Connections between TMD and collinear (twist-3) observables

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TMD Topical Collaboration

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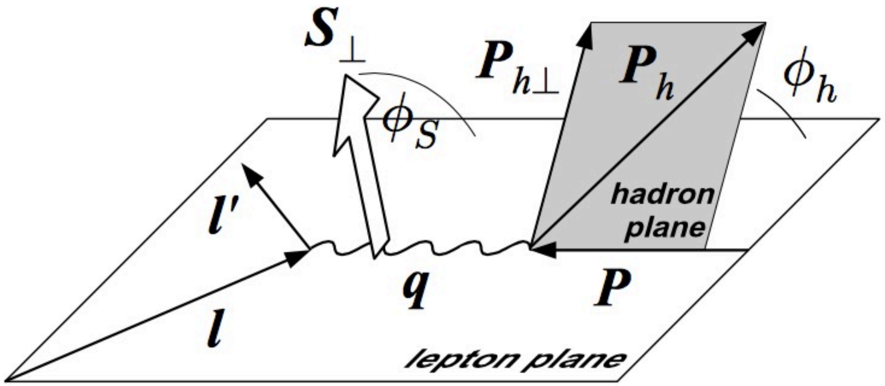
Outline

- Background
  - Transverse single-spin asymmetries
  - TMD and collinear twist-3 (CT3) functions
- TMD and CT3 observables
  - Sivers and Collins effects
  - A_N in $pp \rightarrow \{\gamma , \pi\}\ X$
- Relations between TMD and CT3 functions
- Towards a global analysis of TMD and CT3 observables
- Summary

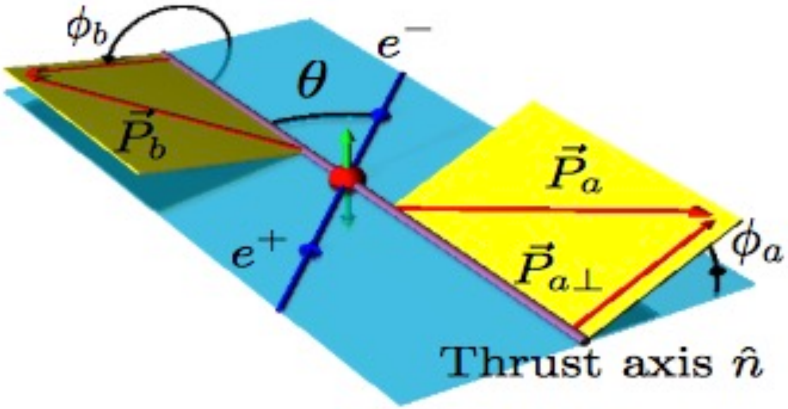
Background

$e\,N \to e'\,h\,X$



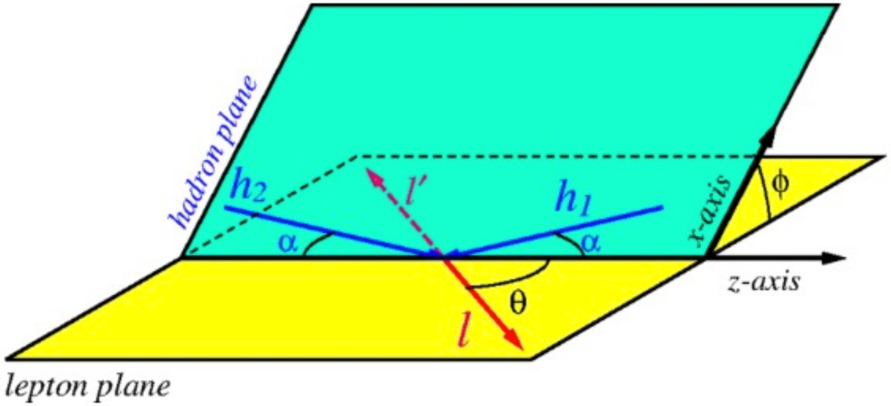
Sivers ~ *sin($\phi_h - \phi_s$)*, Collins ~ *sin($\phi_h + \phi_s$)*, …

$e^+e^- \to h_a h_b X$



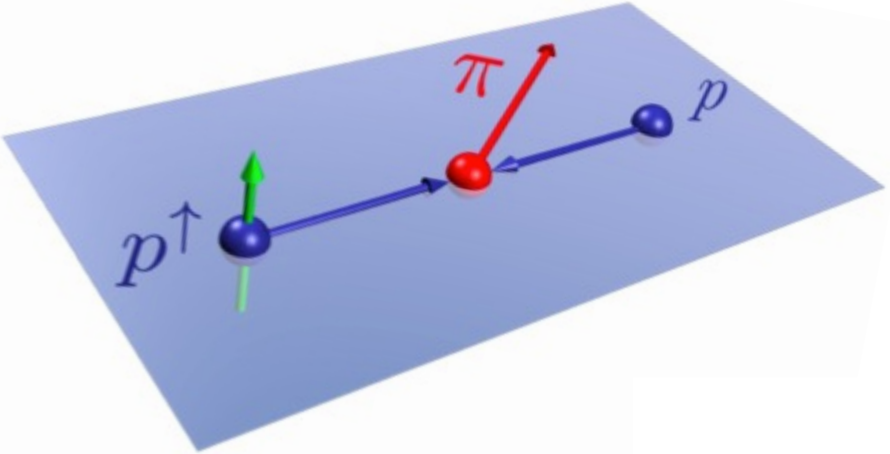
Collins ~ *cos($\phi_a + \phi_b$)*, …

$p^\uparrow\{p,\pi\} \to \{l^+l^-, W/Z\}X$



Sivers ~ *sin(ϕ_s)* (lepton pair) / Sivers ~ *cos($\phi_{W/Z}$)* (boson)

$p^\uparrow\{p,l\} \to \{\pi,\gamma\}X$

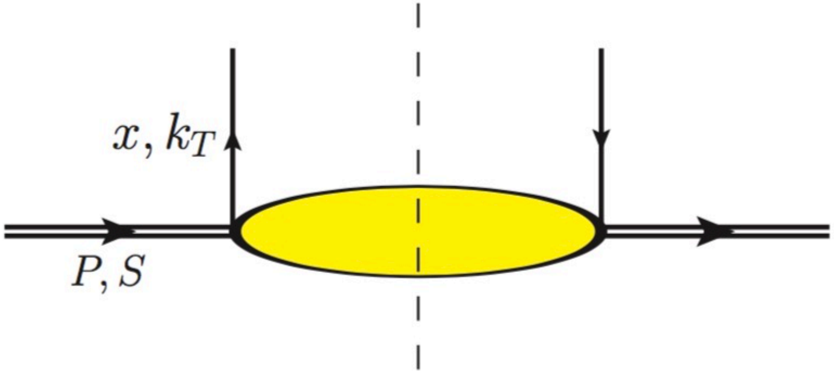


A_N ~ $d\sigma_L - d\sigma_R$

TMD PDFs (x, k_T)

q pol. H pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

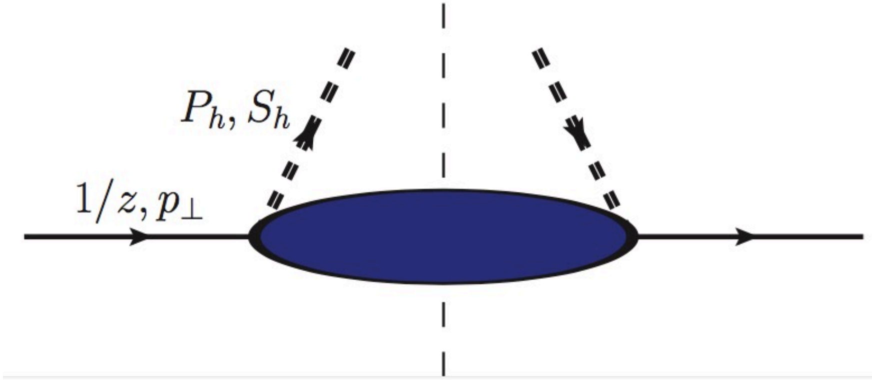
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

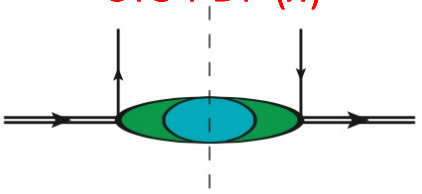
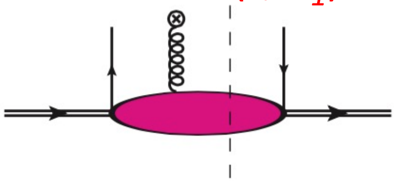
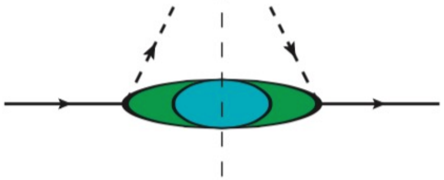
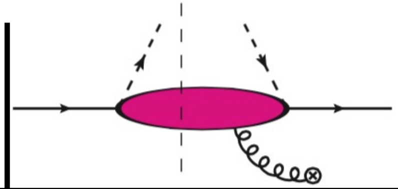


TMD FFs $(z, p_\perp)$

q pol. H pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))

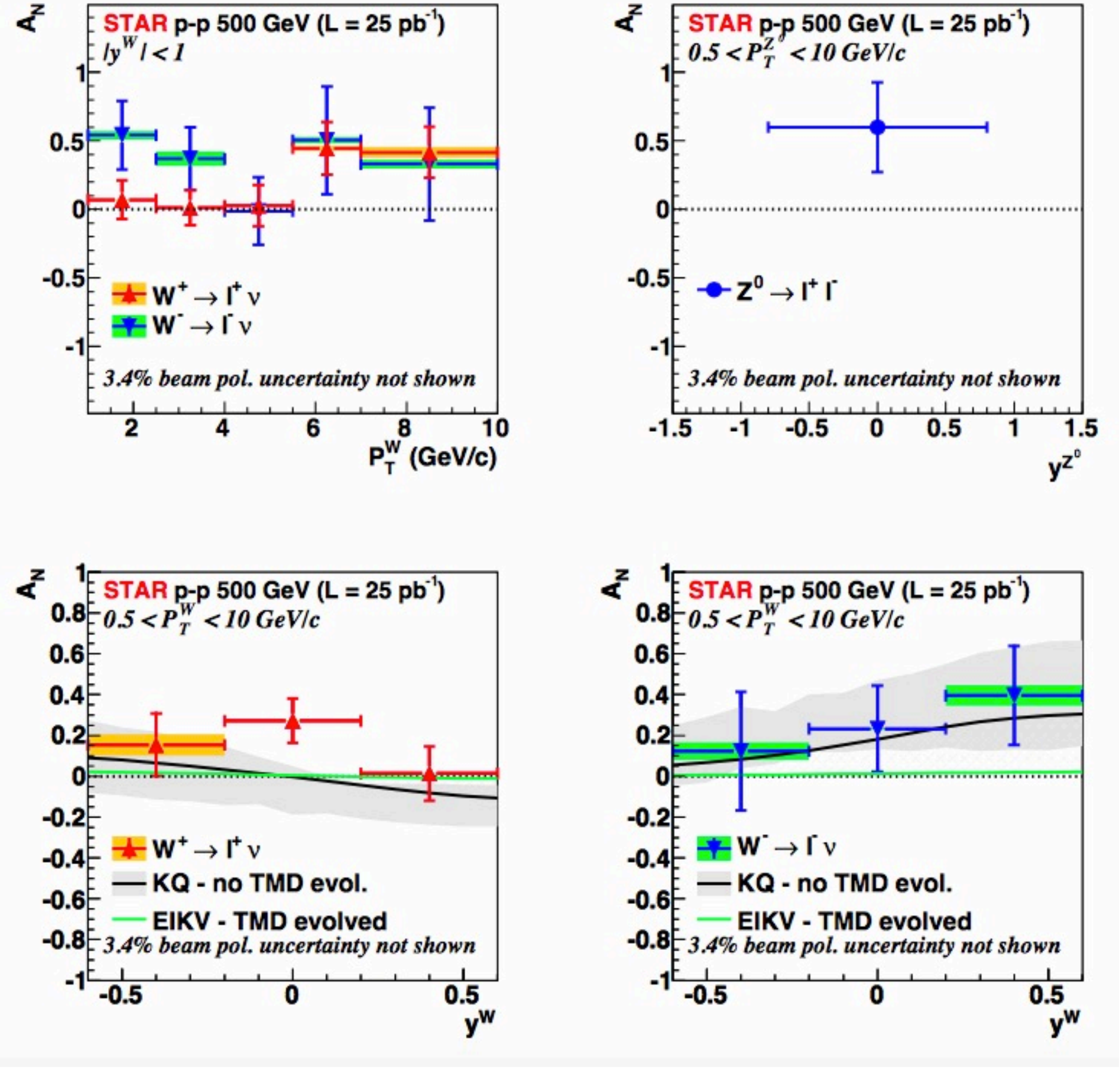


Hadron Pol.	CT3 PDF (x) 		CT3 PDF (x, x_1) 	CT3 FF (z) 		CT3 FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

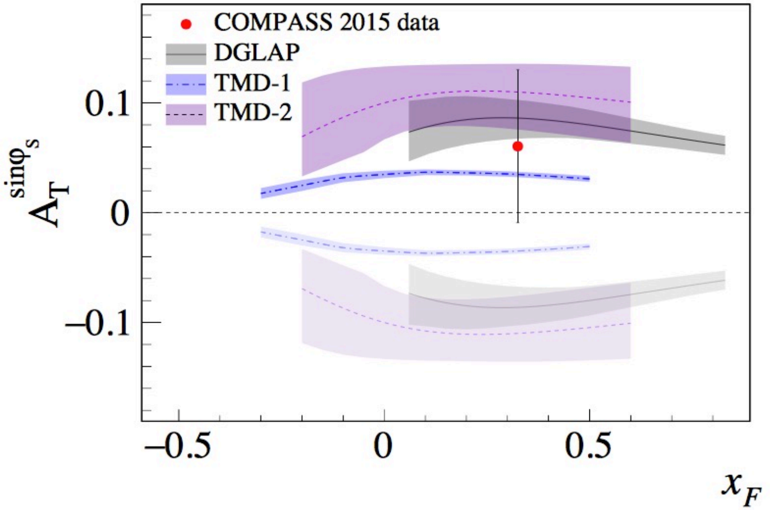
TMD and CT3 Observables

Drell-Yan Sivers effect

RHIC, STAR (2016)

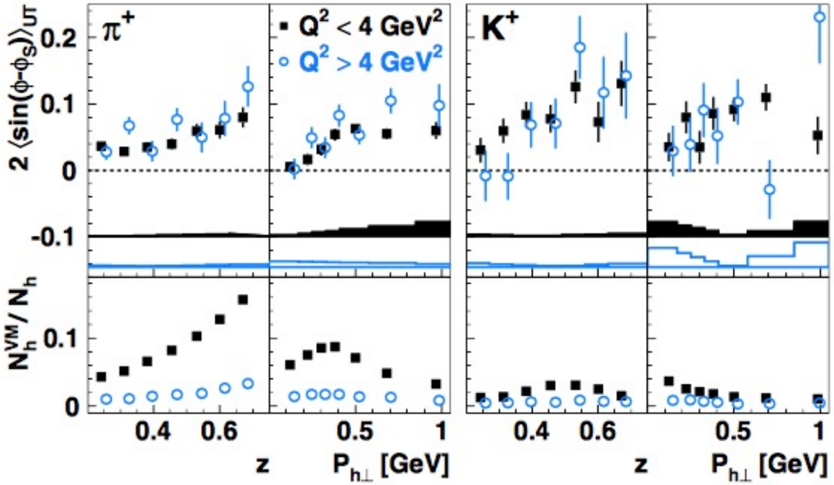


COMPASS (2017)

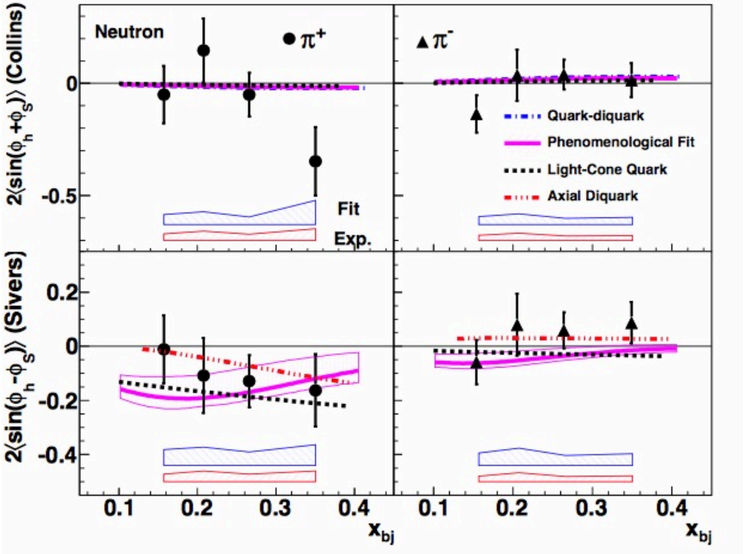


SIDIS Sivers effect (*sin(ϕ_h − ϕ_s)*)

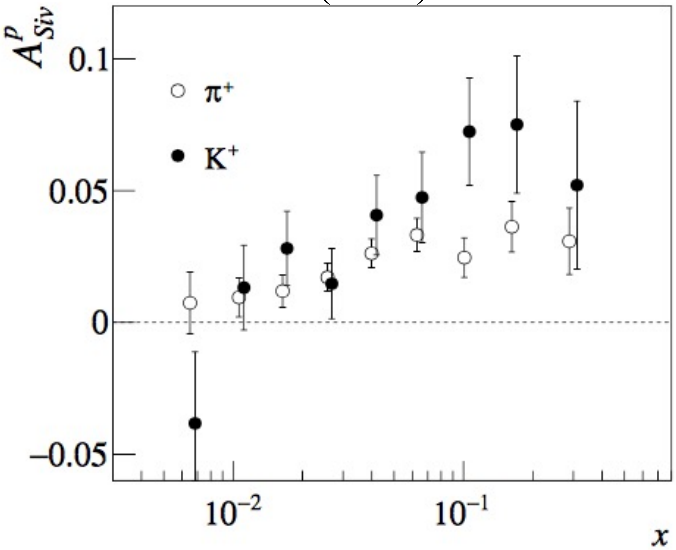
HERMES (2009)



JLab, Hall A (2011)

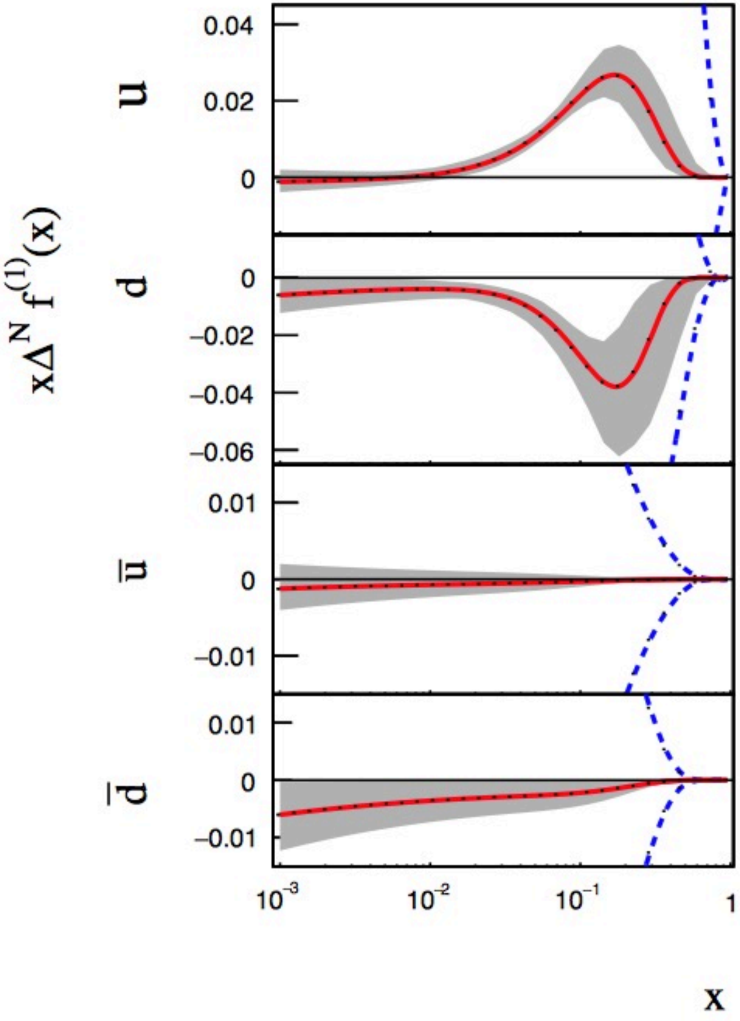


COMPASS (2015)

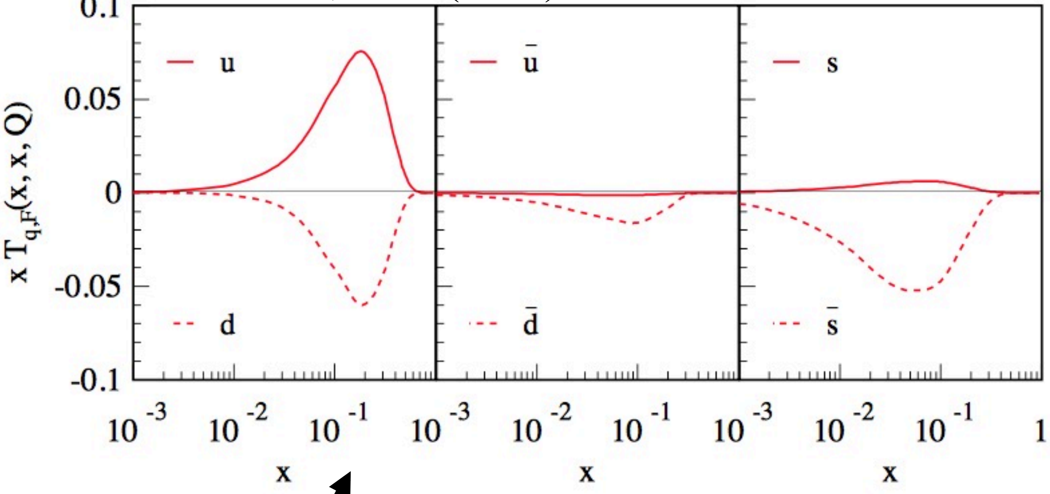


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M}\, \boldsymbol{f_{1T}^{\perp}} D_1\right]$$

Anselmino, et al. (2017)



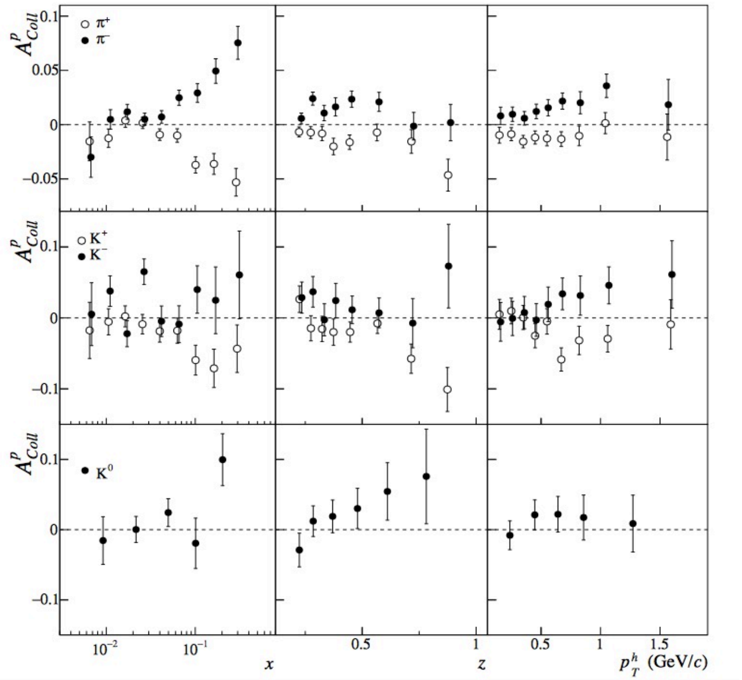
Echevarria, et al. (2014)



TMDs in CSS formalism

SIDIS Collins effect ($\sin(\phi_h + \phi_s)$)

COMPASS (2015)

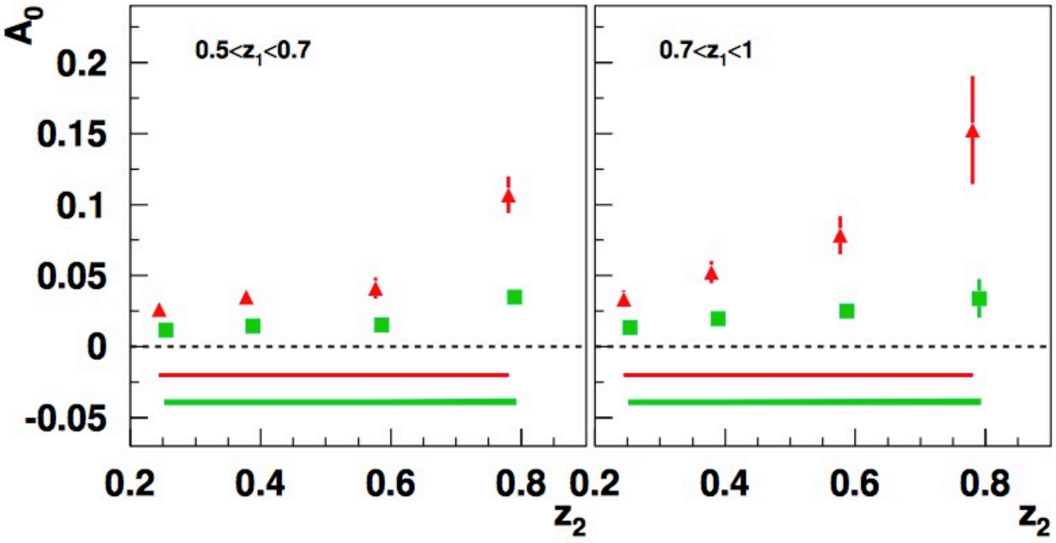


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\,\boldsymbol{H_1^\perp}\right]$$

e^+e^- Collins effect ($\cos(2\phi_0)$)

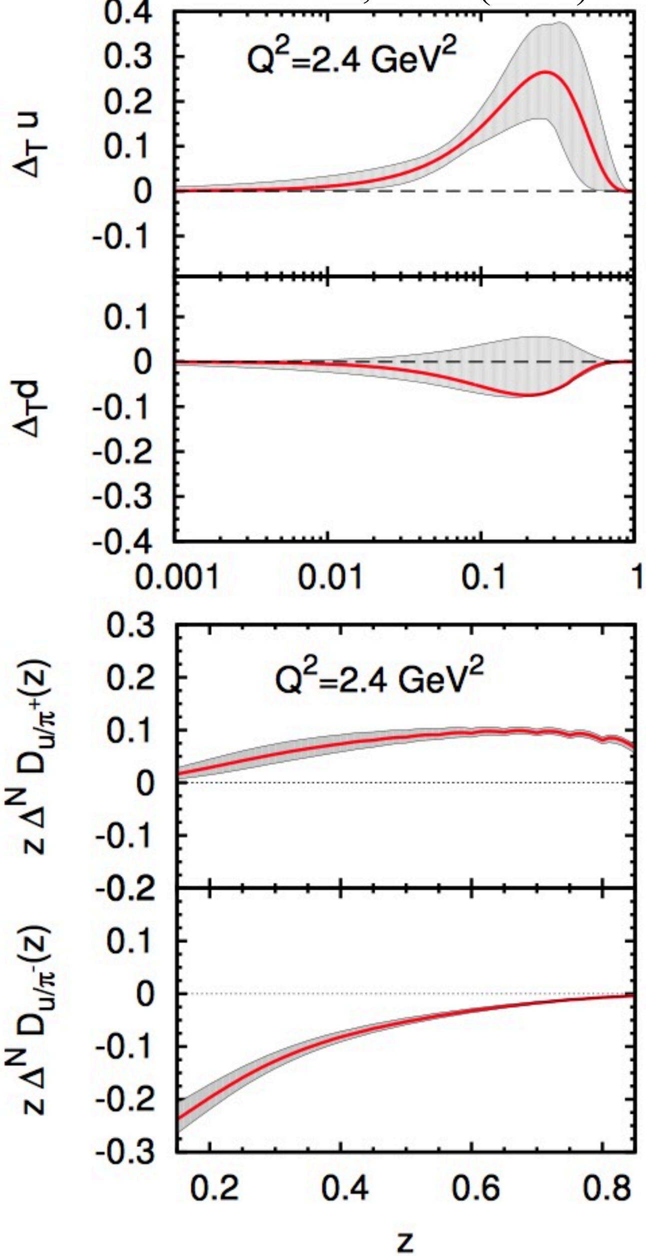
Belle (2008)



Also data from BaBar (2014) and BESIII (2016)

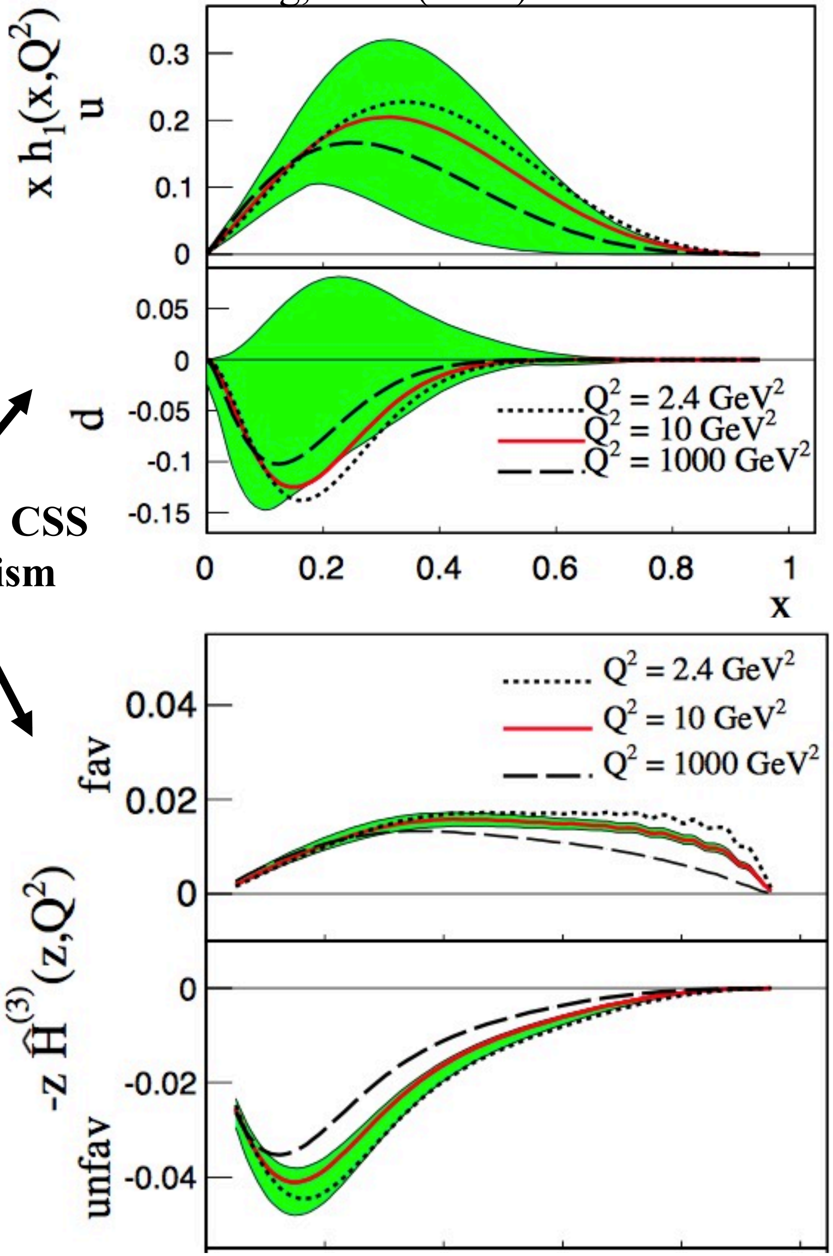
$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_aM_b}\,\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

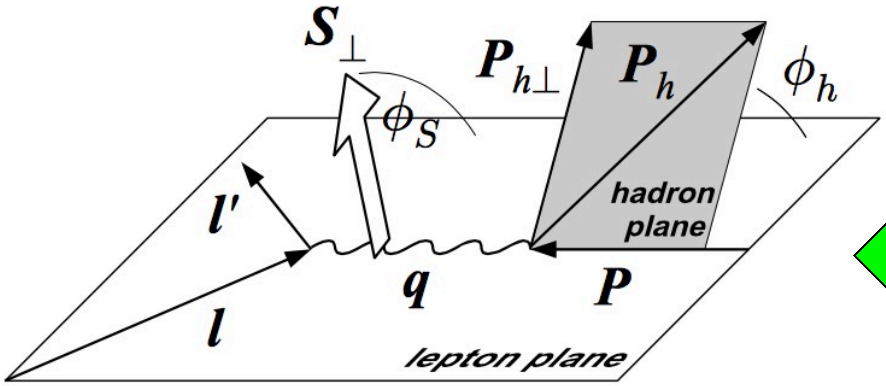
Anselmino, et al. (2015)



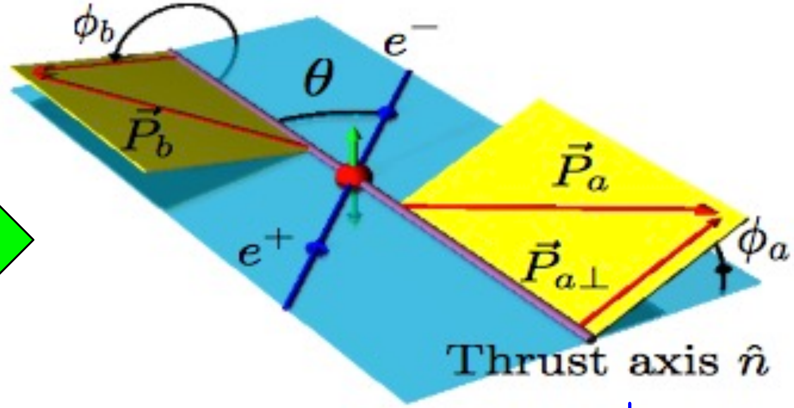
TMDs in CSS formalism

Kang, et al. (2016)

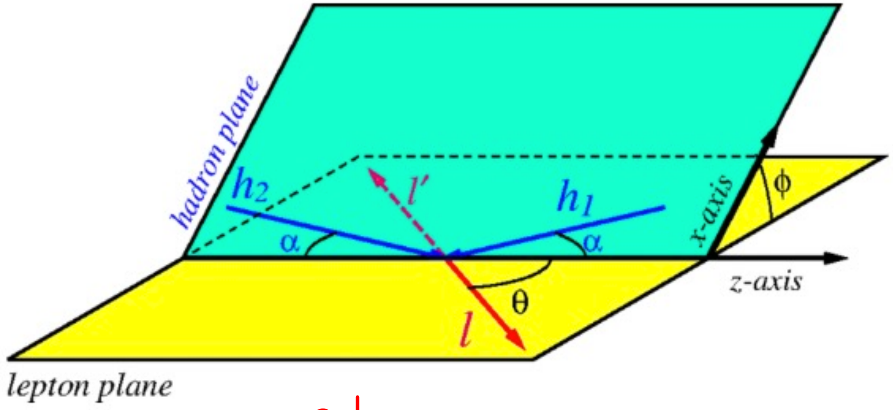




$h_1, f_{1T}^\perp, H_1^\perp$

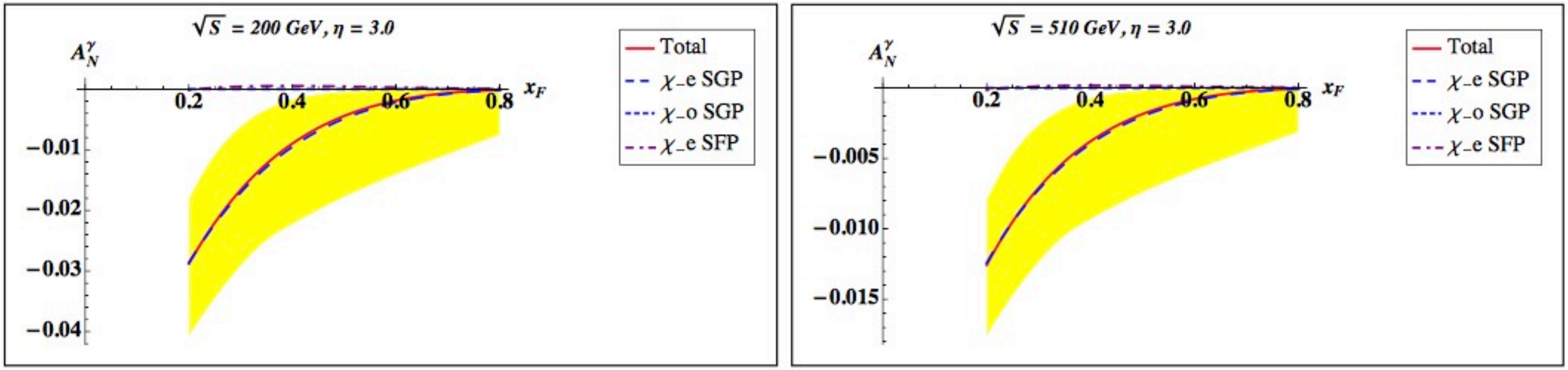


$H_1^\perp$



$f_{1T}^\perp$

A_N in *pp* -> γ *X*



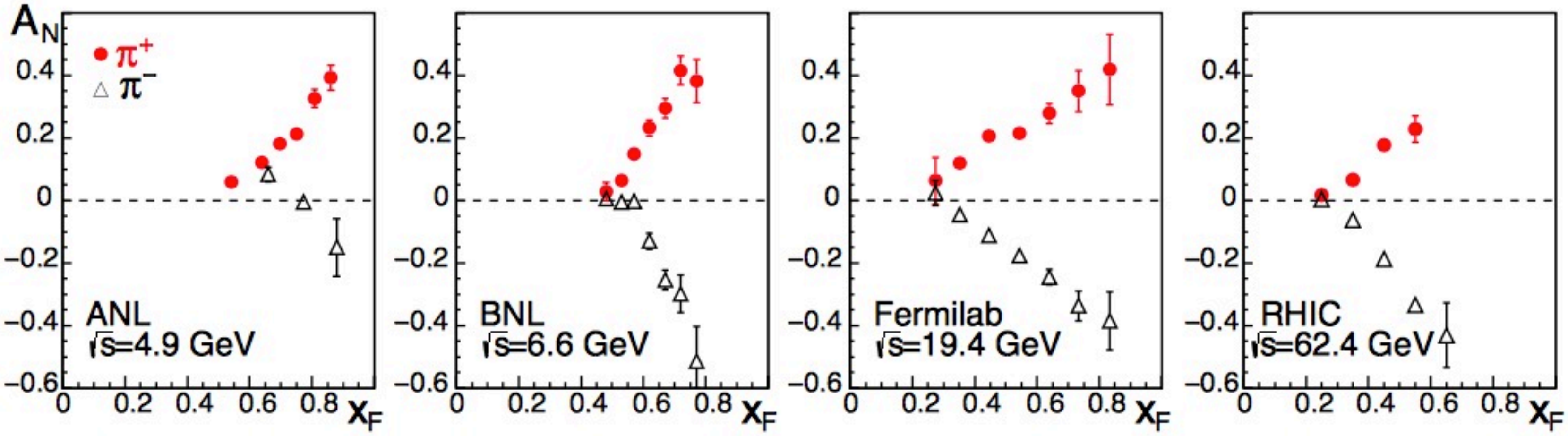
(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
(See also Gamberg, Kang, Prokudin (2013))

Qiu-Sterman term is the main
cause of A_N in *pp* -> γ *X*

$$d\Delta\sigma^\gamma \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

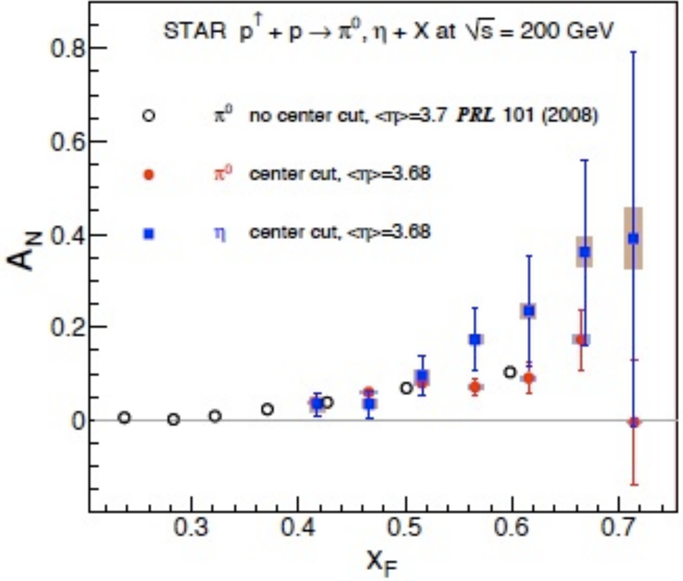
Qiu-Sterman function

A_N in $pp \to \pi X$ – PUZZLE FOR 40+ YEARS!

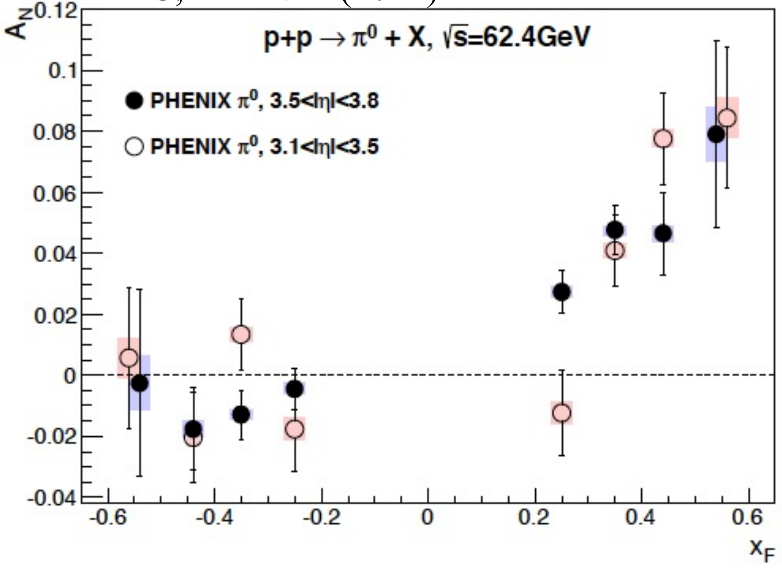


1976 →

RHIC, STAR (2012)



RHIC, PHENIX (2014)



$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^{1} \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^{1} \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

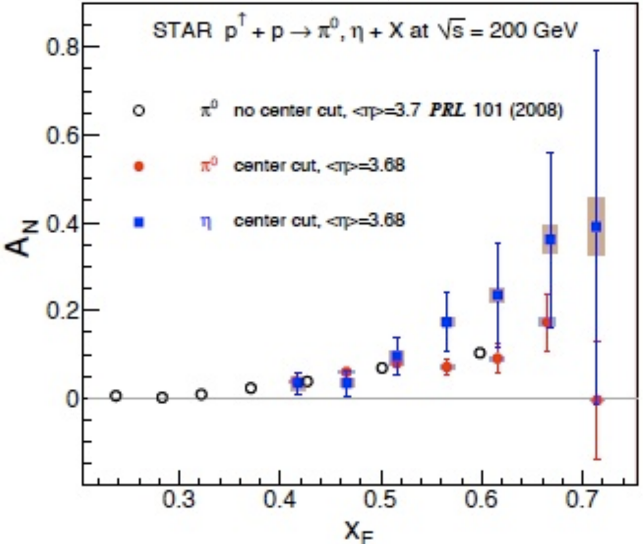
$$\boxed{F_{FT} \sim T_F}$$

(Qiu and Sterman (1999), Kouvaris, et al. (2006))

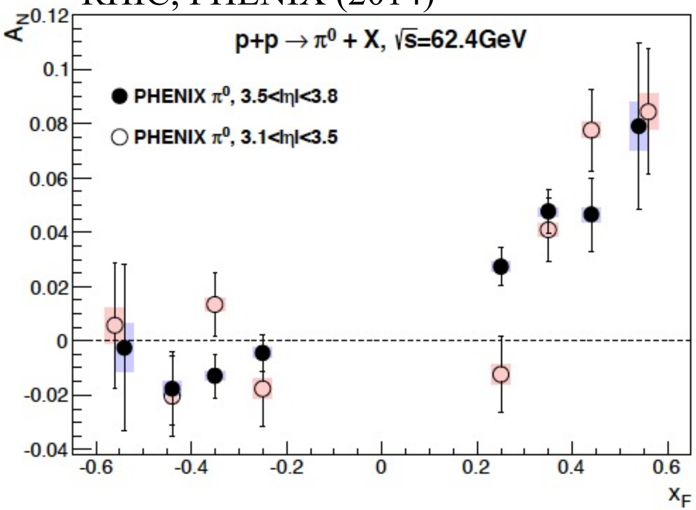
For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $p^{\uparrow}p \longrightarrow \pi\, X$

$$\boxed{p^{\uparrow}p \to h\,X}$$

RHIC, STAR (2012)

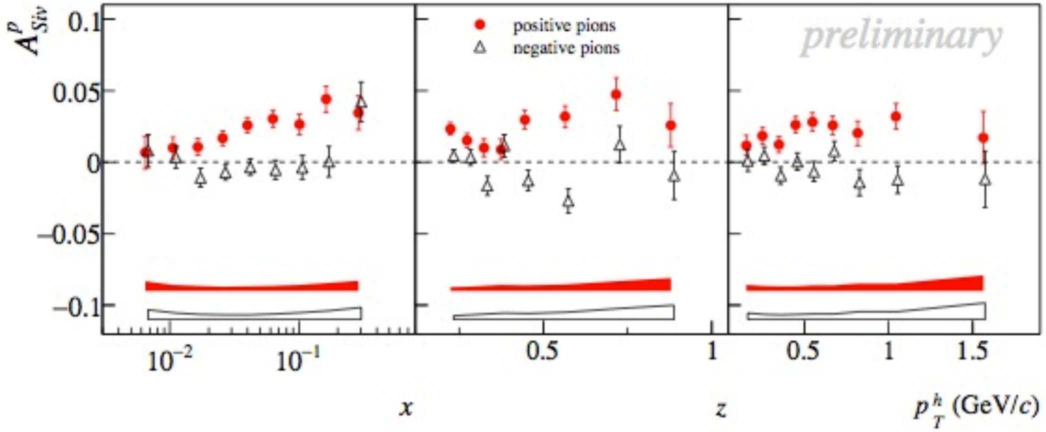


RHIC, PHENIX (2014)



$$\boxed{\ell N^{\uparrow} \to \ell'\,h\,X}$$

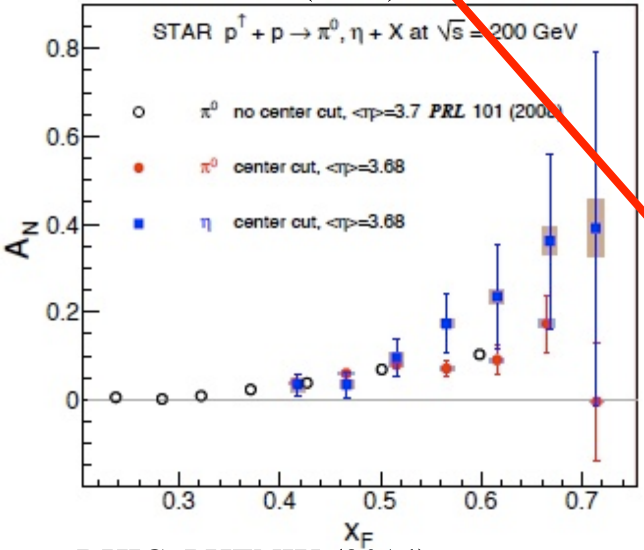
CERN, COMPASS (2013)



$$\pi\,F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

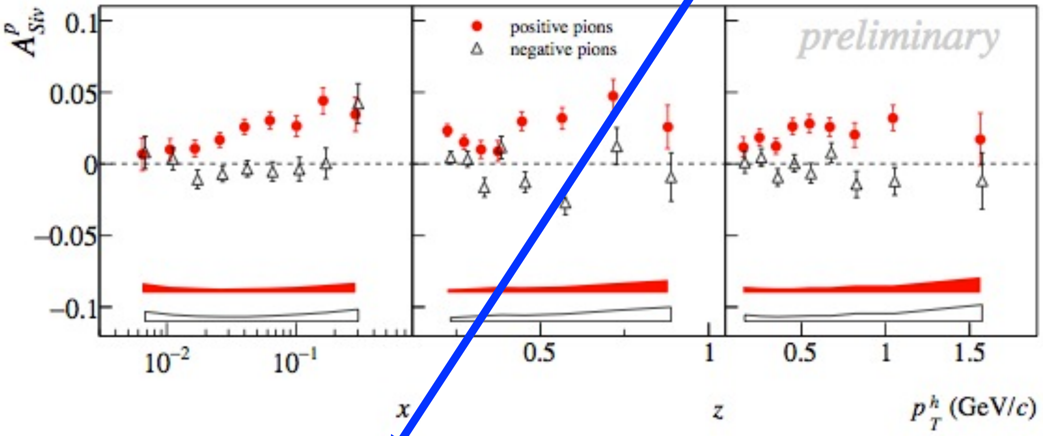
$$\boxed{p^{\uparrow}p \to h\,X}$$

RHIC, STAR (2012)



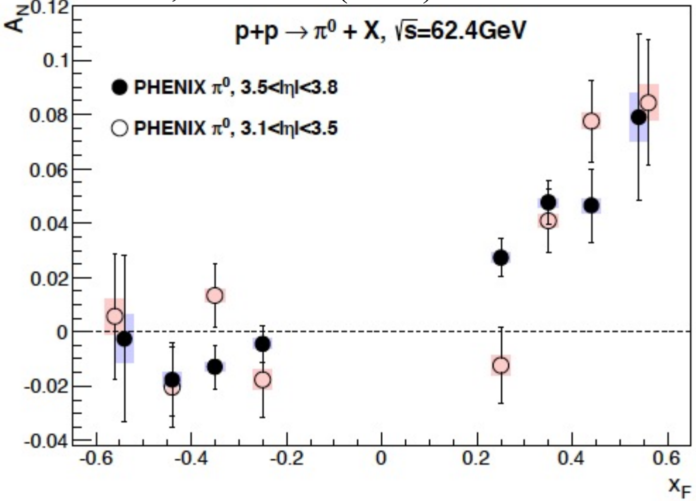
$$\boxed{\ell N^{\uparrow} \to \ell'\,h\,X}$$

CERN, COMPASS (2013)



$$\pi\,F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

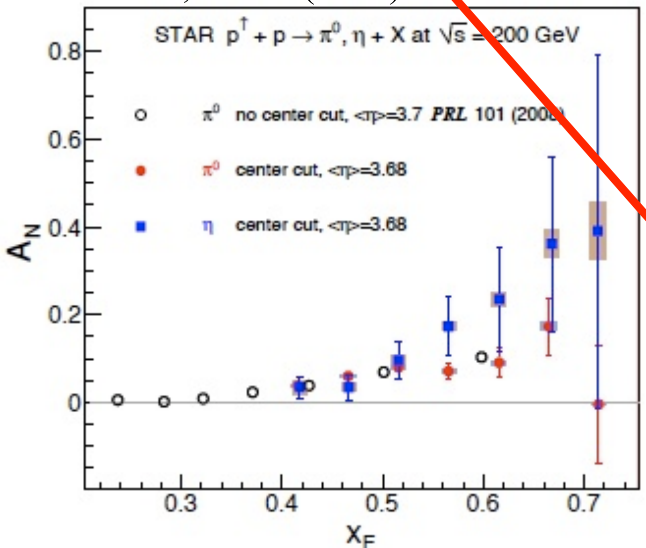
RHIC, PHENIX (2014)



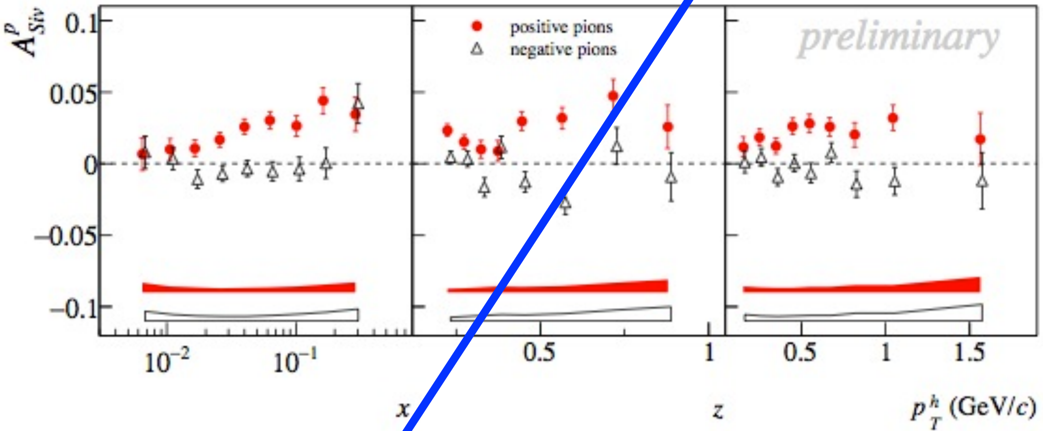
$p^{\uparrow}p \to h\,X$

$\ell N^{\uparrow} \to \ell'\,h\,X$

RHIC, STAR (2012)

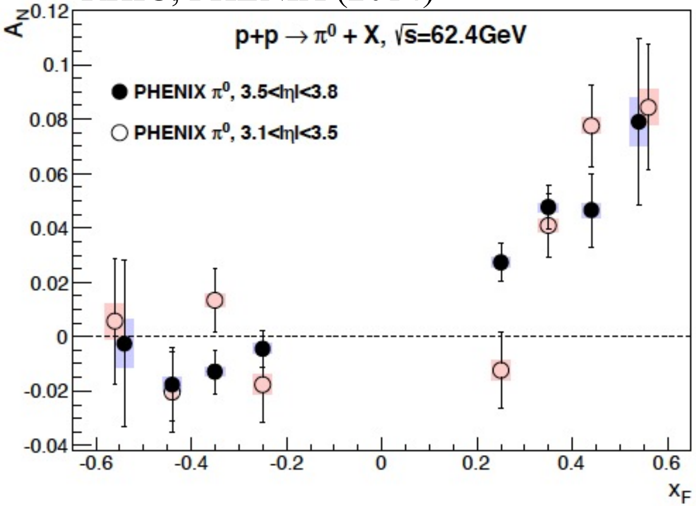


CERN, COMPASS (2013)

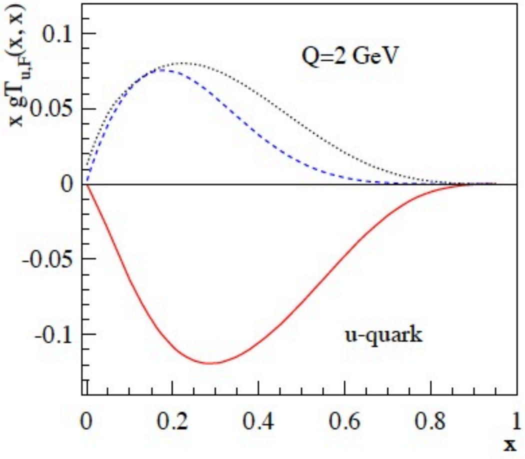


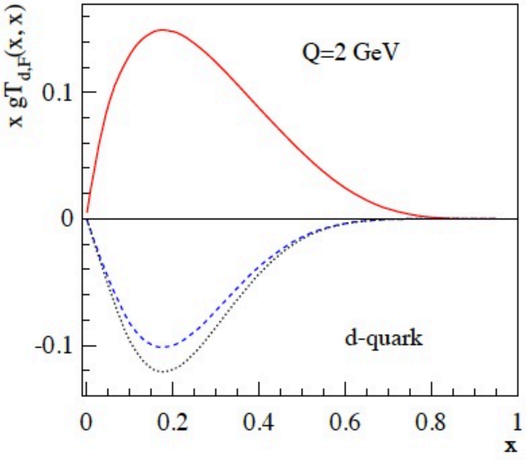
$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

RHIC, PHENIX (2014)

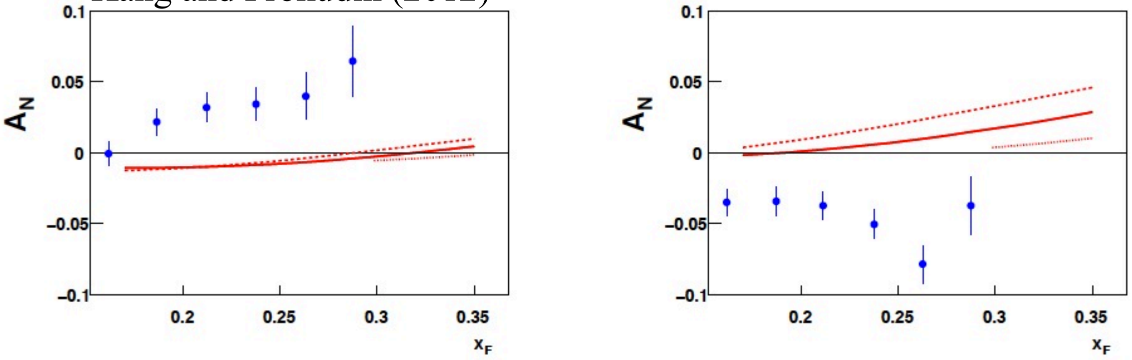


"sign mismatch" (Kang, Qiu, Vogelsang, Yuan (2011))





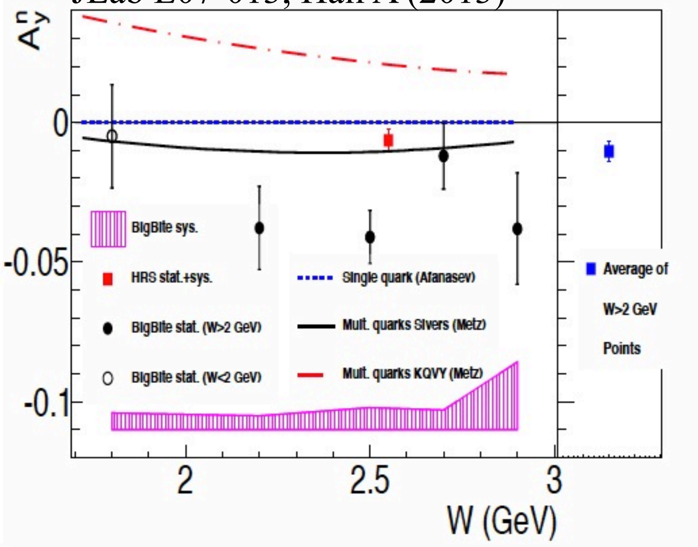
Kang and Prokudin (2012)



Proton-proton data from BRAHMS for π^+ (left) and π^- (right)

Nodes in Sivers cannot resolve issue

JLab E07-013, Hall A (2013)



Neutron TSSA in inclusive DIS

Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou - PRD **86** (2012)

Sivers input agrees reasonably well with the JLab data ➡ FIRST INDICATION on the PROCESS DEPENDENENCE of the Sivers function (see also Gamberg, Kang, Prokudin (2013))

KQVY input gives the wrong sign ➡ Qiu-Sterman function cannot be the main cause of the large TSSAs seen in pion production from *pp* collisions

~~$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x, x)}$~~

$$\cancel{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S^i_{H_1^\perp} + \frac{1}{z} H^c(z)\, S^i_H \right.$$
$$\left. + \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\, \hat{H}_{FU}^{c,\Im}(z,z_1)\, S^i_{\hat{H}_{FU}} \right\}$$

(Metz and DP - PLB **723** (2013))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2}\, \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\boldsymbol{\Im}}}{(1/z - 1/z_1)^2} \right)$$

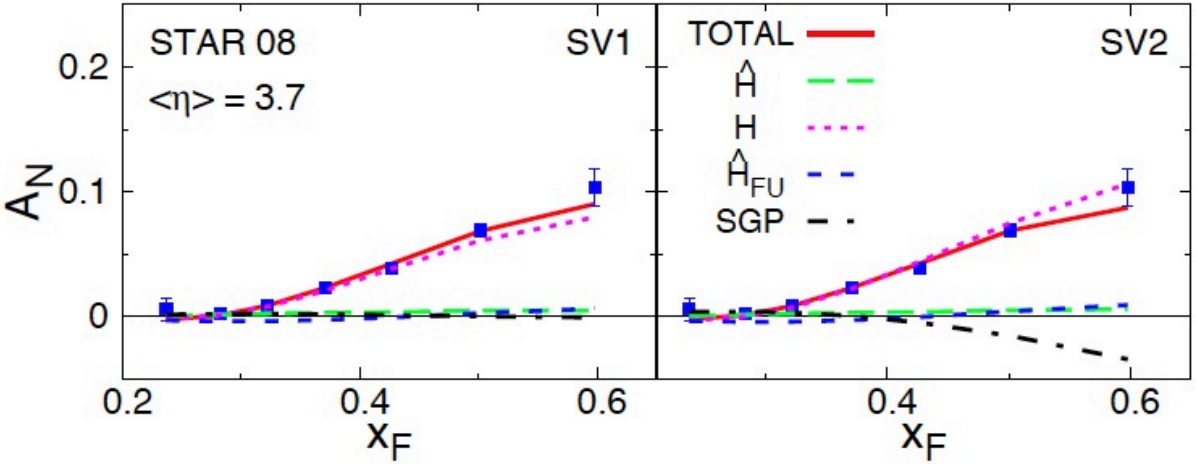
$$H^q(z) \;=\; -2z\, H_1^{\perp(1),q}(z) + \underbrace{\boxed{2z \int_z^{\infty} \frac{dz_1}{z_1^2}\, \frac{1}{\frac{1}{z} - \frac{1}{z_1}}\, \hat{H}_{FU}^{q,\Im}(z, z_1)}}_{\equiv\, \tilde{H}^q(z)}$$

QCD e.o.m.
relation
(EOMR)

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

Also included the Qiu-Sterman term $\pi \boldsymbol{F_{FT}(x,x)} = \boldsymbol{f_{1T}^{\perp(1)}(x)}$



(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

Fragmentation term is the main cause of A_N in *pp -> π X*

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^{\infty} \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$\Downarrow$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

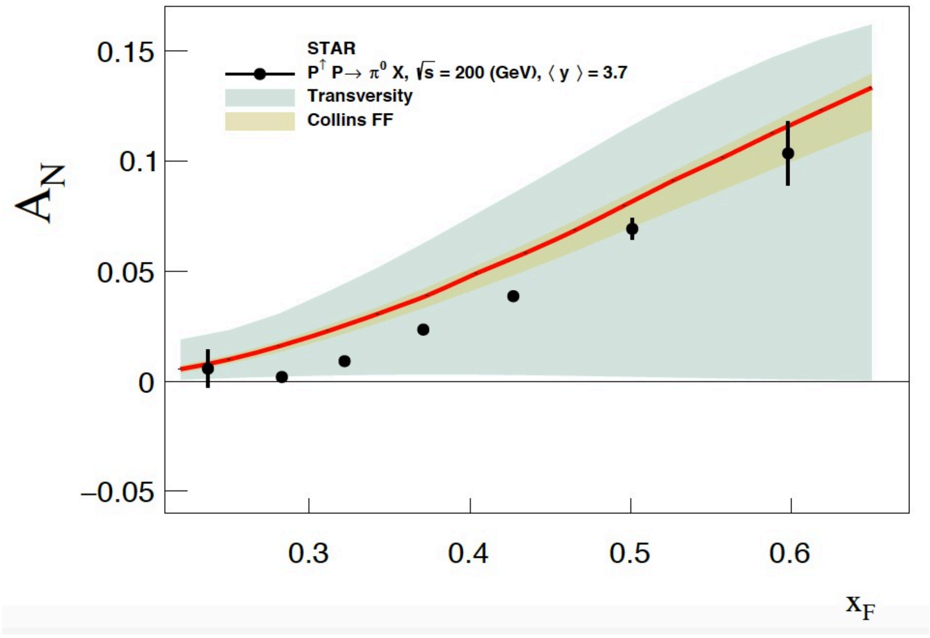
$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\, \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] \tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z) \right] \tilde{S}^i_H \right\}$$

where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(H_1^{\perp(1)}, \tilde{H}, \int \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

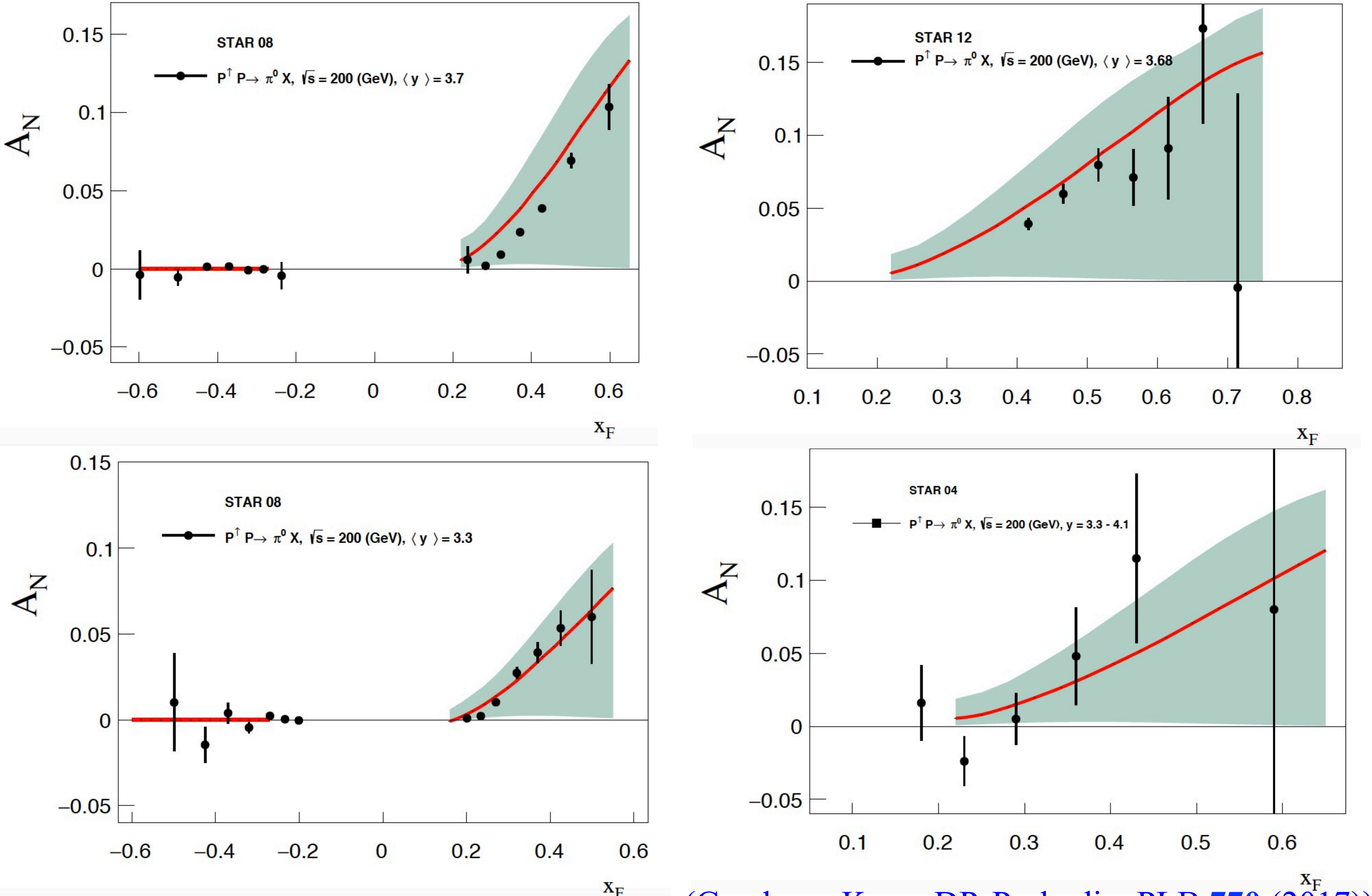
$$d\Delta\sigma^{\pi} \sim \boxed{\boldsymbol{h_1} \otimes \tilde{S} \otimes (H_1^{\perp(1)}}, \tilde{H})$$



Fragmentation term is the main cause of A_N in *pp -> π X*

We can constrain ***transversity at large x*** with A_N data from RHIC!

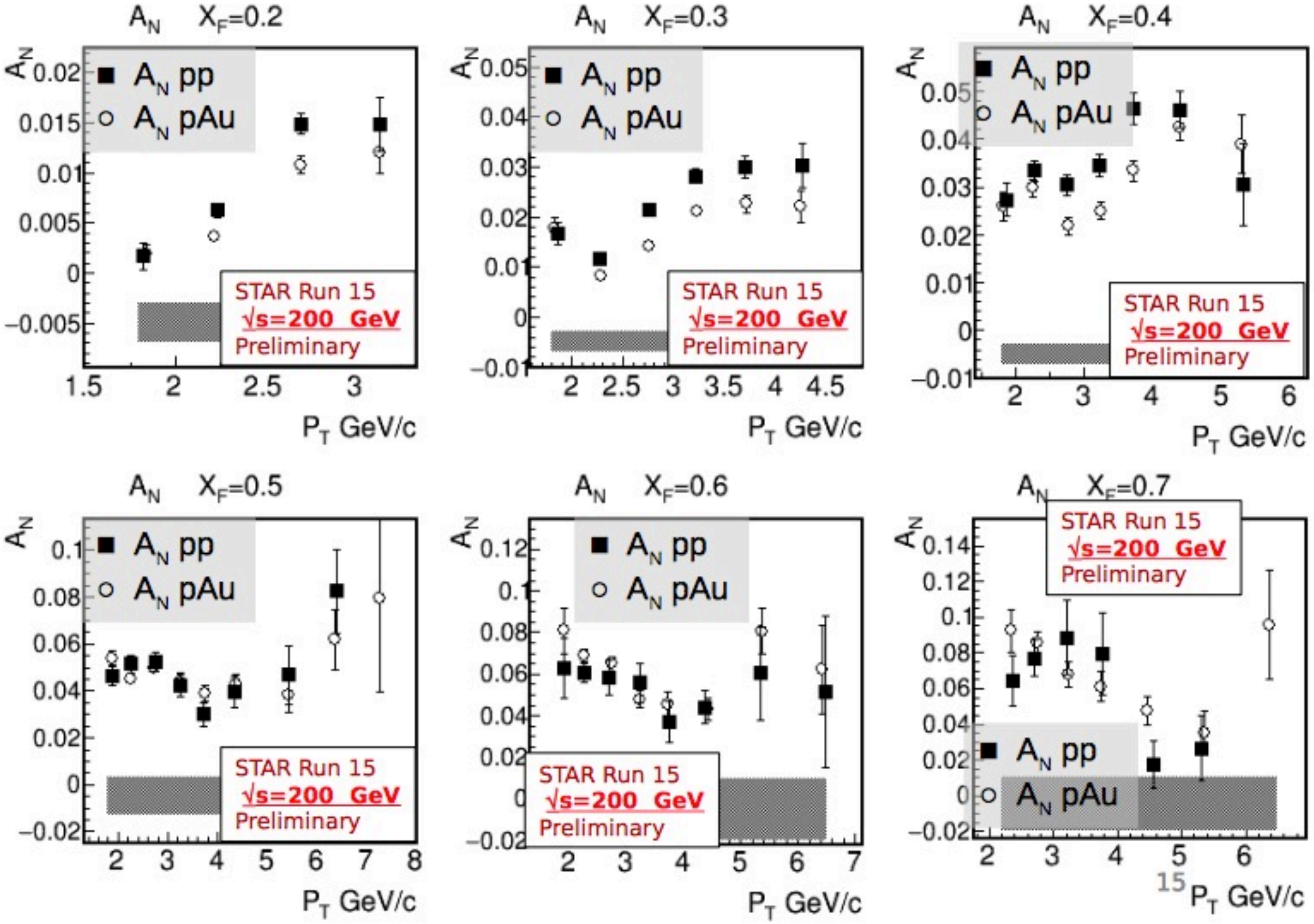
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

A comment on A_N in $pA \to \pi X$

STAR, preliminary



No A dependence observed up to $x_F = 0.7$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\,\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\, f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right] S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

$$\left. +\;\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\,\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\boxed{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]}\,S^i_{H_1^\perp} + \boxed{\frac{1}{z}H^c(z)\,S^i_H} \longrightarrow \sim A^{-1/3}\right.$$

$$\left. + \boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

$\sim A^{-1/3}$

$\sim A^{0}$

Include saturation corrections to calculate *pA* TSSA

(Hatta, Xiao, Yoshida, Yuan (2017))

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

$$\left. + \boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

~ A^0

EOMR + LIR →

$$\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z) + z\frac{dH_1^{\perp(1),c}(z)}{dz} - \frac{1}{z}\tilde{H}^c(z)$$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

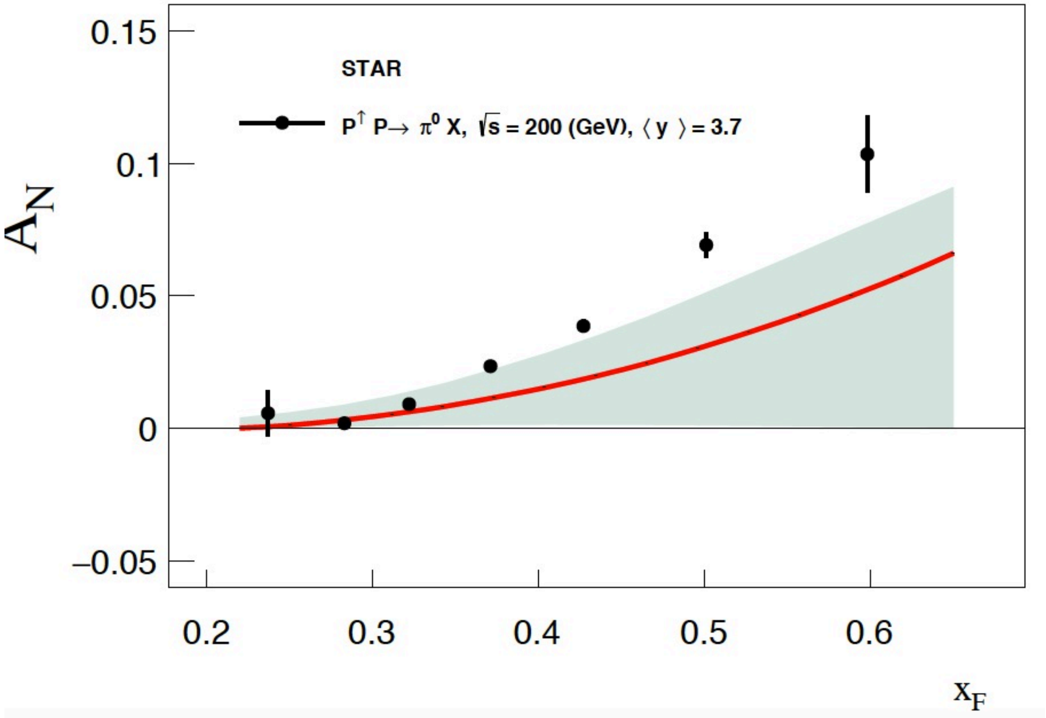
$$\left. + \boxed{\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

~ A^0

EOMR + LIR ➔

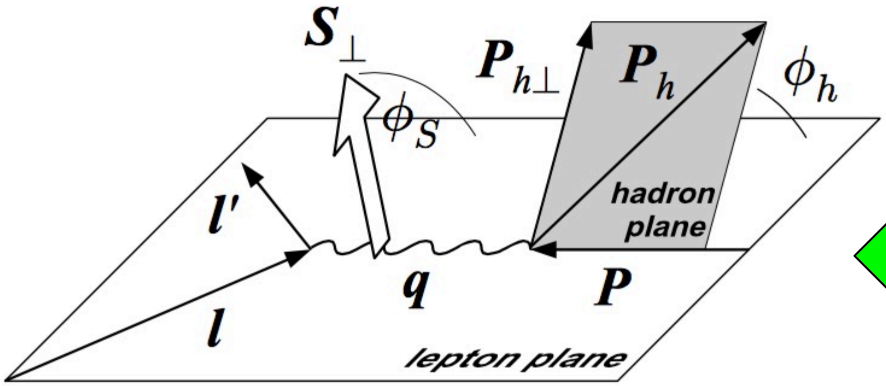
$$\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z) + z\,\frac{dH_1^{\perp(1),c}(z)}{dz} - \frac{1}{z}\tilde{H}^c(z)$$

Calculate pieces involving the (first k_T-moment of the) Collins function to get an updated estimate for the term in blue

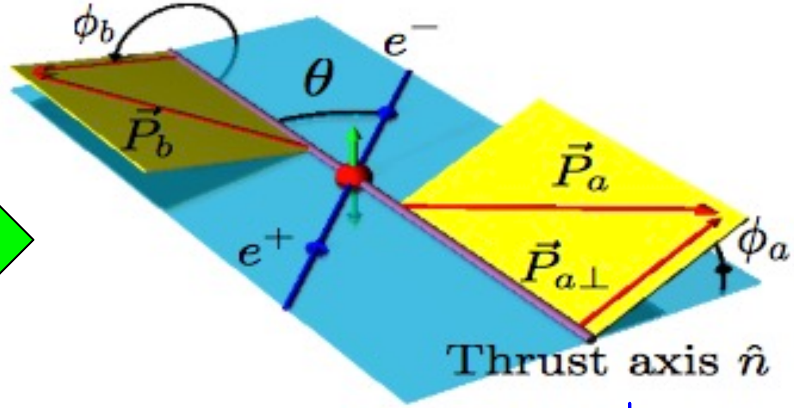


Fragmentation term as the cause of A_N in *pp* collisions is not ruled out by the STAR *pA* TSSA data

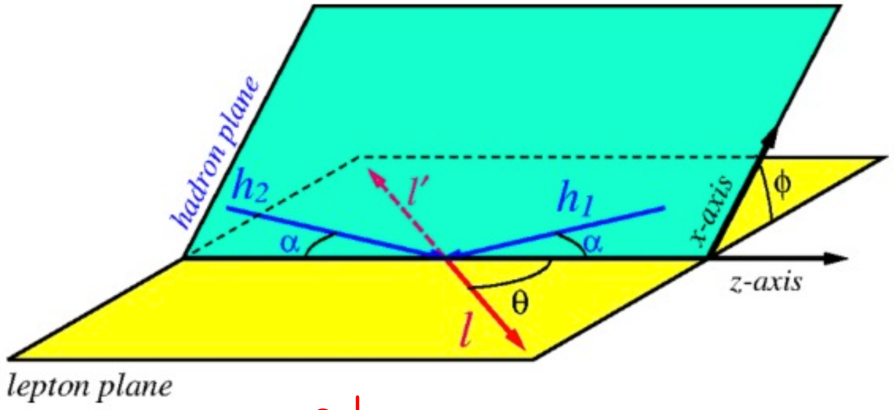
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



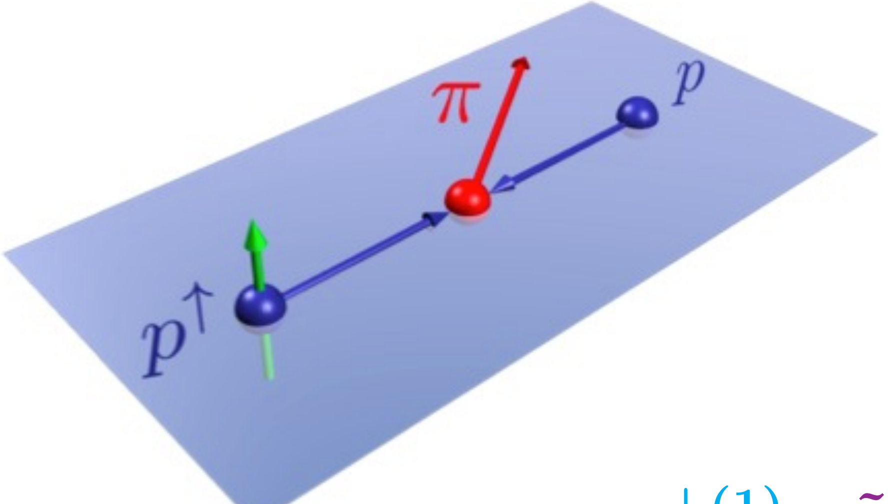
h_1, $f_{1T}^\perp$, $H_1^\perp$



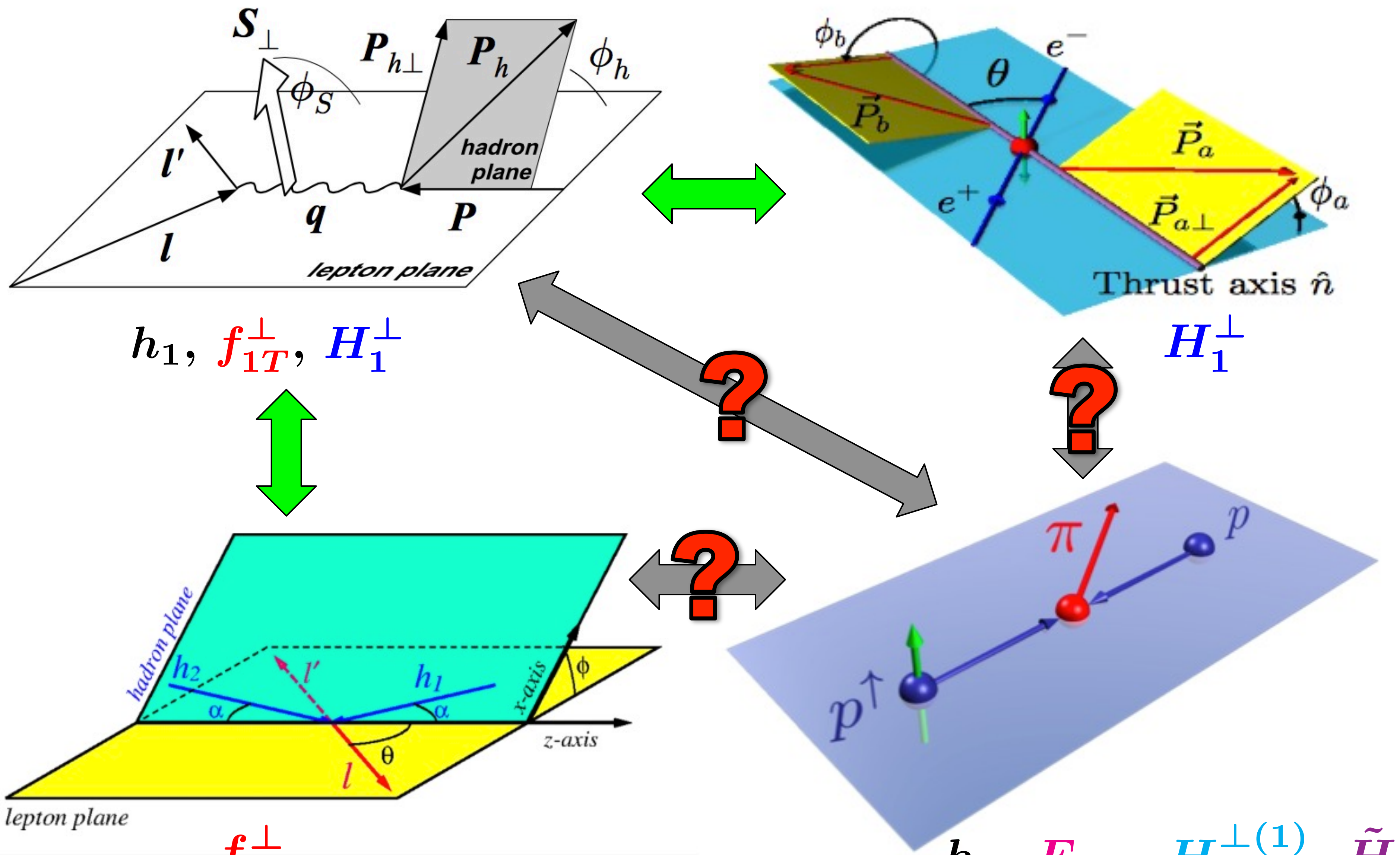
$H_1^\perp$



$f_{1T}^\perp$



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$h_1,\ f_{1T}^{\perp},\ H_1^{\perp}$

$H_1^{\perp}$

$f_{1T}^{\perp}$

$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$

Relations between TMD and CT3 Functions

"Parton Model"

TMD CT2

$$\int d^2\vec{k}_T \quad \boldsymbol{f_1(x,k_T)} \quad = \quad \boldsymbol{f_1(x)}$$

$$\vdots$$

TMD CT2

$$\int d^2\vec{p}_T \quad \boldsymbol{D_1(z,p_T)} \quad = \quad \boldsymbol{D_1(z)}$$

$$\vdots$$

TMD kinematical CT3 dynamical CT3

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2} \quad \boldsymbol{f_{1T}^{\perp}(x,k_T)} \quad = \quad \boldsymbol{f_{1T}^{\perp(1)}(x)} \quad = \quad \pi \boldsymbol{F_{FT}(x,x)}$$

Boer, Mulder, Pijlman (2003); Meissner (2009); ...

$$\vdots$$

TMD kinematical CT3

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2M_h^2} \quad \boldsymbol{H_1^{\perp}(z,p_T)} \quad = \quad \boldsymbol{H_1^{\perp(1)}(z)}$$

Yuan and Zhou (2009)

$$\vdots$$

“Parton Model”

TMD CT2

$$\int d^2\vec{k}_T \quad \boldsymbol{f_1(x,k_T)} \quad = \quad \boldsymbol{f_1(x)}$$

⋮

Ignore UV divergences and effects from soft-gluon radiation

TMD CT2

$$\int d^2\vec{p}_T \quad \boldsymbol{D_1(z,p_T)} \quad = \quad \boldsymbol{D_1(z)}$$

⋮

TMD kinematical CT3 dynamical CT3

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \quad \boldsymbol{f_{1T}^{\perp}(x,k_T)} \quad = \quad \boldsymbol{f_{1T}^{\perp(1)}(x)} \quad = \quad \pi \boldsymbol{F_{FT}(x,x)}$$

Boer, Mulder, Pijlman (2003); Meissner (2009); ...

⋮

TMD kinematical CT3

$$\int d^2\vec{p}_T \, \frac{\vec{p}_T^{\,2}}{2z^2M_h^2} \quad \boldsymbol{H_1^{\perp}(z,p_T)} \quad = \quad \boldsymbol{H_1^{\perp(1)}(z)}$$

Yuan and Zhou (2009)

⋮

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\boldsymbol{\tilde{f}_1(x,b_T;Q^2,\mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right)$$

$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_1}(b_T,Q)\right]$$

“b-space” functions

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)$$

$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)$$

$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

g_{f_1}: different for each TMD

g_K: universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

Note: $b_*(0) = 0$ and $(\mu_{b_*})_{b_* \to 0} = \infty$

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right) \times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\rm max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

Note: $\boldsymbol{b_*(0) = 0 \text{ and } (\mu_{b_*})_{b_* \to 0} = \infty}$

➡ Leads to problematic large logarithms in S_{pert}

(Bozzi, Catani, de Florian, Grazzini (2006); Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

“Improved CSS” (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))*

Place a lower cut-off on b_T: $b_T \rightarrow b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \rightarrow \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

*Other modifications are discussed in this reference that attempt to improve the agreement of the CSS $W+Y$ formulation with the differential cross section over all transverse momentum regions.

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\tilde{\boldsymbol{f}}_1(\boldsymbol{x}, \boldsymbol{b_c(b_T)}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right)$$

$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

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$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, Rogers, in preparation)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} - iM\epsilon^{ij} b_T^i S_T^j \left[\boldsymbol{-\frac{1}{M^2}\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x, b_T; Q^2, \mu_Q)}\right]$$

Boer, Gamberg, Musch, Prokudin (2011)

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Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\tilde{\boldsymbol{f}}_{\boldsymbol{1}}(\boldsymbol{x}, \boldsymbol{b_c(b_T)}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

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Boer, Gamberg, Musch, Prokudin (2011)

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➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

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➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, Rogers, in preparation)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} - iM\epsilon^{ij} b_T^i S_T^j \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)}$$

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1, \hat{x}_2, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{F_{FT}(\hat{x}_1, \hat{x}_2; \mu_{b_*})}\right) \times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes f_1(\hat{x}; \bar{\mu})\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, Rogers, in preparation)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{f}_1(x, b_T; Q^2, \mu_Q) - iM\epsilon^{ij} b_T^i S_T^j \tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)$$

b_T -> $b_c(b_T)$

NO b_T -> $b_c(b_T)$ replacement – kinematic factor NOT associated with the scale evolution

b_T -> $b_c(b_T)$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, Rogers, in preparation)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T, b_c(b_T); Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} - iM\epsilon^{ij} b_T^i S_T^j \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{F_{FT}(\hat{x}_1, \hat{x}_2; \bar{\mu})}\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_c(b_T), Q)\right]$$

Analogous modification for fragmentation functions...

$$\boldsymbol{\tilde{D}_1(z, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{D_1}(z/\hat{z}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{D_1(\hat{z}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{D_1}(b_c(b_T), Q)\right]$$

$$\boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{H_1^\perp}(z/\hat{z}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{H_1^{\perp(1)}(\hat{z}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_c(b_T), Q)\right]$$

We then *define* the momentum-space functions...

$$f_1(x, k_T; Q^2, \mu_Q) \equiv \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{-i\vec{k}_T\cdot\vec{b}_T}\, \tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)$$

$$D_1(z, p_T; Q^2, \mu_Q) \equiv \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{p}_T\cdot\vec{b}_T}\, \tilde{D}_1(z, b_c(b_T); Q^2, \mu_Q)$$

$$\vdots$$

$$\frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q) \equiv \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{-i\vec{k}_T\cdot\vec{b}_T}\, \tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)$$

$$\frac{\vec{p}_T^2}{2z^2M_h^2}\, H_1^{\perp}(z, p_T; Q^2, \mu_Q) \equiv \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{p}_T\cdot\vec{b}_T}\, \tilde{H}_1^{\perp(1)}(z, b_c(b_T); Q^2, \mu_Q)$$

$$\vdots$$

(Gamberg, Metz, DP, Prokudin, Rogers, in preparation) which leads to...

$$\int d^2\vec{k}_T\, \boldsymbol{f_1(x, k_T; Q^2, \mu_Q)} = \boldsymbol{\tilde{f}_1(x, b_c(0); Q^2, \mu_Q)} = \boldsymbol{f_1(x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, \boldsymbol{D_1(z, p_T; Q^2, \mu_Q)} = \boldsymbol{\tilde{D}_1(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{D_1(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\vdots$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q)} = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(0); Q^2, \mu_Q)} = \pi\, \boldsymbol{F_{FT}(x, x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p'}$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2M_h^2} \boldsymbol{H_1^{\perp}(z, p_T; Q^2, \mu_Q)} = \boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{H_1^{\perp(1)}(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

$$\vdots$$

At LO in the "Improved CSS" we recover the parton model relations

(Gamberg, Metz, DP, Prokudin, Rogers, in preparation)

Moreover, from a phenomenology standpoint with TMD observables...

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \quad \sim \quad \boxed{F_{FT}(x, x; \mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$

$$\swarrow$$

$$\boxed{g_{f_{1T}^{\perp}}(x, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

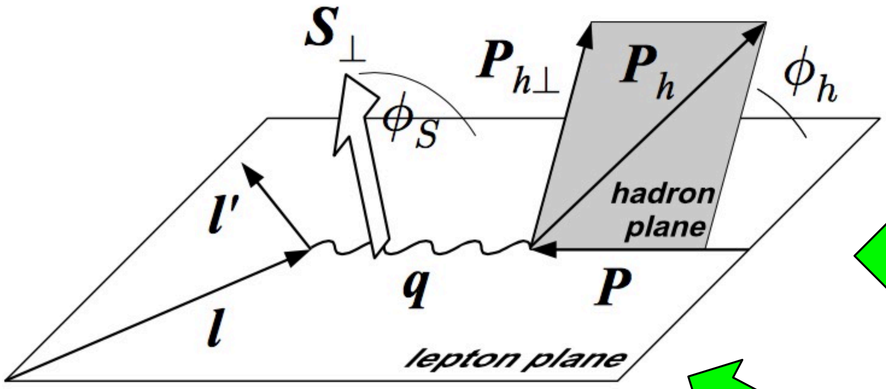
$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \quad \sim \quad \boxed{H_1^{\perp(1)}(z; \mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^{\perp}}(b_T, Q)\right]$$

$$\swarrow$$

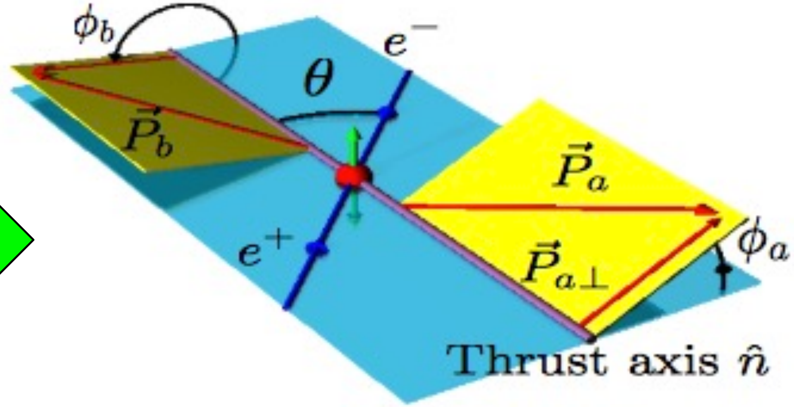
$$\boxed{g_{H_1^{\perp}}(z, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

The **CT3 functions** (along with the NP *g*-functions) are what get extracted in analyses of TSSAs in ***TMD processes*** that use CSS evolution!

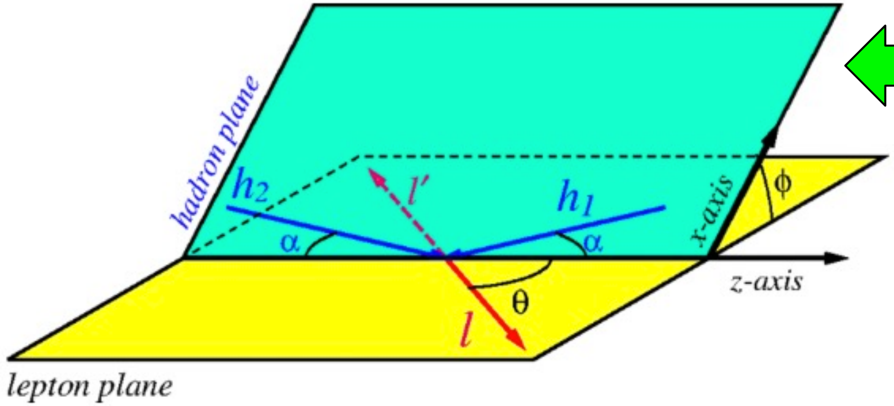
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



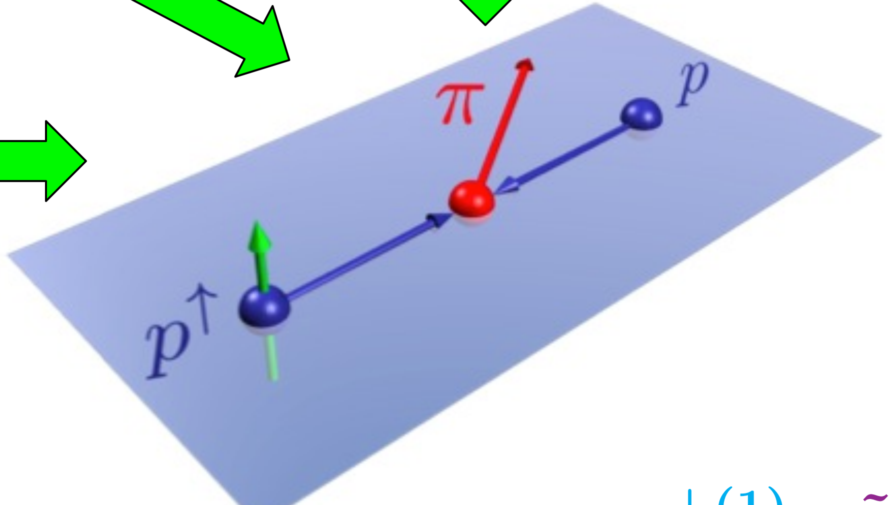
h_1, F_{FT}, $H_1^{\perp(1)}$



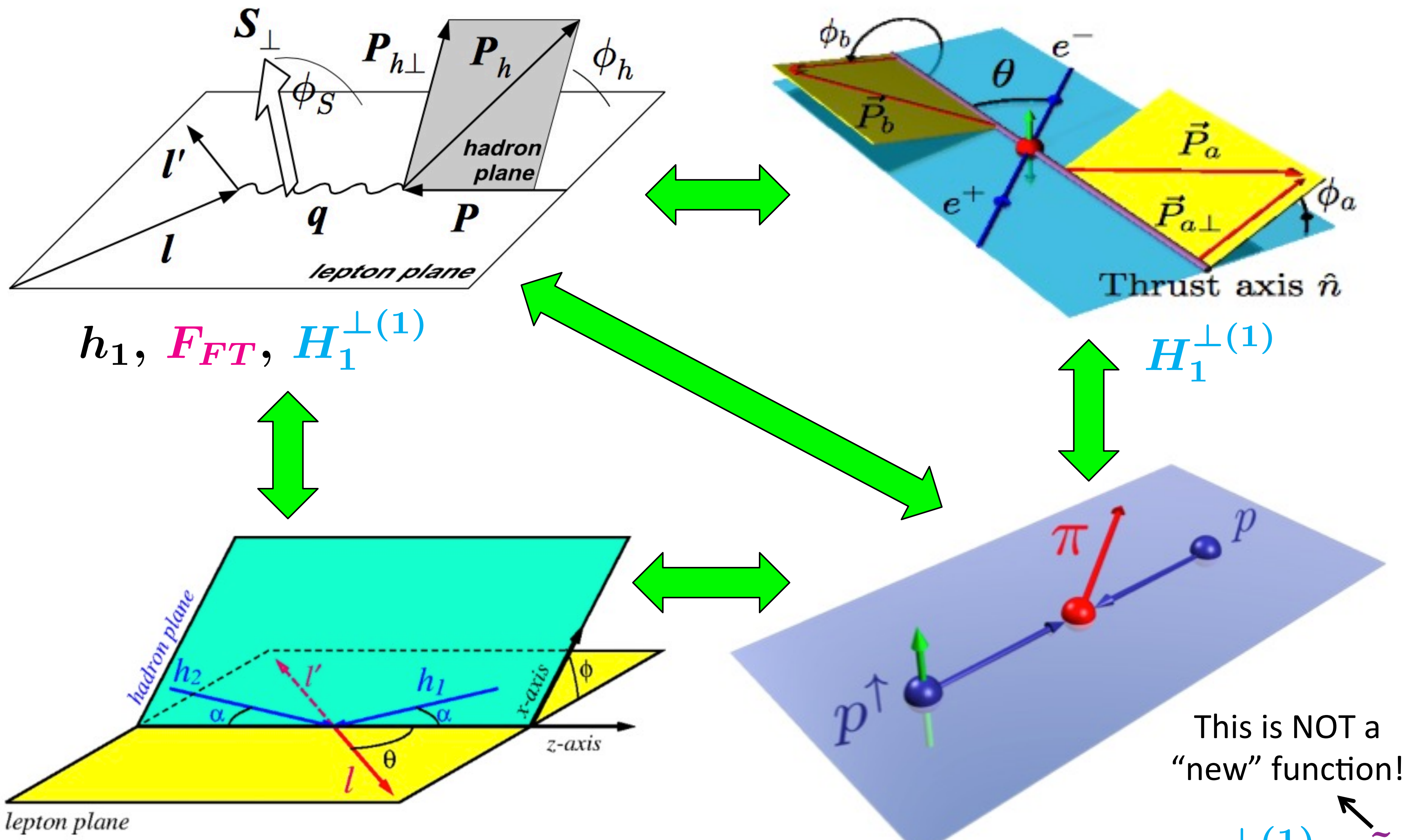
$H_1^{\perp(1)}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$h_1,\ F_{FT},\ H_1^{\perp(1)}$

$H_1^{\perp(1)}$

F_{FT}

This is NOT a "new" function!

$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T (Mulders, Tangerman (1996); Bacchetta, et al. (2007))

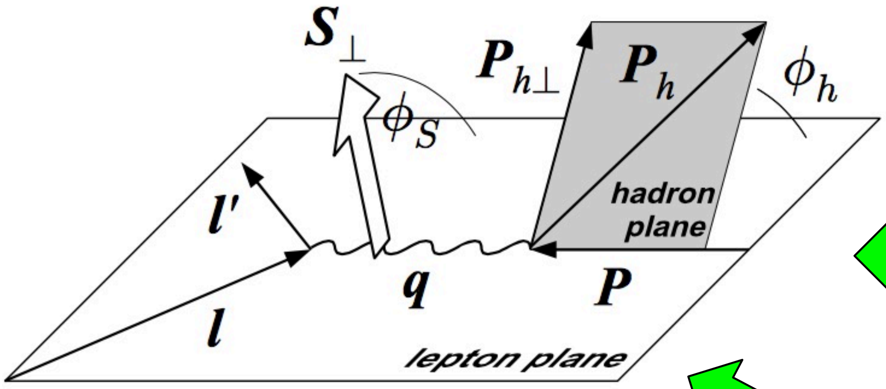
$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{\boldsymbol{H}}^{\boldsymbol{a}}(\boldsymbol{z})}{z}$$

$A_{UT}^{\sin\phi_S}$ in $e^+e^- \rightarrow h_1 h_2 X$ integrated over q_T (Boer, Jakob, Mulders (1997))

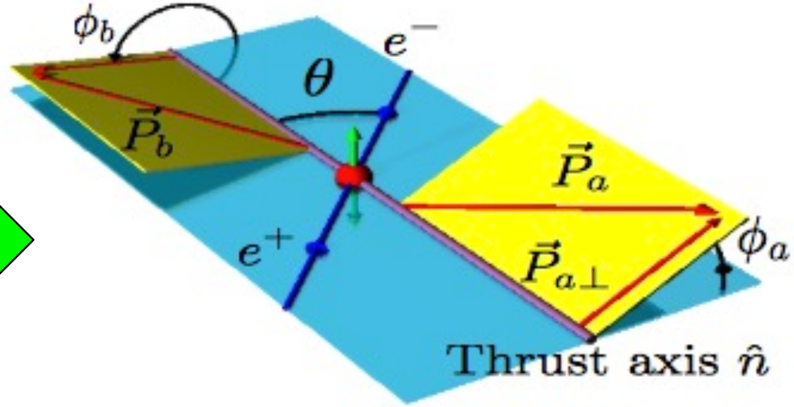
$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \frac{\tilde{\boldsymbol{H}}(\boldsymbol{z}_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

And also the TMD version of these (and other) observables (but with many more terms)

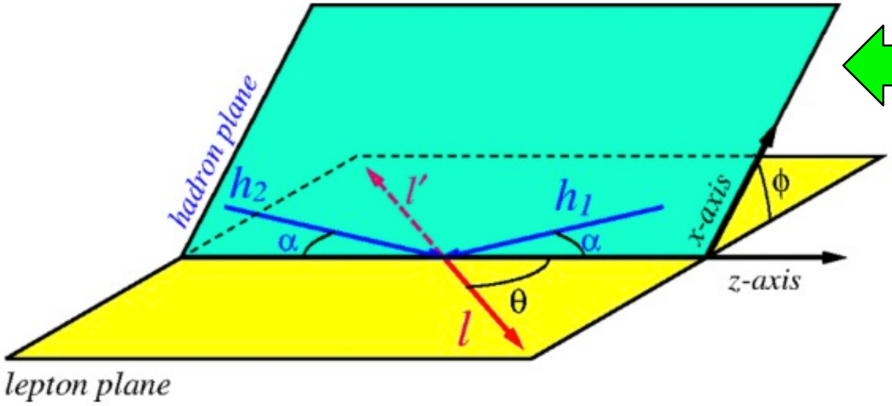
-Note: data from COMPASS, HERMES, and Belle show nonzero effects for the unintegrated version of the above asymmetries



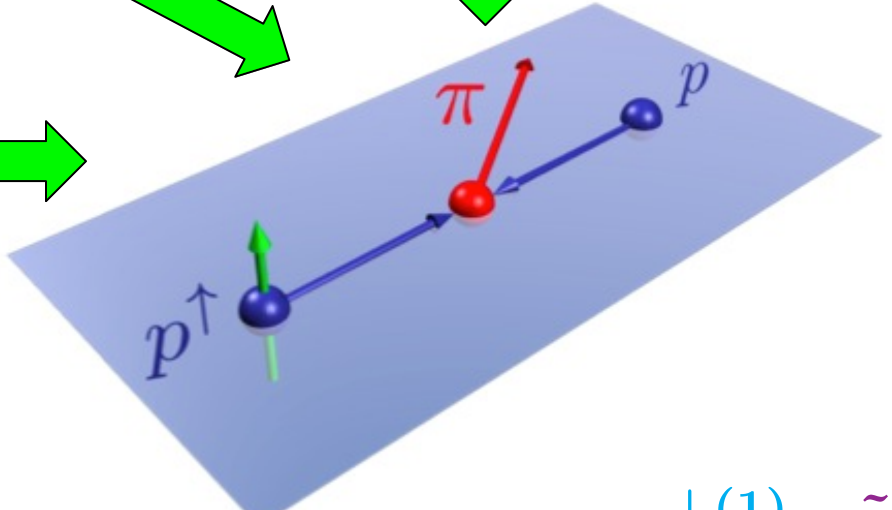
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Towards a Global Analysis of TMD and CT3 Observables

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

QCD e.o.m. relation (EOMR)

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$H^q(z) = -2z\,H_1^{\perp(1),q}(z) + 2z\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{q,\Im}(z,z_1)$$

QCD e.o.m.
relation
(EOMR)

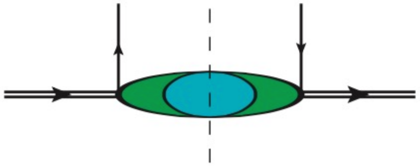
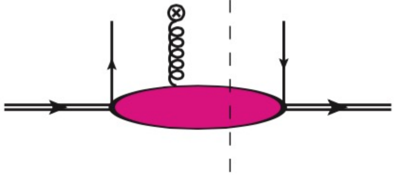
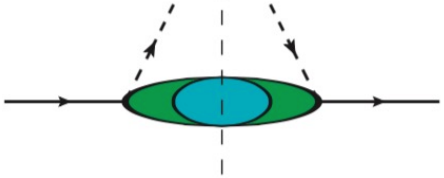
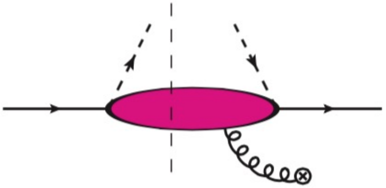
$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right)H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z-1/z_1)^2}$$

Lorentz
invariance
relation (LIR)

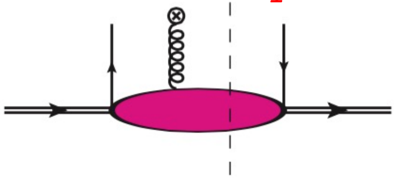
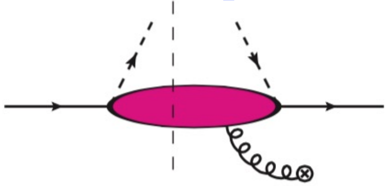
$$\boldsymbol{H(z)} = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\,2\left[\frac{\left(2(\frac{2}{z_1}-\frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)\delta\left(\frac{1}{z_1}-\frac{1}{z}\right)\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\boldsymbol{\hat{H}_{FU}^{\Im}(z_1,z_2)}\right]$$

$$\boldsymbol{H_1^{\perp(1)}(z)} = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\,\frac{\left(\frac{2}{z_1}-\frac{1}{z_2}\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\boldsymbol{\hat{H}_{FU}^{\Im}(z_1,z_2)}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

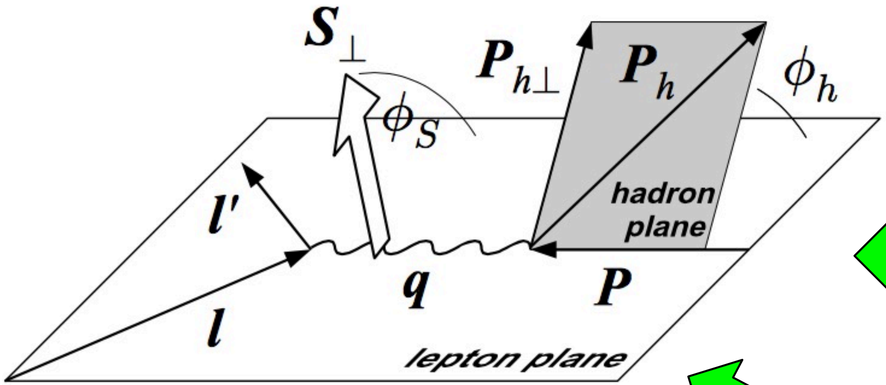
Hadron Pol.	PDF (x) 		PDF (x, x_1) 	FF (z) 		FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	$\xcancel{e}$	$\xcancel{h_1^{\perp(1)}}$	H_{FU}	$\xcancel{E}, \xcancel{H}$	$\xcancel{H_1^{\perp(1)}}$	$\hat{H}_{FU}^{\Re,\Im}$
L	$\xcancel{h_L}$	$\xcancel{h_{1L}^{\perp(1)}}$	H_{FL}	$\xcancel{E_L}, \xcancel{H_L}$	$\xcancel{H_{1L}^{\perp(1)}}$	$\hat{H}_{FL}^{\Re,\Im}$
T	$\xcancel{g_T}$	$\xcancel{f_{1T}^{\perp(1)}}$, $\xcancel{g_{1T}^{\perp(1)}}$	F_{FT}, G_{FT}	$\xcancel{D_T}, \xcancel{G_T}$	$\xcancel{D_{1T}^{\perp(1)}}$, $\xcancel{G_{1T}^{\perp(1)}}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

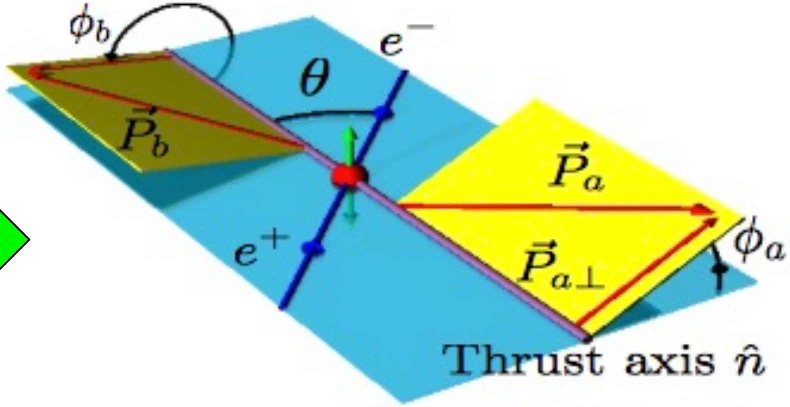
Hadron Pol.	PDF (x, x_1) 	FF (z, z_1) 
U	dynamical H_{FU}	dynamical $\hat{H}_{FU}^{\Re,\Im}$
L	H_{FL}	$\hat{H}_{FL}^{\Re,\Im}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

***ALL** transverse spin observables* are driven by *multi-parton correlations*

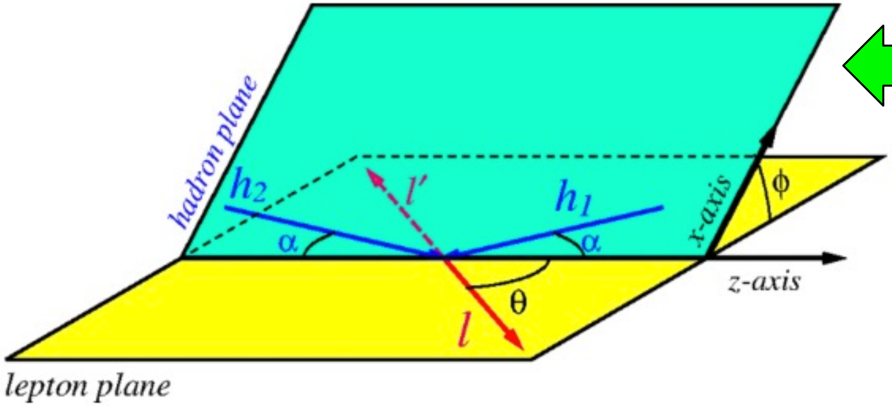
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))



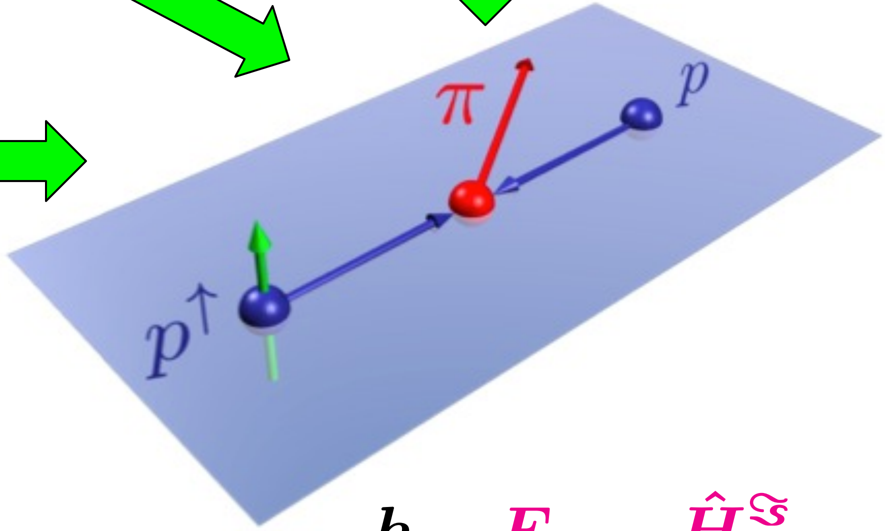
h_1, F_{FT}, $\hat{H}_{FU}^{\Im}$



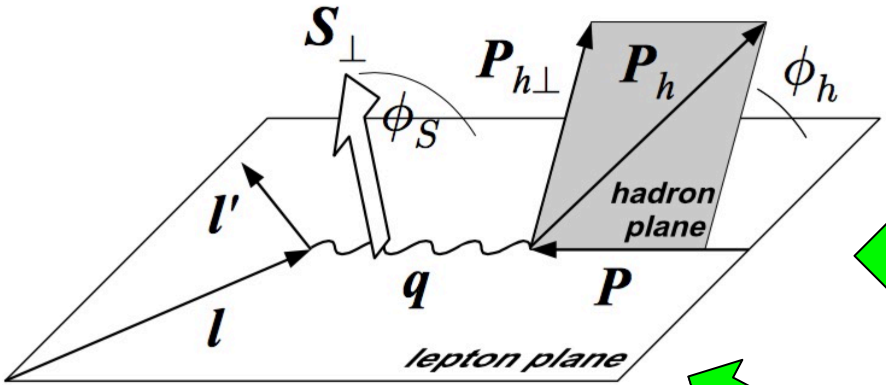
$\hat{H}_{FU}^{\Im}$



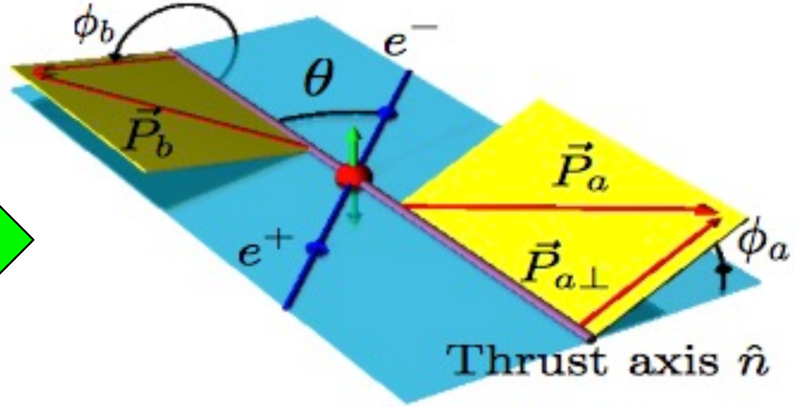
F_{FT}



h_1, F_{FT}, $\hat{H}_{FU}^{\Im}$

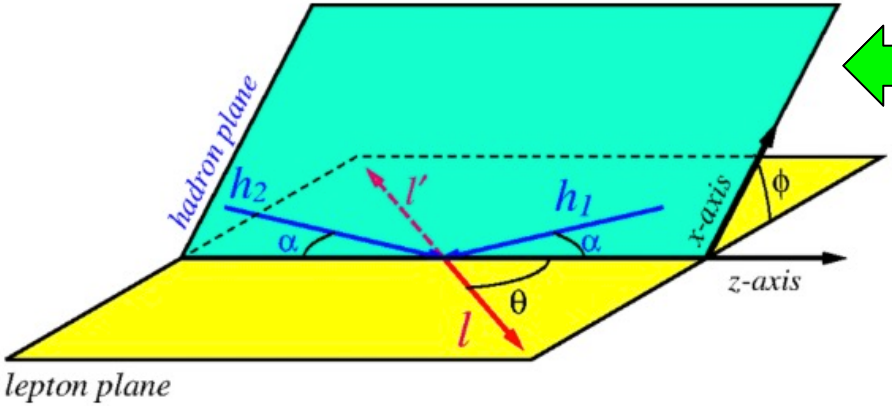


h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

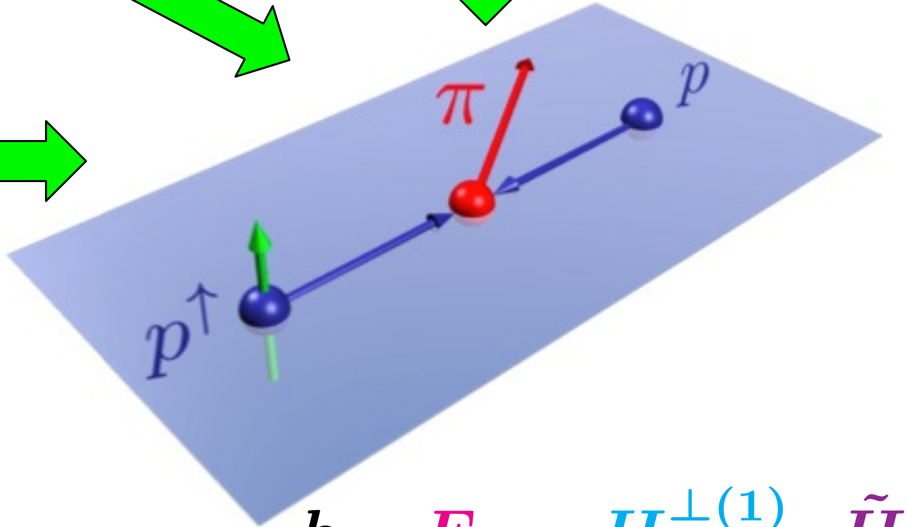


$H_1^{\perp(1)}$, $\tilde{H}$

Need to perform a "global" analysis!



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Summary

- TSSAs have been studied in both TMD processes (SIDIS, e^+e^-, DY) and collinear processes (A_N in *pp* & *lp* collisions).
- The current TMD formalism using improved CSS (iCSS) allows one to rigorously connect these two different types of observables. We have extended the original work on the unpolarized cross section to now include TSSAs.
- (LIRs + EOMRs + iCSS) = ***ALL*** transverse spin observables are driven by 3-parton (dynamical) functions.
- A global analysis of TMD *AND* collinear twist-3 transverse-spin observables is now possible.