The entangled 3D structure of the proton

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Abstract

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The entangled 3D structure of the proton.

Light-front quantized quark and gluon states (partons) play a dominant role in high energy scattering processes. The initial pure proton state in these processes appears as a mixed ensemble of partons, while any produced pure partonic state appears as a mixed ensemble in the 3D world of the detector. The transition from collinear hard physics to the 3D structure including partonic transverse momenta is related to confinement and might hint at a more fundamental link between color and spatial degrees of freedom. Wilson loops, including Wilson lines along light-like directions such as used in the studies of transverse momentum dependent distribution functions (TMDs) might play a role here, establishing a direct link between transverse spatial degrees of freedom and gluonic degrees of freedom. They lead to many peculiarities among them single spin asymmetries in the physics of TMDs but they also unify and simplify our picture for gluons in the low-x domain.

Introduction

- Color & QCD
  - Distinct part of Standard Model, decoupling strong interactions
  - Color unvisible: local gauge invariance! No free quarks or gluons!
  - Color visible: valence quarks, N vs 1/N, f x D (distribution x fragmentation), color flow (future and past pointing gauge links), …
- Pragmatic approach: Front form quantization with good fields dominating in OPE

  $\frac{1}{2}\gamma^-\gamma^+\psi \quad \text{and} \quad g_T^{\alpha\mu}A_\mu^a$

- (1) A different view (entanglement & less dimensions)

  PJM – 1601.00300

- (2) Impact for strongly interacting matter
  - Example: Wilson loops and gluon TMDs

A different view, why?

- Entanglement
  - Entangled pure multipartite states → ensembles in reduced Hilbert space
    Kharzeev & Levin (1702.03489): how to get from a proton to parton ensemble
  - Also the other way: pure partonic state → ensemble of hadrons (fragmentation)
  - Maximal entanglement (MaxEnt)
    Cervera-Lierta, Latorre, Rojo & Rottoli (1703.02989): maximally entangled chiral left/right two-particle states are consistent with QED ($g_A=0$) & electroweak ($g_V=0$), at least if $\sin \Theta_W = ½$
  - Classical/quantum physics ('t Hooft – 1405.1548)

- Less dimensions (1+3 → 1+1) advantageous
  - Convergence in field theory: $d[\phi] = (d-2)/2 \to 0$, $d[\psi] = (d-1)/2 \to ½$.
    Stojkovic – 1406.2696: naturalness, …
  - Chirality (R/L) corresponding to right- and left-movers, P^+, P^- eigenstates

Multipartite states (QIT)

- Multipartites live in a Hilbert space: $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \ldots$ (possibly identical spaces!)

- For entangled bipartite states there is just one class of entangled (Bell) states (using qubits: R/L)
  $|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}(|RL\rangle + |LR\rangle) \quad or \quad \frac{1}{\sqrt{2}}(|RR\rangle + |LL\rangle)$

Class: Equivalence under local unitary transformations:
$U \otimes U \otimes \ldots$

- For entangled tripartite states there are two classes (Dur, Vidal, Cirac 2000)
  - aligned: $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$ (fragile)
  - mingled: $|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$ (robust)

- Multipartites and R/L basis states relevant for our purposes:
  - 1D field theory: $\mathcal{H} = \mathcal{H}^{\otimes x} \leftarrow \mathcal{H}^R \times \mathcal{H}^L$ with right/left-movers (chiral states)
  - Tripartite states: $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$
    - space (3D): leptons, electroweak (GHZ-class)
    - color: quarks in 1D, strong (W-class)

Multipartites, space-time and internal degrees of freedom

- Harmonic oscillator levels (SO(3) ←→ internal symmetry & more symmetry)

level	degeneracy	(n_x, n_y, n_z)	SO(3) (ℓ)	SU(3) ($\underline{n}$)
0	1	(0,0,0)	0	$\underline{1}$
1	3	(1,0,0), ...	1	$\underline{3}$
2	6	(2,0,0), (1,1,0), ...	$0 \oplus 2$	$\underline{6}$
3	10	(3,0,0), (2,1,0), (1,1,1), ...	$1 \oplus 3$	$\underline{10}$
4	15	...	$0 \oplus 2 \oplus 4$	$\underline{15}_s$

- Quark model: SU(6) x O(3)

N	configuration	SU(6) × O(3) multiplets
0	$(0s)^3$	$[56, 0^+]$
1	$(0s)^2(1p)$	$(56, 1^-)$ $[70, 1^-]$
2	$(0s)^2(2s)$	$(56, 0^+)$ $[70, 0^+]$
	$(0s)^2(2d)$	$(56, 2^+)$ $[70, 2^+]$
	$(0s)(1p)^2$	$[56, 0^+]$ $[56, 2^+]$ $(70, 0^+)$ $(70, 1^+)$ $(70, 2^+)$ $[20, 1^+]$

- Problematic at a fundamental level

All Possible Symmetries of the S Matrix*

Sidney Coleman† and Jeffrey Mandula‡
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 16 March 1967)

We prove a new theorem on the impossibility of combining space-time and internal symmetries in any but a trivial way. The theorem is an improvement on known results in that it is applicable to infinite-parameter groups, instead of just to Lie groups. This improvement is gained by using information about the S matrix; previous investigations used only information about the single-particle spectrum. We define a symmetry group of the S matrix as a group of unitary operators which turn one-particle states into one-particle states, transform many-particle states as if they were tensor products, and commute with the S matrix. Let G be a connected symmetry group of the S matrix, and let the following five conditions hold: (1) G contains a subgroup locally isomorphic to the Poincaré group. (2) For any $M>0$, there are only a finite number of one-particle states with mass less than M. (3) Elastic scattering amplitudes are analytic functions of s and t, in some neighborhood of the physical region. (4) The S matrix is nontrivial in the sense that any two one-particle momentum eigenstates scatter (into something), except perhaps at isolated values of s. (5) The generators of G, written as integral operators in momentum space, have distributions for their kernels. Then, we show that G is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

I. INTRODUCTION

UNTIL a few years ago, most physicists believed that the exact or approximate symmetry groups of the world were (locally) isomorphic to direct products of the Poincaré group and compact Lie groups. This world-view changed drastically with the publication of the first papers on $SU(6)$[1]; these raised the dazzling possibility of a relativistic symmetry group which was not simply such a direct product. Unfortunately, all attempts to find such a group came to disastrous ends, and the situation was finally settled by the discovery of

symmetry group of the S matrix, which contains the Poincaré group and which puts a finite number of particles in a supermultiplet. Let the S matrix be nontrivial and let elastic scattering amplitudes be analytic functions of s and t in some neighborhood of the physical region. Finally, let the generators of G be representable as integral operators in momentum space, with kernels that are distributions. Then G is locally isomorphic to the direct product of the Poincaré group and an internal symmetry group. (This is a loose statement of the theorem; a more precise one follows below.)

Basic symmetries including SUSY

- Hilbert space

$$\{(a^\dagger)^n|0\rangle, b^\dagger|0\rangle\}$$

$$[a, a^\dagger] = 1, \;\; \{b, b^\dagger\} = 1$$

- Supercharges

$$Q^\dagger_{ik} = b_i\, a^\dagger_k \text{ and } Q_{ik} = b^\dagger_i a_k$$

$$\{Q^\dagger_{ik}, Q_{jl}\} = \tfrac{1}{2}\,\delta_{ij}\{a^\dagger_l, a_k\} + \tfrac{1}{2}\,\delta_{kl}[b^\dagger_i, b_j]$$

$$a^\dagger_k \xrightarrow{Q_{ik}} b^\dagger_i \qquad a^\dagger_k \xleftarrow{Q^\dagger_{ik}} b^\dagger_i$$

hamiltonian/number operators (i=j, k=l)
& unitary rotations

- For boson and fermion fields

$$\varphi = \frac{1}{\sqrt{2\omega}}\,(a + a^\dagger) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}\,(b + b^\dagger)$$

$$Q = \sqrt{\omega}(a^\dagger b - b^\dagger a)$$

Free fields

$$F = [\varphi, H] = M\varphi$$

$$iD\varphi = M\varphi = i\dot{\varphi}$$

$$[Q, \varphi] = \xi \qquad \{Q, \xi\} = \{Q, [Q, \varphi]\} = F = iD\varphi$$

$$[Q, F] = [Q, \{Q, \xi\}] = iD\xi$$

$$iD = i\partial + gA$$

unitary rotations

- Implement symmetries via constraints F

... and a nontrivial vacuum

$$\phi(x) = \exp(-i\int_0^x ds^\mu D_\mu)\phi$$

Emerging symmetries of standard model

Fields

- Real/Majorana: $\phi \quad \xi$ and $\langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L}$ and $\langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (1D Wess-Zumino → 1D gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$

$$\boxed{iD_\sigma\phi^i = i\partial_\sigma\phi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \phi^j}$$

- 3D: $\phi_S \quad A_k^a \quad \psi$

$$\boxed{iD_\mu\phi^i = i\partial_\mu\phi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \phi^j}$$

- and ....

$$n_\pm^\sigma \longrightarrow n_\alpha^\mu \quad \gamma^\sigma = \begin{bmatrix} 0 & n_-^\sigma \\ n_+^\sigma & 0 \end{bmatrix} \longrightarrow \gamma^\mu = \begin{bmatrix} 0 & \bar\sigma^\mu \\ \sigma^\mu & 0 \end{bmatrix}$$

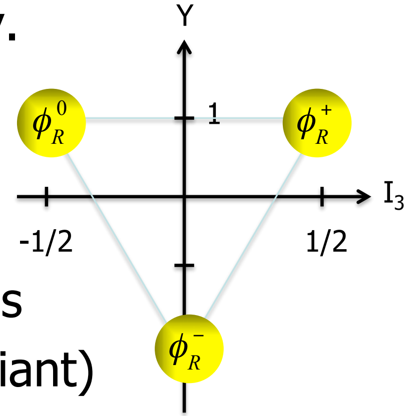
in order to match space-time and field symmetries (Haag-Luposzanski-Sohnius) and avoid Coleman-Mandula when moving K into P(1,1) and SO(3) into P(1,3)

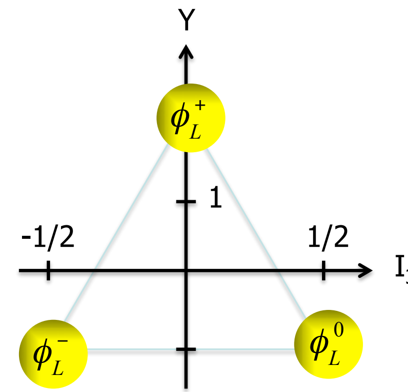
Generators
Space-time & Internal

- H
- P⁺, P⁻ K, SU(3)
- H, P, K SU(3) = [SO(3),SU(2)xU(1)]
- H, **P**, **K**, **J** SU(2)xU(1)

1D bosonic basis states and aligned tripartites

- Basis of each of the Hilbert spaces: P(1,1) x SU(3): $\vec{\phi} = (\phi^1, \phi^2, \phi^3), \ldots$
- Assign Y-I_3 using the SU(3) symmetry.



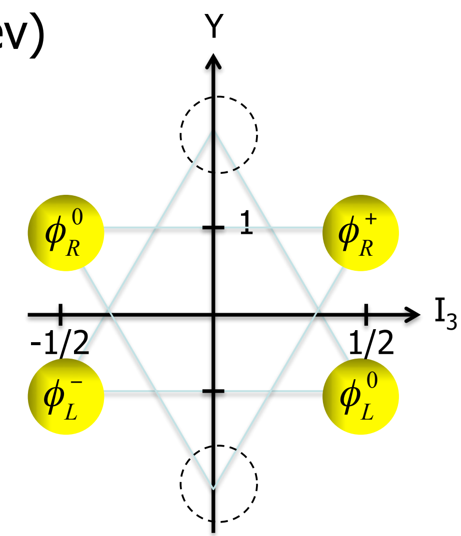


- 3D and P(1,3): aligned tripartite states built on vacuum |0,0,0> (SO(3) invariant)

- generated by SU(2) x U(1) from vacuum (nonzero vev)

$$\phi_R = \frac{1}{\sqrt{2}} \exp\left(+\tfrac{i}{2} \sum_{a=1,2,3,8} \theta^a \lambda_a\right) \begin{bmatrix} 1+\varphi_H \\ 0 \\ 0 \end{bmatrix}$$

$$\phi_L = \frac{1}{\sqrt{2}} \exp\left(-\tfrac{i}{2} \sum_{a=1,2,3,8} \theta^a \lambda_a\right) \begin{bmatrix} 0 \\ 1+\varphi_H \\ 0 \end{bmatrix}$$



Bosonic excitations electroweak symmetry breaking and XQCD

- 3D Electroweak symmetry breaking is SU(2) x U(1) → $U(1)_{QED}$

$$iD_\mu \phi = i\partial_\mu \phi + \frac{g}{2}\left(\sum_{i=1}^{3} W_\mu^i \lambda_i + B_\mu \lambda_8\right)\phi$$
$$= i\partial_\mu \phi + \frac{g}{\sqrt{2}}(W_\mu^+ I_- + W_\mu^- I_+)\phi + \left(g\, W_\mu^0 I_3 + \frac{g}{2\sqrt{3}} B_\mu Y\right)\phi$$

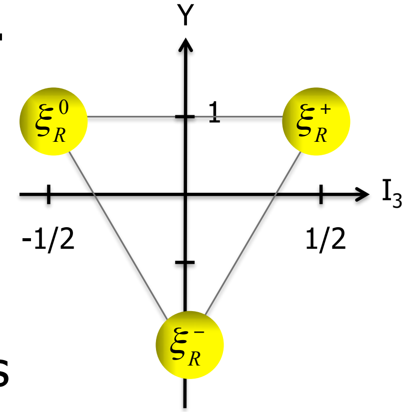
- SU(3) embedding for electroweak gives embarrasingly good ‘zeroth order’ results:
  - Implies weak mixing angle $\sin\theta_W = 1/2$ (Weinberg 1972)
  - $g_0 = M/2$, $g^2 = 3/8$ (use $M = M_{top}$)
  - gives $M_H^2 = M^2/2$, $M_W^2 = 3M_Z^2/4$, $M_Z = M/2$
  - $e = g/2 = (3/32)^{1/2}$ or $1/\alpha = 134$

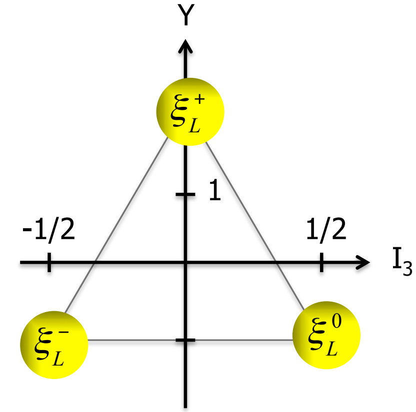
- 1D strong sector: $\mathcal{L} = \frac{1}{2}\partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \overline{\psi}(i\not{D} - M - g_0\, \varphi_S)\psi$
8 instantaneous gluons and a scalar field, resembling $XQCD_{1+1}$ (Kaplan 1306.5818)
and dynamics governed by $gF_{\tau\sigma} = \delta W[C]/\delta\sigma^{\tau\sigma}$ via Wilson loop

$$W[C] = \exp\left(-ig \oint_C ds^\mu A_\mu(s)\right)$$

1D fermionic states & aligned tripartites

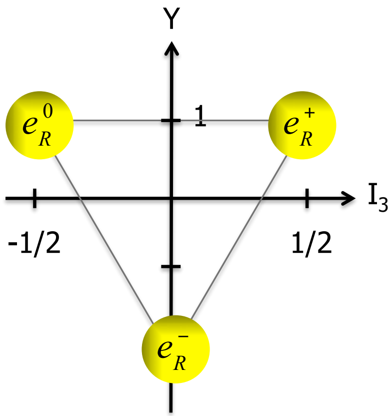
- Basis of each of the Hilbert spaces: P(1,1) x SU(3): $\vec{\xi} = (\xi^1, \xi^2, \xi^3), \ldots$
- Assign Y-I_3 using the SU(3) symmetry.

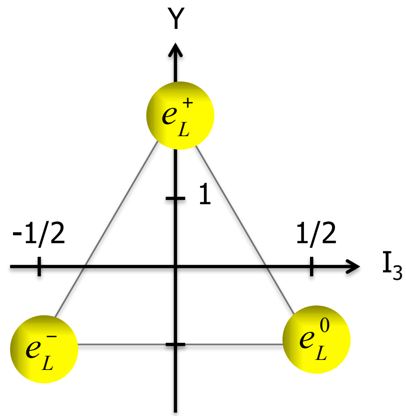




- 3D and P(1,3): aligned tripartite states

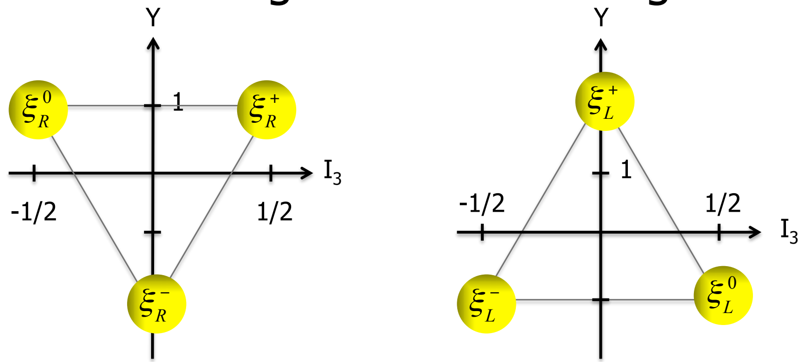
  leptons: electrons & neutrinos





Bosonic and fermionic excitations: lepton families

- 3D embedding fermions is straightforward:



$$\gamma^\mu = \begin{bmatrix} 0 & \overline{\sigma}^\mu \\ \sigma^\mu & 0 \end{bmatrix}$$

$$\gamma_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- Families linked to three singlets of Z(3):

$$Q_{\mathrm{ew}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad Q_{\mathrm{mass}} = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$$

$$Q_{\mathrm{mass}} = U_Q^\dagger \, Q_{\mathrm{ew}} \, U_Q \qquad U_Q^\dagger = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{i}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}} \end{bmatrix}$$

$$\langle \widehat{\phi} \rangle_{\mathrm{fam}} = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \Leftrightarrow \langle \widehat{\phi} \rangle_{\mathrm{ew}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\langle \widehat{\phi} \rangle_{\mathrm{fam}} = W \langle \widehat{\phi} \rangle_{\mathrm{ew}}$$

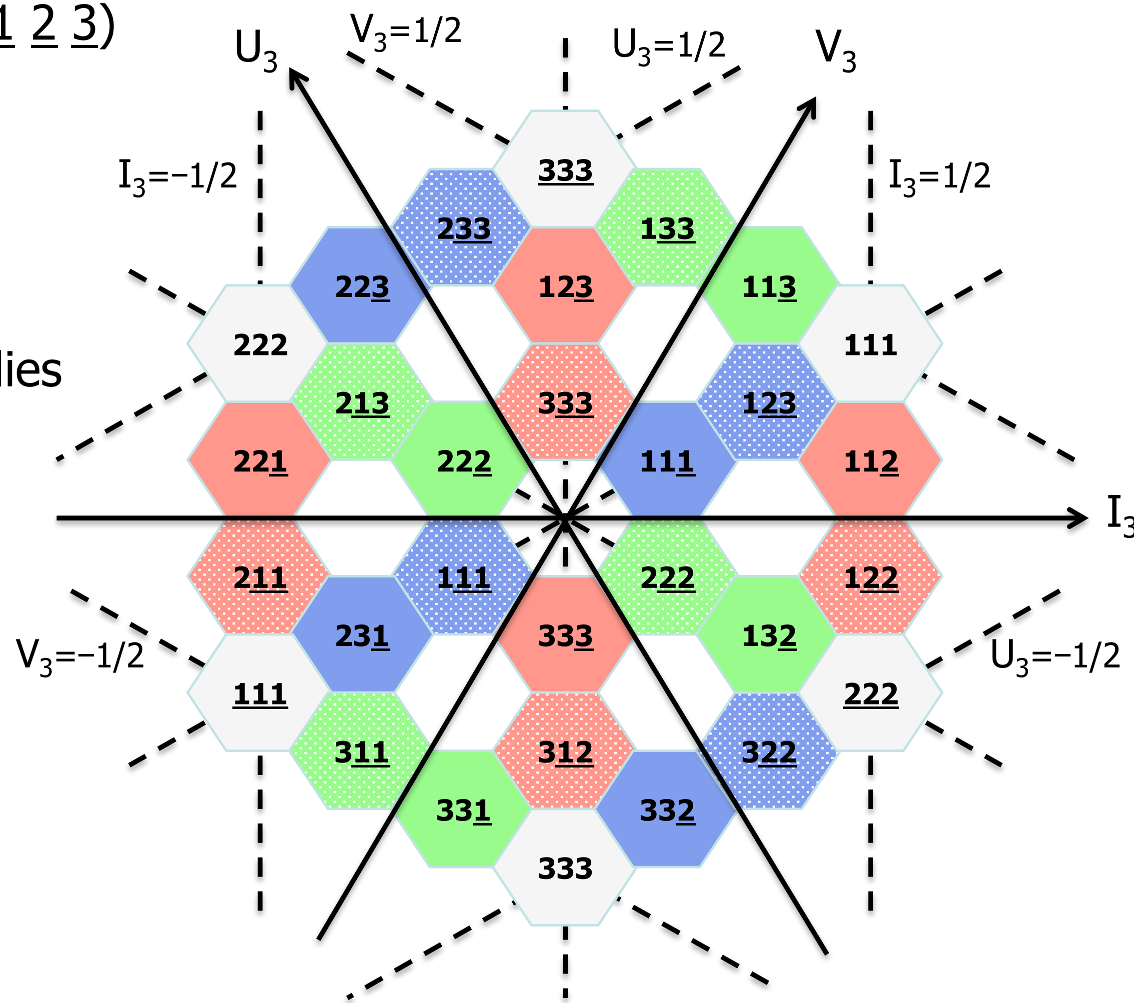
$$W = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ \omega^2 & 1 & \omega \\ \omega & 1 & \omega^2 \end{bmatrix}$$

$$Q_{\mathrm{fam}} = W Q_{\mathrm{ew}} W^\dagger = W U_Q \, Q_{\mathrm{mass}} \, U_Q^\dagger W^\dagger \qquad U_{\mathrm{HPS}} = W U_Q = \begin{bmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{bmatrix}$$

- Lepton masses? Just note: $M/8\pi^2 = 2$ GeV (factor from SO(3) group measure)

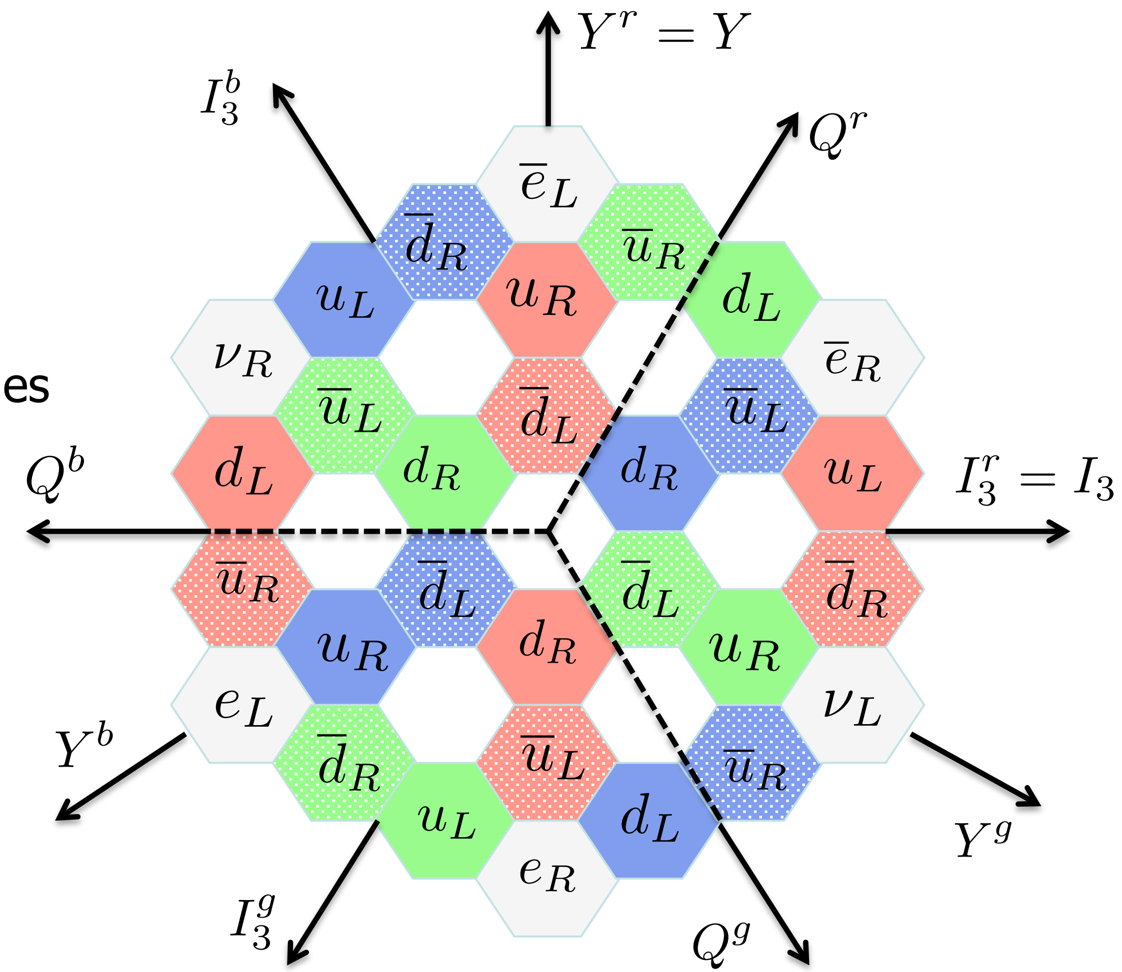
Fermionic excitations: leptons and quarks

- Tripartite states (R: 1 2 3 & L: $\underline{1}$ $\underline{2}$ $\underline{3}$)
- Aligned (RRR, LLL) GHZ states
  - SO(3) → asymptotic/space
    I, U, *and* V allowed
  - Three A(4) singlets → families
- Mingled (RRL, RLL) W-states
  - non-asymptotic
    I, U, *or* V allowed
  - Three A(4) triplets



Fermionic excitations: electroweak quantum numbers

- LEPTONS
  Aligned (RRR, LLL)
  - SO(3) → asymptotic/space
    I, U, *and* V allowed
  - Three A(4) singlets → families
  - Family mixing is
    tri-bimaximal
- QUARKS
  Mingled (RRL, RLL)
  - non-asymptotic
    I, U, *or* V allowed
  - Three A(4) triplets
  - Just one heavy quark!

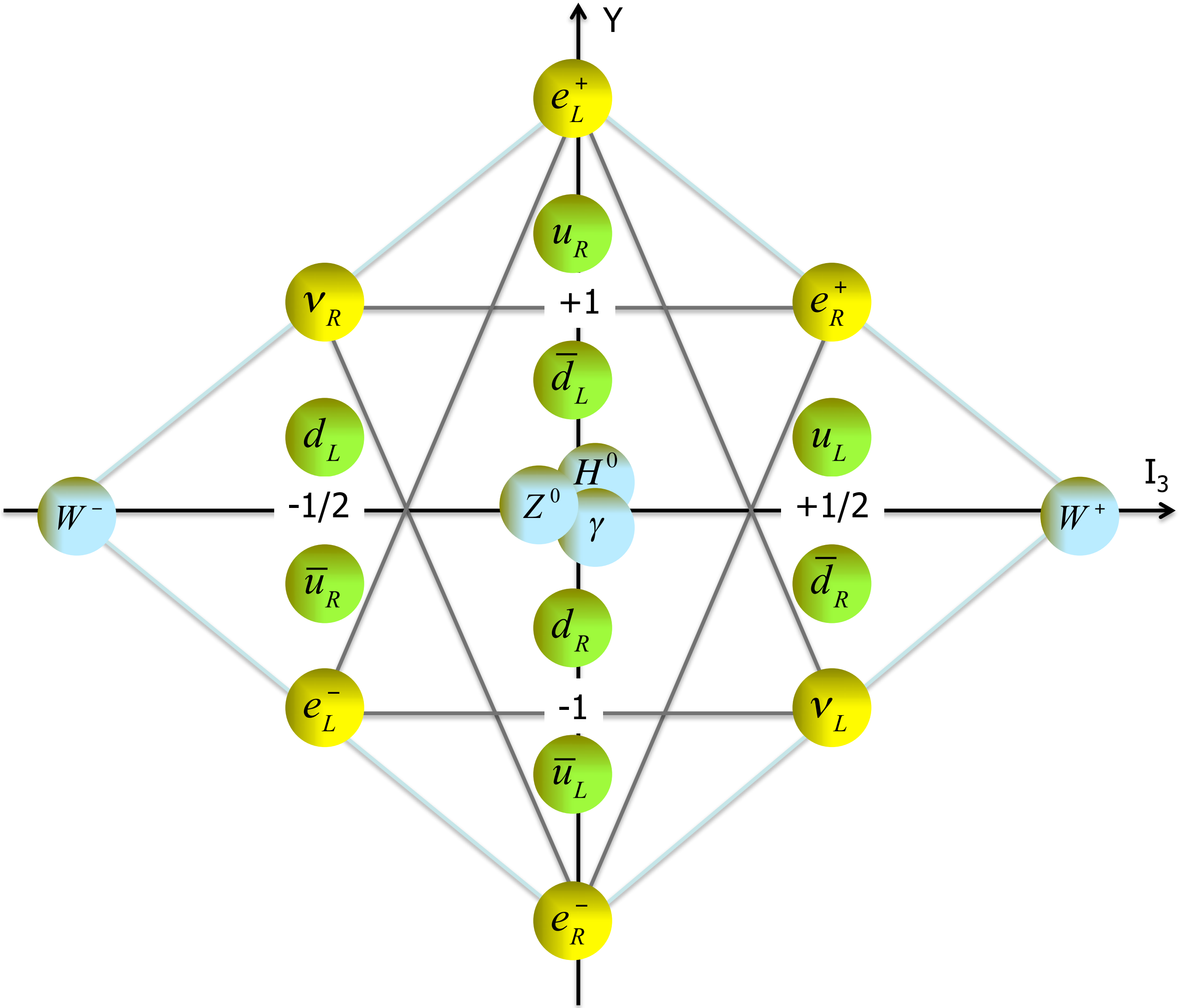


Electroweak particle content of standard model

particle	space			isospin		hypercharge	charge	color
	L	T_1	T_2	I	I_3	Y	Q	$\underline{c}$
ν_L	ξ_L^0	ξ_L^0	ξ_L^0	$1/2$	$+1/2$	-1	0	$\underline{1}$
e_L^-	ξ_L^-	ξ_L^-	ξ_L^-	$1/2$	$-1/2$	-1	-1	$\underline{1}$
e_L^+	ξ_L^+	ξ_L^+	ξ_L^+	0	0	$+2$	$+1$	$\underline{1}$
ν_R	ξ_R^0	ξ_R^0	ξ_R^0	$1/2$	$-1/2$	$+1$	0	$\underline{1}$
e_R^+	ξ_R^+	ξ_R^+	ξ_R^+	$1/2$	$+1/2$	$+1$	$+1$	$\underline{1}$
e_R^-	ξ_R^-	ξ_R^-	ξ_R^-	0	0	-2	-1	$\underline{1}$
u_L	ξ_L^0	$(\xi_R^+$	$\xi_R^+)$	$1/2$	$+1/2$	$+1/3$	$+2/3$	$\underline{3}$
d_L	ξ_L^-	$(\xi_R^0$	$\xi_R^0)$	$1/2$	$-1/2$	$+1/3$	$-1/3$	$\underline{3}$
$\overline{u}_L$	ξ_L^0	$(\xi_L^-$	$\xi_R^-)$	0	0	$-4/3$	$-2/3$	$\underline{3}^*$
$\overline{d}_L$	ξ_L^+	$(\xi_L^0$	$\xi_R^0)$	0	0	$+2/3$	$+1/3$	$\underline{3}^*$
$\overline{u}_R$	ξ_R^0	$(\xi_L^-$	$\xi_L^-)$	$1/2$	$-1/2$	$-1/3$	$-2/3$	$\underline{3}^*$
$\overline{d}_R$	ξ_R^+	$(\xi_L^0$	$\xi_L^0)$	$1/2$	$+1/2$	$-1/3$	$+1/3$	$\underline{3}^*$
u_R	ξ_R^0	$(\xi_L^+$	$\xi_R^+)$	0	0	$+4/3$	$+2/3$	$\underline{3}$
d_R	ξ_R^-	$(\xi_L^0$	$\xi_R^0)$	0	0	$-2/3$	$-1/3$	$\underline{3}$

Resembles rishon model (Harari & Seiberg 1982), but no compositeness!

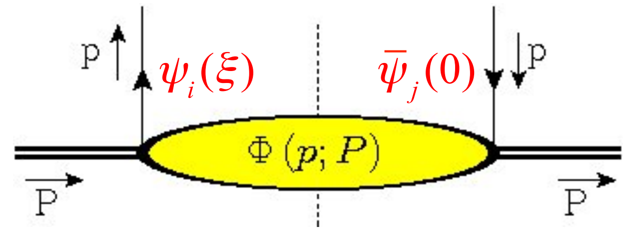
Standard model particle content



TMDs: matrix elements (with gauge links)

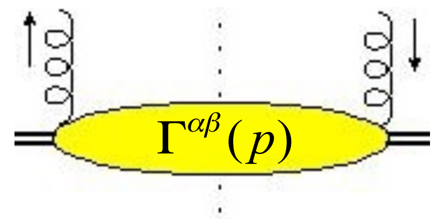
- quark-quark: $u_i(k)\overline{u}_j(k) \Longrightarrow$

$$\Phi^{[U]}_{ij}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\langle P,S|\overline{\psi}_j(0)\,U_{[0,\xi]}\psi_i(\xi)|P,S\rangle\,\big|_{\xi\cdot n=0}$$



- gluon-gluon: $\epsilon^{\alpha}(k)\epsilon^{\beta *}(k) \Longrightarrow$

$$\Gamma^{[U,U']\,\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\;e^{ip\cdot\xi}\,\langle P,S|\,F^{n\mu}(0)\,U_{[0,\xi]}\,F^{n\nu}(\xi)\,U'_{[\xi,0]}\,|P,S\rangle\big|_{\xi\cdot n=0}$$



- Relevant matrix elements at high energies project on ‘good’ fermion and transverse gauge fields and naturally represent densities of these.
- Noncollinearity requires nontrivial Wilson lines / gauge links.

Using 1D to 3D transition (plan)

- Parton distribution functions: collinear PDFs to TMDs and role of Wilson loop (with Daniel Boer, Tom van Daal, Sabrina Cotogno)
- Dominance of gluons at low x (dipole picture, color glass condensate, ...) (with Elena Petreska)

TMDs and color gauge invariance (gauge links)

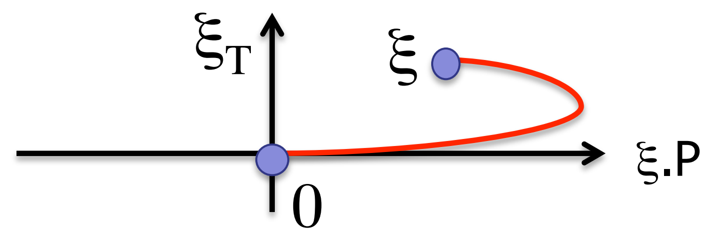
- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

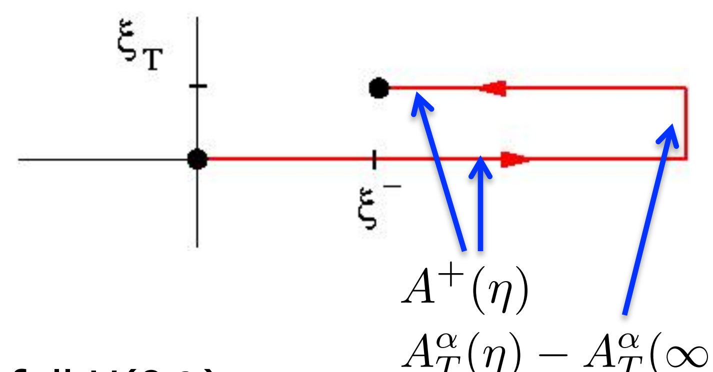
$$U(0,\xi) = \mathcal{P}\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence in $\Phi^{[U]}(x,p_T)$



- 'Dominant' paths: along lightcone connected at lightcone infinity (staples)
- Reduces to 'straight line' for Φ(x)
  $\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$
  (no gluon dynamics)



- Be aware that one needs all orders in g to obtain full U(0,ξ)

Non-universality because of process dependent gauge links

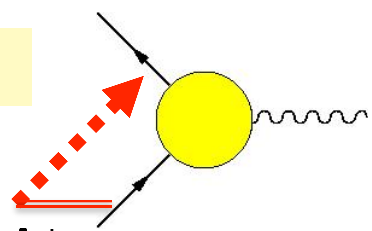
$$\Phi_{ij}^{q[C]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\middle|\bar{\psi}_j(0)\,U_{[0,\xi]}^{[C]}\,\psi_i(\xi)\middle|P\right\rangle_{\xi.n=0}$$

TMD

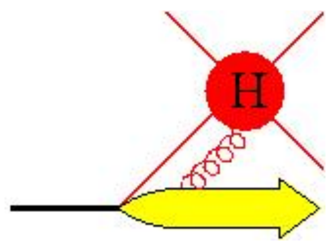
path dependent gauge link

- Gauge links associated with dimension zero (not suppressed!) collinear $A^n = A^+$ gluons, leading for TMD correlators to process-dependence:

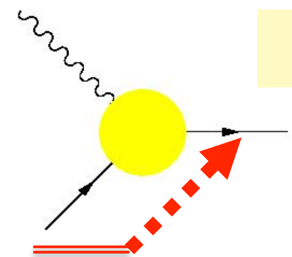
DY



... A^+ ...
(resummation)

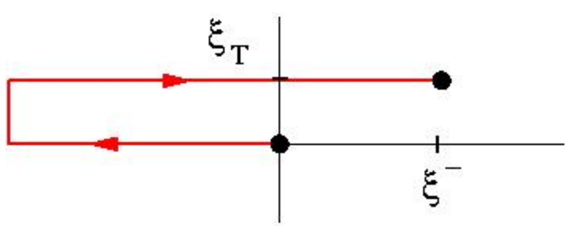


SIDIS

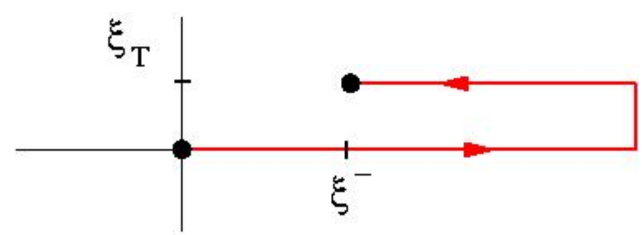


... A^+ ...

$\Phi^{[-]}$



Time reversal



$\Phi^{[+]}$

Belitsky, Ji, Yuan, 2003; Boer, M, Pijlman, 2003

Non-universality because of process dependent gauge links

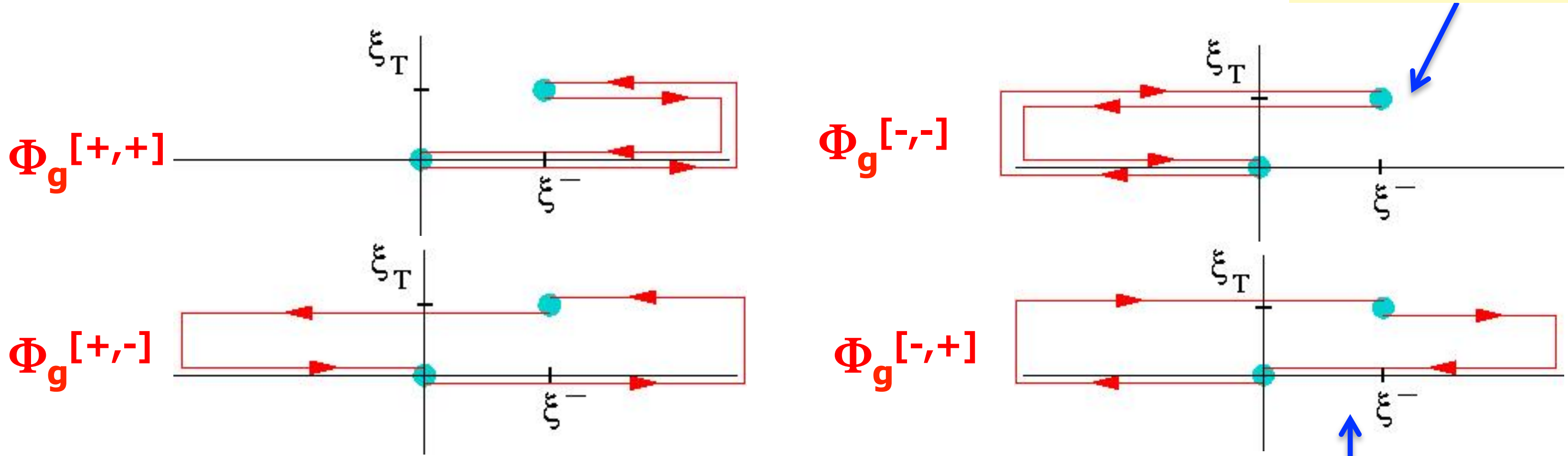
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| U_{[\xi,0]}^{[C]} F^{n\alpha}(0) U_{[0,\xi]}^{[C']} F^{n\beta}(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi) \rightarrow U_{[\eta,\xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

gg → H



in gg → $Q\bar{Q}$

- Collinear gluon PDFs: straight line 'octet' link

Bomhof, M, Pijlman, 2006; Dominguez, Xiao, Yuan, 2011

Gluon correlators in a polarized target (up to spin ½)

- Unpolarized target

$$\Gamma^{ij[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{ij}\, f_1^{[U]}(x,k_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp\,[U]}(x,k_T^2)\right\}$$

- Vector polarized target

$$\Gamma_L^{ij[U]}(x,k_T) = \frac{x}{2}\left\{i\epsilon_T^{ij}\, S_L g_1^{[U]}(x,k_T^2) + \frac{\epsilon_{T\;\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, S_L h_{1L}^{\perp[U]}(x,k_T^2)\right\}$$

$$\Gamma_T^{ij[U]}(x,k_T) = \frac{x}{2}\left\{\frac{g_T^{ij}\,\epsilon_T^{kS_T}}{M}\, f_{1T}^{\perp[U]}(x,k_T^2) - \frac{i\epsilon_T^{ij}\,k_T\!\cdot\!S_T}{M}\, g_{1T}^{[U]}(x,k_T^2) - \frac{\epsilon_T^{k\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,k_T^2) - \frac{\epsilon_{T\;\alpha}^{\{i}k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp}(x,k_T^2)\right\}$$

- ......

(talk of Sabrina Cotogno)

Structure of gluon TMDs in targets (up to spin 1)

TARGET SPIN	PARTON SPIN		
GLUONS	$-g_T^{\alpha\beta}$	$\varepsilon_T^{\alpha\beta}$	$p_T^{\alpha\beta}, \ldots$
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g $h_{1T}^{\perp g}$
LL	f_{1LL}^g		$h_{1LL}^{\perp g}$
LT	f_{1LT}^g	g_{1LT}^g	h_{1LT}^g $h_{1LT}^{\perp g}$
TT	f_{1TT}^g	g_{1TT}^g	h_{1TT}^g $h_{1TT}^{\perp g}$ $h_{1TT}^{\perp\perp g}$

Jaffe & Manohar, Nuclear gluonometry, PL B223 (1989) 218

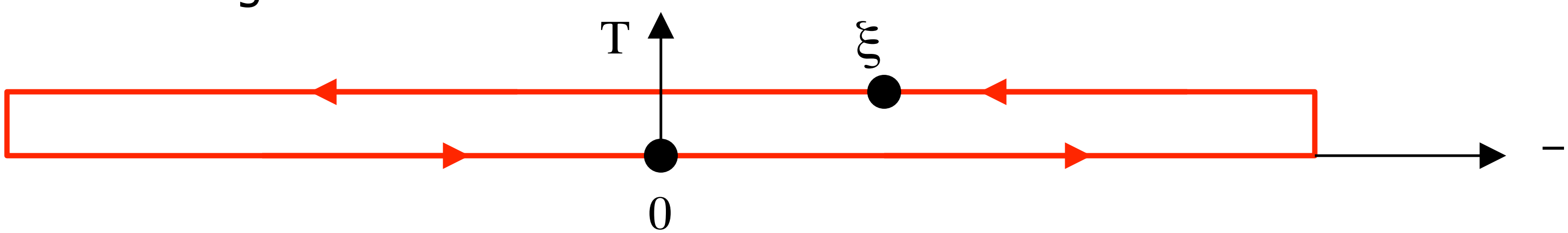
PJM & Rodrigues, PR D63 (2001) 094021

Meissner, Metz and Goeke, PR D76 (2007) 034002

D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

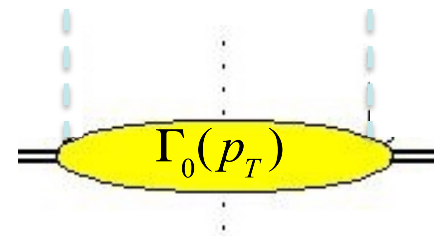
Gluons at low x – link color-space

- Interestingly, there is a Wilson loop linking transverse spatial structure and transverse gluons



$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta\sigma_{\alpha\beta}} \qquad A_T^\alpha \longleftrightarrow F^{+\alpha} = \frac{\delta W[C]}{\delta\sigma_{\alpha+}}$$

- Matrix element of single Wilson loop correlator just represents a 'TMD'



- Relevant for diffraction and as x→0 limit of TMDs
  Even without color exchange interactions can be induced
  Link with dipole picture used at small x
- Differentiation gives $\Gamma^{[+-]}$ gluon TMD for zero momentum (x = 0)

Hatta, Xiao, Yuan, PRL 116 (2016) 202301, ArXiv 1601.01585

D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Small x physics in terms of TMDs

- Note limit x → 0 for gluon TMDs linked to Wilson loop correlator Γ_0

$$\Gamma_0(k_T) = \frac{1}{2M^2}\left\{e(k_T^2) - \frac{\epsilon^{k S_T}}{M}\, e_T(k_T^2)\right\}$$

- Dipole correlators: at small x only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

$$x\, f_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\, e^{[+,-]}(k_T^2)$$

$$x\, h_1^{\perp[+,-]}(x,k_T^2) \longrightarrow e^{[+,-]}(k_T^2)$$

$$x\, f_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\, e_T^{[+,-]}(k_T^2)$$

$$x\, h_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\, e_T^{[+,-]}(k_T^2)$$

$$x\, h_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow e_T^{[+,-]}(k_T^2)$$

Dominguez, Xiao, Yuan 2011
D Boer, MG Echevarria, PJM, J Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485
D Boer, S Cotogno, T van Daal, PJM, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Summary and implications for standard model & QCD

- ‘Different view’ does not invalidate the standard model field theoretical results
  - it may affect way that (QCD+EW) loop corrections are implemented
  - It does away with the confinement issue: quarks are not asymptotic states.
  - Only for color singlet composites, rotational invariance can be employed in analogy to the lepton sector, implying that for valence quarks and antiquarks in hadrons a swap has to be made from $SU(3)_{local}$ in 1D to $SU(3)_{global}$ in 3D
- Provides a new view for many phenomena in QCD (confinement, Bloom-Gilman duality, separation of hard/soft modes in SCET, jet physics, color-kinematic duality, multitude of effective models for QCD, CFT approaches a la Brodsky, de Téramond, Dosch, Lorcé getting to effective SUSY for baryons/mesons)
- It could shed light on the transition from collinear → 3D picture
  - At level of partons/good fields: transition from PDFs to TMDs with staple gauge links
  - Role of Wilson loops in unifying dipole and TMD pictures at small x
- Many open ends remain!

The Portal