

# Transversity and Sivers function extraction from SIDIS COMPASS p and d data

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Phys. Rev. D 91 (2015) 014034

Phys. Rev. D 95 (2017) 094024



INT Program 17-3 Workshop

“Hadron imaging at Jefferson Lab and at a future EIC”

Seattle, September 25 - 29, 2017

- the COMPASS experiment
- nucleon structure and TMDs
- extraction of transversity
- extraction of the Sivers function
- conclusions

# COMPASS

fixed target experiment at the CERN SPS



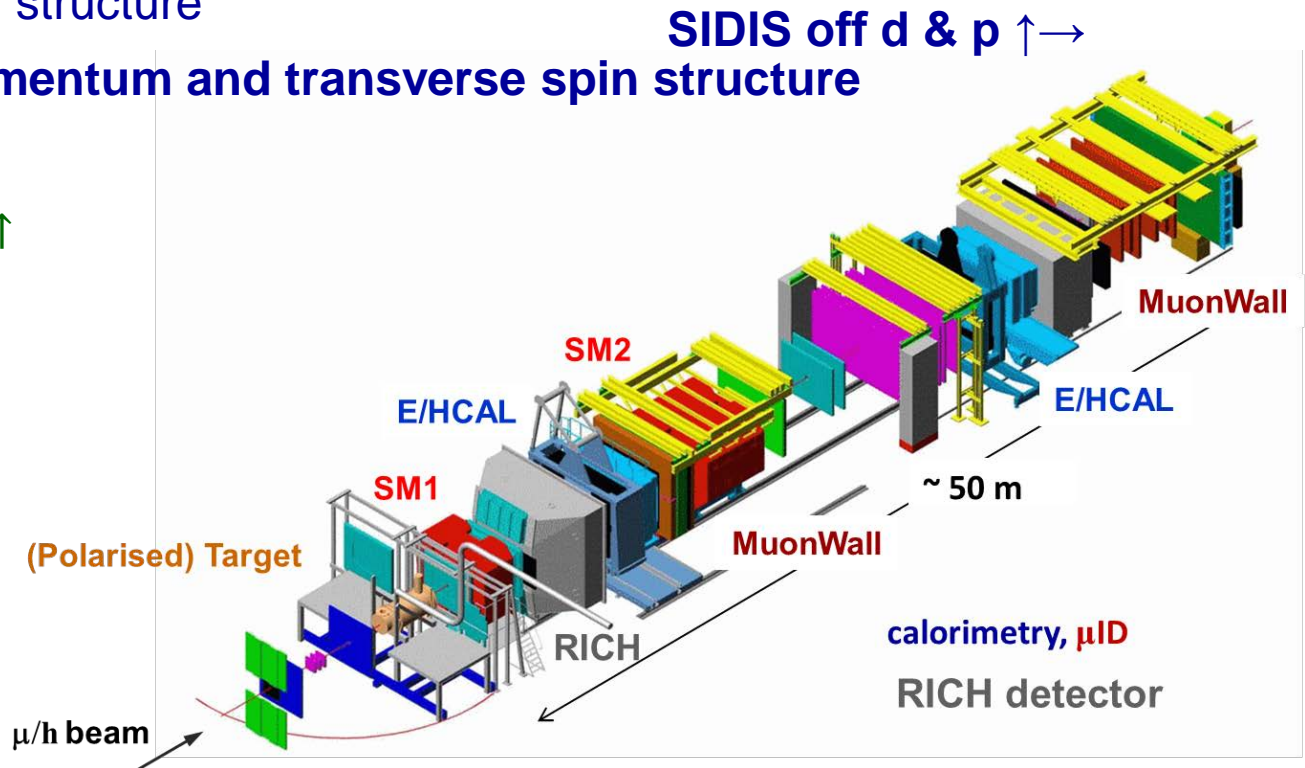
## physics programme:

### hadron spectroscopy ( $\rho$ , $\pi$ , $K$ )

- light mesons, glue-balls, exotic mesons
- polarisability of pion and kaon
















### nucleon structure ( $\mu$ )

- longitudinal spin structure
- **transverse momentum and transverse spin structure**
- DVCS  $\mu p$
- Drell-Yan  $\pi^- p \uparrow$



# the structure of the nucleon

at leading order, 3 parton distribution functions (PDFs) are needed  
in the **collinear case**

		nucleon polarisation			
		U	L	T	
quark polarisation	U	$f_1$  number density $q$		$f_{1T}^\perp$  -  Sivers	$\Delta_{0T}^T q$
	L		$g_1$  -  helicity $\Delta q$	$g_{1T}$  - 	
	T	$h_1^\perp$  -  Boer Mulders	$h_{1L}^\perp$  - 	$h_1$  -  transversity $h_{1T}^\perp$  - 	$\Delta_{TT} q$

number density  $f_1^q$   
 $q$ , well known

helicity  $g_1^q$   
 $\Delta q$ , known

transversity  $h_1^q$   
 $\Delta_T q$ , new

as important as the previous ones  
give access to the tensor charge  
first experimental information in 2005

# the structure of the nucleon

taking into account the quark **intrinsic transverse momentum**  $k_T$   
 at leading order other new 5 TMD PDFs are needed for a full description  
 of the nucleon structure

all interesting
















correlations among spins and  
 transverse quark momentum

**Sivers function**  $f_{1T}^{\perp q}$

correlation between nucleon  
 transverse spin and quark  
 transverse momentum

SIDIS gives access to all of them  
 by measuring the azimuthal asymmetries,  
 i.e. the amplitudes of the different  
 modulations in the azimuthal distributions  
 of the final state hadrons

the use of different targets (proton and deuteron/neutron) and the  
 identification of the final state hadrons allow for flavor separation

		nucleon polarisation			
		U	L	T	
quark polarisation	U	$f_1$  number density $\mathbf{q}$		$f_{1T}^{\perp}$  -  Sivers	$\Delta_0^T \mathbf{q}$
	L		$g_1$  -  helicity $\Delta \mathbf{q}$	$g_{1T}$  - 	
	T	$h_1^{\perp}$  -  Boer Mulders	$h_{1L}^{\perp}$  - 	$h_1$  -  transversity $h_{1T}^{\perp}$  - 	$\Delta_T \mathbf{q}$

# the structure of the nucleon

in SIDIS off transversely polarised nucleons the

**transversity PDF**  $h_1^q$

**Sivers function**  $f_{1T}^{\perp q}$

are responsible for the

**Collins asymmetry**

$$A_{Coll}^h = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q \otimes H_{1q}^{\perp h}}{\sum_{q,\bar{q}} e_q^2 x f_1^q \otimes D_{1q}^h}$$

**Sivers asymmetry**

$$A_{Siv}^h = \frac{\sum_{q,\bar{q}} e_q^2 x f_{1T}^{\perp q} \otimes D_{1q}^h}{\sum_{q,\bar{q}} e_q^2 x f_1^q \otimes D_{1q}^h}$$

**di-hadron asymmetry**

$$A^{hh} = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q H_{1q}^{\angle}}{\sum_{q,\bar{q}} e_q^2 x f_1^q D_{1q}^{hh}}$$

the use of different targets (proton and deuteron/neutron) and the identification of the final state hadrons allow for flavor separation



# transverse target spin effects at COMPASS

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studied measuring SIDIS with

- 160 GeV muon beam
- a transversely polarised **deuteron** ( ${}^6\text{LiD}$ ) target  
(2002, 2003 and 2004, ~20% of running time)
- a transversely polarised **proton** ( $\text{NH}_3$ ) target  
(2007, ~50%, and 2010, 100%, larger geometrical acceptance)

the spin asymmetries have been extracted using the same  $x, z, P_T^h$  binning

→ the mean values of  $x, Q^2$  are almost the same for p and d in each  $x$  bin

results:

the **Collins, the di-hadron asymmetries and the Sivers asymmetries are**

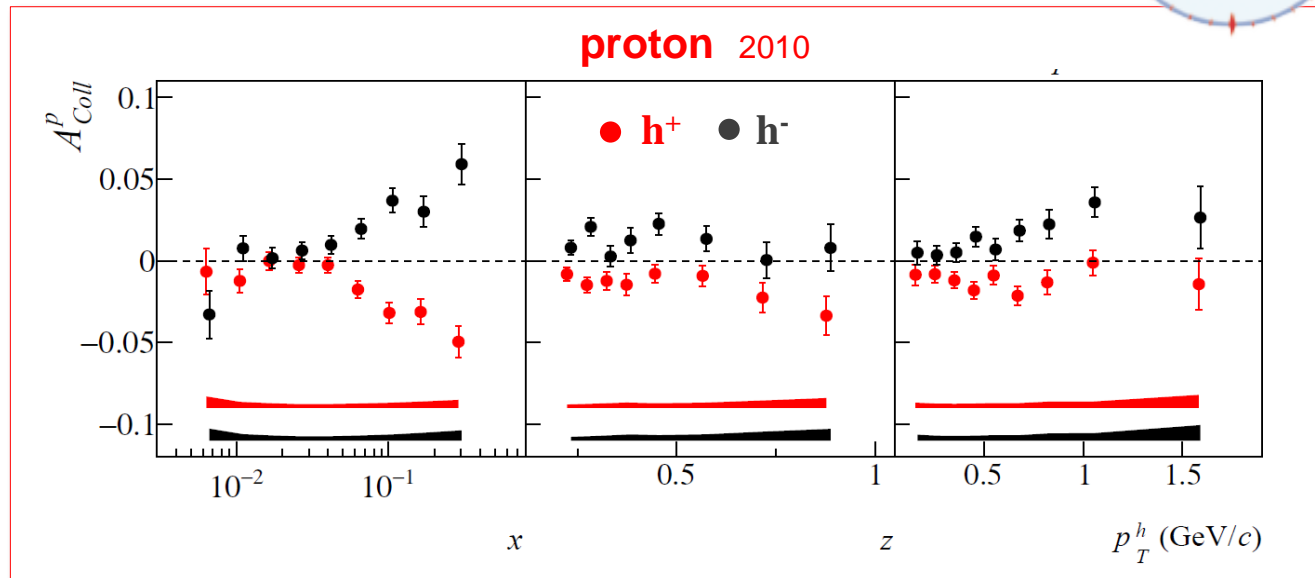
- **different from zero on proton**, as seen by HERMES
- **compatible with zero on deuteron** within the large statistical errors

# Collins asymmetry

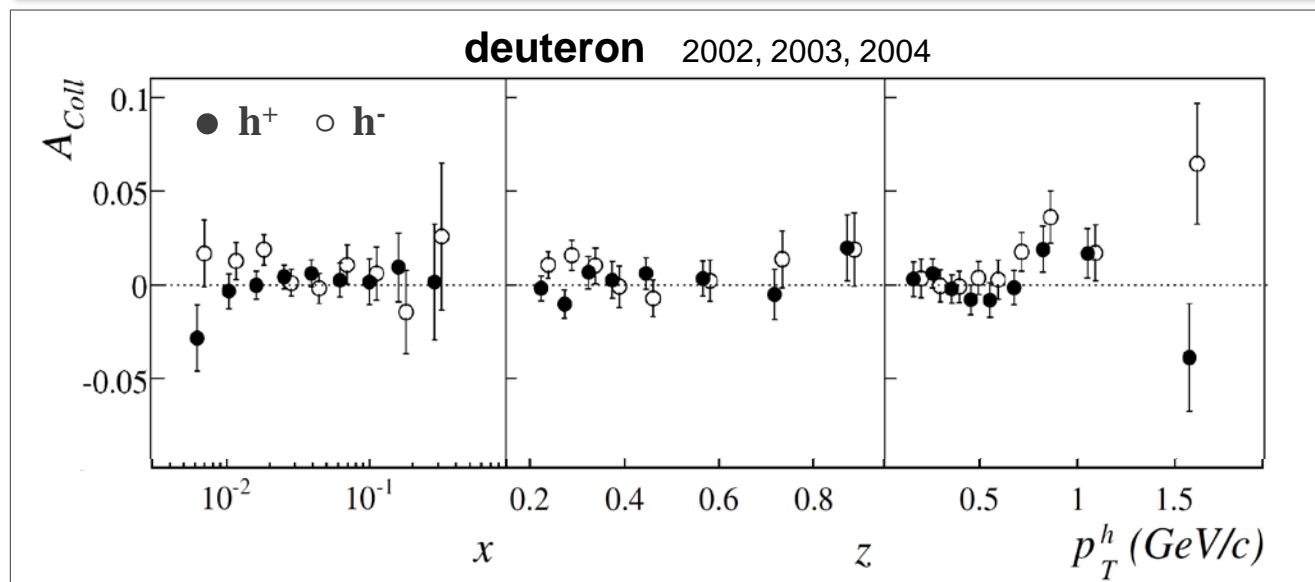
$$A_{Coll}^h = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q \otimes H_{1q}^{\perp h}}{\sum_{q,\bar{q}} e_q^2 x f_1^q \otimes D_{1q}^h}$$



Phys. Lett. B 717 (2012) 376



Nucl. Phys. B765 (2007) 31



Seattle, 25 September 2017

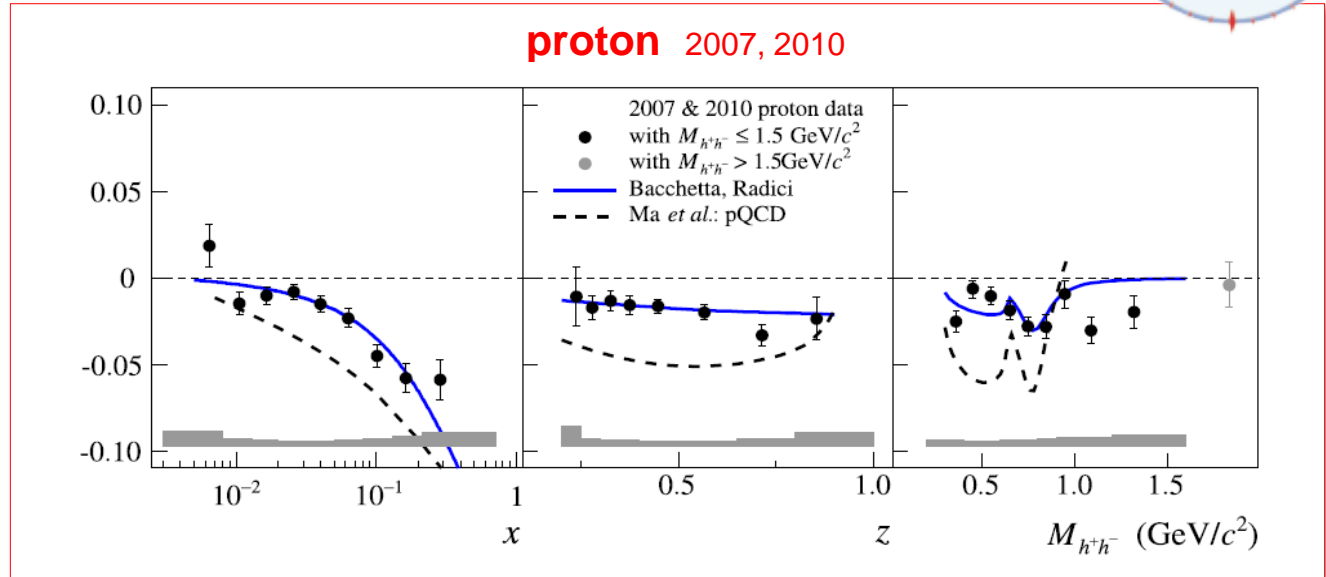


# di-hadron asymmetry

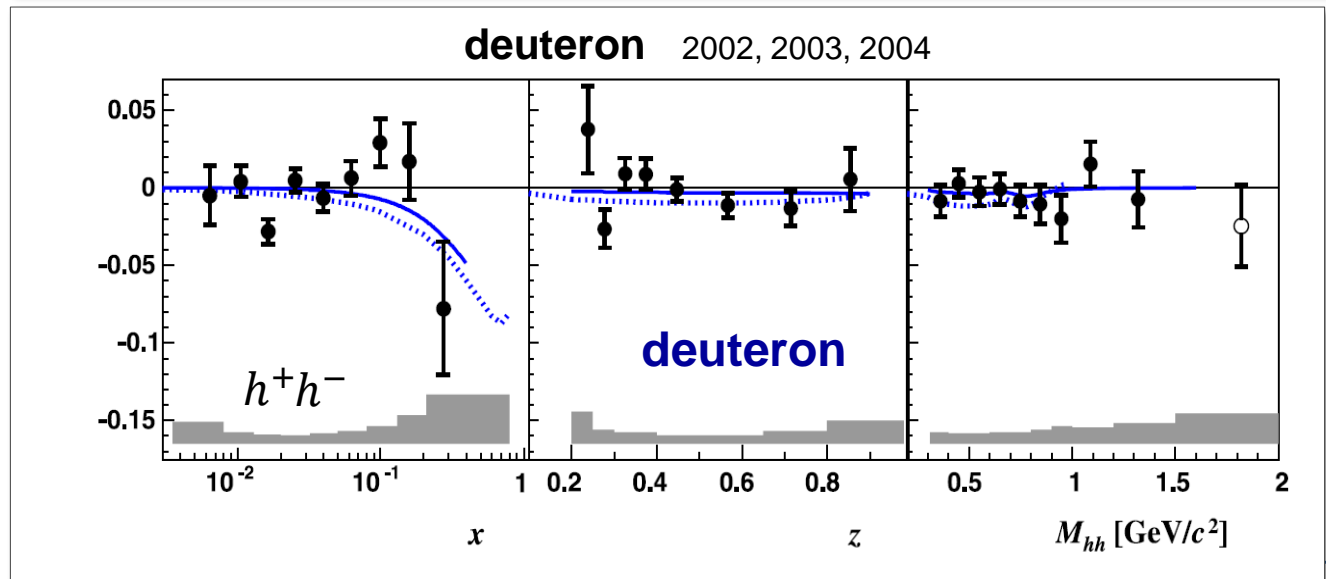
$$A^{hh} = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q H_{1q}^{\prime\prime}}{\sum_{q,\bar{q}} e_q^2 x f_1^q D_{1q}^{hh}}$$



Phys. Lett. B 736 (2014) 124



Phys. Lett. B 713 (2012) 10

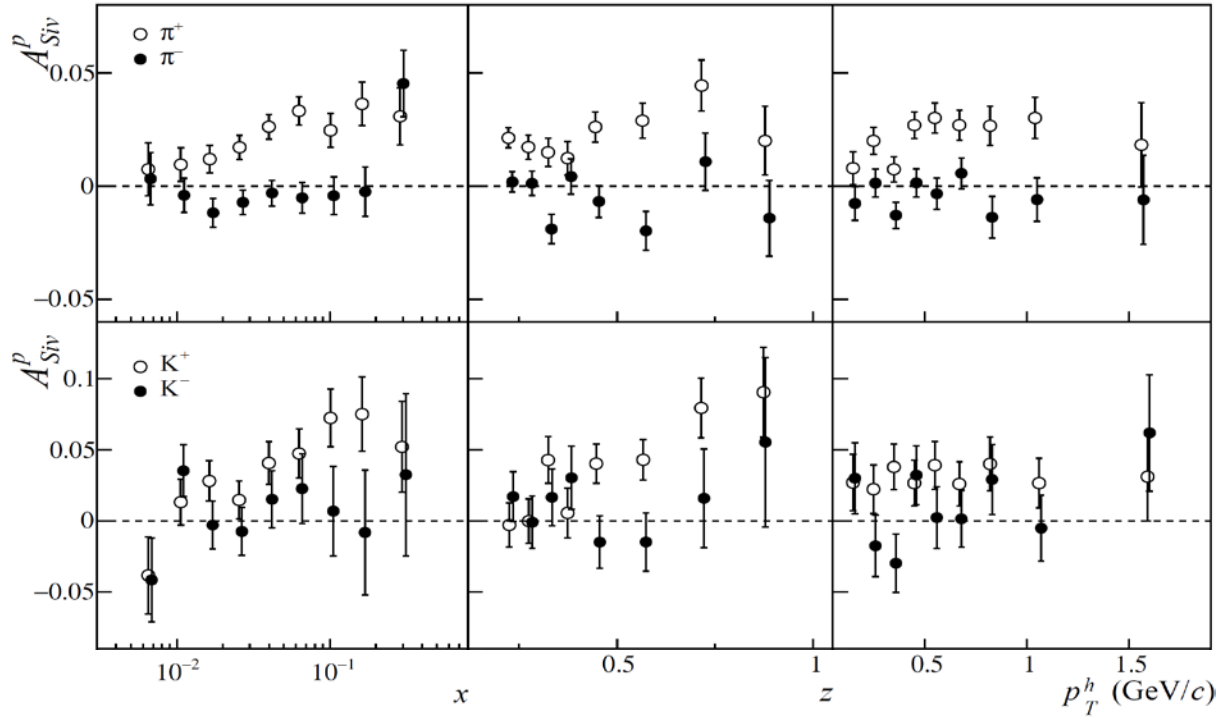


# Sivers asymmetry

$$A_{Siv}^h = \frac{\sum_{q,\bar{q}} e_q^2 x f_{1T}^{\perp q} \otimes D_{1q}^h}{\sum_{q,\bar{q}} e_q^2 x f_1^q \otimes D_{1q}^h}$$



proton 2007,2010



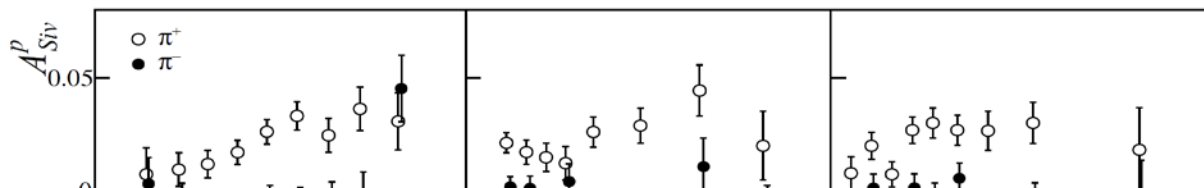
Phys. Lett. B 744 (2015) 250

# Sivers asymmetry

$$A_{Siv}^h = \frac{\sum_{q,\bar{q}} e_q^2 x f_{1T}^{\perp q} \otimes D_{1q}^h}{\sum_{q,\bar{q}} e_q^2 x f_1^q \otimes D_{1q}^h}$$



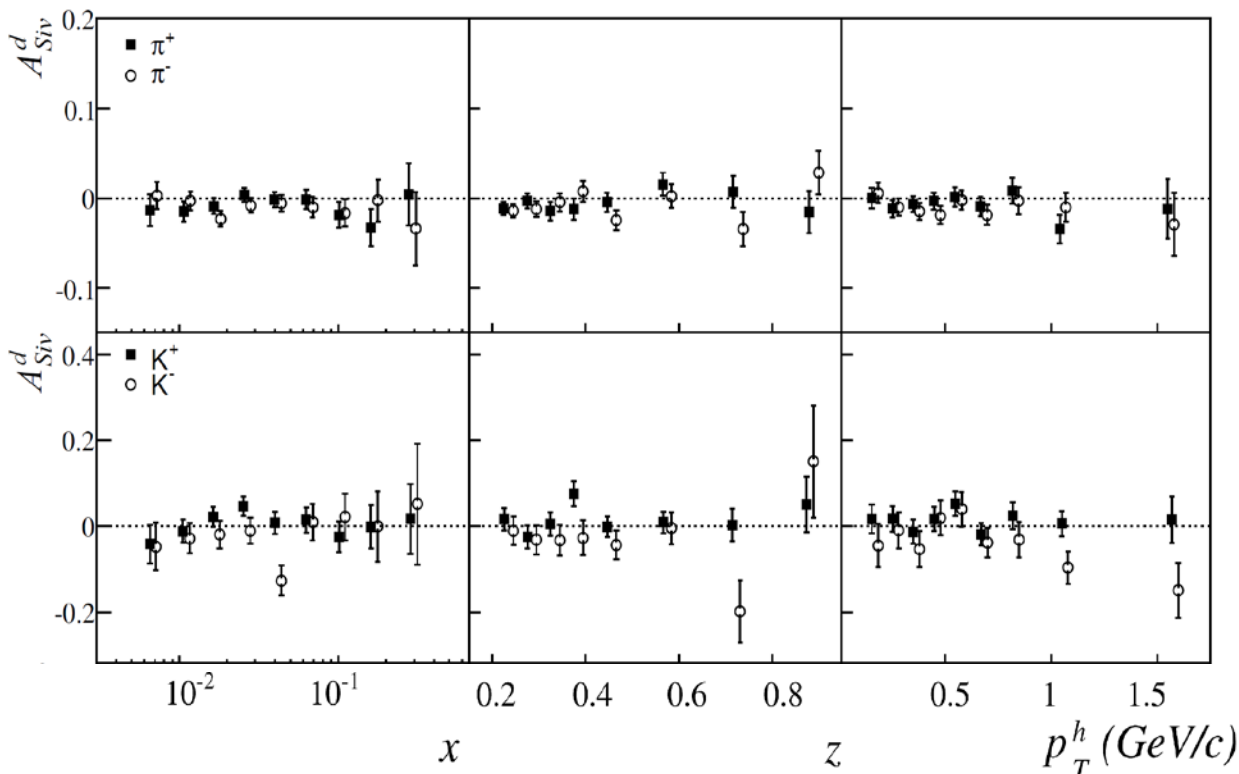
proton 2007,2010



Phys. Lett. B 744 (2015) 250

Phys. Lett. B 673 (2009) 127

deuteron 2002, 2003, 2004



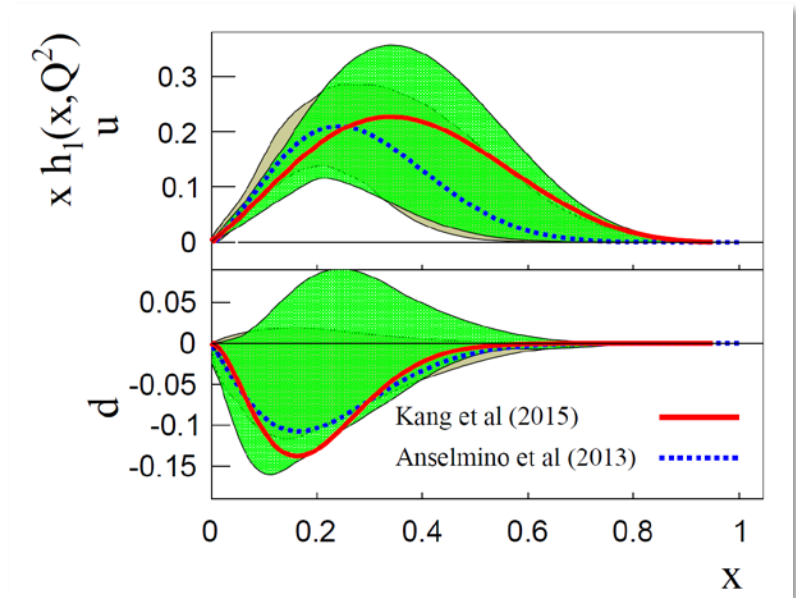
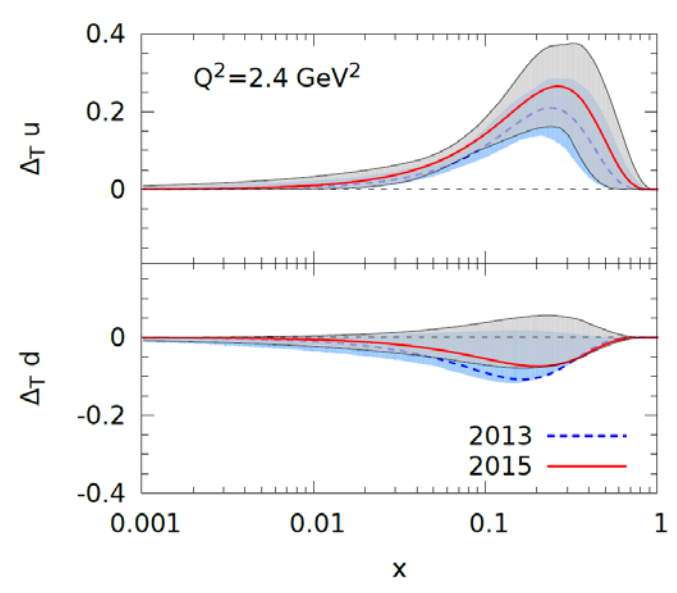
# extraction of the Transversity and Sivers functions

the COMPASS results already used to extract these PDFs

## Transversity:

- **Collins asymmetries**

HERMES p, COMPASS p and d, Belle/Babar  $e^+e^- \rightarrow \text{hadrons}$  data  
Anselmino et al. 2013, 2015, Kang et al. 2015, ...



using parametrisations for transversity and spin-dependent fragmentation functions

# extraction of the Transversity and Sivers functions

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## Transversity:

- **Collins asymmetries**

HERMES p, COMPASS p and d, Belle/Babar  $e^+e^- \rightarrow \text{hadrons}$  data  
Anselmino et al. 2013, 2015, Kang et al. 2015, ...

- **di-hadron asymmetries**

HERMES p, COMPASS p and d, Belle/Babar  $e^+e^- \rightarrow \text{hadrons}$  data  
Bacchetta et al. 2013, Radici et al. 2015, ...

using parametrisations for transversity and spin-dependent fragmentation functions  $(x, z)$

## Sivers function:

- **Sivers asymmetries**

HERMES p, COMPASS p and d  
Anselmino et al. 2012, Echevarria et al. 2014, ...

using parametrisations for the Sivers functions  $(x)$

# extraction of the Transversity and Sivers functions

this work:

use the COMPASS proton and deuteron measurements at the same  $x, Q^2$  values to perform a direct point-by-point extraction of the transversity and Sivers PDFs

- COMPASS results for Phys. Rev. D 91 (2015) 014034
  - p and d Collins asymmetry vs  $x$  (integrated over  $z, p_T$ )  
 $h^+$  and  $h^-$  assuming that all hadrons are pions
  - p and d di-hadron asymmetries vs  $x$  (integrated over  $z, M$ )
- Belle results for the corresponding pion and pion-pair asymmetries  
PRL 107(2011)072004, PRD78(2008)032011 / 86(2012)0399
- unpolarised PDFs and FFs parametrizations → transversity  
PDFs: CTEQ5D      FFs: DSS LO  $h_1^{u_v}, h_1^{d_v}, h_1^{\bar{u}}, h_1^{\bar{d}}$

- COMPASS results for Phys. Rev. D 95 (2017) 094024
  - p and d Sivers asymmetries vs  $x$  (integrated over  $z, M$ )  
 $\pi^\pm$  and  $K^\pm$
- unpolarised PDFs and FFs parametrizations → Sivers function  
PDFs: CTEQ5D      FFs: DSS LO  $f_{1T}^{\perp(1) u_v}, f_{1T}^{\perp(1) d_v}, f_{1T}^{\perp(1) \bar{u}} - f_{1T}^{\perp(1) \bar{d}}$

**Transversity**

# Collins asymmetry – SIDIS

$$A^h(x, z) = \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q(x) \otimes H_{1q}^{\perp h}(z)}{\sum_{q, \bar{q}} e_q^2 x f_1^q(x) \otimes D_{1q}^h(z)}$$

“gaussian ansatz”:

$$A^h(x, z) = C_G \frac{\sum_{q, \bar{q}} e_q^2 x h_1^q(x) H_{1q}^h(z)}{\sum_{q, \bar{q}} e_q^2 x f_1^q(x) D_{1q}^h(z)}$$

Efremov et al., 2006

$$C_G = \frac{1}{\sqrt{1 + z^2 \langle k_{\perp h_1}^2 \rangle / \langle p_{\perp H_1}^2 \rangle}}$$

$$H_{1q}^h \equiv H_{1q}^{\perp(1/2)h} = \frac{\sqrt{\pi \langle p_T^2 \rangle}}{2zM_h} H_{1q}^{\perp h}$$

we have assumed

- $C_G = 1$  most of the statistics at low  $z$
- $H_{1u}^+ = H_{1d}^- = H_{1\bar{d}}^+ = H_{1\bar{u}}^- = H_{1fav}$  and  $H_{1u}^- = H_{1d}^+ = H_{1\bar{d}}^- = H_{1\bar{u}}^+ = H_{1unf}$
- $H_{1s}^{\pm} = H_{1\bar{s}}^{\pm} = 0$  and  $c$  quark contributions to be negligible
- $D_{1u}^+ = D_{1d}^- = D_{1\bar{d}}^+ = D_{1\bar{u}}^- = D_{1fav}$  and  
 $D_{1u}^- = D_{1d}^+ = D_{1\bar{d}}^- = D_{1\bar{u}}^+ = D_{1s}^{\pm} = D_{1\bar{s}}^{\pm} = D_{1unf}$



# Collins asymmetry – SIDIS p and d

in each  $x$  bin the measured asymmetries can be written as

$$A^h(x) = \frac{\sum_{q\bar{q}} e_q^2 x h_1^q(x) \tilde{H}_{1q}^h}{\sum_{q\bar{q}} e_q^2 x f_1^q(x) \tilde{D}_{1q}^h} \quad \tilde{D}_{1q}^h = \int dz D_{1q}^h(z)$$

$$\tilde{H}_{1q}^h = \int dz H_{1q}^h(z)$$

i.e.

$$A_p^+ = \tilde{\alpha}_p^h \frac{4(h_1^u + \tilde{\alpha} h_1^{\bar{u}}) + (\tilde{\alpha} h_1^d + h_1^{\bar{d}})}{f_p^+}$$

$$A_p^- = \tilde{\alpha}_p^h \frac{4(\tilde{\alpha} h_1^u + h_1^{\bar{u}}) + (h_1^d + \tilde{\alpha} h_1^{\bar{d}})}{f_p^-}$$

$$A_d^+ = \tilde{\alpha}_d^h \frac{(4 + \tilde{\alpha})(h_1^u + h_1^d) + (1 + 4\tilde{\alpha})(h_1^{\bar{u}} + h_1^{\bar{d}})}{f_d^+}$$

$$A_d^- = \tilde{\alpha}_d^h \frac{(1 + 4\tilde{\alpha})(h_1^u + h_1^d) + (4 + \tilde{\alpha})(h_1^{\bar{u}} + h_1^{\bar{d}})}{f_d^-}$$

$$\tilde{\alpha}_p^h = \frac{\tilde{H}_{1fav}^h}{\tilde{D}_{1fav}^h} \text{ from } e^+e^- \text{ Belle data}$$

$$\tilde{\alpha} = \frac{\tilde{H}_{1unf}}{\tilde{H}_{1fav}}$$

$$x [4(f_1^u + \tilde{\beta} f_1^{\bar{u}}) + (\tilde{\beta} f_1^d + f_1^{\bar{d}}) + \tilde{\beta}(f_1^s + f_1^{\bar{s}})] \tilde{D}_{1,fav} \equiv x f_p^+ \tilde{D}_{1,fav}$$

$\downarrow$   
 known

$$\tilde{\beta} = \frac{\tilde{D}_{1unf}}{\tilde{D}_{1fav}}$$

# Collins asymmetry – SIDIS p and d

in each  $x$  bin it is

$$xh_1^{u_v} = \frac{1}{5} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha})} \left[ (xf_p^+ A_p^+ - xf_p^- A_p^-) + \frac{1}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) \right]$$

$$xh_1^{d_v} = \frac{1}{5} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha})} \left[ \frac{4}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) - (xf_p^+ A_p^+ - xf_p^- A_p^-) \right]$$

$$\tilde{a}_P^h = \frac{\tilde{H}_{1fav}}{\tilde{D}_{1fav}}$$

from  $e^+e^-$  Belle data

$$\tilde{\alpha} = \frac{\tilde{H}_{1unf}}{\tilde{H}_{1fav}} = \begin{cases} -1 & \mathbf{a1} \\ -\frac{\tilde{D}_{1unf}}{\tilde{D}_{1fav}} = -\tilde{\beta} & \mathbf{a2} \end{cases}$$

both reasonable and in agreement with the considerations on the “interplay between the Collins and the di-hadron FFs”

# Collins asymmetry – SIDIS p and d

in each  $x$  bin it is

$$xh_1^{u_v} = \frac{1}{5} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha})} \left[ (xf_p^+ A_p^+ - xf_p^- A_p^-) + \frac{1}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) \right]$$

$$xh_1^{d_v} = \frac{1}{5} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha})} \left[ \frac{4}{3} (xf_d^+ A_d^+ - xf_d^- A_d^-) - (xf_p^+ A_p^+ - xf_p^- A_p^-) \right]$$

$$\tilde{a}_P^h = \frac{\tilde{H}_{1fav}}{\tilde{D}_{1fav}}$$

from  $e^+e^-$  Belle data

$$\tilde{\alpha} = \frac{\tilde{H}_{1unf}}{\tilde{H}_{1fav}} = \begin{cases} -1 & \mathbf{a1} \\ -\frac{\tilde{D}_{1unf}}{\tilde{D}_{1fav}} = -\tilde{\beta} & \mathbf{a2} \end{cases}$$

$$xh_1^{\bar{u}} = \frac{1}{15} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha}^2)} \left[ (1 - 4\tilde{\alpha}) xf_p^+ A_p^+ + (4 - \tilde{\alpha}) xf_p^- A_p^- - xf_d^+ A_d^+ + \tilde{\alpha} xf_d^- A_d^- \right],$$

$$xh_1^{\bar{d}} = \frac{1}{15} \frac{1}{\tilde{a}_P^h (1 - \tilde{\alpha}^2)} \left[ (4\tilde{\alpha} - 1) xf_p^+ A_p^+ - (4 - \tilde{\alpha}) xf_p^- A_p^- - 4\tilde{\alpha} xf_d^+ A_d^+ + 4 xf_d^- A_d^- \right]$$

# Collins asymmetry – $e^+e^-$ Belle data

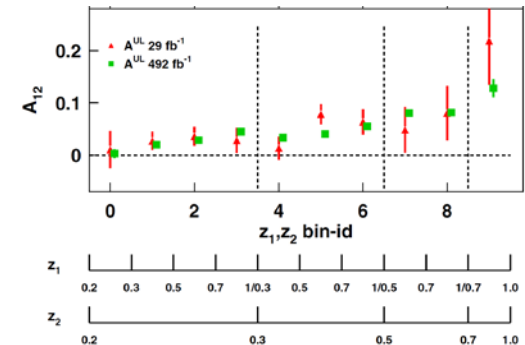
we have used the asymmetries (corrected for charm contribution)

$$A_{e^+e^-}^{UL}(z_1, z_2) = \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} [A_U(z_1, z_2) - A_L(z_1, z_2)]$$

integrated over  $M_1, M_2$

Phys. Rev D 78, 032011 (2008)

with  $z_1 = z_2 = z$



with the previous assumptions on the FFs one obtains

$$|a_p^h(z)| = \left| \frac{H_{1fav}(z)}{D_{1fav}(z)} \right| = \left[ \frac{1}{B(z)} \frac{\langle 1 + \cos^2 \theta \rangle}{\langle \sin^2 \theta \rangle} A_{e^+e^-}^{UL}(z) \right]^{\frac{1}{2}}$$

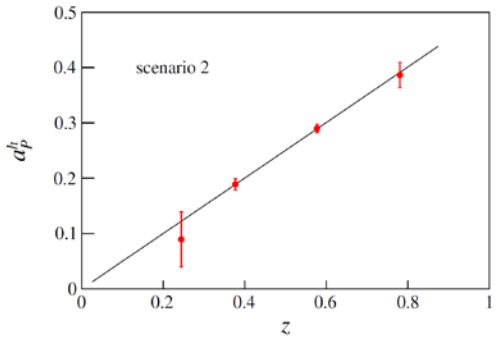
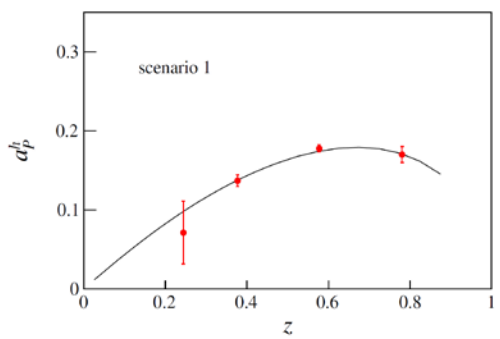
where 
$$B = \frac{5 + 5\alpha^2}{5 + 7\beta^2} - \frac{5\alpha}{5\beta + \beta^2}$$

$$\alpha(z) = \frac{H_{1unf}(z)}{H_{1fav}(z)} = \begin{cases} -1 & \mathbf{a1} \\ -\frac{D_{1unf}(z)}{D_{1fav}(z)} = -\beta & \mathbf{a2} \end{cases}$$

# Collins asymmetry – $e^+e^-$ Belle data

the values of the analysing power  $|a_P^h(z)|$  have been calculated in the four  $z$  bins with both the assumptions

$$\alpha(z) = \frac{H_{1\text{unf}}(z)}{H_{1\text{fav}}(z)} = \begin{cases} -1 & \mathbf{a1} \\ -\frac{D_{1\text{unf}}(z)}{D_{1\text{fav}}(z)} = -\beta & \mathbf{a2} \end{cases}$$



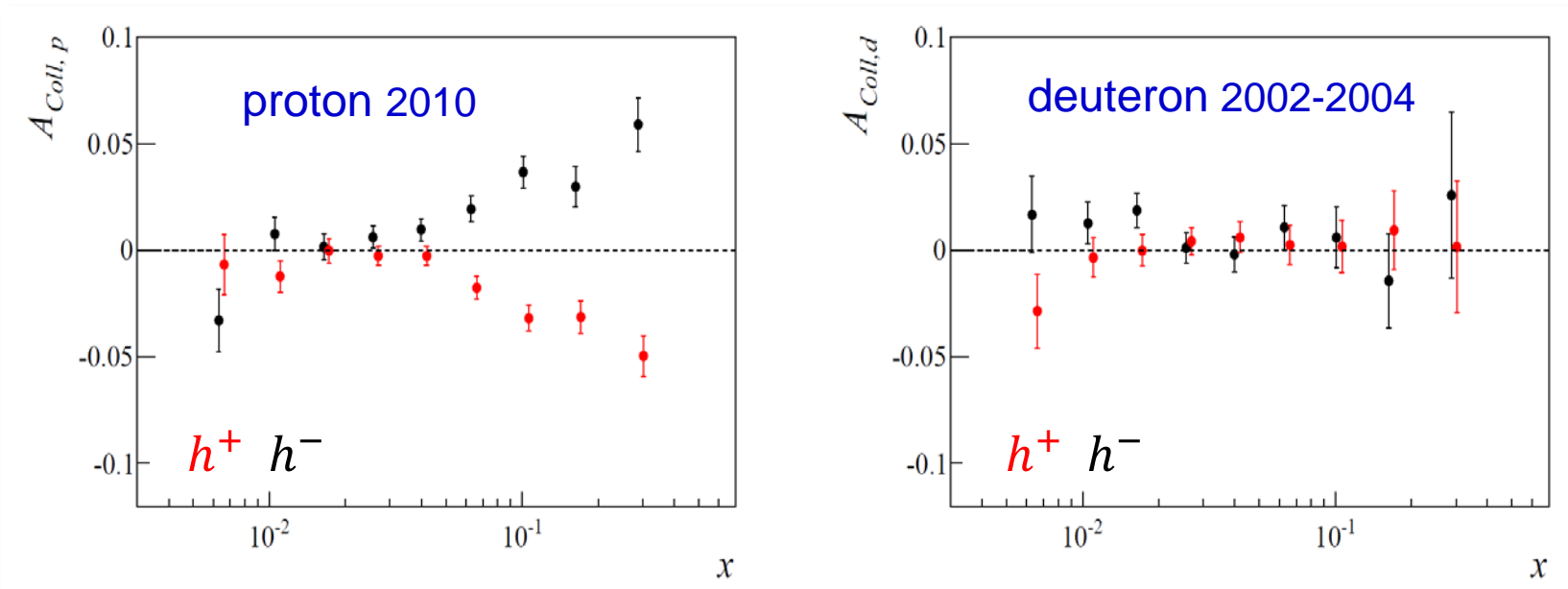
and then fitted with two different functions of  $z$

results:  $\mathbf{a1}$   $|\tilde{a}_P^h| = 0.12$   
 $\mathbf{a2}$   $|\tilde{a}_P^h| = 0.17$

→ same values for the transversity functions!  
 compensated by  $\alpha$

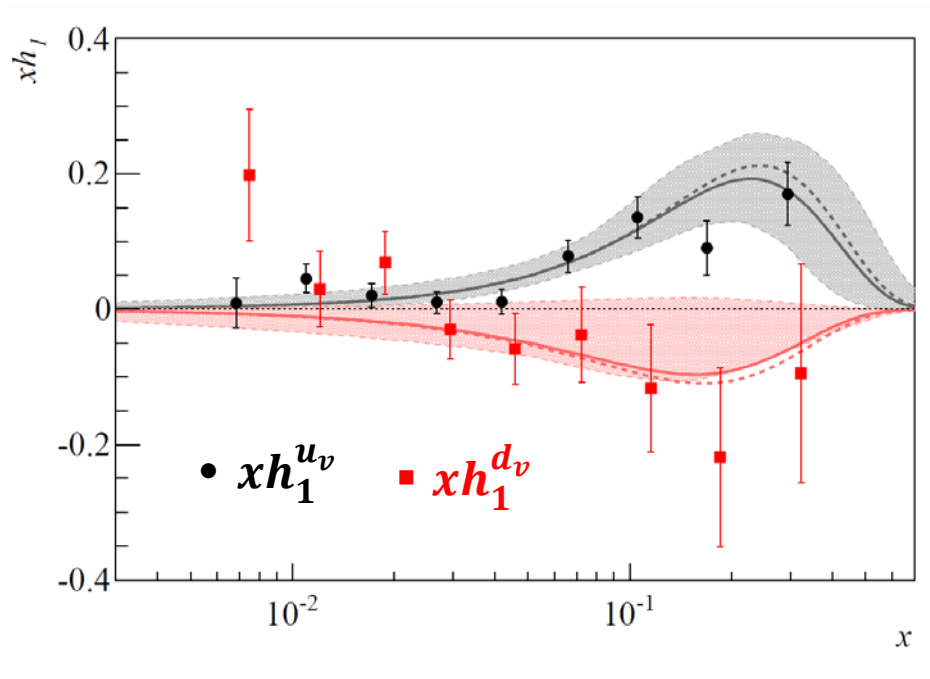
we have assumed  $|a_P^h(z)|$  constant in  $Q^2$  i.e. same evolution for  $H_{1\text{fav}}$  and  $D_{1\text{fav}}$   
 if the evolution of  $H_{1\text{fav}}$  is negligible, the analysing power at COMPASS  $Q^2$  decreases by  $\sim 10\%$

# Collins asymmetry – COMPASS data



charged hadrons ~ charged pions

# Transversity from the Collins asymmetry



error bars:  $1\sigma$  stat. only

$xh_1^{u_v}$  clearly different from zero

$xh_1^{d_v}$  opposite sign

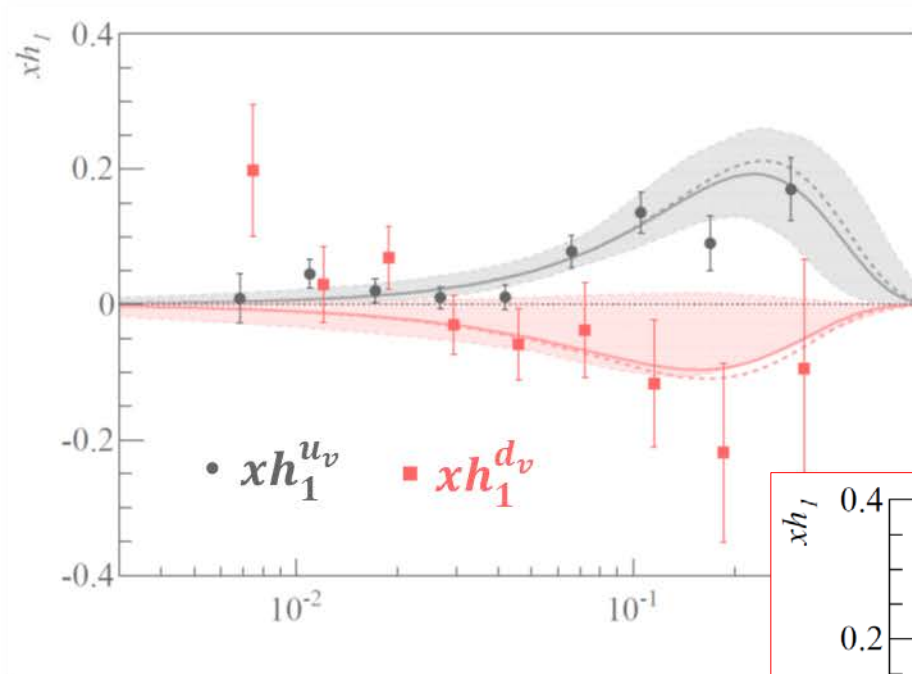
larger errors because of the  
low statistics of the d data

curves:

Anselmino et al., 2013

*Soffer bound*

# Transversity from the Collins asymmetry

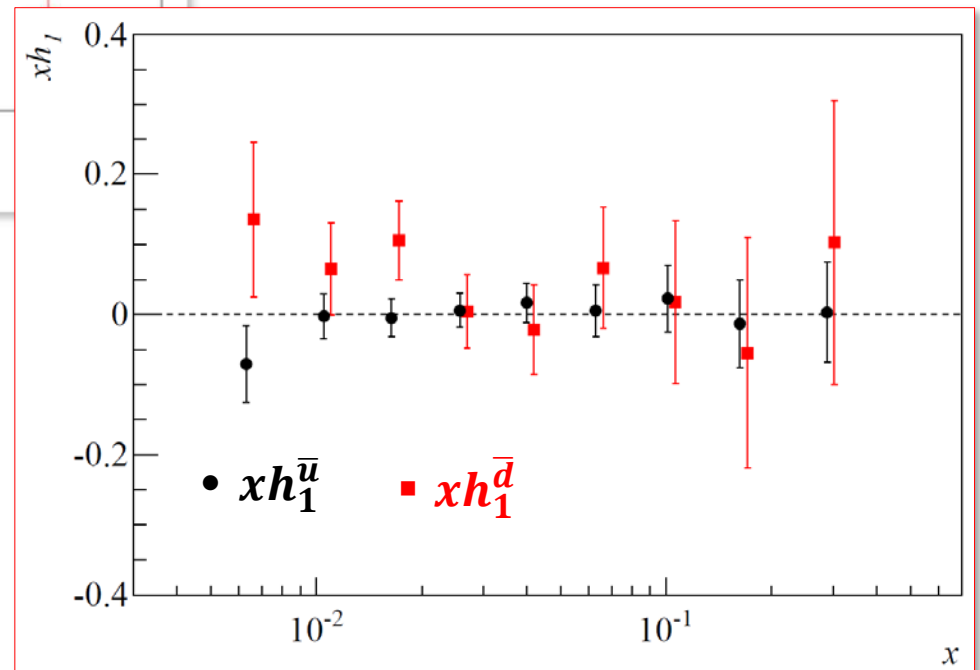


errors:  $1\sigma$  stat. only

$xh_1^{u_v}$  clearly different from zero

$xh_1^{d_v}$  opposite sign

larger errors because of the low statistics of the d data



$xh_1^{\bar{u}}, xh_1^{\bar{d}}$

compatible with zero

same errors as for valence quarks



# Transversity from the di-hadron asymmetry

SIDIS 
$$A^{hh}(x) = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q(x) \tilde{H}_{1q}^\zeta}{\sum_{q,\bar{q}} e_q^2 x f_1^q(x) \tilde{D}_{1q}^{hh}}$$

assuming 
$$\begin{aligned} \tilde{D}_{1u}^{hh} = \tilde{D}_{1d}^{hh} = \tilde{D}_{1\bar{u}}^{hh} = \tilde{D}_{1\bar{d}}^{hh} & \quad \tilde{D}_{1s}^{hh} = \tilde{D}_{1\bar{s}}^{hh} = \lambda \tilde{D}_{1u}^{hh} \quad \lambda = 0.5 \quad (\tilde{D}_{1c}^{hh} = \tilde{D}_{1\bar{c}}^{hh}) \\ \tilde{H}_{1u}^\zeta = -\tilde{H}_{1d}^\zeta = -\tilde{H}_{1\bar{u}}^\zeta = \tilde{H}_{1\bar{d}}^\zeta & \quad \tilde{H}_{1s}^\zeta = -\tilde{H}_{1\bar{s}}^\zeta = 0 \quad (\tilde{H}_{1c}^\zeta = -\tilde{H}_{1\bar{c}}^\zeta = 0) \end{aligned}$$

Bacchetta, Courtoy, Radici 2011 Courtoy, Bacchetta, Radici, Bianconi 2012

$$\begin{aligned} x h_1^{u_v} &= \frac{1}{15} \frac{1}{\tilde{\alpha}_p^{hh}} [3(4 x f_1^{u+\bar{u}} + x f_1^{d+\bar{d}} + 0.5 x f_1^{s+\bar{s}}) A_p^{hh} & f_1^{q+\bar{q}} = f_1^q + f_1^{\bar{q}} \\ & \quad + (5 x f_1^{u+\bar{u}} + 5 x f_1^{d+\bar{d}} + x f_1^{s+\bar{s}}) A_d^{hh}] \end{aligned}$$

$$\begin{aligned} x h_1^{d_v} &= \frac{1}{15} \frac{1}{\tilde{\alpha}_p^{hh}} [-3(4 x f_1^{u+\bar{u}} + x f_1^{d+\bar{d}} + 0.5 x f_1^{s+\bar{s}}) A_p^{hh} \\ & \quad + 4(5 x f_1^{u+\bar{u}} + 5 x f_1^{d+\bar{d}} + x f_1^{s+\bar{s}}) A_d^{hh}] \end{aligned}$$

$$\tilde{\alpha}_p^{hh} = \frac{\tilde{H}_{1u}^\zeta}{\tilde{D}_{1u}^{hh}} \quad \text{from } e^+e^- \text{ Belle data}$$

# Transversity from the di-hadron asymmetry

$$\tilde{\alpha}_P^{hh} = \frac{\tilde{H}_{1u}^\angle}{\tilde{D}_{1u}^{hh}} \quad \text{from } e^+e^- \text{ Belle data}$$

Phys. Rev. Lett. 107, 072004 (2011)

$$|\tilde{\alpha}_P^{hh}| = \left| \frac{\tilde{H}_{1u}^\angle}{\tilde{D}_{1u}^{hh}} \right| = \left[ -\frac{1}{5} (1 + \mu^2)(5 + \lambda^2) \frac{\langle 1 + \cos^2 \theta \rangle}{\langle \sin^2 \theta \rangle} A_{e^+e^-}^{hh} \right]^{\frac{1}{2}}$$

$\mu^2 = 0.5$  charm yield  $\sim$  one half the uds

$$e_c^2 \tilde{D}_{1c}^{hh} \tilde{D}_{1\bar{c}}^{hh} = \mu^2 \left( e_u^2 \tilde{D}_{1u}^{hh} \tilde{D}_{1\bar{u}}^{hh} + e_d^2 \tilde{D}_{1d}^{hh} \tilde{D}_{1\bar{d}}^{hh} + e_s^2 \tilde{D}_{1s}^{hh} \tilde{D}_{1\bar{s}}^{hh} \right)$$

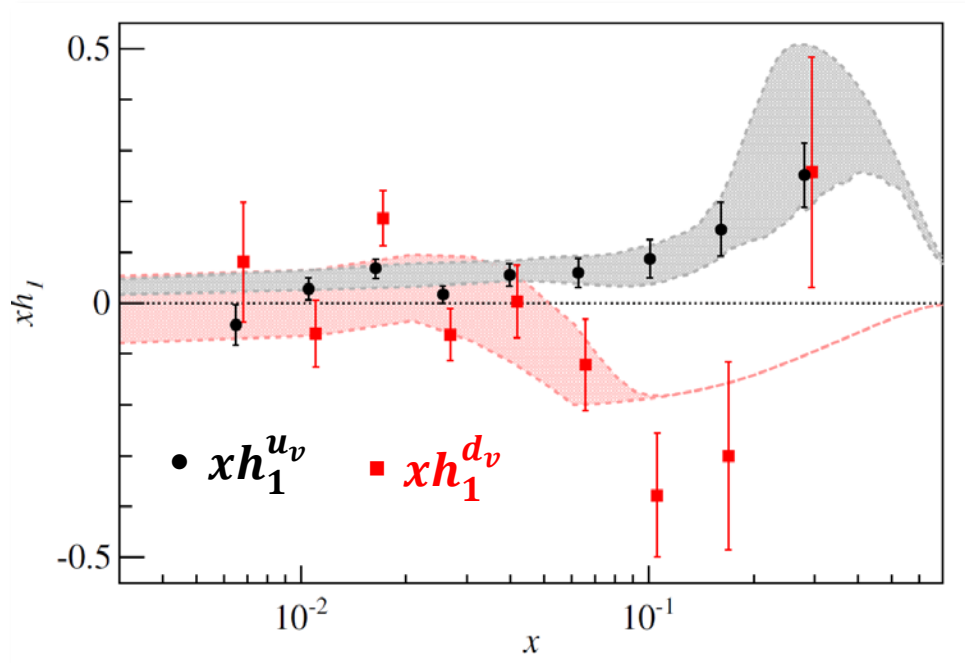
$$A_{e^+e^-}^{hh} = -0.196$$



$$|\tilde{\alpha}_P^{hh}| = 0.201 \quad \text{at } Q_B^2 \cong 110 \text{ GeV}^2/c^2$$

assumed constant in  $Q^2$

# Transversity from the di-hadron asymmetries



error bars:  $1\sigma$  stat. only

$xh_1^{u_v}$  clearly different from zero

$xh_1^{d_v}$  opposite sign

larger errors because of the low statistics of the d data

curves:

Bacchetta et al, 2013

*Soffer bound*

# Transversity

from the **Collins** and from the **di-hadron** asymmetries

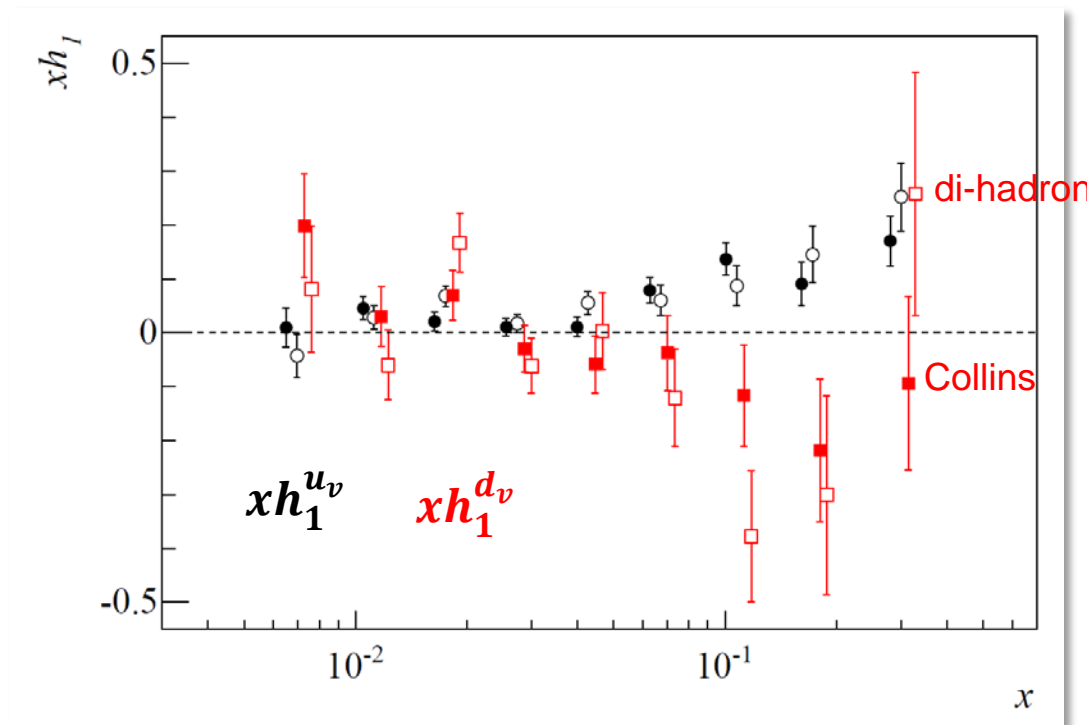
closed points

Gaussian ansatz  
evolution not known

open points

collinear  
known evolution

different assumptions



# Sivers function

# Sivers functions from the SIDIS Sivers asymmetries

a similar method has been used to extract the Sivers function from the COMPASS p and d Sivers asymmetries for charged pions and kaons

$$A_{Siv}^h(x, z) = \frac{\sum_{q, \bar{q}} e_q^2 x f_{1T}^{\perp q}(x) \otimes D_{1q}^h(z)}{\sum_{q, \bar{q}} e_q^2 x f_1^q(x) \otimes D_{1q}^h(z)}$$

“gaussian ansatz”:

$$A_{Siv}^h(x, z) = G z \frac{\sum_{q, \bar{q}} e_q^2 x f_{1T}^{\perp(1)q}(x) D_{1q}^h(z)}{\sum_{q, \bar{q}} e_q^2 x f_1^q(x) D_{1q}^h(z)} \quad f_{1T}^{\perp(1)q} = \int d^2 \vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(k_T^2)$$

assuming  $G = \frac{\sqrt{\pi} M}{[z^2 \langle k_T^2 \rangle_S + \langle p_T^2 \rangle]^{1/2}} \simeq \frac{\pi M}{2 \langle P_T \rangle}$  known and integrating over  $z$

$$A_{Siv}^h(x) = G \frac{\sum_q e_q^2 \cdot x f_{1T}^{\perp(1)q}(x) \cdot \tilde{D}_{1q}^{(1)h}}{\sum_q e_q^2 \cdot x f_1^q(x) \cdot \tilde{D}_{1q}^h}$$

$$\tilde{D}_{1q}^h = \int dz D_{1q}^h(z)$$

$$\tilde{D}_{1q}^{(1)h} = \int dz z D_{1q}^h(z)$$

# Sivers functions from the SIDIS Sivers asymmetries

writing explicitly the Sivers asymmetries one finds, for pions

$$x f_{1T}^{\perp(1)u_v} = \frac{1}{5G\rho_\pi(1 - \beta_\pi^{(1)})} \left[ (x f_p^{\pi^+} A_p^{\pi^+} - x f_p^{\pi^-} A_p^{\pi^-}) + \frac{1}{3} (x f_d^{\pi^+} A_d^{\pi^+} - x f_d^{\pi^-} A_d^{\pi^-}) \right]$$

$$x f_{1T}^{\perp(1)d_v} = \frac{1}{5G\rho_\pi(1 - \beta_\pi^{(1)})} \left[ \frac{4}{3} (x f_d^{\pi^+} A_d^{\pi^+} - x f_d^{\pi^-} A_d^{\pi^-}) - (x f_p^{\pi^+} A_p^{\pi^+} - x f_p^{\pi^-} A_p^{\pi^-}) \right]$$

$$x [4(f_1^u + \beta_\pi f_1^{\bar{u}}) + (\beta_\pi f_1^d + f_1^d) + N\beta_\pi(f_1^s + f_1^{\bar{s}})] \tilde{D}_{1,\text{fav}}^\pi \equiv x f_p^{\pi^+} \tilde{D}_{1,\text{fav}}^\pi$$

$$\beta_\pi = \frac{\tilde{D}_{1,\text{unf}}^\pi}{\tilde{D}_{1,\text{fav}}^\pi} \quad \beta_\pi^{(1)} = \frac{\tilde{D}_{1,\text{unf}}^{(1)\pi}}{\tilde{D}_{1,\text{fav}}^{(1)\pi}} \quad \rho_\pi = \frac{\tilde{D}_{1,\text{fav}}^{(1)\pi}}{\tilde{D}_{1,\text{fav}}^\pi} \quad \tilde{D}_1^\pi = \int dz D_1^\pi(z) \quad \tilde{D}_1^{(1)\pi} = \int dz z D_1^\pi(z)$$

$$x f_{1T}^{\perp(1)\bar{u}} - x f_{1T}^{\perp(1)\bar{d}} = \frac{1}{15G\rho_\pi(1 - \beta_\pi^{(1)2})} \left[ 2(1 - 4\beta_\pi^{(1)})x f_p^{\pi^+} A_p^{\pi^+} + 2(4 - \beta_\pi^{(1)})x f_p^{\pi^-} A_p^{\pi^-} \right. \\ \left. - (1 - 4\beta_\pi^{(1)})x f_d^{\pi^+} A_d^{\pi^+} - (4 - \beta_\pi^{(1)})x f_d^{\pi^-} A_d^{\pi^-} \right].$$

can be obtained directly from the p and d measured asymmetries in each  $x$  bin, at the corresponding  $\langle Q^2 \rangle$

# Sivers functions from the SIDIS Sivers asymmetries

writing explicitly the Sivers asymmetries one finds, for kaons

$$xf_{1T}^{\perp(1)u_v} = \frac{1}{4G\rho_K(1 - \beta_K^{(1)})} [xf_p^{K^+} A_p^{K^+} - xf_p^{K^-} A_p^{K^-}]$$

assuming  $f_{1T}^{\perp(1)s} = f_{1T}^{\perp(1)\bar{s}}$

$$xf_{1T}^{\perp(1)d_v} = -\frac{1}{4G\rho_K(1 - \beta_K^{(1)})} [xf_p^{K^+} A_p^{K^+} - xf_p^{K^-} A_p^{K^-} - (xf_d^{K^+} A_d^{K^+} - xf_d^{K^-} A_d^{K^-})]$$

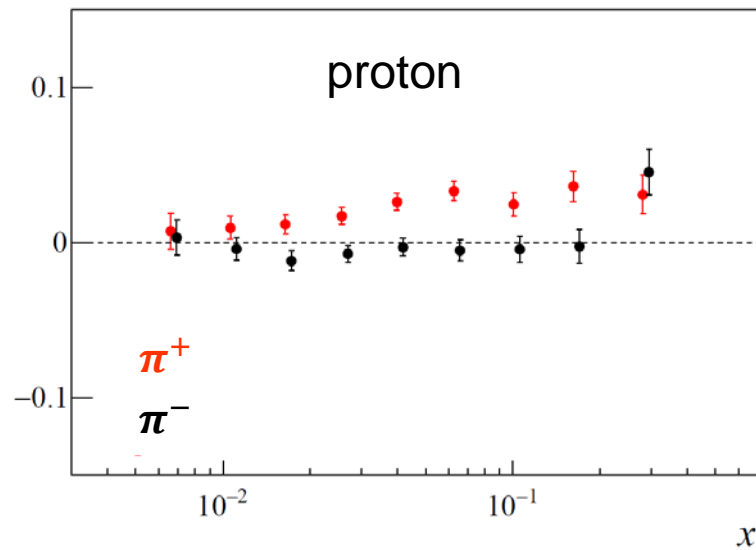
no linear combination give access to  $f_{1T}^{\perp(1)s,\bar{s}}$



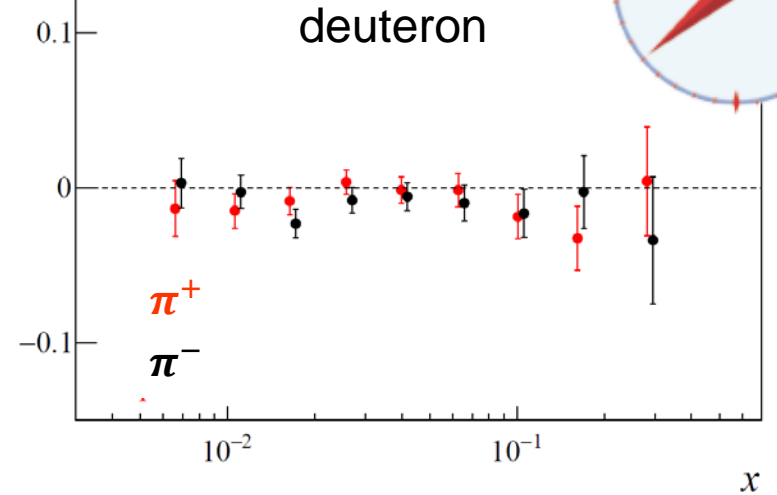
# Sivers functions from the SIDIS Sivers asymmetries



$A_{Siv}$

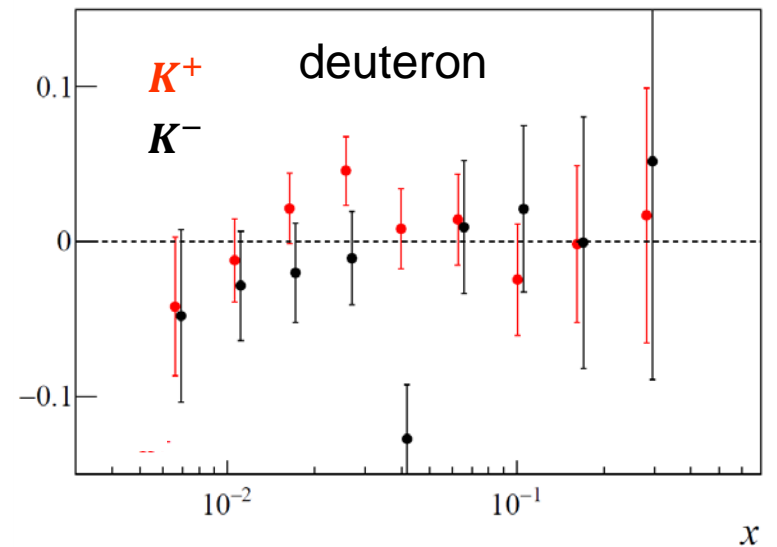
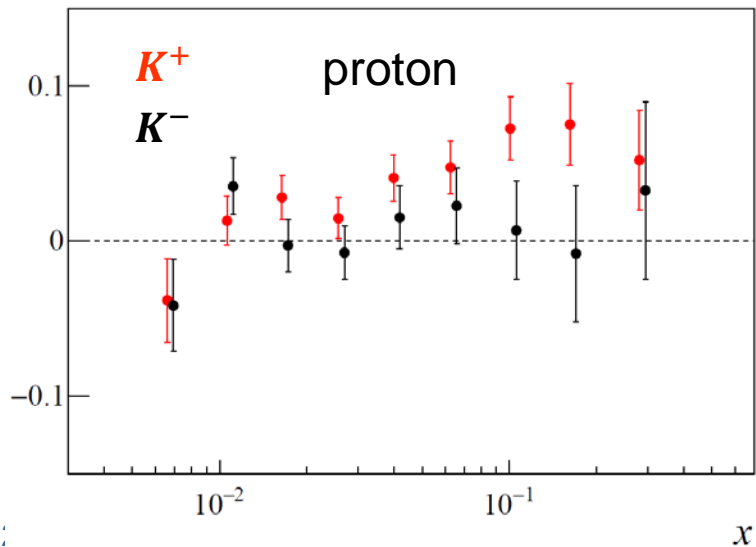


PLB 744 (2015) 250

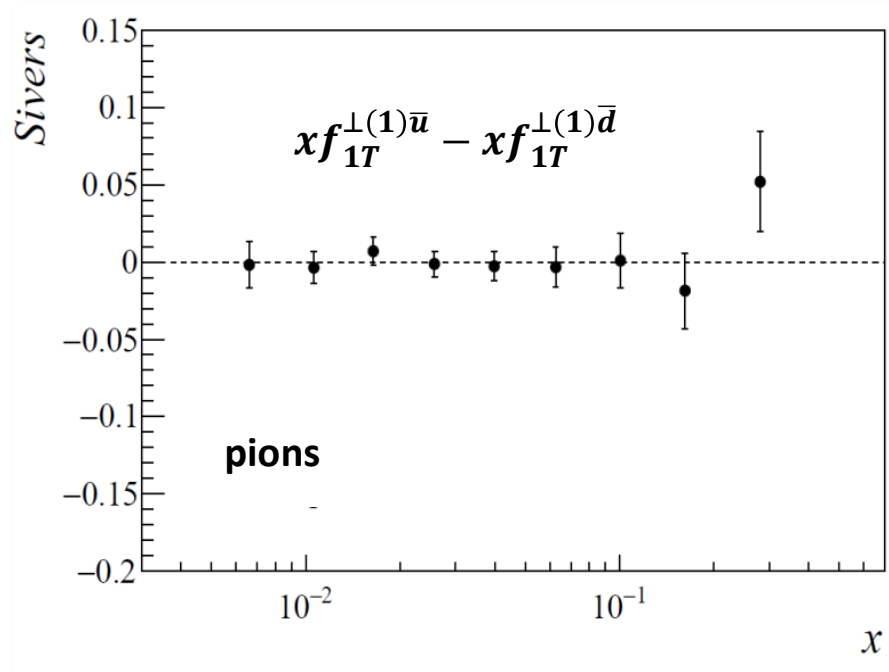


PLB 673 (2009) 127

$A_{Siv}$



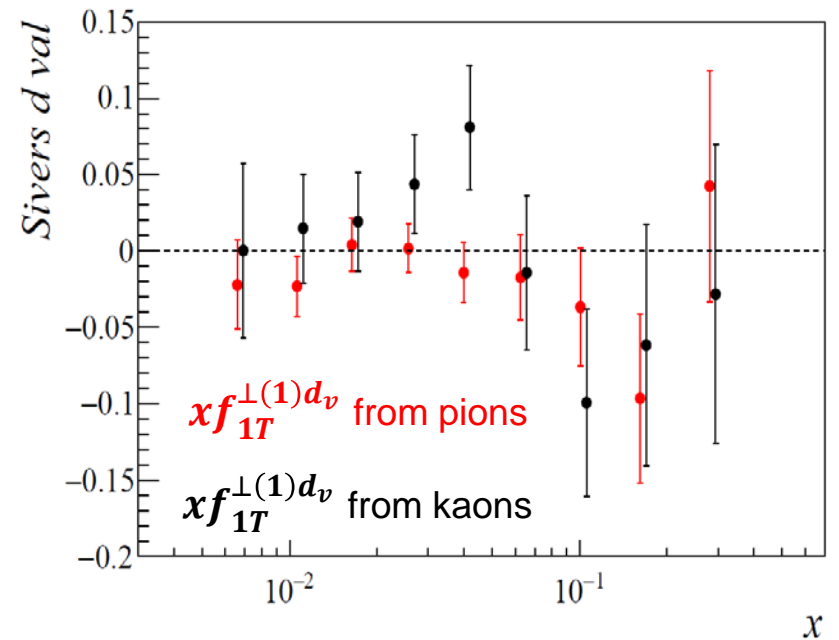
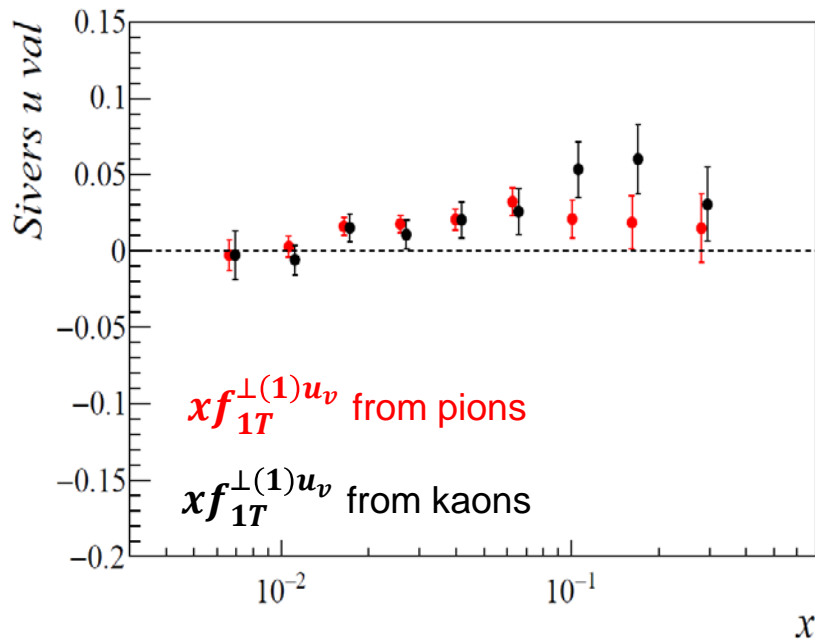
# Sivers functions from the SIDIS Sivers asymmetries



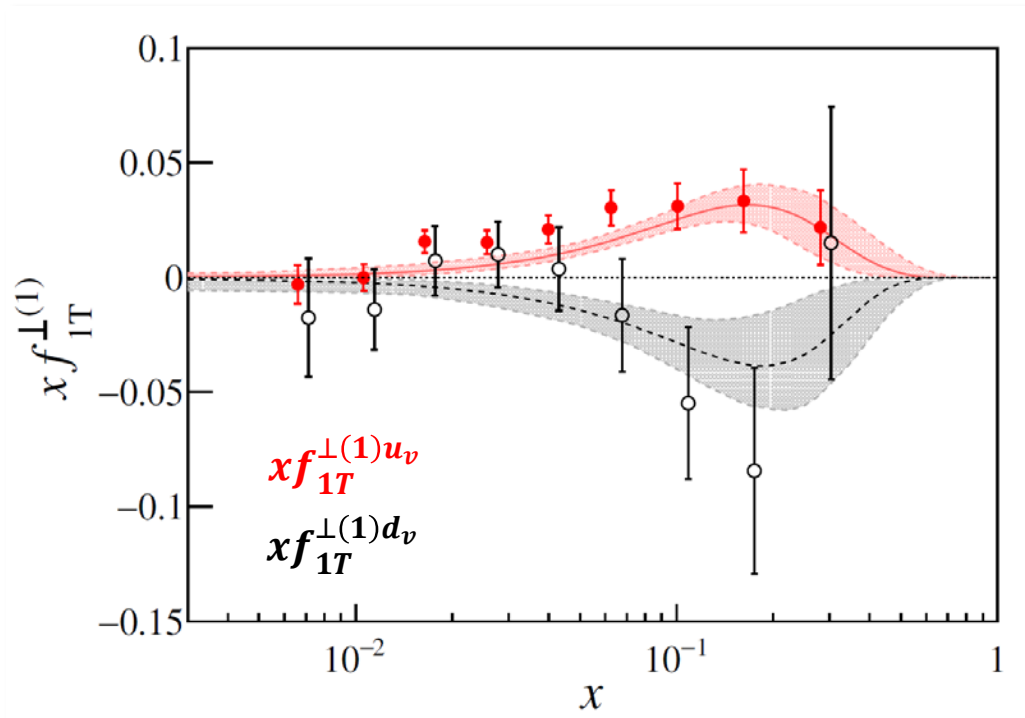
compatible with zero  
→  $x f_{1T}^{\perp(1)\bar{u}}$  and  $x f_{1T}^{\perp(1)\bar{d}}$  small

In the large  $N_c$  limit, the isotriplet ( $\bar{u} - \bar{d}$ ) Sivers combination is expected to dominate over the isosinglet one ( $\bar{u} + \bar{d}$ )

# Sivers functions from the SIDIS Sivers asymmetries



# Sivers functions from the SIDIS Sivers asymmetries



curves:

Anselmino Boglione Melis, 2012,  $Q^2 = 4 \text{ GeV}^2$

# Summary

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the point-by-point extraction of PDFs looks promising

- interesting results for the transversity and the Sivers functions
- no parametrisation of the unknown PDFs and FFs is needed
- can be used also for the weighted asymmetries, ...
- needs proton and deuteron/neutron data at the same  $\langle x \rangle, \langle Q^2 \rangle$  values

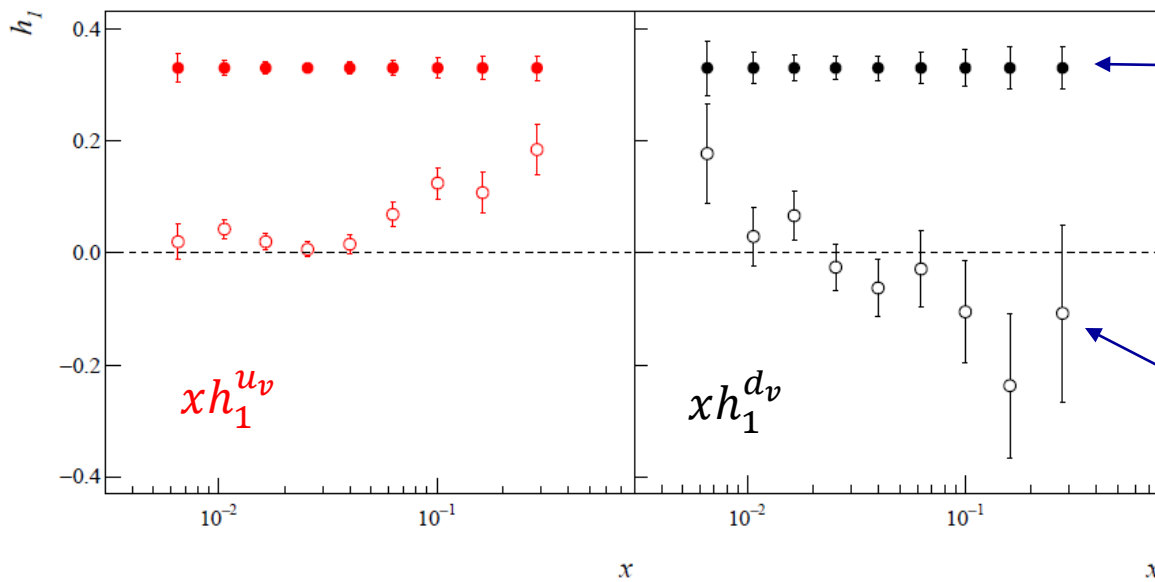
to be kept in mind in planning future experiments

presently, large uncertainties for the d quark due to the  
low statistics deuteron data

we have repeated the extraction of transversity assuming one full year  
of data taking in COMPASS with the transversely polarised d target  
in the same conditions of the 2010 proton run

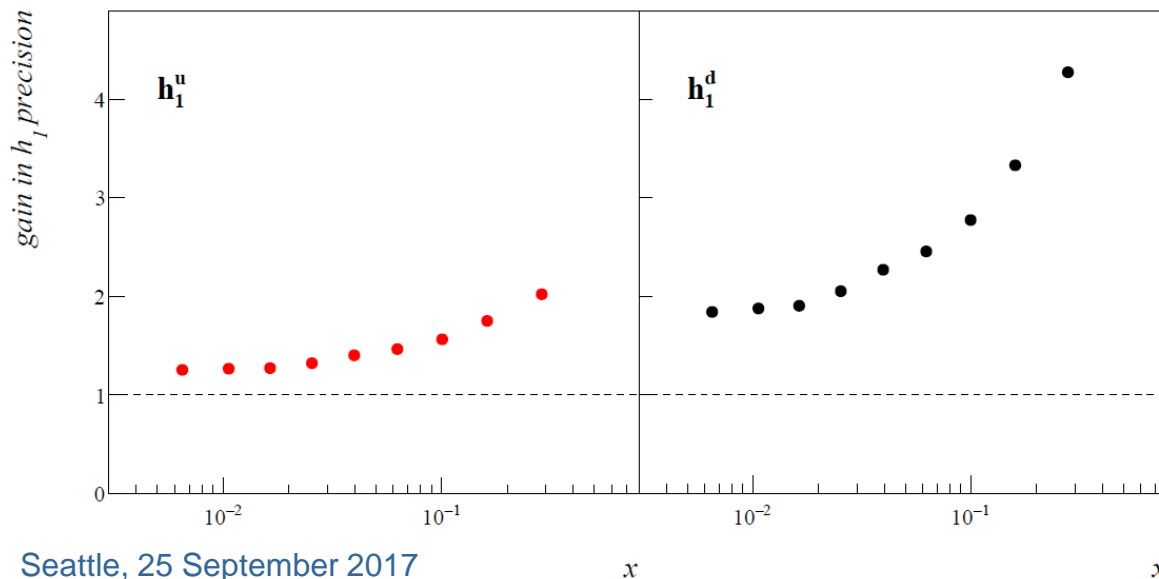


# Transversity



statistical errors with one year of data taking with the  ${}^6\text{LiD}$  transversely polarized target at COMPASS

present results



ratios of the statistical errors  
*present / present+one year d*

*F. Bradamante  
EICUG meeting  
Trieste, July 2017*

# Summary

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the point-by-point extraction of PDFs looks promising

- interesting results for the transversity and the Sivers functions
- no parametrisation of the unknown PDFs and FFs is needed
- can be used also for the weighted asymmetries, ...
- needs proton and deuteron/neutron data at the same  $\langle x \rangle, \langle Q^2 \rangle$  values

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we have repeated the extraction of transversity assuming one full year of data taking in COMPASS with the transversely polarised d target in the same conditions of the 2010 proton run

→ remarkable improvement

unique opportunity to improve our knowledge at “small” x and “large”  $Q^2$ , in a range complementary to the JLab12 measurements

COMPASS can produce other relevant results, useful for EIC

**thank you**