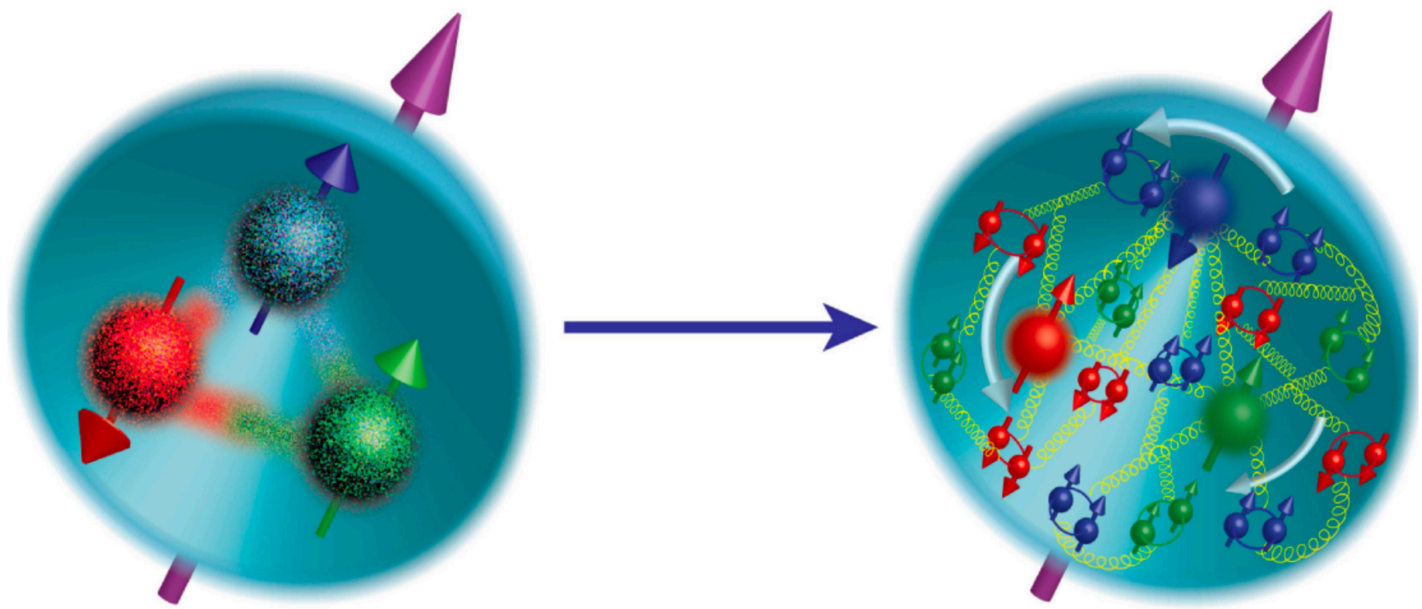
Relating TMD & Collinear Factorization

"3-D nucleon structure"

INT Program INT–17–3
Spatial and Momentum Tomography of Hadrons and Nuclei,
8/28 – 9/29, 2017



Leonard Gamberg

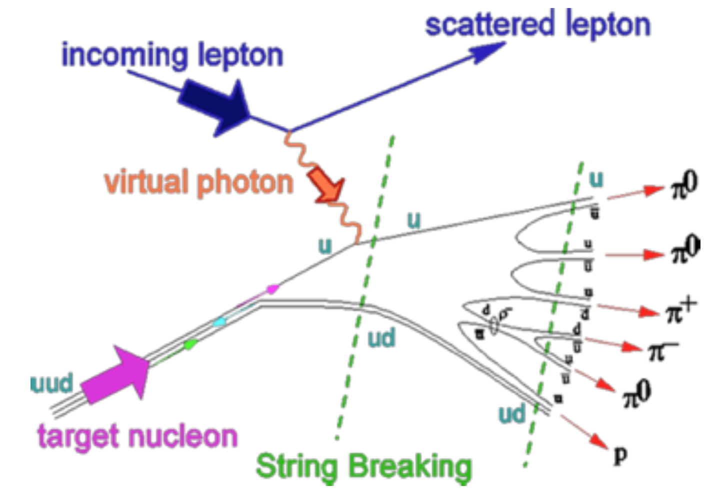
Combination of 2 talks Sept 22 & 25, 2017

U.S. DEPARTMENT OF ENERGY | Office of Science

TMD Collaboration

PennState Berks

Overview comments



- Present an implementation combining TMD factorization and collinear factorization for studying nucleon structure in SIDIS

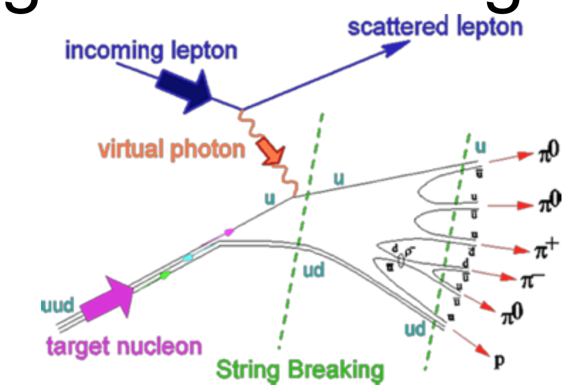
  Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

- This entails a modification of the so called “W+Y” construction of the SIDIS cross section

- Address “*standard matching prescription*” traditionally used in CSS formalism relating low & high q_T behavior cross section @ moderate Q

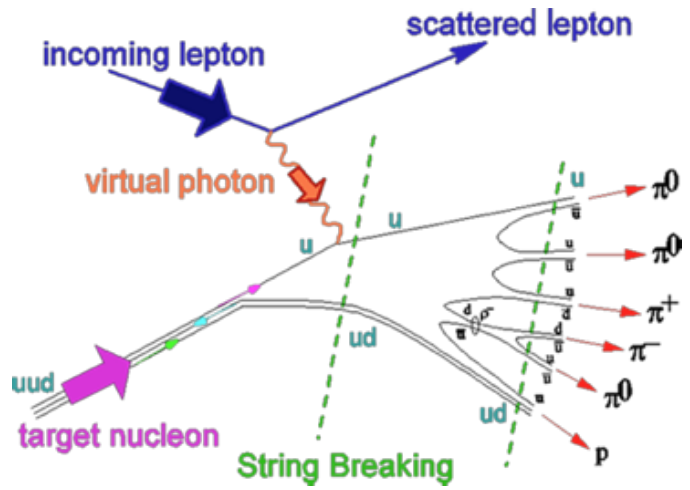
Overview comments

- Addressing the role of so called "*Y* term " matching of low and high q_T behavior of cross section @ moderate Q



  - *Collins Soper Sterman NPB 1985, Altarelli et al, NPB 1985*
  - *Bozzi, Catani et al. NPB 2006, JHEP 2015*
  - *Davies Webber, Stirling, NPB 1985, Arnold and Kauffman NPB 1991*
  - *A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)*
  - *Boglione, Gonzoles, Melis, Prokudin JHEP 2014*
  - *Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang*
- In progress: an extended treatment transversely polarized case, the Sivers Effect
  - *Transverse case, Ji et al. 2006, Kang et al. 2011, Eguchi et al. 2007 …*
  - *new …L.Gamberg, A. Metz, D. Pitonyak, A. Prokudin, T. Rogers … 2017*
- We are able to recover the well-known relations between TMD and collinear quantities one expects from the parton model.
- We recover the LO collinear twist 3 result from a weighted q_T integral of the differential cross section and derive the well known relation between the TMD Sivers function and the collinear twist 3 Qiu Sterman function

Overview comments



- There are a number of pieces to this:
- From matching cross section point by point in q_T (especially @ relatively low Q)
- To improved methods relating TMD & collinear factorization for unpolz.
- To relating the twist 2 and twist 3 formulations of TSSAs

Review of Resummation

Last week 9/22/17

The "W +Y" prescription to describing the q_T dependent cross section now being applied to SIDIS in the language of TMD factorization has its origin in the study of generic high mass systems (vector bosons, Higgs particles, . . .) produced in Drell Yan collisions (e.g. at the Tevatron and now at the LHC)

- *CollinsSoperSterman NPB 1985,*
- *Altarelli et al, NPB 1984*
- *Davies Webber, Stirling, NPB 1985,*
- *Arnold and Kauffman NPB 1991*
- *Nadolsky, Stump, Yuan zPRD 2000*
- *J.-W. Qiu, Zhang, PRL 2001,*
- *Berger, J.-W. Qiu, PRD 2003*
- *A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)*
- *Boglione, Gonzoles, Melis, Prokudin JHEP 2014*
- *Bozzi, Catani et al. NPB 2006, JHEP 2015 …*
- *Collins, Gamberg, Prokudin, Sato, Rogers, Wang, PRD (2016)*

Review of Resummation

Last week 9/22/17

e.g., to obtain a precise measurement of the W mass it is important to have accurate theoretical calculations of the W and Z bosons q_T spectra (…talk of Andrea Signori)

In the large-q_T region ($q_T \sim m_V$), where the transverse momentum is of the order of the vector boson mass m_V, one applies conventional perturbation theory to get at the q_T dependent cross section QCD corrections are known up to $O(\alpha_S{}^2$) and in some case beyond…

Review of Resummation

Last week 9/22/17

However, the bulk of the vector boson cross section is produced in small-q_T region ($q_T \ll m_V$), where reliability of the fixed-order expansion is spoiled by the presence of large logarithmic corrections, $\alpha_S^n (m^2_V / q_T^2) \ln^m(m^2_V / q_T^2)$ of soft & collinear origin

Review of Resummation

Last week 9/22/17

To obtain reliable predictions, these logarithmically-enhanced terms have to be evaluated and systematically "resummed" to all orders in perturbation theory

For large energy and Q^2 the "resummed" and fixed-order calculations, valid at small and large q_T, respectively, can be consistently matched at intermediate values of q_T to achieve a uniform theoretical accuracy for the entire range of transverse momenta

However at lower phenomenologically interesting values of Q, neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic

It is this matching that I will focus on in the context of TMD factorization physics and its connection to collinear limit.

In recent years, the resummation of small-q_T logarithms has been reformulated by using SCET & and TMD factorization

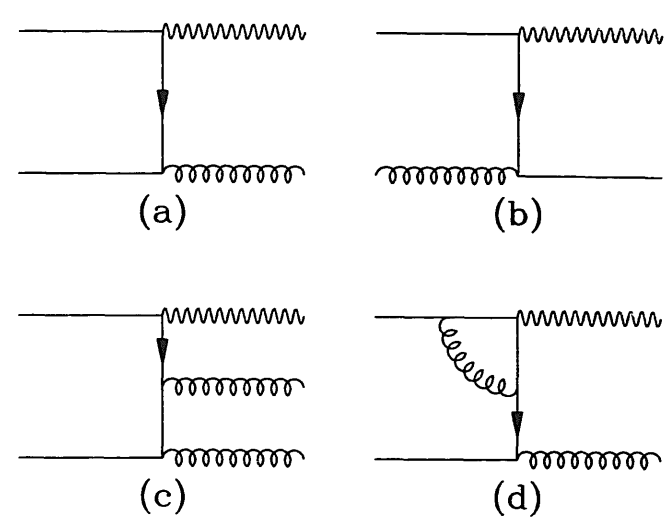
Review of Resummation

Last week 9/22/17

At large transverse momentum q_T one calculates the cross section for W & Z production by factorized conventional pert. theory

$$\frac{d\sigma_F}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, \frac{d\hat{\sigma}_{Fab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\frac{d\hat{\sigma}}{dq_T^2} = \alpha_W \alpha_s \left(u_1 + \alpha_s u_2 + \alpha_s^2 u_3 + \ldots\right)$$



Some examples of Feynman diagrams contributing to W or Z production at non-zero q_T: (a, d) $q\bar{q} \to Wg$, (b) $qg \to Wq$, (c) $q\bar{q} \to Wgg$.

Review of Resummation

Last week 9/22/17

At low q_T , however, the convergence of the perturbation series deteriorates as dominant contributions have the form $\alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right)$

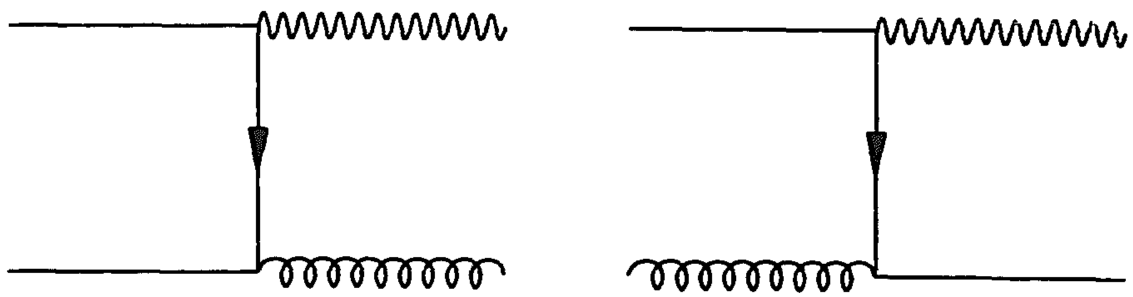
The convergence of the series is governed by $\alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right)$ rather than simply α_s

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_w \alpha_s}{q_T^2} \ln\left(\frac{Q^2}{q_T^2}\right)\left[v_1 + v_2 \alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right) + v_3 \alpha_s^2 \ln^4\left(\frac{Q^2}{q_T^2}\right) + \ldots\right]$$

Fortunately, the coefficients v_i of the "leading-logarithm" approximation are not independent and it is possible to sum the series exactly so that it may be applied even when $\alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right)$ is large

Fixed order theory calculation "asymptotically" diverges at low q_T *cannot by itself describe data*

Last week 9/22/17



$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} \to \frac{\alpha_s}{q_T^2}\ln\left(\frac{M^2}{q_T^2}-\frac{3}{2}\right)$$

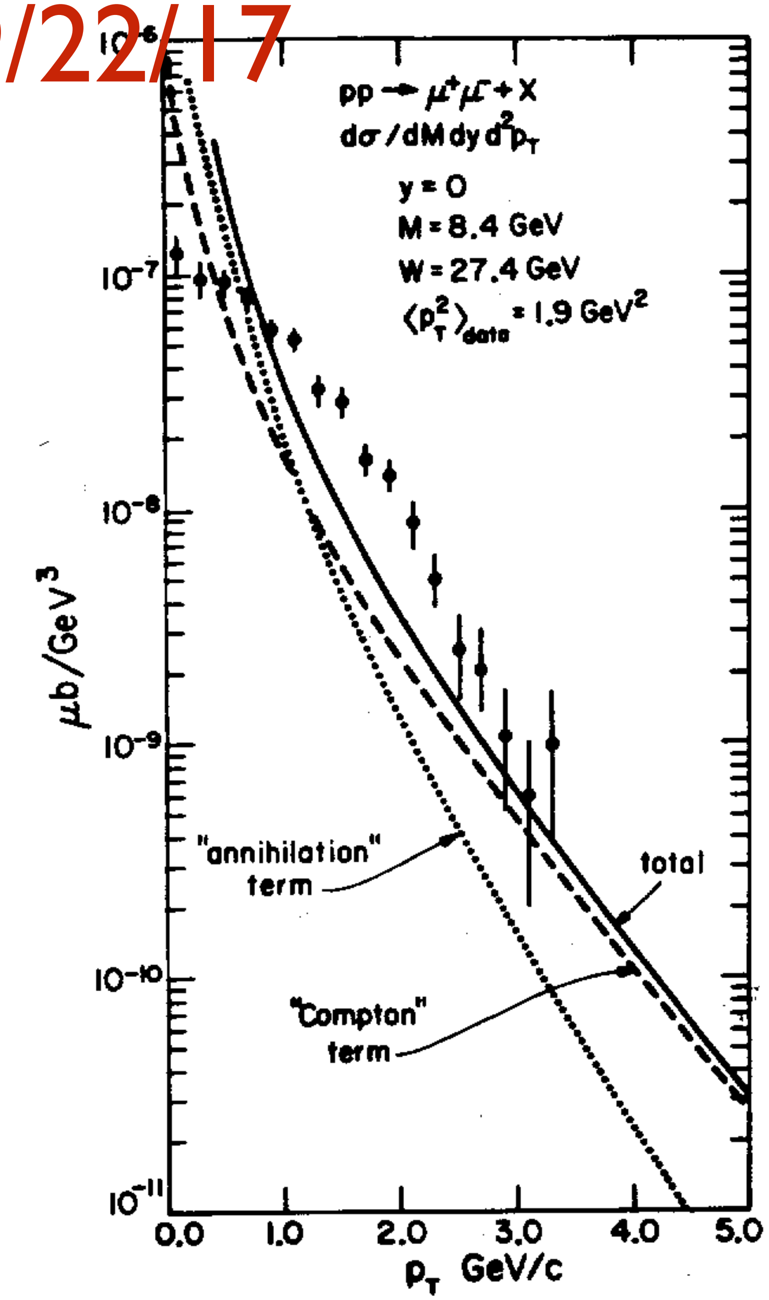


Figure 5.8 The distribution in transverse momentum, p_T, of muon pairs, $\mu^+\mu^-$ produced in pp collisions at $W = \sqrt{s} = 27.4$ GeV compared with the leading order perturbative QCD result. The "Compton" and "annihilation" contributions are given by the dashed and dotted curves, respectively (*taken from Ref. 9*).

From Resummation to CSS

This reorganization and resummation was carried out by Collins and Soper in b space; the result is

✦ *Collins Soper, NPB 1982*

✦ *CSS NPB 1985*

$$\frac{d\sigma}{dq_T^2 dQ^2}(resum) \approx \frac{4\pi^3\alpha_w}{3s} e^2 \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T\cdot b_T} \sum_i \tilde{W}_i(b_T, Q)$$

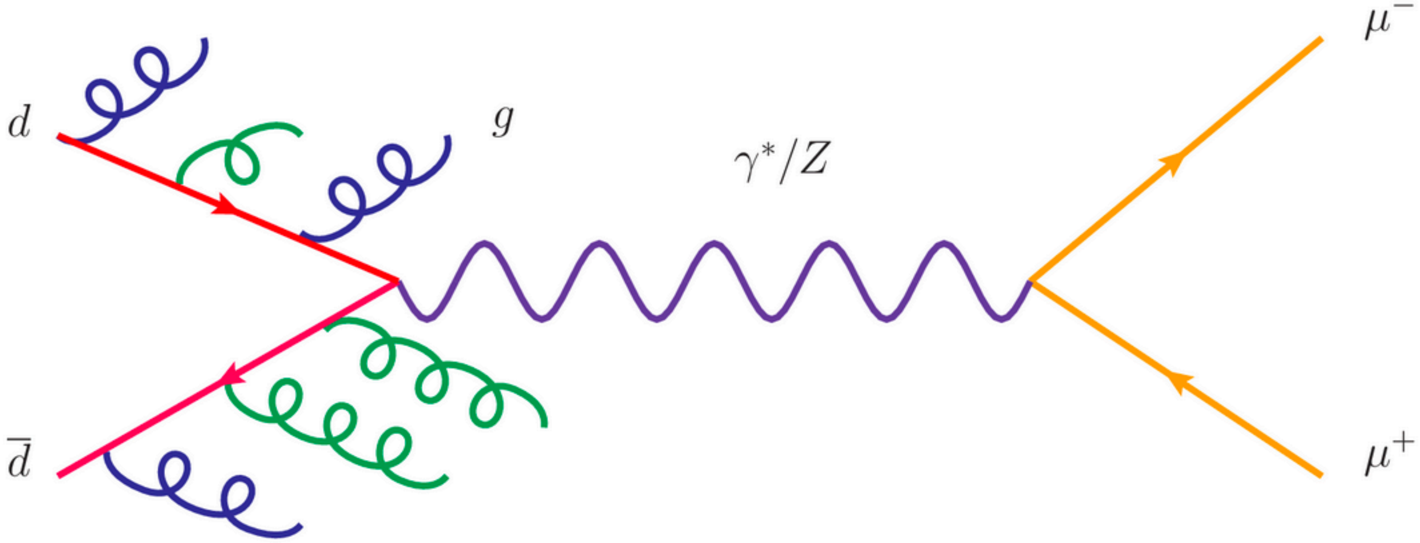
$$\tilde{W}_i(b_T, Q) = H_i(Q) \left(\tilde{C}_i^{pdf}(x_A/\hat{x}, b_T) \otimes \tilde{f}_{i/A}(\hat{x}, \mu_b)\right) \left(\tilde{C}_j^{pdf}(x_B/\hat{z}, b_T) \otimes \tilde{\bar{f}}_{j/B}(\hat{x}, \mu_b)\right) e^{-S(b_T, Q)}$$

... TMD factorization

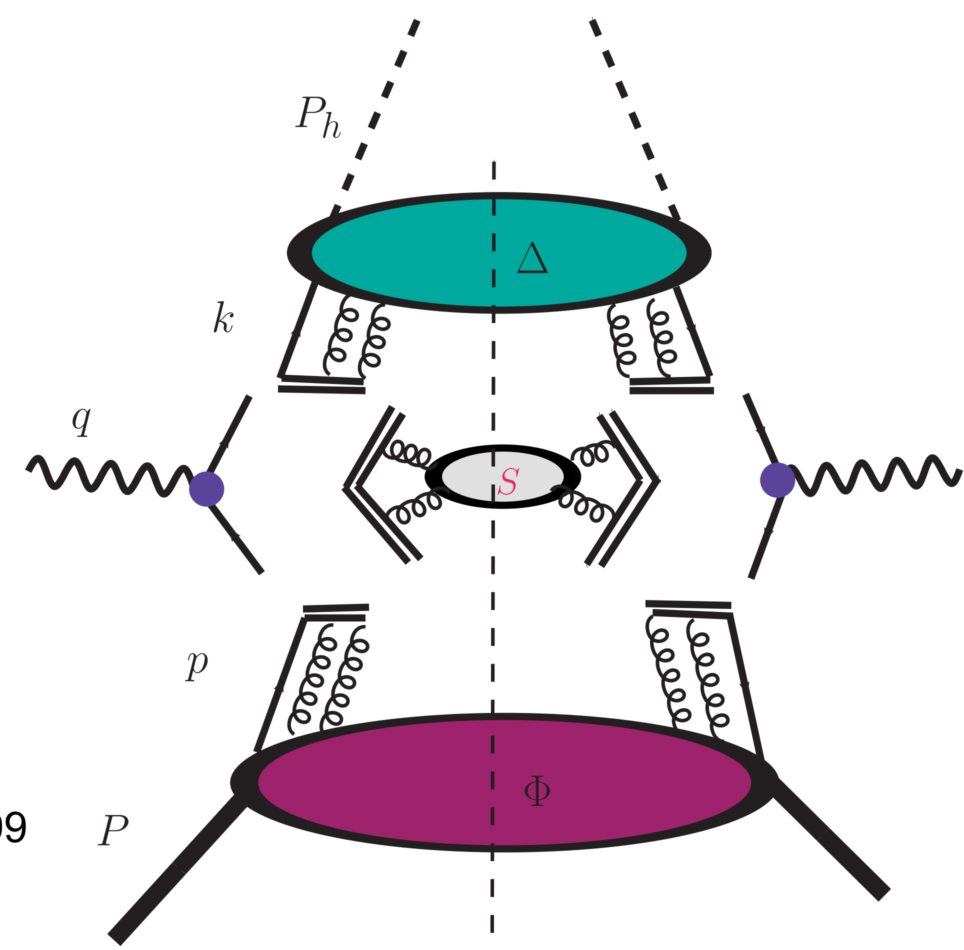
This expression contains the OPE on the Fourier transforms of the TMDs with soft gluon resummation in exponent. We will unpack this!

“resummation” Soft gluons

Small q_T effects factorize into the Soft Factor



Associated with rapidity divergences
Effects factorizes into the Soft Factor



Switch now to SIDIS

✦ Nadolsky Stump C.P. Yuan PRD 1999

Y-term & Matching

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\mathrm{SIDIS}}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\ \tilde{f}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2) \ + \ Y_{\mathrm{SIDIS}}$$

In full QCD, the auxiliary parameters μ and ζ are exactly arbitrary and this is reflected in the the Collins-Soper (CS) equations for the TMD PDF, and the renormalization group (RG) equations

JCC Cambridge Press 2011, Collins arXiv: 1212.5974, Collins, Gamberg, Prokudin, Roger, Sato, Wang PRD 2016

Factorized Evolved TMDs

- Separate small b_T -Perturbative
- $\&$ Large b_T -non-perturbative

$$b_*(b_T) = \sqrt{\frac{b_T^2}{1 + b_T^2/b_{max}}}$$

$$f_1(x, k_T; \mu, \zeta_F) = \int \frac{d^2 b_T}{(2\pi)^2} e^{-ik_T \cdot b_T} f_1(x, b_T; \mu, \zeta_F)$$

Summary of elements of TMD factorization

$$\tilde{W}_{UU}(x,z,b,Q^2) = H_{UU}(Q,\mu=Q)\sum_q e_q^2 \tilde{f}_1^q(x,b,\mu,\zeta_F)\tilde{D}_1^q(z_h,b,\mu,\zeta_D)$$
$$= H_{UU}(Q,\mu=Q)\sum_q e_q^2 \tilde{f}_1^q(x,b_*,\mu,\zeta_F)\tilde{D}_1^q(z_h,b_*,\mu,\zeta_D)e^{-S_{\rm pert}(b_*,Q)-S_{UU}^{NP}(b,Q)}$$
$$= H_{UU}(Q,\mu=Q)\sum_q e_q^2\, C_{q\leftarrow i}^{\rm SIDIS}\otimes \tilde{f}_1^i(x,\mu_b)\hat{C}_{j\leftarrow q}^{\rm SIDIS}\otimes \tilde{D}_{h/j}^q(x,\mu_b)e^{-S_{\rm pert}(b_*,Q)-S_{UU}^{NP}(b,Q)}$$

Formalism expresses evolution of TMDS via OPE in terms of collinear pdfs in b-space

Evolution of Collinear PDFs and multiparton correlation functions relevant single transverse-spin asymmetry through DGLAP and its generalization

Summary of elements of TMD factorization

With $\mu_b = C_1/b_*$ as hard scale, the b dependence of TMDs is calculated in perturbation theory and related to their collinear parton distribution (PDFs), fragmentation functions (FFs), or multiparton correlation functions , … OPE

$$\tilde{f}_1^i(x, b; Q) = C_{q\leftarrow i}^{f_1} \otimes f_1^i(x, \mu_b) e^{\frac{1}{2} S_{pert}(Q, b_*) - S_{NP}^{f_1}(Q, b)}$$

$$C_{q\leftarrow i} \otimes f_1^i(x_B, \mu_b) \equiv \sum_i \int_{x_B}^1 \frac{dx}{x} C_{q\leftarrow i}\left(\frac{x_B}{x}, \mu_b\right) f_1^i(x, \mu_b)$$

$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi}\right)^n C^{(n)}$$ Wilson coefficient

TMD factorization & evolution from b-space rep of SIDIS cross section interpret as a multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate $\boldsymbol{P}_{h\perp}$

$\tilde{W}_{UU}(x,z,b,Q^2)$

$$\frac{d\sigma}{dx_B\,dy\,d\phi_S\,dz_h\,d\phi_h\,|\boldsymbol{P}_{h\perp}|d|\boldsymbol{P}_{h\perp}|} =$$

$$\frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|\Big\{J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,T}+\varepsilon\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU,L}$$

$$+\quad\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos\phi_h}+\varepsilon\cos(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UU}^{\cos(2\phi_h)}$$

$$+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LU}^{\sin\phi_h}$$

Boer, Gamberg, Musch, Prokudin, **JHEP (2011)**

$$+\quad S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UL}^{\sin 2\phi_h}\right]$$

$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{LL}^{\cos\phi_h}\right]$$

$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,\mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\ \varepsilon\,\sin(\phi_h+\phi_S)\,J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+\ \varepsilon\,\sin(3\phi_h-\phi_S)\,J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\,\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)}=-\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\tilde{D}_1]$$

$$\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)}=-\mathcal{P}[\tilde{h}_1\tilde{H}_1^{\perp(1)}]$$

$$\ldots\qquad+\qquad Y$$

Matching TMD & large q_T cross section

- With this insight on connection of TMD factorization and collinear/FO factorization theorems, one can study matching of large q_T cross section & the TMD

- ✦ *Collins Soper Sterman NPB 1982*
- ✦ *A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)*
- ✦ *Boglione, Gonzalez, Melis, Prokudin JHEP 2014*
- ✦ *Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang*
- ✦ *new …Gamberg, Metz, Pitonyak, Prokudin, Rogers … 2017*

Y-term & Matching

Last week 9/22/17

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\text{SIDIS}}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\; \tilde{F}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2) \;+\; Y_{\text{SIDIS}}$$

The *Y* term ???Remainder/correction ???

JCC Cambridge Press 2011, Collins arXiv: 1212.5974, Collins, Gamberg, Prokudin, Roger, Sato, Wang PRD 2016

Y-term & Matching

Last week 9/22/17

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\mathrm{SIDIS}}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\ \tilde{F}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2) \ +\ Y_{\mathrm{SIDIS}}$$

Consider the full q_T spectrum of the DY process: one resums the low q_T contribution to get sensible result @ low q_T: but we still have the FO which describes reasonably well the large q_T CS.

Should we just add the FO and the W term?

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + FO(q_T, Q) + ??\ O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)\ ??$$

JCC Cambridge Press 2011, Collins arXiv: 1212.5974, Collins, Gamberg, Prokudin, Roger, Sato, Wang PRD 2016

Y-term & Matching

$$\frac{d\sigma}{dP_T^2} \propto \sum_{jj'} \mathcal{H}_{jj',\,\mathrm{SIDIS}}(\alpha_s(\mu), \mu/Q) \int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{P}_T}\ \tilde{F}_{j/H_1}(x, b_T; \mu, \zeta_1)\, \tilde{D}_{H_2/j'}(z, b_T; \mu, \zeta_2)\ +\ Y_{\mathrm{SIDIS}}$$

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + FO(q_T, Q) + ??\ O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)\ ??$$

If we do we double count

We add & subtract out the double counting such that the cross section is matched (SIDIS,DY, $e^+ e^-$) in the "overlap region":Designed s.t. valid to leading order in *m/Q* uniformly in q_T (*see role of "approximations" in TMD factorization*)

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

JCC Cambridge Press 2011, Collins arXiv: 1212.5974, Catani et al. NPB 06, 15, Collins, Gamberg, Prokudin, Roger, Sato, Wang PRD 2016 ……

"Matching-1" $W + Y$-schematic

Last week 9/22/17

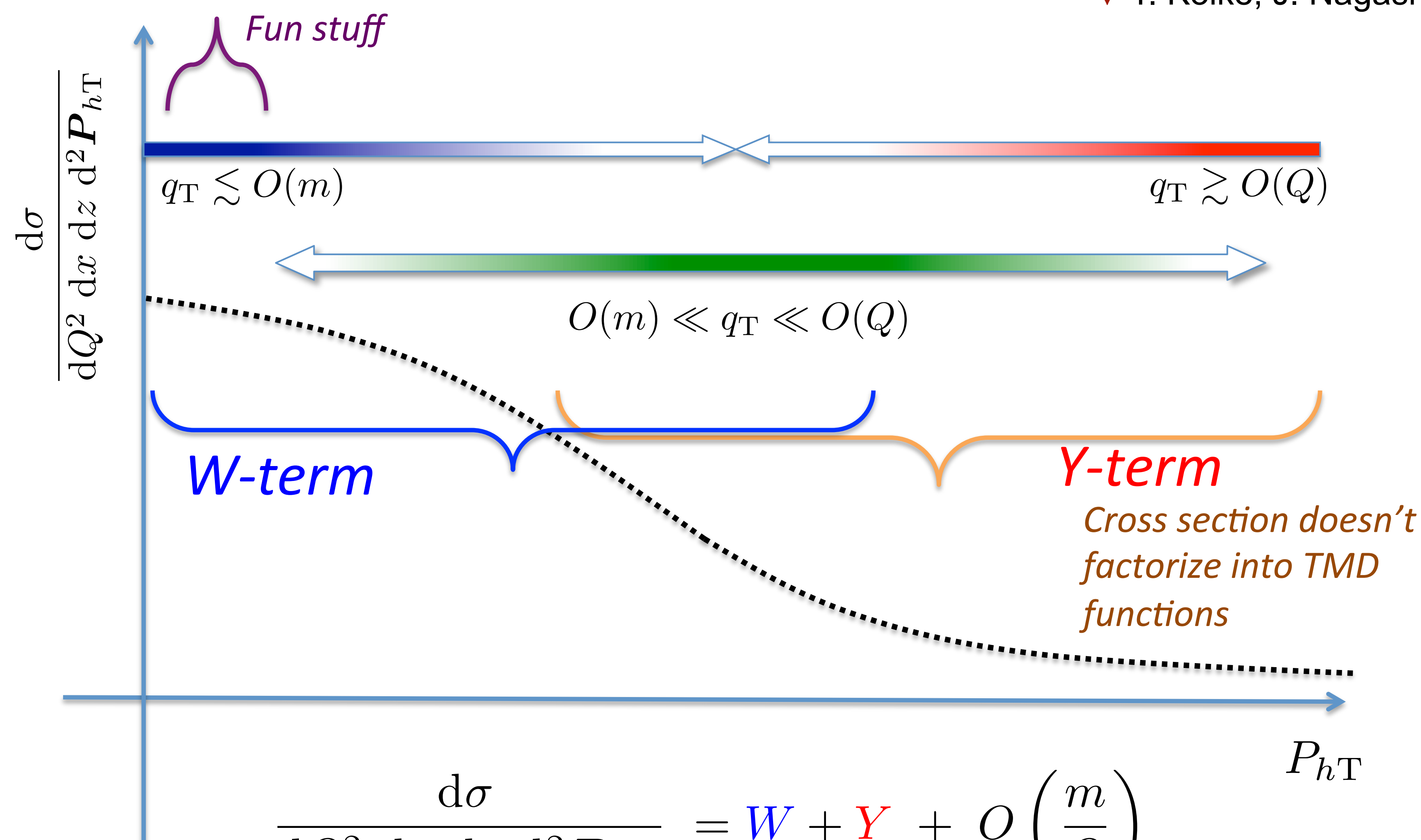
- Was *designed* with the aim to have a formalism that is valid to leading power in m/Q uniformly in q_T, where m is a typical hadronic mass scale
- and where there is a broad intermediate range of transverse momentum characterized by $m \ll q_T \ll Q$

Implementations/studies

✦ Nadolsky Stump C.P. Yuan PRD 1999 **HERA data**

✦ Y. Koike, J. Nagashima, W. Vogelsang NPB (2006) **eRHIC**

From Ted Rogers **W + Y**



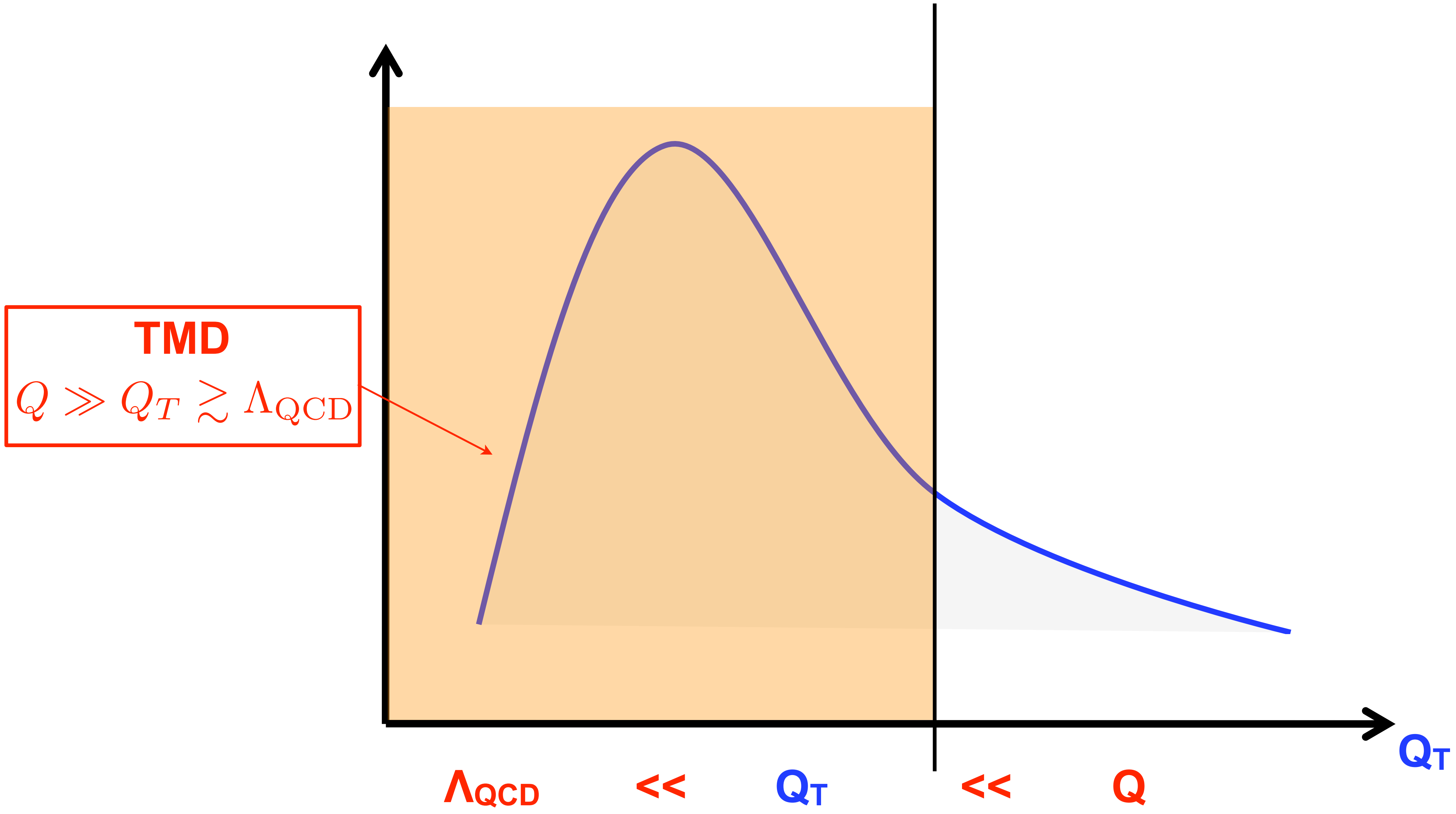
$$\frac{d\sigma}{dQ^2\ dx\ dz\ d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

note $P_{hT} = zq_T$

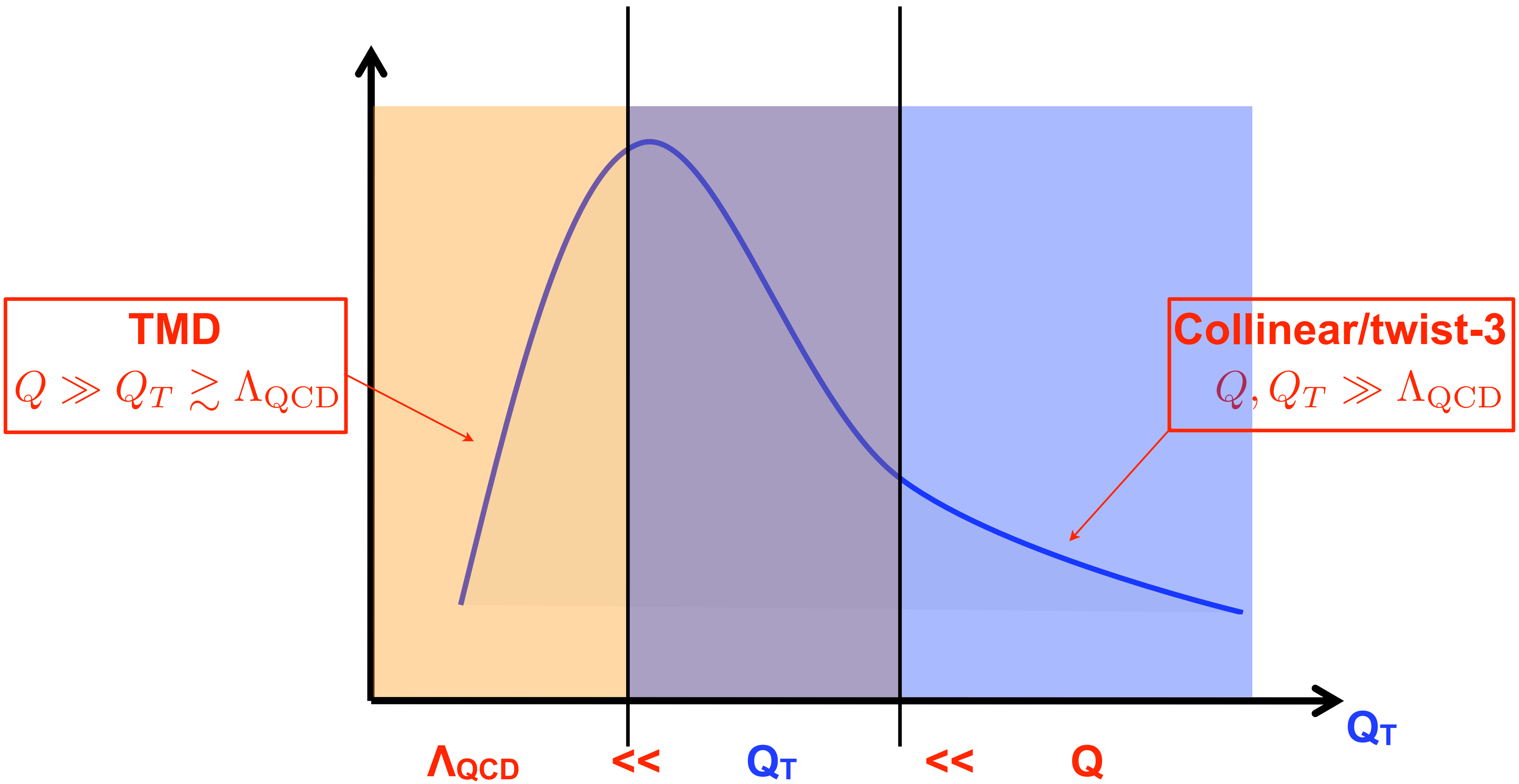
Comments Message

- The standard $W + Y$ prescription was arranged to apply for large Q situations where there is *a broad range* of transverse momentum *s.t.* $m \ll q_T \ll Q$
  - That is where q_T/Q is small s.t. *TMD factorization* is valid & …
  - m/q_T is sufficiently small (i.e. $q_T \sim Q$) s.t. *collinear factorization* is valid
- N.B. keeping full accuracy when $m \ll q_T \ll Q$, give rise to situation where both pure TMD and pure collinear factorization ***have degraded accuracy*** "***outside design regions***"
  - TMD factorization degrade as q_T increases $q_T/Q \sim O(1)$ or $q_T \sim Q$
  - Other hand, as q_T decreases, $m/q_T \sim O(1)$ or $q_T \sim m$
- *Generally get results valid over all q_T need to combine info TMD & collinear factorization*

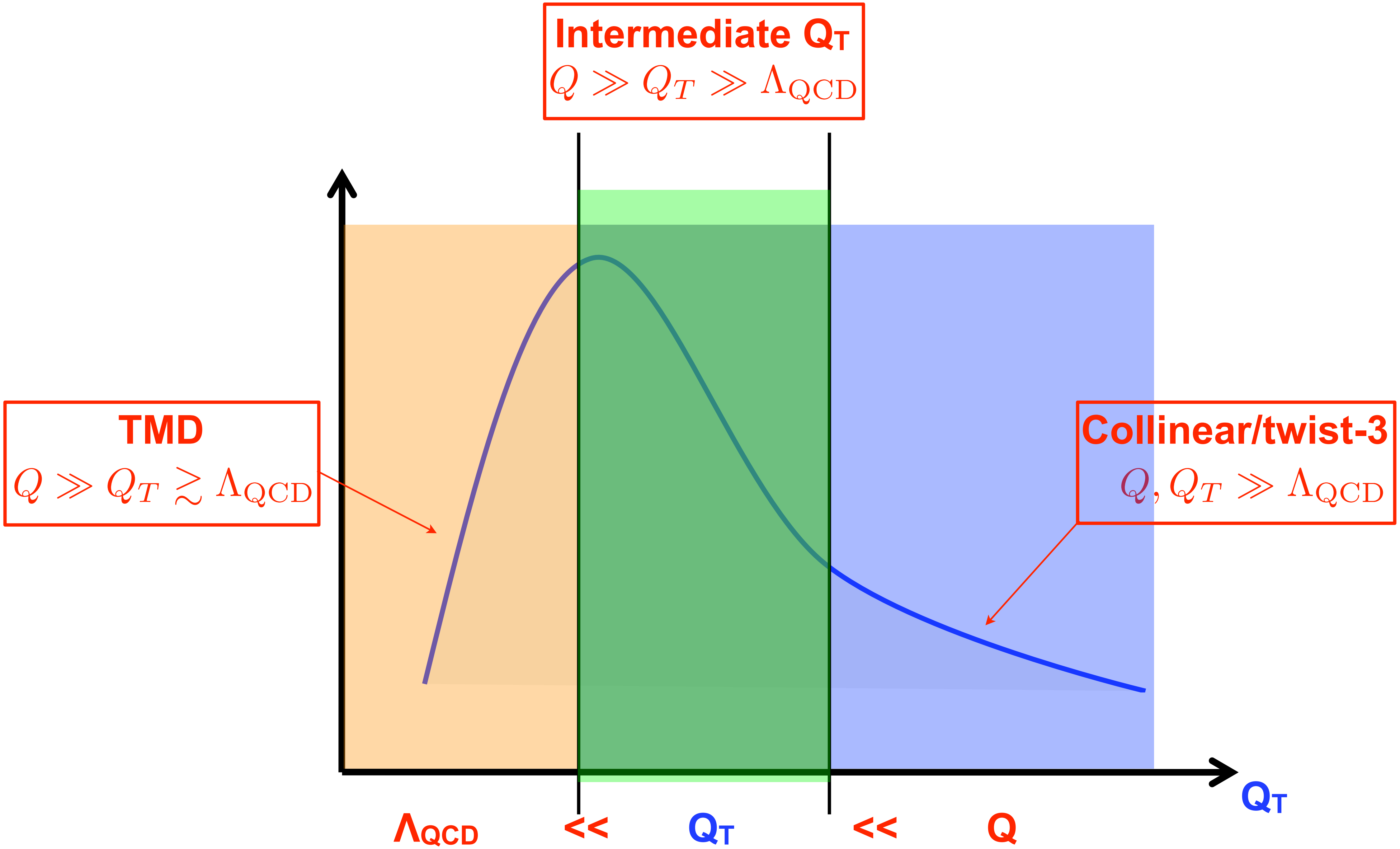
Matching: A unified picture of TMD & conventional FO factorization over the entire range ("point by point") in q_T for SIDIS/Drell Yan



Matching: A unified picture of TMD & conventional FO factorization over the entire range ("point by point") in q_T for SIDIS/Drell Yan

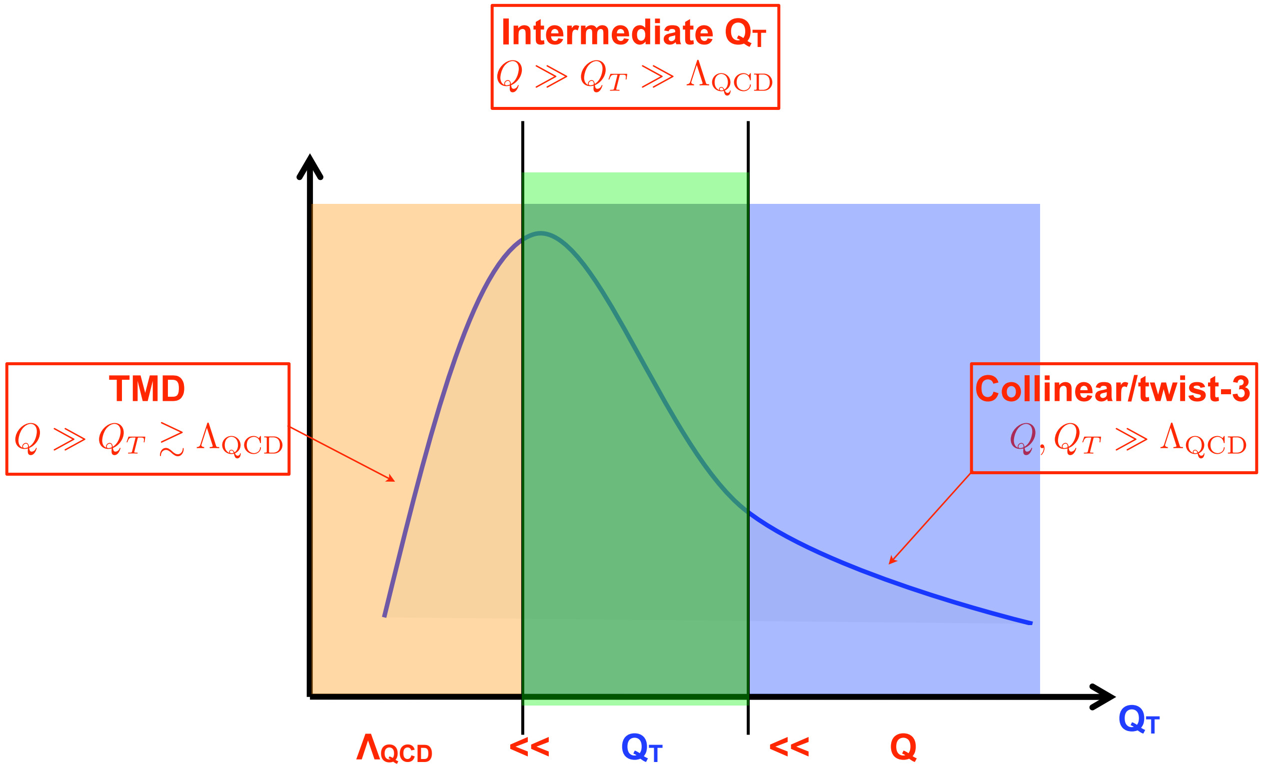


A unified picture of TMD and Collinear SIDIS/Drell Yan



Connection of twist 3 and twist 2 approach for Sivers Effect: "overlap regime"

Ji, Qiu, Vogelsang, Yuan PRL 2006 ...Bacchetta, Boer, Diehl, Mulders JHEP 2008



- Same mechanism in both approaches ISI/FSI ???

- Explore role parton model processes in twist-2&3 approaches
  Gamberg, Kang, PLB(2010,2011,2012) Sivers & Collins, Gamberg, Kang, Prokudin PRL2013 …

- Or just match the fixed order to the TMD twist-2 contribution which dominates $q_T \sim Q$ Collins TMD formalism $d\sigma(Q, q_T)$ = $W+Y$ Collins, Gamberg, Prokudin, Rogers, Sato, Wang PRD 2016

“Matching-1” $W + Y$ studies

Last week 9/22/17

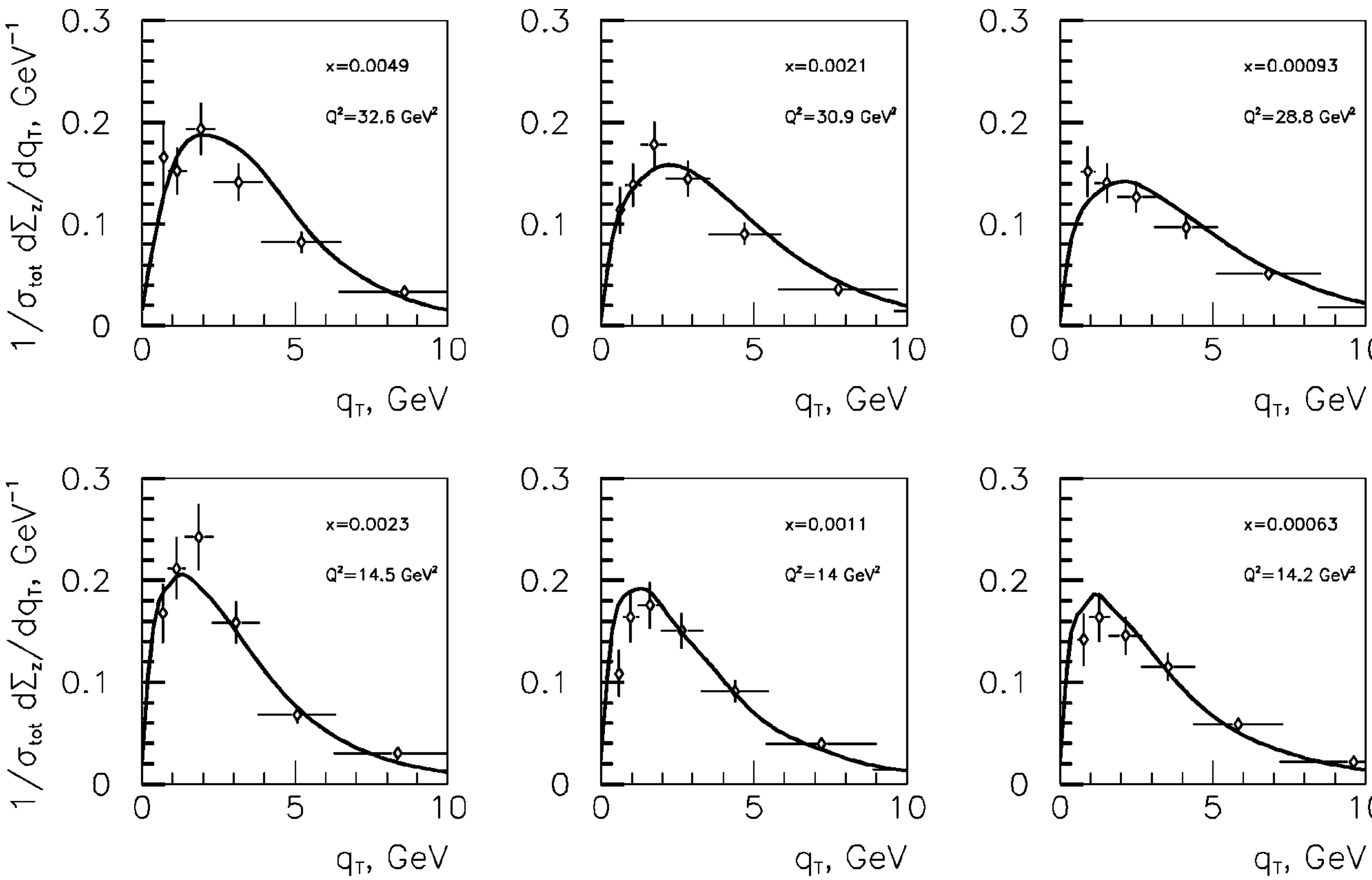
- This was designed with the aim to have a formalism that is valid to leading power in *m*/*Q* uniformly in q_T, where *m* is a typical hadronic mass scale
- and where there is a broad intermediate range of transverse momentum characterized by $m \ll q_T \ll Q$

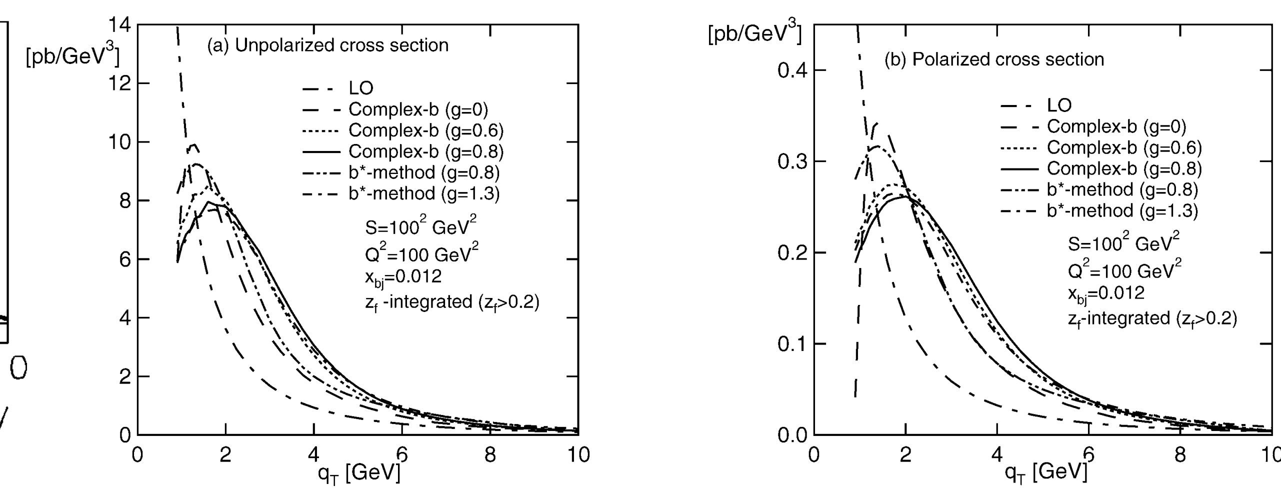
$$\frac{d\Sigma_z}{dx\, dQ^2\, dq_T^2} = \sum_B \int_{z_{min}}^{1} z \frac{d\sigma(e+A \to e+B+X)}{dx\, dz\, dQ^2\, dq_T^2} dz$$

$$\frac{1}{d\sigma_{tot}/(dx dQ^2)} \frac{d\Sigma_z}{dx dQ^2 dq_T}$$

Implementations/studies

- ✦ “z-flow” Nadolsky Stump C.P. Yuan PRD 1999 **HERA data**
- ✦ SIDIS Y. Koike, J. Nagashima, W. Vogelsang NPB (2006) **eRHIC**





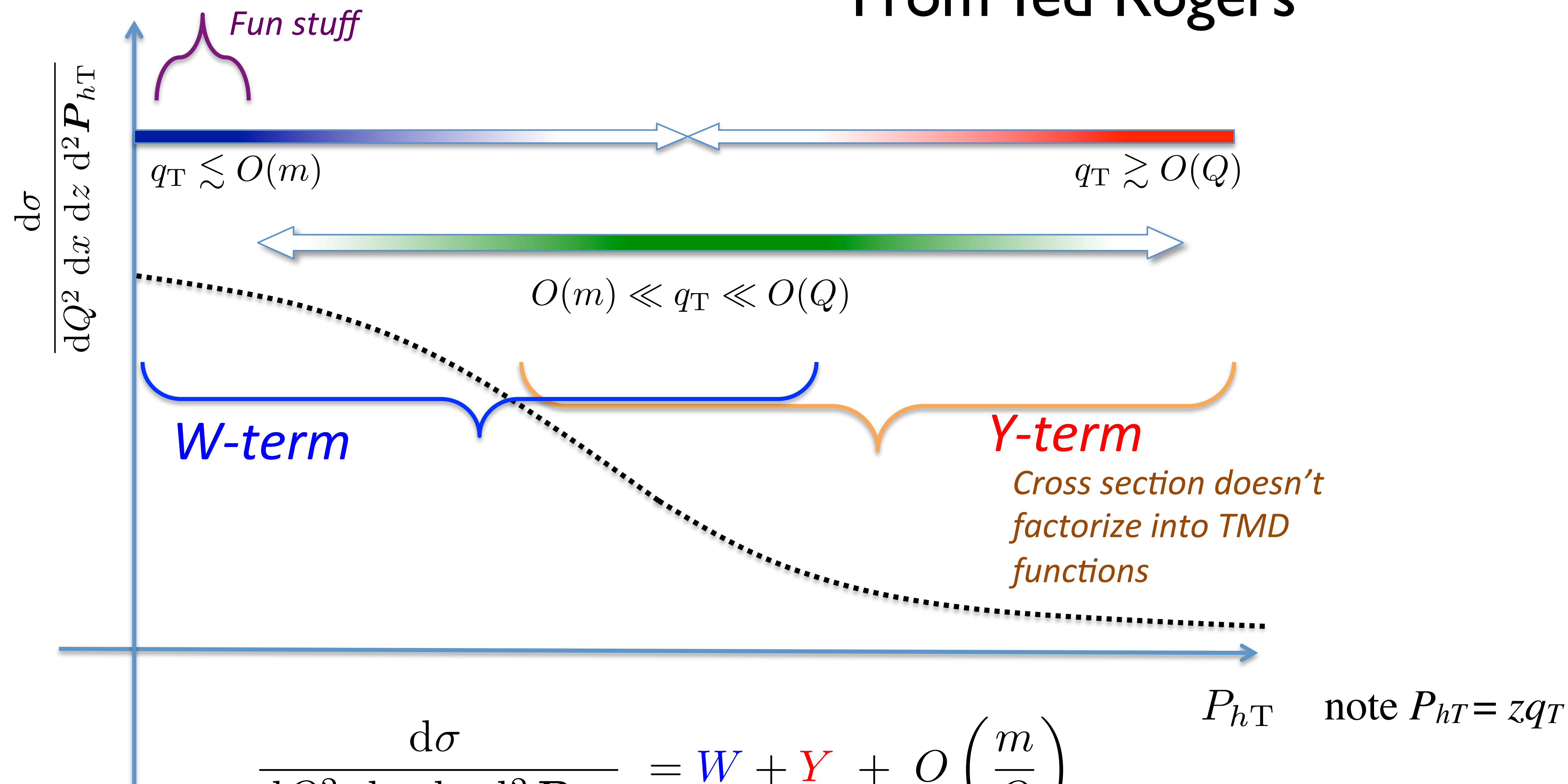
“Matching-1” and $W + Y$-schematic

Last week 9/22/17

- However at lower phenomenologically interesting values of Q, neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic

W + Y

From Ted Rogers



$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

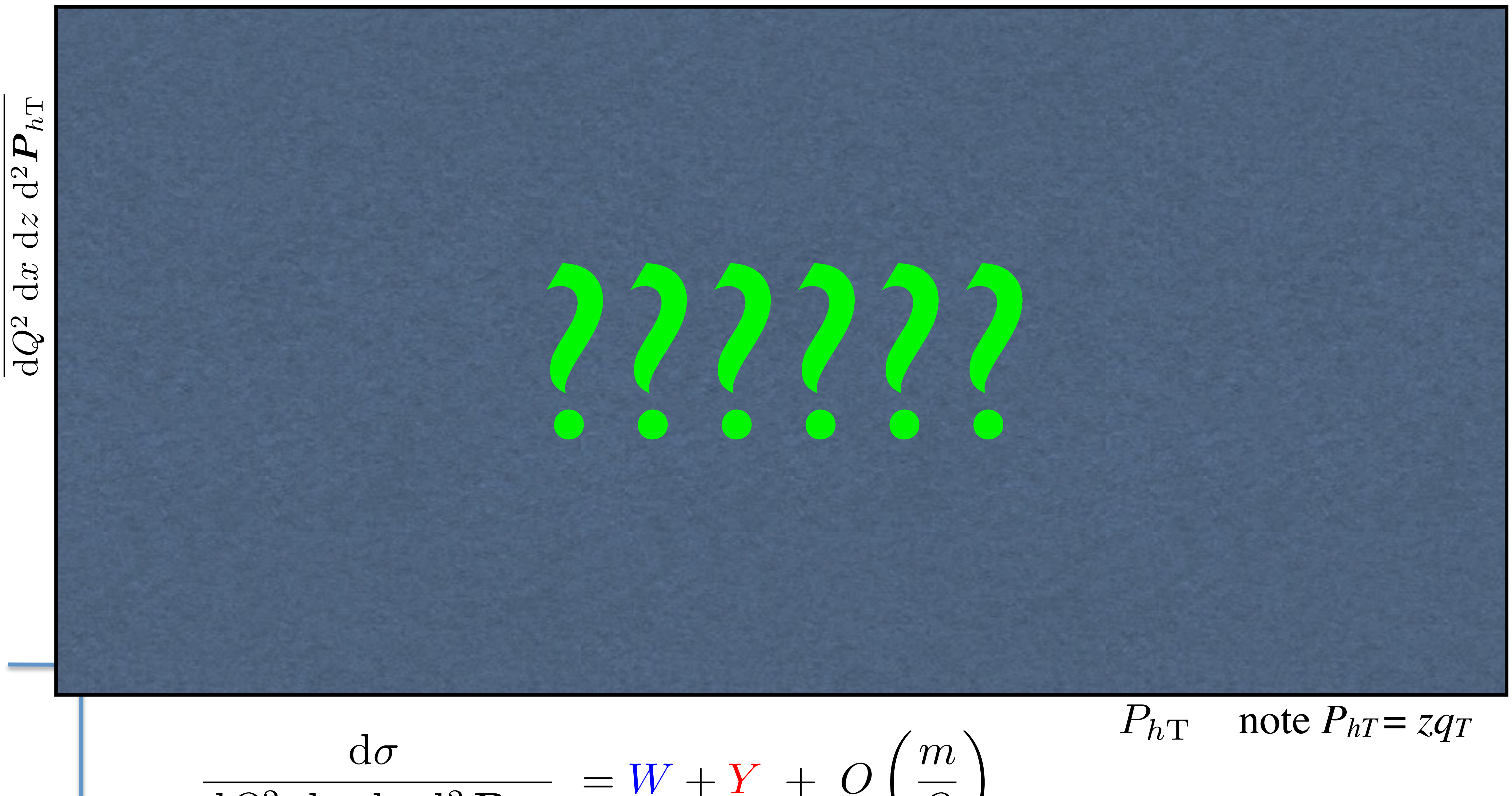
Matching and $W + Y$-schematic

Last week 9/22/17

- However at lower phenomenologically interesting values of Q, neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic

W + Y

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{hT}}$$



P_{hT} note $P_{hT} = zq_T$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\;\mathrm{d}x\;\mathrm{d}z\;\mathrm{d}^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

Matching and $W + Y$ -studies

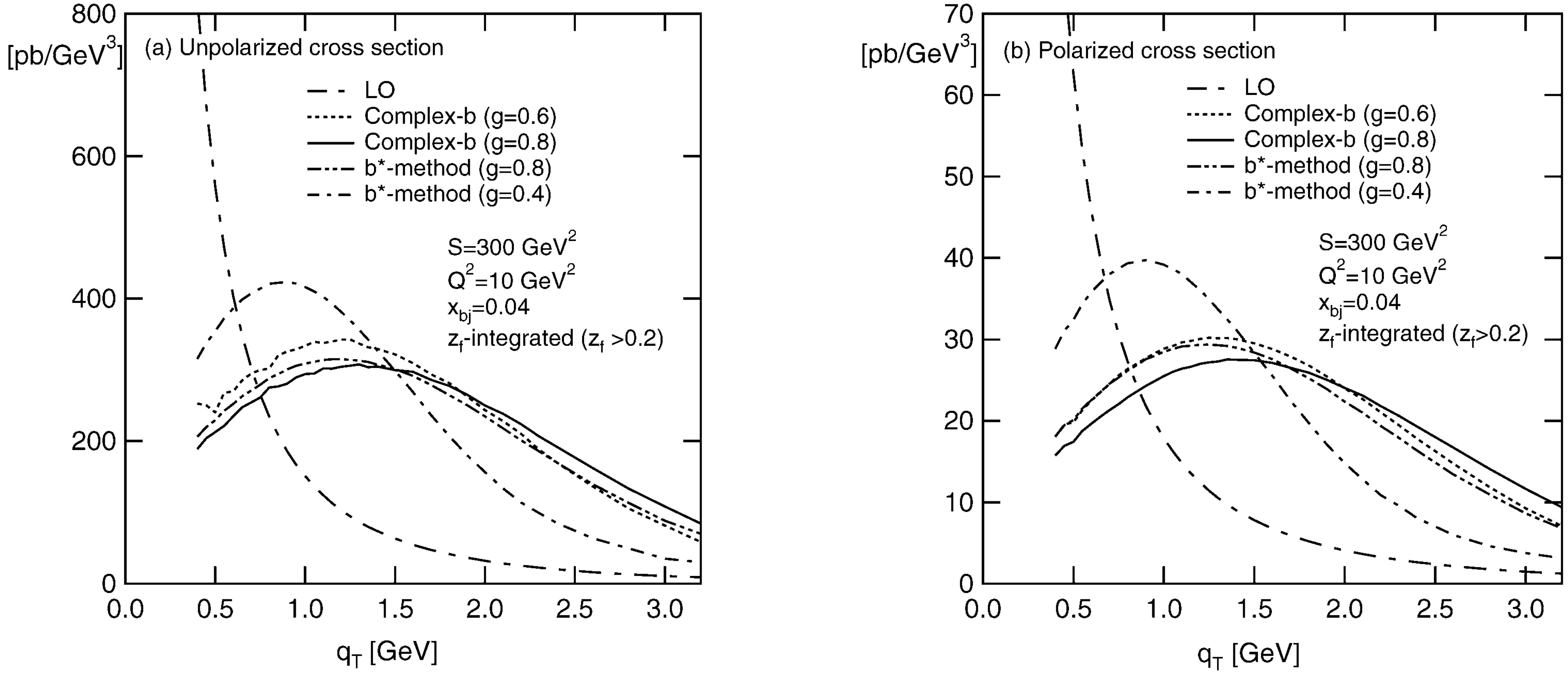
Last week 9/22/17

This impacts studies of non-perturbative nucleon structure @ COMPASS & JLAB !!!

$$m \lesssim q_T \lesssim Q$$

Implementations

✦ Y. Koike, J. Nagashima, W. Vogelsang NPB (2006). "…***COMPASS no data at the time…***"



Matching and $W + Y$ -studies

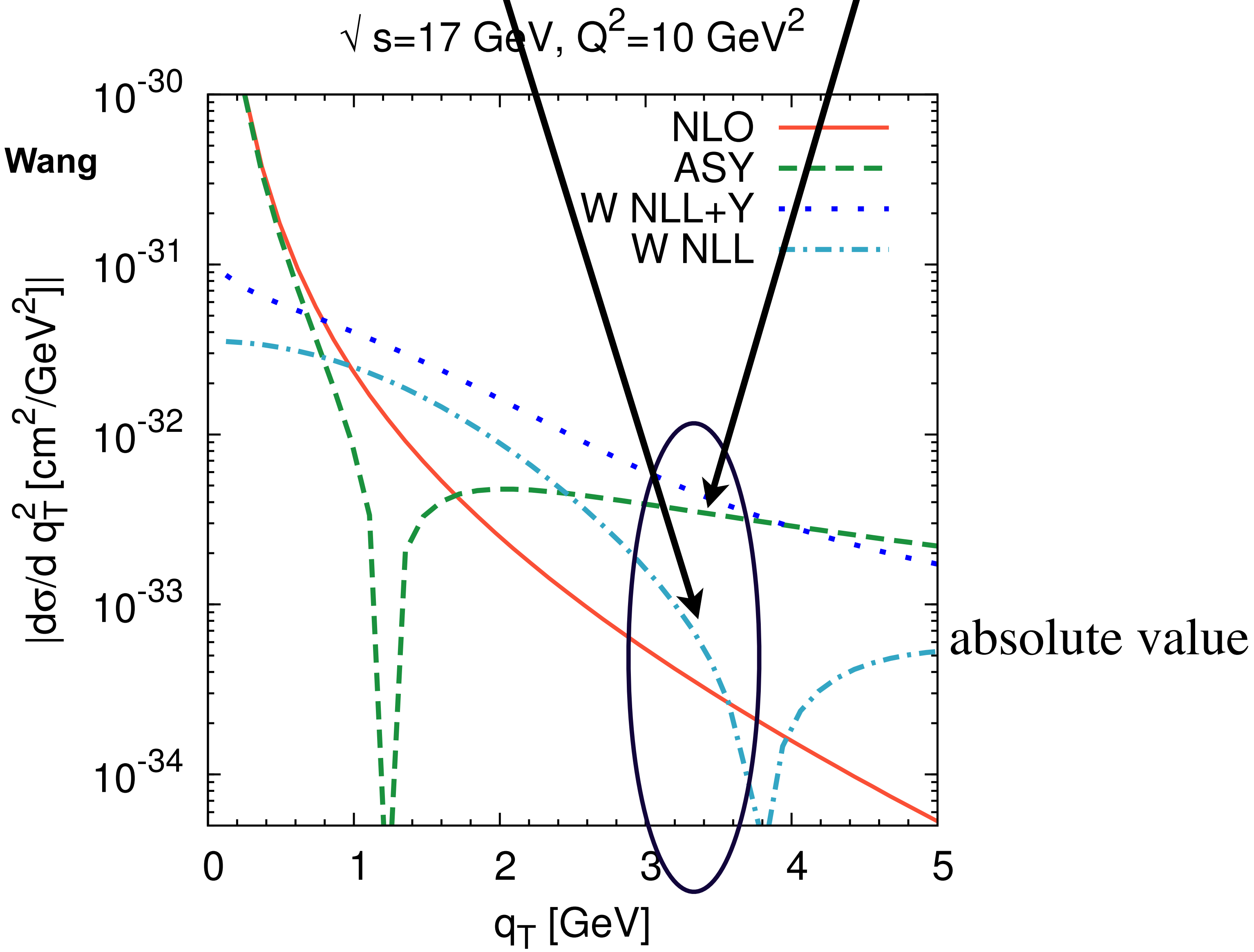
Compass Example

Boglione Prokudin et al. JHEP 2015

- When q_T is above some small fraction of Q, W deviates alot from $d\sigma(q_T, Q)$
- Then it becomes negative and "asymptotes" to $\frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$
  Nadolsky et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, NPB744, 59 (2006)
- At large q_T $W+Y$ is then difference of large terms and *truncation errors* can be augmented (ASY!)

PRD 94 2016 Collins, Gamberg, Prokudin, Sato, Rogers, Wang

Matching becomes a challenge COMPASS/Jlab like energies



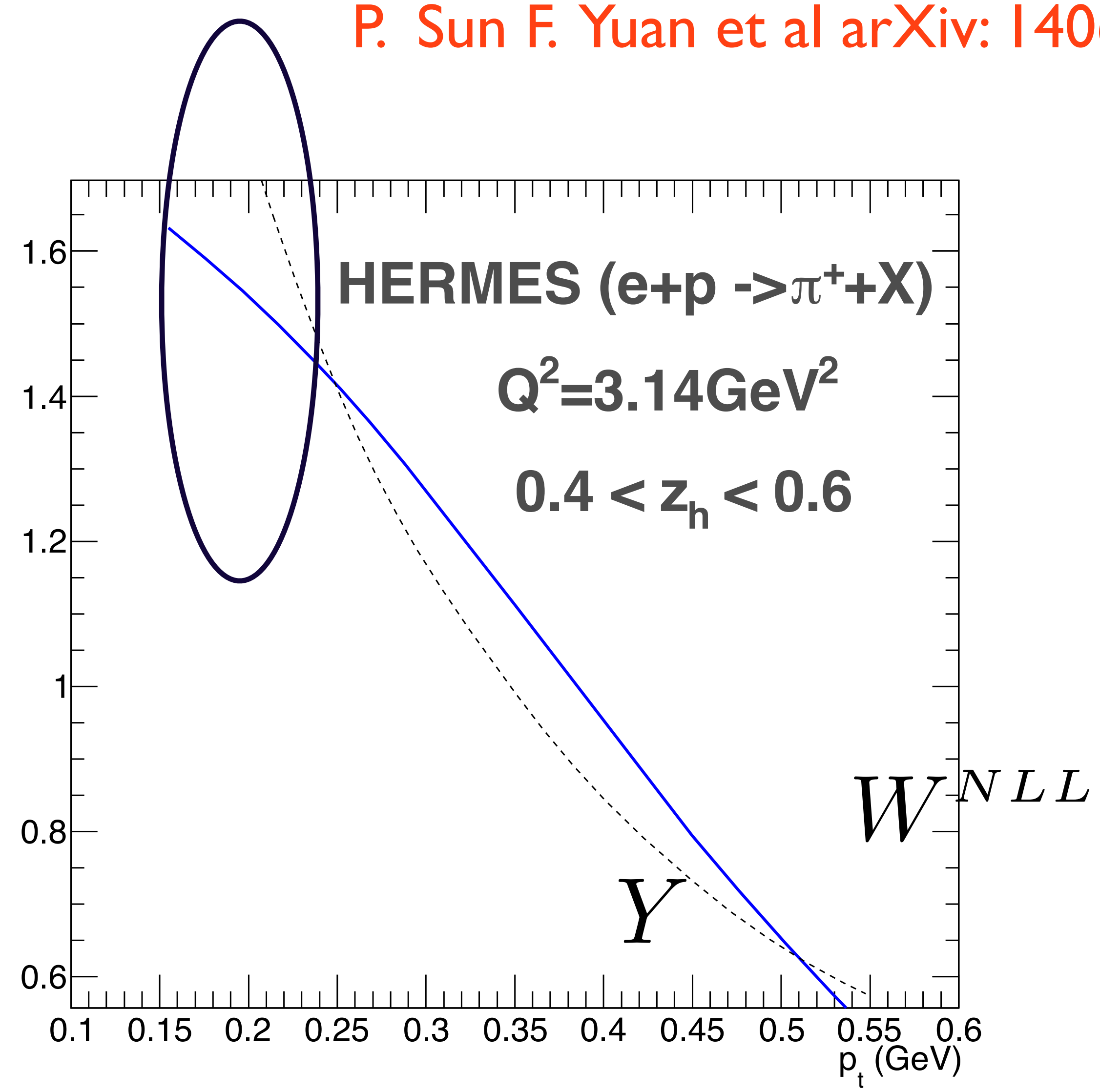
Matching and $W + Y$ -studies

Last week 9/22/17

- At small q_T the Y term is in principle suppressed: it is the difference of the FO perturbative calculation of the cross section and the asymptotic contribution of W for small q_T

- But there can be a difference of of large terms and truncation errors are augmented: **Here the *Y* term is larger than *W* ?!**

P. Sun F. Yuan et al arXiv: 1406.3073



$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

Matching and $W + Y$-enhanced

- Thus the region *between* large and small q_T needs special treatment if errors are to be strictly power suppressed point-by-point in q_T

We address & extend formalism

Phys.Rev. D 94 Collins, L.G, Prokudin, Sato, Rogers, Wang

$$q_T \lesssim m \quad \text{and} \quad q_T \gtrsim Q$$

Extend/enhanced formalism

Phys.Rev. D 94 Collins, L.G, Prokudin, Sato, Rogers, Wang

$$q_T \lesssim m$$

- For $q_T \lesssim m$ collinear factorization is not applicable for the differential cross section. But this region is actually where the *W-term* has its highest validity. So one simply must ensure that the *Y -term* is sufficiently suppressed in Eq. (10) for $q_T \lesssim m$
- Modify *Y*

$$Y(q_T, Q) = \{FO(q_T, Q) - ASY(q_T, Q)\}\, X(q_T/\lambda)$$

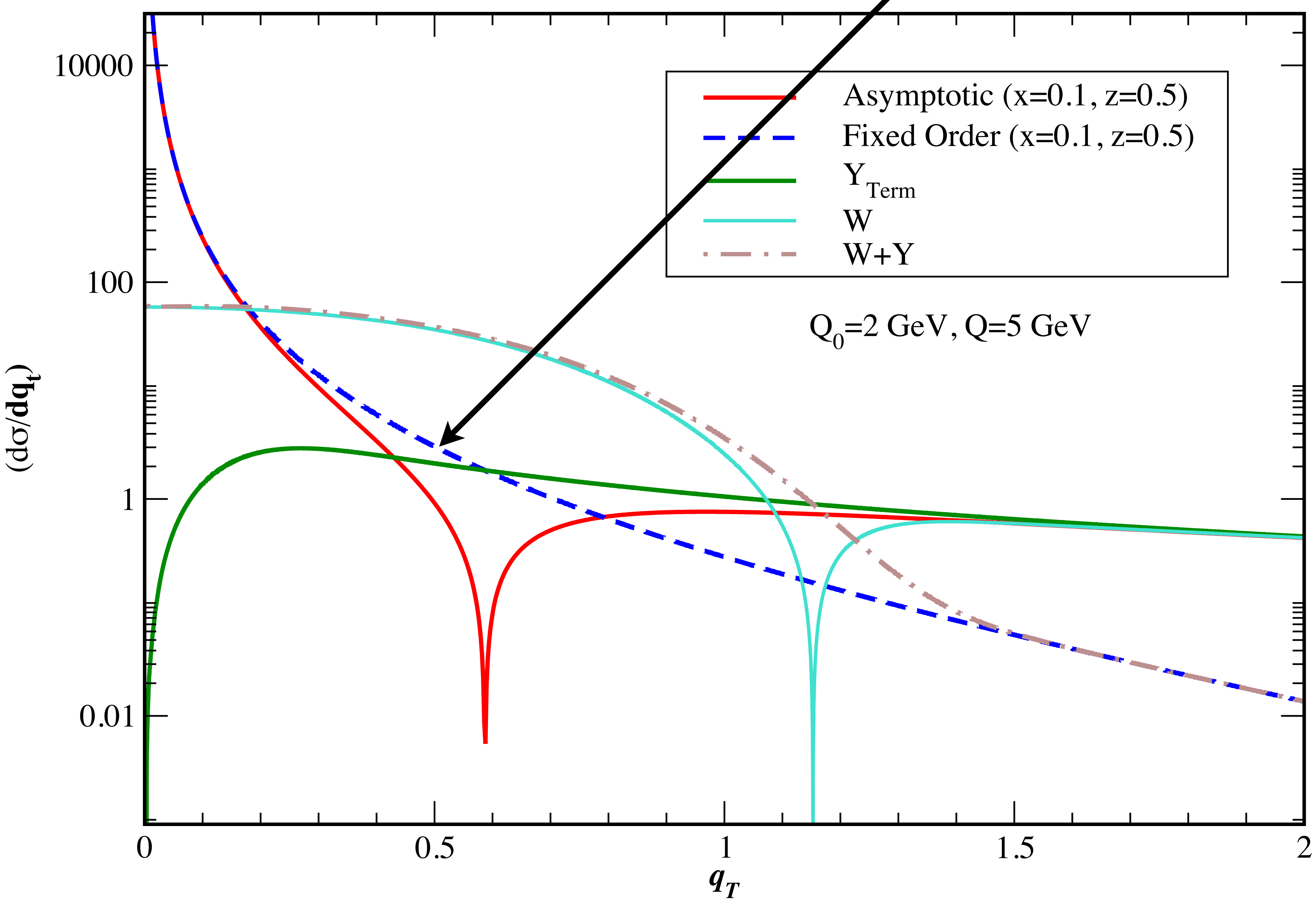
with "switching function at small q_T

Pointed out by Collins 2011 Cambridge press

$$X(q_{\rm T}/\lambda) = 1 - \exp\{-(q_{\rm T}/\lambda)^{a_X}\}$$

- Now we can extend the power suppression error estimate down to $q_T = 0$ to get

$$d\sigma(q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$



Use analytic expressions for the collinear correlation functions, from GRV ZPC 1992 for up-quark pdf and from KKP NPB 2001 for the up-quark-to-pion ffs.

Extend/enhanced formalism

Phys.Rev. D 94 Collins, L.G, Prokudin, Sato, Rogers, Wang

$$q_T \gtrsim Q$$

Modification of the cross section leaves the standard treatment of TMD factorization only slightly modified

In particular the op. definitions along with evolution properties are the same as in the usual formalism

We do this in two steps however now we need explicit expression for *W from JCC* formalism

Many sources ….see Collins Rogers PRD 2015

Summary of elements of TMD factorization

$$W(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}(b_T, Q)$$

Collins 2011 QCD
Aybat Rogers PRD 2011

- Factorization and TMD evolution in b_T *space*
- Solve the CSS & RG evolution eqs. for *W* term in SIDIS with “boundary condition” to freeze b_T above some b_{max}

$$b_*(b_T) = \sqrt{\frac{b_T^2}{1 + b_T^2/b_{max}}}$$

$$\tilde{W}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}^{OPE}(b_*(b_T), Q)\, \tilde{W}_{NP}(b_T, Q; b_{max})$$

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Collins, L.G, Prokudin, Sato, Rogers, Wang
Phys.Rev. D 94

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$$\tilde{W}_i^{OPE}(b_*(b_T), Q) = H_i(Q)\ \tilde{C}_{i/i'}^{pdf}(x_A/\hat{x}, b_* b_\star) \otimes \tilde{f}_{i'/A}(\hat{x}, \mu_{b_\star})\ \tilde{C}_{j'/i}^{ff}(z_B/\hat{z}, b_*) \otimes \tilde{d}_{B/i'}(\hat{z}, \mu_b) e^{-S^{pert}(b_*, Q)}$$

Collinear pdfs

$$e^{-S_{pert}} \equiv \exp\left\{ \ln \frac{Q^2}{\mu_{b_*}^2} \tilde{K}(b_*(b_T); \mu_{b_*}) + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_k(\alpha_s(\mu')) \right] \right\}$$

Evolution kernel

Summary of elements of TMD factorization

$$W(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}(b_T, Q)$$

Collins 2011 QCD
Aybat Rogers PRD 2011

- Factorization and TMD evolution in b_T *space*
- Solve the CSS & RG evolution eqs. for *W* term in SIDIS with “boundary condition” to freeze b_T above some b_{max}

$$b_*(b_T) = \sqrt{\frac{b_T^2}{1 + b_T^2/b_{max}}}$$

$$\tilde{W}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}^{OPE}(b_*(b_T), Q)\, \tilde{W}_{NP}(b_T, Q; b_{max})$$

$$\tilde{W}_{NP}(b_T, Q; b_{max}) = e^{-S_{NP}(b_T, Q; b_{max})}$$

$$S_{NP}(b_T, Q; b_{max}) = g_A(x_A, b_T; b_{max}) + g_B(z_B, b_T; b_{max}) - 2 g_K(b_T; b_{max}) \ln\left(\frac{Q}{Q_0}\right)$$

Aidala, Field, Gamberg, Rogers PRD 2015 $\quad g_K(b_T; b_{\rm max}) = \frac{g_2(b_{\rm max}) b_{\rm NP}^2}{2} \ln\left(1 + \frac{b_T^2}{b_{\rm NP}^2}\right)$

Fourier Transforms of TMDs and universal soft function g_K

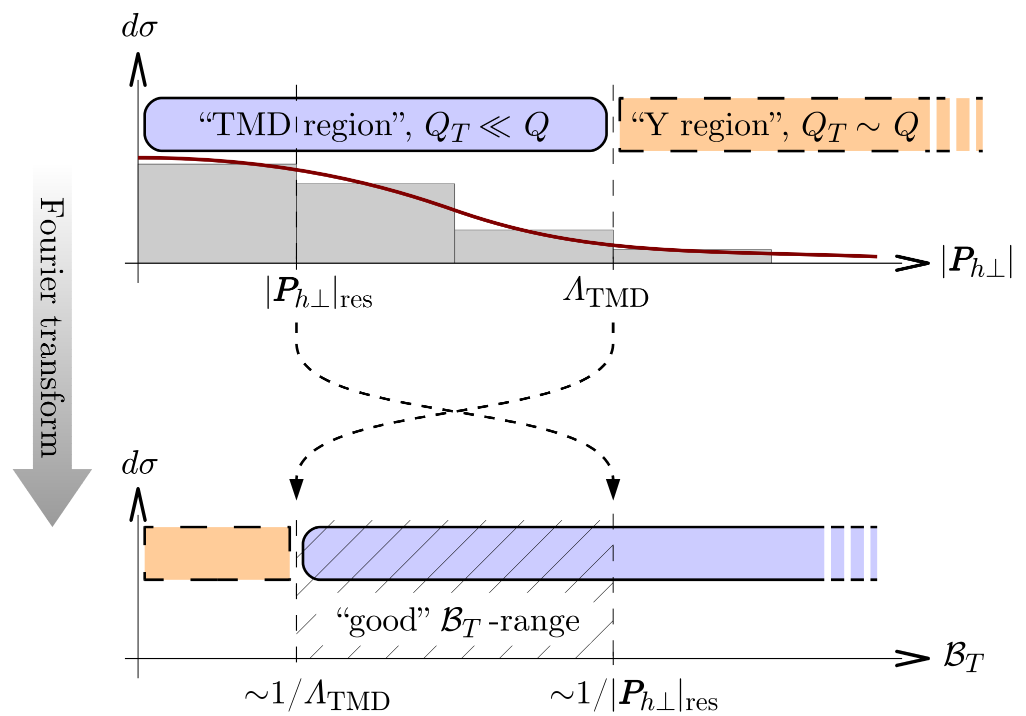
Two modifications

a) B.C. Introduce small b-cuttoff

Similar to Catani et al. NPB 2006,
Bessel Weighting-Boer LG Musch Prokudin JHEP 2011

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$

Regulate unphysical divergences from in W term



b) Introduce large q_T-switching s.t. that $W_{\rm New}$ vanishes at large $q_{\rm T}$

Similar to Nadolsky et al. PRD 1999,
Bozzi & Catani et al NPB 2015

$$\Xi\left(\frac{q_T}{Q}, \eta\right) = \exp\left[-\left(\frac{q_T}{\eta Q}\right)^{a_\Xi}\right]$$

$$\tilde{W}_{New}(q_T, Q; \eta, C_5) = \Xi\left(\frac{q_T}{Q}, \eta\right) \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}^{OPE}\left(b_*(b_c(b_T)), Q\right) \tilde{W}_{NP}(b_c(b_T)), Q; b_{max})$$

Generalized B.C.

$$b_*(b_c(b_{\rm T})) \longrightarrow \begin{cases} b_{\rm min} & b_{\rm T} \ll b_{\rm min} \\ b_{\rm T} & b_{\rm min} \ll b_{\rm T} \ll b_{\rm max} \\ b_{\rm max} & b_{\rm T} \gg b_{\rm max}\,. \end{cases}$$

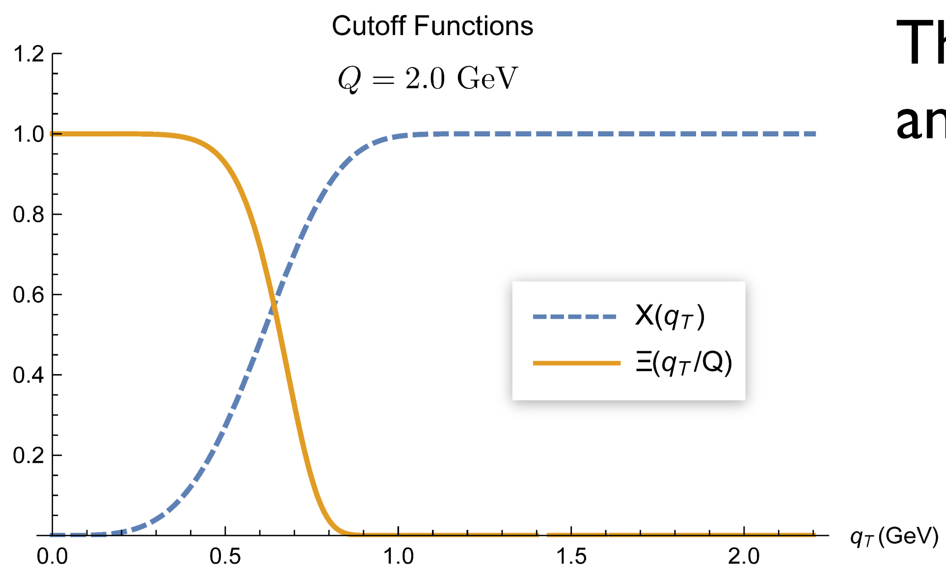
Now Y term is further modified

$$Y_{New}(q_T, Q) = [T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}^{New}\, d\sigma(q_T, Q)]\, X(q_T/\lambda)$$

$$= [FO(q_T, Q) - ASY_{New}(q_T, Q)]\, X(q_T/\lambda)$$

Method of "approximators" in factorization
Collins PRD, 58, 1998 JCC ch 8, summarized in our paper
PRD 2016

Switching functions



(a)

The cutoff functions in for low q_T/lambda (blue dashed line) and large q_T/Q (brown solid line) for Q = 20.0 GeV

See also Altarelli et al NPB1984, Bozzi, Catani et al NPB 2015 Arnold and Kauffman 1991, Alternative approach Qiu & Zhang PRL 2001

Putting all together

$$d\sigma(q_T,Q) \approx T_{TMD}^{New} d\sigma(q_T,Q) + T_{coll}\left[d\sigma(q_T,Q) - T_{TMD}^{New}\, d\sigma(q_T,Q)\right]$$

$$+ O\left(\frac{m}{Q}\right)^c d\sigma(q_T,Q)$$

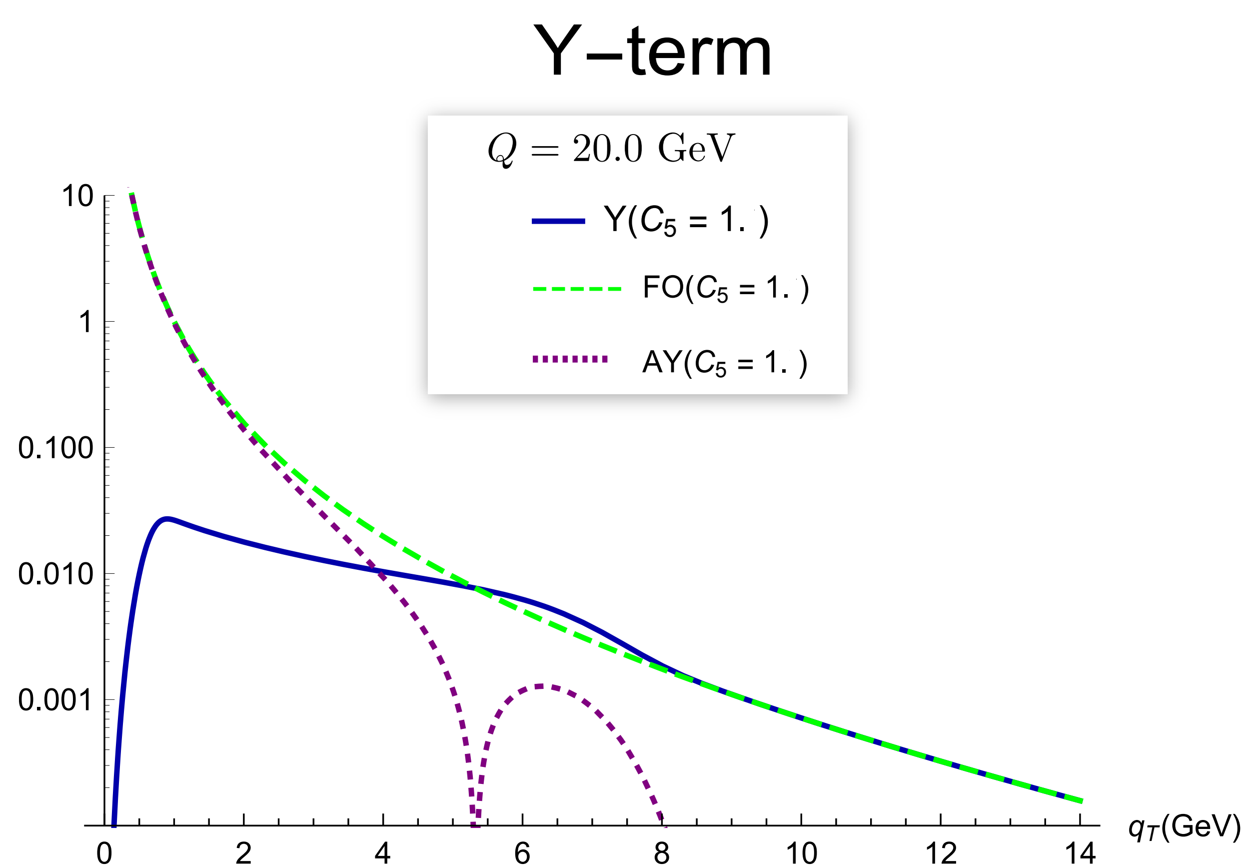
or

$$d\sigma(q_T,Q) \approx W_{New}(q_T,Q) + Y_{New}(q_T,Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T,Q)$$

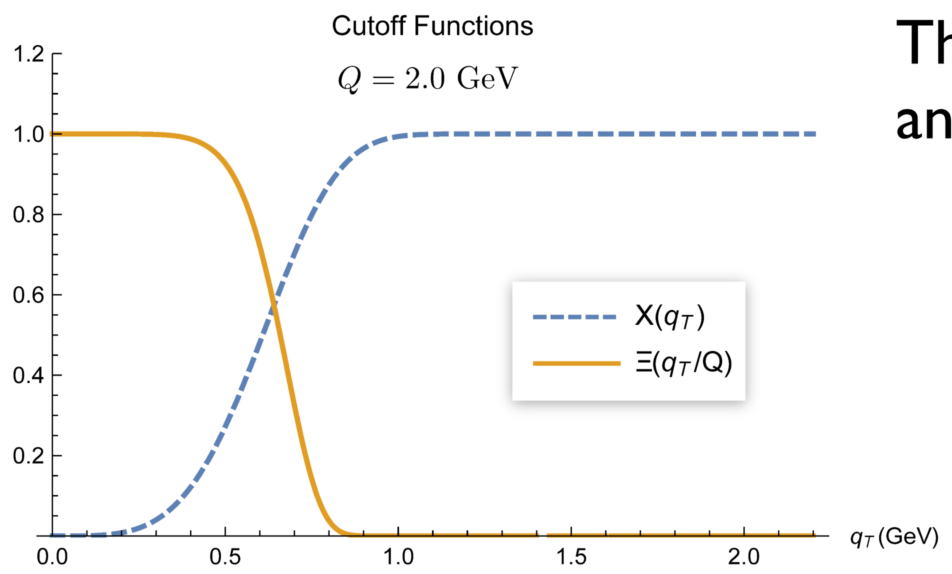
Putting all together demonstration

Illustration: we have performed sample calculations of the Y-term using analytic approximations for the collinear pdfs and collinear ffs. We consider only the target up-quark gamma *q -> q+g* channel, and for the running $alpha_s$ we use the two-loop beta function f = 3 since we are mainly interested in the transition to low Q.
Thus we use $\Lambda_{QCD} = 0.330$

To further simplify our calculations, we use analytic expressions for the collinear correlation functions, taken from appendix A1 of GRV ZPC 1992 for the up-quark pdf and from Eq. (A4) of KKP NPB 2001 for the up-quark-to-pion fragmentation function



Putting all together demonstration



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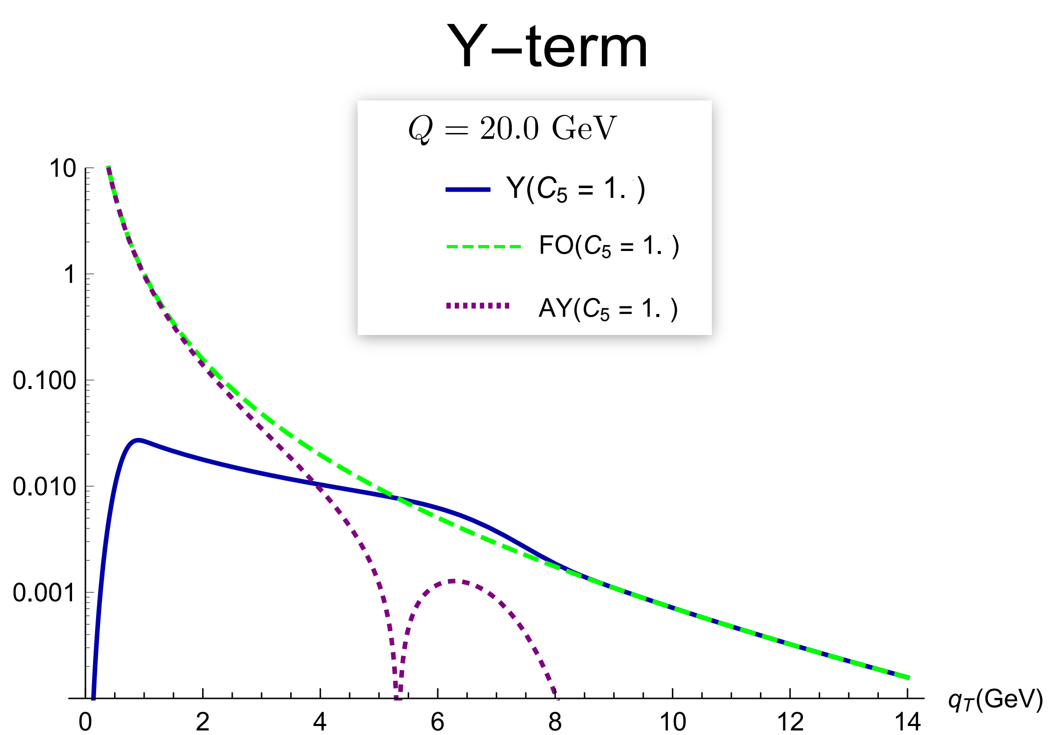
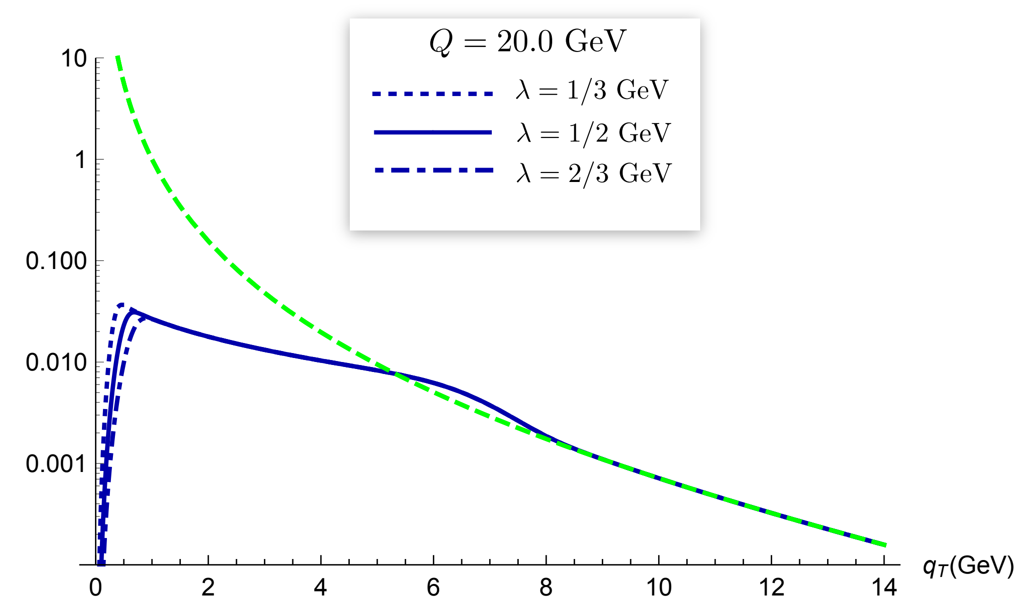


Insensitive to λ



Semi-inclusive to Collinear integrate over q_T

- Parton Model (expectation) *W*-term

$$W_{PM}(q_T,Q) = H_{LO,j',i'}(Q_0)\int d^2k_T f_{j'/A}(x,k_T)d_{B/i'}(z,q_T+k_T)$$

$$\int d^2q_T\, W_{PM}(q_T,Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

Underlies Model building
w/ and w/o evolution using TMD
and collinear evolution approach
Anselmino et al. 2005-2016

- Standard CSS W-term

$$W_{CSS}(q_T,Q) = \int \frac{d^2b_T}{(2\pi)^2} e^{iq_T\cdot b_T}\tilde{W}_{CSS}(b_T,Q)$$

$$\int d^2q_T W_{CSS}(q_T,Q) = 0 \quad !$$

Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

B.C. Introduce small b-cuttoff

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \Longrightarrow b_c(0) \sim 1/Q$$

Cures this property

A little detail: dependence driven by perturbative part of ev. Kernel

$$W_{CSS}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = \int \delta^2(b_T)\, b_T \times \text{logarithmic corrections}$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = 0 \quad !$$

See Phys. Rev. D 94 (2016) for details
J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

A little detail: dependence driven by perturbative part of ev. Kernel

$$\exp\left[\int_{\mu_{b*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - 2\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right]$$

$$\tilde{W}(b_T \to 0, Q) \sim \exp\left[\frac{C_F}{\pi\beta_0}\int_{\ln\mu_b^2}^{\ln\mu_Q^2} \ln\mu'^2\right] = \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{\mu_b^2}{\mu_Q^2}\right)\right]$$

$$= \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{C_1^2}{b_T^2\mu_Q^2}\right)\right]$$

$$= b_T^a \quad \text{where, } a = 2C_F/(\pi\beta_0) > 0$$

$$\to 0$$

Phys. Rev. D 94 (2016) for details
Collins, Gamberg, Prokudin, Sato, Rogers, Wang

B.C. Introduce small b-cuttoff

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$

$$W_{New}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{New}(b_T, Q)\,, \qquad b_{min} = b_0/(C_5 Q)$$

$$\int d^2 q_T W_{New}(q_T, Q) = \tilde{W}(b_{min}, Q) \neq 0$$

$$\int d^2 q_T\, W_{New}(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \; + O(\alpha_s(Q))$$

$$\mu_c \approx C_1 C_5 Q/b_0$$

Has a normal collinear factorization in terms of collinear pdfs w/ hard scale

$$\int d^2 q_T\, W_{New}(q_T, Q) + Y(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \; + O(\alpha_s(Q))$$

$$+ \text{ terms dominated by large } q_T \text{ contribution to } Y \text{ term}$$

With modified W+Y we can match to the collinear formalism

Has implications for modeling TMD and fitting

Comments

- With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two being suppressed by powers of 1/Q
- Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used
- **This work has dealt only with unpolarized cross sections**
- **We ~~are~~ studying the analogous topic applied to polarized phenomena**
- **We have a new now applied to transverse polarized phenomena**

Enhanced expression for $\tilde{W}(b_c, Q)$

$$\tilde{W}(b_c(b_{\mathrm{T}}), Q) = H(\mu_Q, Q) \sum_{j'i'} \int_{x_A}^{1} \frac{d\hat{x}}{\hat{x}} \tilde{C}^{\mathrm{pdf}}_{j/j'}(x_A/\hat{x}, b_*(b_c(b_{\mathrm{T}})); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) f_{j'/A}(\hat{x}; \bar{\mu}) \times$$

$$\times \int_{z_B}^{1} \frac{d\hat{z}}{\hat{z}^3} \tilde{C}^{\mathrm{ff}}_{i'/j}(z_B/\hat{z}, b_*(b_c(b_{\mathrm{T}})); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) d_{B/i'}(\hat{z}; \bar{\mu}) \times$$

$$\times \exp\left\{ \ln \frac{Q^2}{\bar{\mu}^2} \tilde{K}(b_*(b_c(b_{\mathrm{T}})); \bar{\mu}) + \int_{\bar{\mu}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$\times \exp\left\{ -g_A(x_A, b_c(b_{\mathrm{T}}); b_{\max}) - g_B(z_B, b_c(b_{\mathrm{T}}); b_{\max}) - 2 g_K(b_c(b_{\mathrm{T}}); b_{\max}) \ln\left(\frac{Q}{Q_0} \right) \right\}$$

Boundary conditions

$$b_*(b_c(b_{\mathrm{T}})) \longrightarrow \begin{cases} b_{\min} & b_{\mathrm{T}} \ll b_{\min} \\ b_{\mathrm{T}} & b_{\min} \ll b_{\mathrm{T}} \ll b_{\max} \\ b_{\max} & b_{\mathrm{T}} \gg b_{\max} \,. \end{cases}$$

Comments

What impact does this have on the collinear limit of the transverse polarization case?

✦**Some observations ...**

Unpolarized and Sivers evolve in same way

Recall the correlator in *b*-space Bessel Transform

$$\tilde{\Phi}^{[\gamma^+]}(x, \boldsymbol{b}_T) \;=\; \tilde{f}_1(x, \boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma}\, b_{T\rho} S_{T\sigma}\, M\, \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)$$

Boer Gamberg Musch Prokudin JHEP 2011

It obeys Collins Soper Equation

$$\frac{\partial\, \tilde{\phi}^i_{f/P}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta_F)\epsilon_{ij} S_T^j}{\partial \ln\sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)\tilde{\phi}^i_{f/P}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta_F)\epsilon_{ij} S_T^j.$$

Aybat Rogers Collins Qiu PRD 2012
also see Kang Yuan Xiao PRL 2011

Transverse spin case

- So it is the derivative of Sivers function or first moment evolves

$$\frac{\partial \ln \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)$$

Consistent Definition

The FT transform of the e.g. Sivers asympt. reduces to first moment of Sivers TMD

Boer, Gamberg, Musch, Prokudin, **JHEP (2011)**

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T) \equiv \frac{2}{M^2} \frac{\partial}{\partial b_T^2} \tilde{f}_{1T}^{\perp}(x, b_T)$$

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T) = \frac{2\pi}{M^2} \int_0^\infty dk_T \, \frac{k_T^2}{b_T} \, J_1(k_T \, b_T) \; f_{1T}^{\perp}(x, k_T)$$

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{\perp(1)}(x, b_T) = \frac{2}{M^2} 2\pi \int_0^\infty dk_T \, \frac{k_T^2}{2 b_T} \, \frac{k_T \, b_T}{2} f_{1T}^{\perp}(x, k_T)$$

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{\perp(1)}(x, 0) = f_{1T}^{\perp(1)}(x)$$

Boer Mulders PRD 1998

This informs us how to study the collinear limit of transversely polarized cross section

CSS Sivers Structure Function

$$\tilde{\mathcal{W}}_{UT}^{\sin(\phi_h-\phi_S)}(x,z,b,Q) = H_{UT}(Q;\mu)\tilde{f}_{1T\,i/P}^{(1)}(x,b_\star;\mu_b)\tilde{D}_{H/j}(z,b_\star;\mu_b)e^{-S^{pert}(b_*,Q)}e^{-S_{UT}^{NP}(b,Q,x,z)}$$

★ Abyat, Collins, Qiu, Rogers PRD (2011),

$$e^{-S_{UT}^{NP}}(b,Q,x,z) = \exp\left\{-\left[g_1(x,b_T;b_{\max}) + g_2(z,b_T;b_{\max}) + 2g_k(b_T)\ln\left(\frac{Q}{Q_0}\right)\right]\right\}_{UT}$$

Non perturbative factor contribution must be fit

CSS NPB 85

Recall

$$W_{CSS}(q_T,Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T\cdot b_T} \tilde{W}_{CSS}(b_T,Q)$$

$$\int d^2 q_T W_{CSS}(q_T,Q) = 0 \quad !$$

Use projection method: for unpolarized trivial

$$\int d^2 q_T W_{UU}(q_T,Q) = \int d^2 b_T \delta^2(b_T)\, b_T^a \times \text{logarithmic corrections}$$

$$\begin{aligned}
\lim_{b'\to 0} \int d^2 q_T J_0(q_T b'_T) W_{UU}(q_T,Q) &= 2\pi \int dq_T q_T \int db_T b_T J_0(q_T b'_T) J_0(q_T b_T) W_{UU}(b_T,Q) \\
&= 2\pi \int db_T \delta(b_T) \tilde{W}_{UU}(b_T,Q) \\
&= \int db_T \delta(b_T)\, b_T^a \times \text{logarithmic corrections} \\
&= 0 \quad !
\end{aligned}$$

Use projection method: for unpolarized trivial Sivers W_{UT} and study collinear limit

$$
\begin{aligned}
W_{UT}^{\sin(\phi_h-\phi_S)}(x,z,Q) &\equiv \lim_{b'\to 0} 2\pi \int dq_T q_T \frac{J_1(q_T b'_T)}{M_p b} W_{UT}^{\sin(\phi_h-\phi_S)}(x,z,q_T,Q) \\
&= H_{UT}(Q;\mu) \int dbb \frac{\delta(b-b')}{b} \tilde{f}_{1T\, i/P}^{(1)}(x,b_\star;\mu_b) \tilde{D}_{H/j}(z,b_\star;\mu_b) e^{-S^{pert}(b_*,Q)} e^{-S_{UT}^{NP}(b,Q,x,z)} \\
&\to \int db\, \delta(b) b^\alpha \times \text{log corrections} \\
&= 0
\end{aligned}
$$

Due to same perturbative evolution kernel as unpolarized:
Not surprising however two surprising consequences

Due to same perturbative evolution kernel as unpolarized:

Not surprising however two surprising consequences

1) The first moment of the Sivers function is not divergent, <u>its zero in the regulated CSS formalism</u>

2) With modification, the first moment of the Sivers function is well defined and the operator structure relation between the the 1st moment and the Qiu-Sterman function is finite.

Matching TMD to Collinear factorization for Transverse Polarization based

$$W_{UT}(Q) = \quad H_j^{\rm siv}(\mu_Q, Q)\left[-2M_P\, f_{1T,\,j/A}^{\perp(1)}(x; Q^2, \mu_Q)\right] d_{B/j}(z_B; Q^2, \mu_Q) \quad + O(\alpha_s(Q))$$

$$= H_{LO}^{\rm siv}(\mu_Q, Q)\left[-\frac{1}{2} T_{F,\,j/A}(x_A, x_A; \mu_c)\right] d_{B/j}(z_B; \mu_c) + O(\alpha_s(Q))\,,$$

$$\tilde{F}_{1T}^{\prime\,\perp\, f}(x, b_T; \mu, \zeta_F) = \sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1\, d\hat{x}_2}{\hat{x}_1\, \hat{x}_2} \tilde{C}_{f/j}^{\rm Sivers}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b))\, T_{F\, j/P}(\hat{x}_1, \hat{x}_2, \mu_b)$$

$$\times \exp\left\{\ln\frac{\sqrt{\zeta_F}}{\mu_b}\tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu'); 1) - \ln\frac{\sqrt{\zeta_F}}{\mu'}\gamma_K(g(\mu'))\right]\right\} \times \exp\left\{-g_{f/P}^{\rm Sivers}(x, b_T) - g_K(b_T)\ln\frac{\sqrt{\zeta_F}}{Q_0}\right\}$$

Matching TMD to Collinear factorization for Transverse Polarization based

“Improved CSS” (Polarized) (Gamberg, Metz, DP, Prokudin, Rogers, in preparation)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{f}_1(x, b_T; Q^2, \mu_Q) - iM\epsilon^{ij} b_T^i S_T^j \tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)$$

b_T -> $b_c(b_T)$

NO b_T -> $b_c(b_T)$ replacement – kinematic factor NOT associated with the scale evolution

b_T -> $b_c(b_T)$

Matching TMD to Collinear factorization for Transverse Polarization based

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, Rogers, in preparation)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T, b_c(b_T); Q^2, \mu_Q) = \tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q) - iM\epsilon^{ij} b_T^i S_T^j \tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)$$

$$\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q) \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes F_{FT}(\hat{x}_1, \hat{x}_2; \bar{\mu}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_c(b_T), Q)\right]$$

Matching TMD to Collinear factorization for Transverse Polarization based

We then *define* the momentum-space functions...

$$\boldsymbol{f_1(x, k_T; Q^2, \mu_Q)} \equiv \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{-i\vec{k}_T \cdot \vec{b}_T} \boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\boldsymbol{D_1(z, p_T; Q^2, \mu_Q)} \equiv \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{p}_T \cdot \vec{b}_T} \boldsymbol{\tilde{D}_1(z, b_c(b_T); Q^2, \mu_Q)}$$

$$\vdots$$

$$\frac{\vec{k}_T^2}{2M^2} \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q)} \equiv \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{-i\vec{k}_T \cdot \vec{b}_T} \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)}$$

Matching TMD to Collinear factorization for Transverse Polarization based

Moreover, from a phenomenology standpoint with TMD observables...

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim \boxed{F_{FT}(x, x; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$

$$S_{NP}^{f_{1T}^{\perp}}(b_T, Q) \rightarrow \boxed{g_{f_{1T}^{\perp}}(x, b_T)} + g_K(b_T) \ln(Q/Q_0)$$

$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim \boxed{H_1^{\perp(1)}(z; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^{\perp}}(b_T, Q)\right]$$

$$S_{NP}^{H_1^{\perp}}(b_T, Q) \rightarrow \boxed{g_{H_1^{\perp}}(z, b_T)} + g_K(b_T) \ln(Q/Q_0)$$

The **CT3 functions** (along with the NP *g*-functions) are what get extracted in analyses of TSSAs in ***TMD processes*** that use CSS evolution!

(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))

Comments

- With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, & for transverse polarization.

Thanks to Ian, Kawtar, Barbara Zein Eddine and/Organizers for invitation !

- To get a sense of these *truncation errors* we further "unpack" *W+ Y* via their "*Approximators*" and its *construction in terms of W, Y, FO, ASY* terms

Comments Message

✦ **Collinear fact. valid in two ways**

1. For cross sections differential in q_T w/ $q_T \sim Q$ (OPE)
2. Also valid when we integrate over q_T

$$\int d^2 q_T \; d\sigma(q_T, Q)$$

✦ However CSS did not specifically address the issue of *matching to collinear factorization* for the cross section integrated over q_T

Comments Message

$$\int d^2q_T \; d\sigma(q_T, Q)$$

- We develop a prescription to which *matches* the integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two which suppressed by powers of 1/Q
- **Importantly, the exact definitions of the TMD PDFs and FFs are unmodified from the usual ones of factorization derivations**
- We preserve transverse-coordinate space version of the W_{TMD} term, but only modify the way in which it is used

Review of Region Analysis "Construction"

Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

- CONSTRUCTION: one starts with smallest-size region which is in a neighborhood of $q_T = 0$, where T_{TMD} gives a very good approximation adding and subtracting the T_{TMD} approx.

$$d\sigma(q_T, Q) = T_{TMD}\, d\sigma(q_T, Q) + [d\sigma(q_T, Q) - T_{TMD}\, d\sigma(q_T, Q)]$$

- The error in the bracket is order $(q_T/Q)^a$ and is only unsuppressed at $q_T >> m$

- **Now, *extend* the range of q_T ...**

e.g. Collins PRD 1998, Cambridge Press 2011

Review of Region Analysis "Construction"
W, Y, FO, ASY Definitions

- Extending q_T, one then applies T_{coll} to the bracket & uses the fixed order (FO) perturbative expansion

The Result is the combination

$$d\sigma(m \lesssim q_T \lesssim Q, Q) \approx T_{TMD}\, d\sigma(q_T, Q) + T_{coll}\left[d\sigma(q_T, Q) - T_{TMD}\, d\sigma(q_T, Q)\right]$$

$$+ O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$$d\sigma(m \lesssim q_T \lesssim Q, Q) \approx W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

$q_T/Q << 1$ $\qquad$ $q_T \sim Q$ or $m/q_T << 1$

Now we see the definition of the Y term via "approximators"

$$Y(q_T, Q) \equiv T_{coll}\, d\sigma(q_T, Q) - T_{coll} T_{TMD}\, d\sigma(q_T, Q)$$

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

- It is the difference of the cross section calculated with collinear pdfs and ffs at fixed order FO and the asymptotic contribution of the cross section

- ***N.B. At small q_T the FO and ASY are dominated by the same diverging terms***

$$\frac{1}{q_T^2} \quad \text{and} \quad \frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$$

- ***Thus its expected that the Y term is small or zero leaving***

$$d\sigma(q_T \ll Q, Q) \approx W(q_T, Q)$$

The Asymptotic piece of the NLO cross section in detail

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

$$\left(\frac{d\sigma_{BA}}{dx\,dz\,dQ^2\,dq_T^2\,d\phi}\right)_{\text{asym}} = \frac{\sigma_0 F_l}{S_{eA}}\,\frac{\alpha_s}{\pi}\,\frac{1}{2q_T^2}\,\frac{A_1(\psi,\phi)}{2\pi}$$

$$\times \sum_j e_j^2 \Big[D_{B/j}(z,\mu)\{(P_{qq}\otimes f_{j/A})(x,\mu) + (P_{qg}\otimes f_{g/A})(x,\mu)\}$$

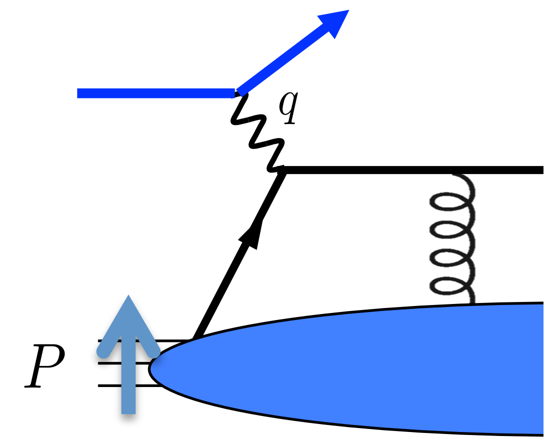
$$+\{(D_{B/j}\otimes P_{qq})(z,\mu) + (D_{B/g}\otimes P_{gq})(z,\mu)\} f_{j/A}(x,\mu)$$

$$+2D_{B/j}(z,\mu) f_{j/A}(x,\mu)\left\{ C_F \log\frac{Q^2}{q_T^2} - \frac{3}{2}C_F \right\} + \mathcal{O}\left(\frac{\alpha_s}{\pi}, q_T^2\right)\Big].$$

- Nadolsky et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. B744, 59 (2006)

The Sivers and Qiu-Sterman functions

- Transverse single spin asymmetry:



- Differential in (small) P_{hT} :
  - Sivers Function: $f_{1T}^{\perp}(x, k_{\mathrm{T}})$
  - Distribution of quarks with transverse momentum k_T inside transversely polarized proton.
  - Sign flip.

- Qiu-Sterman: Collinear but higher twist : $T_F(x_1, x_2)$

- Integrate: $T_{q,F}(x,x) \overset{??}{=} -\int d^2k_\perp \frac{|k_\perp|^2}{M} f_{1T}^{\perp q}(x, k_\perp^2)|_{\mathrm{SIDIS}}$

e.g. BW Example Sivers Function

"Deconvolution"-Structure function simple product " $\mathcal{P}$ "

$$\mathcal{C}\big[w\, f\, D\big] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

"dipole structure"

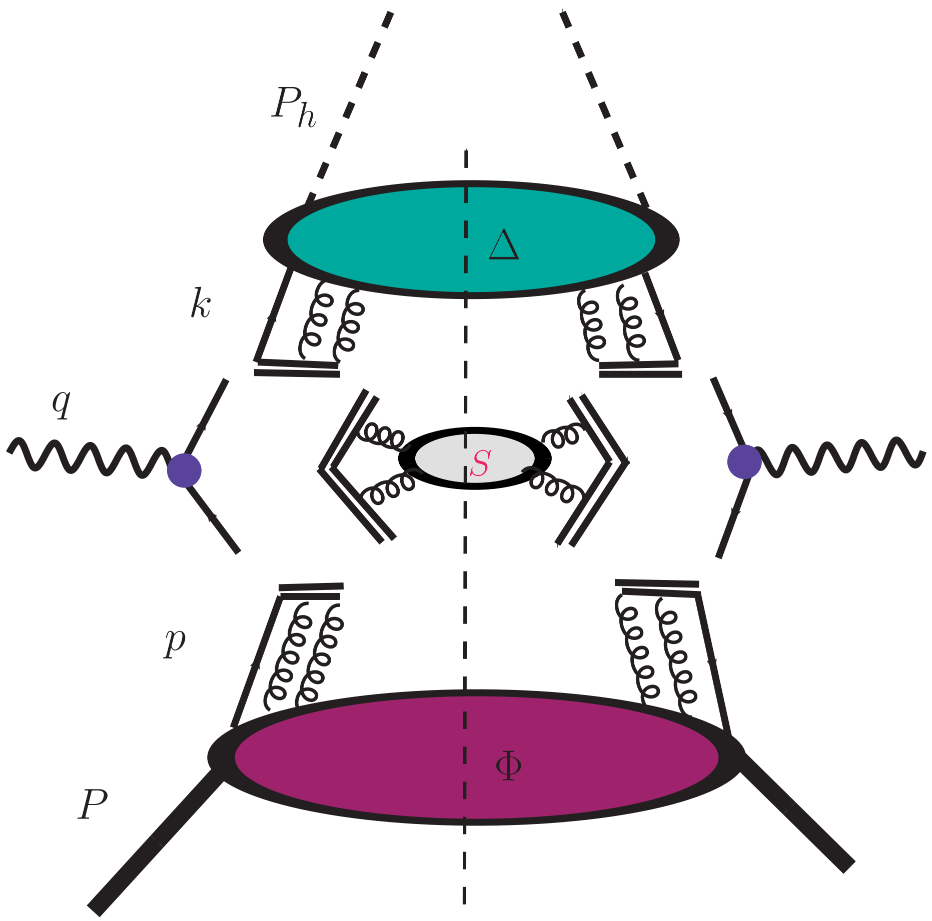
$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|^2 J_1(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\, M z\ \tilde{f}_{1T}^{\perp a(1)}(x, z^2\boldsymbol{b}_T^2)\, \tilde{D}_1^a(z, \boldsymbol{b}_T^2).$$

$\tilde{f}_1,\ \tilde{f}_{1T}^{\perp(1)},$ and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs and finite

Boer, LG, Musch, Prokudin JHEP 2011

Review of TMD factorization

★ Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13), Sun, Yuan (13),Echevarria, Idilbi, Scimemi JHEP 2012, Collins Rogers 2015 ....



•TMDs w/Gauge links: color invariant: emerges from region analysis and Ward Identities

•*In addition Soft factor w/Gauge links*

•*Hard cross section*

•TMD PDFs & Soft factor have rapidity/LC divergences

• Rapidity regulator introduced to regulate these divergences

•Some effects of evolution cancel in Bessel weighted asymmetries

• **Boer, Gamberg, Musch, Prokudin JHEP 2011**

Inconsistency

$$\int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\, \frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_\mathrm{T} \ldots} = \int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\, W + \int \mathrm{d}^2\boldsymbol{q}_\mathrm{T}\, Y$$

From these properties arises a severe problem in getting the integral over qT of the W + Y formula to agree with the collinear factorization results

On the left-hand side, the integral $d\sigma/d^2q_T$ is given by collinear factorization starting at LO, i.e., α_{s0}, up to a power-suppressed error. Fixed-order calculations of the hard scattering are appropriate

On the right-hand side, the integral of W is zero. So the integral of the right-hand side is the integral of Y plus the error term but Y is obtained from collinear factorization starting at NLO, i.e., α_s