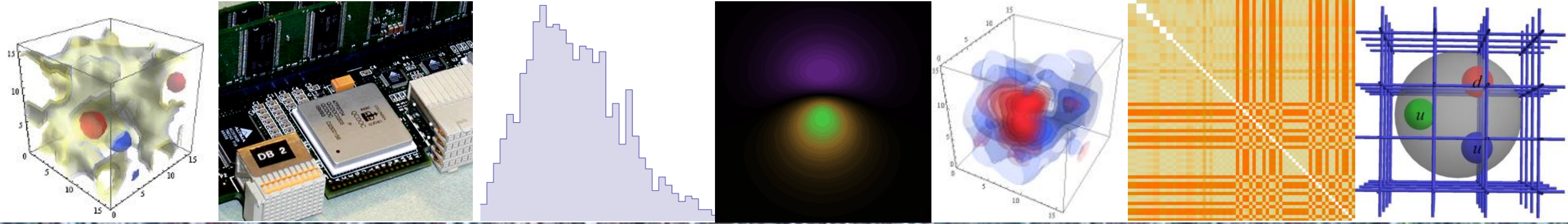
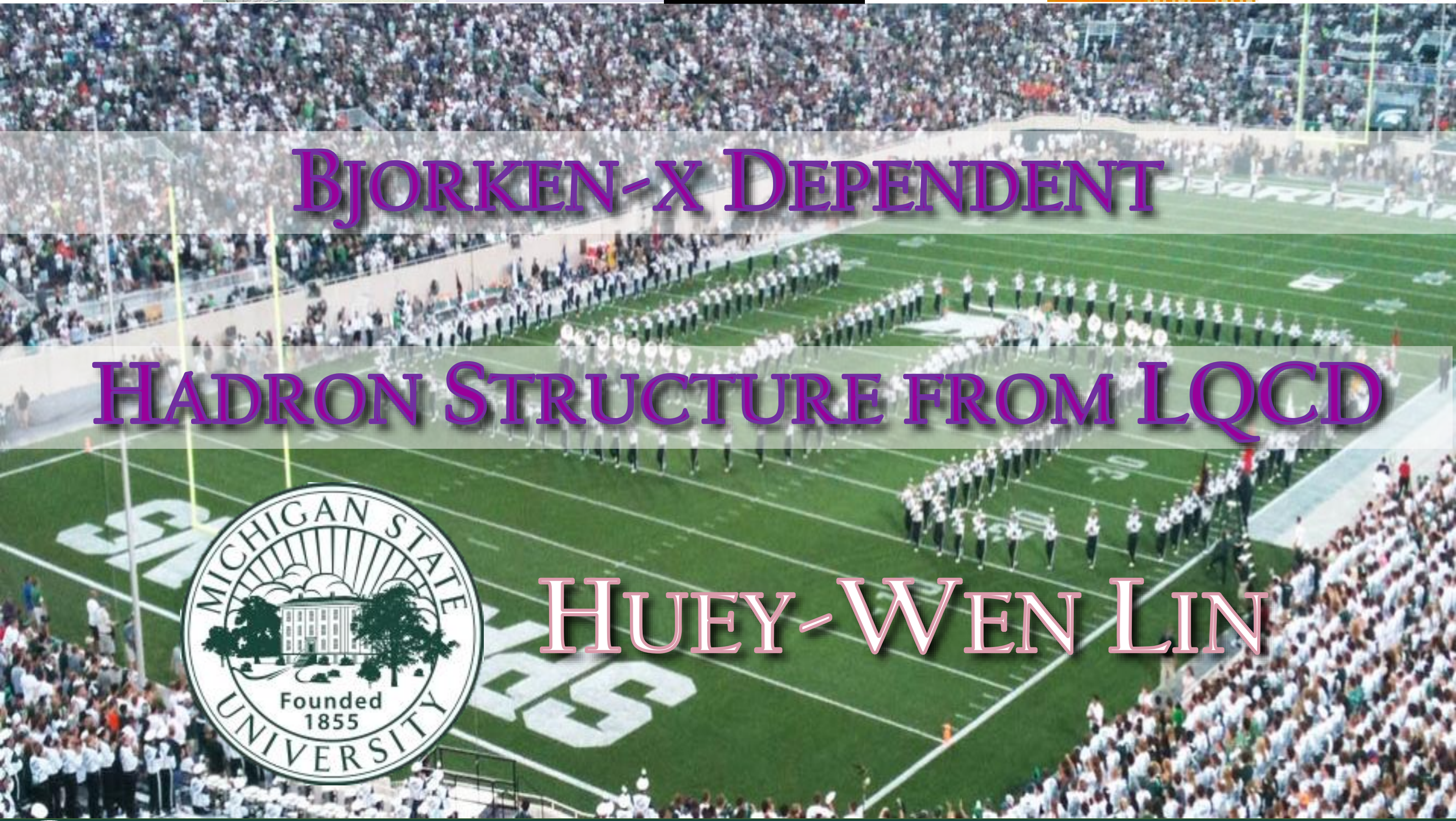


Bjorken-x Dependent Hadron Structure from LQCD

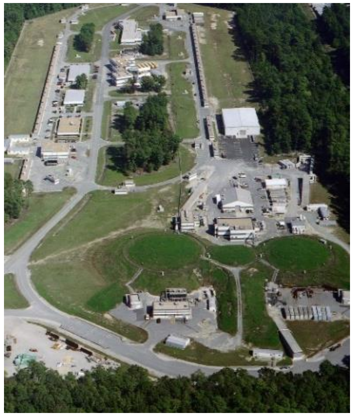
Huey-Wen Lin



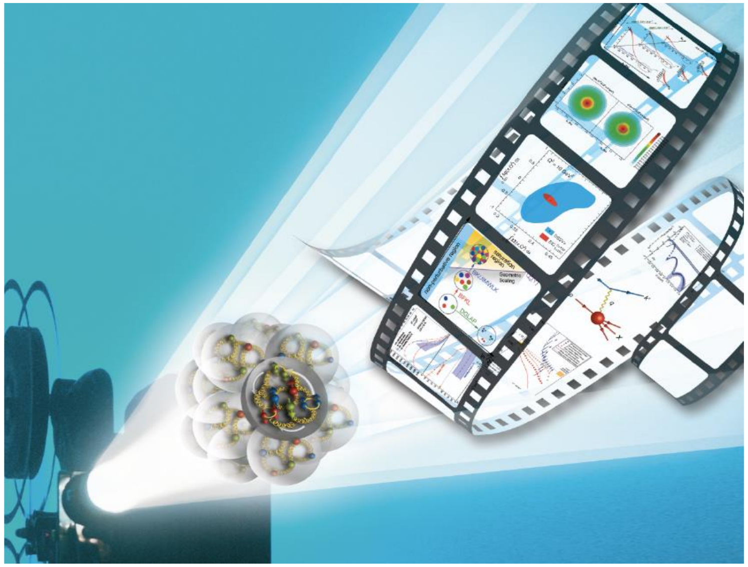
Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)



Electron Ion Collider: The Next QCD Frontier



Imaging of the proton

How are the *sea* *quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?*

EIC White Paper, 1212.1701

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

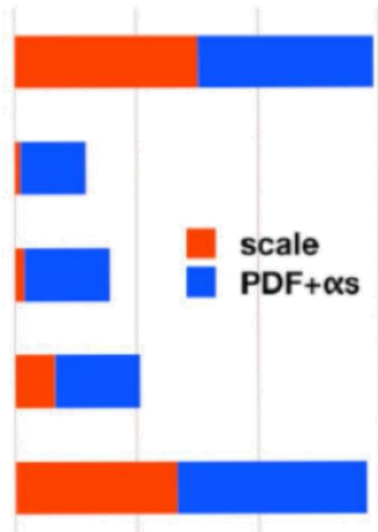
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

§ Important inputs to discern new physics at LHC

- Currently dominate errors in Higgs production

HIGGS PRODUCTION

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
NNLO QCD +NLO EW	VBF	1.56 pb	2.9%
	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%

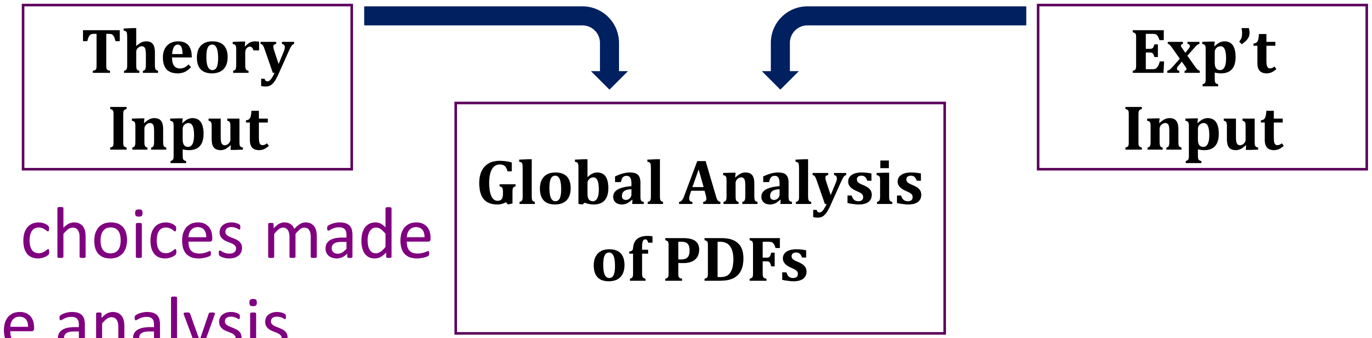


(J. Campbell, HCP2012)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets



§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

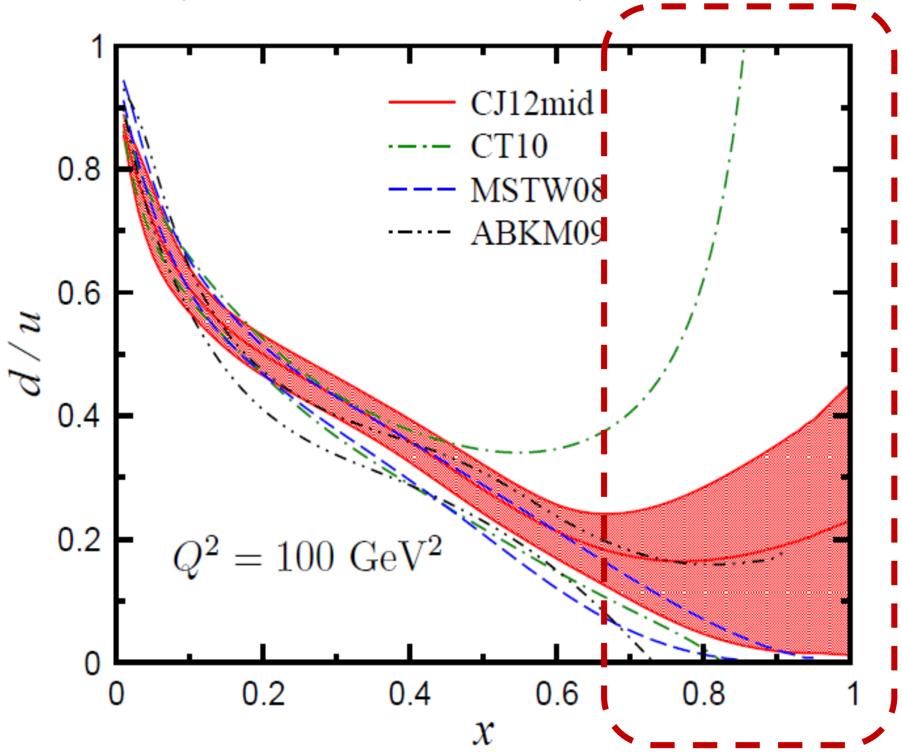
- Assumptions imposed

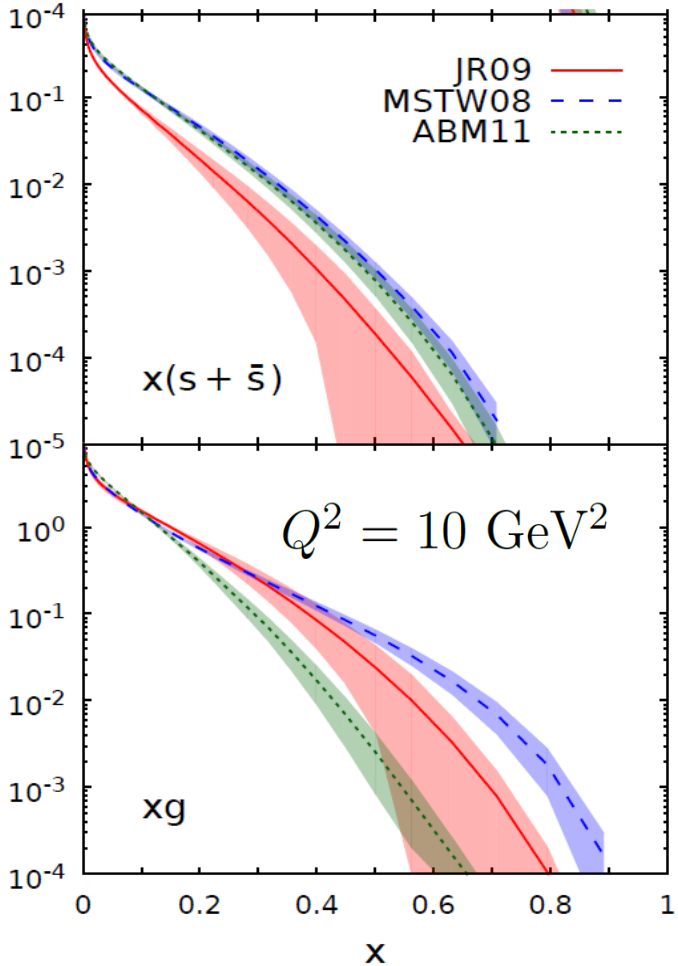
SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.





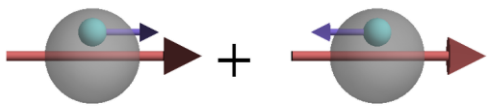
Jimenez-Delgado, Melnitchouk, Owens, J.Phys. G40 (2013) 09310

What can we do on the lattice?

PDFs on the Lattice

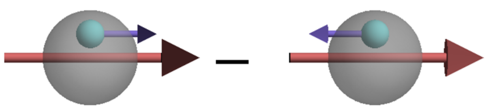
- Lattice calculations rely on operator product expansion, only provide moments

most well known



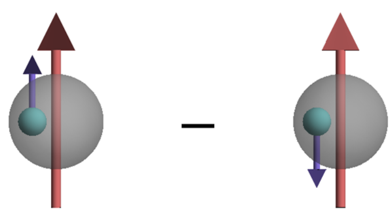
Quark density/unpolarized

$$\langle x^n \rangle_q = \int_{-1}^{1} dx\, x^n q(x)$$



Helicity
longitudinally polarized

$$\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^n \Delta q(x)$$



Transversity
transversely polarized

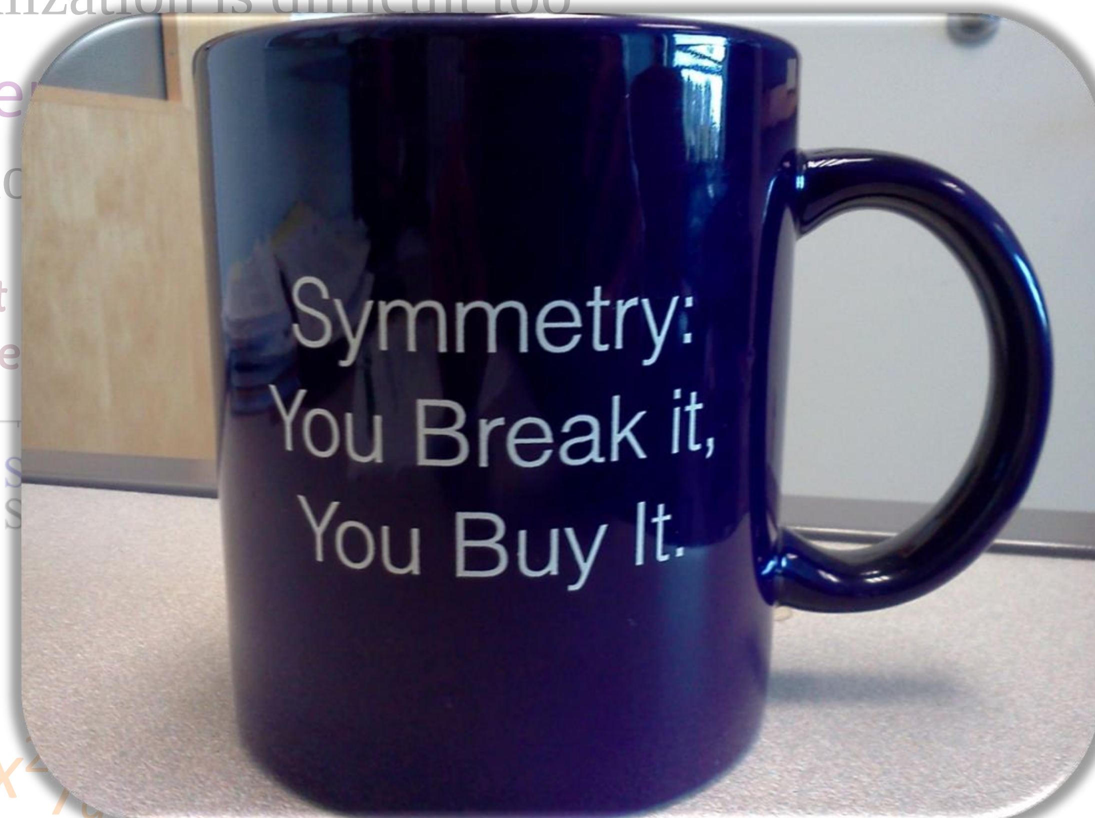
$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

very poorly known

- True distribution can only be recovered with all moments

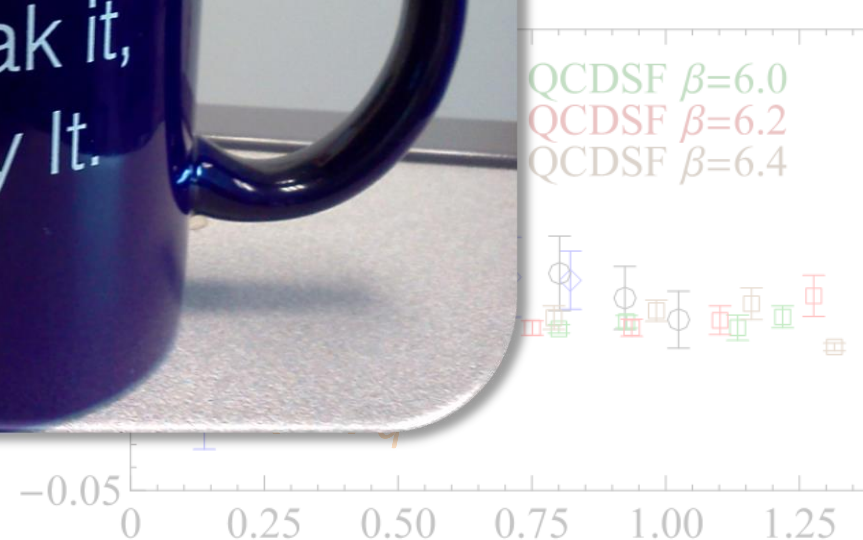
Problem with Moments

- For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- Relative e
  - Calculatio



Dolgov et
Göckeler e

SAM):
clover

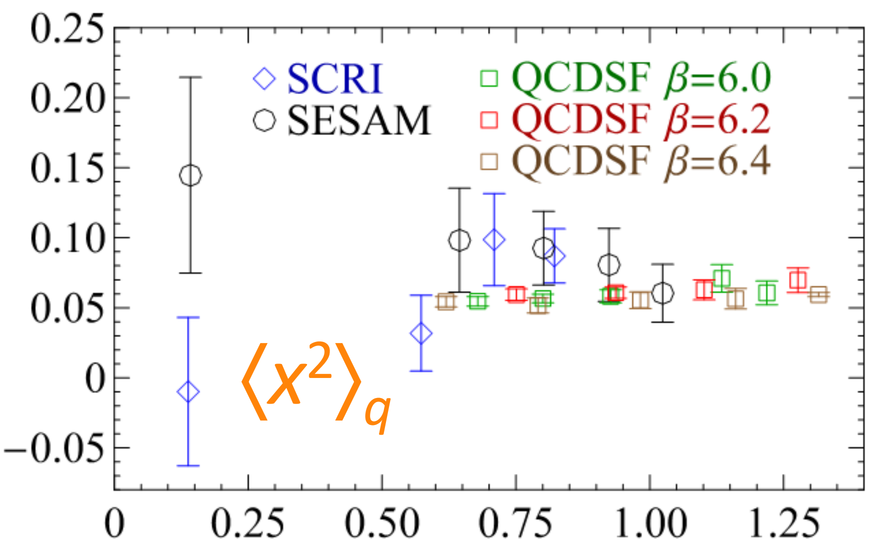


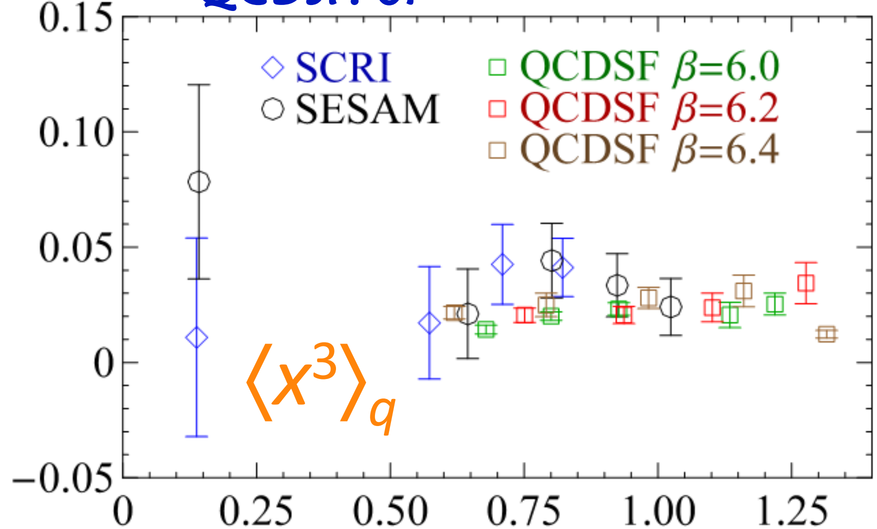
Problem with Moments

- § For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- § Relative error grows in higher moments
  - Calculation would be costly and difficult

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM):
2f, Wilson and clover
QCDSF: 0f



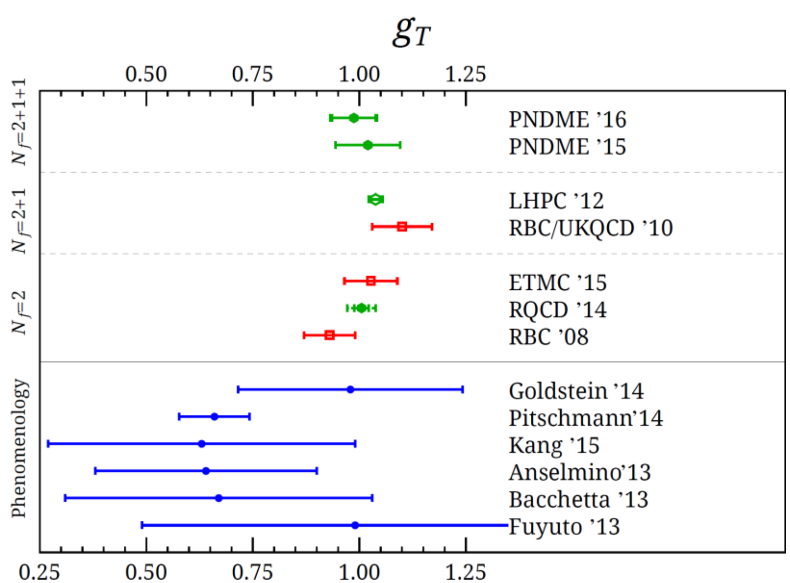


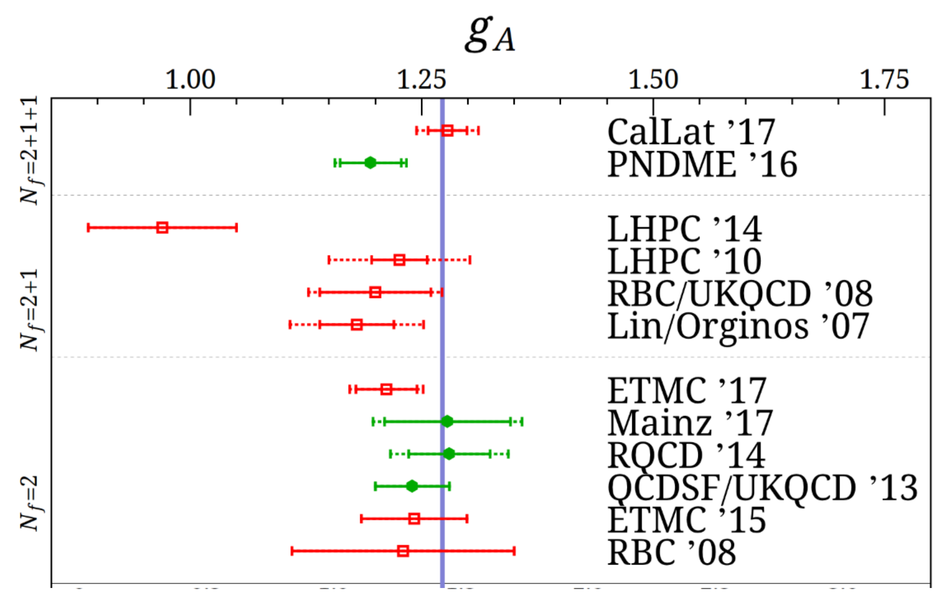
State-of-the-Art Moments

PNDME, 1506.06411; 1606.07049

- FLAG rating system
- New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]

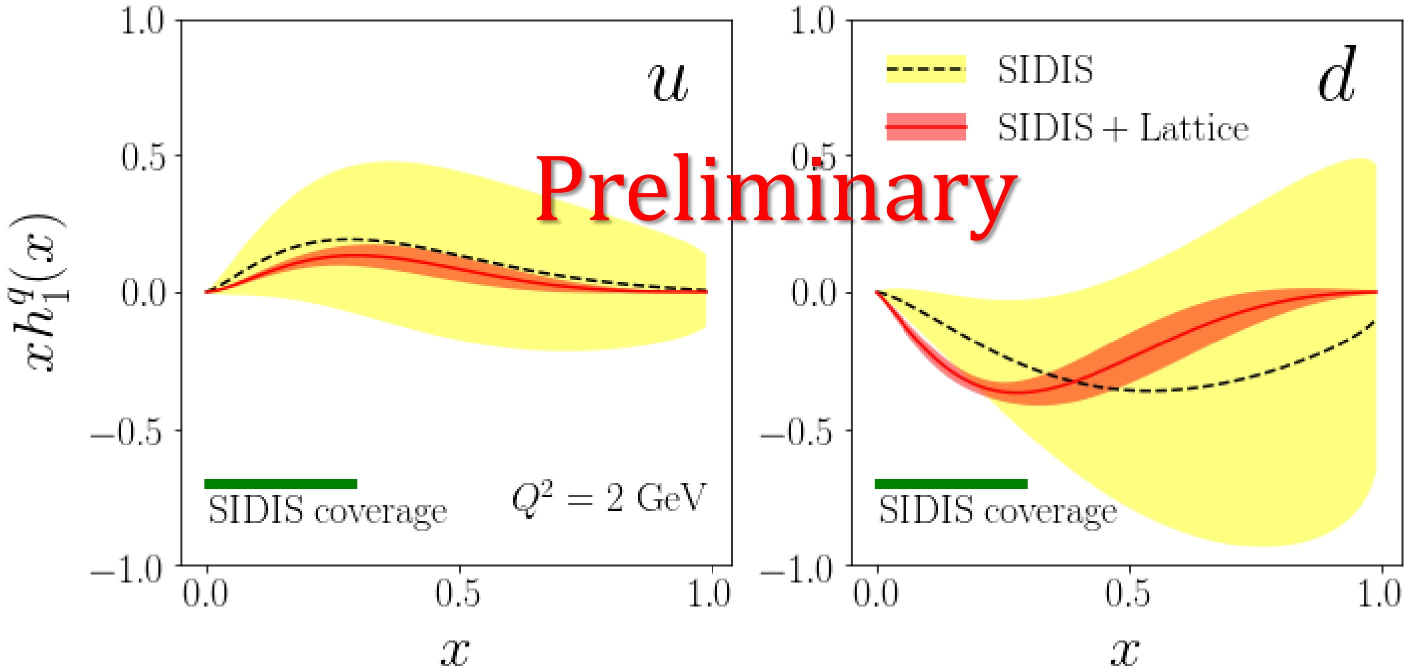




State-of-the-Art Moments

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms, gives $g_T = 1.058$
- Use to constrain the global analysis fits SIDIS $\pi^{\pm}$ production data from proton and deuteron targets



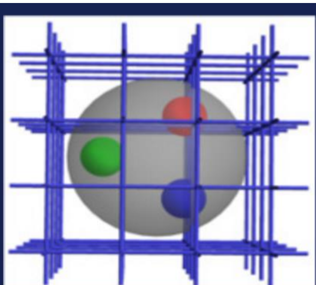
Lin, Melnitchouk, Prokudin, Sato, In preparation

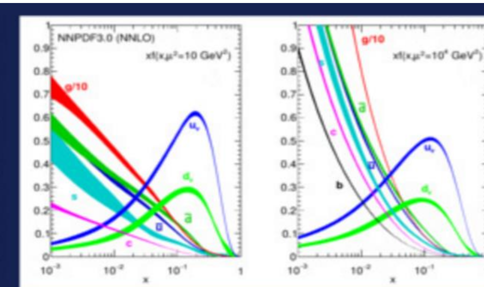
State-of-the-Art Moments

FLAG-like rating system

Community averaging quantities

White paper in progress with representatives from each collaboration





Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK

- Precision moments can be useful to improve PDFs!
  - Inputs as constraint in global analysis, like gA
- Whitepaper will
  - Address precision needed for moments and their impacts
  - Encourage more precision moment calculations in LQCD

Lin et al, In preparation

A Promising New Direction

Large-Momentum Effective Theory (LaMET)

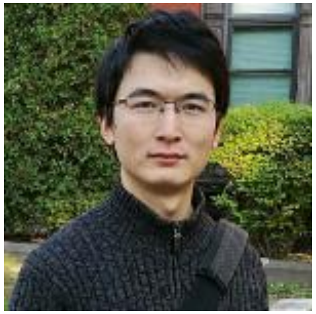
X. Ji, PRL. 111, 262002 (2013)

Lattice Parton Physics Project (LP³)

https://www.pa.msu.edu/~hwlin/LP3/

HWL
(MSU)

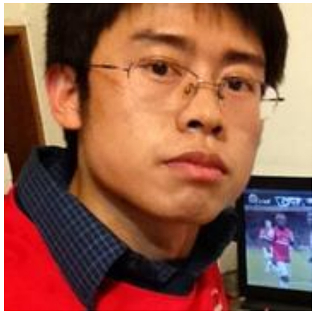
Xiangdong Ji
(UMD)



Luchang Jin
(BNL)

Peng Sun
(MSU)

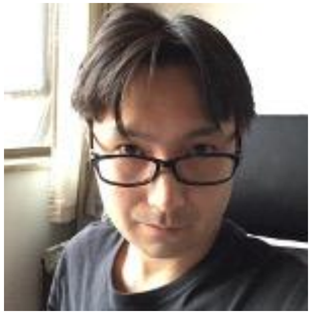
Yi-Bo Yang
(MSU)



Yong Zhao
(MIT)

International collaborators

Jiunn-Wei Chen
(NTU)



Tomomi Ishikawa
(SJTU)

Jian-Hui Zhang
(Regensburg)

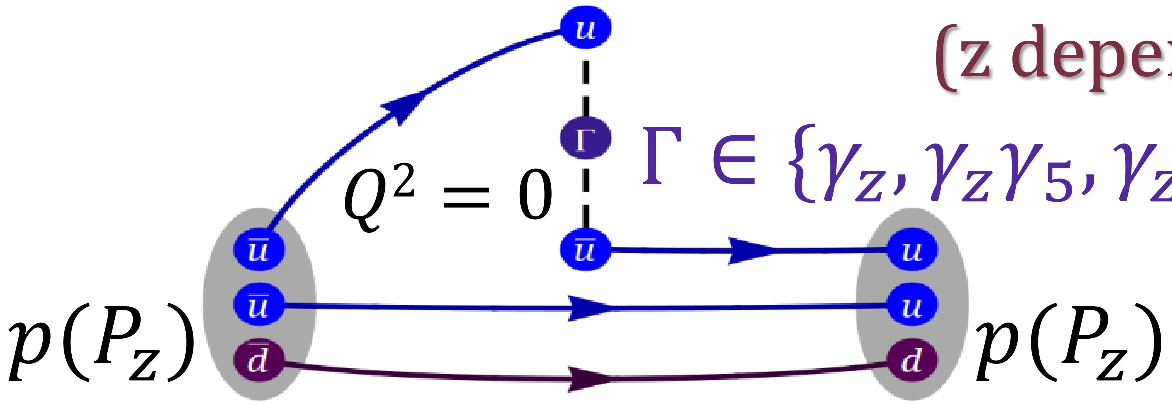
A New Direction

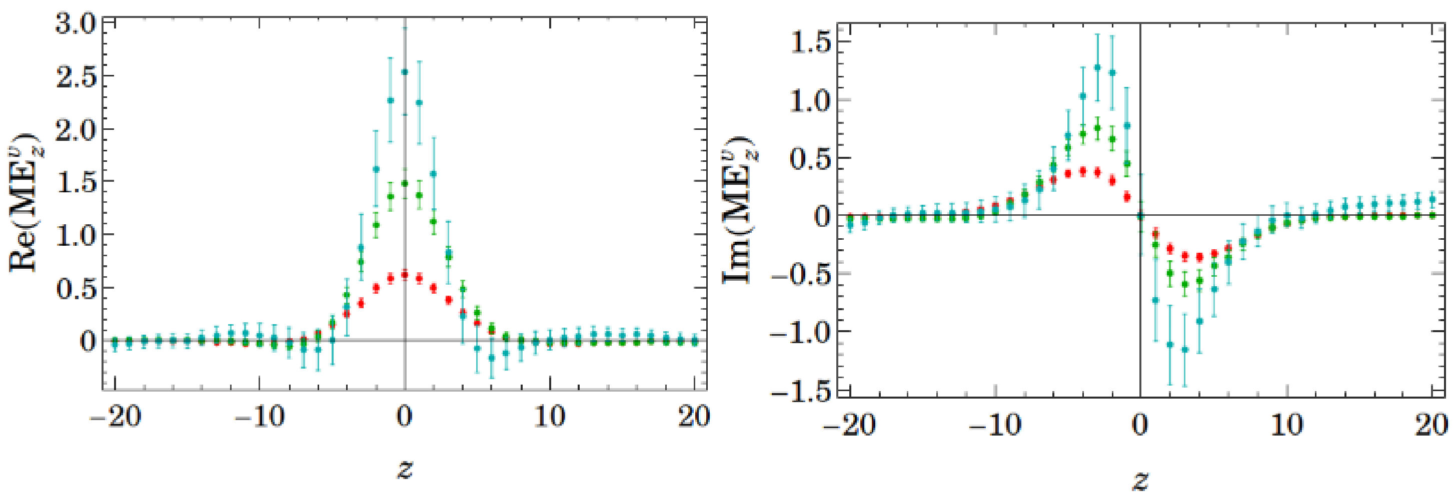
Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

(z dependence)





$P_z \in \{0.43, 0.86, 1.29\}$ GeV

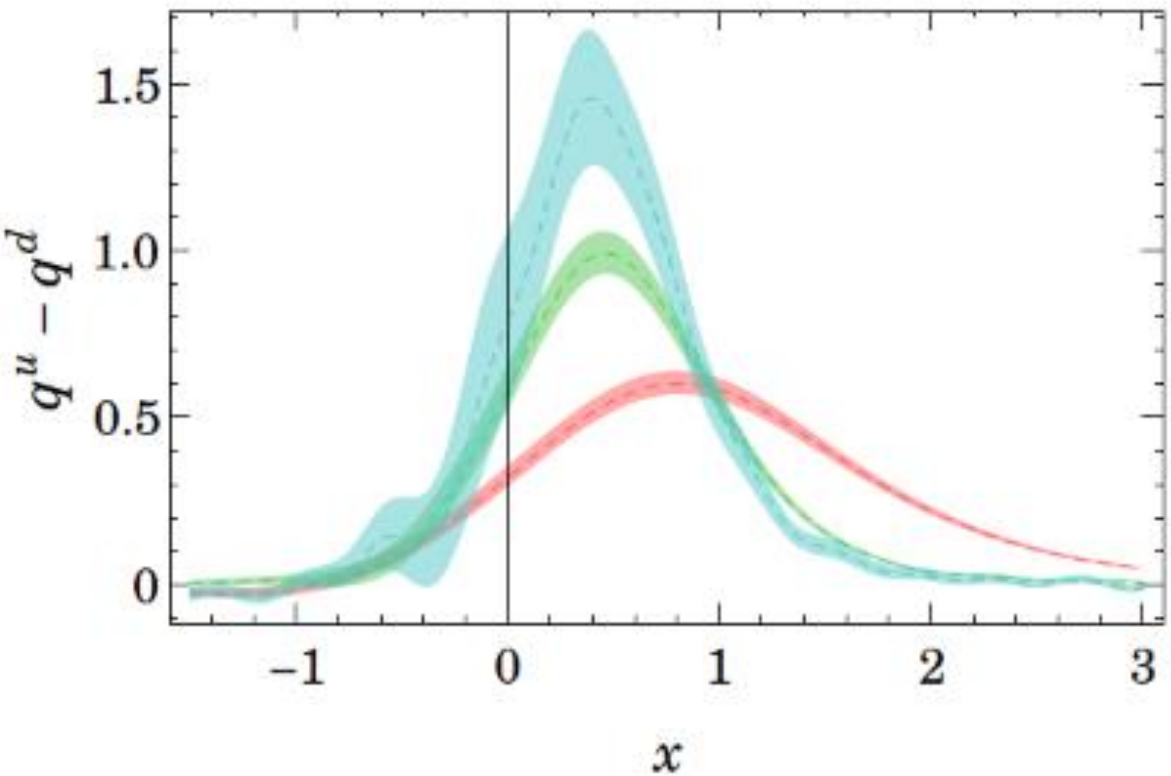
A New Direction

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \bar{\psi}(z) \Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right) \psi(0) \middle| P \right\rangle$$



Uncorrected bare lattice results

$x = k_z / P_z$

Distribution should sharper as P_z increases

Artifacts due to finite P_z on the lattice

A New Direction

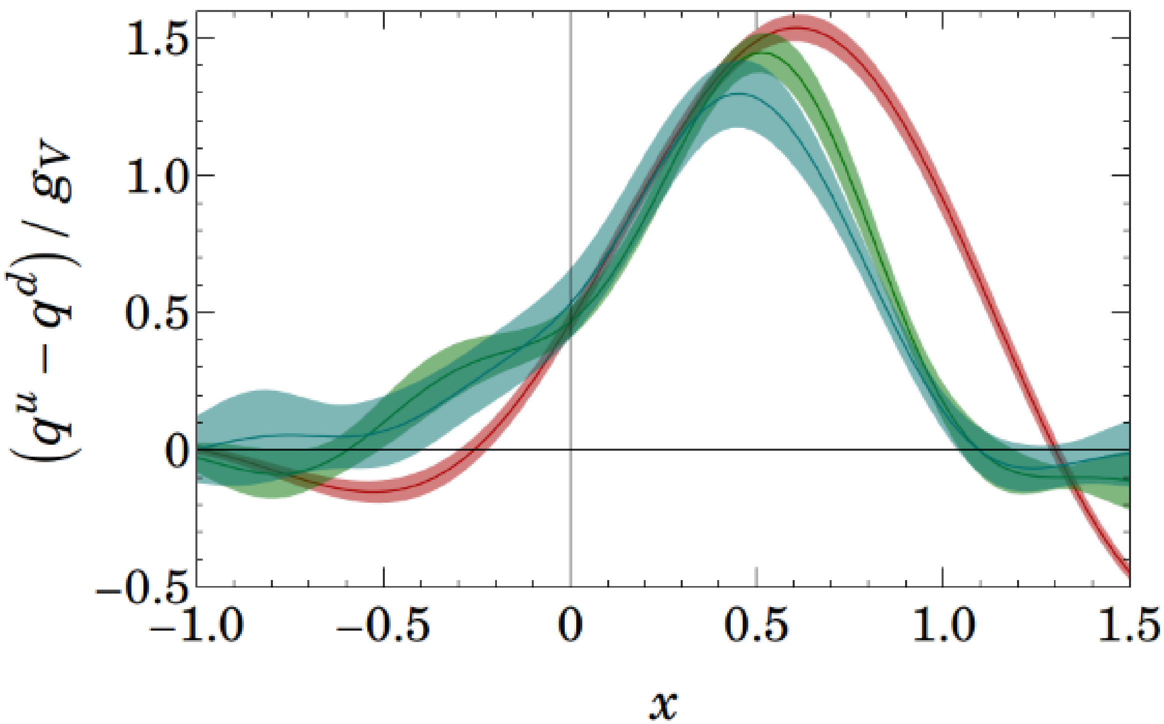
Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \cdots$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

Corrected distributions from the largest 2 P_z show signs of convergence

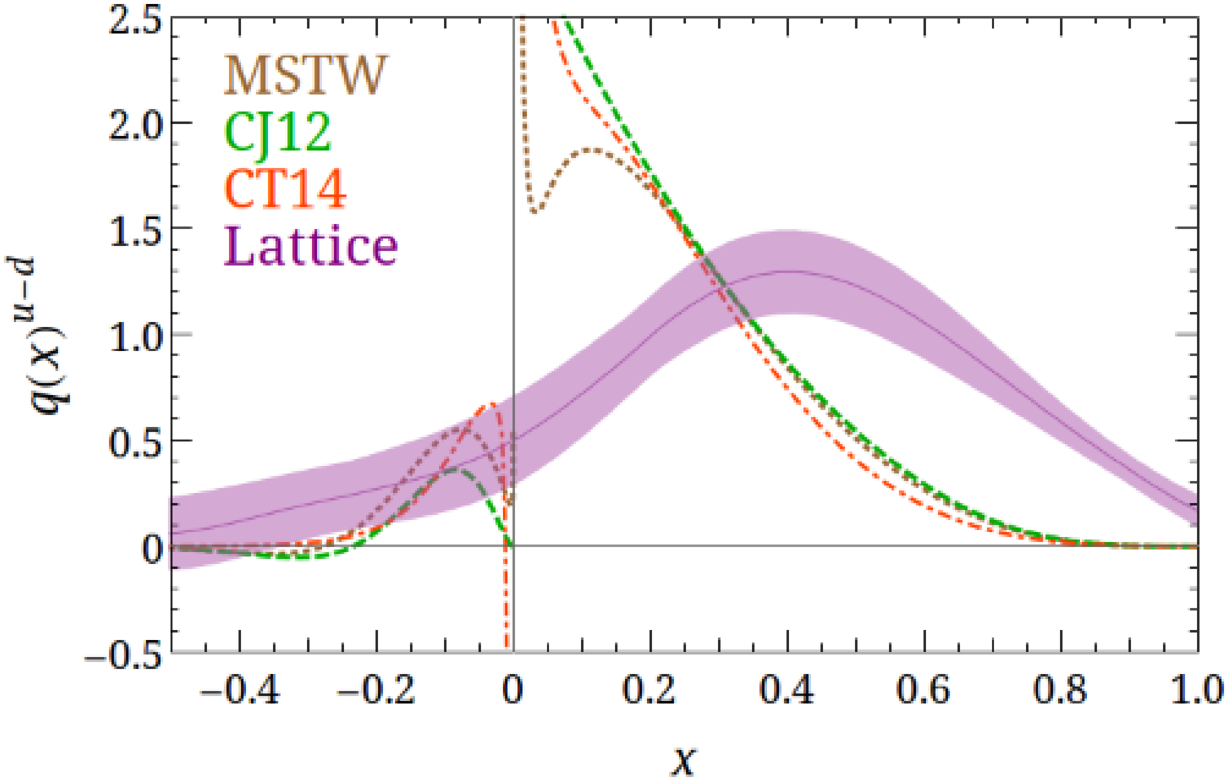
A New Direction

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

4) Remove the leading high-twist effect $\left(\Lambda_{\rm QCD}^2/P_z^2\right)$

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$), O(10^3) measurements

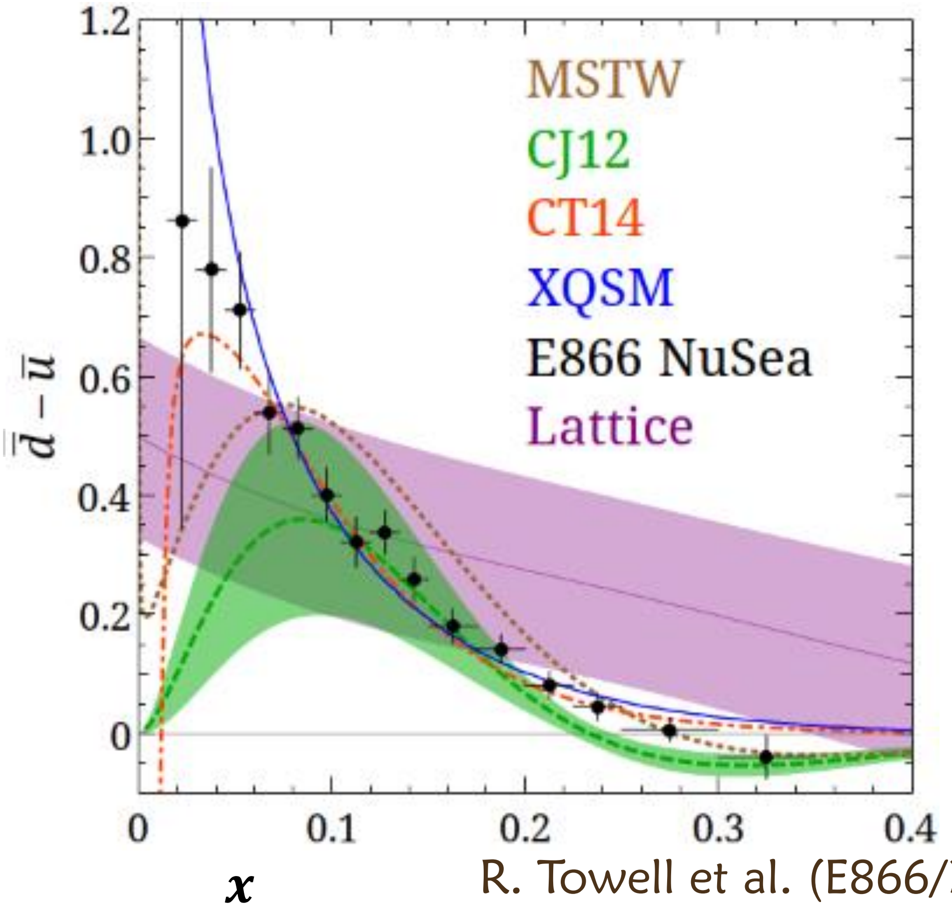


Sea Flavor Asymmetry

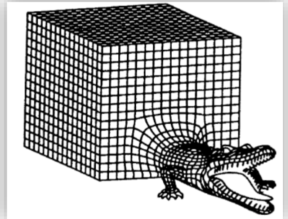
§ First time in LQCD history to study antiquark distribution!

☙ $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**

HWL et al. 1402.1462



$$\bar{q}(x) = -q(-x)$$



Lost resolution in small-x region

Future improvement: larger lattice volume

$$\int dx\, (\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1 [\bar{d}(x) - \bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Sea Flavor Asymmetry

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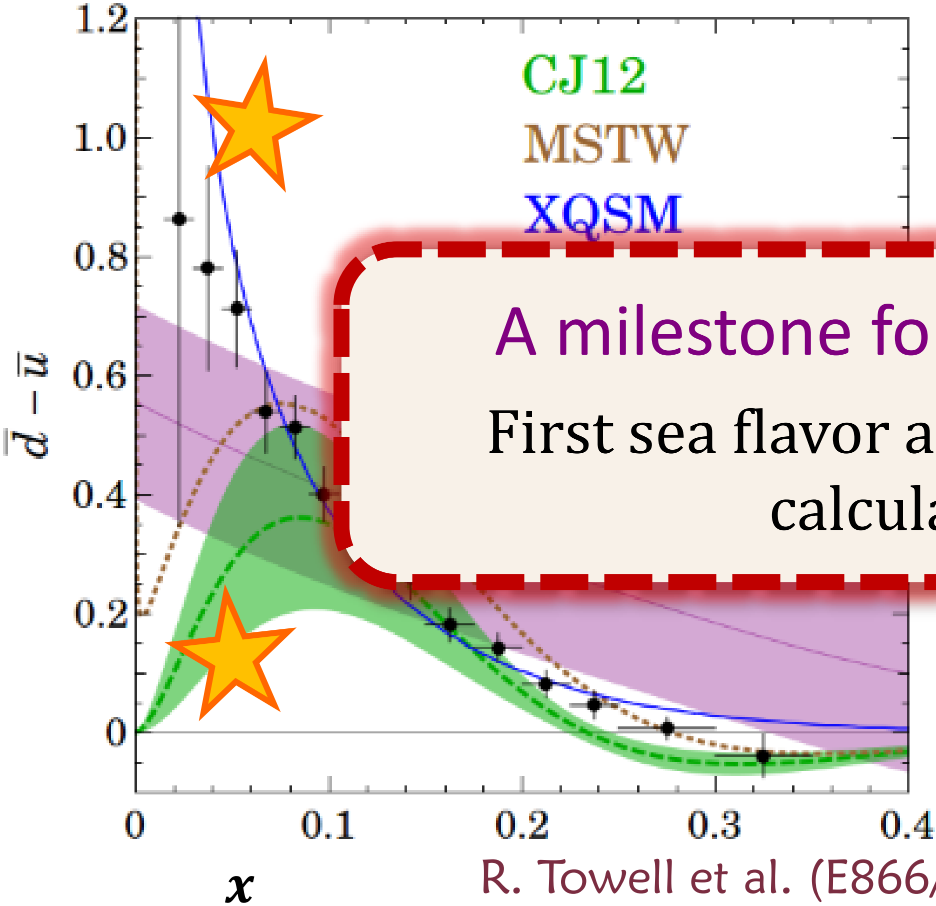
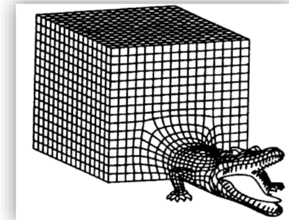
HWL et al. 1402.1462



$\bar{q}(x) = -q(-x)$



Lost resolution in small-x regi…

…ent:

…ume

$\approx -0.16(7)$

A milestone for lattice QCD!

First sea flavor asymmetry ever calculated!

Experiment	x range	$\int_0^1[\ldots - \bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

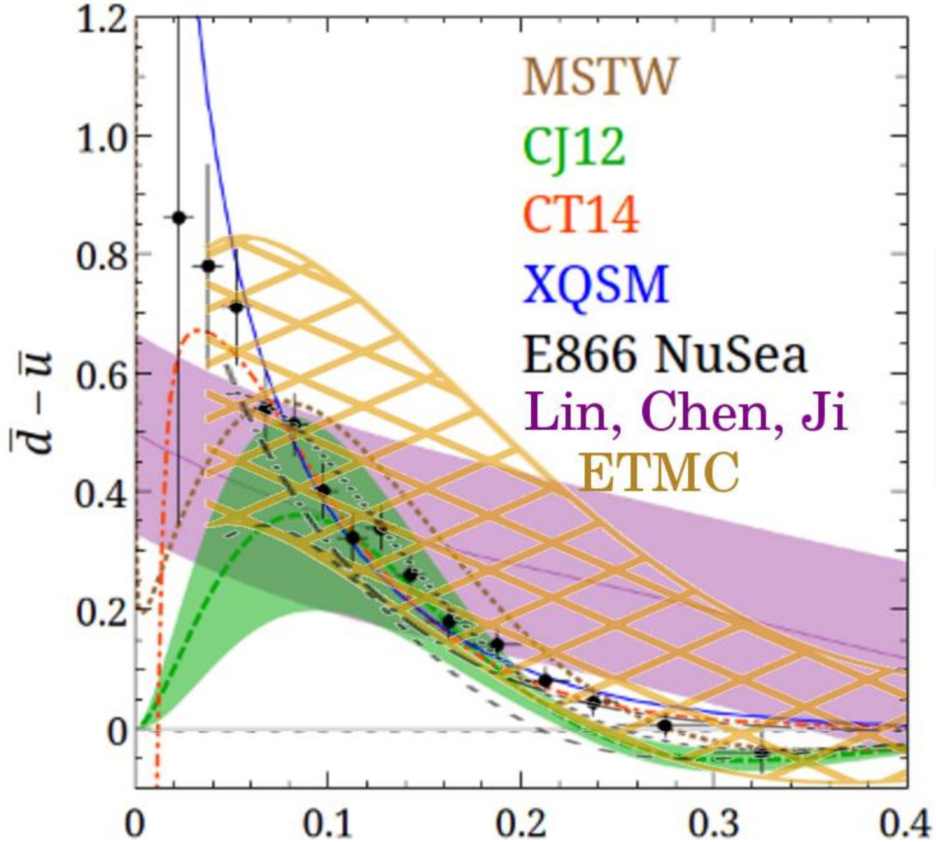
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Sea Flavor Asymmetry

§ Lattice exploratory study

- $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**

HWL et al 1402.1462



Compared with E866

Too good to be true?

Lost resolution in small-*x* region

Similar results repeated by ETMC, at $M_\pi \approx 373$ MeV
ETMC, 1504.07455

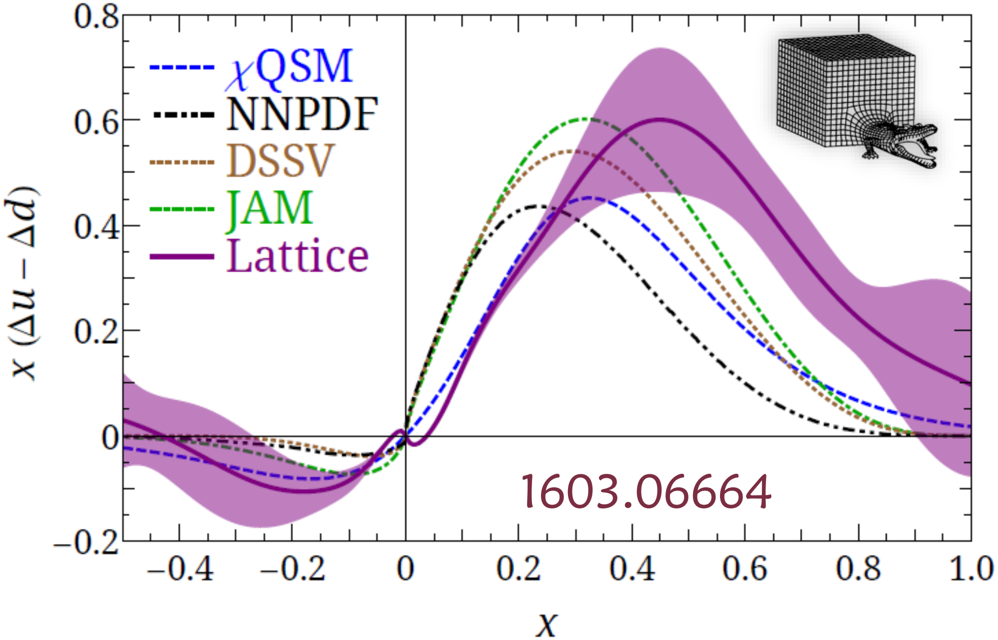
(7)

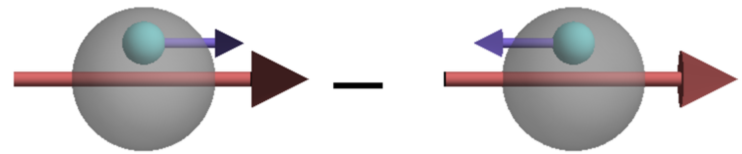
Experiment	x range	$\int_0^1[\overline{d}(x)-\overline{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Helicity Distribution

§ Exploratory study ☙ $M_\pi \approx 310$ MeV





Removing

$O(M_N^n/P_z^n) + O(\Lambda_{QCD}^2/P_z^2)$

errors

☙ We see polarized "sea asymmetry" $\int dx\left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.14(9)$

☙ Both STAR and PHENIX at RHIC see $\Delta\bar{u} > \Delta\bar{d}$

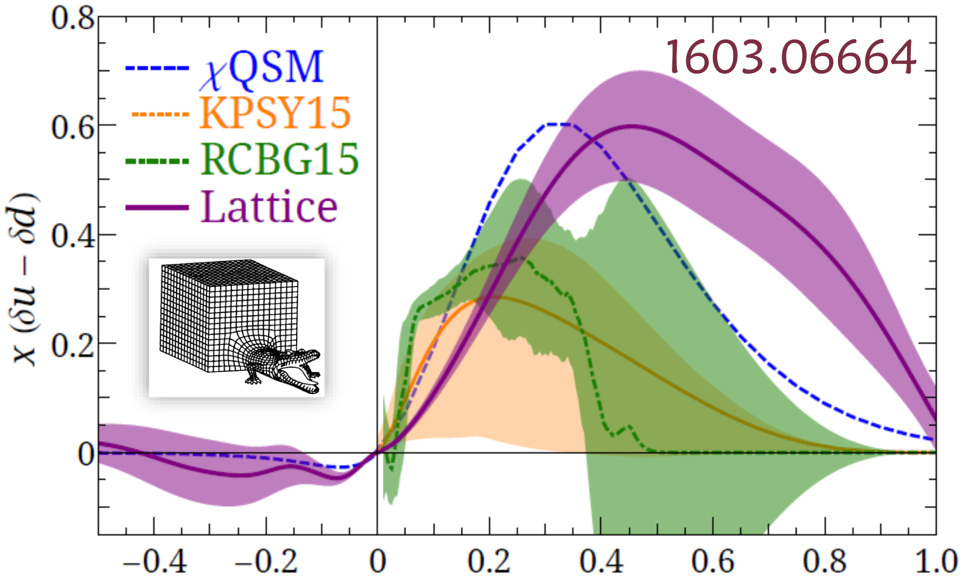
1404.6880 and 1504.07451

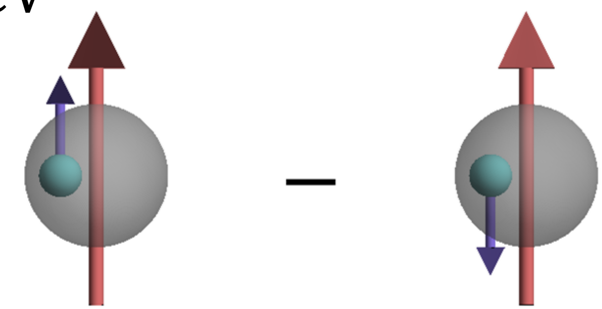
☙ Other experiments, Fermilab DY exp'ts (E1027/E1039), future EIC

Transversity Distribution

§ Exploratory study

- $M_\pi \approx 310$ MeV





Removing $O(M_N^n/P_z^n) + O(\Lambda_{QCD}^2/P_z^2)$ errors

$\delta\bar{q}(x) = -\delta q(-x)$

1505.05589; 1503.03495

- We found sea asymmetry of $\int dx\,\left(\delta\bar{u}(x) - \delta\bar{d}(x)\right) \approx -0.10(8)$
- Chiral quark-soliton model $\int dx\,\left(\delta\bar{u}(x) - \delta\bar{d}(x)\right) \approx -0.082$

P. Schweitzer et al., PRD 64, 034013 (2001)

- SoLID at JLab, Drell-Yan exp't at FNAL (E1027+E1039), EIC, ...

Missing Ingredient: Renormalization (and Updates)

Recent progress:

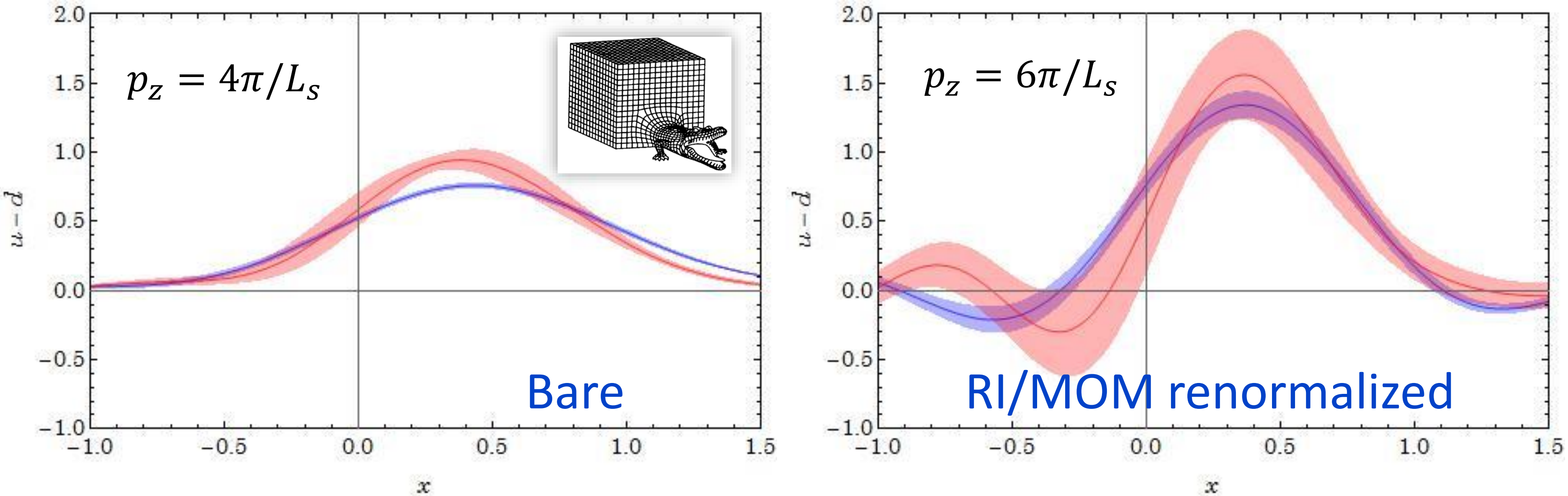
1705.00246, 1705.11193, 1706.00265, 1706.01295, 1706.05373, 1706.08962, 1707.03107, 1707.07152, 1708.02458, 1708.05301 …

Renormalization

§ Effect on quasi-PDFs

$$\tilde{q}_R(x, P_z, \mu_R) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP_z z} \tilde{h}_R(z, P_z, \mu_R)$$

$M_\pi \approx$ **310 MeV,**
$a \approx$ **0.12 fm**



Plot by Jianhui Zhang

§ Avoid mixing using different op for quark distribution

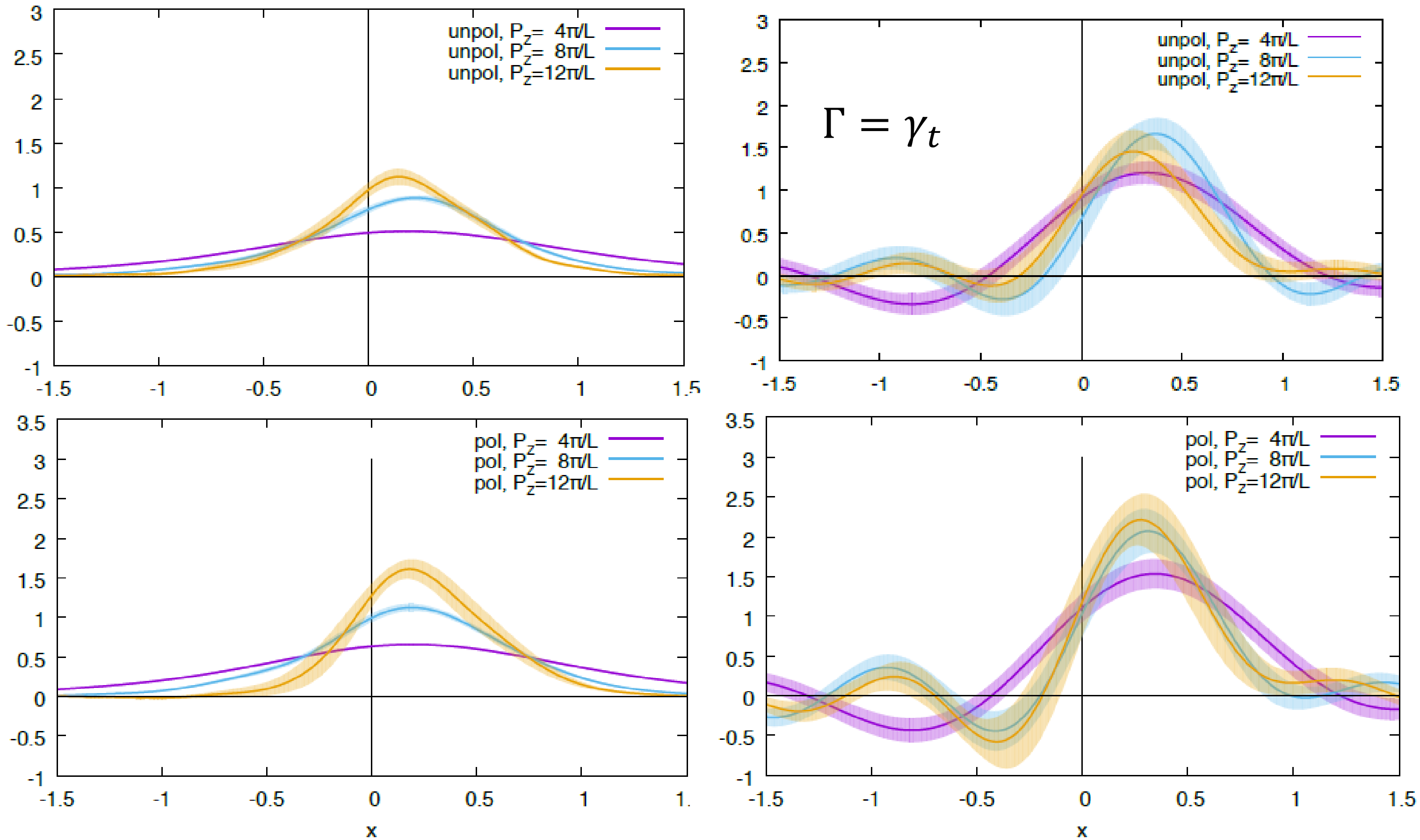
$h_R = Z_V h_{\gamma_t}$

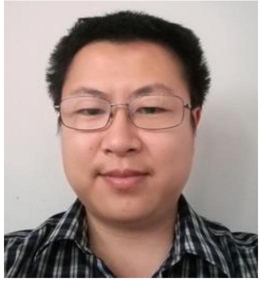
Physical Pion Mass

§ The problem persists/worsens at physical pion mass

$M_\pi \approx$ **135 MeV**, $a \approx$ **0.09 fm** , $L \approx$ **5.6 fm**

Bare | RI/MOM renormalized



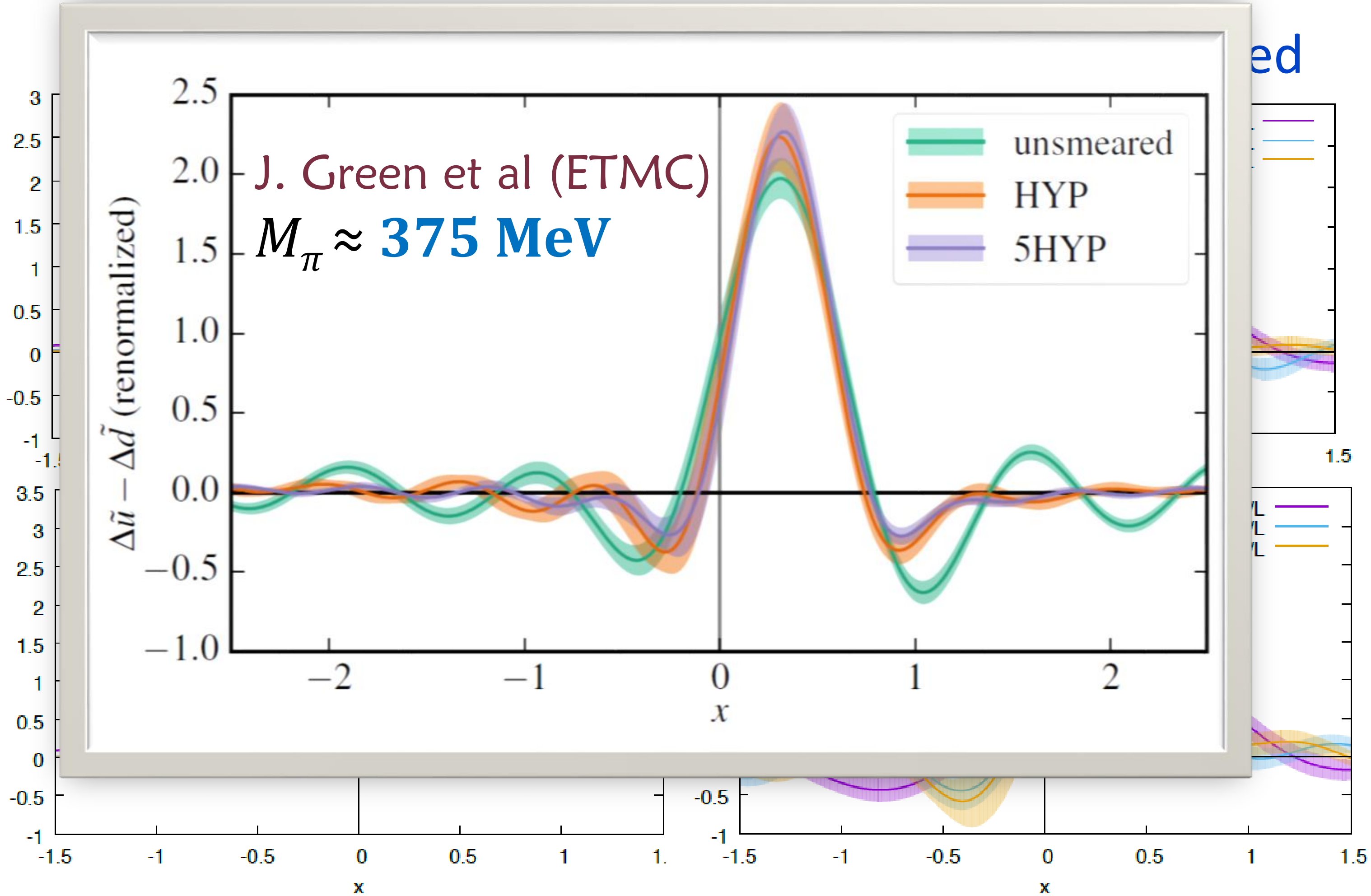


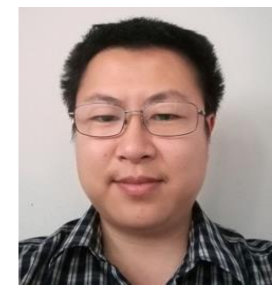
Yi-Bo Yang (MSU)

Physical Pion Mass

§ The problem persists/worsens at physical pion mass

$M_\pi \approx$ **135 MeV**, $a \approx$ **0.09 fm** , $L \approx$ **5.6 fm**

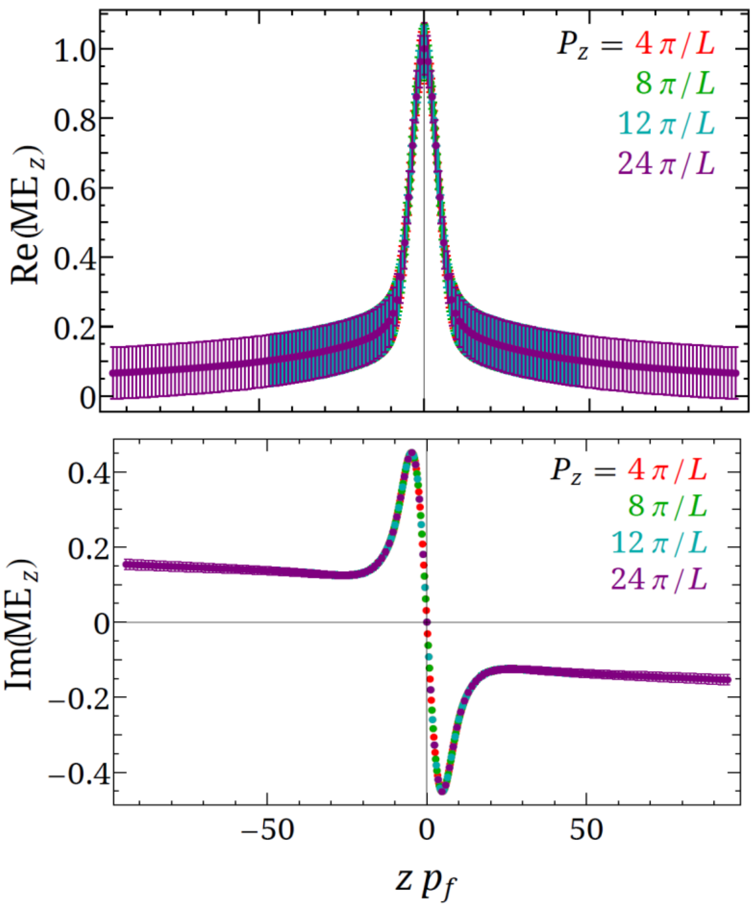


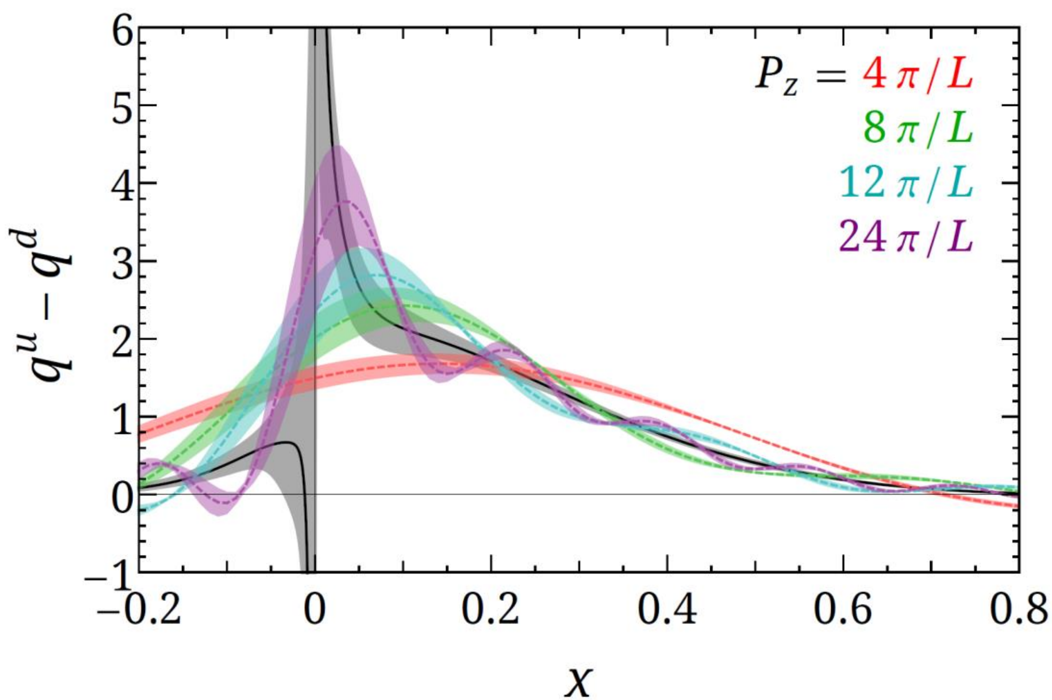


Yi-Bo Yang (MSU)

Physical Pion Mass

- Not a lattice problem but Fourier transform issue
- Simple exercise with CT14 PDF 1506.07443



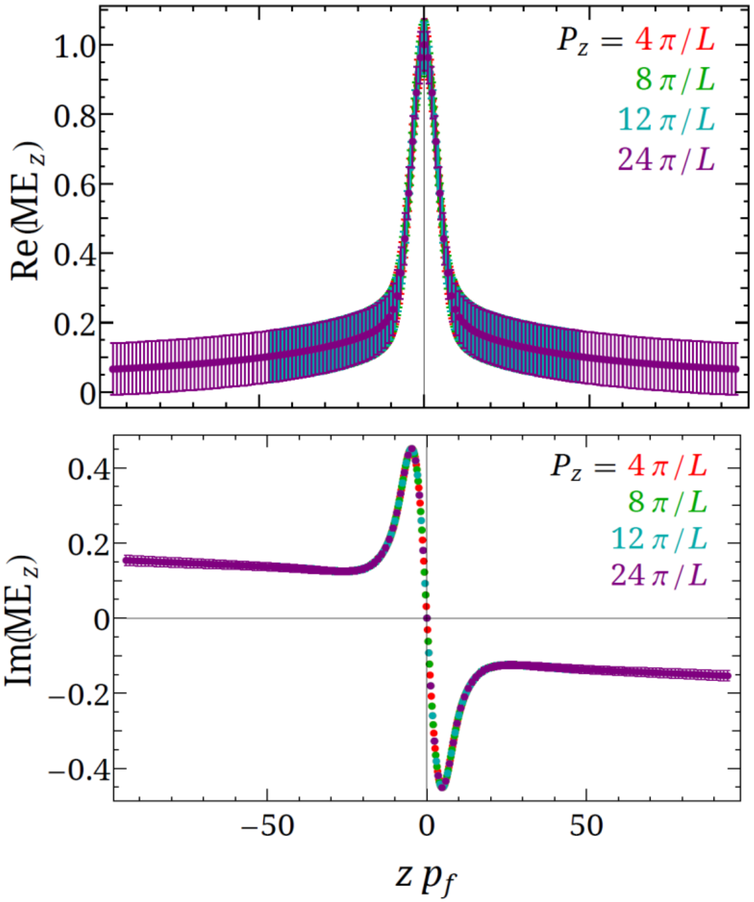


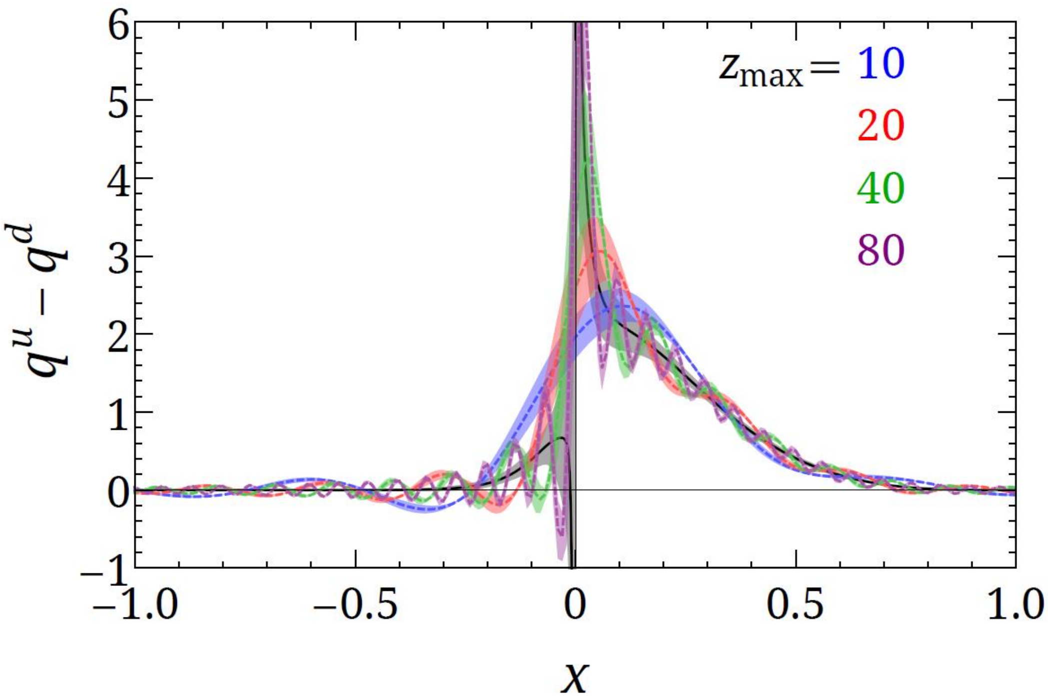
1708.05301 (LP3)

Fixed $L_z = 32$

Physical Pion Mass

- Not a lattice problem but Fourier transform issue
- Simple exercise with CT14 PDF 1506.07443





1708.05301 (LP3)

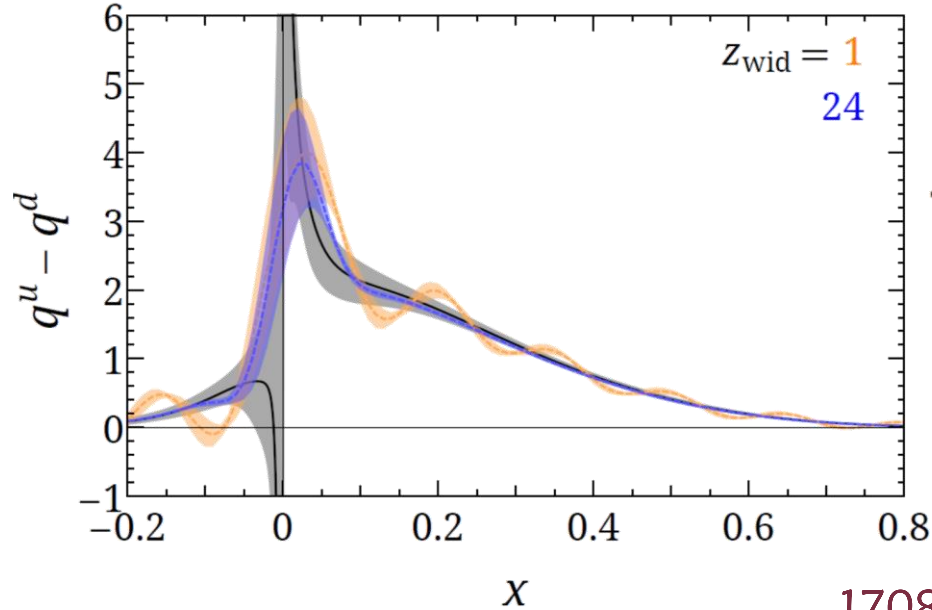
Fixed $P_z = 24\pi/L$

Physical Pion Mass

- Not a lattice problem but Fourier transform issue
- Two possible solutions proposed (likely more)

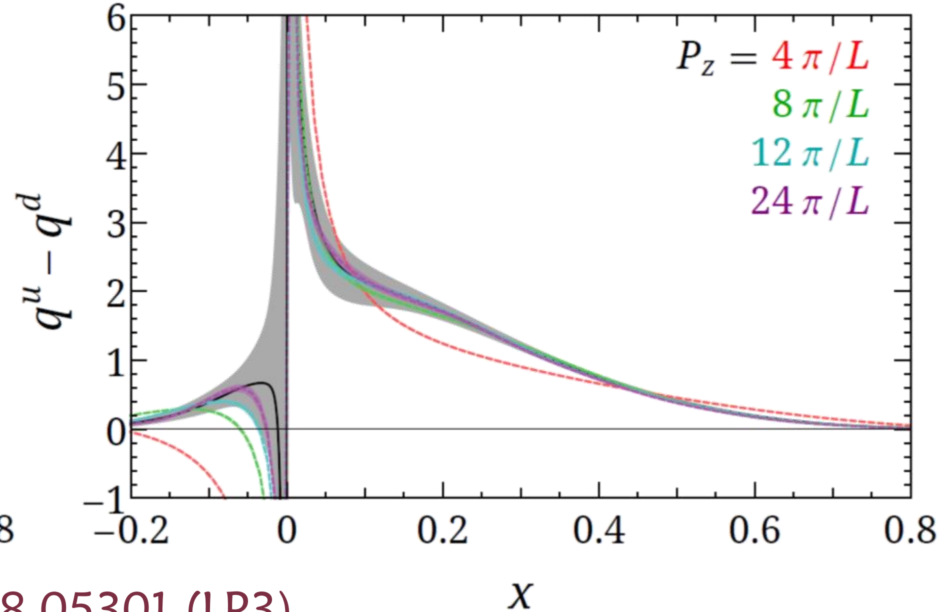
Filter approach

$$F(z_{\rm lim}, z_{\rm wid}) = \frac{1+\mathrm{erf}\left(\frac{z+z_{\rm lim}}{z_{\rm wid}}\right)}{2}\,\frac{1-\mathrm{erf}\left(\frac{z-z_{\rm lim}}{z_{\rm wid}}\right)}{2}$$



Derivative approach

$$q(x) = \int_{-z_{\max}}^{+z_{\max}} dz \frac{-P_z}{2\pi} \frac{e^{ixP_z z}}{iP_z x} h'(z)$$



1708.05301 (LP3)

Physical Pion Mass

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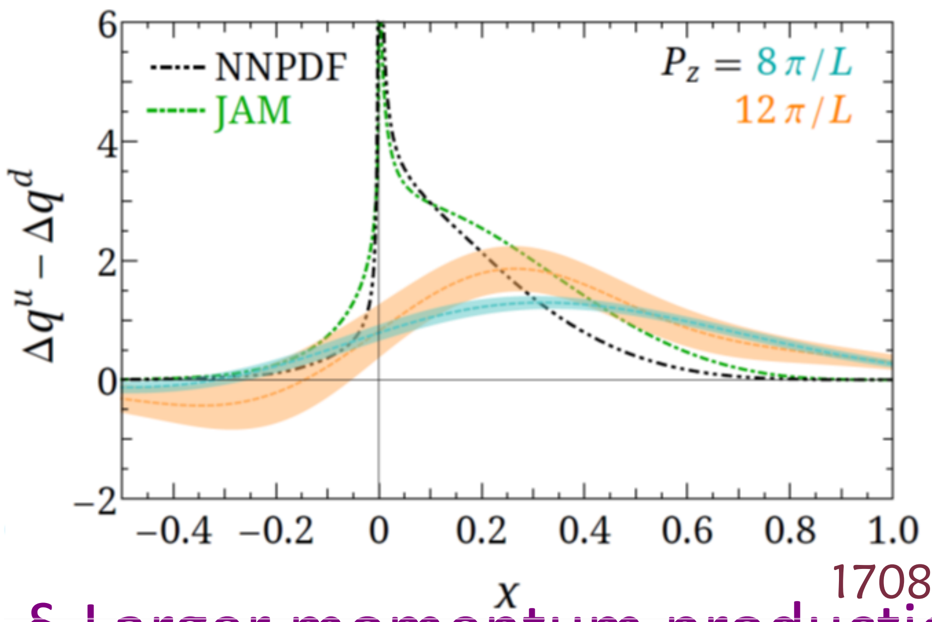
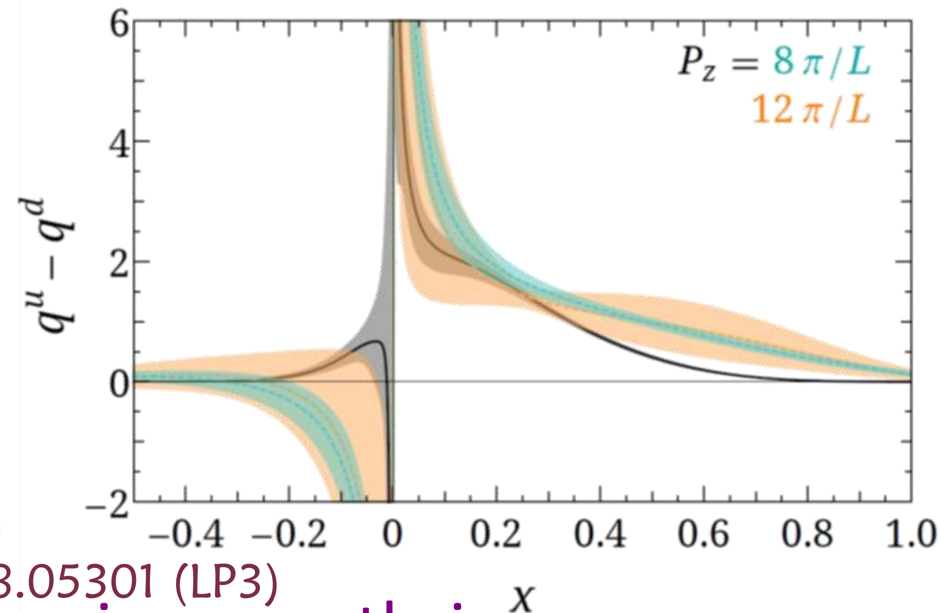
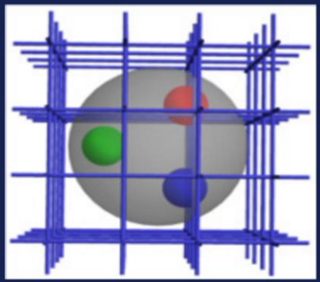
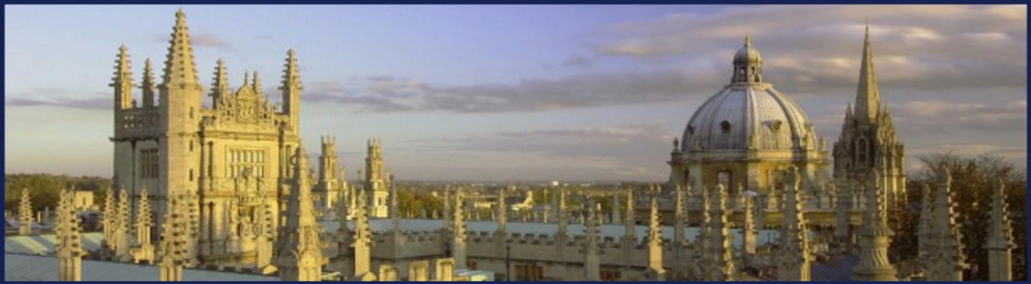


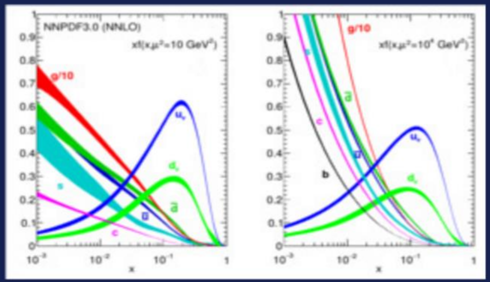


1708.05301 (LP3)

- Larger momentum production is currently in progress

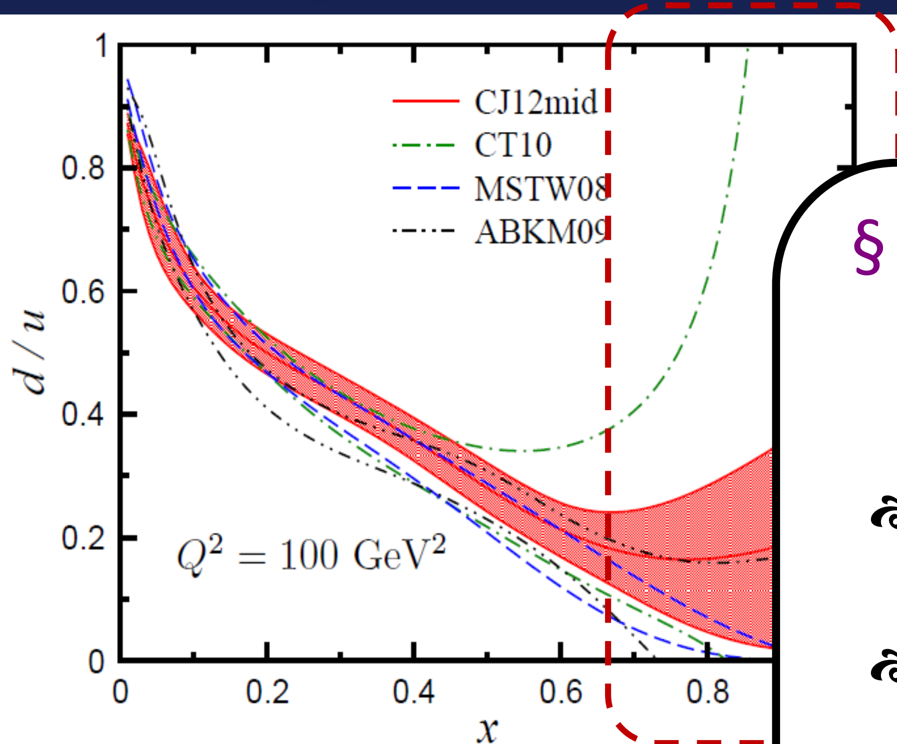






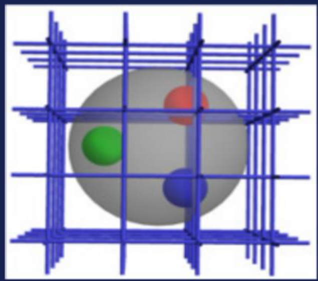
Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

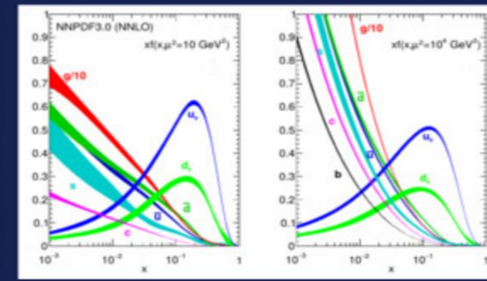
22-24 March 2017, Oxford, UK



Jimenez-Delgado, Melnitchouk, O
J.Phys. G40 (2013) 09310

- A first joint workshop with global-fitting community to address key LQCD inputs
  - http://www.physics.ox.ac.uk/confs/PDFlattice2017
  - Whitepaper study the needed precision of lattice PDFs in the large-x region

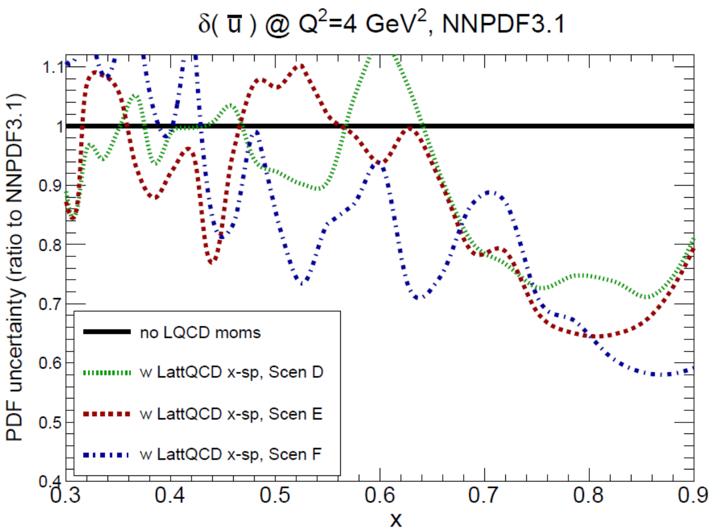


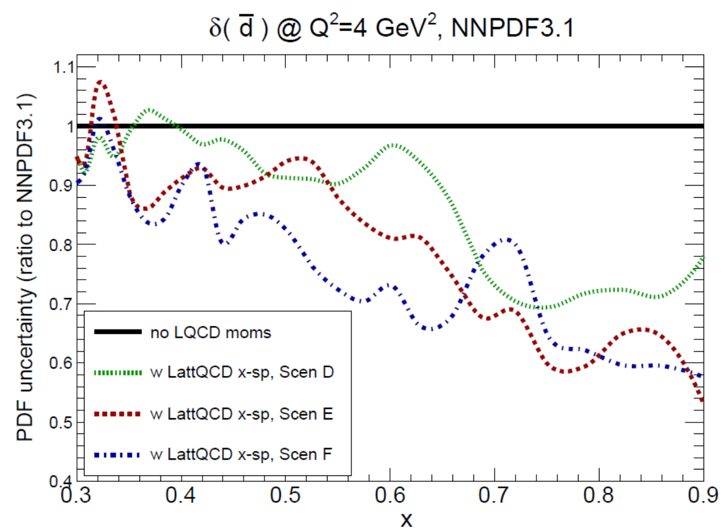


§ Implementing the pseudo-data from LQCD with x=0.7–0.9

$$u(x_i, Q^2) - d(x_i, Q^2) \text{ and } \bar{u}(x_i, Q^2) - \bar{d}(x_i, Q^2)$$

D: 12%
E: 6%
F: 3%





Lin et al, In preparation

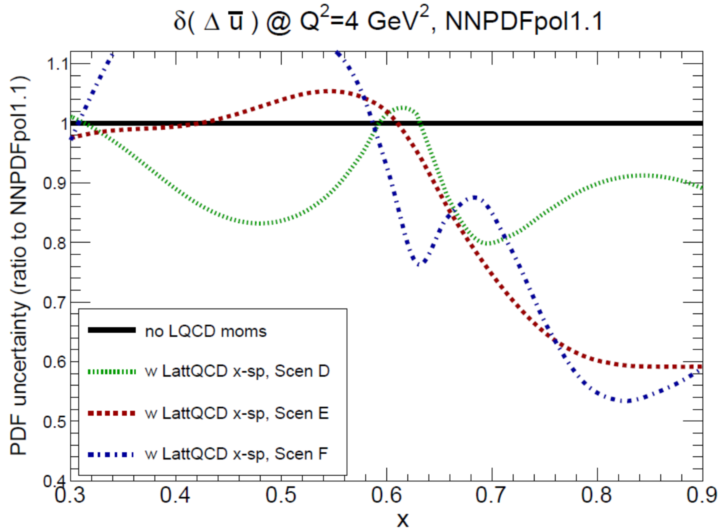
§ Implementing the pseudo-data from LQCD with $x = 0.7$–0.9

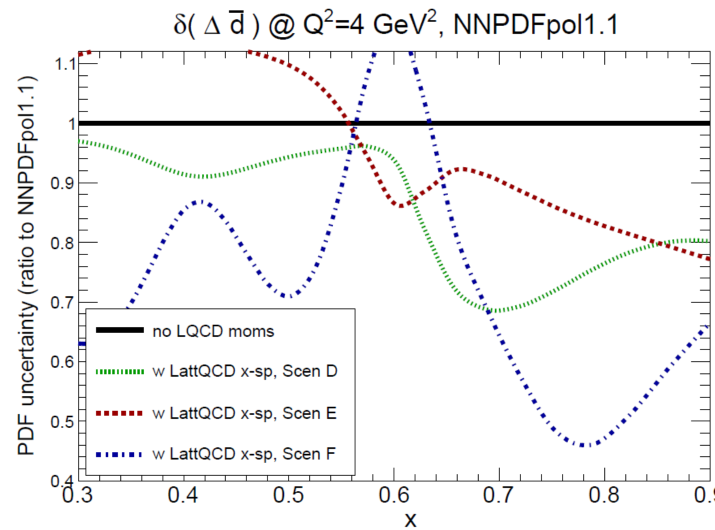
$$\Delta u(x_i, Q^2) - \Delta d(x_i, Q^2) \text{ and} \Delta \bar{u}(x_i, Q^2) - \Delta \bar{d}(x_i, Q^2)$$

D: 12%

E: 6%

F: 3%





Lin et al, In preparation

Summary & Outlook

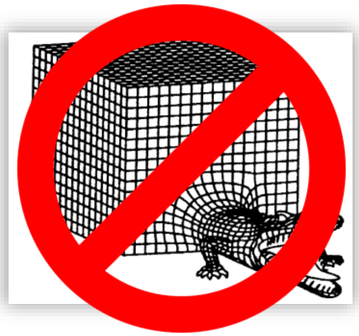
Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full *x*-distribution

- Most importantly, this can be done with today's computers
- First lattice approach to study sea asymmetry
- First look into PDA 1702.00008

§ Moving on to remove the systematics of earlier study

- Working on **renormalization**,
  statistics (all-mode averaging?),
  larger momentum boost, finer lattice spacing, …
- Long-term future for lattice hadronic physics



§ Small-*x* physics for EIC

- Combined analysis with precision moments

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Backup Slides

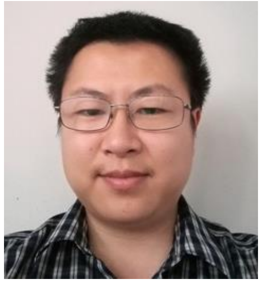
Renormalization

§ Long-link operator

$$O_\Gamma(z) = \bar{\psi}(z)\Gamma W_z(z,0)\psi(0)$$

§ Vector operator mixing with scalar ones 1706.01295 (LP3)

$$\begin{pmatrix} O^R_{\gamma_z}(z) \\ O^R_{\mathbb{I}}(z) \end{pmatrix} = \begin{pmatrix} Z_{VV}(z) & Z_{VS}(z) \\ Z_{SV}(z) & Z_{SS}(z) \end{pmatrix} \begin{pmatrix} O_{\gamma_z}(z) \\ O_{\mathbb{I}}(z) \end{pmatrix}$$



Yi-Bo Yang (MSU)

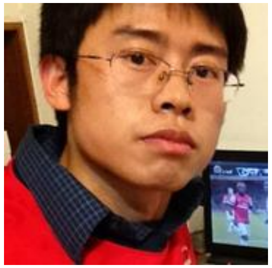
§ RI/MOM renormalization scheme 1706.01295 (LP3)

- $Z^{-1} =$

$$\frac{1}{12e^{-ip_z z}} \begin{pmatrix} \mathrm{Tr}[\tilde{\Gamma}\Lambda(p,z,\gamma_z)] & \mathrm{Tr}[\tilde{\Gamma}\Lambda(p,z,\mathbb{I})] \\ \mathrm{Tr}[\Lambda(p,z,\gamma_z)] & \mathrm{Tr}[\Lambda(p,z,\mathbb{I})] \end{pmatrix}_{p^2=\mu_R^2,\, p_z=P_z}$$

$$\Lambda(p,z,\Gamma) = S(p)^{-1} \left\langle \sum_w S^\dagger(p, w+zn)\Gamma W_z(w+zn) S(p,w) \right\rangle S(p)^{-1}$$

projected with $\tilde{\Gamma} = \not{p}/p_z$



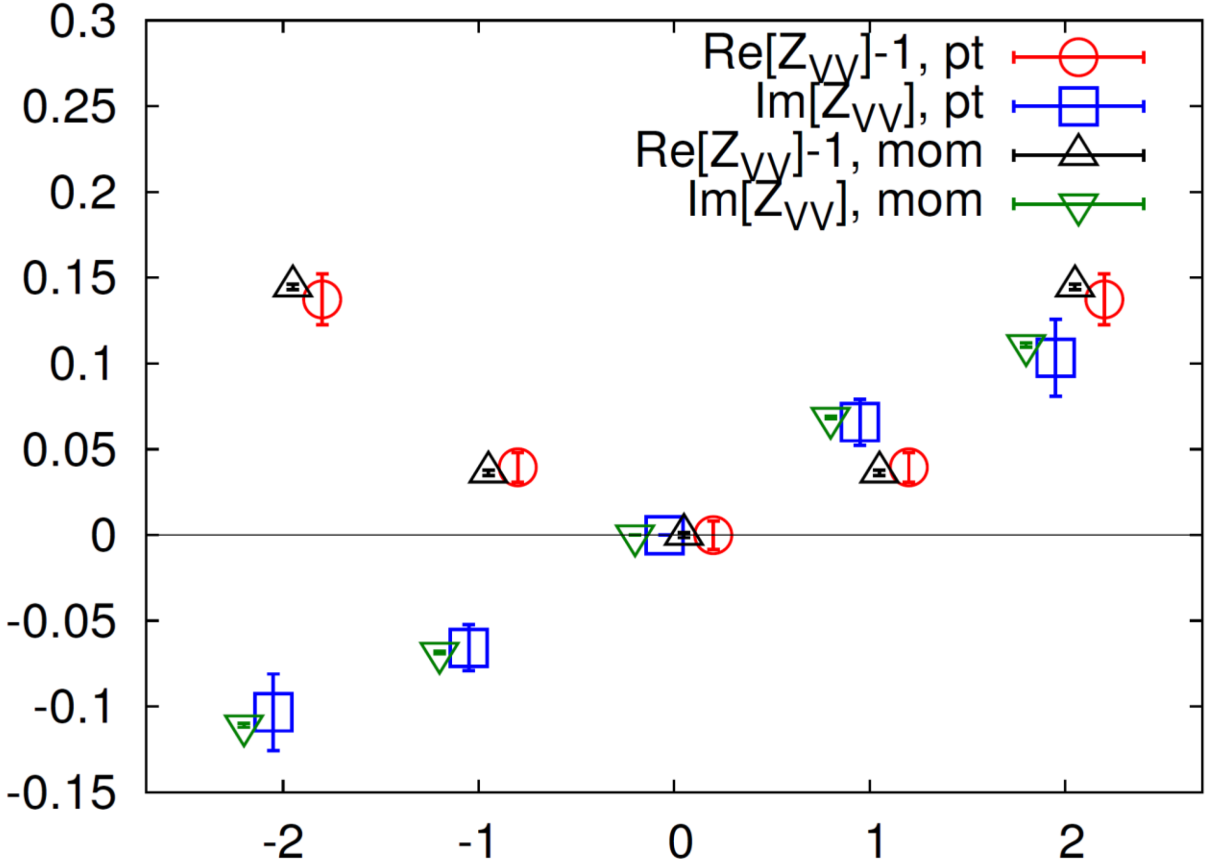
Yong Zhao (MIT)

- Test case: $a \approx 0.12$ fm, $M_\pi \approx 310$ MeV, clover/HISQ

Renormalization

§ RI/MOM renormalization scheme

Momentum source vs point source for $|z| \leq 2$



$M_\pi \approx$ **310 MeV**

$a \approx$ **0.12 fm**

$p_z = 6\pi/L_s$

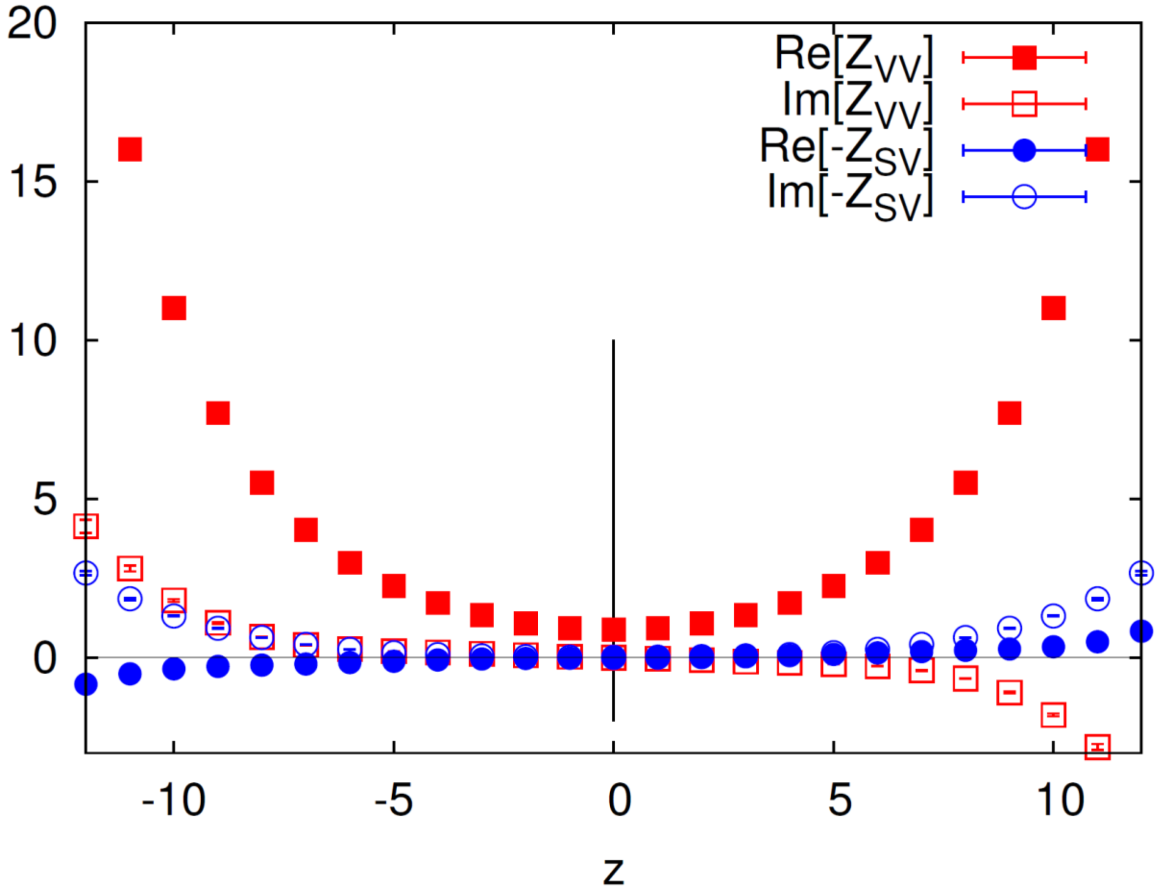
$\mu_R^2 = p^2$

$= 5.74\ \text{GeV}^2$

Plot by Yi-Bo Yang; 1706.01295 (LP3)

Renormalization

§ RI/MOM renormalization scheme



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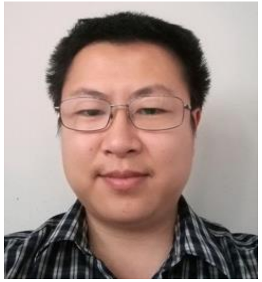


Yi-Bo Yang (MSU)

$$h_R = Z_{VV} h_{\gamma_z} + Z_{SV} h_{\mathbb{I}}$$

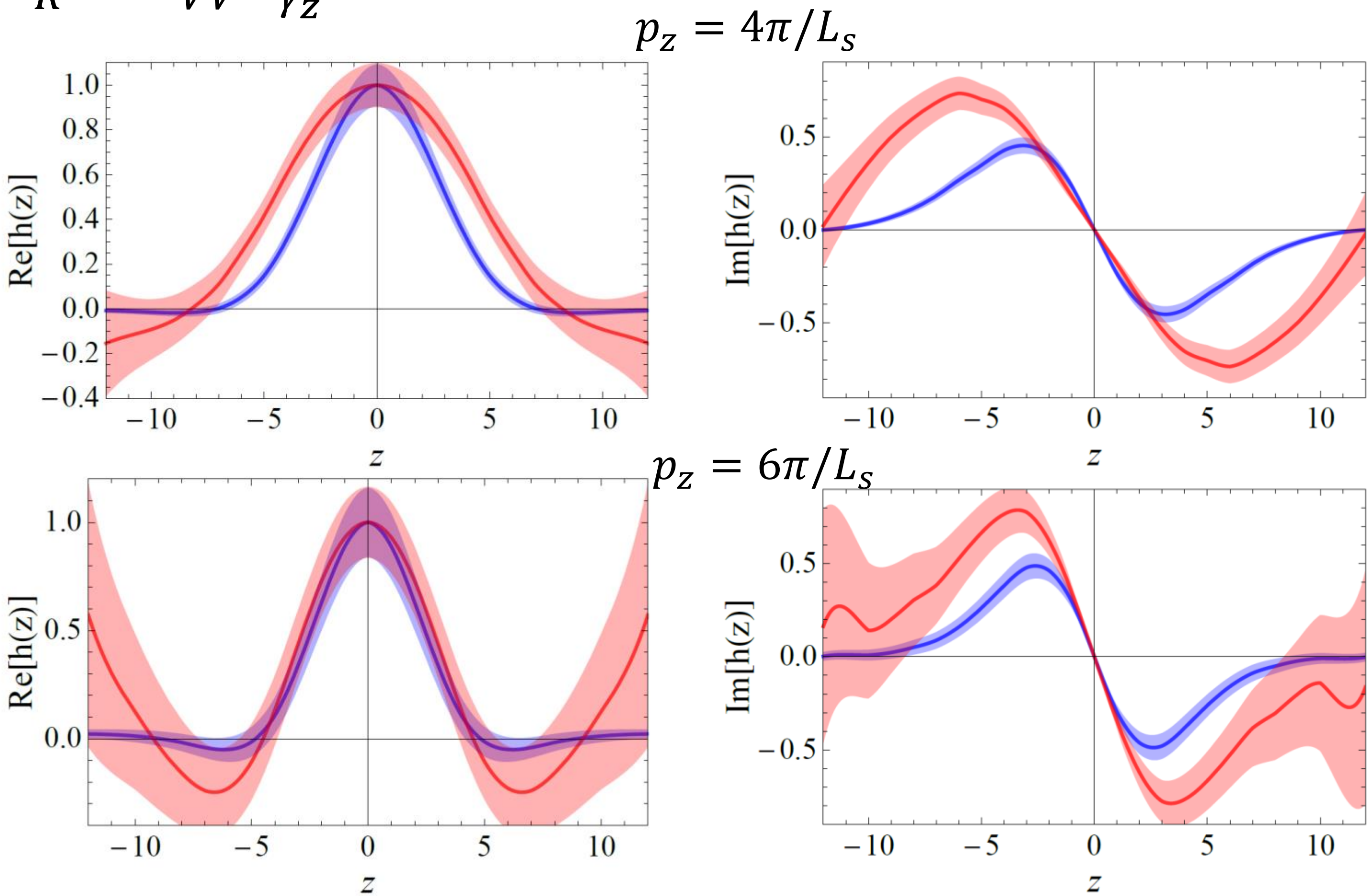
Plot by Yi-Bo Yang; 1706.01295 (LP3)

Renormalization

§ Effect on nucleon matrix elements as function of z

- $h_R \approx Z_{VV} h_{\gamma_z}$

$M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**



Jian-Hui Zhang (Regensburg)

Plot by Jianhui Zhang; 1706.01295 (LP3)