Towards a covariant calculation of nuclear GPDs

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GPDs and Lorentz covariance

- GPDs are, by definition, covariant.
- However, nuclear calculations are not traditionally covariant.
  - Non-relativistic potentials are used, or
  - Spectator models project some particles onto their mass shells.
- Covariance is what gives **polynomiality**:

$$\int \frac{dx}{x} x^{n+1} H(x,\xi,t) = \sum_{k \text{ even}}^{n} A_{n+1,k}(t)(2\xi)^k + \text{mod}(n,2) C_{n+1}(t)(2\xi)^{n+1}$$

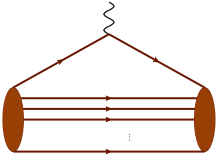
Actually, some GPDs are odd in ξ rather than even; I'm using even here as an illustrative example.

- Also, different calculations are easier in different frames:
  - **Photon frame** best to account for target mass and finite-t effects (*cf.* literature by Braun, Manashov, and Pirnay).
  - **Lab frame** best to calculate DVCS cross sections.
  - Need covariance to reliably transform between frames.

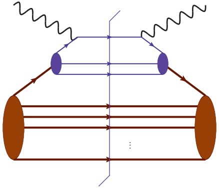
The impulse approximation

- Assume the **impulse approximation**:
  - Virtual photon interacts with one nucleon.
  - The active nucleon behaves like a free nucleon.
  - There are no final state interactions.

$$j_A^\mu = \sum_{\text{nucleons}}$$



$$H_A = \sum_{\text{nucleons}}$$



- The impulse approximation is incomplete.
- It's still useful, as a first approximation, and because deviations indicate nuclear effects (EMC effect, shadowing, ...)

Convolution matrix equation

As the GPD is a sort of PDF-form factor "hybrid," the convolution relation will also be a hybrid:

- The form factor "convolution" relation is just matrix multiplication:

$$\begin{bmatrix} F_{1A}(Q^2) \\ F_{2A}(Q^2) \\ \vdots \end{bmatrix} = \begin{bmatrix} F_{1V}(Q^2) & F_{1T}(Q^2) \\ F_{2V}(Q^2) & F_{2T}(Q^2) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} ZF_{1p}(Q^2) + (A-Z)F_{1n}(Q^2) \\ ZF_{2p}(Q^2) + (A-Z)F_{2n}(Q^2) \end{bmatrix}$$

  The number of form factors depends on the target's spin.

- PDF convolution relation is an integral equation:

$$f_{i/A}(x,\mu) = \int_x^A \frac{dy}{y} \left[Z f_{i/p}\left(\frac{x}{y},\mu\right) f_{p/A}(y) + (A-Z) f_{i/n}\left(\frac{x}{y},\mu\right) f_{n/A}(y) \right]$$

- The GPD convolution equation is a hybrid integral matrix equation:

$$\begin{bmatrix} H_{1A}(x,\xi,t;\mu) \\ H_{2A}(x,\xi,t;\mu) \\ \vdots \end{bmatrix} = \int \frac{dy}{y} \begin{bmatrix} H_{1V}(y,\xi,t) & H_{1T}(y,\xi,t) \\ H_{2V}(y,\xi,t) & H_{2T}(y,\xi,t) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} ZH_p\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + (A-Z)H_n\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) \\ ZE_p\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + (A-Z)E_n\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) \end{bmatrix}$$

  Number of GPDs depends on the spin of the target.

About the matrix

$$\begin{bmatrix} H_{1A}(x,\xi,t;\mu) \\ H_{2A}(x,\xi,t;\mu) \\ \vdots \end{bmatrix} = \int \frac{dy}{y} \begin{bmatrix} H_{1V}(y,\xi,t) & H_{1T}(y,\xi,t) \\ H_{2V}(y,\xi,t) & H_{2T}(y,\xi,t) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} ZH_p\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + (A-Z)H_n\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) \\ ZE_p\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + (A-Z)E_n\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) \end{bmatrix}$$

- The convolution matrix comes from the matrix elements:

$$\langle p'_A, s \mid \left(\not{n} H_N + \frac{i n_\mu \Delta_\nu \sigma^{\mu\nu}}{2m_N} E_N \right) \mid p_A, s \rangle$$

- This is an on-shell nucleon electromagnetic current operator.
- H_{jV} are coefficients multiplying H_N in the convolution relation for producing H_{1A}.
- H_{jT} are the coefficients multiplying E_N in the convolution relation for producing H_{1A}.

Convolution for scalar nucleus

- Let's look at a scalar nucleus (*e.g.*, ^{4}He) as a simple example.
- Only one chiral-even GPD!

$$H_A(x,\xi,t;\mu) = \sum_{N=p,n} \int \frac{dy}{y} \left[H_V\,(y,\xi,t)\,\zeta_N H_N\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + H_T\,(y,\xi,t)\,\zeta_N E_N\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right)\right]$$

$\zeta_p = Z$ and $\zeta_n = (A - Z)$.

- In this case, H_V and H_T are just the factors that multiply H_N and E_N, respectively, in the convolution equation.
- In general, when a nucleus has multiple GPDs:

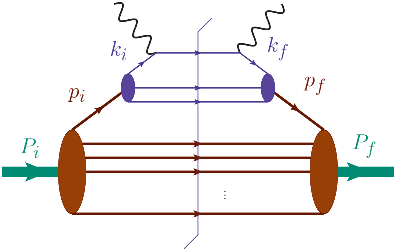
$$H_{jA}(x,\xi,t;\mu) = \sum_{N=p,n} \int \frac{dy}{y} \left[H_{jV}\,(y,\xi,t)\,\zeta_N H_N\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + H_{jT}\,(y,\xi,t)\,\zeta_N E_N\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right)\right]$$

Convolutions and polynomiality

$$P = \frac{1}{2}(P_i + P_f) \qquad x = \frac{k \cdot n}{P \cdot n} \qquad \xi = -\frac{\Delta \cdot n}{2P \cdot n}$$

$$p = \frac{1}{2}(p_i + p_f) \qquad y = \frac{p \cdot n}{P \cdot n} \qquad \xi_N = -\frac{\Delta \cdot n}{2p \cdot n} = \frac{\xi}{y}$$

$$k = \frac{1}{2}(k_i + k_f)$$



- Both nucleon GPD and nuclear coefficient matrix should satisfy polynomiality:

$$\int \frac{dx}{x} x^{n+1} H_N(x,\xi,t) = \sum_{k \text{ even}}^{n} A^{(N)}_{n+1,k}(t)(2\xi)^k + \text{mod}(n,2) C^{(N)}_{n+1}(t)(2\xi)^{n+1}$$

$$\int \frac{dx}{x} x^{n+1} h_{N/A}(x,\xi,t) = \sum_{k \text{ even}}^{n} A^{(A)}_{n+1,k}(t)(2\xi)^k + \text{mod}(n,2) C^{(A)}_{n+1}(t)(2\xi)^{n+1}$$

  $h_{N/A}$ and H_N are rectangular and column matrices here!
- Actually, some GPDs are odd in ξ rather than even; I'm using even here as an illustrative example.

Convolutions and polynomiality

Mellin moment of *total* nuclear GPD:

$$\mathcal{M}_n(\xi,t) = \int \frac{dx}{x} x^{n+1} \int \frac{dy}{y} h_{N/A}(y,\xi,t) H_N\left(\frac{x}{y},\frac{\xi}{y},t\right)$$
$$= \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \int \frac{dz}{z} z^{n+1} H_N\left(z,\frac{\xi}{y},t\right)$$

If $\xi = 0$, we get a product of Mellin moments, but not in general.

$$\mathcal{M}_n(\xi,t) = \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \left[\sum_{k \text{ even}}^{n} A^{(N)}_{n+1,k}(t)\left(2\frac{\xi}{y}\right)^k + \text{mod}(n,2) C^{(N)}_{n+1}(t)\left(2\frac{\xi}{y}\right)^{n+1}\right]$$

Important: z integral produced a polynomial in ξ/y. After some maths:

$$\mathcal{M}_n(\xi,t) = \sum_{l \text{ even}}^{n} (2\xi)^l \sum_{k \text{ even}}^{l} A^{(A)}_{n+1-l,l-k}(t) A^{(N)}_{n+1,k}(t)$$
$$+ \text{mod}(n,2)\,(2\xi)^{n+1}\left\{\sum_{k \text{ even}}^{n} C^{(A)}_{n-k+1}(t) A^{(N)}_{n+1,k}(t) + \int \frac{dy}{y} h_{N/A}(y,\xi,t) C^{(N)}_{n+1}(t)\right\}$$

Discrete convolution relations

Mellin moments / Polynomiality : Discrete convolution relations are obeyed for generalized form factors:

$$A_{n+1,l}^{(q/A)}(t) = \sum_{\substack{k=0 \\ \text{even}}}^{l} A_{n+1-l,l-k}^{(N/A)}(t) A_{n+1,k}^{(q/N)}(t) \xrightarrow[n=0]{} F^{(q/A)}(t) = F^{(N/A)}(t) F^{(q/N)}(t)$$

$$C_{n+1}^{(q/A)}(t) = \sum_{\substack{k=0 \\ \text{even}}}^{n} C_{n-k+1}^{(N/A)}(t) A_{n+1,k}^{(q/N)}(t) + \int \frac{dy}{y} h_{N/A}(y,\xi,t) C_{n+1}^{(q/N)}(t)$$

- Strictly, these are matrices.
- $A^{(q/A)}(t)$ and $A^{(q/N)}(t)$ are column matrices.
- $A^{(N/A)}(t)$ is a rectangular matrix.
- Get form factor matrix equation in $n = 0$ case.
- Unsure the meaning (or convergence) of the $\int \frac{dy}{y} h_{N/A}(y,\xi,t)$ term.

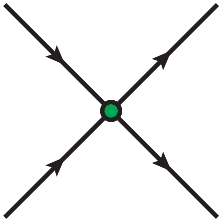
Nuclear GPDs in the impulse approximation

- Three limiting factors to knowledge of nuclear GPDs:
    1. The nucleon GPDs.
    2. The coefficient functions H_{1V}, *etc.*
    3. What happens beyond the impulse approximation.
- We focus on issue #2 here.
- The limiting factor to calculation of the coefficient functions is **knowledge of the nuclear wave function**.
- **Good news**: High-precision nuclear wave functions exist for light nuclei! (AV18+Illinois7)
- **Bad news**: These wave functions are not Lorentz covariant.
- Lorentz covariance is needed for polynomiality.

A covariant nuclear vertex

- The raison d'etre of this work is to find **Lorentz covariant nuclear wave functions**.
- Light nuclei are the obvious (easiest) starting point; first case, the deuteron.
- Cano and Pire (Eur Phys J A19 (2004) 423) give a theoretical treatment, but their numerical results violate polynomiality.
- A worthwhile investigation may be: a simple, *exactly-solvable* model of the deuteron as two nucleons. Simplify the problem with a simple potential.
- Keep Lorentz invariance manifest from the start.

Relativistic contact interactions



- The Nambu-Jona-Lasinio (NJL) model has been extremely successful in describing hadron structure.
- Contact interactions give a simple starting point for exact, *Lorentz-invariant* calculations.

- Contact interaction Lagrangian for *nucleon-nucleon* interactions:

$$\mathcal{L} = \sum_I G_I \left(\psi^T C^{-1} \tau_2 \Omega_I \psi\right) \left(\bar{\psi} \Omega_I C \tau_2 \bar{\psi}^T\right)$$

- It is always possible to write the contact interaction Lagrangian in this form, via Fierz rearrangement.

Two-nucleon bilinears

Contact interaction Lagrangian:

$$\mathcal{L} = \sum_I G_I \left(\psi^T C^{-1} \tau_2 \Omega_I \psi\right) \left(\bar{\psi} \Omega_I C \tau_2 \bar{\psi}^T\right)$$

The matrices Ω_I are tensor products of Clifford algebra matrices, isospin matrices, and derivatives.

- Fermion fields are classically Grassmann numbers:
  - $\psi_1 \psi_2 = -\psi_2 \psi_1$
  - $\psi_1^2 = 0$
  - $\psi^T \psi = 0$
  - $\psi^T M \psi = -\psi^T M^T \psi$ for any matrix M.
- All bilinears in our Lagrangian should use antisymmetric matrices.

Two-nucleon bilinears

Contact interaction Lagrangian:

$$\mathcal{L} = \sum_I G_I \left(\psi^T C^{-1} \tau_2 \Omega_I \psi\right) \left(\bar{\psi} \Omega_I C \tau_2 \bar{\psi}^T\right)$$

For simplicity, consider only first-order derivatives.

$$\partial_\mu^\pm = \frac{\overrightarrow{\partial}_\mu \pm \overleftarrow{\partial}_\mu}{2}$$

$$(\partial_\mu^\pm)^T = \pm \partial_\mu^\pm$$

	Symmetric	Antisymmetric
Clifford	$\gamma^\mu C$, $\sigma^{\mu\nu} C$	C, $\gamma^5 C$, $\gamma^5 \gamma^\mu C$
Isospin	$\tau_j \tau_2$	τ_2
Derivative	1, ∂_μ^+	∂_μ^-

- Matrices $\Omega_I C \tau_2$ are made by mixing and matching, to get an overall antisymmetric matrix.
- A total of **21 terms** available for Lagrangian.
- 10 of these terms are isoscalar ($I = 0$). Focus on these (relevant to deuteron).

Isoscalar Lagrangian

Isoscalar contact Lagrangian:

$$\mathcal{L}_{I=0} = \mathcal{L}_0 + \mathcal{L}_k + \mathcal{L}_p$$

No-deritatives terms:

$$\mathcal{L}_0 = G_V \left(\bar{\psi}\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right) + \frac{1}{2}G_T\left(\bar{\psi}\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\sigma_{\mu\nu}\psi\right)$$

Minus-derivative terms:

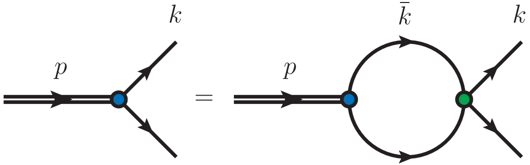
$$\mathcal{L}_k = G_1\left(\bar{\psi}\partial^{-\mu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^-\psi\right) + G_2\left(\bar{\psi}\partial^{-\mu}\gamma_5 C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^-\gamma_5\psi\right)$$
$$+ G_3\left(\bar{\psi}\partial^{-\mu}\gamma_5\gamma^\nu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu\gamma_5\gamma_\nu\psi\right) + G_4\left(\bar{\psi}\gamma_5\not{\partial}^- C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_5\not{\partial}^-\psi\right)$$

Plus-derivative terms:

$$\mathcal{L}_p = G_5\left(\bar{\psi}\partial^{+\mu}\gamma^\nu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^+\gamma_\nu\psi\right) + \frac{1}{2}G_6\left(\bar{\psi}\partial^{+\mu}\sigma^{\nu\pi}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^+\sigma_{\nu\pi}\psi\right)$$
$$+ G_7\left(\bar{\psi}\not{\partial}^+ C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\not{\partial}^+\psi\right) + G_8\left(\bar{\psi}\partial_\nu^+\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial^{+\pi}\sigma_{\mu\pi}\psi\right)$$

Bethe-Salpeter equation for the deuteron

- Apply our contact interaction to the deuteron.
- Deuteron obeys the Bethe-Salpeter equation:

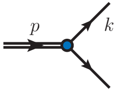


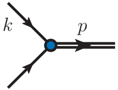
- Derivatives in momentum space:

$$\partial^+_\mu \mapsto \frac{i}{2} p_\mu$$
$$\partial^-_\mu \mapsto i k_\mu$$

- The contact potential is *separable*, so the deuteron vertex is linear in k.

Deuteron vertex

$\Gamma_d^\mu(p,k) =$ 

$\bar{\Gamma}_d^\mu(p,k) =$ 

Most general deuteron vertex compatible with our Lagrangian:

$$\Gamma_d^\mu(p,k) = \left[\alpha_V\left(\gamma^\mu - \frac{\not{p}p^\mu}{p^2}\right) + i\alpha_T\frac{p_\nu\sigma^{\mu\nu}}{M_d} + \frac{\alpha_E}{M_d}\left(k^\mu - \frac{k\cdot p}{p^2}p^\mu\right) + \alpha_D\left(\frac{\not{p}\gamma^\mu\not{k} - \not{k}\gamma^\mu\not{p}}{2p^2}\right)\right]C\tau_2$$

Several Lagrangian terms either vanish or become redundant in BSE.

$$\mathcal{L}_{\text{effective}} = G_V\left(\bar\psi\gamma^\mu C\tau_2\bar\psi^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right) + \frac{1}{2}G_T\left(\bar\psi\sigma^{\mu\nu}C\tau_2\bar\psi^T\right)\left(\psi^T C^{-1}\tau_2\sigma_{\mu\nu}\psi\right)$$
$$+ G_E\left(\bar\psi\partial^{-\mu}C\tau_2\bar\psi^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^-\psi\right) + G_D\left(\bar\psi\partial^{-\mu}\gamma_5\gamma^\nu C\tau_2\bar\psi^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu\gamma_5\gamma_\nu\psi\right)$$

Now only four terms!

Also, $C = -1$ vector meson

- Odd-C vector mesons also require antisymmetric vertices.
- If we set H_N and E_N instead to GPDs of the dressed quark, and replace m_N with a dressed quark mass, our “deuteron GPD” becomes a ρ meson GPD!
- Thus, by finding the dueteorn GPDs in the contact formalism, we are also calculating the ρ meson GPDs in the NJL model.

Matrix form of the BSE

- The BSE can be thought of as a matrix equation.

$$\Gamma_d(p,k) = \mathcal{M}_{\mathrm{BSE}}\Gamma_d(p,k)$$

$$\begin{bmatrix} \alpha_V \\ \alpha_T \\ \alpha_E \\ \alpha_D \end{bmatrix} = 4 \begin{bmatrix} G_V\Pi_{VV} & G_V\Pi_{VT} & G_V\Pi_{VE} & G_V\Pi_{VD} \\ G_T\Pi_{TV} & G_T\Pi_{TT} & G_T\Pi_{TE} & G_T\Pi_{TD} \\ G_E\Pi_{EV} & G_E\Pi_{ET} & G_E\Pi_{EE} & G_E\Pi_{ED} \\ G_D\Pi_{DV} & G_D\Pi_{DT} & G_D\Pi_{DE} & G_D\Pi_{DD} \end{bmatrix} \begin{bmatrix} \alpha_V \\ \alpha_T \\ \alpha_E \\ \alpha_D \end{bmatrix}$$

- The interactions mix up components of the vertex.
- The bubble diagrams Π_{VV} *etc.* contain all the difficulties (UV divergences, *etc.*).
- Once the bubbles are known, solving the BSE is simply linear algebra.
- Theory is non-renormalizable, so cutoff is an additional parameter.

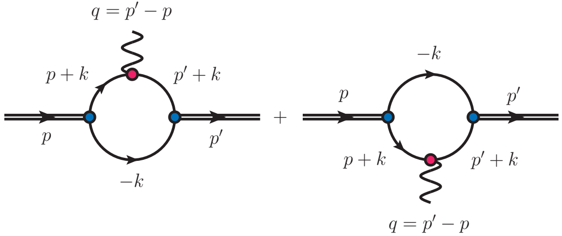
Deuteron form factors

- To determine the G's (or α's), empirical input is needed.
- Electromagnetic properties of the deuteron are well-known.
- Deuteron current decomposes into three Lorentz-invariant form factors:

$$j_d^{\mu;\alpha\beta}(p';p) = (p+p')^\mu g^{\alpha\beta} F_{1d}(Q^2) - (q^\alpha g^{\beta\mu} - q^\beta g^{\alpha\mu}) F_{2d}(Q^2) - (p+p')^\mu \frac{q^\alpha q^\beta}{2M_d^2} F_{3d}(Q^2)$$

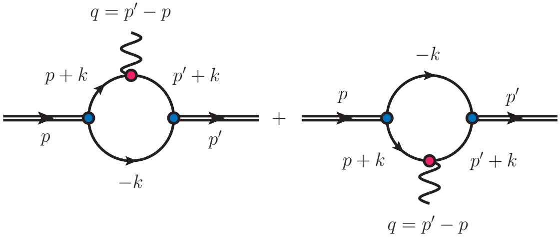
- This can be calculated in the covariant contact model:

$$j_d^{\mu;\alpha\beta}(p';p) =$$



Deuteron form factors

$$j_d^{\mu;\alpha\beta}(p';p) =$$



Using nucleon form factors for the photon-nucleon coupling, we get a matrix equation:

$$\begin{bmatrix} F_{1d}(Q^2) \\ F_{2d}(Q^2) \\ F_{3d}(Q^2) \end{bmatrix} = \begin{bmatrix} F_{1V}(Q^2) & F_{1T}(Q^2) \\ F_{2V}(Q^2) & F_{2T}(Q^2) \\ F_{3V}(Q^2) & F_{3T}(Q^2) \end{bmatrix} \begin{bmatrix} F_{1p}(Q^2) + F_{1n}(Q^2) \\ F_{2p}(Q^2) + F_{2n}(Q^2) \end{bmatrix}$$

F_{1V}, F_{1T}, *etc.* are the body form factors.

Sachs-like form factors

Sachs-like form factors are closer to empirical observation.

$$G_Q(Q^2) = F_{1d}(Q^2) - F_{2d}(Q^2) + (1+\eta)F_{3d}(Q^2)$$
$$G_M(Q^2) = F_{2d}(Q^2)$$
$$G_C(Q^2) = F_{1d}(Q^2) - \frac{2}{3}\eta G_Q(Q^2)$$

$$\langle r_E \rangle_{\text{rms}} = \sqrt{-6\frac{\partial G_C(Q^2=0)}{\partial Q^2}}$$
$$\mu_d = \frac{m_N}{M_d} G_M(Q^2=0)$$
$$\mathcal{Q} = \frac{1}{M_d^2} G_Q(Q^2=0)$$

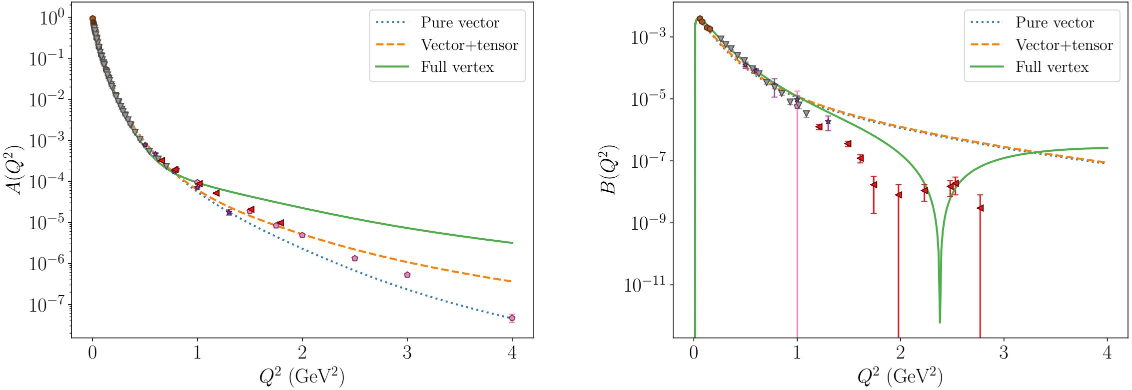
where $\eta = \frac{Q^2}{4M_d^2}$.

They are related to electromagnetic structure functions:

$$A(Q^2) = G_C^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2)$$
$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2(Q^2).$$

Full contact model

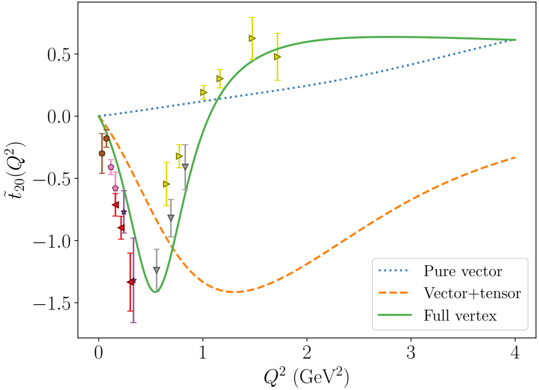
- Attempt fit to data up to $Q^2 = 1$ GeV^2.
- Fit fails when higher-Q^2 data are used: necessity of long-range pion exchange?



Contact model

	Model	Empirical
r_{rms} (fm)	2.15	2.1413(25)
μ_d	0.91	0.8574382311(48)
$\mathcal{Q}_d$ (fm^2)	0.122	0.2859(3)

- Contact model is imperfect.
- The static quadrupole moment is off by a factor of 2.
- Otherwise, quite good description for a contact model.



Long-range pion exchange is likely necessary for a perfect description.

Contact model

- For now, proceed with relativistic contact model.
- The UV cutoff is close to the pion mass, suggesting a breakdown of the contact model when pion exchange becomes relevant.
- The close values of the α's suggests a finely-tuned cancellation between attractive and repulsive forces.

G_V	$-(6.14\text{ fm})^2$
G_T	$(6.28\text{ fm})^2$
G_E	$(3.60\text{ fm})^4$
G_D	$-(2.63\text{ fm})^4$
Λ	142 MeV
α_V	46
α_T	-48
α_E	-45
α_D	18

$$\mathcal{L}_{\text{effective}} = G_V \left(\bar{\psi}\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right) + \frac{1}{2}G_T\left(\bar{\psi}\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\sigma_{\mu\nu}\psi\right) + G_E\left(\bar{\psi}\partial^{-\mu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial^-_\mu\psi\right) + G_D\left(\bar{\psi}\partial^{-\mu}\gamma_5\gamma^\nu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu\gamma_5\gamma_\nu\psi\right)$$

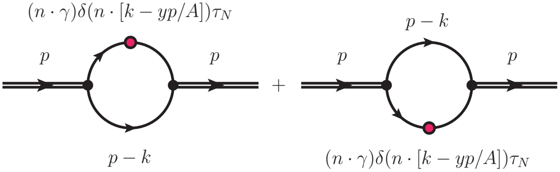
Deuteron PDFs

- Deuteron PDFs related to nucleon PDFs by convolution formula:

$$q^{(\lambda)}_{i/d}(x_A, Q^2) = \sum_{N=p,n} \int_{x_A}^{2} \frac{dy}{y} q_{i/N}\left(\frac{x_A}{y}, Q^2\right) f^{(\lambda)}_{N/d}(y)$$

- The deuteron light cone density (LCD) $f^{(\lambda)}_{N/d}(y)$ can be found by Feynman rules:

$$f^{(\lambda)}_{N/d}(y) =$$



Deuteron LCD

- We find exact expressions for the LCD.
- For example, the "pure vector" (α_V-only) part:

$$f_d^{(\mathrm{unpol})}(y) = \alpha_V^2 \frac{1}{48\pi^2} \int d\tau e^{-\Delta(y)\tau} \left(\frac{1}{\tau} + \frac{M_d^2}{8} y(2-y)[4 - y(2-y)] + \frac{3}{4} m^2 y(2-y) \right)$$

$$f_d^{(\mathrm{tensor})}(y) = -\alpha_V^2 \frac{1}{32\pi^2} \int d\tau e^{-\Delta(y)\tau} \left(\frac{2 - 3y(2-y)}{\tau} - \frac{M_d^2}{2} y(2-y)(y-1)^2 \right)$$

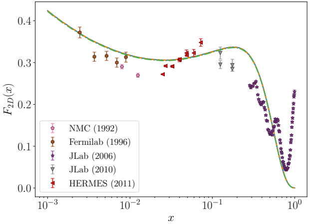
$$\Delta(y) = m_N^2 - \frac{M_d^2}{4} y(2-y)$$

- Full expressions available in upcoming paper.

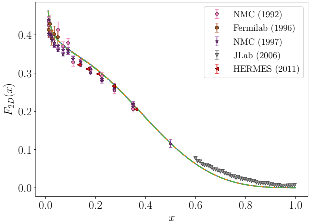
Structure function calculations

We get a good agreement with DIS data for the deuteron.

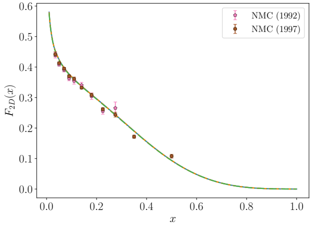
$Q^2 = 1$ GeV^2

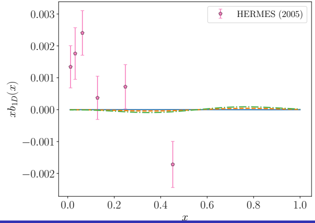


$Q^2 = 5$ GeV^2



$Q^2 = 15$ GeV^2





- $Q^2 = 1$ GeV^2 exhibits quark-hadron duality.
- We can't describe HERMES b_1 data, but no impulse approximation calculation can.

Deuteron GPD

- Unfortunately, the deuteron (and ρ meson) GPDs are still a work in progress.
- But we're close.
- The formalism is in place: it's just a technical matter of calculating some inverse Mellin transforms.

Outlook

- Deuteron GPDs in a relativistic contact model are on the horizon.
- These GPDs **do** satisfy polynomiality.
- These GPDs are also ρ GPDs in the NJL model (with some constants changed).
- It will also be straightforward to generalize to GTMDs.
- The contact formalism will be applied to ^{3}He and ^{4}He next.
- **Lorentz-invariant inclusion of pion exchange would be ideal**
  ... but our primary focus is the convolution formalism itself.
- Contact formalism should work better for helium and ρ, since these are not loosely bound.
- ^{4}He is higher priority, since data is being collected for it now.

The End

Thank you for listening!