

# **Hadron tomography by generalized parton distributions and generalized distribution amplitudes**

**Shunzo Kumano**

**High Energy Accelerator Research Organization (KEK)**

**J-PARC Center (J-PARC)**

**Graduate University for Advanced Studies (SOKENDAI)**

**<http://research.kek.jp/people/kumanos/>**

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**<http://www.int.washington.edu/PROGRAMS/17-3/>**

**September 29, 2017**

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GPD (Generalized Parton Distribution)
- **GDA and possible KEKB/ILC/LHC project**  
GDA (Generalized Distribution Amplitude)  
Gravitational radii for hadrons
- **Summary**

# Motivations

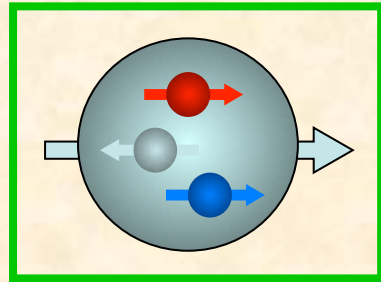
- **3D structure of hadrons**
- **Nucleon spin structure**
- **Exotic hadrons**
- **Gravitational properties of quarks and gluons  
(hadrons)?**

...

**Hadron tomography: 3D structure functions are (can be) investigated at high-energy lepton and hadron facilities (BNL, JLab, Fermilab, CERN, J-PARC, KEKB, GSI, IHEP@China & Russia, EIC, LHeC, ILC, ...).**

# Recent progress on origin of nucleon spin

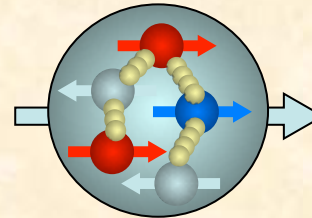
“old” standard model



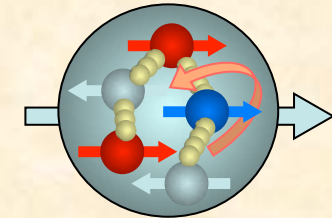
$$p_{\uparrow} = \frac{1}{3\sqrt{2}} \left( uud [2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow] + \text{permutations} \right)$$

$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

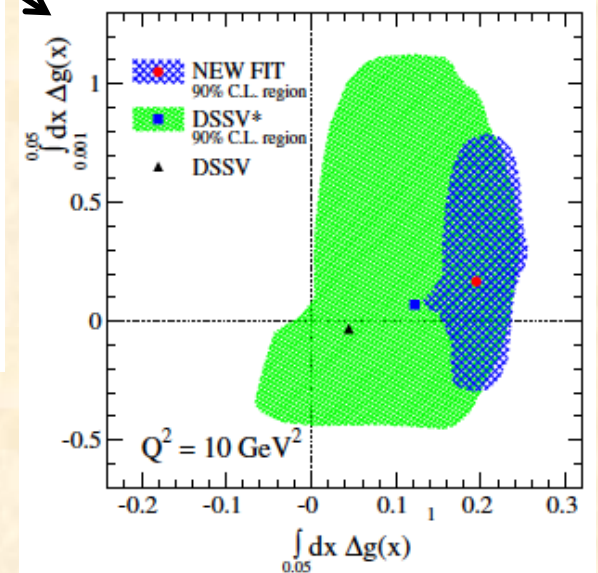
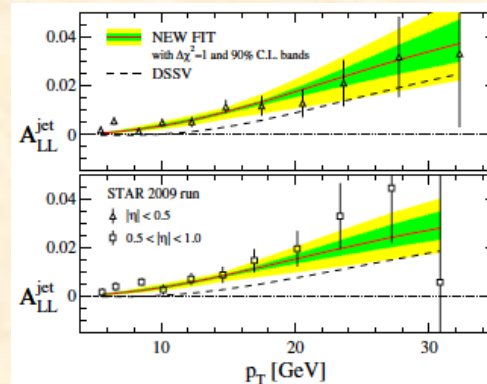
$$\Delta\Sigma = \sum_i \int dx [\Delta q_i(x) + \Delta \bar{q}_i(x)] \rightarrow 1 \text{ (100\%)}$$



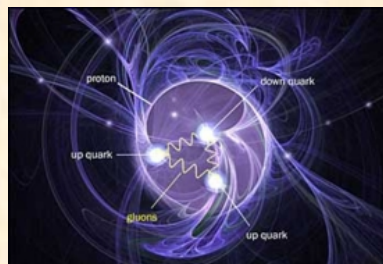
gluon spin



angular momentum



CNN (2014)

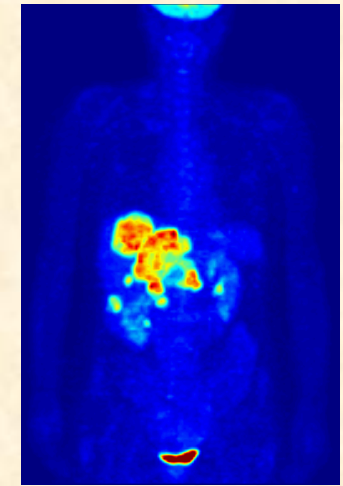


Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L_{q,g}$$

# Nucleon (hadron) tomography

PET (Positron Emission Tomography)

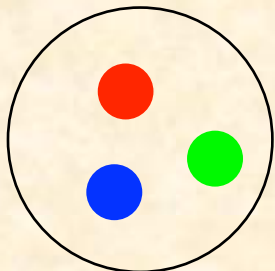


Classical density distribution

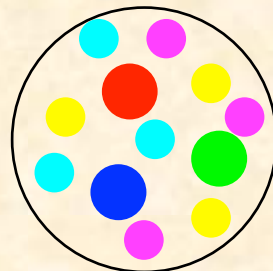
**3D picture of nucleon**  
(Density distribution of quantum system:  
Quantum tomography)

1D(Bjorken-x) picture@HERA

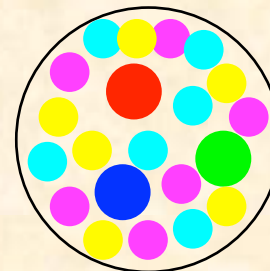
Low energy



Intermediate energy



High energy



# Progress in exotic hadrons

$q\bar{q}$  Meson  
 $q^3$  Baryon

$q^2\bar{q}^2$  Tetraquark  
 $q^4\bar{q}$  Pentaquark  
 $q^6$  Dibaryon

...  
 $q^{10}\bar{q}$  e.g. Strange tribaryon

...  
 $gg$  Glueball

...

- $\Theta^+(1540)???:$  LEPS

$uudd\bar{s} ?$

Pentaquark?

- **Kaonic nuclei?**: KEK-PS, ...  
 Strange tribaryons, ...

$K^- pnn, K^- ppn ?$   
 $K^- pp ?$

- **X (3872), Y(3940):** Belle  
 Tetraquark,  $D\bar{D}$  molecule

$c\bar{c}$   
 $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$   
 $D^+(c\bar{d})D^-(\bar{c}d) ?$

- **$D_{sJ}(2317), D_{sJ}(2460)$ :** BaBar, CLEO, Belle  
 Tetraquark, DK molecule

$c\bar{s}$   
 $D^0(c\bar{u})K^+(u\bar{s})$   
 $D^+(c\bar{d})K^0(d\bar{s}) ?$

- **Z (4430):** Belle

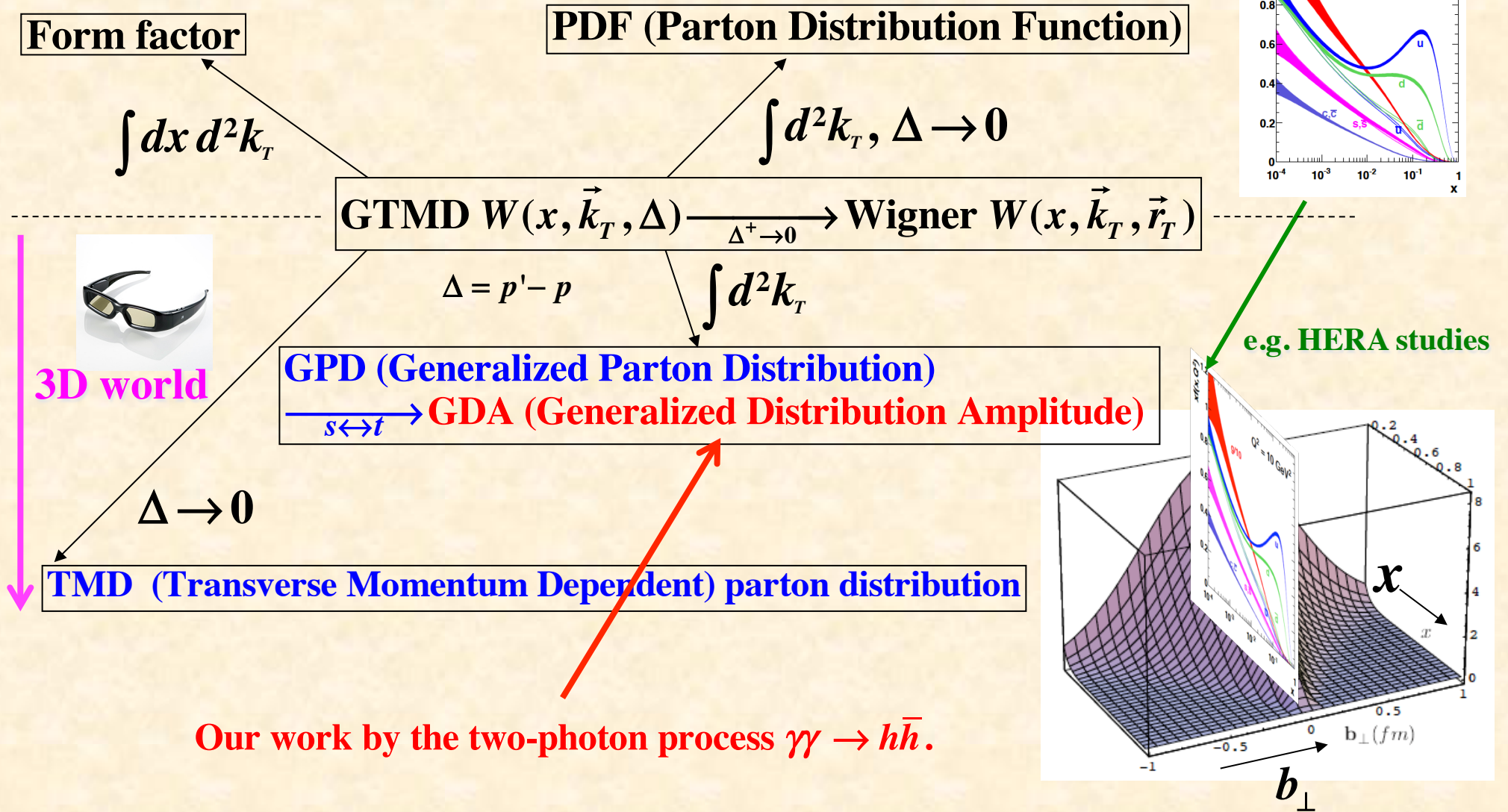
Tetraquark, ...

$c\bar{c}u\bar{d}, D$  molecule?

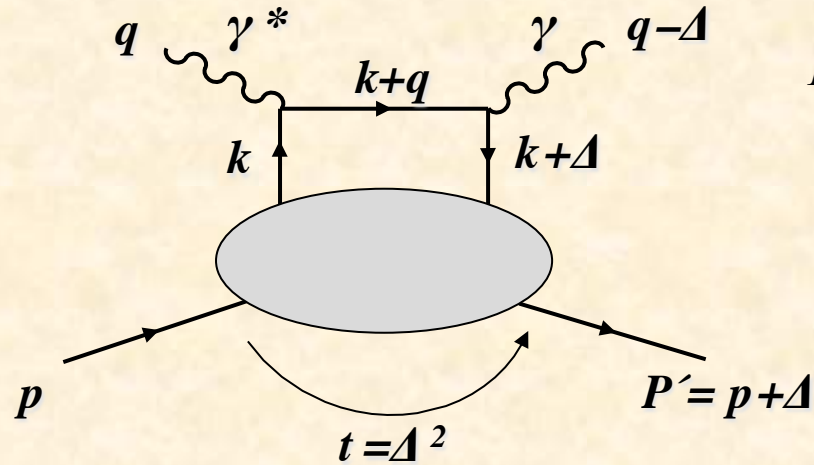
- **$P_c(4380), P_c(4450)$ :** LHCb

- ...  $u\bar{c}udc, \bar{D}(u\bar{c})\Sigma_c^*(udc), \bar{D}^*(u\bar{c})\Sigma_c(udc)$  molecule?

# Wigner distribution and various structure functions



# Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

$$\text{Bjorken variable} \quad x = \frac{Q^2}{2p \cdot q}$$

$$\text{Momentum transfer squared} \quad t = \Delta^2$$

$$\text{Skewness parameter} \quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[ H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[ \tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

**Forward limit: PDFs**  $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

**First moments: Form factors**

$$\text{Dirac and Pauli form factors } F_1, F_2 \quad \int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$$

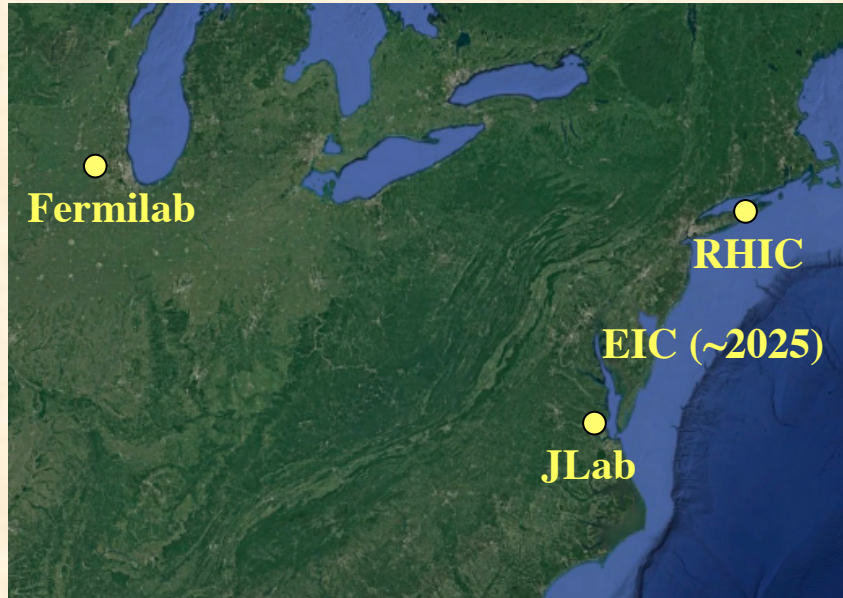
$$\text{Axial and Pseudoscalar form factors } G_A, G_P \quad \int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$$

**Second moments: Angular momenta**

$$\text{Sum rule: } J_q = \frac{1}{2} \int_{-1}^1 dx x \left[ H_q(x, \xi, t=0) + E_q(x, \xi, t=0) \right], \quad J_q = \frac{1}{2} \Delta q + L_q$$



# Hadron-tomography studies in US and Europe



**Fermilab: Main Injector (120 GeV proton),  
Neutrino (Minerva, several GeV)**

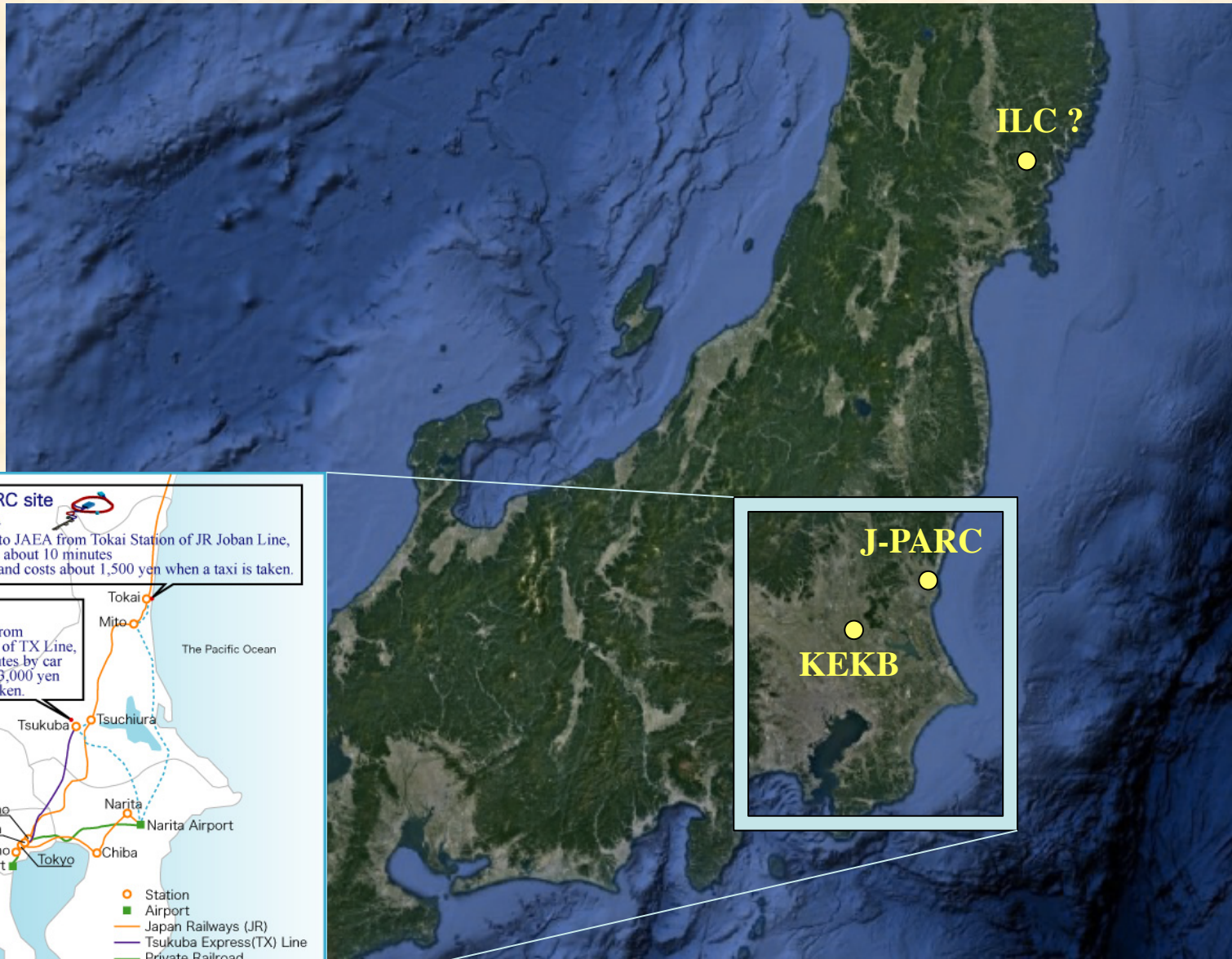
**RHIC: Spin (polarized p + polarized p)  
Heavy ion (e.g. UPC: Ultra-Peripheral Collision)**

**EIC (Electron Ion Collider, ~2025)**

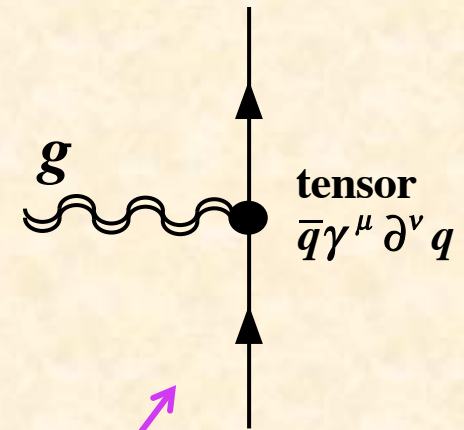
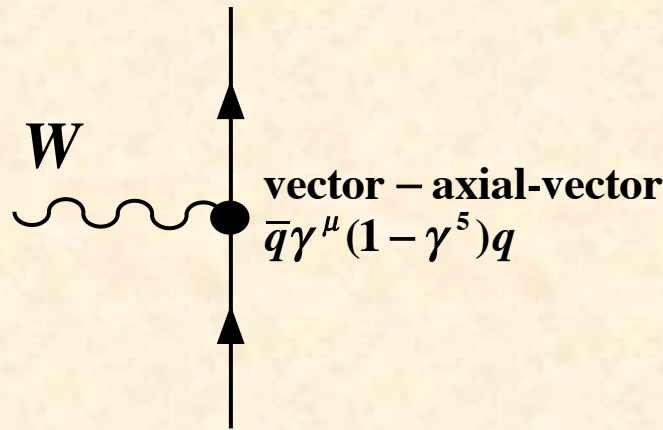
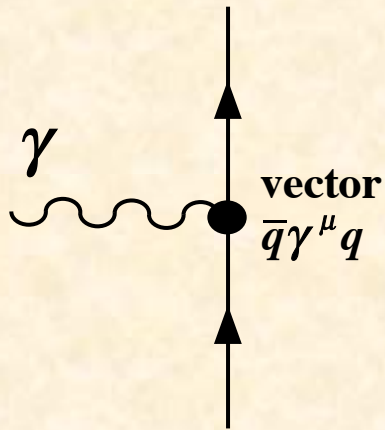


**CERN: COMPASS ( $\mu$ ,  $\pi$  beams)  
LHC Heavy ion  
(e.g. UPC: Ultra-Peripheral Collision)**

# Possible hadron-tomography studies at J-PARC, KEKB, ILC?



# Why gravitational interactions with hadrons ?



Electron-proton elastic scattering cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_f \cos^2 \frac{\theta}{2}}{4E_i^3 \sin^4(\theta/2)} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right], \quad \tau = -\frac{q^2}{4M^2}$$

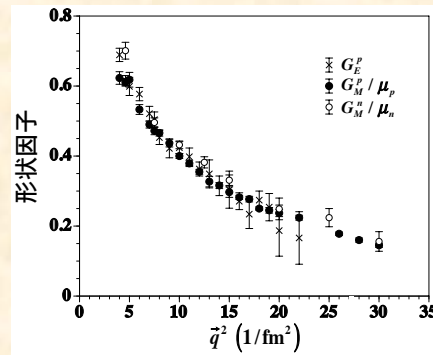
$$F(\vec{q}) = \int d^3x e^{i\vec{q}\cdot\vec{x}} \rho(\vec{x}) = \int d^3x \left[ 1 - \frac{1}{2}(\vec{q}\cdot\vec{x})^2 + \dots \right] \rho(\vec{x})$$

$$\langle r^2 \rangle = \int d^3x r^2 \rho(\vec{x}), \quad r = |\vec{x}|$$

$$\sqrt{\langle r^2 \rangle} = \text{root-mean-square (rms) radius}$$

$$F(\vec{q}) = 1 - \frac{1}{6} \vec{q}^2 \langle r^2 \rangle + \dots, \quad \langle r^2 \rangle = -6 \frac{dF(\vec{q})}{d\vec{q}^2} \Big|_{\vec{q}^2 \rightarrow 0}$$

$$\rho(r) = \frac{\Lambda^3}{8\pi} e^{-\Lambda r} \Leftrightarrow \text{Dipole form: } F(q) = \frac{1}{(1 + |\vec{q}|^2 / \Lambda^2)^2}, \quad \Lambda^2 \approx 0.71 \text{ GeV}^2$$



How about gravitational radius?

Proton-charge-radius puzzle:

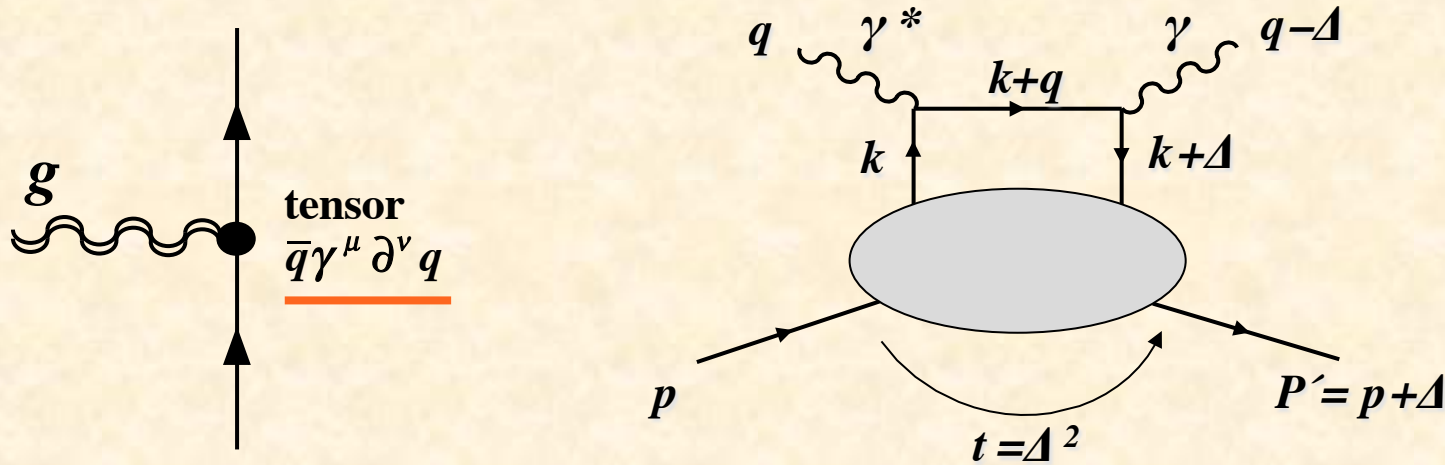
$$R_{\text{electron scattering}} = 0.8775 \text{ fm}$$



$$R_{\text{muonic atom}} = 0.8418 \text{ fm}$$



# Gravitational interactions and 3D structure functions



GPDs: 
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[ H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

Angular momentum: 
$$J_q = \frac{1}{2} \int_{-1}^1 dx x \left[ H_q(x, \xi, t=0) + E_q(x, \xi, t=0) \right], \quad J_q = \frac{1}{2} \Delta q + L_q$$

Non-local operator of GPDs/GDAs:

$$\begin{aligned} (P^+)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[ \bar{q}(-z/2) \gamma^+ q(z/2) \right] \Big|_{z^+=0, \vec{z}_\perp=0} &= \left( i \frac{d}{dz^-} \right)^{n-1} \left[ \bar{q}(-z/2) \gamma^+ q(z/2) \right] \Big|_{z=0} \\ &= \bar{q}(0) \gamma^+ \left( i \vec{\partial}^+ \right)^{n-1} q(0) \end{aligned}$$

= energy-momentum tensor of a quark for  $n = 2$  (electromagnetic for  $n = 1$ )

= source of gravity

# References

## GPDs at J-PARC

**SK, M. Strikman, K. Sudoh, PRD 80 (2009) 074003.**

**T. Sawada, Wen-Chen Chang, S. Kumano, Jen-Chieh Peng,  
S. Sawada, K. Tanaka, PRD 93 (2016) 114034.**

## GPDs and GDAs (including exotic hadrons)

**H. Kawamura, SK, PRD 89 (2014) 054007.**

**SK, Q.-T. Song, O. Teryaev, KEK-TH-1959/J-PARC-TH-0086.**

**Related topics: Constituent counting rule:**

**H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010.**

**W.-C. Chang, SK, T. Sekihara, PRD 93 (2016) 034006.**

# Constituent-counting rule for exotic hadrons

**H. Kawamura, S. Kumano, T. Sekihara,  
Phys. Rev. 88 (2013) 034010.**

**W.-C. Chang, S. Kumano, and T. Sekihara,  
Phys. Rev. D 93 (2016) 034006.**

# Research purposes

It is not easy to find undoubted evidence for exotic hadrons by global observables (mass, spin, parity, decay width) at low energies.

(1) Determination of internal structure of exotic hadrons by high energy processes, where quark-gluon degrees of freedom appear.

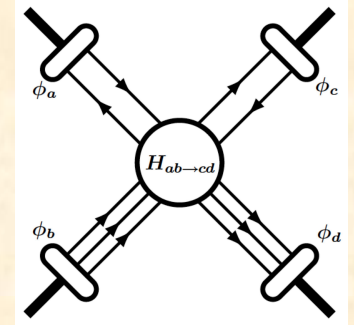
Constituent-counting rule could be used because it counts internal constituents.

(2) Investigation on transition from hadron degrees of freedom to quark-gluon degrees of freedom for exotic hadrons.

$$\frac{d\sigma_{a+b \rightarrow c+d}}{dt} \simeq \frac{1}{16\pi s^2} \sum_{pol} |M_{a+b \rightarrow c+d}|^2 \Rightarrow \frac{d\sigma_{a+b \rightarrow c+d}}{dt} = \frac{1}{s^{n-2}} f_{a+b \rightarrow c+d}(t/s) \quad \text{constituent-counting rule}$$

$n = n_a + n_b + n_c + n_d$

# Constituent-counting rule in perturbative QCD: Hard exclusive processes $a + b \rightarrow c + d$



Consider the hard exclusive hadron reaction  $a + b \rightarrow c + d$

$$M_{ab \rightarrow cd} = \int d[x_a] d[x_b] d[x_c] d[x_d] \phi_c([x_c]) \phi_d([x_d]) H_M([x_a], [x_b], [x_c], [x_d], Q^2) \phi_a([x_a]) \phi_b([x_b])$$

$\phi_p$  = proton distribution amplitude,  $H_M$  = hard amplitude (calculated in pQCD)

Rule for estimating  $M_{ab \rightarrow cd}$

(1) Feynman diagram: Draw leading and connected Feynman diagram by connecting  $n / 2$  quark lines by gluons.

(2) Gluon propagators: The factor  $1/P^2$  is assigned for each gluon propagator.

There are  $n / 2 - 1$  gluon propagators  $\sim 1/(P^2)^{n/2-1}$ .

(3) Quark propagators: The factor  $1/P$  is assigned for each quark propagator.

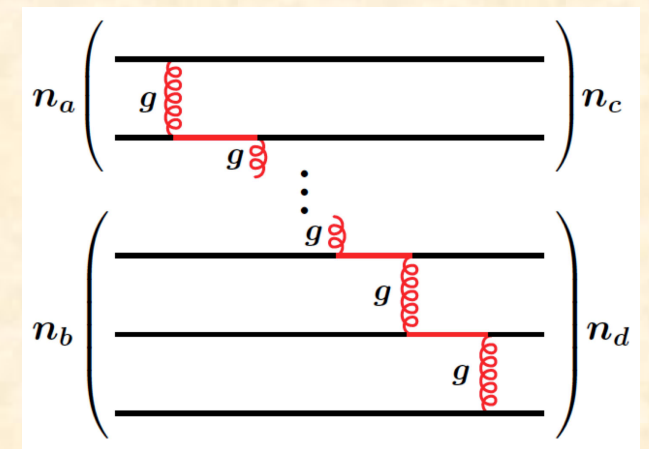
There are  $n / 2 - 2$  gluon propagators  $\sim 1/(P)^{n/2-2}$ .

(4) External quarks: The factor  $\sqrt{P}$  is assigned for each external quark.

There are  $n$  gluon propagators  $\sim (\sqrt{P})^n$ .

$$M_{ab \rightarrow cd} \sim \frac{1}{(P^2)^{n/2-1}} \frac{1}{(P)^{n/2-2}} (\sqrt{P})^n = \frac{(P)^{n/2}}{(P)^{n-2} (P)^{n/2-2}} = \frac{1}{(P)^{n-4}} \sim \frac{1}{s^{n/2-2}}$$

Cross section:  $\frac{d\sigma_{ab \rightarrow cd}}{dt} \simeq \frac{1}{16\pi^2} \sum_{spol} |M_{ab \rightarrow cd}|^2 \sim \frac{1}{s^{n-2}}$





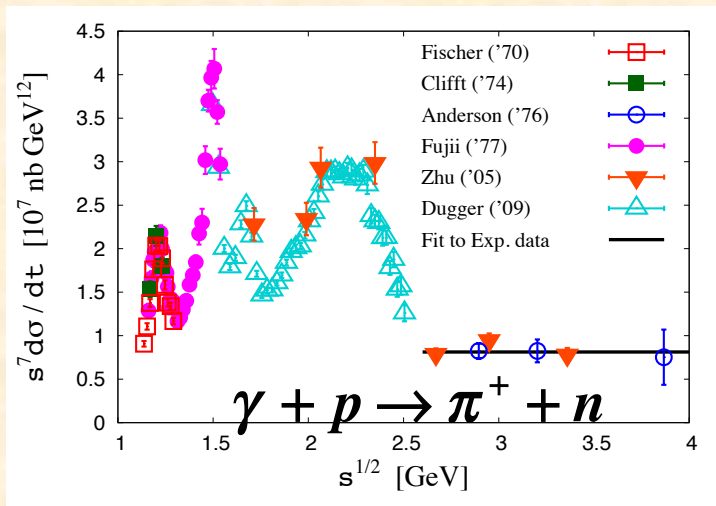
# Constituent-counting rule, Transition from hadron degrees of freedom to quark-gluon ones

## Typical current situation

- Transition from hadron d.o.f to quark d.o.f.
- (Looks like) Constituent-counting scaling

## BNL experiment

C. White et al., PRD 49 (1994) 58.

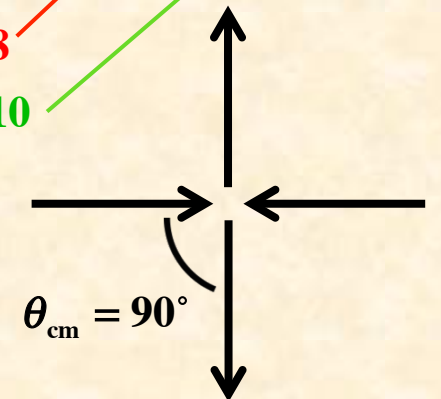


No.	Interaction	Cross section		$n-2$ ( $\frac{d\sigma}{dt} \sim 1/s^{n-2}$ )
		E838	E755	
1	$\pi^+ p \rightarrow p\pi^+$	$132 \pm 10$	$4.6 \pm 0.3$	$6.7 \pm 0.2$
2	$\pi^- p \rightarrow p\pi^-$	$73 \pm 5$	$1.7 \pm 0.2$	$7.5 \pm 0.3$
3	$K^+ p \rightarrow pK^+$	$219 \pm 30$	$3.4 \pm 1.4$	$8.3^{+0.6}_{-1.0}$
4	$K^- p \rightarrow pK^-$	$18 \pm 6$	$0.9 \pm 0.9$	$\geq 3.9$
5	$\pi^+ p \rightarrow p\rho^+$	$214 \pm 30$	$3.4 \pm 0.7$	$8.3 \pm 0.5$
6	$\pi^- p \rightarrow p\rho^-$	$99 \pm 13$	$1.3 \pm 0.6$	$8.7 \pm 1.0$
13	$\pi^+ p \rightarrow \pi^+ \Delta^+$	$45 \pm 10$	$2.0 \pm 0.6$	$6.2 \pm 0.8$
15	$\pi^- p \rightarrow \pi^- \Delta^-$	$24 \pm 5$	$\leq 0.12$	$\geq 10.1$
17	$pp \rightarrow pp$	$3300 \pm 40$	$48 \pm 5$	$9.1 \pm 0.2$
18	$\bar{p}p \rightarrow \bar{p}p$	$75 \pm 8$	$\leq 2.1$	$\geq 7.5$

JLab: L.Y. Zhu *et al.*, PRL 91, 022003 (2003);  
 PRC 71, 044603 (2005);  
 W. Chen *et al.*, PRL 103, 012301 (2009).

$$n-2: (2+3+2+3) - 2 = 8$$

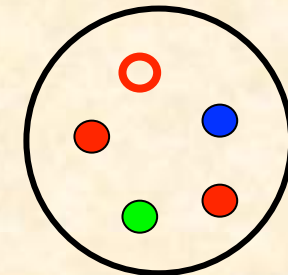
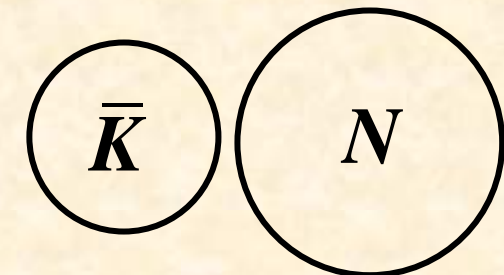
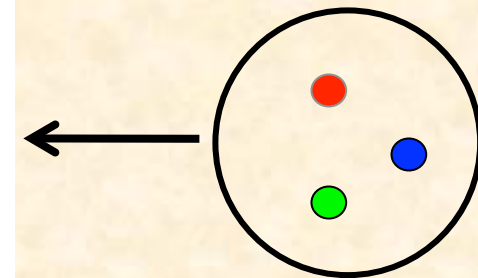
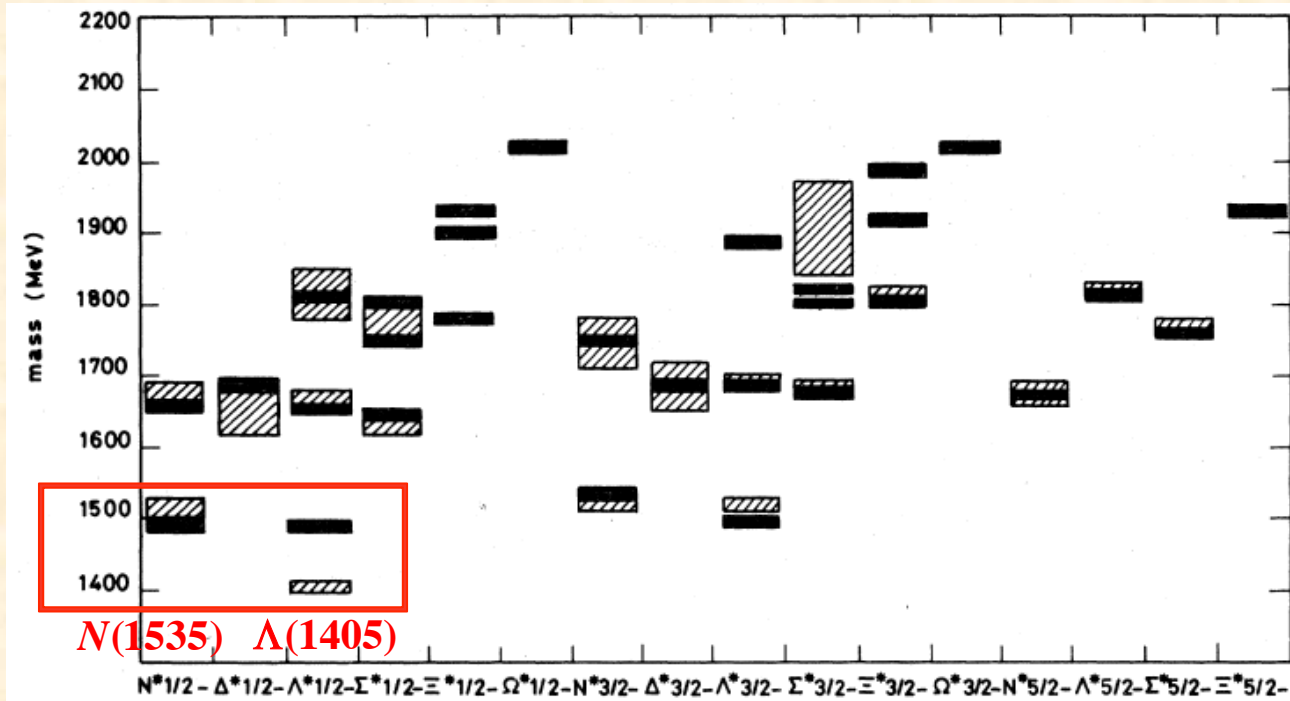
$$(3+3+3+3) - 2 = 10$$



see R. A. Schumacher and M. M. Sargsian,  
 PRC 83 (2011) 025207 for hyperon production

# $\Lambda(1405)$ : exotic hadron?

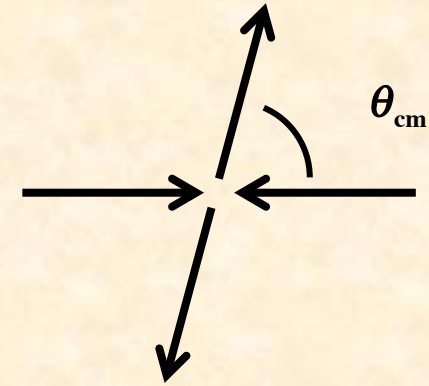
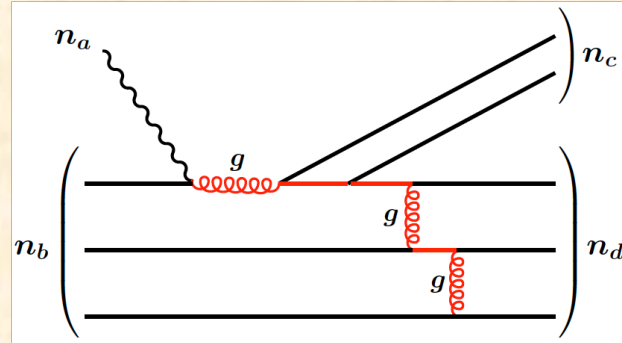
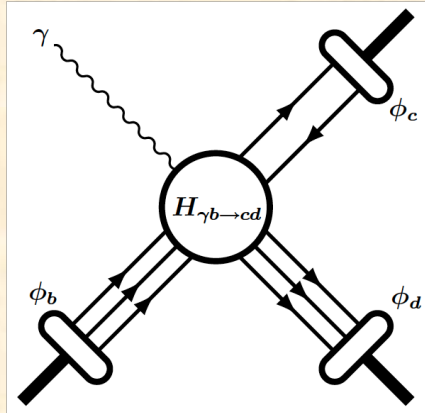
Negative-parity baryons  
N. Isgur and G. Karl,  
PRD 18 (1978) 4187.



Most spectra agree with the ones by a  $3q$ -picture

- Only  $\Lambda(1405)$  deviates from the measurement.
- Difficult to understand the small mass of  $\Lambda(1405)$  in comparison with  $N(1535)$ .  
→  $\bar{K}N$  molecule or penta-quark ( $qqqq\bar{q}$ )?

# JLab hyperon productions



5 bins:  $-0.25 < \cos \theta_{\text{cm}} < -0.15$ , ...,  $0.15 < \cos \theta_{\text{cm}} < 0.25$

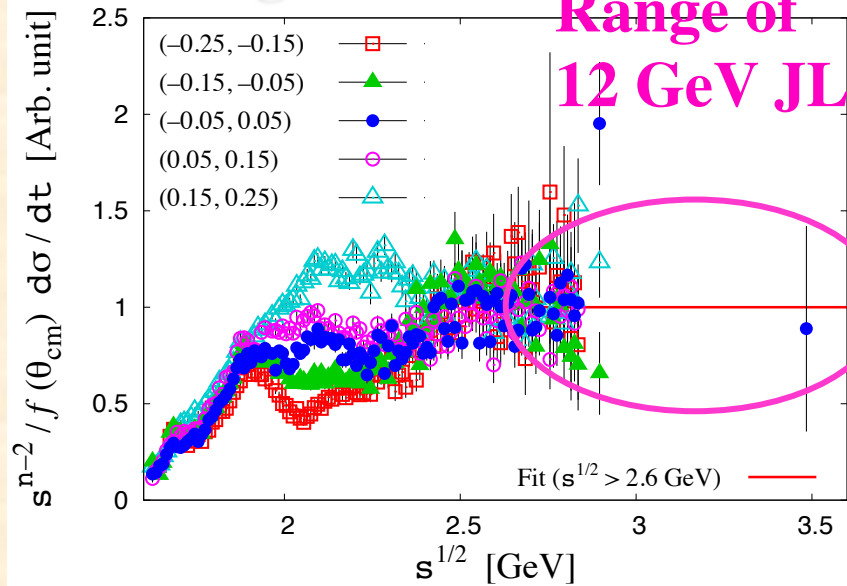
4 bins:  $-0.20 < \cos \theta_{\text{cm}} < -0.10$ , ...,  $0.10 < \cos \theta_{\text{cm}} < 0.20$

...

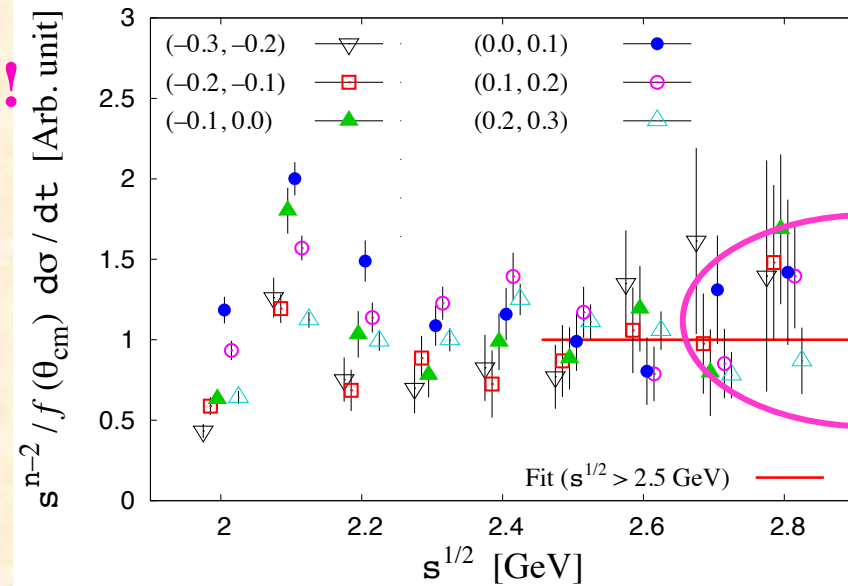
1 bin:  $-0.05 < \cos \theta_{\text{cm}} < +0.05$

ground  $\Lambda$

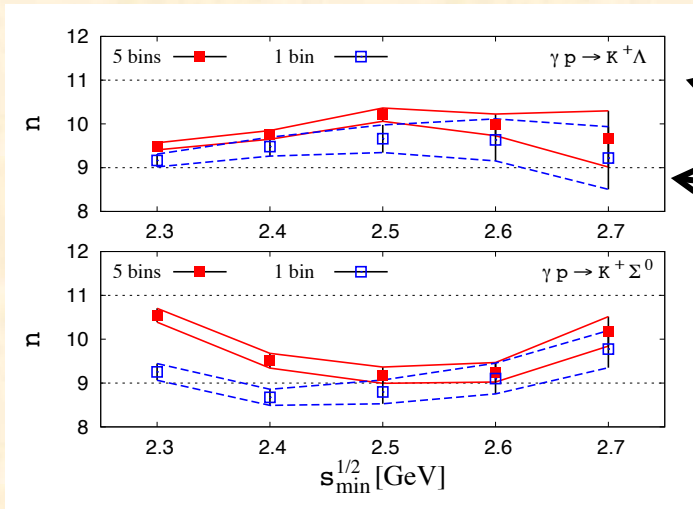
Range of  
12 GeV JLab!



$\Lambda(1405)$

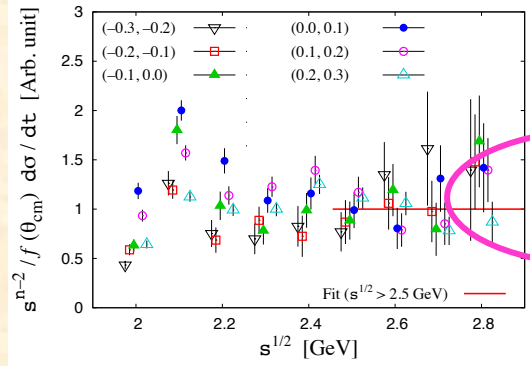
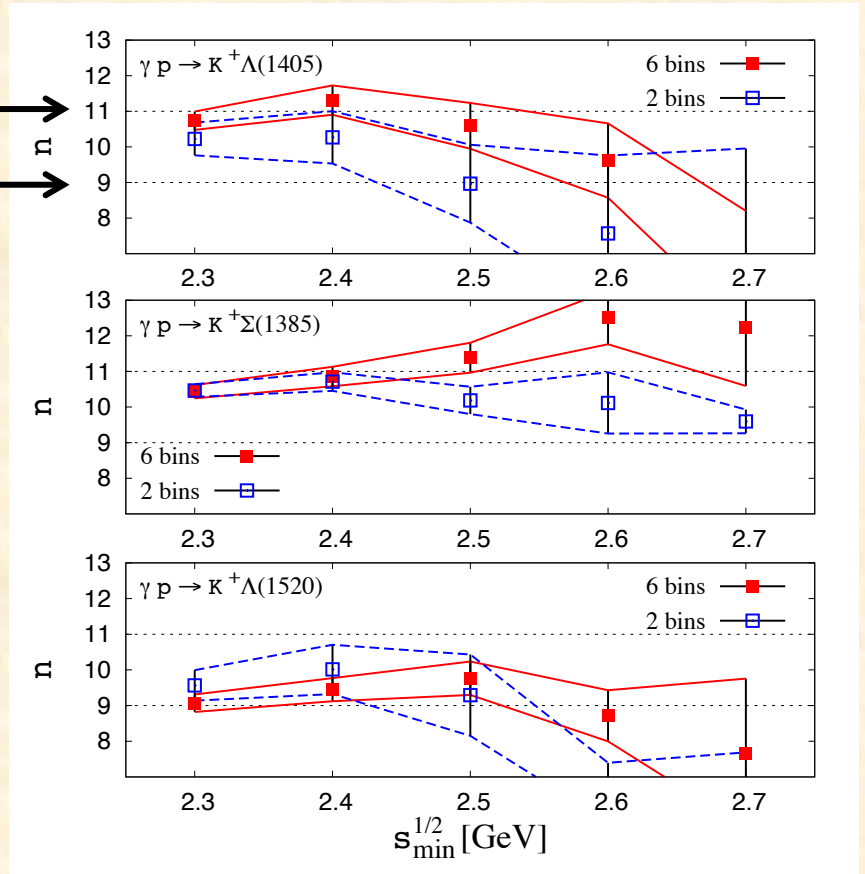


# JLab hyperon productions including $\Lambda(1405)$



$n_{\Lambda} = 5$

$n_{\Lambda} = 3$



Range of  
12 GeV JLab!

- $\Lambda$ .  $\Lambda(1520)$  and  $\Sigma$  seem to be consistent with ordinary baryons with  $n = 3$ .
  - $\Lambda(1405)$  looks penta-quark at low energies but  $n \sim 3$  at high energies???
  - $\Sigma(1385)$ :  $n = 5$  ???
- In order to clarify the nature of  $\Lambda(1405)$  [ $qqq, \bar{K}N, qqqq\bar{q}$ ], the JLab 12-GeV experiment plays an important role!

## Summary on exotic hadron structure by hard exclusive processes

- We propose to use hard exclusive production of exotic hadrons for probing internal quark-gluon structure by the constituent counting rule,  $\frac{d\sigma}{dt} = \frac{\text{const}}{s^{n-2}}$ .
- As an example,  $\pi^- + p \rightarrow K^0 + \Lambda(1405)$  is studied together with  $\pi^- + p \rightarrow K^0 + \Lambda$  as a reference of an ordinary hadron.
- $\gamma + p \rightarrow K^+ + \Lambda(1405)$  is studied.  
 $\Lambda(1405)$  = pentaquark at low energies  
= 3-quark baryon at high energies ???  
→ Measurements of extended kinematical range are necessary (12 GeV JLab).
- Exclusive processes of exotic hadrons can be investigated at many facilities in the world.  
For example, J-PARC, LEP, JLab, COMPASS,  
in general any hadron facilities like GSI, Fermilab, RHIC, LHC, ...

# **Generalized Parton Distributions (GPDs)**

**and J-PARC project**

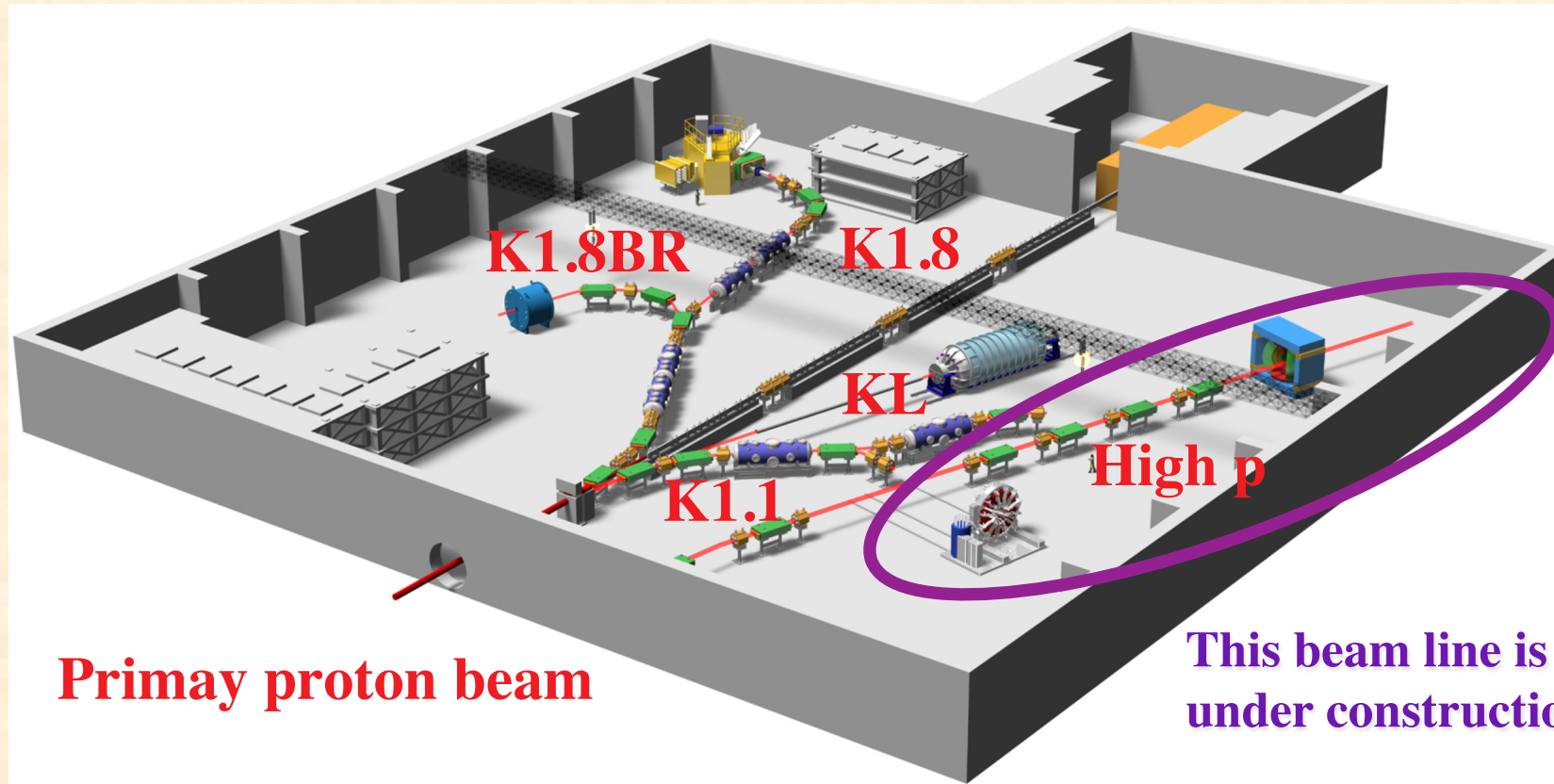
# Comments on J-PARC project

**T. Sawada, W.-C. Chang, S. Kumano,  
J.-C. Peng, S. Sawada, and K. Tanaka,  
Phys. Rev. D93 (2016) 114034.**

**S. Kumano, Int. J. Mod. Phys.:  
Conf. Series, 40 (2016) 1660009.**

# Hadron facility

Workshops on high-momentum beamline physics,  
<http://www-conf.kek.jp/hadron1/j-parc-hm-2013/>  
<http://research.kek.jp/group/hadron10/j-parc-hm-2015/>.



- Proton beam up to 30 GeV
- Unseparated hadron (pion, ...) beam up to 15~20 GeV



# Toward a new proposal at J-PARC

T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng,  
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.

Exclusive Drell-Yan:  $\pi^- + p \rightarrow \mu^+ \mu^- + n$

PHYSICAL REVIEW D 93, 114034 (2016)

Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC

Takahiro Sawada<sup>\*</sup> and Wen-Chen Chang<sup>†</sup>

*Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*

Shunzo Kumano<sup>‡</sup>

*KEK Theory Center, Institute of Particle and Nuclear Studies,  
High Energy Accelerator Research Organization (KEK), 1-1, Oho, Tsukuba, Ibaraki 305-0801, Japan  
and J-PARC Branch, KEK Theory Center, Institute of Particle and Nuclear Studies,  
KEK, 203-1, Shirakata, Tokai, Ibaraki 319-1106, Japan*

Jen-Chieh Peng<sup>§</sup>

*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*

Shinya Sawada<sup>¶</sup>

*High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

Kazuhiro Tanaka<sup>\*\*</sup>

*Department of Physics, Juntendo University, Inzai, Chiba 270-1695, Japan and J-PARC Branch,  
KEK Theory Center, Institute of Particle and Nuclear Studies,  
KEK, 203-1, Shirakata, Tokai, Ibaraki 319-1106, Japan  
(Received 15 May 2016; published 29 June 2016)*

Generalized parton distributions (GPDs) encoding multidimensional information of hadron partonic structure appear as the building blocks in a factorized description of hard exclusive reactions. The nucleon GPDs have been accessed by deeply virtual Compton scattering and deeply virtual meson production with lepton beam. A complementary probe with hadron beam is the exclusive pion-induced Drell-Yan process. In this paper, we discuss recent theoretical advances on describing this process in terms of nucleon GPDs and pion distribution amplitudes. Furthermore, we address the feasibility of measuring the exclusive pion-induced Drell-Yan process  $\pi^- p \rightarrow \mu^+ \mu^- n$  via a spectrometer at the High Momentum Beamline being constructed at J-PARC in Japan. Realization of such measurement at J-PARC will provide a new test of perturbative QCD descriptions of a novel class of hard exclusive reactions. It will also offer the possibility of experimentally accessing nucleon GPDs at large timelike virtuality.

# Exclusive Drell-Yan $\pi^- + p \rightarrow \mu^+ \mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[ (1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$

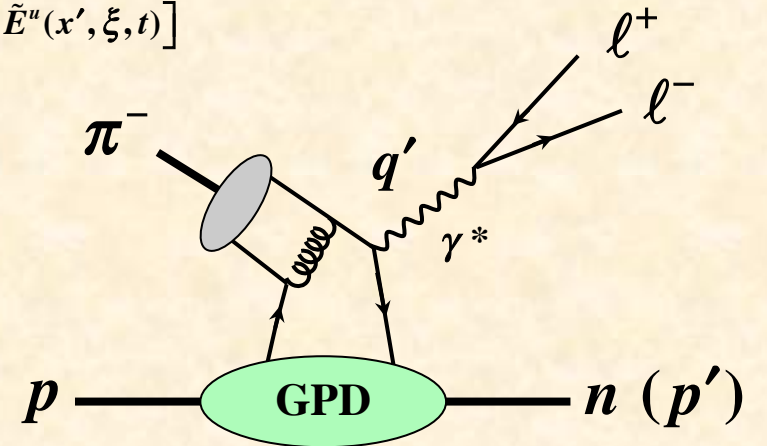
$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle p(p') | \bar{q}(-z/2) \gamma^+ \gamma_5 q(z/2) | p(p) \rangle_{z^+=0, \bar{z}_\perp=0} = \frac{1}{2P^+} \left[ \tilde{H}_p^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_p^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle n(p') | \bar{q}_d(-z/2) \gamma^+ \gamma_5 q_u(z/2) | p(p) \rangle_{z^+=0, \bar{z}_\perp=0} = \frac{1}{2P^+} \left[ \tilde{H}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\tilde{H}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[ \frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{H}^d(x', \xi, t) - \tilde{H}^u(x', \xi, t)]$$

$$\tilde{E}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[ \frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{E}^d(x', \xi, t) - \tilde{E}^u(x', \xi, t)]$$

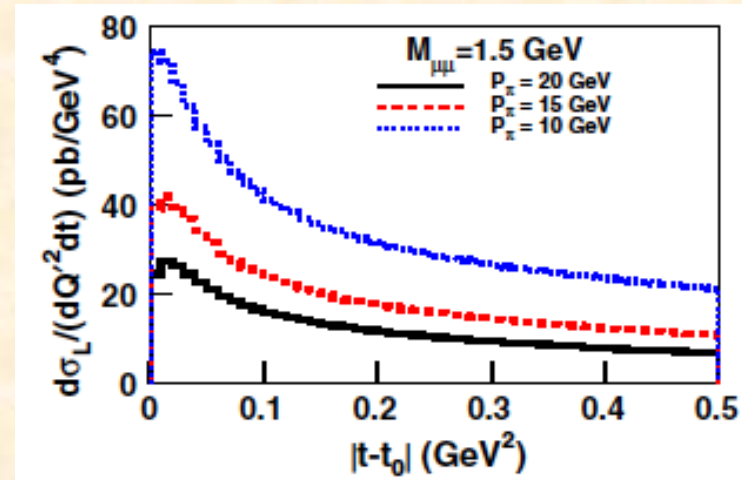
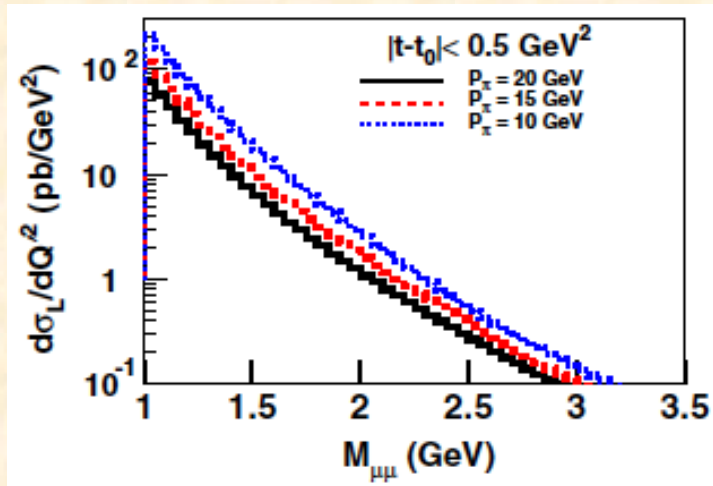


$$\pi^- (\bar{u}d) + p(uud) \rightarrow n(udd) + \gamma^* (\rightarrow \ell^+ \ell^-)$$

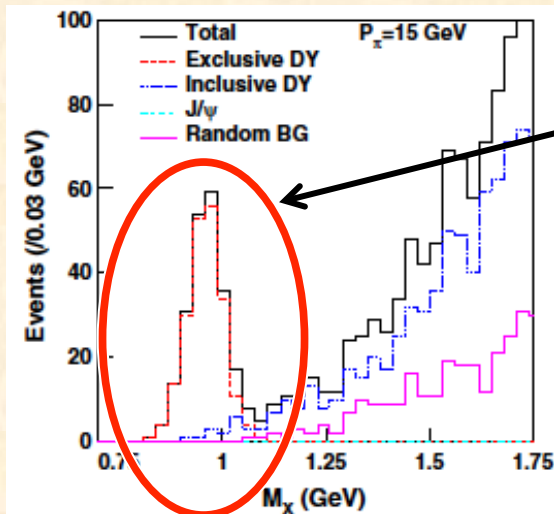
# Expected Drell-Yan events at J-PARC

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

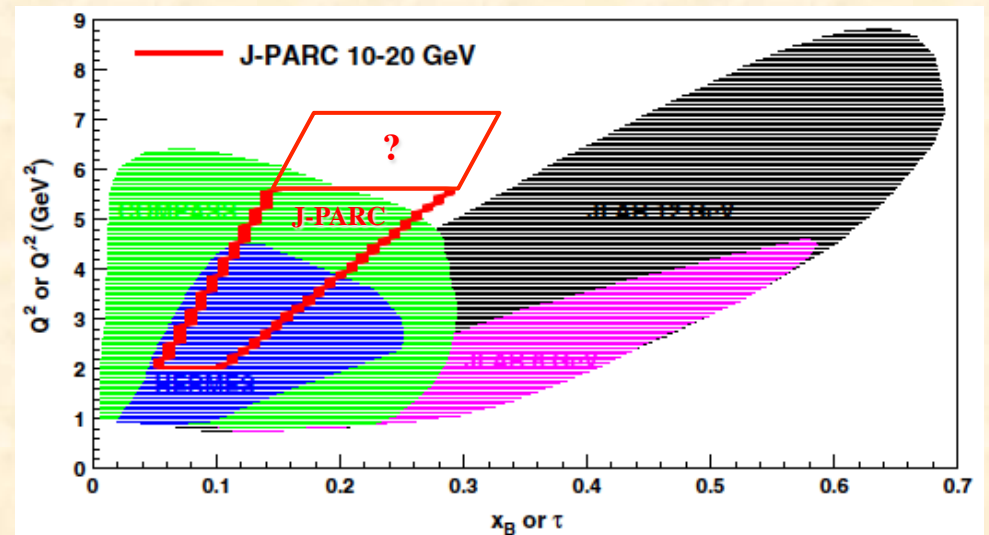
$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[ (1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \operatorname{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$



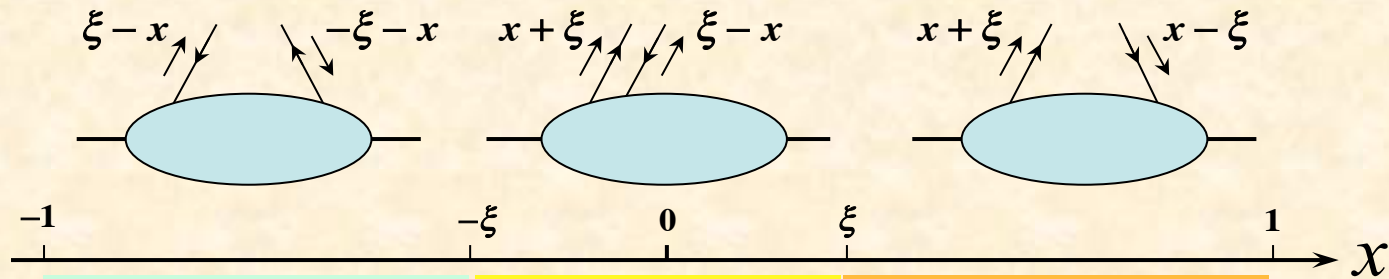
## Missing mass



**Exclusive  
Drell-Yan**



# GPDs in different $x$ regions and GPDs at hadron facilities



$-1 < x < \xi$  ( $x + \xi < 0, x - \xi < 0$ )

$\xi < x < 1$  ( $x + \xi > 0, x - \xi > 0$ )

$-\xi < x < \xi$  ( $x + \xi > 0, x - \xi < 0$ )

## Quark distribution

Emission of quark with momentum fraction  $x + \xi$   
 Absorption of quark with momentum fraction  $x - \xi$

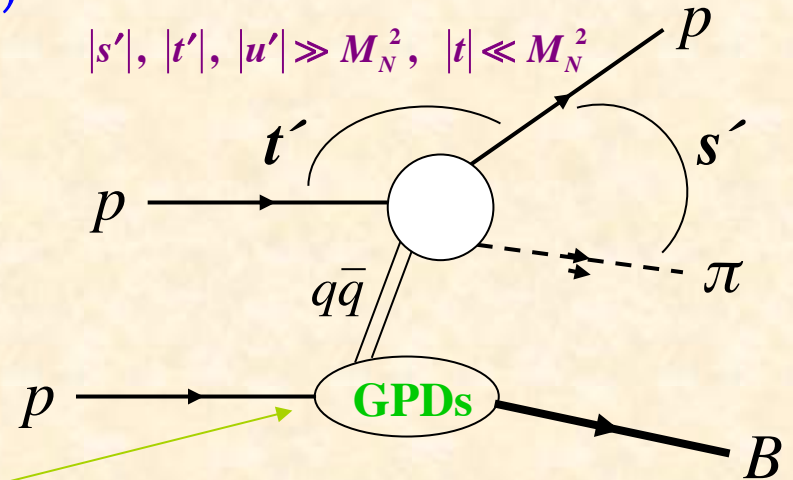
## $q\bar{q}$ (meson)-like distribution amplitude

Emission of quark with momentum fraction  $x + \xi$   
 Emission of antiquark with momentum fraction  $\xi - x$

## Antiquark distribution

Emission of antiquark with momentum fraction  $\xi - x$   
 Absorption of antiquark with momentum fraction  $-\xi - x$

Consider a hard reaction with  
 $|s'|, |t'|, |u'| \gg M_N^2, |t| \ll M_N^2$



GPDs at J-PARC: S. Kumano, M. Strikman, and K. Sudoh, PRD 80 (2009) 074003.

Efremov-Radyushkin  
 -Brodsky-Lepage (ERBL) region

# **GPDs for exotic hadrons at hadron facilities**

**H. Kawamura and S. Kumano  
Phys. Rev. D 89 (2014) 054007.**

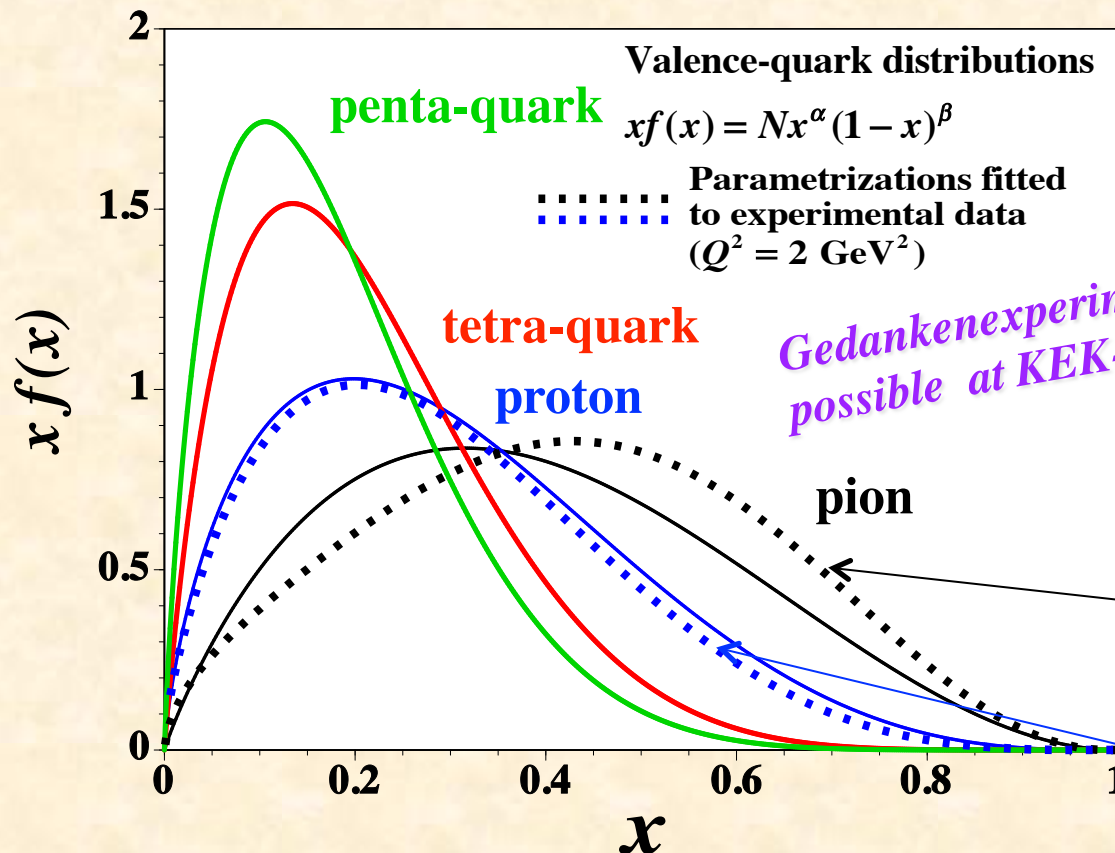
# Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

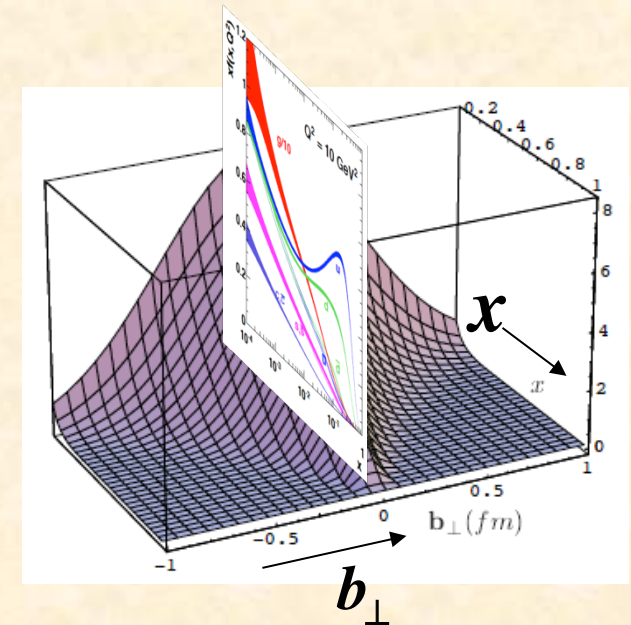
M. Guidal, M.V. Polyakov,  
A.V. Radyushkin, M. Vanderhaeghen,  
PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks:  $f(x) = q_v(x) = c_n x^{\alpha_n} (1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers):  $\int_0^1 dx f(x) = n$
- Constituent counting rule at  $x \rightarrow 1$ :  $\beta_n = 2n - 3 + 2\Delta S$  ( $n$  = number of constituents)
- Momentum carried by quarks  $\langle x \rangle_q \approx \int_0^1 dx x f(x)$



*Gedankenexperiment, but possible at KEK-B, ILC, ...*

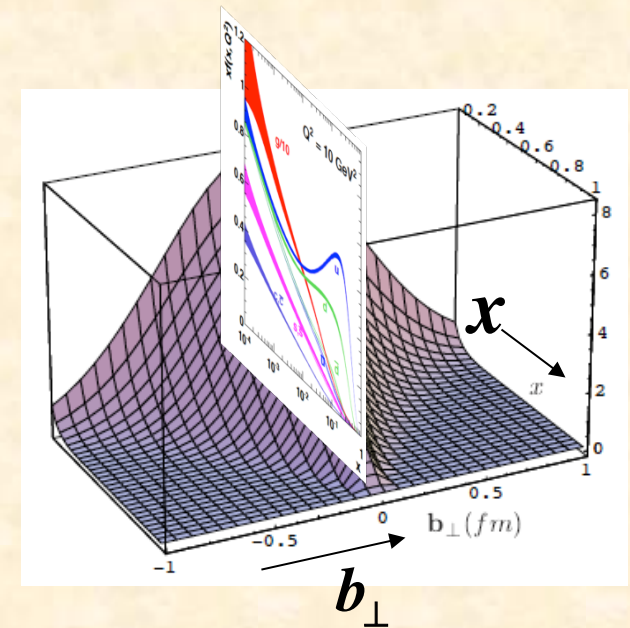
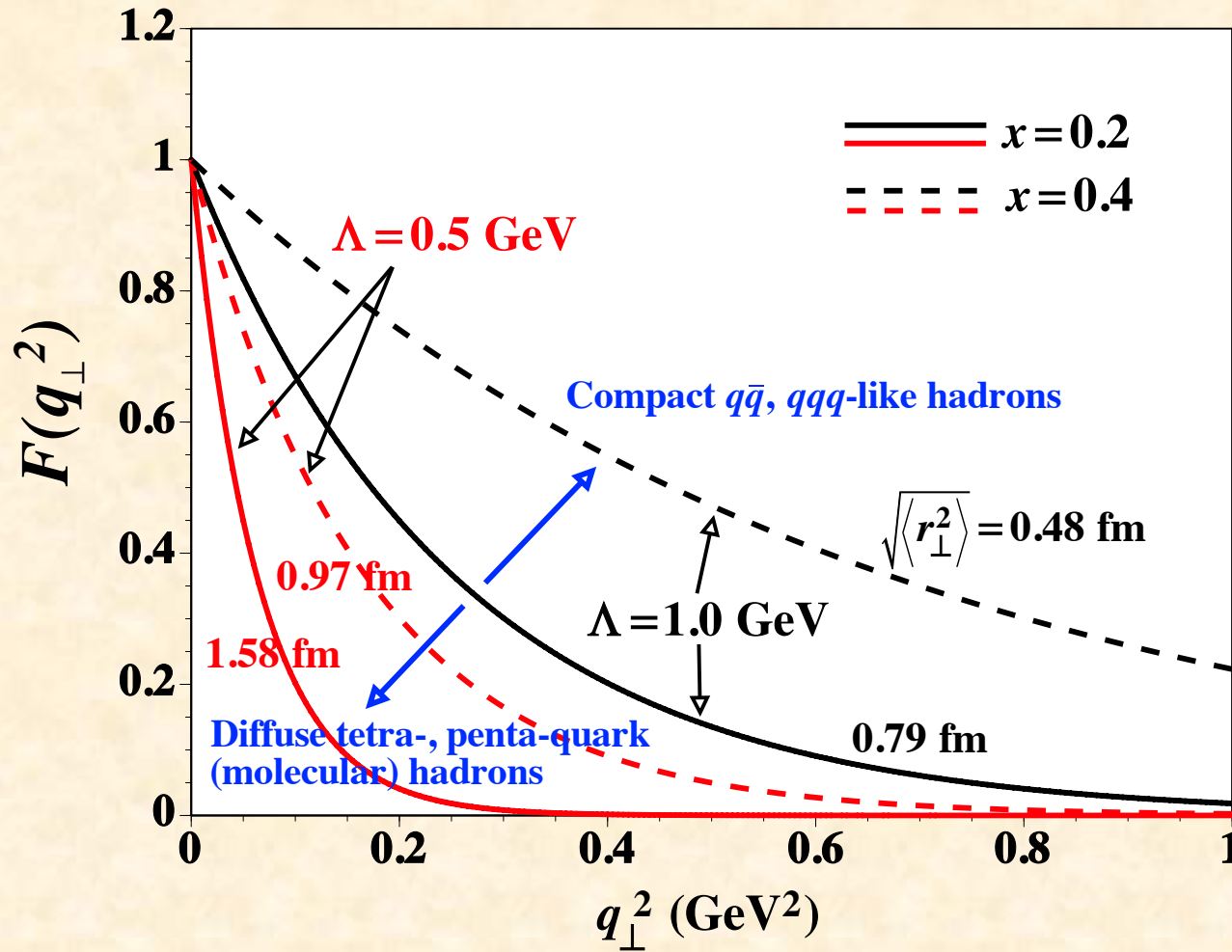


$\pi$ : M. Aicher, A. Schafer, W. Vogelsang,  
PRL 105 (2010) 252003.

$p$ : A. D. Martin, R. G. Roberts,  
W. J. Stirling, PLB 636, 259 (2006)

# Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$

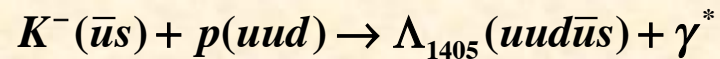
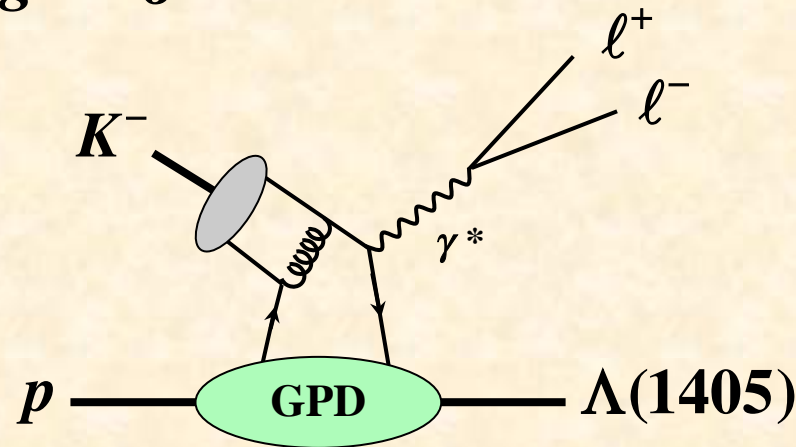


# GPDs for exotic hadrons

Because stable targets do not exist for exotic hadrons, it is not possible to measure their GPDs in a usual way.

→ **Transition GPDs**

*e.g.* at J-PARC





# **Generalized Distribution Amplitudes (GDAs)**

**and KEKB/ILC project**

**H. Kawamura and S. Kumano,  
Phys. Rev. D 89 (2014) 054007.**

**S. Kumano, Q.-T. Song, O. Teryaev,  
KEK-TH-1959, J-PARC-TH-0086.**

# GPDs for exotic hadrons !?

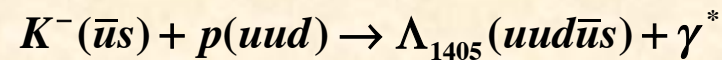
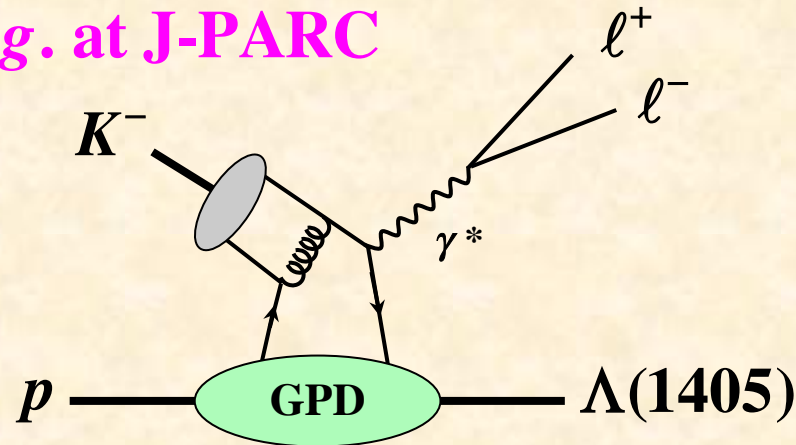
Because stable targets do not exist for exotic hadrons,  
it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

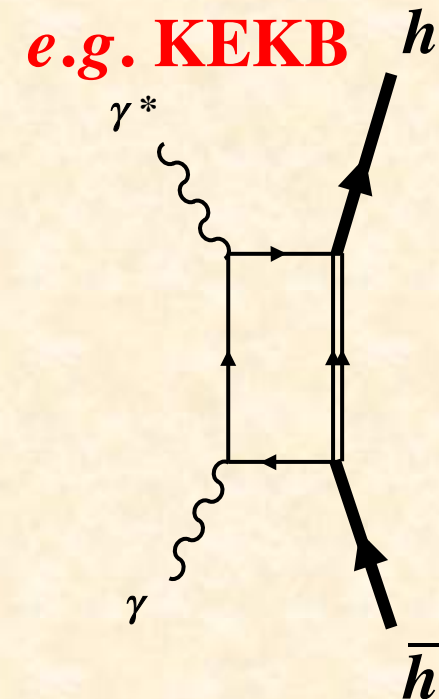
or

→  $s \leftrightarrow t$  crossed quantity = GDAs at KEKB, Linear Collider

*e.g. at J-PARC*



$\Lambda_{1405}$  = pentaquark ( $\bar{K}N$  molecule) candidate



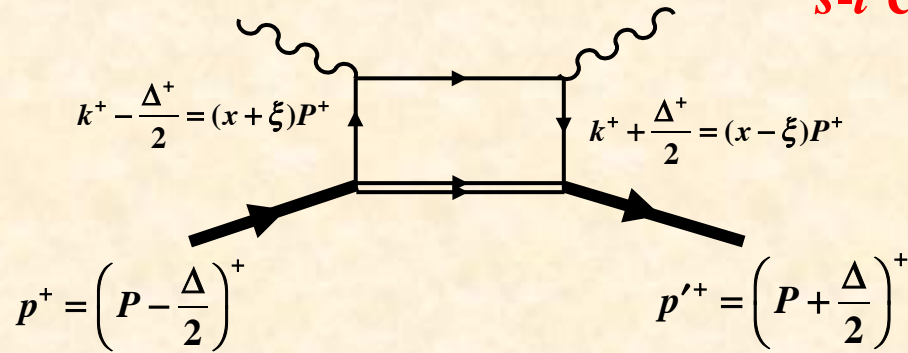
# GPD $H_q^h(x, \xi, t)$ and GDA $\Phi_q^{hh}(z, \zeta, W^2)$

GPD:  $H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$

GDA:  $\Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

DA:  $\Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

$H_q^h(x, \xi, t)$



$P = \frac{p+p'}{2}, \quad \Delta = p' - p$

Bjorken variable:  $x = \frac{Q^2}{2p \cdot q}$

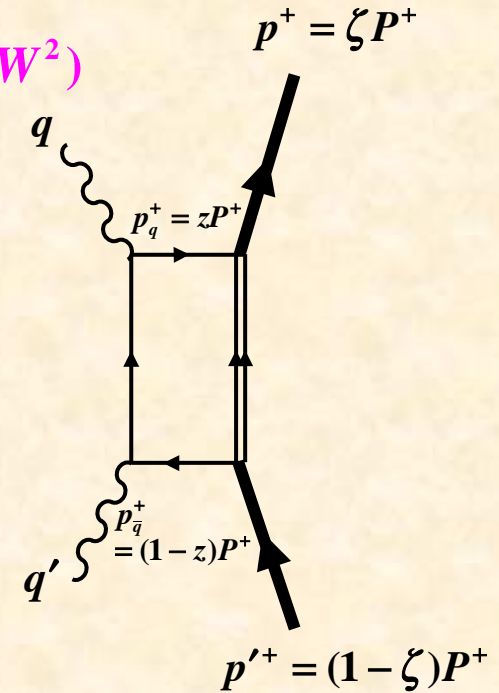
Momentum transfer squared:  $t = \Delta^2$

Skewness parameter:  $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

$\longleftrightarrow$   
**s-t crossing**

$\Phi_q^{hh}(z, \zeta, W^2)$

$z \Leftrightarrow \frac{1-x/\xi}{2}$   
 $\zeta \Leftrightarrow \frac{1-1/\xi}{2}$   
 $W^2 \Leftrightarrow t$



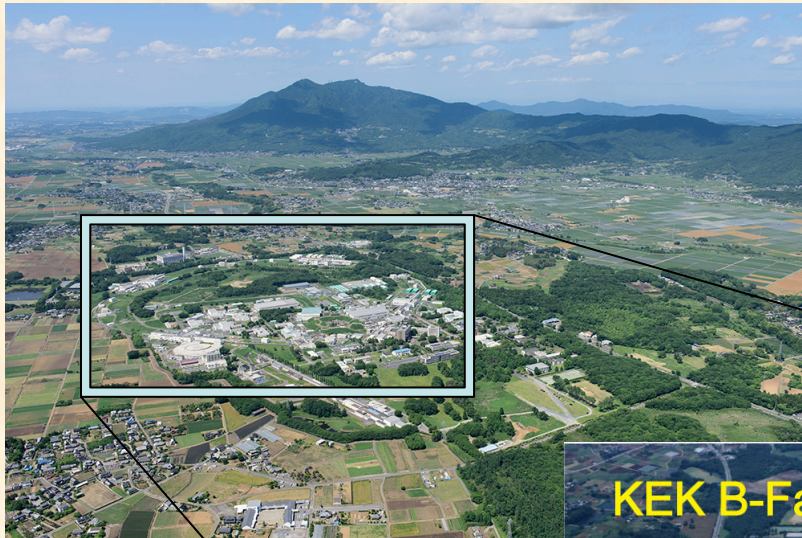
Bjorken variable for  $\gamma\gamma^*$ :  $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in  $h\bar{h}$ :  $\zeta = \frac{p^+}{P^+} = \frac{1 + \beta \cos \theta}{2}$

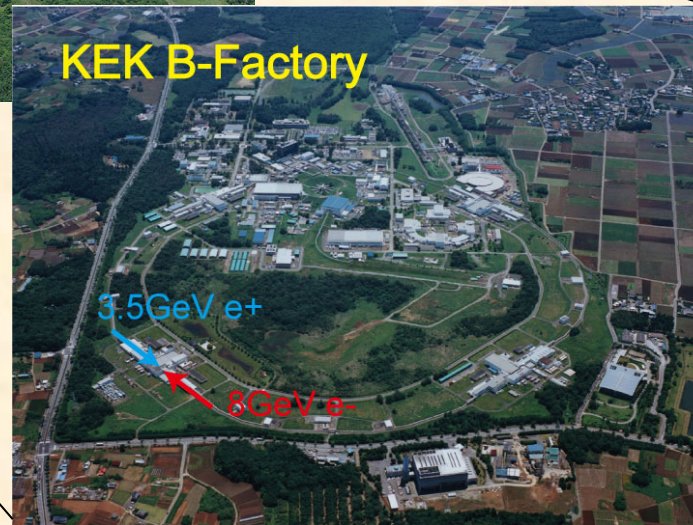
Invariant mass of  $h\bar{h}$ :  $W^2 = (p+p')^2$

# Experimental studies of GDAs in future

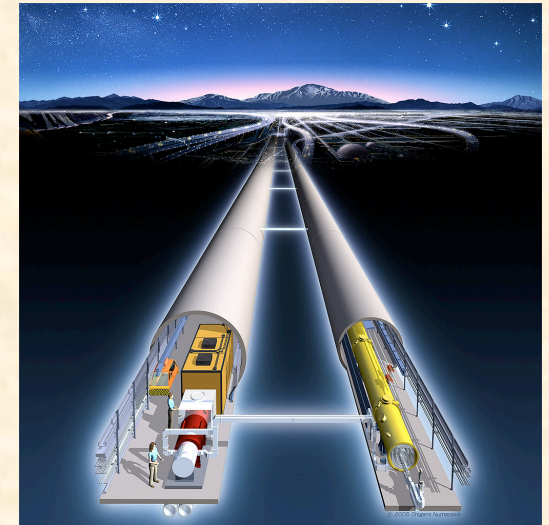
$\gamma\gamma \rightarrow h\bar{h}$  for internal structure of exotic hadron candidate  $h$



**KEK B-factory**

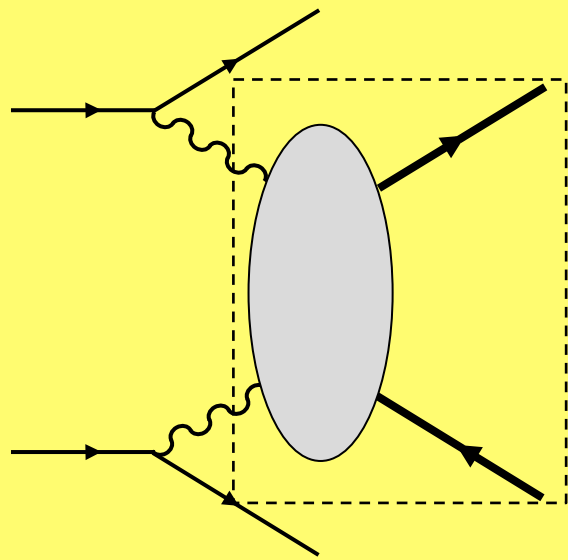


**Linear Collider ?**



# Generalized Distribution Amplitudes (GDAs) for pion

## from KEKB measurements



→ Song's talk at this workshop

$$\gamma\gamma \rightarrow h\bar{h}$$

SK, Q.-T. Song, O. Teryaev,  
KEK-TH-1959, J-PARC-TH-0086

# Cross section for $\gamma \gamma^* \rightarrow \pi^0 \pi^0$

$$d\sigma = \frac{1}{4\sqrt{(q \cdot q')^2 - q^2 q'^2}} (2\pi)^4 \delta^4(q + q' - p - p') \sum_{\lambda, \lambda'} |\overline{\mathcal{M}}|^2 \frac{d^3 p}{(2\pi)^3 2E} \frac{d^3 p'}{(2\pi)^3 2E'}$$

$$q = (q^0, 0, 0, |\vec{q}|), \quad q' = (|\vec{q}|, 0, 0, -|\vec{q}|), \quad q'^2 = 0 \text{ (real photon)}$$

$$p = (p^0, |\vec{p}| \sin \theta, 0, |\vec{p}| \cos \theta), \quad p' = (p^0, -|\vec{p}| \sin \theta, 0, -|\vec{p}| \cos \theta)$$

$$\beta = \frac{|\vec{p}|}{p^0} = \sqrt{1 - \frac{4m_\pi^2}{W^2}}$$

$$\frac{d\sigma}{d(\cos \theta)} = \frac{1}{16\pi(s + Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \sum_{\lambda, \lambda'} |\overline{\mathcal{M}}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu}, \quad T^{\mu\nu} = i \int d^4 \xi e^{-i\xi \cdot q} \langle \pi(p) \pi(p') | T J_{em}^\mu(\xi) J_{em}^\nu(0) | 0 \rangle$$

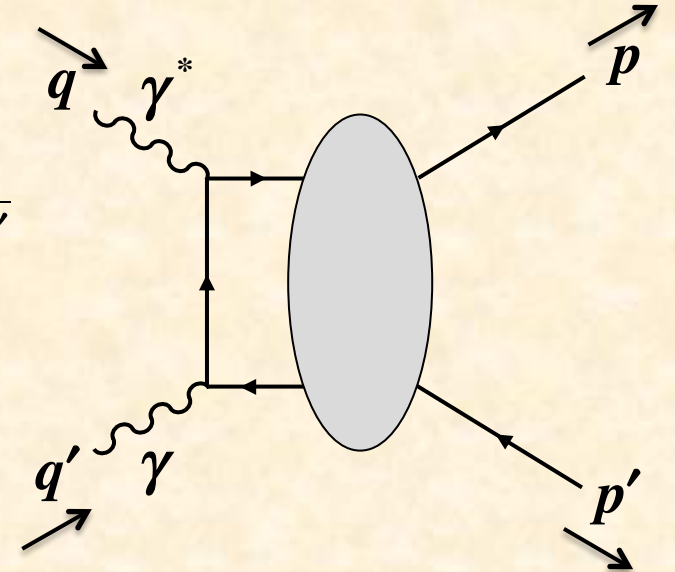
$$\mathcal{M} = e^2 A_{\lambda\lambda'} = 4\pi\alpha A_{\lambda\lambda'}$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

$$\text{GDA: } \Phi_q^{\pi\pi}(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izp^+ y^-} \langle \pi(p) \pi(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle_{y^+=0, \vec{y}_\perp=0}$$

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2), \quad \varepsilon_\mu^+(q) \varepsilon_\nu^+(q') g_T^{\mu\nu} = -1$$

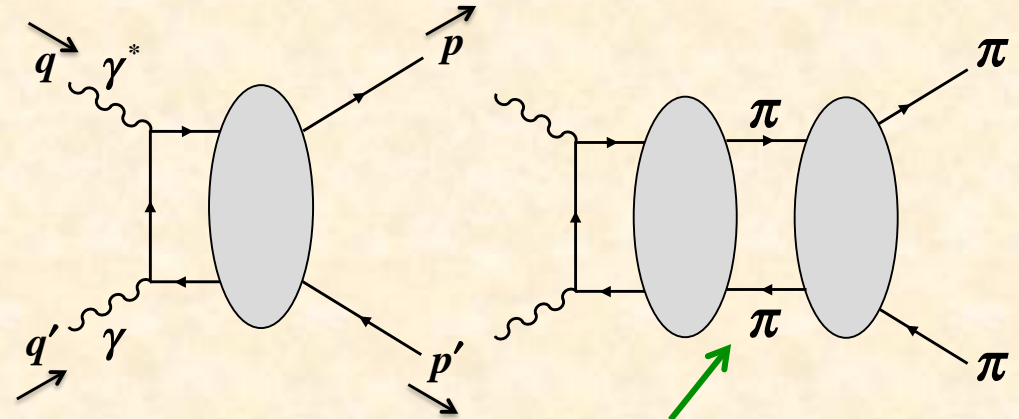
$$\frac{d\sigma}{d(\cos \theta)} \simeq \frac{\pi\alpha^2}{4(s + Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2$$



# GDA parametrization for pion

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m^2}{s}} |A_{++}|^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$



Including intermediate resonance contributions

- **Continuum:** GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = N_\pi z^\alpha (1-z)^\beta (2z-1)\zeta(1-\zeta) F_q^\pi(s)$$

- **Resonances:** There exist resonance contributions to the cross section.

$$\sum_q \Phi_q^{\pi\pi}(z, \zeta, W^2) = 18 N_f z^\alpha (1-z)^\alpha (2z-1) \left[ \tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta) \right]$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$\tilde{B}_{10}(W) = \text{resonance} [f_0(500) \equiv \sigma, f_0(980) \equiv f_0] + \text{continuum}$$

$$\tilde{B}_{12}(W) = \text{resonance} [f_2(1270)] + \text{continuum}$$

$f_0(500)$  or  $\sigma$  [g]  
was  $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

Mass  $m = (400-550)$  MeV  
Full width  $\Gamma = (400-700)$  MeV

$f_0(980)$  [f]

$I^G(J^{PC}) = 0^+(0^{++})$

Mass  $m = 990 \pm 20$  MeV  
Full width  $\Gamma = 10$  to  $100$  MeV

$f_2(1270)$

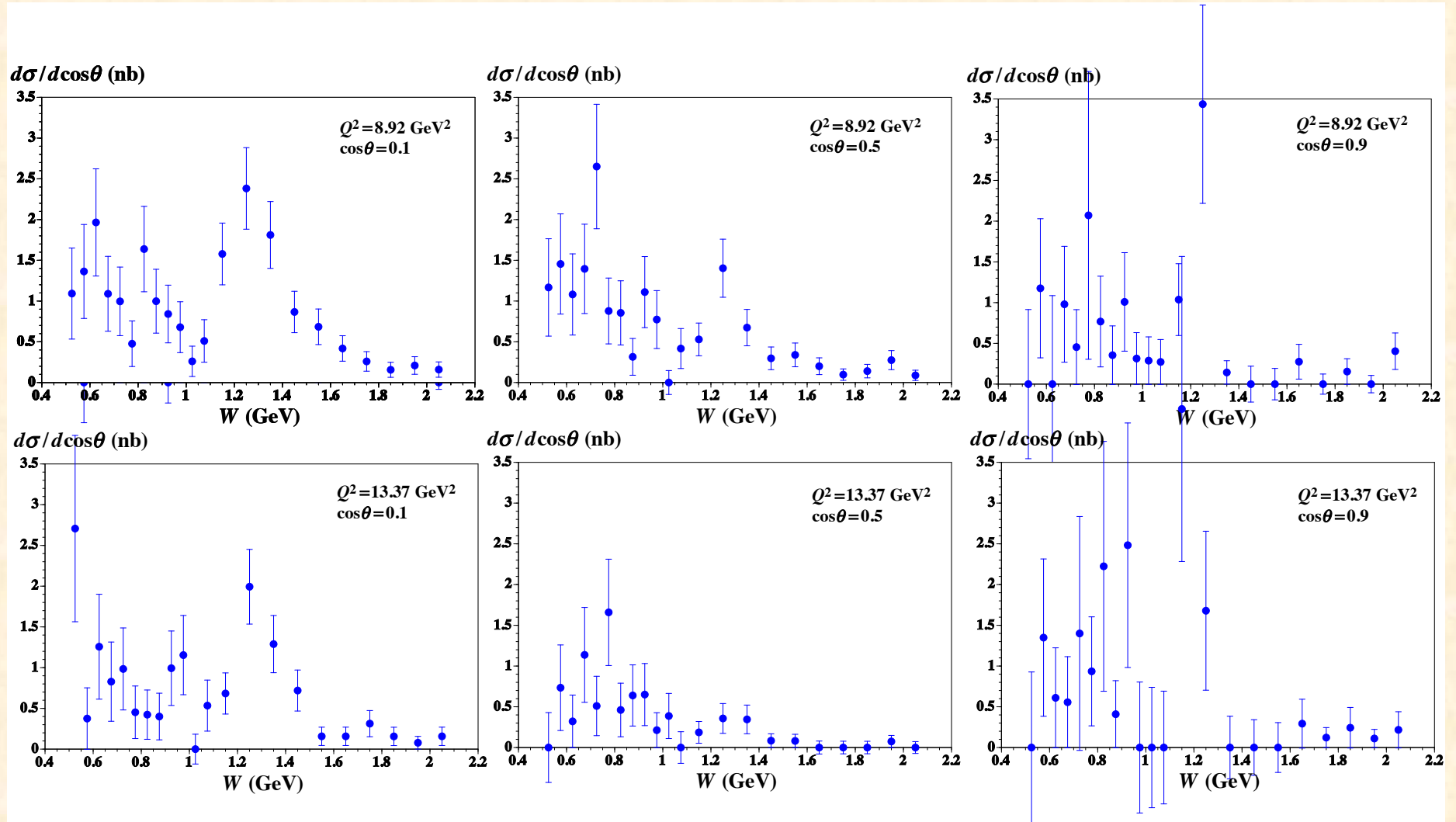
$I^G(J^{PC}) = 0^+(2^{++})$

Mass  $m = 1275.5 \pm 0.8$  MeV  
Full width  $\Gamma = 186.7^{+2.2}_{-2.5}$  MeV ( $S = 1.4$ )

# Analysis of Belle data on $\gamma\gamma^* \rightarrow \pi^0\pi^0$

$Q^2 = 8.92, 13.37 \text{ GeV}^2$

Belle measurements:  
M. Masuda *et al.*,  
PRD93 (2016) 032003.





**Generalized Distribution Amplitudes (GDAs)  
and gravitational radius for pion  
S. Kumano, Q.-T. Song, O. Teryaev,  
KEK-TH-1959, J-PARC-TH-0086,  
to be submitted for publication.**

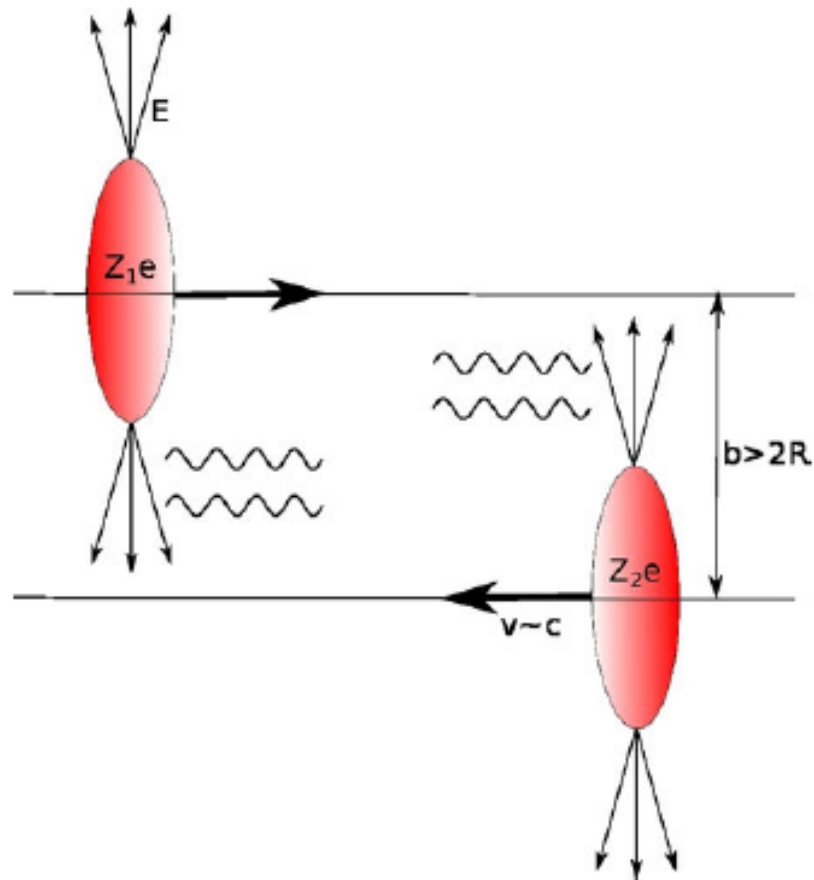
# **Prospects & Summary**

# Ultra-Peripheral Collision (UPC) @ LHC/RHIC

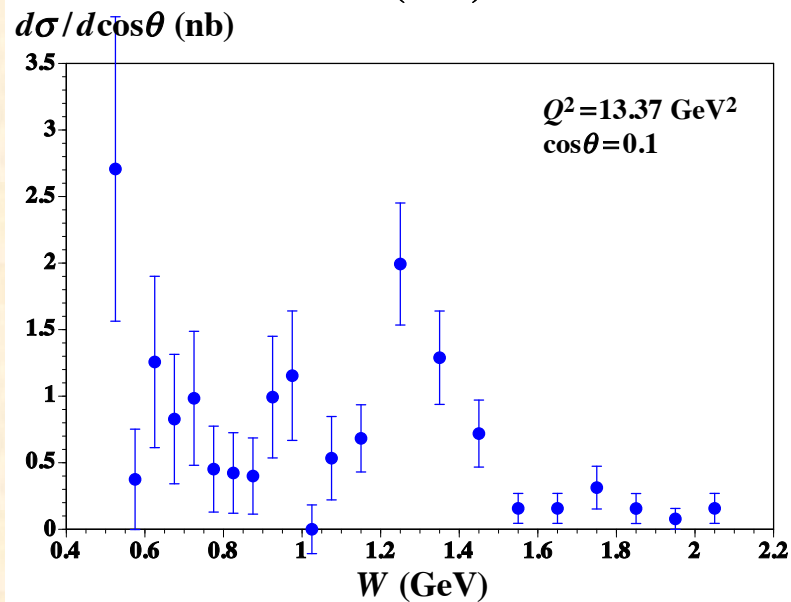
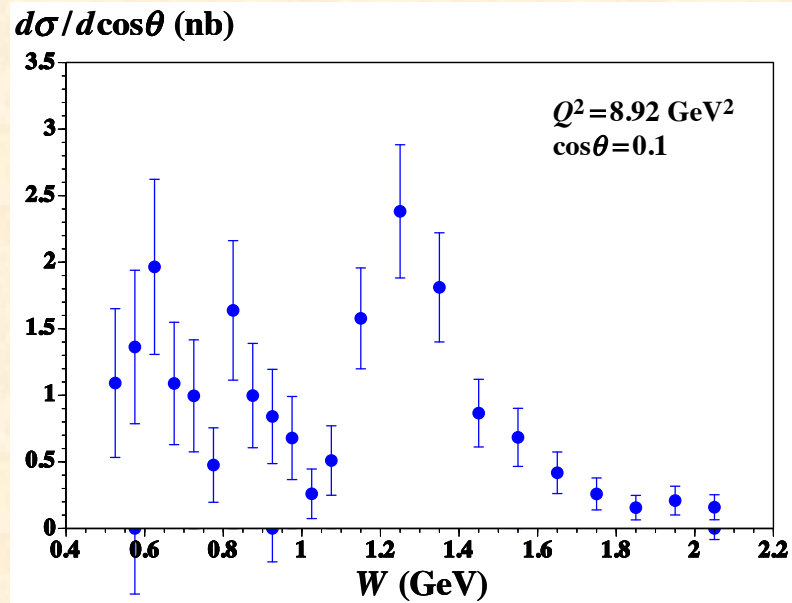
INT Workshop INT-17-65W

Probing QCD in Photon-Nucleus Interactions at RHIC and LHC: the Path to EIC

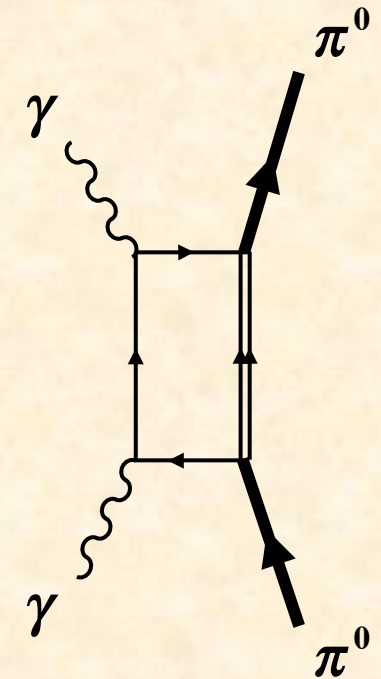
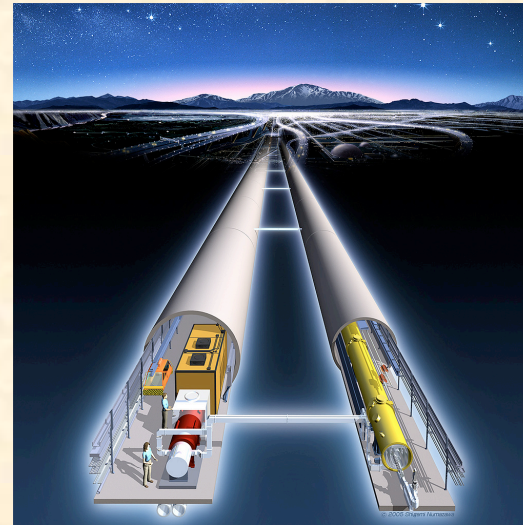
February 13 - 17, 2017



# From KEKB to ILC



## Linear Collider ?



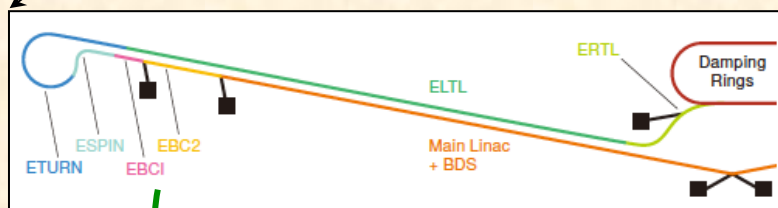
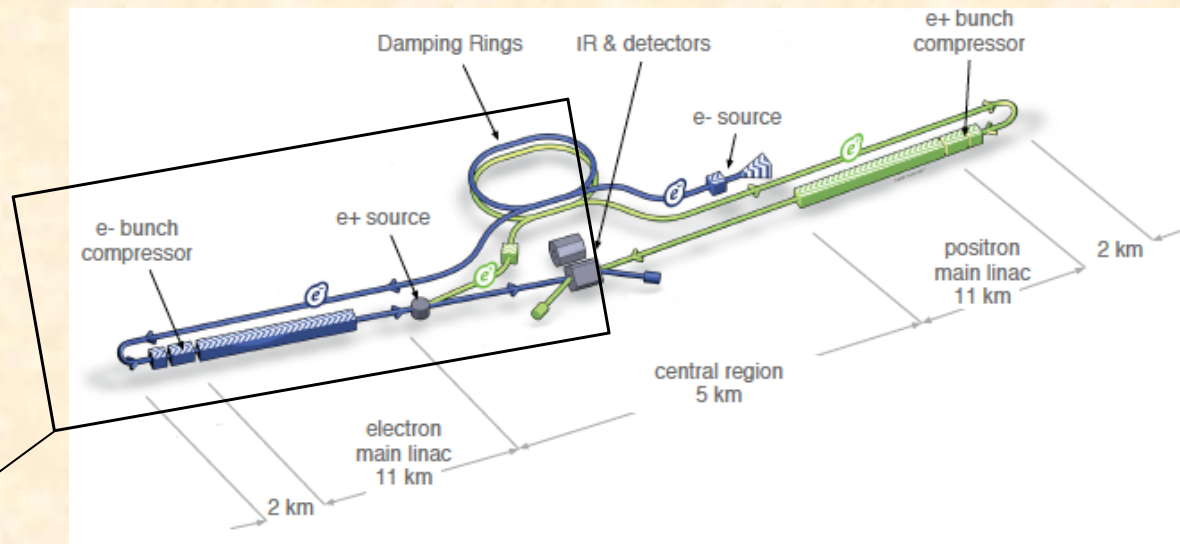
- Very Large  $Q^2$
  - Large  $W^2$
- for extracting GDAs

ILC

# ILC-N (Fixed target option) for hadron physics?

ILC TDR (Technical Design Report)

<https://www.linearcollider.org/ILC/Publications/Technical-Design-Report>



**5 – 250 GeV electron beams  
for fixed target experiments**

**Possibilities for hadron and nuclear physics**

- $e^+e^-$  annihilation processes
  - fixed target experiments  
with 5 – 250 GeV electron beams (ILC-N)
- No serious studies about these feasibilities.

# Electron-ion collider projects and ILC

arXiv:1108.1713 (551 pages)

The EIC Science case: a report on the joint BNL/INT/JLab program

Gluons and the quark sea at high energies:  
distributions, polarization, tomography

BNL

JLab

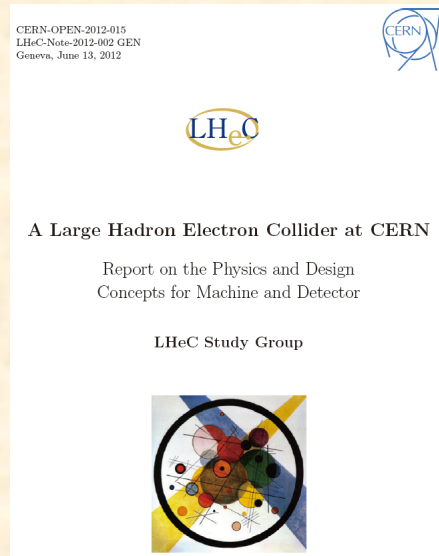
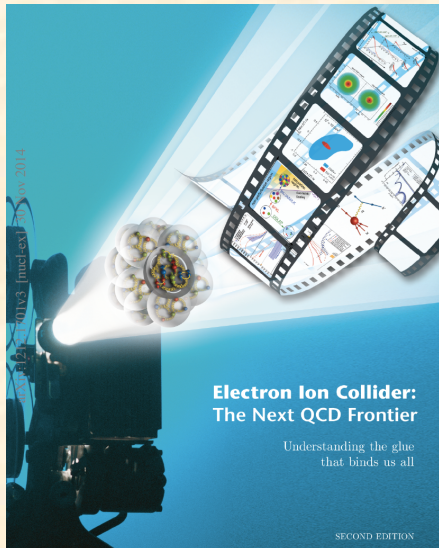
CERN



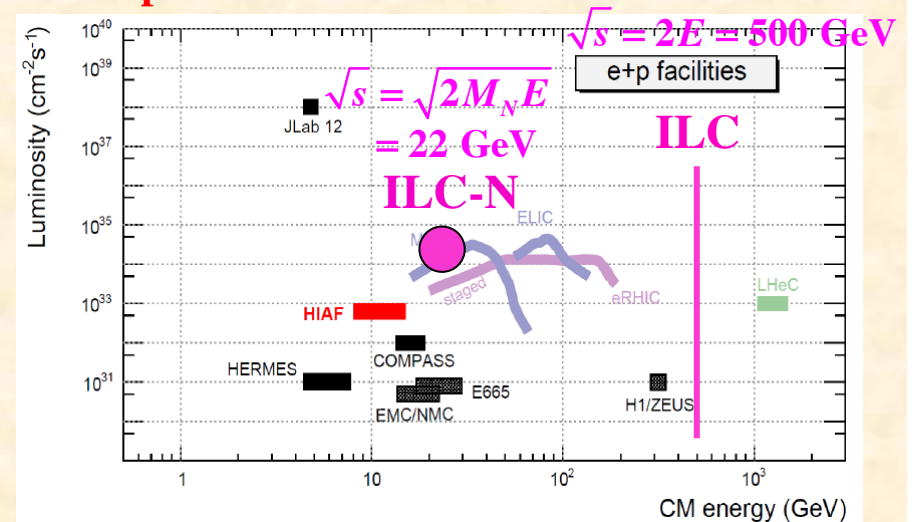
High Intensity Heavy Ion Accelerator Facility (HIAF)

arXiv:1212.1701 (180 pages)

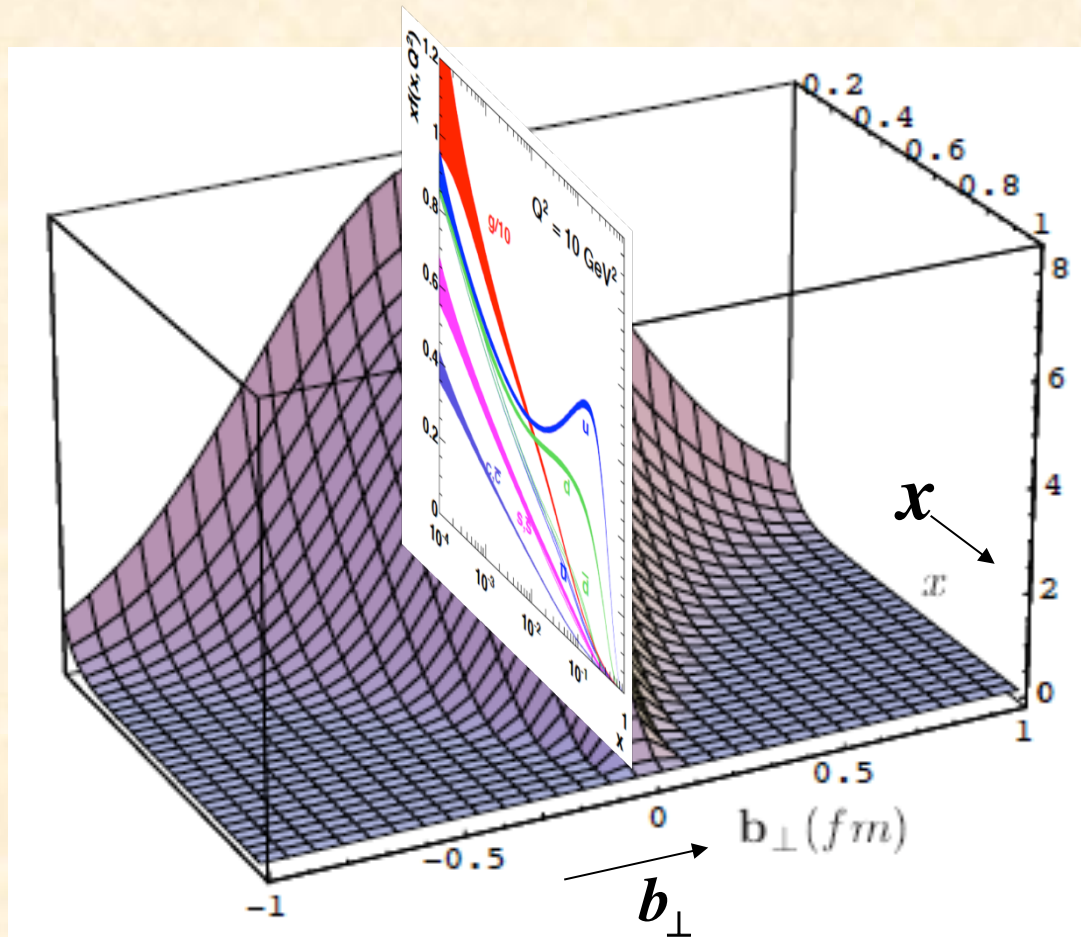
J. Phys. G: Nucl. Part. Phys.  
39 (2012) 075001(632 pages)



**ILC-N is better than on-going COMPASS  
but it is in competition with EIC in 2025 !**



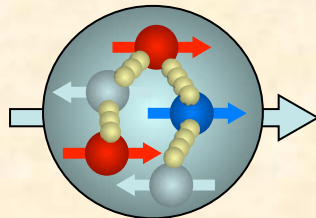
# 3D view of hadrons



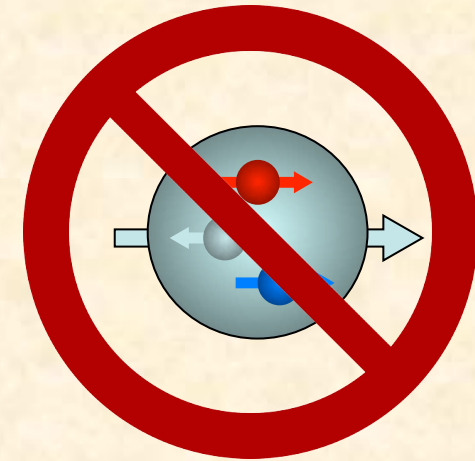
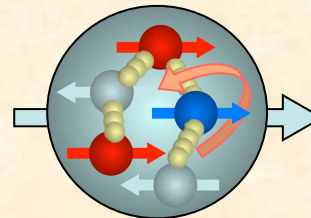
# Origin of nucleon spin ...



By the tomography, we determine

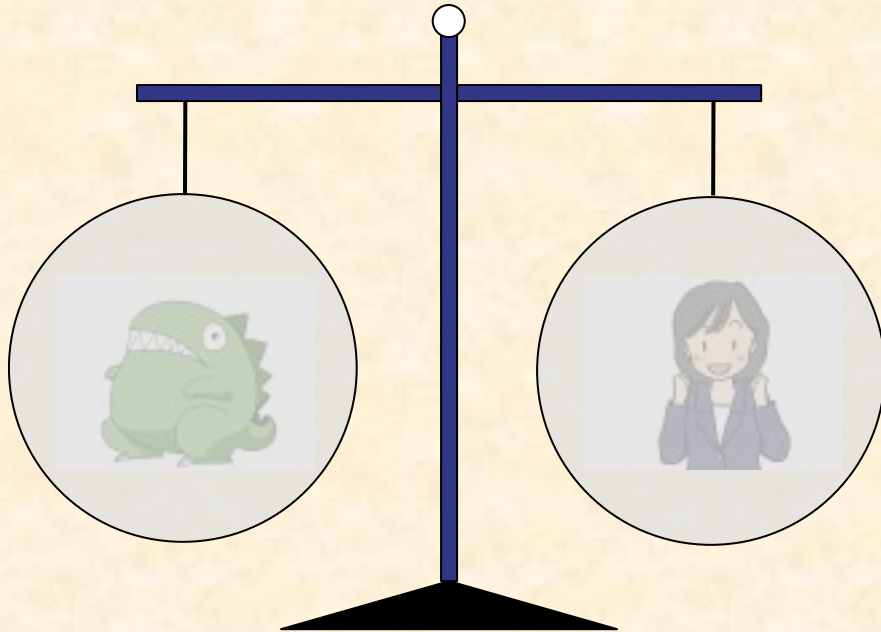


or





# Search for exotic hadrons ...



It is difficult to determine whether or not a hadron is exotic by low-energy observables, masses, decay widths, ...

(Already, history of a half century)



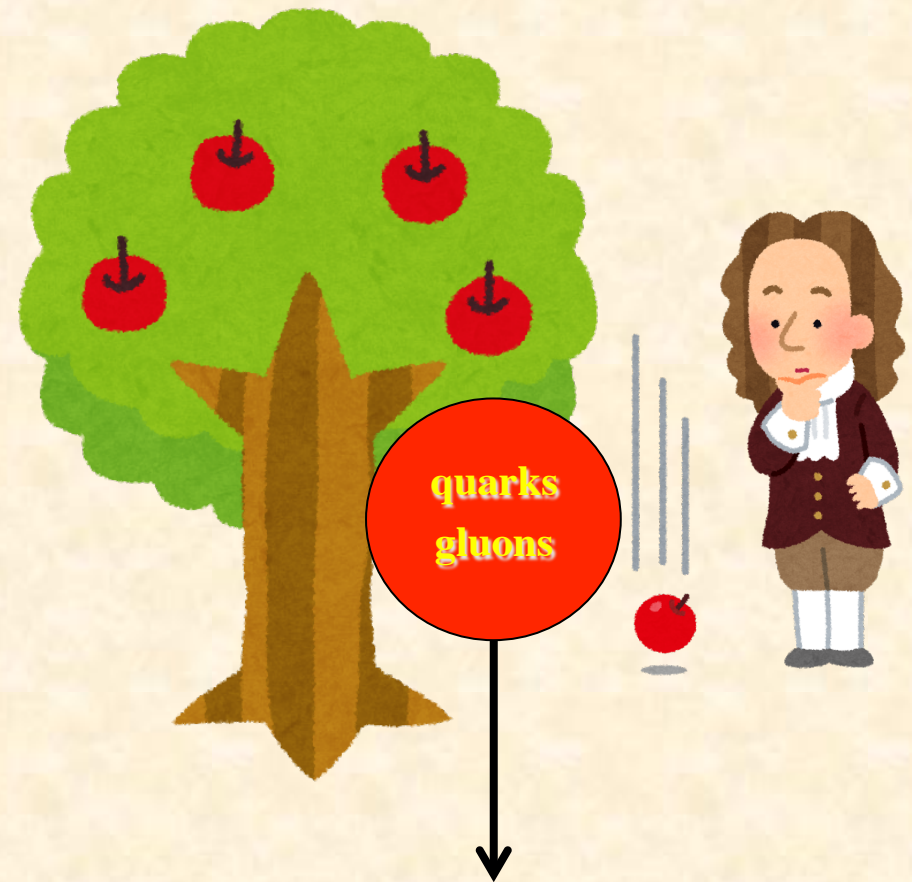
By the tomography, we determine



# Origin of gravity in terms of quarks and gluons...



**By the tomography, we determine gravitational sources (interactions) with quarks and gluons.**



# 8th International Conference on Quarks and Nuclear Physics

November 13-17, 2018, Tsukuba, Japan

<http://www-conf.kek.jp/qnp2018/>

## Quark and gluon structure of hadrons:

- parton distribution functions, generalized parton distributions,
- transverse momentum distributions, high-energy hadron reactions, ...

## Hadron spectroscopy:

- heavy quark physics, exotics,  $N^*$ , ...

## Hadron interactions and nuclear structure:

- hypernuclear physics, kaonic nuclei, baryon interactions, ...

## Hot and cold dense matter:

- quark-gluon plasma, color glass condensate, dense stars,
- strong magnetic field, mesons in nuclear medium, hadronization, ...



# Summary

Hadron tomography studies are important  
for solving **the origin of the nucleon spin**,  
for probing **internal structure of exotic hadrons**,  
for probing **gravitational sources in quark/gluon level**.

## GPDs at J-PARC

GPDs can be investigated by not only DVCS at lepton facilities  
but also exclusive reactions at hadron facilities.

## GDA at KEKB

3D structure of hadrons can be investigated by GDAs ( $s \leftrightarrow t$ ).

## Related experimental projects

RHIC, Fermilab, CERN-COMPASS, JLab, BES, ILC,  
LHC (UPC), GSI, EIC, LHeC, ...

**Gravitational radii can be obtained for hadrons!**

**The End**

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