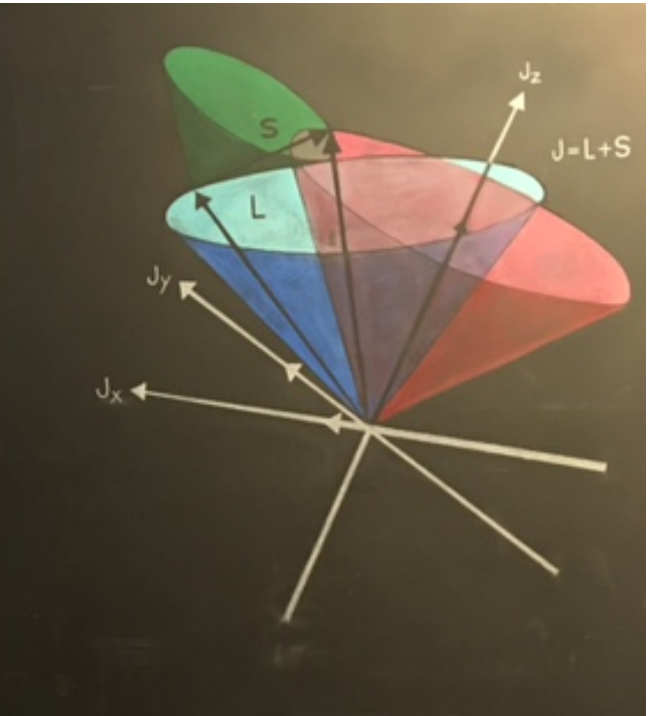
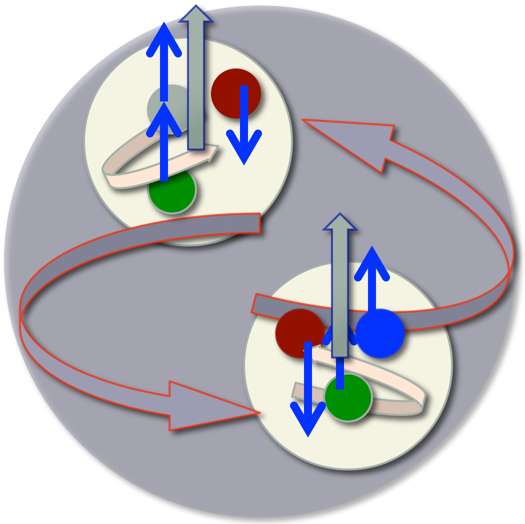
PARTON OAM: EXPERIMENTAL LEADS

3D STRUCTURE OF THE NUCLEON
AUGUST 29, 2017
INT UNIVERSITY OF WASHINGTON



Simonetta Liuti
University of Virginia



Based on

PHYSICAL REVIEW D **94**, 034041 (2016)

Parton transverse momentum and orbital angular momentum distributions

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The quark orbital angular momentum component of proton spin, L_q, can be defined in QCD as the integral of a Wigner phase space distribution weighting the cross product of the quark's transverse position and momentum. It can also be independently defined from the operator product expansion for the off-forward Compton amplitude in terms of a twist-three generalized parton distribution. We provide an explicit link between the two definitions, connecting them through their dependence on partonic intrinsic transverse momentum. Connecting the definitions provides the key for correlating direct experimental determinations of L_q and evaluations through lattice QCD calculations. The direct observation of quark orbital angular momentum does not require transverse spin polarization but can occur using longitudinally polarized targets.

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… and on

Lorentz Invariance and QCD Equation of Motion Relations for Generalized Parton Distributions and the Dynamical Origin of Proton Orbital Angular Momentum

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We derive new Lorentz Invariance and Equation of Motion Relations between twist-three Generalized Parton Distributions (GPDs) and moments in the parton transverse momentum, k_T, of twist-two Generalized Transverse Momentum-Dependent Distributions (GTMDs), as a function of the parton longitudinal momentum fraction x. Although GTMDs in principle define the observables for partonic orbital motion, experiments that can unambiguously detect them appear remote at present. The relations presented here provide a solution to this impasse in that, e.g., the orbital angular momentum density is connected to directly measurable twist-three GPDs. Out of 16 possible Equation of Motion relations that can be written in the T-even sector, we focus on three helicity configurations that can be detected analyzing specific spin asymmetries: two correspond to longitudinal proton polarization and are associated with quark orbital angular momentum and spin-orbit correlations; the third, obtained for transverse proton polarization, is a generalization of the relation obeyed by the g_2 structure function. We also exhibit an additional relation connecting the off-forward extension of the Sivers function to an off-forward Qiu-Sterman term.

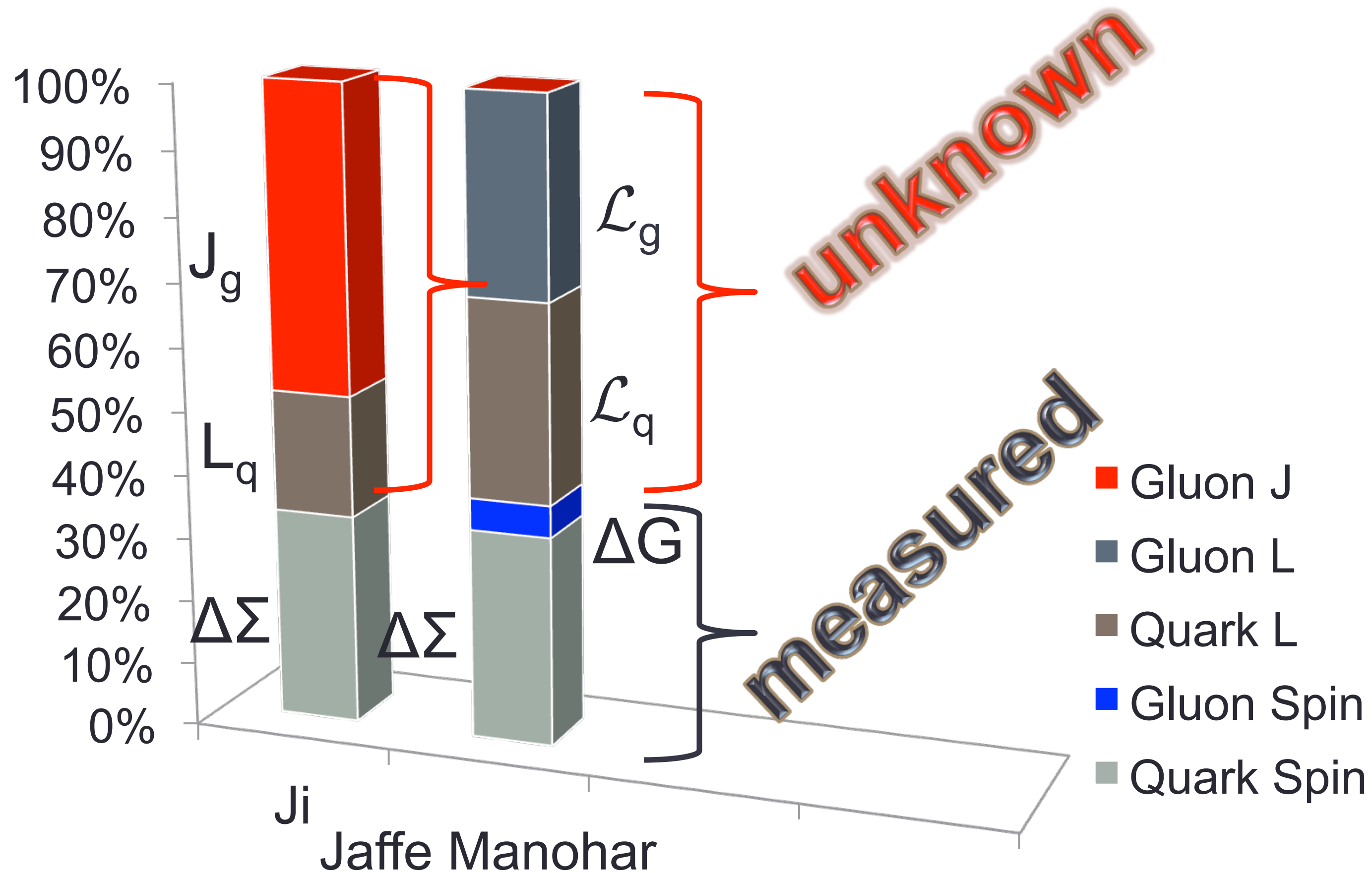
arXiv:1709.xxxx [hep-ph]

Outline

1. Introduction
2. Definitions
3. Lorentz Invariant Relations → twist three GPD $\tilde{E}_{2T}$
4. Equations of Motion Relations
5. A probe of QCD at the amplitude level: FSI and genuine tw 3
6. Nuclei
7. A glimpse on Gluon Angular Momentum
8. DVCS Analysis
9. Conclusions

1. INTRODUCTION

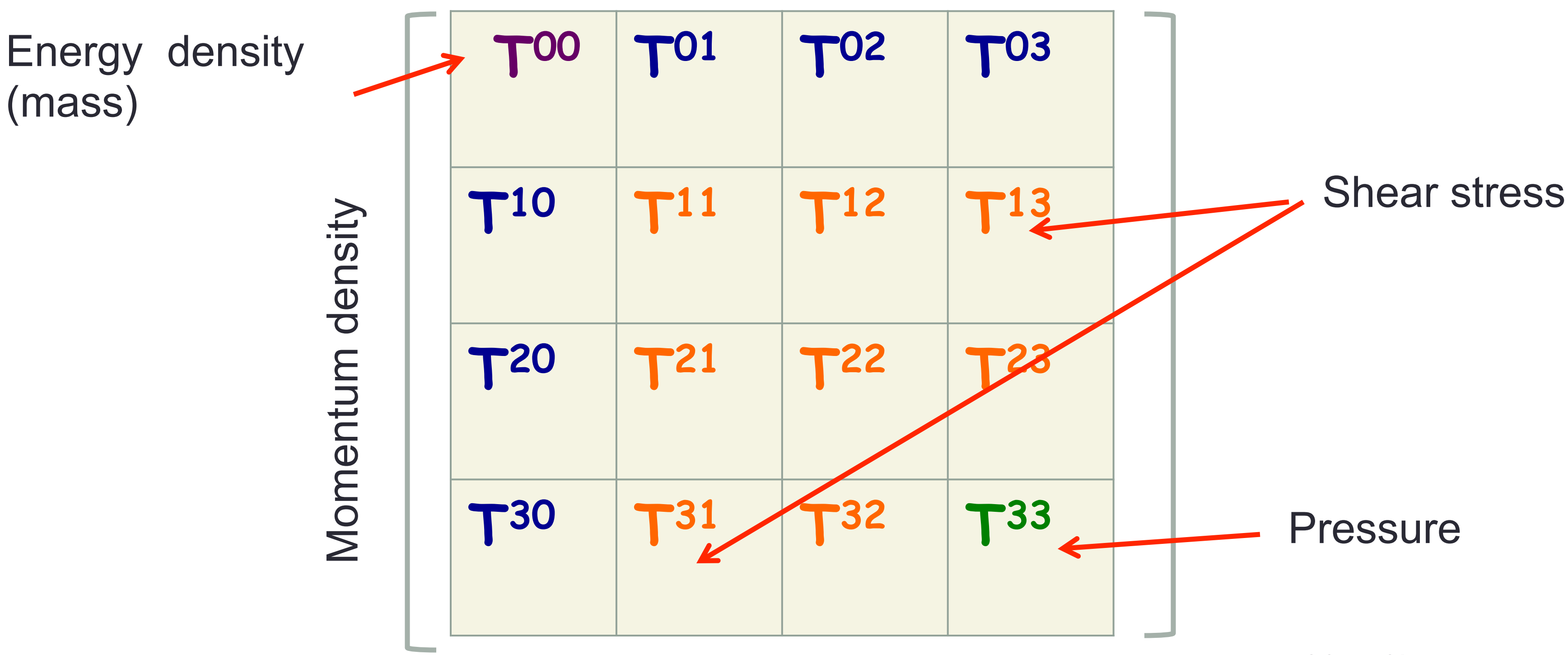
Angular Momentum Budget



The Starting Point

Jaffe and Manohar's field theoretical description of the quark and gluon orbital angular momentum through its relation to the QCD Energy Momentum Tensor,

$$T^{\mu\nu} \rightarrow M^{\mu\nu\lambda} = x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\nu}$$ **Angular Momentum density**



- Hadronic observables have been studied precisely only for T^{00}, T^{0i} (4-momentum and OAM)
- EMT is studied much more extensively in nuclei (RHIC)

In QCD

$$T^{\mu\nu} = \frac{1}{4} i q \bar{\psi}\left(\gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)\psi + Tr\left\{F^{\mu\alpha}F_{\alpha}^{\nu} - \frac{1}{2} g^{\mu\nu} F^{2}\right\}$$

Jaffe Manohar: *

$$M^{+12} = \psi^{\dagger}\sigma^{12}\psi + \psi^{\dagger}\left[\vec{x}\times(-i\partial)\right]^{3}\psi + Tr(\varepsilon^{+-ij}F^{+j}A^{j}) + 2iTrF^{+j}(\vec{x}\times\partial)A^{j}$$

$\Delta\Sigma$ $\mathcal{L}_q$ ΔG $\mathcal{L}_g$

Ji:

$$M^{+12} = \psi^{\dagger}\sigma^{12}\psi + \psi^{\dagger}\left[\vec{x}\times\left(-i\vec{D}\right)\right]^{3}\psi + \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^{3}$$

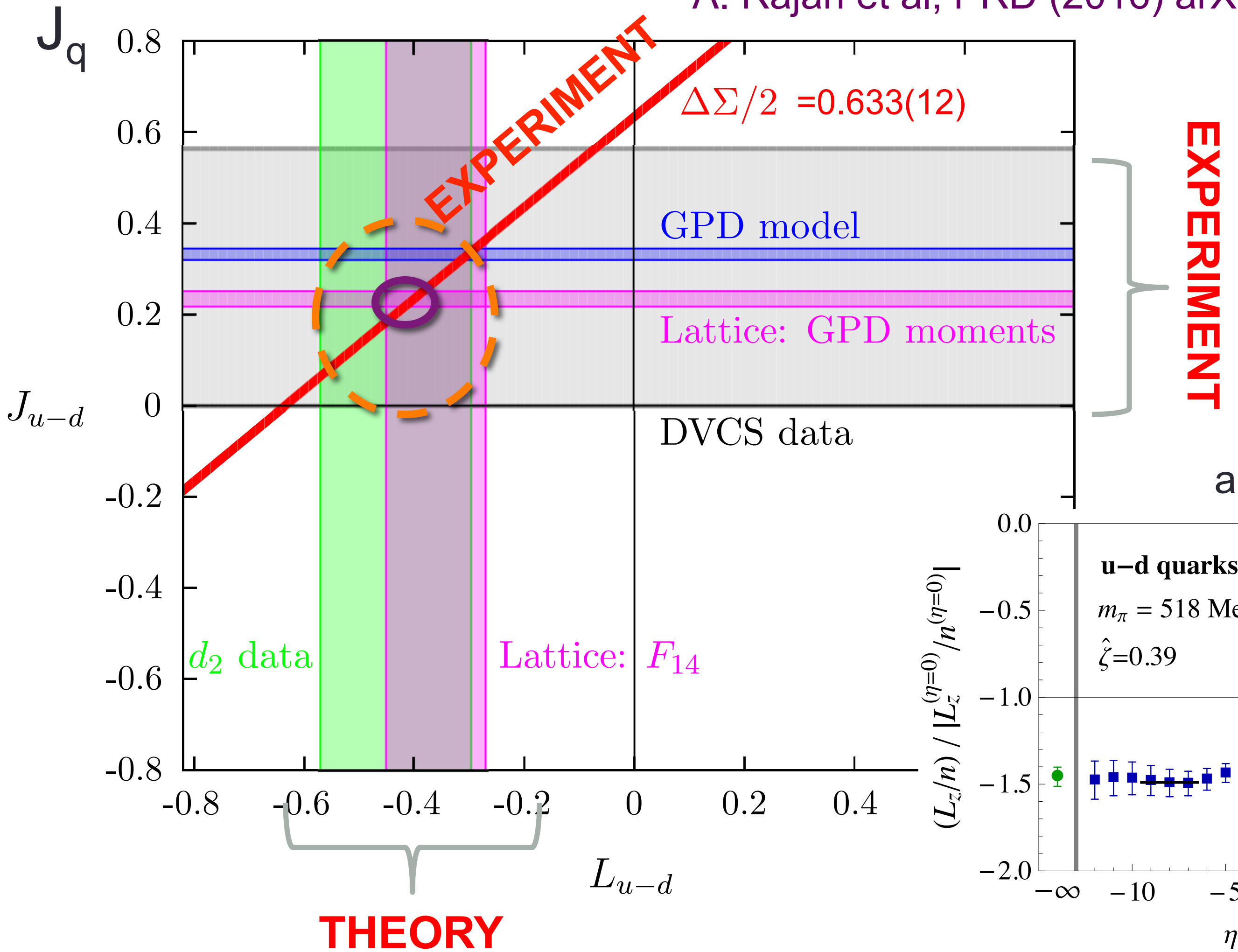
$\Delta\Sigma$ L_q J_g

J_q (= $\Delta\Sigma$ + L_q)

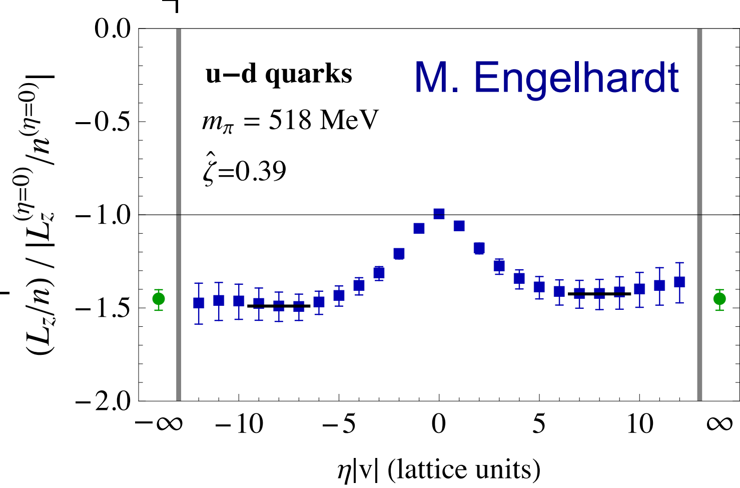
*Chen, Goldman et al., are consistent with this definition (see K.F.Liu et al.)

Quark sector : $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$

A. Rajan et al, PRD (2016) arXiv:1601.06117



arXiv:1701.01536



IS IT POSSIBLE TO TEST OAM IN EXPERIMENTS?

Proposed Observables for L_q

$$\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0) = \langle b_T \times k_T\rangle_3(x) \qquad L_q(x)$$

k_T moment of a GTMD (lacks “proof of observability”)
(Lorce and Pasquini)

$$\int_0^1 dx\, xG_2 \equiv \underbrace{\int_0^1 dx\, x(\tilde{E}_{2T}+H+E)}_{-L_q} = \underbrace{-\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}}_{-J_q+S_q}$$

x moment of twist 3 GPD
(Polyakov)

Are the two connected and how?

Yes, they are connected through a generalized LIR

$$\frac{1}{M}\int d^2k_T\, k_T^2\, F_{14}(x,0,k_T^2,0,0) = -\int_x^1 dy\, \underbrace{\left[\widetilde{E}_{2T} + H + E\right]}_{L_q(x)}$$

A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016), arXiv:1601.06117

Measuring twist three GPDs gives us the same information on OAM as measuring k_T integrals GTMDs, but….

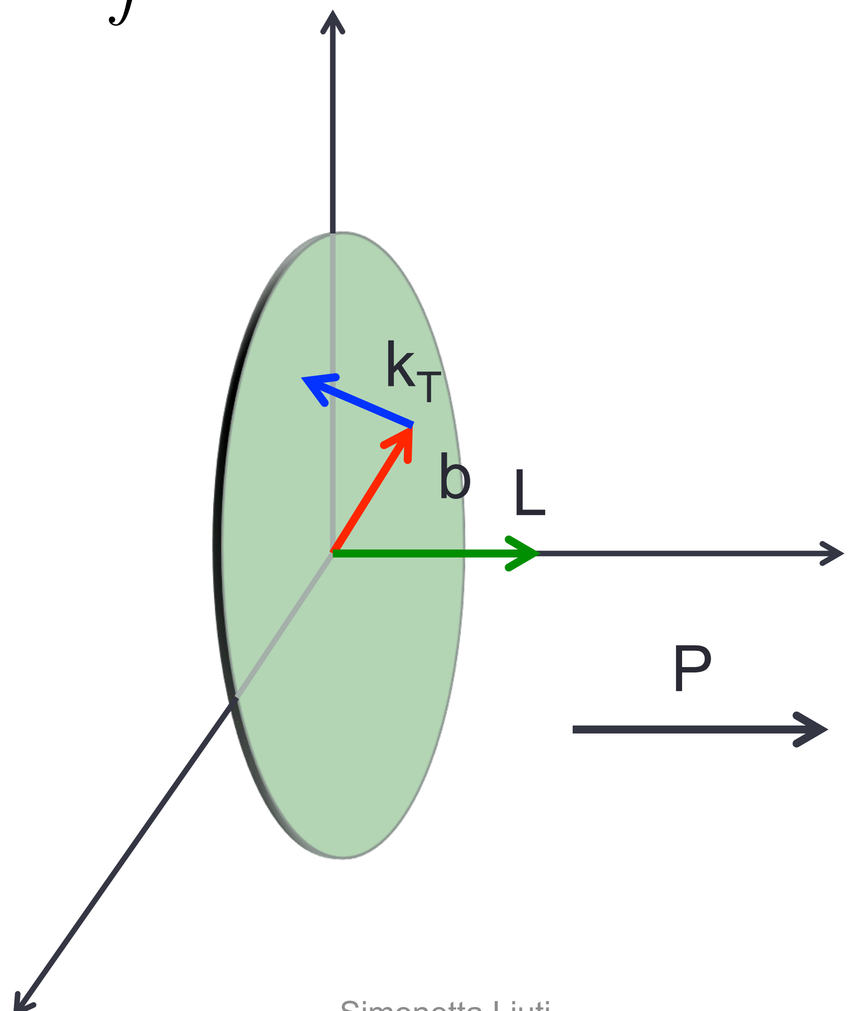
….we have referred so far only to Ji's OAM

2. DEFINITIONS

Partonic OAM: Wigner Distributions

$$L_q^{\mathcal{U}} = \int dx \int d^2\mathbf{k}_T \int d^2\mathbf{b}\,(\mathbf{b}\times\mathbf{k}_T)_z\,\mathcal{W}^{\mathcal{U}}(x,\mathbf{k}_T,\mathbf{b})$$

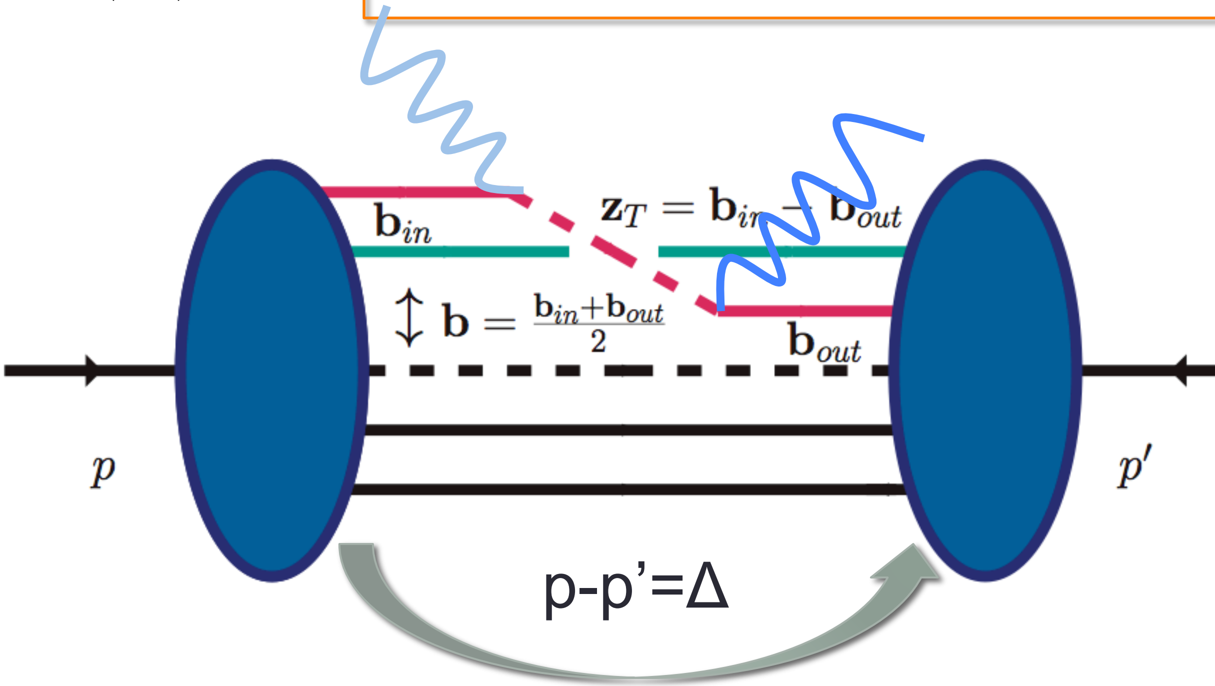
Hatta
Lorce, Pasquini,
Xiong, Yuan
Mukherjee



Wigner Distribution

$$\mathcal{W}^{\mathcal{U}} = \frac{1}{2}\int \frac{d^2\mathbf{\Delta}_T}{(2\pi)^2} e^{i\Delta_T\cdot b} \int dz^- d^2\mathbf{z}_T\, e^{ikz} \langle P-\Delta, \Lambda' \mid \bar{q}(0)\gamma^+\mathcal{U}(0,z)q(z) \mid P, \Lambda\rangle \,|_{z^+=0}$$

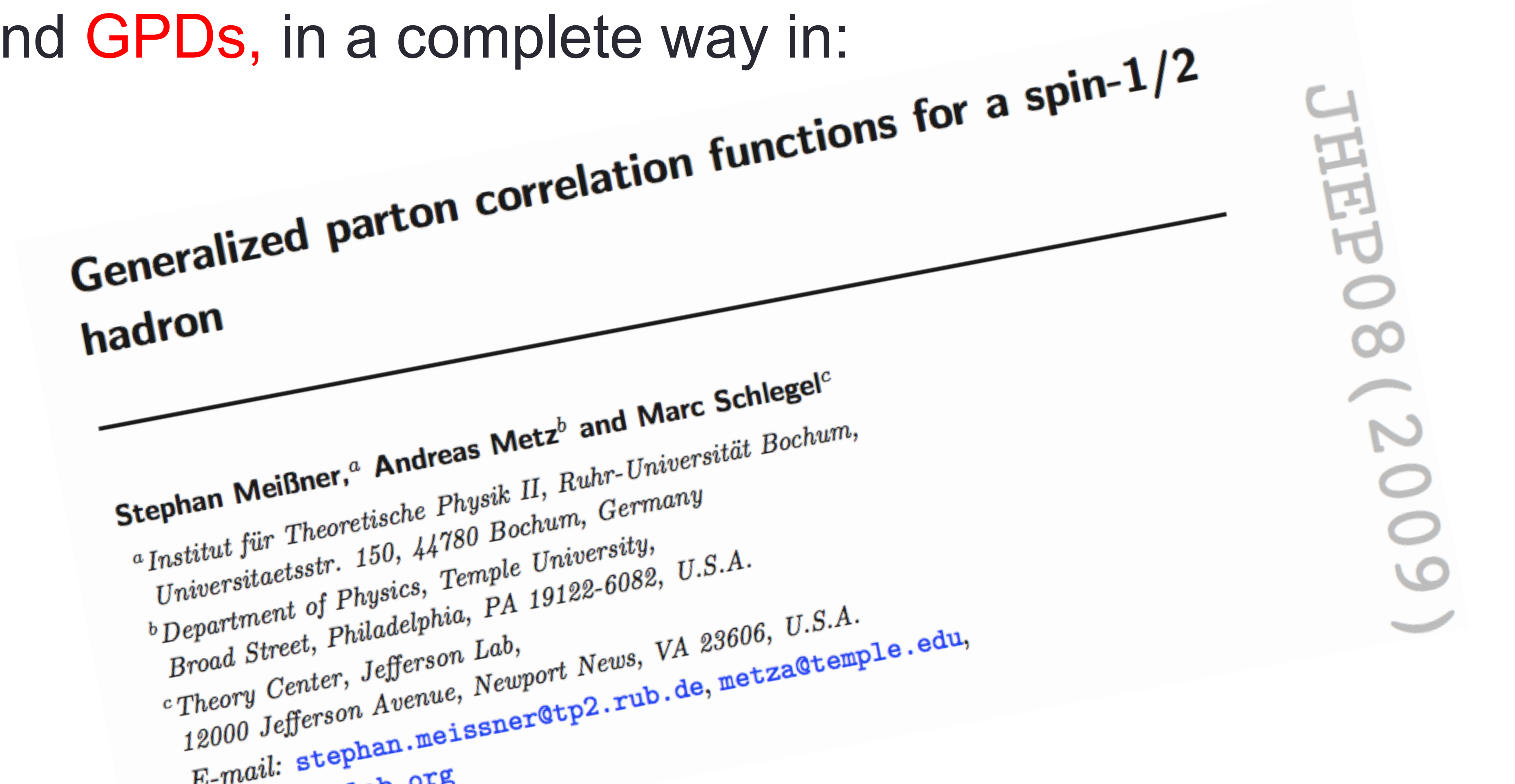
GTMD



- Δ_T Fourier conjugate: **b** = transverse position of the quark inside the proton
- k_T Fourier conjugate: $\mathbf{z}_T$= transverse distance traveled by the struck quark between the initial and final scattering

Which GTMD?

The quark-quark correlator for a spin ½ hadron has been parametrized up to twist four in terms of GTMDs, TMDs and GPDs, in a complete way in:



Generalized parton correlation functions for a spin-1/2 hadron

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JHEP08(2009)

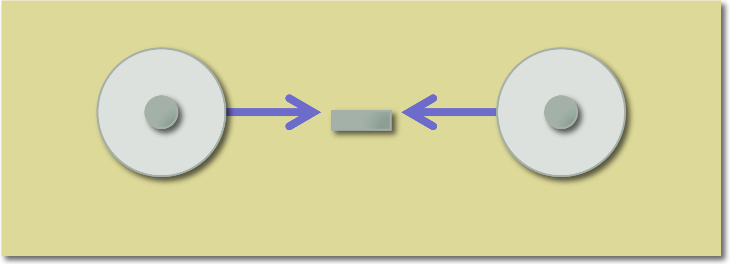
Helicity Structure

F_{14}

$$W_{\Lambda\Lambda'}^{\gamma^+} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\gamma^+ F_{11} + \frac{i\sigma^{i+}\Delta_T^i}{2M}(2F_{13}-F_{11}) + \frac{i\sigma^{i+}\bar{k}_T^i}{2M}(2F_{12}) + \frac{i\sigma^{ij}\bar{k}_T^i\Delta_T^j}{M^2}F_{14}\right]U(p,\Lambda)$$

$$= \delta_{\Lambda,\Lambda'}F_{11} + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\Delta_1 - i\Delta_2}{2M}(2F_{13}-F_{11}) + \delta_{\Lambda,-\Lambda'}\frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M}(2F_{12}) + \delta_{\Lambda,\Lambda'}i\Lambda\frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2}F_{14}$$

helicity non-flip



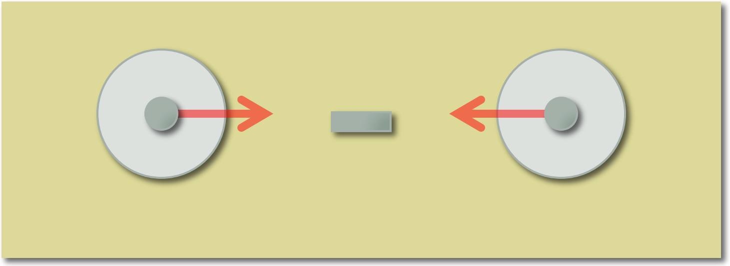
UL correlation: unpolarized quark density in a longitudinally polarized proton

G_{11} $\quad (L\cdot S) \rightarrow \frac{1}{2}\int d^2\mathbf{b}\,(\mathbf{b}\times\mathbf{k}_T)_z\,\langle \bar{q}(0)\gamma^+\gamma_5 q(z)\rangle$

$$W^{\gamma^+\gamma_5}_{\Lambda\Lambda'} = \frac{1}{2M}\overline{U}(p',\Lambda')\left[-\frac{i\epsilon^{ij}_T k^i_T \Delta^j_T}{M^2}G_{11} + \frac{i\sigma^{i+}\gamma^5 k^i_T}{P^+}G_{12} + \frac{i\sigma^{i+}\gamma^5\Delta^i_T}{P^+}G_{13} + i\sigma^{+-}\gamma^5 G_{14}\right]U(p,\Lambda)$$

$$= \left[-\frac{i(k_1\Delta_2 - k_2\Delta_1)}{M^2}G_{11} + \Lambda G_{14}\right]\delta_{\Lambda\Lambda'} + \left[\frac{\Delta_1 + i\Lambda\Delta_2}{M}\left(G_{13} + \frac{i\Lambda(k_1\Delta_2 - k_2\Delta_1)}{2M^2}G_{11}\right) + \frac{k_1 + i\Lambda k_2}{M}G_{12}\right]\delta_{-\Lambda,\Lambda'}$$

helicity non-flip



UL correlation: longitudinally polarized quark density in an unpolarized proton

Integral relations

$$L_q = -\int_0^1 dx \int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14} = -\int_0^1 dx \, F_{14}^{(1)}$$

$$L_q \cdot S_q = -\int_0^1 dx \int d^2k_T \, \frac{k_T^2}{M^2} \, G_{11} = -\int_0^1 dx \, G_{11}^{(1)}$$

Lorce, Pasquini, Xiong, Yuan
Hatta, Yoshida
Ji, Xiong, Yuan

3. LIR

Generalized Lorentz Invariance Relations (LIR)

- LIR are relations between twist-3 PDFs and k_T moments of TMDs (Metz, Pitoniak, Schlegel, Mulders, Goeke, …)
- LIR in the off-forward sector: relations between twist-3 GPDs (→PDFs) and k_T moments of GTMDs (→TMDs)
- Based on the most general Lorentz invariant decomposition of the fully unintegrated quark-quark correlator
- LIRs are a consequence of there being a smaller number of independent unintegrated terms in the decomposition than the number of GTMDs

The completely unintegrated off-forward correlator

$$W^{\Gamma}_{\Lambda'\Lambda}(k,\Delta;\mathcal{U}) = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4} e^{ikz} \langle p',\Lambda' \mid \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{U}\left(-\frac{z}{2},\frac{z}{2}\right) \psi\left(\frac{z}{2}\right) \mid p,\Lambda\rangle$$

➔ parametrized in terms of invariant functions A_1, A_2, …

The unintegrated (over k_T) off-forward correlator

$$\begin{aligned} W^{\Gamma}_{\Lambda'\Lambda}(x,k_T,\xi,\Delta;\mathcal{U}) &= \int dk^- W^{\Gamma}_{\Lambda'\Lambda}(P,k,\Delta;\mathcal{U}) \\ &= \frac{1}{2}\int \frac{dz^-\, d^2\mathbf{z}_T}{(2\pi)^3} e^{ixP^+z^- - i\mathbf{k}_T\cdot\mathbf{z}_T} \langle p',\Lambda' \mid \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{U}_{-z/2,z/2} \psi\left(\frac{z}{2}\right) \mid p,\Lambda\rangle \Big|_{z^+=0} \end{aligned}$$

➔ parametrized in terms of GTMDs: F_{11}, F_{12}, …, F_{21}, F_{22}…

$$
\begin{aligned}
F_{11} &= 2P^+ \int dk^- \left[A_1^F + x A_2^F - \frac{x\Delta_T^2}{2M^2}(A_8^F + x A_9^F) \right] \\
F_{12} &= 2P^+ \int dk^- \left[A_5^F \right] \\
F_{13} &= 2P^+ \int dk^- \left[A_6^F + \frac{P \cdot k - xP^2}{M^2}(A_8^F + x A_9^F) \right] \\
F_{14} &= 2P^+ \int dk^- \left[A_8^F + x A_9^F \right]
\end{aligned}
$$

$$
\frac{d}{dx} F_{14}^{(1)} = \frac{4P^+}{M^2} \int d^2k_T \int dk^- \left[(k \cdot P - xP^2)(A_8^F + x A_9^F) + \frac{k_T^2 \Delta_T^2 - (k_T \cdot \Delta_T)^2}{\Delta_T^2} A_9^F \right]
$$

For any combination of A amplitudes one has:

$$\frac{d}{dx}\int d^2k_T \int dk^- \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}\,\mathcal{X}[A;x] = \int d^2k_T \int dk^-(k\cdot P - xP^2)\mathcal{X}[A;x] + \int d^2k_T \int dk^- \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}\frac{\partial \mathcal{X}}{\partial x}[A;x]$$

k_T moment of GTMD

Extension of Mulders Tangerman relation to off-forward configuration

Now look for GPDs combinations that give the *rhs*

$$H + E = 2P^+ \int d^2k_T \int dk^- \, 2 \left(\frac{k_T \cdot \Delta_T}{\Delta_T^2} A_5^F + A_6^F + \frac{P \cdot k - xP^2}{M^2} (A_8^F + x A_9^F) \right)$$

$$\widetilde{E}_{2T} = 2P^+ \int d^2k_T \int dk^- \, (-2) \left(\frac{k_T \cdot \Delta_T}{\Delta_T^2} A_5^F + A_6^F + \frac{(k_T \cdot \Delta_T)^2 - k_T^2 \Delta_T^2}{M^2 \Delta_T^2} A_9^F \right)$$

OAM distribution emerges from LIR

$L_q(x)$ → density $\qquad$ L_q → integrated

$$F_{14}^{(1)} = -\int_x^1 dy\,(\tilde{E}_{2T} + H + E) \;\Rightarrow\; -L_q = \int_0^1 dx F_{14}^{(1)} = \int_0^1 dx x G_2$$

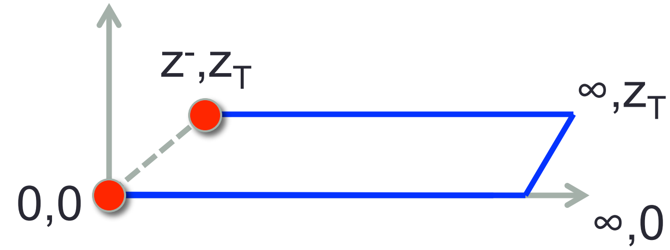
- $F^{(1)}_{14}$ and $\tilde{E}_{2T}$ give us the same information on the distribution in x of OAM!
- "In addition": we confirm and corroborate the global/integrated OAM result deducible from Ji et al

Different notation!

$$G_2 \to \tilde{E}_{2T} + H + E$$

Polyakov et al. $\qquad$ Meissner, Metz and Schlegel, JHEP(2009)

Generalized LIR for a staple link



$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \tilde{E}_{2T} + H + E + \mathcal{A}$$

LIR violating term

$$\mathcal{A}_{F_{14}} = v^- \frac{(2P^+)^2}{M^2}\int d^2k_T \int dk^- \left[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11}^F + xA_{12}^F) + A_{14}^F + \frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8^F}{\partial(k\cdot v)} + x\frac{\partial A_9^F}{\partial(k\cdot v)}\right)\right]$$

4. EQUATIONS OF MOTION

Equations of Motion (EoM) relation

EoM in correlator for $\Gamma = i\,\sigma^{+i}\,\gamma_5$

$$\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T\cdot z_T} \langle p', \Lambda' \mid \overline{\psi}(-z/2)(\Gamma\,\mathcal{U}\, i\overrightarrow{\not{D}} + i\overleftarrow{\not{D}}\,\Gamma\,\mathcal{U})\psi(z/2) \mid p, \Lambda\rangle_{z^+=0} = 0$$

symmetrized with respect to the argument

$$-\frac{\Delta^+}{2} W_{\Lambda'\Lambda}^{[\gamma^i \gamma^5]} + ik^+ \epsilon^{ij} W_{\Lambda'\Lambda}^{[\gamma^j]} + \frac{\Delta^i}{2} W_{\Lambda'\Lambda}^{[\gamma^+ \gamma^5]} - i\epsilon^{ij} k_T^j W_{\Lambda'\Lambda}^{[\gamma^+]} + \mathcal{M}_{\Lambda'\Lambda}^{i,S} = 0$$

GTMDs in helicity combination for OAM: (++) - (- -)

$$-2x\left(\frac{k_T\cdot\Delta_T}{\Delta_T^2} F_{27} + F_{28}\right) + G_{14} - 2\frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{M^2\Delta_T^2} F_{14} + \frac{\Delta^i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,S} - \mathcal{M}_{--}^{i,S}\right) = 0$$

Integrate over k_T

$$x\widetilde{E}_{2T}(x) = -\widetilde{H}(x) + F_{14}^{(1)}(x) - \mathcal{M}_{F_{14}}$$

Insert LIR

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \tilde{E}_{2T} + H + E + \mathcal{A}$$

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

Angular Momentum Sum Rule

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

L = J - S + 0

Integrating over x we re-obtain the OPE based relation

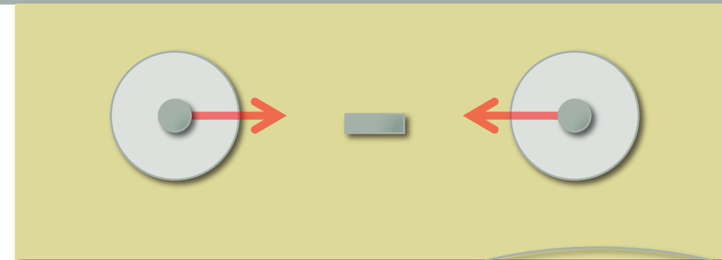
Polyakov et al.(2000), Hatta(2012)

$$\int_0^1 dx\, xG_2 = -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H}$$

$$J_q = L_q + \frac{1}{2}\Delta\Sigma_q$$

A generalized Wandzura Wilczek relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

Other integrated relations



$$\int dx\, x \left(E'_{2T} + 2\tilde{H}'_{2T}\right) = -\frac{1}{2}\int dx\, x\tilde{H} - \frac{1}{2}\int dx H + \frac{m}{2M}\int dx (E_T + 2\tilde{H}_T)$$

$$(L_z S_z)_q = \int dx\, x \left(E'_{2T} + 2\tilde{H}'_{2T} + \tilde{H}\right), \quad \kappa_T = \int dx\, (E_T + 2\tilde{H}_T), \quad e_q = \int dx\, H$$

$$\frac{1}{2}\int dx\, x\tilde{H} = (L_z S_z)_q + \frac{1}{2} e_q - \frac{m_q}{2M}\kappa_T^q$$

- Integral relation without connecting to spin-orbit Polyakov et al. (2000)
- Integral relation with “educated guess” for spin-orbit Lorce (2015)

Chiral symmetry breaking test!

Transverse proton spin (unpolarized quark)

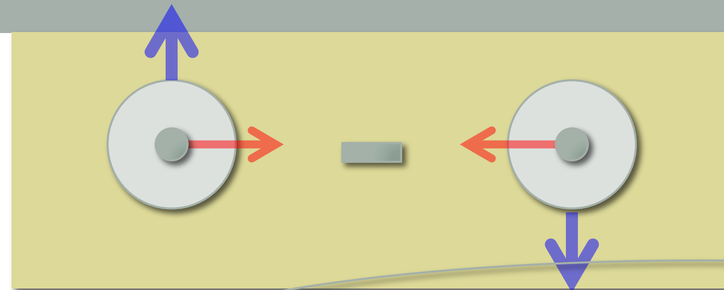
$$-x\left(F_{23}+\frac{k_T^2\Delta_T^2-(k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{24}\right)+\quad\frac{1}{2M^2}\left(\Delta_T^2G_{13}+k_T\cdot\Delta_TG_{12}\right)+\frac{k_T^2\Delta_T^2-(k_T\cdot\Delta_T)^2}{M^2\Delta_T^2}F_{12}$$

$$+\quad\frac{\Delta_T^i}{2M\Delta_T^2}\left((\Delta_1-i\Delta_2)\mathcal{M}^{i,S}_{-+}+(\Delta_1+i\Delta_2)\mathcal{M}^{i,S}_{+-}\right)=0.$$

$$f_{1T}^{\perp(1)}=-F_{12}^{o(1)}=\mathcal{M}_{F_{12}}|_{\Delta_T=0}$$

Sivers function

Qiu-Sterman term



$$H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} = \frac{P^2}{M^2}\int_x^1 \frac{dy}{y}\tilde{H} + \frac{m}{M}\left[\frac{1}{x}\left(H_T - \frac{\Delta_T^2}{4M^2}E_T\right) - \int_x^1 \frac{dy}{y^2}\left(H_T - \frac{\Delta_T^2}{4M^2}E_T\right)\right]$$

$$+\frac{\Delta_T^2}{4M^2}\left[\frac{1}{x}(H+E) - \int_x^1 \frac{dy}{y^2}(H+E)\right] + \left[\frac{\mathcal{M}_{G_{12}}}{x} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{G_{12}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{G_1}$$

$$g_2 = -\left(g_1 - \int_x^1 \frac{dy}{y} g_1\right) + \frac{m}{M}\left(\frac{1}{x}h_1 - \int_x^1 \frac{dy}{y^2}h_1\right) + \left(\widetilde{g}_T - \int_x^1 \frac{dy}{y}\widetilde{g}_T\right) + \int_x^1 \frac{dy}{y}\widehat{g}_T$$

Original Wandzura Wilczek relation in forward limit

Chiral symmetry breaking test!

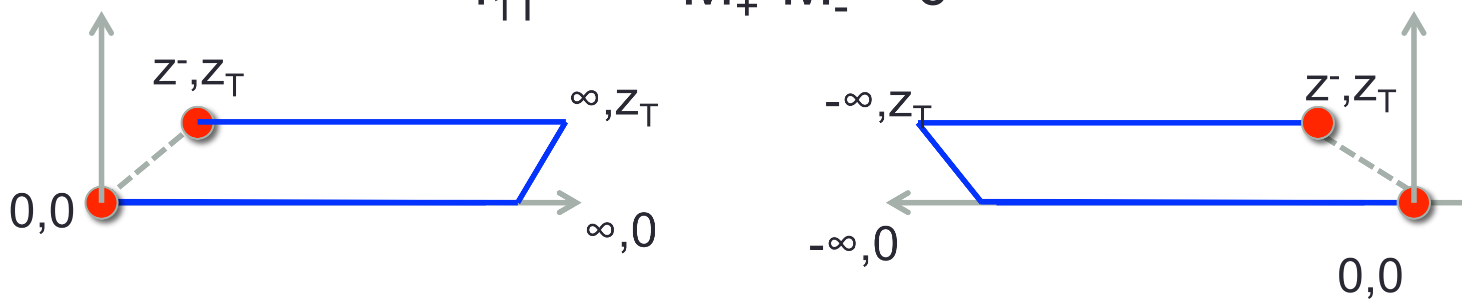
5. A PROBE OF QCD AT THE AMPLITUDE LEVEL

PT transformation

Forward case: Sivers function (J. Collins, 2002)

PT: $$\langle P, S \mid \bar{\psi}(0)\gamma^{+}\psi(z) \mid P, S\rangle = \langle P, -S \mid \bar{\psi}(0)\gamma^{+}\psi(z) \mid P, -S\rangle$$

M_+ $\quad f_{1T}^{perp} = M_+ - M_- = 0$ $\quad M_-$



PT:

$$\langle P, S \mid \bar{\psi}(0)\gamma^{+}U(v,z)\psi(z) \mid P, S\rangle = \langle P, -S \mid \bar{\psi}(0)\gamma^{+}U(-v,z)\psi(z) \mid P, -S\rangle$$

$M_+^{v} - M_-^{-v} = 0$ → $f_{1T}^{perp,SIDIS} = M_+^{v} - M_-^{v} = -f_{1T}^{perp,DY} = -M_+^{-v} + M_-^{-v}$

Off forward case: F_{14}

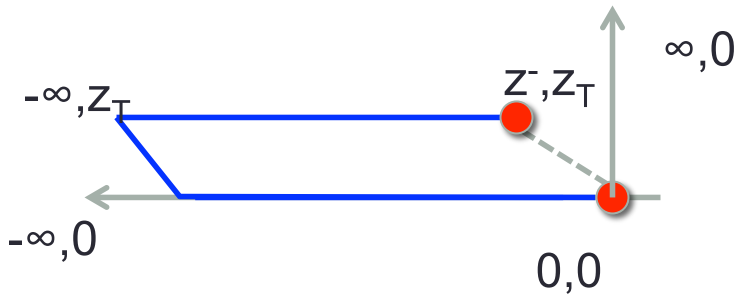
PT:

$$\langle P-\Delta, S \mid \bar{\psi}(0)\gamma^+ U(v,z)\psi(z) \mid P, S\rangle = \langle P, -S \mid \bar{\psi}(0)\gamma^+ U(-v,z)\psi(z) \mid P-\Delta, -S\rangle$$

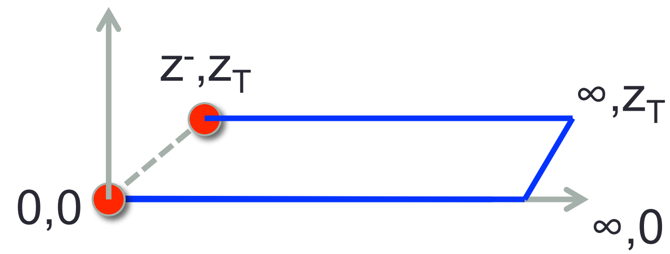
$$\underbrace{}_{L_+^{v,\Delta}} \qquad \underbrace{}_{L_-^{-v,-\Delta}}$$

$L_+^{v,\Delta} - L_-^{-v,-\Delta} = 0$

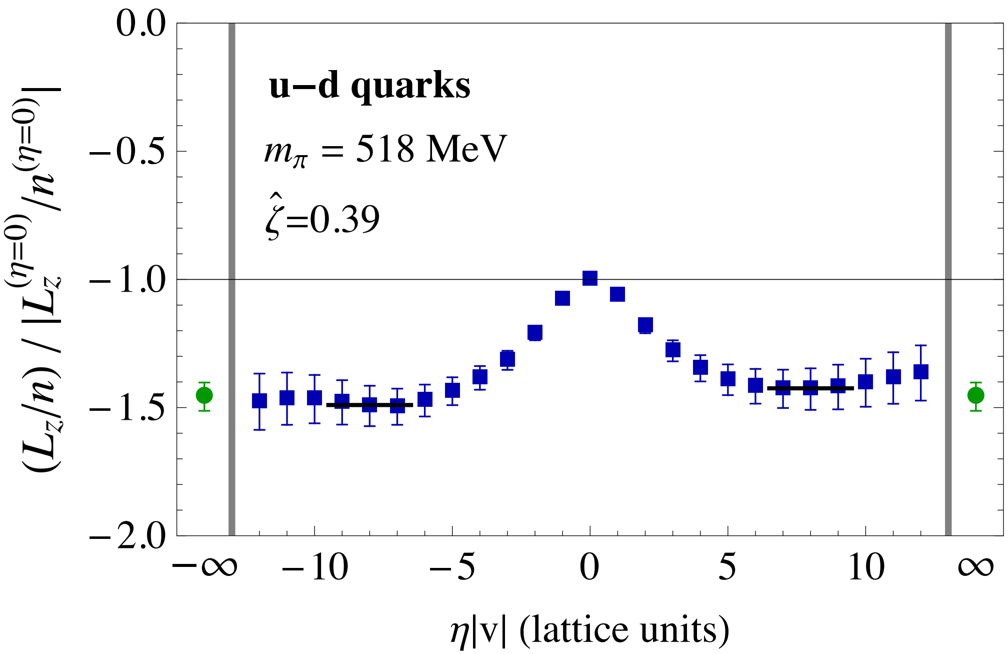
$(k_T x \Delta_T)\, F_{14}^{\text{"SIDIS"}} = L_+^{v,\Delta} - L_-^{v,\Delta} = (k_T x \Delta_T)\, F_{14}^{\text{"DY"}} = L_+^{-v,\Delta} - L_-^{-v,\Delta}$



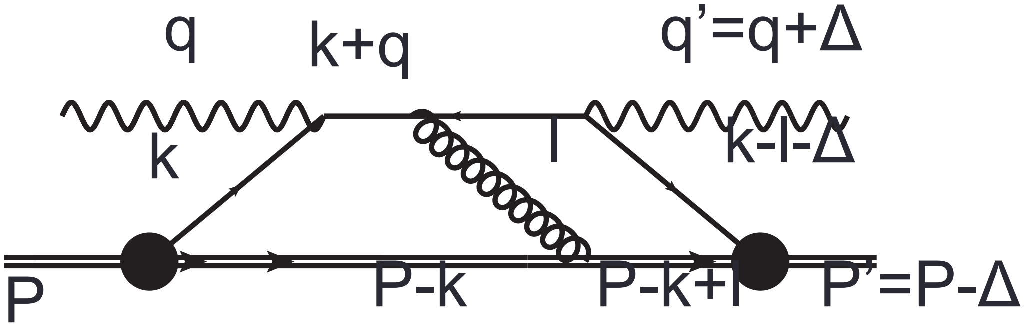
-∞,z_T
z⁻,z_T
∞,0
-∞,0
0,0



z⁻,z_T
∞,z_T
0,0
∞,0

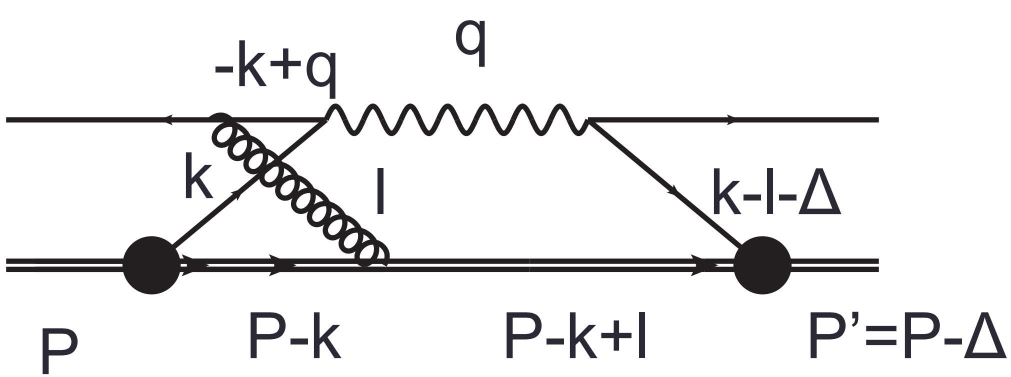


u−d quarks
m_π = 518 MeV
$\hat{\zeta}$=0.39
$(L_z/n) / |L_z^{(\eta=0)}/n^{(\eta=0)}|$
η|v| (lattice units)
0.0
−0.5
−1.0
−1.5
−2.0
−∞
−10
−5
0
5
10
∞



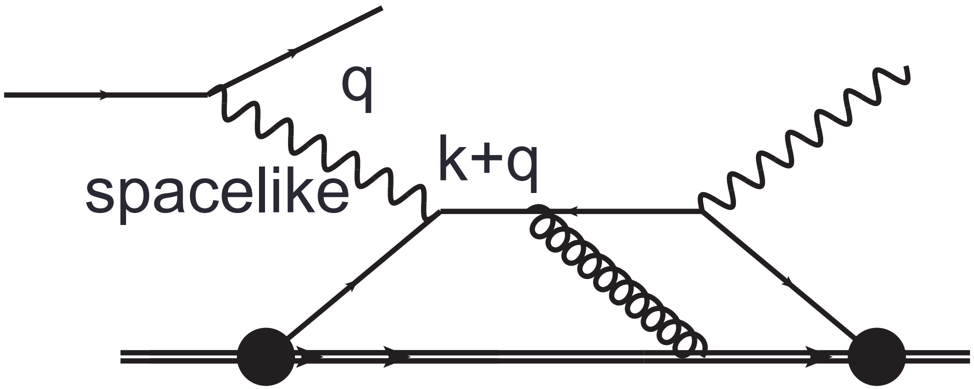
"SIDIS-like"

$$F_{1,4} = \int \frac{d^2 l}{(2\pi)^2} \frac{e_c^2 g_s^2 M^2 2P^+ (1-x)^2 \left(1 + \frac{l_T}{k_T}\cos\phi_l\right)}{2x(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2((k-\Delta)^2 - M_\Lambda^2)^2}$$

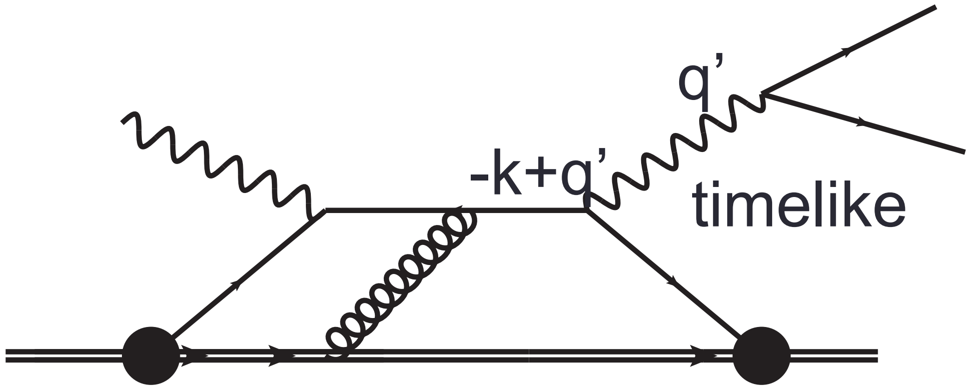


"DrellYan-like"

Two additional processes: DVCS and TCS twist three contributions



DVCS



TCS

Extracting twist 3 GPDs from these processes will allow us to zoom into aspects of the "sign change"

Genuine/intrinsic twist-three terms

$$\mathcal{M}^i_{\Lambda\Lambda'} = \frac{1}{4}\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T\cdot z_T}$$

$$\langle p', \Lambda' \mid \overline{\psi}(-z/2) \left[(\overrightarrow{\partial\!\!\!/} - ig A\!\!\!/)\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial\!\!\!/} + ig A\!\!\!/)\Big|_{z/2} \right] \psi(z/2) \mid p, \Lambda \rangle_{z^+=0}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{staple}} + \mathcal{M}_{F_{14}}^{\text{staple}}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{straight}} + \mathcal{M}_{F_{14}}^{\text{straight}}$$

By subtracting the two expressions

$$F_{14}^{(1)}\Big|_{\text{staple}} - F_{14}^{(1)}\Big|_{\text{staple}} = \mathcal{M}_{F_{14}}|_{\text{staple}} - \mathcal{M}_{F_{14}}|_{\text{straight}}$$

integrating ⬇

$$-\int dx\ F_{14}^{(1)}\Big|_{\text{diff}}\Big|_{\Delta_T=0} = -\frac{\partial}{\partial\Delta_i} i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\,\langle p',+|\bar{\psi}(0)\gamma^+ U(0,sv)F^{+j}(sv)U(sv,0)\psi(0)|p,+\rangle\Big|_{\Delta_T=0}$$

Difference between Jaffe-Manohar and Ji
(Hatta, Burkardt, 2013)

$$\mathcal{A} = \frac{d}{dx}\left(\mathcal{M}^{\text{staple}} - \mathcal{M}^{\text{straight}}\right)$$

LIR violating term is the difference between JM and Ji

Generalized Qiu Sterman term

$$\int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14}^{JM} - \int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14}^{Ji} = T_F(x, x, \Delta)$$

Use the connection with twist-three GPD!

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

$$\tilde{E}_{2T} = \tilde{E}_{2T}^{WW} + \tilde{E}_{2T}^{(3)} + \tilde{E}_{2T}^{LIR}$$

$$\tilde{E}_{2T}^{WW} = -\int_x^1 \frac{dy}{y}(H+E) - \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right]$$

$$-\left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

Can we measure this and how?

Direct measurements cannot distinguish between LIR-generated and EoM-generated genuine twist-three/qgq terms so in principle we cannot measure the difference between Jaffe-Manohar and Ji through DVCS… but….

Caveat

"Assuming that twist-three GPDs are process independent, or that they behave like twist-two collinear objects"

x-Moments

M_0 $$\int dx \tilde{E}_{2T} = -\int dx (H+E) \quad \Rightarrow \int dx \left(\tilde{E}_{2T} + H + E\right) = 0$$

M_1 OAM Sum Rule $$\int dx x \tilde{E}_{2T} = -\frac{1}{2}\int dx x (H+E) - \frac{1}{2}\int dx \tilde{H}$$

M_2 $$\int dx x^2 \tilde{E}_{2T} = -\frac{1}{3}\int dx x^2 (H+E) - \frac{2}{3}\int dx x \tilde{H} - \frac{2}{3}\int dx x \mathcal{M}_{F_{14}}$$

Interpretation

M1

Force acting on quark

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \quad i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0,sv) F^{+j}(sv) U(sv,0)\psi(0)|p,\Lambda\rangle$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \quad -g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 U(0,sv) F^{+i}(sv) U(sv,0)\psi(0)|p,\Lambda\rangle$$

Non zero only for staple link

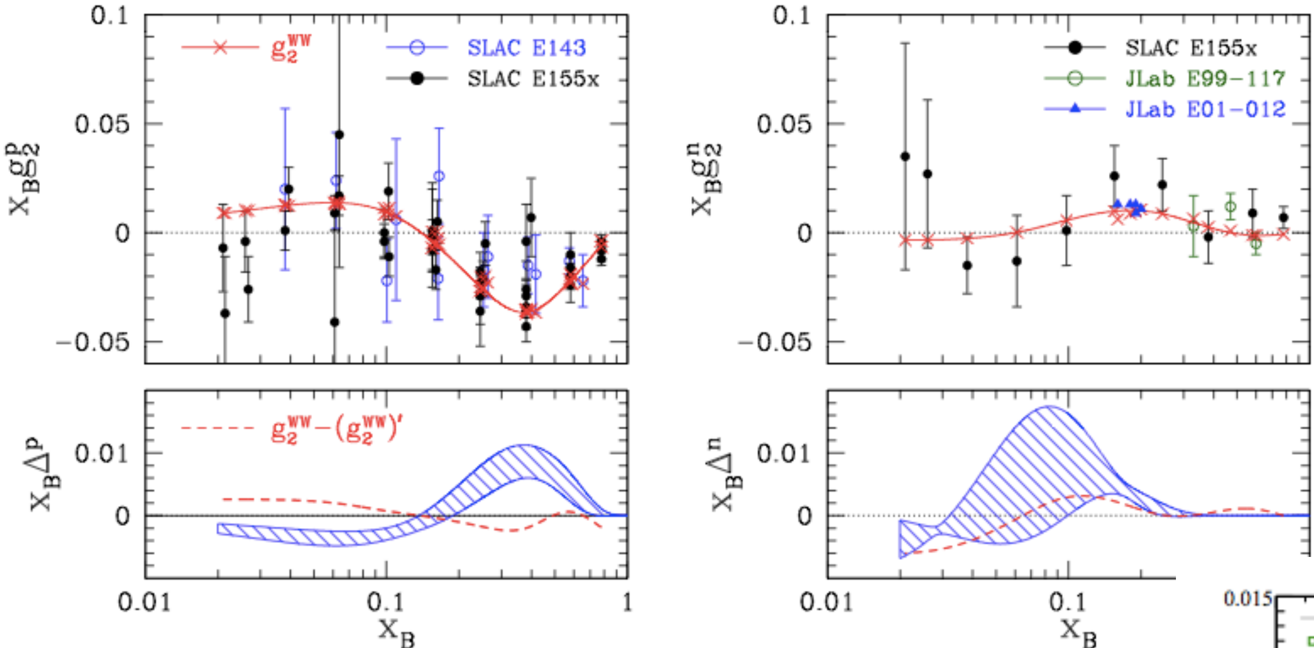
M2

$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 F^{+i}(0)\psi(0)|p,\Lambda\rangle$$

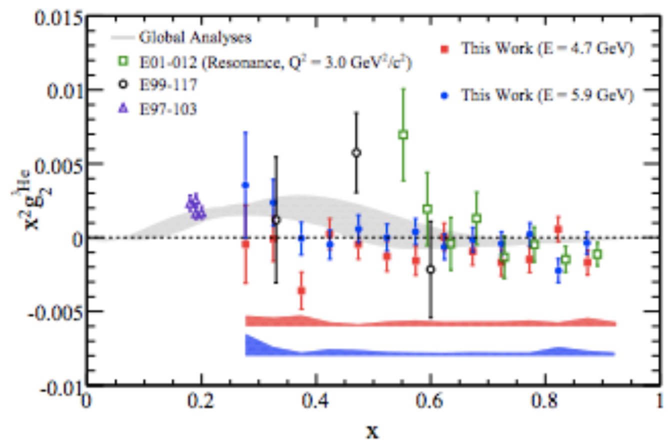
$$\int dx\, x \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2} \epsilon^{ij} \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0)|p,\Lambda\rangle$$

“rest frame” interaction ≠ d_2

Situation is analogous to studies of g_2



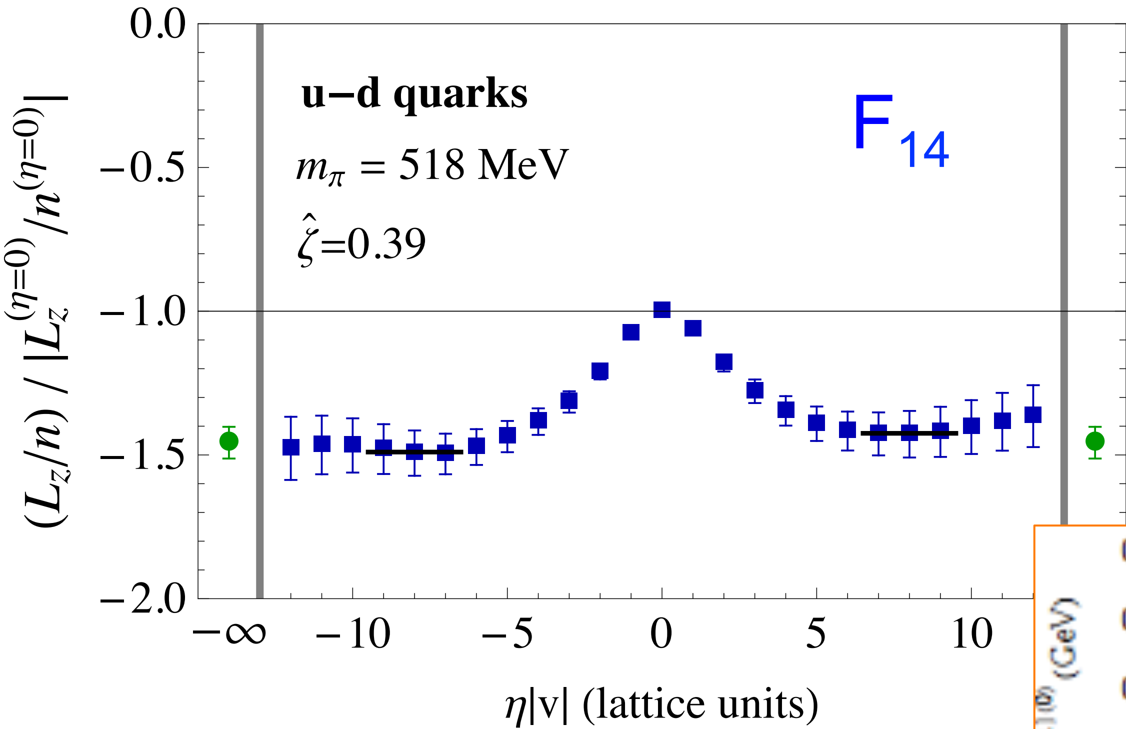
Accardi, Bacchetta, Melnitchouk, Schlegel JHEP (2009)



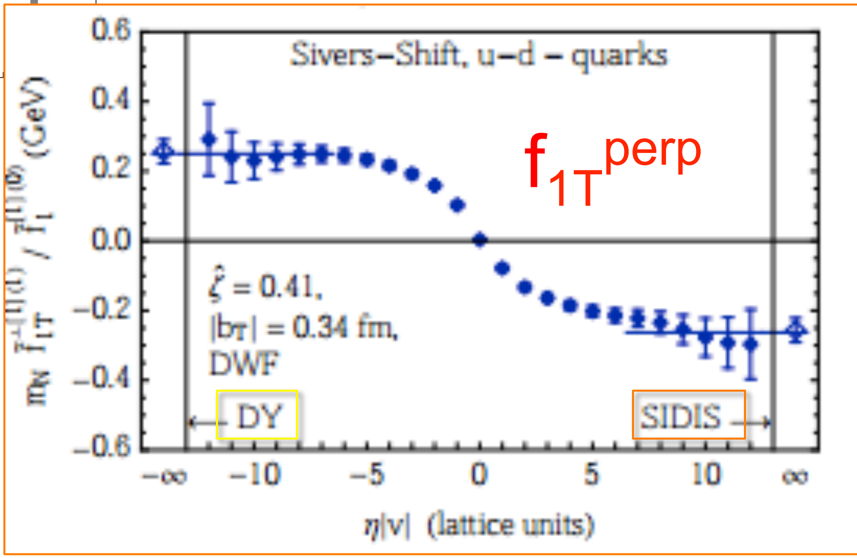
D. Flay et al, PRC 2016

The effect of the two twist-three terms combined might be small but each individual contribution can be large

Large effect from lattice (M. Engelhardt, arXiv:1701.01536)



PRD, arXiv:1111.4249



Direct evaluation of quark orbital angular momentum

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T \, (r_T \times k_T)_3 \, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T)$$

Wigner distribution

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}$$

n : Number of valence quarks

$p' = P + \Delta_T/2$, $p = P - \Delta_T/2$, P, S in 3-direction, $P \to \infty$

This is the same type of operator as used in TMD studies – generalization to off-forward matrix element adds transverse position information

Direct evaluation of quark orbital angular momentum

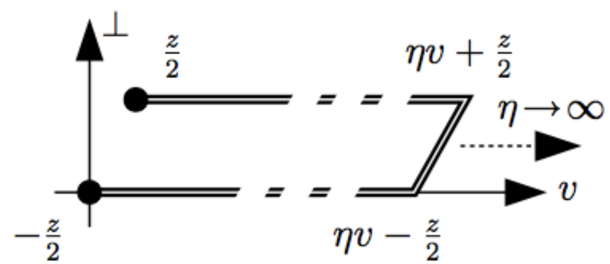
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

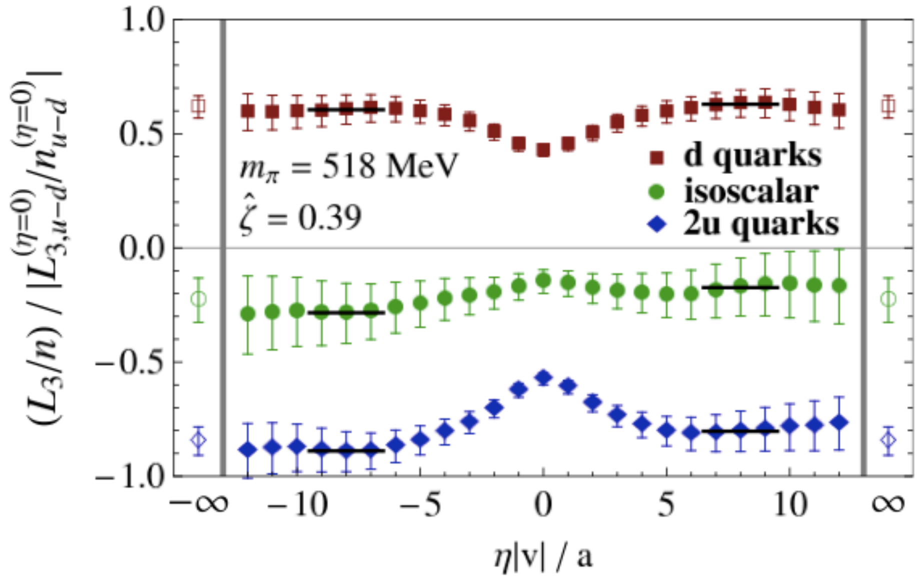
Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\hat{\zeta} \equiv \frac{P\cdot v}{|P||v|}$

Are interested in $\hat{\zeta} \longrightarrow \infty$; synonymous with $P \longrightarrow \infty$ in the frame of the lattice calculation $(v = e_3)$



Flavor separation – from Ji to Jaffe-Manohar quark orbital angular momentum

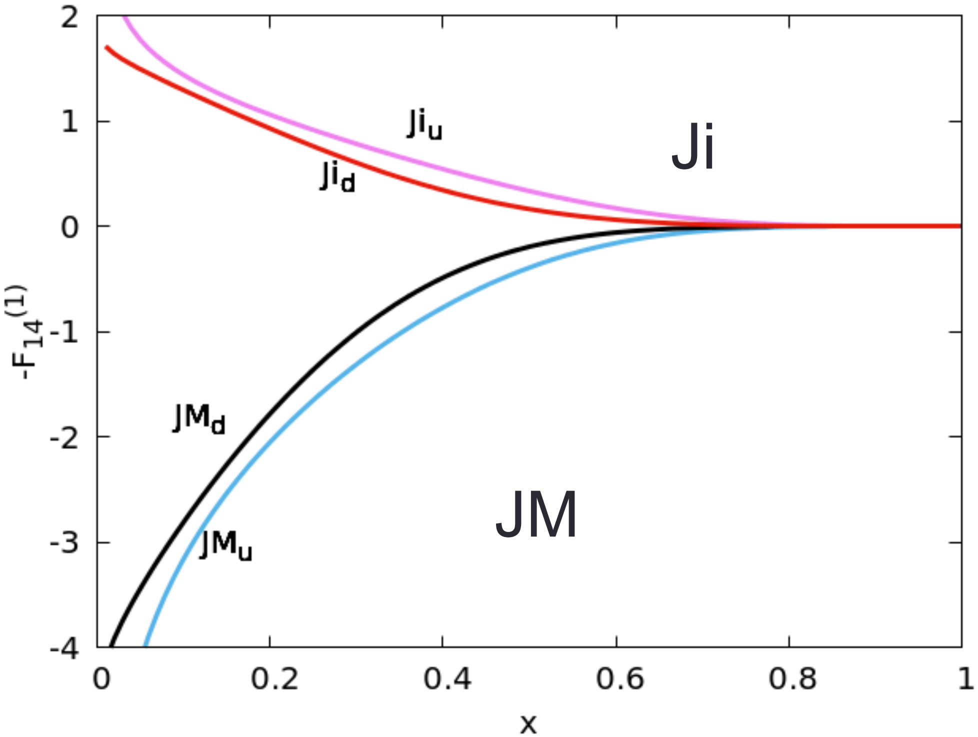


✓ Cancellation of L_u and L_d is found consistently with previous calculations

✓ It persists for Jaffe Manohar OAM!

$F_{14}^{(1)}$

with B. Kriesten and A. Rajan, using diquark model

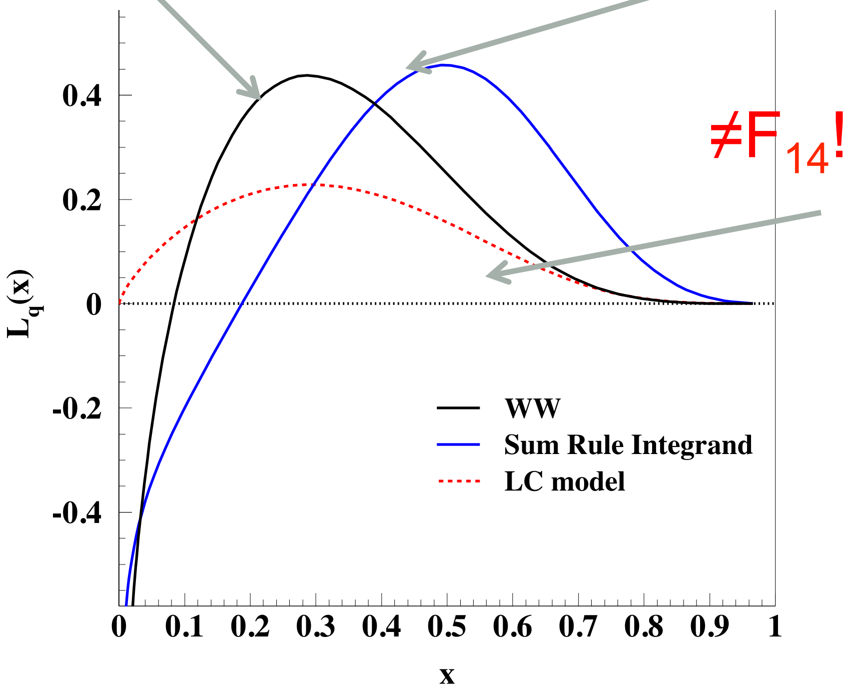


Ratio L_{Ji}/L_{JM}=0.72

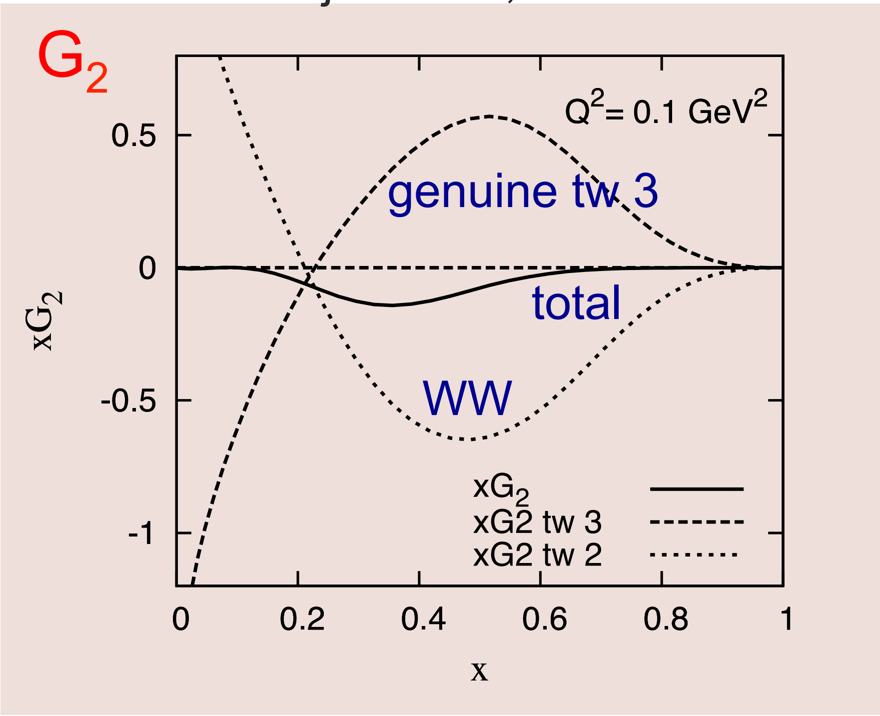
Lattice Ratio L_{Ji}/L_{JM}=0.62±0.16(stat) (extrapolated at $\zeta=\infty$)

$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

Low x!



Abha Rajan et al., arXiv:1601.06117



6. NUCLEI

Finally, nuclei

First Exclusive Measurement of Deeply Virtual Compton Scattering off ^{4}He: Toward the 3D Tomography of Nuclei

M. Hattawy,[1,2] N.A. Baltzell,[1,3] R. Dupré,[1,2,*] K. Hafidi,[1] S. Stepanyan,[3]
S. Bultmann,[4] R. De Vita,[5] A. El Alaoui,[1,6] L. El Fassi,[7] H. Egiyan,[3] F.X. Girod,[3]
M. Guidal,[2] D. Jenkins,[8] S. Liuti,[9] Y. Perrin,[10] B. Torayev,[4] and E. Voutier[10,2]
(The CLAS Collaboration)

Original studies of spin 0 GPDs:

SL & Taneja, PRD70 (2004), PRC72 (2005), PRC72R (2005)

$$W^{\gamma^+}_{\Lambda'\Lambda} = A_{\Lambda'+,\Lambda+} + A_{\Lambda'-,\Lambda-} \qquad \rightarrow \qquad H_A$$

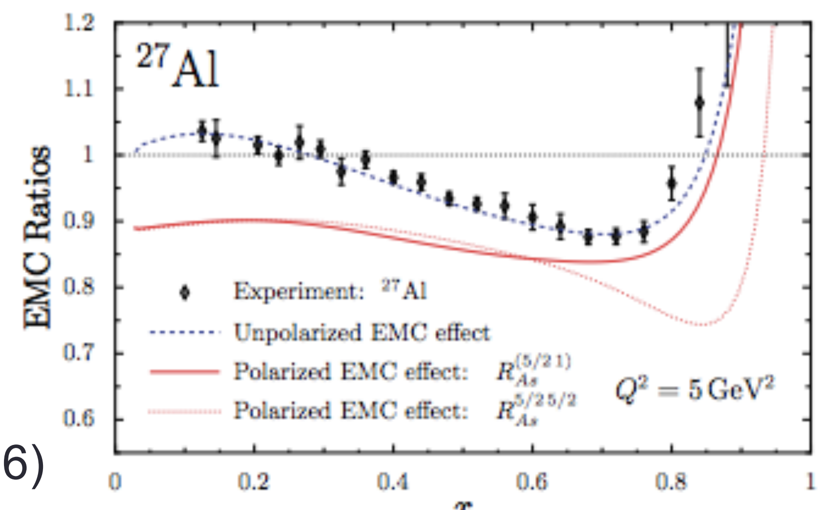
$$W^{i\sigma^{+i}\gamma_5}_{\Lambda'\Lambda} = A_{\Lambda'+,\Lambda-} + A_{\Lambda'-,\Lambda+}. \qquad \rightarrow \qquad H_T^A$$

Twist-three:

$$W^{\gamma^i}_{\Lambda'\Lambda} = A^{(3)}_{\Lambda'+*,\Lambda-} + A^{(3)}_{\Lambda'+,\Lambda-*} + A^{(3)}_{\Lambda'-*,\Lambda+} + A^{(3)}_{\Lambda'-,\Lambda+*} \qquad \rightarrow \qquad H_A^{(3)}$$

$$W^{\gamma_i\gamma_5}_{\Lambda'\Lambda} = A^{(3)}_{\Lambda'+*,\Lambda-} - A^{(3)}_{\Lambda'+,\Lambda-*} + A^{(3)}_{\Lambda'+*,\Lambda-} - A^{(3)}_{\Lambda'+,\Lambda-*} \qquad \rightarrow \qquad \tilde{H}_A^{(3)}$$

The spin-orbit term can shed light on the origin of the polarized EMC effect (work in progress with I. Cloet)



I. Cloet et al PLB (2006)

Physics of the D-term

$$\int_{-A}^{A} dx H^A(x,\xi,t) = F^A(t)$$
$$\int_{-A}^{A} dx x H^A(x,\xi,t) = M_2^A(t) + \frac{4}{5} d_1^A(t)\xi^2,$$

d represents the spatial distribution of the shears forces (Polyakov Shuvaev)

$$d^Q(0) = -\frac{m_N}{2} \int d^3r \; T_{ij}^Q(\vec{r}) \left(r^i r^j - \frac{1}{3} \delta^{ij} r^2 \right)$$

From S.L. and S.K. Taneja, PRC72(2005)

$$F^A(t) = F^{A,point}(t)F^N(t) \quad (54)$$

$$M_2^A(\xi,t) = M_2^{A,point}(t)M_2^N(t) + M_0^{A,point}(t)\frac{4}{5}d_1^N(t)\xi^2, \quad (55)$$

with $M_n^{A,point}(t) = \int dy y^{n-1} f_A(y,t)$, the nuclear moment obtained by considering "point-like" nucleons. At $\xi = 0$ one has:

$$M_2^A(t) = M_2^{A,point}(t)M_2^N(t), \quad (56)$$

related to the average value of the longitudinal momentum carried by the quarks in a nucleus:

$$\langle x(t)\rangle_A = \frac{M_2^A(t)}{F^A(t)} = \frac{M_2^{A,point}(t)}{F^{A,point}(t)}\frac{M_2^N(t)}{F^N(t)} = \langle y(t)\rangle_A\langle x(t)\rangle_N, \quad (57)$$

The D-term in a nucleus reads:

$$d_1^A(t) = M_0^{A,point}(t)d_1^N(t). \quad (58)$$

Is this factorization broken? First signature of non-nucleonic effects

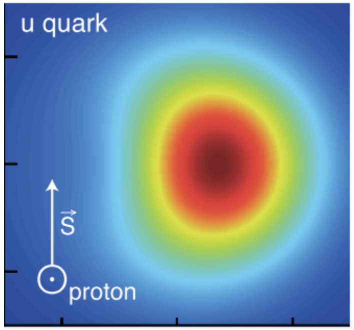
In liquid drop model

$$d_1^A(0) \propto \boxed{A^{7/3}}$$

$$d_1^A(0) \approx \frac{1}{1 - \frac{\langle E \rangle_A}{M} + \frac{2}{3}\frac{\langle p_\mu^2 \rangle_A}{M^2}} \propto \boxed{A \ln A}$$

Nuclear model taking into account virtuality

➢ Spin and 3D structure of Deuteron



$$\frac{1}{2}\int_{-1}^{1} dx x\,[H_q(x,0,0)+E_q(x,0,0)] = J_q$$

Nucleon (Ji, 1997)

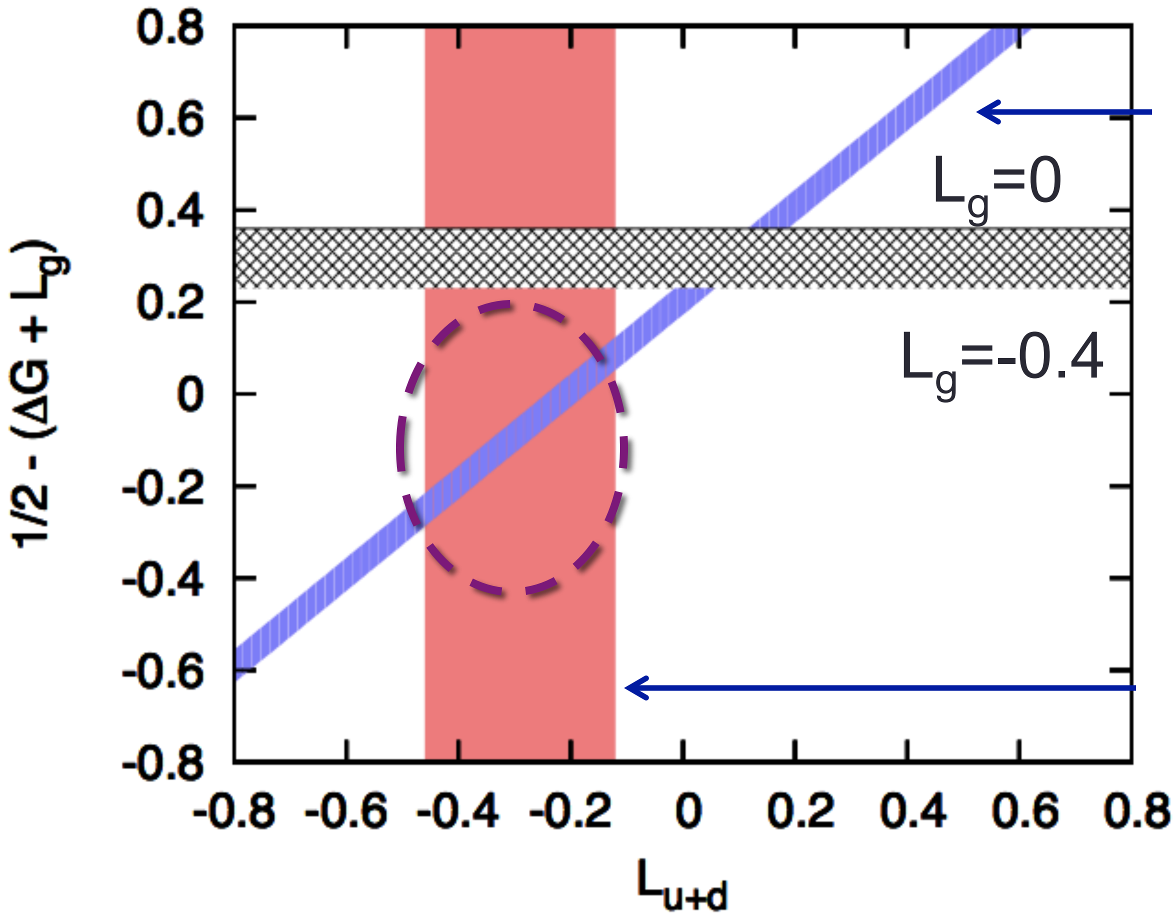
$$\longrightarrow \frac{1}{2}\int_{-1}^{1} dx x\, H_2^q(x,0,0) = J_q$$

Deuteron (S.K. Taneja et al, 2012)

7. GLUON SPIN AND HELICITY IN THE PROTON

Jaffe Manohar's Sum Rule: $\frac{1}{2} - (\Delta G + L_g^{JM}) = L_q^{JM} + \frac{1}{2}\Delta\Sigma_q$

➔ four independently measured quantities



Using the measured value of $\Delta\Sigma_{u+d}$

Using the measured value of ΔG

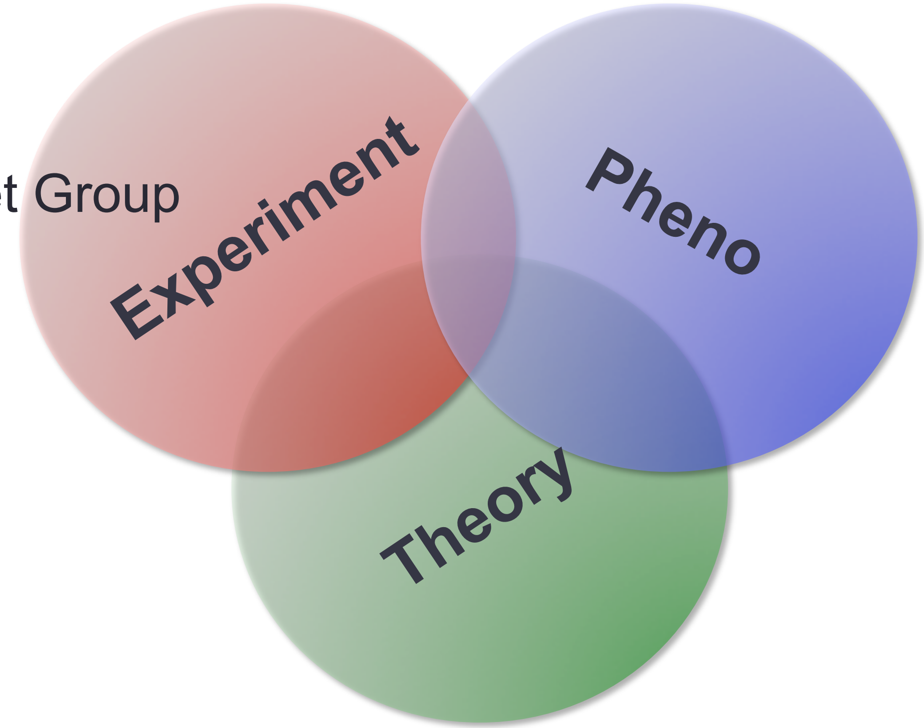
M. Engelhardt, preliminary

Lattice QCD evaluation of GTMD F_{14}+ gauge link

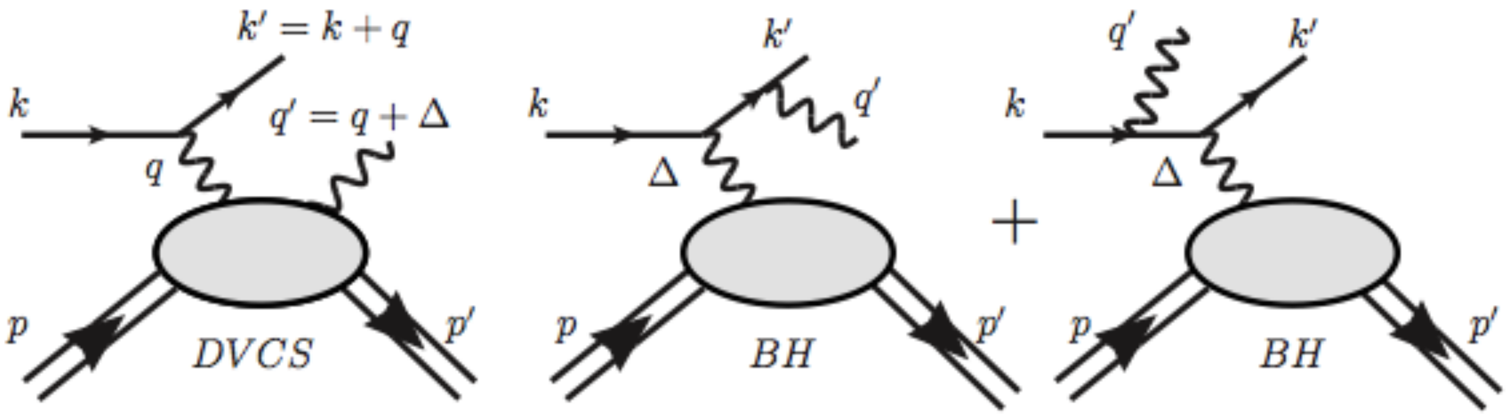
8. DVCS ANALYSIS

How do we detect all this?

Dustin Keller & U.Va. Polarized Target Group



Deeply Virtual Exclusive Photoproduction



$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}\left|T\right|^2 ,$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

From BKM formalism to "exact" Rosenbluth-like separation

Example 1

BH unpolarized cross section

$$\sigma_{BH} = \Gamma \left[A(y,t,\gamma,Q^2,\phi) \frac{F_1 + \tau F_2^2}{M^2} + B(y,t,\gamma,Q^2,\phi) \tau G_M^2(t) \right]$$

Example 2
DVCS unpolarized cross section

$$\sigma^{UU} = \frac{\Gamma}{Q^2(1-\epsilon)}\left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi}\right]$$

$$F_{UU,T} = 2(F_{++}^{11} + F_{+-}^{11} + F_{-+}^{11} + F_{--}^{11}),$$
Twist 2

$$F_{UU,L} = 2F_{++}^{00}$$
Twist 4

$$F_{UU}^{\cos\phi} = \mathrm{Re}\left[F_{++}^{01} + F_{--}^{01}\right]$$
Twist 3

$$F_{UU}^{\cos 2\phi} = \mathrm{Re}\left[F_{++}^{1-1} + F_{+-}^{1-1} + F_{-+}^{1-1} + F_{--}^{1-1}\right]$$
Photon helicity flip: transverse gluons

Helicity amplitudes

Virtual Photon helicities

$$F_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda^{(2)}_{\gamma^*}} = \sum_{\Lambda_{\gamma'}} \left(f_{\Lambda\Lambda'}^{\Lambda^{(1)}_{\gamma^*}\Lambda_{\gamma'}} \right)^* f_{\Lambda\Lambda'}^{\Lambda^{(2)}_{\gamma^*}\Lambda_{\gamma'}}$$

Initial and final proton helicities

Phase dependence

$$f \rightarrow e^{i\left[\Lambda_{\gamma*} - \Lambda_{\gamma'} - (\Lambda - \Lambda')\right]\phi}$$

The phase is determined by the virtual photon helicity which can be different for the amplitude and its conjugate

BASIC MODULE (based on helicity amplitudes)

$$\sum_{\Lambda'_\gamma,\Lambda'} \left(T^{h\Lambda'_\gamma}_{DVCS,\Lambda\Lambda'}\right)^* T^{h\Lambda'_\gamma}_{DVCS,\Lambda\Lambda'} =$$

$$\frac{1}{Q^2}\frac{1}{1-\epsilon}\Big\{(F^{11}_{\Lambda+} + F^{11}_{\Lambda-} + F^{-1-1}_{\Lambda+} + F^{-1-1}_{\Lambda-}) + \epsilon(F^{00}_{\Lambda+} + F^{00}_{\Lambda-})$$

$$+2\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\,(-F^{01}_{\Lambda+} - F^{01}_{\Lambda-} + F^{0-1}_{\Lambda+} + F^{0-1}_{\Lambda-}) + 2\epsilon\,\mathrm{Re}\,(F^{1-1}_{\Lambda+} + F^{1-1}_{\Lambda-})$$

$$+(2h)\Big[\sqrt{1-\epsilon^2}\,(F^{11}_{\Lambda+} + F^{11}_{\Lambda-} - F^{-1-1}_{\Lambda+} - F^{-1-1}_{\Lambda-})$$

polarized lepton

$$-\ 2\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,(F^{01}_{\Lambda+} + F^{01}_{\Lambda-} + F^{0-1}_{\Lambda+} + F^{0-1}_{\Lambda-})\Big]\Big\}$$

$$F^{11}_{++} = (1-\xi^2) \mid \mathcal{H} + \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[(\mathcal{H}^* + \widetilde{\mathcal{H}})^* (\mathcal{E} + \widetilde{\mathcal{E}}) + (\mathcal{H} + \widetilde{\mathcal{H}})(\mathcal{E}^* + \widetilde{\mathcal{E}}^*) \right]$$

$$F^{11}_{--} = (1-\xi^2) \mid \mathcal{H} - \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[(\mathcal{H}^* - \widetilde{\mathcal{H}})^* (\mathcal{E} - \widetilde{\mathcal{E}}) + (\mathcal{H} - \widetilde{\mathcal{H}})(\mathcal{E}^* - \widetilde{\mathcal{E}}^*) \right]$$

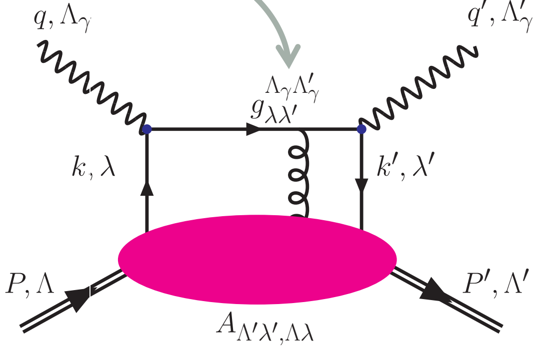
$$F^{11}_{+-} = \frac{t_0 - t}{4M^2} \mid \mathcal{E} + \xi \widetilde{\mathcal{E}} \mid^2$$

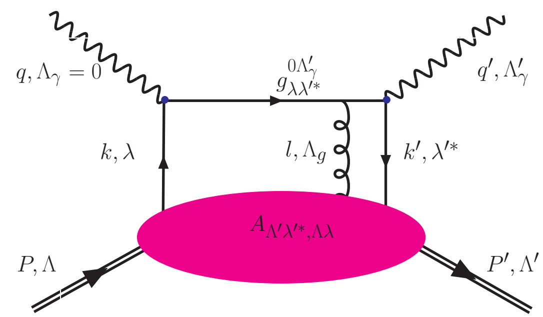
$$F^{11}_{-+} = \frac{t_0 - t}{4M^2} \mid \mathcal{E} - \xi \widetilde{\mathcal{E}} \mid^2$$

Twist 3

$$f^{01}_{\Lambda\Lambda'} = g^{01}_{-*+} \otimes A_{\Lambda'+,\Lambda-*} + g^{01}_{-+*} \otimes A_{\Lambda'+*,\Lambda-} + g^{01}_{+*-} \otimes A_{\Lambda'-,\Lambda+*} + g^{01}_{+-*} \otimes A_{\Lambda'-*,\Lambda+}$$

“Bad” component (exchanged gluon flips the quark chirality)





Connecting the DVCS formalism with the TMD/GPD/GTMD comprehensive parametrizations

Bacchetta et al JHEP02 (2007), Meissner Metz and Schlegel, JHEP08 (2009)

Example

$$A_{+-,++^*} = \frac{1}{2}\left(\tilde{E}_{2T} - \overline{E}_{2T} + \tilde{E}'_{2T} + \overline{E}'_{2T}\right)$$

$$A_{+-^*,++} = \frac{1}{2}\left(-\tilde{E}_{2T} + \overline{E}_{2T} + \tilde{E}'_{2T} + \overline{E}'_{2T}\right)$$

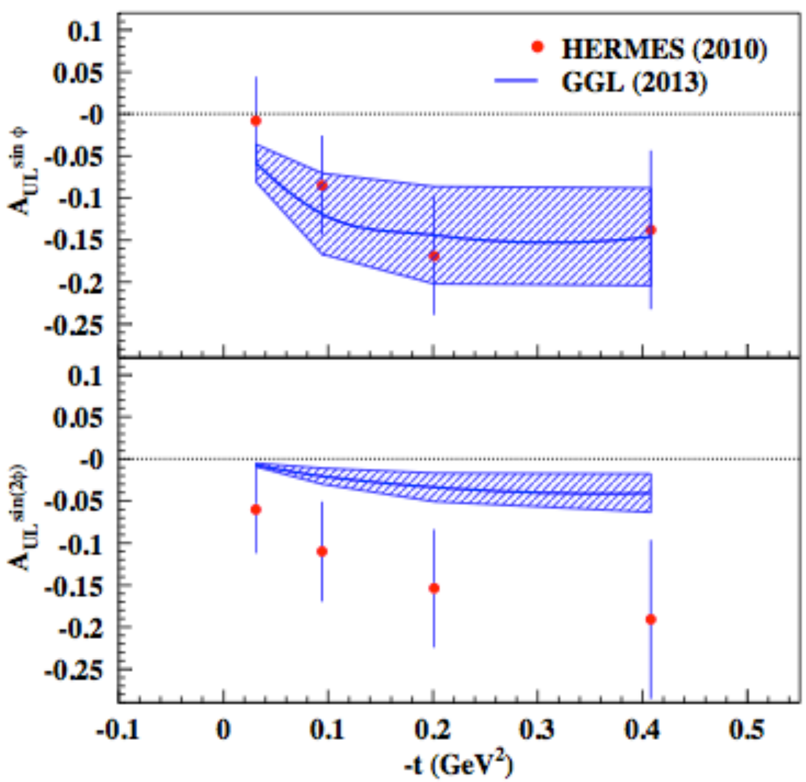
$$\vdots$$

Spin Orbit interaction

Orbital angular momentum

DVCS: bilinears of tw 2 and tw 3 CFFs

$$F_{++}^{01} = \mathcal{P}\left[\mathcal{H}^*(\tilde{\mathcal{E}}_{2T} - \bar{\mathcal{E}}_{2T} + \ldots), \ldots\right]$$

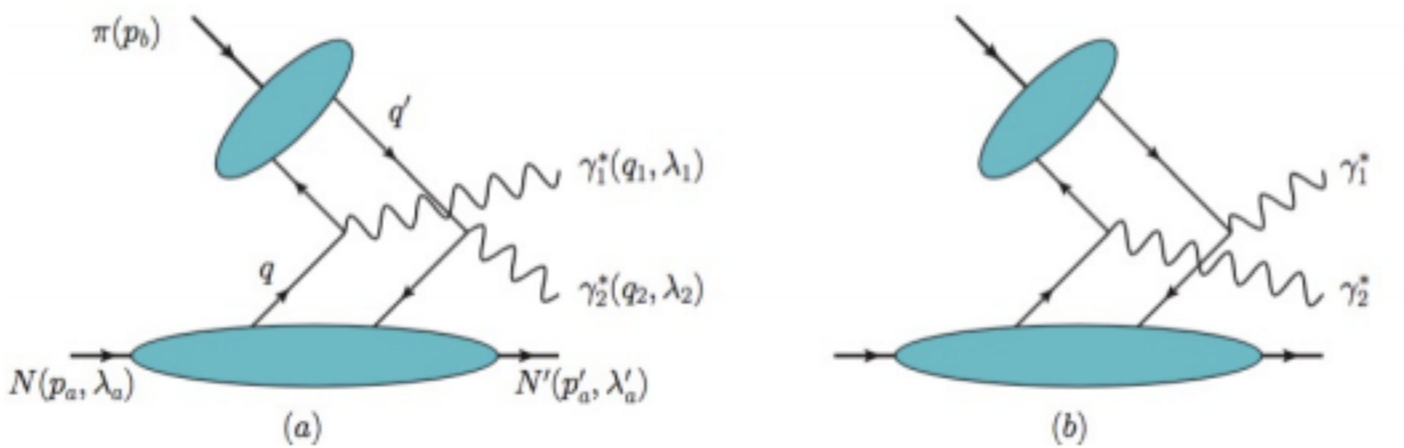


Extraction from experiment using Wandzura Wilczek approximation

A.Courtoy, G.Goldstein, O.Gonzalez Hernandez, S.L. and A.Rajan, PLB 731(2014)

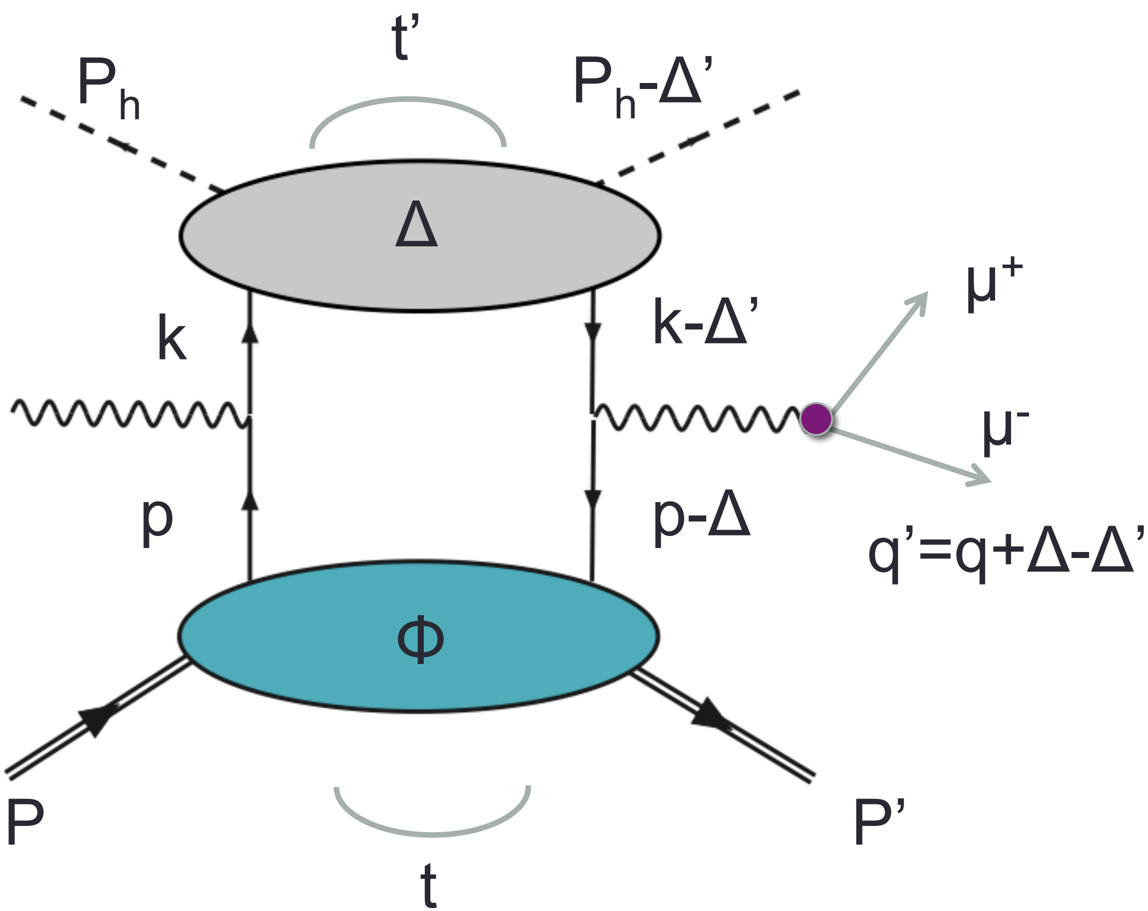
Measuring directly GTMDs

Double DVCS (Bhattacharya, Metz, Zhou, PLB 2017)



GTMDs from Double DVCS hadron production (off-forward SIDIS)

$$ep \rightarrow e'\pi^+\pi^-\mu^+\mu^-p'$$

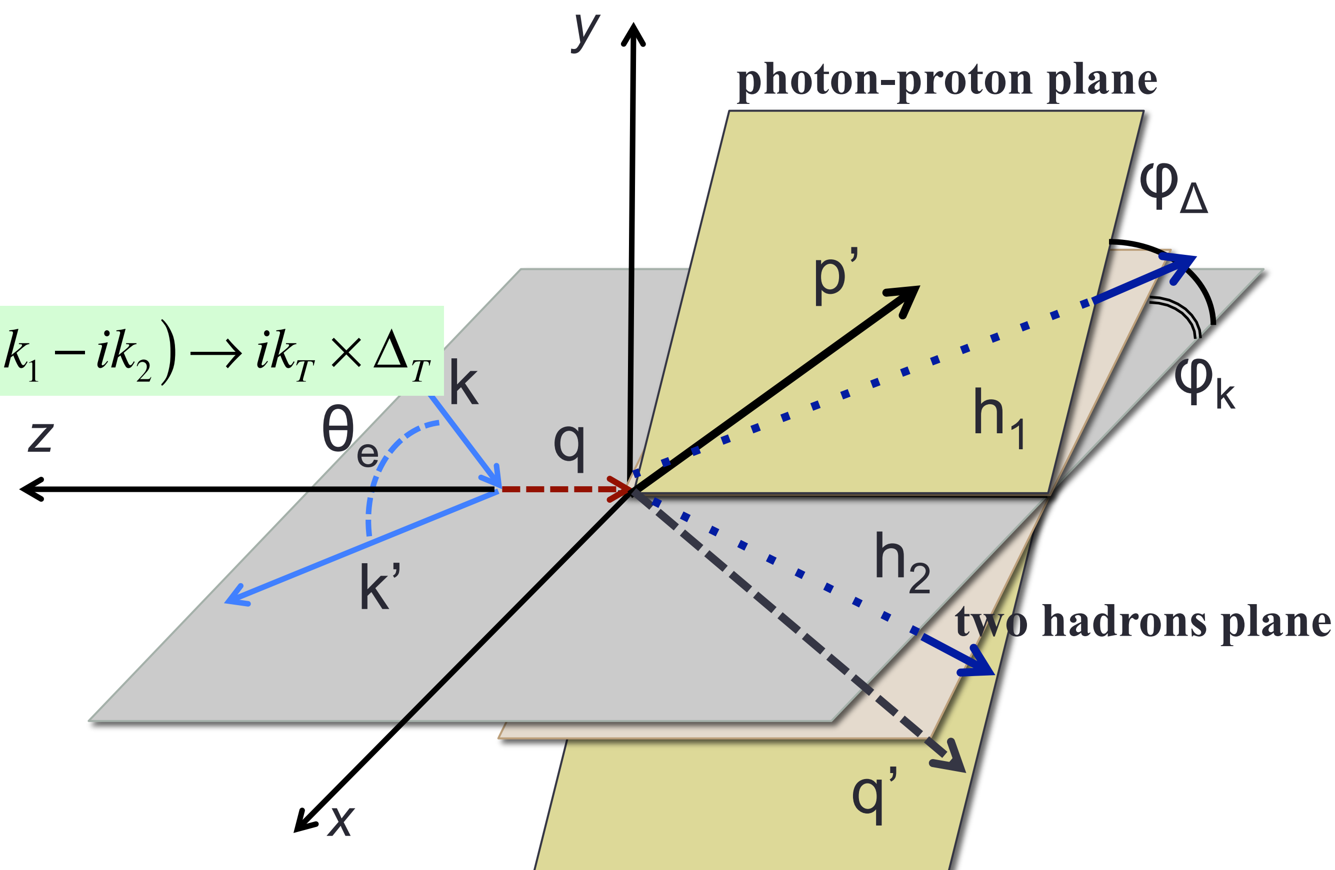


$t, t', P_h^2 < Q^2, Q'^2$

Helicity amplitude formalism for DDVCS hadron production

- To measure F_{14} one has to be in a frame where the reaction cannot be viewed as a two-body quark-proton scattering
- In the CoM the amplitudes are imaginary ➔UL term goes to 0 unless one defines two hadronic planes

$$|\Delta| e^{i\varphi_\Delta} |k_T| e^{i\varphi_k} (\Delta_1 - i\Delta_2)(k_1 - ik_2) \rightarrow ik_T \times \Delta_T$$



Conclusions and Outlook

The connection we established through the new relations between (G)TMDs and Twist 3 GPDs, not only allows us to evaluate the angular momentum sum rule, it also opens many interesting avenues:

- It allows us to study in detail the role of quark-gluon correlations, in a framework where the role of k_T and off-shellness, k^2, is manifest.
- OAM was obtained so far by subtraction (also in lattice). We can now both calculate OAM on the lattice (GTMD) and validate this through measurements (twist 3 GPD)
- It provides an ideal setting to test renormalization issues, evolution etc…
- QCD studies at the amplitude level shed light on chiral symmetry breaking
- TWIST THREE GPDs ARE CRUCIAL TO STUDY QCD AT THE AMPLITUDE LEVEL

Hopefully experimental studies of the hard exclusive processes will fill the gap in our understanding of the strong forces creating our world as we see it.

Maxim Polyakov (hep-ph/0210165)

Back up

Helicity and Transverse Spin Structures of H+E

$$\left(A_{++,++}+A_{+-,+-}+A_{-+,-+}+A_{--,--}\right)+\left(A_{++,-+}+A_{+-,--}-A_{--,+-}-A_{-+,++}\right)$$

$$\left(A^X_{++,++}+A^X_{+-,+-}+A^X_{-+,-+}+A^X_{--,--}\right)+\left(A^X_{++,++}+A^X_{+-,+-}-A^X_{-+,-+}-A^X_{--,--}\right)$$

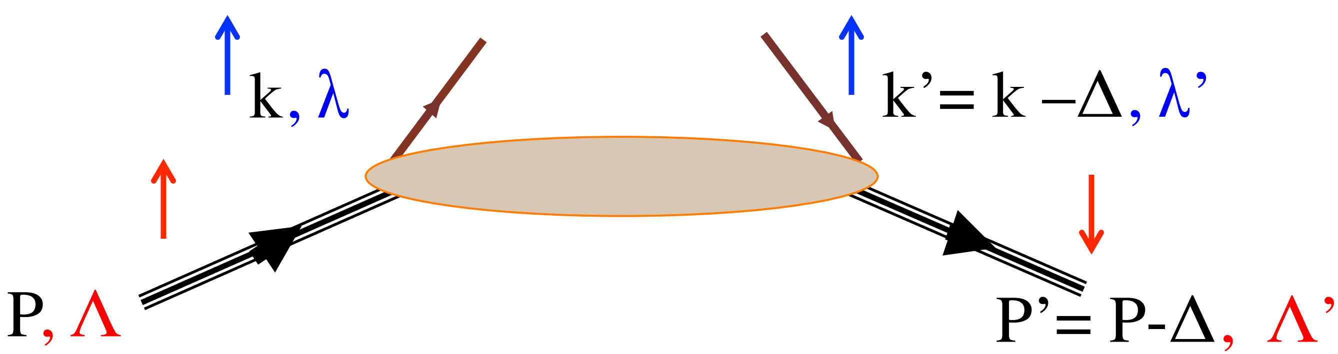
$$\approx H - i\Delta_2 E$$

= Helicity flips

Transv. spin conserved "non flip"

E measures J, not L, but a change of one unit of L (because of the helicity flip)

$$S_z = -1/2 \to 1/2 \Rightarrow \Delta L_z = 1$$ at fixed J



Brodsky and Drell '80s, Belitsky, Ji and Yuan, '90's