Nucleon tomography at small-x

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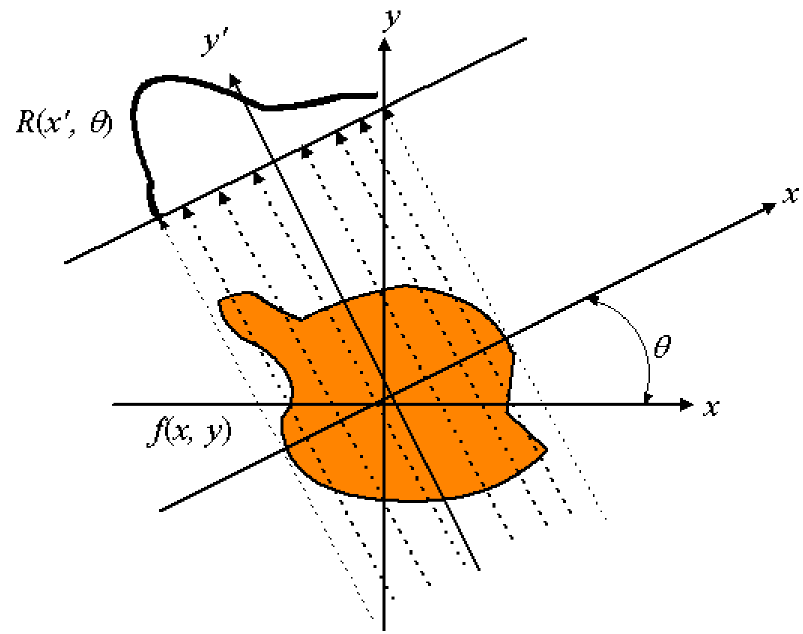
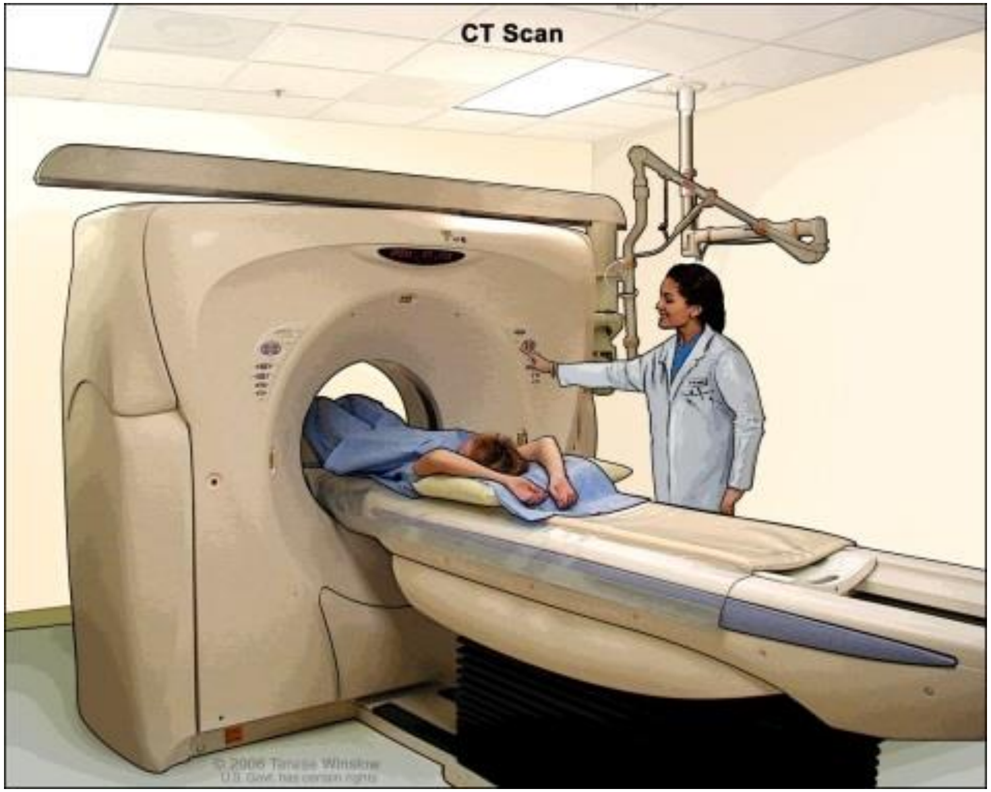
Contents

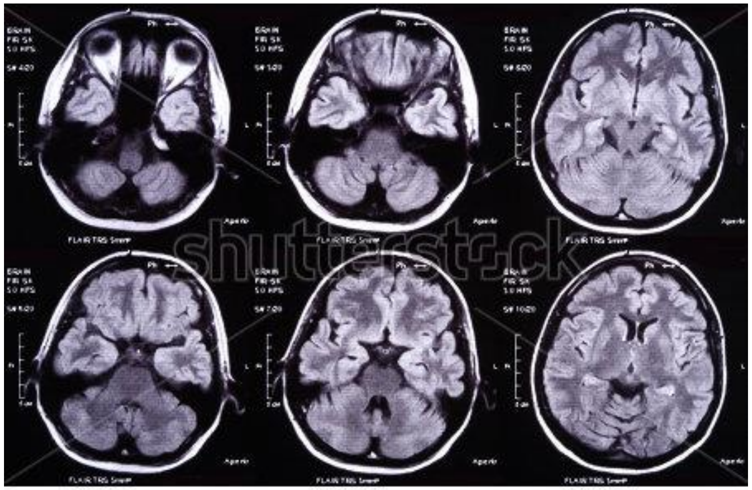
Tomography

CT = Computed Tomography

See inside an object without cutting





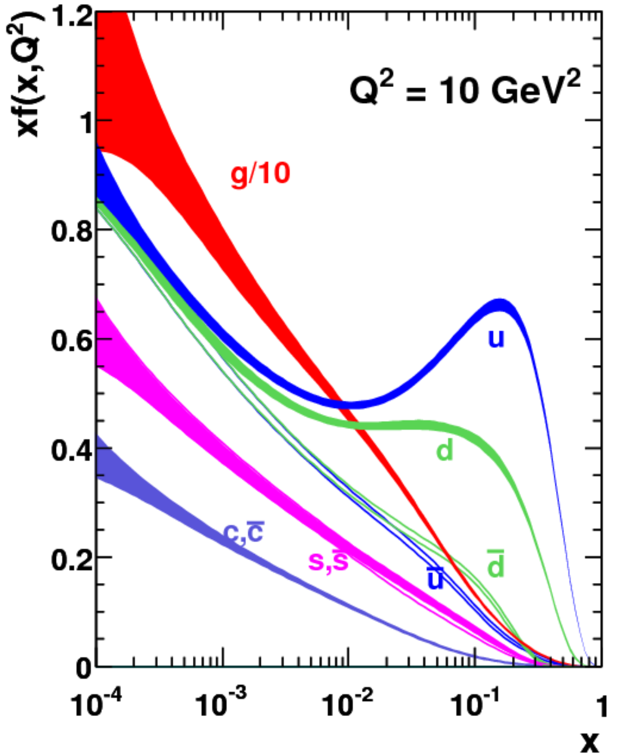


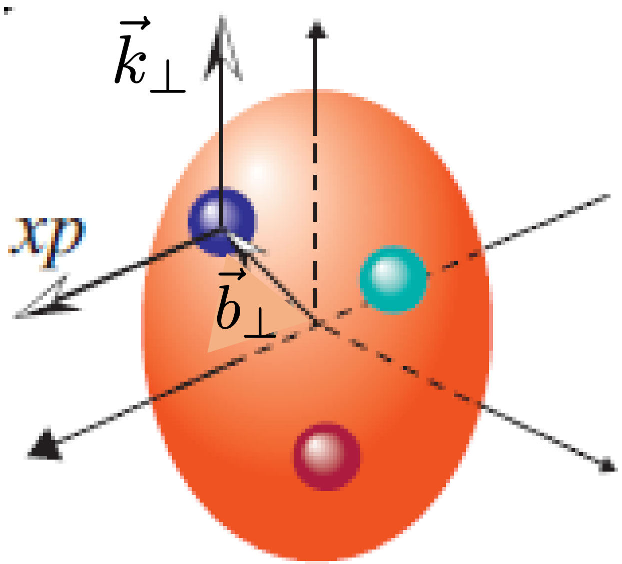
Nucleon tomography

1D tomography: Parton distribution function (PDF)

$$f(x) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P|\bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2})|P\rangle$$

Probability distribution of quarks and gluons with longitudinal momentum fraction $x = \frac{p^+_{parton}}{P^+_{proton}}$



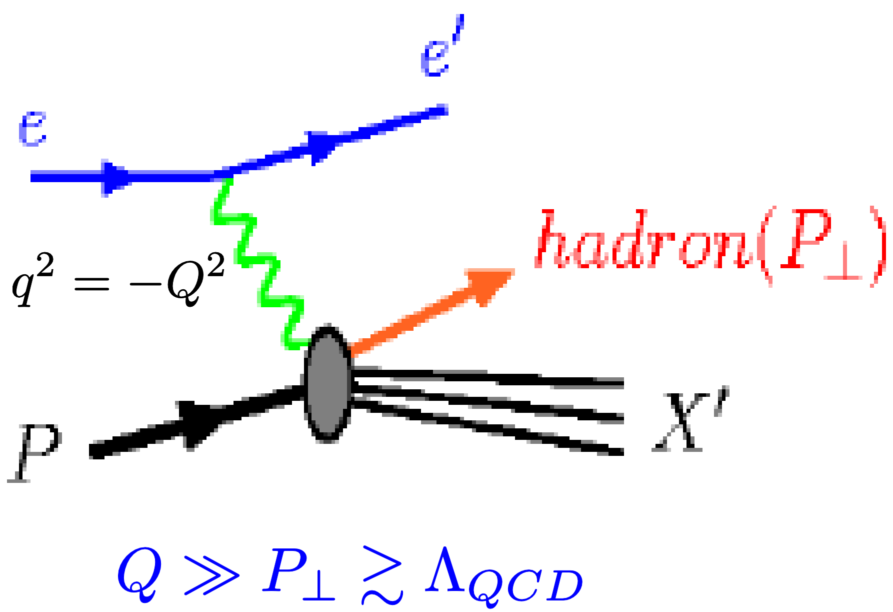


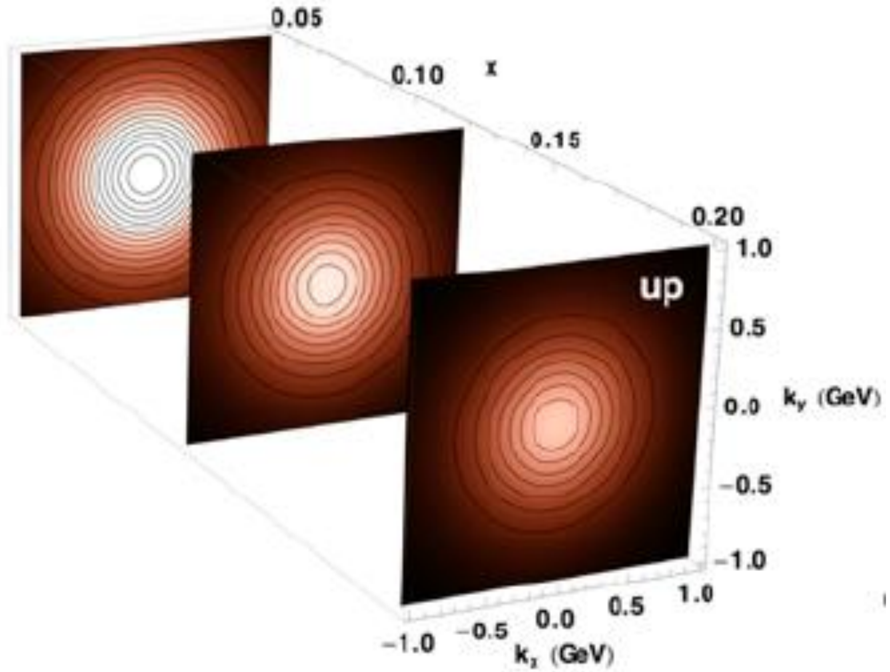
The nucleon is much more complicated!
Partons also have transverse momentum $\vec{k}_\perp$
and are spread in impact parameter space $\vec{b}_\perp$

3D tomography:
Transverse momentum dependent distributions (TMD)

$$f(x,\vec{k}_\perp) = \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P|\bar{q}(-z/2)\gamma^+ W q(z/2)|P\rangle$$

Relevant in semi-inclusive DIS (SIDIS), etc.





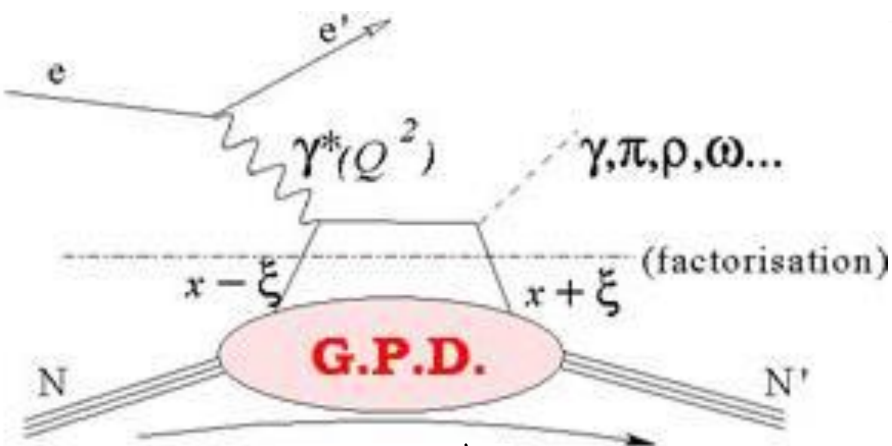
3D tomography:
Generalized parton distributions (GPD)

$$f(x,\vec{\Delta}_\perp) \sim \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P - \frac{\Delta}{2} | \bar{q}(-z/2)\gamma^+ q(z/2) | P + \frac{\Delta}{2} \rangle$$

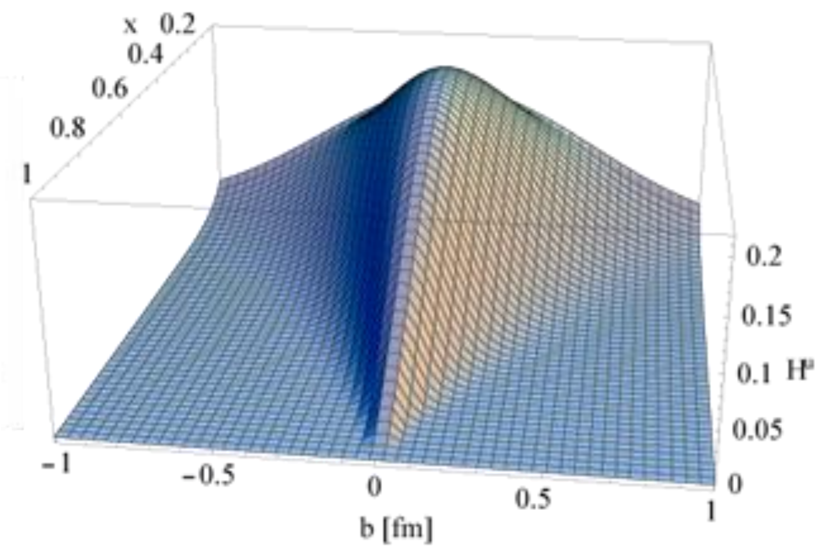
⟺ $f(x,\vec{b}_\perp)$ distribution of partons in impact parameter space

Fourier transform

Deeply Virtual Compton Scattering (DVCS)



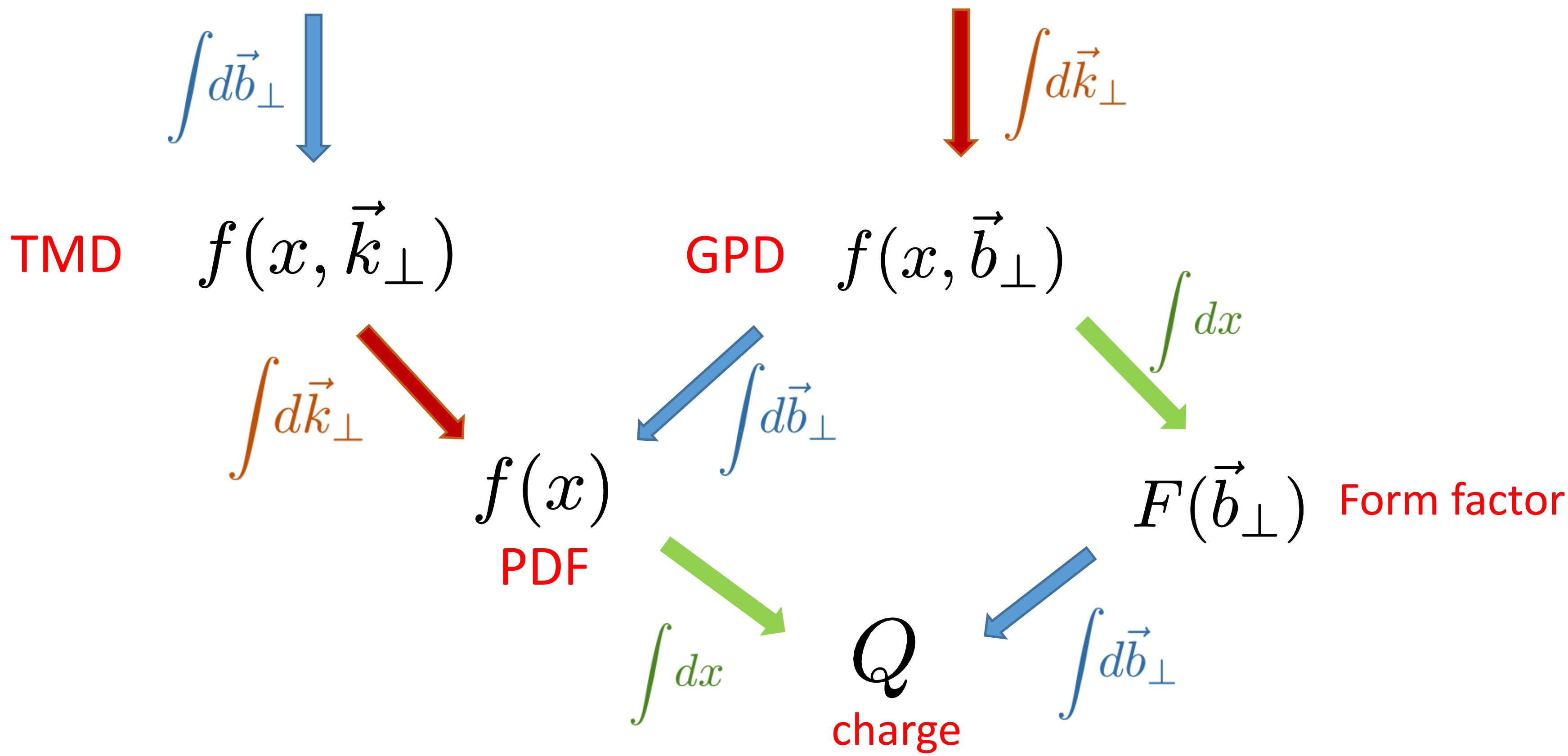
$$t = -\vec{\Delta}_\perp^2$$



5D tomography:
Wigner distribution— the "mother distribution"

Belitsky, Ji, Yuan (2003);
Lorce, Pasquini (2011)

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P - \frac{\Delta}{2} | \bar{q}(-z/2)\gamma^+ q(z/2) | P + \frac{\Delta}{2} \rangle$$



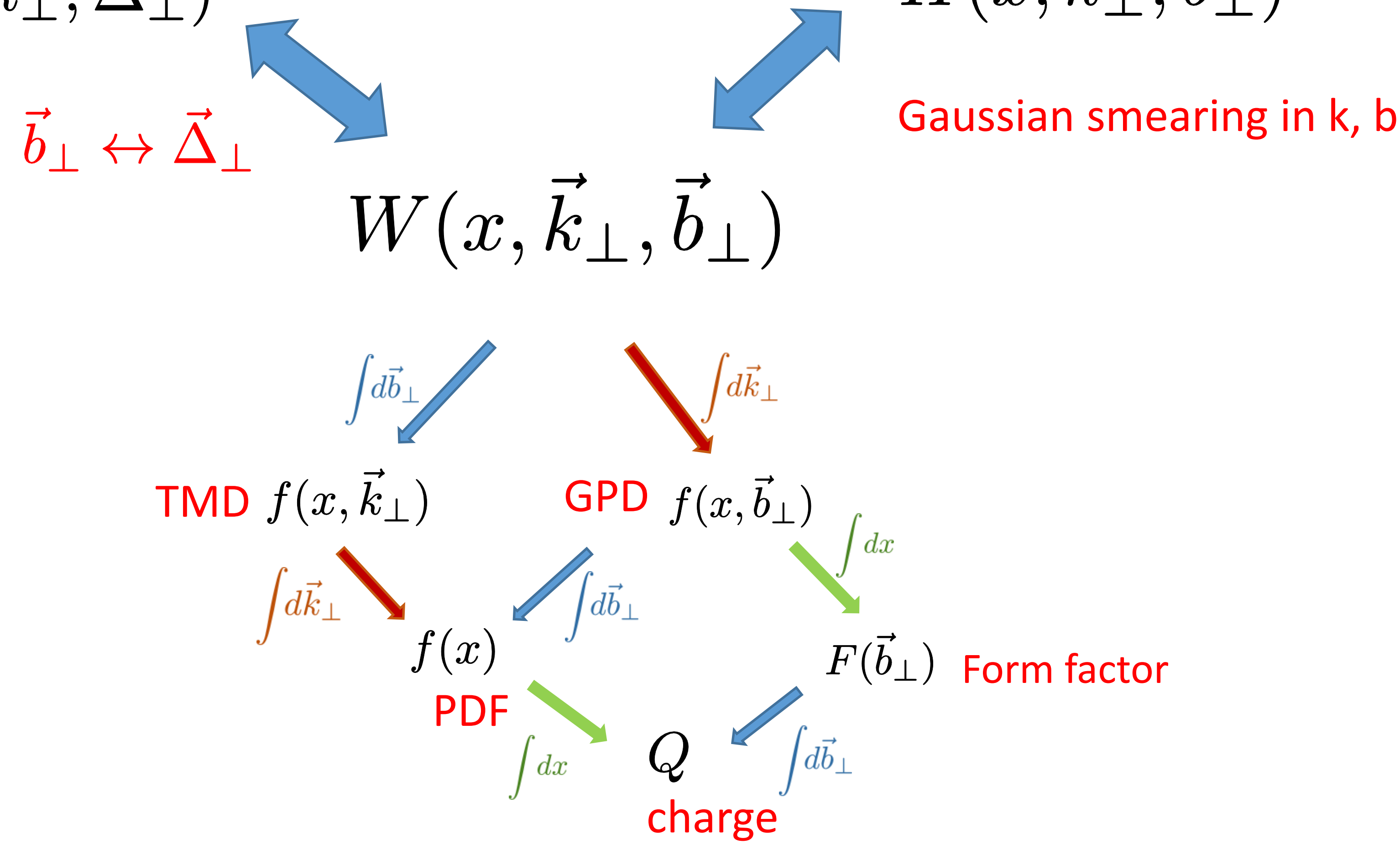
5D tomography: GTMD and Husimi

GTMD Meissner, Metz, Schlegel (2009)

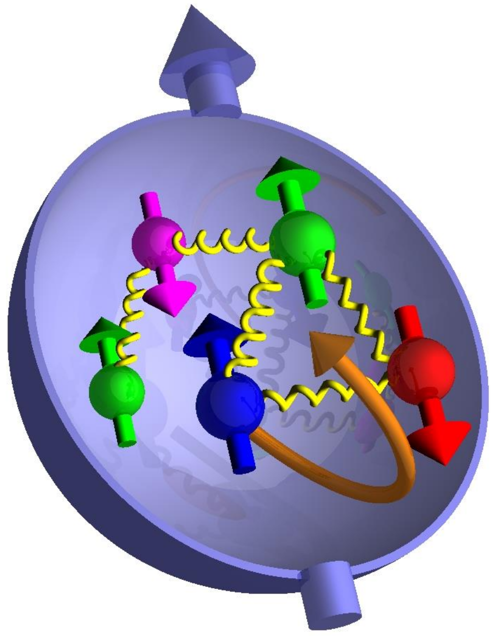
$$W(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

Husimi Hagiwara, YH (2015)

$$H(x, \vec{k}_\perp, \vec{b}_\perp)$$



Wigner distribution and orbital angular momentum



Nucleon spin decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Canonical Orbital angular momentum

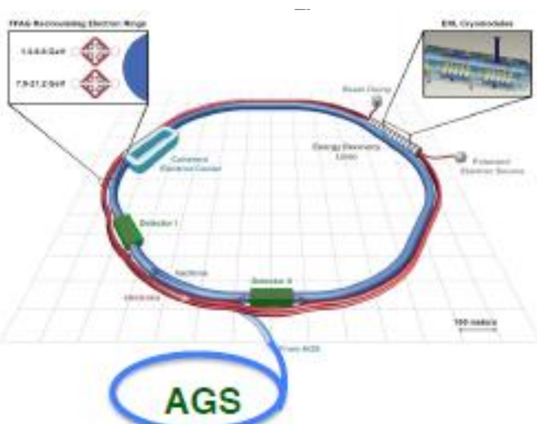
$$L^{q,g} = \int dx \int d^2 b_\perp d^2 k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \begin{cases} W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \\ H^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp) \end{cases}$$

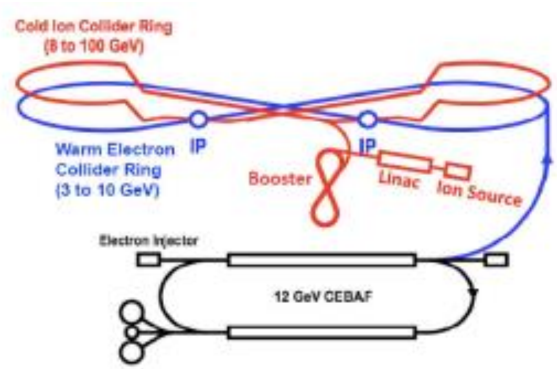
Lorce, Pasquini, (2011);
YH (2011)

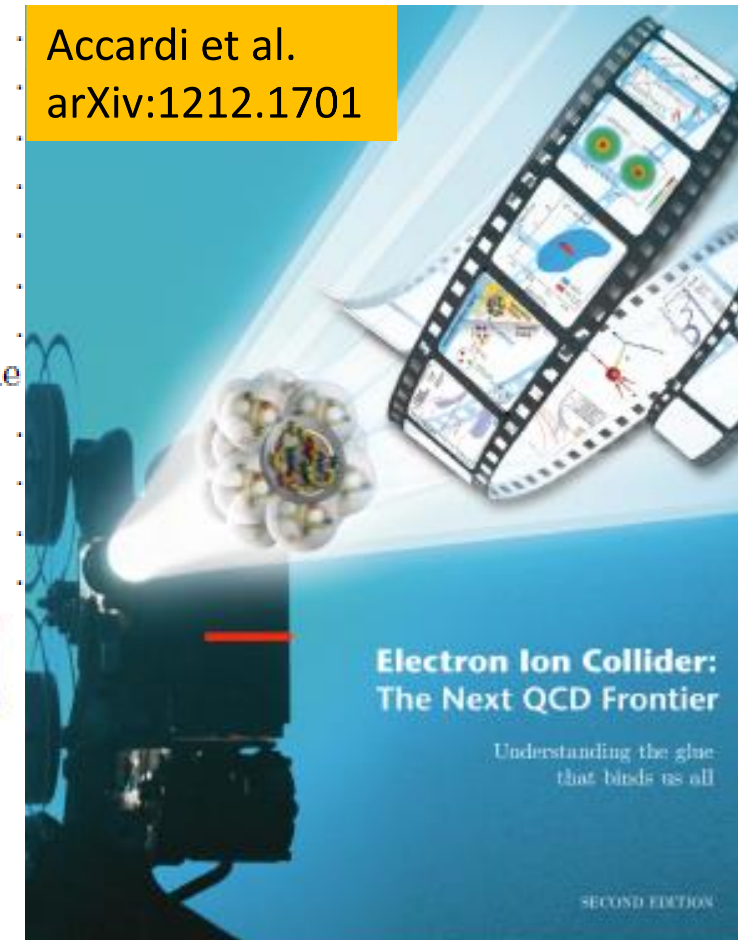
Electron-Ion Collider (EIC)

A future (2025~?), high-luminosity ep, eA experiment dedicated to the study of nucleon structure.

Accardi et al.
arXiv:1212.1701







Wigner distribution: Is it measurable?

In quantum optics, yes!

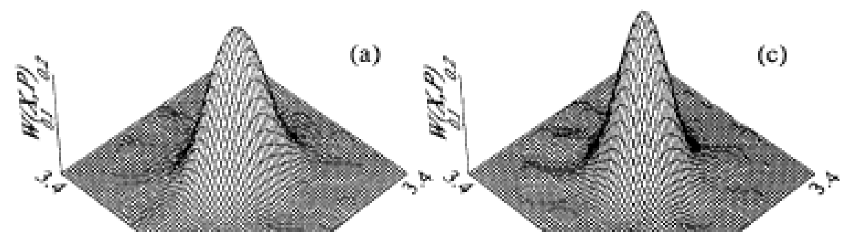
VOLUME 70, NUMBER 9 PHYSICAL REVIEW LETTERS 1 MARCH 1993

Measurement of the Wigner Distribution and the Density Matrix of a Light Mode Using Optical Homodyne Tomography: Application to Squeezed States and the Vacuum

D. T. Smithey, M. Beck, and M. G. Raymer
Department of Physics and Chemical Physics Institute, U

A. Faridani
Department of Mathematics, Oregon State Uni

(Received 16 Novembe



What about in QCD? Go to small-x!

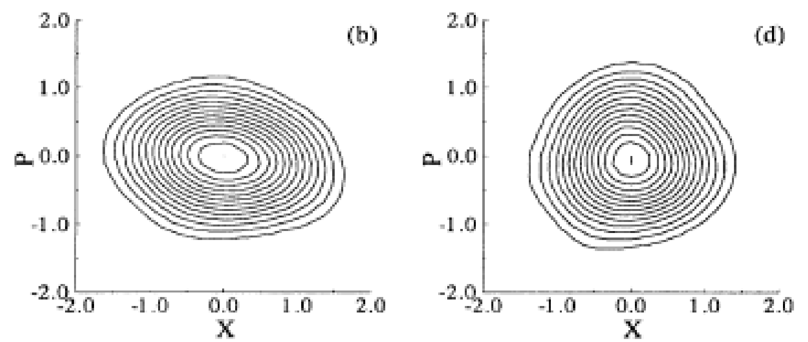


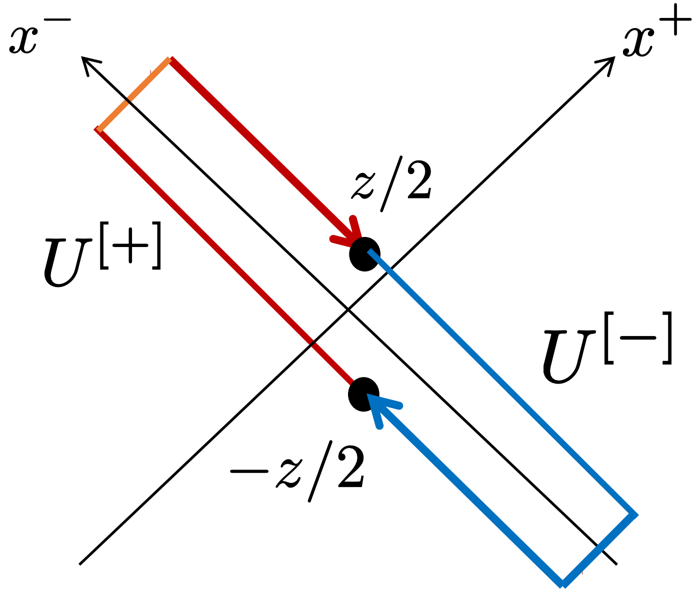
FIG. 1. Measured Wigner distributions for (a),(b) a squeezed state and (c),(d) a vacuum state, viewed in 3D and as contour plots, with equal numbers of constant-height contours. Squeezing of the noise distribution is clearly seen in (b).

Gluon Wigner distribution

$$xW(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int \frac{dz^- d^2z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P-\Delta/2|F^{+i}(-z/2)F^{+}_{i}(z/2)|P+\Delta/2\rangle$$

There are two ways to make it gauge invariant

Dominguez, Marquet, Xiao, Yuan (2011)



Dipole distribution

$$\mathrm{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[-]}]$$

Weizsacker-Williams distribution

$$\mathrm{Tr}[F(-z/2)U^{[+]}F(z/2)U^{[+]}]$$

Dipole gluon Wigner distribution at small-x

YH, Xiao, Yuan (2016)

Approximate $e^{ixP^+z^-} \approx 1$

$$xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s}\int\frac{d^2\vec{r}_\perp}{(2\pi)^2}e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\frac{1}{4}\vec{\nabla}_b^2-\vec{\nabla}_r^2\right)S_x(\vec{b}_\perp,\vec{r}_\perp)$$

``Dipole S-matrix''

$$S_x(\vec{b}_\perp,\vec{r}_\perp)=\left\langle\frac{1}{N_c}\mathrm{Tr}\,U\left(\vec{b}_\perp-\frac{\vec{r}_\perp}{2}\right)U^\dagger\left(\vec{b}_\perp+\frac{\vec{r}_\perp}{2}\right)\right\rangle_x$$

$\cos 2\phi$ correlation expected

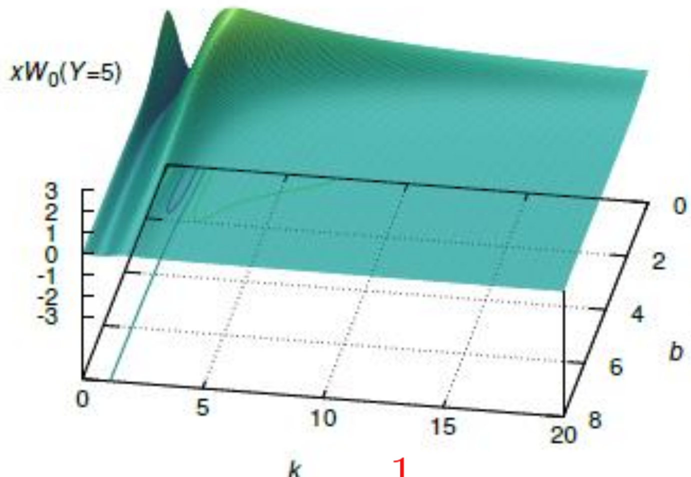
$$W(x,\vec{k}_\perp,\vec{b}_\perp)=W_0(x,k_\perp,b_\perp)+2\cos 2(\phi_k-\phi_b)W_1(x,k_\perp,b_\perp)+\cdots$$

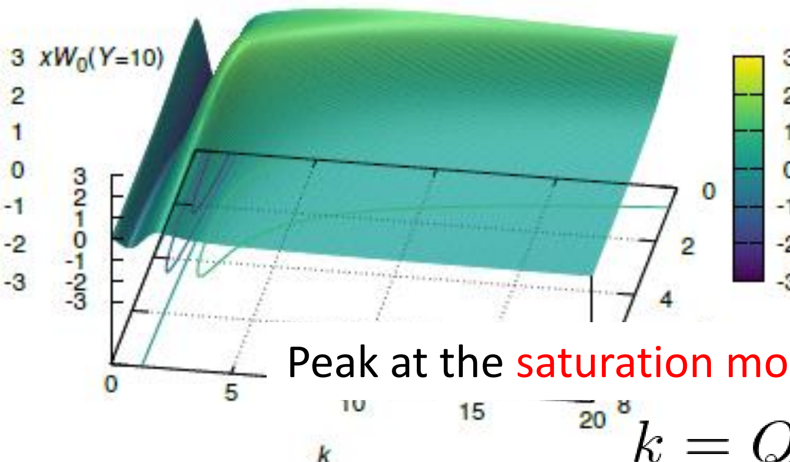
``Elliptic Wigner distribution''

Gluon Wigner from Balitsky-Kovchegov equation

Hagiwara, YH, Ueda (2016)

W_0





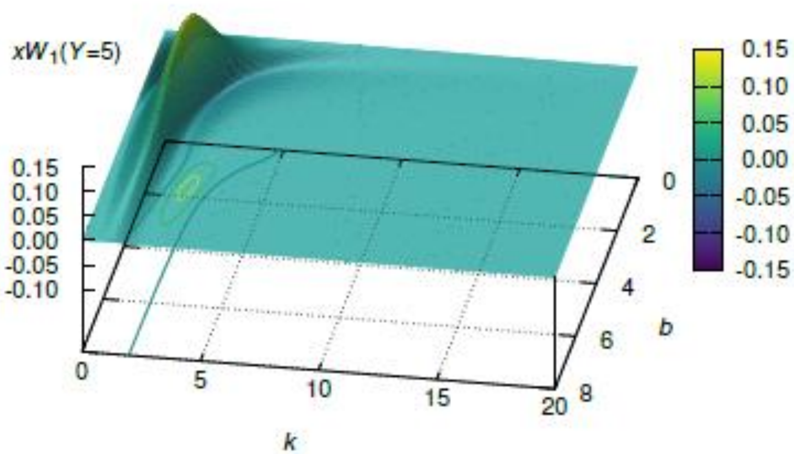
$Y = \ln \frac{1}{x} = 5$

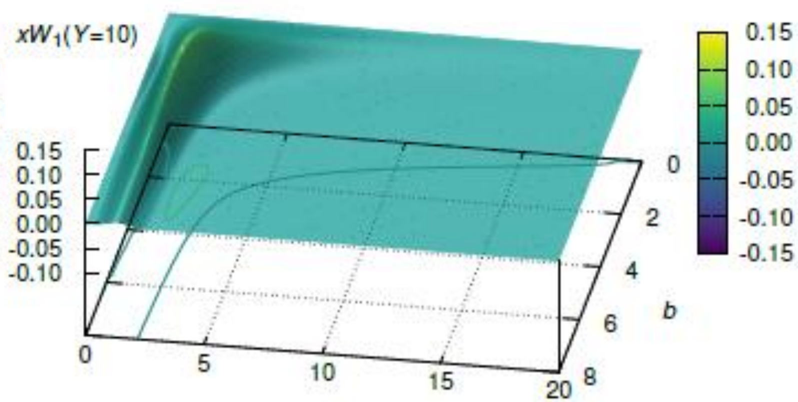
$Y = 10$

Peak at the saturation momentum

$k = Q_s(Y, b)$

W_1





Small in magnitude (a few percent effect)

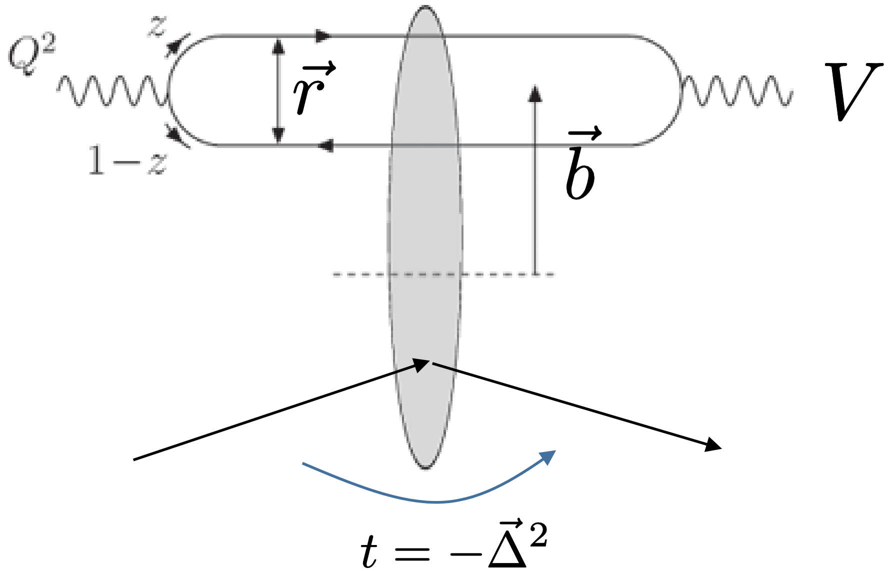
Distinct functional dependence on $x, k_\perp$

How to measure the Wigner distribution?

Find a process sensitive to both $\vec{b}_\perp$ and $\vec{k}_\perp$ .... Nontrivial!

cf. Vector meson production

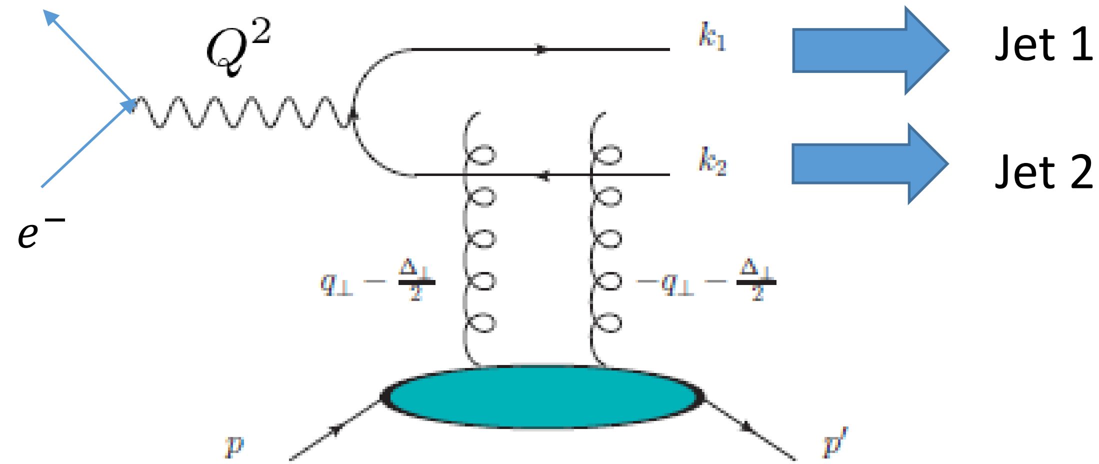
$$\frac{d\sigma}{dt} = \frac{1}{4\pi}\left|\int d^2b\, e^{-i\vec{\Delta}\cdot\vec{b}} A(\vec{b})\right|^2 \qquad A(\vec{b}) = i\int d^2\vec{r}\int dz \Psi^{\gamma^*}(Q,z,\vec{r})(1-S(\vec{r},\vec{b}))\Psi^V(\vec{r},z)$$



One can study the dependence on $\vec{b}$ $(\leftrightarrow \vec{\Delta})$ Munier, Stasto, Mueller, 2001
but not on $\vec{k}$ $(\leftrightarrow \vec{r})$

Probing Wigner (GTMD) in diffractive dijet production

YH, Xiao, Yuan (2016)



$$\vec{\Delta}_\perp = -(\vec{k}_{1\perp} + \vec{k}_{2\perp})$$

$$\vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

Fourier transform of $S(\vec{r}_\perp, \vec{b}_\perp)$

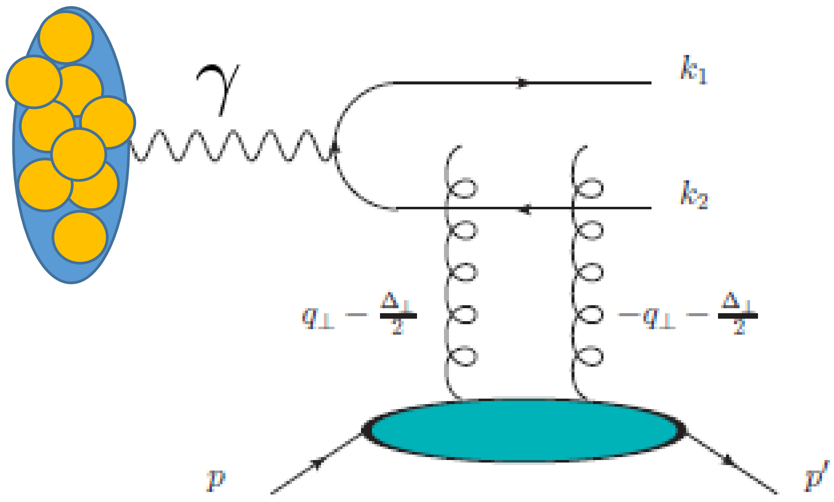
$$\frac{d\sigma^{\gamma_T^* A \to q\bar{q}X}}{dy_1 d^2k_{1\perp} dy_2 d^2k_{2\perp}} \propto z(1-z)[z^2 + (1-z)^2] \int d^2q_\perp d^2q'_\perp S(q_\perp, \Delta_\perp) S(q'_\perp, \Delta_\perp)$$

$$\times \left[\frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{q}_\perp}{(P_\perp - q_\perp)^2 + \epsilon^2}\right] \cdot \left[\frac{\vec{P}_\perp}{P_\perp^2 + \epsilon^2} - \frac{\vec{P}_\perp - \vec{q}'_\perp}{(P_\perp - q'_\perp)^2 + \epsilon^2}\right]$$

$$\sim d\sigma_0 + 2\cos 2(\phi_P - \phi_\Delta) d\tilde{\sigma}$$

Ultra-peripheral pA collisions!

Hagiwara, YH, Pasechnik, Tasevsky, Teryaev, 1706.01765



Q^2 preferably small YH, Xiao, Yuan (2016)

↓

Use the Weiszacker-Williams photons in UPC!

$$\frac{d\sigma^{pA}}{dy_1 dy_2 d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp}} = \omega\frac{dN}{d\omega}\frac{N_c\alpha_{em}(2\pi)^4}{P_\perp^2}\sum_f e_f^2 2z(1-z)(z^2+(1-z)^2)\left(A^2 + 2\cos 2(\phi_P - \phi_\Delta)AB\right)$$

photon flux $\propto Z^2$

$$S_0(P_\perp, \Delta_\perp) = \frac{1}{P_\perp}\frac{\partial}{\partial P_\perp}A(P_\perp, \Delta_\perp).$$

$$S_1(P_\perp, \Delta_\perp) = \frac{\partial B(P_\perp, \Delta_\perp)}{\partial P_\perp^2} - \frac{2}{P_\perp^2}\int^{P_\perp^2}\frac{dP_\perp'^2}{P_\perp'^2}B(P_\perp', \Delta_\perp)$$

Elliptic Wigner in DVCS

YH, Xiao, Yuan (2017)

Gluon transversity GPD

$$\frac{1}{P^+}\int\frac{d\zeta^-}{2\pi}e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$
$$=\frac{\delta^{ij}}{2}xH_g(x,\Delta_\perp)+\frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j-\frac{\delta^{ij}\Delta_\perp^2}{2}\right)+\cdots,$$

$$xE_{Tg}(x,\Delta_\perp)=\frac{4N_cM^2}{\alpha_s\Delta_\perp^2}\int d^2q_\perp q_\perp^2 S_1$$

$$\frac{d\sigma(ep\to e'\gamma p')}{dx_BdQ^2d^2\Delta_\perp}=\frac{\alpha_{em}^3}{\pi x_{Bj}Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)(\mathcal{A}_0^2+\mathcal{A}_2^2)+2(1-y)\mathcal{A}_0\mathcal{A}_2\cos(2\phi_{\Delta l})\right.$$
$$\left.+(2-y)\sqrt{1-y}(\mathcal{A}_0+\mathcal{A}_2)\mathcal{A}_L\cos\phi_{\Delta l}+(1-y)\mathcal{A}_L^2\right\}$$

Elliptic Wigner also relevant to:

\`elliptic flow' v_2 in pA and pp collisions — Hagiwara, YH, Xiao, Yuan (2017)

$\cos 2\phi$ correlation in quasielastic scattering $\gamma_T^* p\to p'X$ — Zhou (2016)

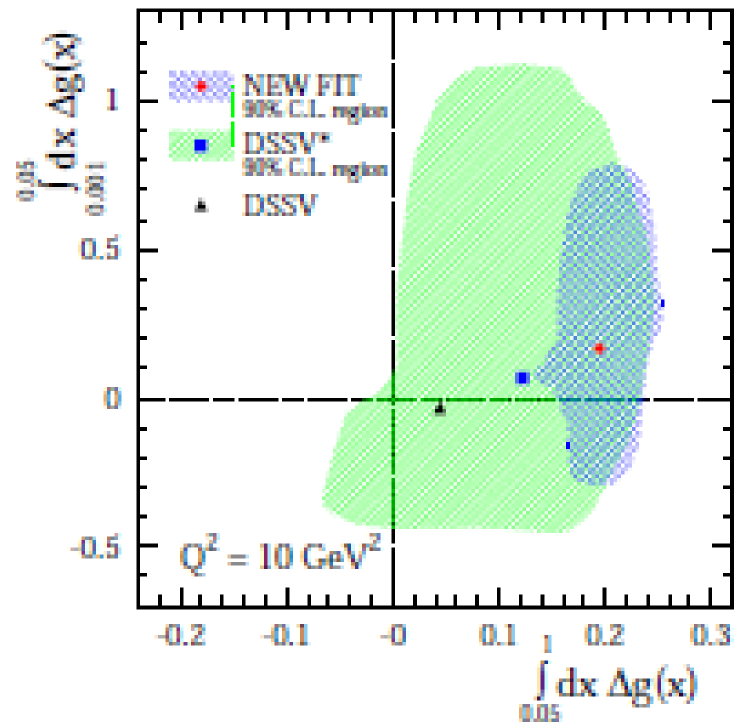
Nucleon spin puzzle and small-x

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Nonzero contribution from `large'-x region

$$\int_{0.05}^{1} dx \Delta G(x, Q^2) \approx 0.2 \pm^{0.06}_{0.07}$$

de Florian, Sassot, Stratmann,Vogelsang (2014)



Huge uncertainty in ΔG from the small-x region.

Maybe $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G$ after including the small-x contribution. OAM not needed?

Can we measure L_{can} ?

A big challenge for the whole community.
No observable proposed so far…although OAM is the future of spin physics!

Hint1: We need to introduce the x-distribution $L_{can} = \int dx L_{can}(x)$ for OAMs

cf. $\Delta\Sigma = \int dx \Delta q(x), \quad \Delta G = \int dx \Delta G(x)$

Hagler, Schafer (1998)
Harindranath, Kundu (1999)
YH, Yoshida (2012)

Hint2: L_{can} is related to the Wigner distribution.
The gluon Wigner distribution is measurable at low-x. YH, Xiao, Yuan (2016)

Recent progress

small-x YH, Nakagawa, Yuan, Xiao, Zhao arXiv:1612.02445
moderate-x Ji, Yuan, Zhao arXiv:1612.02438
quark OAM Bhattacharya, Metz, Zhou arXiv:1702.04387
Liuti, talk in this workshop

OAM from the Wigner distribution

Wigner distribution in QCD

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\int \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle P-\frac{\Delta}{2}|\bar{q}(b-\frac{z}{2})\gamma^+ q(b+\frac{z}{2})|P+\frac{\Delta}{2}\rangle$$

Need a Wilson line !

Define

Lorce, Pasquini (2011);
YH (2011);

$$L^q = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x,\vec{b}_\perp,\vec{k}_\perp)$$

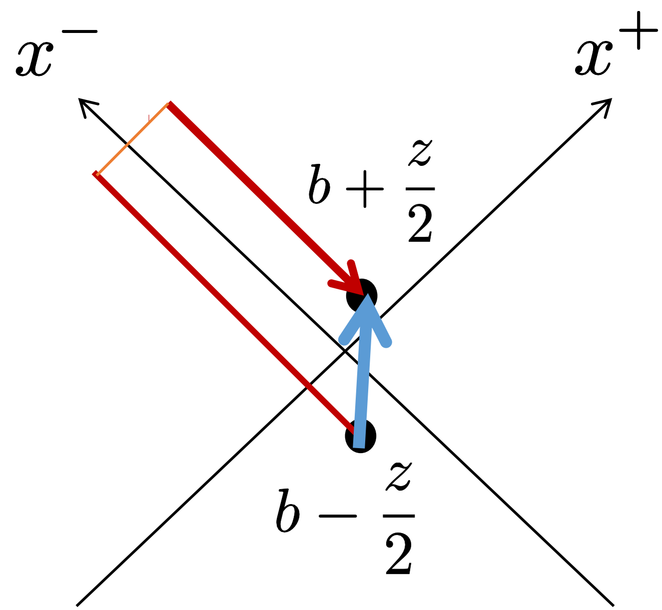
Which OAM is this??

Canonical OAM from the light-cone Wilson line

YH (2011)

$$\int \vec{b} \times \vec{k} W_{light-cone}(\vec{b}, \vec{k}) = \langle \bar{\psi} \vec{b} \times i \overleftrightarrow{D}_{pure} \psi \rangle$$

Kinetic (Ji's) OAM from the straight Wilson line



Ji, Xiong, Yuan (2012)

$$\int \vec{b} \times \vec{k} W_{straight}(\vec{b}, \vec{k})$$

$$= \langle \bar{\psi} \vec{b} \times i \overleftrightarrow{D} \psi \rangle$$

`Potential' OAM

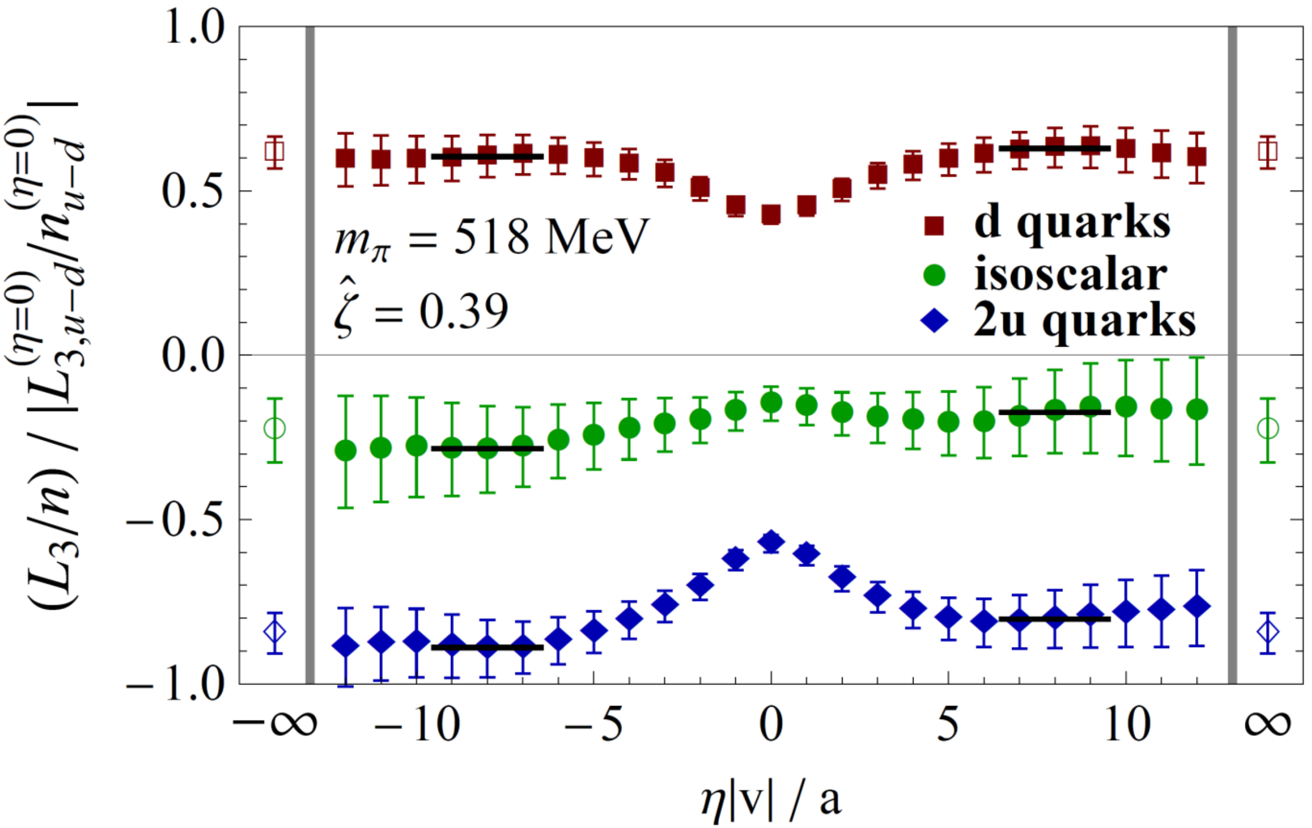
$$L_{pot} \equiv L^q_{\rm Ji} - L^q_{can} = \int dx^- \langle \vec{b} \times \vec{F} \rangle$$

Torque acting on a quark

Burkardt (2012)

Jaffe-Manohar vs. Ji   First lattice result

Engelhardt, 1701.01536



Staple length

OAM parton distribution function

Define the x-distribution $L_{can} = \int dx L_{can}(x)$. YH, Yoshida (2012)

Natural, because Jaffe-Manohar decomposition has a partonic interpretation.

$$L^q_{can} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$\Rightarrow \quad L^q_{can}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp) \qquad ??$$

Go to the momentm space $b_\perp \to \Delta_\perp$ and look for the component

$$W^{q,g} = i\frac{S^+}{P^+}\epsilon^{ij}k^i_\perp \Delta^j_\perp f^{q,g}(x, k_\perp) + \cdots$$

Then

$$L^{q,g}_{can}(x) = \int d^2k_\perp k^2_\perp f^{q,g}(x, k_\perp)$$

Deconstructing OAM

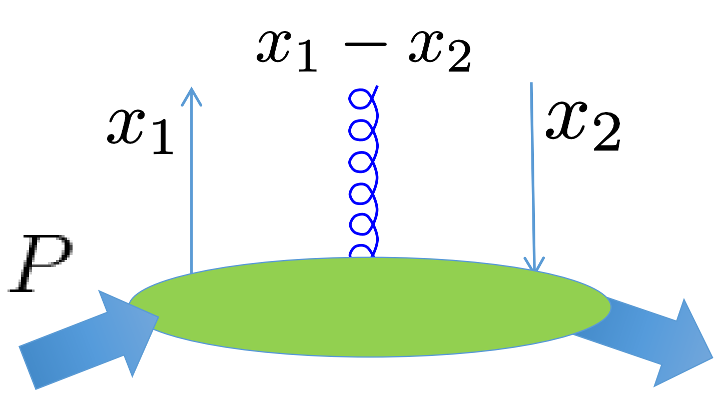
Ji's OAM | canonical OAM | `potential OAM'

$$\langle \bar{\psi}\vec{b}\times\vec{D}\psi\rangle = \langle \bar{\psi}\vec{b}\times\vec{D}_{pure}\psi\rangle + \langle \bar{\psi}\vec{b}\times ig\vec{A}_{phys}\psi\rangle$$

$$A^{\mu}_{phys} = \frac{1}{D^+}F^{+\mu}$$

For a 3-body operator, it is natural to define the double density.

$$\int d\lambda d\mu e^{i\frac{\lambda}{2}(x_1+x_2)+i\mu(x_1-x_2)}\langle P'S'|\bar{\psi}(-\lambda/2)D^i(\mu)\psi(\lambda/2)|PS\rangle$$
$$= \epsilon^{ij}\Delta_j S^+\Phi_D(x_1,x_2)+\cdots$$



Ji's OAM   canonical OAM   `potential OAM'

$$\langle \bar{\psi} \vec{b} \times \vec{D} \psi \rangle = \langle \bar{\psi} \vec{b} \times \vec{D}_{pure} \psi \rangle + \langle \bar{\psi} \vec{b} \times ig \vec{A}_{phys} \psi \rangle$$

doubly-unintegrate

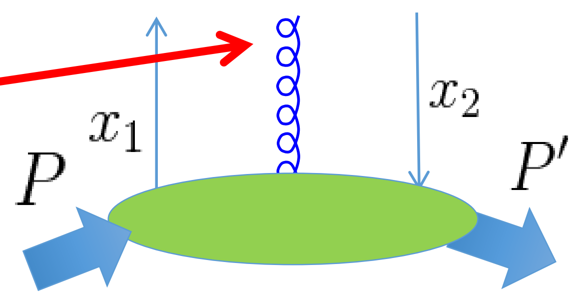
$$\Phi_D(x_1, x_2) = \delta(x_1 - x_2) L^q_{can}(x_1) + \mathcal{P} \frac{1}{x_1 - x_2} \Phi_F(x_1, x_2)$$

The gluon has zero energy, partonic interpretation!

Canonical OAM density

YH, Yoshida (2012)

$$x_1 - x_2 = 0$$



It coincides with $L_{can}(x)$ defined via the Wigner distribution

Twist structure of OAM distributions

YH, Yoshida (2012)

Wandzura-Wilczek part

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1,x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1-x_2)^2}$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^2(x_1-x_2)}\,.$$

Genuine twist-three part

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x') + E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\, \Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$
$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1 - x_2}{x_1^3(x_1-x_2)^2}$$

`DGLAP' equation of OAM PDF

Hagler, Schafer (1998)
YH, Nakagawa, Xiao, Yuan, Zhao (2016)

$$\frac{d}{d\ln Q^2}\begin{pmatrix} L_q(x) \\ L_g(x) \end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \hat{P}_{qq}(z) & \hat{P}_{qg}(z) & \Delta\hat{P}_{qq}(z) & \Delta\hat{P}_{qg}(z) \\ \hat{P}_{gq}(z) & \hat{P}_{gg}(z) & \Delta\hat{P}_{gq}(z) & \Delta\hat{P}_{gg}(z) \end{pmatrix}\begin{pmatrix} L_q(x/z) \\ L_g(x/z) \\ \Delta q(x/z) \\ \Delta G(x/z) \end{pmatrix},$$

$$\hat{P}_{qq}(z) = C_F\left(\frac{z(1+z^2)}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right),$$

$$\hat{P}_{qg}(z) = n_f z(z^2+(1-z)^2),$$

$$\hat{P}_{gq}(z) = C_F(1+(1-z)^2),$$

$$\hat{P}_{gg}(z) = 6\frac{(z^2-z+1)^2}{(1-z)_+} + \frac{\beta_0}{2}\delta(z-1),$$

$$\Delta\hat{P}_{qq}(z) = C_F(z^2-1),$$

$$\Delta\hat{P}_{qg}(z) = n_f(1-3z+4z^2-2z^3),$$

$$\Delta\hat{P}_{gq}(z) = C_F(-z^2+3z-2),$$

$$\Delta\hat{P}_{gg}(z) = 6(z-1)(z^2-z+2),$$

Spin dependence at small-x?

$$S(x,\Delta_\perp,q_\perp) \equiv \int d^2x_\perp d^2y_\perp e^{iq_\perp\cdot(x_\perp-y_\perp)+i(x_\perp+y_\perp)\cdot\frac{\Delta_\perp}{2}} \left\langle P+\frac{\Delta}{2}\left|\frac{1}{N_c}\mathrm{Tr}\left[U(x_\perp)U^\dagger(y_\perp)\right]\right|P-\frac{\Delta}{2}\right\rangle$$

$$= P(x,\Delta_\perp,q_\perp) + iq_\perp\cdot\Delta_\perp O(x,|q_\perp|)$$

"Pomeron" "odderon"

$S(x,\Delta_\perp,q_\perp)$ cannot contain the structure

$$W = i\frac{S^+}{P^+}\epsilon^{ij}q_\perp^i\Delta_\perp^j f(x,q_\perp) + \cdots$$ forbidden by PT symmetry

Lesson: All information about spin is lost in the eikonal approximation.

$$e^{ixP^+z^-} \approx 1$$

OAM as a next-to-eikonal effect

YH, Nakagawa, Xiao, Yuan, Zhao (2016)

Go to next-to-eikonal

$$e^{ixP^+z^-} \approx 1 + ixP^+z^- \quad \Longrightarrow \quad W = W_0 + \delta W$$

$$\delta W(x, \Delta_\perp, q_\perp, S) = \frac{4P^+}{g^2(2\pi)^3} \int d^2x_\perp d^2y_\perp e^{i(q_\perp + \frac{\Delta_\perp}{2})\cdot x_\perp + i(-q_\perp + \frac{\Delta_\perp}{2})\cdot y_\perp}$$
$$\times \left\{ \int_{-T}^{T} dz^- \left(q_\perp^i - \frac{\Delta_\perp^i}{2}\right) \left\langle \mathrm{Tr}\left[U_{Tz^-}(x_\perp) \overleftarrow{D}_i U_{z^- -T}(x_\perp) U^\dagger(y_\perp) \right] \right\rangle \right.$$
$$\left. + \int_{-T}^{T} dz^- \left(q_\perp^i + \frac{\Delta_\perp^i}{2}\right) \langle \mathrm{Tr}\left[U(x_\perp) U_{-Tz^-}(y_\perp) D_i U_{z^- -T}(y_\perp) \right] \rangle \right\}.$$

Can have spin-dependent matrix element. Involves half-infinite Wilson lines

Polarized gluon TMD

$$ix\Delta G(x, q_\perp)\frac{S^+}{P^+} \equiv 2\int \frac{d^2z_\perp dz^-}{(2\pi)^3 P^+} e^{-ixP^+z^- + iq_\perp\cdot z_\perp} \left\langle PS \left| \epsilon_{ij} F^{+i}\left(\tfrac{z}{2}\right) U_- F^{+j}\left(-\tfrac{z}{2}\right) U_+ \right| PS \right\rangle$$

$$\approx \frac{4P^+}{g^2(2\pi)^3}\int d^2x_\perp d^2y_\perp e^{i(q_\perp + \frac{\Delta_\perp}{2})\cdot x_\perp + i(-q_\perp + \frac{\Delta_\perp}{2})\cdot y_\perp}$$

$$\times \epsilon_{ij}\left\{ q_\perp^j \int_{-\infty}^{\infty} dz^- \left\langle \mathrm{Tr}\left[U_{\infty z^-}(x_\perp) \overleftarrow{D}_i U_{z^- -\infty}(x_\perp) U^\dagger(y_\perp) \right] \right\rangle \right.$$

$$\left. + q_\perp^i \int_{-\infty}^{\infty} dz^- \langle \mathrm{Tr}\left[U(x_\perp) U_{-\infty z^-}(y_\perp) D_j U_{z^- \infty}(y_\perp) \right] \rangle \right\}$$

Exactly the same matrix element appears.
→ Linear relation between $\Delta G(x)$ and $L_g(x)$

Our conjecture: $$L_g(x) \approx -2\Delta G(x)$$

YH, Nakagawa, Xiao, Yuan, Zhao (2017)

DLA limit of the DGLAP equation

$$\frac{d}{d\ln Q^2}\Delta G(x) \approx \frac{2C_A\alpha_s}{\pi}\int_x^1 \frac{dz}{z}\Delta G(z),$$

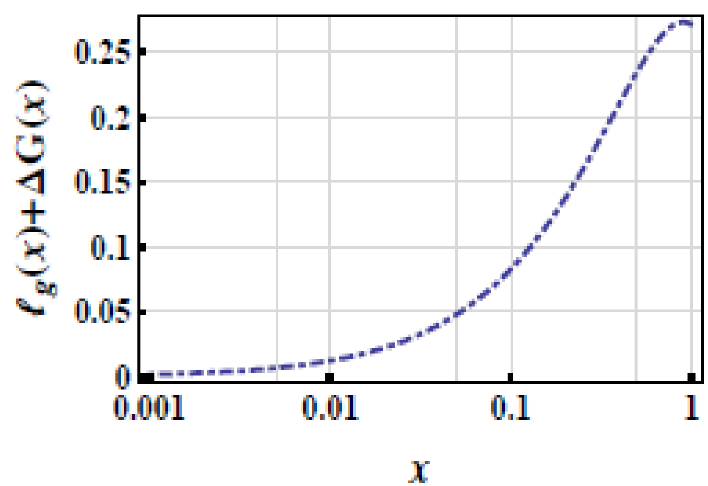
$$\frac{d}{d\ln Q^2}L_g(x) \approx \frac{C_A\alpha_s}{\pi}\int_x^1 \frac{dz}{z}(L_g(z) - 2\Delta G(z)).$$

$$\Rightarrow \quad \frac{d}{d\ln Q^2}(L_g(x)+2\Delta G(x)) \approx \frac{C_A\alpha_s}{\pi}\int_x^1 \frac{dz}{z}(L_g(z)+2\Delta G(z))$$

$$|L_g(x) + 2\Delta G(x)| \ll |\Delta G(x)|, |L_g(x)|,$$

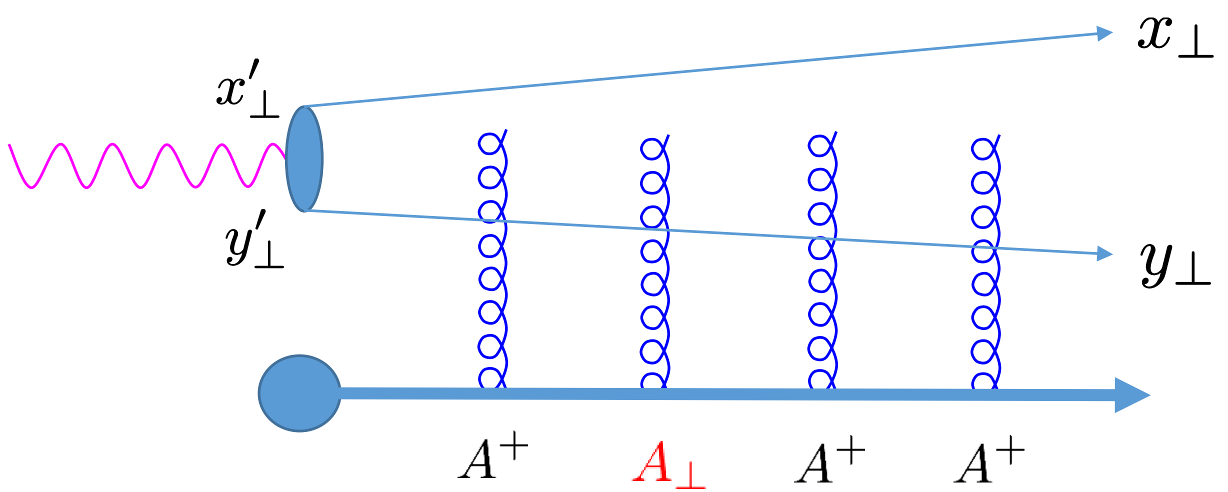
Model calculation

More, Mukherjee, Nair, 1709.00943



Dijet production at next-to-eikonal

$$\frac{1}{N_c}\mathrm{Tr}[U(x_\perp)U^\dagger(y_\perp)] \Rightarrow \frac{1}{N_c}\mathrm{Tr}[U(x_\perp,x'_\perp)U^\dagger(y_\perp,y'_\perp)]$$



Green's function

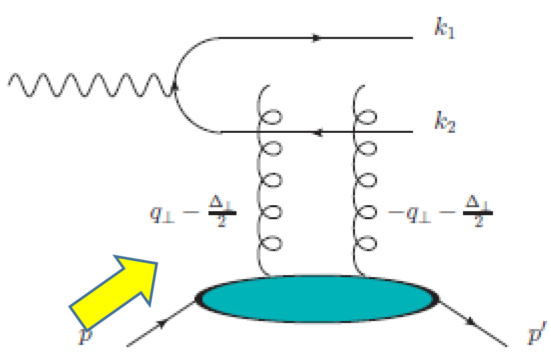
$$\left[i\frac{\partial}{\partial x^-}+\frac{1}{2k^-}D^2_{x_\perp}-gA^+(x^-,x_\perp)\right]G_{k^-}(x^-,x_\perp,x'^-,x'_\perp)=i\delta(x^--x'^-)\delta^{(2)}(x_\perp-x'_\perp)$$

$$\int d^2x_\perp e^{-ik_\perp\cdot(x_\perp-x'_\perp)}U(x_\perp,x'_\perp)=U(x'_\perp)+\frac{i}{2k^-}\int_{-\infty}^{\infty}dz^-U_{\infty z^-}(x'_\perp)(\overleftarrow{D}^2_{x'_\perp}-2ik^i_\perp\overleftarrow{D}_{x'^i_\perp}-k^2_\perp)U_{z^--\infty}(x'_\perp)$$

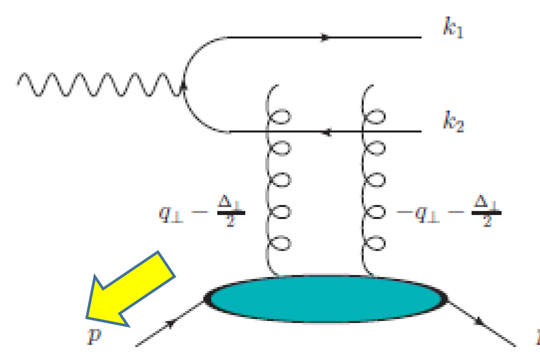
cf. Altinoluk, Armesto, Beuf, Martinez, Salgado (2014)

Longitudinal single spin asymmetry in dijet production

$d\Delta\sigma \sim$ |



$|^2$ **-** |



$|^2$

Interference between eikonal (Pomeron, Odderon) and next-to-eikonal (OAM) contributions.

OAM interferes with odderon, but Pomeron usually dominates….

Pomeron contributions suppressed in the kinematic regions

$$P_\perp \gg q_\perp, Q \qquad Q \gg q_\perp, P_\perp \qquad \vec{P}_\perp = \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$$

$$\int d^2 q_\perp P(x, q_\perp, \Delta_\perp) \propto \delta(\Delta_\perp)$$

$$\frac{d\Delta\sigma}{dy_1 d^2 k_{1\perp} dy_2 d^2 k_{2\perp}}$$
$$\approx 4\pi^4 \alpha_s N_c \alpha_{em} x \sum_f e_f^2 \delta(x_{\gamma^*} - 1)(1-2z)\frac{z^2+(1-z)^2}{z^2(1-z)^2}$$
$$\times \frac{P_\perp \Delta_\perp}{Q^6} \sin\phi_{P\Delta} \left\{ \begin{matrix} -2\Delta G(x) \\ L_g(x) \end{matrix} \right\} \int d^2 q_\perp q_\perp^2 O(x, q_\perp).$$

Measurable at the EIC!

Conclusions

- Let's get 5-dimensional. Even richer physics than GPD and TMD combined.
  Nice addition to the EIC agenda!

- OAM: Holy grail in spin physics. Connection to Wigner crucial.
  3D distributions do not give direct access to (canonical) OAM.

- Diffractive dijet events in ep (photoproduction) and pA (UPC)
  → Promising channel to study Wigner. Look at $\cos 2\phi$ and $\sin\phi$ dependences

- Progress in the small-x evolution of helicity distributions. Kovchegov, Pitonyak, Sievert
  Possibly large spin content in the small-x region?