Minkowski space approach to hadrons: the pion example

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Motivation

Our physical space-time = Minkowski space

LQCD Euclidean space-time: $t \rightarrow -i\,\tau$

Continuous Bethe-Salpeter/ Dysons-Schwinger for QCD:

Wick-rotation $k^0 \rightarrow -ik^0_E$

Euclidean & Wick-Rotation ~ $k\,.\,x \rightarrow -ik^0\,\tau = -ik^0_E\,t$

Observables: spectrum and space-like momentum region

$q^2 < 0$

Problems:

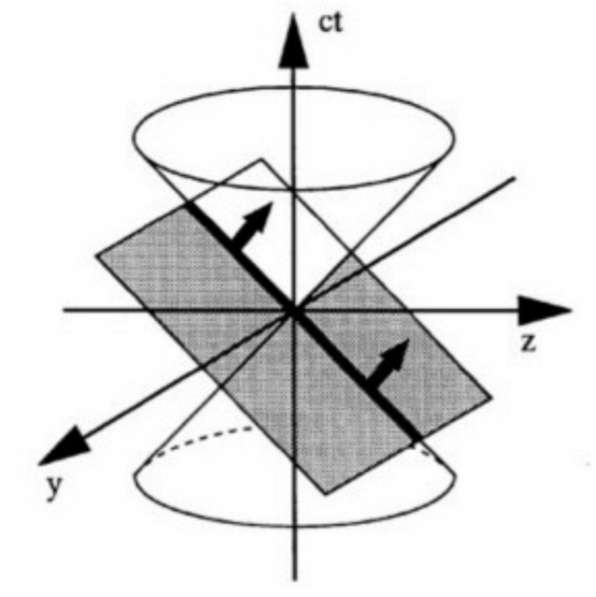
Observables associated with the hadron structure in Minkowski:

- parton distributions (pdfs)
- generalized parton distributions
- transverse momentum distributions (TMDs)

Lattice QCD
first moments; Feynman-Hellman (talk by R. Young);
LQCD quasi-pdfs (talk by M. Constantinou)
[X. Ji PRL 110 (2013) 262002
see also critique Rossi&Testa PRD 96 (2017) 014507]

- **time-like form factors (**$q^2>0$**)**

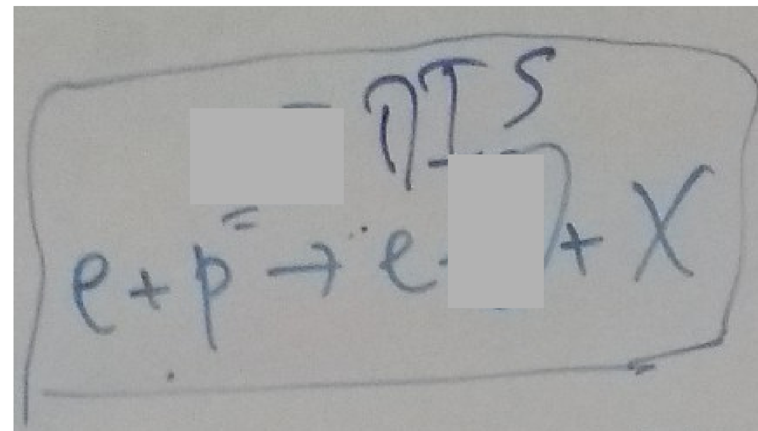
Valence Light-Front WF (LFWF)

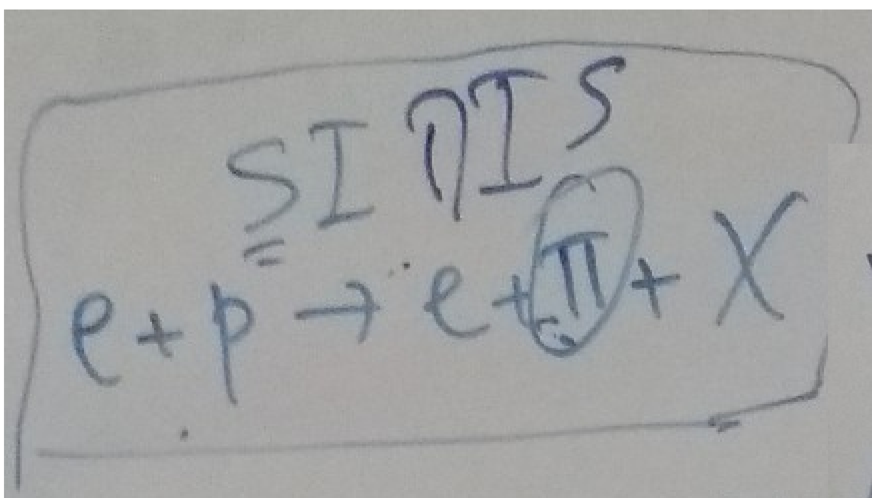


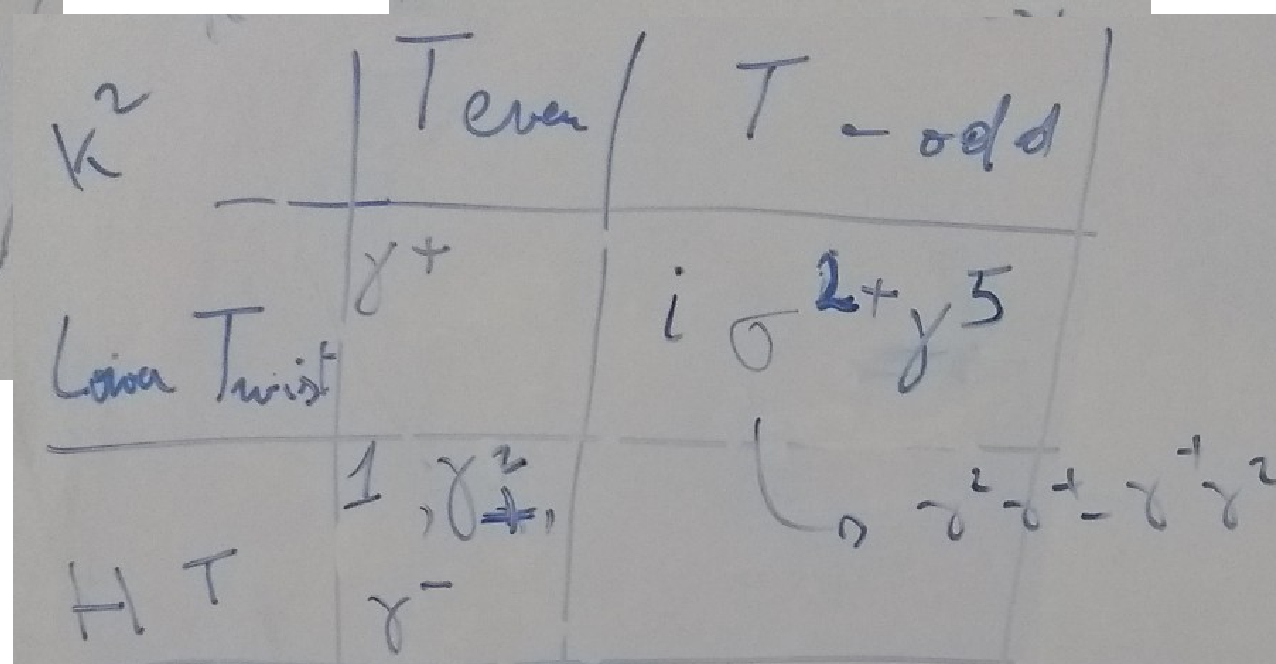
- basic ingredient in PDFs, GPDs and TMDs
- not directly obtained by Euclidean approaches.
- **Nakanishi Integral Representation (NIR)** of the Euclidean BS amplitude [C. D. Roberts, I. Cloet and collaborators]
- **Inversion NIR:** Carbonell, TF, Karmanov EPJC C77 (2017) 58

NUCLEON TOMOGRAPHY (Map 3D structure)

- DIS (deeply virtual inclusive electron scattering) **(PDFs)**
- DVCS **(GPD's)**
- Semi Inclusive DIS…

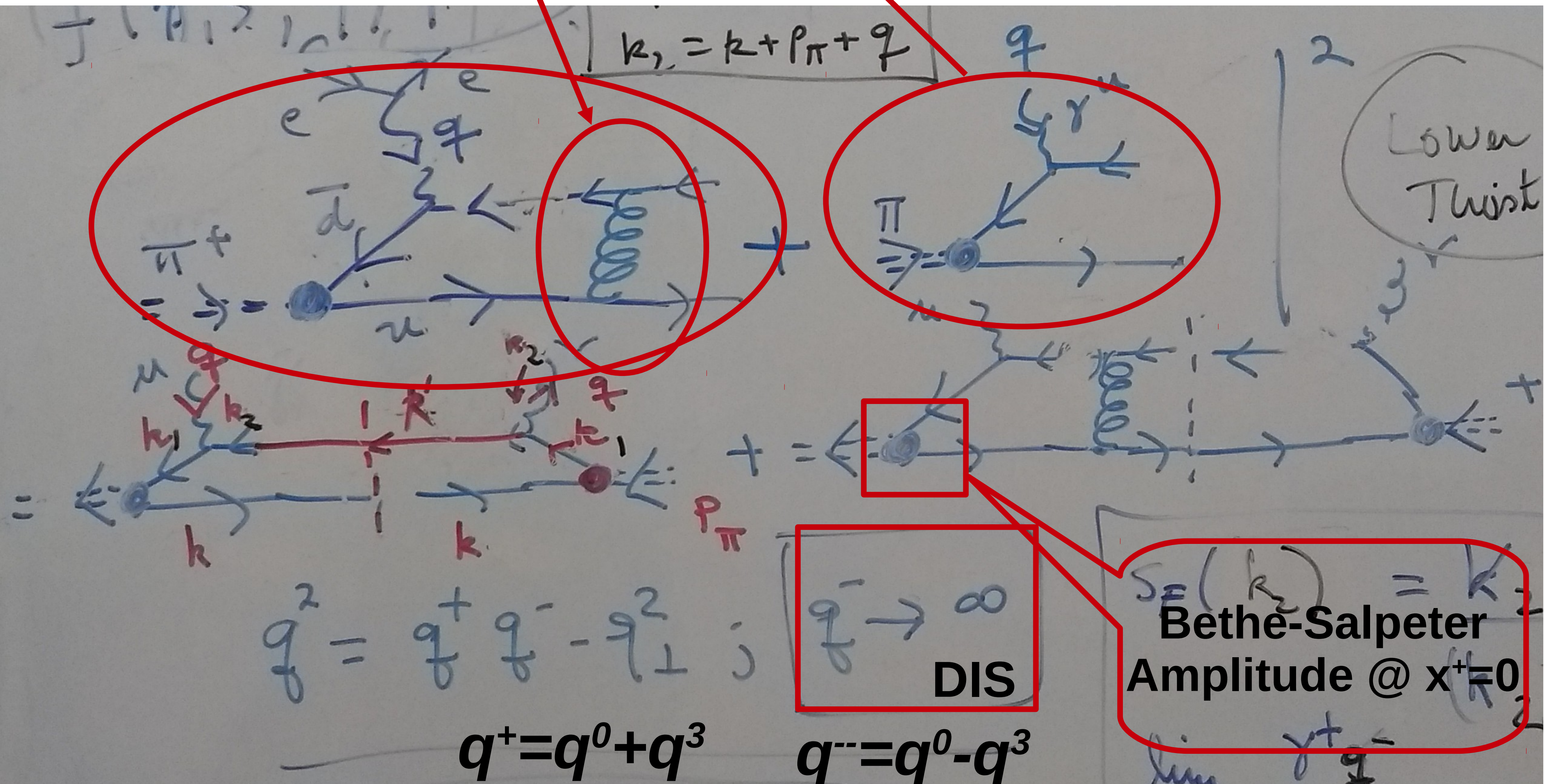






TMDs & PDFs

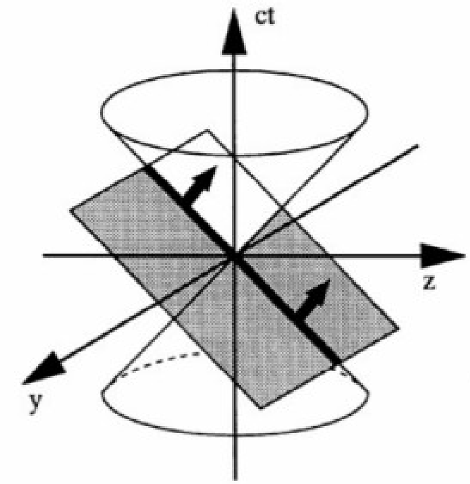
FSI gluon exchange: T-odd



Light-Front Time Evolution

$$\tilde{\Phi}(x, p) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\Phi(k, p)$$

$$p^\mu = p_1^\mu + p_2^\mu \qquad k^\mu = \frac{p_1^\mu - p_2^\mu}{2}$$



$$\tilde{\Phi}(x, p) = \langle 0|T\{\varphi_H(x^\mu/2)\varphi_H(-x^\mu/2)\}|p\rangle$$

$$= \theta(x^+)\langle 0|\varphi(\tilde{x}/2)e^{-iP^-x^+/2}\varphi(-\tilde{x}/2)|p\rangle e^{ip^-x^+/4} + \cdots$$

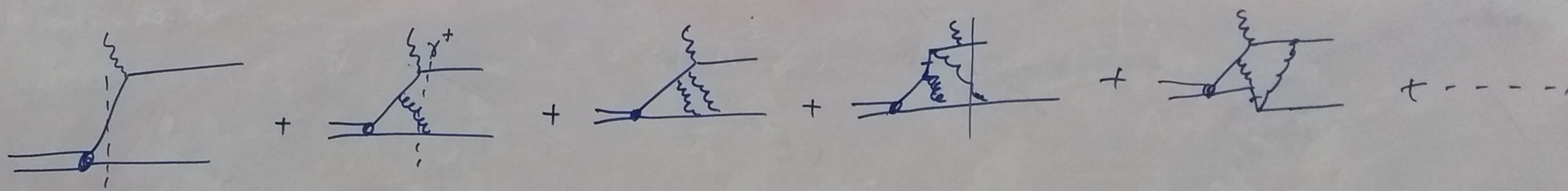
$$= \theta(x^+)\sum_{n,n'} e^{ip^-x^+/4}\langle 0|\varphi(\tilde{x}/2)|n'\rangle\langle n'|e^{-iP^-x^+/2}|n\rangle\langle n|\varphi(-\tilde{x}/2)|p\rangle + \cdots$$

$x^+ = 0$ only valence state remains! How to rebuilt the full BS amplitude?

Iterated Resolvents: Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998)

Yes! Sales, et al. PRC61, 044003 (2000)

Beyond the valence ….



• what is the overlap of the covariant calculation with evolution? GREAT !

Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space" (Nakanishi 1962)

Bethe-Salpeter amplitude

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

BSE in Minkowski space with NIR for bosons

Kusaka and Williams, PRD 51 (1995) 7026

LF projection of the homogeneous BSE: two-boson system

$$\Phi(k,p) = G_0(k,p) \int d^4k' \, \mathcal{K}_{BS}(k,k',p) \, \Phi(k',p)$$

Carbonell, TF, Karmanov, PLB769 (2017) 418

$\Rightarrow$

Stieljes transform

$$\int_0^\infty d\gamma' \frac{g_b(\gamma', z; \kappa^2)}{[\gamma' + \gamma + z^2 m^2 + (1-z^2)\kappa^2 - i\epsilon]^2} =$$

$$= \int_0^\infty d\gamma' \int_{-1}^{1} dz' \; V_b^{LF}(\gamma, z; \gamma', z') g_b(\gamma', z'; \kappa^2).$$

with $V_b^{LF}(\gamma, z; \gamma', z')$ determined by the irreducible kernel $\mathcal{I}(k,k',p)$!

Ladder approx. by Carbonell and Karmanov within the explicitly-covariant LF framework (EPJA 27 (2006) 1 (also x-ladder in EPJA 27 (2006) 11). FSV PRD 89 (2014) 016010, non explicitly covariant version.

Very good agreement for both eigenvalues (the coupling constants at given binding energies) and LF distributions.

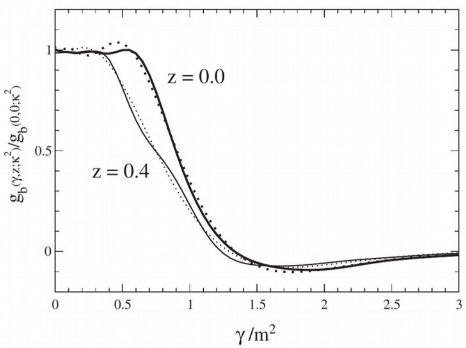
Wide phenomenology: (i) Scattering lengths in FVS EPJC 75 (2015) 398, (ii) spectra of excited states and LF momentum distributions in Gutierrez et al PLB 759 (2016) 131.

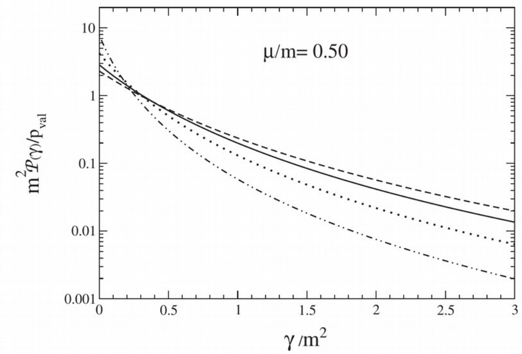
Two-Boson System: ground-state

Building a solvable model…

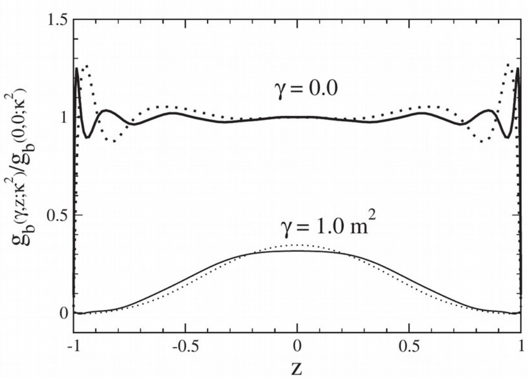
Nakanishi weight function | Valence wave function

3+1 n=1 | LADDER KERNEL | 3+1 n=1





$\mu = 0.5 \;\; B/M = 1$



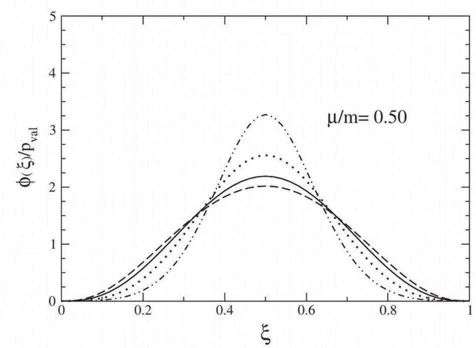


FIG. 3. The longitudinal LF distribution $\phi(\xi)$ for the valence component Eq. (34) vs the longitudinal-momentum fraction ξ for $\mu/m = 0.05,\ 0.15,\ 0.50$. Dash-double-dotted line: $B/m = 0.20$. Dotted line: $B/m = 0.50$. Solid line: $B/m = 1.0$. Dashed line: $B/m = 2.0$. Recall that $\int_0^1 d\xi \phi(\xi) = P_{val}$ (cf. Table III).

Karmanov, Carbonell, EPJA 27, 1 (2006)
Frederico, Salmè, Viviani PRD89, 016010 (2014)

Two-Boson System: Spectrum and BSE

134 C. Gutierrez et al. / Physics Letters B 759 (2016) 131–137

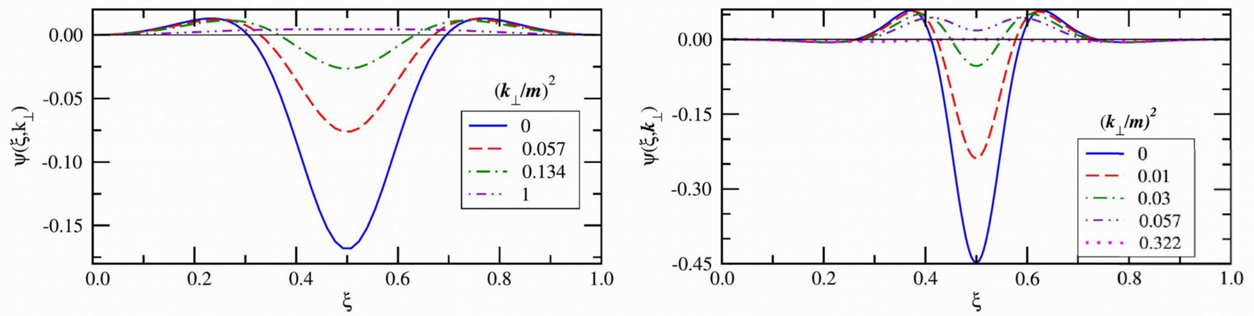


Fig. 2. The valence wave functions vs ξ with fixed values of $(k_\perp/m)^2$, for the first (left panel) and second (right panel) excited states, with $B(1)/m = 0.22$ and $B(2)/m = 0.05$, respectively, obtained from (10) with $\mu/m = 0.1$ and $\alpha = 6.437$.

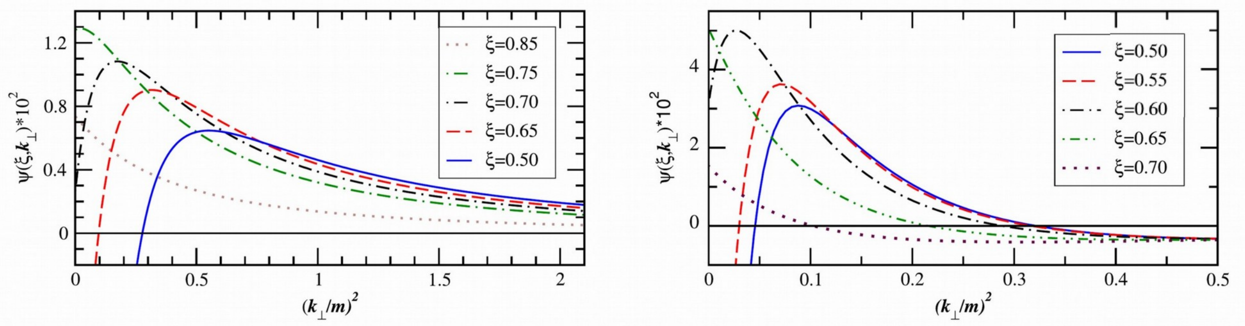


Fig. 3. The valence wave functions vs $(k_\perp/m)^2$ with fixed values of ξ, for the first (left panel) and second (right panel) excited states, with $B(1)/m = 0.22$ and $B(2)/m = 0.05$, respectively, obtained from (10) with $\mu/m = 0.1$ and $\alpha = 6.437$.

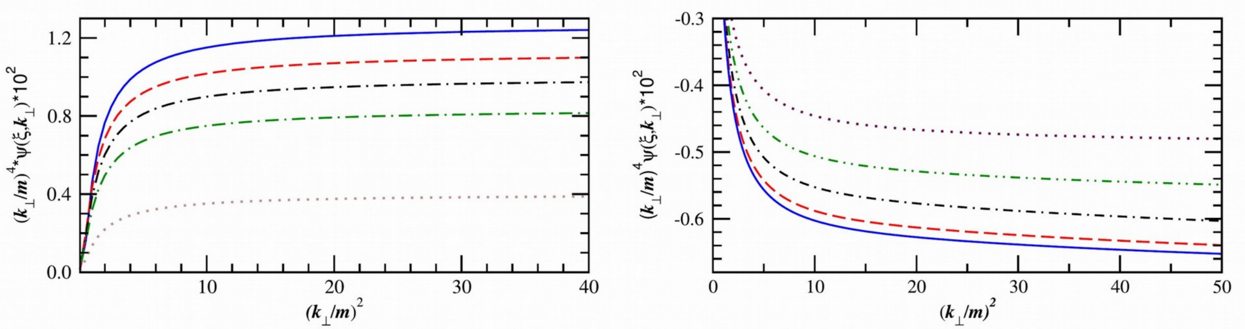


Fig. 4. The asymptotic $k_\perp$ behaviors of the first (left frame) and second (right frame) excited states are shown, using the same label convention as given in Fig. 3.

Gutierrez, Gigante, TF, Salmè,Viviani, Tomio PLB759 (2016) 131

Transverse distribution: Euclidean and Minkowski

$$\phi_M^T(\mathbf{k}_\perp) \equiv \int dk^0 dk^3 \Phi(k,p) = \frac{1}{2}\int dk^+ dk^- \Phi(k,p) \text{ and}$$

$$\phi_E^T(\mathbf{k}_\perp) \equiv \mathrm{i} \int dk_E^0 dk^3 \Phi_E(k_E,p),$$

136 C. Gutierrez et al. / Physics Letters B 759 (2016) 131–137

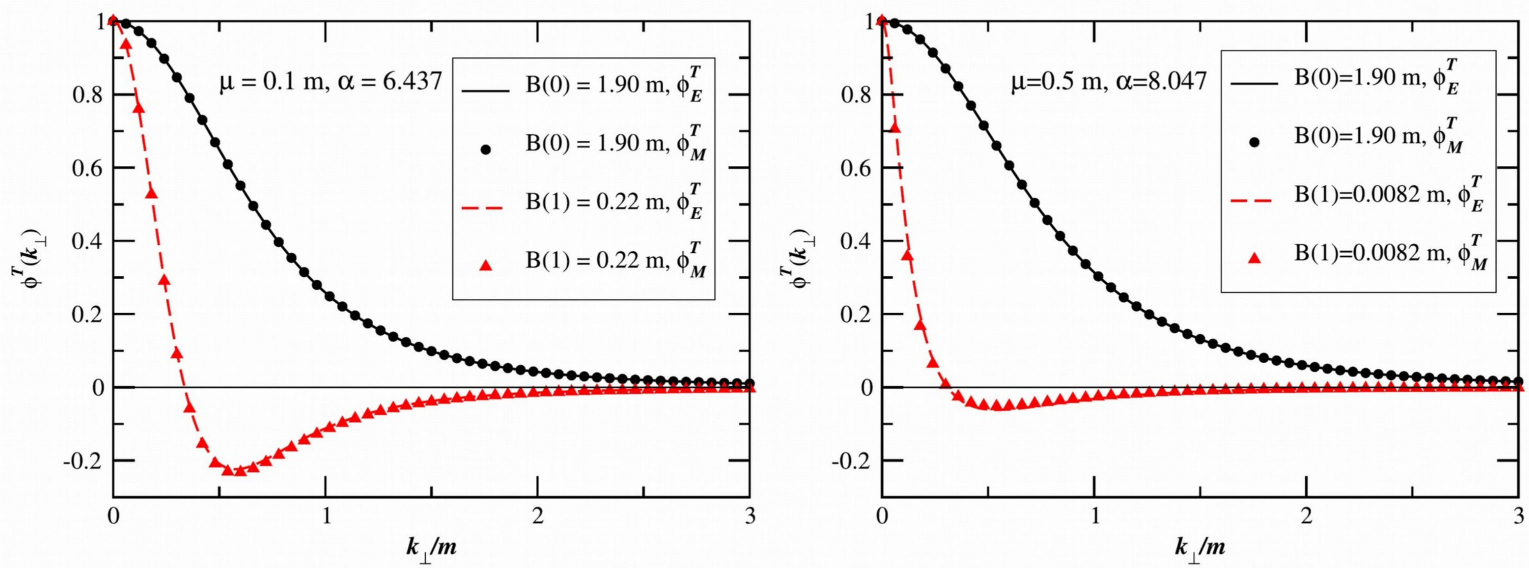


Fig. 6. Transverse momentum amplitudes s-wave states, in Euclidean and Minkowski spaces, vs $k_\perp$, for both ground- and first-excited states, and two values of μ/m and α_{gr} (as indicated in the insets). The amplitudes ϕ_E^T and ϕ_M^T, arbitrarily normalized to 1 at the origin, are not easily distinguishable.

(II) Valence LF wave function in impact parameter space

$$F(\xi, b)|_{b\to\infty} \to e^{-b\sqrt{\kappa^2+(\xi-1/2)^2 M^2}} f(\xi, b)$$

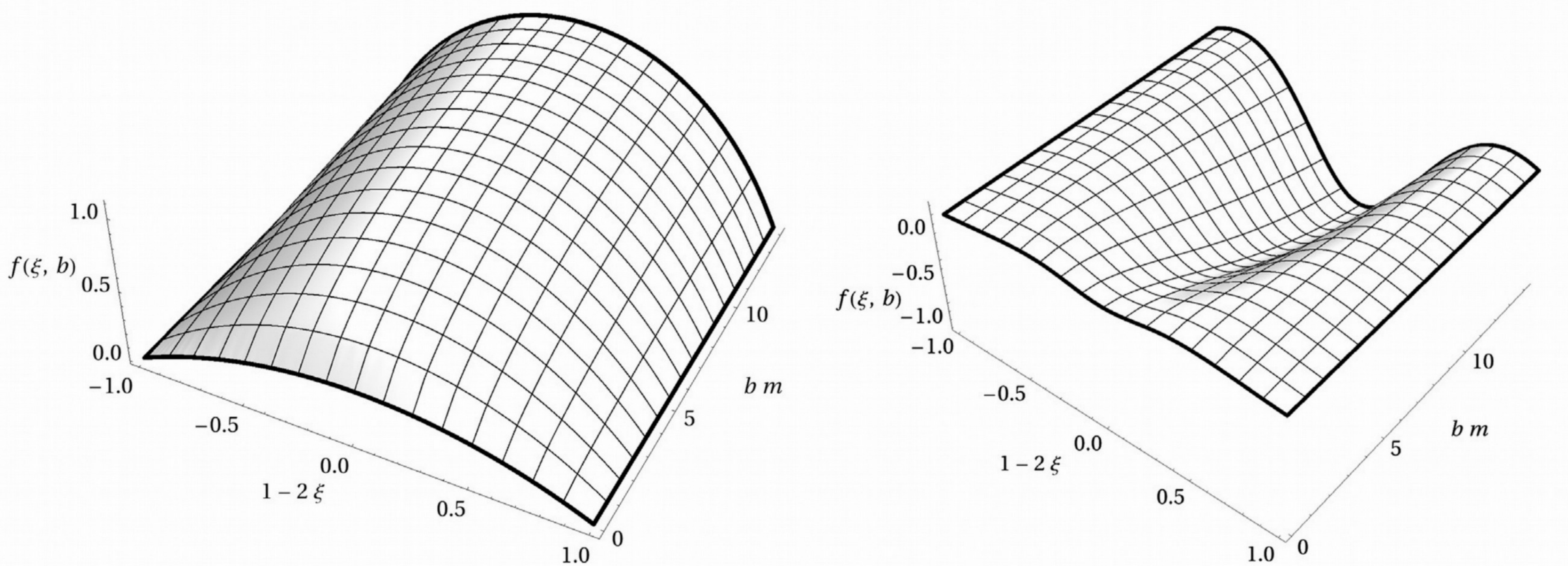


Fig. 7. The valence functions $f(\xi, b)$ in the impact parameter space. Left panel: the ground state, corresponding to $B(0) = 1.9m$, $\mu = 0.1m$ and $\alpha_{gr} = 6.437$. Right panel: first-excited state, corresponding to $B(1) = 0.22m$, $\mu = 0.1m$ and $\alpha_{gr} = 6.437$.

Light-front valence wave function: L+ XL

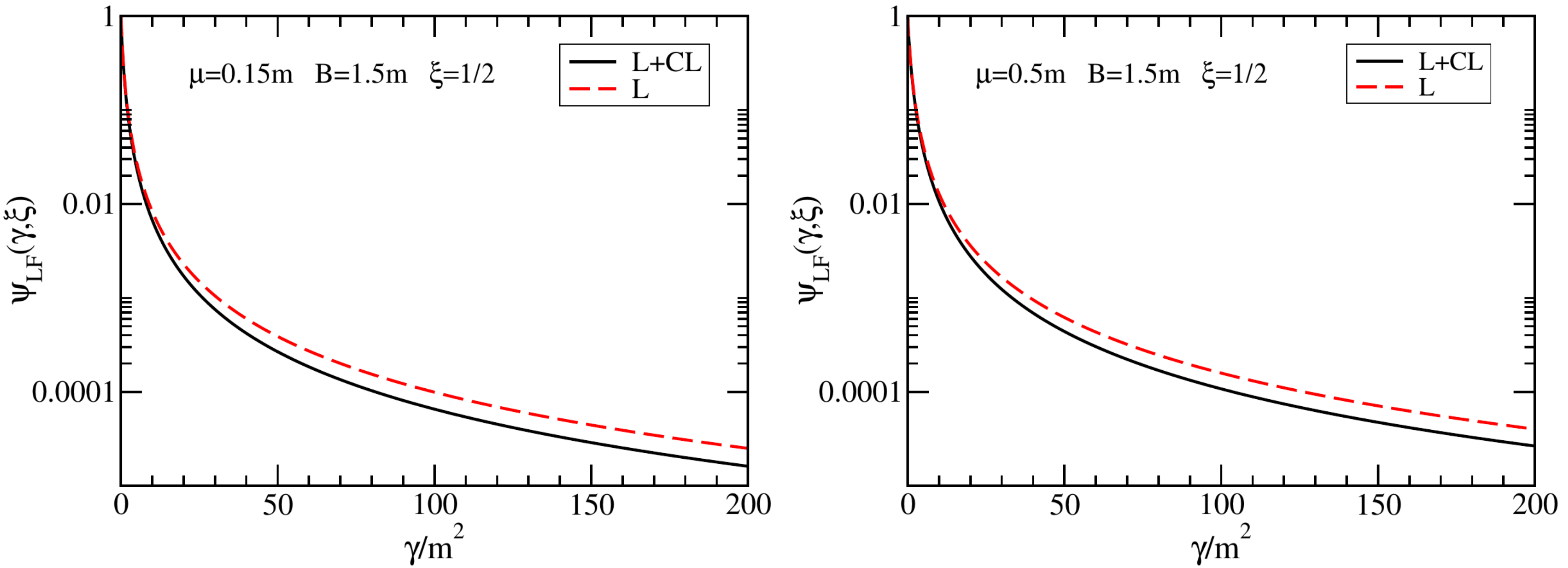


Fig. 1. LF wave function vs. γ for $\xi = 1/2$ with ladder (L) (dashed lines) and ladder plus cross-ladder (L+CL) (solid lines) interaction kernels for $B = 1.5\,m$ and $\mu = 0.15\,m$ (left-frame) and $\mu = 0.5\,m$ (right-frame).

$$\psi_{LF}(\gamma,\xi) \to \alpha\ \gamma^{-2}\, C(\xi) \qquad \gamma=(k_T)^2$$

Gigante, Nogueira, Ydrefors, Gutierrez, Karmanov, TF, PRD95(2017)056012.

$$\psi_{LF}(\gamma,\xi) \to \alpha\, \gamma^{-2}\, C(\xi)$$

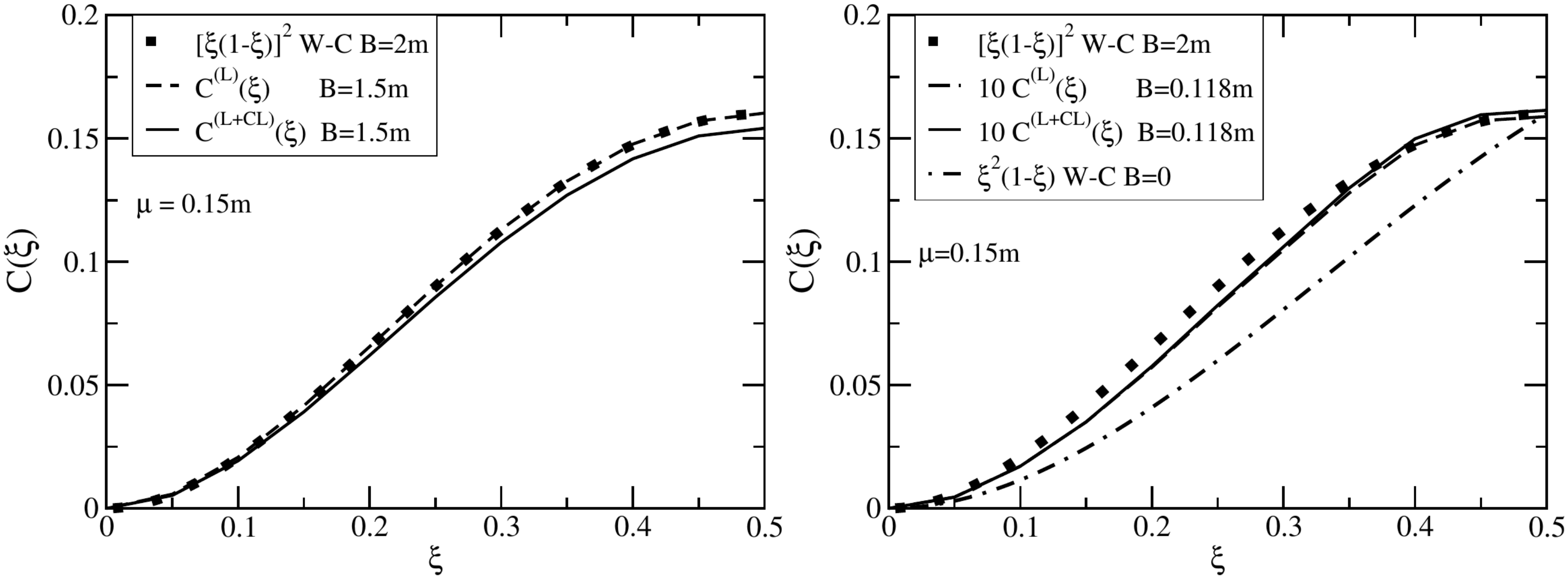
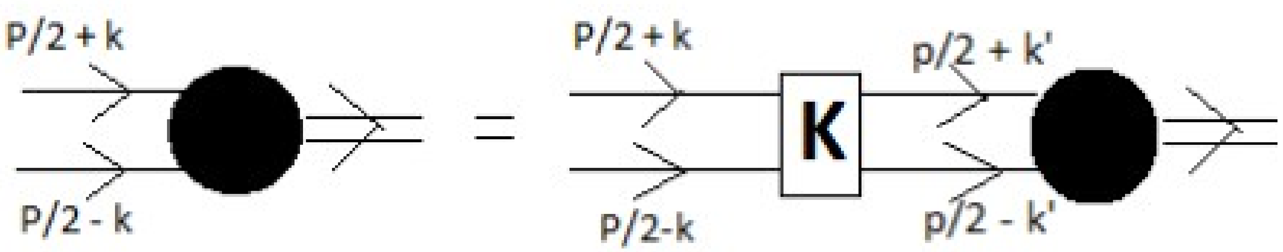


Fig. 2. Asymptotic function $C(\xi)$ defined from the LF wave function for $\gamma \to \infty$ (6) computed for the ladder kernel, $C^{(L)}(\xi)$ (dashed line), and ladder plus cross-ladder kernel, $C^{(L+CL)}(\xi)$ (solid line), with exchanged boson mass of $\mu = 0.15\, m$. Calculations are performed for $B = 1.5\, m$ (left frame) and $B = 0.118\, m$ (right frame). A comparison with the analytical forms of $C(\xi)$ valid for the Wick-Cutkosky model for $B = 2m$ (full box) and $B \to 0$ (dash-dotted line) both arbitrarily normalized.

BSE for qqbar: mesons

Carbonell and Karmanov EPJA 46 (2010) 387;
de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901



$$\Phi(k,p) = S(k+p/2) \int \frac{d^4k'}{(2\pi)^4} F^2(k-k')\, i\mathcal{K}(k,k')\, \Gamma_1\, \Phi(k',p)\, \bar{\Gamma}_2\, S(k-p/2)$$

Ladder approximation (L): suppression of XL for Large Nc [Alvarenga Nogueira, TF, CR Ji in preparation]

Scalar

$$i\mathcal{K}_S^{(Ld)}(k,k') = -ig^2 \frac{1}{(k-k')^2 - \mu^2 + i\epsilon}$$

Pseudo-scalar

$$i\mathcal{K}_{PS}^{(Ld)}(k,k') = ig^2 \frac{1}{(k-k')^2 - \mu^2 + i\epsilon}$$

Vector

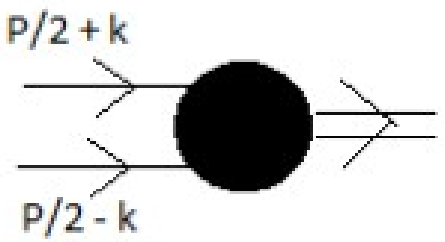
$$i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}$$

Vertex Form-Factor

$$F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$$

NIR for fermion-antifermion: 0⁻ (pion)

BS amplitude



$$\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu\gamma_5$$

Multiplying BSE by S_i and taking the trace

$$\phi_i(k,p) = \frac{i}{\left((p/2+k)^2 - m^2 + i\epsilon\right)}\frac{i}{\left((p/2-k)^2 - m^2 + i\epsilon\right)}$$

$$\times \int \frac{d^4k'}{(2\pi)^4}\frac{(-ig)^2F^2(k-k')}{(k-k')^2-\mu^2+i\epsilon}\sum_{j=1}^{4} c_{ij}(k,k',p)\phi_j(k',p)$$

Light-Front projection

Using the NIR

$$\phi_i(k,p) = \int_{-1}^{+1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{(k^2 + p\cdot k\ z' + M^2/4 - m^2 - \gamma' + i\epsilon)^3}$$

System of coupled integral equations

$$\int_{-1}^{1} dz' \int_0^\infty d\gamma' \frac{g_i(\gamma',z')}{[k^2 + z'p\cdot k - \gamma' - \kappa^2 + i\epsilon]^3} = \sum_j \int_{-1}^{1} dz' \int_0^\infty d\gamma'\ \mathcal{K}_{ij}(k,p;\gamma',z')\ g_j(\gamma',z')$$

We project onto the null plane

$$\psi_i(\gamma,\xi) = \int \frac{dk^-}{2\pi}\ \phi_i(k,p) = -\frac{i}{M}\int_0^\infty d\gamma' \frac{g_i(\gamma',z)}{[\gamma + \gamma' + m^2 z^2 + (1-z^2)\kappa^2]^2}$$

$$\mathcal{L}_{ij}(\gamma,z;\gamma',z') = iM \int \frac{dk^-}{2\pi} \mathcal{K}_{ij}(k,p;\gamma',z')\big|_{k_\perp=\sqrt{\gamma},\, k^+=-\frac{M}{2}z}$$

$$\gamma = k_\perp^2\ ;\quad \kappa^2 = m^2 - M^2/4\ ;\quad M = 2m - B$$

B is the Binding energy

LF singularities

For two-fermion BSE

$$\mathcal{C}_j = \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} (k^-)^j \; \mathcal{S}(k^-, v, z, z', \gamma, \gamma')$$

$$\mathcal{S}(k^-, v, z, z', \gamma, \gamma') \sim \frac{1}{[k^-]^2} \quad \text{for} \quad k^- \to \infty$$

with j=1,2,3 and in the worst case

Then one can not close the arc at the infinity .

The singularities (power j) does not depend on the NIR

We obtain the singular contribution using

$$\int_{-\infty}^{\infty} dx \; \frac{1}{\left[\beta\, x - y \mp i\epsilon\right]^2} = \; \pm\, (2\pi) i \; \frac{\delta(\beta)}{\left[-y \mp i\epsilon\right]}$$

Yan PRD 7 (1973) 1780

Differently, Carbonell and Karmanov introduced a smoothing function to perform the integration (EPJA 46, 387 (2010)).

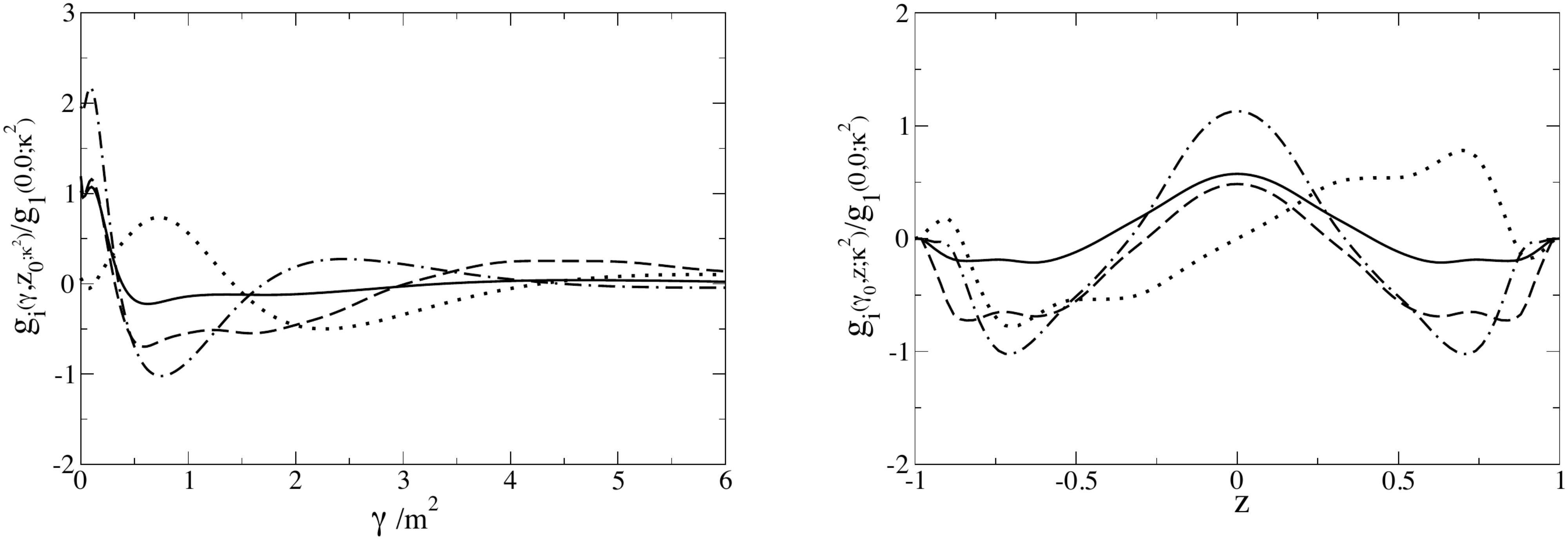
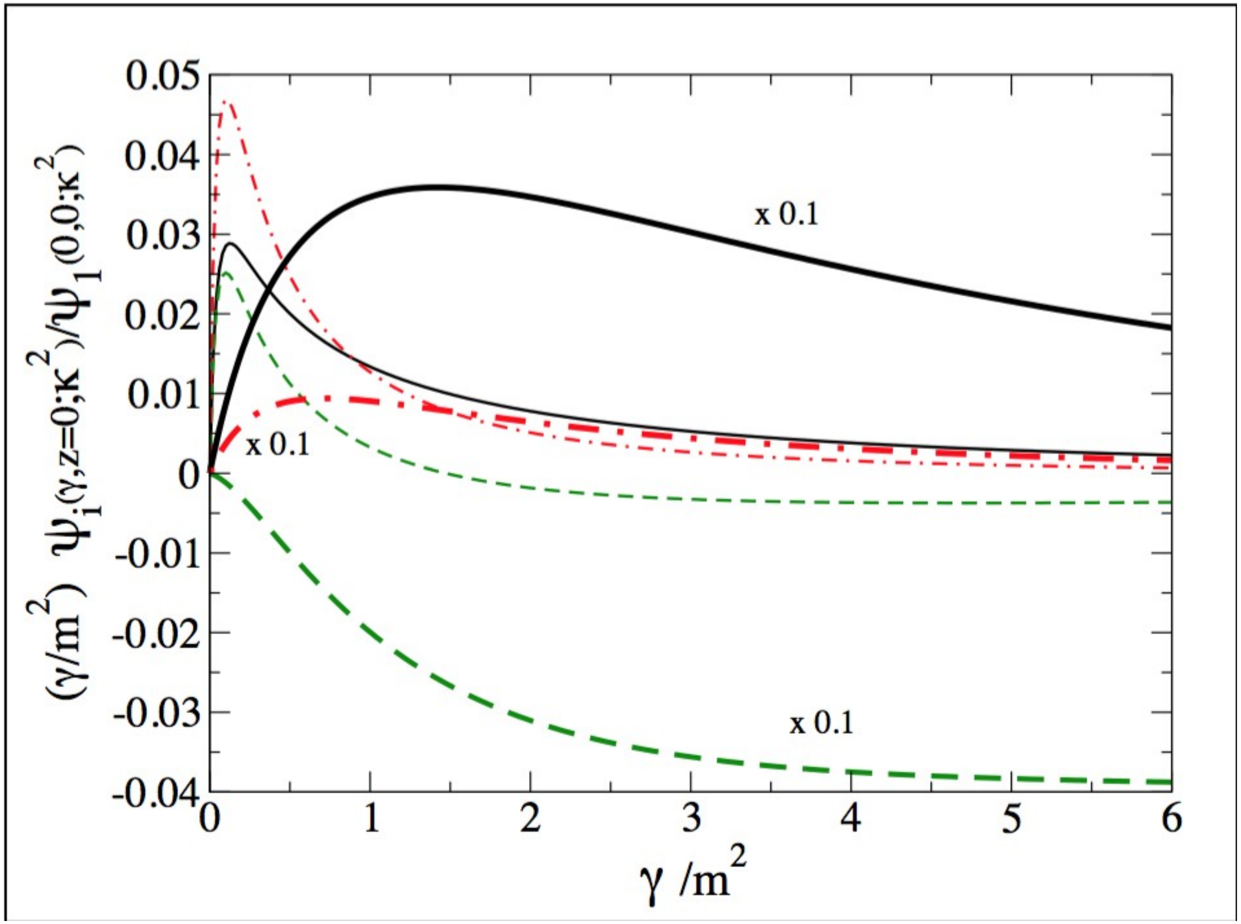


Figure 2. Nakanishi weight-functions $g_i(\gamma,\, z; \kappa^2)$, Eqs. 3.1 and 3.2 evaluated for the 0^+ two-fermion system with a scalar boson exchange such that $\mu/m = 0.5$ and $B/m = 0.1$ (the corresponding coupling is $g^2 = 52.817$ [17]). The vertex form-factor cutoff is $\Lambda/m = 2$. Left panel: $g_i(\gamma,\, z_0; \kappa^2)$ with $z_0 = 0.6$ and running γ/m^2. Right panel: $g_i(\gamma_0,\, z; \kappa^2)$ with $\gamma_0/m^2 = 0.54$ and running z, The Nakanishi weight-functions are normalized with respect to $g_1(0, 0; \kappa^2)$. Solid line: g_1. Dashed line: g_2. Dotted line: g_3. Dot-dashed line: g_4.

Massless vector exchange: high-momentum tails



LF amplitudes ψ_i times γ/m^2 at fixed $z = 0$, for the vector coupling.
$B/m = 0.1$ (thin lines) and 1.0 (thick lines).

—— : $(\gamma/m^2)\ \psi_1$.
— —: $(\gamma/m^2)\ \psi_2$.
— • : $(\gamma/m^2)\ \psi_4$.
$\psi_3 = 0$ for $z = 0$

Power one is expected for the pion valence amplitude:

X Ji et al, PRL 90 (2003) 241601.

MOCK PION

W. de Paula, TF, Pimentel, Salmè, Viviani arXiv:1707.06946v1 [hep-ph]

- **Gluon effective mass ~ 500 MeV – Landau Gauge LQCD**
[Oliveira, Bicudo, JPG 38 (2011) 045003;
Duarte, Oliveira, Silva, Phys. Rev. D 94 (2016) 01450240]

- **Mquark = 250 MeV**
[Parappilly, et al, PR D73 (2006) 054504]

- **Λ/m = 3 and Λ/m = 8**

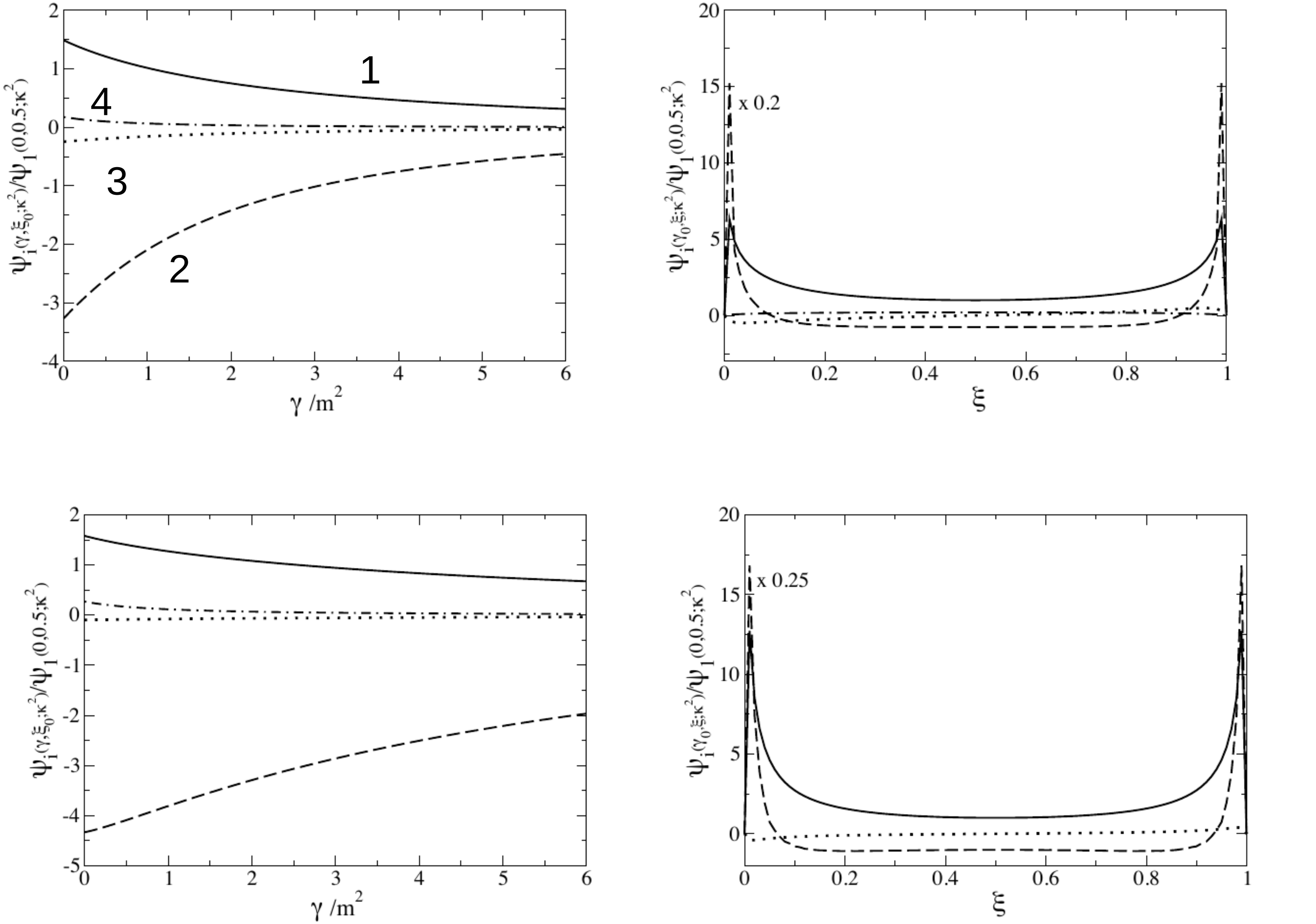


Figure 6. Light-front amplitudes $\psi_i(\gamma,\,\zeta)$, Eq. 3.11, for the pion-like system with a heavy-vector exchange ($\mu/m = 2$), binding energy of $B/m = 1.44$ and constituent mass $m = 250$ MeV. Upper panel: vertex form-factor cutoff $\Lambda/m = 3$ and $g^2 = 435.0$, corresponding to $\alpha_s = 10.68$ (see text for the definition of α_s). Lower panel: vertex form-factor cutoff $\Lambda/m = 8$ and $g^2 = 53.0$, corresponding to $\alpha_s = 3.71$. The value of the longitudinal variable is $\xi_0 = 0.2$ and $\gamma_0 = 0$. Solid line: ψ_1 . Dashed line: ψ_2. Dotted line: ψ_3. Dot-dashed line: ψ_4

Conclusions and Perspectives

- **A method for solving the fermionic BSE: singularities**

LF framework to investigate the fermionic bound state system

- **Our numerical results confirm the robustness of the Nakanishi Integral Representation for solving the BSE.**

- **More realism: self-energies, vertex corrections, Landau gauge, ingredients from LQCD….**

- **Confinement?**

- **Beyond the pion, kaon, D, B, rho…, and the nucleon**

- **Form-Factors, PDFs, TMDs, Fragmentation Functions...**

Collaborators

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THANK YOU!