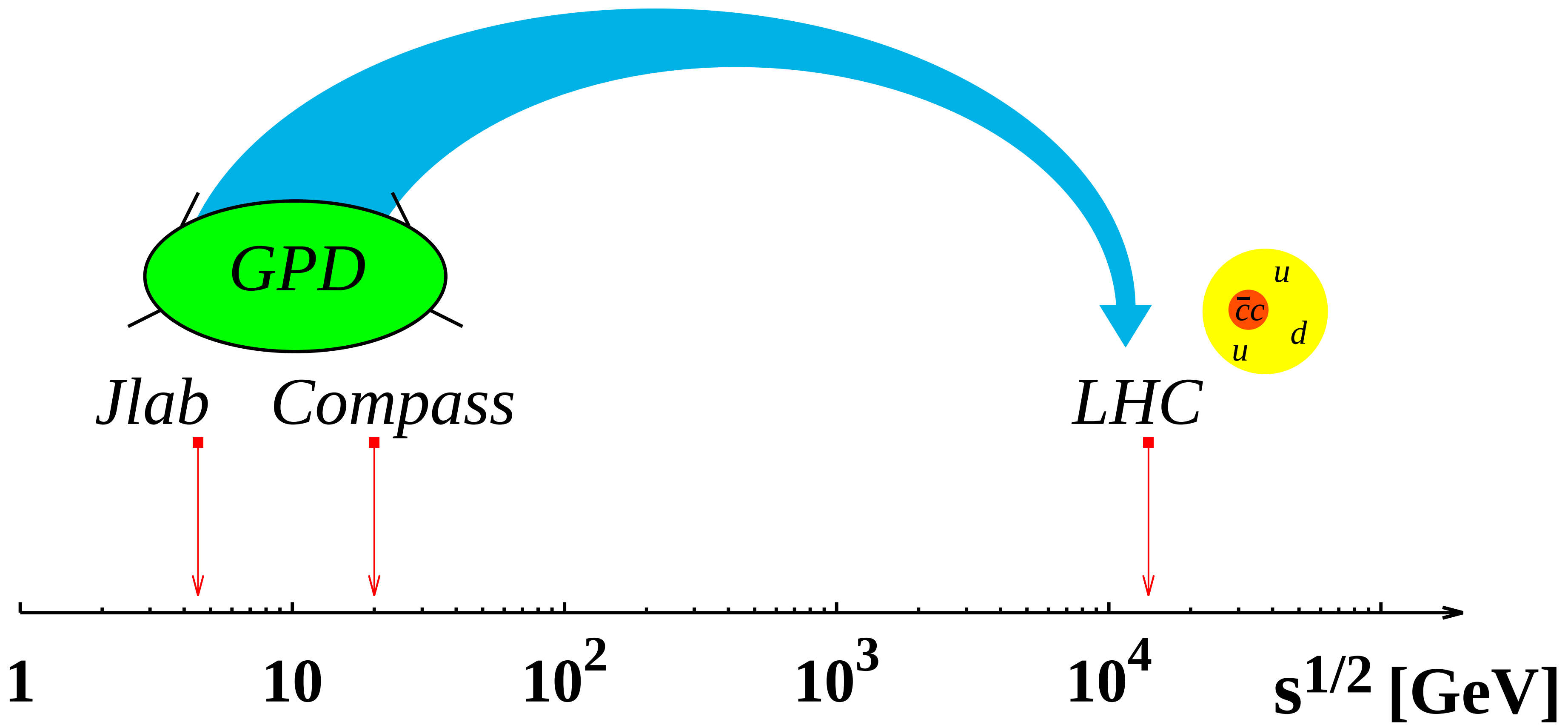
stability needs more:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

mechanical stability
$T^{ij}\, da^i \geq 0$
$\Leftrightarrow$ $\frac{2}{3}\, s(r) + p(r) \geq 0$
artifacts do not satisfy!
$\Rightarrow$ **have positive D-term!!**
That's why they do not exist!
EMT: dynamical understanding
Perevalova et al (2016)

$\Rightarrow$ particles with positive D unphysical!!!

Application III: from hard-exclusive reactions at JLab, COMPASS ...

... to spectroscopy of $\bar{c}c$-pentaquarks at LHCb



not usual hadrons, not just any exotic hadron, $\bar{c}c$-baryon bound states $\rightarrow$ rich potential!

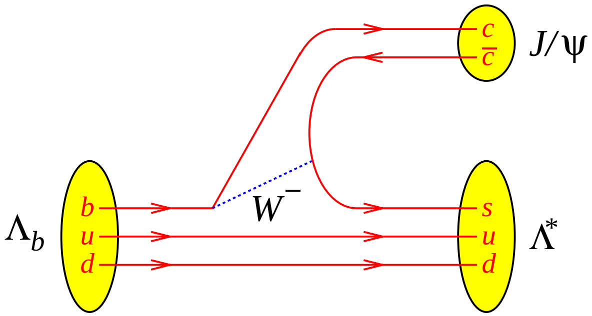
- **hidden-charm pentaquarks in Λ_b^0 decays at LHCb**

Aaij *et al.* PRL 115, 072001 (2015)

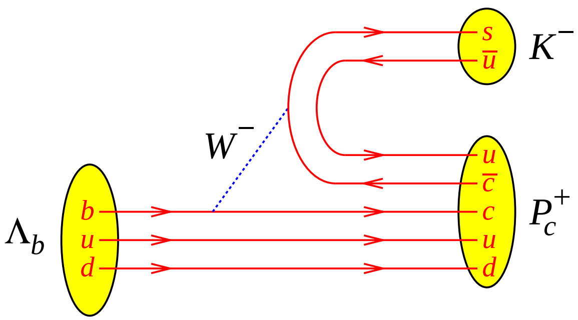
Λ_b^0 $m = 5.6$ GeV, $\tau = 1.5$ ps
J/Ψ $m = 3.1$ GeV, $\Gamma = 93$ keV, $\Gamma_{\mu^+\mu^-} = 6\,\%$
Λ^* $m = 1.4$ GeV or more, $\Lambda^* \to K^- p$ in 10^{-23}s

$\boldsymbol{\Lambda_b^0 \longrightarrow J/\Psi\, p\, K^-}$ seen

$\boldsymbol{\longrightarrow J/\Psi\, \Lambda^*}$



$\boldsymbol{\longrightarrow J/\Psi\, P_c^+}$



state	m [MeV]	Γ [MeV]	$\Gamma_{\rm rel}$	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1)\,\%$	$J/\psi\, p$	$\frac{3}{2}^{\mp}$ or $\frac{5}{2}^{+}$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2)\,\%$	$J/\psi\, p$	$\frac{5}{2}^{\pm}$ or $\frac{3}{2}^{-}$

appealing approach to new pentaquarks

M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

- **theoretical approach**

  $R_{J/\psi} \ll R_N \;\Rightarrow$ non-relativistic multipole expansion Gottfried, PRL 40 (1978) 598
  baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

  $V_{\text{eff}} = -\frac{1}{2}\,\alpha\,\vec{E}^2$ Voloshin, Yad. Fiz. **36**, 247 (1982)

- **chromoelectric polarizability**

$$\alpha(1S) \approx 0.2\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S) \approx 12\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S \to 1S) \approx \begin{cases} -0.6\,\text{GeV}^{-3}\ \text{(pert)}, \\ \pm 2\,\text{GeV}^{-3}\ \text{(pheno)}, \end{cases}$$

in heavy quark mass limit & large-N_c limit
$\rightsquigarrow$ "perturbative result" Peskin, NPB 156 (1979) 365

value for $2S \to 1S$ transition from
phenomenological analysis of $\psi' \to J/\psi\,\pi\,\pi$ data
Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455

- **chromoelectric field strength:**

$$\vec{E}^2 = g^2\left(\frac{8\pi^2}{bg^2}\,T^{\mu}{}_{\mu} + T^{G}_{00}\right)$$

$b = \frac{11}{3}\,N_c - \frac{2}{3}\,N_F$ leading coeff. of β-function
$g =$ strong coupling at low (nucleon) scale $\lesssim 1$ GeV
$g_s =$ strong coupling at scale of heavy quark ($g_s \neq g$)
$T^{G}_{00} = \xi T_{00}$ with $\xi =$ fractional contributions of gluon to M_N
$T^{\mu}{}_{\mu} = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i}{r}\frac{r^j}{r} - \frac{1}{3}\,\delta^{ij}\right)s(r) + \delta^{ij}\,p(r)$

- **universal effective potential**

$$\boldsymbol{V_{\text{eff}} = -\tfrac{1}{2}\,\alpha\,\frac{8\pi^2}{b}\,\frac{g^2}{g_s^2}\left[\nu\,T_{00}(r) + 3p(r)\right], \quad \nu = 1 + \xi_s\,\frac{b\,g_s^2}{8\pi^2}}$$

$\nu \approx 1.5$ estimate by Eides et al, op. cit.
Novikov & Shifman, Z.Phys.C8, 43 (1981);
X. D. Ji, Phys. Rev. Lett. **74**, 1071 (1995)

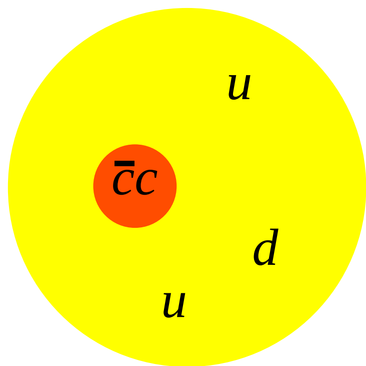
- **in future GPDs can help:** GPDs ⇒ EMT form factors ⇒ EMT densities ⇒ universal potential $V_{\rm eff}$ for quarkonium-baryon interaction!

- **currently:** use chiral quark soliton model (Eides et al, 2015); Skyrme (Perevalova et al 2016)

- **compute quarkonium-nucleon bound state**

solve $\left(-\frac{\vec{\nabla}^2}{2\mu} + V_{\rm eff}(r)\right)\psi = E_{\rm bind}\,\psi$

$\mu =$ reduced quarkonium-baryon mass



- **results:**

nucleon and J/ψ form no bound state

nucleon and $\psi(2S)$ form 2 bound states with nearly degenerate masses around 4450 MeV in $L = 0$ channel, $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ if $\alpha(2S) \approx 17$ GeV^{-3} (consistent with guideline from pert. calc.)

- **decay**

  $M_{\psi(2S)} + M_N > 4450\,\text{MeV}$ so no decay to N and $\Psi(2S)$ possible

  $\Gamma_{\Psi(2S)} \sim 300\,\text{keV}$, "wait" for transition $(2S) \to (1S)$ governed by same $\boldsymbol{V_{\text{eff}}}$
  but with small $\alpha(2S \to 1S)$ transition polarizability

  transition "takes time," after "completed,"
  prompt decay to J/ψ + nucleon (observed final states)
  estimated width is tens of MeV $\to$ compatible with data!

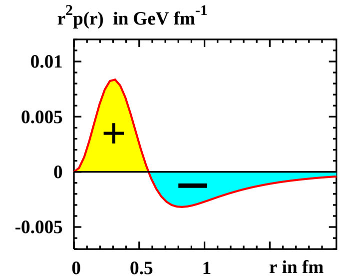
- new prediction:
  also **Δ and $\psi(2S)$ form a bound state!**
  isospin $\frac{3}{2}$, mass $= 4.5\,\text{GeV}$, $\Gamma_{\Delta\bar{c}c} \sim 60$ MeV
  positive parity, spin $|\frac{3}{2} - 1| \leq J \leq \frac{3}{2} + 1$
  (states degenerate in heavy quark limit)
  decay $\boldsymbol{P_c(4500)} \to J/\psi + \underbrace{\text{nucleon} + \text{pion}}_{\Delta\text{-resonance}}$ $\leftarrow$ test the approach
  JLab, Meziani et al arXiv:1609.00676

- what about $P_c^+(4380)$?
  broader, not charmonium-nucleon bound-state, different mechanism

Summary & Outlook

- **GPDs** important objects, we learn a lot!!

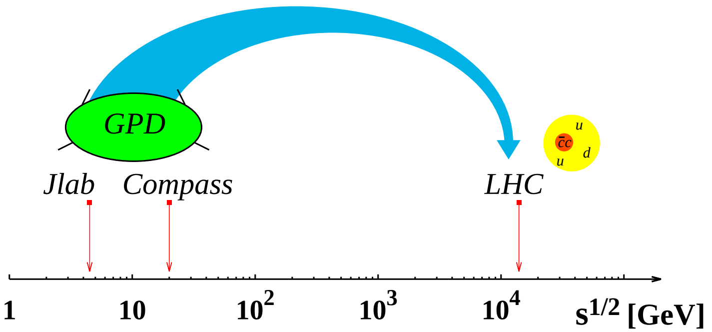
- $\hookrightarrow$ form factors of **energy momentum tensor**
  mass decomposition, spin decomposition, and D-term!



- **D-term**: last unknown global property, related to forces
  attractive and physically appealling $\rightarrow$ "motivation"

- recent development: knowledge of internal forces and energy density
  $\rightarrow$ **quarkonium-baryon interaction** $V_{\rm eff}$

- naturally explains properties of $P_c^+(4450)$ observed at LHCb
  rich potential, new predictions, ongoing work



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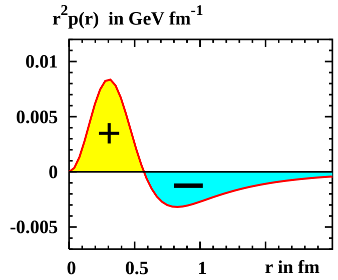

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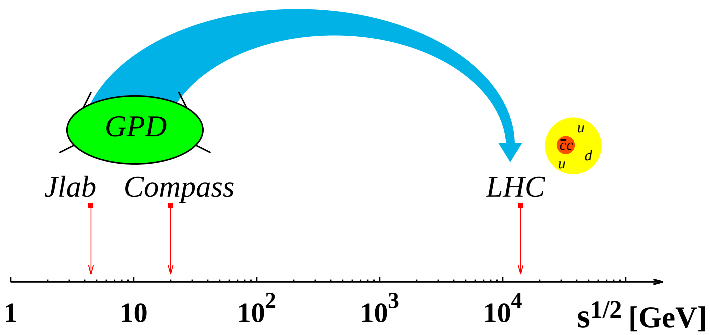

Thank you!