Baryon and Meson Distribution Amplitudes from LQCD

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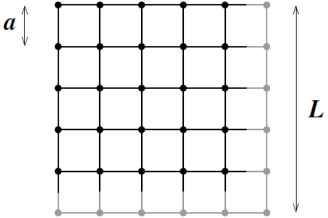
Lattice QCD

PART 0: QCD in a box

Lattice QCD



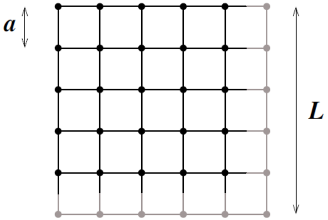
$x = a\,(n_0, n_1, n_2, n_3)\,, \qquad n_\mu \in \mathbb{Z}$

$a \equiv$ lattice spacing

$L/a \equiv$ lattice size

$L \leq \infty$, possibly $T \neq L$

Lattice QCD



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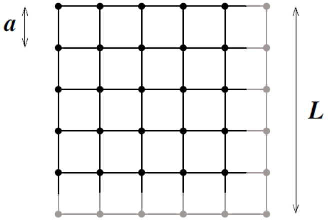

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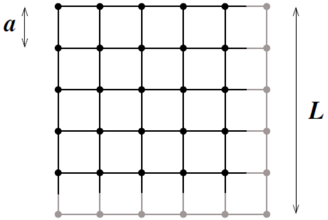

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- $\$\$\$ \propto L^{z_L} a^{-z_a} m_\pi^{-z_\pi}$
- $z_L \approx 5,\ z_a \approx 7,\ z_\pi \approx 6$

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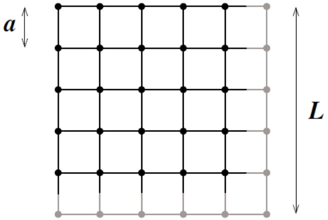

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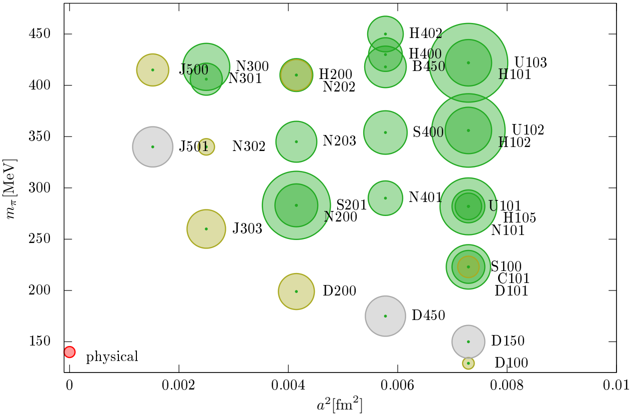
- Need small lattice spacings a to reduce cutoff-effects
- $\$\$\$ \propto L^{z_L} a^{-z_a} m_\pi^{-z_\pi}$
- $z_L \approx 5$, $z_a \approx 7$, $z_\pi \approx 6$
- For $a \leq 0.05$ fm MC algorithms get trapped in sectors of fixed topological charge

CLS effort

Lattices fulfilling TrM =const.



- area $\propto$ # configurations
- Cost (so far!): 165 Mcore-h superMUC equivalent, 350 TB of data
- (1 supermuc core-h $\approx$ 3.7 Juqueen core-h)

PART I: Baryons

Baryon Distribution Amplitudes

- What are Distribution Amplitudes (DAs)?
- Full baryonic wave functions very complex $\Rightarrow$ reduce complexity by introducing DAs

Bethe-Salpeter wave function

$$\Psi_{BS}(x, k_\perp) = \langle 0|\epsilon^{ijk} f^i(x_1, k_{1\perp}) g^j(x_2, k_{2\perp}) h^k(x_3, k_{3\perp})|B\rangle$$

$$\Phi(x, \mu) = Z(\mu) \int_{|k_\perp| \leq \mu} [d^2 k_\perp] \; \Psi_{BS}(x, k_\perp)$$

- Three-quark DAs contain information about the momentum distribution of valence quarks at small transverse separations

Simplest case: Nucleon

- Leading twist decomposition of the nucleon-to-vacuum matrix element

$$
\begin{aligned}
&4\langle 0|u_\alpha(a_1 n)u_\beta(a_2 n)d_\gamma(a_3 n)|N(p,\lambda)\rangle \\
&= \int [dx] e^{-i\,n\cdot p \sum_i a_i x_i} \Big[V^N(x_i)(\not{\tilde{n}}C)_{\alpha\beta}(\gamma_5 u_N^+(p,\lambda))_\gamma + A^N(x_i)(\not{\tilde{n}}\gamma_5 C)_{\alpha\beta}(u_N^+(p,\lambda))_\gamma \\
&\qquad + T^N(x_i)(i\sigma_{\perp\tilde{n}}C)_{\alpha\beta}(\gamma^\perp\gamma_5 u_N^+(p,\lambda))_\gamma + \ldots \Big]
\end{aligned}
$$

- "…" contain 21 DAs of higher twist
- One independent leading twist distribution amplitude

$$\Phi^N(x_1,x_2,x_3) = V^N(x_1,x_2,x_3) - A^N(x_1,x_2,x_3)$$

- Due to isopsin symmetry

$$2T^N(x_1,x_3,x_2) = \Phi^N(x_1,x_2,x_3) + \Phi^N(x_3,x_2,x_1)$$

Nucleon wave function

- Consider the three-quark Fock state in the infinite momentum frame
- At leading twist with transverse momentum components integrated out the nucleon wave function can be written as

$$
\begin{aligned}
|N^\uparrow\rangle &= \int \frac{[dx]}{8\sqrt{6x_1x_2x_3}} |uud\rangle \otimes \Big\{ [V+A]^N(x_1,x_2,x_3)|\downarrow\uparrow\uparrow\rangle + [V-A]^N(x_1,x_2,x_3)|\uparrow\downarrow\uparrow\rangle \\
&\qquad -2T^N(x_1,x_2,x_3)|\uparrow\uparrow\downarrow\rangle \Big\} \\
&= \int \frac{[dx]}{8\sqrt{3x_1x_2x_3}} |\uparrow\uparrow\downarrow\rangle \otimes \Big\{ -\sqrt{3}\Phi_+^N(x_1,x_3,x_2)|\mathsf{MS},N\rangle + \Phi_-^N(x_1,x_3,x_2)|\mathsf{MA},N\rangle \Big\}
\end{aligned}
$$

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\end{aligned}
$$

- Mixed symmetric and mixed antisymmetric flavor wave function

$$|\mathsf{MS}, N\rangle = (2|uud\rangle - |udu\rangle - |duu\rangle)/\sqrt{6} \qquad |\mathsf{MA}, N\rangle = (|udu\rangle - |duu\rangle)/\sqrt{2}$$

- One can combine $\Phi^N = \Phi_+^N + \Phi_-^N$
- Wave function at the origin $f^N = \int [dx]\ \Phi^N(x_i)$

Octet baryon wave functions

- Since SU(3) symmetry is broken Π^B is now an independent DA
- Totally symmetric (decuplet-like) and antisymmetric (singlet-like) flavor functions appear in the helicity ordered octet baryon wave functions

$$
\begin{aligned}
|(B \neq \Lambda)^{\uparrow}\rangle = \int \frac{[dx]}{8\sqrt{3x_1x_2x_3}} |{\uparrow\uparrow\downarrow}\rangle \otimes \Big\{ & -\sqrt{3}\Phi_+^B(x_1,x_3,x_2)\big(|\mathsf{MS},B\rangle - \sqrt{2}|\mathsf{S},B\rangle\big)/3 \\
& -\sqrt{3}\Pi^B(x_1,x_3,x_2)\big(2|\mathsf{MS},B\rangle + \sqrt{2}|\mathsf{S},B\rangle\big)/3 \\
& + \Phi_-^B(x_1,x_3,x_2)|\mathsf{MA},B\rangle\Big\} \\
|\Lambda^{\uparrow}\rangle = \int \frac{[dx]}{8\sqrt{3x_1x_2x_3}} |{\uparrow\uparrow\downarrow}\rangle \otimes \Big\{ & -\sqrt{3}\Phi_+^\Lambda(x_1,x_3,x_2)|\mathsf{MS},\Lambda\rangle \\
& + \Pi^\Lambda(x_1,x_3,x_2)\big(2|\mathsf{MA},\Lambda\rangle - \sqrt{2}|\mathsf{A},\Lambda\rangle\big)/3 \\
& + \Phi_-^\Lambda(x_1,x_3,x_2)\big(|\mathsf{MA},\Lambda\rangle + \sqrt{2}|\mathsf{A},\Lambda\rangle\big)/3\Big\}
\end{aligned}
$$

- The well-defined SU(3) symmetric limit is analogous to the nucleon case

$$
|B^{\uparrow}\rangle^{\star} = \int \frac{[dx]}{8\sqrt{3x_1x_2x_3}} |{\uparrow\uparrow\downarrow}\rangle \otimes \Big\{ -\sqrt{3}\Phi_+^{\star}(x_1,x_3,x_2)|\mathsf{MS},B\rangle + \Phi_-^{\star}(x_1,x_3,x_2)|\mathsf{MA},B\rangle\Big\}
$$

Moments $\longleftrightarrow$ matrix elements of local operators

Moments of DAs

$$V^B_{lmn} = \int [dx]\; x_1^l x_2^m x_3^n V^B(x_1, x_2, x_3)$$

- Unlike the full DAs the moments can be directly evaluated on the lattice
- For that purpose we define local operators such as

$$\mathcal{V}^{B,000}_{\rho} = \epsilon^{ijk}\Big(f^{T\,i}(0) C\gamma_\rho g^j(0)\Big)\gamma_5 h^k(0)$$
$$\mathcal{V}^{B,001}_{\rho\nu} = \epsilon^{ijk}\Big(f^{T\,i}(0) C\gamma_\rho g^j(0)\Big)\gamma_5 \Big[iD_\nu h(0)\Big]^k$$

- These operators clearly have the desired Dirac-matrix, flavor and color structure
- However, they are not pure twist 3 operators

Normalization and shape parameters

- Distribution amplitudes can be expanded in a set of orthogonal polynomials

$$\Phi_+^B = 120x_1x_2x_3\left(\varphi_{00}^B\mathcal{P}_{00} + \varphi_{11}^B\mathcal{P}_{11} + \ldots\right)$$
$$\Phi_-^B = 120x_1x_2x_3\left(\varphi_{10}^B\mathcal{P}_{10} + \ldots\right)$$
$$\Pi^{B\neq\Lambda} = 120x_1x_2x_3\left(\pi_{00}^B\mathcal{P}_{00} + \pi_{11}^B\mathcal{P}_{11} + \ldots\right)$$
$$\Pi^{\Lambda} = 120x_1x_2x_3\left(\pi_{10}^\Lambda\mathcal{P}_{10} + \ldots\right)$$

$$\mathcal{P}_{00} = 1 \qquad \mathcal{P}_{11} = 7(x_1 - 2x_2 + x_3) \qquad \mathcal{P}_{10} = 21(x_1 - x_3)$$

- Couplings and shape parameters can be reexpressed as moments of V^B, A^B and T^B

$$f^{B\neq\Lambda} = \varphi_{00}^B = V_{000}^B$$
$$f_T^{B\neq\Lambda} = \pi_{00}^B = T_{000}^B$$
$$f^{\Lambda} = \varphi_{00}^\Lambda = -\sqrt{\tfrac{2}{3}}A_{000}^\Lambda$$

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$$\varphi_{11}^{B\neq\Lambda} = \tfrac{1}{2}\left([V-A]_{100}^B - 2[V-A]_{010}^B + [V-A]_{001}^B\right)$$
$$\varphi_{10}^{B\neq\Lambda} = \tfrac{1}{2}\left([V-A]_{100}^B - [V-A]_{001}^B\right)$$
$$\pi_{11}^{B\neq\Lambda} = \tfrac{1}{2}\left(T_{100}^B + T_{010}^B - 2T_{001}^B\right)$$

Normalization and shape parameters

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- Couplings and shape parameters can be reexpressed as moments of V^B, A^B and T^B

$$\varphi_{11}^\Lambda = \frac{1}{\sqrt{6}}\big([V-A]_{100}^\Lambda - 2[V-A]_{010}^\Lambda + [V-A]_{001}^\Lambda\big)$$
$$\varphi_{10}^\Lambda = -\sqrt{\frac{3}{2}}\big([V-A]_{100}^\Lambda - [V-A]_{001}^\Lambda\big)$$
$$\pi_{10}^\Lambda = \sqrt{\frac{3}{2}}\big(T_{100}^\Lambda - T_{010}^\Lambda\big)$$

Leading twist projection

- Our specific choice of linear combinations is distinguished by the fact that they are directly equivalent to $\overline{\mathrm{H}(4)}$ operators (introduced in renormalization section)
- For the normalization constants we use

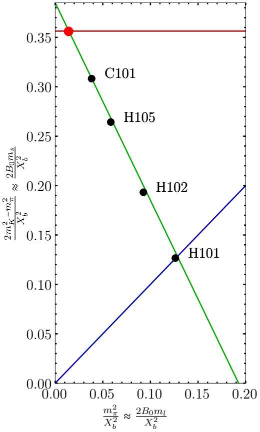
$$\mathcal{O}^{B,000}_{\mathcal{V},\mathfrak{A}} = -\gamma_1 \mathcal{V}^{B,000}_1 + \gamma_2 \mathcal{V}^{B,000}_2$$
$$\mathcal{O}^{B,000}_{\mathcal{V},\mathfrak{B}} = -\gamma_3 \mathcal{V}^{B,000}_3 + \gamma_4 \mathcal{V}^{B,000}_4$$
$$\mathcal{O}^{B,000}_{\mathcal{V},\mathfrak{C}} = -\gamma_1 \mathcal{V}^{B,000}_1 - \gamma_2 \mathcal{V}^{B,000}_2 + \gamma_3 \mathcal{V}^{B,000}_3 + \gamma_4 \mathcal{V}^{B,000}_4$$

- For the first moments we use $(l + m + n = 1)$

$$\mathcal{O}^{B,lmn}_{\mathcal{V},\mathfrak{A}} = +\gamma_1\gamma_3 \mathcal{V}^{B,lmn}_{\{13\}} + \gamma_1\gamma_4 \mathcal{V}^{B,lmn}_{\{14\}} - \gamma_2\gamma_3 \mathcal{V}^{B,lmn}_{\{23\}} - \gamma_2\gamma_4 \mathcal{V}^{B,lmn}_{\{24\}} - 2\gamma_1\gamma_2 \mathcal{V}^{B,lmn}_{\{12\}}$$
$$\mathcal{O}^{B,lmn}_{\mathcal{V},\mathfrak{B}} = +\gamma_1\gamma_3 \mathcal{V}^{B,lmn}_{\{13\}} - \gamma_1\gamma_4 \mathcal{V}^{B,lmn}_{\{14\}} + \gamma_2\gamma_3 \mathcal{V}^{B,lmn}_{\{23\}} - \gamma_2\gamma_4 \mathcal{V}^{B,lmn}_{\{24\}} + 2\gamma_3\gamma_4 \mathcal{V}^{B,lmn}_{\{34\}}$$
$$\mathcal{O}^{B,lmn}_{\mathcal{V},\mathfrak{C}} = -\gamma_1\gamma_3 \mathcal{V}^{B,lmn}_{\{13\}} + \gamma_1\gamma_4 \mathcal{V}^{B,lmn}_{\{14\}} + \gamma_2\gamma_3 \mathcal{V}^{B,lmn}_{\{23\}} - \gamma_2\gamma_4 \mathcal{V}^{B,lmn}_{\{24\}}$$

- We can now proceed to evaluating two-point functions involving these operators

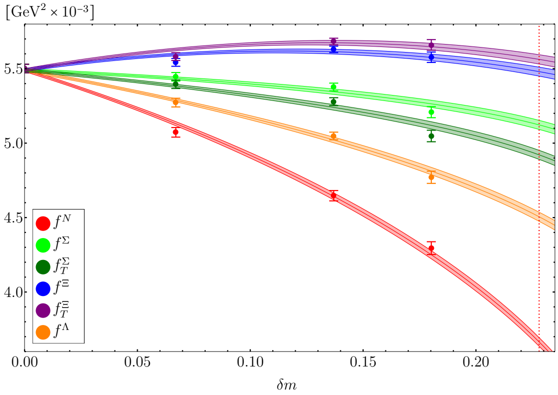
Simulation details



id	β	N_s	N_t	κ_u	κ_s	m_π [MeV]	m_K [MeV]	$m_\pi L$	#conf.
H101	3.40	32	96	0.13675962	0.13675962	421	421	5.8	2000
H102	3.40	32	96	0.136865	0.136549339	355	442	4.9	1997
H105	3.40	32	96	0.136970	0.136340790	281	466	3.9	2833
C101	3.40	48	96	0.137030	0.136222041	223	476	4.6	1552

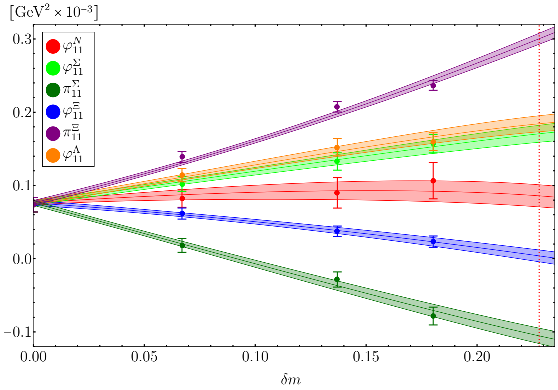
- CLS $N_f = 2 + 1$
- Open boundary conditions in time direction
- Twisted-mass determinant reweighting
- Consistency check at the flavor symmetric point
- Source positions $t_{\text{src}} = 30$, 47 and 65

Chiral extrapolation: f^B and f_T^Σ, f_T^Ξ



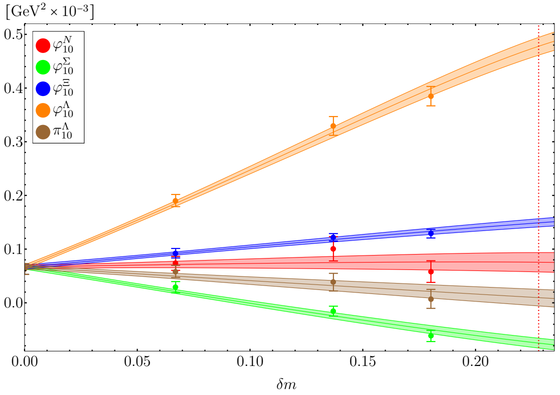
- $f^\star = f^{N\star} = f^{\Sigma\star} = f^{\Xi\star} = f^{\Lambda\star} = f_T^{\Sigma\star} = f_T^{\Xi\star}$ is fulfilled exactly
- SU(3) breaking $\sim 50\%$: $f_T^\Xi \approx 1.5 f^N$

Chiral extrapolation: first moments φ_{11}^{B} and π_{11}^{Σ}, π_{11}^{Ξ}



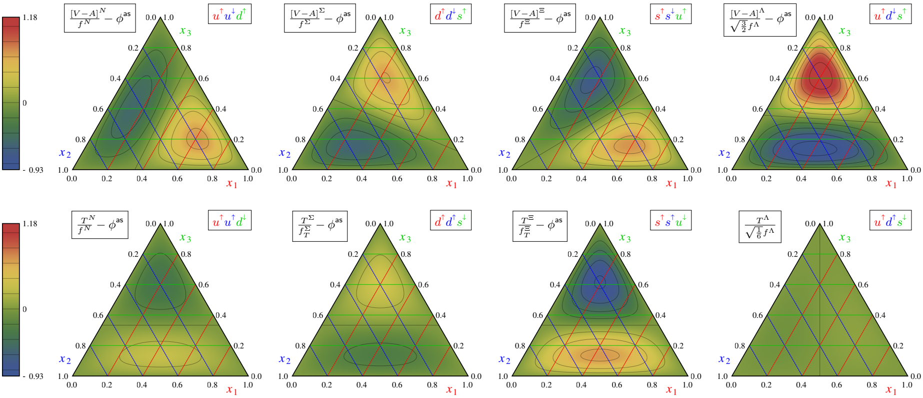
- $\varphi_{11}^{\star} = \varphi_{11}^{N\star} = \varphi_{11}^{\Sigma\star} = \varphi_{11}^{\Xi\star} = \varphi_{11}^{\Lambda\star} = \pi_{11}^{\Sigma\star} = \pi_{11}^{\Xi\star}$ is fulfilled exactly
- Very large SU(3) breaking $\sim 200\%$: $\pi_{11}^{\Xi} \approx 3\varphi_{11}^{N}$
- The moment π_{11}^{Σ} changes its sign!

Chiral extrapolation: first moments φ_{10}^B and π_{10}^Λ



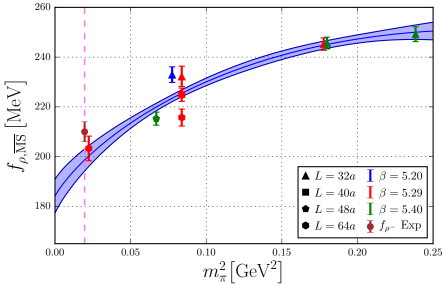
- $\varphi_{10}^\star = \varphi_{10}^{N\star} = \varphi_{10}^{\Sigma\star} = \varphi_{10}^{\Xi\star} = \varphi_{10}^{\Lambda\star} = \pi_{10}^{\Lambda\star}$ is fulfilled exactly
- Very large SU(3) breaking $\sim 500\%$: $\varphi_{10}^\Lambda \approx 6\varphi_{10}^N$
- The moment φ_{10}^Σ changes its sign!

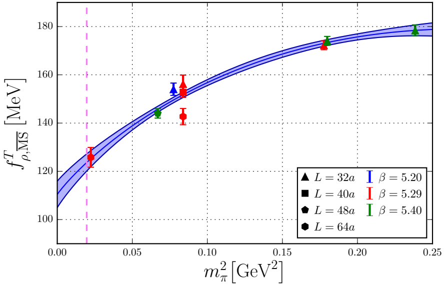
Barycentric plots



- Deviations of $[V-A]^B$ (top) and T^B (bottom) from asymptotic shape
- From left to right the plots show the baryons N, Σ, Ξ, Λ
- $B \neq \Lambda$: shift towards strange quarks and towards the leading quark
- T^Λ: asymptotic limit vanishes by construction; also deviations are very small

PART II: Mesons





Pion Distribution Amplitude

- Symmetry Group: collinear conformal transformations SL$(2, \mathbb{R})$
- Expansion in Gegenbauer polynomials (analogous to $Y^{lm}(\theta, \varphi)$ with $O(3)$)

The difference ξ of the momentum fractions contains all nontrivial information:

$$\phi(x, \mu^2) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n(\mu^2) C_n^{3/2}(2x-1)\right]$$

$$\phi(x, \mu \to \infty) = \phi^{\text{as}}(x) = 6x(1-x).$$

- Odd moments vanish $\to$ a_2 first nontrivial Gegenbauer coefficient

Lattice framework

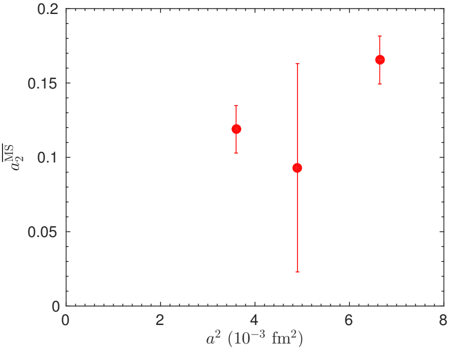
- In order to calculate the second moments of the pion DA ($n = 2$) we define the bare operators:

$$
\begin{aligned}
\mathcal{P}(x) &= \bar{d}(x)\gamma_5 u(x), \\
\mathcal{A}_\rho(x) &= \bar{d}(x)\gamma_\rho\gamma_5 u(x), \\
\mathcal{O}^-_{\rho\mu\nu}(x) &= \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_\nu - 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_\nu + \overrightarrow{D}_{(\mu}\overrightarrow{D}_\nu\right]\gamma_{\rho)}\gamma_5 u(x), \\
\mathcal{O}^+_{\rho\mu\nu}(x) &= \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_\nu + 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_\nu + \overrightarrow{D}_{(\mu}\overrightarrow{D}_\nu\right]\gamma_{\rho)}\gamma_5 u(x),
\end{aligned}
$$

- where D_μ is the covariant derivative, which will be replaced by a symmetric discretized version on the lattice.
- Obtain leading twist projection by symmetrizing over all Lorentz indices and subtract all traces. $(\dots)$, e.g., $\mathcal{O}_{(\mu\nu)} = \frac{1}{2}\left(\mathcal{O}_{\mu\nu} + \mathcal{O}_{\nu\mu}\right) - \frac{1}{4}\delta_{\mu\nu}\mathcal{O}_{\lambda\lambda}$.

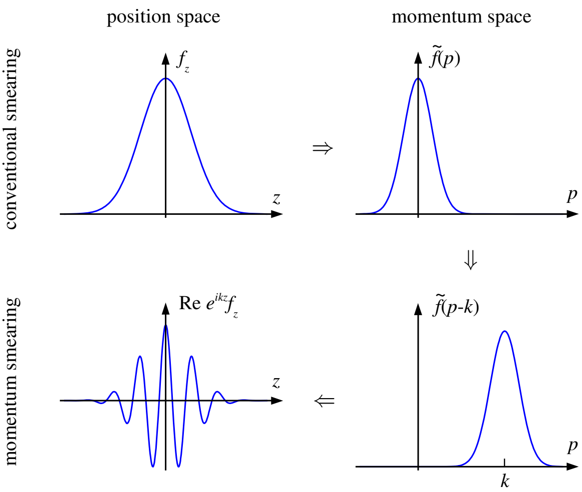
Pion Distribution Amplitude

- Continuum extrapolations in the past not reliable due to big errors:



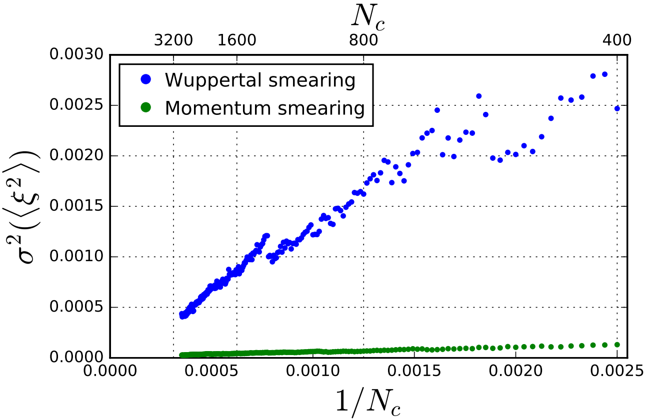
- ϟϟϟ Second moment of pion DAs require at least two non-vanishing momentum components, e.g. $\vec{p} = (110)$
- ϟϟϟ Additionally: Employing two derivatives considerably deteriorates the signal-to-noise ratio

Momentum Smearing

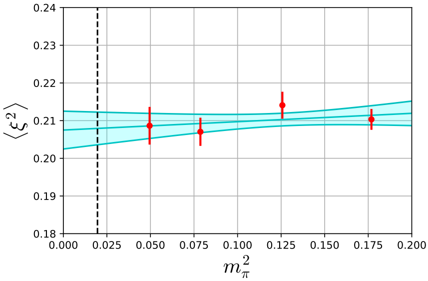


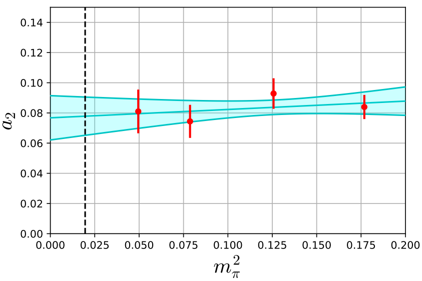
Momentum Smearing

- For example H105 for $\vec{n}_{\vec{p}} = (110), (101), (011)$
- Each momentum requires 2 inversions for the momentum smearing (6 inversions)
- Additional momenta only require additional Fourier sums for the Wuppertal smearing (1 inversion)



Chiral Extrapolation

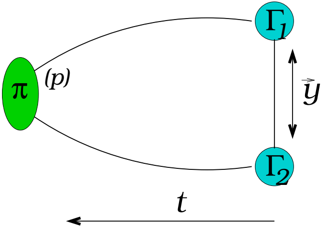




Outlook: Position space approach

- Exploratory
- Difficult to access higher moments due to mixing of lower dimensional operators
- compute DA directly by realizing a near light-front situation with a high lattice momentum

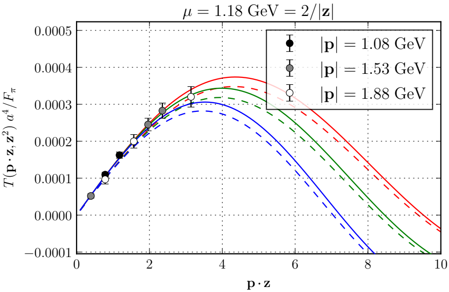
$$T_{12} = \langle 0|\bar{d}(0,\mathbf{y}/2)\Gamma_1 q(0,\mathbf{y}/2)\,\bar{q}(0,-\mathbf{y}/2)\Gamma_2 u(0,-\mathbf{y}/2)|\pi^+\rangle$$

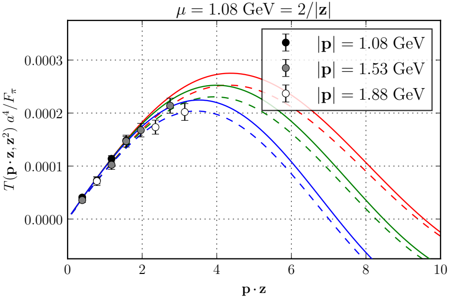


$$T_{SP}^{\overline{\rm MS}}(\mu_0^2) = \frac{p\cdot y}{\pi^2 y^4}\mathcal{F}^{SP}(\mathbf{p}\cdot\mathbf{y}, 4/\mathbf{y}^2, \mu_0^2),$$

Outlook: Position space approach

- Exploratory





Thank you for your attention!