From Baryon Distribution Amplitudes to Generalised Parton Distributions.

Cédric Mezrag

Argonne National Laboratory

August 30^{th}, 2017

Chapter 1: Baryon Distribution Amplitudes

Cédric Mezrag, Craig Roberts and Jorge Segovia

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle$$

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk}u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\Big[(\not{p}C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-) + (\not{p}\gamma_5 C)_{\alpha\beta}\left(N^+\right)_\gamma A(z_i^-) - (ip^\mu\sigma_{\mu\nu}C)_{\alpha\beta}\left(\gamma^\nu\gamma_5 N^+\right)_\gamma T(z_i^-)\Big]$$

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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

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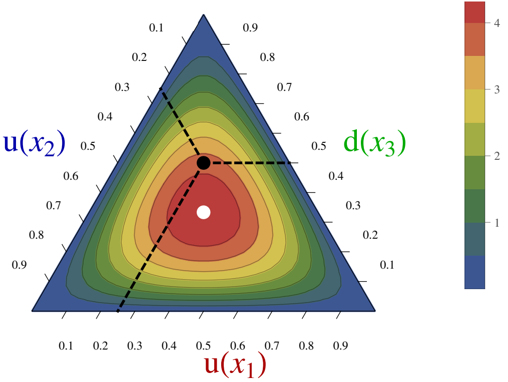
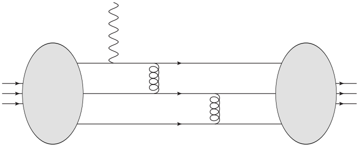
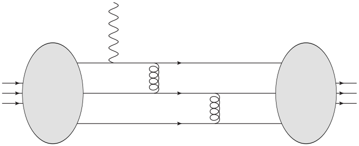
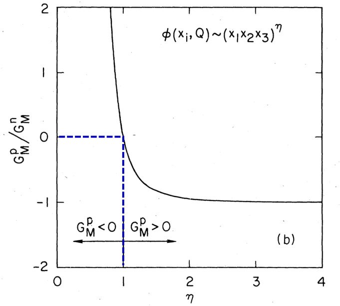
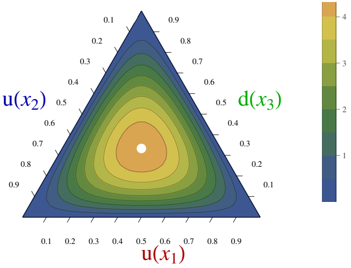


Form Factors: Nucleon case









$\eta = 1$

S. Brodsky and G. Lepage, PRD 22, (1980)

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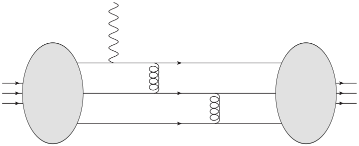

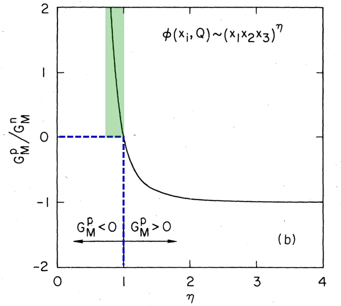

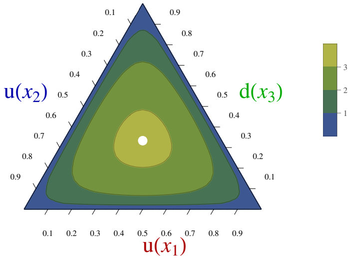

$\eta = 0.5$

S. Brodsky and G. Lepage, PRD 22, (1980)

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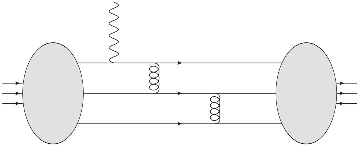

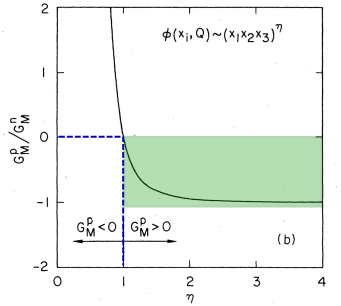

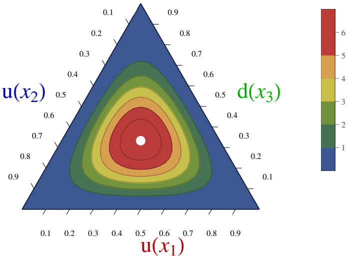

$\eta = 2$

S. Brodsky and G. Lepage, PRD 22, (1980)

Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation (See F. Hutzler talk)
  - G. Bali *et al.*, JHEP 2016 02

Faddeev Equation and diquark picture

- The Faddeev equation provides a covariant framework to describe the nucleon as a bound state of three dressed quarks.

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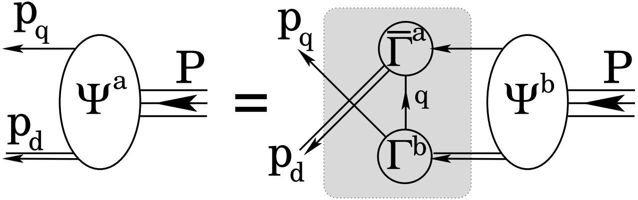

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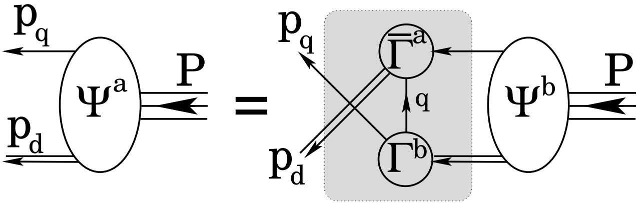

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks, whose mass is roughly 2/3 of the nucleon mass,
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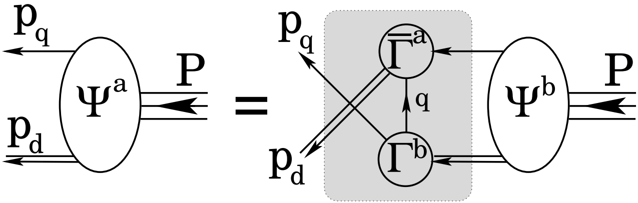

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- Can we understand the nucleon DA in terms of quark-diquarks correlations?

Nucleon DA and DSE

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

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Braun *et al.*, Nucl.Phys. B589 (2000)

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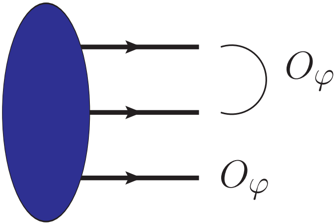

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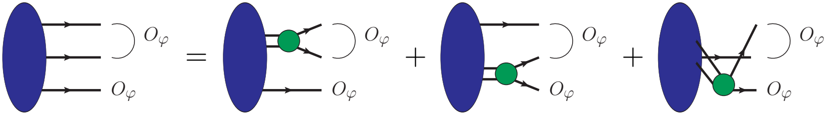

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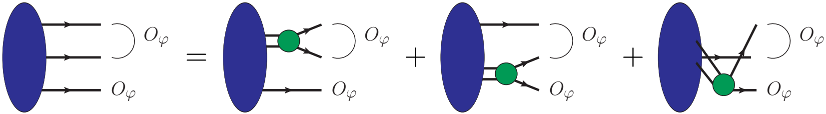
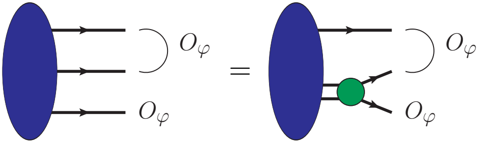


- The operator then selects the relevant component of the wave function.

Scalar diquark contribution

- In the scalar diquark case, only one contribution remains (φ case):



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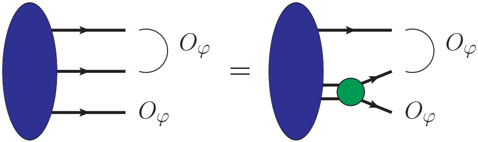

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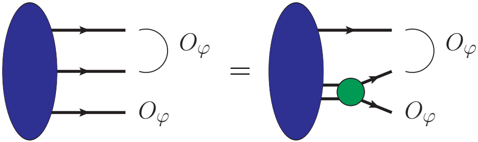

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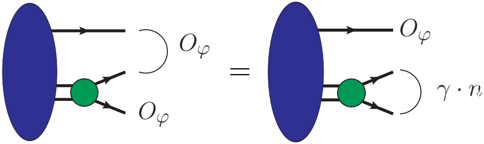

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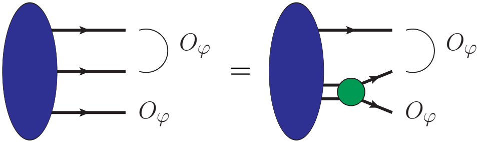

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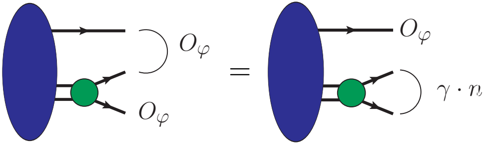

We recognise the leading twist DA of a scalar diquark

AV Contributions

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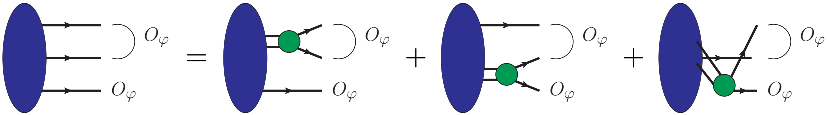

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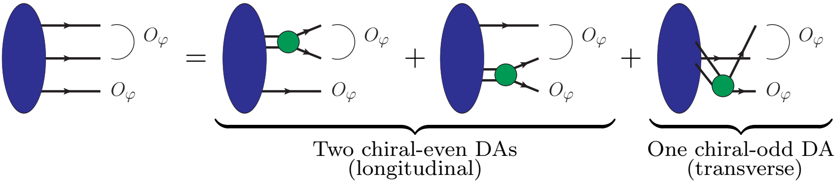

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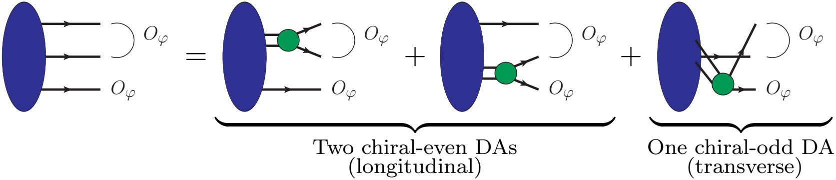

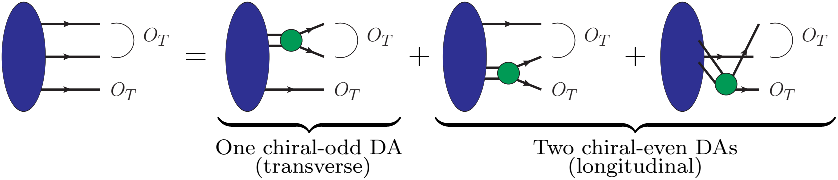

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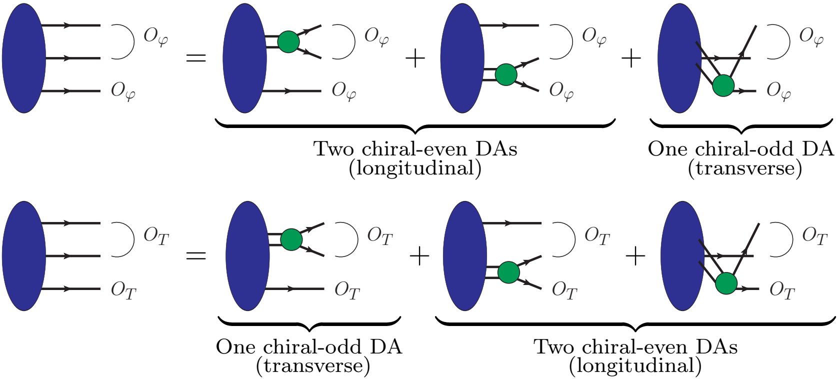

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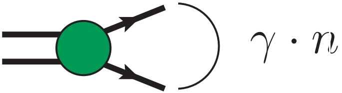
Modeling the Diquarks

Scalar diquark I: the point-like case



- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

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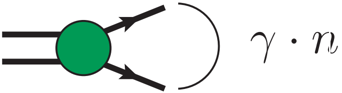

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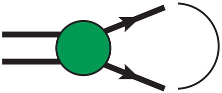

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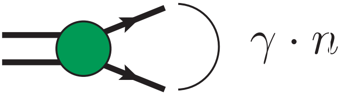
- This point-like case leads to a flat DA:

$$\phi_{\mathrm{PL}}(x) = 1$$



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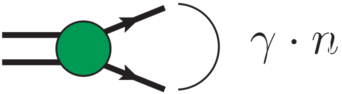


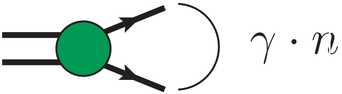
- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

- Bethe-Salpeter amplitude (1 out of 4 structures):

$$\Gamma^{0+}_{\mathrm{PL}}(q, K) = i\gamma_5 C \mathcal{N}^{0+} \int_{-1}^{1} \mathrm{d}z \frac{(1 - z^2)}{\left[\left(q - \frac{1-z}{2}K\right)^2 + \Lambda_q^2\right]}$$

Scalar diquark II: the Nakanishi case



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- The Nakanishi case leads to a non trivial DA:

$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

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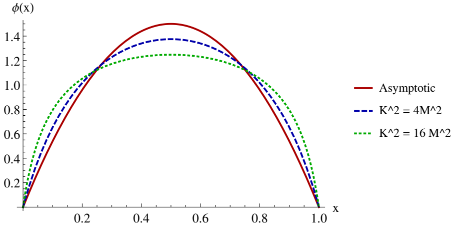
Scalar diquark



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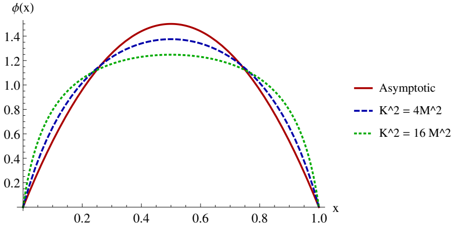
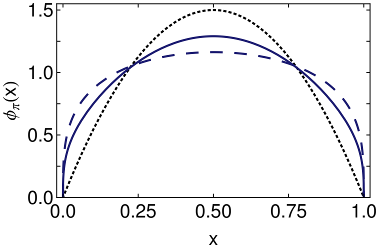
Scalar diquark



Pion

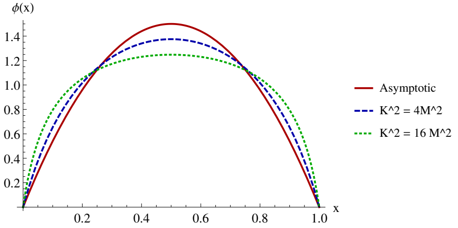


Pion figure from L. Chang *et al.*, PRL 110 (2013)

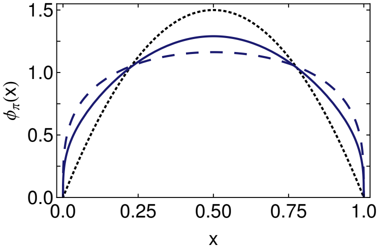
Scalar DA behaviour

$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2} x(1-x)\right]}{x(1-x)}$$

Scalar diquark



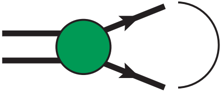
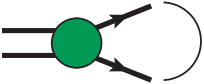
Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

This extended version of the DA seems promising!

AV diquark DA

 $\gamma \cdot n$  $\sigma^{\mu\nu} n_\nu$

- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

AV diquark DA

- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

- Bethe-Salpeter amplitude (2 out of 8 structures):

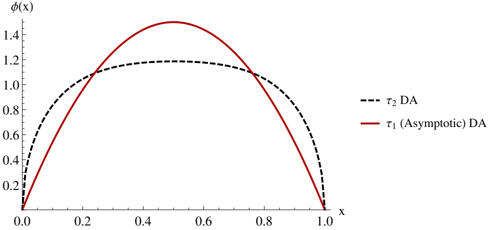
$$\Gamma^{\mu}_{\rm PL}(q,K) = (\mathcal{N}_1 \tau_1^\mu + \mathcal{N}_2 \tau_2^\mu)\, C \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(q - \frac{1-z}{2}K\right)^2 + \Lambda_q^2\right]}$$

$$\tau_1^\mu = i\left(\gamma^\mu - K^\mu \frac{\not{K}}{K^2}\right) \to \text{Chiral even}$$

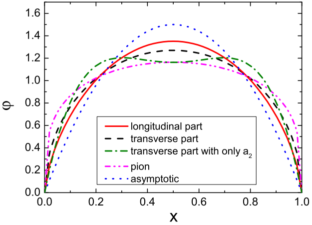
$$\tau_2^\mu = \frac{K\cdot q}{\sqrt{q^2(K-q)^2}\sqrt{K^2}} \left(-i\tau_1^\mu \not{q} + i\not{q}\tau_1^\mu\right) \to \text{Chiral odd}$$

Comparison with the ρ meson

AV diquark

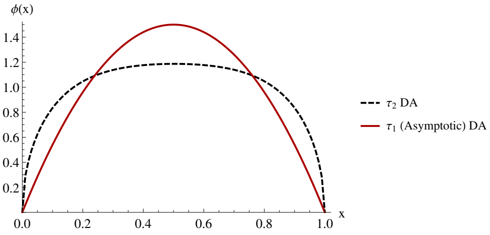


ρ meson

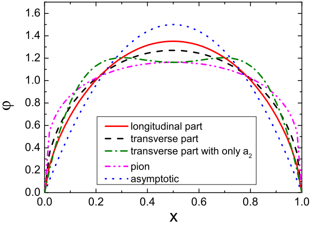


ρ figure from F. Gao *et al.*, PRD 90 (2014)

AV diquark



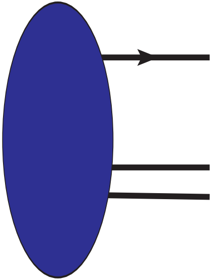
ρ meson



ρ figure from F. Gao *et al.*, PRD 90 (2014)

- Same "shape ordering" $\rightarrow \phi_\perp$ is flatter in both cases.
- Farther apart compared to the ρ meson case.

Modeling the Faddeev Amplitude



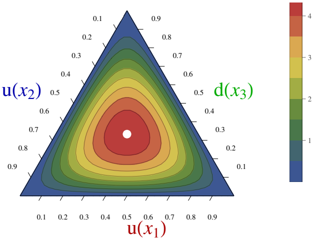
- Scalar case:

$$s_1(K,P) = \mathcal{N}_1 \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

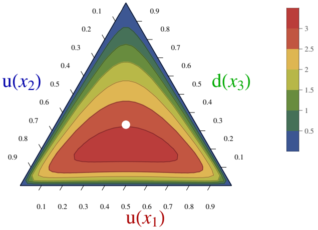
- AV case (2 out of 6 structures):

$$A^\mu(K,P) = \left(\gamma_5\gamma^\mu - i\gamma_5\hat{P}^\mu\right) \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

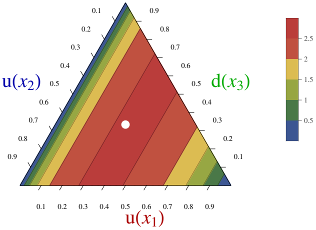
Results in the scalar channel



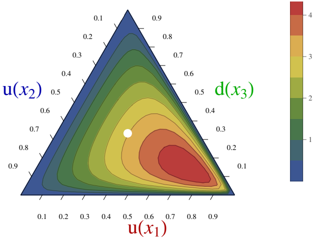
Asymptotic DA



Extended case: T



Point-like case: φ



Extended case: φ

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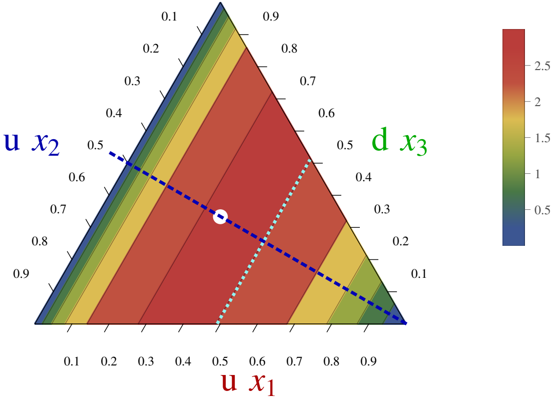
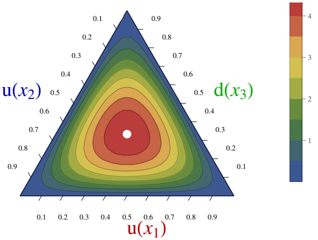
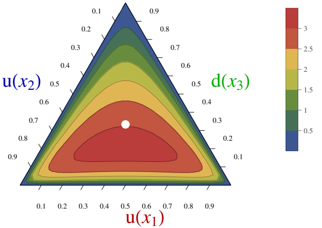


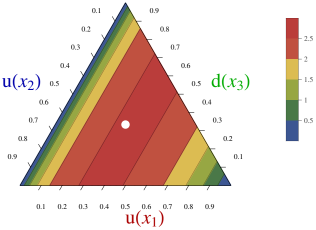
Point-like case: φ



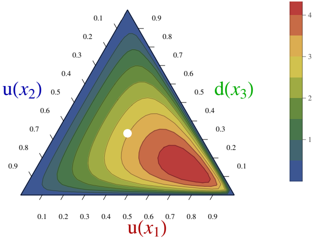
Asymptotic DA



Extended case: T

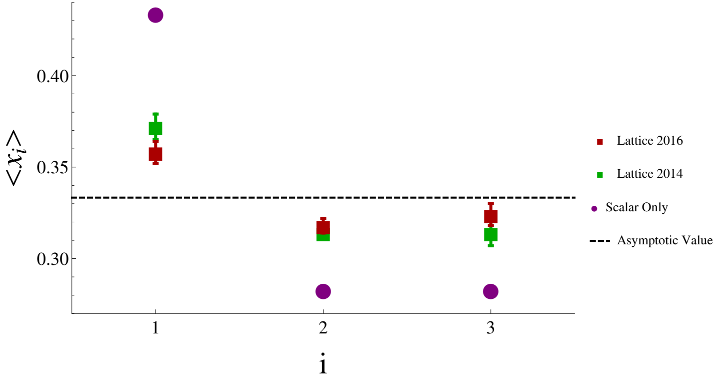


Point-like case: φ



Extended case: φ

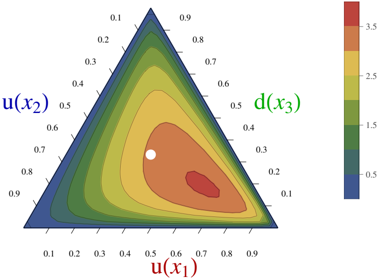
$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$

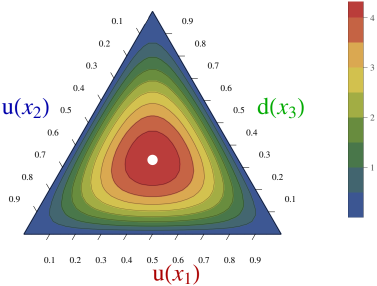


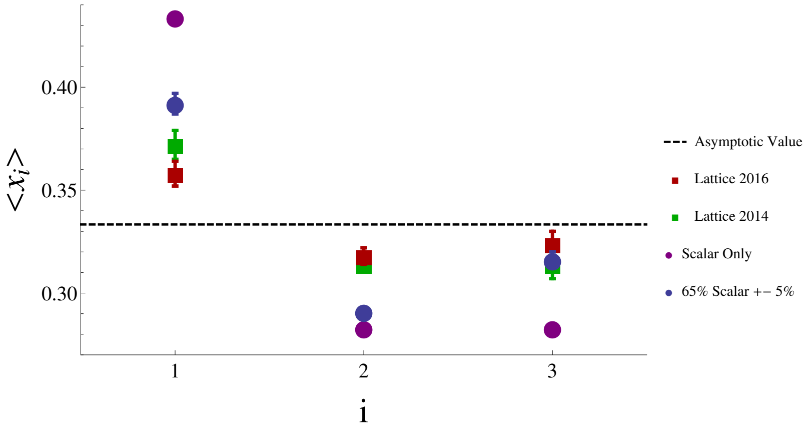
Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

- We use the prediction from the Faddeev equation to weight the scalar and AV contributions 65/35:



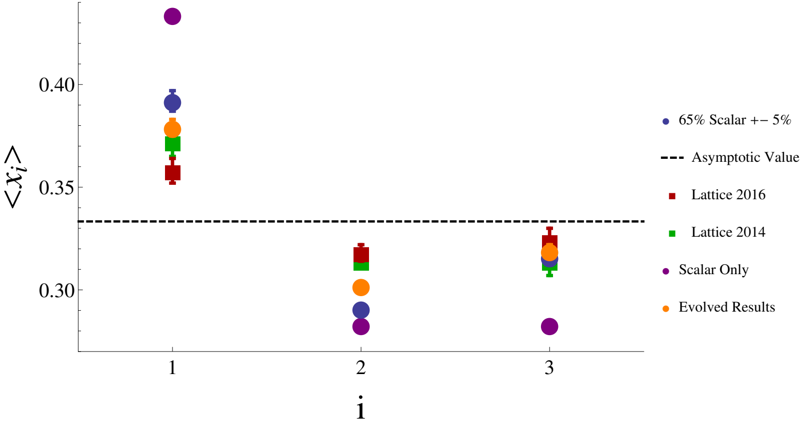




Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

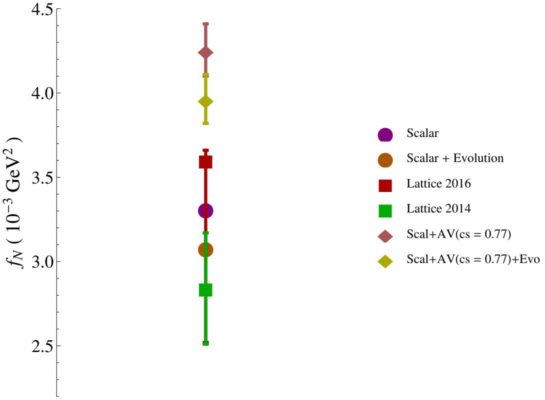
Comparison with lattice II



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

Comparison with lattice III



Computations done by J. Segovia

Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

The Roper Resonnance

Extension to the Roper

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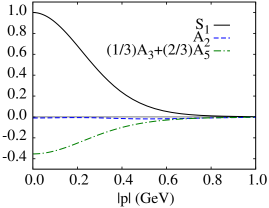
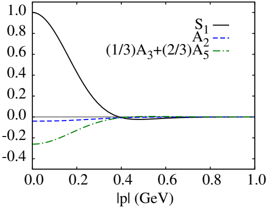
- In particular in the Chebychev moments:





figures from J. Segovia *et al.*,PRL 115 (2015)

Extension to the Roper

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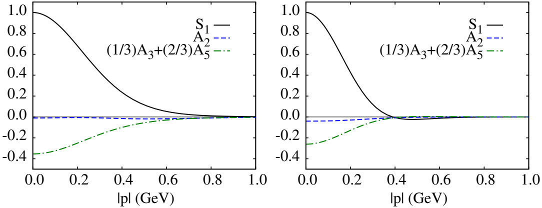
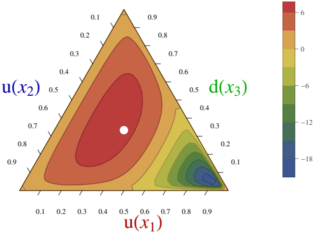


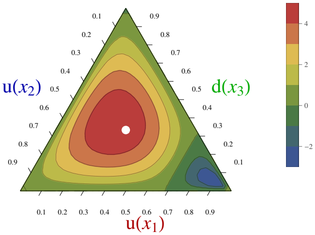
figures from J. Segovia *et al.*,PRL 115 (2015)

- This behaviour can be obtained by adding a zero in the Faddeev amplitude through:

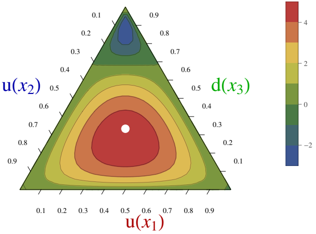
$$\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]} \rightarrow \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)(z-\kappa)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_R^2\right]}$$



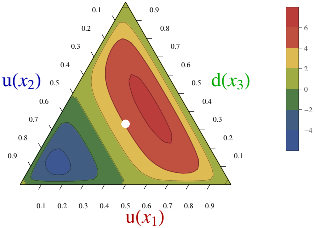
Scalar



AV Long.



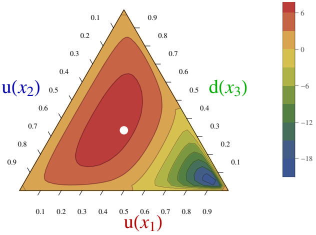
AV Long.



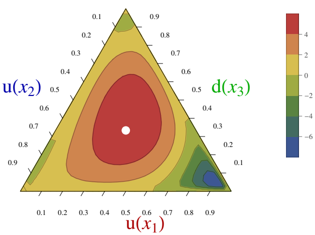
AV Trans.

Complete results for φ_R

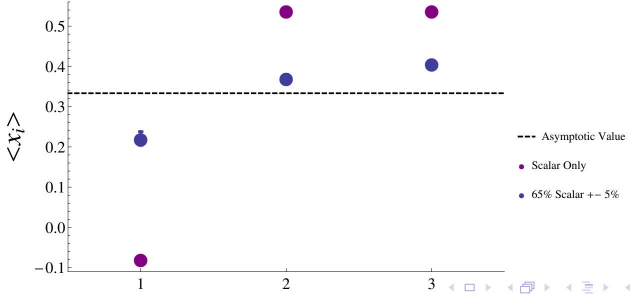
Argonne NATIONAL LABORATORY



100 % Scalar



65% Scalar



Summary of Chapter 1

- Both nucleon DAs φ and T can be described using a quark-diquark approximation.
- We show how the diquark types and diquarks polarisations were selected.
- The comparison with lattice computation explains how the different diquarks contribute to the total DAs, and the respective sensitivity of the latter to the AV-diquarks.
- The comparison with the lattice data is encouraging.
- It is possible to extend the work on the nucleon to the Roper case.
- In the Roper case, the results of individual diquarks contributions seem to be consistent with a $n = 1$ excited state.
- Working on an Evolution code.
- Computations of the Form Factors are in progress.

Chapter 2: Generalised Parton Distributions and Lightfront Wave Functions

N. Chouika, C. Mezrag, H. Moutarde, J. Rodriguez-Quintero

A quick reminder on GPDs

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- Generalised Parton Distributions (GPDs):
  - are defined according to a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$=\frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u+E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,z=0}$$
$$=\frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5u+\tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

A quick reminder on GPDs

- Generalised Parton Distributions (GPDs):
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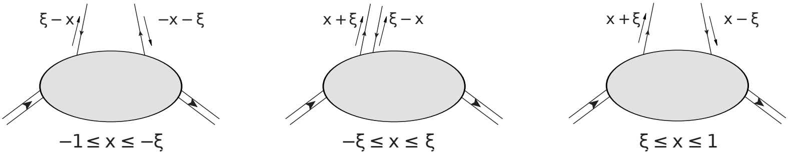
  - depend on three variables (x, ξ, t),



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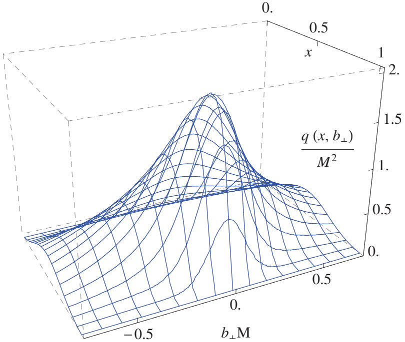
  - can be related to the 2+1D parton number density when $\xi \to 0$.

M. Burkardt, Phys. Rev. **D62**, 071503 (2000)



Pion GPD in Impact parameter space from:
CM *et al.*, Phys. Lett. **B741**, 190-196 (2015)

A quick reminder on GPDs

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  - can split in terms of quark flavour and gluon contributions,
  - can be related to the 2+1D parton number density when $\xi \to 0$.
  - are univeral, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions:

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$

- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x \; x^m H^q(x,\xi,t) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C^q_{2j}(t) + mod(m,2)\xi^{m+1} C^q_{m+1}(t)$$

Lorentz Covariance

GPDs: Theoretical Constraints

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

GPDs: Theoretical Constraints

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Soft pion theorem (pion GPDs only)

M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999)
CM *et al.*, Phys. Lett. **B741**, 190 (2015)

Dynamical Chiral Symmetry Breaking

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

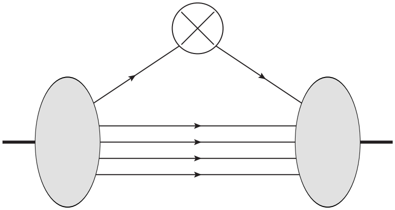
- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x, k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

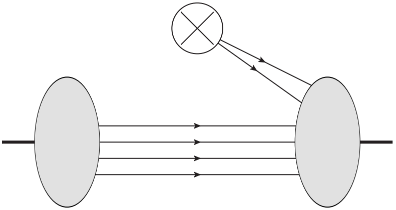
GPDs as overlap of LFWFs

DGLAP: $|x| > |\xi|$



- Same N LFWFs
- Truncation unambiguous

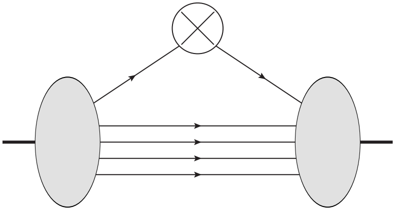
ERBL: $|x| < |\xi|$



- N and $N+2$ LFWFs
- Ambiguity in truncation

GPDs as overlap of LFWFs

DGLAP: $|x| > |\xi|$



- Same N LFWFs
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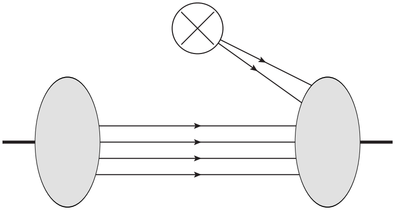
ERBL: $|x| < |\xi|$

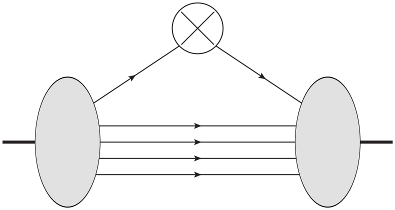


- N and $N+2$ LFWFs
- Ambiguity in truncation

LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

GPDs as overlap of LFWFs

DGLAP: $|x| > |\xi|$



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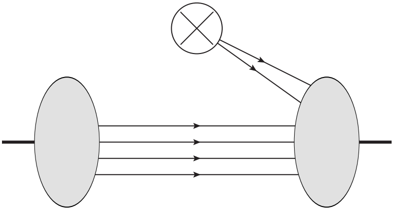

- N and $N+2$ LFWFs
- Ambiguity in truncation

LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

Covariant extension of the DGLAP region (see **Nabil Chouika** talk).

Chapter 3:

The PARTONS Project

B. Berthou *et al.* arXiv:1512.06174
Will be updated soon

Why PARTONS?

- From GPDs to observables
  - Flexibility in the choice of models
  - Flexibility in the scale of GPDs (evolution)
  - Computation of CFFs
  - Flexibility in the choice of pertubative approximation (α_s)
  - Flexibility in changing twist approximations ($1/Q$)
  - Computations of a given set of observables

PARTONS contains the tools to compare your GPD model to available data

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PARTONS contains the tools to compare your GPD model to available data

- From observables to GPDs:
  - Flexibility in the choice of observables
  - Extraction of CFFs
  - Flexibility in changing twist approximations ($1/Q$)
  - Extraction of GPDs from CFFs at a given scale (evolution)
  - Flexibility in the choice of pertubative approximation (α_s)

PARTONS allows you to extract GPDs from your favourite data set.

Recipes on xml

```xml
<!-- Indicate service and its methods to be used and indicate if the result should be stored in the database -->
<task service="ObservableService" method="computeObservable" storeInDB="0">

   <!-- Define DVCS observable kinematics -->
   <kinematics type="ObservableKinematic">
      <param name="xB" value="0.2" />
      <param name="t" value="-0.1" />
      <param name="Q2" value="2." />
      <param name="E" value="6." />
   </kinematics>

   <!-- Define physics assumptions -->
   <computation_configuration>

      <!-- Select DVCS observable -->
      <module type="Observable" name="DVCSAllMinus">

         <!-- Select DVCS process model -->
         <module type="ProcessModule" name="DVCSProcessGV08">

            <!-- Select scales module -->
            <!-- (it is used to evaluate factorization and renormalization scales out of kinematics) -->
            <module type="ScalesModule" name="ScalesQ2Multiplier">

               <!-- Configure this module -->
               <param name="lambda" value="1." />
            </module>

            <!-- Select xi-converter module -->
            <!-- (it is used to evaluate GPD variable xi out of kinematics) -->
            <module type="XiConverterModule" name="XiConverterXBToXi">
            </module>

            <!-- Select DVCS CFF model -->
            <module type="ConvolCoeffFunctionModule" name="DVCSCFFStandard">

               <!-- Indicate pQCD order of calculation -->
               <param name="qcd_order_type" value="NLO" />

               <!-- Select GPD model -->
               <module type="GPDModule" name="GPDMMS13">
               </module>

            </module>

         </module>

      </module>

   </computation_configuration>
```

Recipes on xml

```xml
<!-- Define DVCS observable kinematics -->
<kinematics type="ObservableKinematic">
    <param name="xB" value="0.2" />
    <param name="t" value="-0.1" />
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```

Recipes on xml

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      </module>

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        <!-- Indicate pQCD order of calculation -->
        <param name="qcd_order_type" value="NLO" />

        <!-- Select GPD model -->
        <module type="GPDModule" name="GPDMMS13">
        </module>

      </module>

    </module>

  </module>
```

Provided Examples

Argonne NATIONAL LABORATORY

- At GPD level:
  - How to get a given set of GPD at one defined $(x, \xi, t, \mu_R, \mu_F)$ kinematics
  - How to get a list of results from a file containing multiples kinematics.
  - How to plot the results stored in the data base.
  - How to use evolution equations
  - How to change integration routines
- at CFF level:
  - How to get a set of CFF at one defined (x_B, t, Q^2) kinematics
  - How to get multiple results from multiple kinematic stored in a given file.
  - How to plot the results from the database.
  - How to change integration routines
- at Observable level:
  - Same thing than CFF with additionnal angular dependence.

A live show

C++ Users

```
void computeSingleKinematicsForGPD() {

    // Retrieve GPD service
    PARTONS::GPDService* pGPDService =
            PARTONS::Partons::getInstance()->getServiceObjectRegistry()->getGPDService();

    // Create GPD module with the BaseModuleFactory
    PARTONS::GPDModule* pGPDModel =
            PARTONS::Partons::getInstance()->getModuleObjectFactory()->newGPDModule(
                    PARTONS::GPDMMS13::classId);

    // Create a GPDKinematic(x, xi, t, MuF, MuR) to compute
    PARTONS::GPDKinematic gpdKinematic(0.1, 0.2, -0.1, 2., 2.);

    // Run computation
    PARTONS::GPDResult gpdResult = pGPDService->computeGPDModel(gpdKinematic,
            pGPDModel);

    // Print results
    PARTONS::Partons::getInstance()->getLoggerManager()->info("main", __func__,
            gpdResult.toString());

    // Remove pointer reference ; Module pointers are managed by PARTONS.
    PARTONS::Partons::getInstance()->getModuleObjectFactory()->updateModulePointerReference(
            pGPDModel, 0);
    pGPDModel = 0;
}
```

Website

Argonne NATIONAL LABORATORY

PARTONS

PARtonic Tomography Of Nucleon Software

Main Page Reference documentation + Search

Main Page

What is PARTONS?

PARTONS is a C++ software framework dedicated to the phenomenology of Generalized Parton Distributions (GPDs). GPDs provide a comprehensive description of the partonic structure of the nucleon and contain a wealth of new information. In particular, GPDs provide a description of the nucleon as an extended object, referred to as 3-dimensional nucleon tomography, and give an access to the orbital angular momentum of quarks.

PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments. A detailed description of the project can be found here.

Table of Contents

- What is PARTONS?
- Get PARTONS
- Configure PARTONS
- How to use PARTONS
- Publications and talks
- Acknowledgments
- License
- Contact and newsletter

Get PARTONS

Here you can learn how to get your own version of PARTONS. We offer two ways.

You can use our provided virtual machine with an out-of-the-box PARTONS runtime and development environment. This is the easiest way to start your experience with PARTONS.

Using PARTONS with our provided Virtual Machine

You can also build PARTONS by your own on either GNU/Linux or Mac OS X. This is useful if you want to have PARTONS on your computer without using the virtualization technology or if you want to use PARTONS on computing farms.

Using PARTONS on GNU/Linux

Using PARTONS on Mac OS X

Configure PARTONS

If you are using **our virtual machine**, you will find all configuration files set up and ready to be used. However, if you want to tune the configuration or if you have installed PARTONS by your own, this tutorial will be helpful for

www.partons.cea.fr

Website

PARTONS — PARtonic Tomography Of Nucleon Software

Main Page | Reference documentation + | Search

Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

[detail level 1 2]

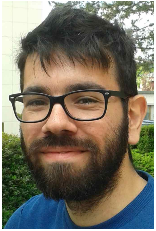
▼ N PARTONS	
C ActiveFlavorsThresholds	Interval of factorization scale with fixed number of flavors
C ActiveFlavorsThresholdsModule	Abstract class for modules defining number of quark flavors intervals
C ActiveFlavorsThresholdsQuarkMasses	Number of active quark flavors intervals corresponding to quark masses
C AutomationService	Automation service is designed to dynamically run complex tasks (by calling service object methods) or to create some complex C++ objects, all described by an XML file
C BaseObject	BaseObject is the "zeroth-level-object" of the architecture
C BaseObjectData	Container to store data to be used by base objects
C BaseObjectFactory	Provides a clone (returned as a BaseObject pointer) of an object identified by its class name and previously stored in the BaseObjectRegistry
C BaseObjectRegistry	The Registry is the analog of a phonebook, which lists all available objects (modules or services most of the time) identified by a unique integer identifier or by a unique string (class name) for translation
C BaseType	
C BCSimplifiedVertex	Simplified Ball-Chiu Vertex
C BCVertex	Ball-Chiu Vertex
C CCFModuleNullPointerException	Exception to indicate missing CFF module
C CompareUtils	Set of utility tools to perform comparisons
C ComparisonData	Comparison report for single data point
C ComparisonMode	Definition of comparison modes
C ComparisonReport	Comparison report
C ComparisonService	
C Computation	Class to store computation information
C ComputationDao	Computation information Data Access Object (DAO)
C ComputationDaoService	Computation information Data Access Object (DAO) service
C ConvolCoeffFunctionKinematicDao	Compton form factor (CFF) kinematics Data Access Object (DAO)
C ConvolCoeffFunctionKinematicDaoService	Compton form factor (CFF) kinematics Data Access Object (DAO) service
C ConvolCoeffFunctionModule	Abstract class that provides a skeleton to implement a Convolution of Coefficient Function module

www.partons.cea.fr

A tribute to our postdocs and student

P. Sznajder
NCJB Warsaw

N. Chouika
IRFU/DPhN



L. Colaneri
IPNO

Conclusion

Conclusion

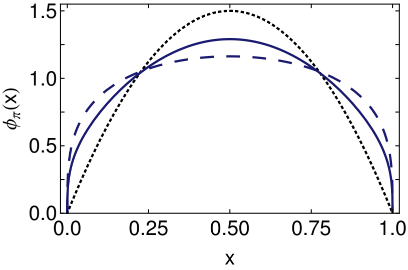
- Our results on the Baryon PDA are very encouraging within our simple assumptions as they almost match the lattice one. This bring us confidence in the computations of LFWFs.
- There is now a clear path to compute GPDs **fulfilling all the required theoretical constraints** through continuum QCD techniques:
  - Solve the Dyson-Schwinger and Faddeev equations
  - Parametrise the solutions using the Nakanishi representation
  - Project the results to get LFWFs
  - Use the overlap representation to compute the GPDs in the DGLAP region
  - Extend to the ERBL region through the Radon Inverse transform
  - Use PARTONS to compute observables related to different channel
- Even if we know the path, every step remains difficult and technical, and it will still probably take several years before we achieve it.

Thank you for your attention

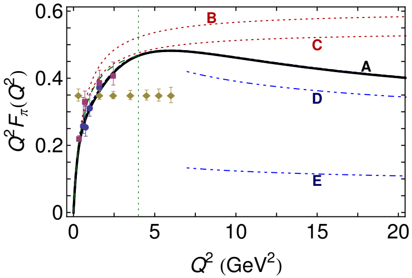
Back up slides

Pion distribution amplitude

$$\phi_{As}(x) = 6x(1-x)$$



L. Chang *et al.* (2013)



L. Chang *et al.* (2013)

- Broad DSE pion DA is much more consistent with the form factor than the asymptotic one.
- The scale when the asymptotic DA become relevant is huge.

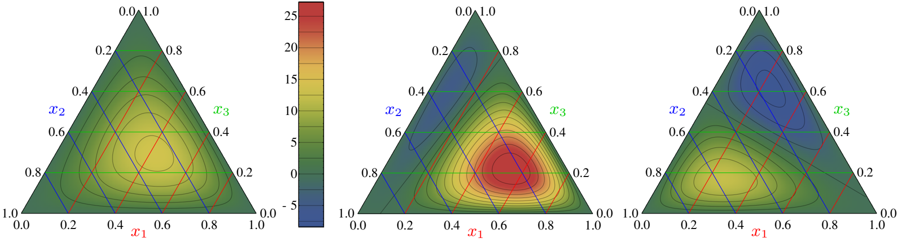


Figure from V. Braun *et la.*,Phys. Rev. **D89**, 094511 (2014)

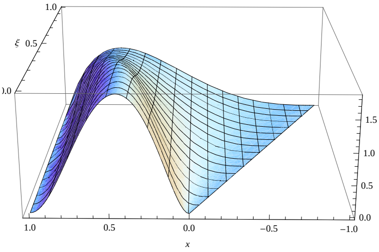
- The form factor is only the first Mellin Moment of GPDs and GDAs.
- The perturbative formula have been generalised to GPDs at large t and GDAs at large s for mesons and baryons.

M. Diehl *et al.*, PRD 61, (2000) 074029
C. Vogt, PRD 64, (2001), 057501
P. Hoodboy *et al.*, PRL 92 (2004) 012003
B. Pire *et al.*, PLB 639, (2006) 642-651

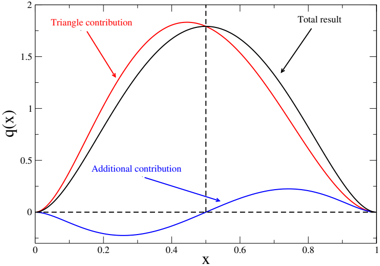
Can we use our DA models to get relevant information on GPDs and GDAs for mesons and baryons?

Issues with the impulse approximation

Violation of Positivity



Violation of discrete symmetries



These issues might be related, but no solution has been found yet.