Recreating Generalized Parton Distribution Functions From Their Coordinate Space Behavior

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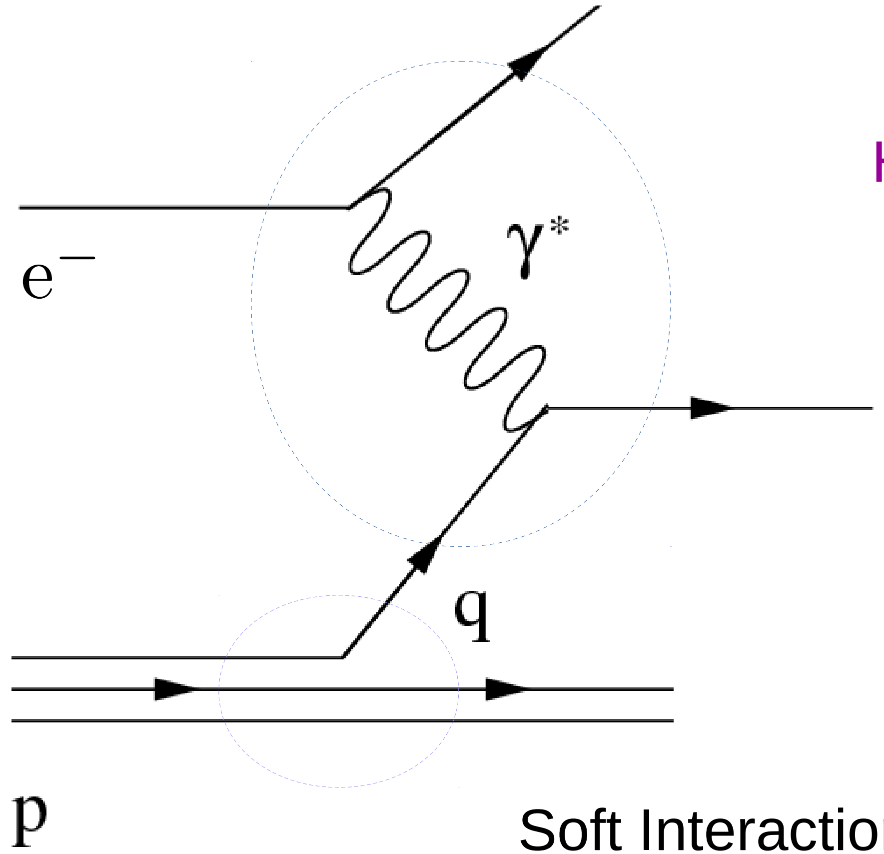
Outline

- Parton Distribution Functions and Generalized Parton Distributions: definitions and symmetries
- Ioffe time
- Reconstructing Parton Distributions using Lattice QCD moments
- Extending to Generalized Parton Distributions
- Pseudo PDFs

Hard and Soft Parts



Hard Interaction

Deep Inelastic Scattering

Soft Interaction

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p, \Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0} = f_1(x)$$

$$a^\pm = \frac{a^0 \pm a^3}{\sqrt{2}}$$

Symmetries

$$q(x) = -\bar{q}(-x)$$

$$q^{+}(x) = q(x) + \bar{q}(x)$$

$$\Rightarrow q^{+}(x) = q^{+}(-x)$$

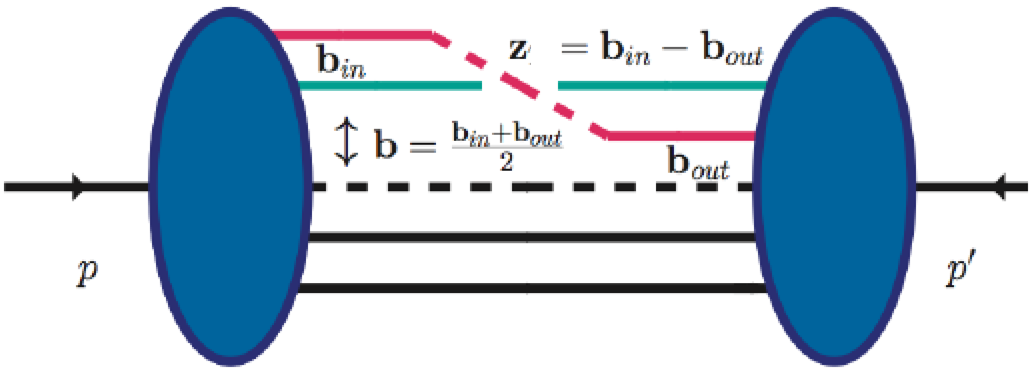
Symmetric in x

$$q^{-}(x) = q(x) - \bar{q}(x)$$

$$\Rightarrow q^{-}(x) = -q^{-}(-x)$$

Anti symmetric in x

Coordinate Space



Ioffe Time and z_T

$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Impact Parameter

Coordinate space picture, Ioffe Time

- At small z^- use Mellin moments to describe the correlator in z^- space.
- At large z^-, large compared to proton size, use Regge behavior .

Braun, Gornicki and Mankiewicz, Phys. Rev. D 1995

Fourier transform to z- space

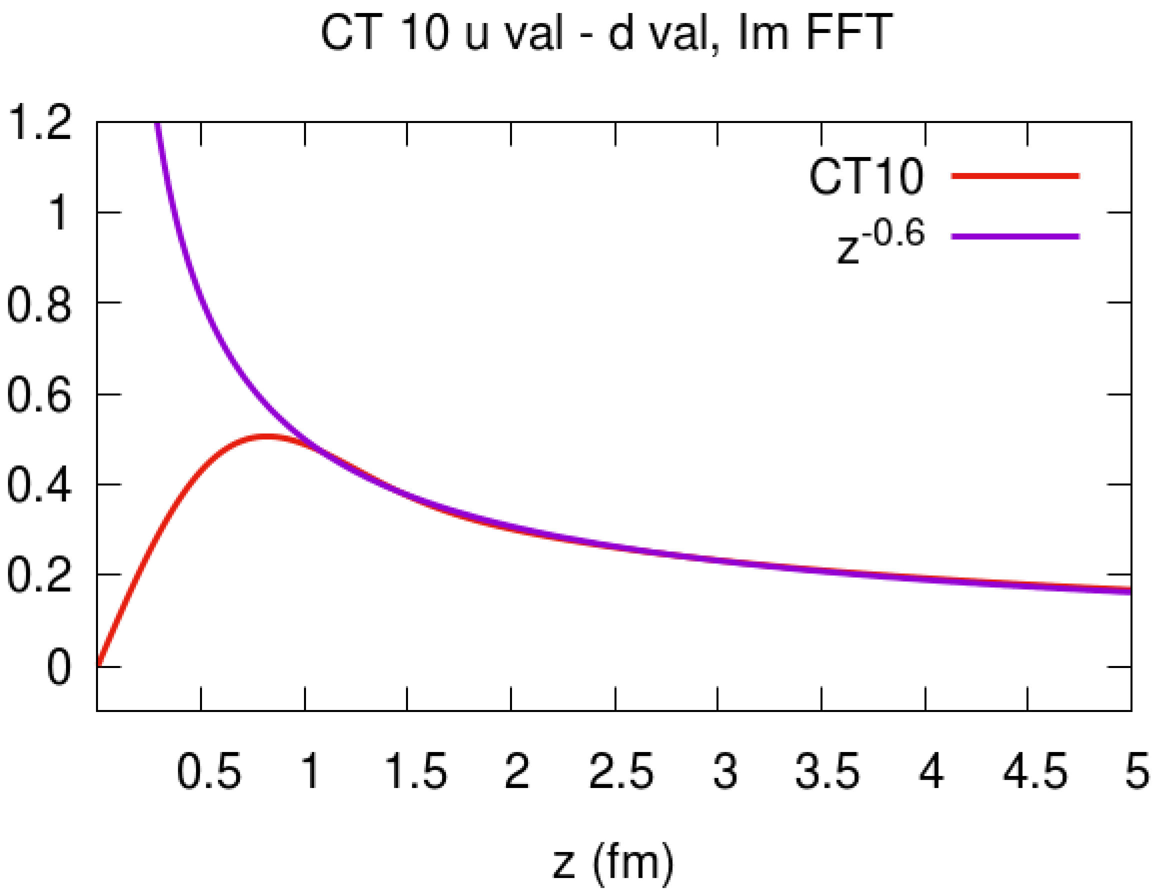
$$\mathcal{M}(z) = \int_0^1 dx e^{ixz} q(x)$$

$$= \int_0^1 dx \cos(xz) q(x) + i \int_0^1 dx \sin(xz) q(x)$$

$$M_0 - M_2 \frac{z^2}{2!} + M_4 \frac{z^4}{4!} + \ldots \qquad M_1 z - M_3 \frac{z^3}{3!} + M_4 \frac{z^5}{5!} + \ldots$$

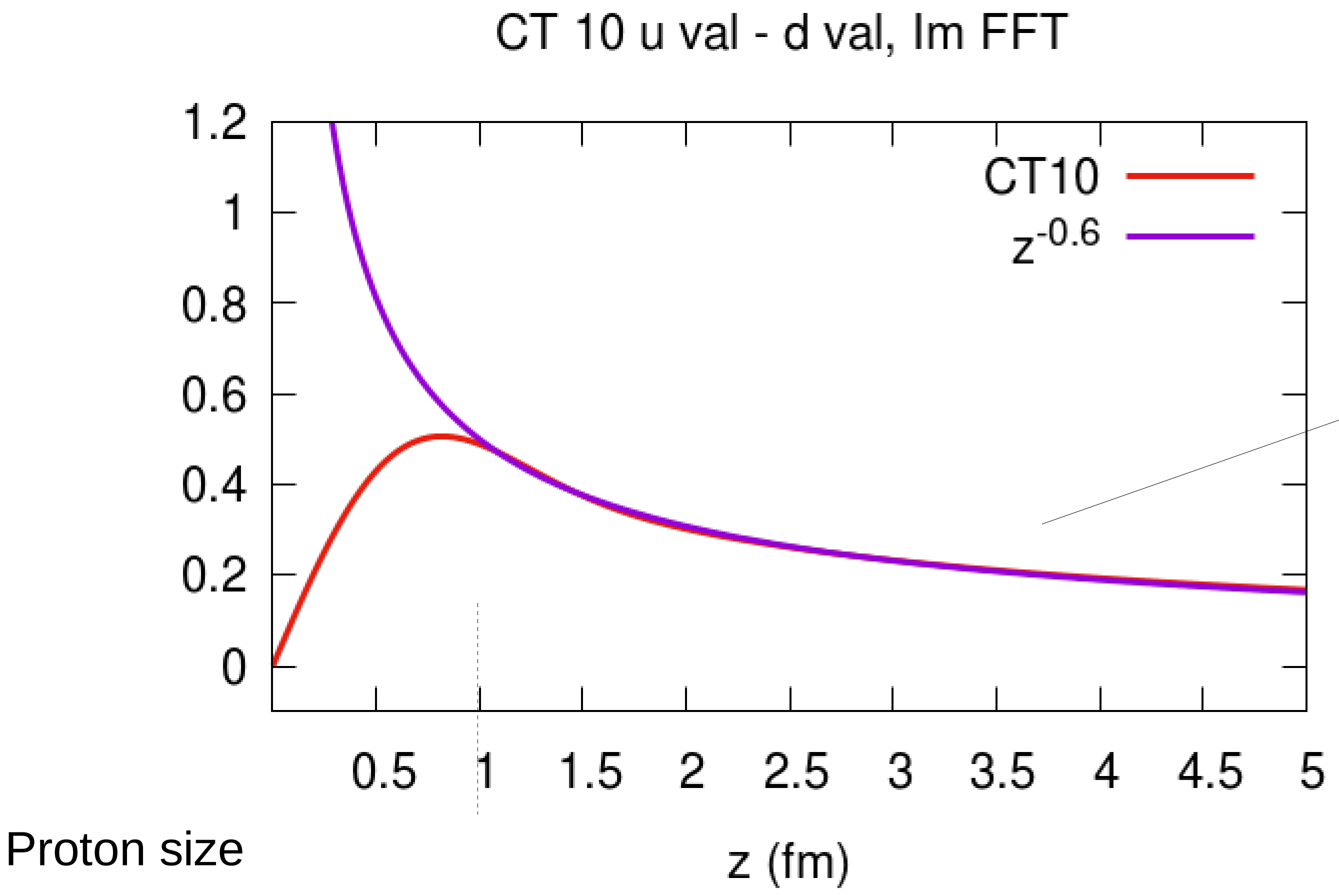
Can be approximated using Mellin moments for small z.

Large z-



Imaginary part of Fourier transform of PDF described by Regge part for large z

Large z-

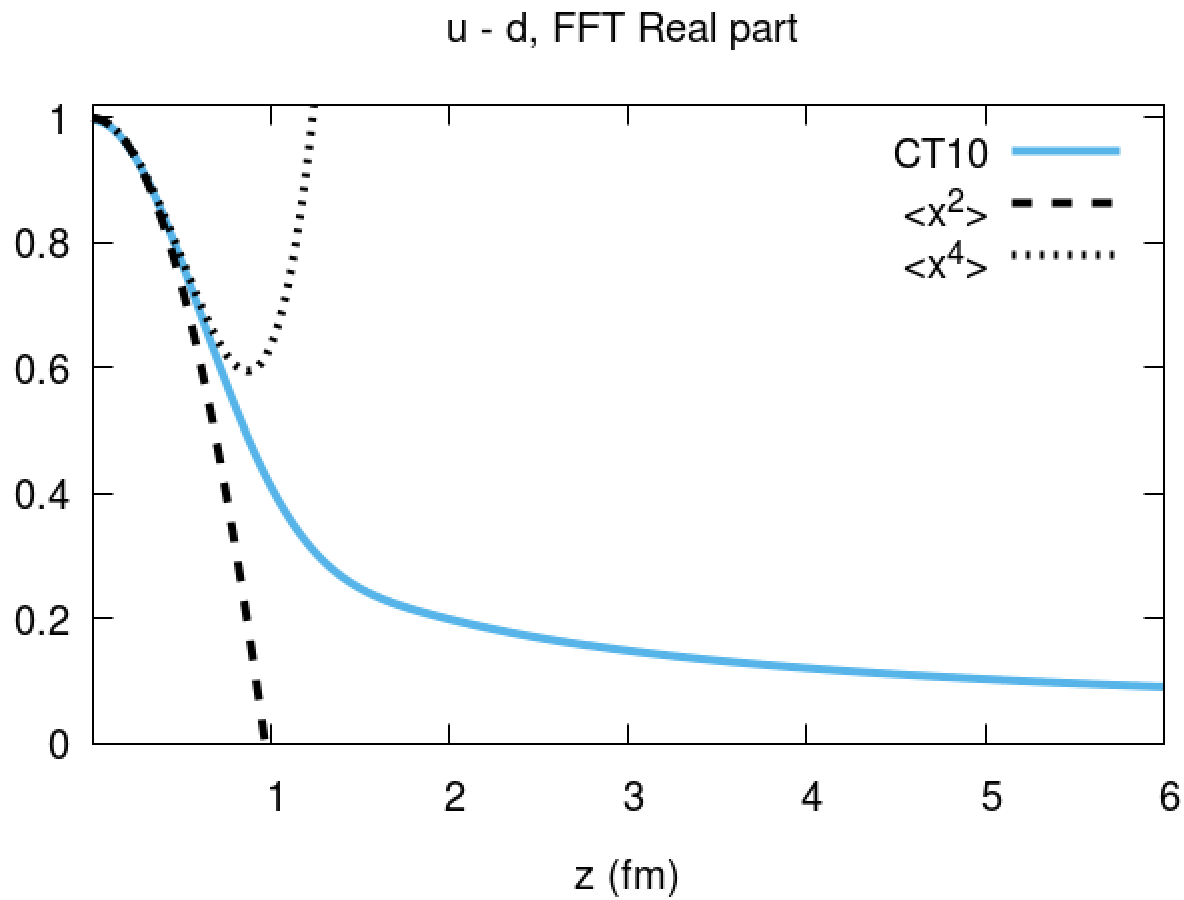


$z^{-\alpha+1}$

Imaginary part of Fourier transform of PDF described by Regge part for large z

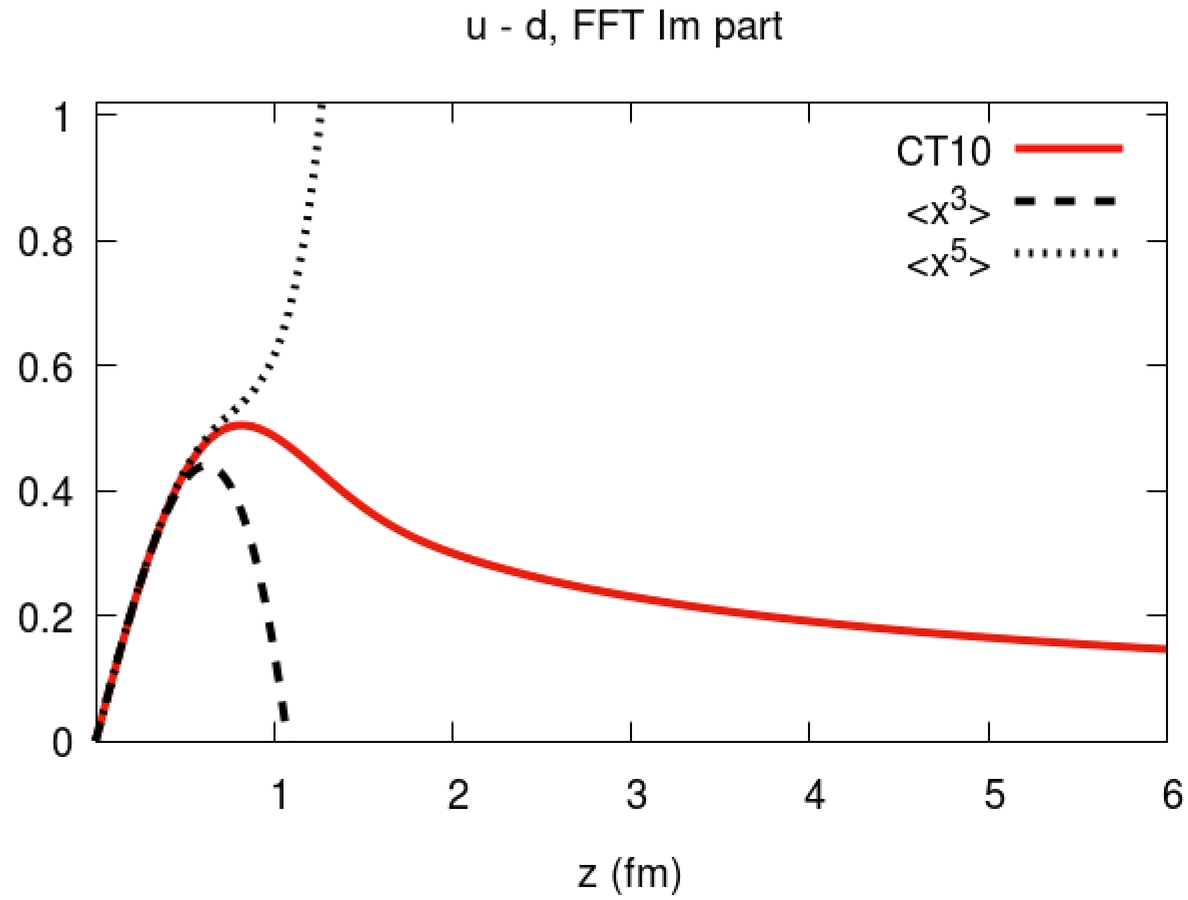
Small z-

$$\int_0^1 dx \cos(xz) q(x)$$



Small z$^-$

$$\int_0^1 dx \sin(xz) q(x)$$



Mellin Moments from Lattice

u - d

W. Detmold et al., Eur.Phys.J.3 (2001), Mod.Phys.Lett. A18 (2003)

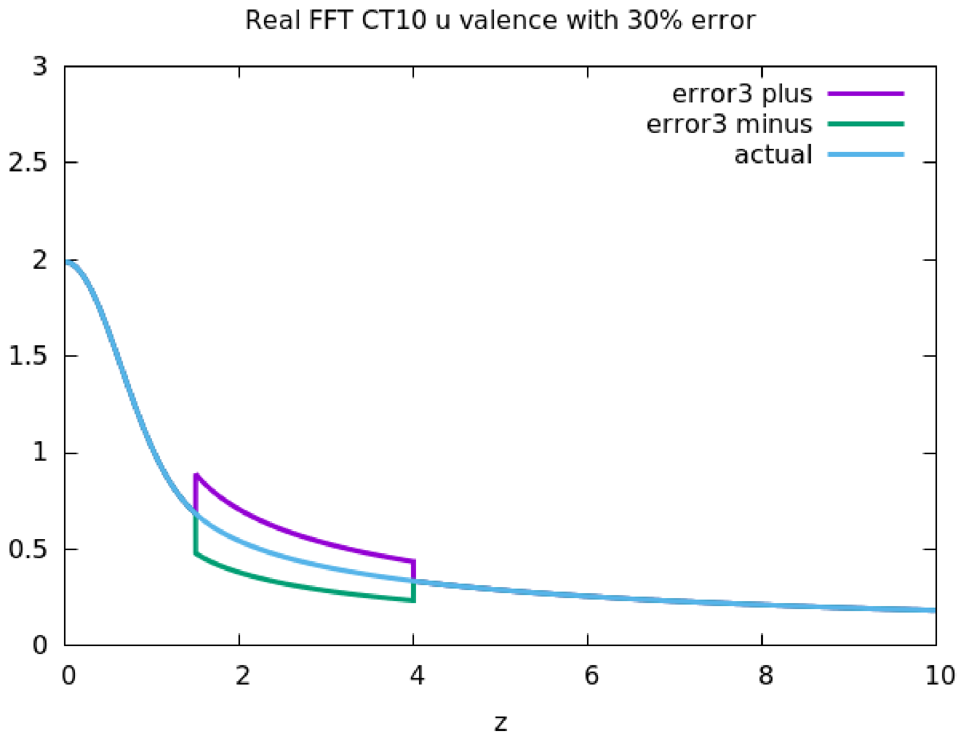
Moment u-d μ^2= 4 GeV^2	**Linear extrapolation**	**Chiral extrapolation**	**Phenomenology CT10**
M_1	1	1	1
M_2	0.262	0.18(3)	0.169
M_3	0.0843	0.05(2)	0.0536
M_4	0.0340	0.02(1)	0.0221

LHPC (Ph. Hägler et al.) Phys.Rev. D77 (2008)

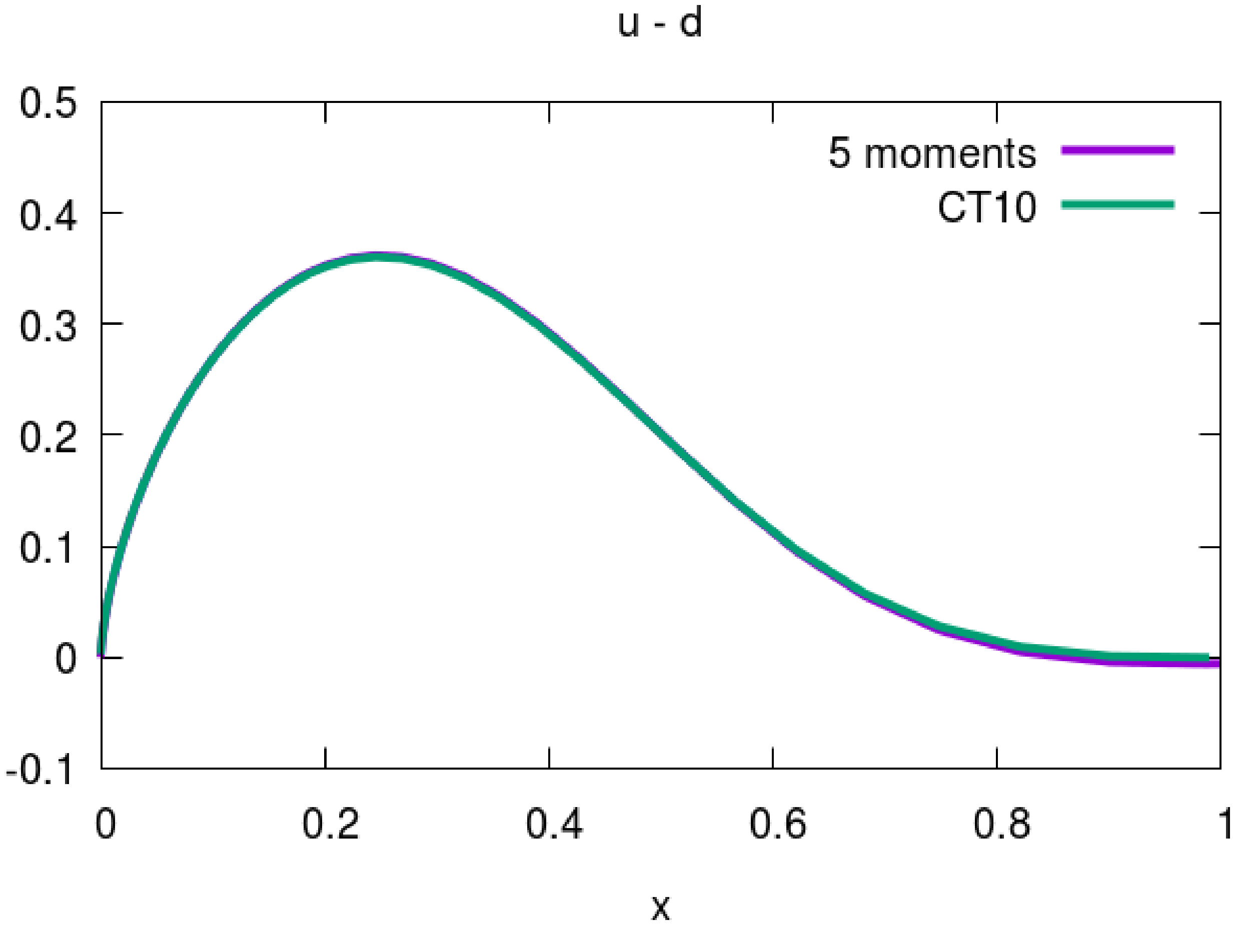
Moment u-d μ^2= 4 GeV^2	**m_π=352 MeV**	**Chiral extrapolation**	**Phenomenology CT10**
M_1	1	1	1
M_2	0.206(14)	0.157(10)	0.169
M_3	0.078(16)	/	0.0536
M_4	/	/	0.0221

Note

- The region between small z and large z is a source of uncertainity, more moments make a big difference.

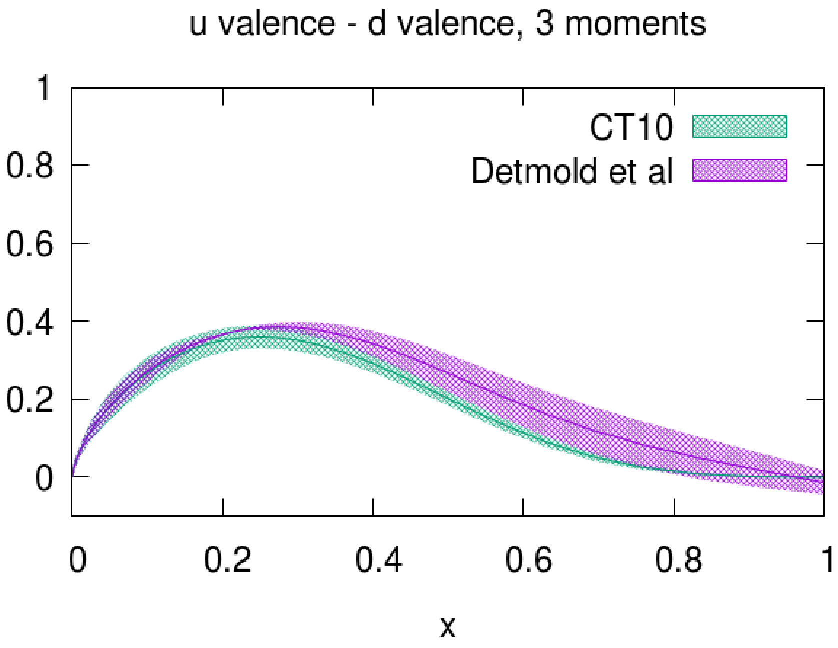


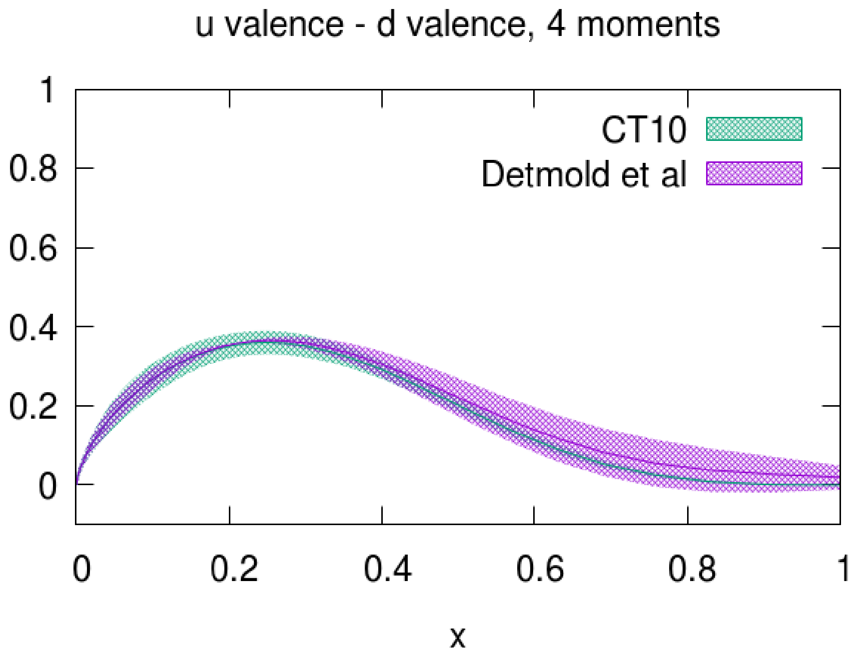
Reconstructing PDFs using its own moments



Reconstructing CT10 using 5 of its own moments.

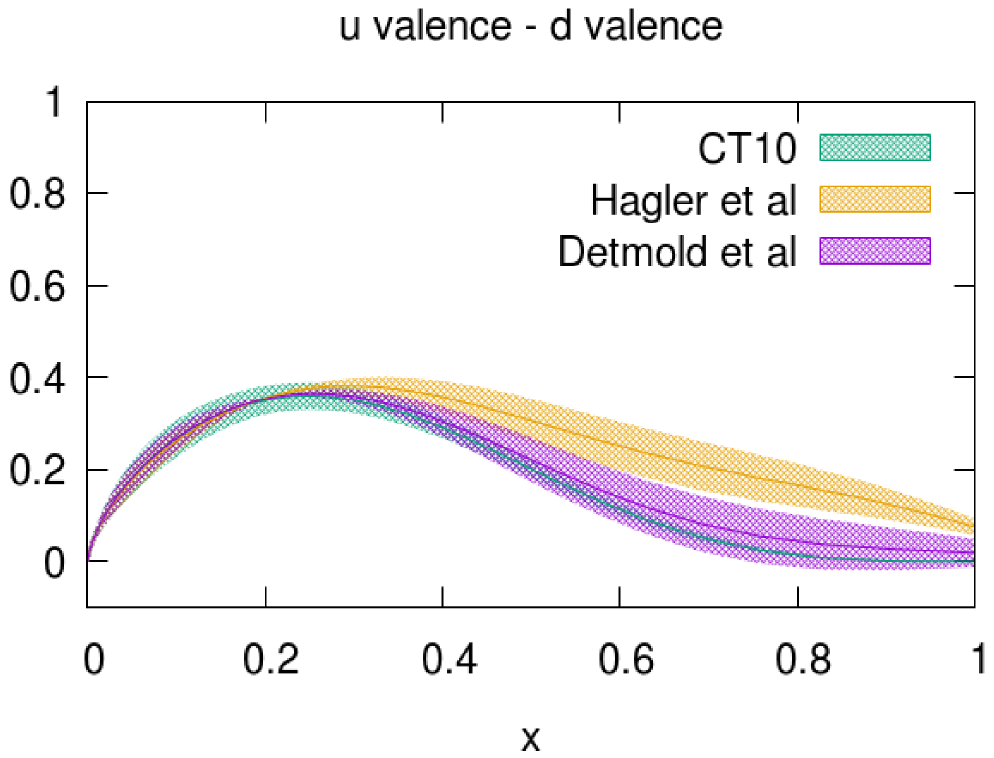
Reconstructing using Lattice QCD moments





More moments give higher precision !

Reconstructing using Lattice QCD moments

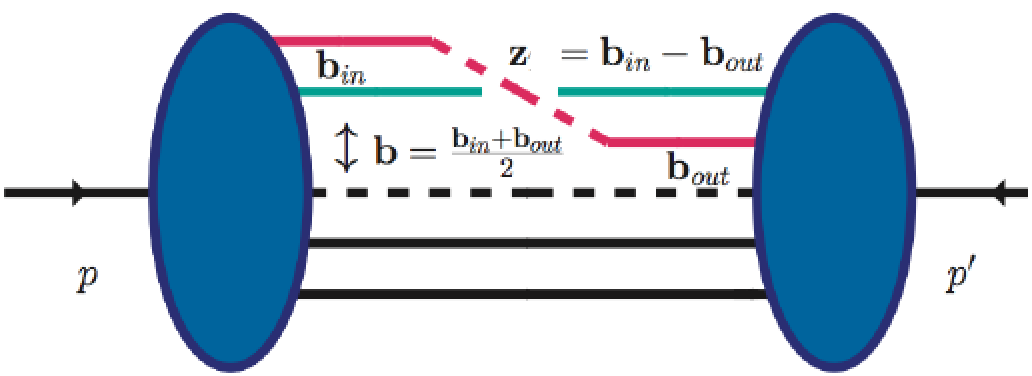


Comparison Detmold and Hägler

Generalized Parton Distribution Functions

$$W^{\gamma^+}_{\Lambda\Lambda'}(x,\xi,t) = \int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p',\Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p,\Lambda\rangle_{z^+=z_T=0}$$

$$\xi = \frac{\Delta^+}{P^+} \qquad t = \Delta^2 \qquad \Delta = p' - p$$



$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Ioffe Time and z_T

Impact Parameter

Polynomiality Property of GPDs

$$H^{(n)}(t) = \int_{-1}^{1} dx x^n H(x,\xi,t)$$

The x moments of the GPDs depend on ξ and the Generalized Form Factors.

$$H^{(n)}(t) = \sum_{i=0,even}^{n} (2\xi)^i A_{n+1,i}(t) + mod(n,2)(2\xi)^{n+1} C_{n+1}(t)$$

$$H^{(0)}(t) = A_{1,0}(t)$$

$$H^{(1)}(t) = A_{2,0}(t) + (2\xi)^2 C_2(t)$$

We can generate the x ξ plane if we know the GFFs.

Lattice Calculations of Generalized From Factors

Hagler et al, Phys. Rev. D (2008)

$-t[\mathrm{GeV}^2]$	A_{10}^{u-d}	B_{10}^{u-d}	A_{20}^{u-d}	B_{20}^{u-d}	C_{20}^{u-d}	A_{30}^{u-d}	B_{30}^{u-d}
0.000	1.000(4)	⋯	0.206(14)	⋯	⋯	0.078(16)	⋯
0.107	1.035(192)	3.055(997)	0.190(45)	0.032(253)	−0.083(435)	0.060(32)	−0.303(221)
0.124	0.850(19)	2.756(233)	0.197(11)	0.331(59)	−0.097(54)	0.070(10)	0.099(46)
0.125	0.829(84)	2.262(376)	0.192(20)	0.284(110)	−0.017(113)	0.072(12)	0.094(62)
0.219	0.868(190)	2.749(823)	0.167(42)	0.182(169)	0.026(130)	0.063(24)	−0.084(109)
0.244	0.767(28)	2.330(193)	0.177(11)	0.298(49)	−0.015(28)	0.063(10)	0.091(34)
0.245	0.733(79)	1.883(328)	0.179(20)	0.137(93)	−0.164(73)	0.068(12)	0.099(64)
0.254	0.678(39)	2.074(219)	0.179(14)	0.273(61)	0.004(43)	0.081(13)	0.067(42)
0.359	0.693(36)	1.910(170)	0.161(12)	0.251(50)	0.005(33)	0.061(12)	0.087(36)
0.379	0.584(40)	1.786(192)	0.158(14)	0.250(52)	0.010(39)	0.067(11)	0.040(36)
0.471	0.617(42)	1.625(185)	0.162(15)	0.140(57)	−0.009(35)	0.064(14)	0.019(50)
0.473	0.551(91)	1.910(448)	0.155(28)	0.116(107)	−0.030(56)	0.077(17)	0.065(67)
0.508	0.602(56)	1.533(172)	0.185(21)	0.227(64)	−0.024(46)	⋯	0.044(26)
0.578	0.561(45)	1.447(150)	0.155(15)	0.188(43)	−0.028(20)	0.068(13)	0.031(34)
0.615	0.478(49)	1.324(186)	0.135(17)	0.117(59)	−0.060(34)	0.068(13)	0.012(37)
0.632	0.512(45)	1.268(131)	0.173(18)	0.193(46)	−0.040(25)	0.076(17)	0.002(36)

Lattice Calculations of Coefficients

Hagler et al, Phys. Rev. D (2008)

$-t[\mathrm{GeV}^2]$	A_{10}^{u-d}	B_{10}^{u-d}	A_{20}^{u-d}	B_{20}^{u-d}	C_{20}^{u-d}	A_{30}^{u-d}	B_{30}^{u-d}
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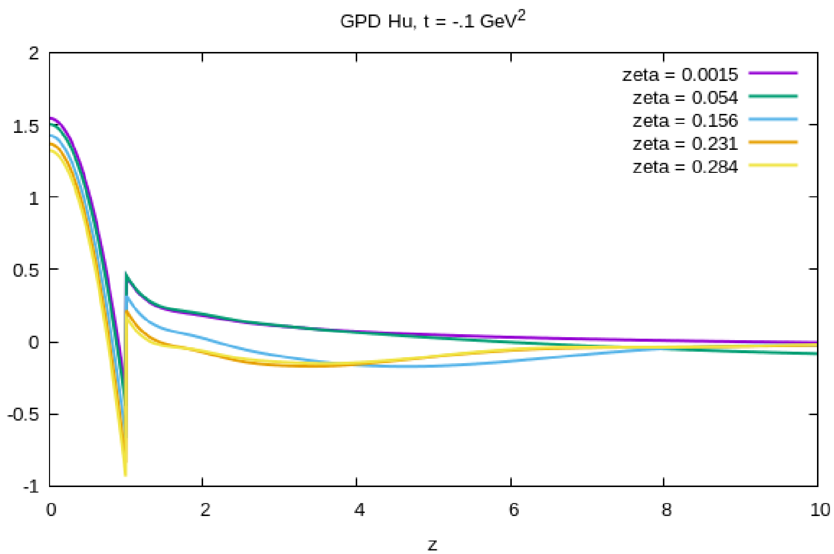
Can be used to calculate the x and ξ dependence of the GPD H. Also use A32.

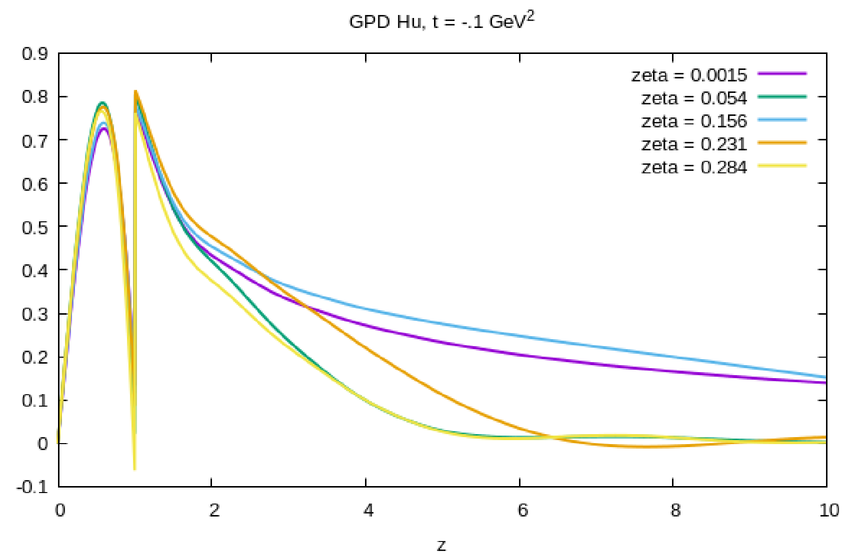
Inputs For Reconstructing GPDs

- Reconstruct the Ioffe time dependence using lattice calculations of GFFs and Regge behavior.

Goldstein, Gonzalez and Liuti Phys. Rev. D (2011)

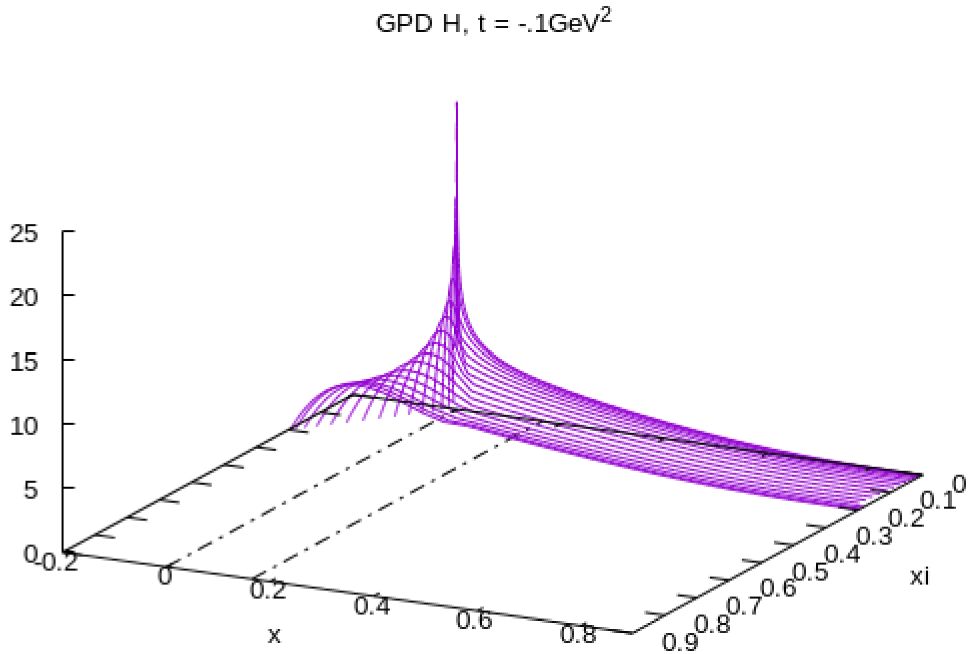


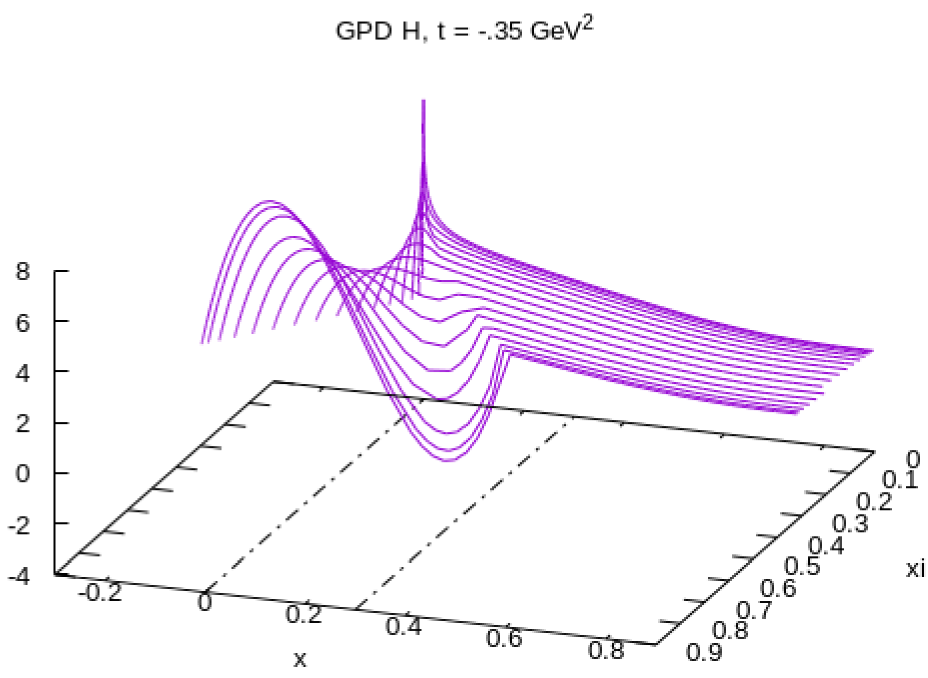
FFT cos



FFT sin

Reconstructed GPDs

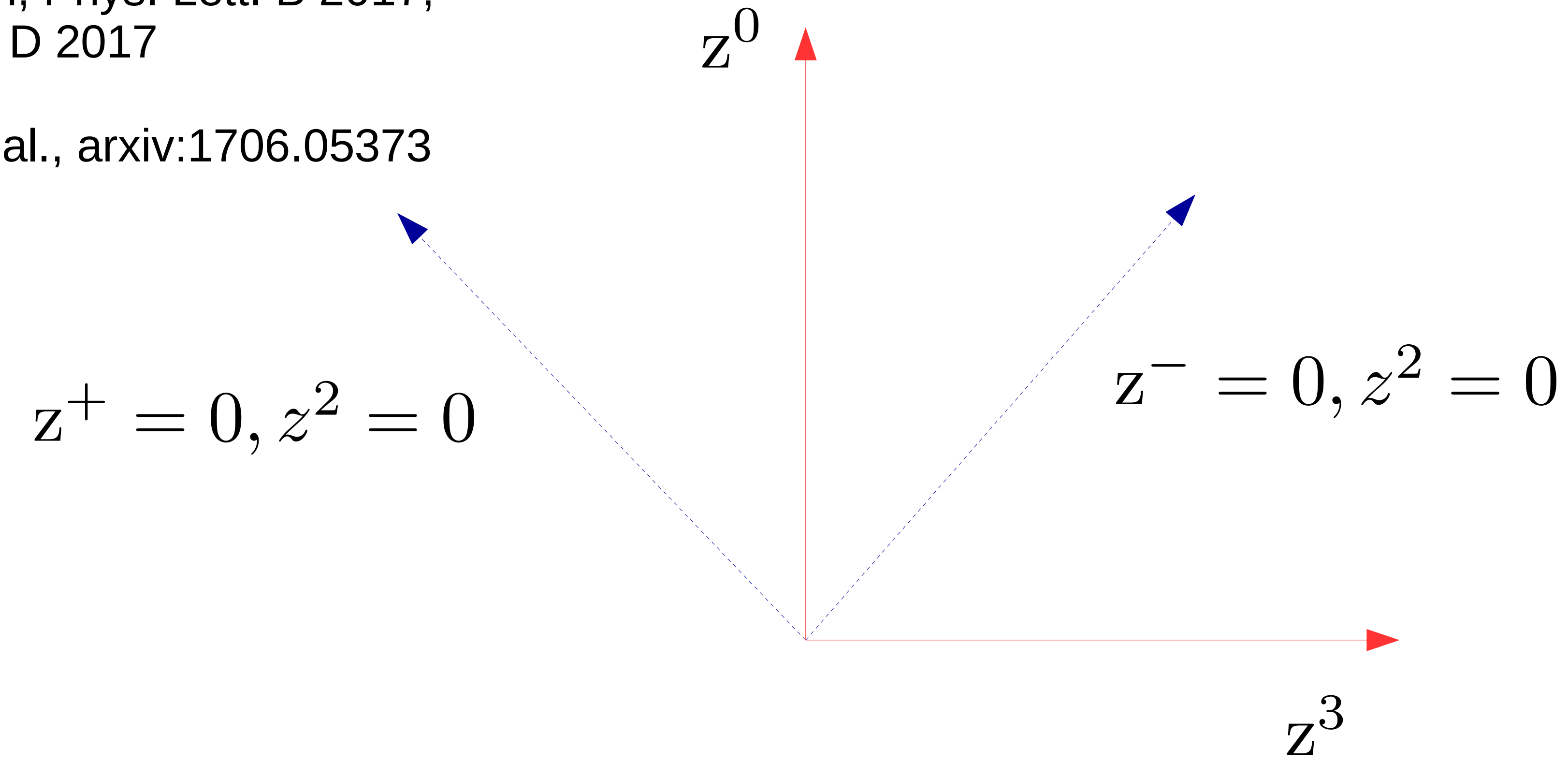




Pseudo PDFs : Going of the light cone

Radyushkin, Phys. Lett. B 2017,
Phys. Rev. D 2017

Orginos et al., arxiv:1706.05373



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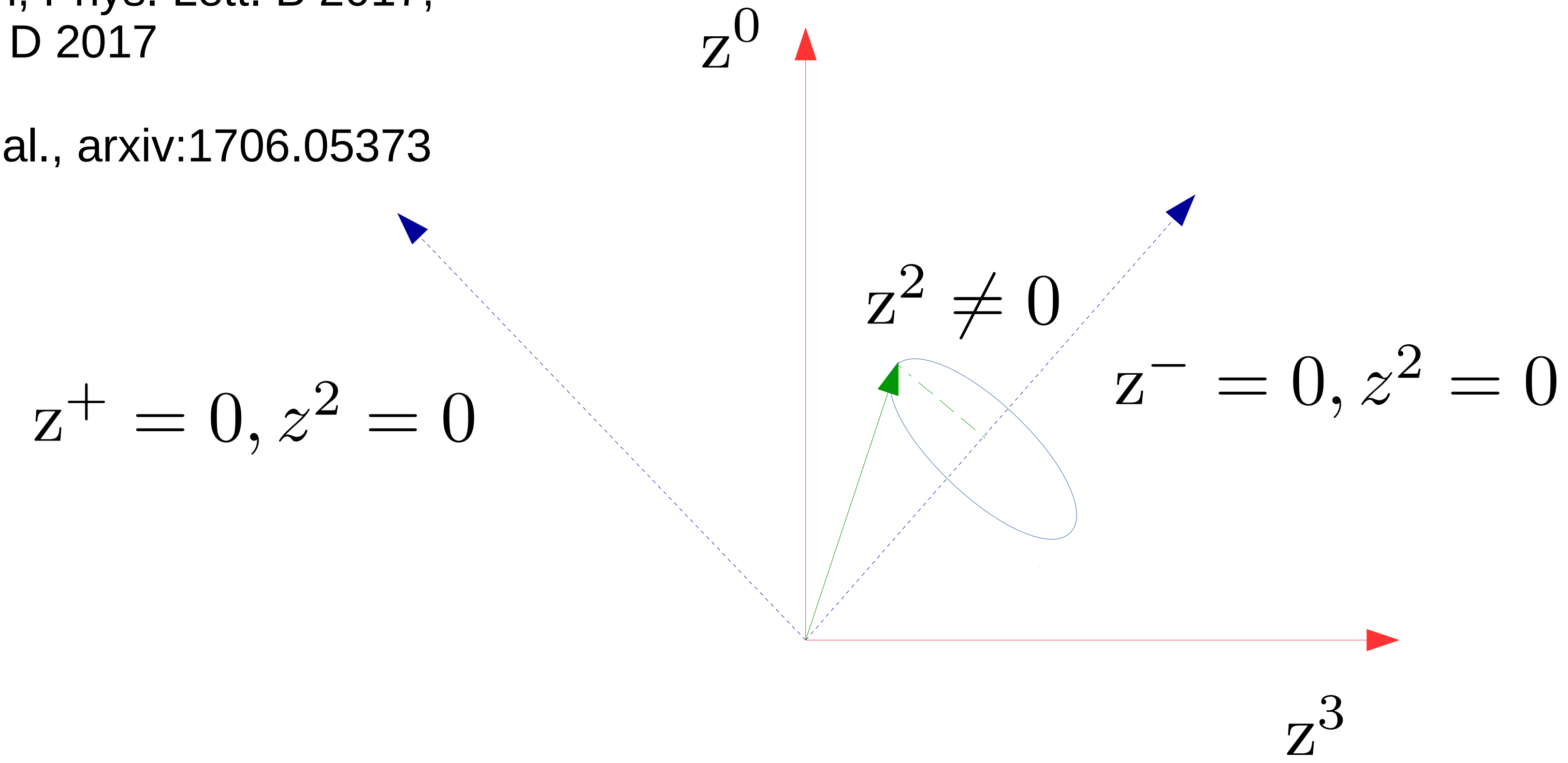

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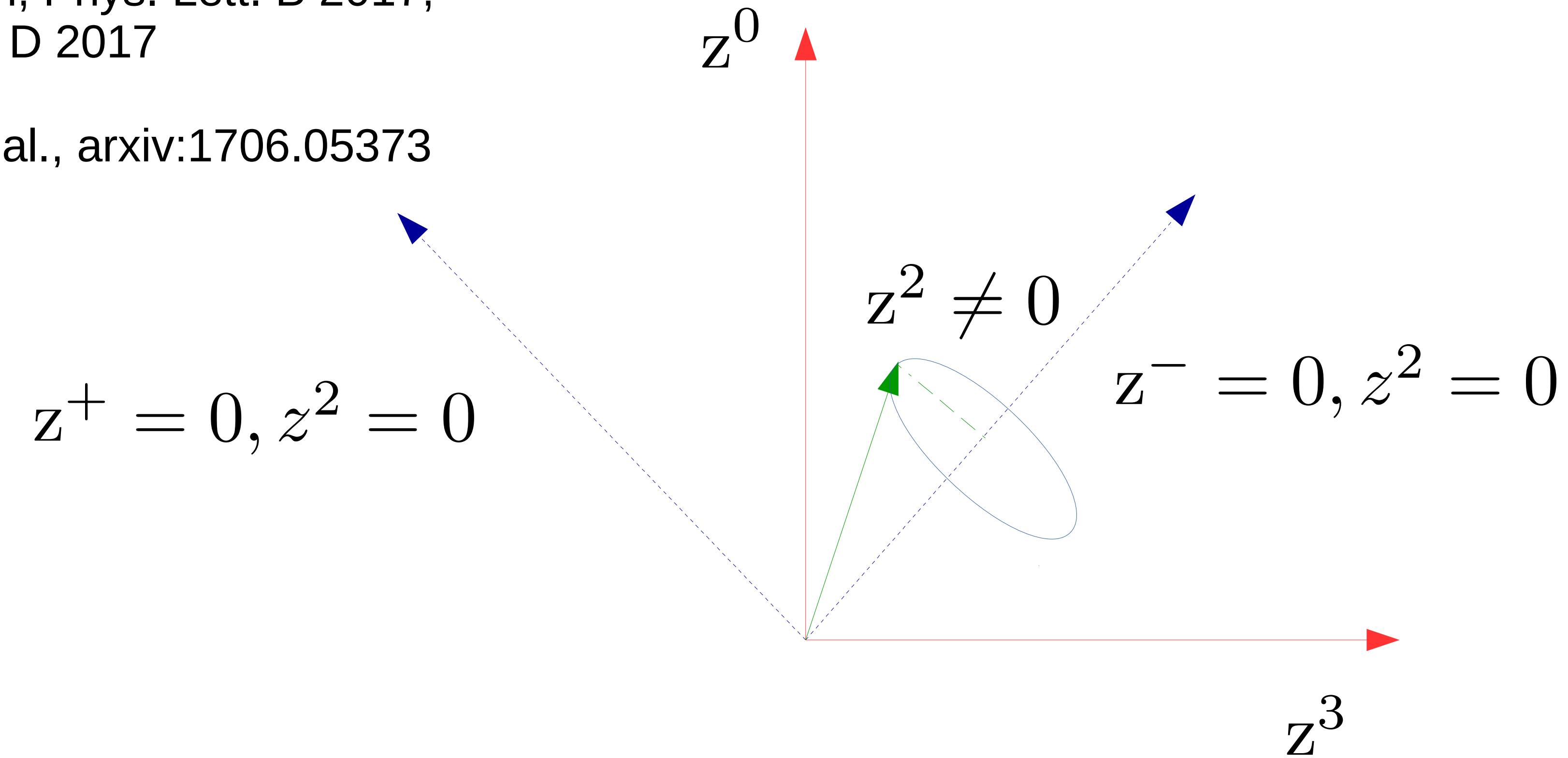


Pseudo PDFs go off the light cone, $z^- \to z^3 \to z_T$

$z_T \to 0$ PDF limit

Pseudo PDFs in z- space

$$\mathcal{M}(z,0) \to \mathcal{M}(z,z^2) \to \mathcal{M}(z,z_T^2)$$

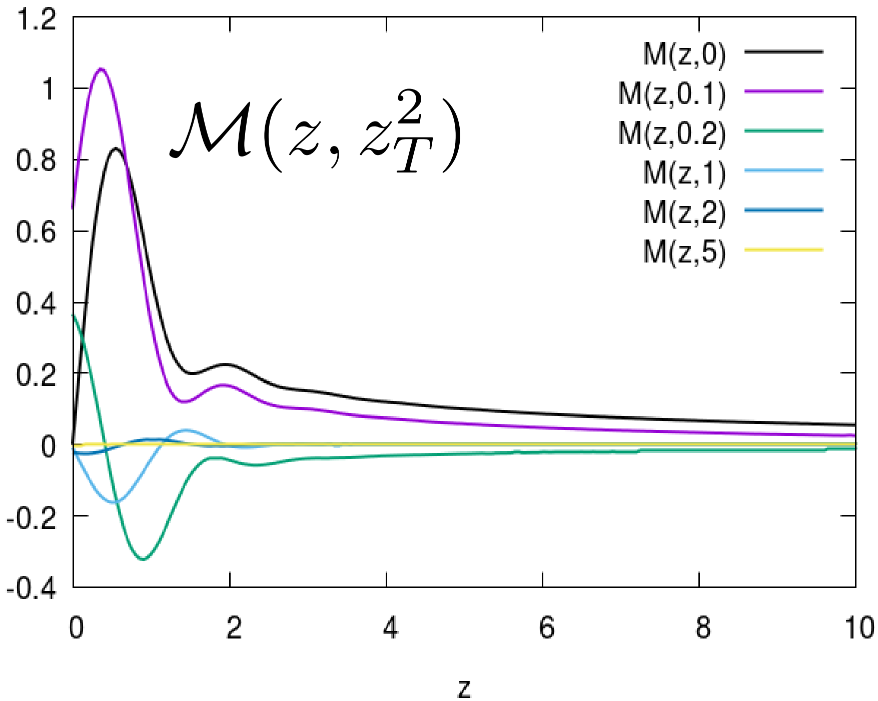
- z_T is conjugate to k_T
- If the k_T dependence factors out, taking a ratio will leave us with the Ioffe time distribution which is the Fourier transform of PDFs.

$$\mathcal{M}_R = \frac{\mathcal{M}(z,z^2)}{\mathcal{M}(0,z^2)}$$

$$\mathcal{F}(x,k_T) = f(x)K(k_T)$$

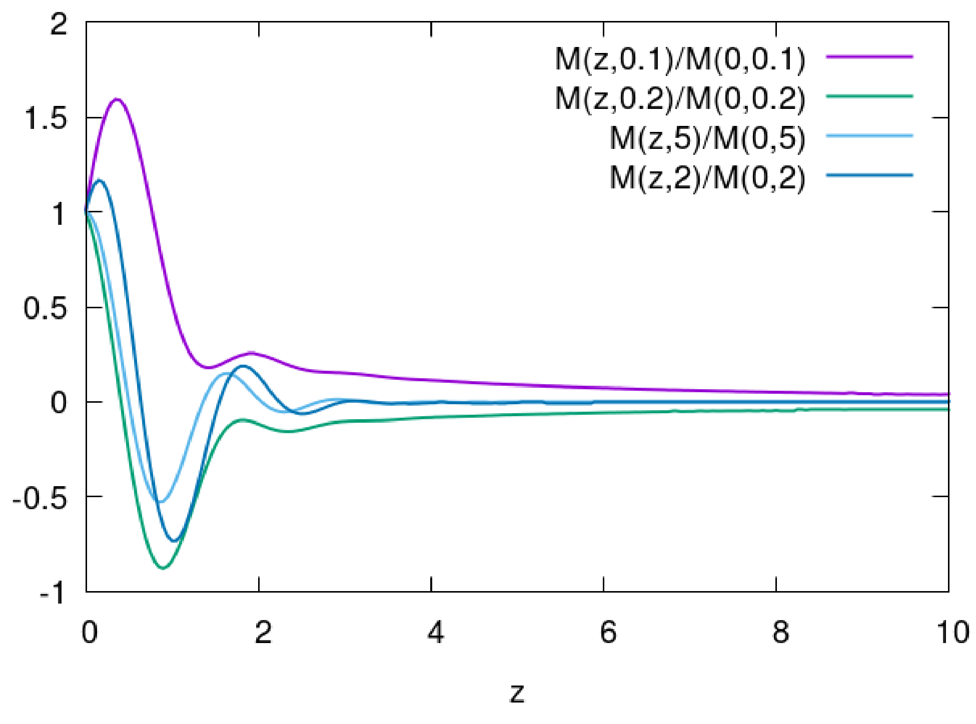
Does this actually happen ?

Checking in diquark model

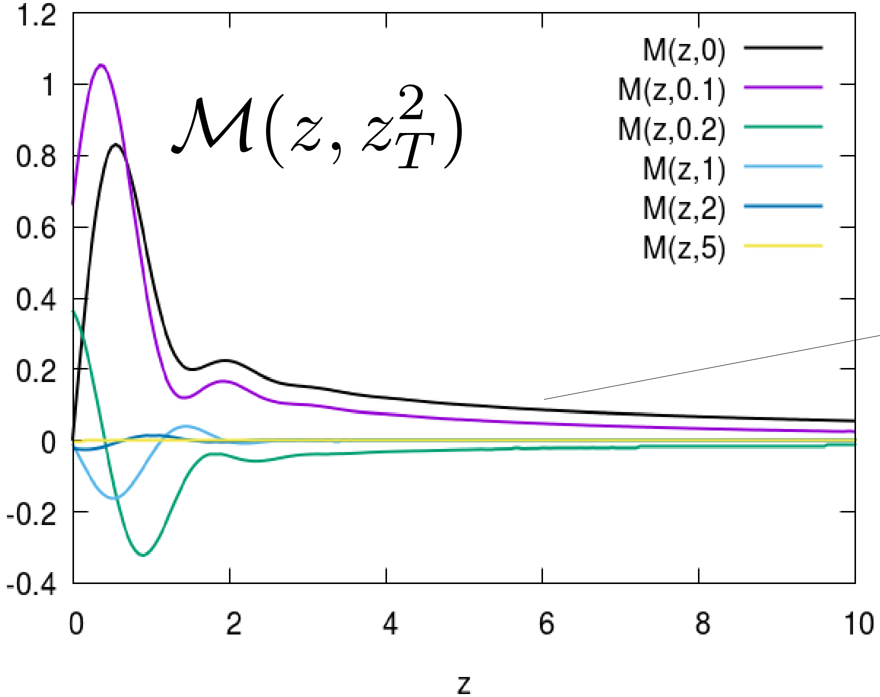


Dependence on zT

$$\mathcal{M}_R = \frac{\mathcal{M}(z, z^2)}{\mathcal{M}(0, z^2)}$$



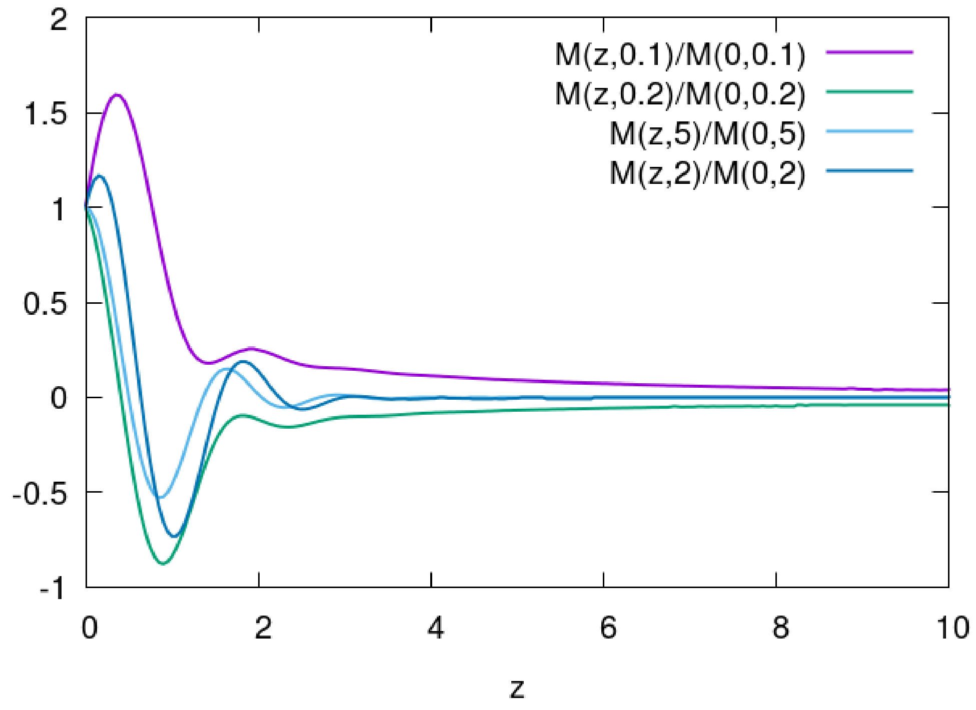
Checking in diquark model



Dependence on zT

$\mathcal{M}(z, 0)$

$$\mathcal{M}_R = \frac{\mathcal{M}(z, z^2)}{\mathcal{M}(0, z^2)}$$



Summary

- Lattice QCD moments provide crucial information about PDFs and GPDs, more moments will be very helpful.
- Going to z space allows us to map out which part of x dependence comes from inside the proton and which from the large z.
- What is the significance of some of the GFFs being so small ?
- Pseudo PDFs are a provide a new way of obtaining PDFs from the lattice, we study how well z_T dependence factors out.