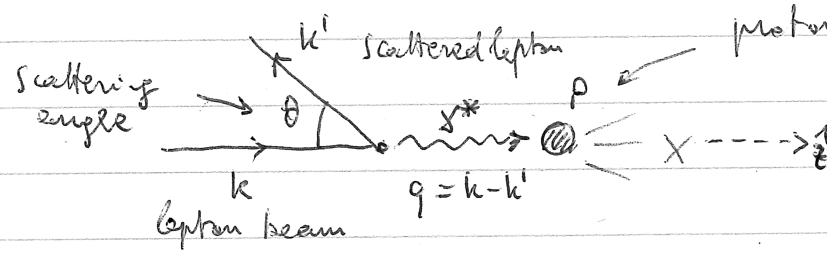
Reminder Investigate the internal structure of hadrons in the so-called "Deep-Inelastic" kinematical regime

Example: lepton - proton Scattering



proton initially at rest
$P = (M, 0, 0, 0)$

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if we assume $q = (\nu, \vec{q})$ and $\vec{q} = (0, 0, |\vec{q}|)$ then the proton final momentum becomes $P' = \left(\sqrt{M^2 + P_z'^2}, 0, 0, P_z'\right)$ with $P_z' = |\vec{q}|$

Deep - Inelastic (DIS) regime means $Q^2 = -q^2 \to \infty$

$x_B = \dfrac{Q^2}{2P \cdot q}$ finite

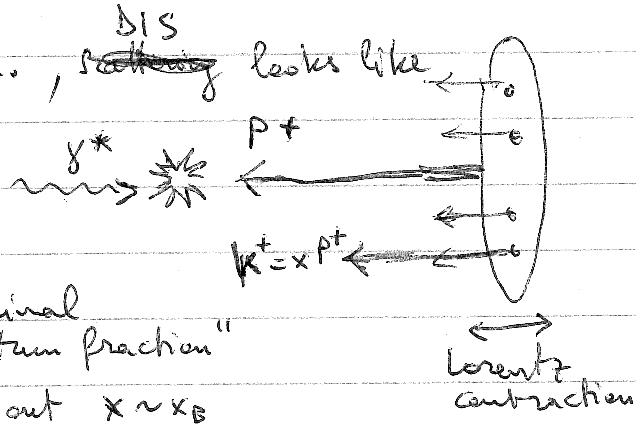
In this limit, $P_z' = |\vec{q}| >> M$ hence

$P' \sim (P_z', 0, 0, P_z')$ or using Light-Cone (LC) coordinates $P^\pm = \dfrac{P^0 \pm P_z}{\sqrt{2}}$

$= \left(P'^+ = \sqrt{2}\, P_z',\ P'^- = 0,\ \vec{0}_\perp\right)$ i.e. $P'^+ >> P'^-$ ← neglected

So, DIS means that the dominant component of proton momentum is P^+ (and the problem has 1 less dimension)

In c.m., ~~scattering~~ DIS looks like



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x called "longitudinal momentum fraction"

It turns out $x \sim x_B$

Basic idea of Feynman parton model.
Factorization between hard scattering in ✳ and Parton Distribution Function PDF(x) that describes the assembly of collinear partons before collision

General picture

If we release the approximation of all partons moving collinearly, we can define a new set of nonperturbative objects containing a wider information.

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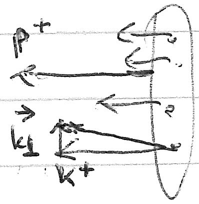
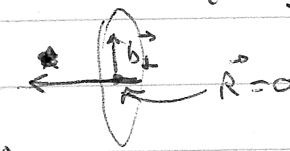


$\vec{b}_\perp$ distance in $\perp$ plane from ~~transverse~~ transverse center of momentum $\vec{R} = \sum_i x_i \vec{b}_{\perp i}$



$$
\begin{tikzcd}
 & PDF(x) & \\
TMD(x, \vec{k}_\perp) \arrow[ur, "\int d\vec{k}_\perp"] & & \rho(x, \vec{b}_\perp) \arrow[ul, "\int d\vec{b}_\perp"'] \\
 & W(x, \vec{k}_\perp, \vec{b}_\perp) \arrow[ul, "\int d\vec{b}_\perp"] \arrow[ur, "\int d\vec{k}_\perp"'] \arrow[d, leftrightarrow, "F.T."] & \\
 & GTMD(x, \vec{k}_\perp, \xi=0, \vec{\Delta}_\perp) &
\end{tikzcd}
$$

Transverse Momentum Distributions

impact parameter distribution

Wigner distributions

Generalized TMD

$\vec{b}_\perp \longrightarrow$ Fourier conjugated $\vec{\Delta}_\perp$

$\vec{\Delta}_\perp = \vec{P}'_\perp - \vec{P}_\perp$

ρ is Fourier ~~defin~~ transformed to Generalized Parton Distributions

$GPD(x, \xi=0, \vec{\Delta}_\perp)$

with $\xi = \frac{P^+ - P'^+}{P^+ + P'^+}$

Wigner distributions (or GTMD) contain the most complete information on parton dynamics. But we are not so sure about which process we can extract them from (talk from A. Metz next week).

Hence, it's important to explore 3-dimensional motion of partons independently through TMD's & GPD's (this is possible because $\vec{k}_\perp$ is not Fourier-conjugate of $\vec{b}_\perp$)

Here, we focus on TMD, i.e. on 3-dim ~~motion~~ structure of ~~partons~~ hadrons in momentum space.

Evidences beyond collinear picture

Evidences of ~~features~~ inadequacy of collinear picture from experiments:

1 – "Spin crisis"

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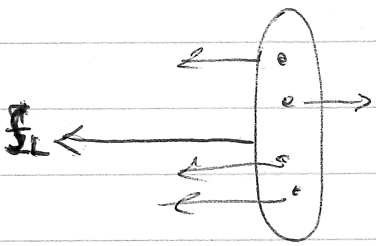
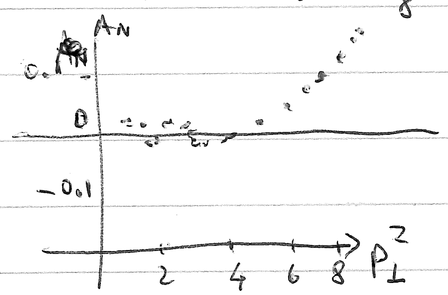


The helicity of the proton S_L (projection of its spin along direction of motion) is not saturated by the sum of the helicities of quarks Δq

The contribution of Δq depends on the energy scale of the considered process, but it can be $\sim 25\%$.
We do not have yet full control of the contribution of gluons Δg. But it is unlikely that is $\Delta g \sim 75\%$.
In which case, ~~we need~~ in order to saturate the sum rule we need an extra contribution from the Orbital Angular Momentum (OAM) of partons $\rightarrow$ indication that orbital (non collinear) motion matters

2 – elastic scattering $p^\uparrow p \to pp$ build Asymmetry $\dfrac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = A_N$



$P_\perp^2$ is the transv. momentum of scattered proton
Data from CERN (24 GeV beam)
AGS @ BNL (24 & 28 GeV)

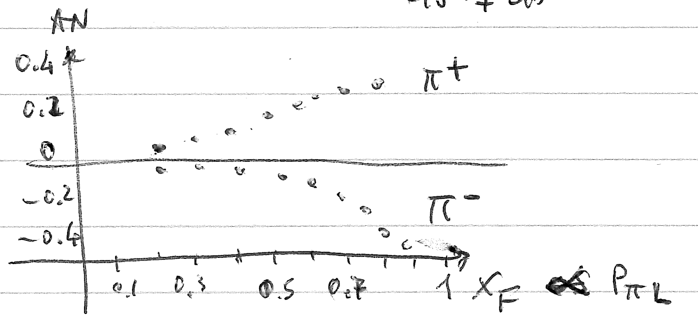
Review Krisch, E.P.J. A31 (07) 417

The perturbative QCD (pQCD) predicts $A_N = 0$.
$A_N \neq 0$ is an indication of correlation between S_T of p and $\vec{k}_\perp$ of partons. Persist (and large) at high energies.

4/16

3 - Semi-inclusive proton collisions $p^\uparrow p \to \pi X$

again $A_N = \dfrac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$



in pQCD A_N is suppressed because transv. polarization mixes different helicities.
But data show $|A_N| \lesssim 40\%$
Correlation between S_T of proton and $\vec{k}_\perp$ and flavor of partons?

E704 data at $\sqrt{s} = 20$ GeV
Persists up to $\sqrt{s} = 200$ GeV (Adams et al. (STAR), P.R.L. 92 (04) 171801)

— o —

Multi-scale factorization theorem

TMD's need two scales:

- hard scale Q^2 to "see" partons
- soft scale $\sim M_p$ to be sensitive to the motion of partons inside hadrons, i.e. to confinement

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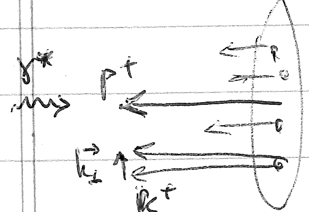

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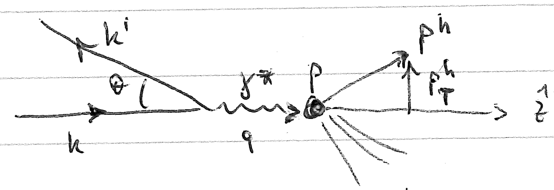
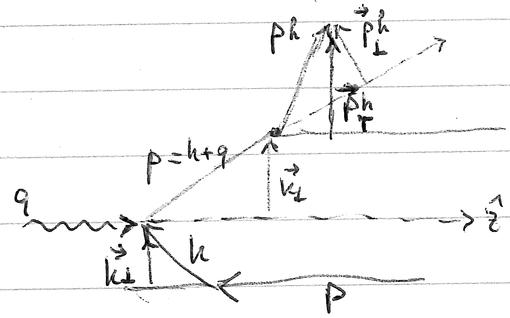


in order to be sensitive to $\vec{k}_\perp$ of partons, we need to measure a final-state hadron with $\vec{P}^h_T \neq 0$

$\Rightarrow$ Semi-Inclusive DIS (SIDIS)

In fact, the kinematics is



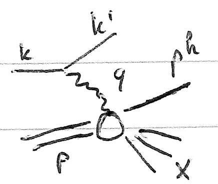
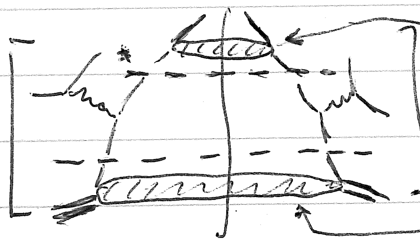
and we have $\vec{P}^h_T = z\vec{k}_\perp + \vec{P}^h_\perp + O\left(\frac{k_\perp^2}{Q^2}\right)$

measured — internal

The variable z is the fractional energy carried by the hadron.
It is $z = \frac{p^{h-}}{k^-}$, it is the analogue of x for the fragmentation process.

So, for $|P^h_T| \sim M \ll Q$ we can factorise the SIDIS cross section:

keep figure

$\sum_X \Big|$  $\Big|^2$ $\overset{\text{optical theorem + factoriz. theorem}}{=}$ $\mathrm{Im}\Big[$  $\Big] + O\left(\frac{1}{Q^2}\right)$

$$d\sigma = \int d\vec{k}_\perp \, d\vec{P}^h_\perp \, \delta\left(z\vec{k}_\perp + \vec{P}^h_\perp - \vec{P}^h_T\right) w(\vec{k}_\perp, \vec{P}^h_\perp) \cdot \mathrm{TMDPDF}(x, \vec{k}_\perp) \, d\hat{\sigma}_{hard} \, \mathrm{TMDFF}(z, \vec{P}^h_\perp) + O\left(\frac{P^{h\,2}_\perp}{Q^2}\right)$$

$$= \mathrm{TMDPDF} \otimes_w d\hat{\sigma}_{hard} \otimes_w \mathrm{TMDFF} + O\left(\frac{P^{h\,2}_\perp}{Q^2}\right)$$

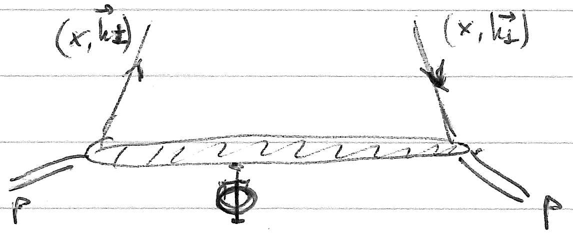
N.B.
- $w(\vec{k}_\perp, \vec{P}^h_\perp)$ can be simple as $=1$
- at $|P^h_\perp| \ll Q$ the $d\hat{\sigma}_{hard}$ can be factorized out of convolution since $d\hat{\sigma}(Q^2)$

6/16

The TMD "Zoo"

Factorization theorem for SIDIS allow us to isolate the non local correlator

$\Longrightarrow$ keep figure



OPE expansion $\rightarrow$ $\Phi(x, \vec{k}_\perp) = \sum_a \Phi^{[\Gamma_a]}(x, \vec{k}_\perp)$

$$= \frac{1}{2}\int \frac{dz^- d\vec{z}_\perp}{(2\pi)^3}\, e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \cdot \langle PS|\bar{\psi}(-\tfrac{z}{2})\,\Gamma_a\, W(-\tfrac{z}{2},\tfrac{z}{2})\,\psi(\tfrac{z}{2})|PS\rangle$$

where the gauge link $W = \exp\left[\int_{-\frac{z}{2}}^{\frac{z}{2}} dy^- A^+(y^-)\right]$ grants color-gauge invariance for the non local operator acting on $-\frac{z}{2}$ and $\frac{z}{2}$.
The W describes the color-FSI of active quark and spectators through the exchange of infinite soft gluons along the "$-$" direction

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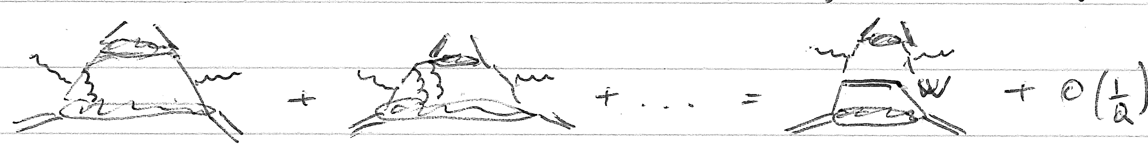

Γ_a is a Dirac operator that projects a certain polarization state of the parton: $\gamma^+ \rightarrow$ unpolarized, $\gamma^+\gamma_5$ longitudinally polarized, ...
Some of this projections appear unsuppressed (leading twist) some suppressed as $\frac{1}{P^+} \equiv \frac{1}{Q}$ (subleading twist), etc...
If we add the possibility for the proton to be polarized (S) we deduce the following table of TMD PDF's at leading twist

proton polarization \ quark polariz.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1\ h_{1T}^\perp$

7/16

We get an analogous table for TMDFF

hadron polarization \ quark polariz.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_1, H_{1T}^\perp$

Comments | ~~Comments~~

1- if we integrate $\int d\vec{k}_\perp$ only diagonal elements of table survive

TMDPDF $\rightarrow$ PDF
TMDFF $\rightarrow$ FF

2- LL box $\rightarrow g_{1L}$ is the TMDPDF involved in the ~~TMDPDF~~ calculation of parton contribution to the proton helicity ("spin sum rule")

3- The T column contains correlator involving transversely polarized partons. ~~For quarks (spin $=\frac{1}{2}$), the~~ It means that these matrix elements do not conserve helicity $\left(\langle\uparrow|\ldots|\uparrow\rangle = \langle +|\ldots|+\rangle + \langle -|\ldots|-\rangle + \langle +|\ldots|-\rangle + \langle -|\ldots|+\rangle \right)$
Since for quarks (spin $=\frac{1}{2}$) helicity = chirality, these TMDPDF's are named "chiral-odd". For pQCD, their ~~are suppressed~~ contribution is suppressed as m_q/Q. But they appear in spin asymmetries at leading twist, and the size of the asymmetry can be as large as 10%!

4- The UT and TU boxes survive only thanks to the ~~the~~ gauge link $\mathcal{W}$, which provides the necessary phase difference to generate interference of different channels (color FSI).

5- In particular, the UT box contains the so-called Sivers functions that is produced by the interference of LC wave functions with different OAM. The Sivers effect has been measured: it gives us the size of distortion of $\vec{k}_\perp$-distribution of an unpolarized quark because of the transv. polarization S_T of the proton.

6 – the UT box (Sivers effect) could be represented as the nonperturbative correlation $S_T \cdot (\vec{k}_\perp \times \vec{P})$.
Similarly, the TU box (Boer-Mulders effect) is $\vec{s}_T^{\,q} \cdot (\vec{k}_\perp \times \vec{P})$.
The empty boxes UL and LU are forbidden by Parity-invariance: $\vec{s}_L^{\,q} \cdot (\vec{k}_\perp \times \vec{P}) = 0$ and $\vec{S}_L \cdot (\vec{k}_\perp \times \vec{P}) = 0$

7 – data on azimuthal / spin asymmetries related to all boxes have been obtained in the last years (Hermes, Compass, JLab). Some clear evidences (Sivers effect, transversity h_1 ..), some of them are consistent with 0 ($h_{1T}^\perp$, $h_{1L}^\perp$).
In general, the data set is not yet so large to allow to perform precision physics (in the sense applied to PDF extraction).

8 – As for the TMDFF table, since most of the time the final state is made of mesons (π, K), we basically have info only on the U row (D_1, $H_1^\perp$). Surprisingly, we have more information on the $P_\perp^h$-dependence of $H_1^\perp$ (thanks to data from BaBar and BESIII), while data suitable for studying the $P_\perp^h$-dependence of D_1 have been announced by the Belle Collaboration.

Data for the UV box

Let's concentrate on the unpolarized case, the UV box $f_1(x, \vec{k}_\perp)$ and $D_1(z, \vec{k}^h_\perp)$

If we consider the SIDIS process, we face the problem of having 2 unknowns and 1 ~~process~~ source of information only. ~~But the TF~~ We have seen that the TMD FF can be individually extracted by looking at the semi-inclusive production of hadrons in e^+e^- annihilation. But these data have not been released yet.
The TMDPDF's can be isolated independently in the so-called Drell-Yan processes, where lepton pairs are produced by the decay of vector bosons (γ^*, Z) ~~prod~~ arising from proton-proton collisions.
Because of universality of TMDPDF's:

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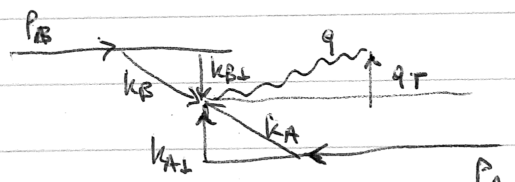
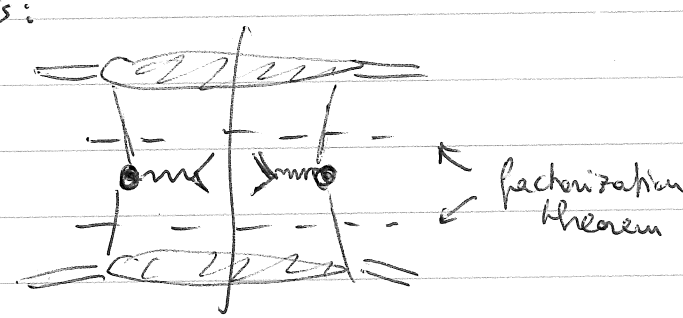
kinematics



$\delta(\vec{q_T} - \vec{k}_{A\perp} - \vec{k}_{B\perp})$

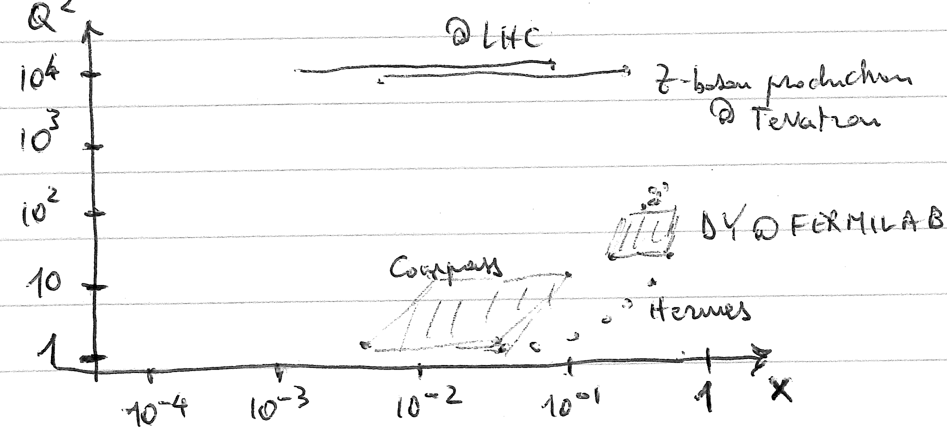


The hard and soft scales necessary for factorization theorem are:
- hard Q = invariant mass of vector boson
- soft q_T transv. momentum " "

Then $d\sigma = \text{TMDPDF}_A \otimes d\hat{\sigma}_{hard} \otimes \text{TMDPDF}_B$

Useful data for TMDPDF extraction

$\Longrightarrow$ keep figure



Several groups have analyzed these data using different data subsets and different levels of sophistication in :

- calculation of $d\hat{\sigma}_{hard}$ (Leading Order LO in α_s)
  Next-to- " " NLO
  ⋮ ⋮

- resummation of large logarithms deriving from calculation of radiation effects at low $\vec{k}_\perp$
  (Leading Log approx. LL
  Next-to- " " NLL)
  ⋮ ⋮

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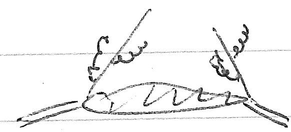


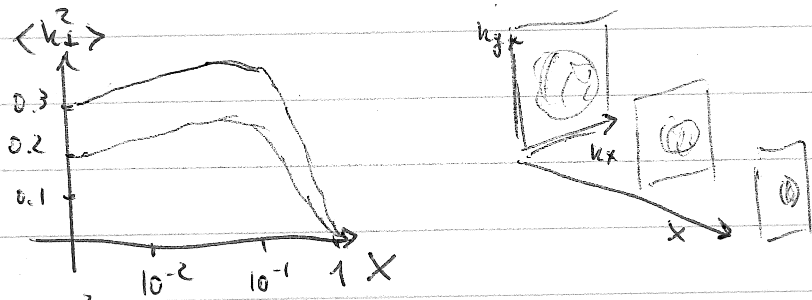
		Hermes	Compass	DY	Z	# points
Konychev & Nadolsky P.L. B633 (06)	NLL/NLO	×	×	V	V	98
Pavia group 2013 JHEP 1311 (13)	no evo	V	×	×	×	1538
Torino group 2014 JHEP 1404 (14)	no evo	V separate fit	V separate fit	×	×	576 6284
D'Alesio, Echevarria, Melis, Scimemi JHEP 1411 (14)	NNLL/NLO	×	×	V	V	223
Echevarria, Idilbi, Kang, Vitev PRD 89 (14)	NLL/LO	1 bin	1 bin	V	V	500
Scimemi & Vladimirov arXiv: 1706.01473	NNLL/NNLO	×	×	V	V	309

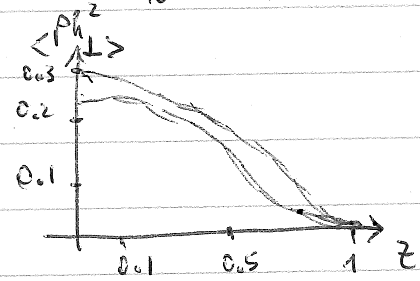
No group has ever considered all available data in one global fit until

Pavia group 2016 JHEP 1706 (17)	NLL/LO	V	V	V	V	8059

First attempt to extract TMD's from global fit → initiate era of precision physics for TMD's much as it happened with PDF
Work still to be improved in theoretical analysis and to include LHC data

Tomography Main outcome of this global fit is the first tomography of proton in momentum space:





Similar info for TMD FF

Open problems

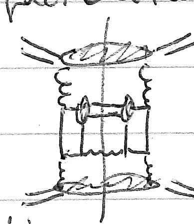
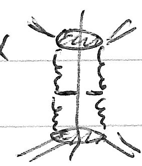
1- extractions of TMDPDF's and TMDFF's from only SIDIS data face the problem of anticorrelation: ~~since~~ $\vec{P}^h_T = z\vec{k}_\perp + \vec{P}^h_\perp$

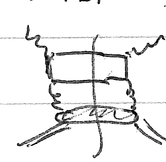
measured — unknowns

we have different anticorrelated pairs of $(\vec{k}_\perp, \vec{P}^h_\perp)$ that reproduce the same measured $\vec{P}^h_T$.

In global fit of Pavia 2016, adding the ~~SIDIS~~ DY data and Z-boson production data slightly reduce the degree of anticorrelation, but it's not yet satisfactory.
The ultimate solution would be the release of data on semi-inclusive hadron production from e^+e^- annihilations including P^h_T information bin by bin. Waiting for these data from Belle to have an independent constraint on TMDFF's.

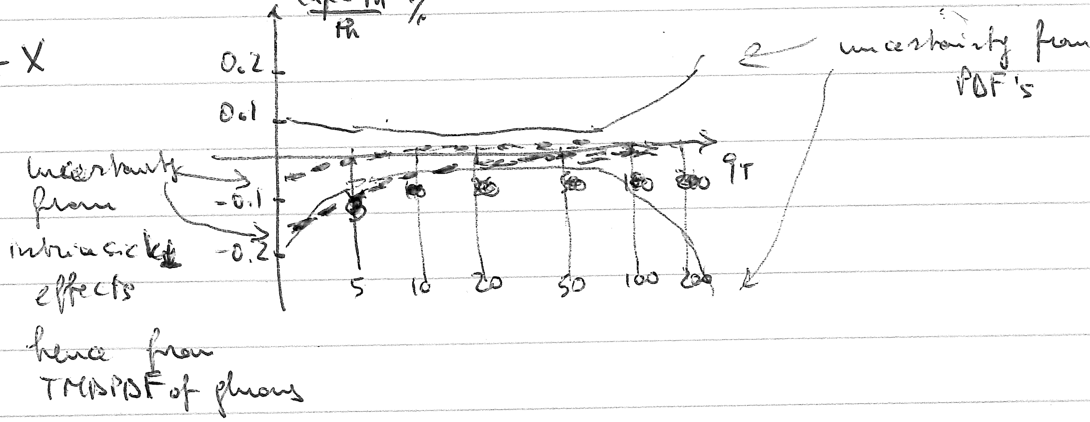
2- present data are not sensitive enough to allow for a statistically relevant analysis of flavor dependence of TMD's. The PAVIA 2016 global fit is performed independently of flavor. Work is in progress, because it is natural to expect, e.g., $\langle k_\perp^u\rangle \neq \langle k_\perp^d\rangle$

3- On equal footing, we don't have any quantitative info on gluon TMD's. This happens also because for hadronic collisions the factorization th. is available ~~only~~ for DY. For processes with 3 hadrons, there is no proof but also there is no counterexample disproving factorization.
Processes like $pp \to J/\psi + \gamma + X$  or $pp \to \eta_c + X$ 

could be useful for TMDPDF^g extraction! Hard scale is $M_{J/\psi}$ or M_{η_c}
Also $ep \to e' + \text{jet} + \text{jet} + X$  but again no fact. proof
(see talk by Pisano on Friday)

TMD PDF^g has important impact on LHC physics.

$pp \to H + X$



4- TMD evolution is different from PDF one!

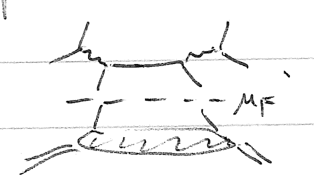
in ~~collinear~~ PDF depend formally on two scales:

- μ_R renormalization scale

  physics is independent of μ_R: $\frac{d\, d\sigma}{d\mu_R^2} = 0 \Rightarrow$ RGE

- μ_F factorization scale,

DIS



OPE $\to d\sigma = \sum_i \int_x^1 \frac{dy}{y}\, C_i\left(\frac{x}{y}, \frac{Q^2}{\mu_F^2}, \alpha_s(\mu_R^2)\right) \mathrm{PDF}(y, \mu_R, \mu_F)$

perturbative calculable

non perturbative

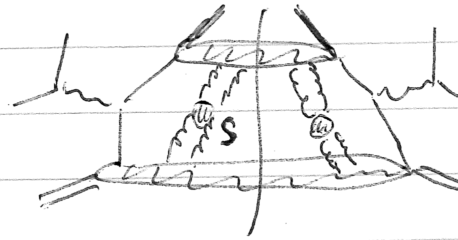
physics independent of μ_F

$\frac{d\, d\sigma}{d\mu_F^2} = 0 \Rightarrow$ DGLAP evolution equations: how the PDF scales with changing μ_F

N.B. usually, we take $\mu_F = \mu_R$

Now, TMD factorization in SIDIS process is more complicated, because there is an additional scale

rename figure



We need to include a so-called soft factor S encoding the emission of soft gluons with only transv. momentum $\sim P_T^h/Q$

Why? to kinematically balance the transverse momentum flowing through the partonic line up to the observed hadron P_T^h.

~~We have therefore~~

$$\cancel{d\sigma = \mathrm{TMDPDF} \otimes d\hat{\sigma}_{hard} \otimes S \otimes \mathrm{TMDFF} + \mathcal{O}(\ldots)}$$

The appearence of S seems to break the factorization. Moreover, S contains divergences related to gluons with infinite rapidity. A new scale ζ is introduced to regroup these divergent contributions partly in TMDPDF and partly in TMDFF restoring factorization:

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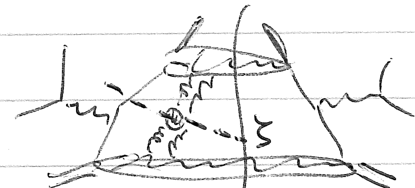


$$d\sigma = \mathrm{TMDPDF} \otimes S_i \otimes d\hat{\sigma}_{hard} \otimes S_f \otimes \mathrm{TMDFF} + \mathcal{O}\left(\frac{P_T^h}{Q}\right)$$

keep formula $\Longrightarrow$

Then, TMDs obey RGE $\quad \frac{d \log \mathrm{TMD}}{d \log \mu_R} = \gamma_D \quad$ anomalous dimension

and obey a new set of eq's $\quad \frac{d \log \mathrm{TMD}}{d \log \zeta} = -K \quad$ kernel process independent

TMD evolution is better analyzed in b_T space.
For $b_T \ll \frac{1}{\Lambda_{QCD}}$ (large k_T), the expression for TMDPDF can be worked out in perturbation theory using OPE. The final result is

keep formula $\Longrightarrow$

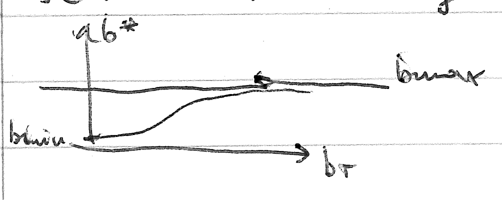
$$\mathrm{TMDPDF}(x, b_T; \mu_R, \mu_F, \zeta) = e^{-S(\mu_R, \mu_b)}\, e^{-K(b_T, \mu_b) \log \zeta/\mu_b^2} \cdot \left[C \otimes \mathrm{PDF}\right](x, b_T; \mu_b^2, \mu_F)$$

where S is the Sudakov factor and $\mu_b = \frac{2e^{-\gamma_E}}{b_T}$

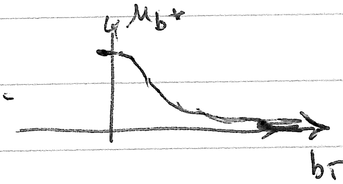
The Wilson coeffs C also obey $\frac{d \log C}{d \log \zeta} = -K$ ~~and~~ $\cancel{\frac{d \log C}{d \log \mu_R}}$

The PDF in the convolution depend on μ_F via DGLAP evolution.
The above formula is valid for $b_T \ll 1/\Lambda_{QCD}$. For larger b_T we enter the non-perturbative domain. ~~It's~~ The transition can be made "smooth" by defining a $b^*(b_T)$ such that $\lim_{b_T \to \infty} b^* = b_{max}$

$\lim_{b_T \to 0} b^* = b_{min}$



Then, the scale $\mu_{b^*} = \dfrac{2e^{-\gamma_E}}{b^*(b_T)}$ gets frozen ~~to~~ a minimum value at large b_T, where non perturbative evolution sets in. And saturates to a max value $< Q$ at small b_T



The kernel K is calculable at low b_T. ~~A general expression valid for all b_T looks like K~~ At large b_T, the non perturbative ~~evol~~ kernel is what is left over: $g_{NP}(b_T) = K(b_T, \mu_b) - K(b^*, \mu_{b^*})$

So, the final expression for TMDPDF is (taking $\mu_R = \mu_F = Q = \sqrt{s}$)

resume formula

$$\mathrm{TMDPDF}(x, b_T; Q) = e^{-S(Q,\mu_{b^*})}\, e^{-K(b^*,\mu_{b^*}) \log Q^2/\mu_{b^*}^2}\, e^{-g_{NP}(b_T) \log Q^2/\mu_{b^*}^2}$$

$$\times \left[C \otimes \mathrm{PDF} \right](x, b_T; \mu_b^2, Q) \times F_{NP}(x, b_T; Q_0)$$

where F_{NP} is the input function that describes the b_T distribution at large b_T and starting scale Q_0.

Various prescriptions are possible for non perturbative components of above formula:

- choice of $b^*(b_T)$ form
- choice of $g_{NP}(b_T)$ function
- choice of $F_{NP}(x, b_T)$ at Q_0

They all affect evolution, i.e. how $k_\perp$ distribution changes with scale. They need to be constrained by data.

5- The factorization formula for SIDIS $d\sigma$

resume formula

$$d\sigma = \mathrm{TMDPDF} \otimes S_i \otimes d\sigma_{hard} \otimes S_f \otimes \mathrm{TMDFF} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

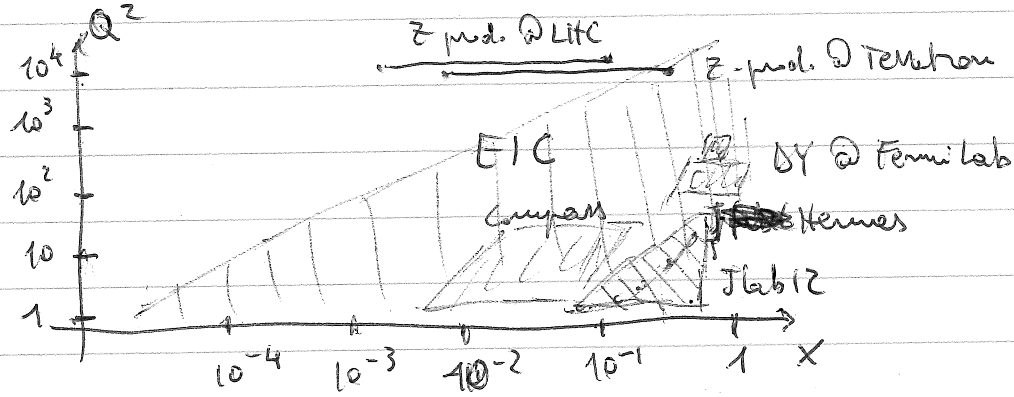
works for $P_T^h/Q \ll 1$. When P_T^h becomes large, it must match the result of fixed-order calculations in pQCD. This is still an open problem

16 / 16

Future All above problems can receive valid input from new, abundant, and precise data.

JLab 12 & EIC are suited for that

resume figure



EIC filling gap in Q^2 between SIDIS and DY data

JLab12 ~~exp~~ looking at Q^2 evolution at high x