

# Gluon TMDs: Universality and Process Dependence

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INT Program  
Spatial and Momentum Tomography  
of Hadrons and Nuclei

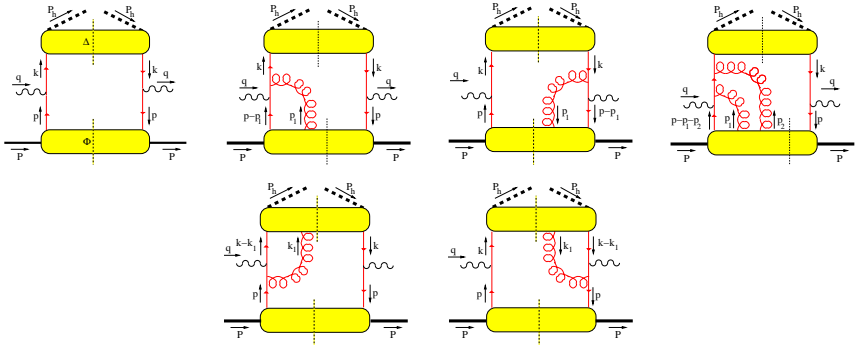
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Seattle (USA)

# TMD factorization and color gauge invariance





Resummation of all gluon exchanges leads to *gauge links* in the correlators  $\Phi$ ,  $\Delta$



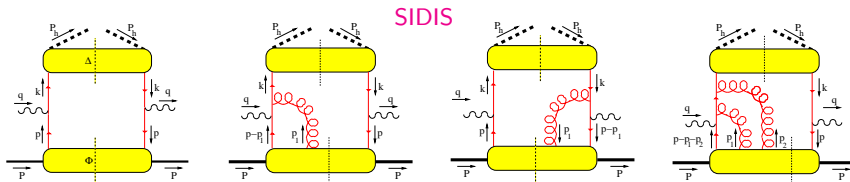
Boer, Mulders, Pijlman, NPB 667 (2003)

$$\mathcal{U}_{[0,\xi]}^C = \mathcal{P} \exp \left( -ig \int_{\mathcal{C}[0,\xi]} ds_\mu A^\mu(s) \right)$$

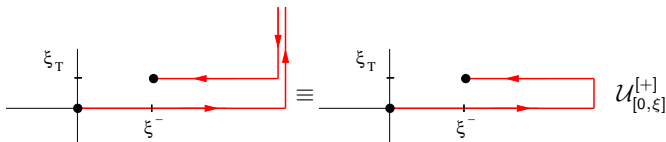
The path  $\mathcal{C}$  depends on the color interactions, *i.e.* on the specific process

Gauge invariant definition of  $\Phi$  (not unique)

$$\Phi^{[U]} \propto \langle P, S | \bar{\psi}(0) \mathcal{U}_{[0, \xi]}^C \psi(\xi) | P, S \rangle$$

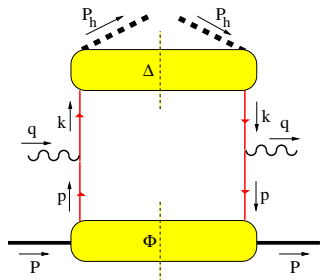


Belitsky, Ji, Yuan, NPB 656 (2003)  
Boer, Mulders, Pijlman, NPB 667 (2003)

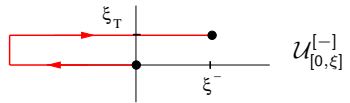
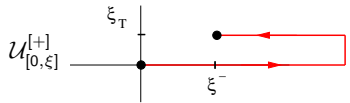
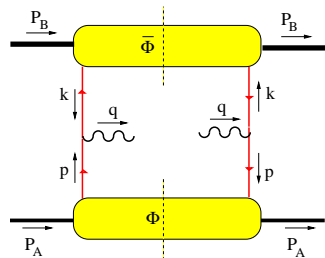


Possible effects in transverse momentum observables ( $\xi_T$  is conjugate to  $k_T$ )

SIDIS



Drell-Yan

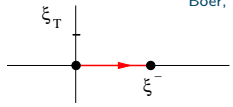


$\mathcal{U}_{[0,\xi]}^{[+]}$

$\mathcal{U}_{[0,\xi]}^{[-]}$

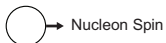
Belitsky, Ji, Yuan, NPB 656 (2003)  
Boer, Mulders, Pijlman, NPB 667 (2003)  
Boer, talk at RBRC Synergies workshop (2017)
















$\int dk_T \rightarrow \xi_T = 0 \rightarrow$



the same in both cases

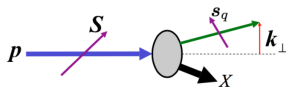
## Leading Twist TMDs



		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$ 		$h_1^\perp =$  -  Boer-Mulders
	L		$g_{1L} =$  -  Helicity	$h_{1L}^\perp =$  - 
	T	$f_{1T}^\perp =$  -  Sivers	$g_{1T}^\perp =$  - 	$h_1 =$  -  Transversity $h_{1T}^\perp =$  - 



Beyond the unpolarized  $f_1$ , helicity  $g_{1L}$  and transversity  $h_1$  surviving the collinear limit, we have five more. In particular the Sivers ( $f_{1T}^\perp$ ) and Boer-Mulders ( $h_1^\perp$ ):



$$\mathbf{S} \cdot (\mathbf{p} \times \mathbf{k}_\perp)$$

Sivers effect

$$\mathbf{s}_q \cdot (\mathbf{p} \times \mathbf{k}_\perp)$$

Boer-Mulders effect

Correlations between (proton or quark) spin and quark transverse momentum

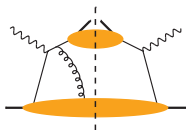
The Sivers effect is expected to give rise to transverse single spin asymmetries

Sivers, PRD 41 (1990)

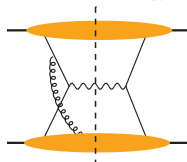
Fundamental test of TMD theory

$$f_{1T}^{\perp [DY]}(x, k_{\perp}^2) = -f_{1T}^{\perp [SIDIS]}(x, k_{\perp}^2) \quad h_1^{\perp [DY]}(x, k_{\perp}^2) = -h_1^{\perp [SIDIS]}(x, k_{\perp}^2)$$

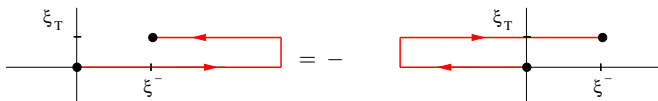
Collins, PLB 536 (2002)



FSI in SIDIS



ISI in DY



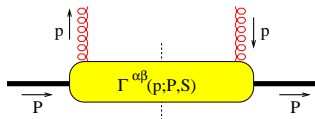
ISI/FSI lead to process dependence of TMDs, could even break factorization

Collins, Qiu, PRD 75 (2007)

Collins, PRD 77 (2007)

Rogers, Mulders, PRD 81 (2010)

# Process dependence of gluon TMDs



### Gauge invariant definition of $\Gamma^{\mu\nu}$

$$\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P, S | \text{Tr}_c [ F^{+\nu}(0) \mathcal{U}_{[0,\xi]}^c F^{+\mu}(\xi) \mathcal{U}_{[\xi,0]}^{c'} ] | P, S \rangle$$

Mulders, Rodrigues, PRD 63 (2001)

Buffing, Mukherjee, Mulders, PRD 88 (2013)

Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

The gluon correlator depends on two path-dependent gauge links

$ep \rightarrow e' Q \bar{Q} X$ ,  $ep \rightarrow e' \text{ jet jet } X$  probe gluon TMDs with  $[++]$  gauge links

$pp \rightarrow \gamma\gamma X$  (and/or other CS final state) probes gluon TMDs with  $[- -]$  gauge links

$pp \rightarrow \gamma \text{ jet } X$  probes an entirely independent gluon TMD:  $[+-]$  links (dipole)

GLUONS	<i>unpolarized</i>	<i>circular</i>	<i>linear</i>
U	$f_1^g$		$h_1^{\perp g}$
L		$g_{1L}^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_{1T}^g, h_{1T}^{\perp g}$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)  
 Mulders, Rodrigues, PRD 63 (2001)  
 Meissner, Metz, Goeke, PRD 76 (2007)

- ▶  $h_1^{\perp g}$ :  $T$ -even distribution of linearly polarized gluons inside an unp. hadron
- ▶  $f_{1T}^{\perp g}$ :  $T$ -odd gluon Sivers function

In contrast to quark TMDs, gluon TMDs are almost unknown

Even unpolarized gluon TMDs are process dependent: *two* relevant types

This was first realized in the small- $x$  framework:

Dominguez, Marquet, Xiao, Yuan, PRD (2011)

- ▶ Weizsäcker-Williams distribution (WW)
- ▶ Dipole distribution (DP)

Unpolarized (and in general  $T$ -even) gluon TMDs

$$\begin{aligned} [++] &= [--] \quad (\text{WW}) \\ [+ -] &= [- +] \quad (\text{DP}) \end{aligned}$$

In general they can differ in magnitude and width. Only constraint:

$$\int d^2\mathbf{k}_T f_1^{[++]}g(x, \mathbf{k}_T^2) = \int d^2\mathbf{k}_T f_1^{[+-]}g(x, \mathbf{k}_T^2)$$

Different processes can probe either types or a mixture of them

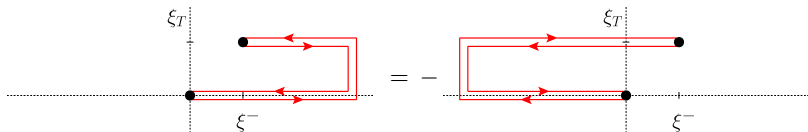
## Related Processes

$ep^\uparrow \rightarrow e' Q\bar{Q}X$ ,  $ep^\uparrow \rightarrow e' \text{jet jet } X$  probe GSF with  $[++]$  gauge links (WW)

$p^\uparrow p \rightarrow \gamma\gamma X$  (and/or other CS final state) probe GSF with  $[--]$  gauge links

Analogue of the sign change of  $f_{1T}^{\perp g}$  between SIDIS and DY (true also for  $h_{1T}^g$  and  $h_{1T}^{\perp g}$ )

$$f_{1T}^{\perp g}[e p^\uparrow \rightarrow e' Q\bar{Q}X] = -f_{1T}^{\perp g}[p^\uparrow p \rightarrow \gamma\gamma X]$$



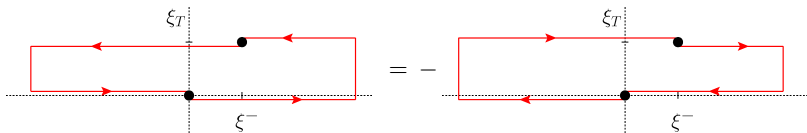
Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC

## Complementary Processes

$ep^\uparrow \rightarrow e' Q \bar{Q} X$  probes a GSF with  $[++]$  gauge links (WW)

$p^\uparrow p \rightarrow \gamma \text{jet} X$  ( $gq \rightarrow \gamma q$ ) probes a gluon TMD with  $[-+]$  links (DP)



At small- $x$  the WW Sivvers function appears to be suppressed by a factor of  $x$  compared to the unpolarized gluon function, unlike the dipole one

The DP gluon Sivvers function at small- $x$  is the **spin dependent odderon** (single spin asymmetries from a single Wilson loop matrix element)

Boer, Echevarria, Mulders, Zhou, PRL 116 (2016)

Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)



The first transverse moments of the WW and DP gluon Sivvers functions

$$f_{1T}^{\perp(1)g(f/d)}(x) = \int d^2\mathbf{k}_T \frac{k_T^2}{2M_p^2} f_{1T}^{\perp g(f/d)}(x, \mathbf{k}_T^2)$$

related to two different trigluon Qiu-Sterman functions  $T_G^{(f/d)}$ , involving the antisymmetric  $f_{abc}$  and symmetric  $d_{abc}$  color structures, respectively

Bomhof, Mulders, JHEP 0702 (2007)  
Buffing, Mukherjee, Mulders, PRD 88 (2013)

The two distributions have a different behavior under charge conjugation

The Burkardt sum rule constraints only the  $f$ -type gluon Sivvers function

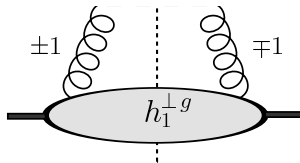
$$\sum_{a=q,\bar{q},g} \int dx f_{1T}^{\perp(1)a}(x) = 0$$

Boer, Lorcé, CP, Zhou, AHEP 2015 (2015)

The distribution of linearly polarized gluons  
inside an unpolarized proton:  $h_1^\perp g$

Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum

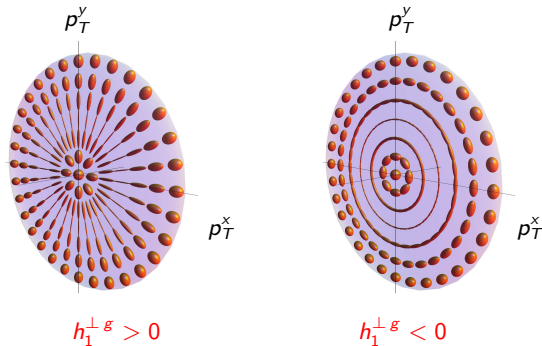


Interference between  $\pm 1$  gluon helicity states

Like the unpolarized gluon TMD, it is  $T$ -even and exists in different versions:

- ▶  $[++] = [--]$  (WW)
- ▶  $[+-] = [-+]$  (DP)

Visualization of the gluon polarization in the transverse momentum plane  
 $h_1^{\perp g}$  is taken to be a Gaussian



The ellipsoid axis lengths are proportional to the probability of finding a gluon with a linear polarization in that direction

$f_1^g$  and  $h_1^{\perp g} \sim \ln 1/x$  as  $x \rightarrow 0$ : computable in a saturation models (CGC)

Dominguez *et al*, PRD **85** (2012)  
Metz, Zhou, PRD **84** (2011)

- ▶ The DP  $h_1^{\perp g}$  saturates its positivity bound
- ▶ The WW  $h_1^{\perp g}$  is moderately suppressed at small  $p_T$ , bound saturated at large  $p_T \gg Q_s$

$k_T$ -factorization approach predicts maximal polarization (no process dependence)

Catani, Ciafaloni, Hautmann, NPB 336 (1991)

In the TMD formalism the DP  $h_1^{\perp g}$  becomes maximal when  $x \rightarrow 0$

Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

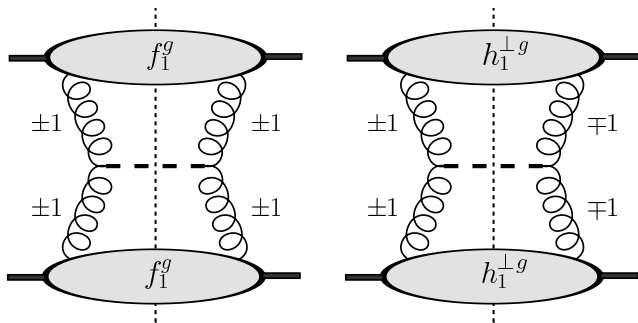
EIC can probe the WW  $h_1^{\perp g}$ , while RHIC/LHC can probe both the DP and WW

TMDs considered in the following in  $pp$  collisions are of the WW-type at small- $x$

Higgs boson production happens mainly via  $gg \rightarrow H$

Pol. gluons affect the Higgs transverse spectrum at NNLO pQCD

Catani, Grazzini, NPB 845 (2011)



The nonperturbative distribution can be present at tree level and would contribute to Higgs production at low  $q_T$

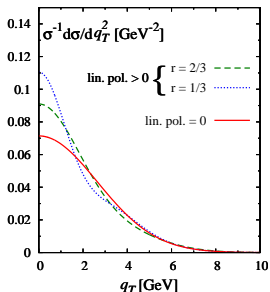
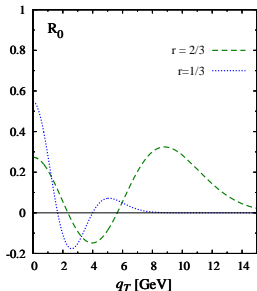
Boer, den Dunnen, CP, Schlegel, Vogelsang, PRL 108 (2012)

Boer, den Dunnen, CP, Schlegel, PRL 111 (2013)

### $q_T$ -distribution of the Higgs boson

$$\frac{1}{\sigma} \frac{d\sigma}{dq_T^2} \propto 1 + R(q_T^2) \quad R = \frac{h_1^{\perp g} \otimes h_1^{\perp g}}{f_1^g \otimes f_1^g} \quad |h_1^{\perp g}(x, \mathbf{p}_T^2)| \leq \frac{2M_p^2}{\mathbf{p}_T^2} f_1^g(x, \mathbf{p}_T^2)$$

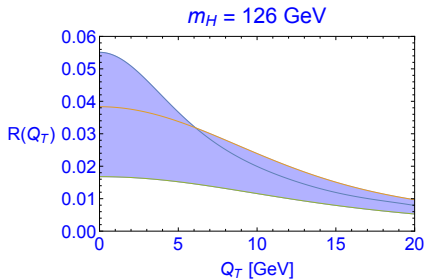
### Gaussian Model



### Gaussian model for $f_1^g$ and $h_1^{\perp g}$

$$f_1^g(x, \mathbf{p}_T^2) = \frac{f_1^g(x)}{\pi \langle \mathbf{p}_T^2 \rangle} \exp\left(-\frac{\mathbf{p}_T^2}{\langle \mathbf{p}_T^2 \rangle}\right); \quad f_{1T}^{\perp g}(x, \mathbf{p}_T^2) = \frac{M f_1^g(x)}{\pi \langle \mathbf{p}_T^2 \rangle^{3/2}} \sqrt{\frac{2e(1-r)}{r}} \exp\left(-\frac{1}{r} \frac{\mathbf{p}_T^2}{\langle \mathbf{p}_T^2 \rangle}\right) \quad 0 < r < 1$$

TMD evolution, at NNLL accuracy, suppresses the ratio  $R$  with increasing energy



Boer, den Dunnen, NPB 886 (2014)

Echevarria, Kasemets, Mulders, CP, JHEP 1507 (2015)

At  $Q \approx 9 \text{ GeV}$  ( $\Upsilon$  mass),  $R$  will be much larger, up to 60%



# Gluon TMDs and quarkonium production

Color Octet (CO) production in  $pp$  collisions involves a complicated link structure

Color Singlet (CS) production of  $C$ -even quarkonia from two gluons is possible

This is not allowed for  $J/\psi$  or  $\Upsilon$  because of the Landau-Yang theorem

$$pp \rightarrow [Q\bar{Q}]X \quad (gg \rightarrow [Q\bar{Q}])$$

Hard scale can only be the particle mass:  $Q = M$

TMD Factorization requires the resulting particle  $Q$  to have small  $q_T$  ( $q_T \ll M$ )

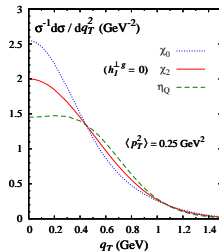
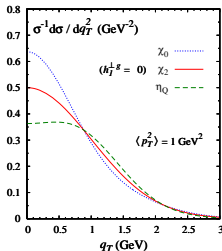
$q_T$ -distribution of  $\eta_Q$  and  $\chi_{QJ}$  ( $Q = c, b$ ) in the kinematic region  $q_T \ll 2M_Q$

$$\frac{1}{\sigma(\eta_Q)} \frac{d\sigma(\eta_Q)}{dq_T^2} \propto f_1^g \otimes f_1^g [1 - R(q_T^2)] \quad [\text{pseudoscalar}]$$

$$\frac{1}{\sigma(\chi_{Q0})} \frac{d\sigma(\chi_{Q0})}{dq_T^2} \propto f_1^g \otimes f_1^g [1 + R(q_T^2)] \quad [\text{scalar}]$$

$$\frac{1}{\sigma(\chi_{Q2})} \frac{d\sigma(\chi_{Q2})}{dq_T^2} \propto f_1^g \otimes f_1^g$$

Boer, CP, PRD 86 (2012) 094007



Proof of factorization at NLO for  $pp \rightarrow \eta_Q X$  in the Color Singlet Model (CSM)

Ma, Wang, Zhao, PRD 88 (2013), 014027; PLB 737 (2014) 103

$A_N^{\sin \phi_S}$  for  $p^\uparrow p \rightarrow \eta_Q X$  and  $p^\uparrow p \rightarrow \chi_{QJ} X$

$$A_N^{\sin \phi_S}(\eta_Q) = \frac{|\mathbf{S}_T|}{2(1 - R_0) f_1^g \otimes f_1^g} \left\{ f_1^g \otimes f_{1T}^{\perp g} + h_1^{\perp g} \otimes h_{1T}^g + h_1^{\perp g} \otimes h_{1T}^{\perp g} \right\}$$

$$A_N^{\sin \phi_S}(\chi_{Q0}) = \frac{|\mathbf{S}_T|}{2(1 + R_0) f_1^g \otimes f_1^g} \left\{ f_1^g \otimes f_{1T}^{\perp g} - h_1^{\perp g} \otimes h_{1T}^g - h_1^{\perp g} \otimes h_{1T}^{\perp g} \right\}$$

$$A_N^{\sin \phi_S}(\chi_{Q2}) = \frac{|\mathbf{S}_T|}{2 f_1^g \otimes f_1^g} f_1^g \otimes f_{1T}^{\perp g}$$

Boer, Lansberg, CP, *in preparation*

Quarkonium  $Q \equiv Q\bar{Q}[^3S_1]$  and isolated  $\gamma$  produced almost back to back

den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)

Accessible at the LHC: only the transverse momentum of the  $Q + \gamma$  pair needs to be small, not the individual ones

Study of TMD evolution by tuning the invariant mass of  $Q + \gamma$  (evolution scale)

Color octet (CO) contributions to  $Q + \gamma$  likely smaller than for inclusive  $Q$

Kim, Lee, Song, PRD 55 (1997)

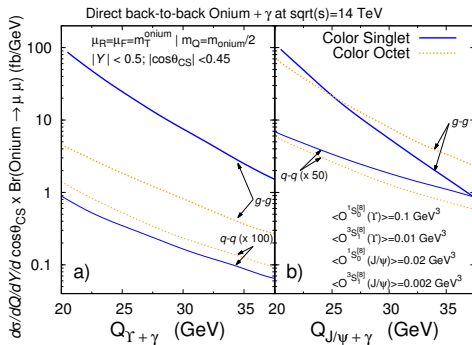
Li, Wang, PLB 672 (2009)

Lansberg, PLB 679 (2009)

CO further suppressed w.r.t. CS contributions when  $Q$  and  $\gamma$  are back to back

Mathews, Sridhar, Basu, PRD 60 (1999) 014009

Process dominated by  $gg$  fusion



CS yield is clearly dominant for the  $\Upsilon$ , above the CO one for  $J/\psi$  at low  $Q$

Further suppression of CO contributions by isolating  $Q$  (not needed for  $\Upsilon$ )

Kraan, AIP Conf. Proc. 1038 (2008) 45  
 Nikola, NP Proc. Suppl. 214 (2011) 177

$$\frac{d\sigma}{dQdYd^2q_Td\Omega} \propto A f_1^g \otimes f_1^g + B f_1^g \otimes h_1^{\perp g} \cos(2\phi_{CS}) + C h_1^{\perp g} \otimes h_1^{\perp g} \cos(4\phi_{CS})$$

- ▶ valid up to corrections  $\mathcal{O}(q_T/Q)$
- ▶  $Y$ : rapidity of the  $Q + \gamma$  pair, along the beam in the hadronic c.m. frame
- ▶  $d\Omega = d\cos\theta_{CS} d\phi_{CS}$ : solid angle for  $Q - \gamma$  in the Collins-Soper frame

Analysis similar to the one for  $pp \rightarrow \gamma\gamma X$ ,  $pp \rightarrow H \text{ jet } X$

Qiu, Schlegel, Vogelsang, PRL 107 (2011)  
Boer, CP, PRD 91 (2015)

The three contributions can be disentangled by defining the transverse moments

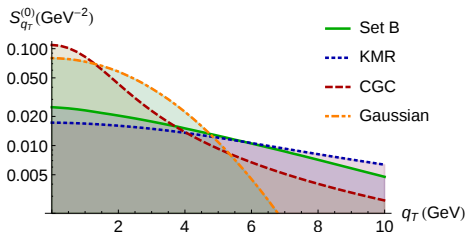
$$S_{q_T}^{(n)} \equiv \frac{\int_0^{2\pi} d\phi_{CS} \cos(n\phi_{CS}) \frac{d\sigma}{dQdYd^2q_Td\Omega}}{\int_0^{q_T^{\max}} dq_T^2 \int_0^{2\pi} d\phi_{CS} \frac{d\sigma}{dQdYd^2q_Td\Omega}} \quad (n = 0, 2, 4) \quad q_T^{\max} = \frac{Q^2}{4}$$

$$\begin{aligned} S_{q_T}^{(0)} &\implies f_1^g \otimes f_1^g \\ S_{q_T}^{(2)} &\implies f_1^g \otimes h_1^{\perp g} \\ S_{q_T}^{(4)} &\implies h_1^{\perp g} \otimes h_1^{\perp g} \end{aligned}$$

$$Q = 20 \text{ GeV},$$

$$Y = 0,$$

$$\theta_{CS} = \pi/2$$



Models for  $f_1^g$ : assumed to be the same as for Unintegrated Gluon Distributions:

- ▶ **Set B:** B0 solution to CCFM equation with input based on HERA data  
Jung et al., EPJC 70 (2010)
- ▶ **KMR:** Formalism embodies both DGLAP and BFKL evolution equations  
Kimber, Martin, Ryskin, PRD 63 (2010)
- ▶ **CGC:** Color Glass Condensate Model

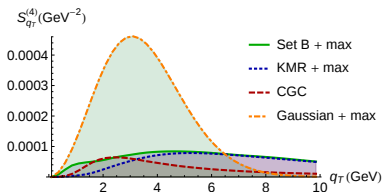
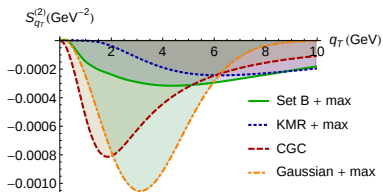
Dominguez, Qiu, Xiao, Yuan, PRD 85 (2012)  
Metz, Zhou, PRD 84 (2011)



$$Q = 20 \text{ GeV},$$

$$Y = 0,$$

$$\theta_{CS} = \pi/2$$



$h_1^{\perp g}$ : predictions only in the CGC: in the other models saturated to upper bound

$S_{q_T}^{(2,4)}$  smaller than  $S_{q_T}^{(0)}$ : can be integrated up to  $q_T = 10$  GeV

$$2.0\% \text{ (KMR)} < \left| \int dq_T^2 S_{q_T}^{(2)} \right| < 2.9\% \text{ (Gauss)}$$

$$0.3\% \text{ (CGC)} < \int dq_T^2 S_{q_T}^{(4)} < 1.2\% \text{ (Gauss)}$$

Possible determination of the shape of  $f_1^g$  and verification of a non-zero  $h_1^{\perp g}$

## Angular structure of the cross section for $p^\uparrow p \rightarrow J/\psi + \gamma X$

$$\begin{aligned} \frac{d\sigma_{UT}}{dy_\psi dy_\gamma d^2\mathbf{K}_\perp d^2\mathbf{q}_T} &\propto \sin\phi_S f_1^g \otimes f_{1T}^{\perp g} + B \left\{ \sin(\phi_S - 2\phi) f_1^g \otimes h_{1T}^g \right. \\ &+ \sin\phi_S \cos 2\phi [f_1^g \otimes h_{1T}^{\perp g} + h_{1T}^{\perp g} \otimes f_{1T}^{\perp g}] \\ &+ \sin\phi_S \cos 4\phi [h_{1T}^{\perp g} \otimes h_{1T}^g + h_{1T}^{\perp g} \otimes h_{1T}^{\perp g}] \\ &+ \cos\phi_S \sin 2\phi [f_1^g \bar{\otimes} h_{1T}^{\perp g} + h_{1T}^{\perp g} \bar{\otimes} f_{1T}^{\perp g}] \\ &\left. + \cos\phi_S \sin 4\phi [h_{1T}^{\perp g} \bar{\otimes} h_{1T}^g + h_{1T}^{\perp g} \bar{\otimes} h_{1T}^{\perp g}] \right\} \quad \phi \equiv \phi_T - \phi_\perp \end{aligned}$$

Lansberg, CP, Schlegel, in preparation

$$A_N^{\sin\phi_S} \equiv \frac{\int d\phi_S \sin\phi_S [d\sigma(\phi_S) - d\sigma(\phi_S + \pi)]}{\int d\phi_S [d\sigma(\phi_S) + d\sigma(\phi_S + \pi)]} = \frac{\int d\phi_S \sin\phi_S d\sigma_{UT}}{\int d\phi_S d\sigma_{UU}}$$

$$A_N^{\sin\phi_S} = |\mathbf{S}_T| \frac{f_1^g \otimes f_{1T}^{\perp g}}{2 f_1^g \otimes f_1^g}$$

- ▶ TMD distributions are affected by ISI/FSI, encoded in the gauge links, which render them gauge invariant but process dependent
- ▶ Different behavior of **unpolarized WW and dipole gluon TMDs**, both accessible at RHIC, could be tested experimentally
- ▶ Same considerations apply to **linearly polarized gluons**, which affect transverse spectra of (pseudo) scalar particles, and generate azimuthal asymmetries in associated quarkonium production at the LHC
- ▶ Two distinct **gluon Sivers functions** can be measured in  $pp$  collisions (RHIC and AFTER@LHC); the WW-type allows for a sign-change test (EIC)
- ▶ Observables related to gluon TMDs could be part of both the *spin* and the *small-x* program at a future EIC

CP, talk next Thursday  
see also: P. Mulders, talk on Monday  
S. Cotogno, talk on Tuesday