

lessons learnt at HERMES on technical aspects of TMD measurements

- the devil is in the detail -

discuss

Disclaimers

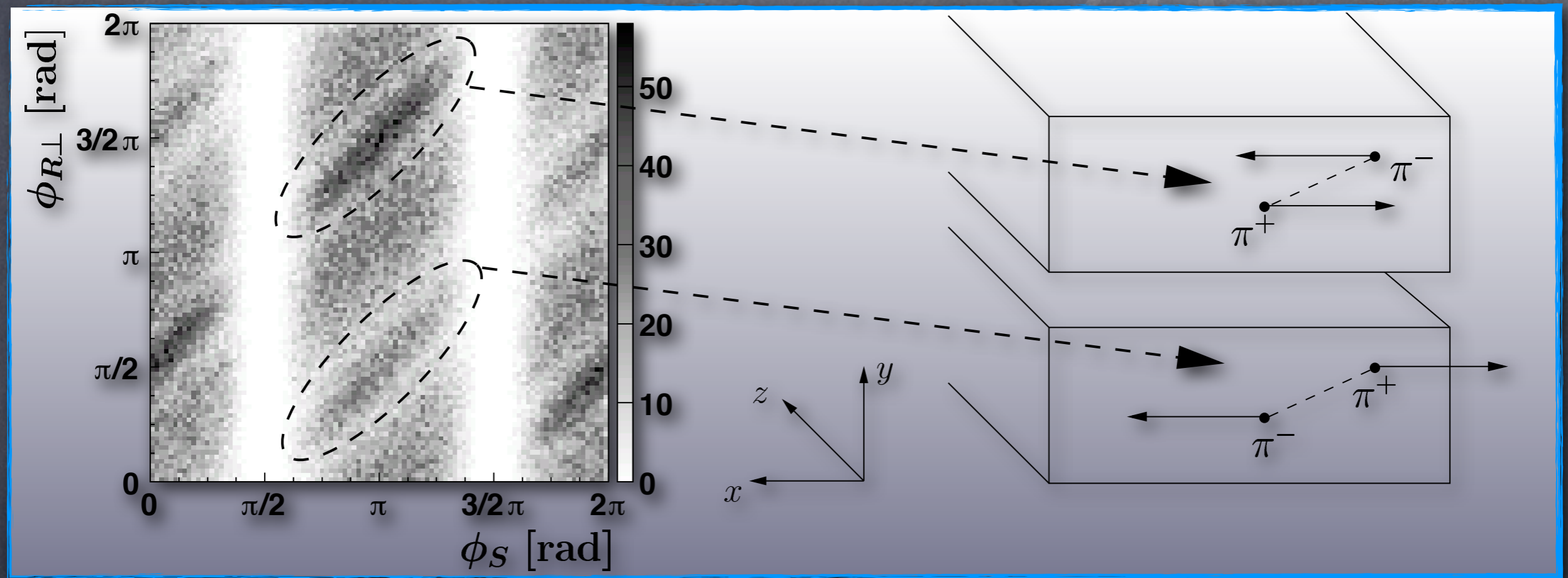
- contains a number of trivial, but hopefully still useful, statements
- can not offer a general recipe, though hopefully some guidance

Prelude: role of acceptance in experiments

an unfortunate Lemma

- “No particle-physics experiment has a perfect acceptance!”
- obvious for detectors with gaps/holes
- but also for “ 4π ”, especially when looking at complicated final states

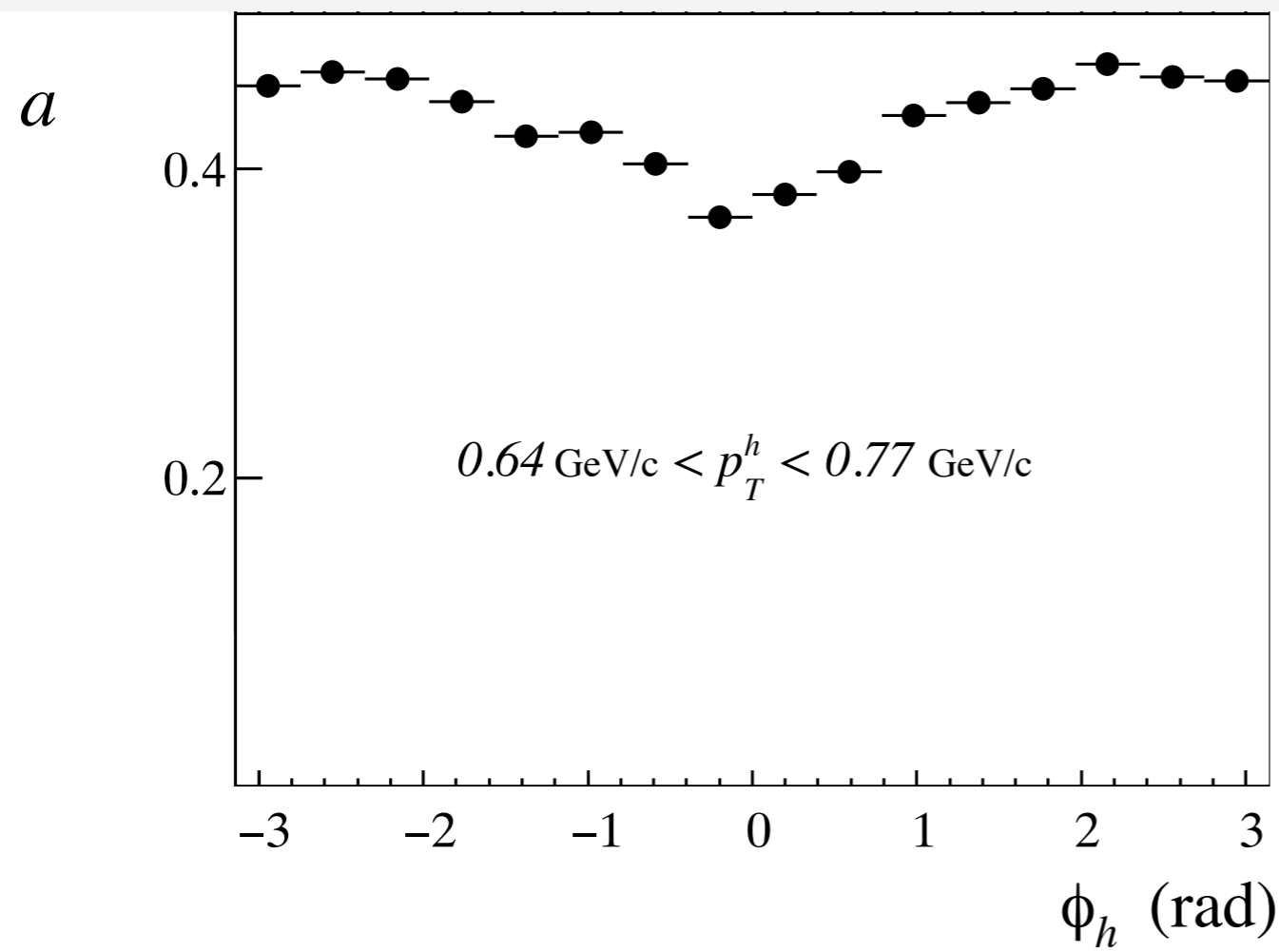
an unfortunate Lemma



HERMES azimuthal acceptance for 2-hadron production

[P. van der Nat, Ph.D. thesis, Vrije Universiteit (2007)]

an unfortunate Lemma

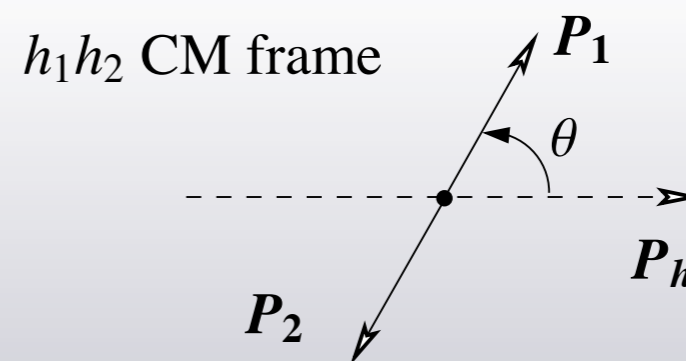
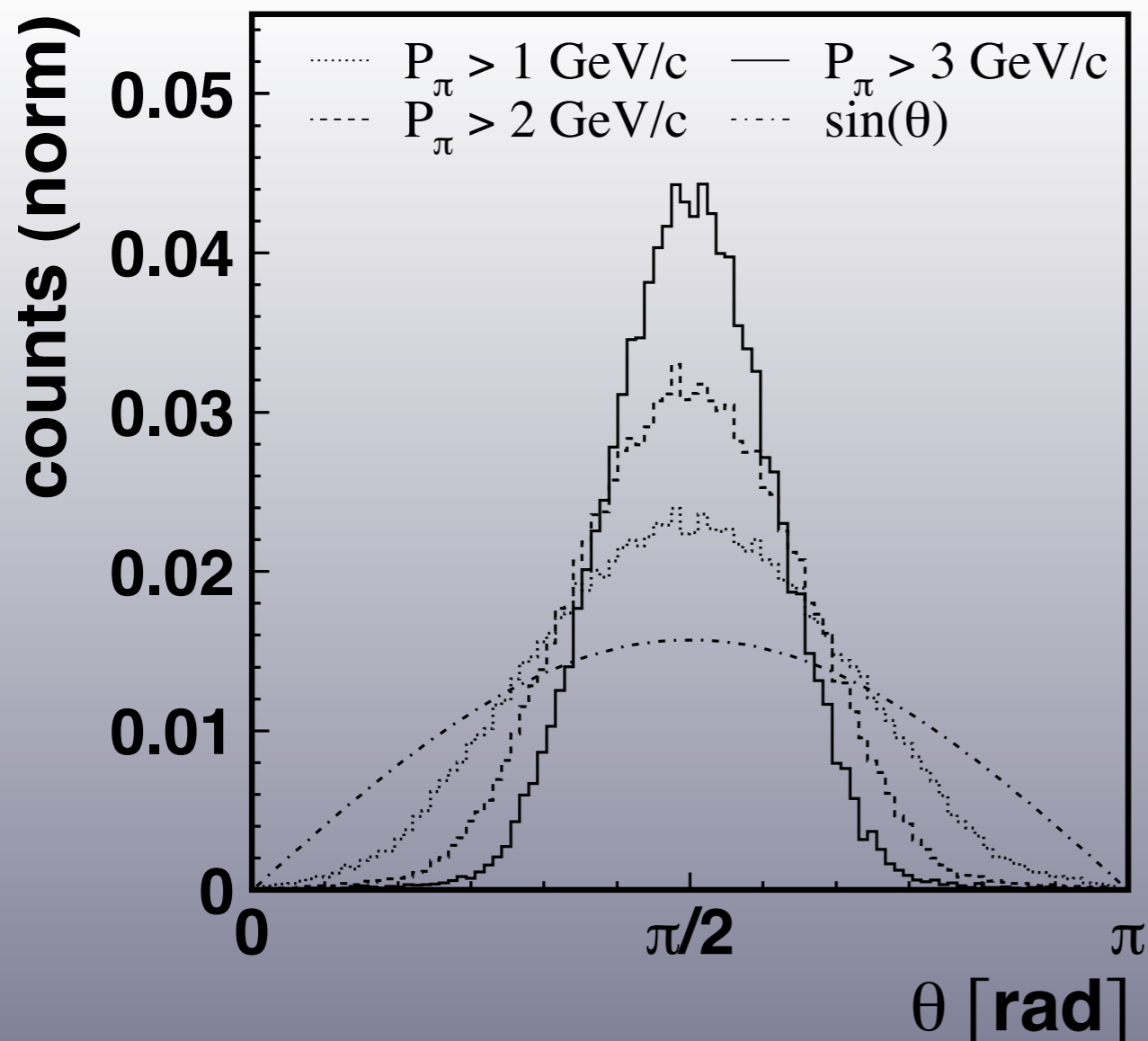


maybe " 2π " around beam axis, but not around virtual-photon axis because of lower limit on θ

[see also A. Bianconi et al., Eur.Phys.J. A49 (2013) 42]

[C. Adolph, [arXiv:1401.6284](https://arxiv.org/abs/1401.6284)]

an unfortunate Lemma



**momentum requirements
strongly distort kinematic
distributions even for
"4 π " acceptance**

[P. van der Nat, Ph.D. thesis,
Vrije Universiteit (2007)]

an unfortunate Lemma

- “No particle-physics experiment has a perfect acceptance!”
 - obvious for detectors with gaps/holes
 - but also for “ 4π ”, especially when looking at complicated final states
- How acceptance effects are handled is one of the essential questions in experiments!

some acceptance effects

- acceptance in kinematic variable studied, e.g., azimuthal coverage in extraction of azimuthal moments
- acceptance in kinematic variables integrated over, e.g., due to limited statistics not being able to do fully differential analysis
- event migration due to smearing

a common misconception

- "acceptance cancels in asymmetries"

discussion slides

a common misconception

- “acceptance cancels in asymmetries”

$$\begin{aligned} A_{UT}(\phi, \Omega) &= \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} && \Omega = x, y, z, \dots \\ &= \frac{\sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)} && \epsilon : \text{detection efficiency} \\ &\neq \frac{\int d\Omega \sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)} \neq \frac{\int d\Omega \sigma_{UT}(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega)} \equiv A_{UT}(\phi) \end{aligned}$$

Acceptance does not cancel in general when integrating numerator and denominator over (large) ranges in kinematic variables!

... geometric acceptance

extract acceptance from Monte Carlo simulation? ...

$$\epsilon(\phi, \Omega) = \frac{\epsilon(\phi, \Omega) \sigma_{UU}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \quad \Omega = x, y, z, \dots$$

simulated acceptance

simulated cross section

discuss

... geometric acceptance

extract acceptance from Monte Carlo simulation? ...

$$\epsilon(\phi, \Omega) = \frac{\epsilon(\phi, \Omega) \sigma_{UU}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)}$$

$$\Omega = x, y, z, \dots$$

$$\neq \frac{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega)}$$

"Aus Differenzen und Summen kürzen nur die Dummen."

discuss

... geometric acceptance

extract acceptance from Monte Carlo simulation? ...

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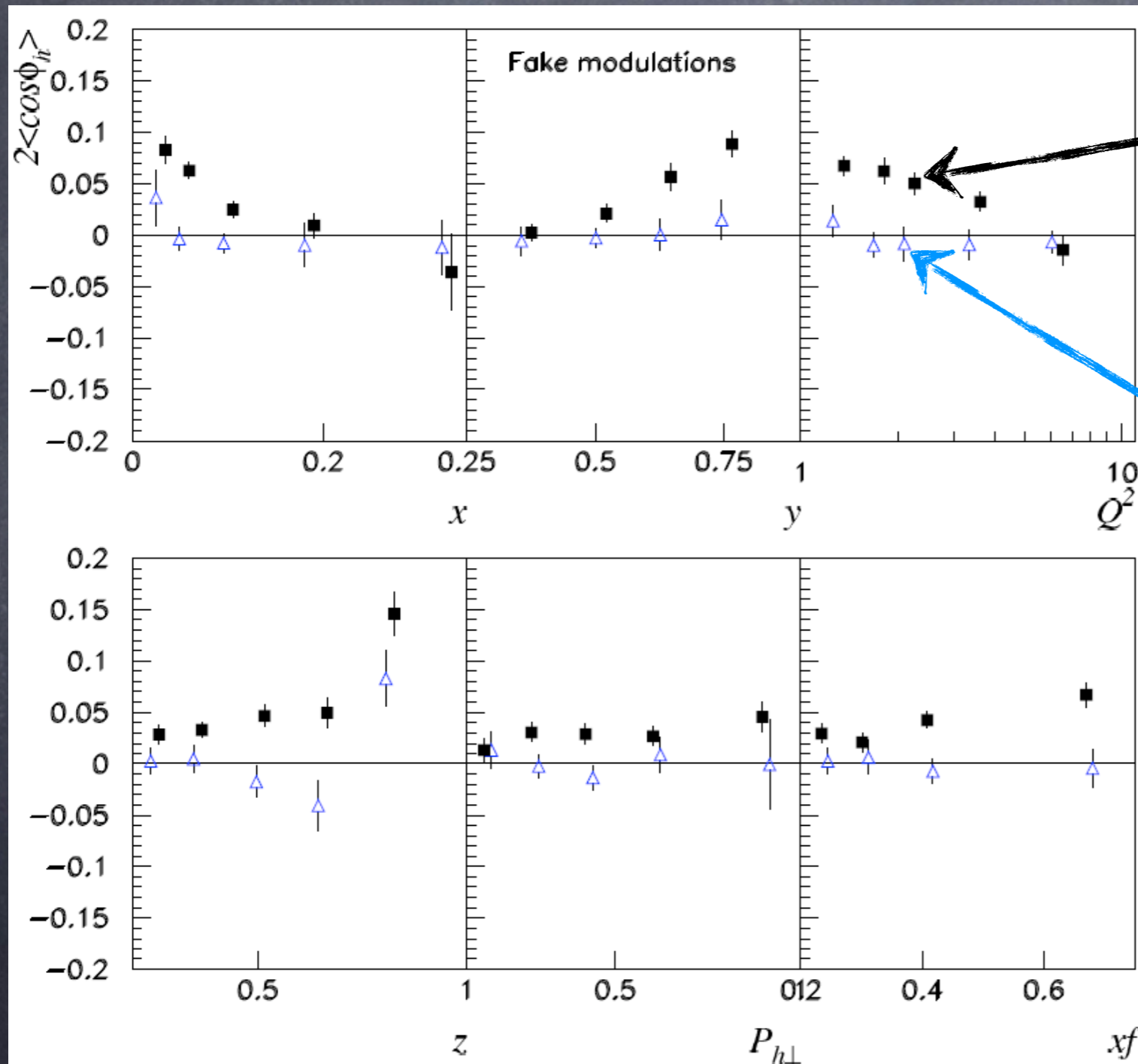
$$\neq \frac{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega)}$$

"Aus Differenzen und Summen kürzen nur die Dummen."

$$\neq \int d\Omega \epsilon(\phi, \Omega) \equiv \epsilon(\phi)$$

Cross-section model does NOT CANCEL in general when integrating numerator and denominator over (large) ranges in kinematic variables!

"Classique" Example: $\langle \cos\phi \rangle_{UU}$



1D correction

(input: MC without azimuthal modulation)

5D correction

[F. Giordano, Transversity 2008, Ferrara]

... averaging ...

often enough one has to average observables over available phase space:

$$\langle A(\Omega) \rangle_{\epsilon} \equiv \int d\Omega A(\Omega) \epsilon(\Omega)$$

properly normalized for simplicity

discussion

... averaging ...

often enough one has to average observables over available phase space:

$$\langle A(\Omega) \rangle_{\epsilon} \equiv \int d\Omega A(\Omega) \epsilon(\Omega)$$

$$\neq \int d\Omega A(\Omega) \equiv \langle A(\Omega) \rangle_{\text{“}4\pi\text{”}}$$

... averaging ...

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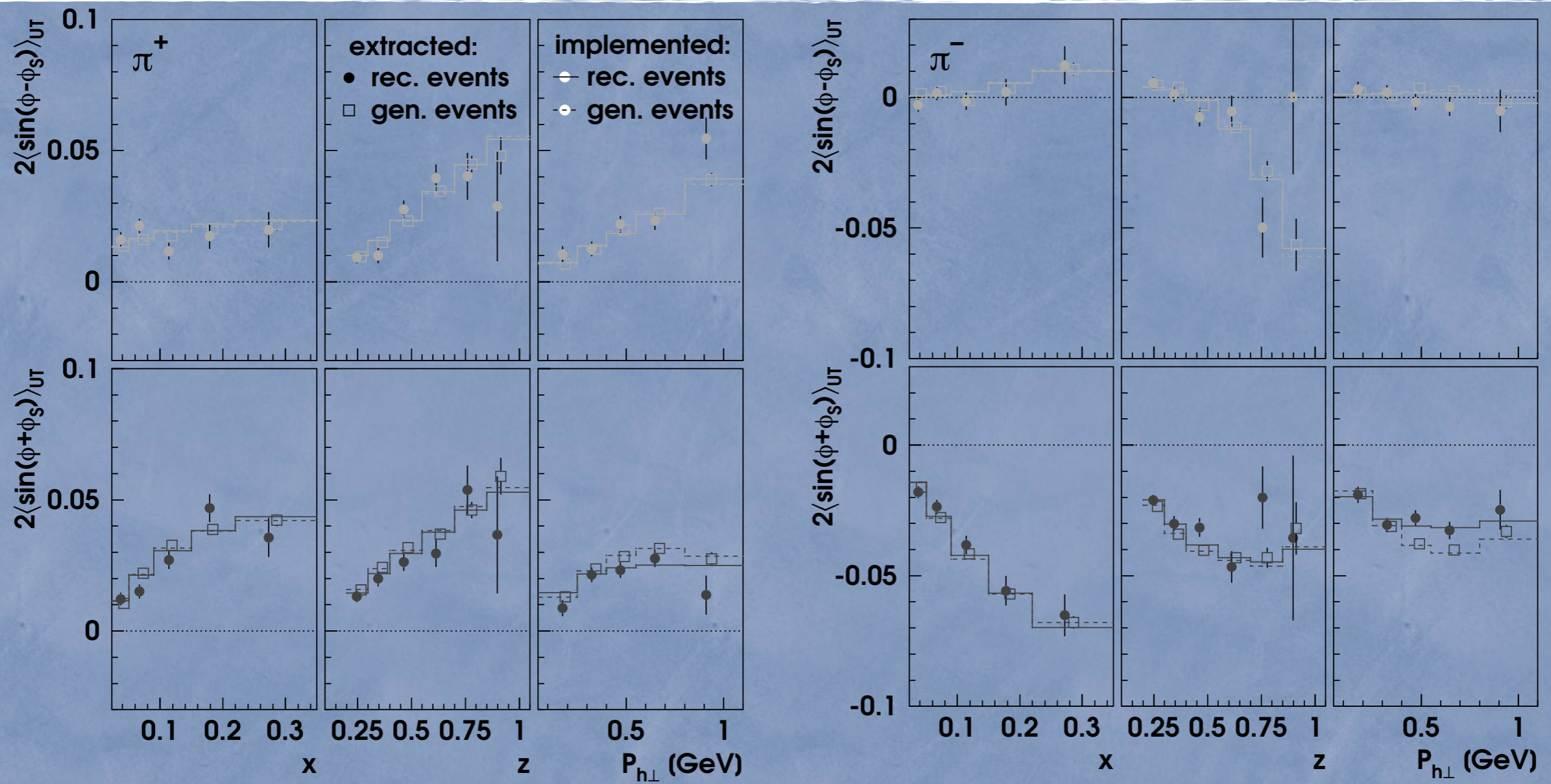
$$\langle A(\Omega) \rangle_{\epsilon} \equiv \int d\Omega A(\Omega) \epsilon(\Omega)$$

$$\neq \int d\Omega A(\Omega) \equiv \langle A(\Omega) \rangle_{\text{“}4\pi\text{”}}$$

life (of the experimentalist) simplifies if asymmetries are weakly (i.e. not more than linearly) dependent on kinematics:

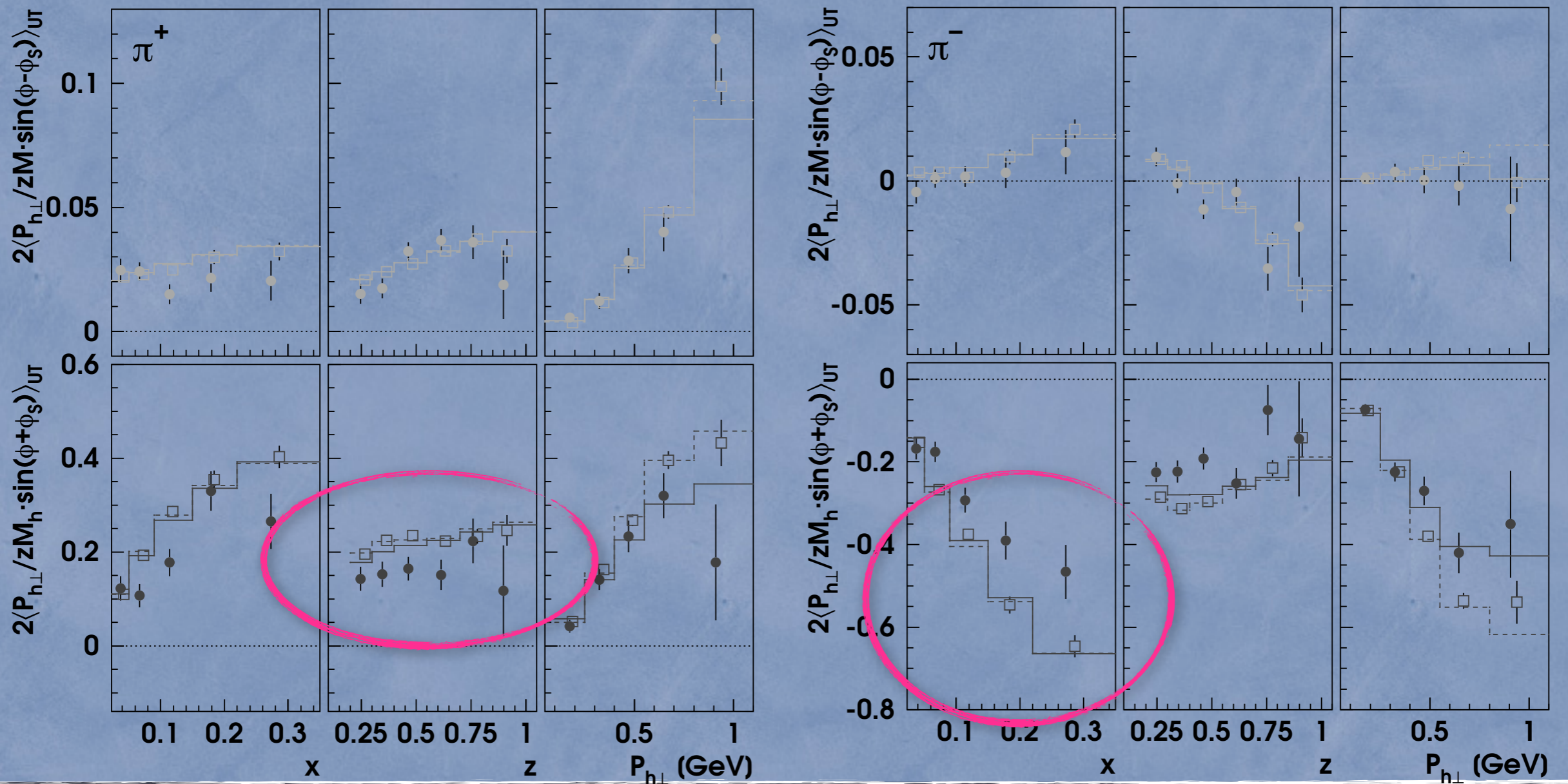
$$\langle A(\Omega) \rangle_{\epsilon} = A(\langle \Omega \rangle_{\epsilon}) \quad \text{for} \quad A(\Omega) = A_0 + A_1 \Omega$$

generated vs. extracted A_{UT}



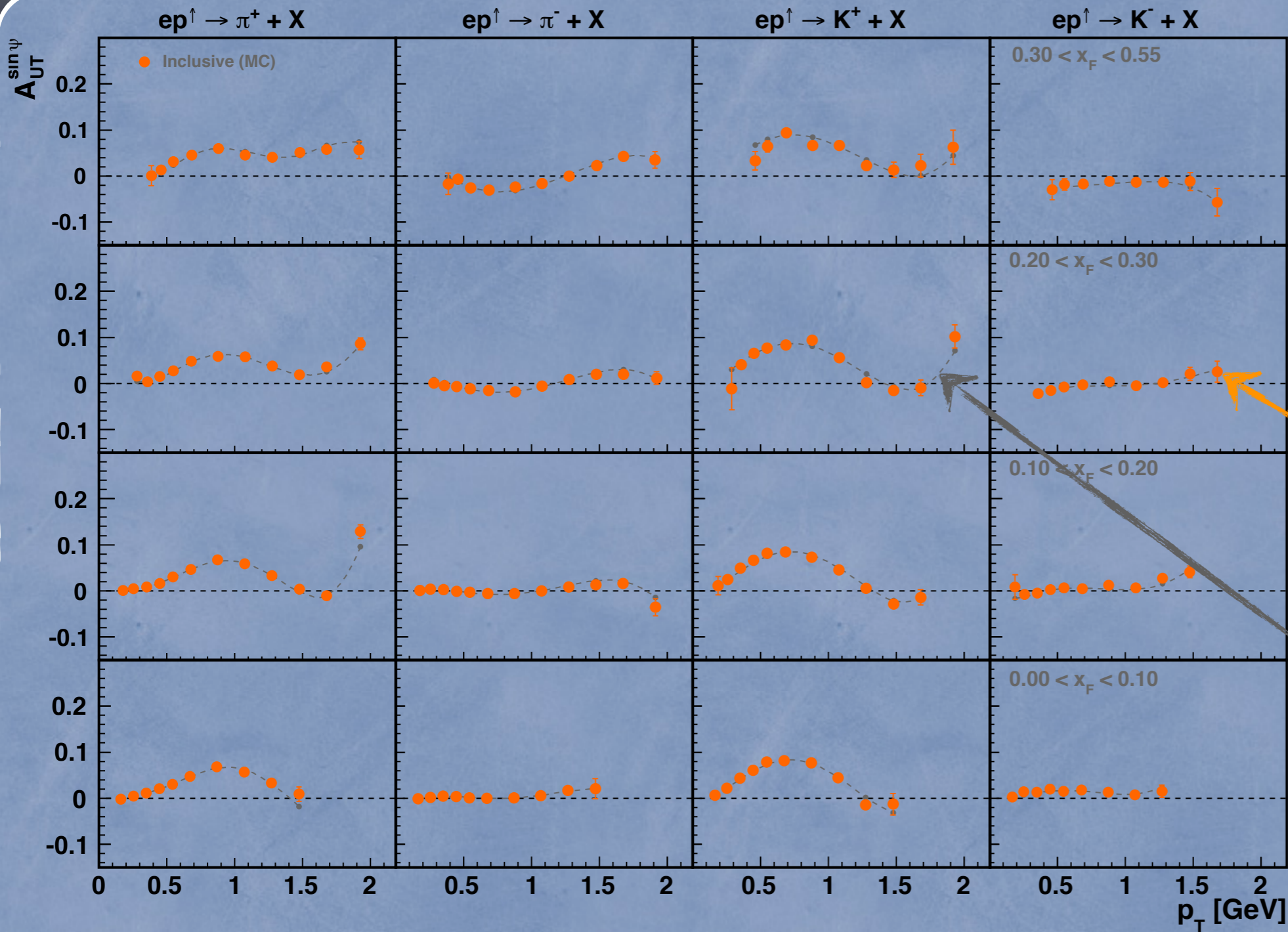
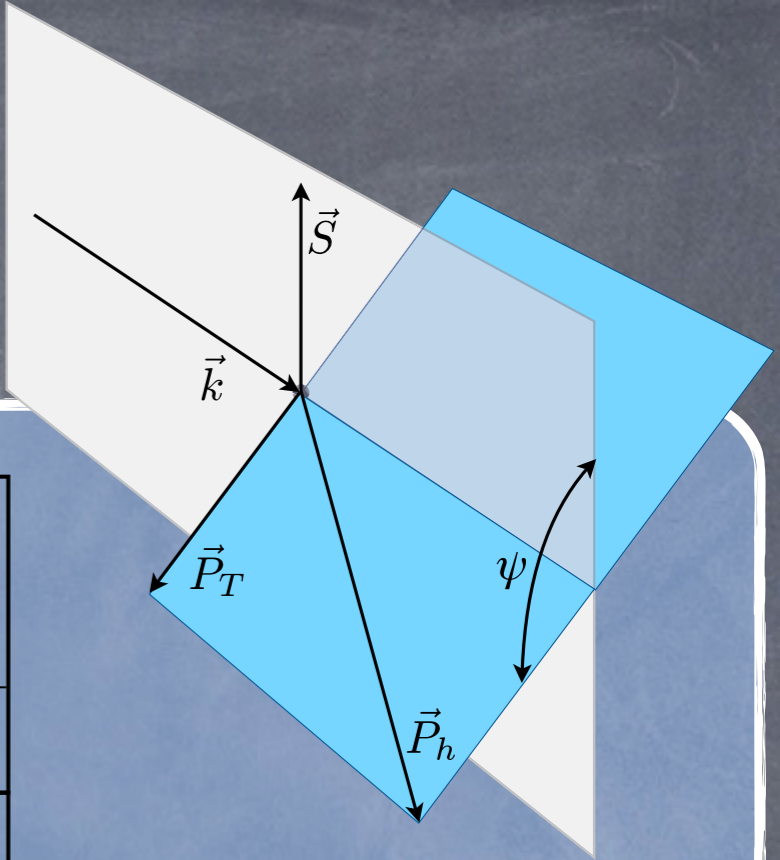
Extraction method works well!

what about weighted moments



Not so good news for weighted moments!

$A_{UT}^{\sin\psi}$ inclusive hadrons

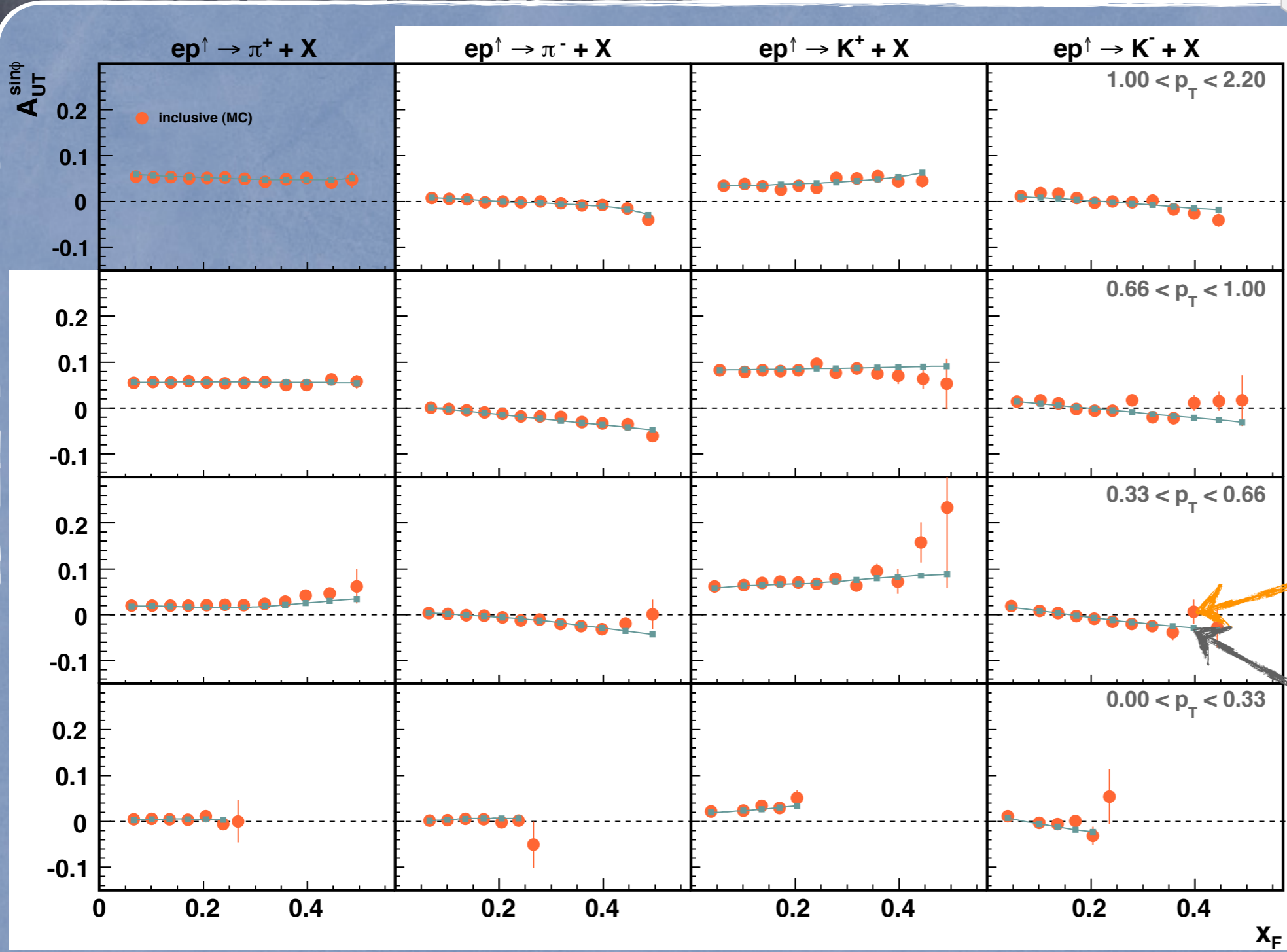
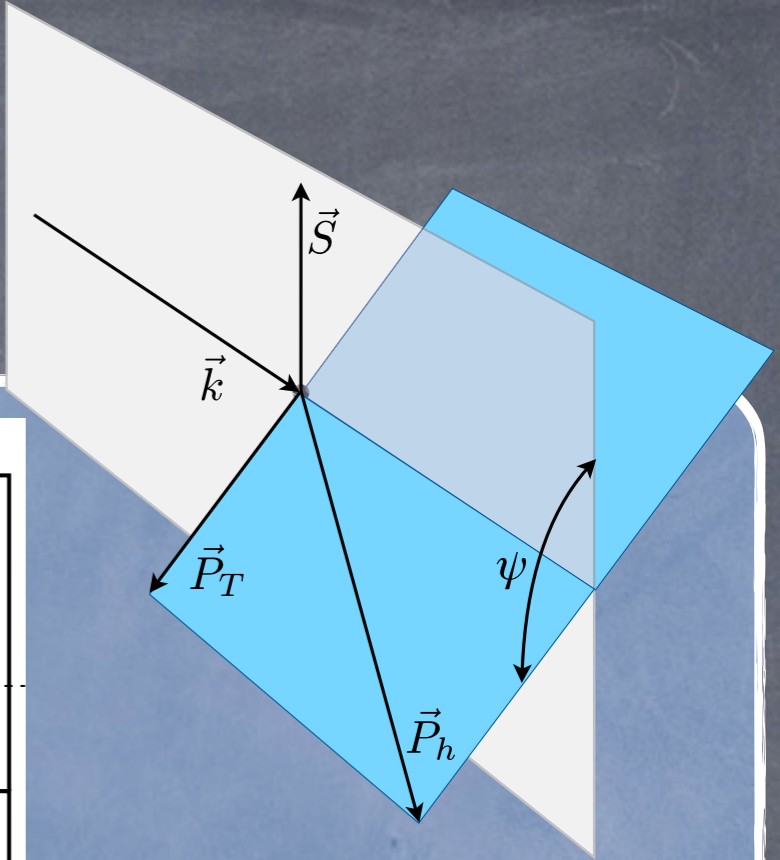


reconstructed
MC

input model
(fit to data)

small detector effects in fully differential analysis

A_{UT} inclusive hadrons

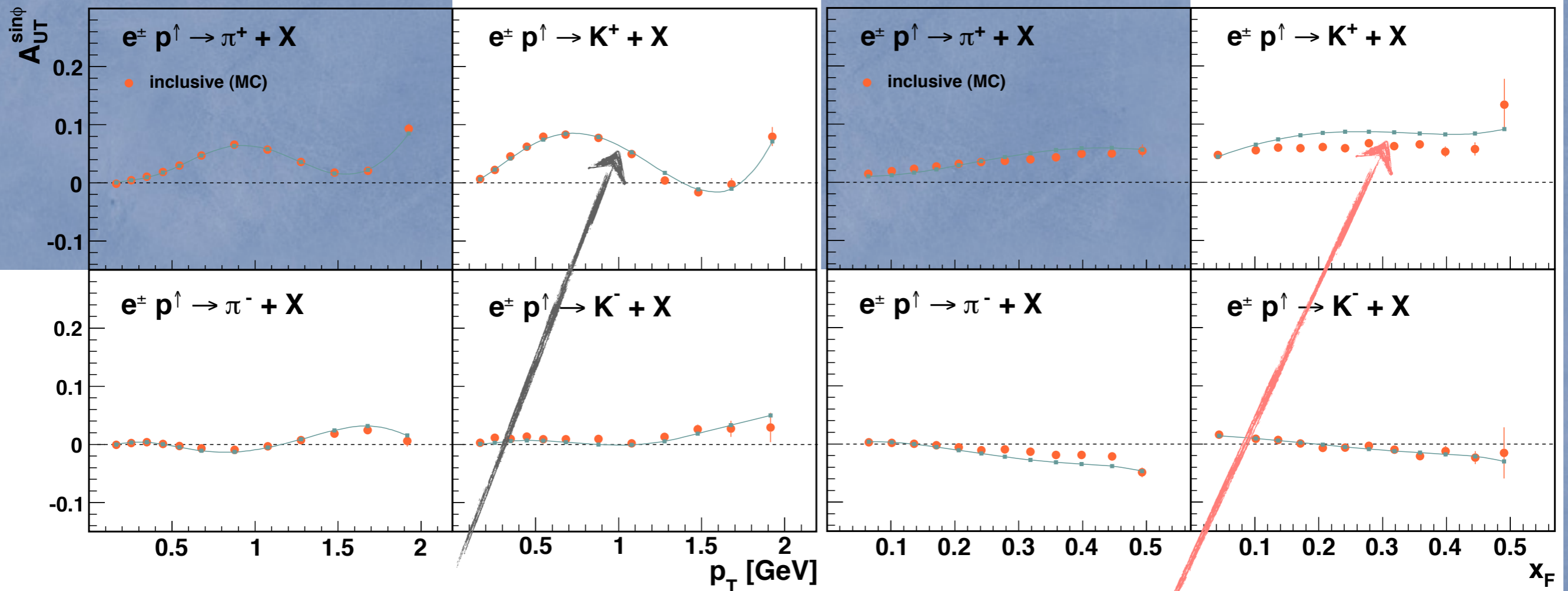


reconstructed MC

input model (fit to data)

small detector effects in fully differential analysis

A_{UT} inclusive hadrons



strong kinematic dependence can lead to large systematic effects if integrated over

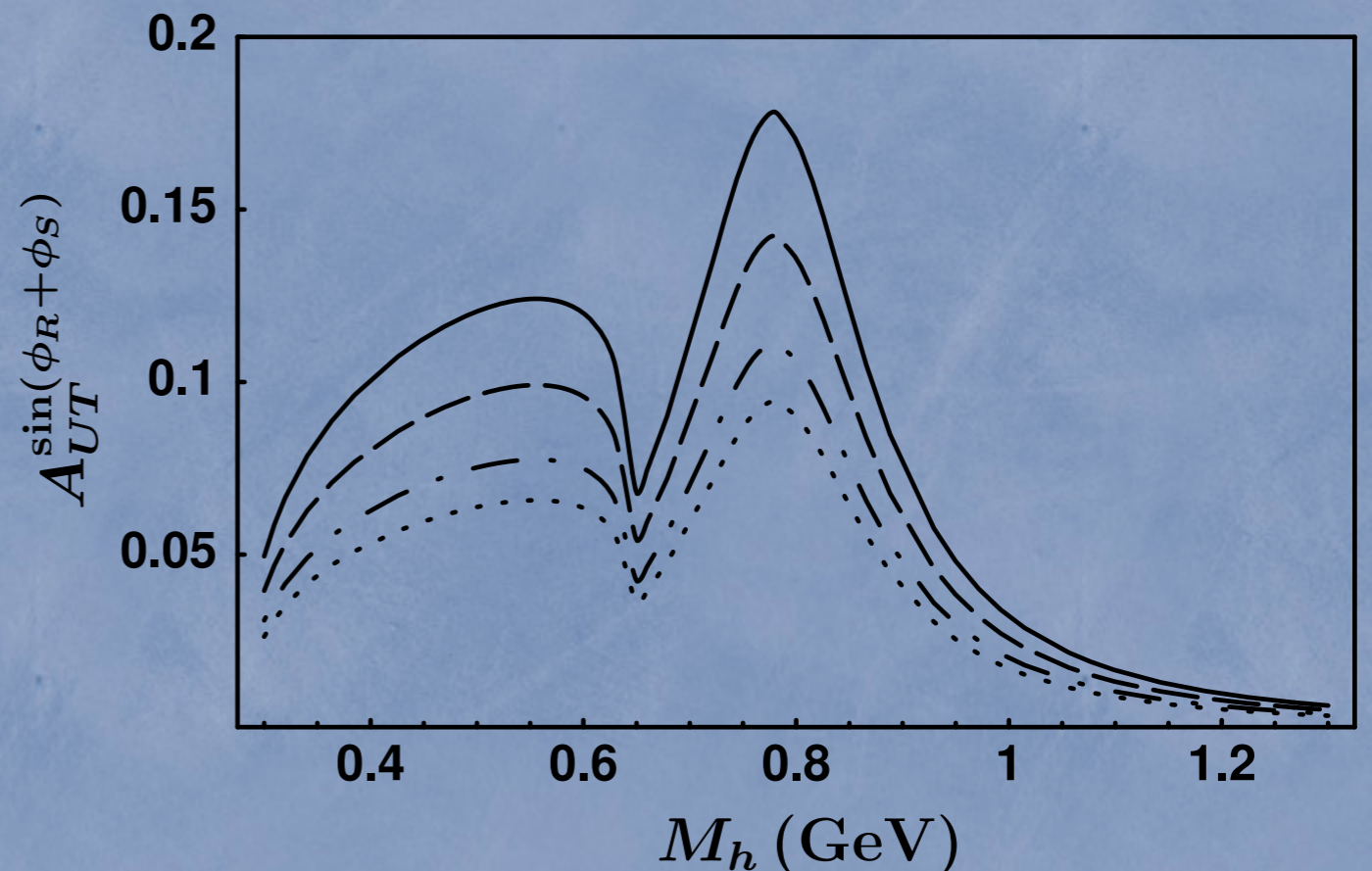
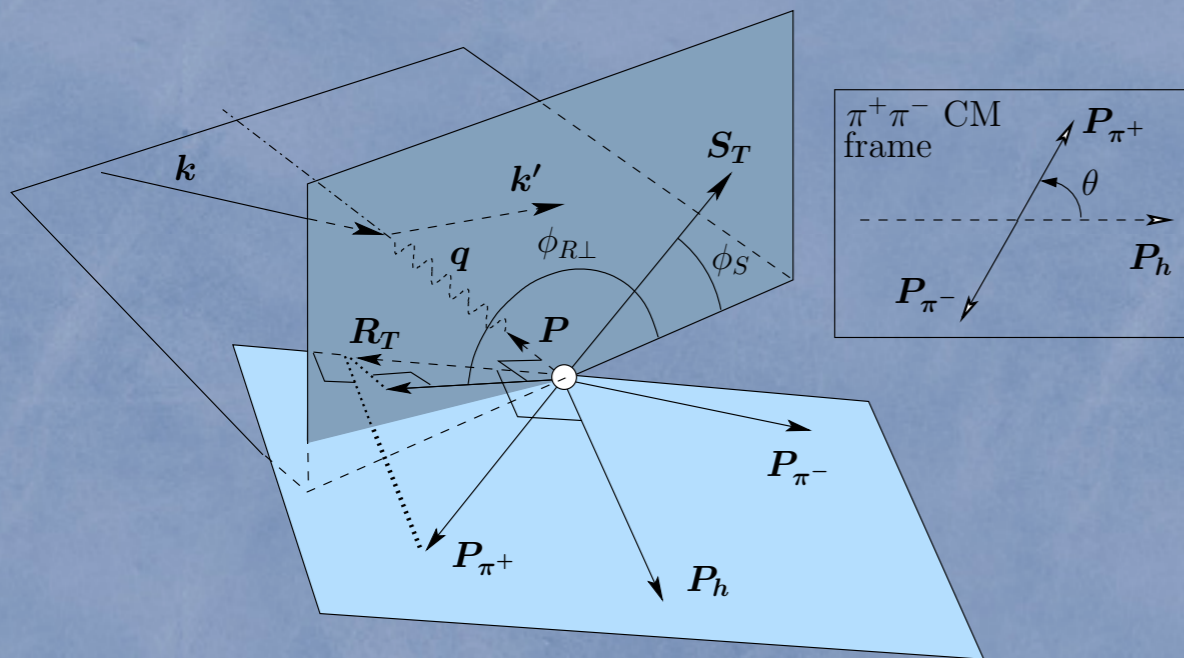
not so small detector effects in 1D analysis

similar problem: di-hadron A_{UT}

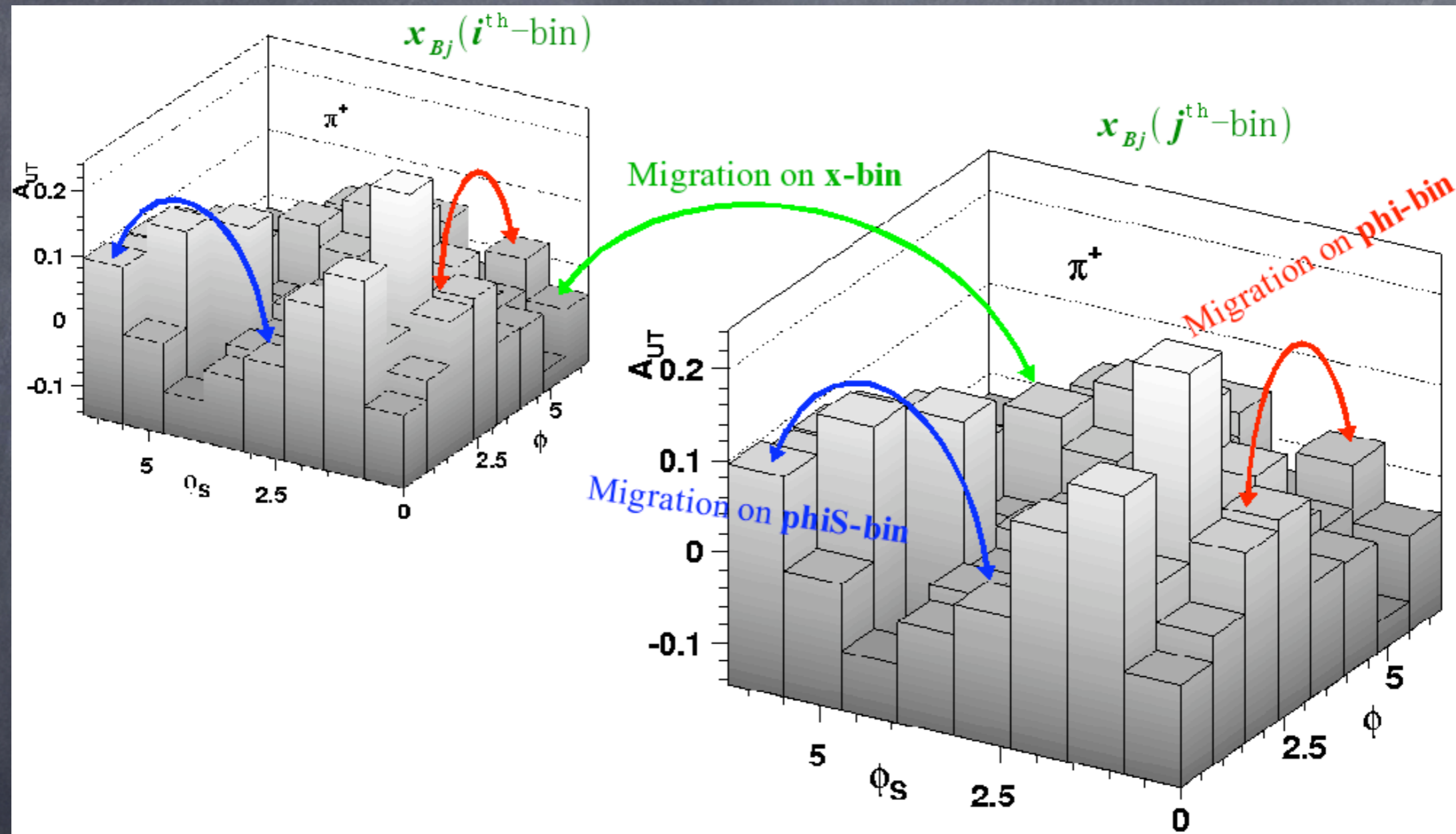
$$N^{\uparrow(\downarrow)}(\phi_{R\perp}, \phi_S, \theta, M_{\pi\pi}) \propto \int dx dy dz d^2 \mathbf{P}_{h\perp} \epsilon(x, y, z, \mathbf{P}_{h\perp}, \phi_{R\perp}, \phi_S, \theta, M_{\pi\pi}) \times \\ \times \sigma_{U\uparrow(\downarrow)}(x, y, z, \mathbf{P}_{h\perp}, \phi_{R\perp}, \phi_S, \theta, M_{\pi\pi}),$$

- many kinematic variables needed to describe process
- at least for one of them strong dependence expected:

[A. Bacchetta and M. Radici, Phys. Rev. D74 (2006) 114007]



... event migration ...



- migration correlates yields in different bins
- can't be corrected properly in bin-by-bin approach

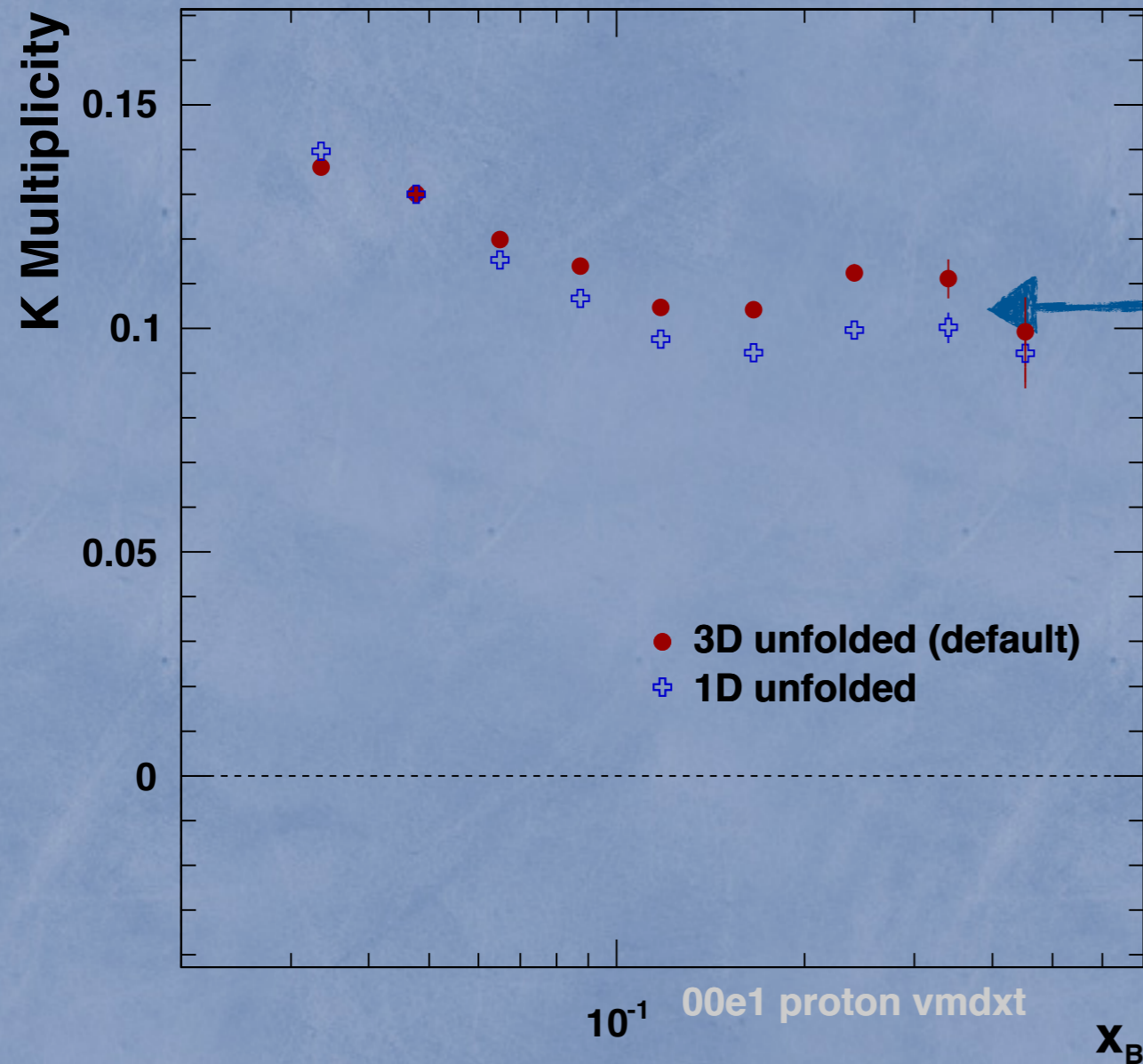
... event migration → unfolding

$$Y^{\text{exp}}(\Omega_i) \propto \sum_{j=1}^N S_{ij} \int_j d\Omega d\sigma(\Omega) + \mathcal{B}(\Omega_i)$$

- experimental yield in i^{th} bin depends on all Born bins j ...
- ... and on BG entering kinematic range from outside region
- smearing matrix S_{ij} embeds information on migration
 - determined from Monte Carlo - independent of physics model in limit of infinitesimally small bins and/or flat acceptance/cross-section in every bin
 - in real life: dependence on BG and physics model due to finite bin sizes
- inversion of relation gives Born cross section from measured yields

Multi-D vs. 1D unfolding

[S.J. Joosten, PhD thesis UIUC (2013)]

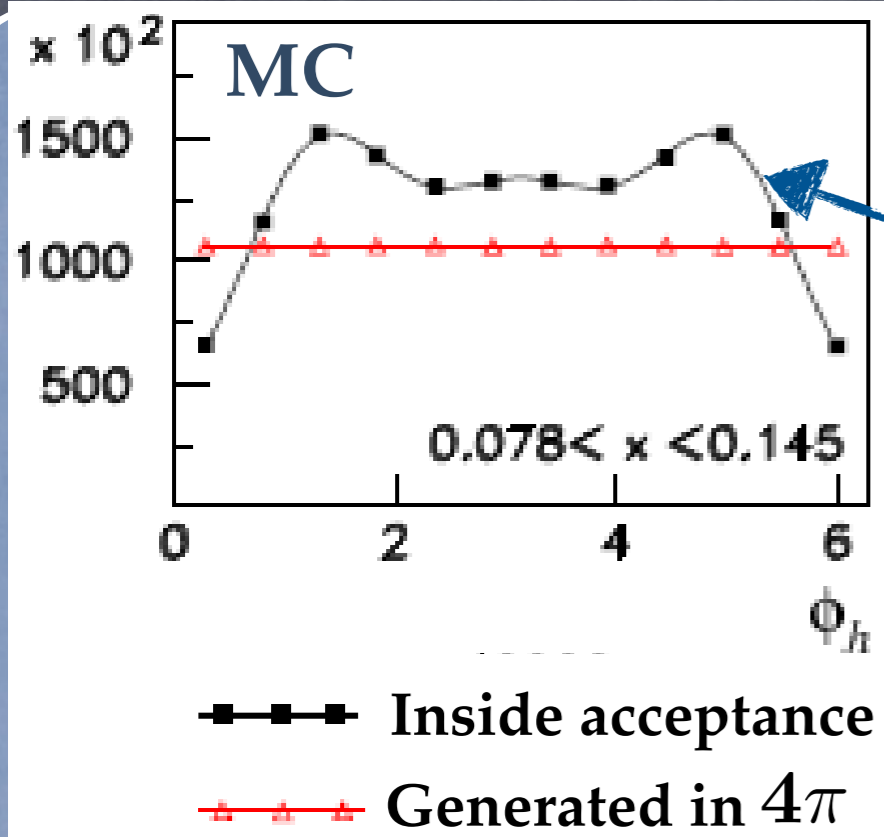


Neglecting to unfold in z changes x dependence dramatically

➔ 1D unfolding clearly insufficient

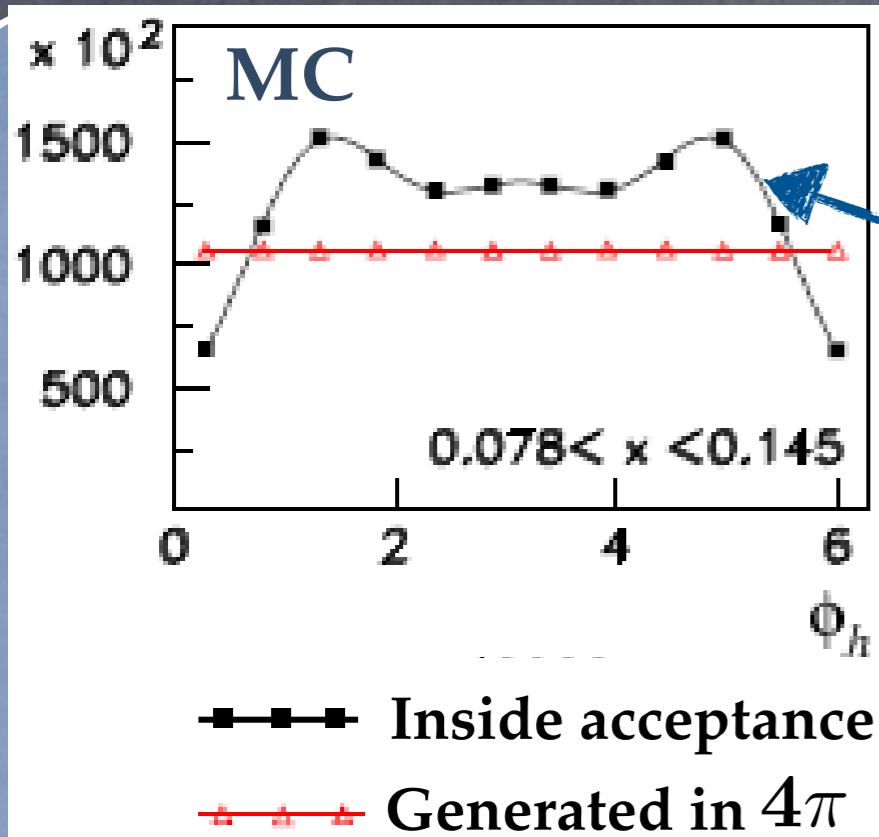
even though only interested in collinear observable, need to carefully consider transverse d.o.f.

Multi-D vs. 1D unfolding



fully simulated yield with clear cosine modulations from migration and acceptance

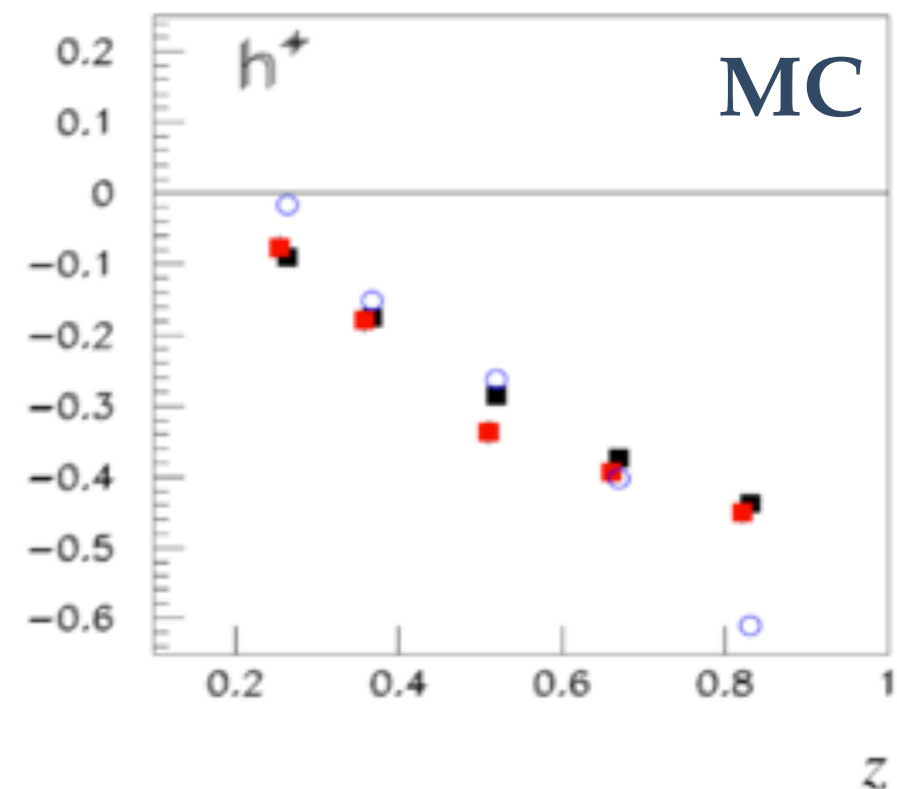
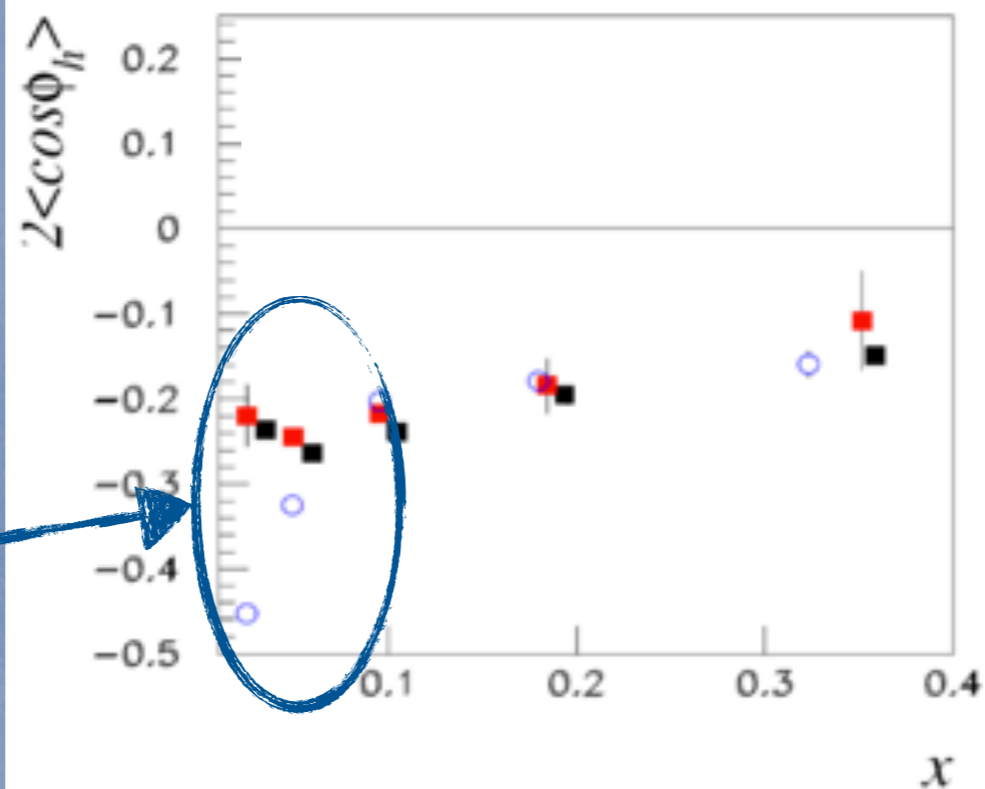
Multi-D vs. 1D unfolding



fully simulated yield with clear cosine modulations from migration and acceptance

■ Model ■ 4D ○ 1D

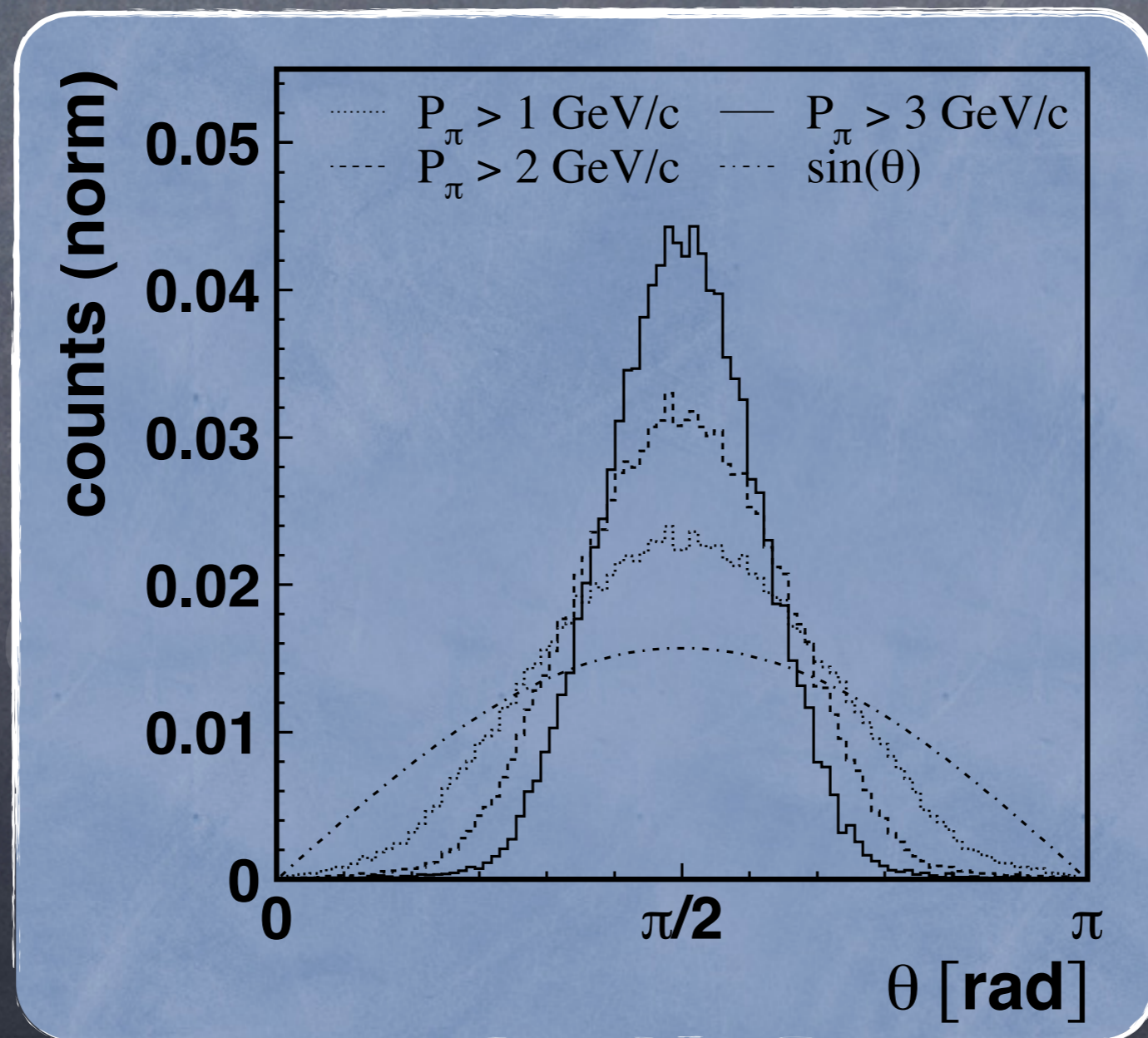
1D clearly not sufficient

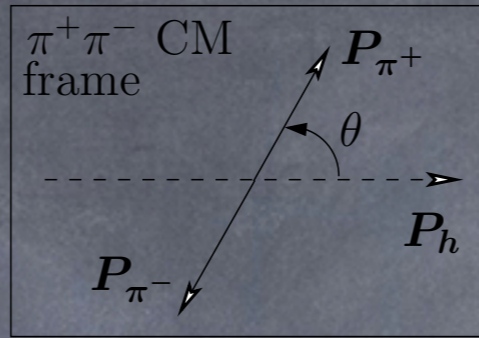
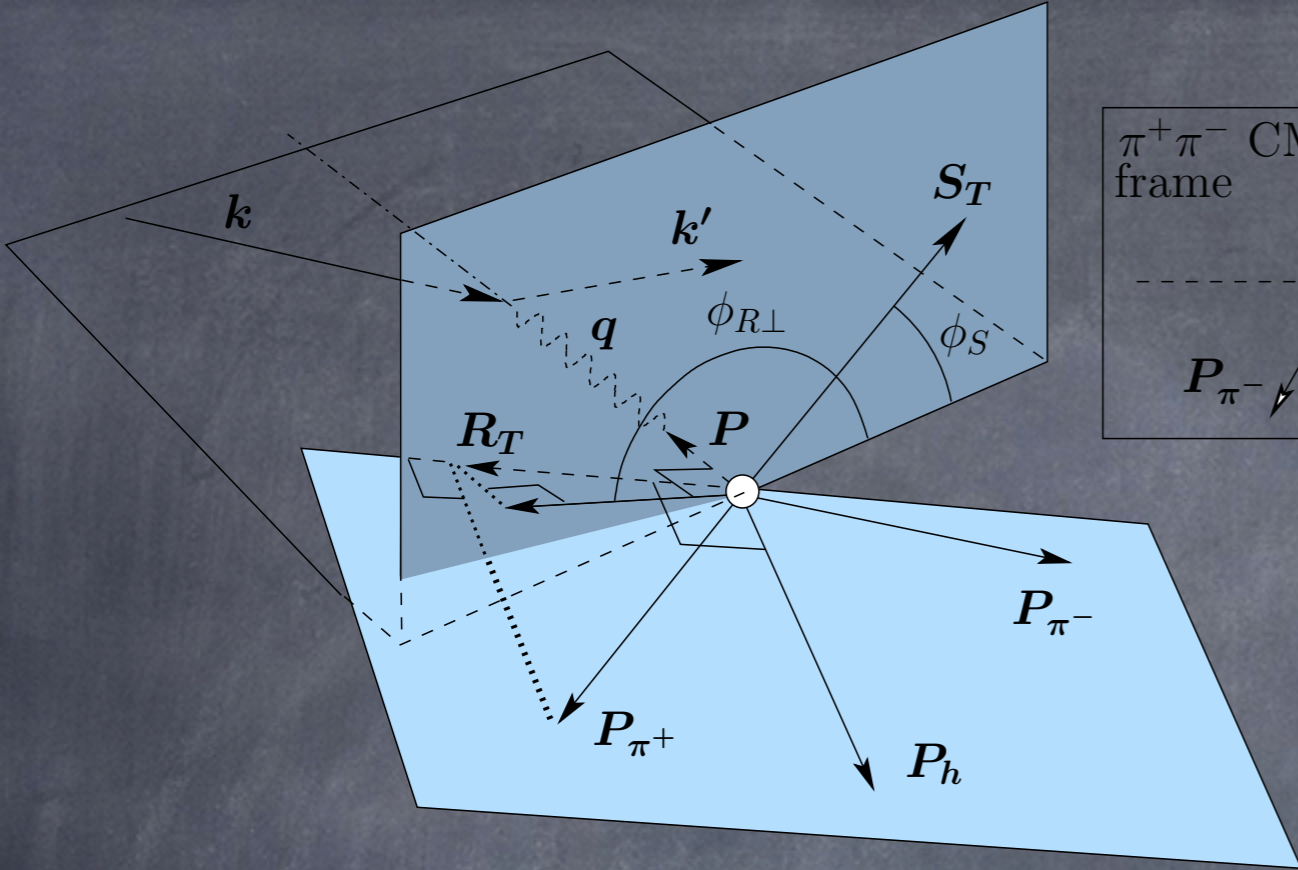


define your measurement wisely and clearly!

- data point interpreted as asymmetry/multiplicity
 - at the average kinematics given
 - integrated over kinematic ranges
- results in different systematics → select the one with smallest systematics?
- try to go fully differential to minimize biases

more "pitfalls" in dihadron fragmentation

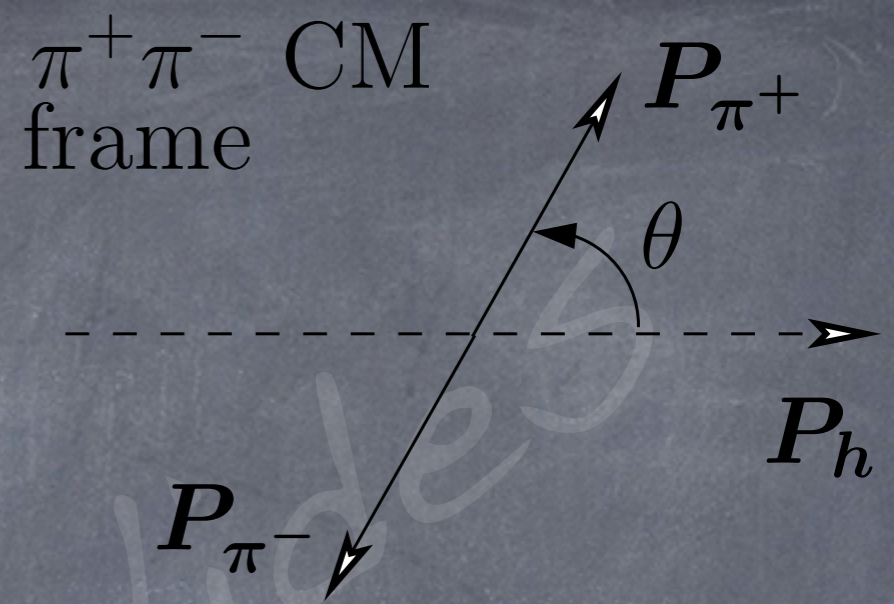




"appetizer"

- dihadron FFs: alternative path to extract (collinear) transversity
 - exploit orientation of hadron's relative momentum, correlate with target polarization
- complication: SIDIS cross section now differential in 9(!) variables
- integration over polar angle eliminates, in theory, a number of contributing FFs (partial waves)
- experimental constraints limit acceptance in polar angle, most prominently the minimum-momentum requirements

simple case study



basic assumptions:

- dihadron pair with equal-mass hadrons; here: pions
- e^+e^- annihilation, thus energy fractions z translates directly to energy/momentum of particles/system as primary energy is "fixed"
(\rightarrow simplifies Lorentz boost)
- without loss of generality, focus on B factory and use primary quark energy $E_0 = 5.79\text{GeV}$
- minimum energy of each pion in lab frame: $0.1 E_0$
(i.e., $z_{\min} = 0.1$)

application of Lorentz boost

- can easily apply Lorentz boost using the invariant mass of the dihadron M and its energy zE_0 to arrive at condition on θ , e.g., polar angle of pions in center-of-mass frame:

$$\cos \theta \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M^2](M^2 - 4m_\pi^2)}} E_0 M$$

- as both pions have to fulfill the constraint on the minimum energy:

$$\cos(\pi - \theta) = -\cos \theta \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M^2](M^2 - 4m_\pi^2)}} E_0 M$$

thus:

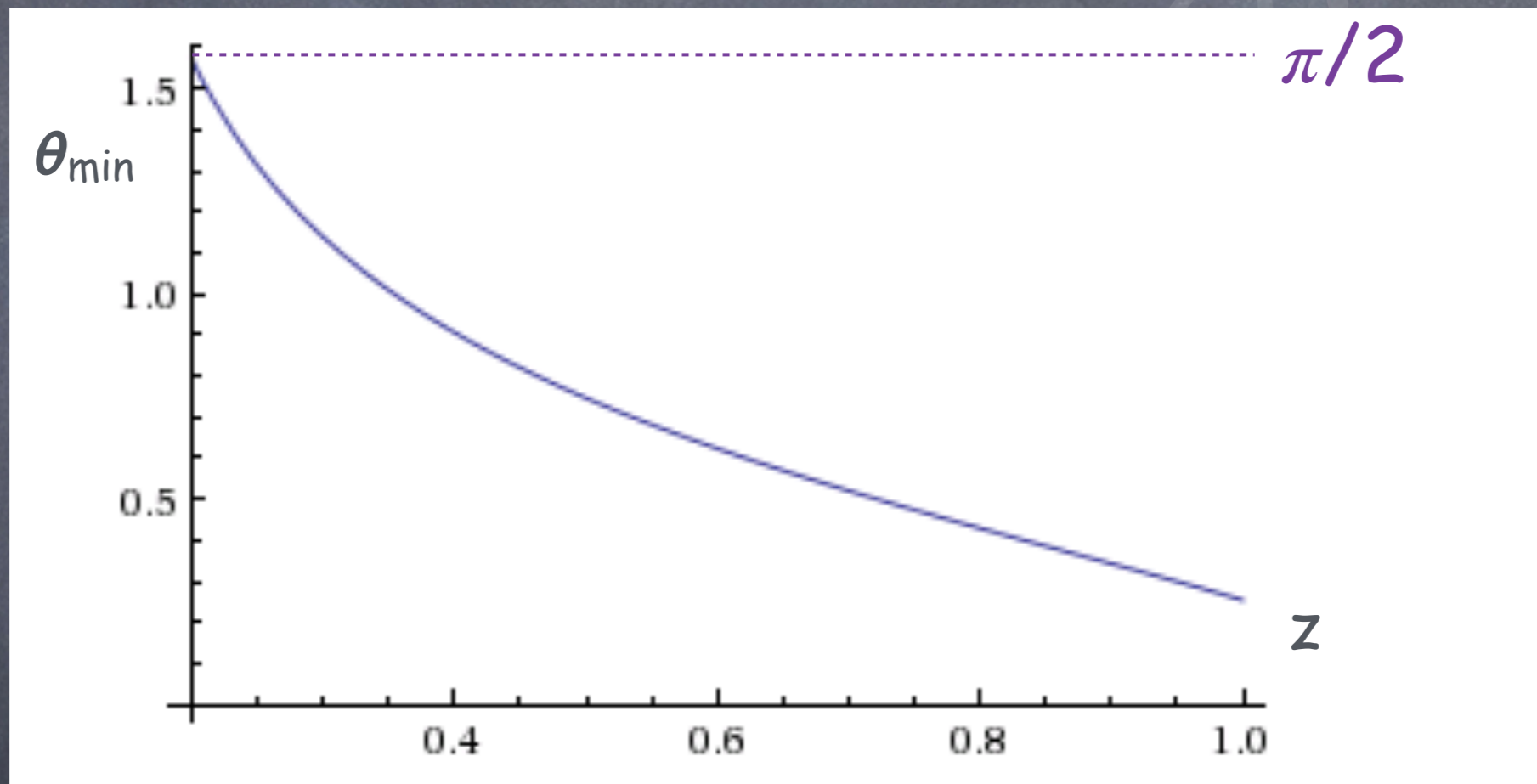
$$|\cos \theta| \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M^2](M^2 - 4m_\pi^2)}} E_0 M$$

- translates to a symmetric range around $\pi/2$

(can be easily understood because at $\pi/2$ the pions will have both the same energy in the lab and easily pass the z_{\min} requirement, while in the case of one pion going backward in the CMS, that pion will have less energy in the lab frame ... and maybe too little)

impact of $z_{\min}=0.1$ on accepted polar range

- (again without loss of generality) let's assume $M=0.5$ GeV :



- all theta below curve (and above its mirror curve relative to dashed line) are excluded
- clearly limited, especially at low z

partial-wave expansion

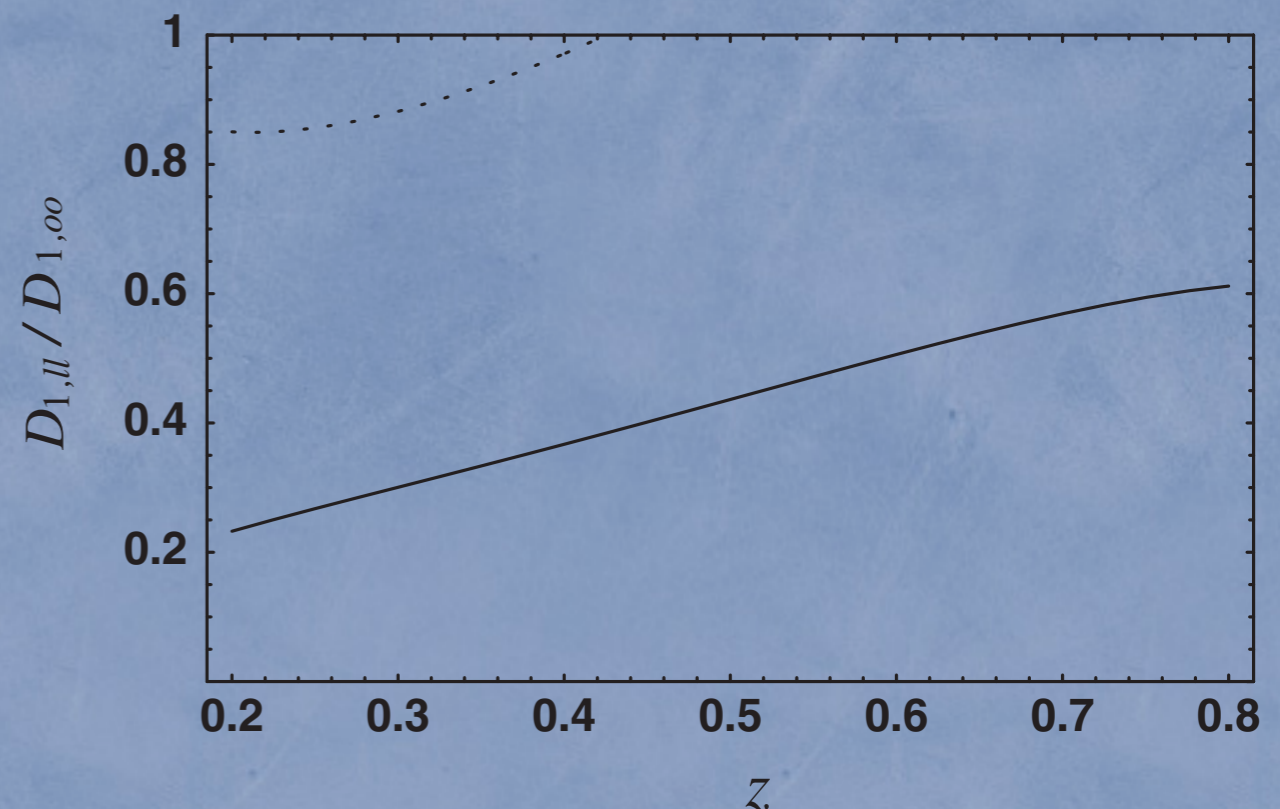
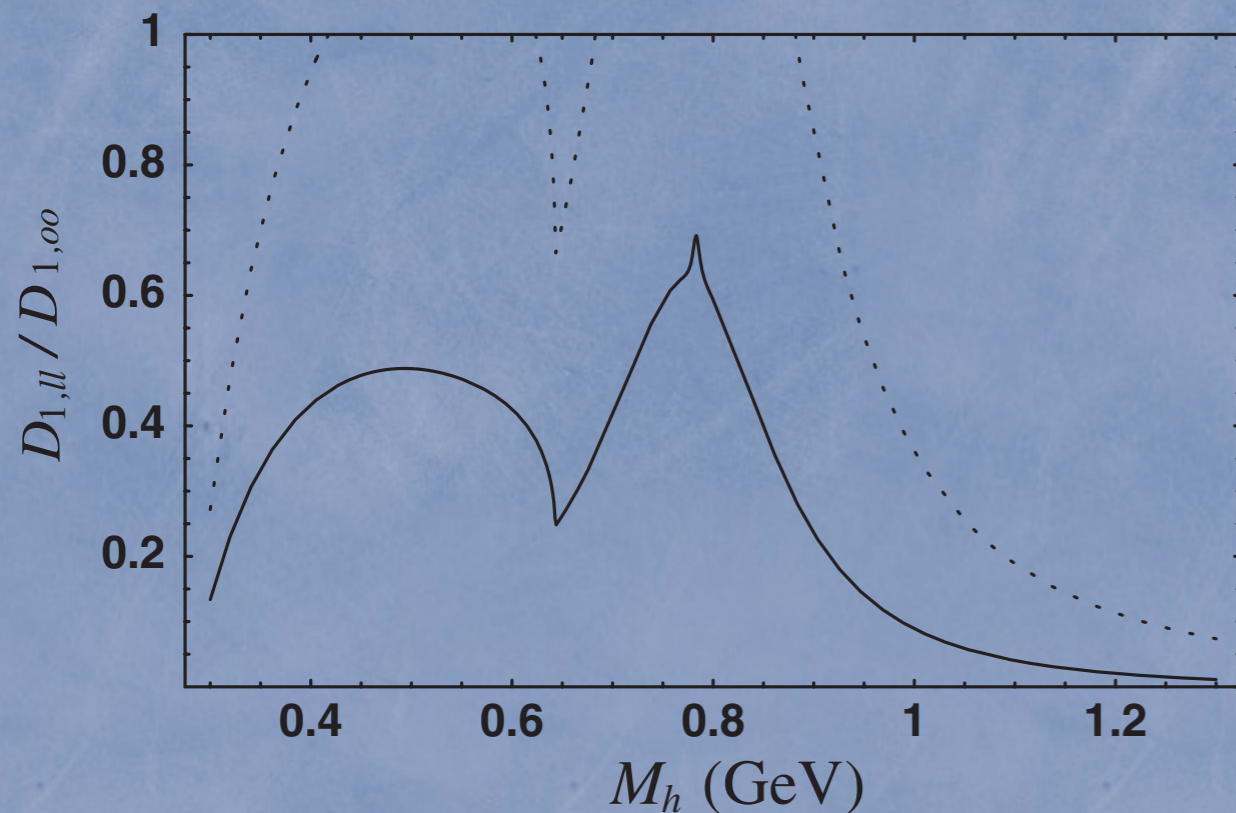
- partial-wave expansion worked out in [Phys. Rev. D67 \(2003\) 094002](#)
- for the particular case here, use [Phys. Rev. D74 \(2006\) 114007](#), in particular Eq. (12), and (later on) Figure 5:

$$D_1^q(z, \cos\theta, M_h^2) \approx D_{1,oo}^q(z, M_h^2) + D_{1,ol}^q(z, M_h^2) \cos\theta + D_{1,ll}^q(z, M_h^2) \frac{1}{4}(3\cos^2\theta - 1), \quad (12)$$

- it is the first contribution ($D_{1,oo}$) that is used in “collinear extraction” of transversity (and subject of a current Belle analysis)
 - it is also the only one surviving the integration over θ
- the $D_{1,ol}$ contribution vanishes upon integration over θ as long as the theta range is symmetric around $\pi/2$ (as it is the case here)
- the $D_{1,ll}$ term, however, will in general contribute in case of only partial integration over θ — the question is how much?

$D_{1,1}$ contribution to DiFF

- $D_{1,1}$ is unknown and can't be calculated using first principles
- it can not be extracted from cross sections integrated over θ
- upon (partial) integration there is no way to disentangle the two contributions
- in [PRD74 \(2006\) 114007](#), a model for dihadron fragmentation was tuned to PYTHIA and used to estimate the various partial-wave contributions
- its Figure 5 gives an indication about the relative size of $D_{1,1}$ vs. $D_{1,00}$:



effect of partial integration

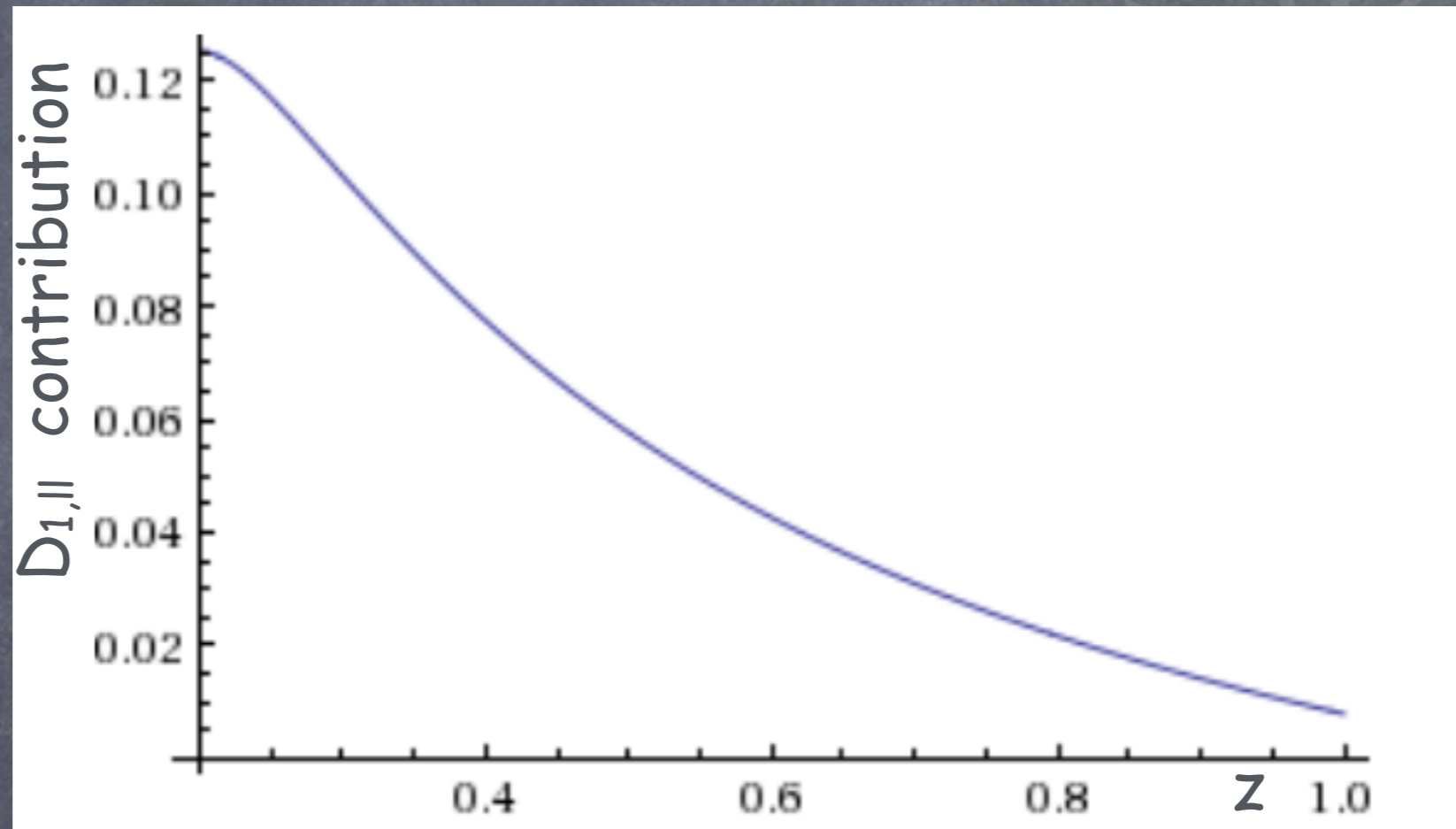
- as both contributions — $D_{1,\parallel}$ and $D_{1,00}$ — will be affected by the partial integration, look at relative size of the $D_{1,\parallel}$ to $D_{1,00}$ modulations when subjected to integration:

$$\frac{D_{1,\parallel}}{D_{1,00}} \frac{\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} d\cos\theta \frac{1}{4}(3\cos^2\theta - 1)}{\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} d\cos\theta} = -\frac{1}{4}(1 - \cos^2\theta_0) \frac{D_{1,\parallel}}{D_{1,00}}$$

- without limit in the polar-angular range ($\theta_0 = 0$) → no contribution from $D_{1,\parallel}$ (sanity check!)
- the relative size of the partial integrals reaches a maximum of 25% for $z=0.2$ (i.e., pions at 90 degrees in center-of-mass system)
- in order to estimate the $D_{1,\parallel}$ contribution, one “just” needs the relative size of $D_{1,\parallel}$ vs. $D_{1,00}$, e.g., Figure 5 of [PRD74 \(2006\) 114007](#)
 - let's take for that size 0.5 (rough value for $M=0.5$ GeV)

effect of partial integration

- ... $D_{1,||} / D_{1,00} \sim 0.5$ results in an up to $O(10\%)$ effect on the measured cross section:

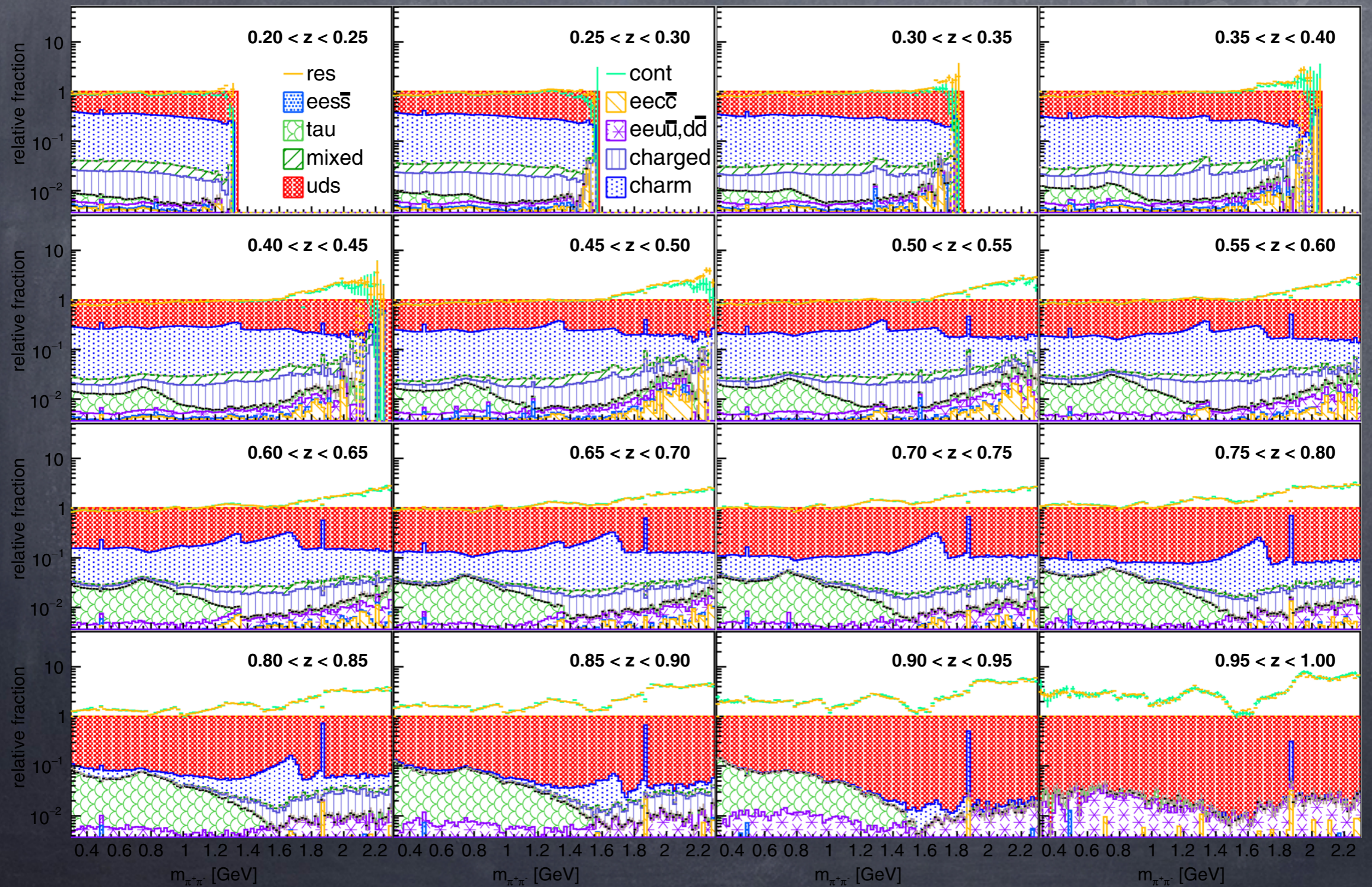


- depending on the sign of $D_{1,||}$, the partial integration thus leads to a systematic **underestimation** (positive $D_{1,||}$) or **overestimation** (negative $D_{1,||}$) of the “integrated” dihadron cross section
 - leads to **overestimate/underestimate** of extracted transversity

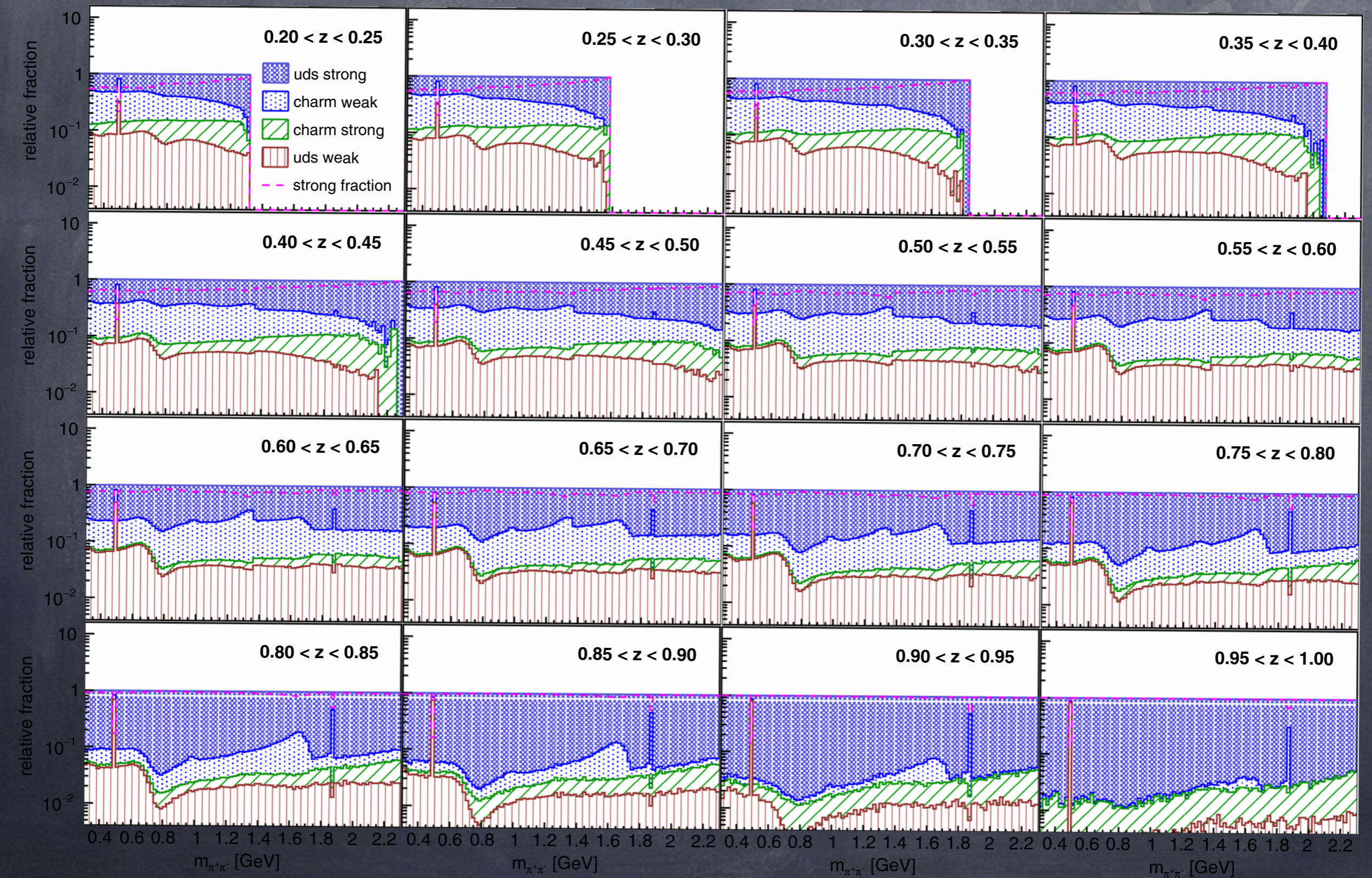
back to experiment

- how precisely can we measure FFs, e.g., how precisely can we deal with other-than-uds contributions

subprocess contributions



weak contributions



resonance contributions

