

**INT-17-3 week 5**  
**Hadron Imaging at JLab and at future EIC**  
Sept. 25-29, 2017

**Transversity and tensor charge**

**Marco Radici**

INFN - Pavia

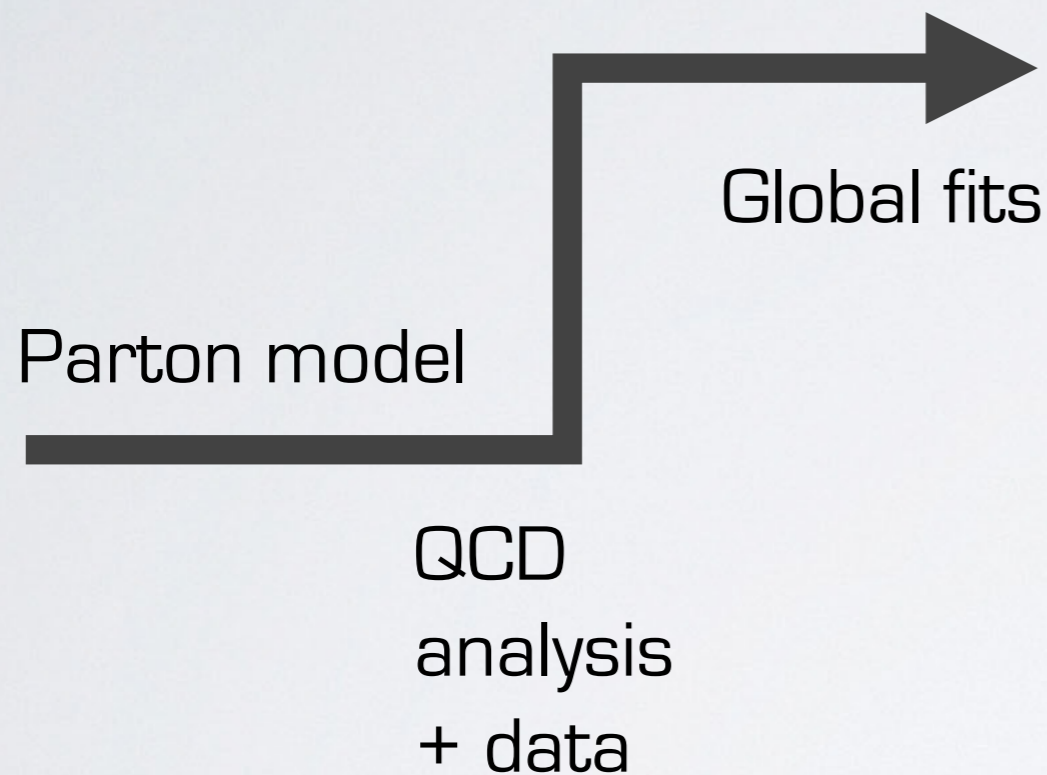
in collaboration with  
A. Bacchetta (Univ. Pavia)



# a phase transition in 3D studies

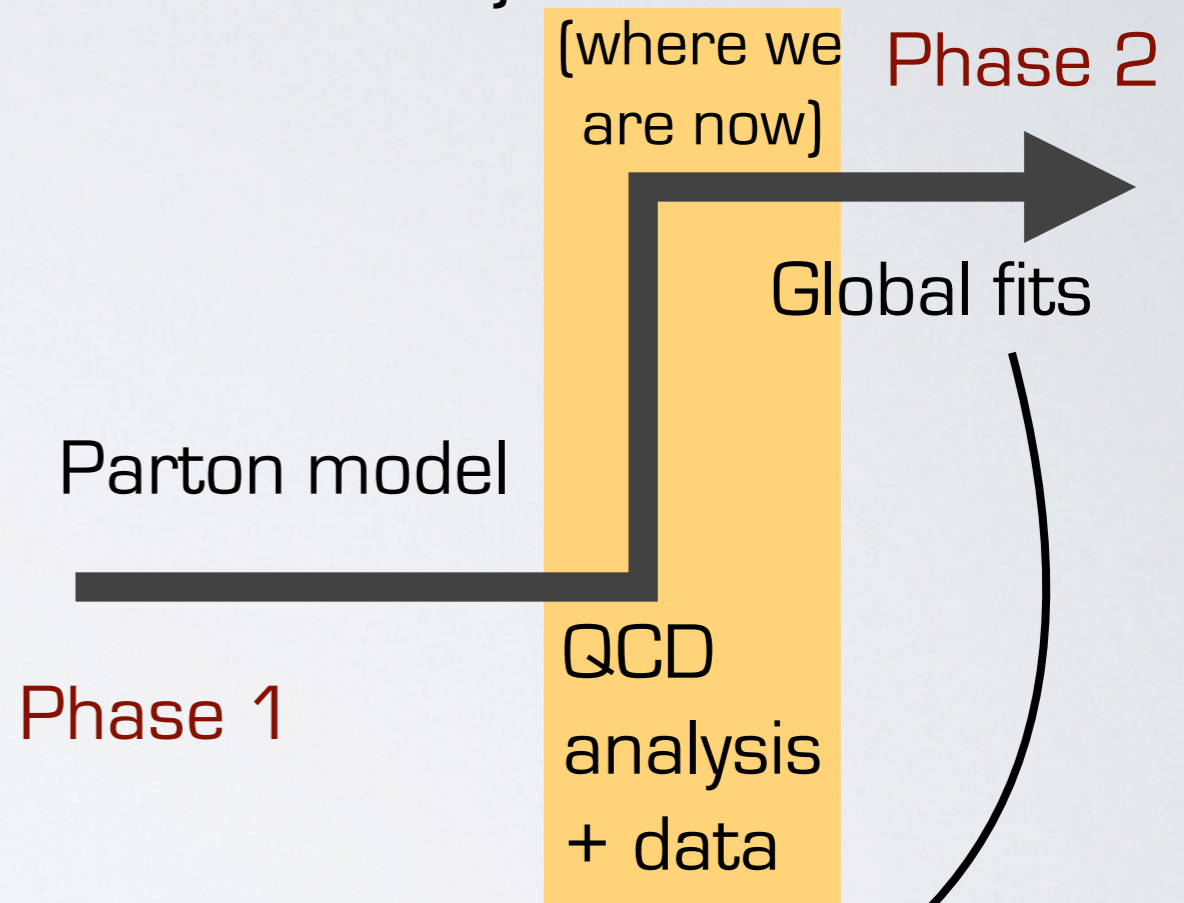
1D

(standard parton distribution functions - PDFs)



3D

(transverse momentum distributions - TMDs)



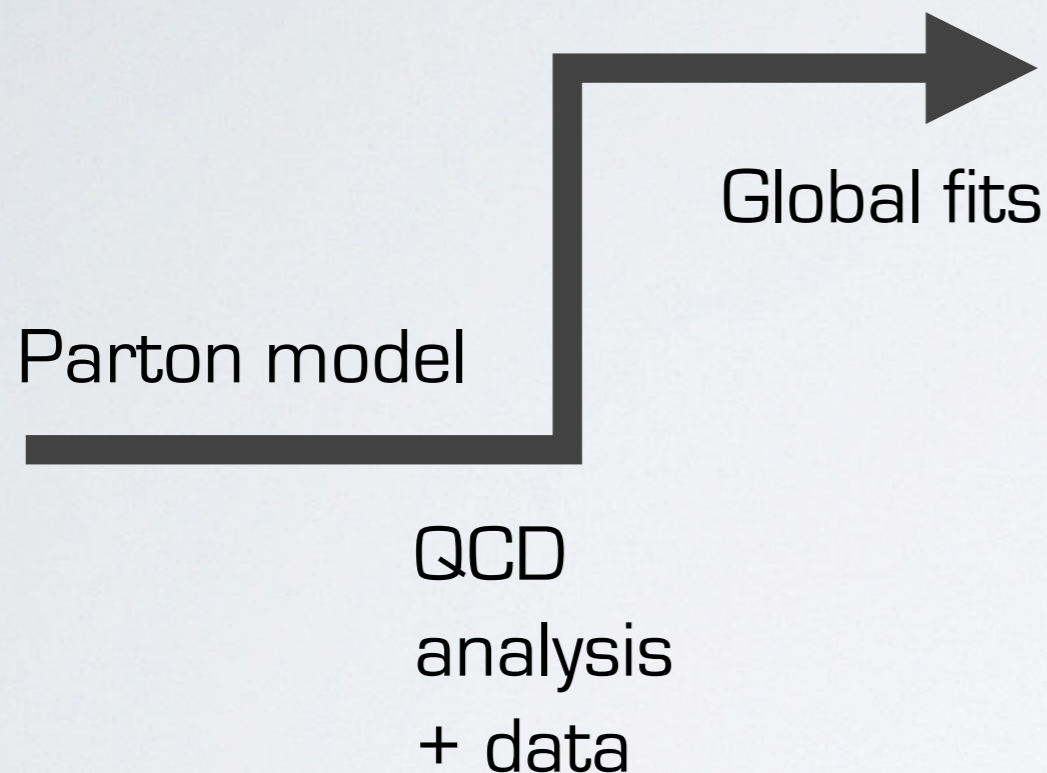
first global fit of  $f_1(x, \mathbf{k}_\perp)$

*Bacchetta et al.,  
JHEP 1706 (17) 081*

# a phase transition in 3D studies

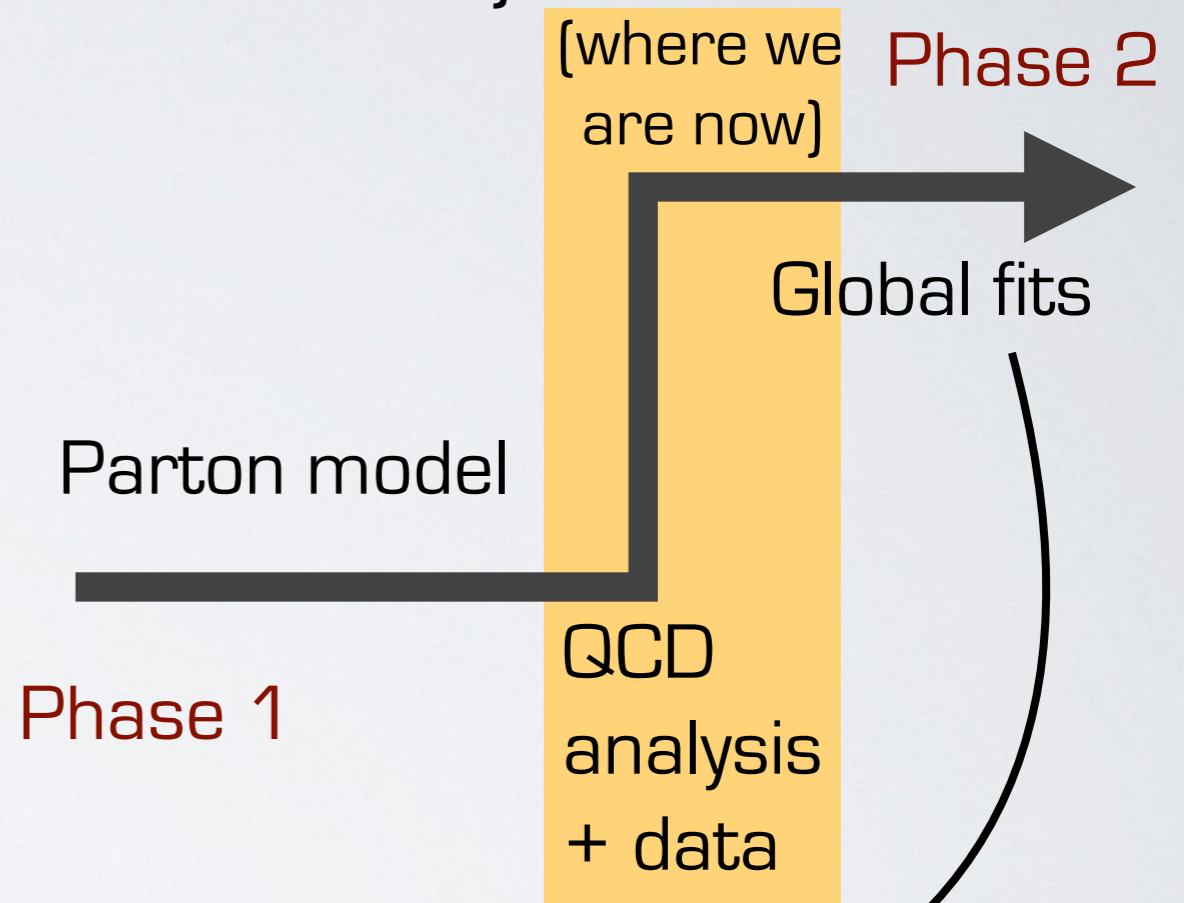
1D

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first global fit of  $f_1(x, \mathbf{k}_\perp)$

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JHEP 1706 (17) 081*

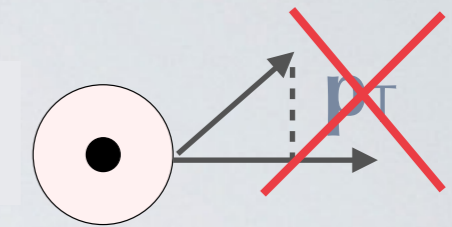
but there is another missing global fit for leading-order PDFs:  
the transversity!

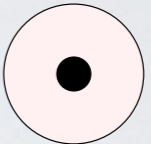
# leading-twist PDF map

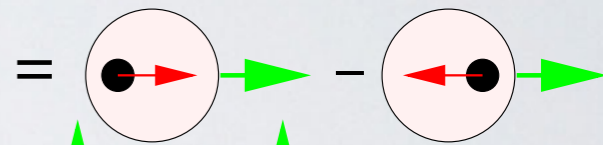
quark polarization

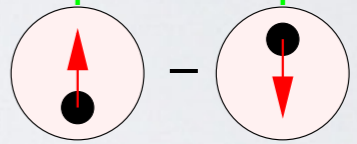
	U	L	T
U	<b><math>f_1</math></b>		$h_1^\perp$
L		<b><math>g_{1L}</math></b>	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	<b><math>h_1</math></b> $h_{1T}^\perp$

nucleon polarization



$f_1 =$  

$g_1 =$  

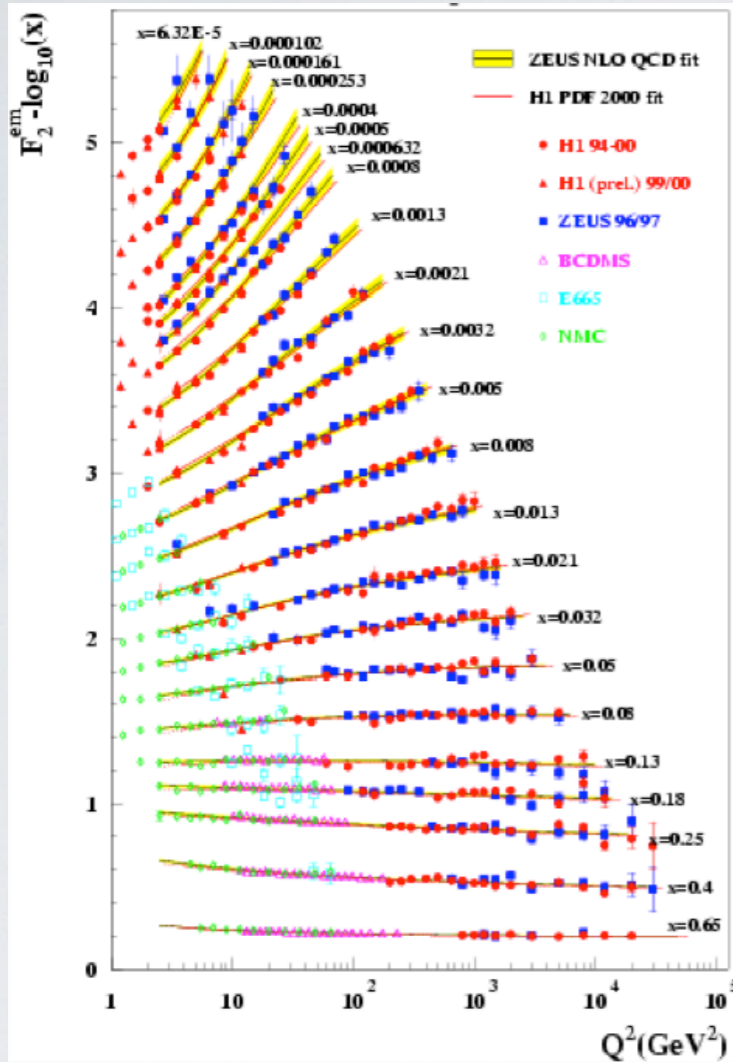
$h_1 =$  

flips helicity (chiral-odd)  
→ suppressed in inclusive DIS

**all three PDFs** needed for a complete description of proton (spin) structure at leading order

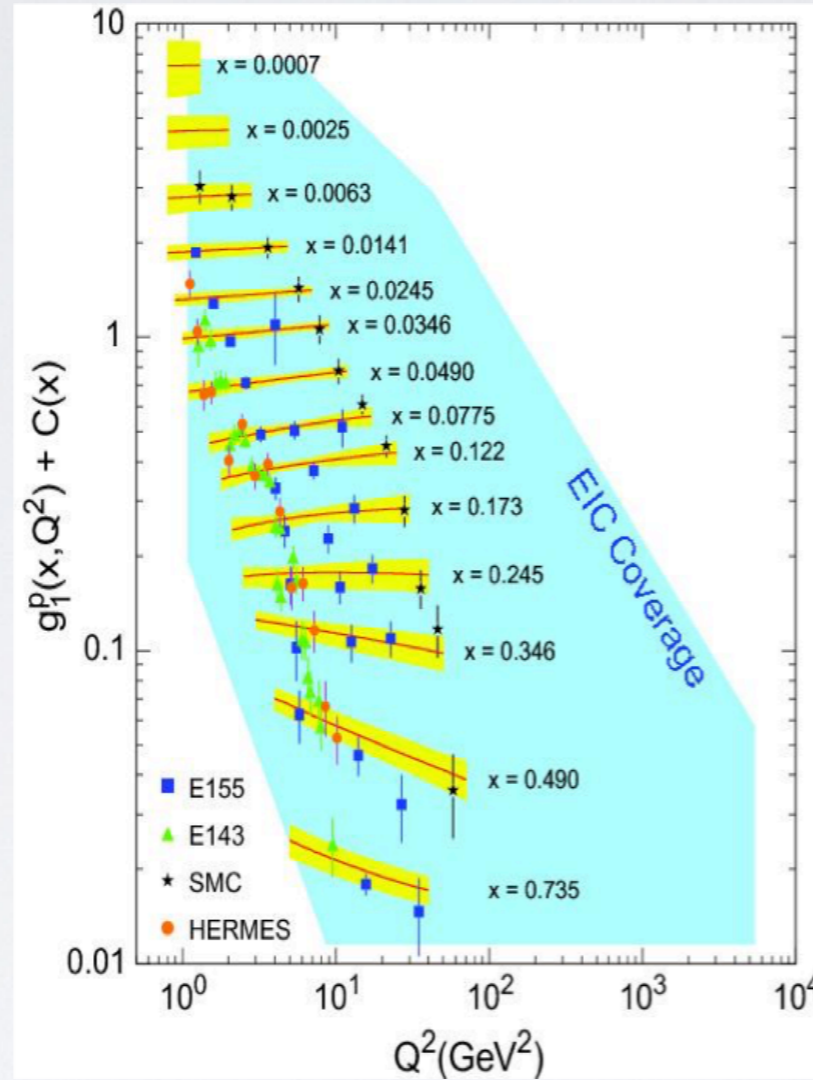
# Transversity poorly known

World data for  $F_2^p$



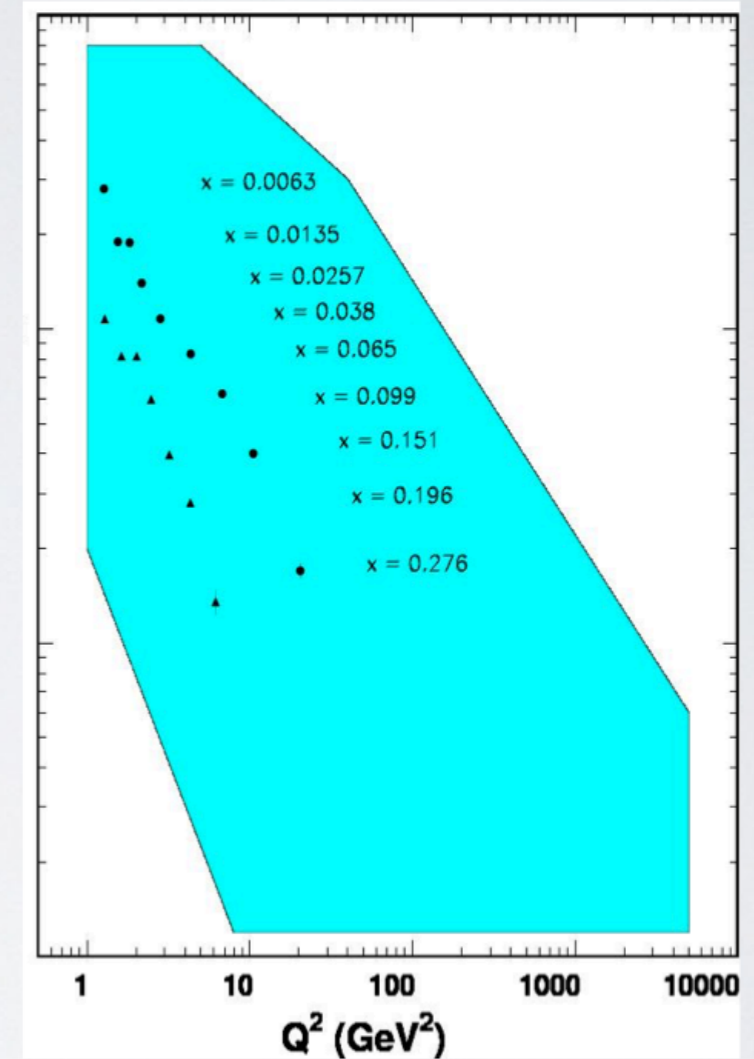
$f_1$  from fits of  
thousands data

World data for  $g_1^p$



$g_1$  from fits of  
hundreds data

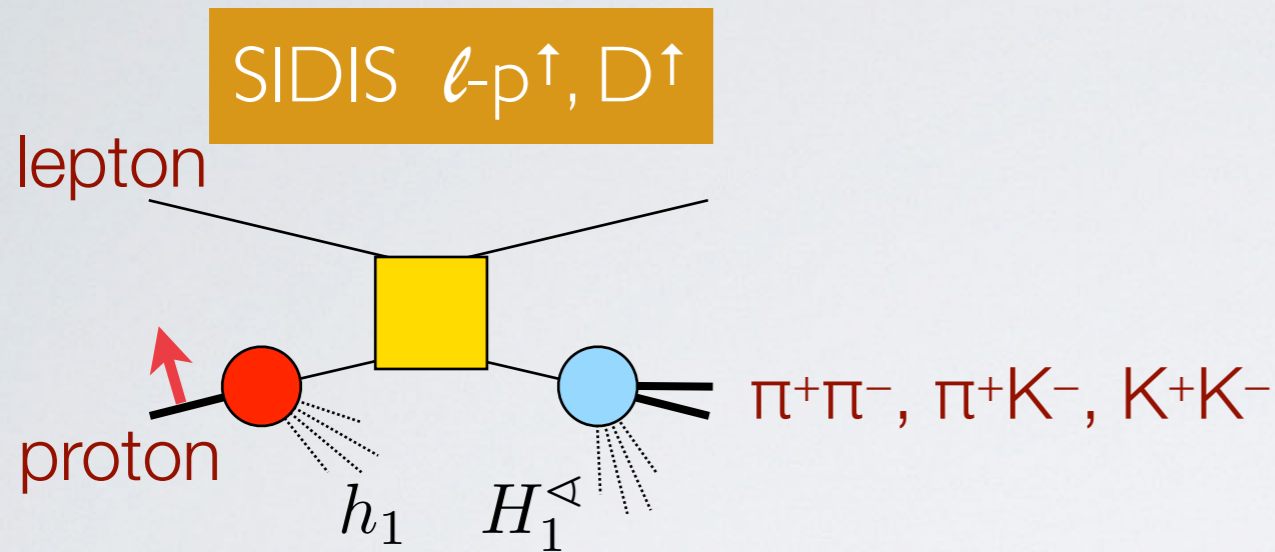
World data for  $h_1$



$h_1$  from fits of  
tens data

*slide from H. Montgomery,  
QCD Evolution 2016*

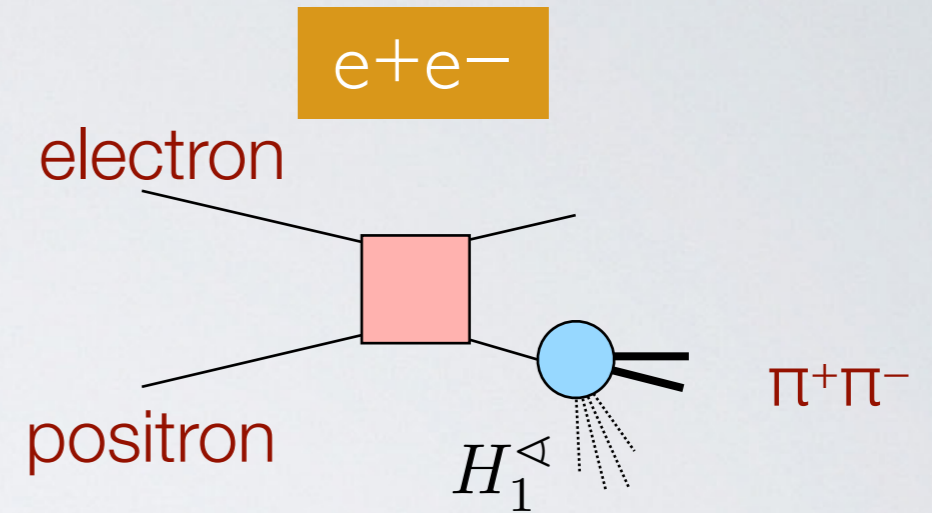
# extraction from **2-hadron**-inclusive data



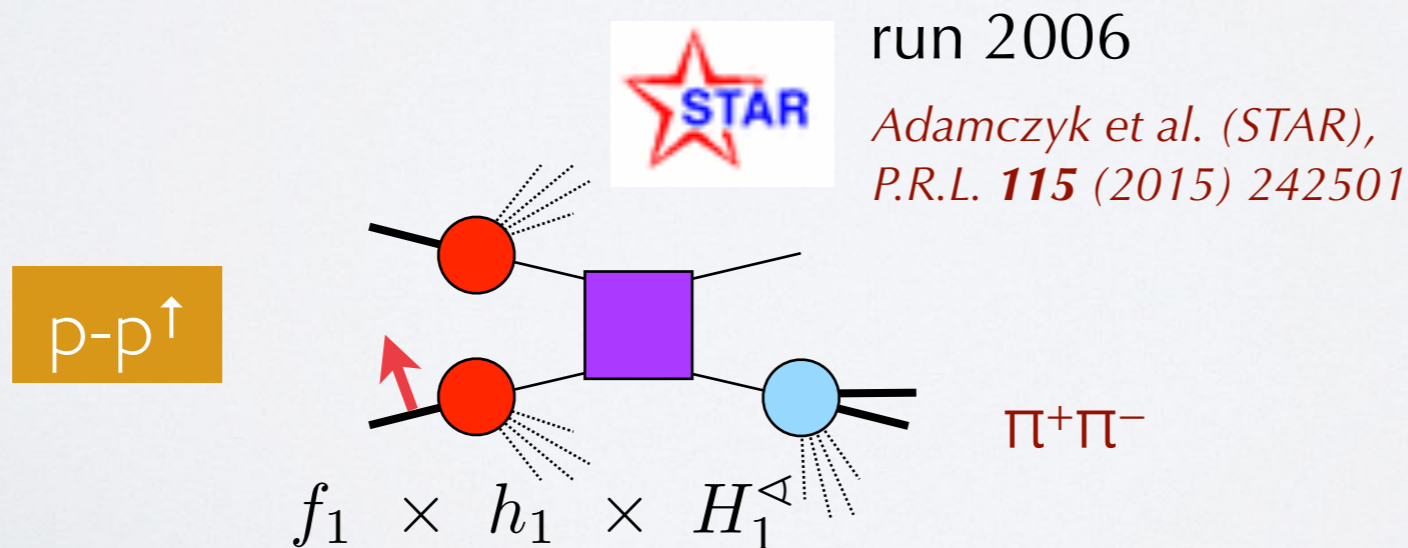
Airapetian et al.,  
JHEP **0806** (08) 017



Adolph et al., P.L. **B713** (12)  
Braun et al., E.P.J. Web Conf. **85** (15) 02018

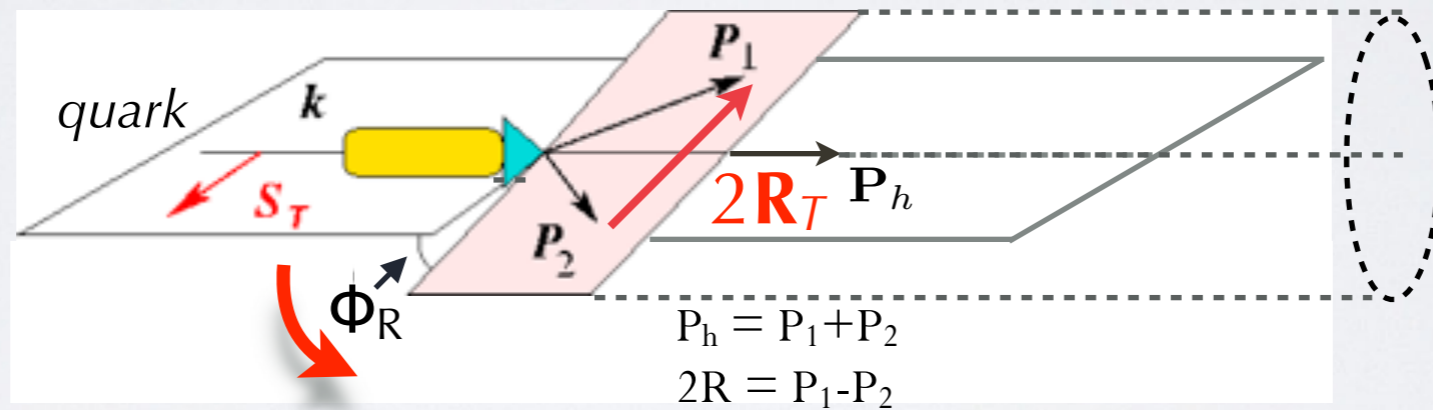


Vossen et al.,  
P.R.L. **107** (11) 072004



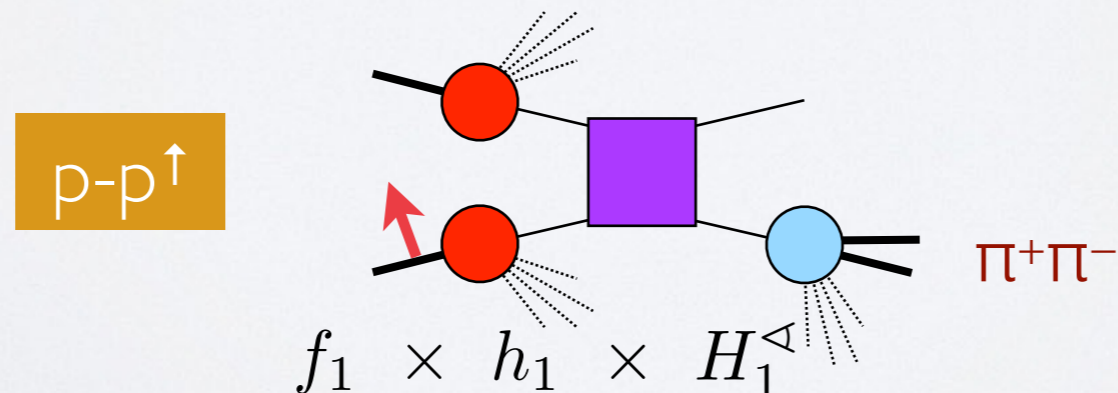
more data  
(hopefully) soon  
to be released

# extraction from **2-hadron**-inclusive data

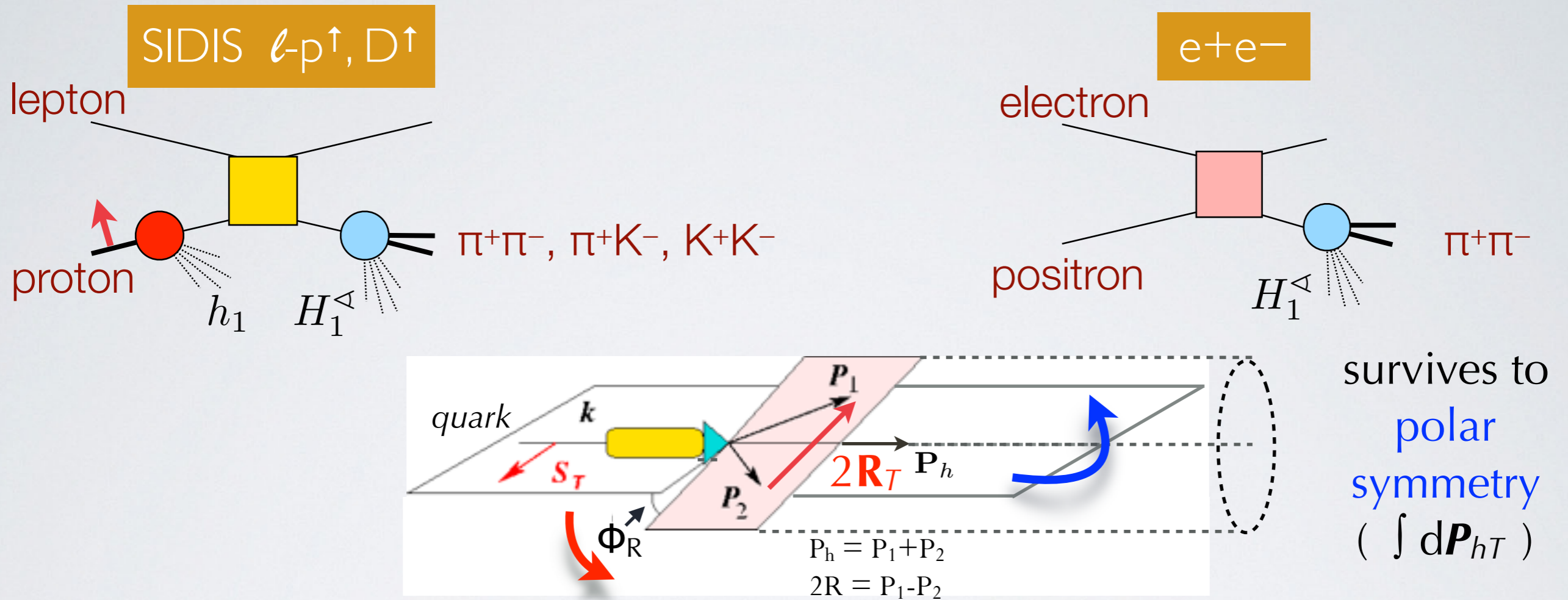


Collins, Heppelman, Ladinsky,  
N.P. **B420** (94)

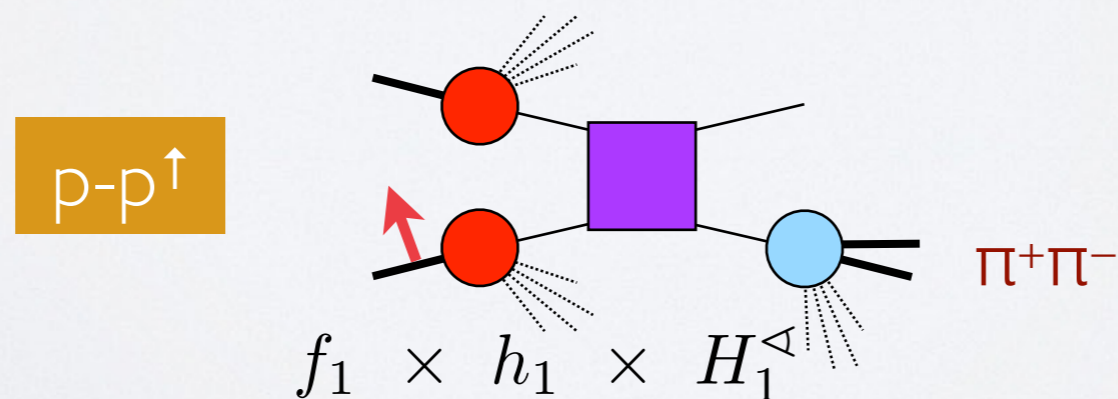
correlation  $S_T$  and  $R_T \rightarrow$  azimuthal asymmetry



# extraction from **2-hadron**-inclusive data



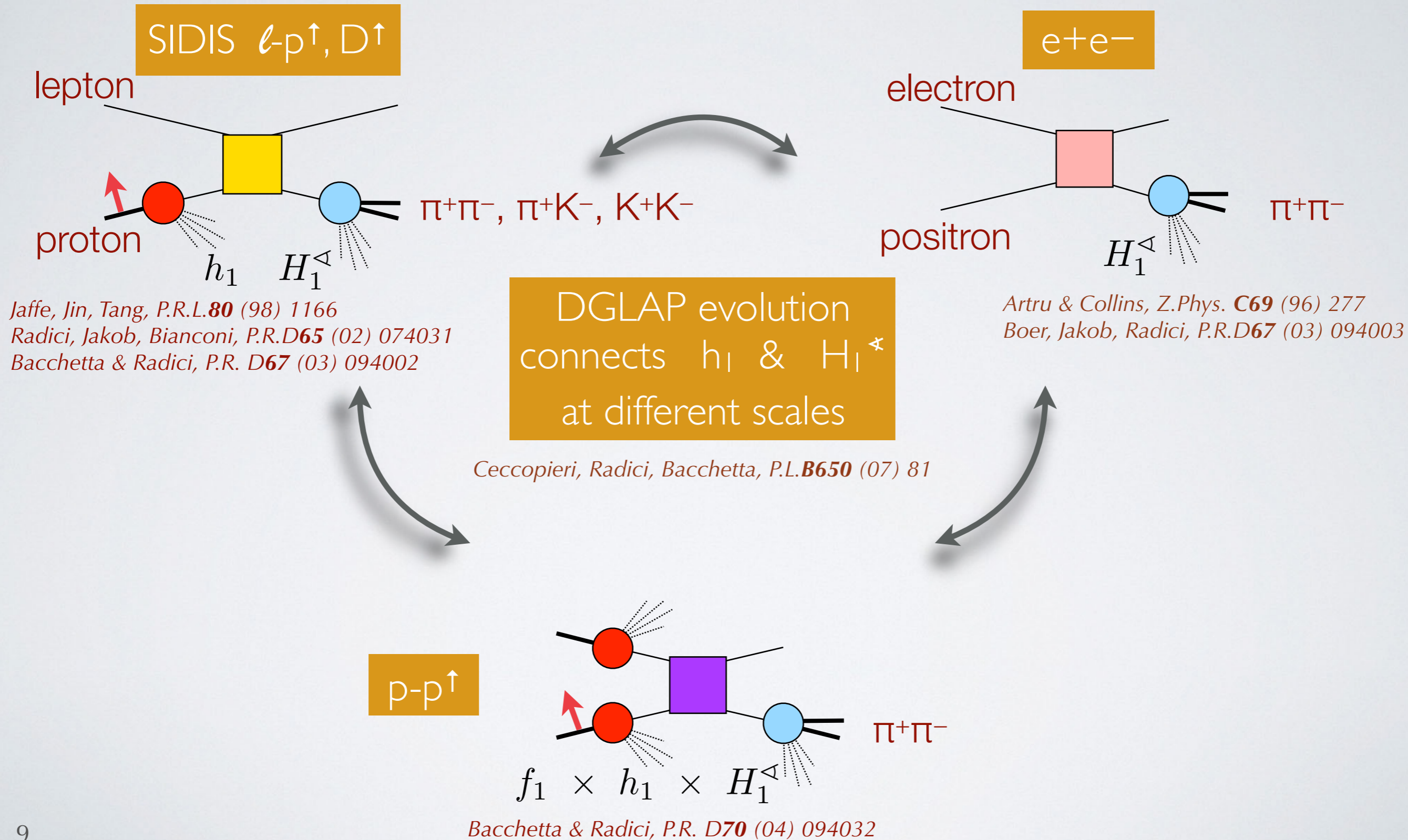
correlation  $S_T$  and  $R_T \rightarrow$  azimuthal asymmetry



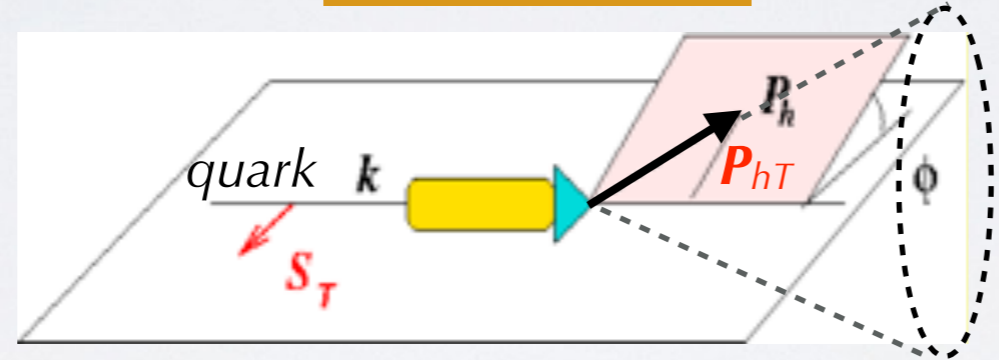
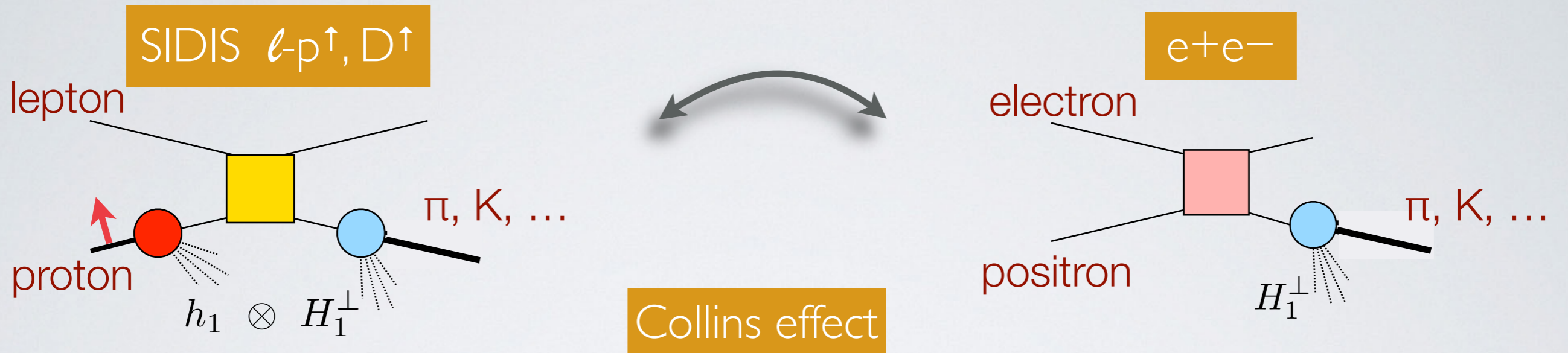
framework  
collinear  
factorization  
 $R_T \ll Q$



# extraction from **2-hadron**-inclusive data

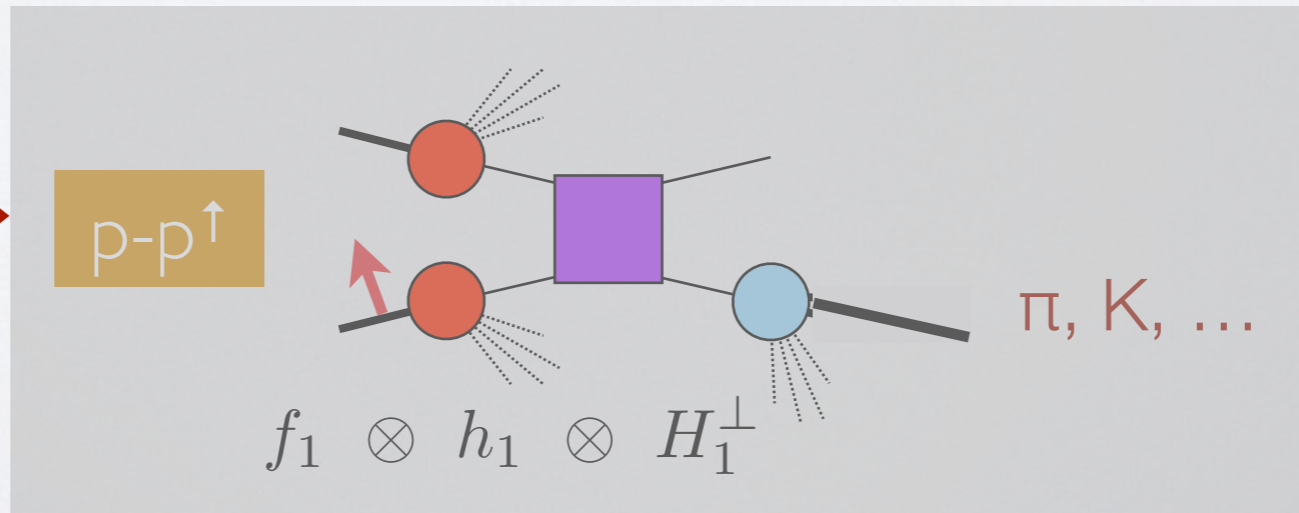


# not possible for 1-hadron-inclusive data

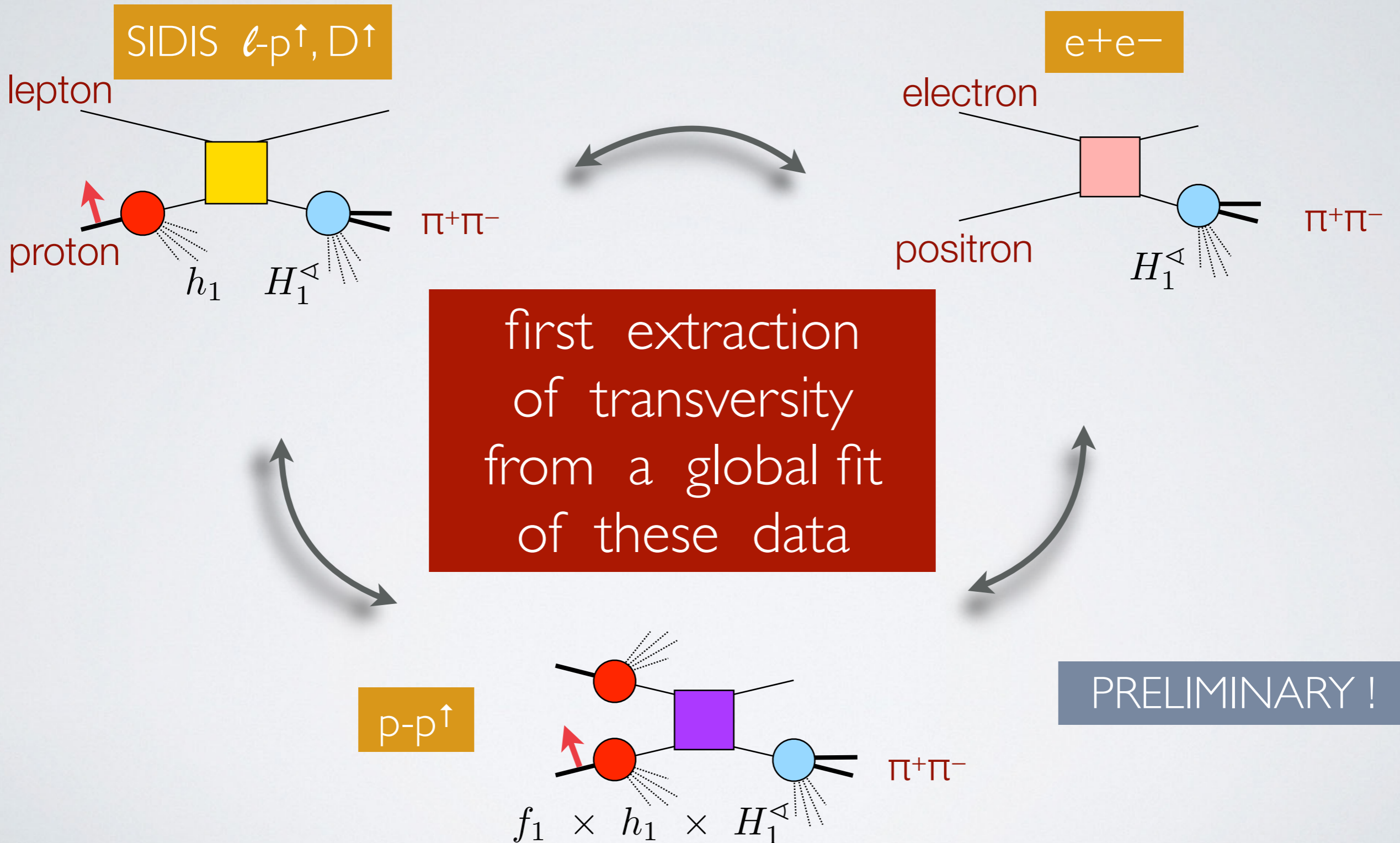


correlation  $S_T$  and  $P_{hT}$   $\rightarrow$  azimuthal asymmetry

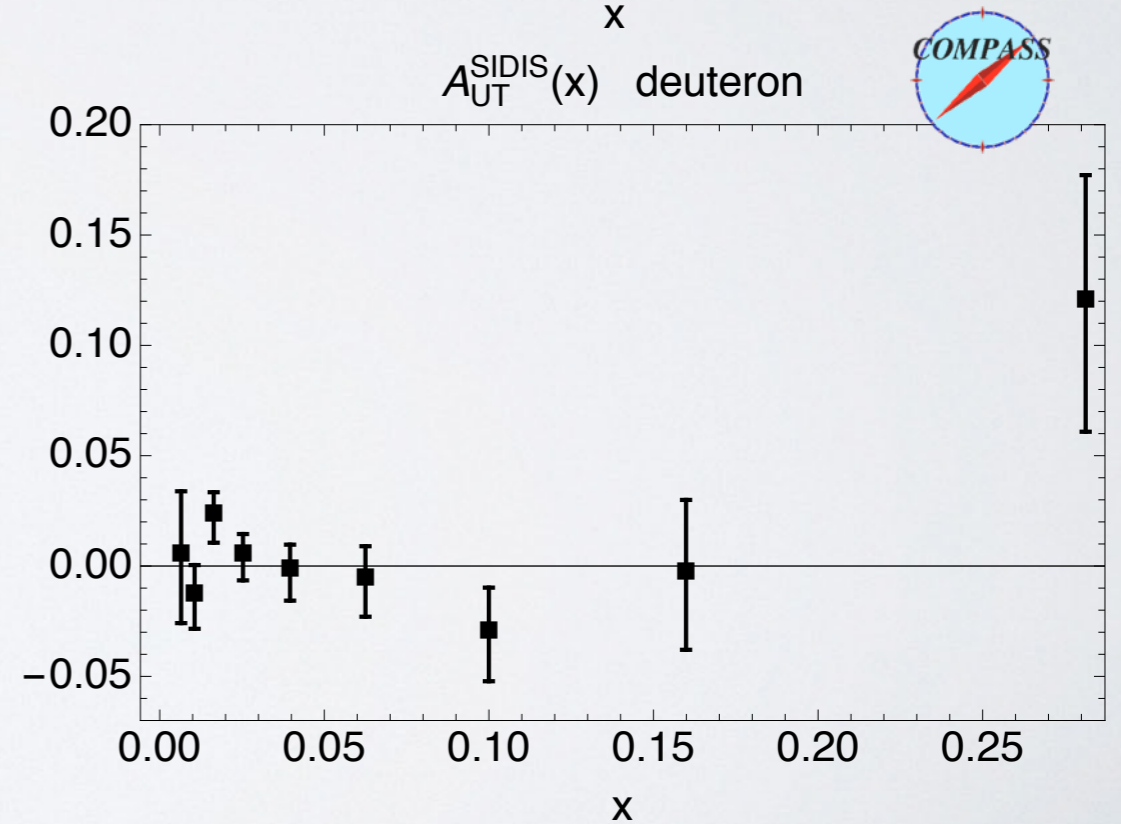
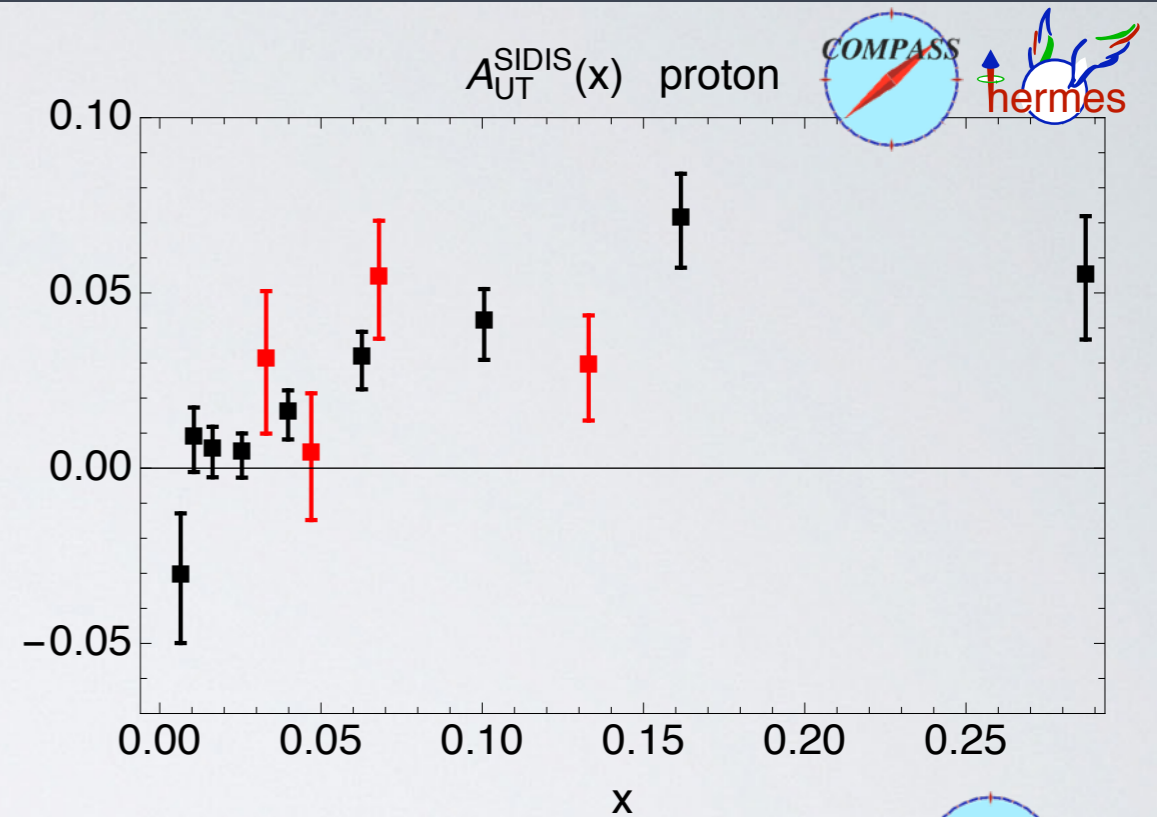
factorization theorem not yet proved



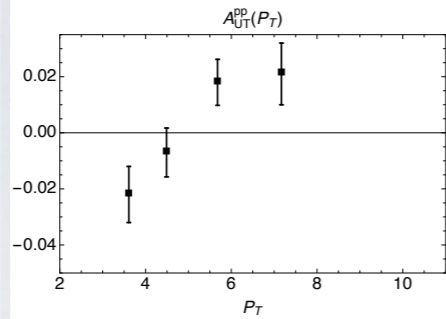
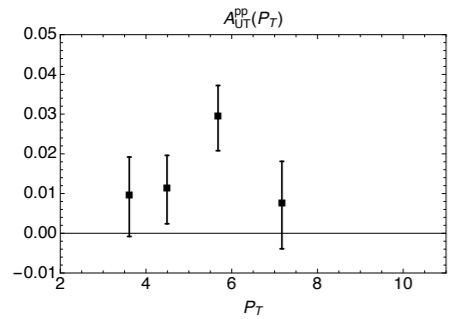
# take-away message



# the data set in more detail

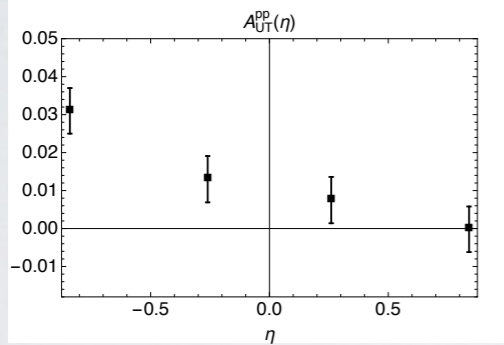


# the data set in more detail



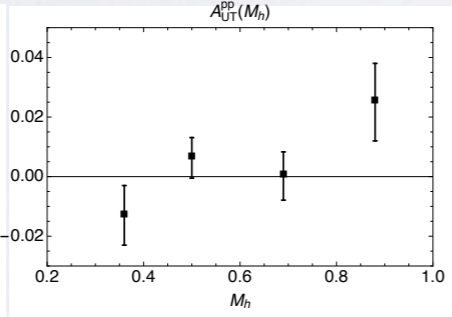
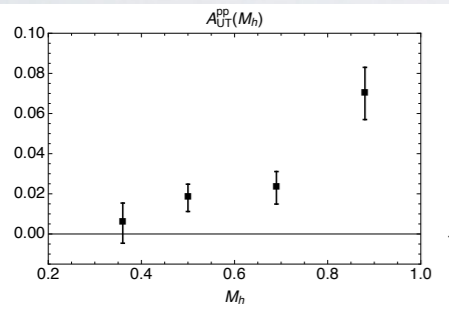
$A_{ut}^{pp}(P_T)$

$\eta < 0$



$\eta > 0$

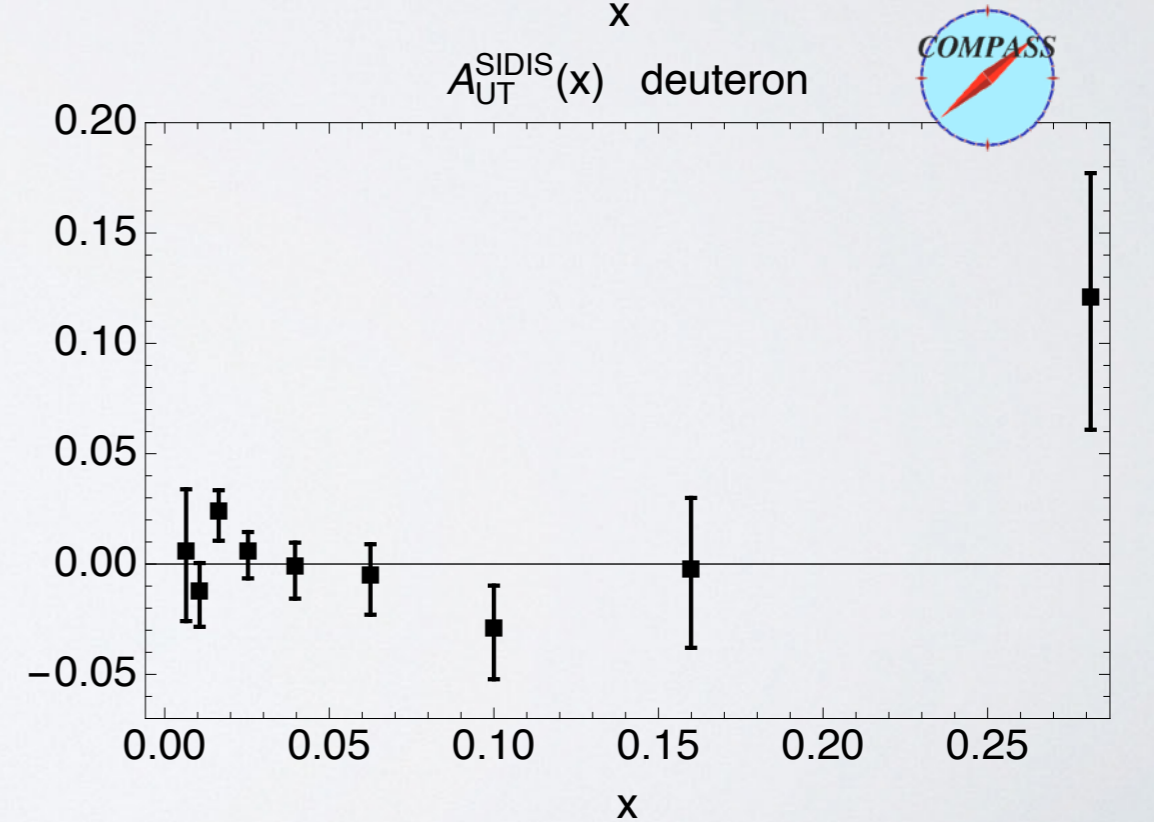
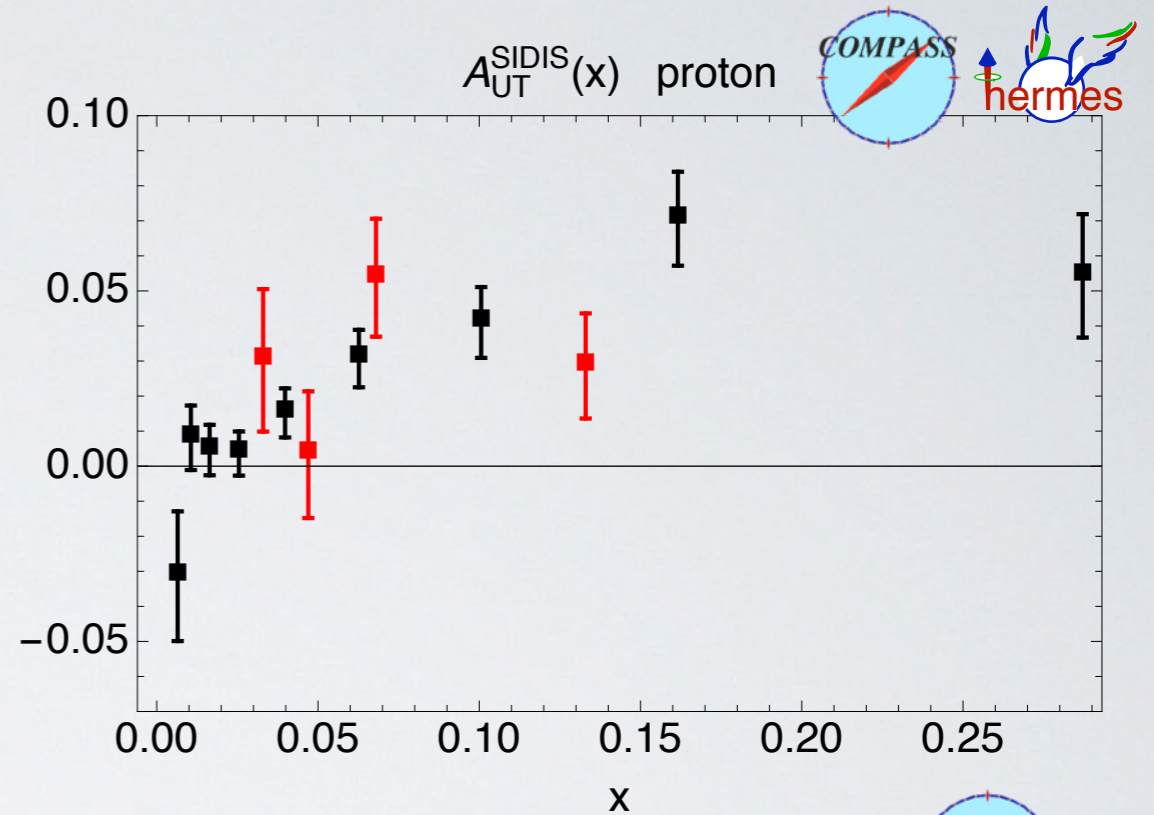
$A_{ut}^{pp}(\eta)$



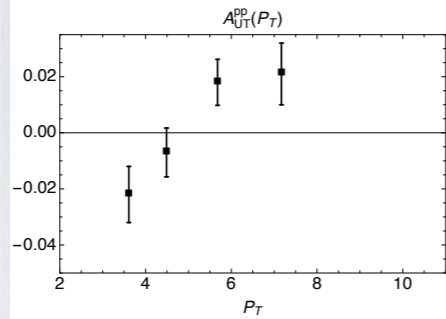
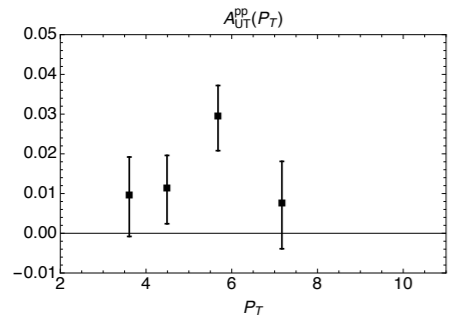
$A_{ut}^{pp}(M_h)$



run 2006  $s=200 \text{ GeV}^2$

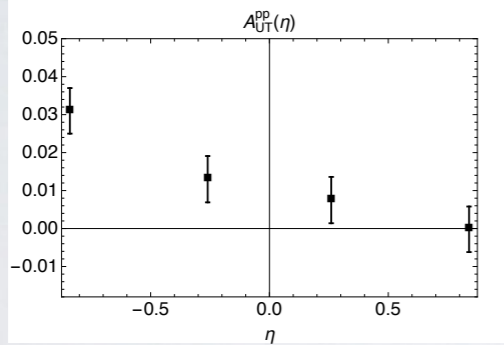


# the data set in more detail



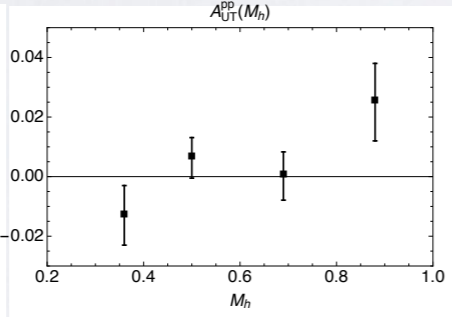
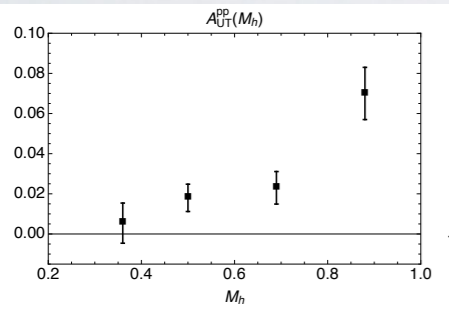
$A_{UT}^{pp}(P_T)$

$\eta < 0$



$\eta > 0$

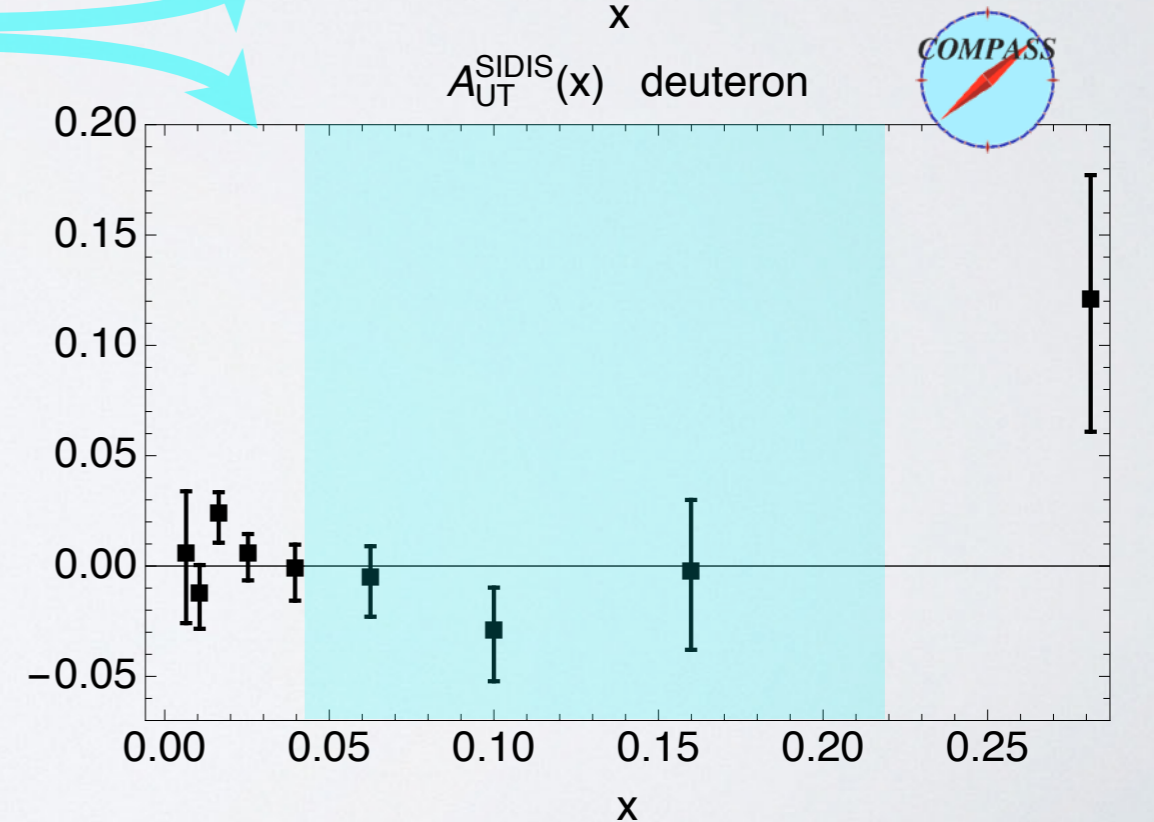
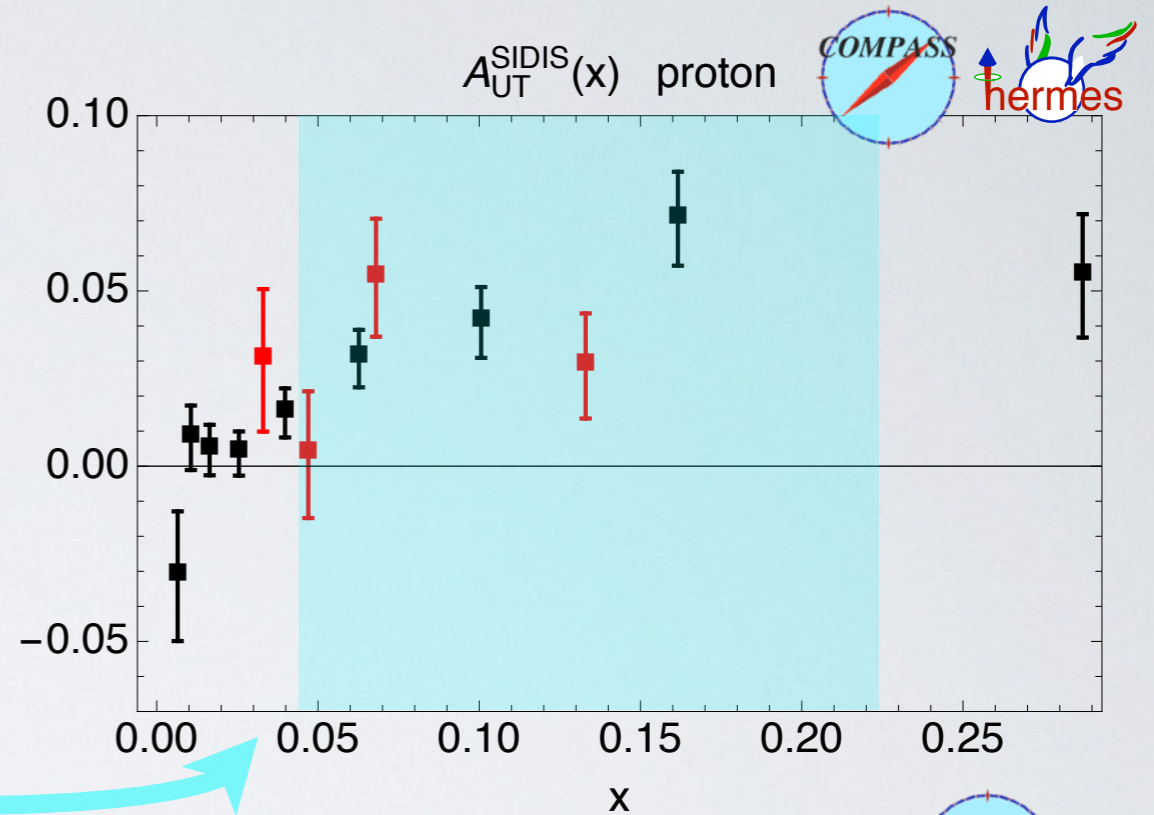
$A_{UT}^{pp}(\eta)$



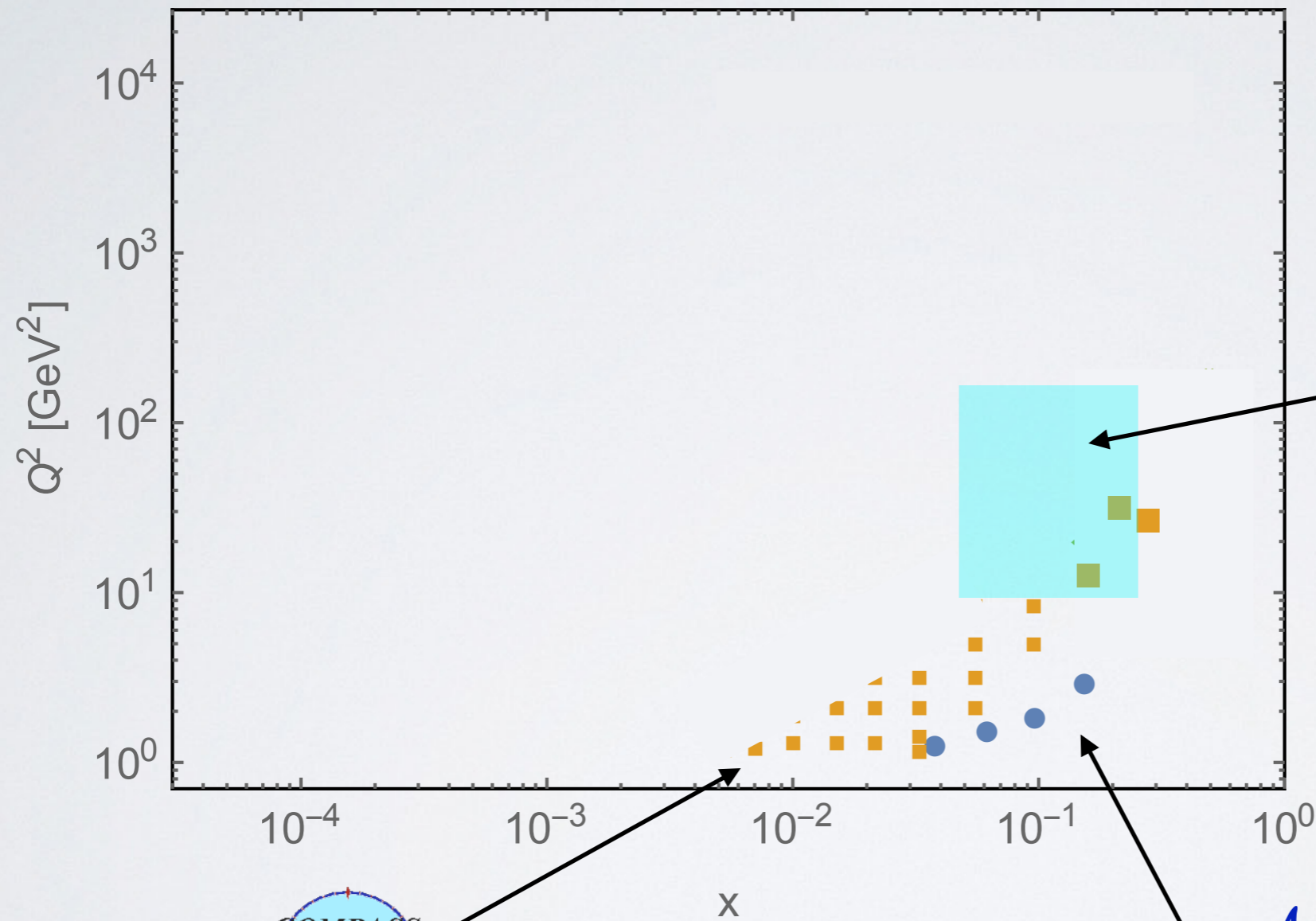
$A_{UT}^{pp}(M_h)$



run 2006  $s=200 \text{ GeV}^2$   
(effective coverage in x        )



# the kinematics



*Adamczyk et al. (STAR),  
P.R.L. **115** (2015) 242501*

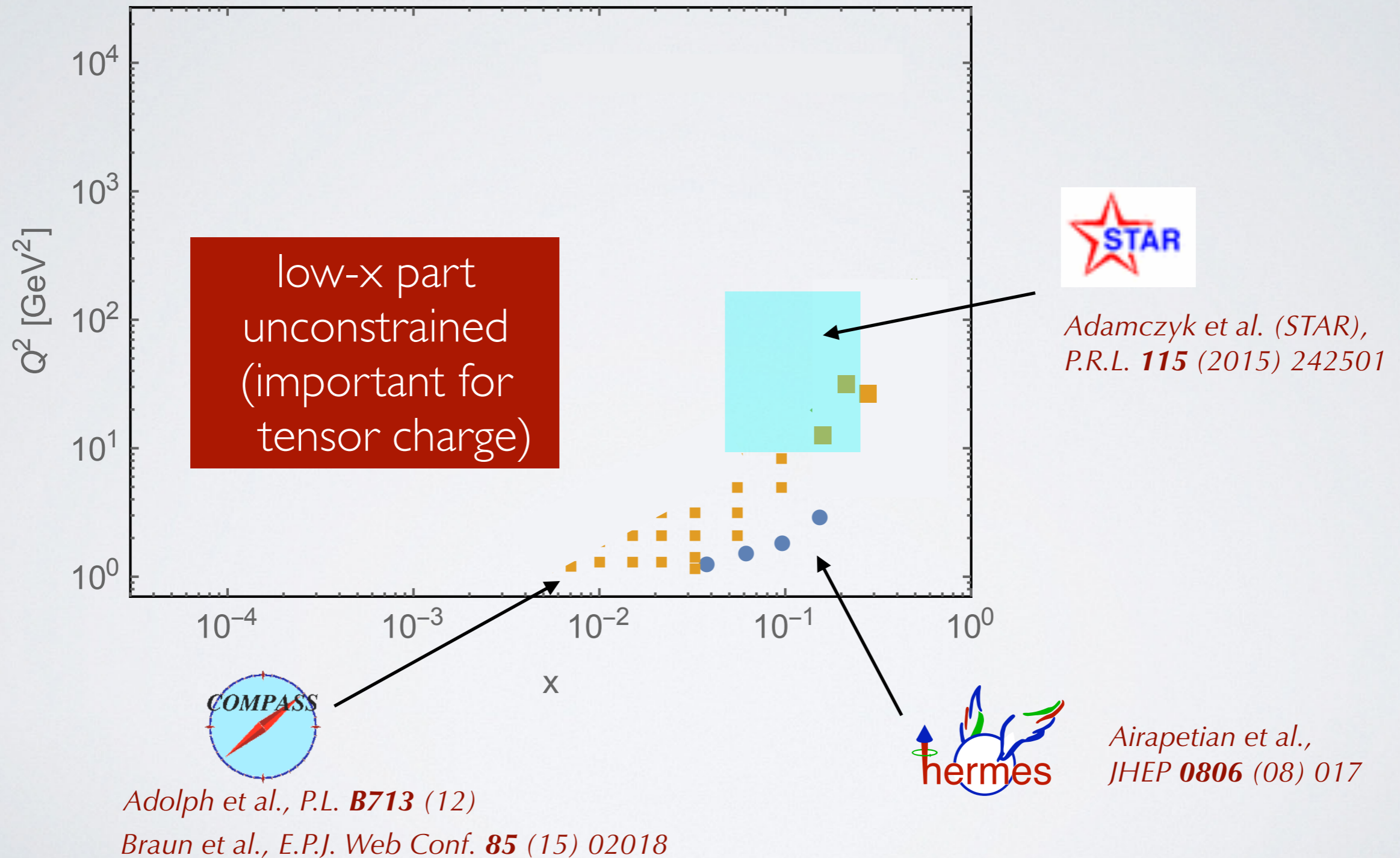


*Adolph et al., P.L. **B713** (12)  
Braun et al., E.P.J. Web Conf. **85** (15) 02018*



*Airapetian et al.,  
JHEP **0806** (08) 017*

# the kinematics





# choice of functional form

$$h_1^{qv}(x; Q_0^2) = F(x) \left[ \text{SB}^q(x) + \overline{\text{SB}}^{\bar{q}}(x) \right]$$



Soffer Bound

$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}^q(x, Q^2) = |f_1^q(x, Q^2) + g_1^q(x, Q^2)|$$

# choice of functional form

$$h_1^{qv}(x; Q_0^2) = F(x) \left[ \text{SB}^q(x) + \overline{\text{SB}}^{\bar{q}}(x) \right]$$

Soffer Bound

$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}^q(x, Q^2) = |f_1^q(x, Q^2) + g_1^q(x, Q^2)|$$

$$F(x) = \frac{N}{\max_x [|F(x)|]} x^A [1 + B \text{Ceb}_1(x) + C \text{Ceb}_2(x) + D \text{Ceb}_3(x)]$$

$$|N| \leq 1 \Rightarrow |F(x)| \leq 1$$

Ceb<sub>n</sub>(x) Chebyshev polynomial

10 fitting parameters

**Soffer Bound satisfied at any Q<sup>2</sup>**

# choice of functional form

$$h_1^{qv}(x; Q_0^2) = F(x) \left[ \text{SB}^q(x) + \overline{\text{SB}}^{\bar{q}}(x) \right]$$

Soffer Bound

$$2|h_1^q(x, Q^2)| \leq 2 \text{SB}^q(x, Q^2) = |f_1^q(x, Q^2) + g_1^q(x, Q^2)|$$

$$F(x) = \frac{N}{\max_x [|F(x)|]} x^A [1 + B \text{Ceb}_1(x) + C \text{Ceb}_2(x) + D \text{Ceb}_3(x)]$$

$$|N| \leq 1 \Rightarrow |F(x)| \leq 1$$

Ceb<sub>n</sub>(x) Chebyshev polynomial

10 fitting parameters

**Soffer Bound satisfied at any Q<sup>2</sup>**

if  $\lim_{x \rightarrow 0} x \text{SB}(x) \propto x^{\bar{a}}$  then  $A + \bar{a} > 0$  grants  $\int_0^1 dx h_1^q(x; Q^2) \equiv \delta q(Q^2)$  is finite

**this bound drastically constrains the tensor charge**

**with new functional form, Mellin transform can be computed analytically**

# choice of functional form

typical cross section for  $a+b^\uparrow \rightarrow c^\uparrow+d$  process

$$\frac{d\sigma_{UT}}{d\eta} \propto \int d|\mathbf{P}_T| dM_h \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) h_1^b(x_b) \frac{d\hat{\sigma}_{ab^\uparrow \rightarrow c^\uparrow d}}{d\hat{t}} H_1^{\triangleleft c}(\bar{z}, M_h)$$

to be computed thousands times... usual trick: use **Mellin anti-transform**

$$h_1(x, Q^2) = \int_{\mathcal{C}_N} dN x^{-N} h_1^N(Q^2) \quad N \in \mathbb{C}$$

*Stratmann & Vogelsang,  
P.R. D64 (01) 114007*

# choice of functional form

typical cross section for  $a+b^\uparrow \rightarrow c^\uparrow+d$  process

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to be computed thousands times... usual trick: use **Mellin anti-transform**

$$h_1(x, Q^2) = \int_{C_N} dN x^{-N} h_1^N(Q^2) \quad N \in \mathbb{C}$$

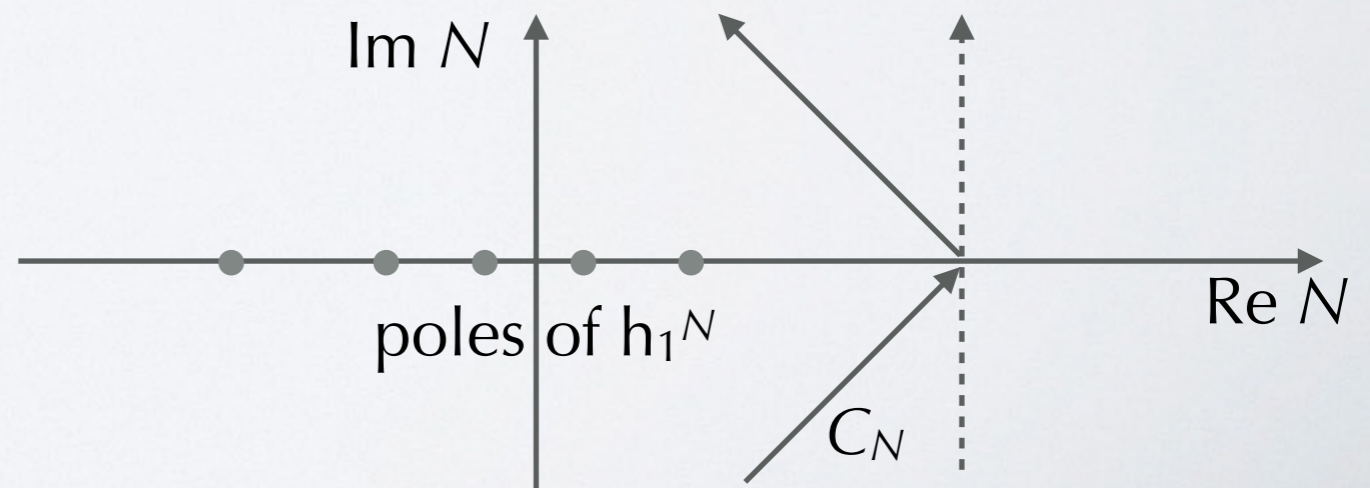
*Stratmann & Vogelsang,  
P.R. D64 (01) 114007*

$$\frac{d\sigma_{UT}}{d\eta} \propto \sum_b \int_{C_N} dN \int d|\mathbf{P}_T| h_{1b}^N(P_T^2) \int dM_h \sum_{a,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) x_b^{-N} \frac{d\hat{\sigma}_{ab^\uparrow \rightarrow c^\uparrow d}}{d\hat{t}} H_1^{\triangleleft c}(\bar{z}, M_h)$$

$F_b(N, \eta, |\mathbf{P}_T|, M_h)$

pre-compute  $F_b$  only one time  
on contour  $C_N$

this speeds up convergence  
and facilitates  $\int dN$ , provided  
that  $h_1^N$  is known analytically



# fit SIDIS asymmetry



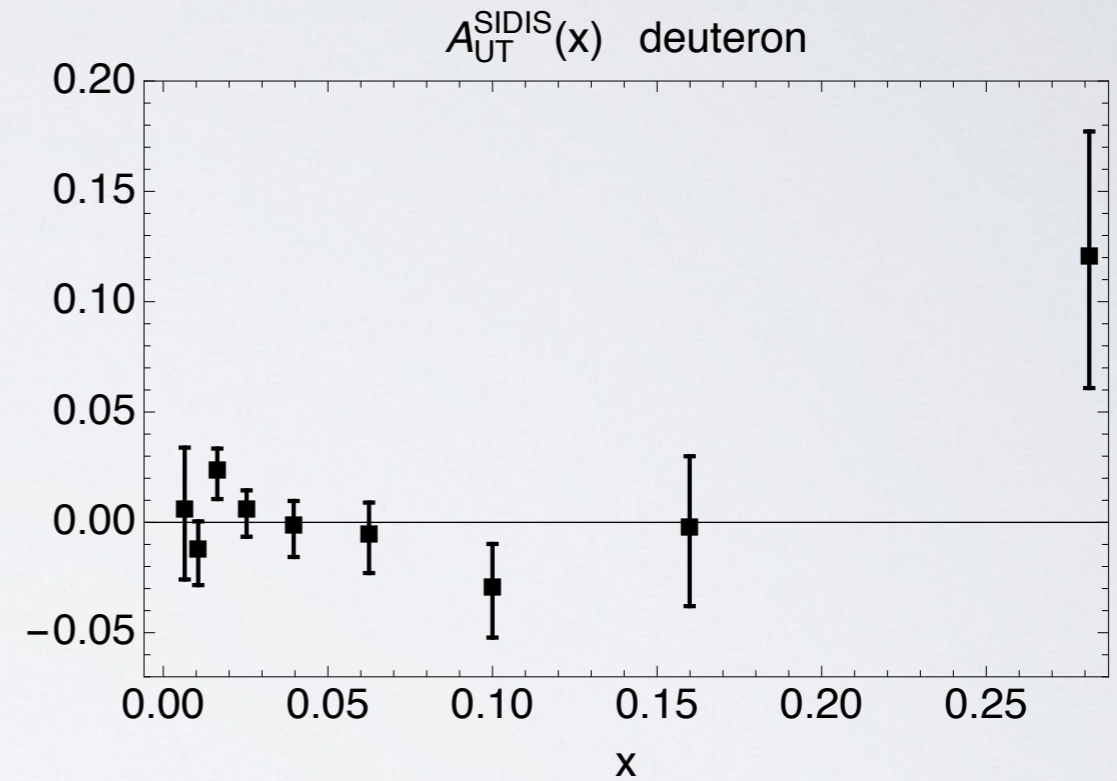
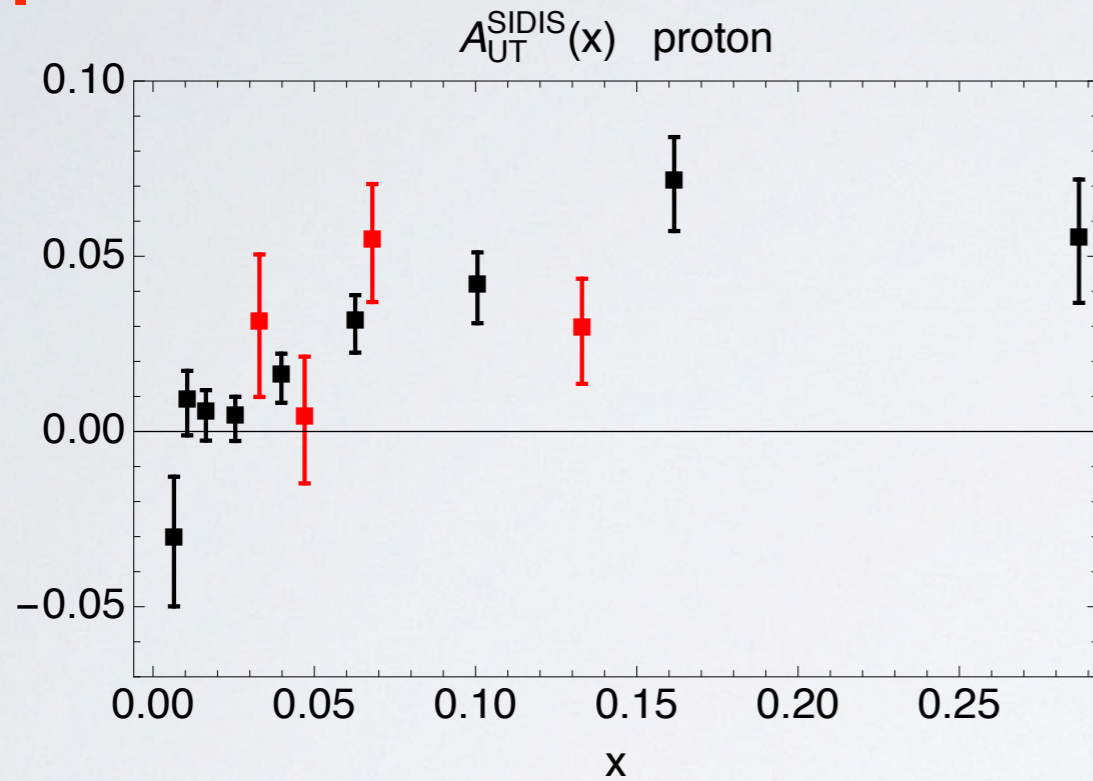
Braun et al., *E.P.J. Web Conf.* **85** (15) 02018



Airapetian et al., *JHEP* **0806** (08) 017

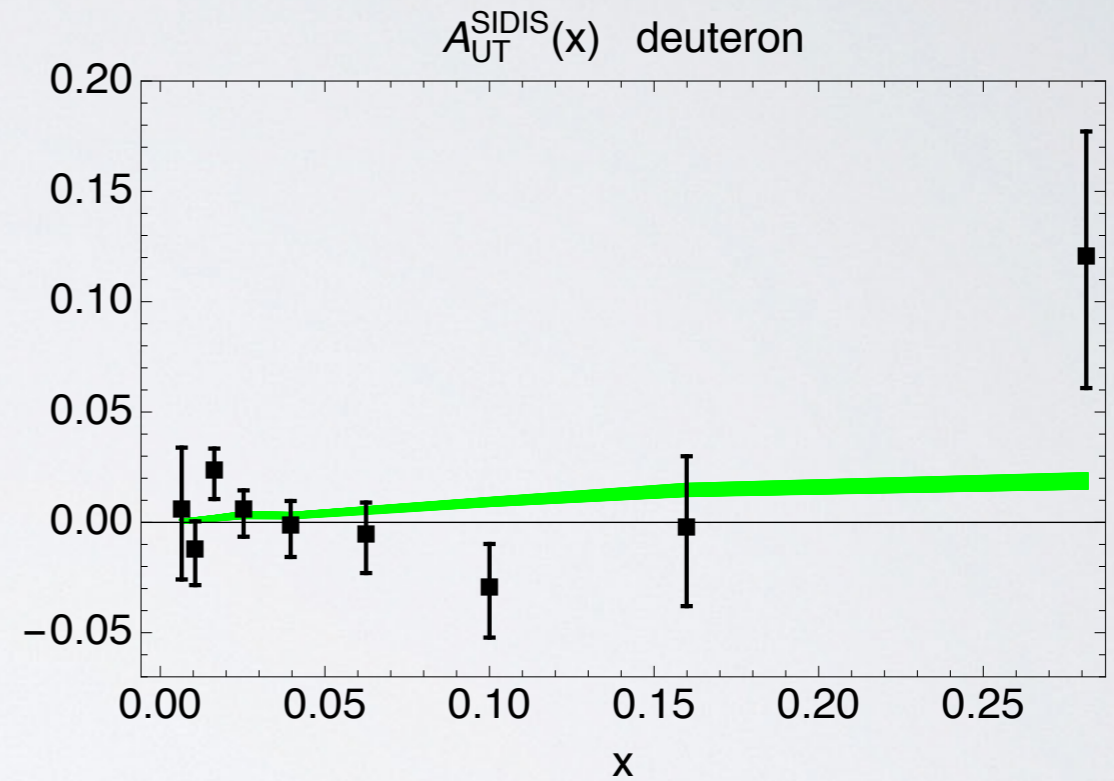
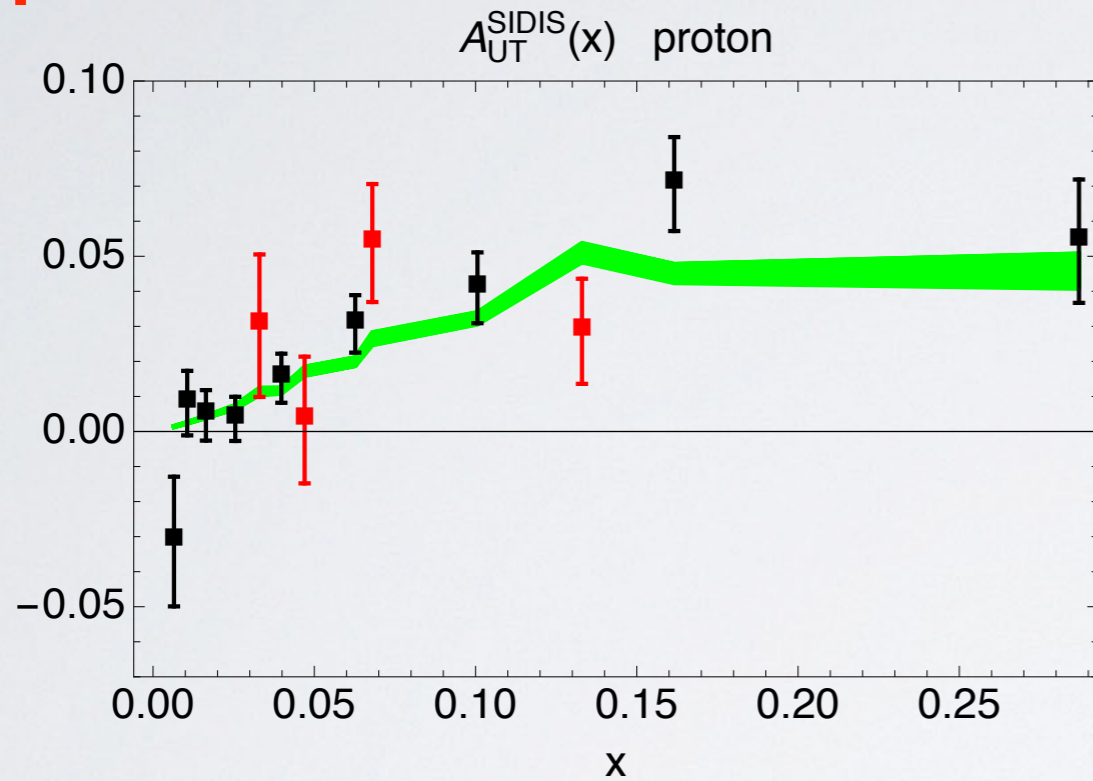
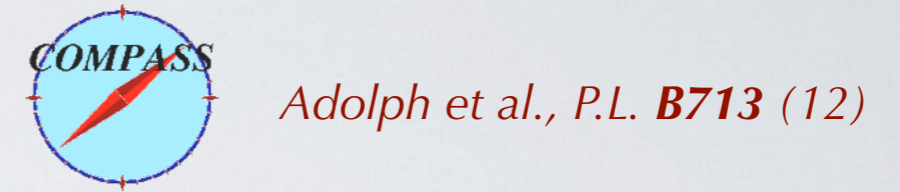
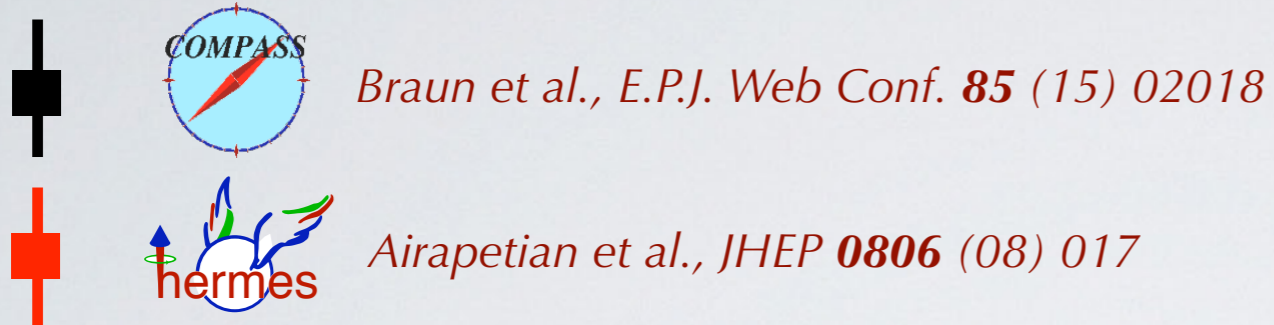


Adolph et al., *P.L.* **B713** (12)



the replica method

# fit SIDIS asymmetry



the replica method (50)

# fit SIDIS asymmetry



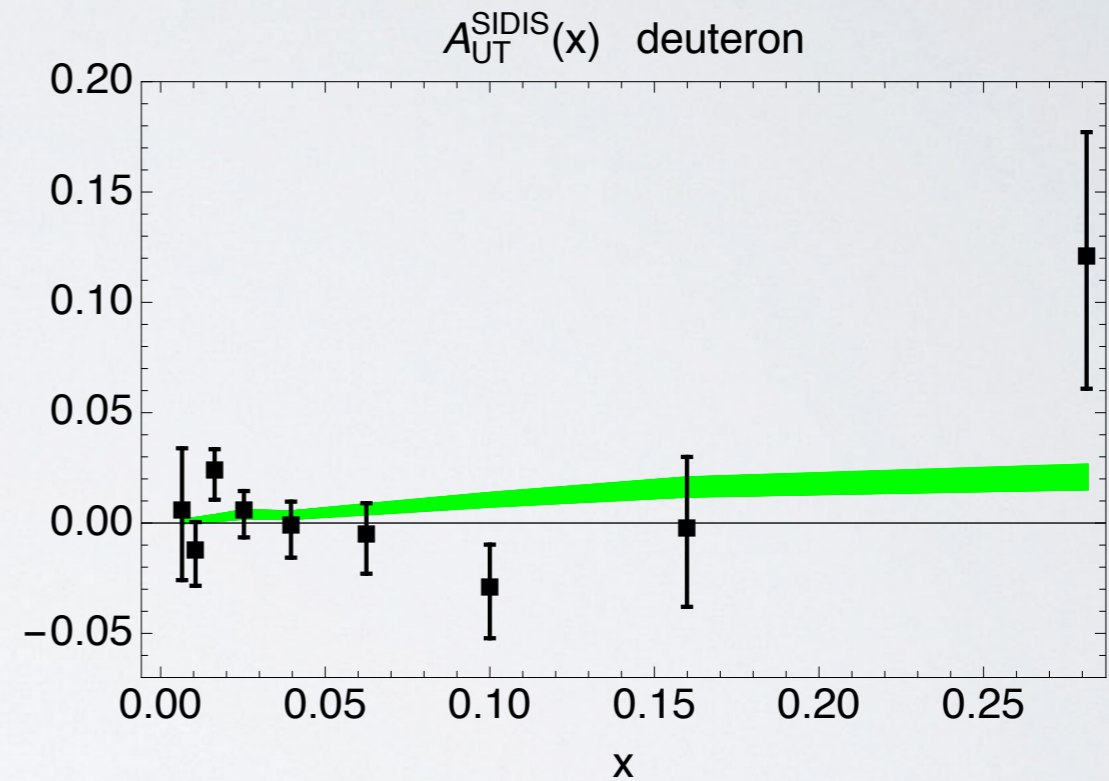
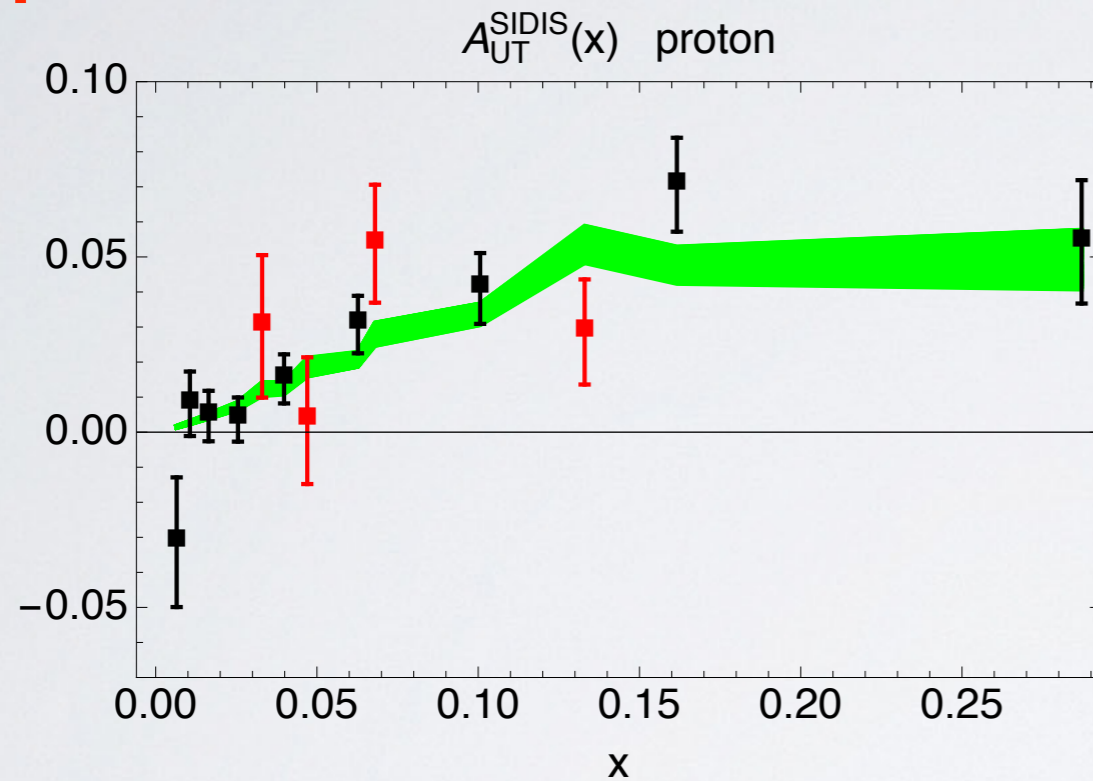
Braun et al., *E.P.J. Web Conf.* **85** (15) 02018



Airapetian et al., *JHEP* **0806** (08) 017



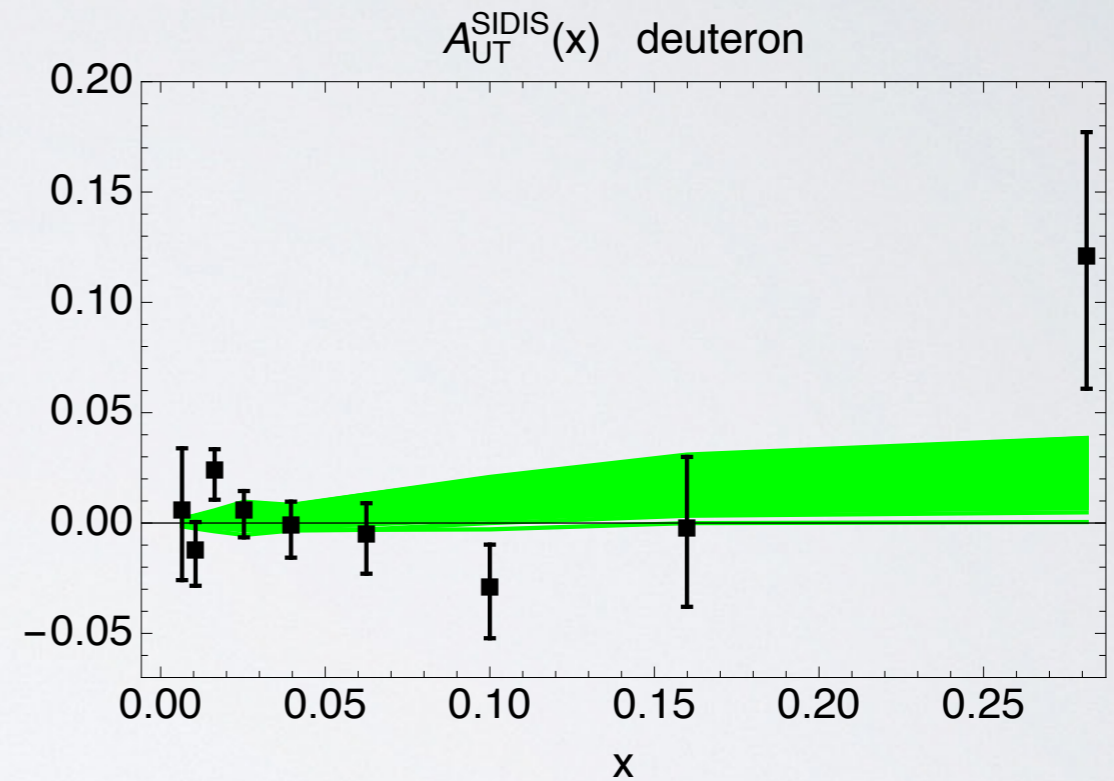
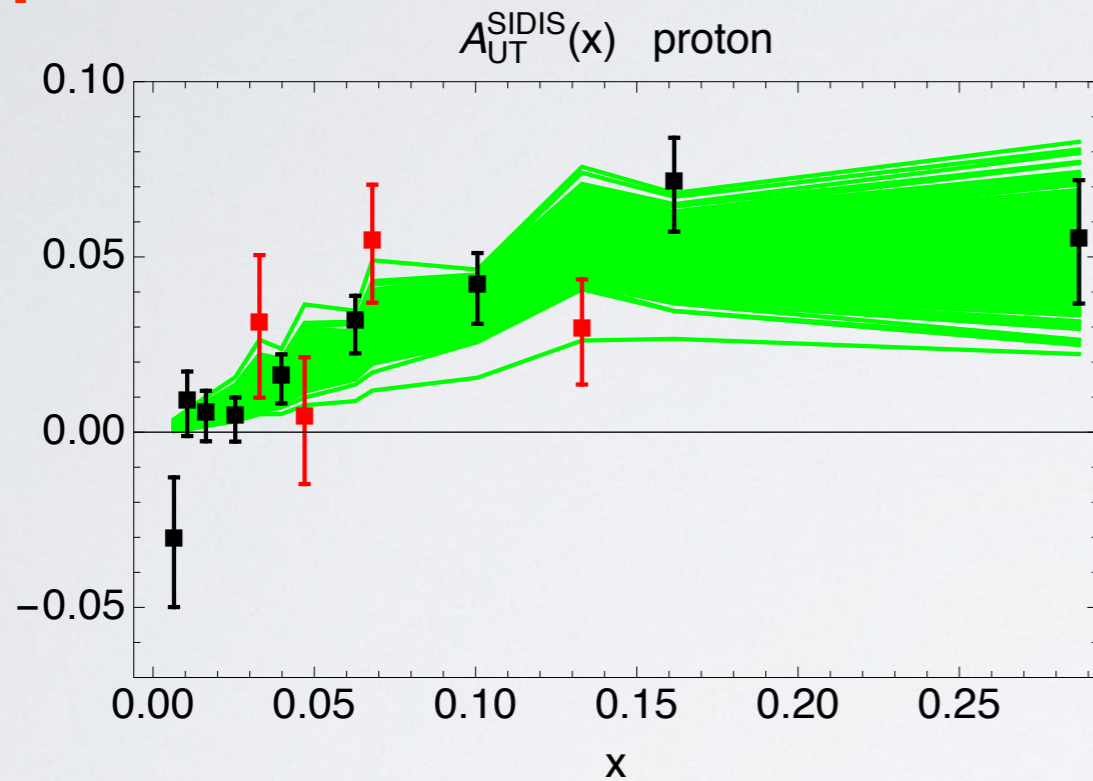
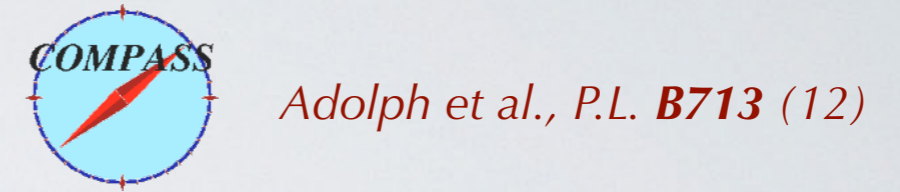
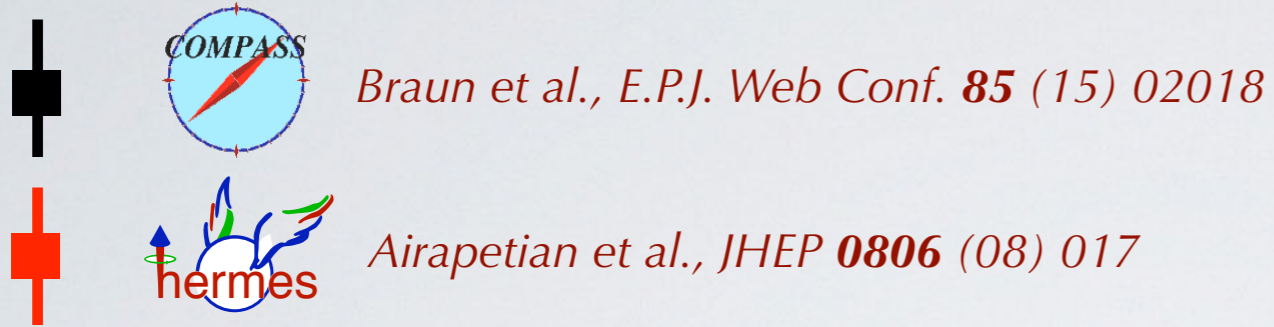
Adolph et al., *P.L.* **B713** (12)



the replica method (100)

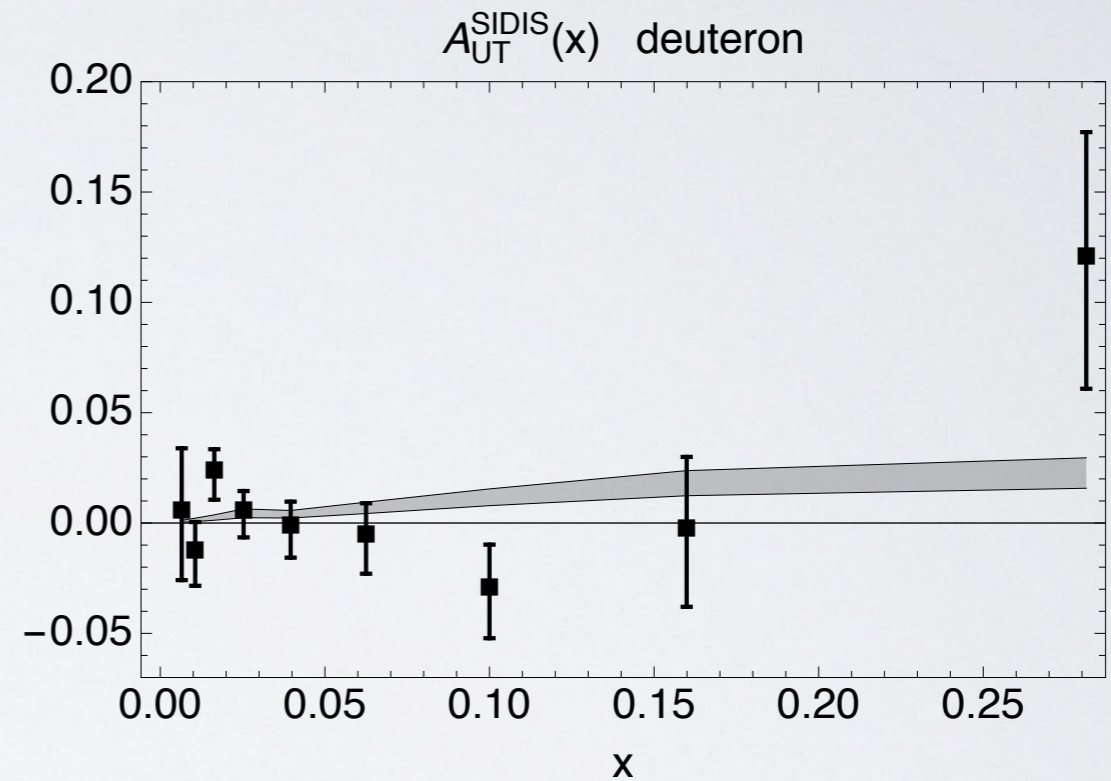
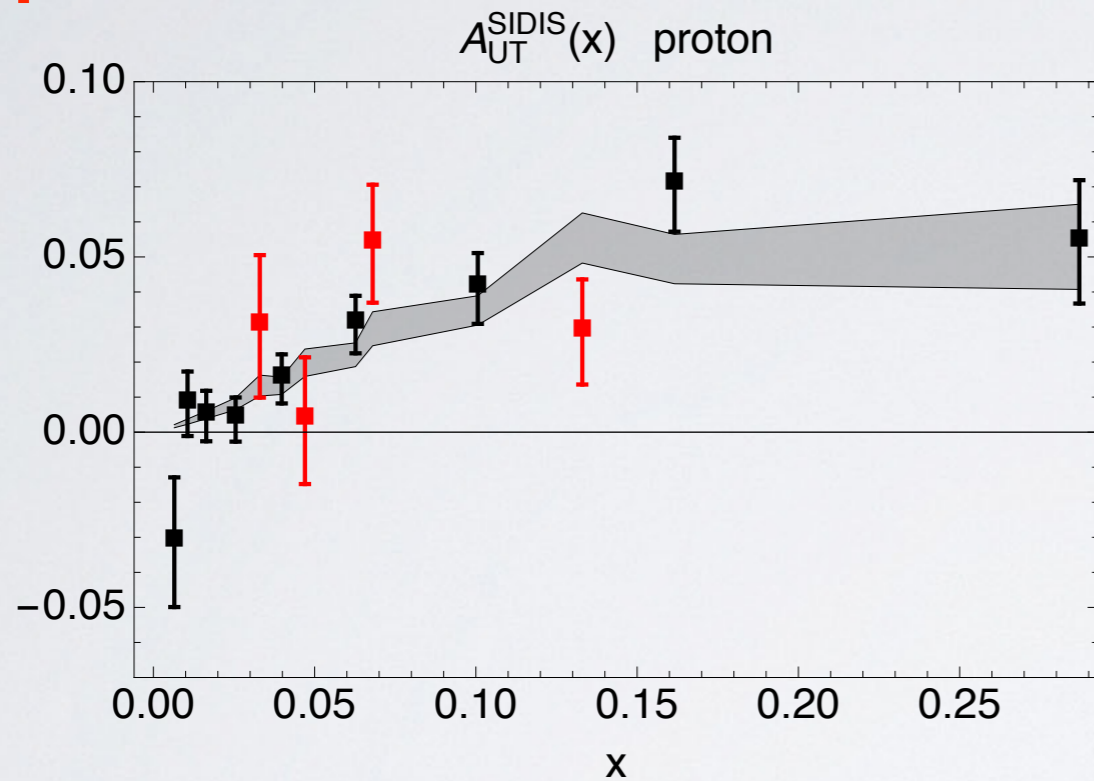
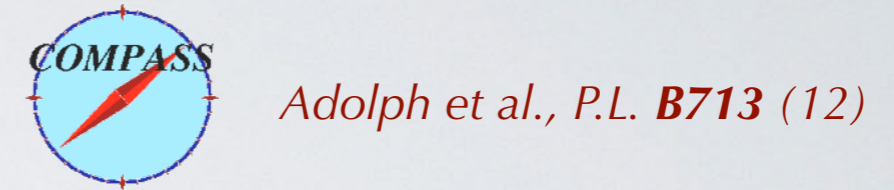
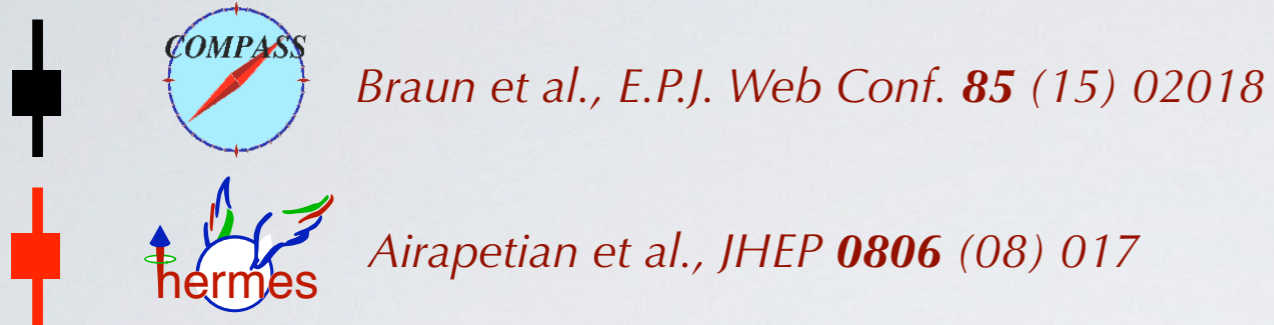


# fit SIDIS asymmetry



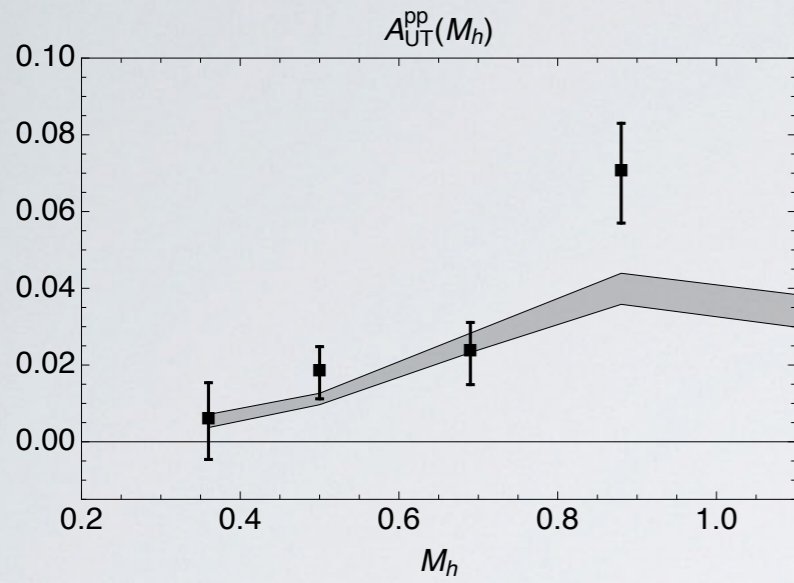
the replica method (200)

# fit SIDIS asymmetry



the replica method (68%)

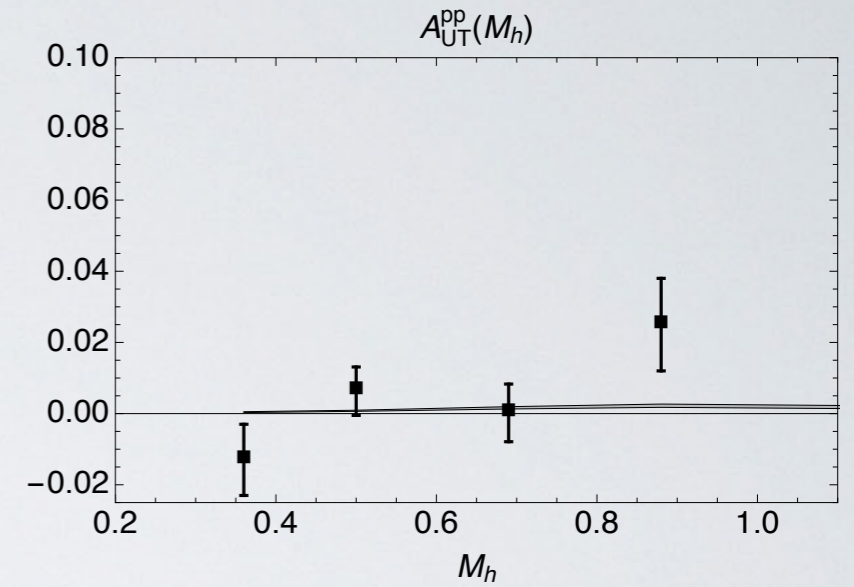
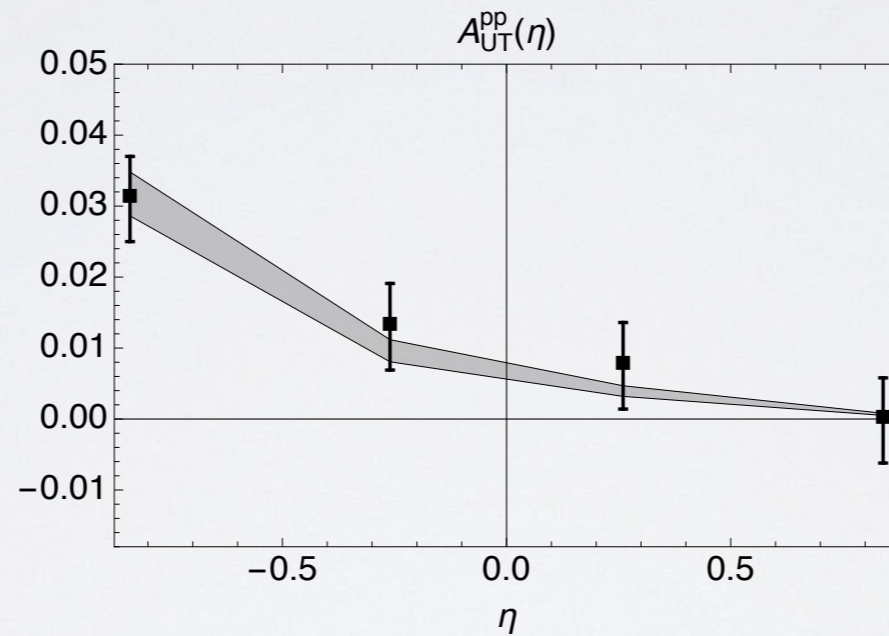
# fit STAR asymmetry



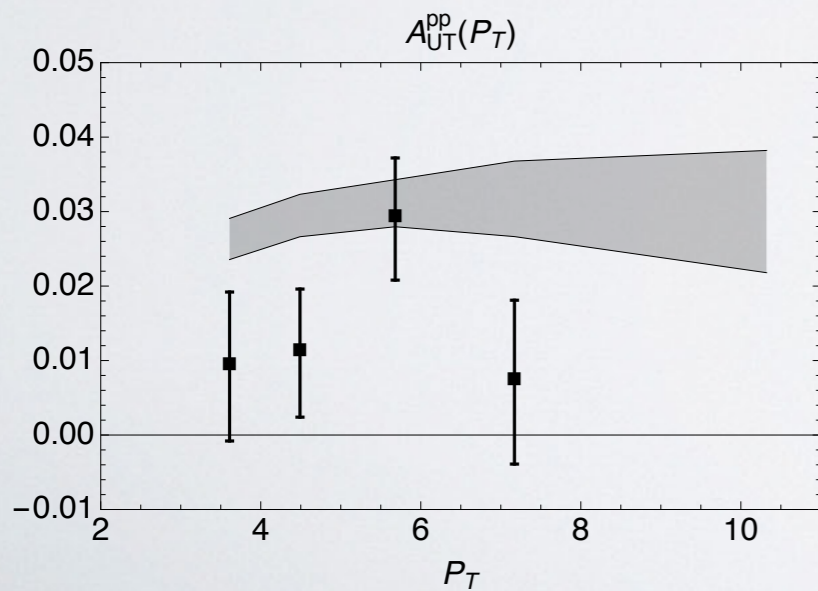
$\eta < 0$



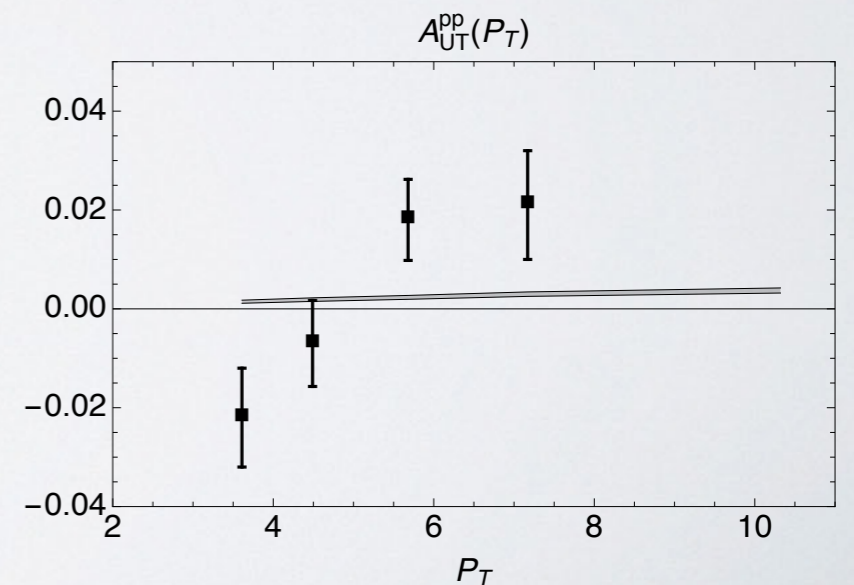
*Adamczyk et al. (STAR),  
P.R.L. 115 (2015) 242501*



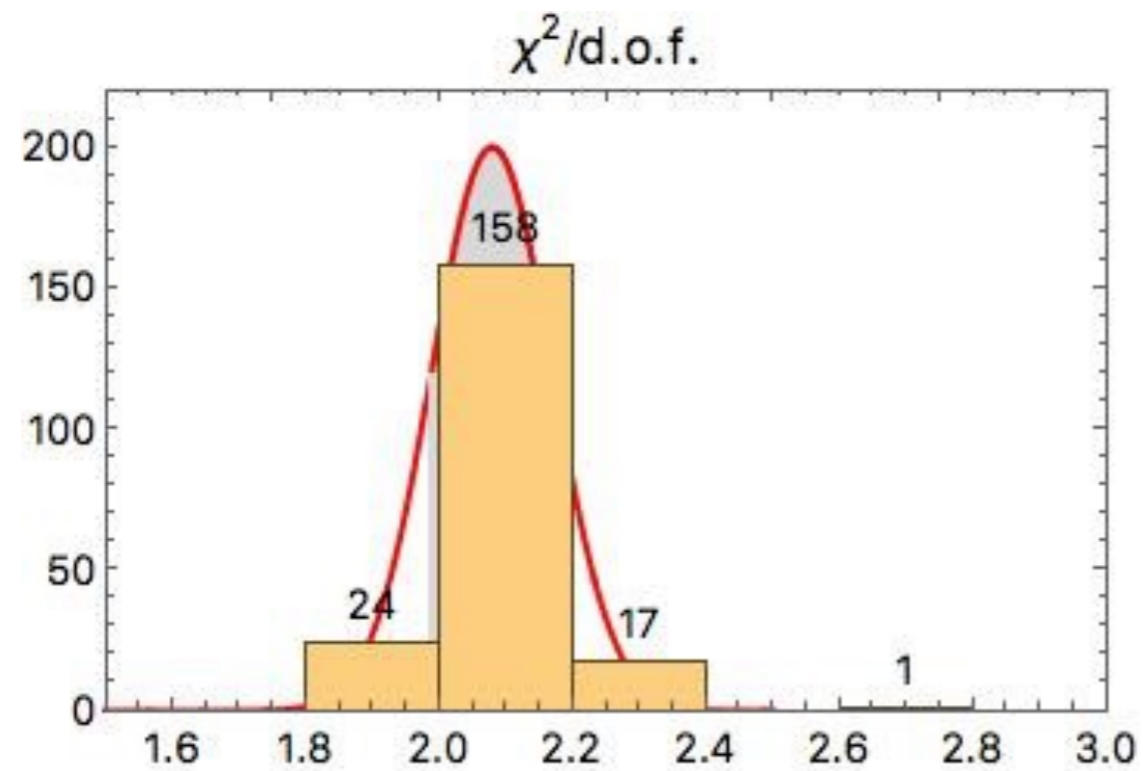
$\eta > 0$



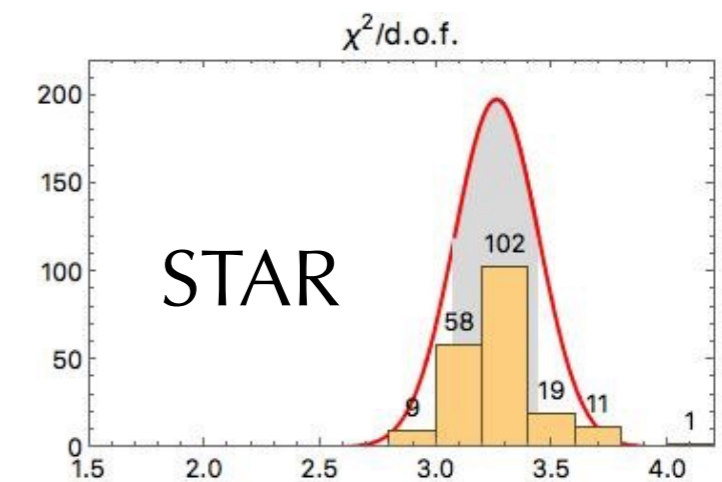
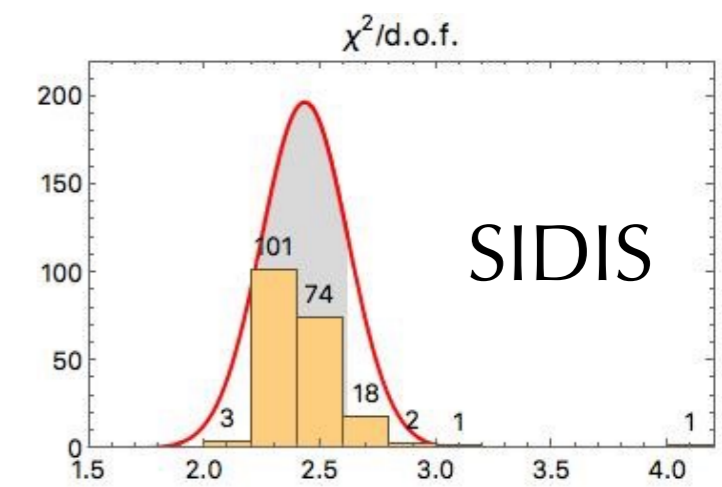
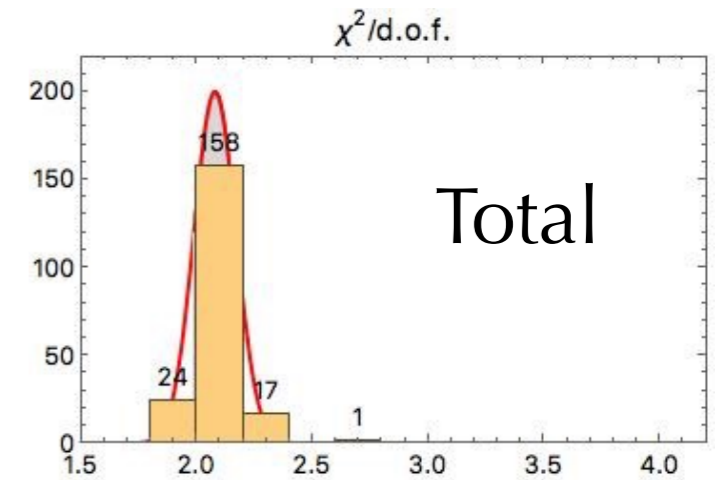
68% uncertainty band



# $\chi^2$ of the fit

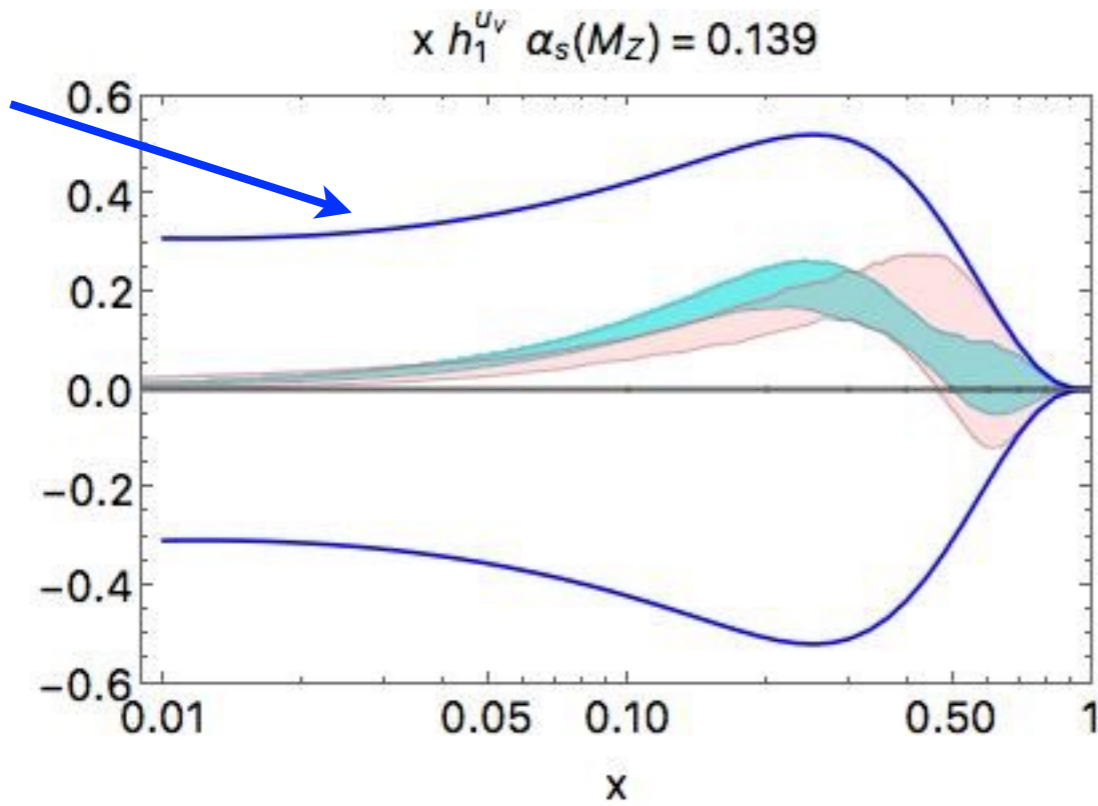


$$\chi^2/\text{dof} = 2.08 \pm 0.09$$



# comparison with previous fit

Soffer bound



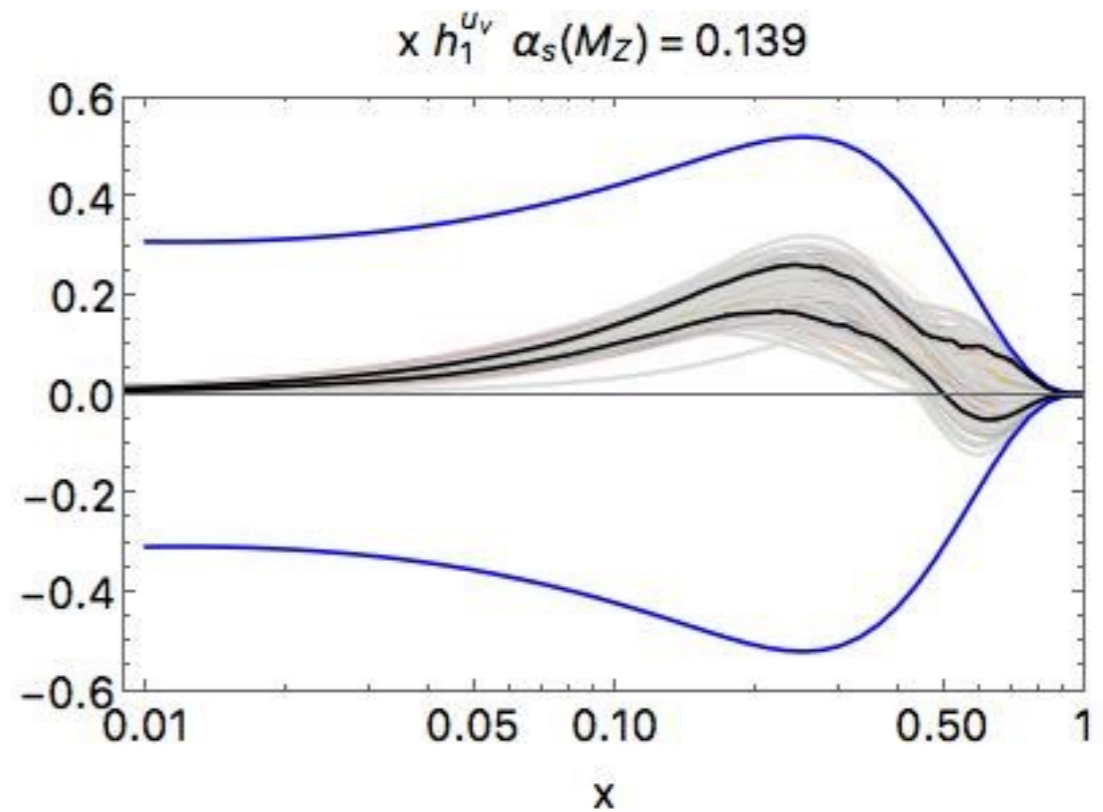
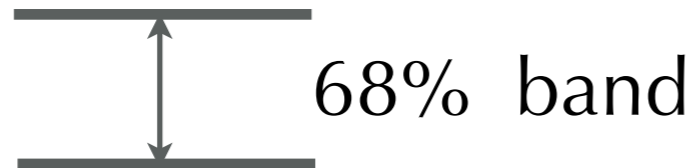
global fit

old fit

up

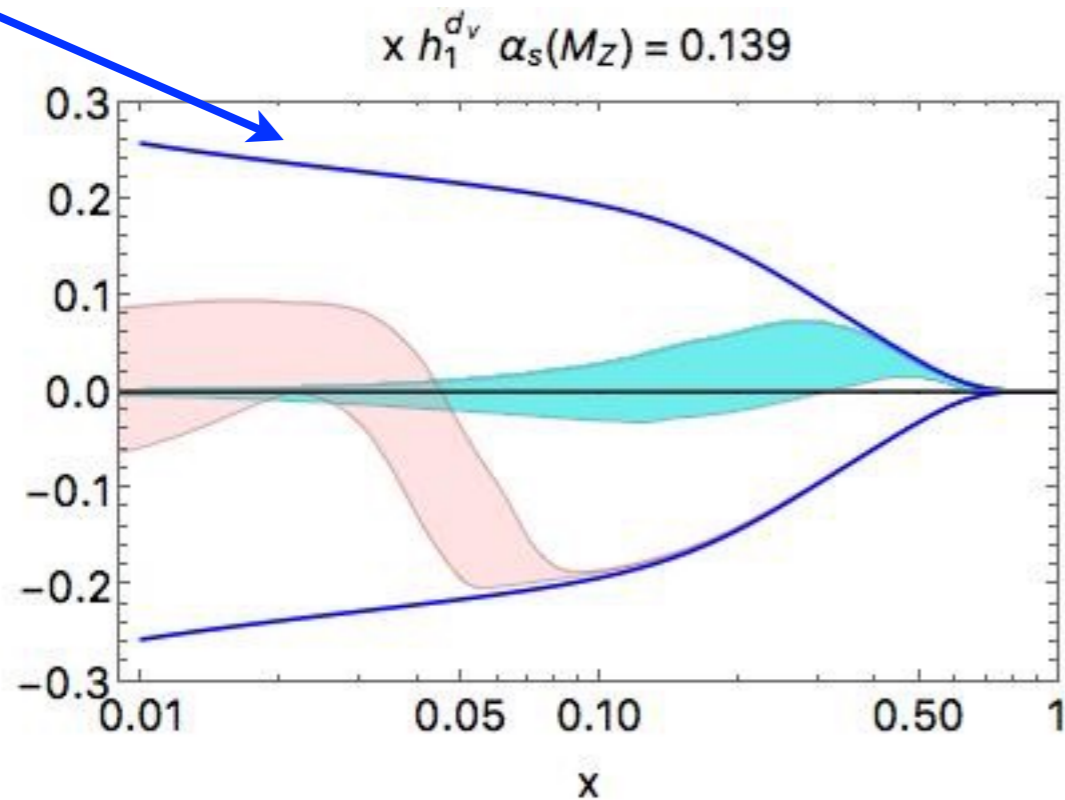
higher precision

all 200 replicas



# comparison with previous fit

Soffer bound



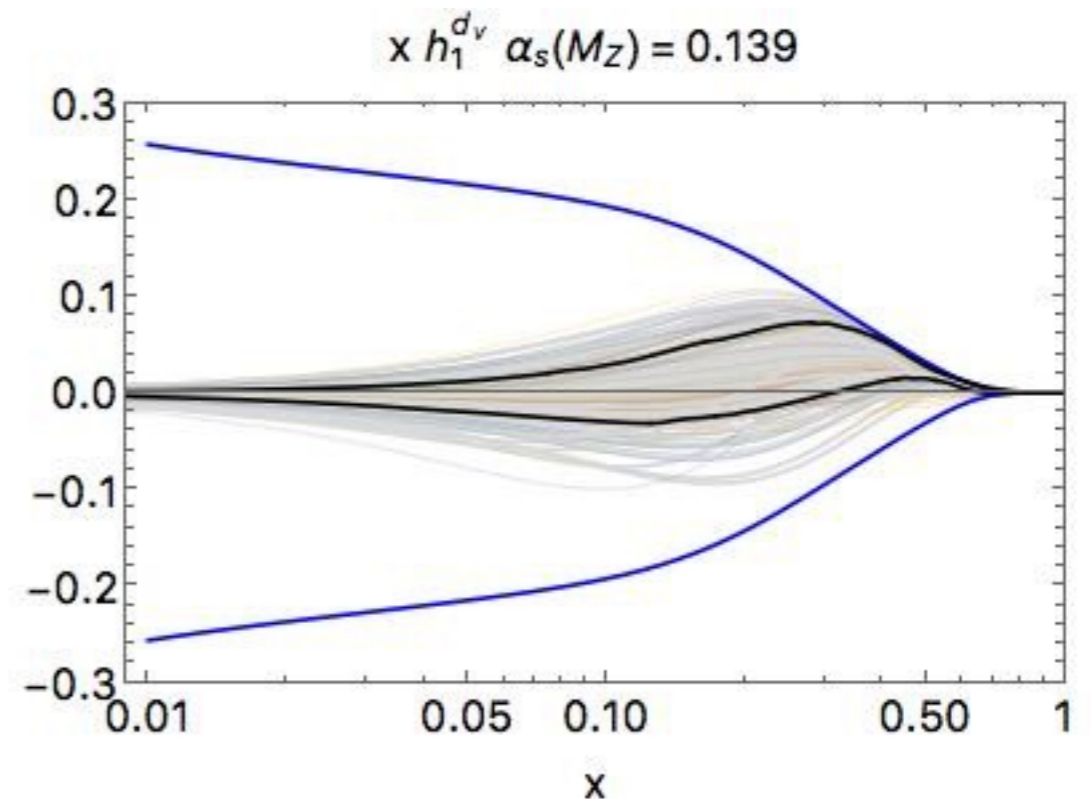
global fit

old fit

down

all 200 replicas

68% band



effect of STAR data :  
saturation of Soffer bound  
practically disappeared !

# origin of saturation of Soffer bound

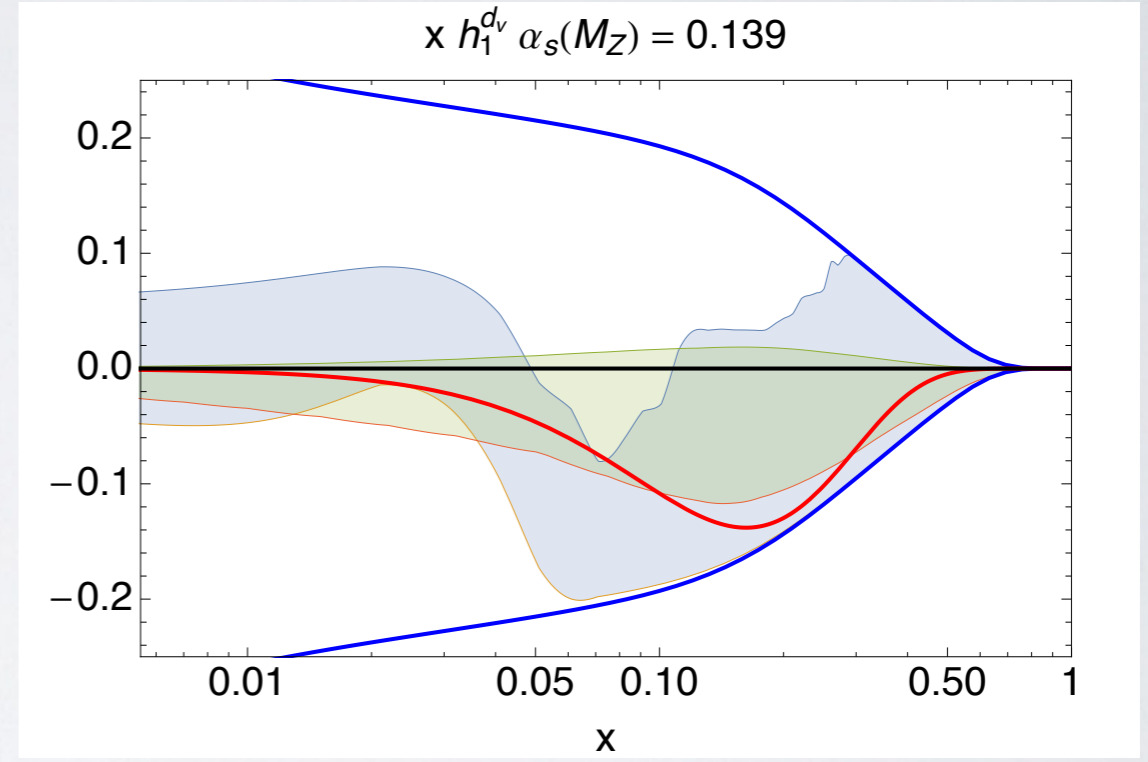
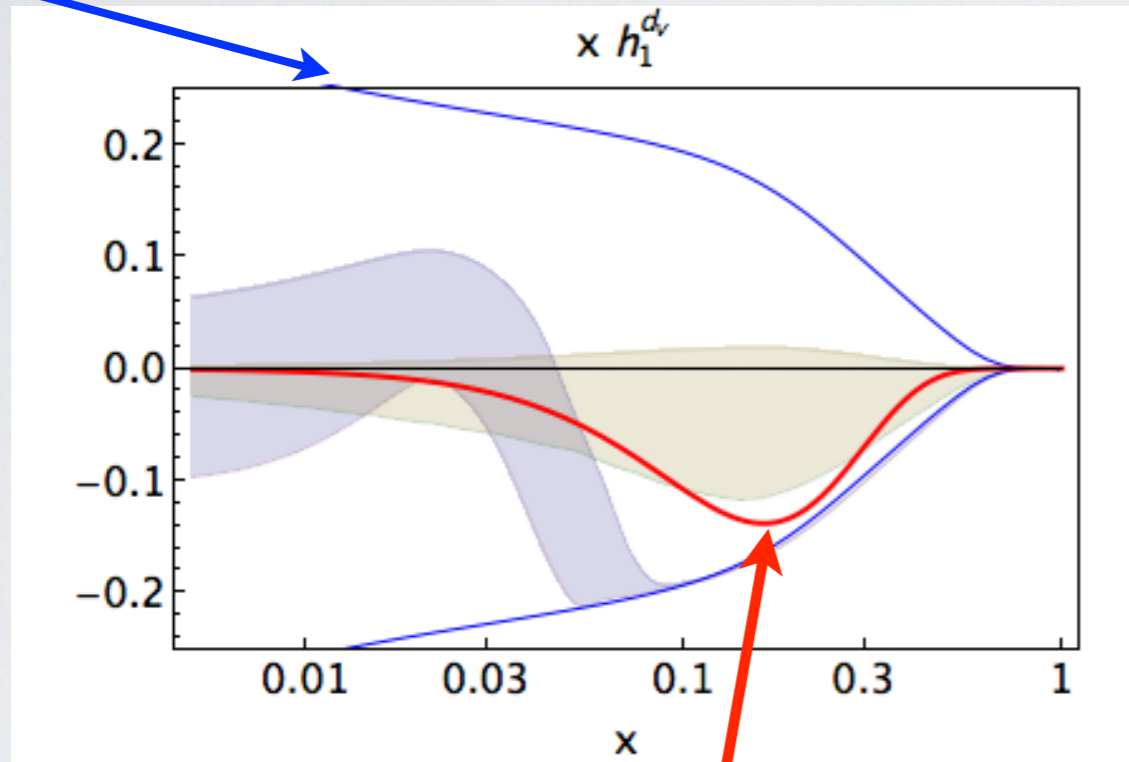
full SIDIS fit

down

“reduced” SIDIS fit :  
no bins #7,8 with deuteron



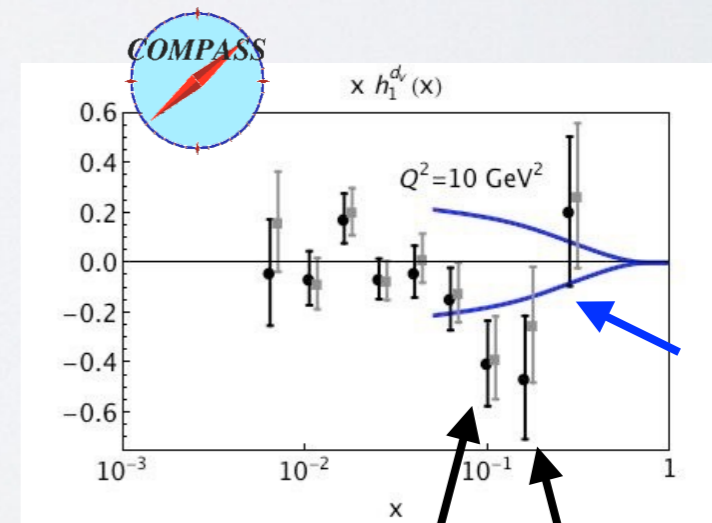
Soffer bound



Radici et al.,  
*JHEP* **1505** (15) 123

Kang et al.,  
*P.R. D93* (16) 014009


Anselmino et al.,  
*P.R. D87* (13) 094019

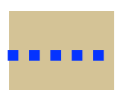


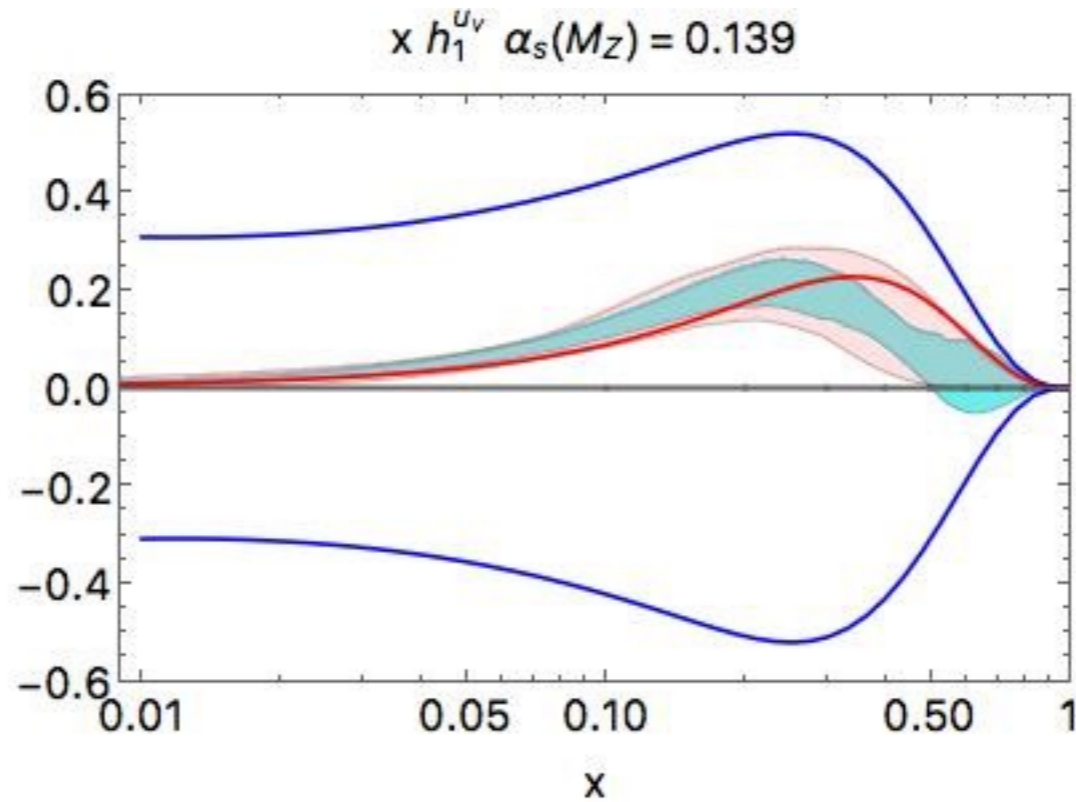
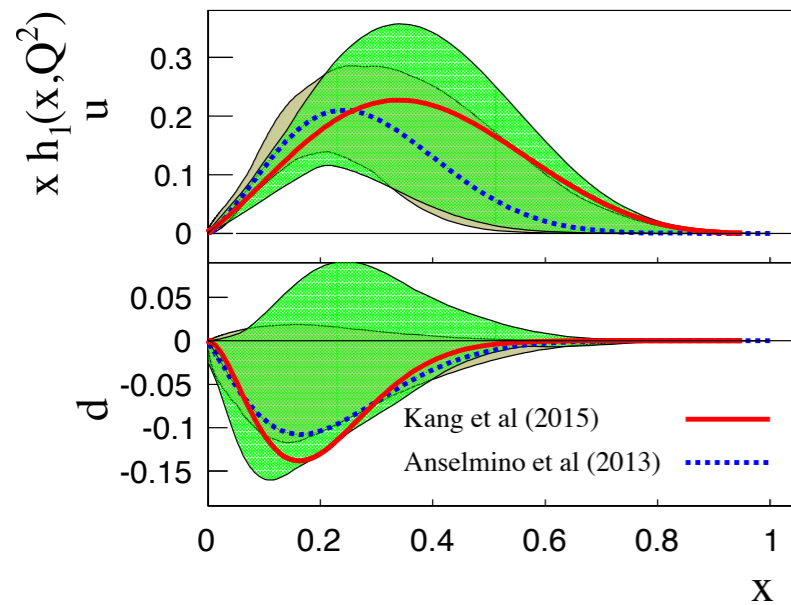
Soffer bound

bins #7,8

# Comparison with Collins effect

 Kang et al. ("**TMDfit**"),  
P.R. D93 (16) 014009

 Anselmino et al. (**Torino**),  
P.R. D87 (13) 094019

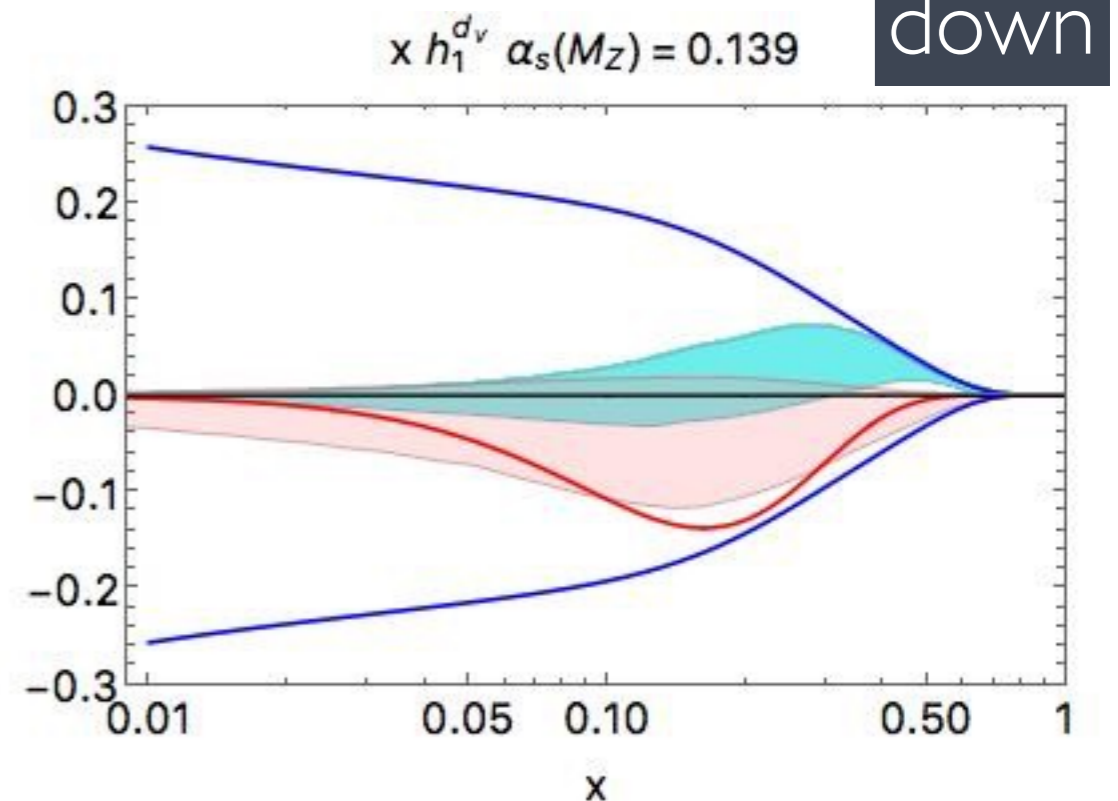


up

global fit

Torino

"TMDfit"

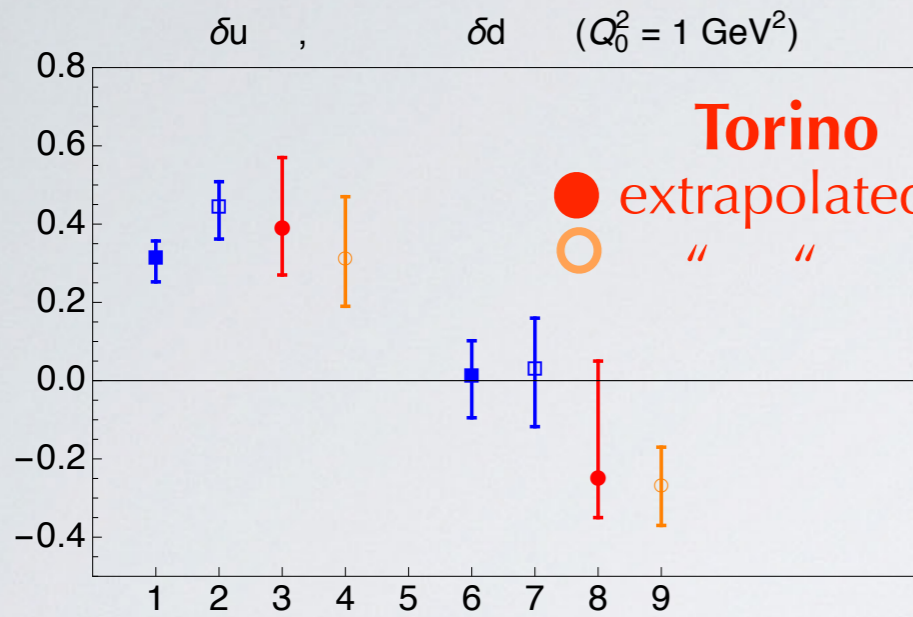


down

global fit : • gain in precision  
• some tension with deuteron

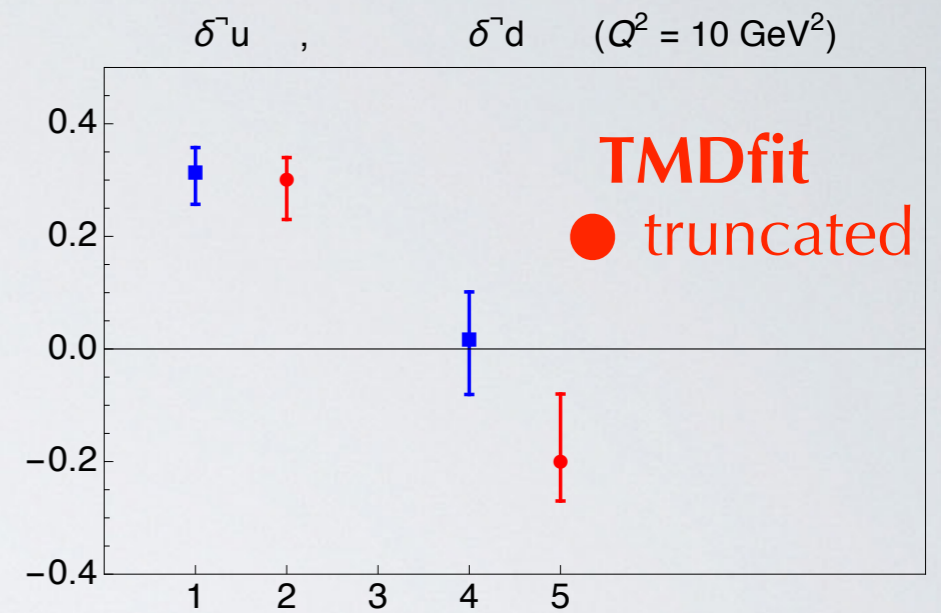


# tensor charge $\delta q(Q^2) = \int dx h_1^{q-\bar{q}}(x, Q^2)$



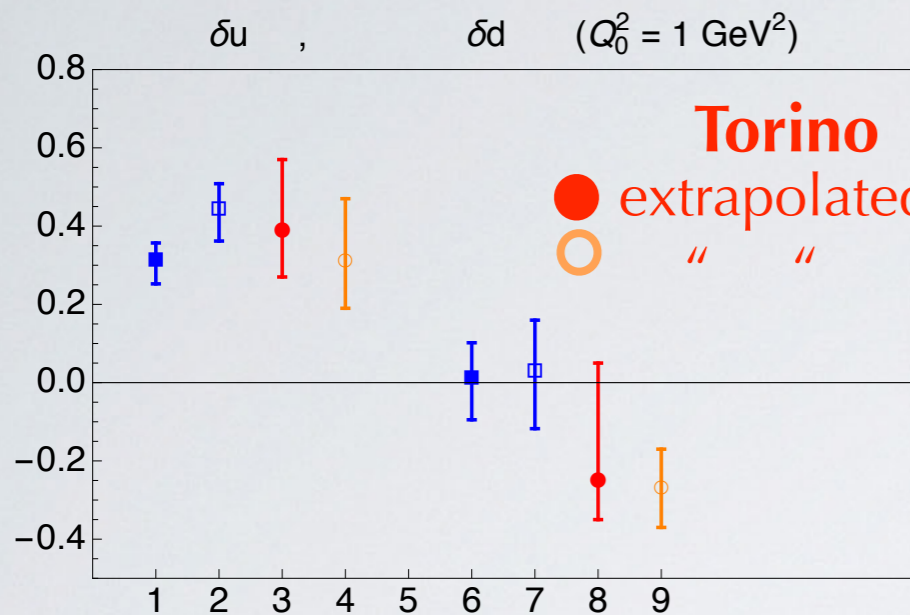
**Torino**  
 ● extrapolated,  $A_{12}$   
 ○ " "

**global fit**  
 ■ truncated  
 □ extrapolated



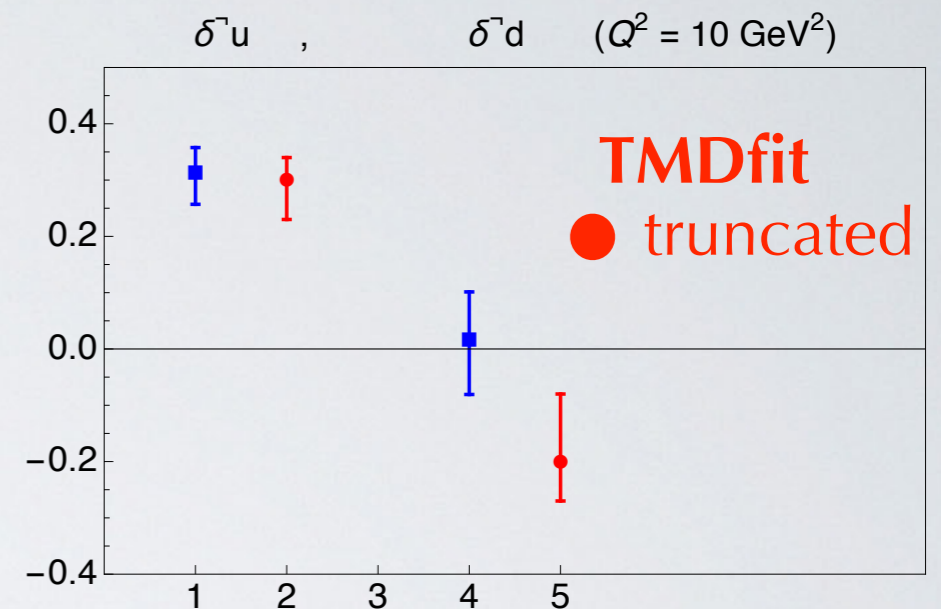
**TMDfit**  
 ● truncated

# tensor charge $\delta q(Q^2) = \int dx h_1^{q-\bar{q}}(x, Q^2)$



**Torino**  
 ● extrapolated,  $A_{12}$   
 ○ " "

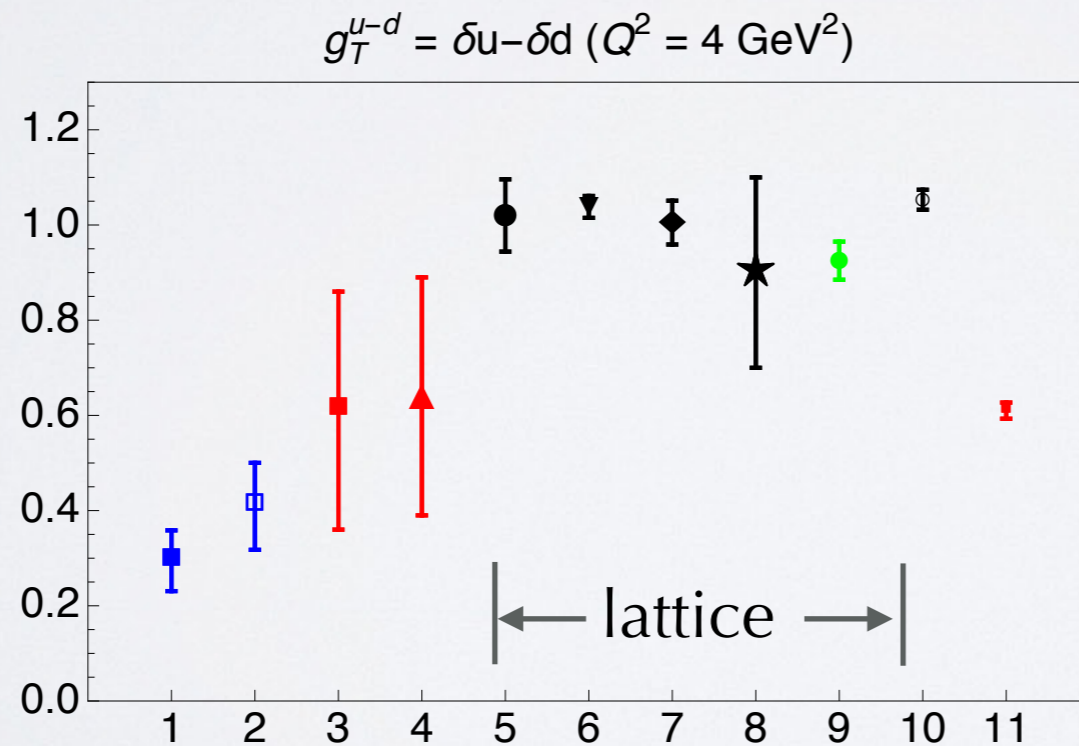
**global fit**  
 ■ truncated  
 □ extrapolated



**TMDfit**  
 ● truncated

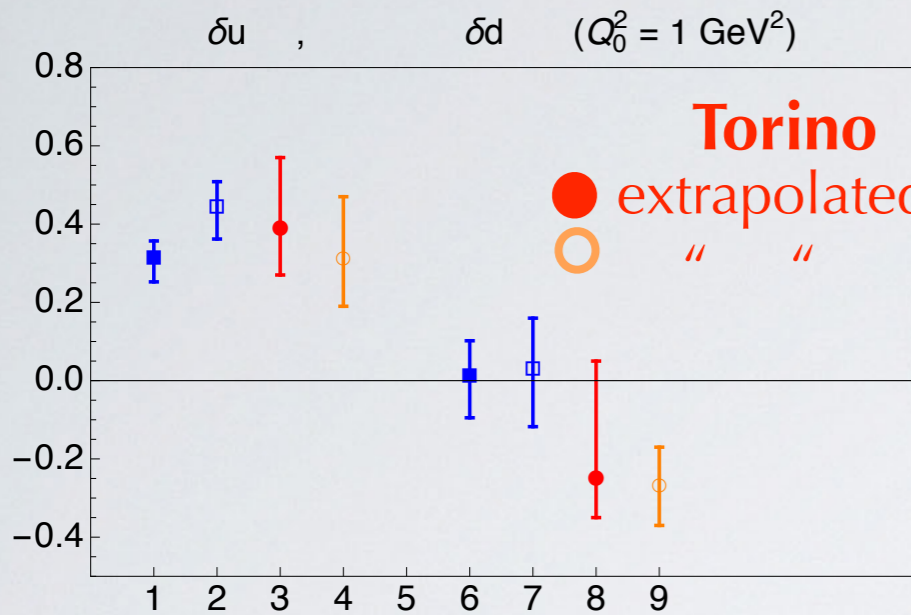
**global fit**

- 1) truncated
- 2) extrapolated
- 3) "TMDfit"
- 4) Torino



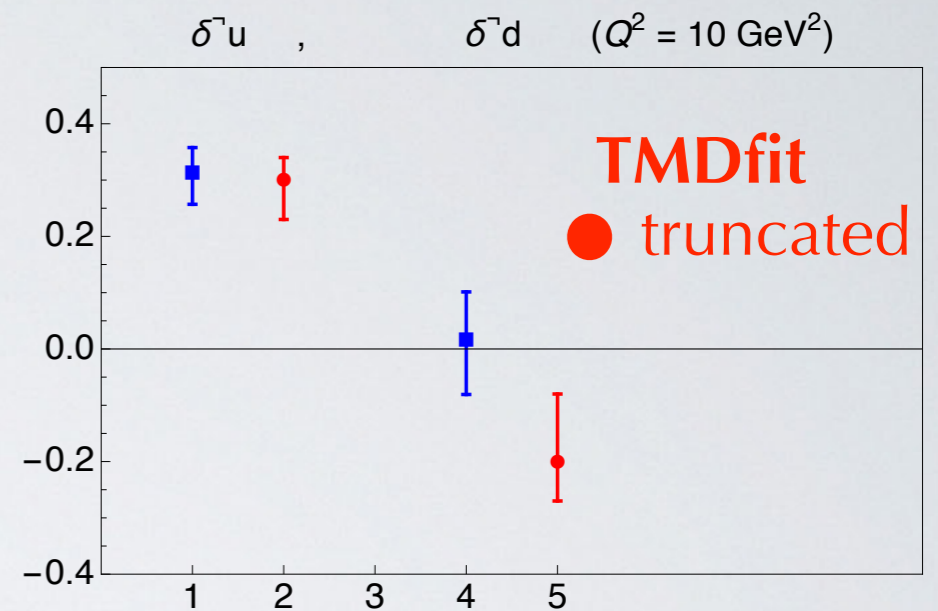
- 5) PNDME '15 *Bhattacharya et al., P.R. D92 (15)*
- 6) LHPC '12 *Green et al., P.R. D86 (12)*
- 7) RQCD '14 *Bali et al., P.R. D91 (15)*
- 8) RBC-UKQCD *Aoki et al., P.R. D82 (10)*
- 9) ETMC '17 *Alexandrou et al., arXiv:1703.08788*
- 10) ETMC '15 *Abdel-Rehim et al., P.R.D92 (15); E P.R.D93 (16)*
- 11) SOLID *Ye et al., P.L. B767 (17) 91*

# tensor charge $\delta q(Q^2) = \int dx h_1^{q-\bar{q}}(x, Q^2)$



**Torino**  
 ● extrapolated,  $A_{12}$   
 ○ " "  $A_0$

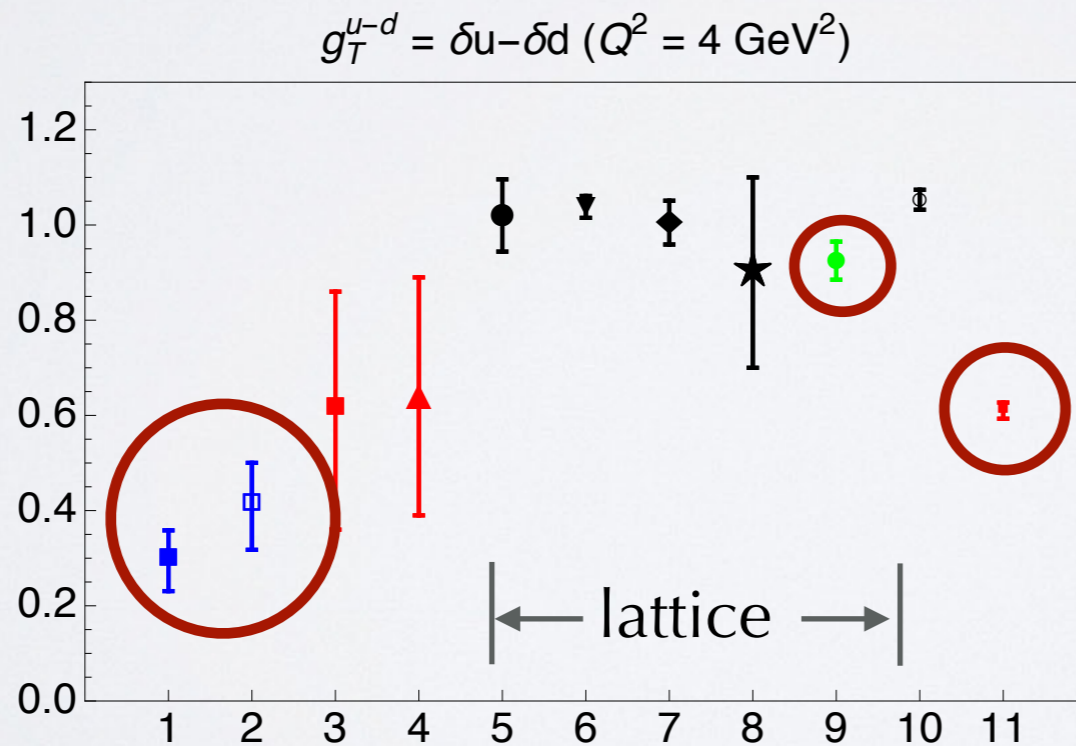
**global fit**  
 ■ truncated  
 □ extrapolated



**TMDfit**  
 ● truncated

**global fit**

- 1) truncated
- 2) extrapolated
- 3) "TMDfit"
- 4) Torino



- 5) PNDME '15 *Bhattacharya et al., P.R. D92 (15)*
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**precision !**

# precision : potential for BSM searches

$$\begin{aligned} P^{[\mu} S^{\nu]} g_T^q(Q^2) &= P^{[\mu} S^{\nu]} \int_0^1 dx [h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2)] \\ &= \langle P, S | \bar{q} \sigma^{\mu\nu} q | P, S \rangle \end{aligned}$$

tensor operator not directly accessible in  $\mathcal{L}_{\text{SM}}$   
low-energy footprint of new physics (BSM) at higher scales ?

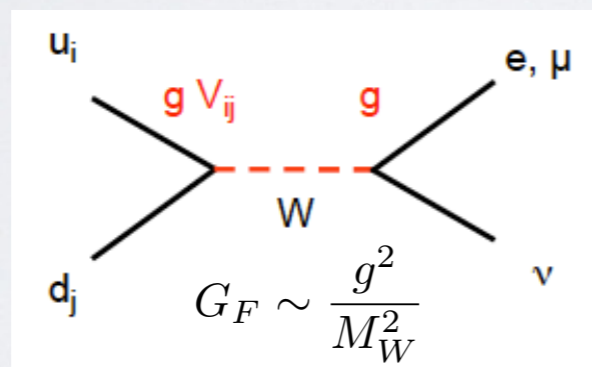
# precision : potential for BSM searches

$$P^{[\mu} S^{\nu]} g_T^q(Q^2) = P^{[\mu} S^{\nu]} \int_0^1 dx [h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2)]$$

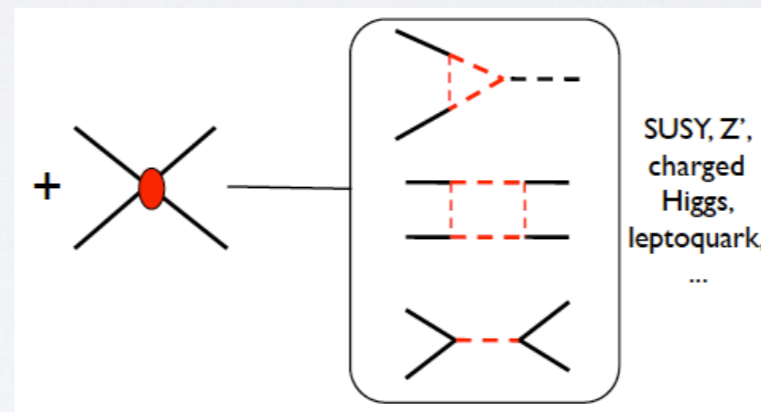
$$= \langle P, S | \bar{q} \sigma^{\mu\nu} q | P, S \rangle$$

tensor operator not directly accessible in  $\mathcal{L}_{SM}$   
 low-energy footprint of new physics (BSM) at higher scales ?

Example: neutron  $\beta$ -decay  $n \rightarrow p e^- \bar{\nu}_e$



$\mathcal{L}_{SM}$  universal V-A



$\mathcal{L}_{BSM}$  new couplings:  $\epsilon_S 1$ ,  $\epsilon_{PS} \gamma_5$ ,  $\epsilon_T \sigma^{\mu\nu}$

$$\epsilon_T g_T \approx M_W^2 / M_{BSM}^2$$

precision of 0.1%  $\Rightarrow$  [3-5] TeV bound for BSM scale

# precision of $g_T^{u-d}$

current most stringent constraints on BSM tensor coupling come from

- Dalitz-plot study of radiative pion decay  $\pi^+ \rightarrow e^+ \nu_e \gamma$

*Bychkov et al. (PIBETA), P.R.L. 103 (09) 051802*

- measurement of correlation parameters in neutron  $\beta$ -decay of various nuclei

*Pattie et al., P.R. C88 (13) 048501*

$$|\epsilon_T g_T| \approx 5 \times 10^{-4}$$

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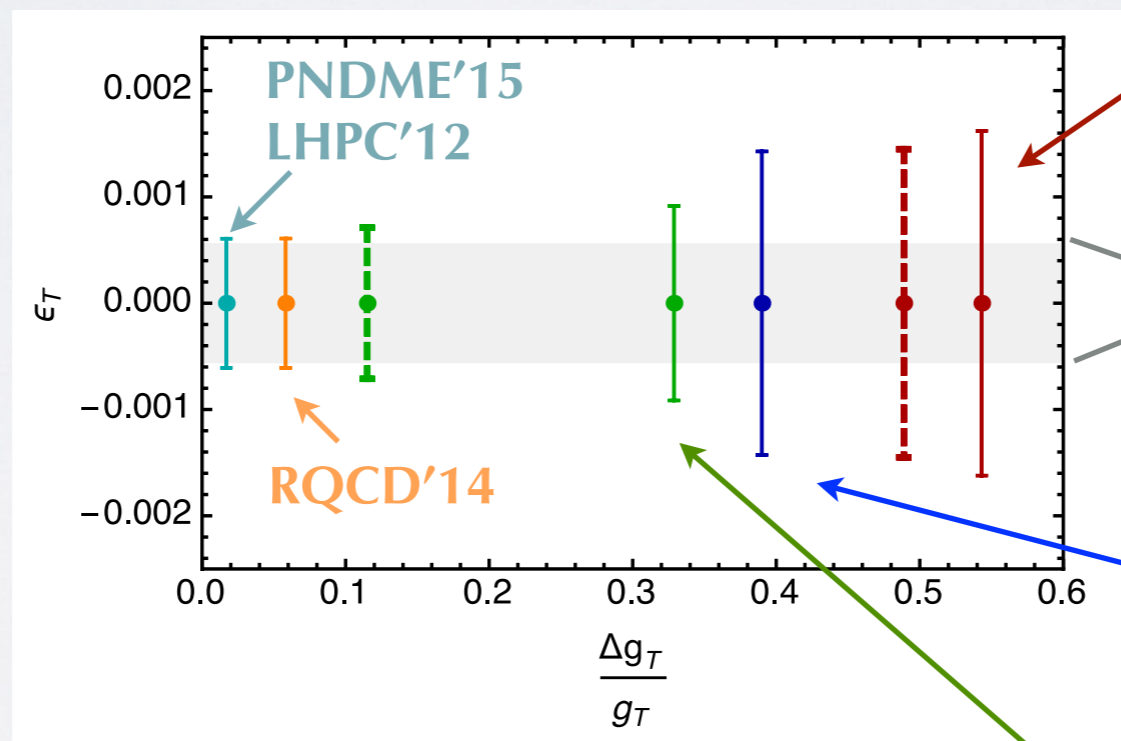
*Bychkov et al. (PIBETA), P.R.L. 103 (09) 051802*

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*Pattie et al., P.R. C88 (13) 048501*

$$|\epsilon_T g_T| \approx 5 \times 10^{-4}$$

$\Delta\epsilon_T$  from *Radici et al., JHEP 1505 (15) 123*



$\Delta\epsilon_T$  assuming  $\Delta g_T=0$

$\Delta\epsilon_T$  from Torino

*Goldstein et al., arXiv:1401.0438*

*Courtoy et al., P.R.L. 115 (2015) 162001*

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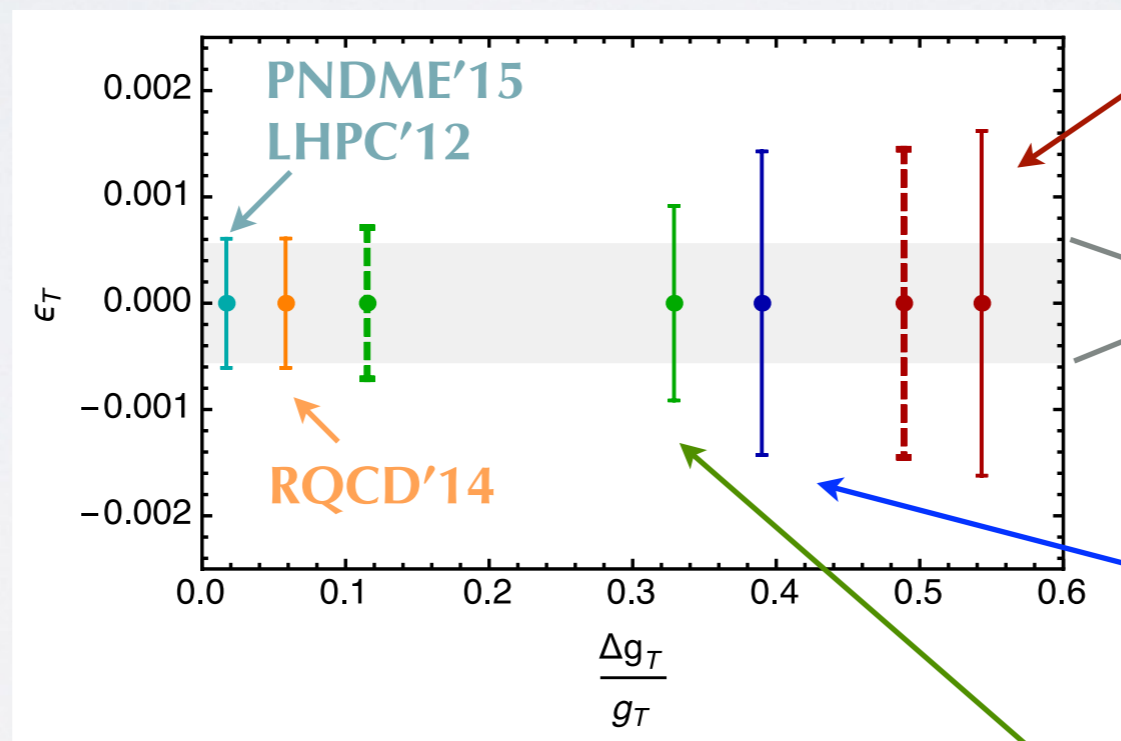
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$$|\epsilon_T g_T| \approx 5 \times 10^{-4}$$

$\Delta\epsilon_T$  from *Radici et al., JHEP 1505 (15) 123*

need more data  
to adapt  
phenomenology  
to precision of  
measurements  
and lattice



(to be improved  
by global fit)

$\Delta\epsilon_T$  assuming  $\Delta g_T=0$

$\Delta\epsilon_T$  from Torino

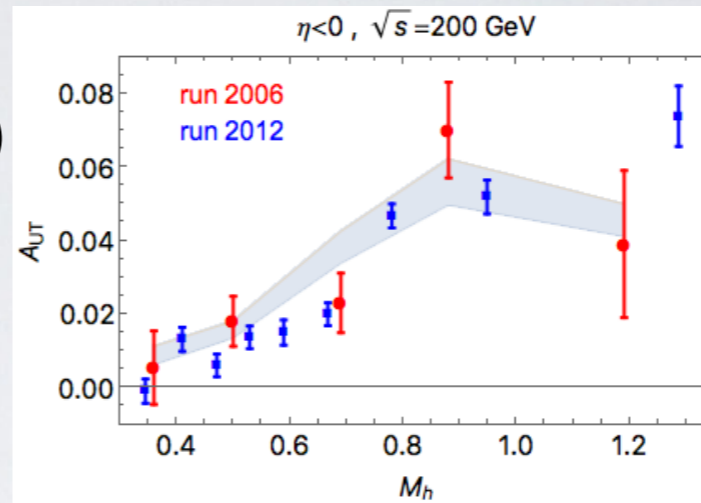
*Goldstein et al., arXiv:1401.0438*

*Courtoy et al., P.R.L. 115 (2015) 162001*

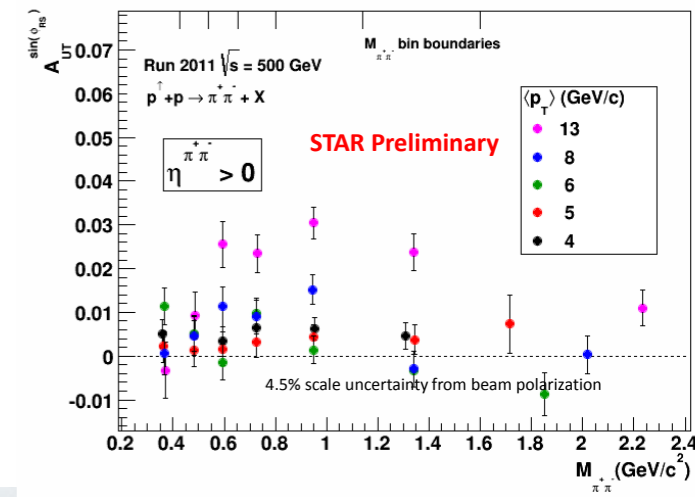


# To do list

- use also other (multi-dimensional) data from STAR run 2012 ( $s=200$ ) and run 2011 ( $s=500$ )



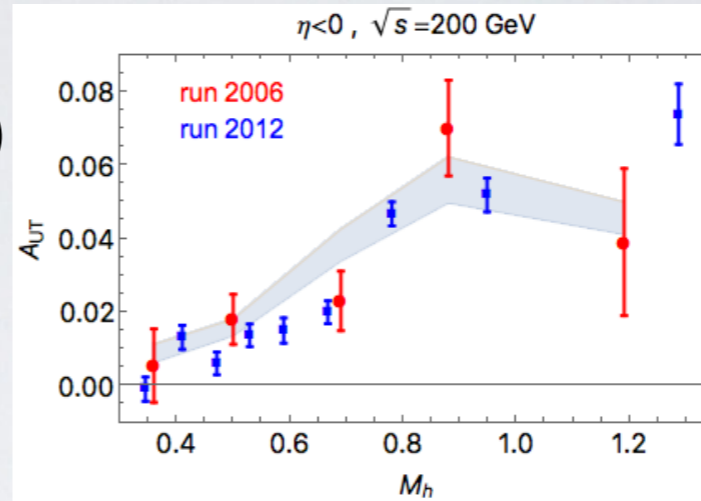
*Radici et al., P.R. D94 (16) 034012*



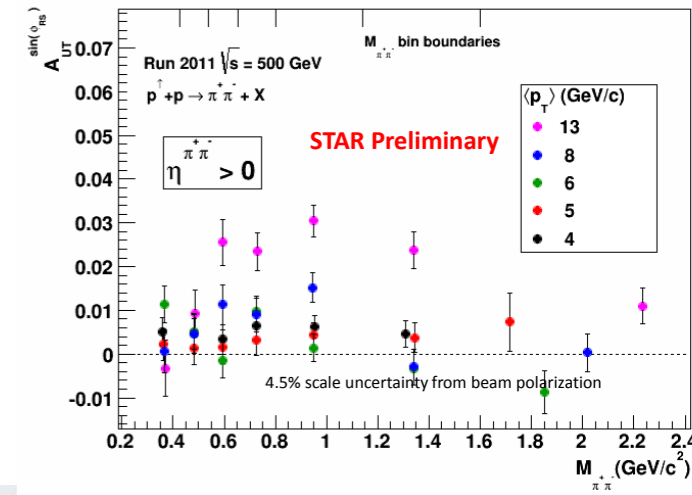
*M. Skoby, SPIN 2014*

# To do list

- use also other (multi-dimensional) data from STAR run 2012 ( $s=200$ ) and run 2011 ( $s=500$ )



*Radici et al., P.R. D94 (16) 034012*



*M. Skoby, SPIN 2014*

- wait for data on unpolarized cross section  $d\sigma^0$  :

$e^+e^- \rightarrow (\pi\pi) X$  constrains  $D_{1q}$

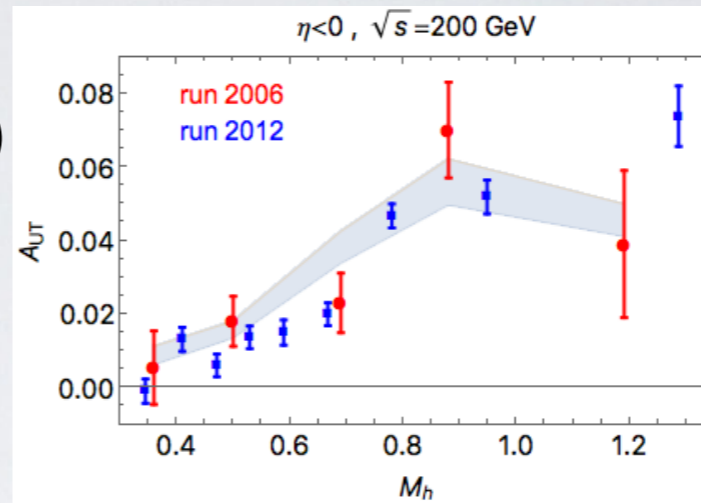
$p+p \rightarrow (\pi\pi) X$  constrains  $D_{1g}$

$$A_{UT} = \frac{d\sigma_{UT}}{d\sigma^0}$$

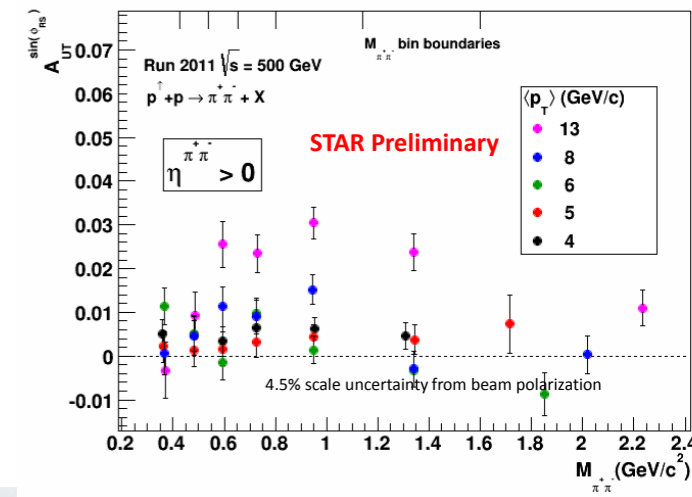
large K factor in  $d\sigma^0$ ? ( but not in  $d\sigma_{UT}$  )  
uncertainty band probably underestimated  
but no K factor can modify  $A_{UT}(M_h)$

# To do list

- use also other (multi-dimensional) data from STAR run 2012 ( $s=200$ ) and run 2011 ( $s=500$ )



*Radici et al., P.R. D94 (16) 034012*



*M. Skoby, SPIN 2014*

- wait for data on unpolarized cross section  $d\sigma^0$  :

$e^+e^- \rightarrow (\pi\pi) X$  constrains  $D_{1^q}$

$p+p \rightarrow (\pi\pi) X$  constrains  $D_{1^g}$

$$A_{UT} = \frac{d\sigma_{UT}}{d\sigma^0}$$

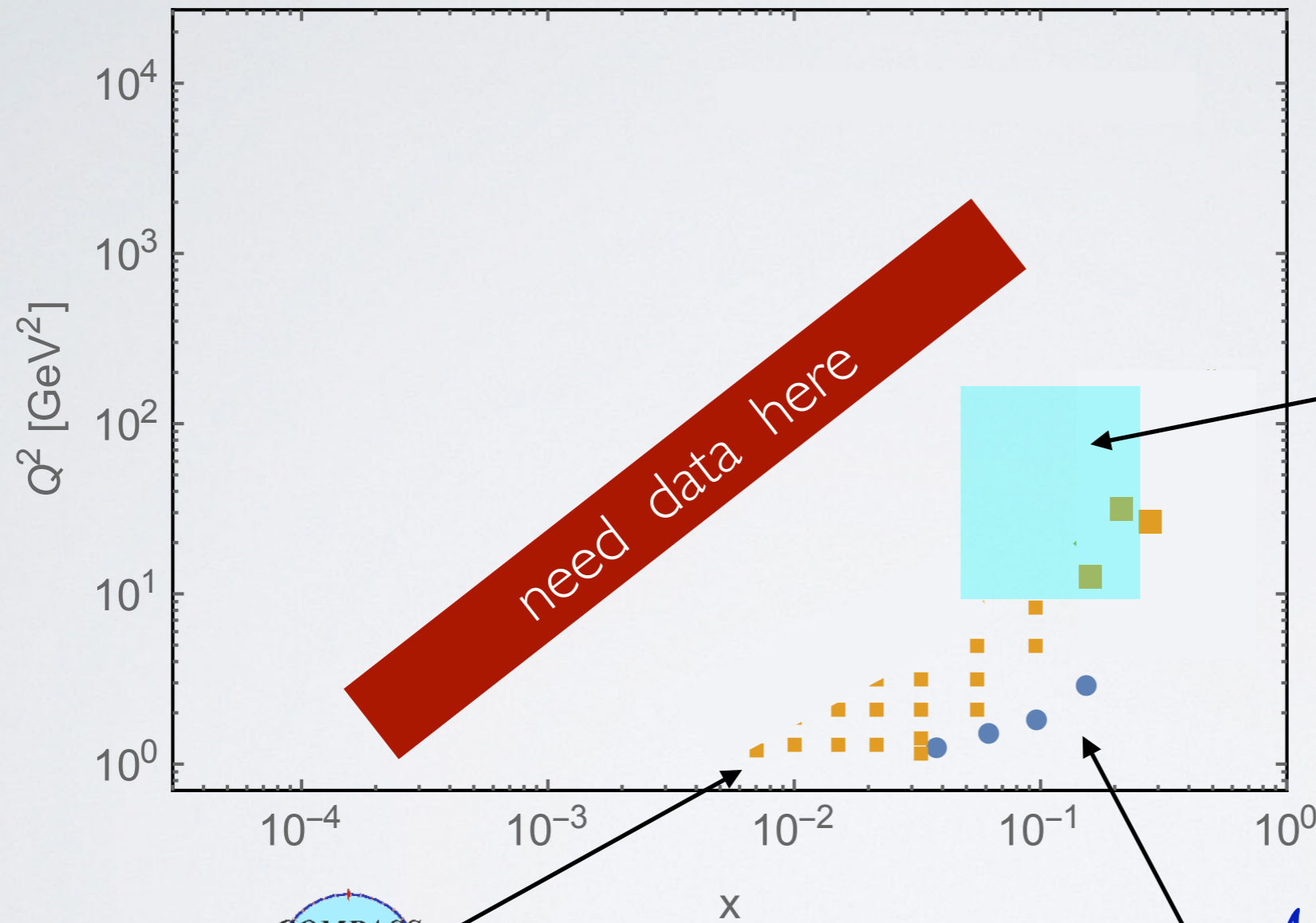
large K factor in  $d\sigma^0$ ? ( but not in  $d\sigma_{UT}$  )  
 uncertainty band probably underestimated  
 but no K factor can modify  $A_{UT}(M_h)$

- use Compass data on  $\pi K$  and  $KK$  channels :  
 constrain strange contribution ?

- explore other channels, like inclusive DIS via Jet fragm. funct.'s

*Accardi and Bacchetta, arXiv:1706.02000*

# the kinematics



Adamczyk et al. (STAR),  
*P.R.L.* **115** (2015) 242501



Adolph et al., *P.L.* **B713** (12)  
Braun et al., *E.P.J. Web Conf.* **85** (15) 02018



Airapetian et al.,  
*JHEP* **0806** (08) 017

# Conclusions

- first global fit of di-hadron inclusive data leading to extraction of transversity in collinear framework (PRELIMINARY!)
- inclusion of STAR  $p$ - $p^\uparrow$  data increases precision of extracted transversity and eliminates suspicious behavior of down channel; some tension with extraction from Collins effect
- tensor charge useful for low-energy explorations of BSM new physics  $\Rightarrow$  precision is an issue. In this respect, the global fit is a significant step forward

**THANK YOU**