Generalized parton distributions of light nuclei

Adam Freese

Argonne National Laboratory

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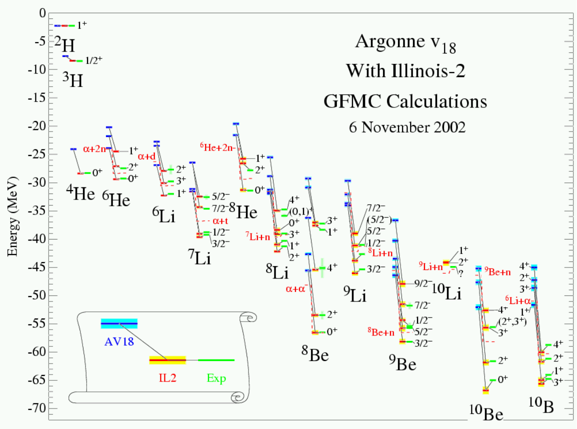
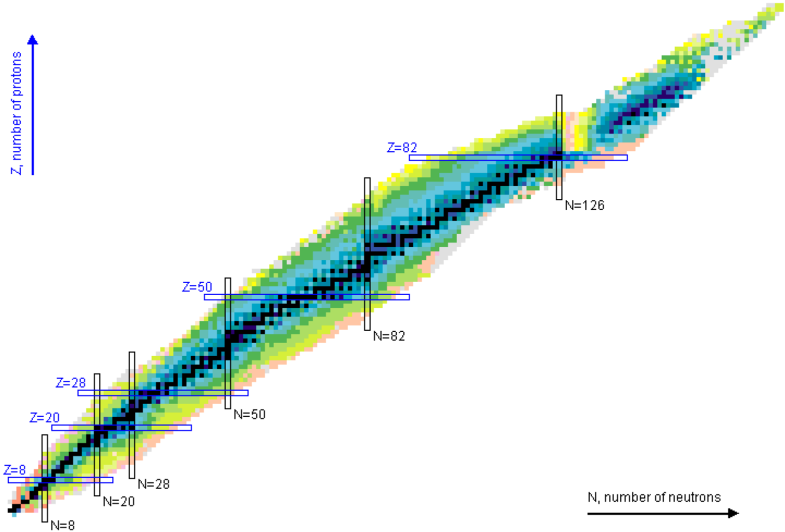
The nucleus as nucleons

- Quarks and gluons are the *real* constituents of nuclei.
- But there are many nuclei.
- It has proved useful in many cases (nuclear structure calculations) to assume the nucleus is made of nucleons.
- Science proceeds by making assumptions, and then discoveries when these assumptions are wrong.



Organizational principles are helpful



Besides, what would be the point in trying to find GPDs of all of these without a concrete reason?

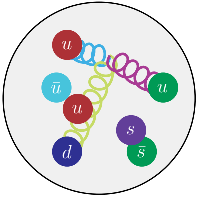
We have our hands full with even the proton.

A bold assumption

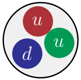
- Let's *assume* that nuclei are made of nucleons, and then proceed to image them.
- But what if we make wrong predictions? ... that would be good!
    - Partonic structure might get modified in the nuclear medium. (Learn more about QCD, possibly the phase diagram.)
    - There might be *non-nucleonic components* to nuclei. (Hidden color, six quark bags, Delta-Delta components.) Imaging could tell us more.
    - Nuclear tomography can give us extra information for a better handle on the EMC effect.

For example, color screening

- Frankfurt and Strikman hypothesize that the EMC effect comes from suppression of small-sized configurations.
- On average, a nucleon inside a nucleus should be bigger than a free nucleon.
- Tagged/incoherent DVCS can study tomography of bound nucleons. Do they actually swell?



Average-sized configuration

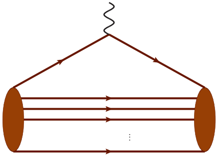


Point-like configuration
Is this suppressed in nuclei?

Form factors in a nucleonic model

- *Assume*: nucleus is made of (unmodified) nucleons.

$$j^\mu(A) = \sum_{\text{nucleons}}$$



- Nuclear form factor defined by a matrix equation.

$$\begin{bmatrix} F_{1A}(Q^2) \\ F_{2A}(Q^2) \\ \vdots \end{bmatrix} = \begin{bmatrix} F_{1V}(Q^2) & F_{1T}(Q^2) \\ F_{2V}(Q^2) & F_{2T}(Q^2) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} ZF_{1p}(Q^2) + (A-Z)F_{1n}(Q^2) \\ ZF_{2p}(Q^2) + (A-Z)F_{2n}(Q^2) \end{bmatrix}$$

- Number of form factors depends on nuclear spin (1 for spin-0, 2 for spin-half, *etc.*).
- The body form factors F_{1V}, *etc.*, encode nuclear dynamics.

Breakdown of the nucleonic model

Fig. 12

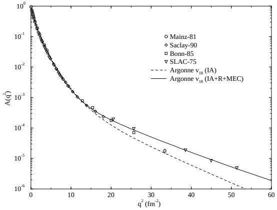


Fig. 13

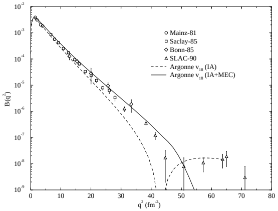
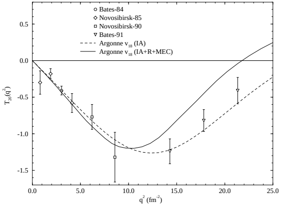


Fig. 14



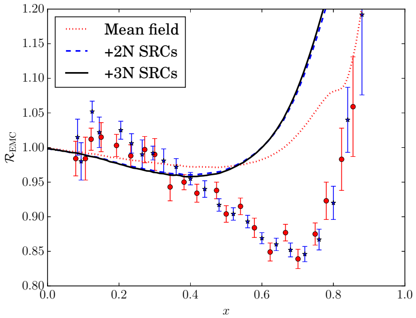
- Various "corrections" contribute to nuclear form factors.
  - Delta-isobar components.
  - Meson exchange currents.
- Corrections show the nucleonic model is incomplete.
- However, nucleonic model is a springboard from which to discover particular new phenomena.

PDF convolution and the EMC effect

- *Assume*: nucleus is made of (unmodified) nucleons.
- Can derive convolution equation:

$$f_{i/A}(x,\mu) = \int_x^A \frac{dy}{y} \left[Z f_{i/p}\left(\frac{x}{y},\mu\right) f_{p/A}(y) + (A-Z) f_{i/n}\left(\frac{x}{y},\mu\right) f_{n/A}(y) \right]$$

- This equation is incomplete: **EMC effect**.

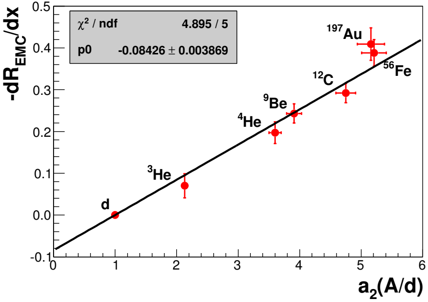


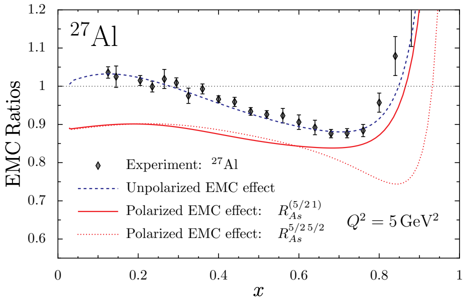
Gerry Miller gave rigorous proof that EMC effect cannot be due to unmodified nucleonic motion.

PRC65 (2002) 015211, 055206

The EMC effect is telling us something

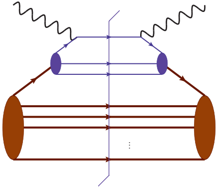
- The EMC effect is *telling us something*.
- The nucleus-as-nucleons model is incomplete, but in a systematic way.
- We don't know what's going on, but several hypotheses that make differing predictions for *polarized* PDFs exist.
- The nucleonic model again serves as a springboard.





GPD convolution

- A nucleonic model of nuclear GPDs can again be a springboard.

$$\langle p', \lambda' | \mathcal{O}_V | p, \lambda \rangle = \sum_{\text{nucleons}}$$ 

- Similar equation for axial operator.
- A hybrid convolution/matrix equation should hold:

$$\begin{bmatrix} H_{1A}(x,\xi,t;\mu) \\ H_{2A}(x,\xi,t;\mu) \\ \vdots \end{bmatrix} = \int \frac{dy}{y} \begin{bmatrix} H_{1V}(y,\xi,t) & H_{1T}(y,\xi,t) \\ H_{2V}(y,\xi,t) & H_{2T}(y,\xi,t) \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} ZH_p\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + (A-Z)H_n\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) \\ ZE_p\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) + (A-Z)E_n\left(\frac{x}{y},\frac{\xi}{y},t;\mu\right) \end{bmatrix}$$

See Sergio's talk.

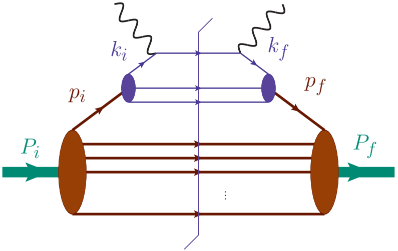
- This equation will be *incomplete*, perhaps because of modification and/or non-nucleonic components.
- It is worth studying *how* and *why* it falls short.

Convolutions and polynomiality

$$P = \frac{1}{2}\left(P_i + P_f\right) \qquad x = \frac{k \cdot n}{P \cdot n} \qquad \xi = -\frac{\Delta \cdot n}{2P \cdot n}$$

$$p = \frac{1}{2}\left(p_i + p_f\right) \qquad y = \frac{p \cdot n}{P \cdot n} \qquad \xi_N = -\frac{\Delta \cdot n}{2p \cdot n} = \frac{\xi}{y}$$

$$k = \frac{1}{2}\left(k_i + k_f\right)$$



- Both nucleon GPD and nuclear "body" GPDs should satisfy polynomiality:

$$\int \frac{dx}{x} x^{n+1} H_N(x,\xi,t) = \sum_{k\ \mathrm{even}}^{n} A^{(N)}_{n+1,k}(t)(2\xi)^k + \mathrm{mod}(n,2) C^{(N)}_{n+1}(t)(2\xi)^{n+1}$$

$$\int \frac{dx}{x} x^{n+1} h_{N/A}(x,\xi,t) = \sum_{k\ \mathrm{even}}^{n} A^{(A)}_{n+1,k}(t)(2\xi)^k + \mathrm{mod}(n,2) C^{(A)}_{n+1}(t)(2\xi)^{n+1}$$

$h_{N/A}$ and H_N are rectangular and column matrices here!

- Actually, some GPDs are odd in ξ rather than even; I'm using even here as an illustrative example.

Convolutions and polynomiality

Mellin moment of *total* nuclear GPD:

$$\begin{aligned}\mathcal{M}_n(\xi,t) &= \int \frac{dx}{x} x^{n+1} \int \frac{dy}{y} h_{N/A}(y,\xi,t) H_N\left(\frac{x}{y},\frac{\xi}{y},t\right) \\ &= \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \int \frac{dz}{z} z^{n+1} H_N\left(z,\frac{\xi}{y},t\right)\end{aligned}$$

If $\xi = 0$, we get a product of Mellin moments, but not in general.

$$\mathcal{M}_n(\xi,t) = \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \left[\sum_{k \text{ even}}^{n} A^{(N)}_{n+1,k}(t)\left(2\frac{\xi}{y}\right)^k + \mathrm{mod}(n,2) C^{(N)}_{n+1}(t)\left(2\frac{\xi}{y}\right)^{n+1}\right]$$

Convolutions and polynomiality

Mellin moment of *total* nuclear GPD:

$$\mathcal{M}_n(\xi,t) = \int \frac{dx}{x} x^{n+1} \int \frac{dy}{y} h_{N/A}(y,\xi,t) H_N\left(\frac{x}{y},\frac{\xi}{y},t\right)$$
$$= \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \int \frac{dz}{z} z^{n+1} H_N\left(z,\frac{\xi}{y},t\right)$$

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$$= \sum_{k \text{ even}}^{n} A^{(N)}_{n+1,k}(t)\,(2\xi)^k \int \frac{dy}{y} y^{n-k+1} h_{N/A}(y,\xi,t) + \mathrm{mod}(n,2) C^{(N)}_{n+1}(t)\,(2\xi)^{n+1} \int \frac{dy}{y} h_{N/A}(y,\xi,t)$$

Convolutions and polynomiality

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$$= \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \int \frac{dz}{z} z^{n+1} H_N\left(z,\frac{\xi}{y},t\right)$$

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$$\mathcal{M}_n(\xi,t) = \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \left[\sum_{k \text{ even}}^{n} A^{(N)}_{n+1,k}(t)\left(2\frac{\xi}{y}\right)^k + \text{mod}(n,2) C^{(N)}_{n+1}(t)\left(2\frac{\xi}{y}\right)^{n+1}\right]$$
$$= \sum_{k \text{ even}}^{n} \left[\sum_{j \text{ even}}^{n-k} A^{(A)}_{n-k+1,j}(t)(2\xi)^j + \text{mod}(n-k,2) C^{(A)}_{n-k+1}(t)(2\xi)^{n-k+1}\right] A^{(N)}_{n+1,k}(t)\,(2\xi)^k$$
$$+ \text{mod}(n,2) \int \frac{dy}{y} h_{N/A}(y,\xi,t) C^{(N)}_{n+1}(t)\,(2\xi)^{n+1}$$

Convolutions and polynomiality

Mellin moment of *total* nuclear GPD:

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$$\mathcal{M}_n(\xi,t) = \int \frac{dy}{y} y^{n+1} h_{N/A}(y,\xi,t) \left[\sum_{k \text{ even}}^{n} A_{n+1,k}^{(N)}(t)\left(2\frac{\xi}{y}\right)^k + \mathrm{mod}(n,2) C_{n+1}^{(N)}(t)\left(2\frac{\xi}{y}\right)^{n+1}\right]$$
$$= \sum_{k \text{ even}}^{n} \sum_{j \text{ even}}^{n-k} A_{n-k+1,j}^{(A)}(t)(2\xi)^j A_{n+1,k}^{(N)}(t)\,(2\xi)^k + \mathrm{mod}(n,2) \sum_{k \text{ even}}^{n} C_{n-k+1}^{(A)}(t)(2\xi)^{n-k+1} A_{n+1,k}^{(N)}(t)\,(2\xi)^k$$
$$+ \mathrm{mod}(n,2) \int \frac{dy}{y} h_{N/A}(y,\xi,t) C_{n+1}^{(N)}(t)\,(2\xi)^{n+1}$$

Convolutions and polynomiality

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$$= \sum_{l \text{ even}}^{n} (2\xi)^l \sum_{k \text{ even}}^{l} A_{n+1-l,l-k}^{(A)}(t) A_{n+1,k}^{(N)}(t)$$
$$+ \mathrm{mod}(n,2)\,(2\xi)^{n+1}\left\{\sum_{k \text{ even}}^{n} C_{n-k+1}^{(A)}(t) A_{n+1,k}^{(N)}(t) + \int \frac{dy}{y} h_{N/A}(y,\xi,t) C_{n+1}^{(N)}(t)\right\}$$

A mircale occurs!

Convolution for generalized form factors

Discrete convolution relations are obeyed for generalized form factors:

$$A_{n+1,l}^{(q/A)}(t) = \sum_{\substack{k=0 \\ \text{even}}}^{l} A_{n+1-l,l-k}^{(N/A)}(t) A_{n+1,k}^{(q/N)}(t) \xrightarrow[n=0]{} F^{(q/A)}(t) = F^{(N/A)}(t) F^{(q/N)}(t)$$

$$C_{n+1}^{(q/A)}(t) = \sum_{\substack{k=0 \\ \text{even}}}^{n} C_{n-k+1}^{(N/A)}(t) A_{n+1,k}^{(q/N)}(t) + \int \frac{dy}{y} h_{N/A}(y,\xi,t) C_{n+1}^{(q/N)}(t)$$

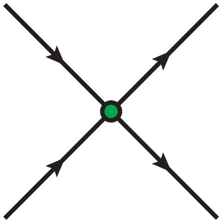
- Strictly, these are matrices.
- $A^{(q/A)}(t)$ and $A^{(q/N)}(t)$ are column matrices.
- $A^{(N/A)}(t)$ is a rectangular matrix.
- Get body form factor equation in $n = 0$ case.
- Unsure the meaning (or convergence) of the $\int \frac{dy}{y} h_{N/A}(y,\xi,t)$ term.

Few-body many-body systems

- Light nuclei are the obvious (easiest) starting point.
- Sergio has given us a **convolution relation!**
- Deuteron should be simplest case.
- Cano and Pire (Eur Phys J A19 (2004) 423) give a theoretical treatment, but their numerical results violate polynomiality.
- This may be due to missing higher Fock components. (They use light cone overlap formalism.)
- A worthwhile investigation may be: a simple, *exactly-solvable* model of the deuteron as two nucleons.
- Keep Lorentz invariance manifest from the start.

Relativistic contact interactions

- The Nambu-Jona-Lasinio (NJL) model has been extremely successful in describing hadron structure.
- Contact interactions give a simple starting point for exact, *Lorentz-invariant* calculations.



- Contact interaction Lagrangian for *nucleon-nucleon* interactions:

$$\mathcal{L} = \sum_I G_I \left(\psi^T C^{-1} \tau_2 \Omega_I \psi\right) \left(\bar{\psi} \Omega_I C \tau_2 \bar{\psi}^T\right)$$

- It is always possible to write the contact interaction Lagrangian in this form, via Fierz rearrangement.

Two-nucleon bilinears

Contact interaction Lagrangian:

$$\mathcal{L} = \sum_I G_I \left(\psi^T C^{-1} \tau_2 \Omega_I \psi\right) \left(\bar{\psi} \Omega_I C \tau_2 \bar{\psi}^T\right)$$

The matrices Ω_I are tensor products of Clifford algebra matrices, isospin matrices, and derivatives.

- Fermion fields are classically Grassmann numbers:
  - $\psi_1 \psi_2 = -\psi_2 \psi_1$
  - $\psi_1^2 = 0$
  - $\psi^T \psi = 0$
  - $\psi^T M \psi = -\psi^T M^T \psi$ for any matrix M.
- All bilinears in our Lagrangian should use antisymmetric matrices.

Two-nucleon bilinears

Contact interaction Lagrangian:

$$\mathcal{L} = \sum_I G_I \left(\psi^T C^{-1} \tau_2 \Omega_I \psi\right) \left(\bar{\psi} \Omega_I C \tau_2 \bar{\psi}^T\right)$$

For simplicity, consider only first-order derivatives.

$$\partial_\mu^\pm = \frac{\overrightarrow{\partial}_\mu \pm \overleftarrow{\partial}_\mu}{2}$$

$$(\partial_\mu^\pm)^T = \pm \partial_\mu^\pm$$

	Symmetric	Antisymmetric
Clifford	$\gamma^\mu C$, $\sigma^{\mu\nu} C$	C, $\gamma^5 C$, $\gamma^5 \gamma^\mu C$
Isospin	$\tau_j \tau_2$	τ_2
Derivative	1, ∂_μ^+	∂_μ^-

- Matrices $\Omega_I C \tau_2$ are made by mixing and matching, to get an overall antisymmetric matrix.
- A total of **21 terms** available for Lagrangian.
- 10 of these terms are isoscalar ($I = 0$). Focus on these (relevant to deuteron).

Isoscalar Lagrangian

Isoscalar contact Lagrangian:

$$\mathcal{L}_{I=0} = \mathcal{L}_0 + \mathcal{L}_k + \mathcal{L}_p$$

No-deritatives terms:

$$\mathcal{L}_0 = G_V \left(\bar{\psi}\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right) + \frac{1}{2}G_T\left(\bar{\psi}\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\sigma_{\mu\nu}\psi\right)$$

Minus-derivative terms:

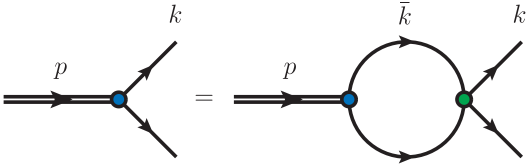
$$\mathcal{L}_k = G_1\left(\bar{\psi}\partial^{-\mu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^-\psi\right) + G_2\left(\bar{\psi}\partial^{-\mu}\gamma_5 C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^-\gamma_5\psi\right)$$
$$+ G_3\left(\bar{\psi}\partial^{-\mu}\gamma_5\gamma^\nu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu\gamma_5\gamma_\nu\psi\right) + G_4\left(\bar{\psi}\gamma_5\not{\partial}^- C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_5\not{\partial}^-\psi\right)$$

Plus-derivative terms:

$$\mathcal{L}_p = G_5\left(\bar{\psi}\partial^{+\mu}\gamma^\nu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^+\gamma_\nu\psi\right) + \frac{1}{2}G_6\left(\bar{\psi}\partial^{+\mu}\sigma^{\nu\pi}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu^+\sigma_{\nu\pi}\psi\right)$$
$$+ G_7\left(\bar{\psi}\not{\partial}^+ C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\not{\partial}^+\psi\right) + G_8\left(\bar{\psi}\partial_\nu^+\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial^{+\pi}\sigma_{\mu\pi}\psi\right)$$

Bethe-Salpeter equation for the deuteron

- Apply our contact interaction to the deuteron.
- Deuteron obeys the Bethe-Salpeter equation:

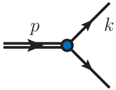


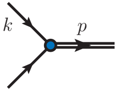
- Derivatives in momentum space:

$$\partial_\mu^+ \mapsto \frac{i}{2} p_\mu$$
$$\partial_\mu^- \mapsto i k_\mu$$

- The contact potential is *separable*, so the deuteron vertex is linear in k.

Deuteron vertex

$\Gamma^\mu_d(p,k) =$ 

$\bar{\Gamma}^\mu_d(p,k) =$ 

Most general deuteron vertex compatible with our Lagrangian:

$$\Gamma^\mu_d(p,k) = \left[\alpha_V\left(\gamma^\mu - \frac{\not{p}p^\mu}{p^2}\right) + i\alpha_T\frac{p_\nu\sigma^{\mu\nu}}{M_d} + \frac{\alpha_E}{M_d}\left(k^\mu - \frac{k\cdot p}{p^2}p^\mu\right) + \alpha_D\left(\frac{\not{p}\gamma^\mu\not{k} - \not{k}\gamma^\mu\not{p}}{2p^2}\right)\right]C\tau_2$$

Several Lagrangian terms either vanish or become redundant in BSE.

$$\mathcal{L}_{\text{effective}} = G_V\left(\bar{\psi}\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right) + \frac{1}{2}G_T\left(\bar{\psi}\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\sigma_{\mu\nu}\psi\right)$$
$$+ G_E\left(\bar{\psi}\partial^{-\mu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial^-_\mu\psi\right) + G_D\left(\bar{\psi}\partial^{-\mu}\gamma_5\gamma^\nu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu\gamma_5\gamma_\nu\psi\right)$$

Now only four terms!

Matrix form of the BSE

- The BSE can be thought of as a matrix equation.

$$\Gamma_d(p,k) = \mathcal{M}_{\mathrm{BSE}}\Gamma_d(p,k)$$

$$\begin{bmatrix} \alpha_V \\ \alpha_T \\ \alpha_E \\ \alpha_D \end{bmatrix} = 4 \begin{bmatrix} G_V\Pi_{VV} & G_V\Pi_{VT} & G_V\Pi_{VE} & G_V\Pi_{VD} \\ G_T\Pi_{TV} & G_T\Pi_{TT} & G_T\Pi_{TE} & G_T\Pi_{TD} \\ G_E\Pi_{EV} & G_E\Pi_{ET} & G_E\Pi_{EE} & G_E\Pi_{ED} \\ G_D\Pi_{DV} & G_D\Pi_{DT} & G_D\Pi_{DE} & G_D\Pi_{DD} \end{bmatrix} \begin{bmatrix} \alpha_V \\ \alpha_T \\ \alpha_E \\ \alpha_D \end{bmatrix}$$

- The interactions mix up components of the vertex.
- The bubble diagrams Π_{VV} *etc.* contain all the difficulties (UV divergences, *etc.*).
- Once the bubbles are known, solving the BSE is simply linear algebra.
- Theory is non-renormalizable, so cutoff is an additional parameter.

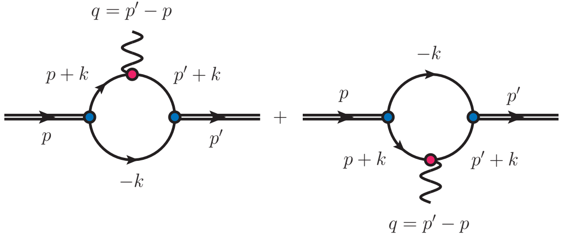
Deuteron form factors

- To determine the G's (or α's), empirical input is needed.
- Electromagnetic properties of the deuteron are well-known.
- Deuteron current decomposes into three Lorentz-invariant form factors:

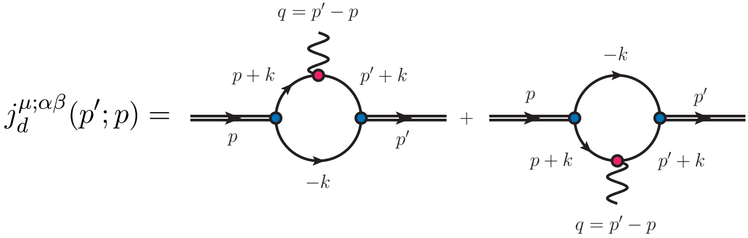
$$j_d^{\mu;\alpha\beta}(p';p) = (p+p')^\mu g^{\alpha\beta} F_{1d}(Q^2) - (q^\alpha g^{\beta\mu} - q^\beta g^{\alpha\mu}) F_{2d}(Q^2) - (p+p')^\mu \frac{q^\alpha q^\beta}{2M_d^2} F_{3d}(Q^2)$$

- This can be calculated in the covariant contact model:

$$j_d^{\mu;\alpha\beta}(p';p) =$$



Deuteron form factors



Using nucleon form factors for the photon-nucleon coupling, we get a matrix equation:

$$\begin{bmatrix} F_{1d}(Q^2) \\ F_{2d}(Q^2) \\ F_{3d}(Q^2) \end{bmatrix} = \begin{bmatrix} F_{1V}(Q^2) & F_{1T}(Q^2) \\ F_{2V}(Q^2) & F_{2T}(Q^2) \\ F_{3V}(Q^2) & F_{3T}(Q^2) \end{bmatrix} \begin{bmatrix} F_{1p}(Q^2) + F_{1n}(Q^2) \\ F_{2p}(Q^2) + F_{2n}(Q^2) \end{bmatrix}$$

F_{1V}, F_{1T}, *etc.* are the body form factors.

Sachs-like form factors

Sachs-like form factors are closer to empirical observation.

$$G_Q(Q^2) = F_{1d}(Q^2) - F_{2d}(Q^2) + (1+\eta)F_{3d}(Q^2)$$
$$G_M(Q^2) = F_{2d}(Q^2)$$
$$G_C(Q^2) = F_{1d}(Q^2) - \frac{2}{3}\eta G_Q(Q^2)$$

where $\eta = \frac{Q^2}{4M_d^2}$.

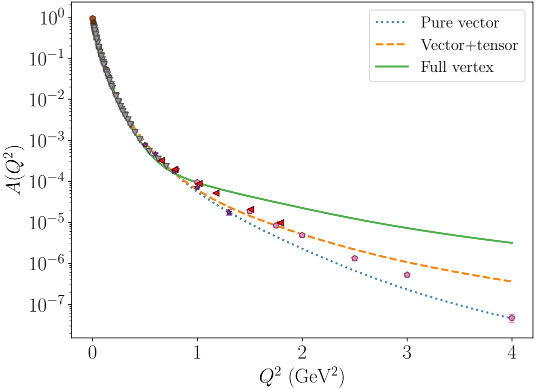
$$\langle r_E \rangle_{\text{rms}} = \sqrt{-6\frac{\partial G_C(Q^2=0)}{\partial Q^2}}$$
$$\mu_d = \frac{m_N}{M_d} G_M(Q^2=0)$$
$$\mathcal{Q} = \frac{1}{M_d^2} G_Q(Q^2=0)$$

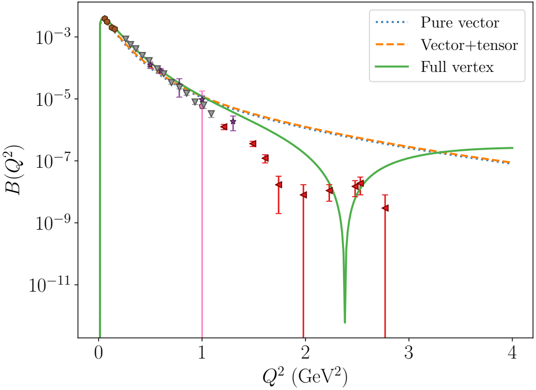
They are related to electromagnetic structure functions:

$$A(Q^2) = G_C^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2)$$
$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2(Q^2).$$

Full contact model

- Attempt fit to data up to $Q^2 = 1$ GeV^2.
- Fit fails when higher-Q^2 data are used: necessity of long-range pion exchange?

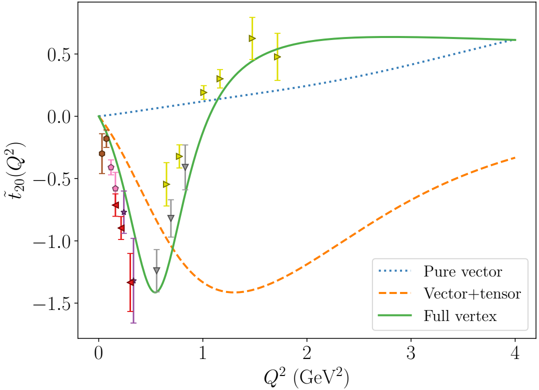




Contact model

	Model	Empirical
$r_{\rm rms}$ (fm)	2.15	2.1413(25)
μ_d	0.91	0.8574382311(48)
$\mathcal{Q}_d$ (fm^2)	0.122	0.2859(3)

- Contact model is imperfect.
- The static quadrupole moment is off by a factor of 2.
- Otherwise, quite good description for a contact model.



Long-range pion exchange is likely necessary for a perfect description.

Contact model

- For now, proceed with relativistic contact model.
- The UV cutoff is close to the pion mass, suggesting a breakdown of the contact model when pion exchange becomes relevant.
- The close values of the α's suggests a finely-tuned cancellation between attractive and repulsive forces.

G_V	$-(6.14\text{ fm})^2$
G_T	$(6.28\text{ fm})^2$
G_E	$(3.60\text{ fm})^4$
G_D	$-(2.63\text{ fm})^4$
Λ	142 MeV
α_V	46
α_T	-48
α_E	-45
α_D	18

$$\mathcal{L}_{\text{effective}} = G_V\left(\bar{\psi}\gamma^\mu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\gamma_\mu\psi\right) + \frac{1}{2}G_T\left(\bar{\psi}\sigma^{\mu\nu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\sigma_{\mu\nu}\psi\right)$$
$$+ G_E\left(\bar{\psi}\partial^{-\mu}C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial^-_\mu\psi\right) + G_D\left(\bar{\psi}\partial^{-\mu}\gamma_5\gamma^\nu C\tau_2\bar{\psi}^T\right)\left(\psi^T C^{-1}\tau_2\partial_\mu\gamma_5\gamma_\nu\psi\right)$$

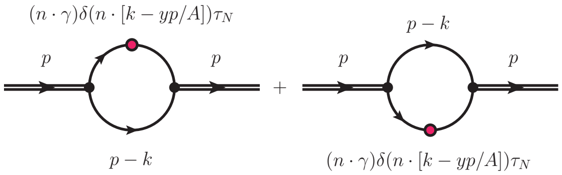
Deuteron PDFs

- Deuteron PDFs related to nucleon PDFs by convolution formula:

$$q_{i/d}^{(\lambda)}(x_A, Q^2) = \sum_{N=p,n} \int_{x_A}^{2} \frac{dy}{y} q_{i/N}\left(\frac{x_A}{y}, Q^2\right) f_{N/d}^{(\lambda)}(y)$$

- The deuteron light cone density (LCD) $f_{N/d}^{(\lambda)}(y)$ can be found by Feynman rules:

$f_{N/d}^{(\lambda)}(y) =$



Deuteron LCD

- We find exact expressions for the LCD.
- For example, the "pure vector" (α_V-only) part:

$$f_d^{(\mathrm{unpol})}(y) = \alpha_V^2 \frac{1}{32\pi^2} \int d\tau e^{-\Delta(y)\tau} \left(\frac{4}{3\tau} + m_N^2 y(2-y) + \frac{M_d^2}{12} y(2-y) \left[4 - y(2-y)\right] \right)$$

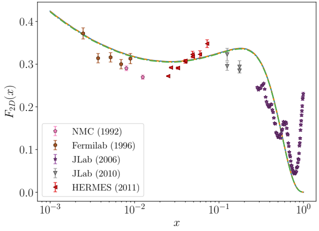
$$f_d^{(\mathrm{tensor})}(y) = -\alpha_V^2 \frac{1}{32\pi^2} \int d\tau e^{-\Delta(y)\tau} \left(\frac{2-3y(2-y)}{\tau} - \frac{M_d^2}{2} y(2-y)(y-1)^2 \right)$$

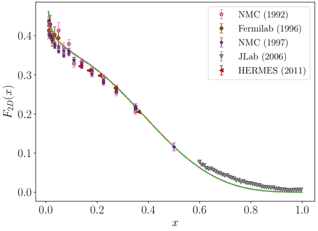
$$\Delta(y) = m_N^2 - \frac{M_d^2}{4} y(2-y)$$

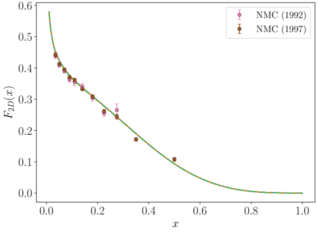
- Full expressions available in upcoming paper.

Structure function calculations

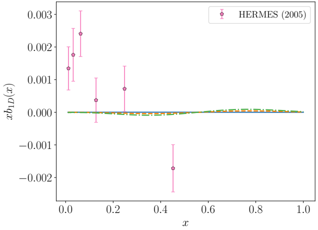
We get a good agreement with DIS data for the deuteron.





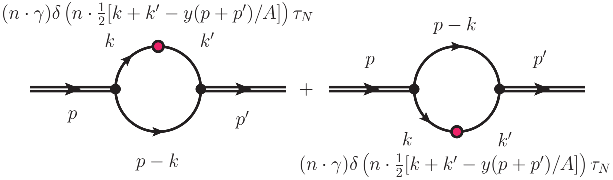


...well, we can't describe the HERMES b_1 data, but this is no surprise.



Deuteron GPDs

- For deuteron GPD, we follow a similar procedure as for the PDF.



- Unfortunately, this is currently a work in progress.
- Currently only have Mellin moments, and in too unwieldy a form for presentation here, but...
- **The deuteron "body" GPDs obey polynomiality.**
- More specifically, the correct body GPDs are either even or odd in ξ, and have the correct highest power.

Outlook

- Deuteron body GPDs in a relativistic contact model are on the horizon.
- These GPDs **will** satisfy polynomiality.
- These GPDs are also ρ GPDs in the NJL model (with some constants changed).
- The contact formalism will be applied to ^{3}He and ^{4}He next.
- Lorentz-invariant inclusion of pion exchange would be ideal, though our primary focus is the convolution formalism.
- Contact formalism should work better for helium and ρ, since these are not loosely bound.
- ^{4}He is higher priority, since data is being collected for it now. (See Mohammed's talk.)
- **Very near term**: will derive polynomiality proofs for GPDs that are odd in ξ too.

The End

Thank you for listening!