INSTITUTE FOR NUCLEAR THEORY

INT Workshop INT-17-3
Spatial and Momentum Tomography of Hadrons and Nuclei
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A new decomposition of the hadron mass

Based on **[C.L., arXiv:1706.05853]**

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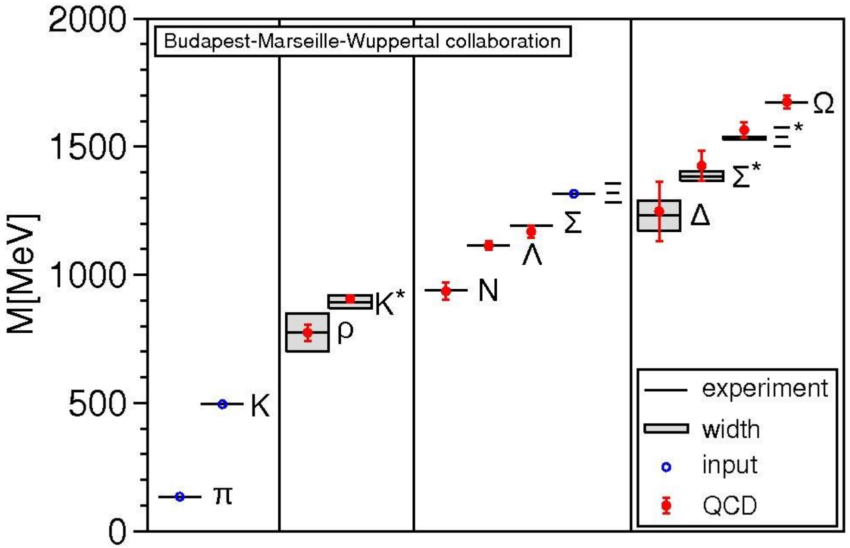
Outline

1. **Hadron mass and energy-momentum tensor**
2. **Trace decomposition**
3. **Ji's decomposition**
4. **New decomposition**
5. **Summary**

Lattice QCD

Ab initio mass calculation based on Euclidean space-time correlators $\sim \sum_n e^{-E_n \tau}$



[Dürr *et al*. (2008)]

Unfortunately, little insight on where mass comes from …

Energy-momentum tensor

Classical QCD energy-momentum tensor

$$T^{\mu\nu} = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu \psi - G^{a\mu\alpha} G^{a\nu}{}_\alpha + \tfrac{1}{4}\,\eta^{\mu\nu} G^2$$

Translation invariance $\Longrightarrow$ **energy-momentum conservation**

$$\partial_\mu T^{\mu\nu} = 0$$

Renormalized trace of the QCD EMT

$$T^\mu{}_\mu = \underbrace{\frac{\beta(g)}{2g}}_{\text{Trace anomaly}} G^2 + (1+\gamma_m)\,\overline{\psi} \underset{\substack{\uparrow \\ \text{Quark mass matrix}}}{m} \psi$$

[Crewther (1972)]
[Chanowitz, Ellis (1972)]
[Nielsen (1975)]
[Adler, Collins, Duncan (1977)]
[Collins, Duncan, Joglekar (1977)]
[Nielsen (1977)]

Textbook decomposition

Forward matrix element

$$\langle P|T^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu$$

$$\langle P'|P\rangle = 2P^0\,(2\pi)^3\,\delta^{(3)}(\vec{P}' - \vec{P})$$

Trace decomposition

$$\begin{aligned} 2M^2 &= \langle P|T^\mu{}_\mu(0)|P\rangle \\ &= \underbrace{\langle P|\frac{\beta(g)}{2g}\,G^2|P\rangle}_{\sim 89\%} + \underbrace{\langle P|(1+\gamma_m)\,\overline{\psi}m\psi|P\rangle}_{\sim 11\%} \end{aligned}$$

[Shifman, Vainshtein, Zakharov (1978)]
[Luke, Manohar, Savage (1992)]
[Donoghue, Golowich, Holstein (1992)]
[Kharzeev (1996)]
[Bressani, Wiedner, Filippi (2005)]
[Roberts (2017)]
[Krein, Thomas, Tsushima (2017)]

- ✓ **Manifestly covariant**
- ✓ **Compatible with Gell-Mann–Oakes–Renner formula for pion**

- ✗ **Depends on state normalization**
- ✗ **No spatial extension**
- ✗ **No clear relation to energy**

Ji's decomposition

[Ji (1995)]
[Gaite (2013)]

Separation of trace and traceless parts

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu} \qquad \hat{T}^{\mu\nu} = \tfrac{1}{4}\,\eta^{\mu\nu}\,T^{\alpha}{}_{\alpha}$$

Forward matrix elements

$$\langle P|\bar{T}^{\mu\nu}(0)|P\rangle = 2\left(P^{\mu}P^{\nu} - \tfrac{1}{4}\,\eta^{\mu\nu}M^2\right)$$
$$\langle P|\hat{T}^{\mu\nu}(0)|P\rangle = \tfrac{1}{2}\,\eta^{\mu\nu}M^2$$

Virial decomposition

$$\begin{aligned} M &= \langle T^{00}\rangle|_{\vec{P}=\vec{0}} \\ &= \underbrace{\langle \bar{T}^{00}\rangle|_{\vec{P}=\vec{0}}}_{\textbf{75\%}} + \underbrace{\langle \hat{T}^{00}\rangle|_{\vec{P}=\vec{0}}}_{\textbf{25\%}} \end{aligned} \qquad \langle O\rangle = \frac{\langle P|\int \mathrm{d}^3r\, O(r)|P\rangle}{\langle P|P\rangle}$$

Ji's decomposition

[Ji (1995)]

Separation of quark and gluon contributions

$$\bar{T}^{\mu\nu} = \bar{T}^{\mu\nu}_q + \bar{T}^{\mu\nu}_g \qquad\qquad \hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_m + \hat{T}^{\mu\nu}_a$$

Forward matrix elements

$$\langle P|\bar{T}^{\mu\nu}_i(0)|P\rangle = 2\,a_i(\mu^2)\left(P^\mu P^\nu - \tfrac{1}{4}\,\eta^{\mu\nu}M^2\right)$$
$$\langle P|\hat{T}^{\mu\nu}_i(0)|P\rangle = \tfrac{1}{2}\,b_i(\mu^2)\,\eta^{\mu\nu}M^2$$

$$\sum_i a_i(\mu^2) = \sum_i b_i(\mu^2) = 1$$

Ji's decomposition [Gao *et al.* (2015)]

$$M = \underbrace{M_q}_{\sim 33\%} + \underbrace{M_g}_{\sim 34\%} + \underbrace{M_m}_{\sim 11\%} + \underbrace{M_a}_{\sim 22\%}$$

$\mu = 2\,\mathrm{GeV}$

$$M_q = \langle\bar{T}^{00}_q\rangle|_{\vec{P}=\vec{0}} - \tfrac{3}{1+\gamma_m}\,\langle\hat{T}^{00}_m\rangle|_{\vec{P}=\vec{0}}$$
$$M_g = \langle\bar{T}^{00}_g\rangle|_{\vec{P}=\vec{0}}$$
$$M_m = \tfrac{4+\gamma_m}{1+\gamma_m}\,\langle\hat{T}^{00}_m\rangle|_{\vec{P}=\vec{0}}$$
$$M_a = \langle\hat{T}^{00}_a\rangle|_{\vec{P}=\vec{0}}$$

- ✓ **Proper normalization**
- ✓ **Clear relation to energy distribution**
- ! **Scale-dependent interpretation in the rest frame**
- ✗ **Pressure effects not taken into account**

Continuum mechanics

Interpretation of the EMT components

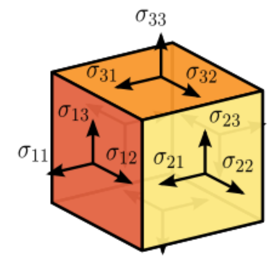
Energy density — Momentum density

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Shear stress

Normal stress (pressure)

Energy flux — Momentum flux



Analogy with relativistic hydrodynamics

Hadron $$\langle P|T_i^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu A_i(0) + 2M^2\eta^{\mu\nu}\bar{C}_i(0)$$

Perfect fluid element $$\Theta_i^{\mu\nu} = (\varepsilon_i + p_i)u^\mu u^\nu - p_i\,\eta^{\mu\nu}$$

$\Rightarrow$

Four-velocity

$$u^\mu = P^\mu/M$$

Energy density

$$\varepsilon_i = [A_i(0) + \bar{C}_i(0)]\,\frac{M}{V}$$

Isotropic pressure

$$p_i = -\bar{C}_i(0)\,\frac{M}{V}$$

New decomposition

Mass as the total internal energy

$$M = \sum_i U_i \qquad\qquad U_i = \varepsilon_i V$$

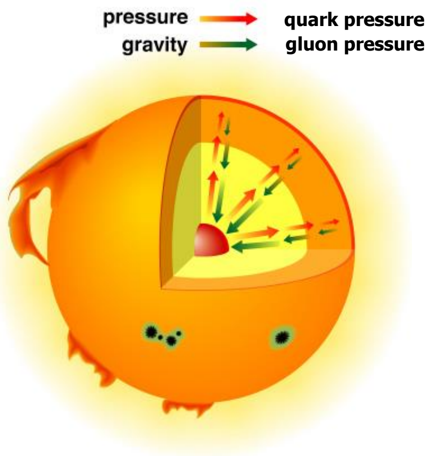
Stability condition → **energy-momentum conservation**

$$\sum_i p_i = 0 \quad \longrightarrow \quad \sum_i \bar{C}_i(0) = 0 \quad \longrightarrow \quad \sum_i A_i(0) = 1$$

Nucleon mass decomposition

$$M = \underbrace{U_q}_{\sim 44\%} + \underbrace{U_g}_{\sim 56\%} \qquad\qquad \underbrace{p_q = -p_g}_{\sim 11\%}$$

$\mu = 2\,\mathrm{GeV}$



Some numerology

At the center of the Sun

$$\rho_\odot \approx 1.6 \times 10^5 \,\mathrm{kg/m^3} \qquad\qquad p_\odot \approx 2.5 \times 10^{16} \,\mathrm{N/m^2}$$

$$\Rightarrow \quad p_\odot/\rho_\odot c^2 \approx 1.7 \times 10^{-6}$$

Inside a nucleon

$$\rho_N \approx 2 \times 10^{17} \,\mathrm{kg/m^3} \qquad\qquad p_N \approx 2 \times 10^{33} \,\mathrm{N/m^2}$$

$$\Rightarrow \quad p_N/\rho_N c^2 \approx 0.11$$

Inside a black hole

$$\rho_{\mathrm{BH}} = \frac{3c^6}{32\pi G^3 M^2} \qquad \rho_{\mathrm{BH}}^{\odot} \approx 1.8 \times 10^{19} \,\mathrm{kg/m^3} \qquad \rho_{\mathrm{BH}}^{N} \approx 2.6 \times 10^{133} \,\mathrm{kg/m^3}$$

Trace anomaly and quark mass contributions

Back to virial decomposition

Energy	$\langle \bar{T}_i^{00} \rangle\vert_{\vec{P}=\vec{0}} = \frac{3}{4}\,(U_i + p_i V)$	$\langle \hat{T}_i^{00} \rangle\vert_{\vec{P}=\vec{0}} = \frac{1}{4}\,(U_i - 3p_i V)$
Pressure	$\langle \bar{T}_i^{33} \rangle\vert_{\vec{P}=\vec{0}} = \frac{1}{4}\,(U_i + p_i V)$	$\langle \hat{T}_i^{33} \rangle\vert_{\vec{P}=\vec{0}} = -\frac{1}{4}\,(U_i - 3p_i V)$

Trace anomaly contribution

$$T_a^{\mu\nu} = \hat{T}_g^{\mu\nu} \quad \Longrightarrow \quad U_a \equiv \frac{1}{4}\,U_g \qquad p_a \equiv \frac{3}{4}\,p_g$$

~ 14% **~ – 8%** $\mu = 2\,\mathrm{GeV}$

Quark mass contribution is trickier

Indistinguishable from $\eta^{\mu\nu}$ **in Ji's approach**

Dust (no pressure)

$$T_{\text{mass}}^{\mu\nu} \equiv \boxed{\frac{P^\mu P^\nu}{M^2}}\,\frac{1}{4}\,(4+\gamma_m)\,\overline{\psi}m\psi$$

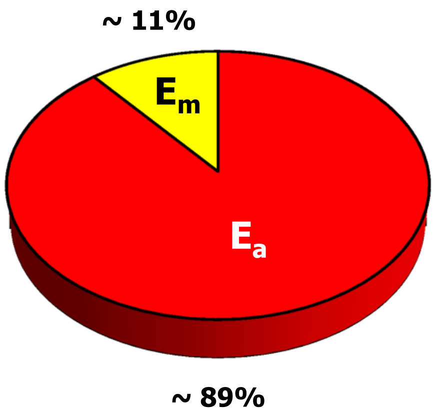
$$U_m = \langle T_{\text{mass}}^{00} \rangle\vert_{\vec{P}=\vec{0}} = \frac{4+\gamma_m}{1+\gamma_m}\,\langle \hat{T}_m^{00} \rangle\vert_{\vec{P}=\vec{0}} = M_m$$

~ 11% $\mu = 2\,\mathrm{GeV}$

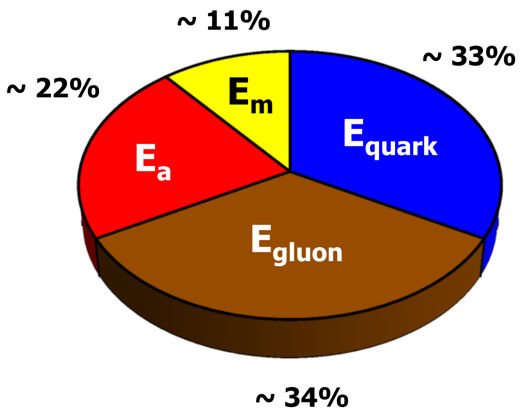
Summary

$\mu = 2\,\mathrm{GeV}$

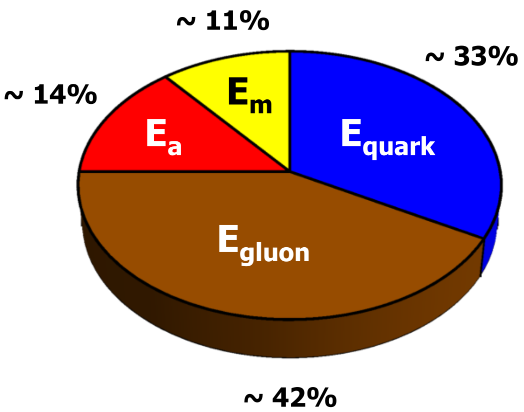
Trace decomposition



Ji's decomposition



New decomposition



Stability