

The transverse momentum distribution of hadrons inside jets

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Outline

- Introduction
- In-jet TMDs
- Collins asymmetries
- Conclusions

Kang, Liu, FR, Xing '17

Kang, Prokudin, FR, Yuan '17

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- Introduction

- In-jet TMDs

Kang, Liu, FR, Xing '17

- Collins asymmetries

Kang, Prokudin, FR, Yuan '17

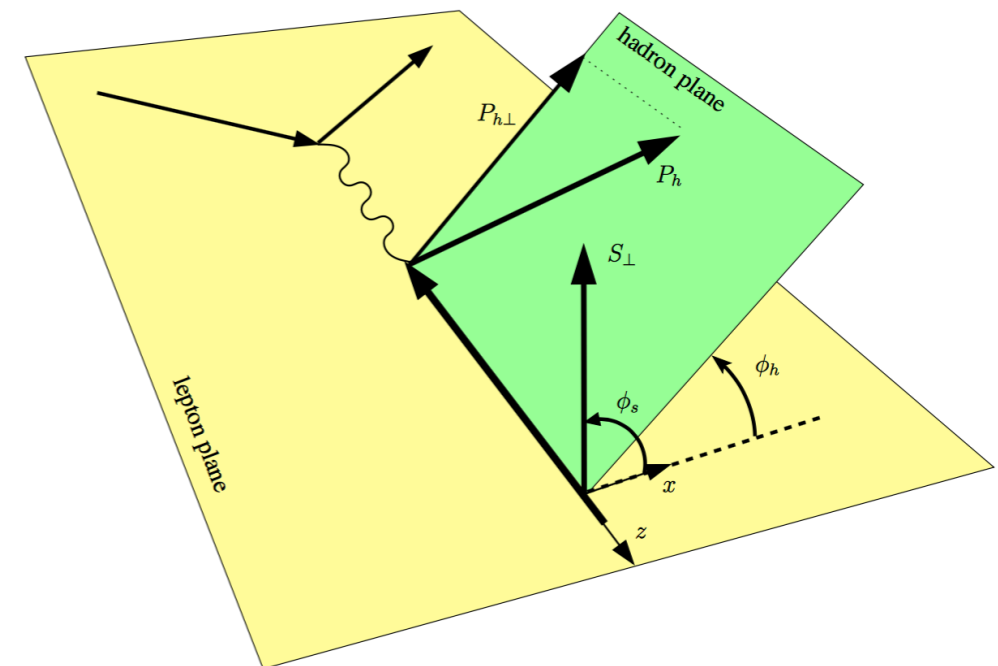
- Conclusions

Accessing Transverse Momentum Distributions

- SIDIS: JLab, HERMES, COMPASS
- Drell-Yan
- Electron-positron: BELLE, BABAR

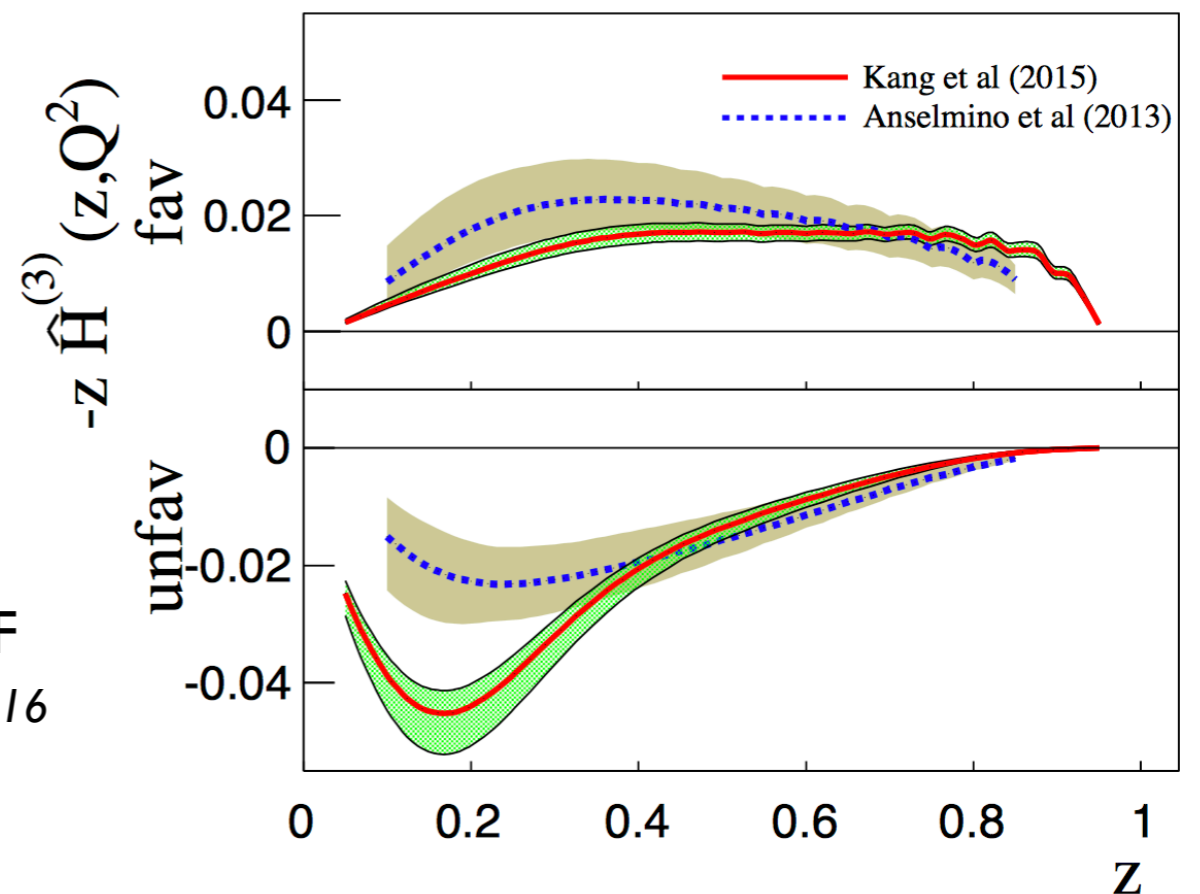
TMD extractions in global fits

Bacchetta et al. '17, Kang et al. '16, ...



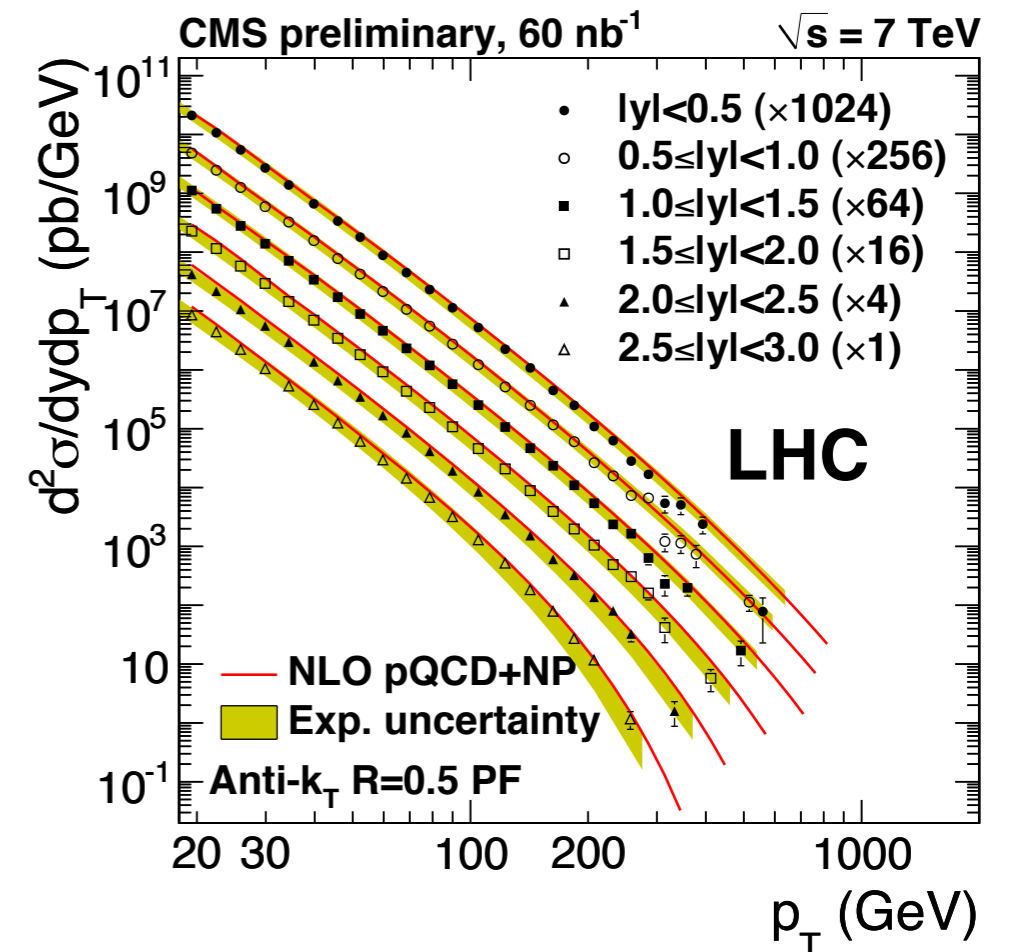
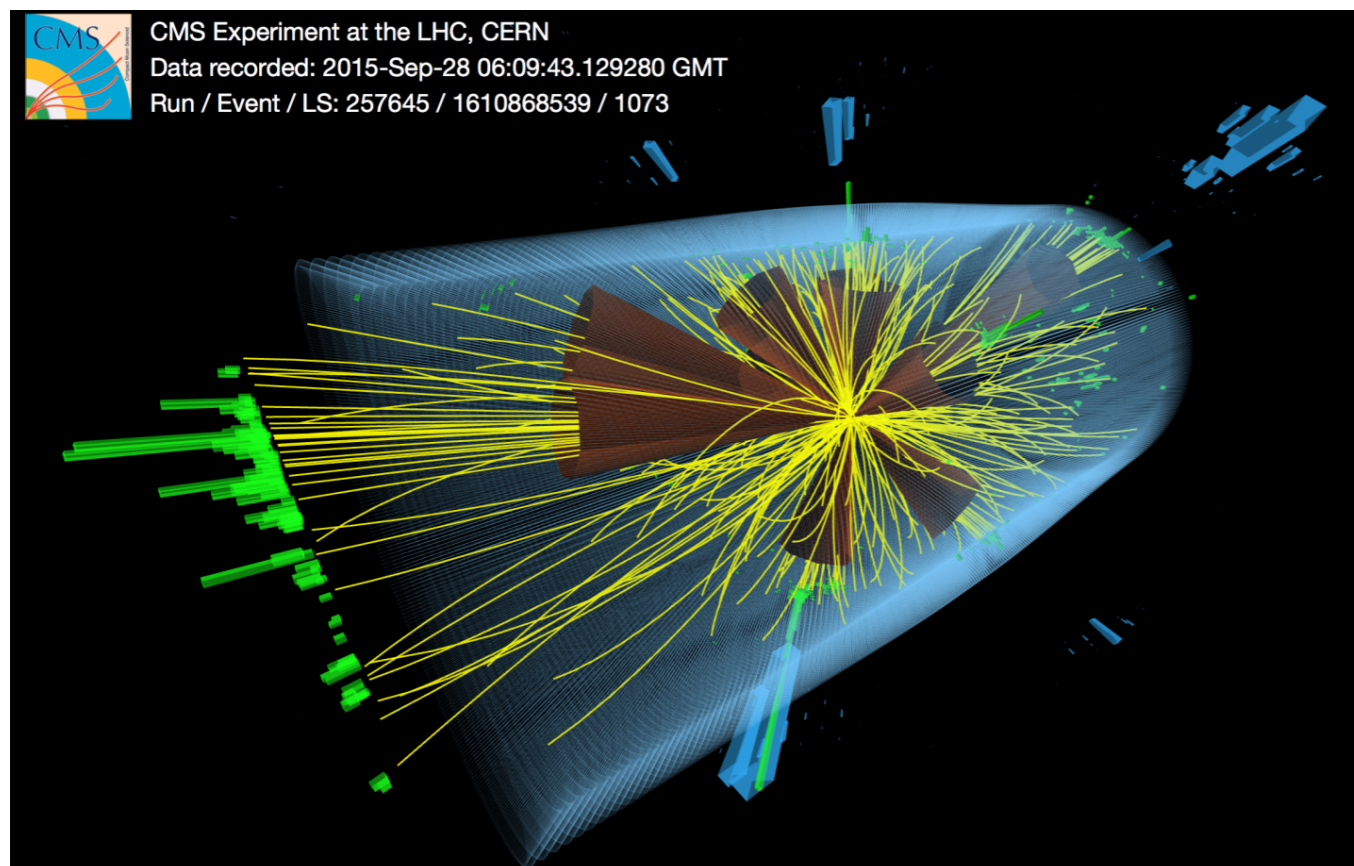
- Factorization *Boglionne et al. '17*
- Test of universality
- TMD evolution

Collins FF
Kang et al. '16



Jets in proton-proton collisions

- PDFs and α_s are constrained by collider jet data
- High p_T jets are a promising observable for the search of BSM physics at the LHC
- Baseline for jet quenching in heavy-ion collisions
- Jet substructure



Hadron distributions inside jets

- Study hadron distributions inside a reconstructed jet

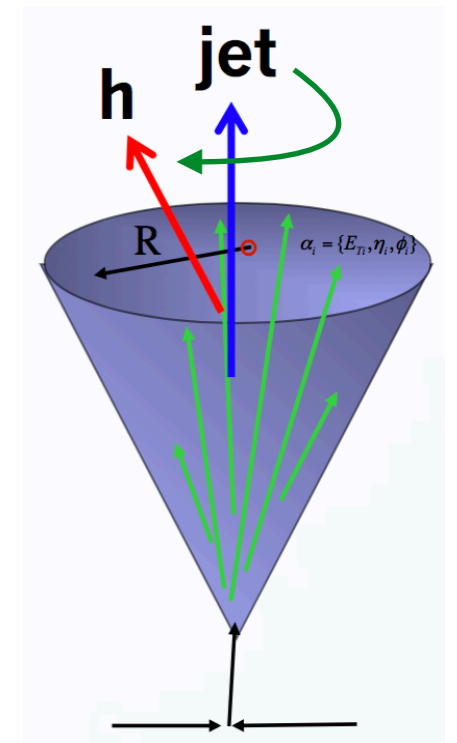
$$F(z_h, p_T) = \frac{d\sigma^{pp \rightarrow (\text{jet}h)X}}{dp_T d\eta dz_h} \bigg/ \frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta}$$

$$F(z_h, j_\perp, p_T) = \frac{d\sigma^{pp \rightarrow (\text{jet}h)X}}{dp_T d\eta dz_h d^2 j_\perp} \bigg/ \frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta}$$

$$z_h = p_T^h / p_T$$

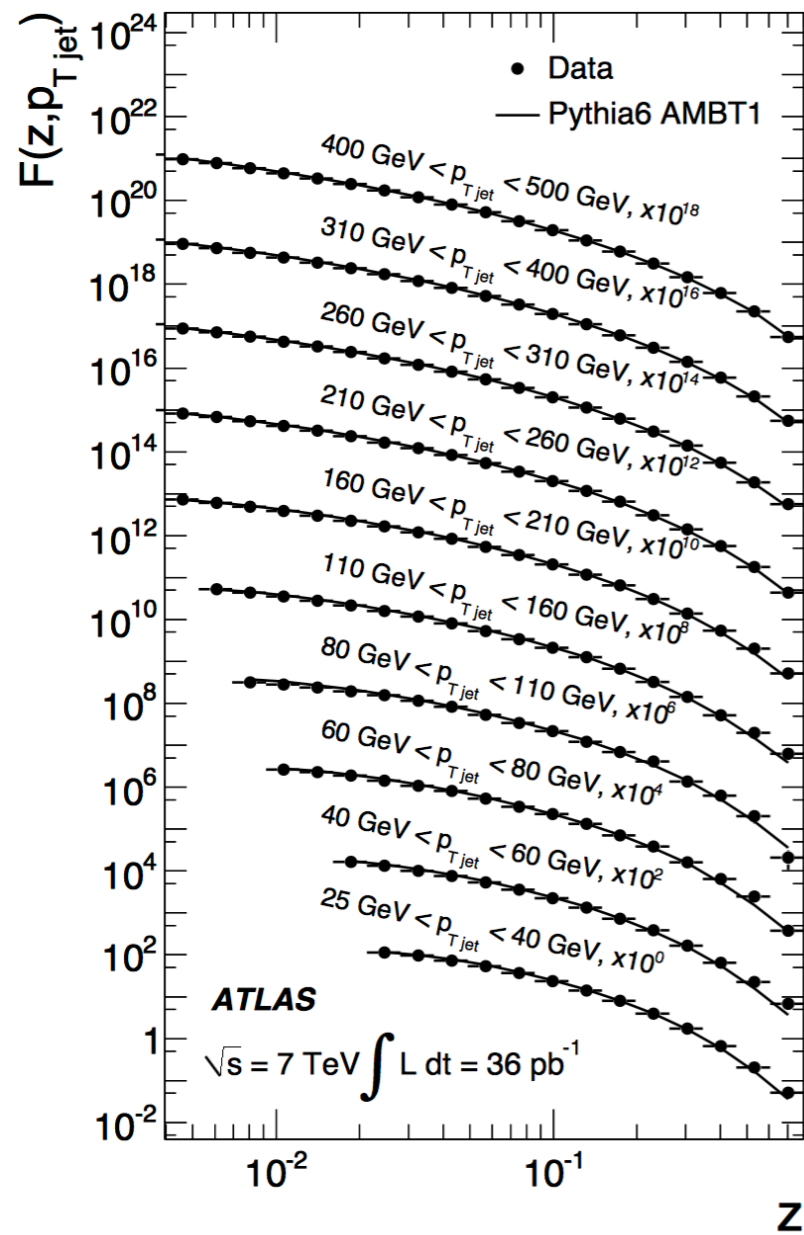
j_\perp : hadron transverse momentum with respect to the (standard) jet axis

- Longitudinal momentum distribution probes collinear FFs
- Transverse momentum distribution probes TMD FFs
- Collins azimuthal asymmetries



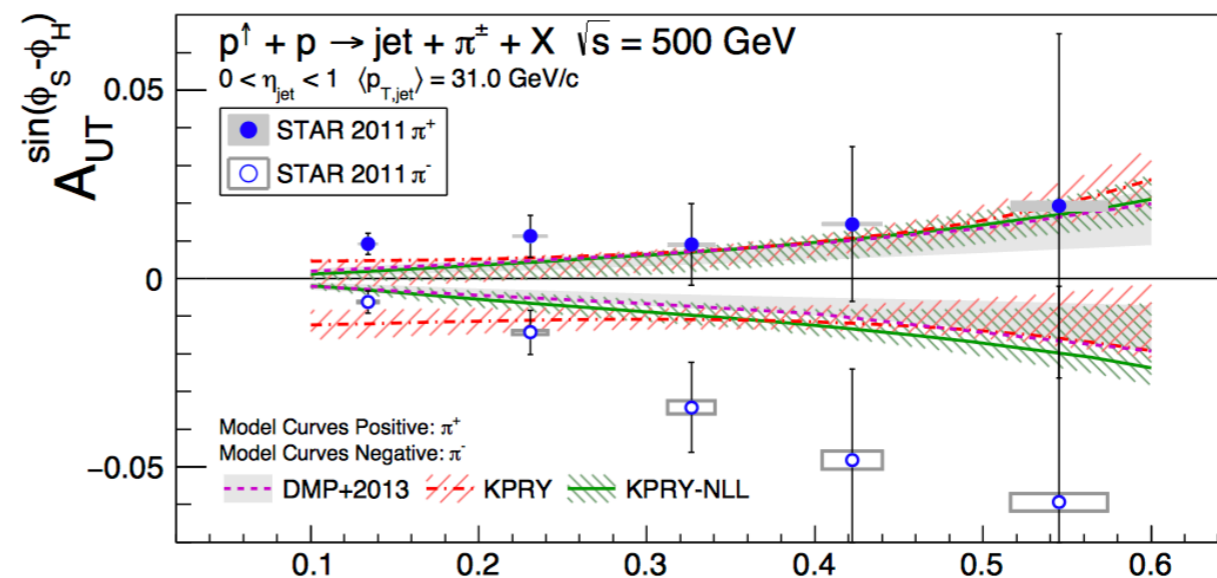
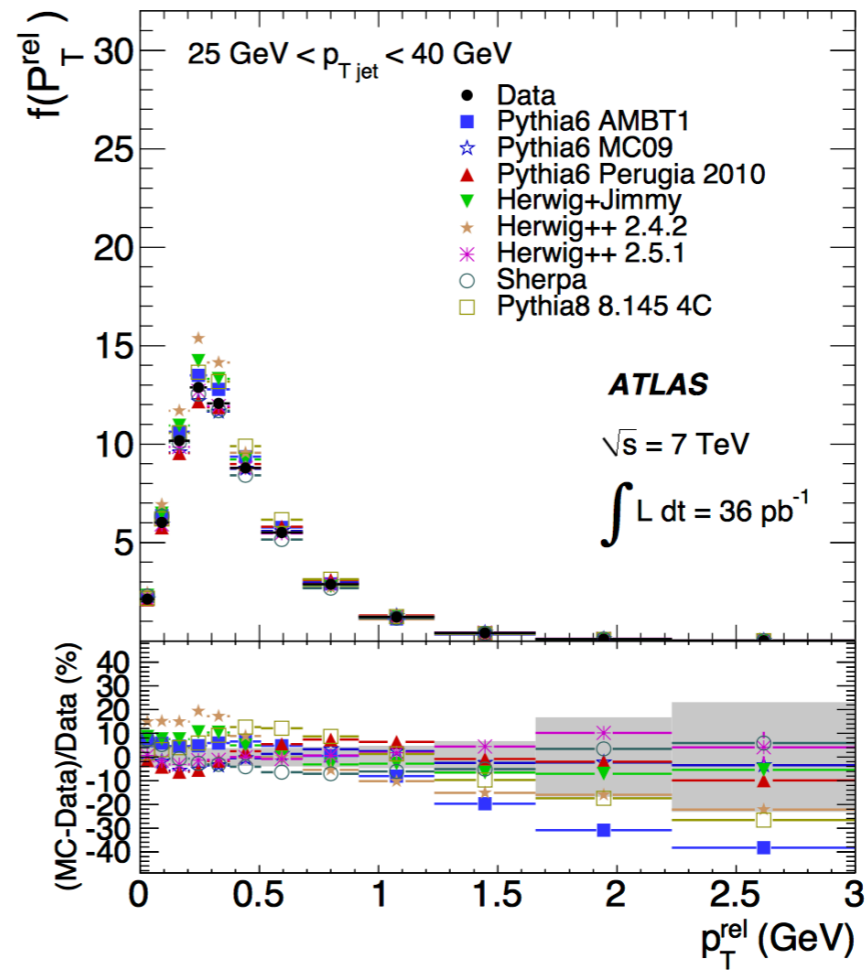
$$pp \rightarrow (\text{jet } h) + X$$

Recent measurements at the LHC and RHIC

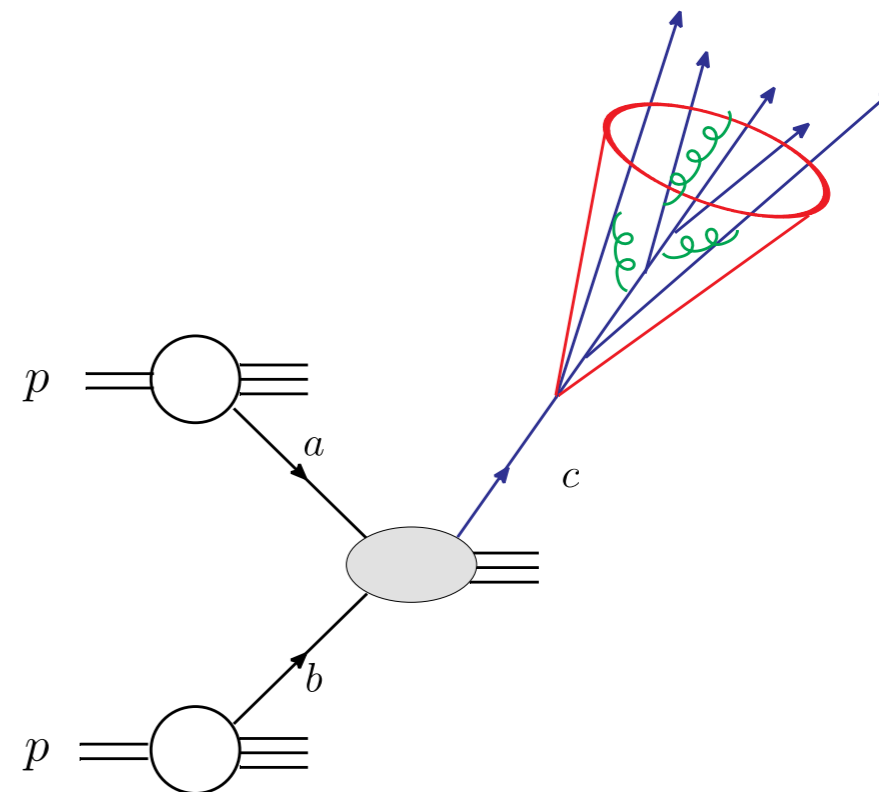
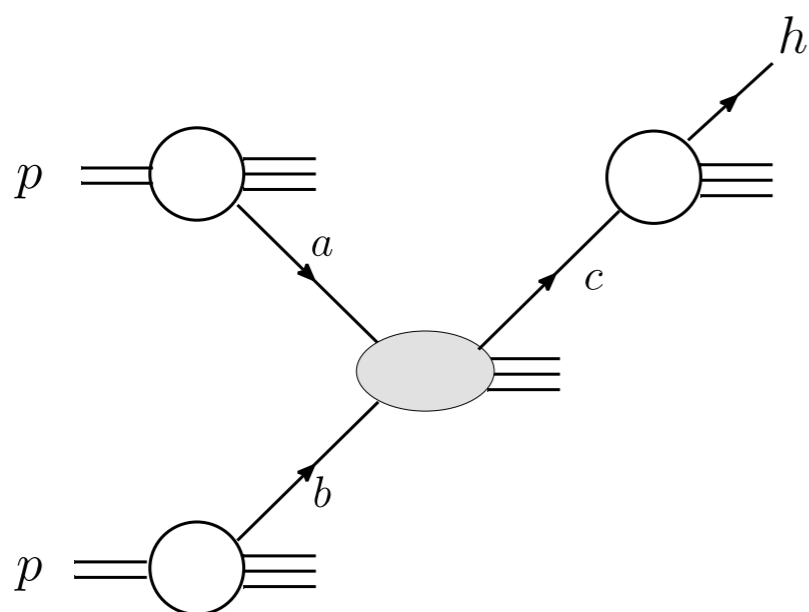


ATLAS *Eur. Phys. J. C*71 (2011) 1795
 arXiv: 1109.5816

STAR Collaboration, arXiv: 1708.0708



Analogy of hadron and jet cross sections



Factorization

Evolution

Jet

$$\frac{d\sigma^{pp \rightarrow \text{jet} X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes J_c$$

$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$

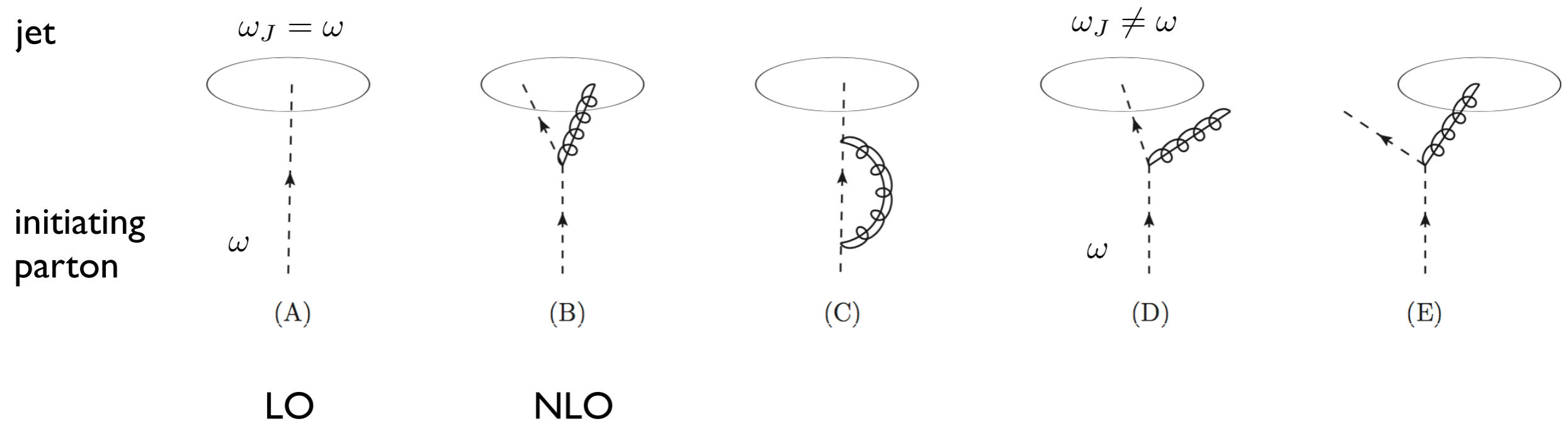
Hadron

$$\frac{d\sigma^{pp \rightarrow h X}}{dp_T d\eta} = \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab}^c \otimes D_c^h$$

$$\mu \frac{d}{d\mu} D_i^h = \sum_j P_{ji} \otimes D_j^h$$

Semi-inclusive jet function in SCET

- The siJFs describe how a parton is transformed into a jet with radius R and carrying an energy fraction z



where $z = \omega_J/\omega$

momentum sum rule: $\int_0^1 dz z J_i(z, \omega R, \mu) = 1$

see also: Kaufmann, Mukherjee, Vogelsang '15
Dai, Kim, Leibovich '16

Semi-inclusive jet function in SCET

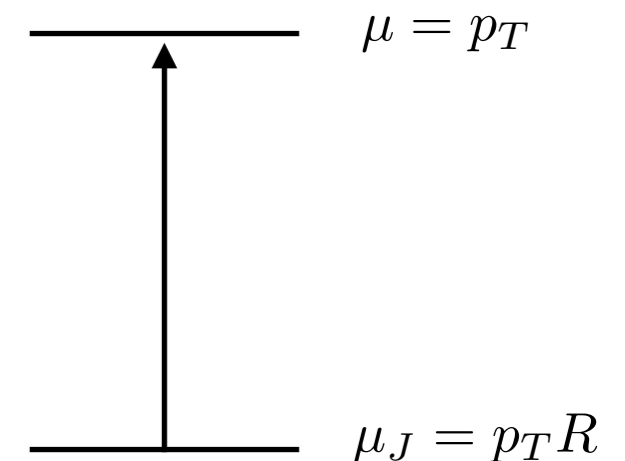
- NLO result

$$\begin{aligned}
 J_q^{(1)}(z, \omega_J) &= \frac{\alpha_s}{2\pi} \left(\frac{1}{\epsilon} \overset{\text{UV}}{\ln} \left(\frac{\mu^2}{p_T^2 R^2} \right) \right) [P_{qq}(z) + P_{gq}(z)] \\
 &\quad - \frac{\alpha_s}{2\pi} \left\{ C_F \left[2(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ + (1-z) \right] - \delta(1-z) d_J^{q,\text{alg}} \right. \\
 &\quad \left. + P_{gq}(z) 2 \ln(1-z) + C_F z \right\},
 \end{aligned}$$

$\overline{\text{MS}}$ scheme

- RG equation
timelike DGLAP for semi-inclusive jet function

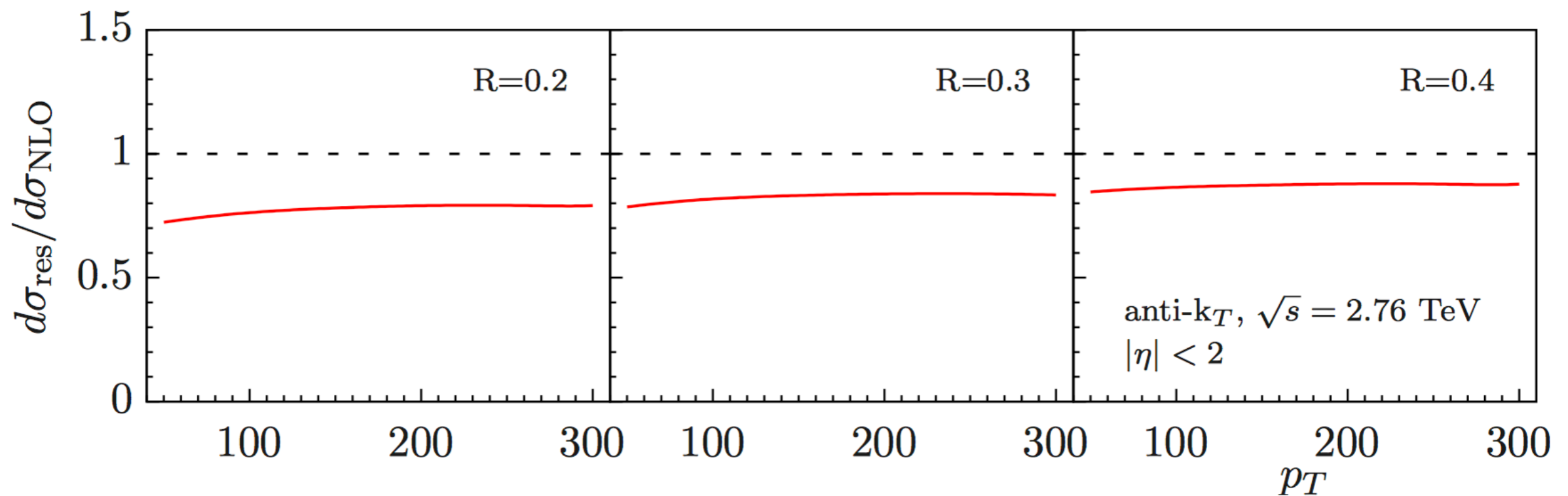
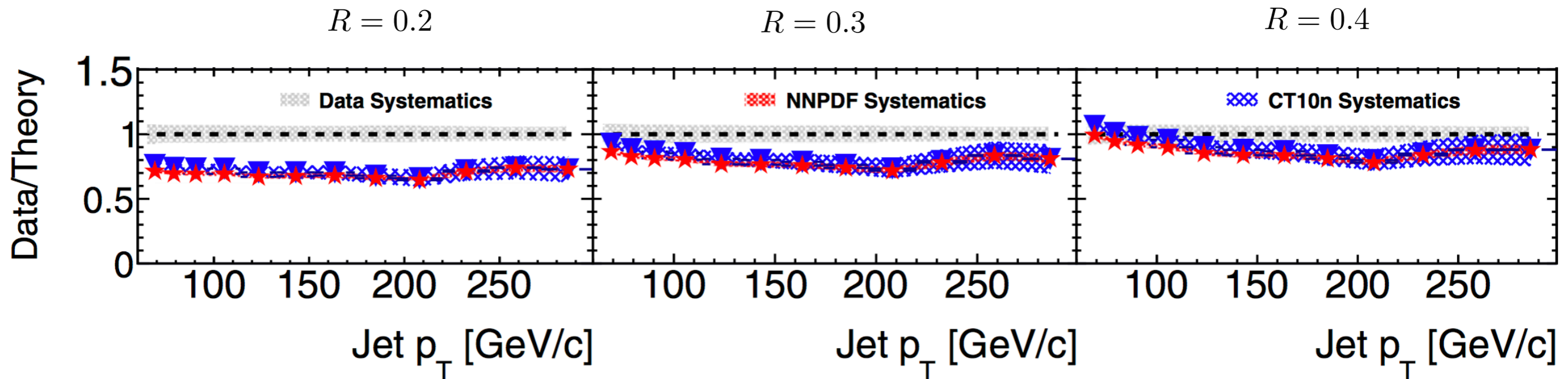
$$\mu \frac{d}{d\mu} J_i = \sum_j P_{ji} \otimes J_j$$



resummation of $\alpha_s^n \ln^n R$

see also: Dasgupta, Dreyer, Salam, Soyez '16

Comparison to LHC data



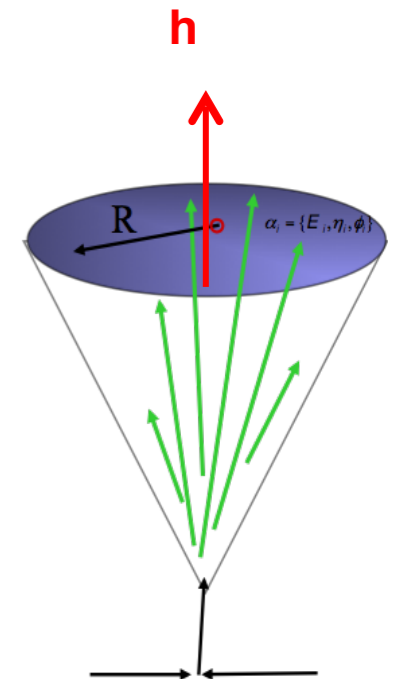
The jet fragmentation function $pp \rightarrow (\text{jet } h) X$

- First reconstruct a jet and then identify hadrons inside that jet

$$F(z_h, p_T) = \frac{d\sigma^{pp \rightarrow (\text{jet } h) X}}{dp_T d\eta dz_h} / \frac{d\sigma^{pp \rightarrow \text{jet } X}}{dp_T d\eta}$$

$$z_h = p_T^h / p_T$$

$$z = p_T / p_T^c$$



- Similar factorization to inclusive jet production

$$\frac{d\sigma^{pp \rightarrow (\text{jet } h) X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_a(x_a, \mu) \otimes f_b(x_b, \mu) \otimes H_{ab}^c(x_a, x_b, \eta, p_T/z, \mu) \otimes \mathcal{G}_c^h(z, z_h, \omega_J, \mu)$$

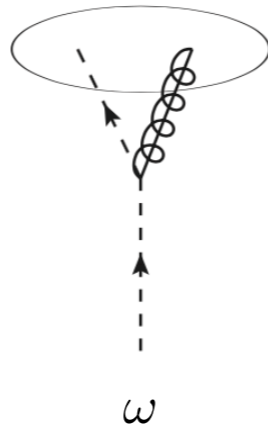
Semi-inclusive fragmenting jet function

Procura, Stewart `10, Liu `11, Jain, Procura, Waalewijn `11, Arleo, Fontannaz, Guillet, Nguyen `14, Kaufmann, Mukherjee, Vogelsang `15, Kang, FR, Vitev `16, ...

Semi-inclusive fragmenting jet function

- Fragmenting parton

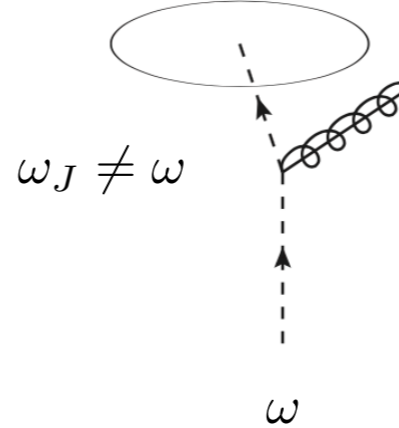
$$\omega_h \neq \omega_J$$



- Jet

$$\omega_J = \omega$$

$$\omega_h = \omega_J$$



$$z_h = \omega_h / \omega_J$$

$$z = \omega_J / \omega$$

- Initiating parton

 ω
 ω

Semi-inclusive fragmenting jet function

- Fragmenting parton

$$\omega_h \neq \omega_J$$

$$\omega_h = \omega_J$$

- Jet

$$\omega_J = \omega$$

$$\omega_J \neq \omega$$

- Initiating parton

$$\omega$$

$$\omega$$

$$z_h = \omega_h / \omega_J$$

$$z = \omega_J / \omega$$

- Matching

$$\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h}{z_h'} \mathcal{J}_{ij}(z, z_h', \omega_J, \mu) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)$$

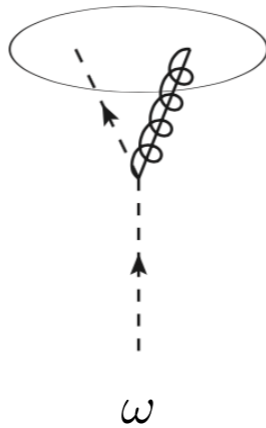
- $\ln R$ resummation

$$\mu \frac{d}{d\mu} \mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right) \mathcal{G}_j^h(z', z_h, \omega_J, \mu)$$

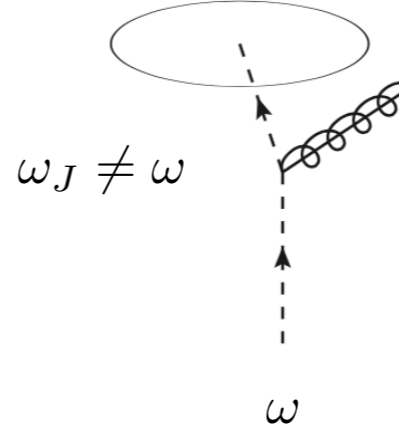
Semi-inclusive fragmenting jet function

- Fragmenting parton

$$\omega_h \neq \omega_J$$



$$\omega_h = \omega_J$$



- Jet

$$\omega_J = \omega$$

$$\omega_J \neq \omega$$

- Initiating parton

$$\omega$$

$$\omega$$

$$z_h = \omega_h / \omega_J$$

$$z = \omega_J / \omega$$

- Matching

$$\mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h}{z_h'} \mathcal{J}_{ij}(z, z_h', \omega_J, \mu) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)$$

$$H_{ab}^i \quad \mu = p_T$$

$$\mathcal{G}_i^h \quad \mu_J = p_T R$$



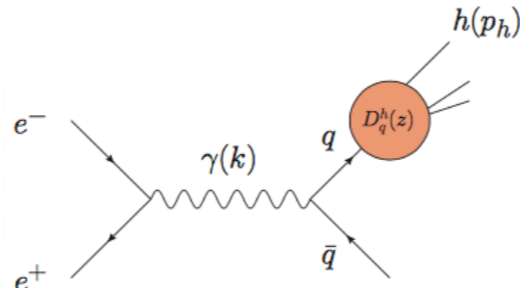


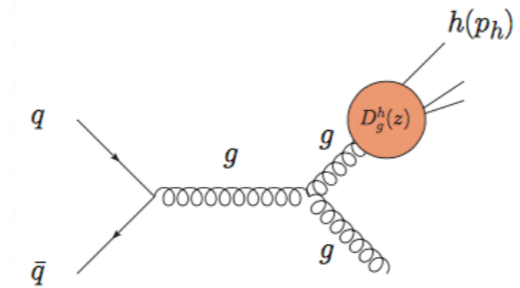


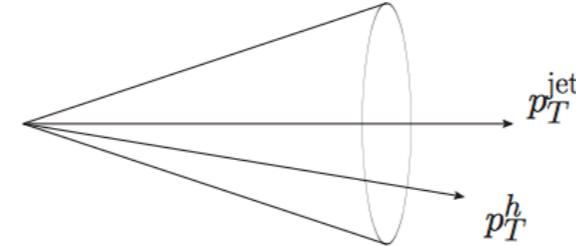
$$D_i^h \quad 1 \text{ GeV}$$

- $\ln R$ resummation

$$\mu \frac{d}{d\mu} \mathcal{G}_i^h(z, z_h, \omega_J, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right) \mathcal{G}_j^h(z', z_h, \omega_J, \mu)$$

... 2 DGLAPs now

- The JFF is an ideal observable to constrain in particular gluon FFs

Process	D_g^h ?	Direct scan?
$e^+e^- \rightarrow hX$ $ep \rightarrow ehX$		  $z = \frac{2p_h \cdot k}{k^2}$
$pp \rightarrow hX$		  $\hat{p}_T = \frac{p_T^h}{z_c} \quad z_c^{\min} = \frac{2p_T^h}{\sqrt{S}} \cosh \eta$ $d\sigma \propto \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} [f_a \otimes f_b \otimes d\hat{\sigma}_{ab}^c(\hat{p}_T, \dots)] D_c^h(z_c)$
$pp \rightarrow (\text{jet } h) X$		  $z = z_h \equiv \frac{p_T^h}{p_T^{\text{jet}}}$

Phenomenology

- Light charged hadrons

Arleo, Fontannaz, Guillet, Nguyen`14

Kaufmann, Mukherjee, Vogelsang`15

Kang, FR, Vitev`16

Neill, Scimemi, Waalewijn`16

- Photons

Kaufmann, Mukherjee, Vogelsang`16

- Heavy flavor mesons

Chien, Kang, FR, Vitev, Xing`15

Bain, Dai, Hornig, Leibovich, Makris, Mehen`16

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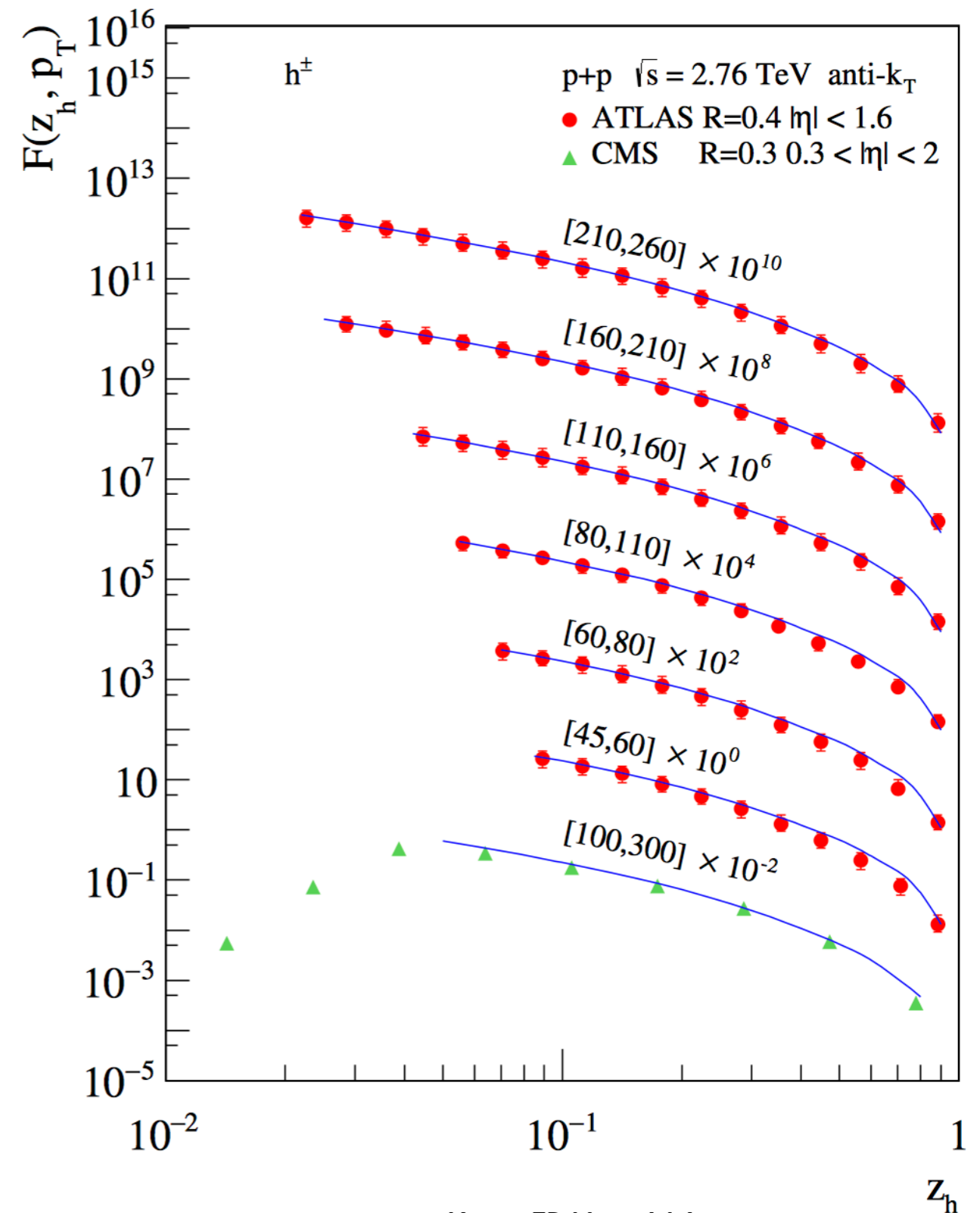
- Quarkonia

Baumgart, Leibovich, Mehen, Rothstein`14

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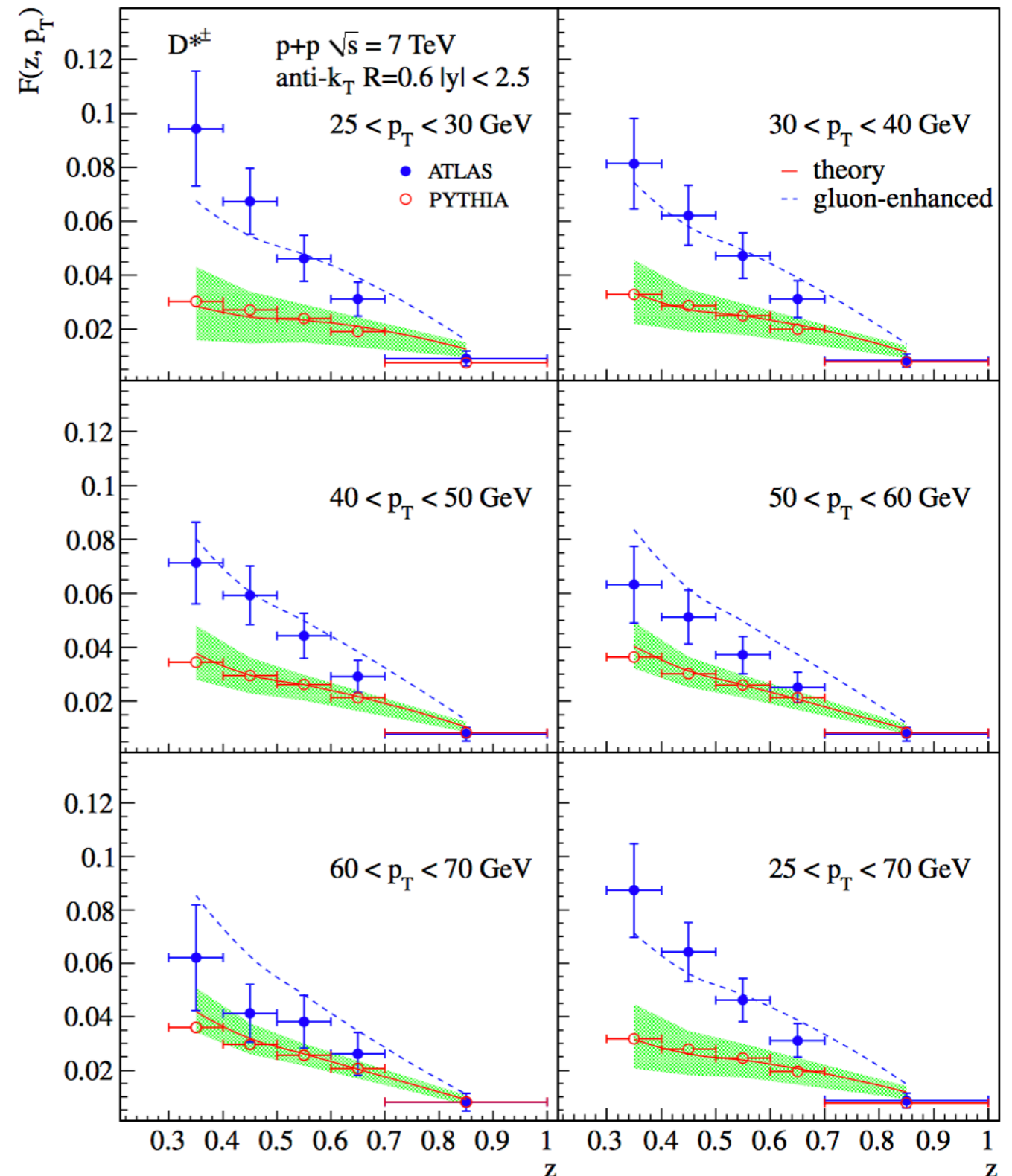
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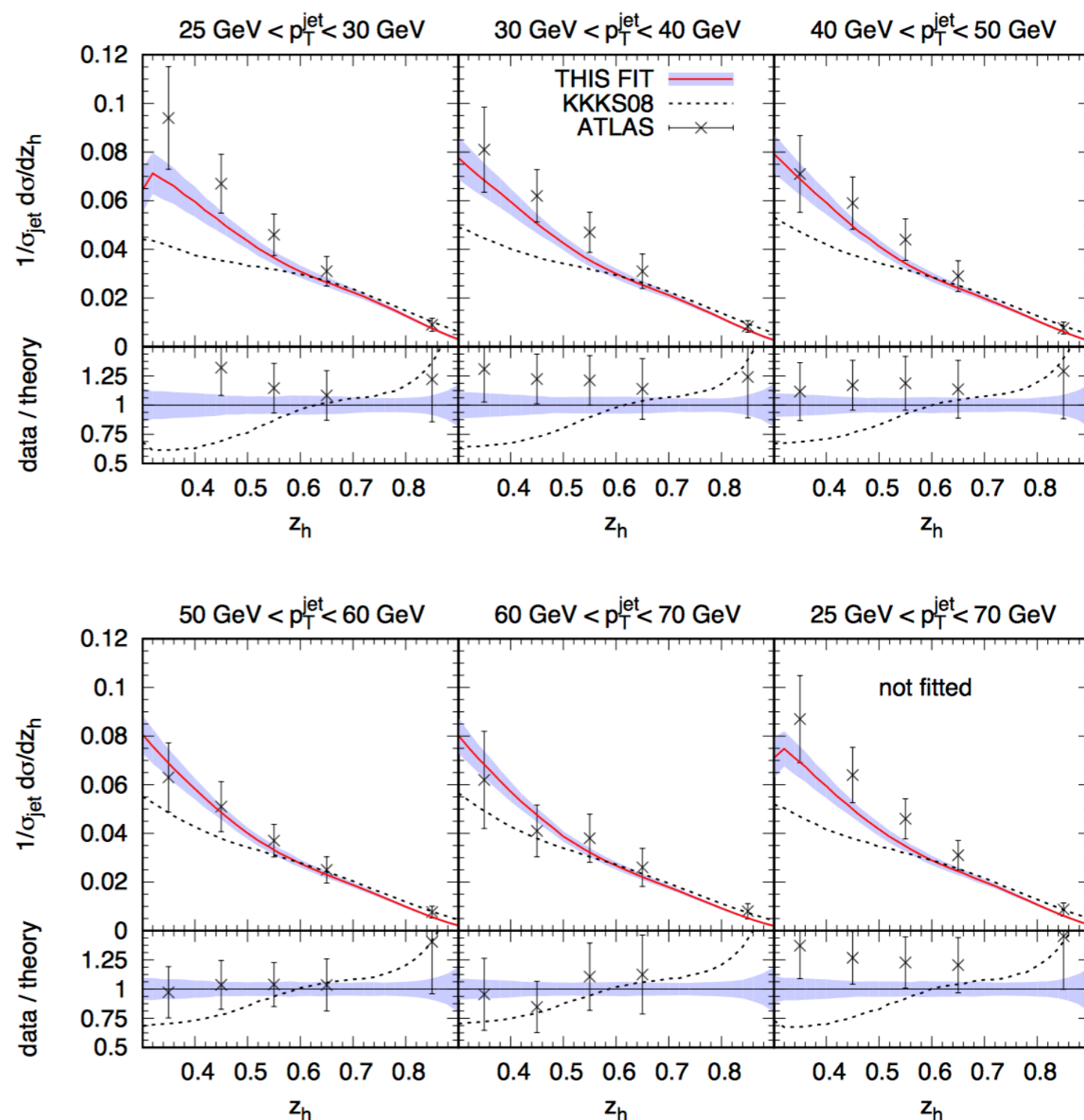
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Phenomenology

$$pp \rightarrow (\text{jet } D^*) X$$



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New global D^* -meson FF fit

Anderle, Kaufmann, Stratmann, FR, Vitev `17

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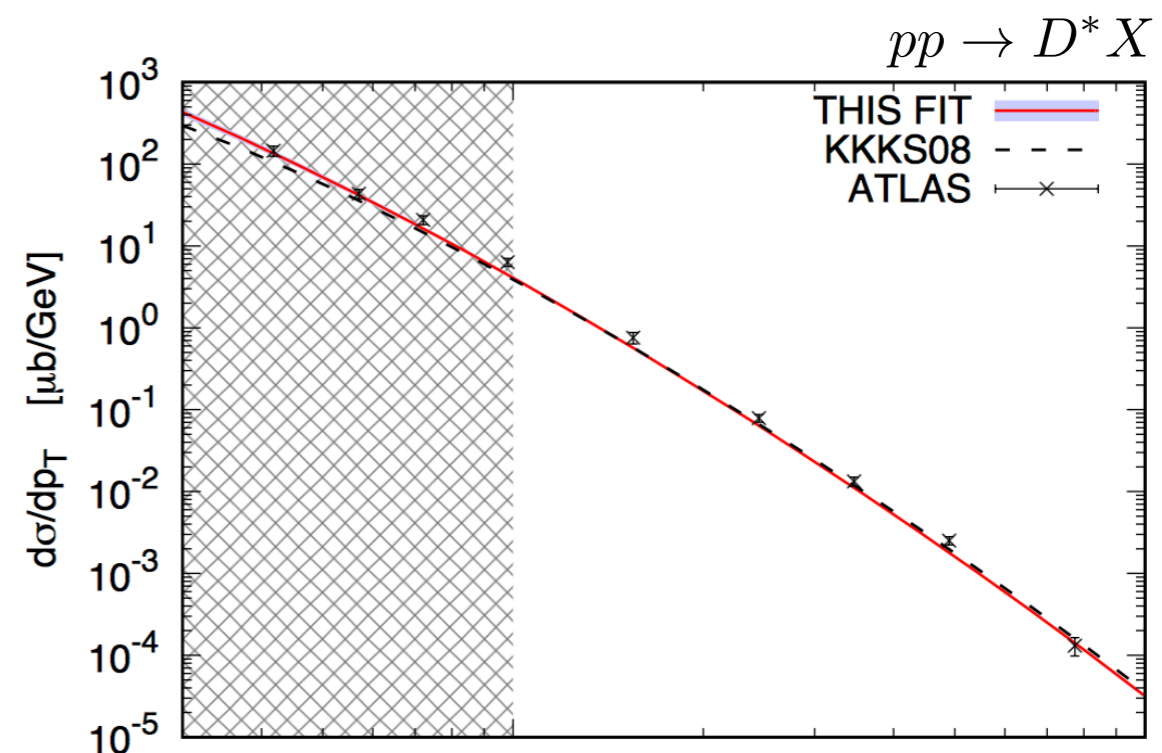
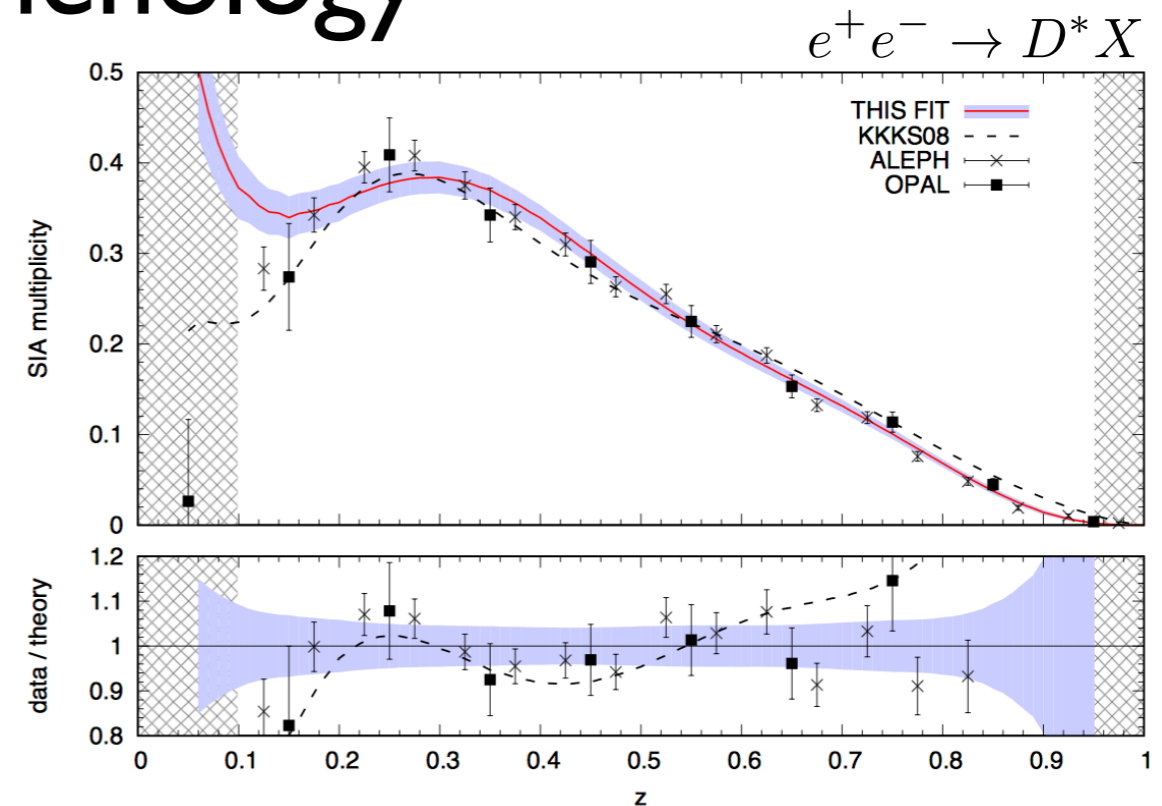
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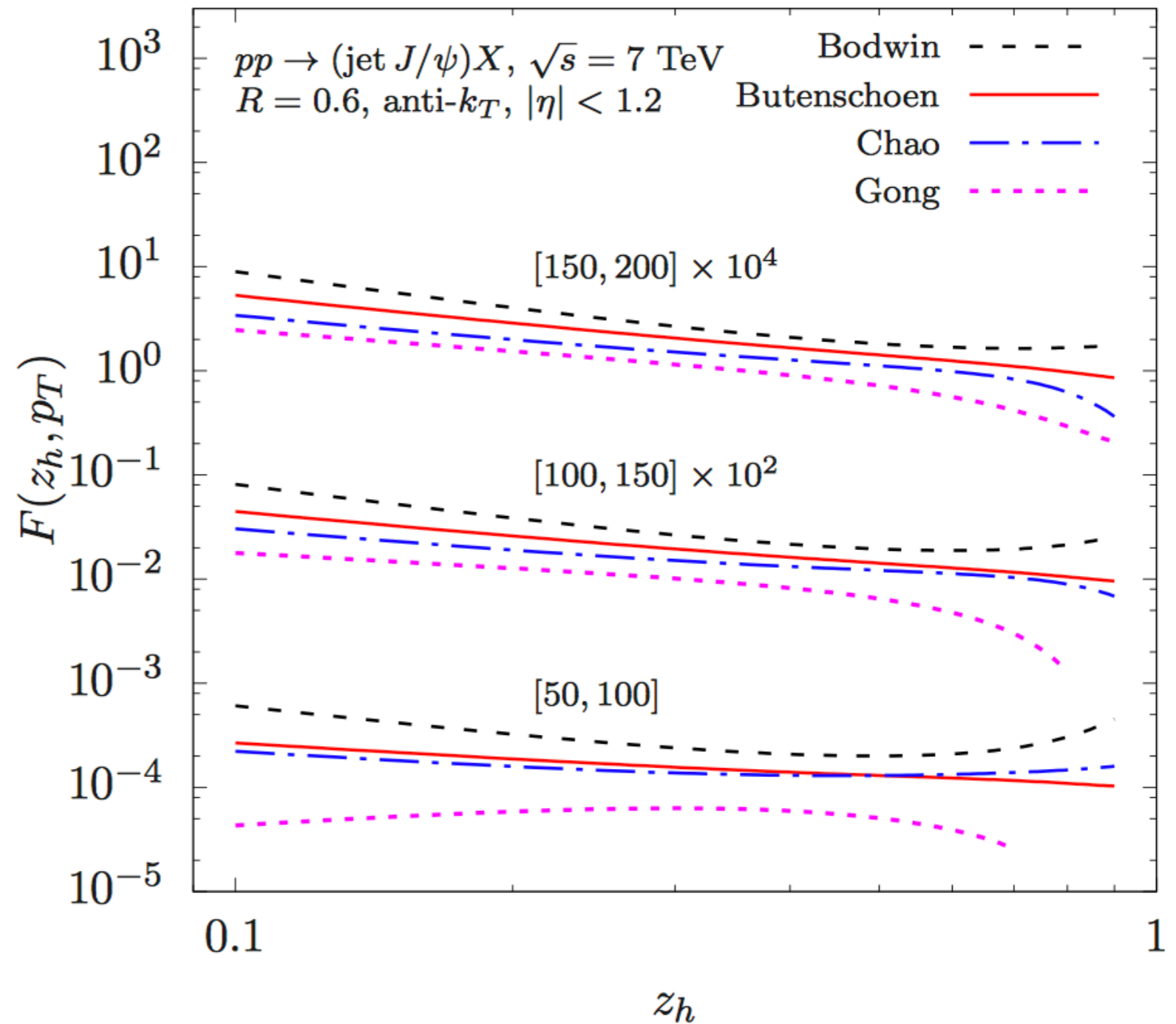
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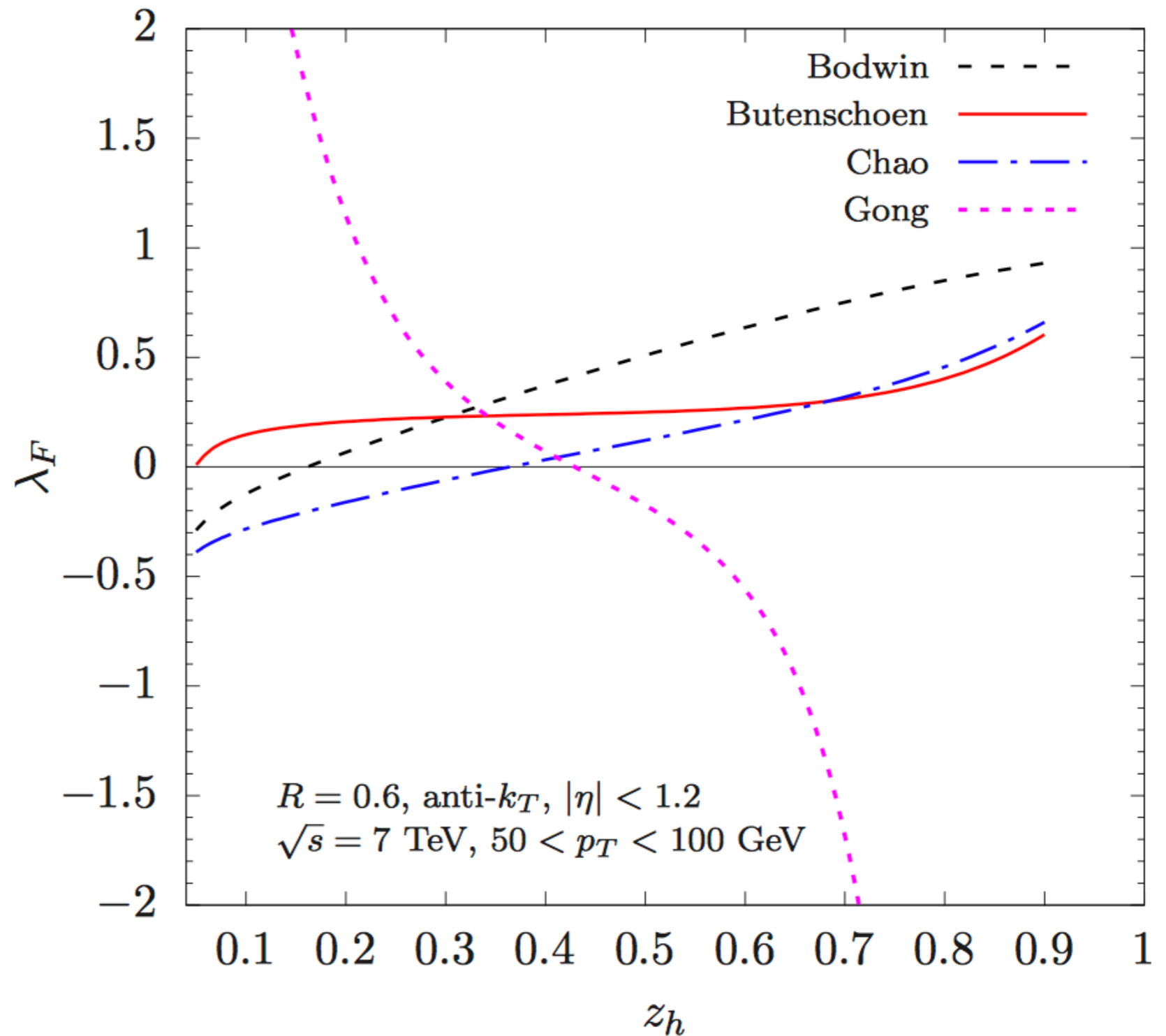
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Kang, Liu, FR, Xing '17

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TMD sensitive jet substructure observables

Kang, Liu, FR, Xing '17

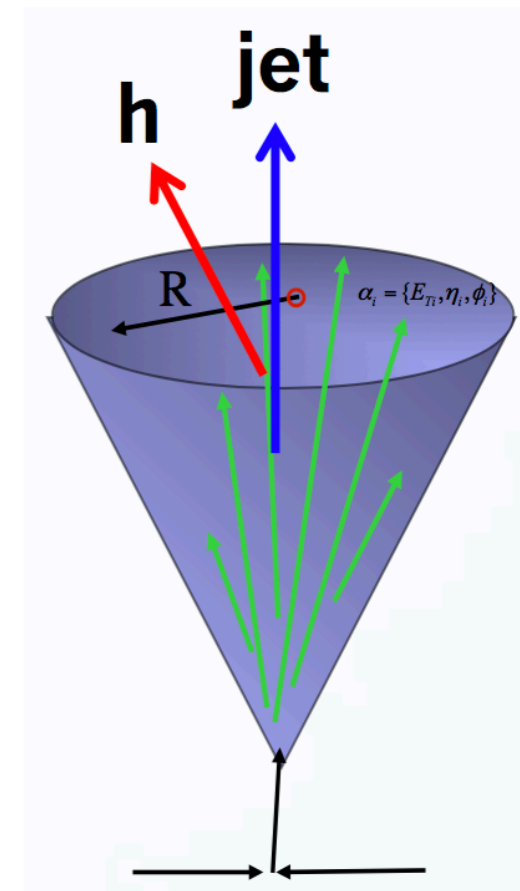
- Measure in addition the relative transverse momentum of the hadron wrt. to the (standard) jet axis

$$\frac{d\sigma^{pp \rightarrow (\text{jet } h)X}}{dp_T d\eta dz_h d^2\mathbf{j}_\perp} = \sum_{a,b,c} f_a(x_a, \mu) \otimes f_b(x_b, \mu) \otimes H_{ab}^c(x_a, x_b, \eta, p_T/z, \mu) \otimes \mathcal{G}_c^h(z, z_h, \mathbf{j}_\perp, \omega_J, \mu)$$

momentum fraction z_h

transverse momentum \mathbf{j}_\perp

- Standard jet axis
- Inclusive jet sample
- Relation to usual TMD evolution and fits
- Light charged hadrons



See also: Bain, Makris, Mehen '16
Neill, Scimemi, Waalewijn '17

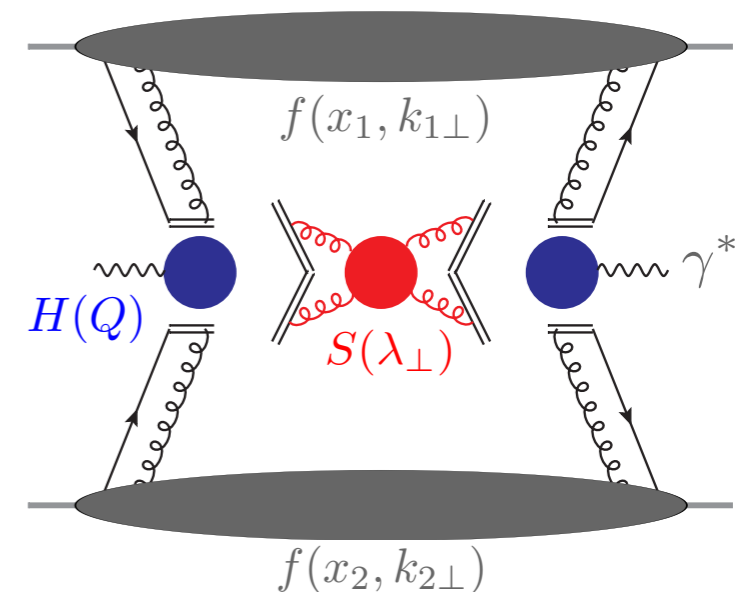
Drell-Yan $pp \rightarrow [\gamma^* \rightarrow \ell^+ \ell^-] X$

Parton model interpretation

$$\begin{aligned} \frac{d\sigma}{dQ^2 dy d^2\mathbf{q}_\perp} &\sim \int d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} d^2\boldsymbol{\lambda}_\perp H(Q) f(x_1, \mathbf{k}_{1\perp}) f(x_2, \mathbf{k}_{2\perp}) S(\boldsymbol{\lambda}_\perp) \delta^2(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp} + \boldsymbol{\lambda}_\perp - \mathbf{q}_\perp) \\ &= \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{q}_\perp \cdot \mathbf{b}} H(Q) f(x_1, \mathbf{b}) f(x_2, \mathbf{b}) S(\mathbf{b}) \\ &= \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{q}_\perp \cdot \mathbf{b}} H(Q) F(x_1, \mathbf{b}) F(x_2, \mathbf{b}) \end{aligned}$$

Rapidity divergences
cancel in redefined TMD

$$F(x, \mathbf{b}) = f(x, \mathbf{b}) \sqrt{S(\mathbf{b})}$$



Jet fragmentation function $pp \rightarrow (\text{jet } h)X$

Factorization formalism

$$\frac{d\sigma^{pp \rightarrow (\text{jet } h)X}}{dp_T d\eta dz_h d^2 \mathbf{j}_\perp} = \sum_{a,b,c} f_a(x_a, \mu) \otimes f_b(x_b, \mu) \otimes H_{ab}^c(x_a, x_b, \eta, p_T/z, \mu) \otimes \mathcal{G}_c^h(z, z_h, \mathbf{j}_\perp, \omega_J, \mu)$$

where for $|\mathbf{j}_\perp| \ll p_T R$

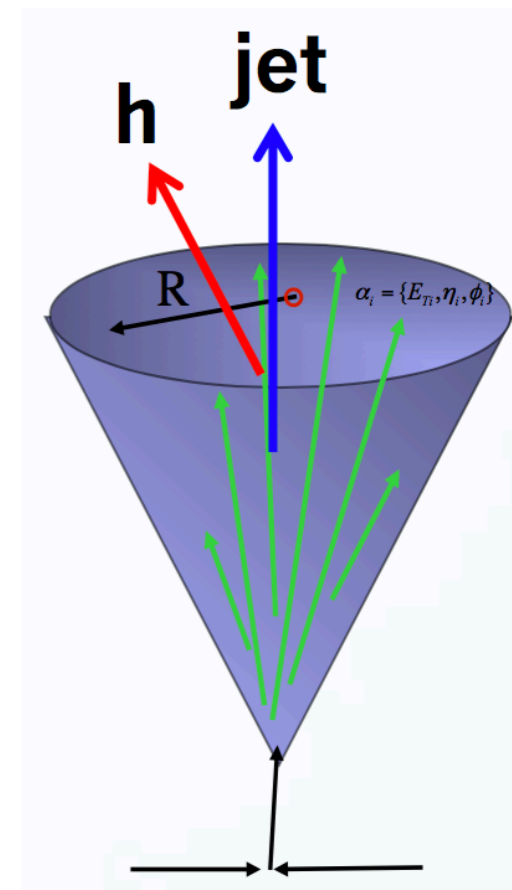
hard matching function

$$\mathcal{G}_c^h(z, z_h, \mathbf{j}_\perp, \omega_J, \mu) = \mathcal{H}_{c \rightarrow i}(z, \omega_J, \mu) \int d^2 \mathbf{k}_\perp d^2 \boldsymbol{\lambda}_\perp \delta^2(z_h \boldsymbol{\lambda}_\perp + \mathbf{k}_\perp - \mathbf{j}_\perp)$$

$$\times D_i^h(z_h, \mathbf{k}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, R, \mu, \nu)$$

↑
TMD FF

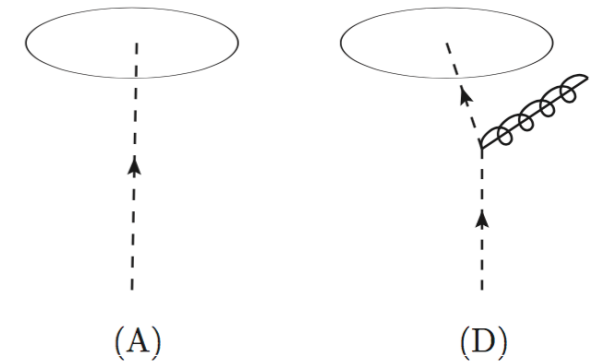
↑
Soft function



Jet fragmentation function $pp \rightarrow (\text{jeth})X$

- Hard matching functions

$$\mathcal{H}_{q \rightarrow q'}(z, \omega_J, \mu) = \delta_{qq'} \delta(1-z) + \delta_{qq'} \frac{\alpha_s}{2\pi} \left[C_F \delta(1-z) \left(-\frac{L^2}{2} - \frac{3}{2}L + \frac{\pi^2}{12} \right) \right. \\ \left. + P_{qq}(z)L - 2C_F(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - C_F(1-z) \right]$$



$$L = \ln \left(\frac{\mu^2}{p_T^2 R^2} \right)$$

- Evolution: modified DGLAP

$$\mu \frac{d}{d\mu} \mathcal{H}_{i \rightarrow j}(z, \omega_R, \mu) = \sum_k \int_z^1 \frac{dz'}{z'} \gamma_{ik} \left(\frac{z}{z'} \right) \mathcal{H}_{k \rightarrow j}(z', \omega_R, \mu)$$

where $\gamma_{ij}(z) = \delta_{ij} \delta(1-z) \Gamma_i + \frac{\alpha_s}{\pi} P_{ji}(z), \quad \Gamma_q = \frac{\alpha_s}{\pi} C_F \left(-L - \frac{3}{2} \right)$

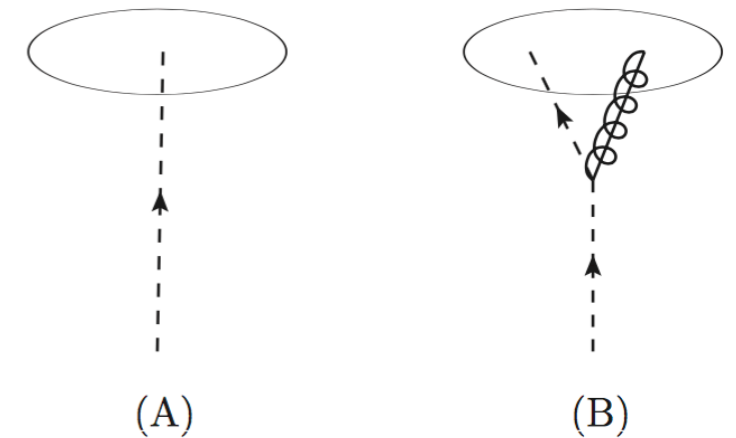
4 coupled equations with double logarithms. Characteristic scale $\mu_{\mathcal{H}} = p_T R$

see also: Kang, FR, Waalewijn '17

Jet fragmentation function $pp \rightarrow (\text{jet}h)X$

- Rapidity regulator η , scale ν *Chiu, Jain, Neill, Rothstein '12*
- (In-jet) quark TMD

$$D_q^q(z_h, \mathbf{k}_\perp, \mu, \nu) = \delta(1 - z_h) \delta^2(\mathbf{k}_\perp) + \frac{\alpha_s}{2\pi^2} C_F \Gamma(1 + \epsilon) e^{\gamma_E \epsilon} \frac{1}{\mu^2} \left(\frac{\mu^2}{\mathbf{k}_\perp^2} \right)^{1 + \epsilon} \\ \times \left[\frac{2z_h}{(1 - z_h)^{1 + \eta}} \left(\frac{\nu}{\omega_J} \right)^\eta + (1 - \epsilon)(1 - z_h) \right]$$



Jet fragmentation function $pp \rightarrow (\text{jeth})X$

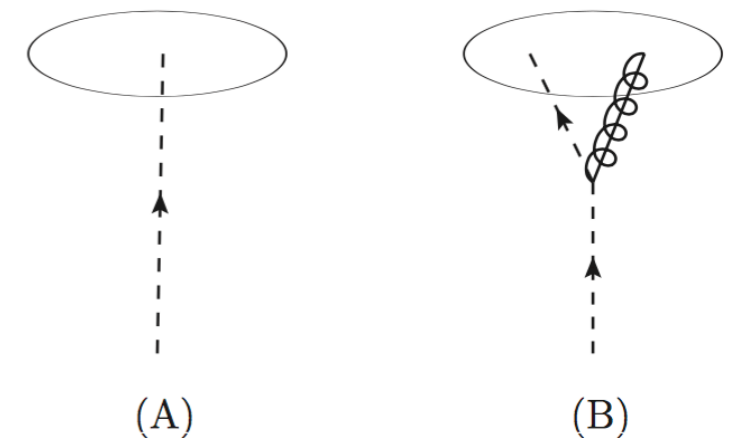
- Rapidity regulator η , scale ν *Chiu, Jain, Neill, Rothstein '12*
- (In-jet) quark TMD

$$D_q^q(z_h, \mathbf{k}_\perp, \mu, \nu) = \delta(1 - z_h) \delta^2(\mathbf{k}_\perp) + \frac{\alpha_s}{2\pi^2} C_F \Gamma(1 + \epsilon) e^{\gamma_E \epsilon} \frac{1}{\mu^2} \left(\frac{\mu^2}{\mathbf{k}_\perp^2} \right)^{1+\epsilon} \\ \times \left[\frac{2z_h}{(1 - z_h)^{1+\eta}} \left(\frac{\nu}{\omega_J} \right)^\eta + (1 - \epsilon)(1 - z_h) \right]$$

b-space and expansion in η, ϵ :

$$D_q^q(z_h, \mathbf{b}, \mu, \nu) = \frac{1}{z_h^2} \left\{ \delta(1 - z_h) \right. \\ \left. + \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\eta} \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{\mu_b^2} \right) \right) + \frac{1}{\epsilon} \left(2 \ln \left(\frac{\nu}{\omega_J} \right) + \frac{3}{2} \right) \right] \delta(1 - z_h) \right. \\ \left. + \frac{\alpha_s}{2\pi} C_F \left[-\frac{1}{\epsilon} - \ln \left(\frac{\mu^2}{z_h^2 \mu_b^2} \right) \right] P_{qq}(z_h) \right. \\ \left. + \frac{\alpha_s}{2\pi} C_F \left[\ln \left(\frac{\mu^2}{\mu_b^2} \right) \left(2 \ln \left(\frac{\nu}{\omega_J} \right) + \frac{3}{2} \right) \delta(1 - z_h) + (1 - z_h) \right] \right\}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b}$$



Jet fragmentation function $pp \rightarrow (\text{jet}h)X$

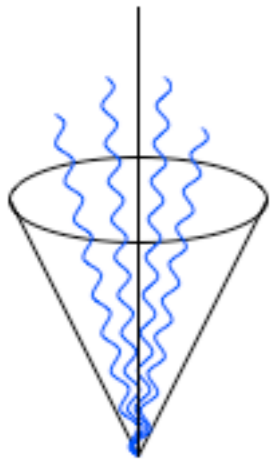
- Rapidity regulator η , scale ν Chiu, Jain, Neill, Rothstein '12

- (In-jet) quark TMD $D_q^q(z_h, \mathbf{b}, \mu, \nu) = \frac{1}{z_h^2} \left\{ \delta(1 - z_h) \right.$

$$+ \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\eta} \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{\mu_b^2} \right) \right) + \frac{1}{\epsilon} \left(2 \ln \left(\frac{\nu}{\omega_J} \right) + \frac{3}{2} \right) \right] \delta(1 - z_h)$$

$$+ \frac{\alpha_s}{2\pi} C_F \left[-\frac{1}{\epsilon} - \ln \left(\frac{\mu^2}{z_h^2 \mu_b^2} \right) \right] P_{qq}(z_h)$$

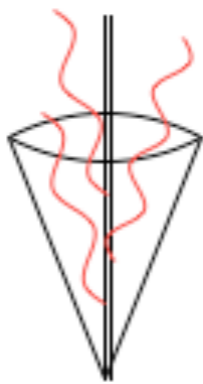
$$\left. + \frac{\alpha_s}{2\pi} C_F \left[\ln \left(\frac{\mu^2}{\mu_b^2} \right) \left(2 \ln \left(\frac{\nu}{\omega_J} \right) + \frac{3}{2} \right) \delta(1 - z_h) + (1 - z_h) \right] \right\}$$



- In-jet soft function $S_q(\mathbf{b}, R, \mu, \nu) = \int d^2 \lambda_\perp e^{-i \lambda_\perp \cdot \mathbf{b}} S_q(\lambda_\perp, R, \mu, \nu)$

$$= 1 + \frac{\alpha_s}{2\pi} C_F \left[-\frac{2}{\eta} \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{\mu_b^2} \right) \right) + \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \left(\frac{\nu^2 \tan^2(R/2)}{\mu^2} \right) \right.$$

$$\left. - \ln \left(\frac{\mu^2}{\mu_b^2} \right) \ln \left(\frac{\nu^2 \tan^2(R/2)}{\mu_b^2} \right) + \frac{1}{2} \ln^2 \left(\frac{\mu^2}{\mu_b^2} \right) - \frac{\pi^2}{12} \right]$$



Jet fragmentation function $pp \rightarrow (\text{jet}h)X$

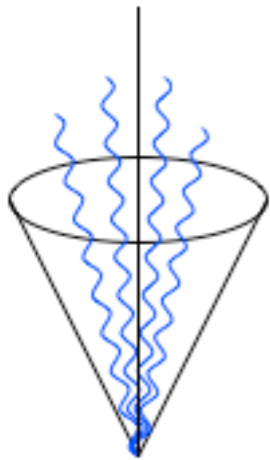
- Rapidity regulator η , scale ν Chiu, Jain, Neill, Rothstein '12

- (In-jet) quark TMD $D_q^q(z_h, \mathbf{b}, \mu, \nu) = \frac{1}{z_h^2} \left\{ \delta(1 - z_h) \right.$

$$+ \frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\eta} \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{\mu_b^2} \right) \right) + \frac{1}{\epsilon} \left(2 \ln \left(\frac{\nu}{\omega_J} \right) + \frac{3}{2} \right) \right] \delta(1 - z_h)$$

$$+ \frac{\alpha_s}{2\pi} C_F \left[-\frac{1}{\epsilon} - \ln \left(\frac{\mu^2}{z_h^2 \mu_b^2} \right) \right] P_{qq}(z_h)$$

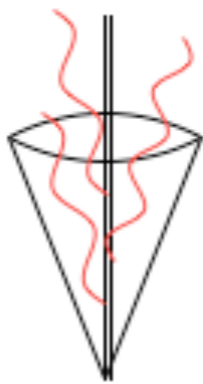
$$\left. + \frac{\alpha_s}{2\pi} C_F \left[\ln \left(\frac{\mu^2}{\mu_b^2} \right) \left(2 \ln \left(\frac{\nu}{\omega_J} \right) + \frac{3}{2} \right) \delta(1 - z_h) + (1 - z_h) \right] \right\}$$



- In-jet soft function $S_q(\mathbf{b}, R, \mu, \nu) = \int d^2 \lambda_{\perp} e^{-i \lambda_{\perp} \cdot \mathbf{b}} S_q(\lambda_{\perp}, R, \mu, \nu)$

$$= 1 + \frac{\alpha_s}{2\pi} C_F \left[-\frac{2}{\eta} \left(\frac{1}{\epsilon} + \ln \left(\frac{\mu^2}{\mu_b^2} \right) \right) + \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \left(\frac{\nu^2 \tan^2(R/2)}{\mu^2} \right) \right.$$

$$\left. - \ln \left(\frac{\mu^2}{\mu_b^2} \right) \ln \left(\frac{\nu^2 \tan^2(R/2)}{\mu_b^2} \right) + \frac{1}{2} \ln^2 \left(\frac{\mu^2}{\mu_b^2} \right) - \frac{\pi^2}{12} \right]$$



Jet fragmentation function $pp \rightarrow (\text{jeth})X$

- Renormalization

$$D_{i,\text{bare}}(z_h, \mathbf{b}, \mu, \nu) = Z_i^D(\mathbf{b}, \mu, \nu) D_{i,\text{ren}}(z_h, \mathbf{b}, \mu, \nu)$$

$$S_{i,\text{bare}}(\mathbf{b}, R, \mu, \nu) = Z_i^S(\mathbf{b}, R, \mu, \nu) S_{i,\text{ren}}(\mathbf{b}, R, \mu, \nu)$$

- Evolution

$$\mu \frac{d}{d\mu} S_i(\mathbf{b}, R, \mu, \nu) = \gamma_{i,\mu}^S(R, \mu, \nu) S_i(\mathbf{b}, R, \mu, \nu)$$

$$\nu \frac{d}{d\nu} S_i(\mathbf{b}, R, \mu, \nu) = \gamma_{i,\nu}^S(\mathbf{b}, \mu) S_i(\mathbf{b}, R, \mu, \nu)$$

$$\mu \frac{d}{d\mu} D_i(z_h, \mathbf{b}, \mu, \nu) = \gamma_{i,\mu}^D(\omega_J, \nu) D_i(z_h, \mathbf{b}, \mu, \nu)$$

$$\nu \frac{d}{d\nu} D_i(z_h, \mathbf{b}, \mu, \nu) = \gamma_{i,\nu}^D(\mathbf{b}, \mu) D_i(z_h, \mathbf{b}, \mu, \nu)$$

anomalous dimensions:

$$\gamma_{q,\mu}^S(R, \mu, \nu) = -\frac{\alpha_s}{\pi} C_F \ln \left(\frac{\nu^2 \tan^2(R/2)}{\mu^2} \right)$$

$$\gamma_{q,\nu}^D(\mathbf{b}, \mu) = -\gamma_{q,\nu}^S(\mathbf{b}, \mu) = \frac{\alpha_s}{\pi} C_F \ln \left(\frac{\mu^2}{\mu_b^2} \right)$$

$$\gamma_{q,\mu}^D(\omega_J, \nu) = \frac{\alpha_s}{\pi} C_F \left[\ln \left(\frac{\nu^2}{\omega_J^2} \right) + \frac{3}{2} \right]$$

Jet fragmentation function $pp \rightarrow (\text{jeth})X$

- Evolution

redefined TMD $\mathcal{D}_{h/i}^R$
and evolution to a common scale

$$\mathcal{D}_{h/i}^R(z_h, \mathbf{b}; \mu_b) \equiv D_{h/i}(z_h, \mathbf{b}, \mu_D, \nu_D) S_i(\mathbf{b}, \mu_S, \nu_S R)$$

$$= \mathcal{D}_{h/i}^R(z_h, \mathbf{b}; \mu_b) \exp \left[- \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left(\Gamma_{\text{cusp}}^i \ln \left(\frac{\mu_J^2}{\mu'^2} \right) + \gamma^i \right) \right]$$

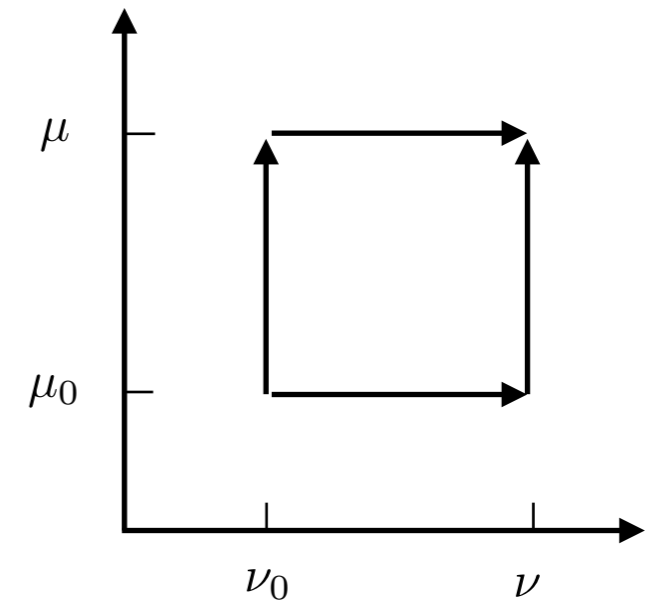
$$= \hat{\mathcal{D}}_{h/i}(z_h, \mathbf{b}; \mu_J) \exp \left[- \int_{\mu_J}^{\mu} \frac{d\mu'}{\mu'} \left(\Gamma_{\text{cusp}}^i \ln \left(\frac{\mu_J^2}{\mu'^2} \right) + \gamma^i \right) \right]$$



standard TMD at μ_J



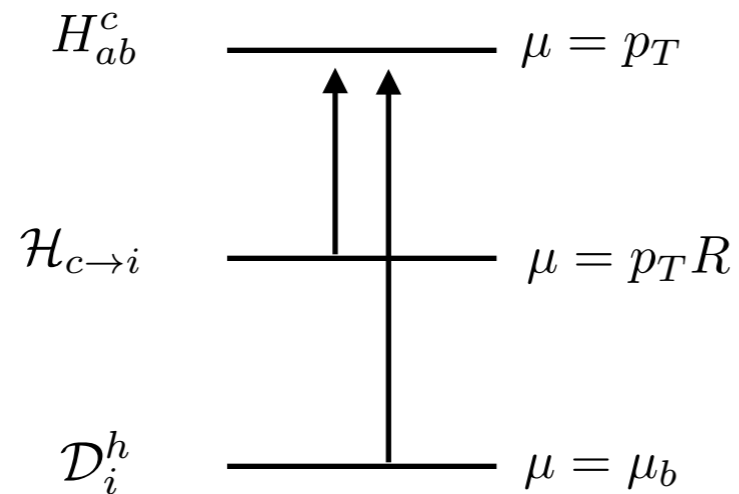
extra evolution factor $\mu_J \rightarrow \mu$



Collins, Soper, Sterman '85

Evolution

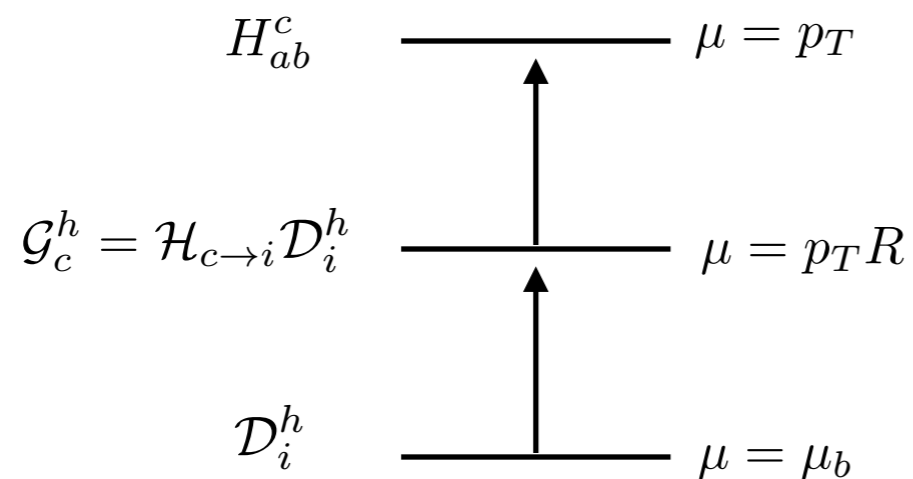
1.



using modified DGLAP for $\mathcal{H}_{c \rightarrow i}$

$$\mu \frac{d}{d\mu} \mathcal{H}_{c \rightarrow i} = \gamma_{ck} \otimes \mathcal{H}_{ki}$$

2.



using DGLAP for siFJs \mathcal{G}_c^h

$$\mu \frac{d}{d\mu} \mathcal{G}_c^h = P_{ic} \otimes \mathcal{G}_i^h$$

$$\gamma_{ii}^{\Gamma} + \gamma_{i,\mu}^S + \gamma_{i,\mu}^D = 0$$

b^* prescription

$$\hat{\mathcal{D}}_{h/i}(z_h, \mathbf{j}_\perp; \mu_J) = \frac{1}{z_h^2} \int \frac{b db}{2\pi} J_0(j_\perp b/z) C_{j \leftarrow i} \otimes D_{h/j}(z_h, \mu_{b^*}) e^{-S_{\text{pert}}^i(b^*, \mu_J) - S_{\text{NP}}^i(b, \mu_J)}$$

- perturbative Sudakov factor: $S_{\text{pert}}^i(b^*, \mu_J) = \int_{\mu_{b^*}}^{\mu_J} \frac{d\mu'}{\mu'} \left(\Gamma_{\text{cusp}}^i \ln \left(\frac{\mu_J^2}{\mu'^2} \right) + \gamma^i \right)$

where: $\mu_{b^*} = 2e^{-\gamma_E}/b^*$, $b^* = b/\sqrt{1 + b^2/b_{\text{max}}^2}$, $b_{\text{max}} < 1/\Lambda_{\text{QCD}}$

Collins, Soper, Sterman '85

- non-perturbative Sudakov factor:

quark TMD $S_{\text{NP}}^q = g_2 \ln \left(\frac{b}{b^*} \right) \ln \left(\frac{\mu}{\mu_0} \right) + \frac{g_h}{z_h^2} b^2$

Sun, Isaacson, Yuan, Yuan '14

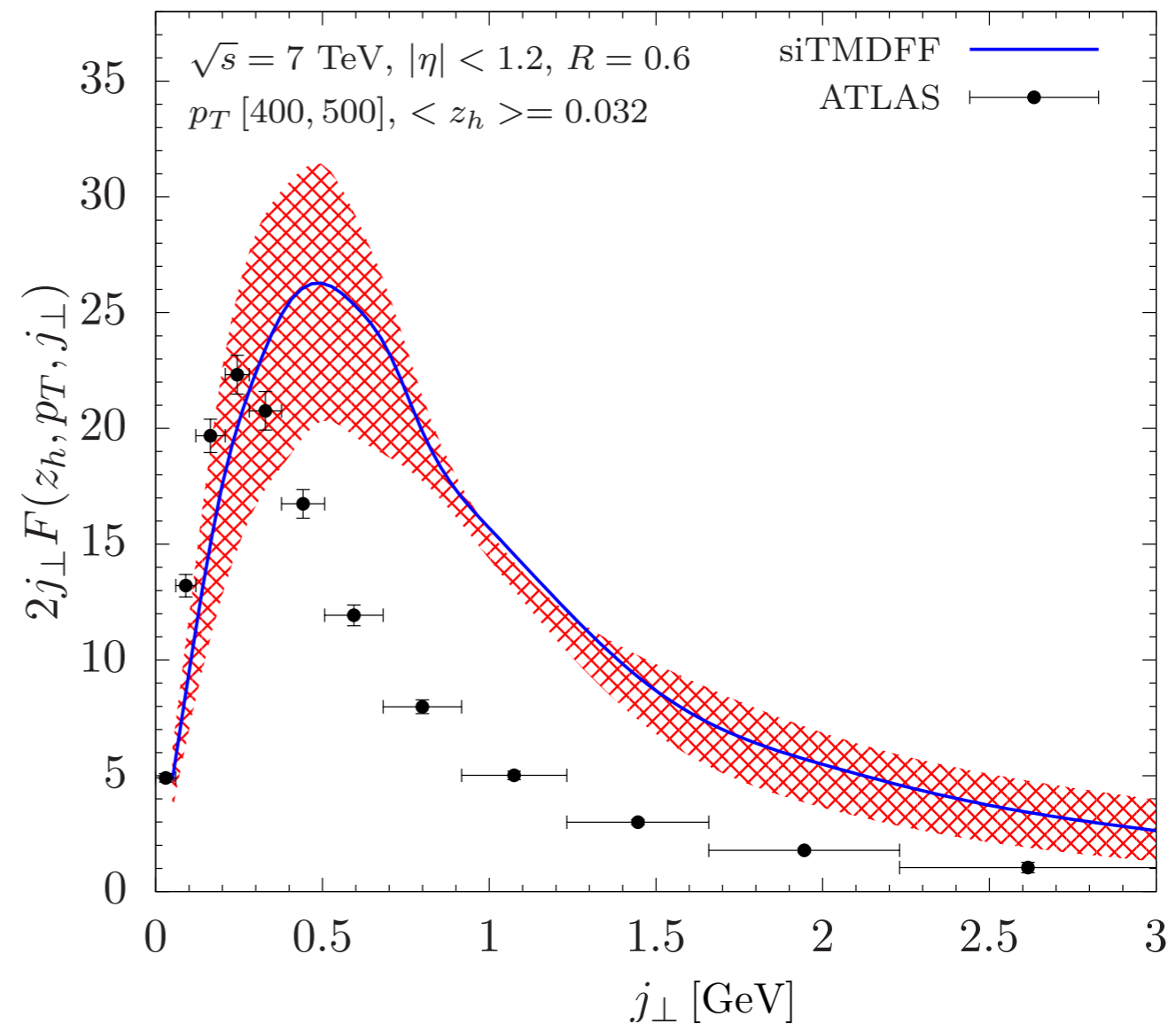
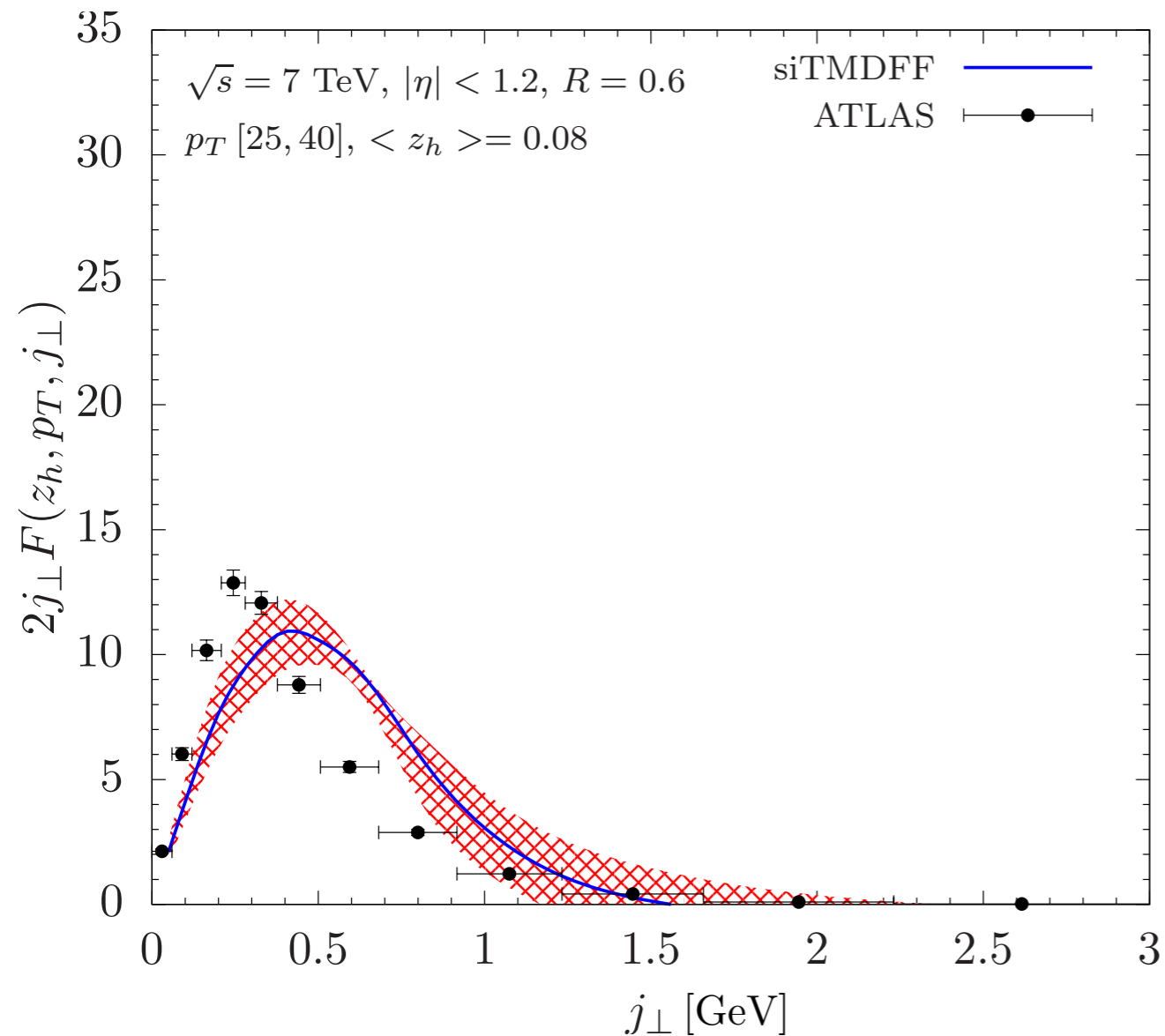
gluon TMD $S_{\text{NP}}^g = \frac{C_A}{C_F} g_2 \ln \left(\frac{b}{b^*} \right) \ln \left(\frac{\mu}{\mu_0} \right) + \frac{g_h}{z_h^2} b^2$

Balazs, Berger, Mrenna, Yuan '14

Balazs, Berger, Nadolsky, Yuan '14

Consistent with the fits of e.g. Sun, Kang, Prokudin, Yuan '16

Comparison to ATLAS data



- Varying μ, μ_J by factors of 2
- Gluon contribution dominates at low p_T

- Still need to include NGLs

Outline

- Introduction
- In-jet TMDs
- **Collins asymmetries**
- Conclusions

Kang, Liu, FR, Xing '17

Kang, Prokudin, FR, Yuan '17

Collins azimuthal asymmetries inside jets

- Transversely polarized pp collisions

$$p^\uparrow(P_A, S_T, \phi_S) + p(P_B) \rightarrow \text{jet}(\eta, p_T) h(z_h, j_\perp, \phi_H) + X$$

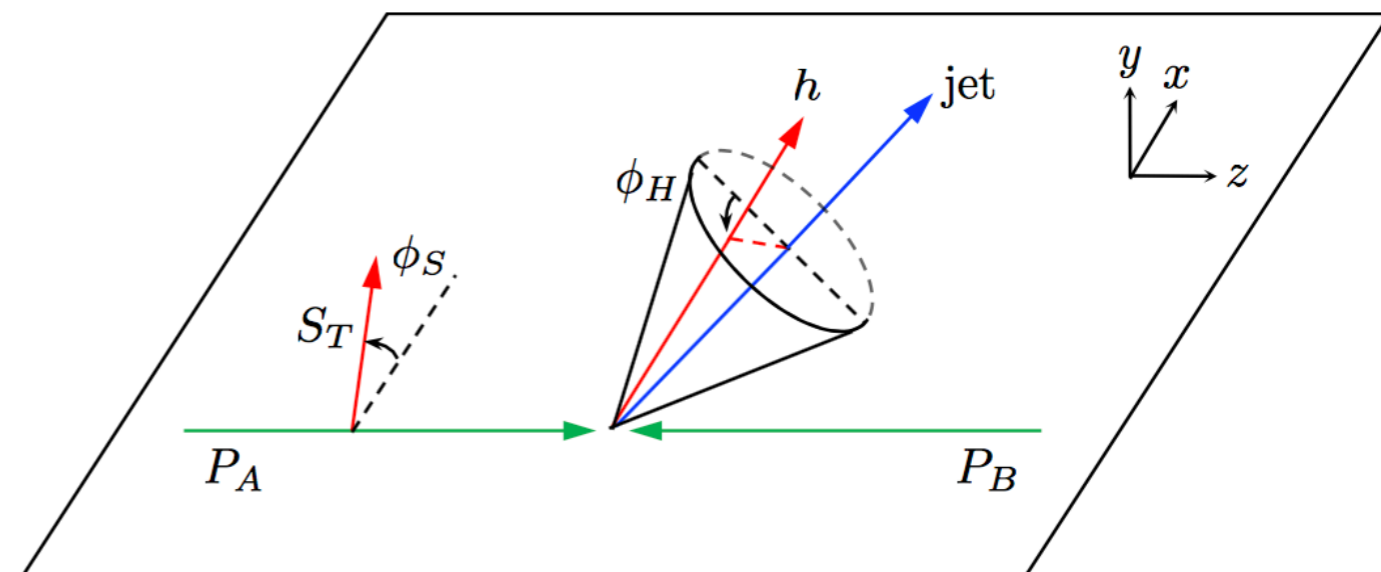
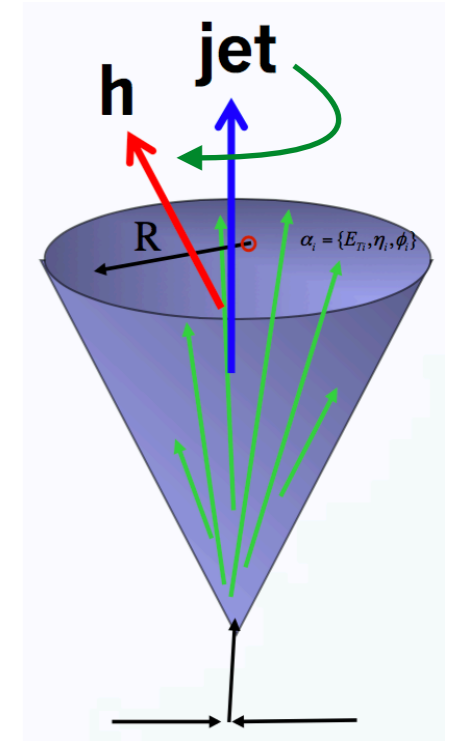
$$\frac{d\sigma}{d\eta d^2p_T dz_h d^2j_\perp} = F_{UU} + \sin(\phi_S - \phi_H) F_{UT}^{\sin(\phi_S - \phi_H)}$$

- Test of the universality of the Collins FF as currently extracted in global fits to SIDIS and electron-positron data
- Test of TMD evolution effects

Collins azimuthal spin asymmetry

$$A_{UT}^{\sin(\phi_S - \phi_H)}(z_h, j_\perp; \eta, p_T) = \frac{F_{UT}^{\sin(\phi_S - \phi_H)}}{F_{UU}}$$

Yuan `08



Collins azimuthal asymmetries inside jets

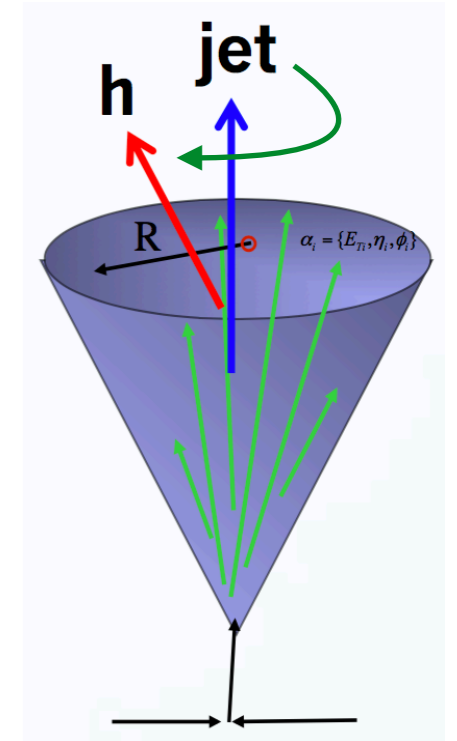
$$A_{UT}^{\sin(\phi_S - \phi_H)}(z_h, j_\perp; \eta, p_T) = \frac{F_{UT}^{\sin(\phi_S - \phi_H)}}{F_{UU}}$$

where

$$F_{UT}^{\sin(\phi_S - \phi_H)}(z_h, j_\perp) = \frac{\alpha_s^2}{s} \sum_{a,b,c} \int_{x_{1\min}}^1 \frac{dx_1}{x_1} h_1^a(x_1, \mu) \int_{x_{2\min}}^1 \frac{dx_2}{x_2} f_{b/B}(x_2, \mu) \frac{j_\perp}{z_h M_h} H_{1h/c}^\perp(z_h, j_\perp^2; Q) \\ \times H_{ab \rightarrow c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u})$$

quark transversity distribution

Collins FF

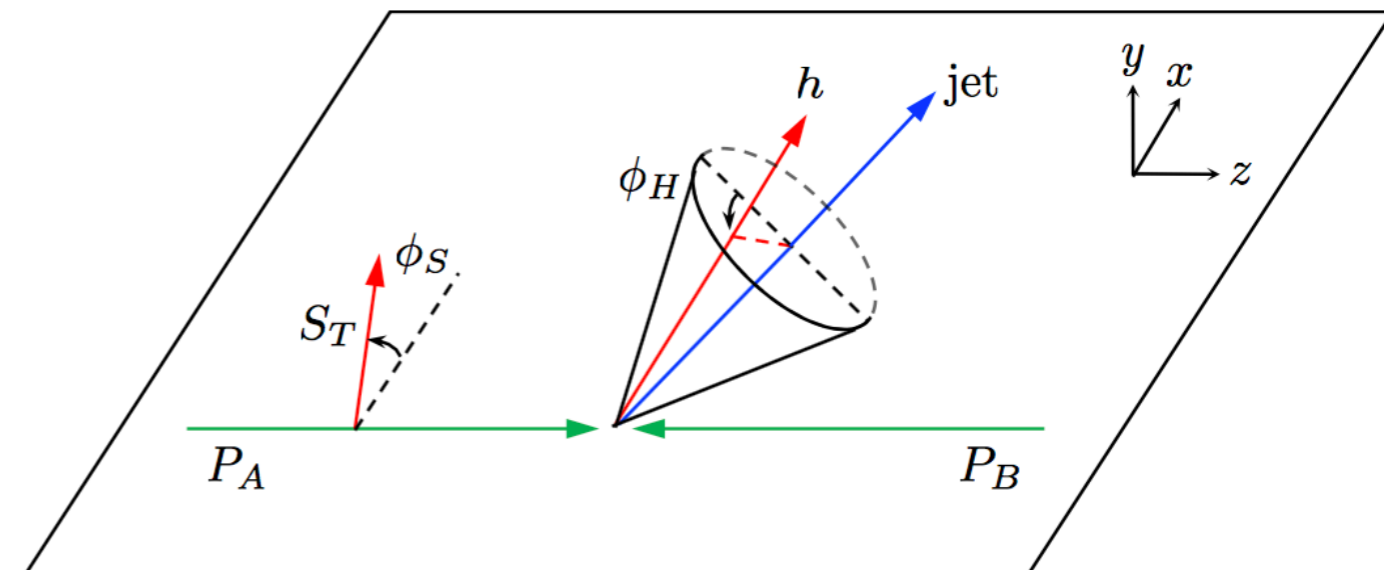


Global fits used for our numerical studies

- with TMD evolution Kang, Prokudin, Sun, Yuan `16
- without Anselmino, Boglione, D'Alesio, Hernandez, Melis, Murgia, Prokudin `15

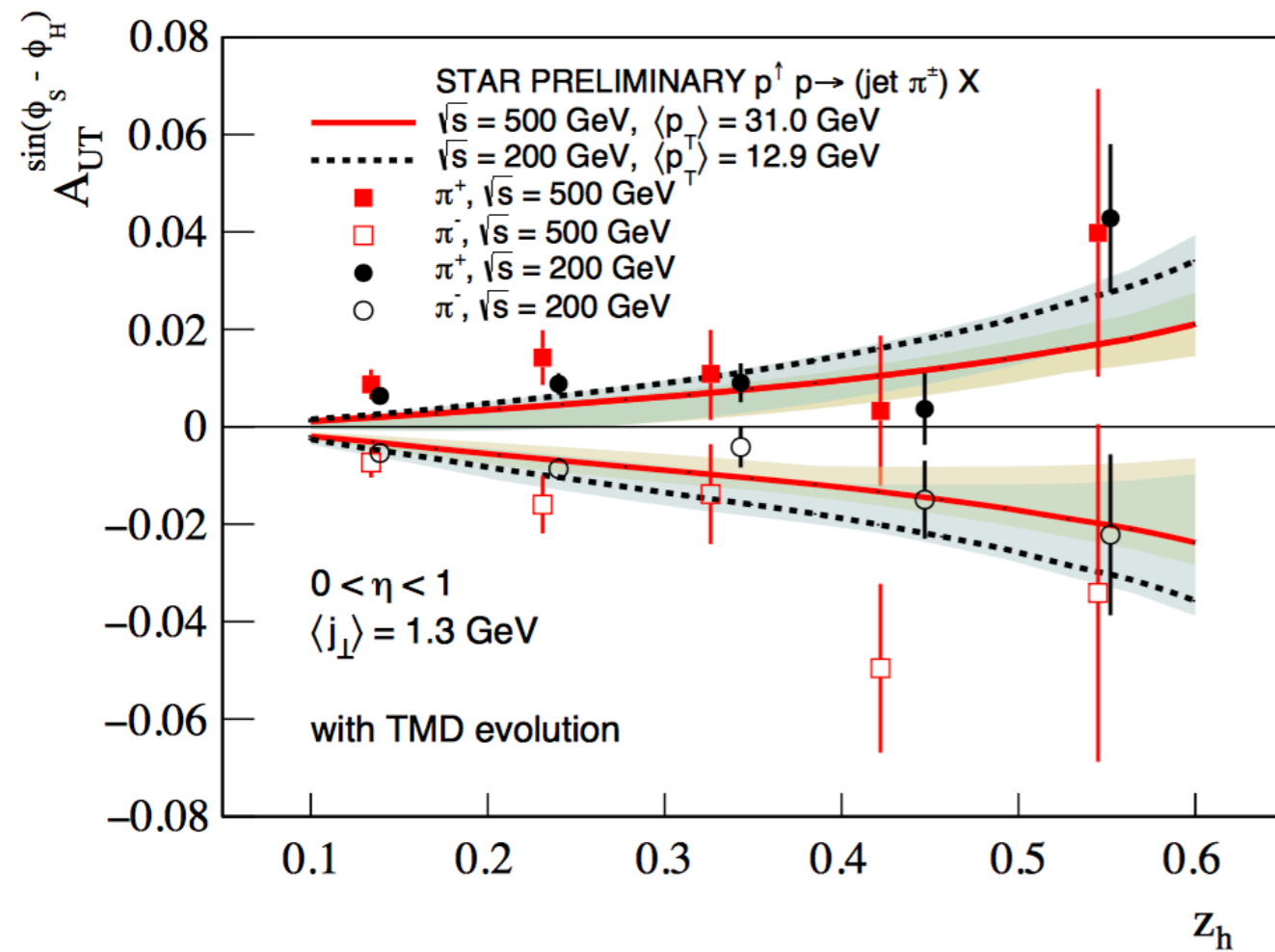
see also: generalized parton model

D'Alesio, Murgia, Pisano `11, `17

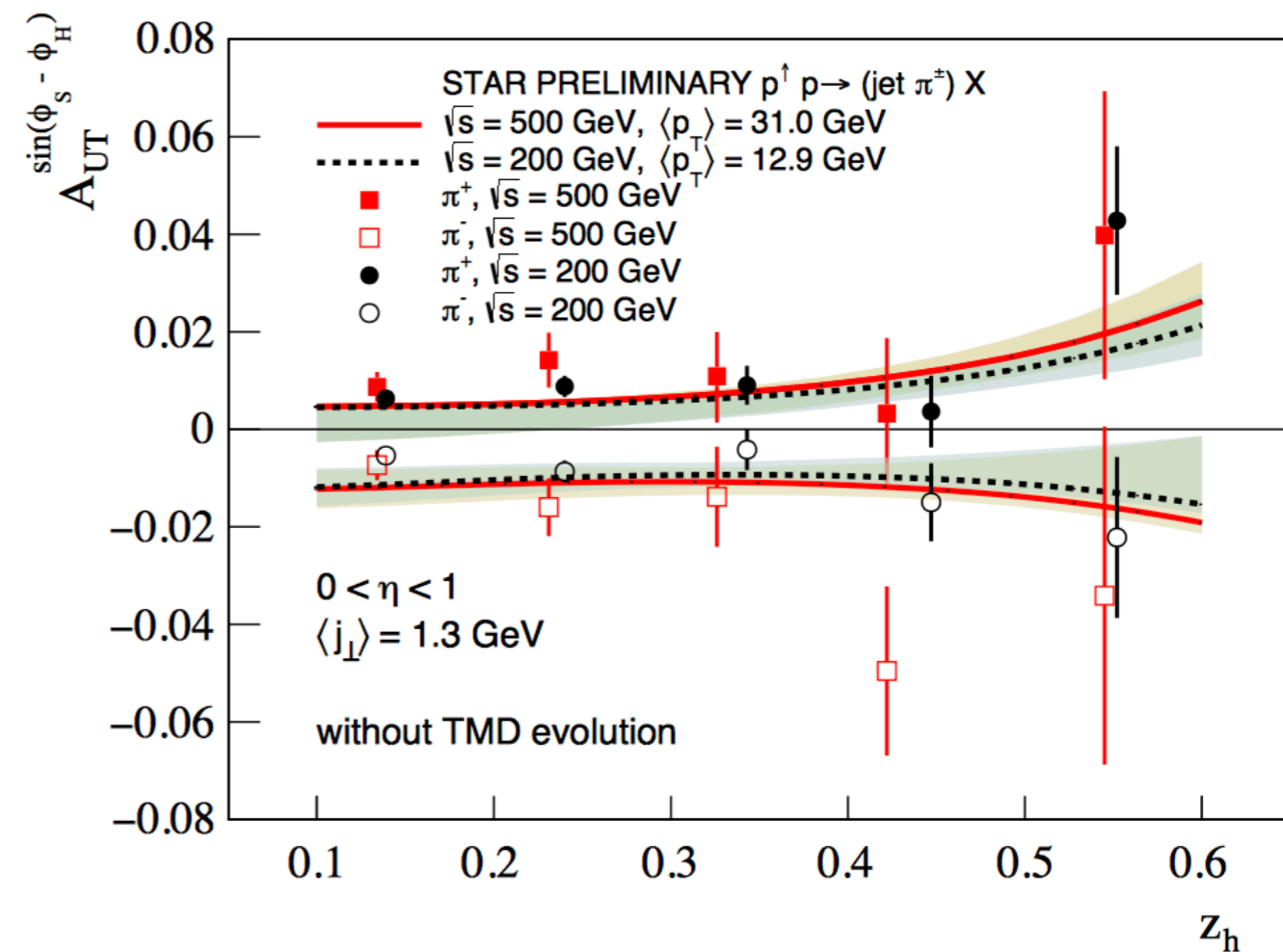


Comparison to RHIC data

with TMD evolution



without



$pp \rightarrow (\text{jet } \pi^\pm) X$

Outline

- Introduction
- In-jet TMDs
- Collins asymmetries
- **Conclusions**

Kang, Liu, FR, Xing '17

Kang, Prokudin, FR, Yuan '17

Conclusions

- Inclusive jets and their substructure
- Identified hadrons within jets:
 - light hadrons, photons, open heavy flavor, quarkonia
- TMD FFs within jets
- Y-term, non-global logarithms, global fits
- $e^+e^- \rightarrow (\text{jet}h)X$

